

Informed Search

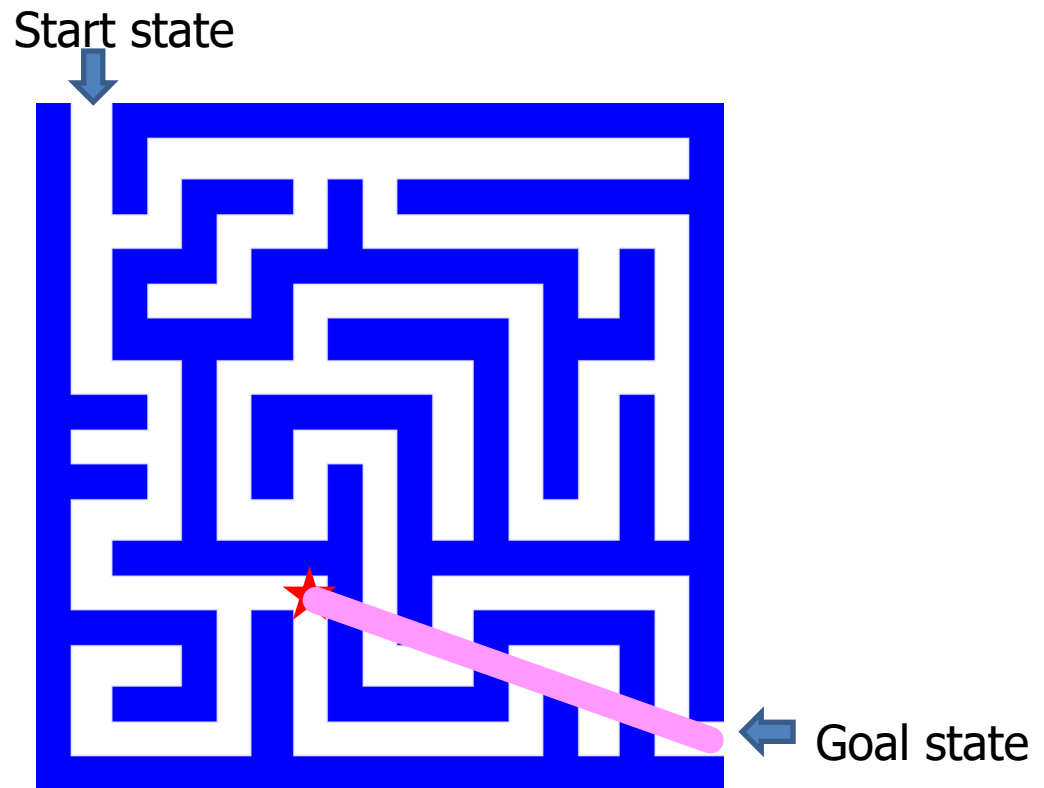
Sanja Lazarova-Molnar

Informed search

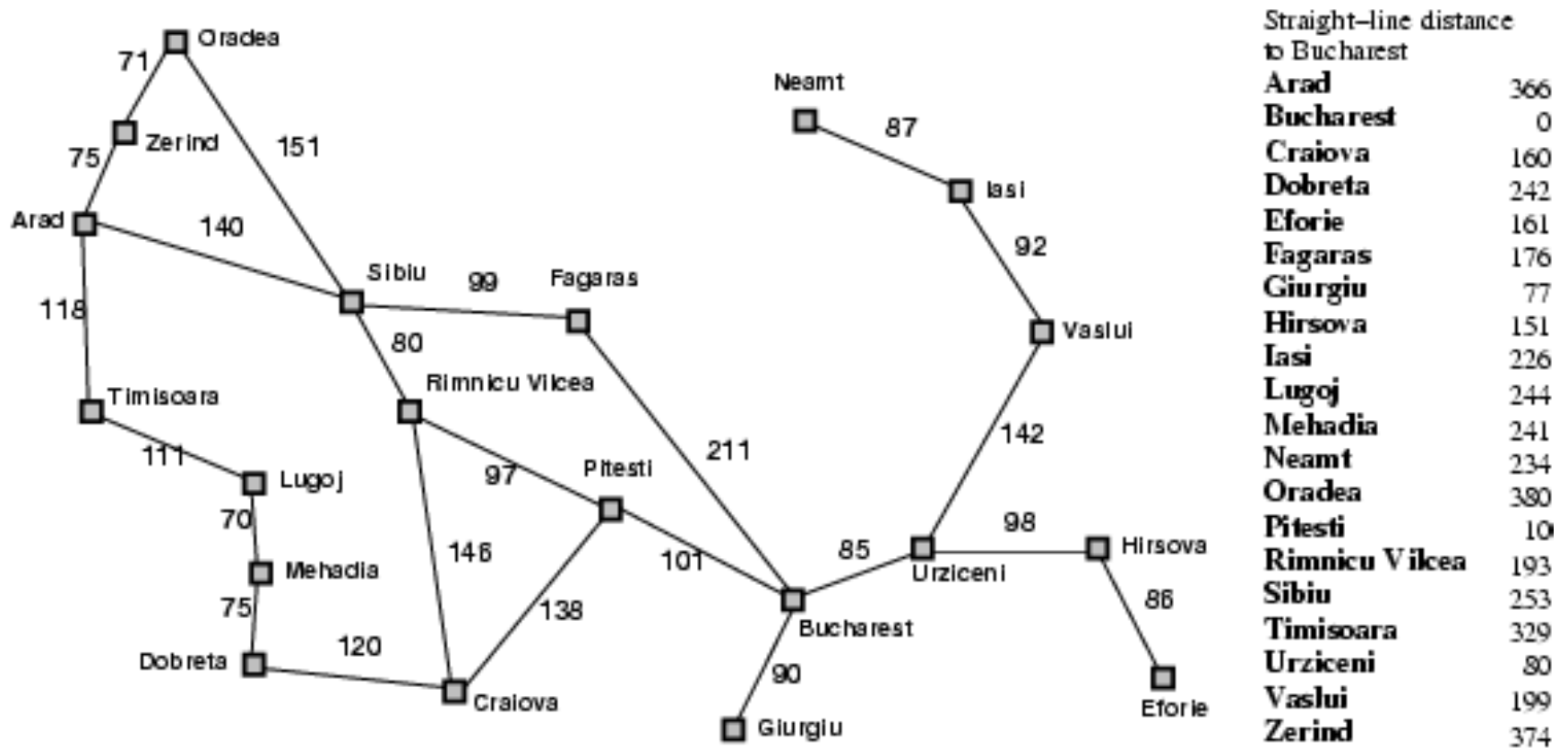
- Idea: give the algorithm “hints” about the desirability of different states
 - Use an *evaluation function* to rank nodes and select the most promising one for expansion
- Greedy best-first search
- A* search

Heuristic function

- **Heuristic function** $h(n)$ estimates the cost of reaching goal from node n
- Example:



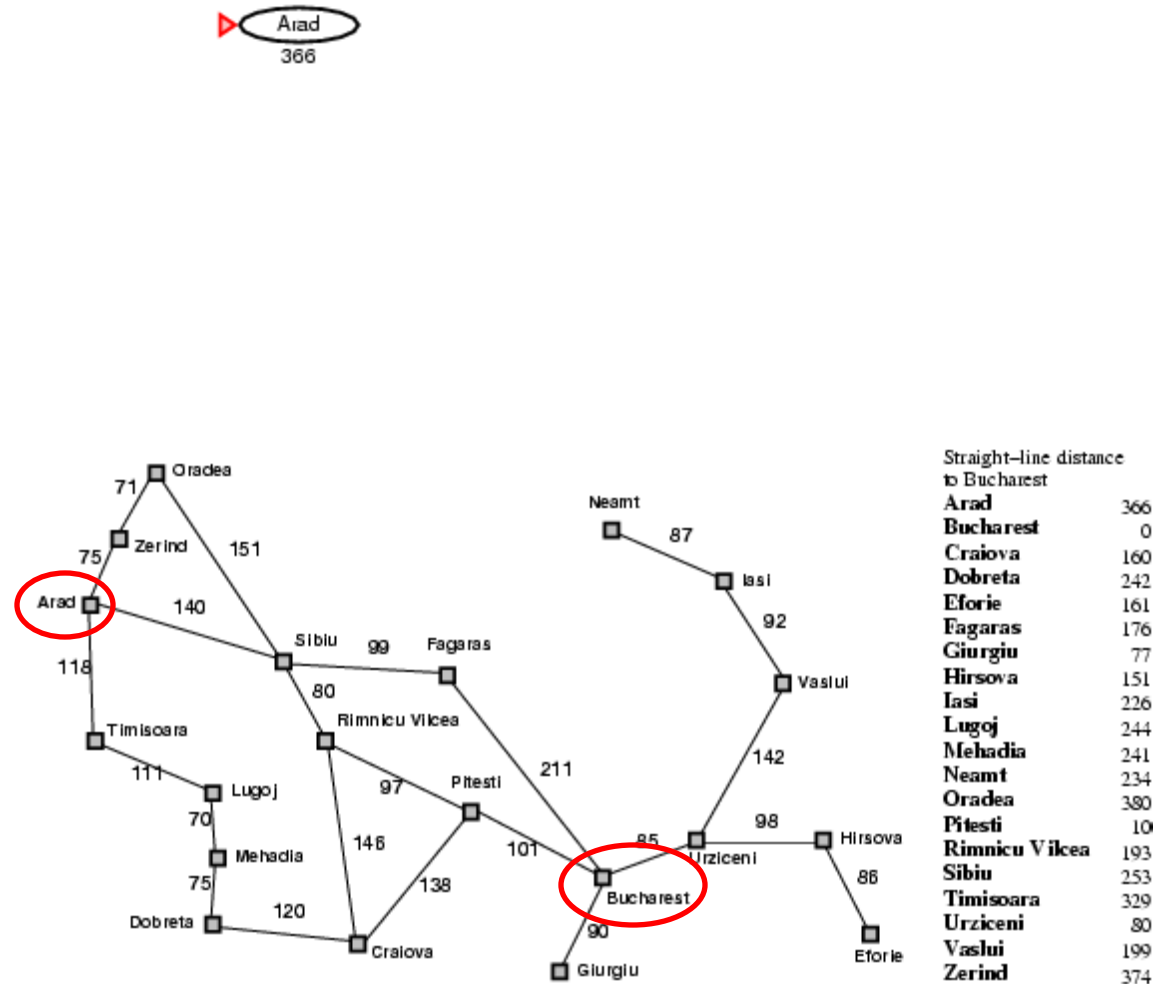
Heuristic for the Romania problem



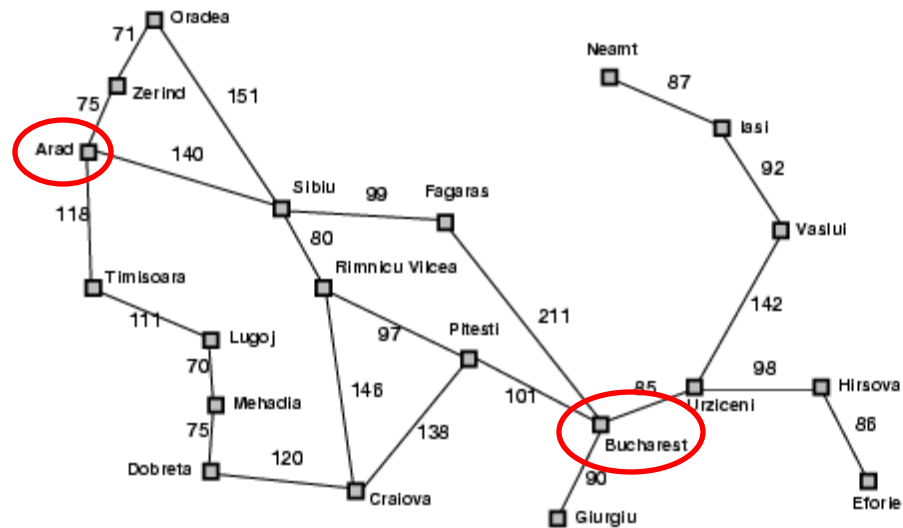
Greedy best-first search

- Expand the node that has the **lowest value** of the heuristic function $h(n)$

Greedy best-first search example



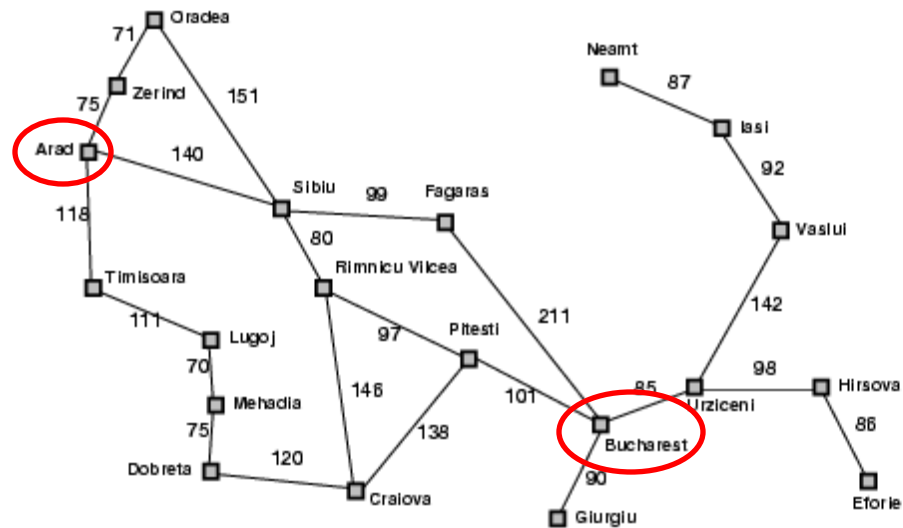
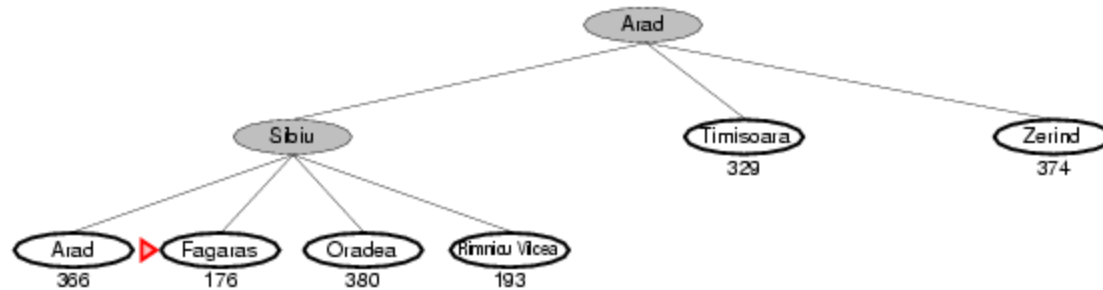
Greedy best-first search example



Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
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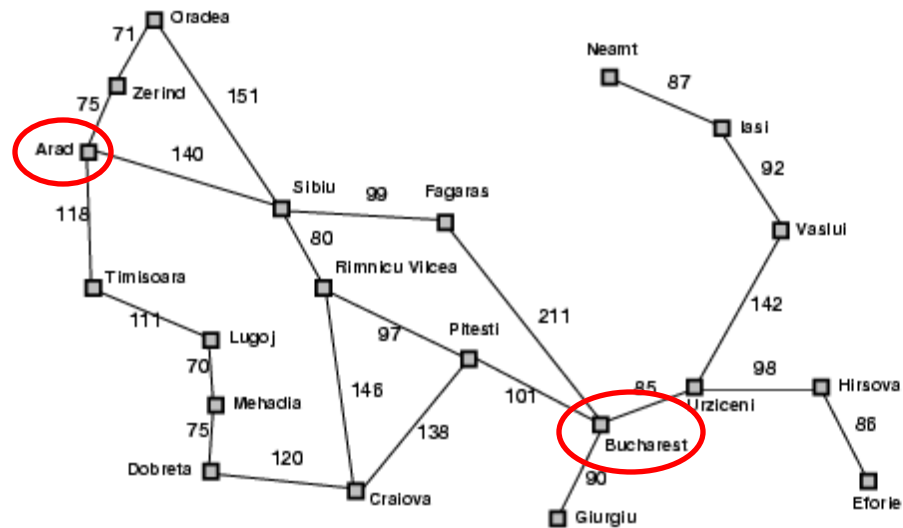
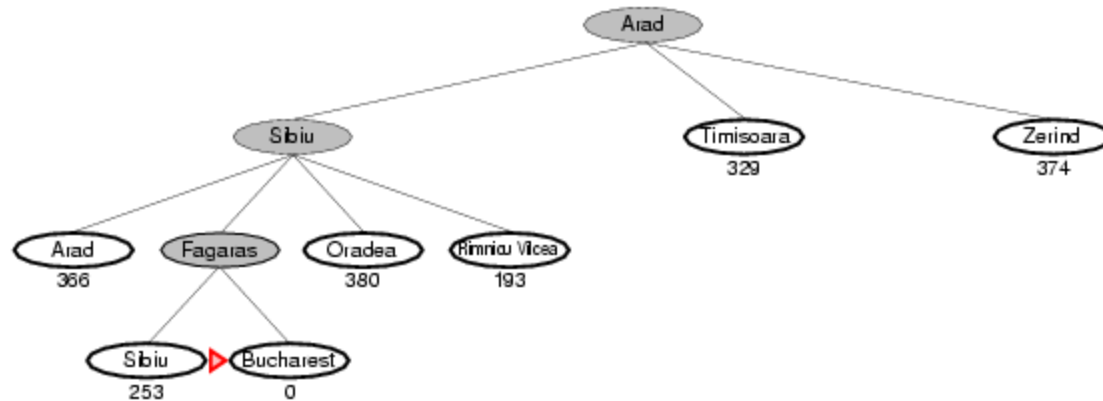
Greedy best-first search example



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Greedy best-first search example



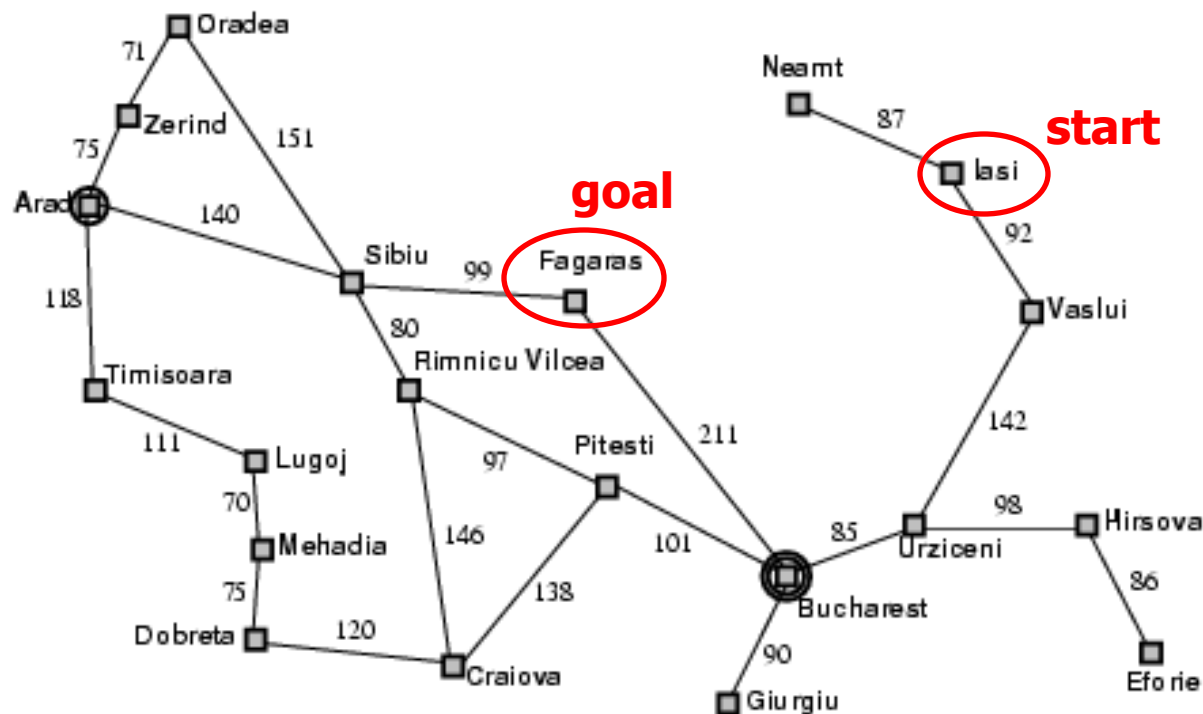
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Properties of greedy best-first search

