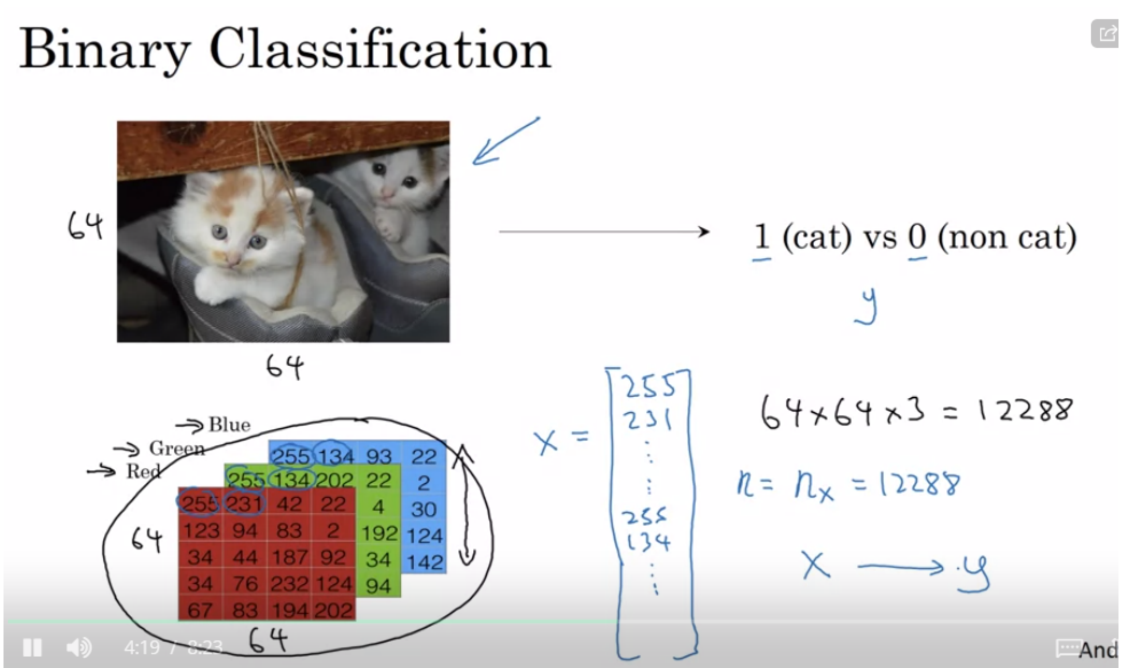
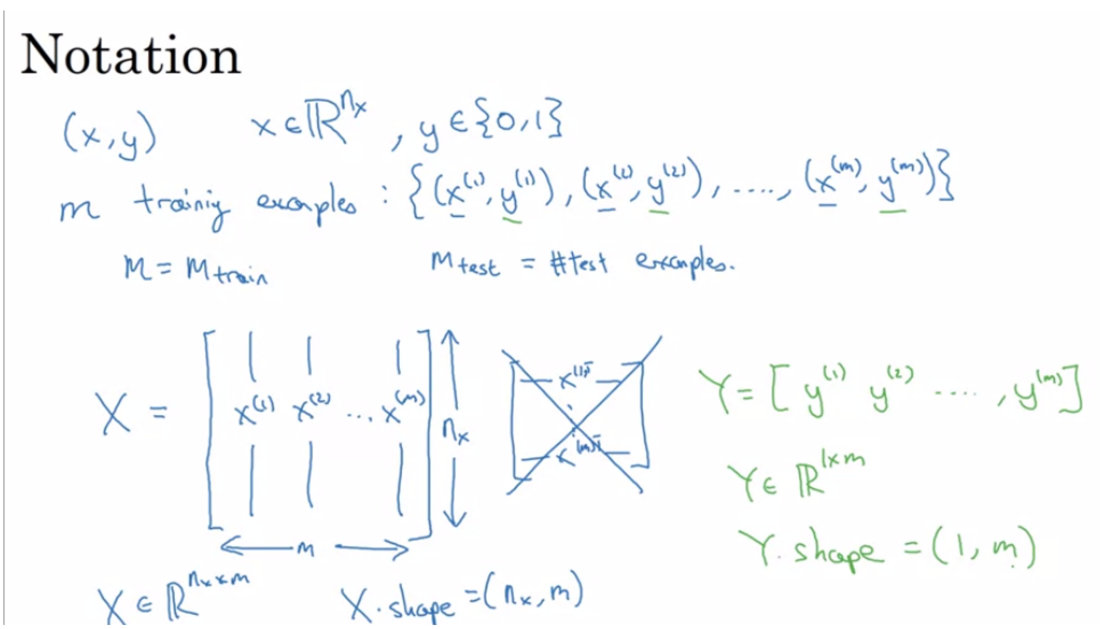
Week 2-1 NN for LR

