(2) $\binom{n}{0} = \frac{n!}{(n-0)!\,0!} = 1$ $\qquad \binom{n}{1} = \frac{n!}{(n-1)!\,1!} = n$ $\qquad \binom{n}{n} = \frac{n!}{(n-n)!\,n!} = 1$

$\binom{n}{k} = \binom{n}{n-k}$ $\qquad \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$,

$\binom{n}{n-k} = \frac{n!}{(n-n+k)!\,(n-k)!} = \frac{n!}{k!\,(n-k)!}$

$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$ $\qquad \frac{n!}{(n-k)!\,k!} + \frac{n!}{(n-k-1)!\,(k+1)!} = \frac{(k+1)n! + (n-k)n!}{(n-k)!\,(k+1)!} =$

$= \frac{n!\,(k+1+n-k)}{(n-k)!\,(k+1)!} = \frac{n!\,(n+1)}{(n-k)!\,(k+1)!} =$

$= \frac{(n+1)!}{(n-k)!\,(k+1)!}$

(3) $p: (m-1)x - (m+2)y + m = 0$

$q: (m+2)x + (m-3)y + 3m - 1 = 0$ $\qquad p \cap q = \{P\}$ $\quad P \in o_y$

$P = [0, y_P]$

$-my_P - 2y_P + m = 0$ $\qquad (m-3)\frac{m}{m+2} + 3m - 1 = 0$

$y_P = \frac{m}{m+2}$ $\qquad m^2 - 3m + 3m^2 + 6m - m - 2 = 0$

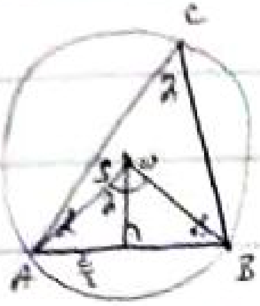
$4m^2 + 2m - 2 = 0$

$2m^2 + m - 1 = 0$

$D = 9$

$m_{1,2} = \frac{-1 \pm 3}{4} = \begin{cases} \underline{\underline{\frac{1}{2}}} \\ \underline{\underline{-1}} \end{cases}$

(2)



ω - stredový $\omega = 2\gamma$

γ - obvodový uhol

$$\sin\gamma = \frac{\frac{c}{2}}{r}$$

$$\sin\gamma = \frac{c}{2r}$$

$$2r = \frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

(3) $f: y = a\sin\left(\frac{\pi}{6} - x\right) + b$ $\quad A = \left[0; -\frac{1}{2}\right]; B = \left[\frac{3\pi}{2}; \frac{4-5\sqrt{3}}{2}\right]$

$A, B \in f$

$$-\frac{1}{2} = a\sin\frac{\pi}{6} + b$$

$$\frac{4-5\sqrt{3}}{2} = a\sin\left(\frac{\pi}{6} - \frac{3\pi}{2}\right) + b$$

$$-\frac{1}{2} = a\cdot\frac{1}{2} + b \quad \rightarrow \cancel{a = -1-2b} \quad b = -\frac{1}{2} - \frac{1}{2}a \Rightarrow b = 2$$

$$\frac{4-5\sqrt{3}}{2} = a\cdot\left(+\frac{\sqrt{3}}{2}\right) + b$$

$$\cancel{b = -\frac{1}{2} - \frac{1}{2}(10-5\sqrt{3})}$$

$$\cancel{b = -\frac{1}{2} - 5 + \frac{5\sqrt{3}}{2}}$$

$$\cancel{b = \frac{5\sqrt{3}-11}{2}}$$

$$\frac{4-5\sqrt{3}}{2} = +\frac{\sqrt{3}}{2}a - \frac{1}{2} - \frac{1}{2}a \;/\cdot 2$$

$$4 - 5\sqrt{3} = +\sqrt{3}a - 1 - a$$

$$5 - 5\sqrt{3} = a(+\sqrt{3} - 1)$$

$$a = \frac{-5\sqrt{3}+5}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{-15-5\sqrt{3}+5\sqrt{3}+5}{2} = \cancel{\frac{20-10\sqrt{3}}{2}} = -\frac{10}{2}$$

$$a = -5 \qquad \cancel{a = 10-5\sqrt{3}} \qquad \cancel{b = \frac{5\sqrt{3}-11}{2}}$$

$\rightarrow O = [0,0] \in f$

$$\cancel{0 = (10-5\sqrt{3})\sin\left(\frac{\pi}{6} - 0\right) + b}$$

$$\cancel{b = -(10-5\sqrt{3})\cdot\frac{1}{2} = -5 + \frac{5}{2}\sqrt{3}}$$

$$\cancel{b = \frac{5\sqrt{3}-10}{2}}$$

$$0 = -5\cdot\sin\left(\frac{\pi}{6} - 0\right) + b$$

$$b = 5\cdot\frac{1}{2}$$

$$b = \frac{5}{2}$$

① a) $\dfrac{tg\,2x \cdot tg\,x}{tg\,2x - tg\,x} = \sin 2x$

$$\frac{\dfrac{2\sin x\cos x}{\cos^2x-\sin^2x}\cdot\dfrac{\sin x}{\cancel{\cos x}}}{\dfrac{2\sin x\cos x}{\cos^2x-\sin^2x}-\dfrac{\sin x}{\cos x}} = \frac{\dfrac{2\sin^2x}{\cancel{\cos^2x-\sin^2x}}}{\dfrac{2\sin x\cos^2x-\sin x\cos^2x+\sin^3x}{\cos x(\cancel{\cos^2x-\sin^2x})}} =$$

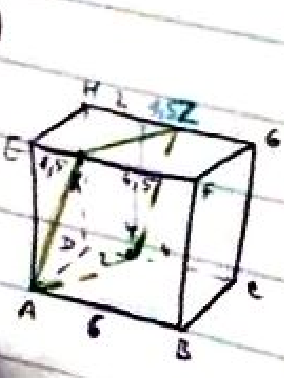
$$= \frac{2\sin^2x\cos x}{\cancel{\sin}\,x(\underbrace{\cos^2x+\sin^2x}_{1})} = \sin 2x$$

b) $\dfrac{1}{\cos^2x} - \dfrac{\sin^3x+\cos^3x}{\cos^2x\sin x+\cos^3x} = tg\,x$

$$\frac{1}{\cos^2x} - \frac{(\cancel{\sin x+\cos x})(\sin^2x-\sin x\cos x+\cos^2x)}{\cos^2x(\cancel{\sin x+\cos x})} = \frac{1-1+\sin x\cos x}{\cos^2x} =$$

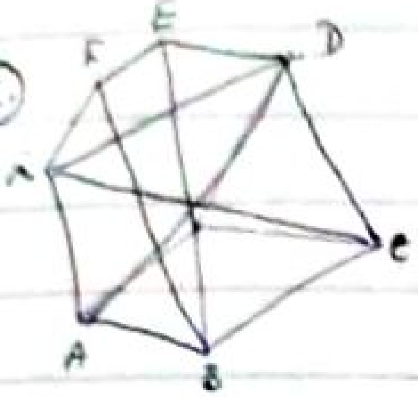
$$= \frac{\sin x}{\cos x} = tg\,x$$

③



$|GZ| = 2{,}5\,cm$

2.