- **Complete?**

No – can get stuck in loops



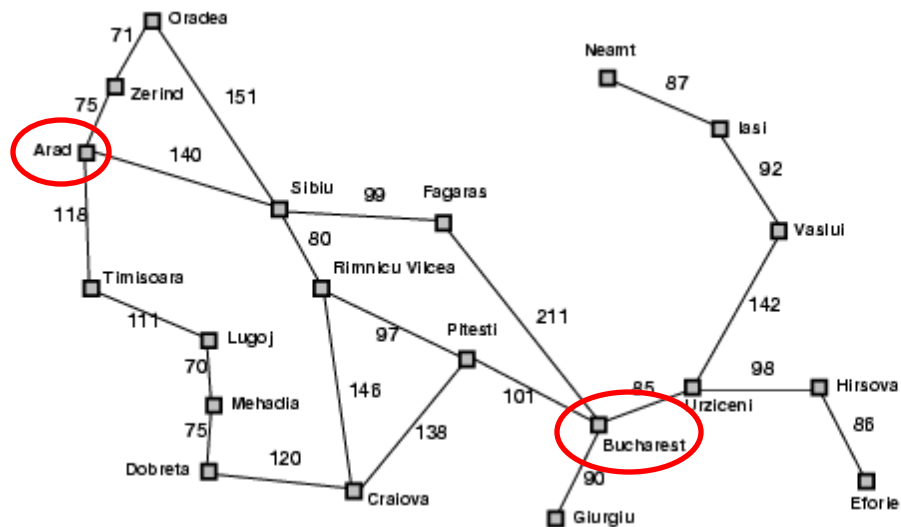
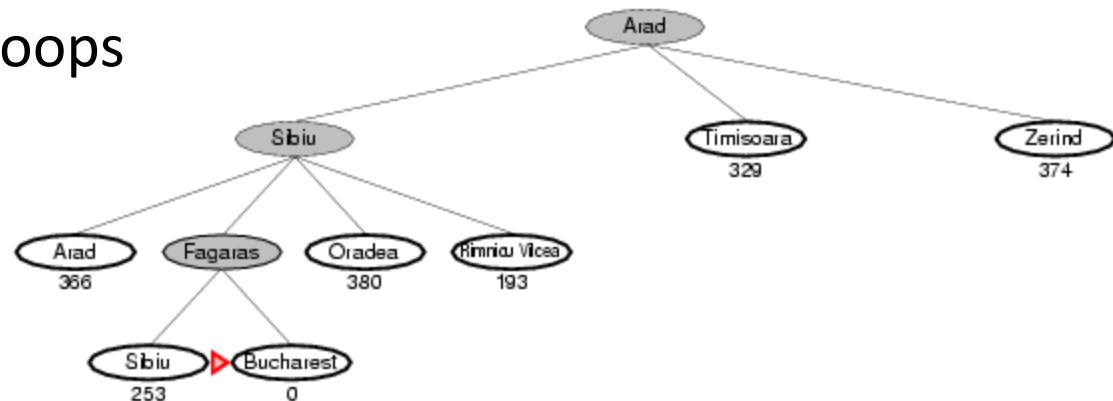
Properties of greedy best-first search

- **Complete?**

No – can get stuck in loops

- **Optimal?**

No



Properties of greedy best-first search

- **Complete?**

No – can get stuck in loops

- **Optimal?**

No

- **Time?**

Worst case: $O(b^m)$

Best case: $O(bd)$ – If $h(n)$ is 100% accurate

- **Space?**

Worst case: $O(b^m)$

How can we fix the greedy problem?

- Add another parameter to evaluate nodes!?

A* search

- Idea: avoid expanding paths that are already expensive
- The evaluation function $f(n)$ is the estimated total cost of the path through node n to the goal:

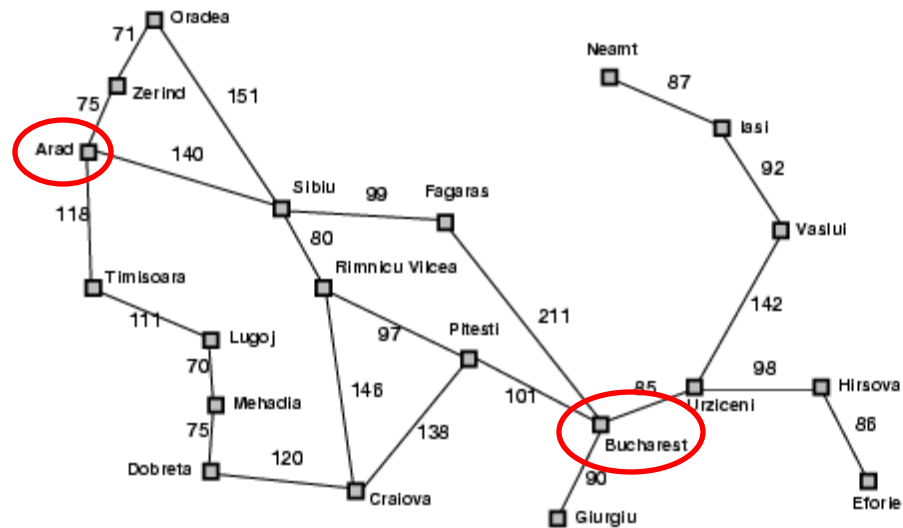
$$f(n) = g(n) + h(n)$$

$g(n)$: cost so far to reach n (path cost)

$h(n)$: estimated cost from n to goal (heuristic)

