Forecasting the Price of Bitcoin Using a Hybrid ARIMA-GARCH Model - Report

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1: Introduction

Bitcoin (BTC) has established itself as a significant and unique asset in today's world, being seen as the first of its kind in a world where thousands of cryptocurrencies now exist. This digital asset was first introduced to the world in 2009, in which an anonymous entity under the pseudonym of "Satoshi Nakamoto" published the famous Bitcoin whitepaper, creating an asset that’s “fully peer-to-peer, with no trusted third party" (Investopedia 2024).

However, despite emerging as a revolutionary and globally adopted asset, Bitcoin’s extreme price volatility continues to present challenges for investors. Traditional models such as Geometric Brownian Motion (GBM) struggle to account for Bitcoin's non-linear and highly volatile nature characterised by volatility clustering (high/low volatility periods are prolonged and clustered together), extreme price swings (sudden rallies and declines) and persistent momentum effects. This shows how techniques used on assets such as gold or stocks cannot be used for Bitcoin as they are generally inadequate to account for Bitcoin’s erratic behaviour This presents gaps within traditional models, leaving investors with a limited set of tools to accurately predict prices and manage risk. Therefore, this causes investors to rely on speculation rather than data-driven strategy, intensifying risks in an already unpredictable market (Reiff 2024).

To address these shortcomings, this project proposes a hybrid ARIMA-GARCH model being implemented with simulation. The objectives of implementing this model are to:

- Predict Bitcoin’s short-term price movements through the simulation of multiple possible price paths.
- Gain valuable risk management insights for investors.

This hybrid approach works better for Bitcoin than techniques such as GBM as it combines the best of both worlds. ARIMA handles the clear price trends while GARCH handles periods where volatility seems to cluster together, matching Bitcoin's behaviour much closer. When we bring these two methods together, we can predict Bitcoin's short-term price swings and simulate where prices might head next much better as it addresses key blind spots that other models miss. This allows traders to estimate Bitcoin's future price movements, providing improvements to managing risk and less dependence on speculation in famously unstable market

2: Brief Literature Review

The challenge of forecasting Bitcoin's future price movements has attracted many individuals to attempt to devise models to help predict possible price paths. However, with the cryptocurrency market being notoriously volatile, traditional financial models struggle to accurately predict Bitcoin's movements. Notable studies found that have contributed to the strategy of using a hybrid ARIMA-GARCH model within this project are outlined in the following literature review.

A study by Towards AI (2020) titled "Statistical Forecasting of Time Series Data Part 4: Forecasting Volatility using GARCH" outlines the use of GARCH models to forecast volatility. The resource describes the effective nature of GARCH models in capturing time-varying volatility often seen within cryptocurrency markets, providing a clear idea of a method to capture the volatility aspect within the project.

The study titled "Estimating and forecasting Bitcoin daily prices using ARIMA-GARCH models" highlights how the use of a hybrid ARIMA-GARCH model outperforms other traditional standalone models. The study outlines how the combination of linear trend analysis (ARIMA) and volatility clustering (GARCH) offer stronger predicting capabilities for the price of Bitcoin, describing how capturing linear trends and volatility clustering addresses the key characteristics of Bitcoin's price movements. Due to this, a significant reduction in forecasting errors and more reliable predictions were produced, allowing for greater risk management for investors.

Another notable resource titled "ARIMA-GARCH forecasting with Python" by Thomas Dierckx, 2020, outlines the implementation of an ARIMA-GARCH model within Python. The resource goes into depth about the process of identifying optimal parameters for both ARIMA (p, d, q) and GARCH (p, q) through the implementation of AIC and BIC. This was implemented within the project allowing for ease of implementation of the hybrid model.

The combination of these studies allowed for a strong theoretical and practical foundation to undertake the task of forecasting future price movements of Bitcoin. This is due to the studies in-depth explanations on what, why and how to complete the tasks needed to accurately predict Bitcoin's price. From these studies, it is clear that an ARIMA-GARCH model is superior compared to other traditional financial modelling techniques as it addresses key characteristics found within the cryptocurrency market as opposed to markets for other assets.

