Non-dimension SWE equations

$$\epsilon U - \frac{1}{2} yV = -ikP \tag{1}$$

$$\epsilon V + \frac{1}{2} yU = -P_y \tag{2}$$

$$\epsilon P + ikU + V_y = -Q \tag{3}$$

Let $\frac{\partial}{\partial y}(1) = ik \times (2)$

$$\frac{\partial}{\partial y}\left(\epsilon U - \frac{1}{2} yV\right) = ik \times \left(\epsilon V + \frac{1}{2} yU\right)$$

$$\epsilon U_y - \frac{1}{2} V - \frac{1}{2} yV_y = ik\epsilon V + \frac{ik}{2} yU$$

$$U_y = ikV + \frac{ik}{2\epsilon} yU + \frac{1}{2\epsilon} V + \frac{1}{2\epsilon} yV_y \tag{4}$$

Let $\epsilon \times (1) - ik \times (3)$

$$\epsilon\left(\epsilon U - \frac{1}{2} yV\right) - ik\left(\epsilon P + ikU + V_y\right) = \epsilon(-ikP) - ik(-Q)$$

$$\epsilon^2 U - \frac{1}{2}\epsilon yV \cancel{- ik\epsilon P} + k^2 U - ikV_y = \cancel{- ik\epsilon P} + ikQ$$

$$\epsilon^2 U - \frac{1}{2}\epsilon yV + k^2 U - ikV_y = ikQ \tag{5}$$

Let $\epsilon \times (2) - \frac{\partial}{\partial y}(3)$

$$\epsilon\left(\epsilon V + \frac{1}{2} yU\right) - \frac{\partial}{\partial y}\left(\epsilon P + ikU + V_y\right) = \epsilon\left(-P_y\right) - \frac{\partial}{\partial y}(-Q)$$

$$\epsilon^2 V + \frac{1}{2}\epsilon yU \cancel{- \epsilon P_y} - ikU_y - V_{yy} = \cancel{- \epsilon P_y} + Q_y$$

$$\epsilon^2 V + \frac{1}{2}\epsilon yU - ikU_y - V_{yy} = Q_y \tag{6}$$

Rewrite (6) using (4)

$$\epsilon^2 V + \frac{1}{2}\epsilon yU - ik\left(ikV + \frac{ik}{2\epsilon} yU + \frac{1}{2\epsilon} V + \frac{1}{2\epsilon} yV_y\right) - V_{yy} = Q_y$$

$$\epsilon^2 V + \frac{1}{2}\epsilon yU + k^2 V + \frac{k^2}{2\epsilon} yU - \frac{ik}{2\epsilon} V - \frac{ik}{2\epsilon} yV_y - V_{yy} = Q_y$$

$$\epsilon^2 V + \frac{1}{2}\epsilon yU + k^2 V + \frac{k^2}{2\epsilon} yU - \frac{ik}{2\epsilon} V - \frac{ik}{2\epsilon} yV_y - V_{yy} = Q_y \tag{7}$$

Let $\frac{1}{2\epsilon}\mathrm{y} \times (5) - (7)$

$$\frac{1}{2\epsilon}y\left(\epsilon^2U-\frac{1}{2}\epsilon yV+k^2U-ikV_y\right)$$

$$-\left(\epsilon^2V+\frac{1}{2}\epsilon yU+k^2V+\frac{k^2}{2\epsilon}yU-\frac{ik}{2\epsilon}V-\frac{ik}{2\epsilon}yV_y-V_{yy}\right)=\frac{1}{2\epsilon}y(ikQ)-Q_y$$

$$\cancel{\frac{1}{2}\epsilon yU}-\frac{1}{4}y^2V\cancel{+\frac{k^2}{2\epsilon}yU}\cancel{-\frac{ik}{2\epsilon}yV_y}-\epsilon^2V\cancel{-\frac{1}{2}\epsilon yU}-k^2V\cancel{-\frac{k^2}{2\epsilon}yU}+\frac{ik}{2\epsilon}V\cancel{+\frac{ik}{2\epsilon}yV_y}+V_{yy}$$

$$=\frac{ik}{2\epsilon}yQ-Q_y$$

$$-\frac{1}{4}y^2V-\epsilon^2V-k^2V+\frac{ik}{2\epsilon}V+V_{yy}=\frac{ik}{2\epsilon}yQ-Q_y$$

$$V_{yy}-\frac{1}{4}y^2V+\frac{ik}{2\epsilon}V-\epsilon^2V-k^2V=-Q_y+\frac{ik}{2\epsilon}yQ$$