A* search example

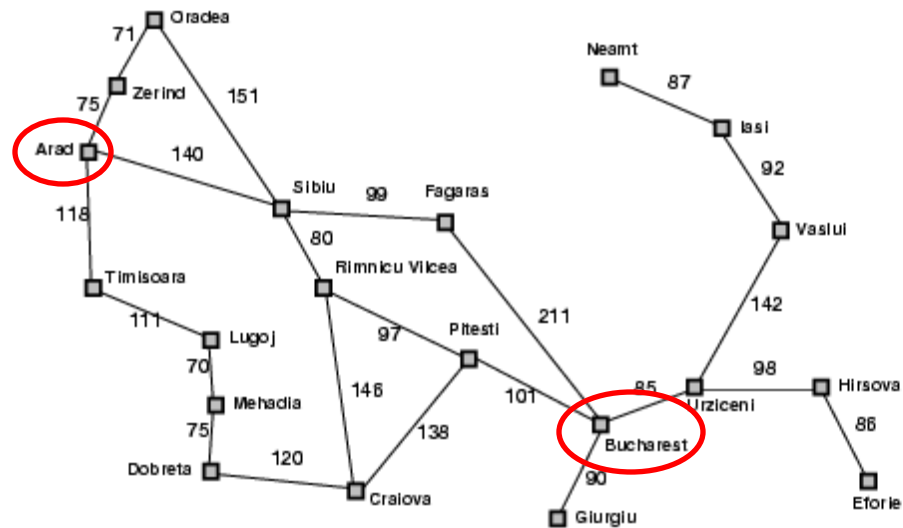
Arad
366=0+366



Straight-line distance
to Bucharest

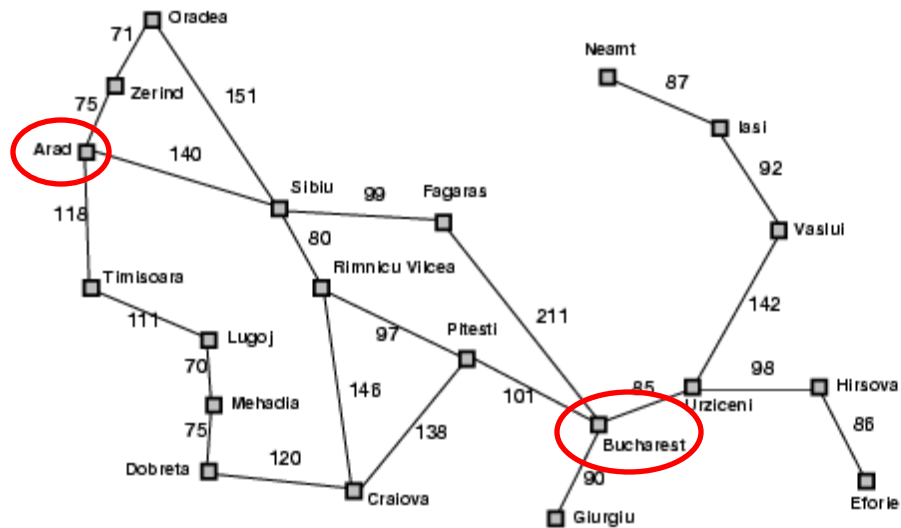
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A* search example



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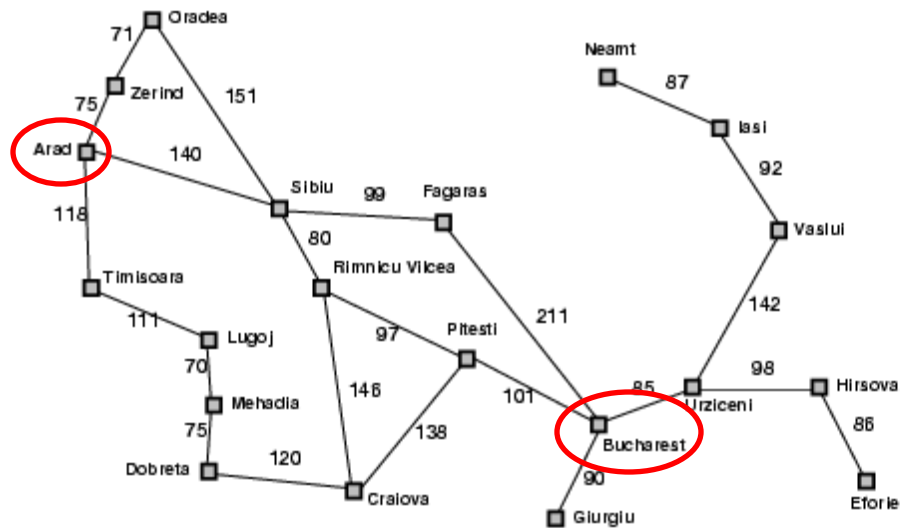
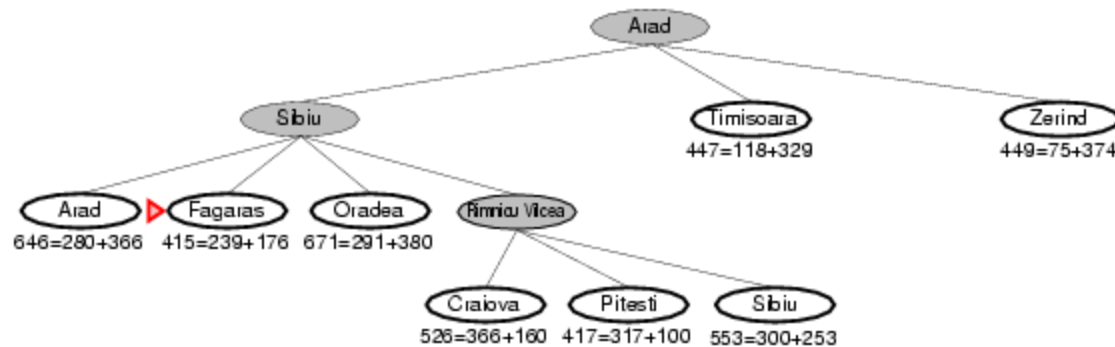
A* search example