3: Methodology and Data

In the project's aim to forecast future price paths of Bitcoin through simulation of an ARIMA-GARCH model, several steps were required to be conducted using Python. The below methodology is simplified and targets the key areas of data collection, model development, simulation and model evaluation. A more in-depth methodology and rationale can be found within the Jupyter Notebook. The methodology is as follows:

Data collection and preparation:

Historic Bitcoin data was gathered from Yahoo Finance covering the period from January 1st, 2020, to the present. The data used is the daily closing price of Bitcoin in USD, incorporating both bull and bear market scenarios, leading to a more comprehensive outlook on the price of Bitcoin.

To ensure the data was sufficient to be used within the project, several steps were taken:

- Handle missing values and irregularities within the dataset (interpolation/removal).
- Convert prices to log returns for stationarity (necessary transformation to stabilise variance). An Augmented Dickey-Fuller (ADF) test was employed to ensure this requirement was met. The conversion to log returns is required for the ARIMA model.
- Split data into an 80:20 ratio. This allows 80% of the data going towards the development/training of the model, with the remaining 20% of it going to testing/validation of the model. This split is important as it allows our model to be trained with an adequate amount of historical data, and be tested with recent data, simulating an environment of forecasting to see if the model performs as desired.
- Decompose the close price and log returns to identify seasonal trends.

$$\boxed{R_t = \log P_t - \log P_{t-1} = \log\left(\frac{P_t}{P_{t-1}}\right)}$$

Figure 1, Log returns equation. Source: Jayati WALIA, "Returns", SimTrade, 2022

ARIMA model development:

Auto-Regressive Integrated Moving Average (ARIMA) captures linear trends found within Bitcoin's price movements. To create this model, the following steps were undertaken:

- ARIMA is specified by three parameters being p (autoregressive order), d (differencing order) and q (moving average order). The optimal parameter values were determined using a grid search based on Akaike Information Criterion (AIC) and Bayesian Information (BIC). These criteria identify the parameter values that best explain the data.
- Once the above is complete, the model is fitted using 'ARIMA' from the 'statsmodel' python library with the optimal parameters incorporated.

- To ensure the ARIMA model properly captured the underlying trends in the data, diagnostic tests were conducted. The Ljung-Box test was used on the residuals to identify hidden correlations, implying the model required adjustments. ACF and PACF plots were created to visually confirm that no significant autocorrelation was left in the residuals.

$$\underset{\text{Integrated}}{y_t^{(d)}} = c+\varepsilon_t+\underbrace{\phi_1 y_{t-1}^{(d)} + \phi_2 y_{t-2}^{(d)} + \dots + \phi_p y_{t-p}^{(d)}}_{\text{Auto-Regressive}}+\underbrace{\theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \dots + \theta_p\varepsilon_{t-q}}_{\text{Moving Average}}$$

Figure 2, ARIMA model equation. Source: David Andres, "Introduction to ARIMA models, Machine Learning Pills, 2022

GARCH model development:

Generalised Autoregressive Conditional Heteroskedasticity (GARCH) captures volatility clustering found within Bitcoin's price movements. To create this model, the following steps were undertaken:

- GARCH is specified by two parameters being p (order of autoregressive component for squared residuals) and q (order of moving average component for conditional variance). The optimal parameter values were determined using a grid search based on Akaike Information Criterion (AIC) and Bayesian Information (BIC). These criteria identify the parameter values that best explain the data.
- Once the above is complete, the model is fitted using 'arch_model' from the 'arch' python package with the optimal parameters incorporated.
- A visualisation of the condition volatility was created to visually represent periods expected to have high volatility for Bitcoin's price.