笔记本: DL 1 - NN and DL
创建时间: 2021/1/5 10:37　　**更新时间:** 2021/1/7 01:09



Notation



(different from ML)

LR

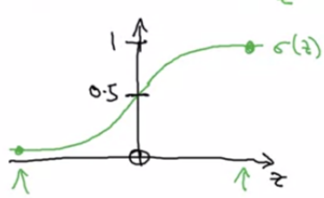
Logistic Regression

Given x, want $\hat{y} = P(y=1 \mid x)$

$x \in \mathbb{R}^{n_x}$ $\qquad 0 \le \hat{y} \le 1$

Parameters: $\boxed{w} \in \mathbb{R}^{n_x}$, $\boxed{b} \in \mathbb{R}$.

Output $\hat{y} = \sigma(\underbrace{w^T x + b}_{z})$



$X_0 = 1$, $\quad x \in \mathbb{R}^{n_x+1}$

$\hat{y} = \sigma(\theta^T x)$

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \begin{matrix} \}\, b \leftarrow \\ \\ \}\, w \leftarrow \\ \\ \\ \end{matrix}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

If z large $\sigma(z) \approx \frac{1}{1+0} = 1$

If z large negative number

$$\sigma(z) = \frac{1}{1+e^{-z}} \approx \frac{1}{1+\text{Bignum}} \approx 0$$

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Cost Func

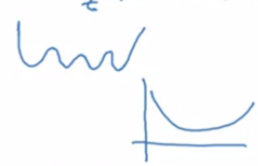
Logistic Regression cost function

$\rightarrow \hat{y}^{(i)} = \sigma(w^T x^{(i)} + b)$, where $\sigma(z^{(i)}) = \frac{1}{1+e^{-z^{(i)}}}$ $\qquad z^{(i)} = w^T x^{(i)} + b$

Given $\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$, want $\hat{y}^{(i)} \approx y^{(i)}$.

$x^{(i)}$, $y^{(i)}$, $z^{(i)}$ $\quad$ i-th example.

$\boxed{\text{Loss}}$ (error) function: $\mathcal{L}(\hat{y}, y) = \frac{1}{2}(\hat{y} - y)^2$



$$\boxed{\mathcal{L}(\hat{y}, y)} = -\left(\boxed{y \log \hat{y}} + (1-y)\log(1-\hat{y})\right) \leftarrow$$

If $y = 1$: $\mathcal{L}(\hat{y}, y) = -\log \hat{y} \leftarrow$ Want $\log \hat{y}$ large, want $\hat{y}$ large.

If $y = 0$: $\mathcal{L}(\hat{y}, y) = -\log(1-\hat{y}) \leftarrow$ Want $\log 1-\hat{y}$ large .... want $\hat{y}$ small

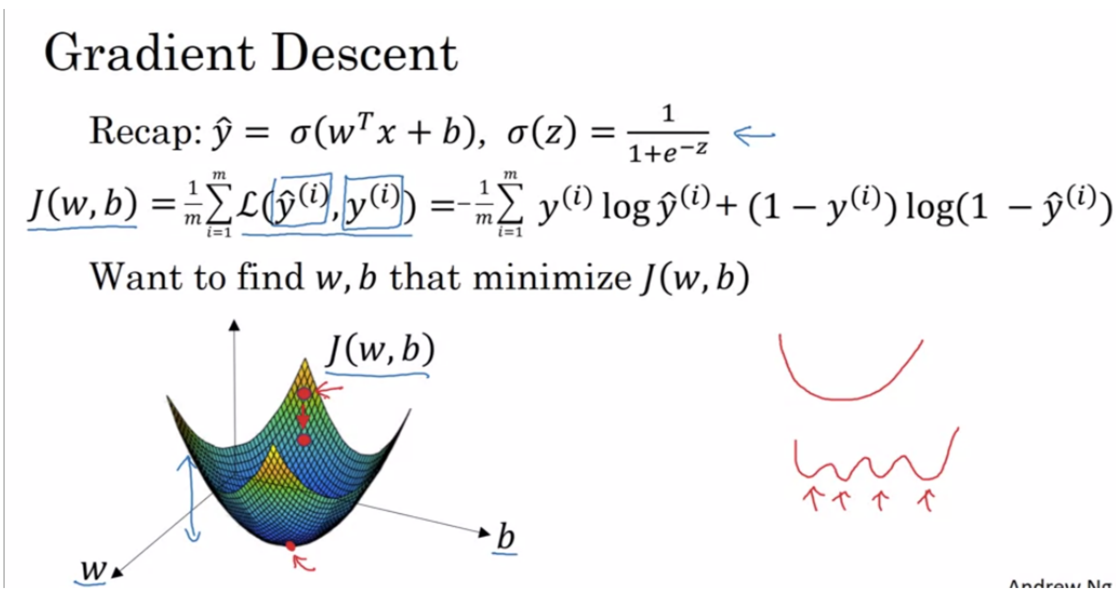
$\boxed{\text{Cost}}$ function: $J(w,b) = \frac{1}{m}\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)}) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log \hat{y}^{(i)} + (1-y^{(i)})\log(1-\hat{y}^{(i)})\right]$

