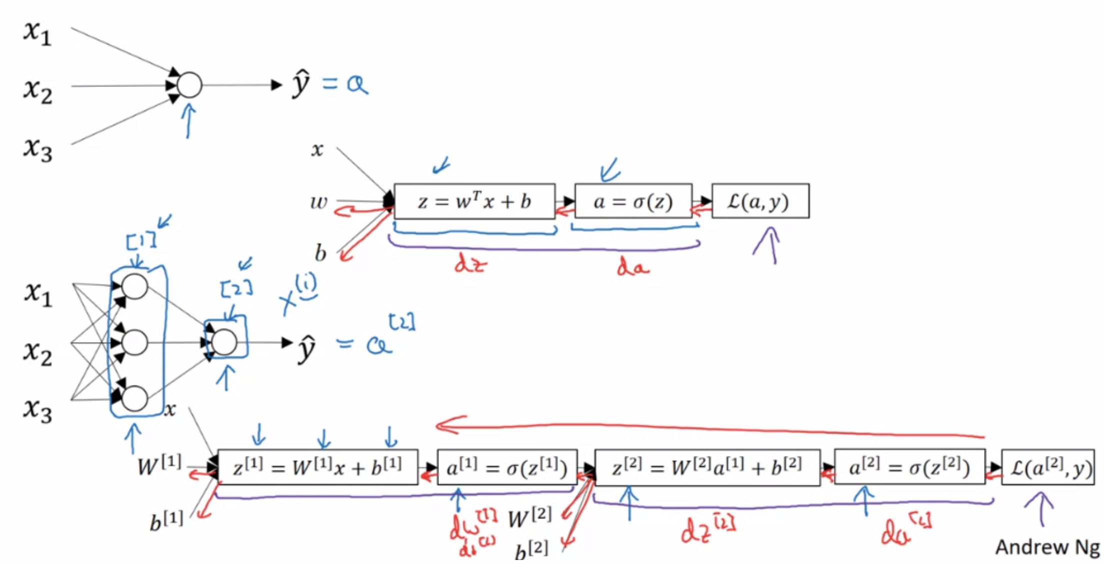
Week 3-1 Vectorizing NN across Multi-examples (Shallow 2-layer NN)

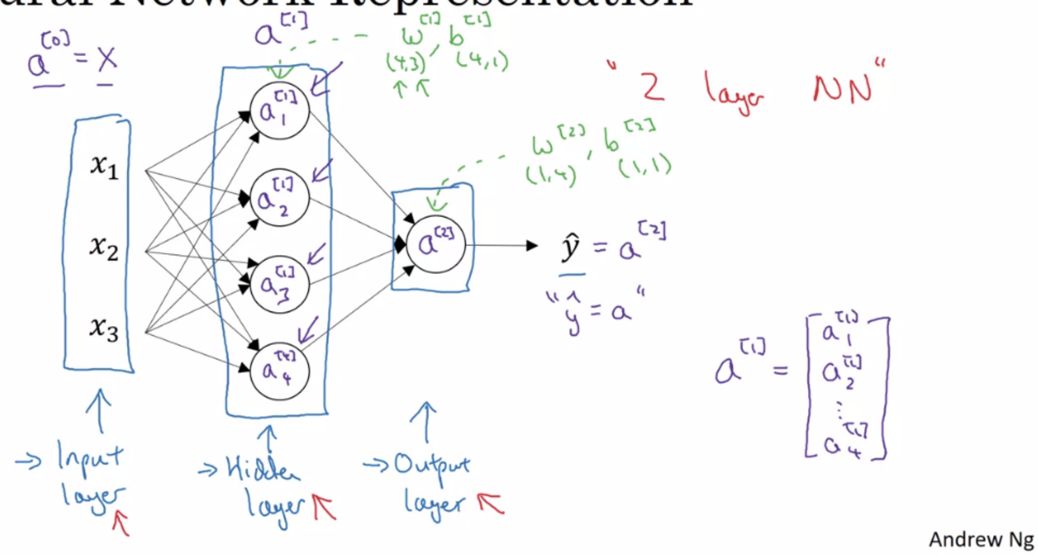
笔记本: DL 1 - NN and DL
创建时间: 2021/1/8 11:12 **更新时间:** 2021/1/8 11:49