$$C_2(n) - n = \frac{n!}{(n-2)!\,2!} - n = \frac{n(n-1)}{2} - n =$$

$$= \frac{n^2 - n - 2n}{2} = \frac{n^2 - 3n}{2} = \frac{n(n-3)}{2}$$

3. $a(n) = \dfrac{15+n}{9-n}$

a) $a(n) \in \langle -1; 0)$

$$\frac{15+n}{9-n} \geq -1 \;|+1 \quad \wedge \quad \frac{15+n}{9-n} < 0$$

$$\frac{15+n+9-n}{9-n} \geq 0 \qquad \frac{15+n}{n-9} > 0$$

$$\frac{24}{9-n} \geq 0 \qquad N.B.\ -15;\ 9$$

$$n \leq 9 \quad \wedge \quad n \in (-\infty, -15) \cup (9, \infty)$$

$$n \in (-\infty, -15)$$

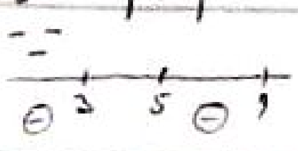
b) $\dfrac{15+n}{9-n} > n \;|-n$

$$\frac{15+n-9n+n^2}{9-n} > 0$$

$$\frac{n^2-8n+15}{n-9} < 0$$

$$\frac{(n-5)(n-3)}{n-9} < 0$$

N.B. $5;\ 3;\ 9$ $\quad n \in (-\infty, 3) \cup (5, 9)$



② a) $\forall n \in N: n^3+5n$ je deliteľné $3 \wedge 6$

$n=3k \quad 27k^3+15k = 3(9k^3+5k)$

$n=3k+1 \quad 27k^3+27k^2+9k+1+15k+5 = 3(9k^3+9k^2+8k+2)$

$n=3k+2 \quad 27k^3+54k^2+36k+8+15k+10 = 3(9k^3+18k^2+17k+6)$

$n(n^2+5)$
nep. → párne
$2/n^3+5n \quad \rightarrow \; 6/n^3+5n$

b) $\forall a,b,c \in N: a/b \wedge a/(b-c) \Rightarrow a/c$

$b = k.a \quad b-c = l.a$

$c = b - l.a = ka - la = a(k-l) \rightarrow a/c$

③

$5.V_2(n) = V_3(n)$

$5 \cdot \frac{n!}{(n-2)!} = \frac{n!}{(n-3)!}$

$5n(n-1) - n(n-1)(n-2) = 0$

$n(n-1)[5-(n-2)] = 0$

$n(n-1)(7-n) = 0$

$n = 0 \,;\, 1 \,;\, \underline{\underline{7}}$

② $\left\{\frac{n^2}{n^2+2n+1}\right\}_{n=1}^{\infty}$ $\quad \frac{1}{4}, \frac{4}{9}, \frac{9}{16}, \frac{16}{25}$

$(n+1)^2$

$\frac{10000}{10201}$

$0,98$

$\frac{1}{4} < a_n < 1$

$\frac{n^2}{n^2+2n+1} > \frac{1}{4} \quad | -\frac{1}{4}$

$\frac{4n^2-n^2-2n-1}{4(n^2+2n+1)} > 0$

$\frac{3n^2-2n-1}{4(n+1)^2} > 0$

$3n^2-2n-1>0$

$D=16$

$n_{1,2} = \frac{2 \pm 4}{6} = \begin{cases} 1 \\ -\frac{1}{3} \end{cases}$

N.B. $-\frac{1}{3}; 1$

$n \in (-\infty, -\frac{1}{3}) \cup (1, \infty)$ ✓

$\frac{n^2}{n^2+2n+1} < 1 \quad | -1$

$\frac{n^2-n^2-2n-1}{n^2+2n+1} < 0$

$\frac{-2n-1}{(n+1)^2} < 0$

$-2n-1<0$

$n > -\frac{1}{2}$

✓

③ a) $2 \cdot 4^x + 5^{x-\frac{1}{2}} = 5^{x+\frac{1}{2}} - 2^{2x-1}$

$2 \cdot 2^{2x} + 5^x \cdot 5^{-\frac{1}{2}} = 5^x \cdot 5^{\frac{1}{2}} - 2^{2x} \cdot 2^{-1}$

$2^{2x}(2+\frac{1}{2}) = 5^x(\sqrt{5} - \frac{1}{\sqrt{5}})$

$4^x \cdot \frac{5}{2} = 5^x \frac{5-1}{\sqrt{5}}$

$\frac{4^x}{5^x} = \frac{4}{\sqrt{5}} \cdot \frac{2}{5} = \sqrt{4}$

$\left(\frac{4}{5}\right)^x = \left(\frac{4}{5}\right)^{\frac{3}{2}}$

$\underline{\underline{x = \frac{3}{2}}}$

$\rightarrow \rightarrow$ 3a)

b) $1 + \log_3(5-x) - \log_3(2x-1) = \log_3(2x-1)$

$\log_3 3$

$$\log_3 \frac{3(5-x)}{2x-1} = \log_3(2x-1)$$

$$\frac{15-3x}{2x-1} = 2x-1 \quad / \cdot (2x-1)$$

$$15 - 3x = 4x^2 - 4x + 1$$

$$0 = 4x^2 - x - 14$$

$$D = 225 \qquad x_{1,2} = \frac{1 \pm 15}{8} = \begin{cases} 2 \\ -\frac{7}{4} \end{cases}$$

$$K = \{2\}$$

3 a) $2^{2x+1} + 4^{x+1} + 16^{\frac{x}{2}} = 28$

$$2^{2x} \cdot 2 + 2^{2x} \cdot 2^2 + 2^{2x} = 28$$

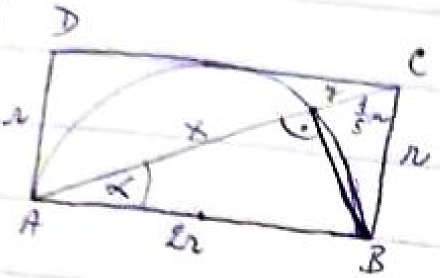
$$2^{2x}(2+4+1) = 28 \quad / : 7$$

$$2^{2x} = 4$$

$$2x = 2$$

$$x = 1 \qquad K = \{1\}$$

(2)



$u^2 = 4r^2 + r^2$

$u = \sqrt{5}\, r$

$\cos\alpha = \frac{2r}{\sqrt{5}\, r} = \frac{x}{2r}$

$x = \frac{4r}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}\, r}{5}$

$y = \sqrt{5}\, r - \frac{4}{5}\sqrt{5}\, r = \frac{1}{5}\sqrt{5}\, r = \frac{1}{5} u$

(3) I. III. II.

$-11;\ 3;\ 3;\ 3;\ 3;\ 8;\ 10;\ 10;\ 10;\ 15$

mod = 3 med = 5,5

$\bar{x} = \frac{54}{10} = 5{,}4$

$\bar{x}$ med

$\frac{46+a}{10} = \frac{3+a}{2} \quad / \cdot 10$

$46 + a = 15 + 5a$

$4a = 31$

$a = \frac{31}{4}$

$\cancel{\frac{51+a}{10} = \frac{a+8}{2} \quad / \cdot 10}$

$\cancel{51 + a = 5a + 40}$

$\cancel{4a = 11}$

$\cancel{a = \frac{11}{4}}$

II. $\frac{46+a}{10} = \frac{10+3}{2} \quad / \cdot 10$

$46 + a = 65$

$a = 19$

III. $\frac{46+a}{10} = 3 \quad / \cdot 10$

$46 + a = 30$

$a = -16$

① $\left\{\frac{n+4}{-n}\right\}_{n=1}^{\infty}$ $\qquad \frac{5}{-1} < \frac{6}{-2} < \frac{7}{-3} \quad \cdots \quad \frac{104}{-100}$