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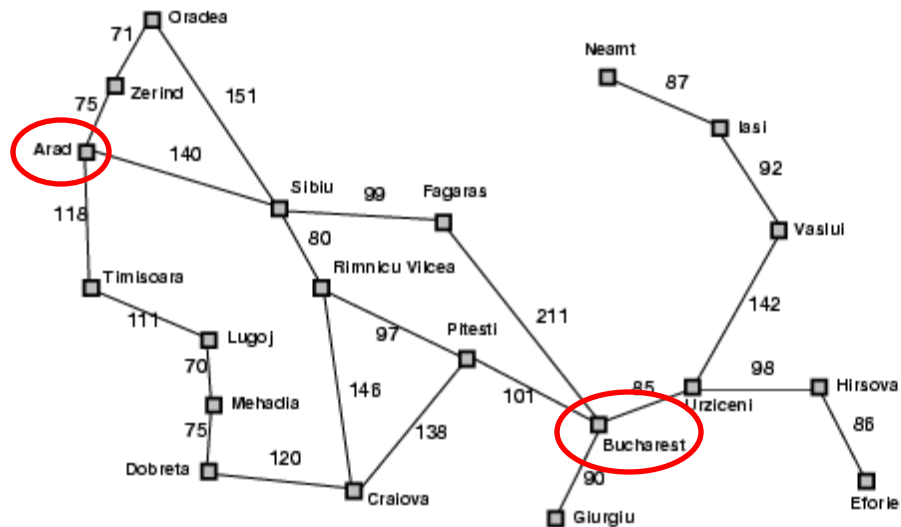
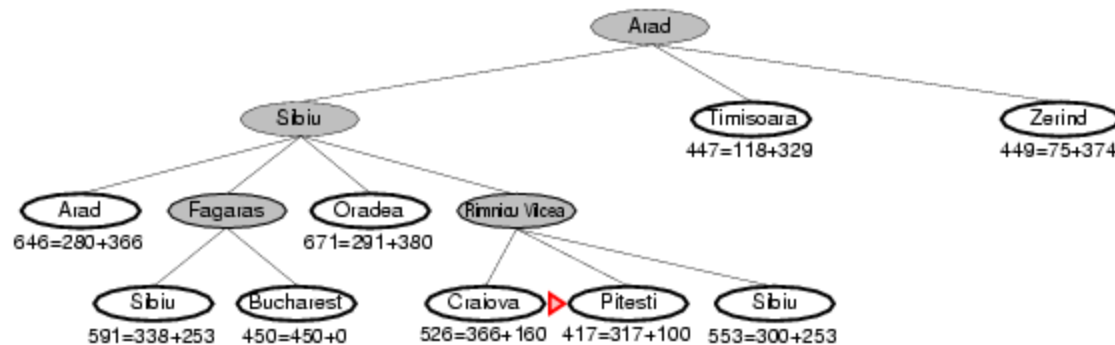
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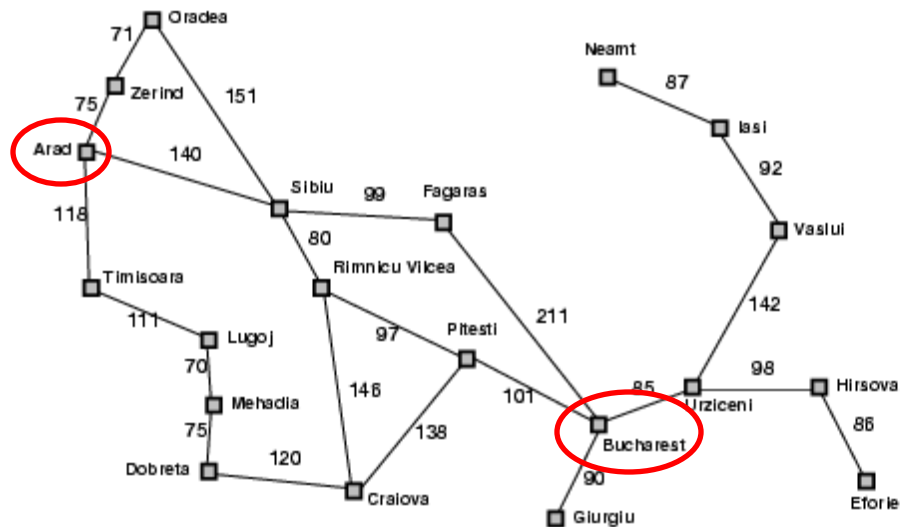
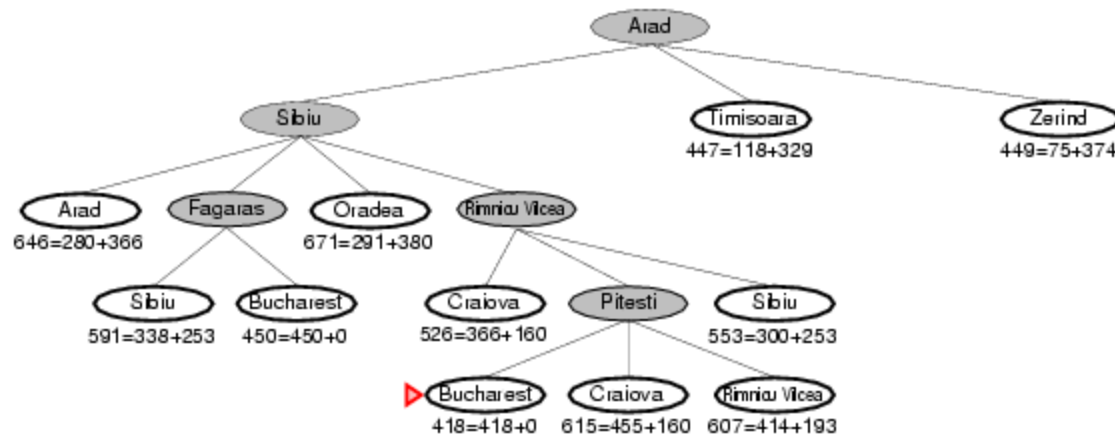
A* search example



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Admissible heuristics

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- A heuristic $h(n)$ is **admissible** if for every node n , $h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n
- Example: straight line distance never overestimates the actual road distance
- **Theorem:** If $h(n)$ is admissible, A^* is optimal

Optimality of A^*

- A^* is *optimally efficient* – no other tree-based algorithm that uses the same heuristic can expand fewer nodes and still be guaranteed to find the optimal solution
 - any algorithm that does not expand all nodes in the contours between the root and the goal contour runs the risk of missing the optimal solution

Properties of A*

- **Complete?**

Yes – unless there are infinitely many nodes with $f(n) \leq C^*$

- **Optimal?**

Yes

- **Time?**

Number of nodes for which $f(n) \leq C^*$ (exponential)

- **Space?**

Exponential

Designing heuristic functions

- Heuristics for the 8-puzzle

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)

7	2	4
5		6
8	3	1

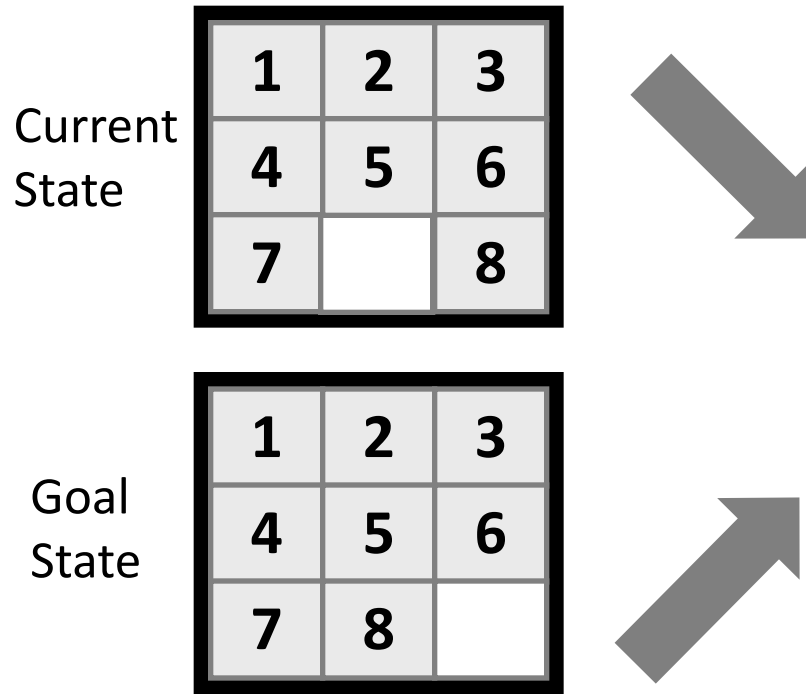
Start State

	1	2
3	4	5
6	7	8

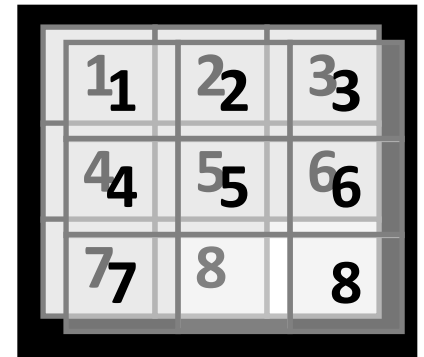
Goal State

- Are h_1 and h_2 admissible?

