

2.6 a

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad A^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$$

b

$$A = \begin{pmatrix} 1+i \\ \sqrt{2}i \end{pmatrix} \quad A^\dagger = (1-i \quad 2-\sqrt{2}i)$$

$$(1-i \quad 2-\sqrt{2}i) \begin{pmatrix} 1+i \\ \sqrt{2}i \end{pmatrix} = 1 - i^2 + 2\sqrt{2}i - 2 = -1 + 2\sqrt{2}i$$

3.1

$$b \quad \begin{pmatrix} 1+i \\ 1-i \end{pmatrix} - \begin{pmatrix} 1+i \\ 1+i \end{pmatrix} = \begin{pmatrix} 0 \\ -2i \end{pmatrix}$$

c

$$\sum_{j=0}^{d-1} |e_j\rangle \langle e_j| \quad e_0 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad e_1 = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix} \quad e_d = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$3.4 \quad |u\rangle = \begin{pmatrix} 3+2i \\ i \\ 0 \end{pmatrix}, |v\rangle = \begin{pmatrix} 1-i \\ 2+3i \\ 5i \end{pmatrix}$$

$$\begin{pmatrix} 3-2i & -i & 0 \end{pmatrix} \cdot \begin{pmatrix} 1-i \\ 2+3i \\ 5i \end{pmatrix} = 3-2i-3i+2i^2-2i-3i^2+0 = 4-7i$$

3.5

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle 1 | 1 \rangle = 1 \quad \text{Vzhledom, že sú kolmé}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0+0=0 \quad \checkmark$$

x tiež sú kolmé

$$b \quad |+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, |-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$2.7 \quad A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$A^\dagger A = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 \quad B^\dagger B = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

$$A A^\dagger = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad B B^\dagger = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A A^\dagger - B B^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$b \quad A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$A^\dagger A = \frac{1}{2} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \quad B^\dagger B = \frac{1}{2} \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 1$$

$$A A^\dagger = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad B B^\dagger = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \frac{1}{2} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$c \quad A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$A^\dagger A = \frac{1}{2} \begin{pmatrix} 1 & -i \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = 1 \quad B^\dagger B = \frac{1}{2} \begin{pmatrix} 1 & i \end{pmatrix} \begin{pmatrix} 1 \\ -i \end{pmatrix} = 1$$

$$A A^\dagger = \frac{1}{2} \begin{pmatrix} 1 \\ i \end{pmatrix} \begin{pmatrix} 1 & -i \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} \quad B B^\dagger = \frac{1}{2} \begin{pmatrix} 1 \\ -i \end{pmatrix} \begin{pmatrix} 1 & i \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$$

$$\frac{1}{2} \begin{pmatrix} 0 & -2i \\ 2i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$\langle + | - \rangle$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2} (1 \cdot 1 - 1 \cdot 1) = 0 \quad \checkmark$$

c)

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix},$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{3} (1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1) = \frac{3}{3} = 1 \quad \times$$

d)

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{2} (1 \cdot 1 + 1 \cdot 1) = 1 \quad \checkmark$$

3.7

$$a \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

d)

$$\begin{pmatrix} -1 \\ 0 \\ 2+i \\ -i \end{pmatrix} = - \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} (2+i) - i \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$|0\rangle \quad |2\rangle \quad |3\rangle$

$$-|0\rangle + (2+i)|2\rangle - i|3\rangle$$