$-5 \le a_n < -1$

$a_{n+1} > a_n$

$\frac{n+1+4}{-(n+1)} > \frac{n+4}{-n} \quad /\cdot(-1)$

$\frac{n+5}{n+1} - \frac{n+4}{n} < 0$

$\frac{n^2+5n-n^2-5n-4}{n(n+1)} < 0$

$\frac{-4}{n(n+1)} < 0$

$n(n+1) > 0$

$0;\ -1$

$n \in (-\infty,-1) \cup (0,\infty)$ ✓

$\frac{n+4}{-n} \ge -5 \quad /\cdot(-1)$

$\frac{n+4-5n}{n} \le 0$

$\frac{4-4n}{n} \le 0$

$\frac{4(n-1)}{n} \ge 0$

$1;\ 0$

$n \in (-\infty,0) \cup \langle 1,\infty)$ ✓

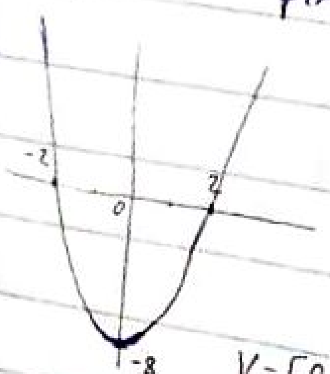
$\frac{n+4}{-n} < -1 \quad /\cdot(-1)$

$\frac{n+4-n}{n} > 0$

$\frac{4}{n} > 0$

$n \in (0,\infty)$ ✓

③ f - párna $\quad f(\min) = -8 \qquad [2;0] \in f$



$V = [0,-8]$

$c = -8 \quad b = 0$

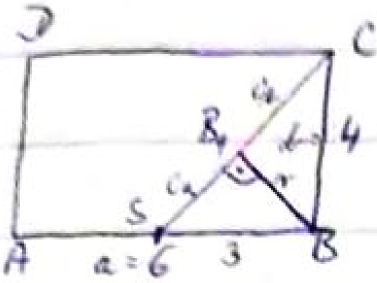
$y = ax^2 - 8$

$0 = 4a - 8$

$a = 2$

$f: y = 2x^2 - 8$

(2)



$|SB_1| : |B_1C| = 9 : 16$

$b^2 = c_b \cdot c$

$16 = c_b \cdot 5 \qquad 9 = c_a \cdot 5$

$c_b = \frac{16}{5} \qquad c_a = \frac{9}{5}$

$\frac{c_a}{c_b} = \frac{\frac{9}{5}}{\frac{16}{5}} = \frac{9}{16}$

(3) 1) $P - 1 \qquad Q - 0 \qquad R - ?$

a) $P \vee (Q \wedge R)$ 1

b) $(P \wedge Q) \Rightarrow R$ 1

c) $P \Rightarrow (Q \vee R)$ X

d) $(P \Leftrightarrow Q) \vee R$ X

e) $[\underset{0}{P'} \Rightarrow \underset{1}{(Q \wedge R)'}]'$ 0

2) a) $\forall x \in R \ \exists y \in R: x + y = 1 \qquad p = 1 \qquad y = 1 - x$

$\exists x \in R \ \nexists y \in R$

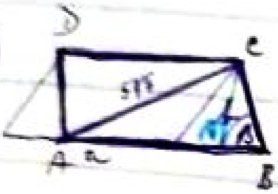
b) $\forall x \in R \ \exists y \in R: x \cdot y = 1 \qquad p = 0 \qquad y = \frac{1}{x} \qquad x = 0 \rightarrow y \ \emptyset$

② $\forall a, b \in \mathbb{R}: \ a^2 + b^2 + 1 \geq ab + a + b \ / \cdot 2$

$$a^2 - 2ab + b^2 + a^2 - 2a + 1 + b^2 - 2b + 1 \geq 0$$

$$(a+b)^2 + (a-1)^2 + (b-1)^2 \geq 0$$

③



$a + b = 25$

$\beta = 60°$

$|AC| = 5\sqrt{7}$

$a, b, S = ?$

$$(5\sqrt{7})^2 = a^2 + (25-a)^2 - 2a(25-a)\cos 60°$$

$$175 = a^2 + 625 - 50a + a^2 - 25a + a^2$$

$$0 = 3a^2 - 75a + 450$$

$$0 = a^2 - 25 + 150$$

$$0 = (a-10)(a-15)$$

$a = 10$; $a = 15$

$b = 15$ $b = 10$

$a = 15$

$\sin 60° = \frac{r}{10}$

$\frac{\sqrt{3}}{2} = \frac{r}{10}$

$r = 5\sqrt{3}$ cm

$S = \frac{15 \cdot 5\sqrt{3}}{2}$

$S = \frac{75\sqrt{3}}{2}$ cm²

([illegible])

(129,9 cm²)

(2) $\forall x \in \langle 1, \infty)$ $\quad 2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} < 2\sqrt{x} - 2\sqrt{x-1}$

$2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} \;/ + 2\sqrt{x} \;/^2 \quad \wedge \quad \frac{1}{\sqrt{x}} < 2\sqrt{x} - 2\sqrt{x-1} \;/ + 2\sqrt{x-1} \;/^2$

P.S. > 0
L.S. > 0

$4(x+1) < \frac{1}{x} + 4 + 4x \quad \wedge \quad \frac{1}{x} + \frac{4\sqrt{x-1}}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} + 4(x-1) < 4x$

$4x + 4 < \frac{1}{x} + 4 + 4x \qquad \frac{1}{x} + \frac{4\sqrt{x}\sqrt{x-1}}{x} - 4 < 0 \;/ \cdot x$

$0 < \frac{1}{x}$ ✓

$4\sqrt{x}\sqrt{x-1} < 4x - 1 \;/^2 \qquad x \in \langle 1, \infty)$

$16x(x-1) < 16x^2 - 8x + 1$

$16x^2 - 16x < 16x^2 - 8x + 1$

$0 < 8x + 1$ ✓ $\qquad x \in \langle 1, \infty)$

(3.) $\underbrace{5 \ldots}_{s = 630} 640$

$s = 630 + 640 + 5 = 1275$

$s_n = a_1 \frac{q^n - 1}{q - 1} \qquad a_n = a_1 q^{n-1}$

$640 = 5 \cdot q^{n-1}$

$1275 = 5 \cdot \frac{q^n - 1}{q - 1} \;/ : 5 \qquad q^n = 128 q \qquad (n = 8)$

$255 = \frac{128q - 1}{q - 1} \;/ (q-1)$

$255q - 255 = 128q - 1$

$q = 2 \qquad 5, \underbrace{10, 20, 40, 80, 160, 320}_{630}, 640$

② a) $\frac{1}{2}\log_4 256 = 2$

$-\log 0{,}0001 = 4$ $\quad q = 2 \quad \frac{8}{4} = \frac{4}{2}$

$\log_2 256 = 8$

b) $\cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$

$\operatorname{tg}\pi = 0$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2} \quad d = \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} - 0 = 0 - \frac{\sqrt{2}}{2}$

③

$x + (a-1)y = 1 \qquad x = 1-(a-1)y$

$(a+1)x + 3y = -1$

$(a+1)\cdot[1-(a-1)y] + 3y = -1$

$(a+1)[1-ay+y] + 3y = -1$

$a - a^2y + ay + 1 - ay + y + 3y = -1$

$4y - a^2y = -2 - a$

$y = \frac{-2-a}{4-a^2} = \frac{-(2+a)}{(2-a)(2+a)} = \frac{1}{a-2}$

$x = 1-(a-1)\cdot\frac{1}{a-2} =$

$= \frac{a-2-a+1}{a-2} = \frac{-1}{a-2}$

$\frac{-1}{a-2} > 0 \quad \wedge \quad \frac{1}{a-2} > 0$

$a-2<0$

$a<2 \quad \wedge \quad a-2>0$

$a>2$

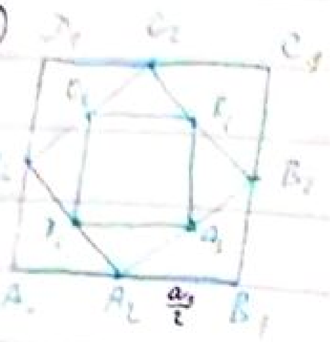
$\emptyset$

$a = -2 \Rightarrow$

$\left[\frac{-1}{a-2}; \frac{1}{a-2}\right]$

opačné hodnoty

(2)



$$a_2^2 = \frac{a_1^2}{4} + \frac{a_1^2}{4}$$

$$a_2 = \frac{a_1}{\sqrt{2}} = \frac{\sqrt{2}\,a_1}{2}$$

$$a_3^2 = \frac{a_1^2}{8} + \frac{a_1^2}{8}$$

$$a_3 = \frac{a_1}{2}$$

$$\frac{a_3}{a_2} = \frac{a_2}{a_1}$$

$$\frac{a_1}{2} : \frac{a_1}{\sqrt{2}} = \frac{a_1}{\sqrt{2}} : a_1$$

$$\frac{2}{\sqrt{2}} = \sqrt{2} \checkmark$$

$$o_1 = 4a_1$$

$$o_2 = 4 \cdot \frac{\sqrt{2}\,a_1}{2} = 2\sqrt{2}\,a_1$$

$$q = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$S_1 = a_1^2$$

$$S_2 = \frac{a_1^2}{2}$$

$$q = \frac{1}{2}$$

(3)