[1] - layer
(1) - training sample

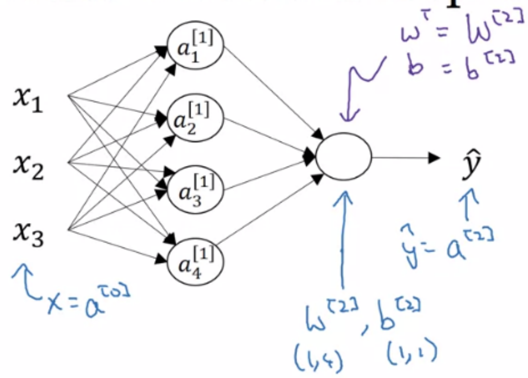
What is a Neural Network?



Neural Network Representation



Neural Network Representation learning



Given input x:

$$\underset{(4,1)}{z^{[1]}} = \underset{(4,3)}{W^{[1]}} \underset{(3,1)}{\cancel{x}^{\,a^{[0]}}} + \underset{(4,1)}{b^{[1]}}$$

$$\underset{(4,1)}{a^{[1]}} = \sigma(\underset{(4,1)}{z^{[1]}})$$

$$\underset{(1,1)}{z^{[2]}} = \underset{(1,4)}{W^{[2]}} \underset{(4,1)}{a^{[1]}} + \underset{(1,1)}{b^{[2]}}$$

$$\underset{(1,1)}{a^{[2]}} = \sigma(\underset{(1,1)}{z^{[2]}})$$

Vectorizing across multiple examples

```
for i = 1 to m:
```

$$\boxed{z^{[1](i)}} = W^{[1]}x^{(i)} + b^{[1]}$$
$$\boxed{a^{[1](i)}} = \sigma(z^{[1](i)})$$
$$\boxed{z^{[2](i)}} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$\boxed{a^{[2](i)}} = \sigma(z^{[2](i)})$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

$$X = \underset{(n_x, m)}{\begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}}$$

$$Z^{[1]} = \begin{bmatrix} | & | & & | \\ z^{[1](1)} & z^{[1](2)} & \cdots & z^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

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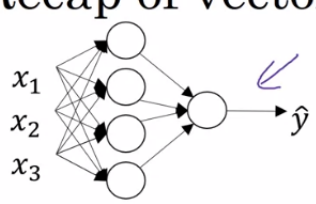
Recap of vectorizing across multiple examples




```
for i = 1 to m
```

$$z^{[1](i)} = W^{[1]}x^{(i)} + b^{[1]}$$
$$a^{[1](i)} = \sigma(z^{[1](i)})$$
$$z^{[2](i)} = W^{[2]}a^{[1](i)} + b^{[2]}$$
$$a^{[2](i)} = \sigma(z^{[2](i)})$$

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \cdots & x^{(m)} \\ | & | & & | \end{bmatrix}$$

$$A^{[1]} = \begin{bmatrix} | & | & & | \\ a^{[1](1)} & a^{[1](2)} & \cdots & a^{[1](m)} \\ | & | & & | \end{bmatrix}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$
$$A^{[1]} = \sigma(Z^{[1]})$$
$$Z^{[2]} = W^{[2]}A^{[1]} + b^{[2]}$$
$$A^{[2]} = \sigma(Z^{[2]})$$

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GD

Gradient descent for neural networks

Parameters: $W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}$ $\quad n_x = n^{[0]}, n^{[1]}, \underline{n^{[2]} = 1}$

$(n^{[1]}, n^{[0]})$ $(n^{[1]}, 1)$ $(n^{[2]}, n^{[1]})$ $(n^{[2]}, 1)$

Cost function: $J(W^{[1]}, b^{[1]}, W^{[2]}, b^{[2]}) = \frac{1}{m} \sum_{i=1}^{n} \mathcal{L}(\hat{y}, y)$ $\quad \hat{y} \leftarrow a^{[2]}$