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (1)$$

Where σ_t^2 is the conditional volatility, and ε_{t-1}^2 are squared unexpected returns for the previous period. ω would be positive always; and α and β would be non-negative (>=0). ε_{t-1}^2 are derived from a conditional mean equation that could be simple random walk model ($r_t = c + \varepsilon_t$), or AR (1) model ($r_t = c + \gamma * r_{t-1} + \varepsilon_t$), or any other ARMA model. But generally conditional mean equations are kept simple as it can cause convergence problems in GARCH estimation (Alexander, 2001). Where r_t is the returns from a financial series.

Figure 3, GARCH model equation. Source: Sarveshwar Kumar Inani, "GARCH Modelling", Sarveshwar Inani's Blog, 2016

Simulation of hybrid model:

The hybrid ARIMA-GARCH model works by combining two key characteristics of Bitcoin's price. ARIMA captures the expected returns whereas GARCH captures the volatility. To simulate this model, the following were completed:

- Generated conditional mean forecasts for log returns using ARIMA. Once completed, conditional variance forecasts of residuals were generated using GARCH. Combining these components creates a complete distribution of possible future returns.
- Once the above is completed, a Monte Carlo simulation can be employed. Using this approach allowed 1000 simulations to be run, simulating the price paths based off the estimated conditional mean and variance gained from the ARIMA-GARCH model. Using these simulated price predictions allows for multiple potential outcomes rather than a singular forecast.
- After simulations were complete, the cumulative summing of returns and exponentiating them over a base price allowed for returns to be converted back to price levels, therefore providing insights into future price movements (mean), fluctuations (volatility) and risk (VaR). This contains valuable information for investors looking to invest in Bitcoin and achieves the objectives set out for the project.

$$\hat{y}_t = \boxed{\mu + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{i=1}^{q} b_i \epsilon_{t-i}} + \boxed{\epsilon_t}$$

$$\epsilon_t = \sqrt{\sigma_t} z_t, \quad \sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2$$

Figure 4, ARIMA GARCH model equation. Source: Thomas Dierckx, "ARIMA-GARCH forecasting with Python", Medium, 2020

Model evaluation:

Once the simulations were completed, the model could be evaluated using key metrics:

- To ensure the model was reliable, MSE, RMSE and MAE were calculated to assess the accuracy of predicted price movements.
- A Diebold Mariano test was employed to compare the hybrid model against the standalone ARIMA and GARCH models in terms of forecast accuracy.
- Visualisations created for predicted future price movements and returns as well as calculated Value-at-Risk information. These visualisations lead to an easier understanding of short-term predictions and risk management.
- Visualisation of Q-Q plot for the GARCH residuals, leading to an understanding of the distribution, kurtosis and skewness within the data.

The above methodology ensures a thorough approach in developing, implementing and evaluating the hybrid ARIMA-GARCH model to forecast Bitcoin's price in the short-term.

Forecasting models	QLIKE	HMSE	HMAE
Mean	5.471***	4.457***	6.853***
Median	2.850***	6.154***	8.030***
Trimmed mean	2.747***	6.089***	7.548***
DMSPE(1)	6.146***	3.655***	6.758***
DMSPE(0.9)	6.091***	2.138**	3.623***

This table reports the Diebold-Mariano (DM) statistics for the paired comparison of the standard and iterated combination approaches. The DM statistic is calculated as

$$DM\ statistic = \frac{\overline{d}}{\sqrt{Var(d)}},$$

where $\overline{d} = \frac{1}{q}\sum_{t=m+1}^{m+q} d_t$, $d_t = L(\widehat{RV}_{c,t}, RV_t) - L(\widehat{RV}_{ic,t}, RV_t)$, $Var(d)$ is the variance of

Figure 5, Diebold Mariano model equation. Source: Yaojie Zhang, "Out-of-sample prediction of the oil futures market volatility, A comparison of new and traditional combination approaches", ResearchGate, 2018

4: Code and Rationale

The implementation of the hybrid ARIMA-GARCH model and necessary simulations were conducted within a Jupyter Notebook (Python code). The code produced utilises libraries created for time series analysis as well as several other essential operations. The Jupyter Notebook is attached within the zip document, outlining the rationale of steps within their respective sections.