$k: (x-2)^2 + (y+1)^2 = 9$ $\qquad o: 2x - y + 5 = 0$

$S = [2; -1] \quad r = 3$

$$k: x + 2y + c = 0$$
$$2 - 2 + c = 0$$
$$c = 0$$
$$k: x + 2y = 0$$

$o \cap k: \quad 2(-2y) - y + 5 = 0$

$$-4y - y + 5 = 0$$
$$y = 1$$
$$x = -2$$

$$X = [-2; 1]$$

$k': (x+6)^2 + (y-3)^2 = 9$

$$S' = [-6; 3]$$

1.5

(2.) $ax^2 + bx + c = 0 \;/ : a$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$

$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$ D

$x + \frac{b}{2a} = \begin{cases} \sqrt{\frac{D}{4a^2}} \\ -\sqrt{\frac{D}{4a^2}} \end{cases}$ $\rightarrow$ $x = \sqrt{\frac{D}{4a^2}} - \frac{b}{2a}$ $\rightarrow$ $x_1 = \frac{-b + \sqrt{D}}{2a}$

$x = -\sqrt{\frac{D}{4a^2}} - \frac{b}{2a}$ $\qquad x_2 = \frac{-b - \sqrt{D}}{2a}$

(3.) z-360 495 972 781 g91 000

a) 8/č

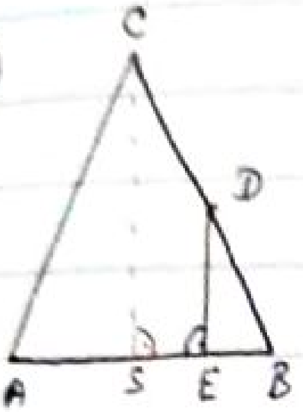
g = 1 → 9/č

72/č

b) 790 461 308 754 021 980 653 021

90

2.



$|AE| = \frac{3}{4}|AB|$

$\triangle SBC \sim \triangle EBD \ (uu)$

$\frac{|BD|}{|BC|} = \frac{|BE|}{|BS|} \rightarrow |BE| = \frac{1}{2}|BS| \rightarrow$

$|BE| = \frac{1}{4}|AB| \rightarrow |AE| = \frac{3}{4}|AB|$

3. $1 \ldots 50$

a) deliteľné 6 : $P(A) = \frac{8}{50} = \frac{4}{25}$

b) deliteľné 8 : $P(B) = \frac{6}{50} = \frac{3}{25}$

c) $6 \wedge 8$: $24, 48$ $\quad P(C) = \frac{2}{50} = \frac{1}{25}$

d) $6 \vee 8$: $P(D) = \frac{14-2}{50} = \frac{12}{50} = \frac{6}{25}$

(2) $\forall x, y \in \mathbb{R}^+ : \quad \dfrac{x^4+y^4}{2} \geq \dfrac{x+y}{2} \cdot \dfrac{x^3+y^3}{2} \quad / \cdot 4$

$$2x^4+2y^4 \geq x^4+xy^3+x^3y+y^4$$

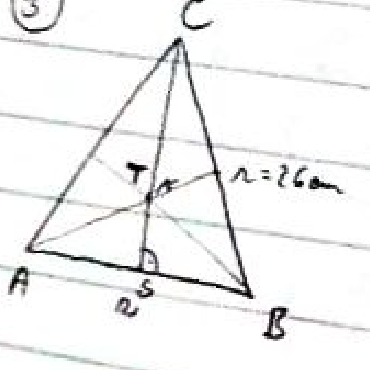
$$x^4-xy^3-x^3y+y^4 \geq 0$$

$$x^3(x-y)-y^3(x-y) \geq 0$$

$$(x-y)(x-y)(x^2+xy+y^2) \geq 0$$

$$\underbrace{(x-y)^2}_{>0}\underbrace{(x^2+xy+y^2)}_{>0} \geq 0$$

(3)



$c : v = 10 : 12$

$S_{\triangle ABT} = ?$

$c = 10x$
$v = 12x$

$$26^2 = 144x^2 + 25x^2$$

$$676 = 169x^2$$

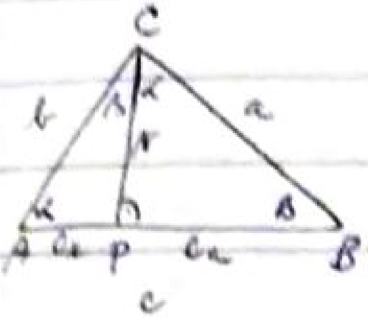
$$\underline{x = 2}$$

$c = 20\,cm$
$v = 24\,cm$

$|ST| = \frac{1}{3} \cdot 24 = 8\,cm$

$$S = \frac{20 \cdot 8}{2} = \underline{\underline{80\,cm^2}}$$

2



$\triangle APC \sim \triangle CPB$ (uu)

$\frac{v}{c_b} = \frac{c_a}{v} \rightarrow v^2 = c_a \cdot c_b$

$\triangle APC \sim \triangle ACB$ (uu)

$\frac{c_b}{b} = \frac{b}{c} \rightarrow b^2 = c_b \cdot c$

$\triangle CPB \sim \triangle ACB$ (uu)

$\frac{c_a}{a} = \frac{a}{c} \rightarrow a^2 = c_a \cdot c$

3 a) $A = \{x \in Z : x^2 < 10\} = \{-3; -2; -1; 0; 1; 2; 3\}$

$B = \{x \in N : 3/x \wedge x < 17\} = \{3; 6; 9; 12; 15\}$

$C = \{x \in Z : x^2 = 1 \vee 2|x| < 5\} = \{-1; 1; -2; 2; 0\}$

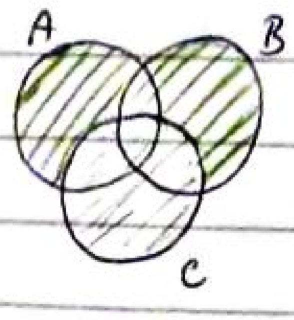
$A \cap B = \{3\}$

$B \cap C = \{\ \}$

$B \cup C = \{-1; 1; -2; 2; 0; 3; 6; 9; 12; 15\}$

$C'_A = \{-3; 3\}$

b)

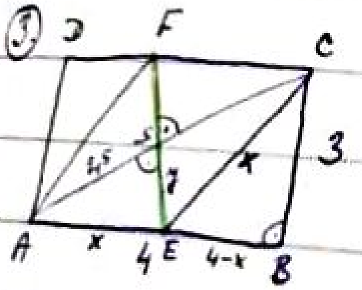


① $\frac{2\cos 2x}{\sin 2x - 2\sin^2 x} - \cot g\, x =$

$= \frac{2(\cos^2 x - \sin^2 x)}{2\sin x\cos x - 2\sin^2 x} - \frac{\cos x}{\sin x} = \frac{2(\cancel{\cos x - \sin x})(\cos x + \sin x)}{2\sin x(\cancel{\cos x - \sin x})} - \frac{\cos x}{\sin x} =$