Gradient descent:

$\rightarrow$ Repeat {

$\rightarrow$ Compute predicts $(\hat{y}^{(i)}, i = 1 \ldots, m)$

$dW^{[1]} = \frac{\partial J}{\partial W^{[1]}}, \quad db^{[1]} = \frac{\partial J}{\partial b^{[1]}}, \ldots$

$W^{[1]} := W^{[1]} - \alpha \, dW^{[1]}$

$b^{[1]} := b^{[1]} - \alpha \, db^{[1]}$

Formulas for computing derivatives

Forward propagation:

$Z^{[1]} = W^{[1]} X + b^{[1]}$

$A^{[1]} = g^{[1]}(Z^{[1]}) \leftarrow$

$Z^{[2]} = W^{[2]} A^{[1]} + b^{[2]}$

$A^{[2]} = g^{[2]}(Z^{[2]}) = \sigma(Z^{[2]})$

Back propagation:

$dZ^{[2]} = A^{[2]} - Y \leftarrow$ $\quad Y = [y^{(1)} \; y^{(2)} \ldots, y^{(m)}]$

$dW^{[2]} = \frac{1}{m} dZ^{[2]} A^{[1]T}$ $\quad (n^{[2]},) \leftarrow$

$\quad (n^{[2]}, 1) \leftarrow$

$db^{[2]} = \frac{1}{m}$ np.sum($dZ^{[2]}$, axis=1, keepdims=True)

$dZ^{[1]} = \underbrace{W^{[2]T} dZ^{[2]}}_{(n^{[1]}, m)} * \underbrace{g^{[1]'}(Z^{[1]})}_{(n^{[1]}, m)}$

$\uparrow$ element-wise product

$dW^{[1]} = \frac{1}{m} dZ^{[1]} X^T$

$db^{[1]} = \frac{1}{m}$ np.sum($dZ^{[1]}$, axis=1, keepdims=True)

$(n^{[1]}, 1)$ $\quad (n^{[1]},)$

Computing gradients

$x_1, x_2, x_3 \rightarrow (\sigma) \rightarrow \hat{y} = a$

Logistic regression

$x, w, b \rightarrow \boxed{z = w^T x + b} \rightarrow \boxed{a = \sigma(z)} \rightarrow \boxed{\mathcal{L}(a, y)}$

dw

db

$dz = a - y$

$da = \frac{d}{da} \mathcal{L}(a, y) = -y \log a - (1 - y) \log(1 - a)$

$= -\frac{y}{a} + \frac{1 - y}{1 - a}$

$\boxed{dz = da \cdot g'(z)}$

$g(z) = \sigma(z)$

$\boxed{\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \mathcal{L}}{\partial a} \cdot \boxed{\frac{da}{dz}}}$