5: Findings and their interpretations

Significant findings were uncovered with the implementation of the ARIMA-GARCH model to predict the future price movements of Bitcoin. The following section outlines the key results obtained, their interpretations, and how they answer the project questions of:

- Can a Monte Carlo simulation of a hybrid ARIMA-GARCH model accurately predict short-term price movements of Bitcoin?
- Can useful risk management insights be obtained through completing this task?

Model performance:

The combined ARIMA-GARCH model presented strong accuracy in predicting Bitcoin's returns. This is shown through error metrics calculated within the code as follows:

- **Mean Squared Error (MSE): 0.000693**
  - Low value = large errors are rare
- **Mean Absolute Error (MAE): 0.018971**
  - Predictions off by 1.9% in log-return terms
- **Root Mean Squared Error (RMSE): 0.026327**
  - Typical error approximately 2.6% in return units

The above metrics represent the models' strong predictive capabilities as all error metrics are relatively low, especially considering the highly unpredictable nature of the cryptocurrency market.

To test the hybrid ARIMA-GARCH model against the standalone ARIMA and standalone GARCH models, a Diebold-Mariano test was implemented. This test allows individuals to statistically compare the predictive capabilities of certain models. The results obtained are as follows:

Comparison	DM Statistic	p-value	Interpretation
ARIMA vs Hybrid	-1.9896	0.0473	Hybrid = better, statistically significant
GARCH vs Hybrid	-0.9949	0.3204	Hybrid = better, not statistically significant
ARIMA vs GARCH	-0.4842	0.6285	GARCH = better, not statistically significant

Figure 6, Diebold Mariano test results

The above table shows the values of the Diebold Mariano statistic and the p-value from each of the comparisons. For the first two rows, the table shows the hybrid model compared with the standalone ARIMA and GARCH models respectively. In both rows, we see a negative DM statistic, indicating that for both comparisons, the hybrid model consistently produced less errors than the standalone models. Also, the small p-value (<0.05) found within the first row indicates strong evidence against the null hypothesis, suggesting the predictive accuracy of the models are not equal. However, as the p-value for the second row is above 0.05, the difference in forecast accuracy is not statistically significant. For the last comparison within the table, we see both the standalone ARIMA and standalone GARCH models being compared, presenting an insignificant DM statistic and large p-value. This therefore indicates that the standalone models present no significant difference in predictive performance.

With these results, it is shown that the predictive capabilities of the hybrid ARIMA-GARCH model is greater than that of the standalone models. This presents the importance of combining linear trends with volatility clustering to improve the prediction accuracy for the future price movements of Bitcoin.

Key results:

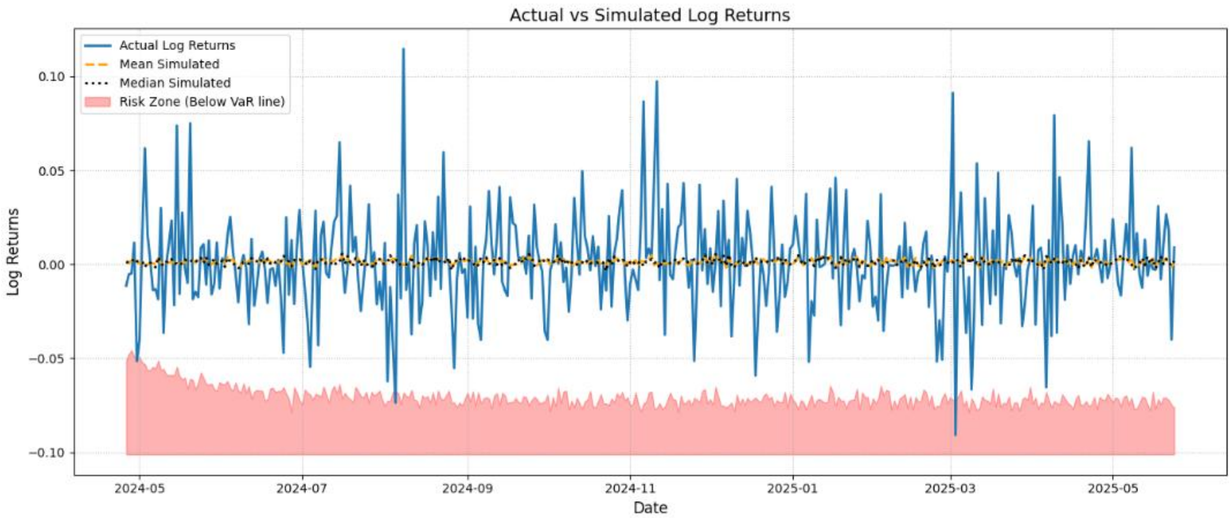

Figure 7, plot of actual vs simulated log returns for Bitcoin

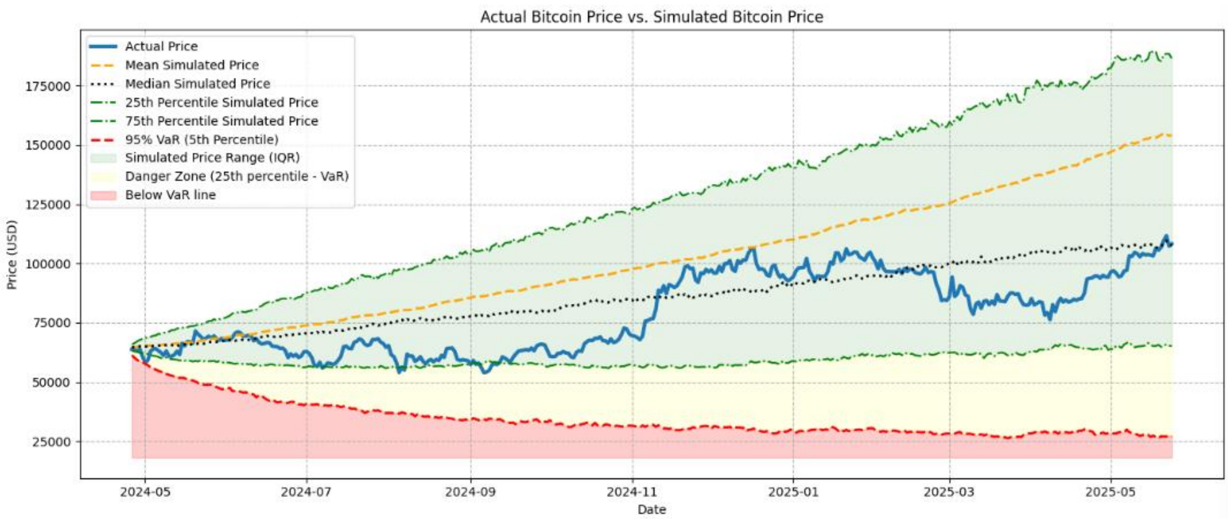

Figure 8, plot of actual vs simulated price for Bitcoin

The above plots present the accurate predictions and valuable risk management insights able to be obtained through the simulation of the hybrid ARIMA-GARCH model.

The first plot outlines the actual (blue line) vs simulated (orange and black) log returns for Bitcoin, including an area (shaded red) presenting when the log returns dip below the 95% VaR. The second plot outlines the actual (blue line) vs simulated (orange and black) price of Bitcoin, including a green (IQR for simulations), yellow (between 25th percentile and VaR line) and red (below VaR line) shaded areas for ease of understanding.

These plots present the predictive capabilities of the hybrid model. This is shown as the simulated log returns are shown to be an accurate depiction of expected returns, being around the average of the actual returns. The accurate predictive capabilities are also shown within the second plot, in which the median simulated price is very close to the actual price of Bitcoin. These insights show how investors can accurately predict the short-term price movements of Bitcoin as required.