$= \frac{\cos x + \sin x - \cos x}{\sin x} = \underline{\underline{1}}$

③



$|EF| = ?$ $|AC|^2 = 16 + 9$

$|AC| = 5$

$x^2 = 9 + 16 - 8x + x^2$

$8x = 25$

$x = \frac{25}{8}$

$y^2 = \left(\frac{25}{8}\right)^2 - 2{,}5^2$

$y = 1{,}875 \rightarrow |EF| = 2 \cdot 1{,}875 = \underline{\underline{3{,}75}}$

pomocou obsahov

$S = \frac{u_1 \cdot u_2}{2}$

(2)

$$\frac{V_K}{V_G} = \frac{9}{4}$$

$$x^2 = 4r^2 - r^2 \qquad R = \frac{1}{3}\sqrt{3}\,r$$

$$x = \sqrt{3}\,r$$

$$V_K = \frac{\pi r^2 \cdot \sqrt{3}\,r}{3} \qquad V_G = \frac{4}{3}\pi\left(\frac{1}{3}\sqrt{3}\,r\right)^3$$

$$\frac{V_K}{V_G} = \frac{\sqrt{3}\pi r^3}{3} : \frac{4\pi 3\sqrt{3}\,r^3}{3^4} = \frac{\cancel{\sqrt{3}}\cancel{\pi}\cancel{r^3}}{\cancel{3}} \cdot \frac{3^{3}}{4\cancel{\pi}\cancel{\sqrt{3}}\cancel{r^3}} = \underline{\underline{\frac{9}{4}}}$$

(3) $2\sin^2 x = 2 - \operatorname{ctg} x$

$$2\sin^2 x = 2 - \frac{\cos x}{\sin x} \quad / \cdot \sin x \qquad [x \neq k \cdot 180°]$$

$$2\sin^3 x = 2\sin x - \cos x$$

$$2\sin x(\sin^2 x - 1) + \cos x = 0$$

$$-2\sin x\cos^2 x + \cos x = 0$$

$$\cos x(1 - 2\sin x\cos x) = 0$$

$$\cos x = 0$$

$$x = 90° + k \cdot 180°$$

$$1 - 2\sin x\cos x = 0$$

$$1 - \sin 2x = 0$$

$$\sin 2x = 1$$

$$2x = 90° + k \cdot 360°$$

$$\underline{\underline{x = 45° + k \cdot 180°}}$$

(2) $V = \log_3 243 + \log_4 \frac{1}{256} + \log_{0,2} 0{,}04 + \log_5 625 =$

$= 5 + (-4) + 2 + 4 = 7$

$U = 2\log_5 \sqrt{25} - \log_7 \frac{1}{49} - \log_3 3 + \log 1000 =$

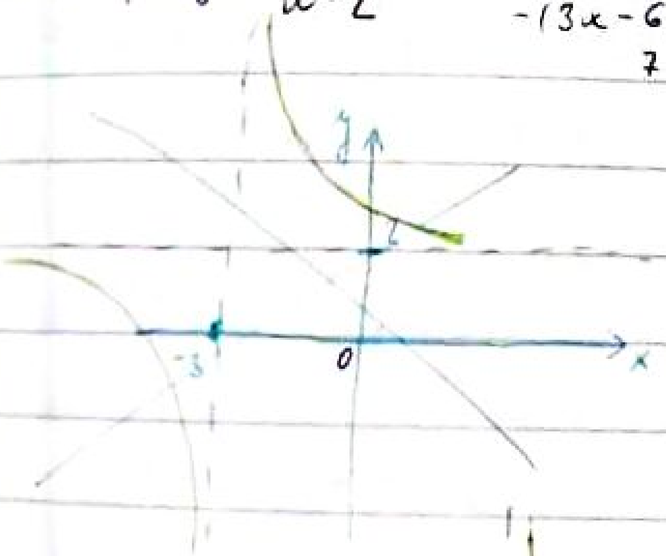
$= 2 - (-2) - 1 + 3 = 6$

$V - U = 7 - 6 = \underline{1}$

(3) $f: y = \frac{3x+1}{x-2}$ $\qquad (3x+1) : (x-2) = 3 + \frac{7}{x-2}$

$-(3x-6)$

7



$x = 0 \rightarrow y = -\frac{1}{2}$

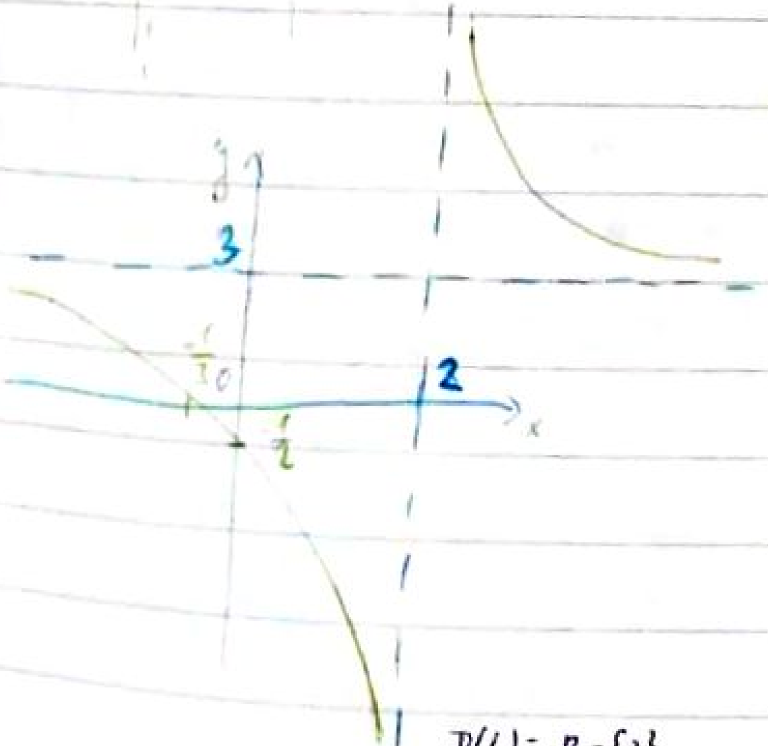
$y = 0 \rightarrow x = -\frac{1}{3}$

$f^{-1}: x = \frac{3y+1}{y-2}$

$xy - 2x = 3y + 1$

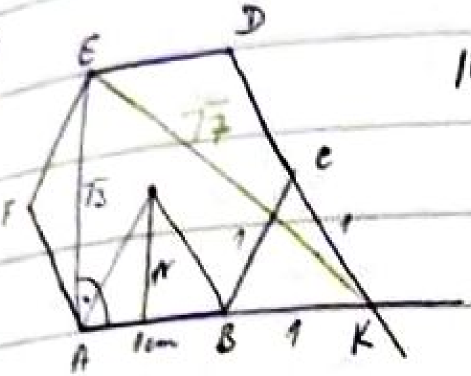
$xy - 3y = 2x + 1$

$y = \frac{2x+1}{x-3} = 2 + \frac{7}{x-3}$



$D(f) = \mathbb{R} - \{2\}$ $\qquad D(f^{-1}) = \mathbb{R} - \{3\}$

$H(f) = \mathbb{R} - \{3\}$ $\qquad H(f^{-1}) = \mathbb{R} - \{2\}$

② 

$|EK| = \sqrt{7}$

$r^2 = 1^2 - 0{,}5^2$ $\qquad |AE| = \sqrt{3}$

$r = \frac{\sqrt{3}}{2}$ $\qquad \Delta EAK: |EK|^2 = 3 + 4$

$\underline{\underline{|EK| = \sqrt{7}}}$

③ $(a-1)x^2 + (1-5a)x + 20 = 0 \qquad x_1 - x_2 = 3$

$a \neq 1 \qquad D = 1 - 10a + 25a^2 - 80(a-1)$

$D = 25a^2 - 90a + 81 = (5a-9)^2$

$$x_{1,2} = \frac{-1+5a \pm (5a-9)}{2(a-1)} = \begin{cases} \frac{-1+5a+5a-9}{2(a-1)} = \frac{10a-10}{2(a-1)} = \frac{10(a-1)}{2(a-1)} = 5 \\ \frac{-1+5a-5a+9}{2(a-1)} = \frac{8}{2(a-1)} = \frac{4}{a-1} \end{cases}$$

