Uloha 4

16 October 2022 16:04

3.6 b

$$\begin{pmatrix} 1 \\ \sqrt{k} \\ \sqrt{m-k-1} \end{pmatrix} \quad k < m$$

$$|v| = \begin{pmatrix} 1 & \sqrt{k} & \sqrt{m-k-1} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \sqrt{k} \\ \sqrt{m-k-1} \end{pmatrix} = (1 + k + (m-k-1)) = \sqrt{m}$$

$$\text{norm} \begin{pmatrix} 1 \\ \sqrt{k} \\ \sqrt{m-k-1} \end{pmatrix} : \sqrt{m}$$

$$a \cdot \begin{pmatrix} 1 \\ \sqrt{k} \\ \sqrt{m-k-1} \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{k} \\ \sqrt{m-k-1} \end{pmatrix} : \sqrt{m}$$

$$a = \frac{1}{\sqrt{m}}$$

3.11 b

$$|\mu_1\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad |\mu_2\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \quad |\mu_3\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \quad |\mu_4\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}.$$

b) $|\psi\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 1-i \\ -1 \\ i \end{pmatrix}$,

$$\frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 1-i \\ -1 \\ i \end{pmatrix} = \sum_{j=1}^{d} \langle \mu_j | \psi \rangle | \mu_j \rangle$$

$$\frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 1-i \\ -1 \\ i \end{pmatrix} = \langle \mu_1 | \psi \rangle | \mu_1 \rangle + \langle \mu_2 | \psi \rangle | \mu_2 \rangle + \langle \mu_3 | \psi \rangle | \mu_3 \rangle + \langle \mu_4 | \psi \rangle | \mu_4 \rangle$$

$$\frac{1}{2}(1\ 1\ 1\ 1)\cdot \frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\1-i\\-1\\i\end{pmatrix} = \frac{1}{2\sqrt{6}}(1+i+1-i-1+i) = \frac{1}{2\sqrt{6}}(1+i)$$

$$\frac{1}{2}(1\ -1\ 1\ -1)\cdot \frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\1-i\\-1\\i\end{pmatrix} = \frac{1}{2\sqrt{6}}(1+i-1+i-1-i) = \frac{1}{2\sqrt{6}}(-1+i)$$

$$\frac{1}{2}(1\ 1\ -1\ -1)\cdot \frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\1-i\\-1\\i\end{pmatrix} = \frac{1}{2\sqrt{6}}(1+i+1-i+1-i) = \frac{1}{2\sqrt{6}}(3-i)$$

$$\frac{1}{2}(1\ -1\ -1\ 1)\cdot \frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\1-i\\-1\\i\end{pmatrix} = \frac{1}{2\sqrt{6}}(1+i-1+i+1+i) = \frac{1}{2\sqrt{6}}(1+3i)$$

$$\frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\1-i\\-1\\i\end{pmatrix} = \frac{1}{2\sqrt{6}}\Big((1+i)|\psi_1\rangle + (-1+i)|\psi_2\rangle + (3-i)|\psi_3\rangle + (1+3i)|\psi_4\rangle\Big)$$

3.14

a

$|0\rangle \begin{pmatrix}1\\0\end{pmatrix}$

$$p_j = |\langle j|v\rangle|^2 = |v_j|^2.$$

$P_0 = |\langle 0|0\rangle|^2 \quad P_0 = (1\ 0)\cdot\begin{pmatrix}1\\0\end{pmatrix} = P_0 = 1+0 = 1$

$P_1 = |\langle 1|0\rangle|^2 \quad P_1 = (0\ 1)\cdot\begin{pmatrix}1\\0\end{pmatrix} = 0$

b $|+\rangle\ \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$P_0\ |\langle 0|+\rangle|^2 \quad P_0 = \left|(1\ 0)\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = \left(\frac{1}{\sqrt{2}}\cdot 1\right)^2 = \frac{1}{2}$

$P_1\ |\langle 1|+\rangle|^2 \quad P_1 = \left|(0\ 1)\ \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = \left|\frac{1}{\sqrt{2}}\cdot 1\right|^2 = \frac{1}{2}$

d $\quad |\psi\rangle = \frac{1}{2}\begin{pmatrix}1\\i\sqrt{3}\end{pmatrix}$

$P_0 = |\langle 0|\psi\rangle|^2 \quad P_0 = \left|(1\ 0)\ \frac{1}{2}\begin{pmatrix}1\\i\sqrt{3}\end{pmatrix}\right|^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$P_1\ |\langle 1|\psi\rangle|^2 \quad P_1 = \left|(0\ 1)\ \frac{1}{2}\begin{pmatrix}1\\i\sqrt{3}\end{pmatrix}\right|^2 = \left(\frac{1}{2}i\sqrt{3}\right) \qquad \left(\frac{i\sqrt{3}}{2}\cdot\frac{-i\sqrt{3}}{2}\right) = \frac{-i^2\,3}{4} = \frac{3}{4}$

$\frac{1}{2}(1\ -i\sqrt{3})\begin{pmatrix}0\\1\end{pmatrix} = \left(\frac{-i\sqrt{3}}{2}\right)$

$$\frac{1}{2}\begin{pmatrix}1 & -i\sqrt{3}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \left(\frac{-i\sqrt{3}}{2}\right)$$

1 2 2 1 4 4

3.75

basis $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

a $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$.

$P_+ = |\langle +|0\rangle|^2$ $P_+ = \left|\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right|^2 = \frac{1}{2}$

$P_- = |\langle -|0\rangle|^2$ $P_- = \left|\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right|^2 = \frac{1}{2}$

b $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$P_+ = |\langle +|+\rangle|^2$ $P_+ = \left|\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = \left|\frac{2}{2}\right|^2 = 1^2 = 1$

$P_- = |\langle -|+\rangle|^2$ $P_- = \left|\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\right|^2 = \left|\frac{1-1}{2}\right|^2 = 0$

d) $|\psi\rangle = \frac{1}{2}\begin{pmatrix}1\\i\sqrt{3}\end{pmatrix}$

$P_+ = |\langle +|\psi\rangle|^2$ $P_+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\end{pmatrix}\frac{1}{2}\begin{pmatrix}1\\i\sqrt{3}\end{pmatrix} = \frac{1+i\sqrt{3}}{2\sqrt{2}}$

$P_+ = \frac{1}{2}\begin{pmatrix}1 & -i\sqrt{3}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \underline{\frac{1-i\sqrt{3}}{2\sqrt{2}}}$

$$\frac{1-i^2 3}{8} = \frac{4}{8} = \frac{1}{2}$$

$P_- = |\langle -|\psi\rangle|^2$ $P_- = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & -1\end{pmatrix}\frac{1}{2}\begin{pmatrix}1\\i\sqrt{3}\end{pmatrix} = \frac{1-i\sqrt{3}}{2\sqrt{2}}$

$P_- = \frac{1}{2}\begin{pmatrix}1 & -i\sqrt{3}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \underline{\frac{1+i\sqrt{3}}{2\sqrt{2}}}$

$$\frac{1-i^2 3}{8} = \frac{1}{2}$$