These plots also present the valuable risk management insights able to be gained from the simulation of this model. Both plots show a red shaded area, which is below the 95% VaR for simulated returns and prices. This presents a 'risk zone' in which investors can quantify downside risk and interpret potential losses, allowing for better decision making during uncertain periods. We see within the log returns plot that the returns briefly dip into this 'risk zone' at one point, presenting the model's practical usefulness.

Other findings:

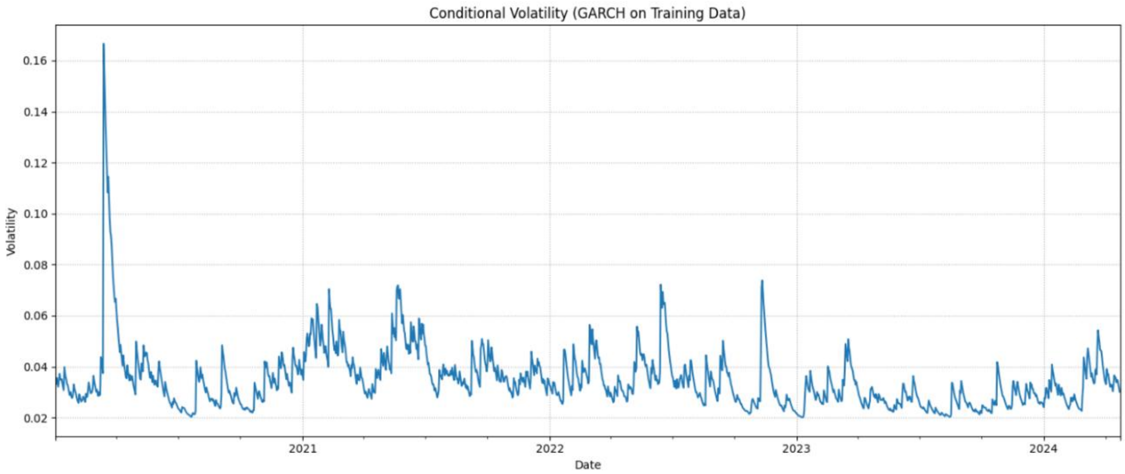

Figure 9, plot of conditional volatility found using GARCH on training data

It was found that the GARCH component of the hybrid model effectively captured Bitcoin's volatility clustering, as found in the above plot. This shows the long-lasting effects of volatility shocks to Bitcoin, a characteristic found within cryptocurrency markets.