$5 - \frac{4}{a-1} = 3 \ /\cdot(a-1)$

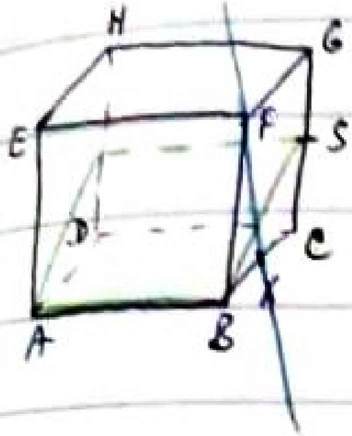
$2a - 2 - 4 = 0$

$\underline{\underline{a = 3}}$

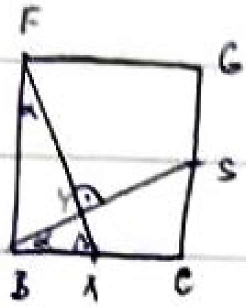
$\frac{4}{a-1} - 5 = 3 \ /\cdot(a-1)$

$4 - 5a + 5 = 3a - 3$

$a = \frac{12}{8} = \underline{\underline{\frac{3}{2}}}$

① 

$AB \perp BCG \Rightarrow AB \perp FX$



$$\left.\begin{array}{l} AB \perp FX \\ BS \perp FX \end{array}\right\} FX \perp ABS$$

$\triangle XYB \sim \triangle XBF$

③ a) $$\frac{\left(15^{\frac{1}{3}} \cdot 27^{-\frac{1}{2}}\right)^{-3}}{\left(25^{\frac{1}{4}} \cdot 9^{\frac{1}{8}}\right)^{-2}} : \frac{\sqrt{\sqrt[3]{9}}}{\sqrt[3]{3\sqrt[4]{27}}} = \frac{\cancel{5^{-1}} \cdot 3^{-1} \cdot 3^{\frac{9}{2}}}{\cancel{5^{-1}} \cdot 3^{-\frac{1}{2}}} \cdot \frac{\cancel{3^{\frac{1}{3}}} \cdot 3^{\frac{1}{4}}}{\cancel{3^{\frac{1}{3}}}} =$$

$$= \underline{\underline{3^{\frac{17}{4}}}}$$

b) $\log_x m = 2\log_x(a-2) + 3\log_x(a+2) - 2\log_x(a^2-4)$

$$\log_x m = \log_x \frac{(a-2)^2 \cdot (a+2)^3}{(a^2-4)^2}$$

$$m = \frac{\cancel{(a-2)^2} \cdot (a+2)^{\cancel{3}}}{\cancel{(a-2)^2}(a+2)^2} = a+2 \qquad \underline{\underline{m = a+2}}$$

(4) $n^3 = 6 \cdot C(3,n) + 6 \cdot C(2,n) + C(1,n)$

$n^3 = 6 \cdot \frac{n!}{(n-3)! \cdot 3!} + \overset{3}{6} \cdot \frac{n!}{(n-2)! \cdot 2!} + n$

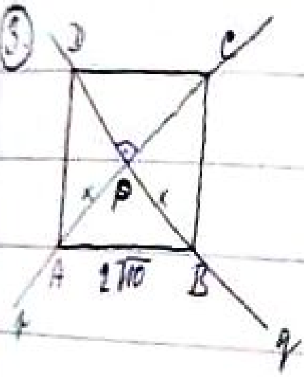
$n^3 = n(n-1)(n-2) + 3n(n-1) + n$

$n^3 = n[n^2 - 3n + 2 + 3n - 3 + 1]$

$n^3 = n[n^2]$

$n^3 = n^3$ ✓ $\qquad n \geq 3$

(5)



$p: x - 2y + 1 = 0$

$q: 2x + y - 3 = 0 \quad / \cdot 2$

$5x - 5 = 0 \qquad 2 \cdot 1 + y - 3 = 0$

$x = 1 \qquad y = 1$

$p \cap q = \quad P = [1, 1]$

$x^2 + x^2 = 4 \cdot 10$

$x = 2\sqrt{5} \qquad |AP| = 2\sqrt{5}$

$A, C: \quad [2y - 1, y] \qquad \left(\sqrt{(\underbrace{2y-1-1}_{2y-2})^2 + (y-1)^2} = 2\sqrt{5}\right)^2$

$4y^2 - 8y + 4 + y^2 - 2y + 1 = 20$

$5y^2 - 10y - 15 = 0 \quad / : 5$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$y_1 = 3 \qquad x_1 = 5 \qquad [5; 3]$

$y_2 = -1 \qquad x_2 = -3 \qquad [-3; -1]$

3) $k: x^2+y^2-8x-y+5=0$

$t \parallel p$; $p: 2x-y+2=0$

$t: 2x-y+c=0$

$y=2x+c$

$k \cap t$: $x^2+4x^2+4xc+c^2-8x-2x-c+5=0$

$5x^2+4xc-10x+c^2-c+5=0$

$D=(4c-10)^2-20(c^2-c+5)=$

$=16c^2-80c+100-20c^2+20c-100=$

$=-4c^2-60c$

$D=0 \quad -4c^2-60c=0 \,/ :(-4)$

$c(c+15)=0$

$c_1=0, c_2=-15$

$t_1 = 2x-y=0$

$t_2 = 2x-y-15=0$

(2)

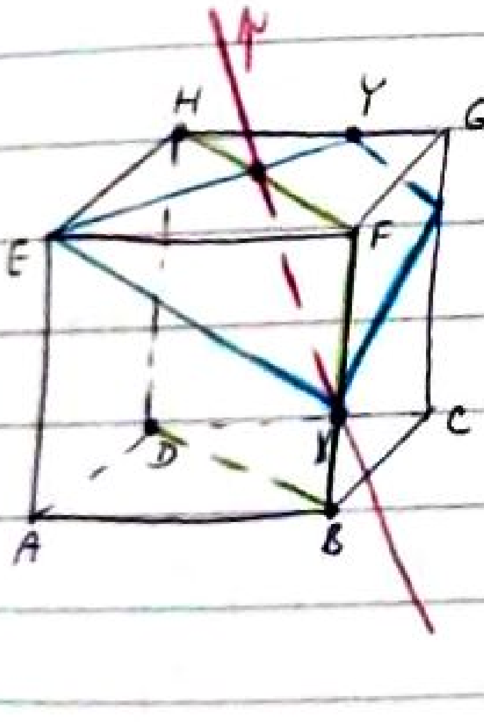
$|DE| = |AD| + |BE|$

$\triangle ASD$, $\triangle SBE$ - rovnoramenné

$|BE| = |SE|$; $|AD| = |DS|$

$|DE|$

(3)



$\overleftrightarrow{BDH} \cap \overleftrightarrow{XYE} = p$

$B, D: \quad [x; 3-2x] \qquad P=[1;1] \qquad x = 2\sqrt{5}$

$$\sqrt{(x-1)^2+(\underbrace{3-2x-1}_{2-2x})^2} = 2\sqrt{5} \quad /^2$$

$$x^2-2x+1+4-8x+4x^2=20$$

$$5x^2-10x-15=0 \quad /:5$$

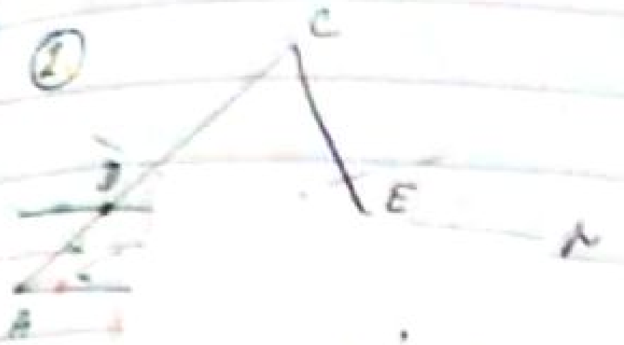
$$x^2-2x-3=0$$

$$(x-3)(x+1)=0$$

$x_1 = 3 \qquad y_1 = -3 \qquad [3;-3]$

$x_2 = -1 \qquad y_2 = 5 \qquad [-1;5]$

①



$|DE| = |AD| + |BE|$

... - rovnostranné

výroky ✓

množiny ✓

④ DÔKAZY ⑪ ⑫

mocniny odmocniny

28.

