

# The error function

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The error function is defined as:

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^x \exp -t^2 dt = \frac{2}{\sqrt{2\pi}} \int_0^x \exp -t^2 dt. \quad (1)$$

The error function can be seen in figure 1.

## 1 Derivative

The derivative of the error function follows from its definition

$$\frac{d}{dz} \operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \exp -z^2 \quad (2)$$

The error function is an entire function - it has no singularities. The higher order derivatives are

$$\operatorname{erf}^{(k)}(z) = \frac{2(-1)^{k-1}}{\sqrt{\pi}} H_{k-1}(z) \exp -z^2, \quad k = 1, 2, \dots \quad (3)$$

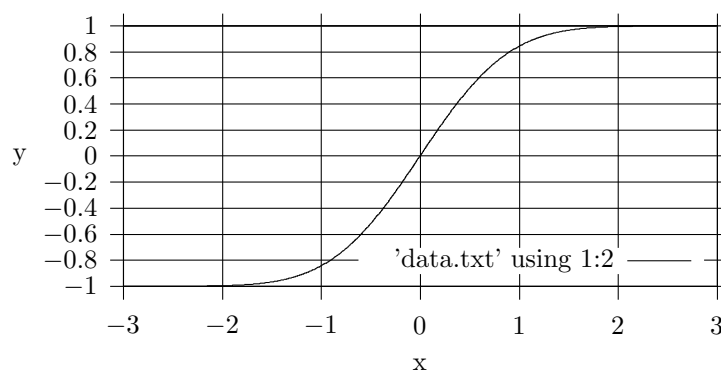


Figure 1: This is a plot of the error function.