"dz" = "da" $\quad \frac{d}{dz} g(z) = \underline{g'(z)}$

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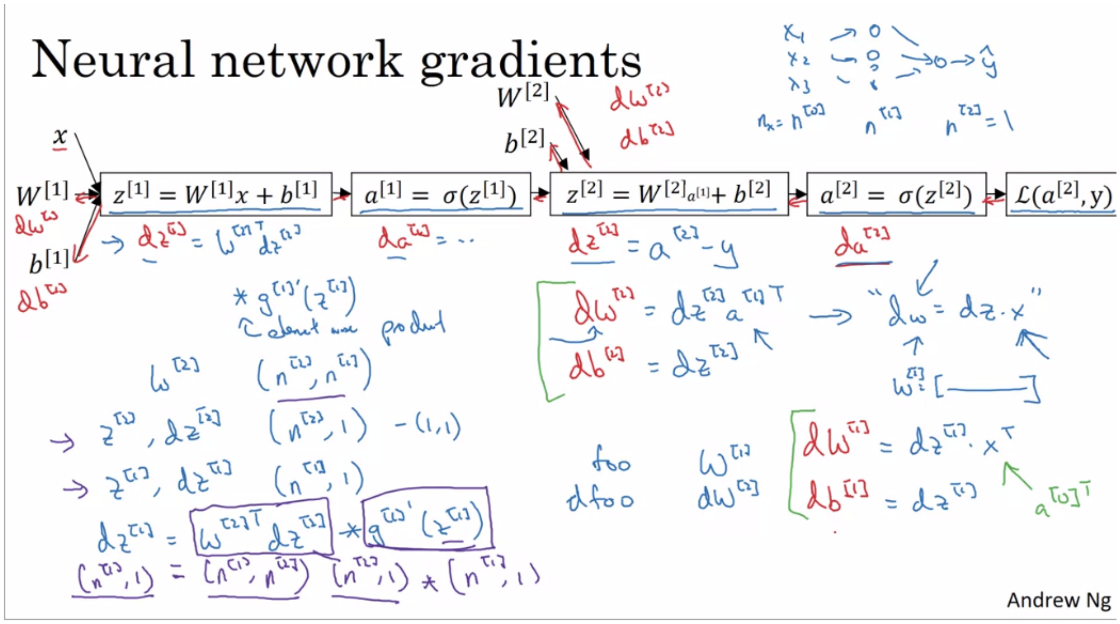
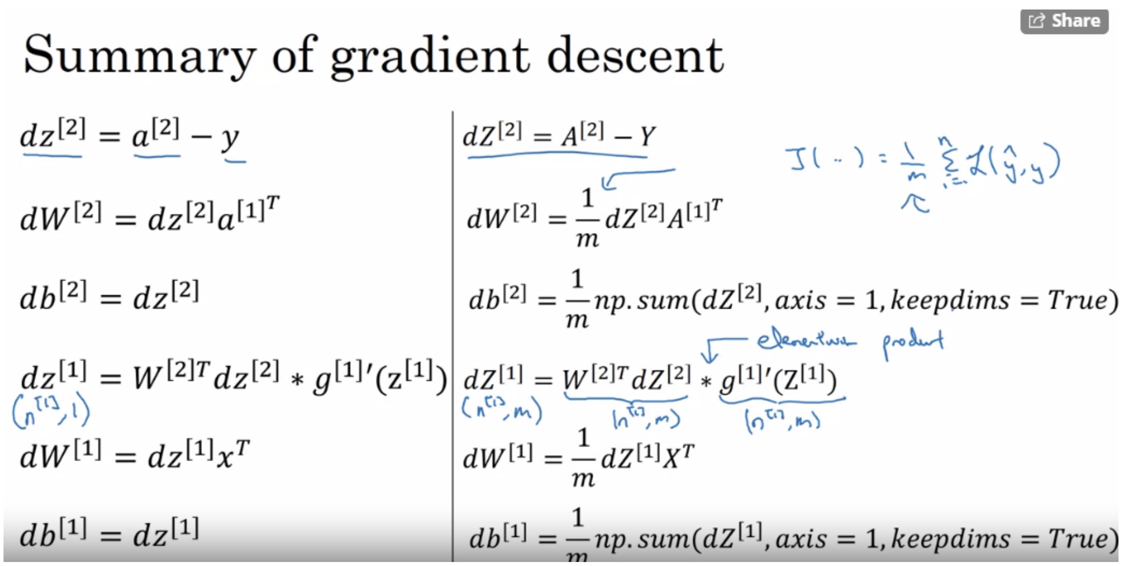
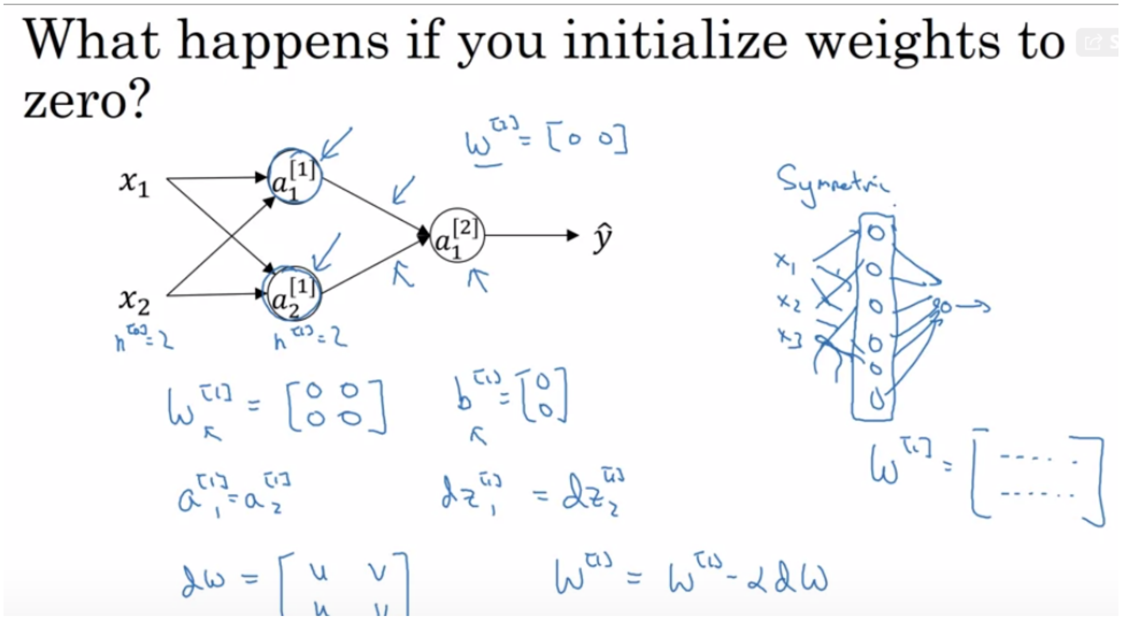