$h_1(n)$ = number of misplaced tiles



*The number of
misplaced tiles
(not including
the blank)*



N	N	N
N	N	N
N	Y	

In this case, only “8” is misplaced, so heuristic function evaluates to 1

In other words, the heuristic *says* that it *thinks* a solution may be available in just 1 more move

$$h_2(n) = \text{total Manhattan distance}$$

Manhattan Distance (not including the blank)

Current State

3	2	8
4	5	6
7	1	

Goal State

1	2	3
4	5	6
7	8	

- The **3**, **8** and **1** tiles are misplaced (by 2, 3, and 3 steps) so the heuristic function evaluates to 8
- Heuristic says that it *thinks* a solution may be available in just 8 more moves.
- The misplaced heuristic's value is 3

3	→	<u>3</u>

2 spaces

	←	8
	↓	
	<u>8</u>	

3 spaces

<u>1</u>	←	
	↑	
	1	

3 spaces

Total 8

Designing heuristic functions

- Heuristics for the 8-puzzle

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- What are the values for h_1 and h_2 for the start state?
- Are h_1 and h_2 admissible?

Designing heuristic functions

- Heuristics for the 8-puzzle

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total Manhattan distance (number of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

$$h_1(\text{start}) = 8$$

$$h_2(\text{start}) = 3+1+2+2+2+3+3+2 = 18$$

- Both h_1 and h_2 are admissible

Dominance

- If h_1 and h_2 are both admissible heuristics and $h_2(n) \geq h_1(n)$ for all n , (both admissible) then h_2 **dominates** h_1
- Which one is better for search?
 - A* search expands every node with $f(n) < C^*$ or $h(n) < C^* - g(n)$
 - Therefore, A* search with h_1 will expand more nodes, so h_2 is better

C^* - optimal cost

Heuristics from relaxed problems

- A problem with fewer restrictions on the actions - **relaxed problem**
- The cost of an optimal solution to a relaxed problem - admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move **anywhere**, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to **any adjacent square**, then $h_2(n)$ gives the shortest solution

Dominance

- Typical search costs for the 8-puzzle (average number of nodes expanded for different solution depths):
 - $d=12$ IDS = 3,644,035 nodes
 $A^*(h_1)$ = 227 nodes
 $A^*(h_2)$ = 73 nodes
 - $d=24$ IDS \approx 54,000,000,000 nodes
 $A^*(h_1)$ = 39,135 nodes
 $A^*(h_2)$ = 1,641 nodes

Combining heuristics

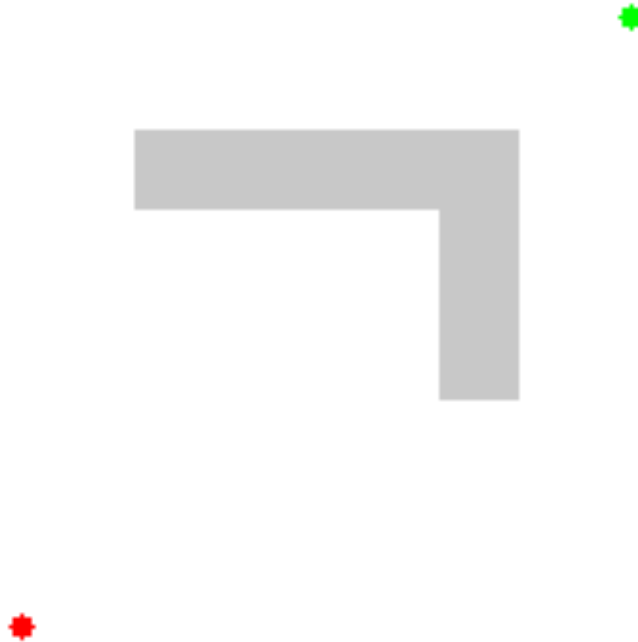
- Suppose we have a collection of admissible heuristics $h_1(n), h_2(n), \dots, h_m(n)$, but none of them dominates the others
- How can we combine them?

$$h(n) = \max\{h_1(n), h_2(n), \dots, h_m(n)\}$$

Weighted A* search

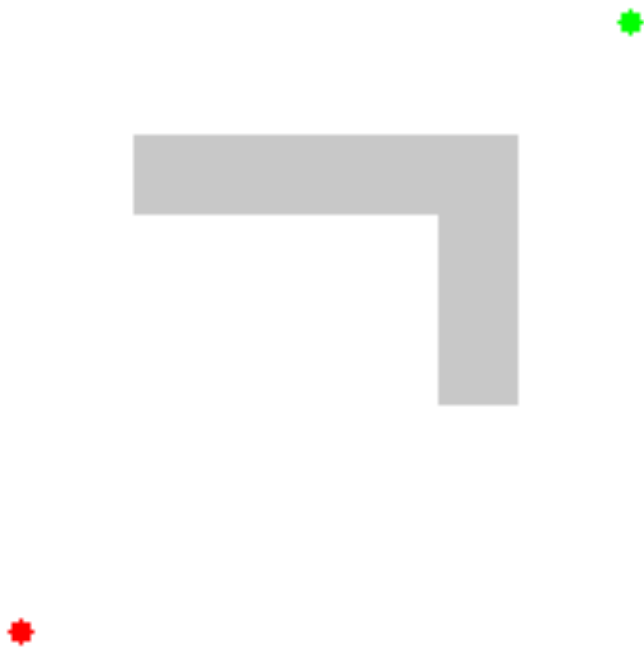
- Idea: speed up search at the expense of optimality
- Take an admissible heuristic, “inflate” it by a multiple $\alpha > 1$, and then perform A* search as usual
- Fewer nodes tend to get expanded, but the resulting solution may be suboptimal (its cost will be at most α times the cost of the optimal solution)

Example of weighted A* search



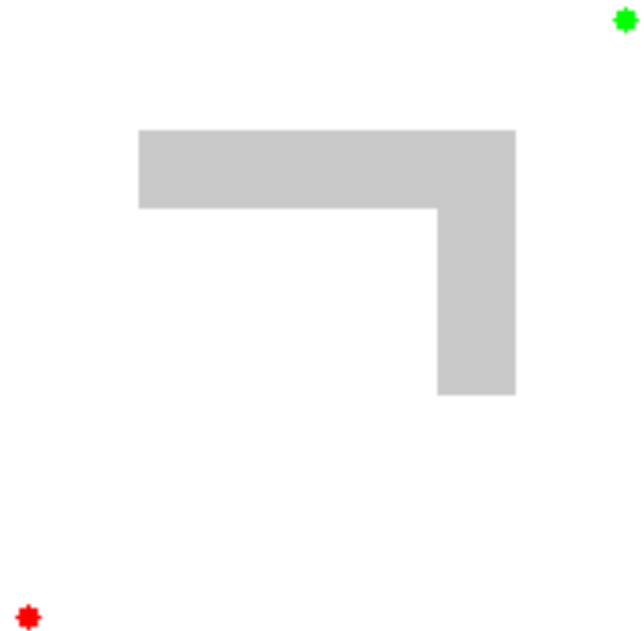
Heuristic: $5 * \text{Euclidean distance from goal}$
Source: [Wikipedia](https://en.wikipedia.org/wiki/A*_search_algorithm)