③

lin. rov., nerov., s neznám., súčin/súčty čísel

3. odhadni počet uhlopriečok

geometria - planimetria úlohy

deliteľnosť NSD, NSN - ťažnica

vzd. bodu od priamky

štvoruholníky

podobnosť △

zhodnosť △

úlohy - S, o

kruhové uhly

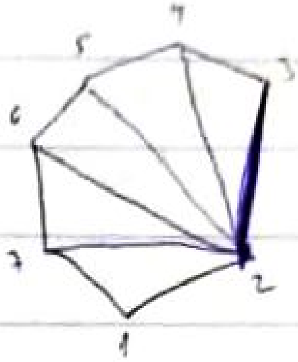
rozdelenie alebo rozdelenie

kružnice - pravý △

$\alpha_1 + \alpha_1' + \alpha_1'' = 180°$

$n \cdot 180° - 360° =$

$= 180°(n-2)$

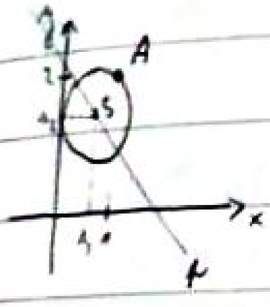


$n-2$ Δ-ov

$(n-2)\,180°$

③ $A = [1;2] \in k$ $\qquad$ k sa dotýka osi y

$S \in p: x + y - 4 = 0$



$k: (x - s_1)^2 + (y - s_2)^2 = s_1^2$

$A \in k: \quad (1 - s_1)^2 + (2 - s_2)^2 = s_1^2$

$S \in p: \quad s_1 + s_2 - 4 = 0$

$s_1 = 4 - s_2$

$\underline{s_1 = r}$

$(\underbrace{1 - 4 + s_2}_{s_2 - 3})^2 + (2 - s_2)^2 = (4 - s_2)^2$

$s_2^2 - 6s_2 + 9 + 4 - 4s_2 + s_2^2 = 16 - 8s_2 + s_2^2$

$s_2^2 - 2s_2 - 3 = 0$

$(s_2 - 3)(s_2 + 1) = 0$

$s_2 = 3 \quad s_2' = -1$

$s_1 = 1 \quad s_1' = 5$

$S = [1;3] \quad S' = [5;-1]$

$k: (x-1)^2 + (y-3)^2 = 1$

$k': (x-5)^2 + (y+1)^2 = 25$

② $\forall x \in \mathbb{R}: \ (4x^2+x-2)^2 \geq 4x(2x-1)^2 - 8x$

$16x^4+4x^3-8x^2+4x^3+x^2-2x-8x^2-2x+4 \geq$

$\geq 4x(4x^2-4x+1)-8x$

$16x^4+8x^3-15x^2-4x+4 \geq 16x^3-16x^2+4x-8x$

$16x^4-8x^3+x^2+4 \geq 0$

$x^2(16x^2-8x+1)+4 \geq 0$

$\underbrace{x^2}_{\geq 0}\underbrace{(4x-1)^2}_{\geq 0}+4 \geq 0$ ✓

③ 8 č 6 B (14B)

a) č, B, č $\quad p = \frac{8}{14}\cdot\frac{6}{14}\cdot\frac{8}{14} = \frac{48}{343} \doteq 0{,}14$

vracím z.

b) č, B, č $\quad p = \frac{8}{14}\cdot\frac{6}{13}\cdot\frac{7}{12} = \frac{2}{13} \doteq 0{,}15$

nevracím z.

c) 2č 1B $\quad p = \frac{C_2(8)\cdot C_1(6)}{C_3(14)} = \frac{\frac{8!}{6!\,2!}\cdot 6}{\frac{14!}{11!\,3!}} = \frac{\frac{8\cdot 7}{2}\cdot 6}{\frac{14\cdot 13\cdot 12}{6}} = \frac{8\cdot 7\cdot 3}{14\cdot 13\cdot 2} = \frac{6}{13} \doteq 0{,}46$

(2) $\forall x, y, z \in R^+: \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq \frac{9}{x+y+z}$

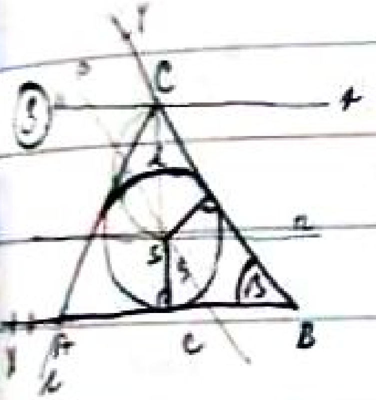
$$\frac{yz(x+y+z) + xz(x+y+z) + xy(x+y+z) - 9xyz}{\underbrace{xyz(x+y+z)}_{>0}} \geq 0$$

$$xyz + y^2z + yz^2 + x^2z + xyz + xz^2 + x^2y + xy^2 + xyz - 9xyz \geq 0$$

$$z(y^2 - 2xy + x^2) + y(z^2 - 2xz + x^2) + x(z^2 - 2yz + y^2) \geq 0 \quad -6xyz$$

$$z(y-x)^2 + y(z-x)^2 + x(z-y)^2 \geq 0$$

(5)



$\rho = 1,5$ cm

$v_c = 4$ cm

$\beta = 60°$

$|\sphericalangle XBY| = 60°$

$p;\ p \parallel \overleftrightarrow{XB};\ |p, \overleftrightarrow{XB}| = 4$ cm

$C;\ C \in p \cap \overrightarrow{BY}$

$r;\ r \parallel XB;\ |r, XB| = 1,5$ cm

$s;\ s \parallel BY;\ |s, BY| = 1,5$ cm

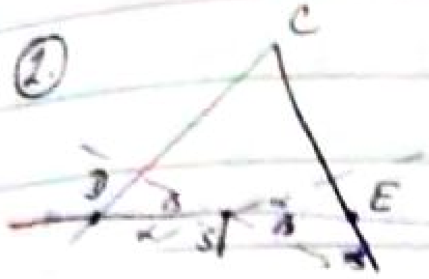
$S;\ S \in r \cap s$

$k;\ k(S; 1,5$ cm$)$

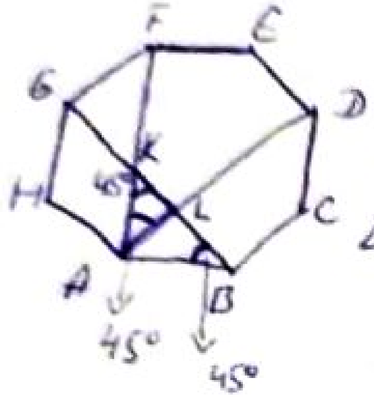
$l;\ l$ - dot. kr. k, $C \in l$

$A;\ A \in l \cap \overrightarrow{BX}$

(2.)