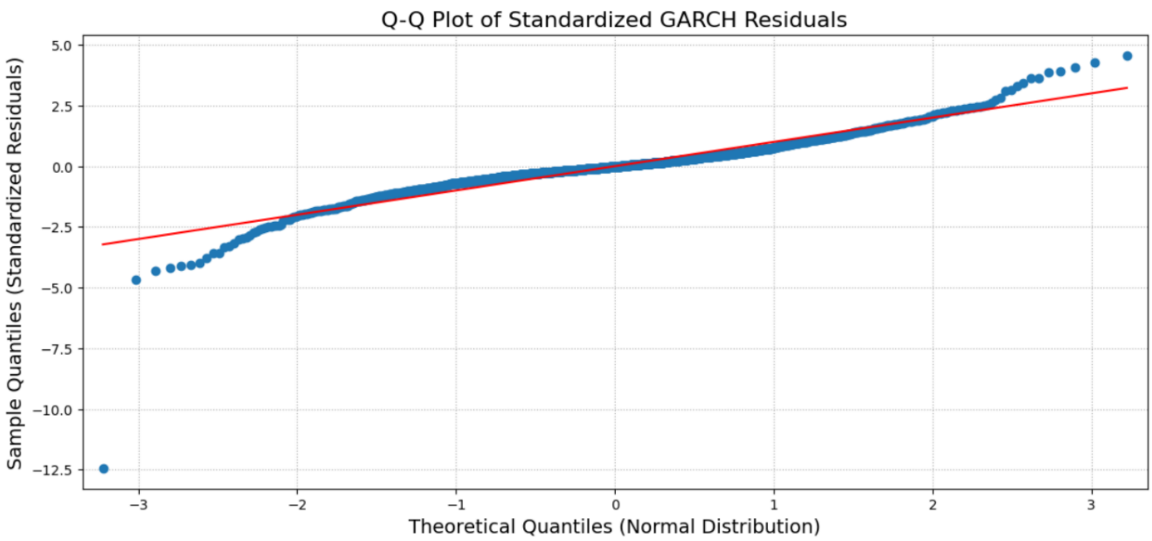

Figure 10, Q-Q plot of standardised GARCH residuals

The Q-Q plot showed Bitcoin's returns don't follow a perfect normal distribution, particularly for extreme values. This is presented through the fatter tails (excess kurtosis) and potential asymmetry within the distribution. This means that Bitcoin's price experiences more frequent large price swings and might require more sophisticated distributional assumptions to improve the model's forecasting ability.

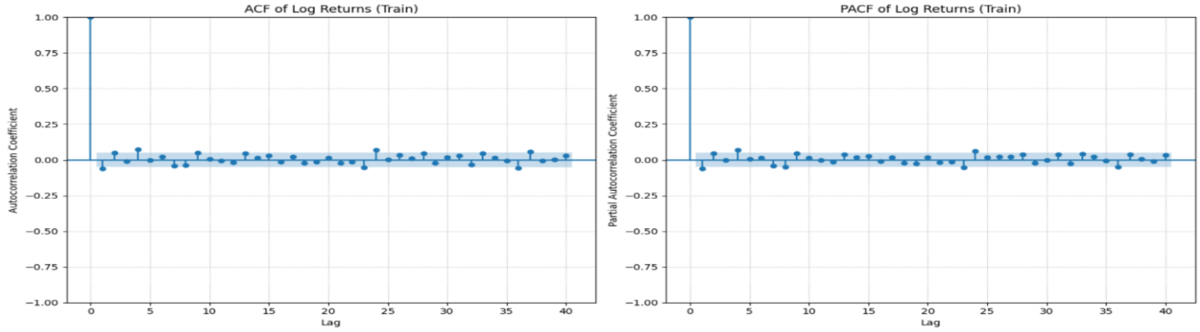

Figure 11, ACF + PACF plots of log returns on Bitcoin training data

The ACF and PACF plots of log returns presented little significant autocorrelation at most lag values, indicating that Bitcoin's future prices aren't strongly affected by past prices. This presents how investors can't reliably predict Bitcoin's price by simply looking at past price movements, as it doesn't account for new information relating to markets and the growing maturity of Bitcoin as an asset.

Limitations of Model:

Despite the hybrid model presenting impressive forecasting capabilities, there are certain constraints that pose challenges to the model's effectiveness.

- **Bias in short term forecasting:** Long term predictions are unreliable due to the increasing uncertainty over extended time frames. Also, as the model is trained from data after 2020, the model may overlook longer term trends and cyclical patterns.
- **Exclusion of external variables:** As the model is only trained on the price data of Bitcoin, other factors such as market sentiment and macroeconomic indicators that have the potential to strongly affect Bitcoin's price are not accounted for. This could also be coupled with cryptocurrency specific regulatory changes to cause uncertainty within the market. This could lead to unreliable price predictions that mislead investors.
- **Data Limitations:** As the model is trained on the daily closing price, the model is unable to forecast intraday volatility. A way in which this could be fixed is to use higher frequency data such as the hourly Bitcoin price accompanied with more complex computational methods.

Conclusion:

Despite limitations present within the simulation of the hybrid ARIMA-GARCH model, the model's performance and insights gathered above outline the powerful predictive capabilities of this model. Therefore, it is shown that the simulation of this hybrid model can equip investors with accurate short-term price movements and valuable risk management information regarding Bitcoin.

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