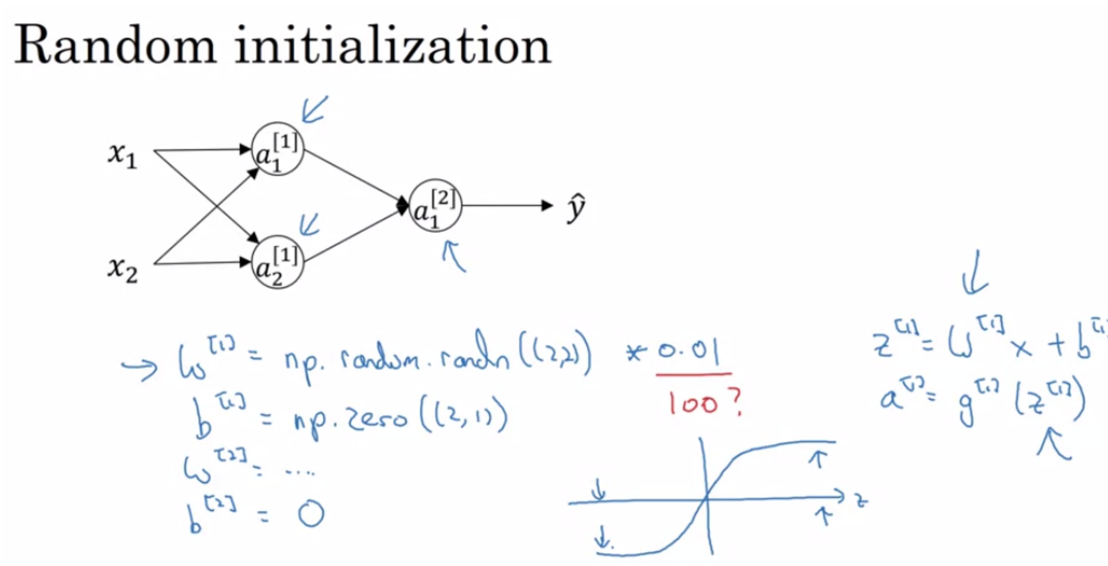


vectorized



Random Initialization - all units calculate the same thing (symmetric)





why 0.01? (Initialize the para to a very small value -> derivative larger)