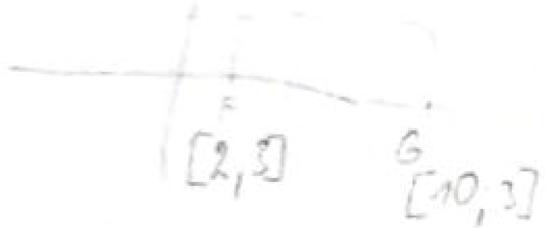
$|DE| = |AD| + |BE|$



$360 : 8 = 45°$

Dok.

$\triangle AKL$ je rovnoramenný

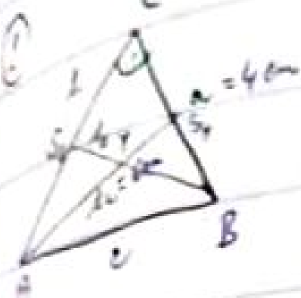


$$\frac{(x+2)^2}{4} - \frac{(y+1)^2}{5} = 1$$

$$5(x^2+4x+4) - 4(y^2-2y+1) = 20$$

$20 - 4$

(2)



$h_a : h_b = \frac{\sqrt{6}}{2}$

$\triangle AB_1C: \; 6^2 = 2^2 + b^2$

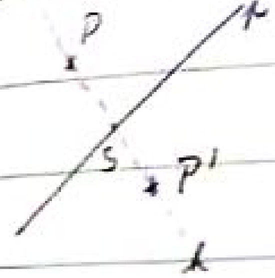
$b = \sqrt{32} = 4\sqrt{2}$ cm

$\triangle S_2BC: \; h_b^2 = \underset{16}{4^2} + \underset{4\cdot 2}{(2\sqrt{2})^2}$

$h_b = \sqrt{24} = 2\sqrt{6}$ cm

$\frac{h_a}{h_b} = \frac{6}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{6\sqrt{6}}{2\cdot 6} = \underline{\underline{\frac{\sqrt{6}}{2}}}$

(3)



$P = [2; -3]$

$p: 2x - y + 3 = 0$

$k: x + 2y + c = 0$

$P \in k \quad 2 - 6 + c = 0$

$c = 4$

$k: x + 2y + 4 = 0$

$k \cap p: \quad x + 2y + 4 = 0$

$2x - y + 3 = 0 \; / \cdot 2$

$5x + 10 = 0$

$x = -2 \quad y = -1$

$S = [-2; -1]$

$P' = [-6; 1]$

④ $f(x) = \dfrac{\sin \frac{1}{x}}{\log(x+2) + \log(2-x) + \log(4-x^2)}$

$f(-x) = \dfrac{\sin\left(\frac{1}{-x}\right)}{\log(-x+2) + \log(2-(-x)) + \log(4-(-x)^2)} =$

$= \dfrac{-\sin \frac{1}{x}}{\log(2-x) + \log(2+x) + \log(4-x^2)}$ $\qquad f(x) = -f(-x)$

⑤ $k: x^2+y^2-4x+12y+27=0$

$k \perp p: \ 3x+2y+5=0$ $\qquad \rightarrow \rightarrow$

$k: 2x-3y+c=0$

$$x = \frac{3y-c}{2}$$

$k \cap k: \ \dfrac{9y^2-6yc+c^2}{4} + y^2 - 4 \cdot \dfrac{3y-c}{2} + 12y + 27 = 0 \quad / \cdot 4$

$9y^2-6yc+c^2+4y^2-24y+8c+48y+108=0$

$13y^2+24y-6yc+c^2+8c+108=0$

$13y^2+y(24-6c)+c^2+8c+108=0$

$D = 576-288c+36c^2-52(c^2+8c+108)$

$D = -16c^2-704c-5040$ $\qquad k: 2x-3y-9=0$

$D=0 \quad -16c^2-704c-5040=0 \ / \cdot(-16)$ $\qquad 2x-3y-35=0$

$c^2+44c+315=0$

$D = 676$

$c_{1,2} = \dfrac{-44 \pm 26}{2} = \begin{cases} -9 \\ -35 \end{cases}$

③₄ $\forall n \in \mathbb{N}:\ 7/(2^n+3^{2n}) \Rightarrow 7/(2^{n+1}+3^{2(n+1)})$ $\qquad 2^n+3^{2n}=7k,\ 2^n=7k-3^{2n}$

$$2^n\cdot 2+3^{2n}\cdot 3^2=(7k-3^{2n})\cdot 2+3^{2n}\cdot 9=14k\underbrace{-2\cdot 3^{2n}+9\cdot 3^{2n}}_{7\cdot 3^{2n}}=7(2k+3^{2n})$$

✓

4) $3/[n^3+(n+1)^3+(n+2)^3]$

$$n^3+n^3+3n^2+3n+1+n^3+6n^2+12n+8=3n^3+9n^2+15n+9n=$$

$$=3(n^3+3n^2+5n+3n) \Rightarrow 3/\ldots$$

✓

⑤ $f: y=a\log_5(3-x)+b$ $\qquad A,B\in f$ $\qquad A=[-2;1]$

$B=[\frac{14}{5};7]$

$C=[-22;-2]\in f$?

$$1=a\log_5(3+2)+b$$
$$7=a\log_5(3-\tfrac{14}{5})+b$$

$$1=a\cdot 1+b$$
$$7=a\cdot(-1)+b$$

$8=2b$ $\qquad 1=a+4$

$\underline{b=4}$ $\qquad \underline{a=-3}$

$f: y=-3\log_5(3-x)+4$

$$-2=-3\log_5(3+22)+4$$
$$-2=-3\cdot 2+4$$
$$-2=-2$$

$C\in f$

$f^{-1}: x=-3\log_5(3-y)+4$

$$\frac{x-4}{-3}=\log_5(3-y)$$

$$3-y=5^{\frac{4-x}{3}}$$

$f^{-1}: y=3-5^{\frac{4-x}{3}}$

4) $P(n+1)$: $7 / (2^n + 3^{2n}) \Rightarrow 7 / (2^{n+1} + 3^{2(n+1)})$ $\qquad 2^n + 3^{2n} = 7k$; $2^n = 7k - 3^{2n}$

$$2^n \cdot 2 + 3^{2n} \cdot 3^2 = (7k - 3^{2n}) \cdot 2 + 3^{2n} \cdot 9 = 14k \underbrace{- 2 \cdot 3^{2n} + 9 \cdot 3^{2n}}_{7 \cdot 3^{2n}} = 7(2k + 3^{2n})$$

4) $3 / [n^3 + (n+1)^3 + (n+2)^3]$

$$n^3 + n^3 + 3n^2 + 3n + 1 + n^3 + 6n^2 + 12n + 8 = 3n^3 + 9n^2 + 15n + 9n =$$

$$= 3(n^3 + 3n^2 + 5n + 3n) \Rightarrow 3 / \ldots$$

5) $f: y = a \log_5 (3 - x) + b$ $\qquad A, B \in f$ $\qquad A = [-2; 1]$

$B = [\frac{14}{5}; 7]$

$C = [-22; -2] \in f$?

$$1 = a \log_5 (3 + 2) + b$$
$$7 = a \log_5 (3 - \tfrac{14}{5}) + b$$

$$1 = a \cdot 1 + b$$
$$7 = a \cdot (-1) + b$$

$8 = 2b$ $\qquad 1 = a + 4$

$\underline{b = 4}$ $\qquad \underline{a = -3}$

$f: y = -3 \log_5 (3 - x) + 4$

$$-2 = -3 \log_5 (3 + 22) + 4$$
$$-2 = -3 \cdot 2 + 4$$
$$-2 = -2 \qquad C \in f$$

f^{-1}: $x = -3 \log_5 (3 - y) + 4$

$$\frac{x - 4}{-3} = \log_5 (3 - y)$$

$$3 - y = 5^{\frac{4 - x}{3}} \qquad f^{-1}: y = 3 - 5^{\frac{4 - x}{3}}$$