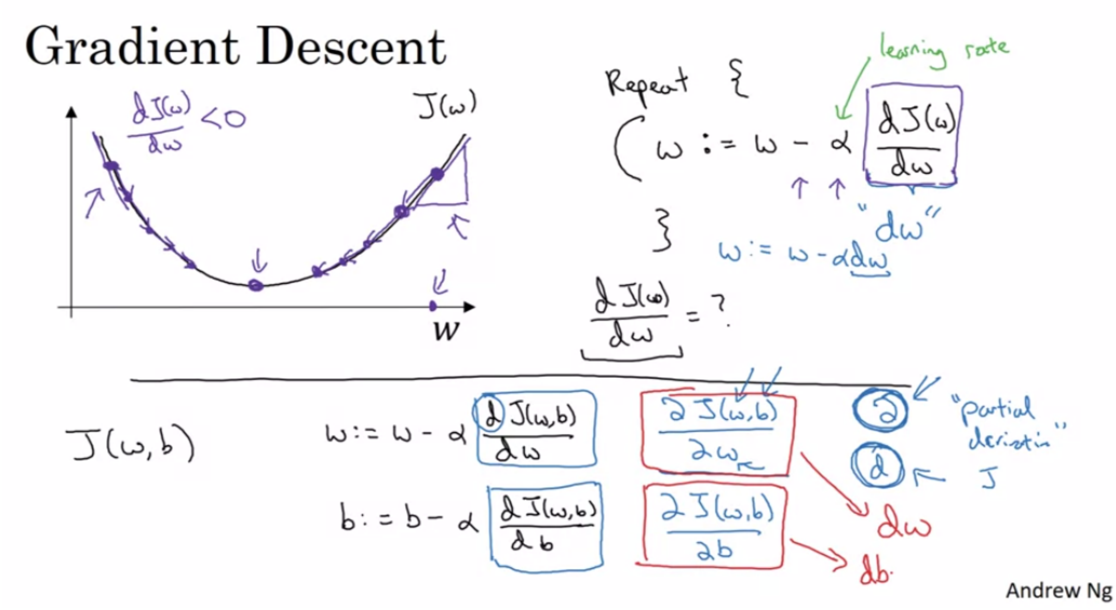
What is the difference between the cost function and the loss function for logistic regression?

- ( ) The cost function computes the error for a single training example; the loss function is the average of the cost functions of the entire training set.
- ( ) They are different names for the same function.
- (x) The loss function computes the error for a single training example; the cost function is the average of the loss functions of the entire training set.

