Uloha 5

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4.2 d) unitary $\rightarrow U^{+} = U^{-1} \rightarrow U^{+} \cdot U = 1$

d) $\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \jmath & \jmath^{*}\\ 1 & \jmath^{*} & \jmath\end{pmatrix}$, where $\jmath = e^{\frac{2\pi i}{3}}$.

$U^{+} = \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \jmath^{*} & \jmath\\ 1 & \jmath & \jmath^{*}\end{pmatrix}$

$$\frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \jmath^{*} & \jmath\\ 1 & \jmath & \jmath^{*}\end{pmatrix} \cdot \frac{1}{\sqrt{3}}\begin{pmatrix}1 & 1 & 1\\ 1 & \jmath & \jmath^{*}\\ 1 & \jmath^{*} & \jmath\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1+1+1 & 1+\jmath+\jmath^{*} & 1+\jmath^{*}+\jmath\\ 1+\jmath^{*}+\jmath & 1+\jmath^{*}\cdot\jmath+\jmath\cdot\jmath^{*} & 1+\jmath^{*2}+\jmath^{2}\\ 1+\jmath+\jmath^{*} & 1+\jmath^{2}+\jmath^{*2} & 1+\jmath\cdot\jmath^{*}+\jmath^{*}\cdot\jmath\end{pmatrix}$$

$U = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

$U^{+} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix} \cdot \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix} \checkmark$$

4.3

c) $\frac{1}{2}\begin{pmatrix}1 & -\sqrt{3}\\ \sqrt{3} & 1\end{pmatrix}$,

$\frac{1}{2}\left(|0\rangle + \sqrt{3}|1\rangle\right), \frac{1}{2}\left(-\sqrt{3}|0\rangle + |1\rangle\right)$

$\frac{1}{2}\begin{pmatrix}1 & \sqrt{3}\end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix}-\sqrt{3}\\ 1\end{pmatrix} = 0$ OG $\checkmark$

$$\frac{1}{2}(1\ \sqrt{3}) \cdot \frac{1}{2}\begin{pmatrix}1\\ \sqrt{3}\end{pmatrix} = \frac{(1+3)}{4} = 1 \quad ONV$$

$$\frac{1}{2}(-\sqrt{3}\ \ 1) \cdot \frac{1}{2}\begin{pmatrix}-\sqrt{3}\\ 1\end{pmatrix} = \frac{3+1}{4} = 1$$

4.7 t $(U|\psi\rangle)^\dagger = \langle\psi| U^\dagger$ where $U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$, and that $(|\psi\rangle\langle+|)^\dagger = |+\rangle\langle\psi|$,

b b) $|\psi\rangle = \frac{1}{2}(|0\rangle + i\sqrt{3}|1\rangle)$,

$$|\psi\rangle\ \frac{1}{2}\begin{pmatrix}1\\ i\sqrt{3}\end{pmatrix}$$

$$U|\psi\rangle = \begin{pmatrix}a & b\\ -b^* & a^*\end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix}1\\ i\sqrt{3}\end{pmatrix} = \begin{pmatrix}a + bi\sqrt{3}\\ -b^* + a^* i\sqrt{3}\end{pmatrix}\frac{1}{2}$$

$$(U|\psi\rangle)^\dagger = (a^* - b^* i\sqrt{3}\ | -b - a\ i\sqrt{3})\frac{1}{2}$$

$$U^\dagger = \begin{pmatrix}a^* & -b\\ b^* & a\end{pmatrix}$$

$$\langle\psi|U^\dagger = \frac{1}{2}(1\ -i\sqrt{3})\begin{pmatrix}a^* & -b\\ b^* & a\end{pmatrix} = \frac{1}{2}(a^* - b^* i\sqrt{3}\ \ | -b\ -a i\sqrt{3})$$

$$(U|\psi\rangle)^\dagger = \langle\psi|U^\dagger \rightarrow (a^* - b^* i\sqrt{3}\ | -b - a\ i\sqrt{3})\frac{1}{2} = \frac{1}{2}(a^* - b^* i\sqrt{3}\ \ | -b\ -a i\sqrt{3})$$

$$|+\rangle\ \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$(|\psi\rangle\langle+|)^\dagger$$

$$|\psi\rangle\langle+| = \frac{1}{2}\begin{pmatrix}1\\ i\sqrt{3}\end{pmatrix}\frac{1}{\sqrt{2}}(1\ \ 1) = \frac{1}{\sqrt{2}\,2}\begin{pmatrix}1 & 1\\ i\sqrt{3} & i\sqrt{3}\end{pmatrix}$$

$$(|\psi\rangle\langle+|)^\dagger = \frac{1}{\sqrt{2}\,2}\begin{pmatrix}1 & -i\sqrt{3}\\ 1 & -i\sqrt{3}\end{pmatrix}$$

$$|+\rangle\langle\psi| = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}\frac{1}{2}(1\ -i\sqrt{3}) = \frac{1}{\sqrt{2}\,2}\begin{pmatrix}1 & -i\sqrt{3}\\ 1 & -i\sqrt{3}\end{pmatrix} \quad \checkmark$$

c $|\psi\rangle = \psi_0|0\rangle + \psi_1|1\rangle$.

$$|\psi\rangle = \begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix}$$

$$U|\psi\rangle = \begin{pmatrix}a & b\\ -b^* & a^*\end{pmatrix}\cdot\begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix} = \begin{pmatrix}a\psi_0 + b\psi_1\\ \ldots\end{pmatrix}$$

$$U|\psi\rangle = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \cdot \begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix} = \begin{pmatrix} a\psi_0 + b\psi_1 \\ -b^*\psi_0 + a^*\psi_1\end{pmatrix}$$

$$(U|\psi\rangle)^\dagger = \begin{pmatrix} a^*\psi_0^* + b^*\psi_1^* & -b\psi_0^* + a\psi_1^* \end{pmatrix}$$

$$\langle\psi| = \begin{pmatrix}\psi_0^* & \psi_1^*\end{pmatrix}$$

$$=$$

$$\langle\psi|U^\dagger = \begin{pmatrix}\psi_0^* & \psi_1^*\end{pmatrix} \cdot \begin{pmatrix} a^* & -b \\ b^* & a\end{pmatrix} = \begin{pmatrix} a^*\psi_0^* + b^*\psi_1^* \mid -b\psi_0^* + a\psi_1^* \end{pmatrix}$$

$$|\psi\rangle\langle +| = \begin{pmatrix}\psi_0\\ \psi_1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_0 & \psi_0\\ \psi_1 & \psi_1\end{pmatrix}$$

$$(|\psi\rangle\langle +|)^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_0^* & \psi_1^* \\ \psi_0^* & \psi_1^*\end{pmatrix}$$

$$=$$

$$|+\rangle\langle\psi| = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} \cdot \begin{pmatrix}\psi_0^* & \psi_1^*\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_0^* & \psi_1^* \\ \psi_0^* & \psi_1^*\end{pmatrix}$$