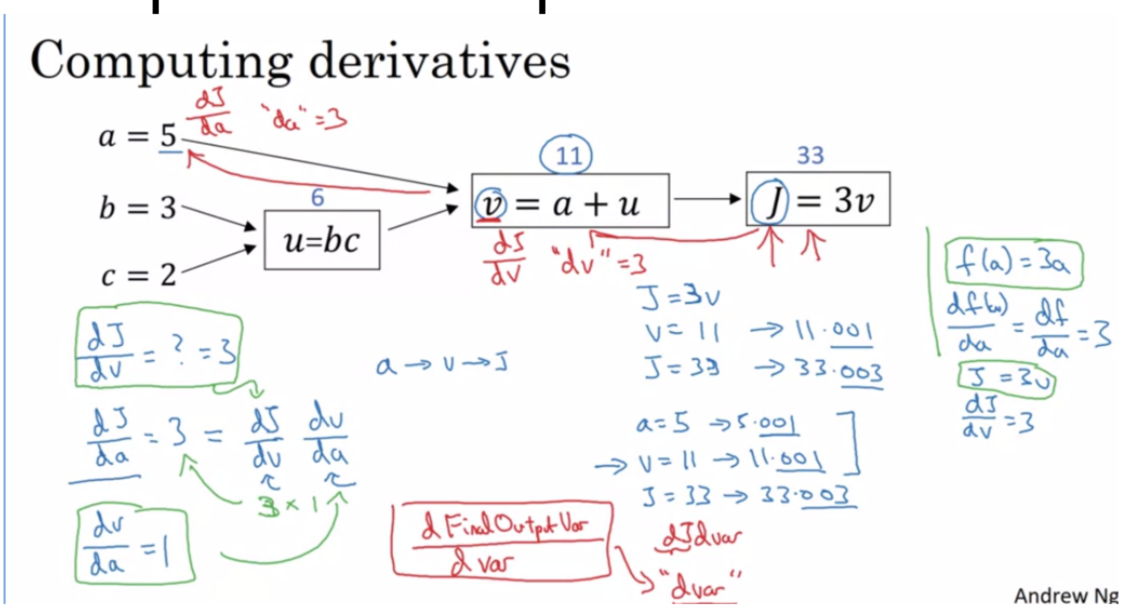
✓ **Correct**

GD





Computation Graph for derivatives



right to left

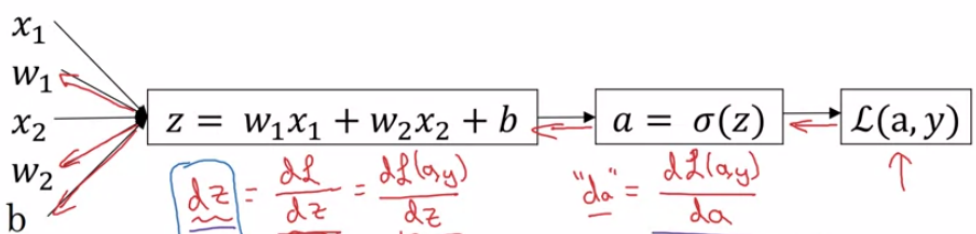
Logistic regression recap

$$z = w^T x + b$$

$$\hat{y} = a = \sigma(z)$$

$$\mathcal{L}(a, y) = -(y \log(a) + (1 - y)\log(1 - a))$$

Logistic regression derivatives



$\boxed{dz} = \frac{d\mathcal{L}}{dz} = \frac{d\mathcal{L}(a,y)}{dz}$

$= a - y$

$= \boxed{\frac{d\mathcal{L}}{da}} \cdot \boxed{\frac{da}{dz}}$

$a(1-a)$

"da" $= \frac{d\mathcal{L}(a,y)}{da}$

$= \boxed{-\frac{y}{a} + \frac{1-y}{1-a}}$

$\frac{\partial \mathcal{L}}{\partial w_1} =$ "dw_1" $= x_1 \cdot dz$. $\quad dw_2 = x_2 \cdot dz$. $\quad db = dz$.

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$.

Logistic regression on m examples

$J = 0$; $dw_1 = 0$; $dw_2 = 0$; $db = 0$

For $i = 1$ to m

$\quad z^{(i)} = w^T x^{(i)} + b$

$\quad a^{(i)} = \sigma(z^{(i)})$

$\quad J += -[y^{(i)} \log a^{(i)} + (1 - y^{(i)}) \log(1 - a^{(i)})]$

$\quad dz^{(i)} = a^{(i)} - y^{(i)}$

$\quad dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$

$\quad dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$ $\quad n = 2$

$\quad db \mathrel{+}= dz^{(i)}$

$J /= m$

$dw_1 /= m$; $dw_2 /= m$; $db /= m$.

$dw_1 = \frac{\partial J}{\partial w_1}$

$w_1 := w_1 - \alpha\, dw_1$

$w_2 := w_2 - \alpha\, dw_2$

$b := b - \alpha\, db$.

Logistic regression cost function

If $y = 1$: $p(y|x) = \hat{y}$

If $y = 0$: $p(y|x) = 1 - \hat{y}$

$p(y|x)$

$$\boxed{p(y|x) = \underbrace{\hat{y}^{y}}\,\underbrace{(1-\hat{y})^{(1-y)}}}$$

If $y=1$: $\underline{p(y|x) = \hat{y}} \quad \underbrace{(1-\hat{y})^{0}}_{=1}$

If $y=0$: $p(y|x) = \underbrace{\hat{y}^{0}}_{=1} \, (1-\hat{y})^{(1-y)} = 1 \times (1-\hat{y}) = \underline{1-\hat{y}}$

$$\uparrow \log \underline{p(y|x)} = \log \hat{y}^{y} (1-\hat{y})^{(1-y)} = y \log \hat{y} + (1-y) \log (1-\hat{y})$$

$$= -\underline{\mathcal{L}(\hat{y}, y)} \downarrow$$

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maximum likelihood method

Cost on m examples

$$\log p(\underline{\text{labels in training set}}) = \log \prod_{i=1}^{m} p(y^{(i)}|x^{(i)}) \leftarrow$$

$$\underline{\log p(\dots)} = \sum_{i=1}^{m} \underbrace{\log p(y^{(i)}|x^{(i)})}_{-\mathcal{L}(\hat{y}^{(i)}, y^{(i)})}$$

Maximum likelihood estimation

$$= -\sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$$

Cost: (minimize) $\underline{J(w,b)} = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}(\hat{y}^{(i)}, y^{(i)})$