Example of weighted A* search



Heuristic: $5 * \text{Euclidean distance from goal}$

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Compare: Exact A*

Dealing with hard problems

- For large problems, A^* may require too much space
- Variations conserve memory: IDA* and SMA*
- IDA*, iterative deepening A^* , uses successive iteration with growing limits on f , e.g.
 - A^* but don't consider a node n where $f(n) > 10$
 - A^* but don't consider a node n where $f(n) > 20$
 - A^* but don't consider a node n where $f(n) > 30, \dots$
- SMA* -- Simplified Memory-Bounded A^*
 - Uses queue of restricted size to limit memory use

Uninformed search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
BFS	Yes	If all step costs are equal	$O(b^d)$	$O(b^d)$
UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$

b: maximum branching factor of the search tree

d: depth of the optimal solution

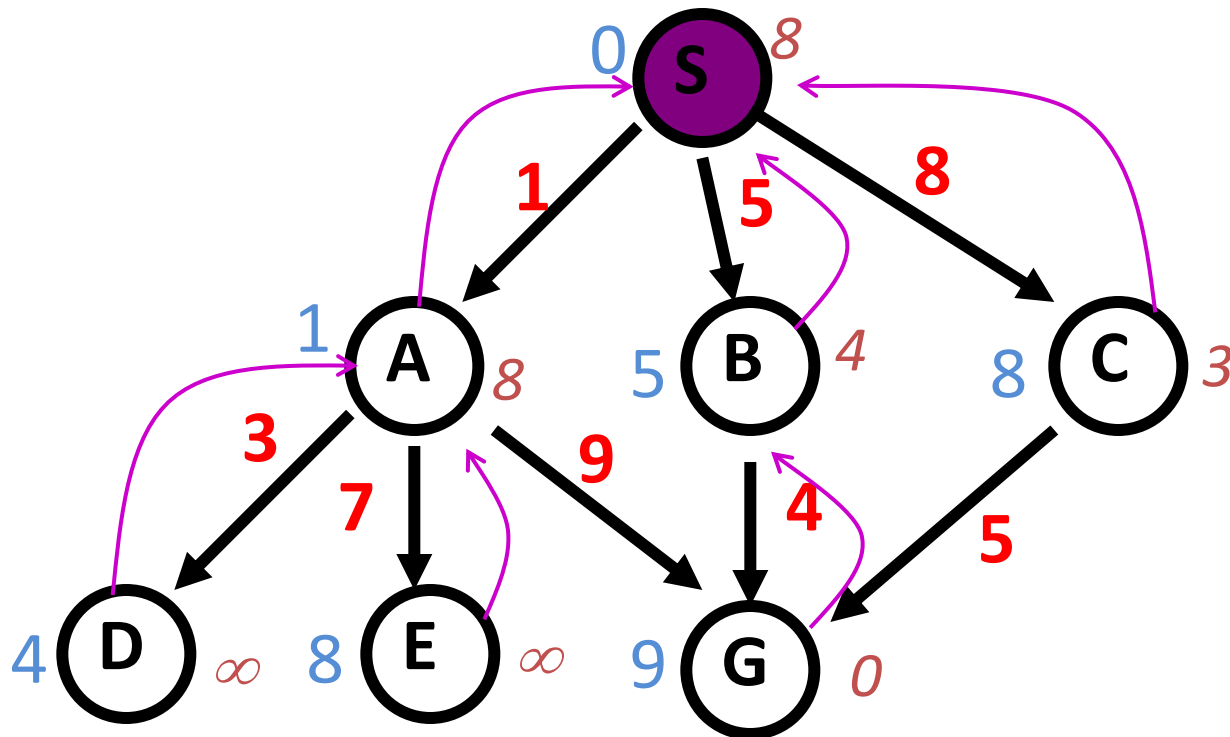
m: maximum length of any path in the state space

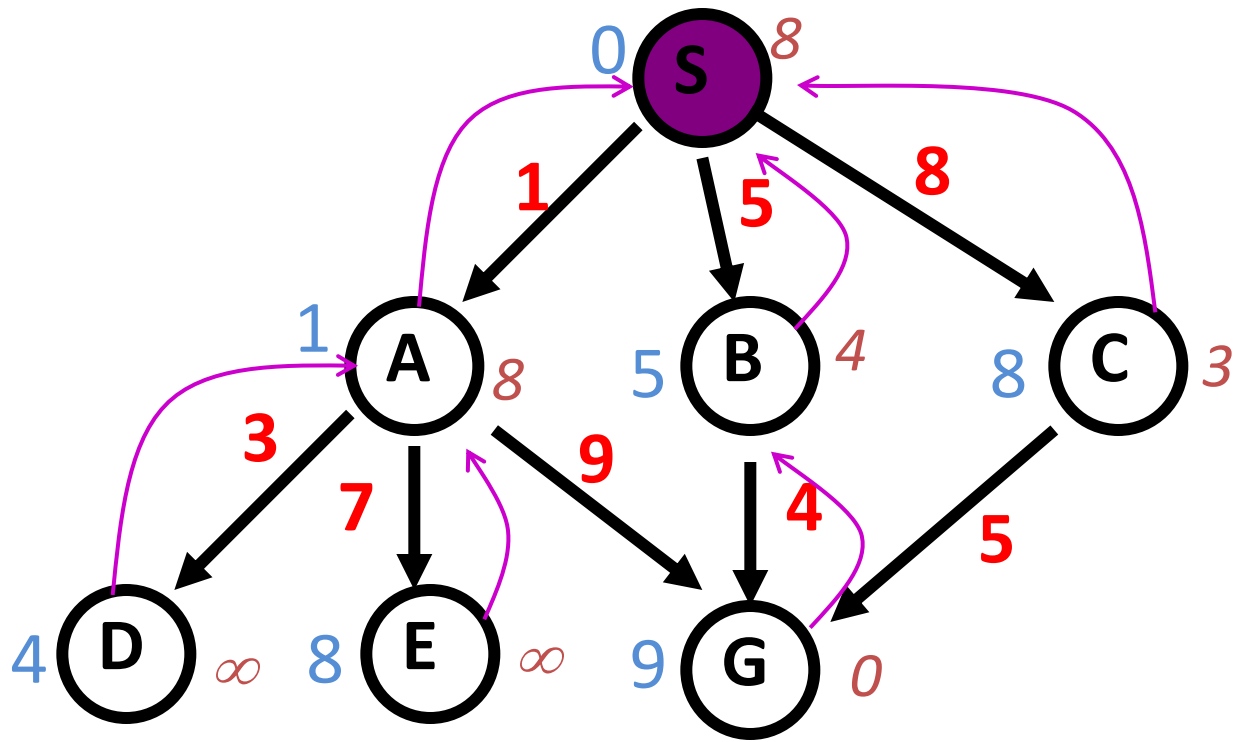
C^* : cost of optimal solution

All search strategies

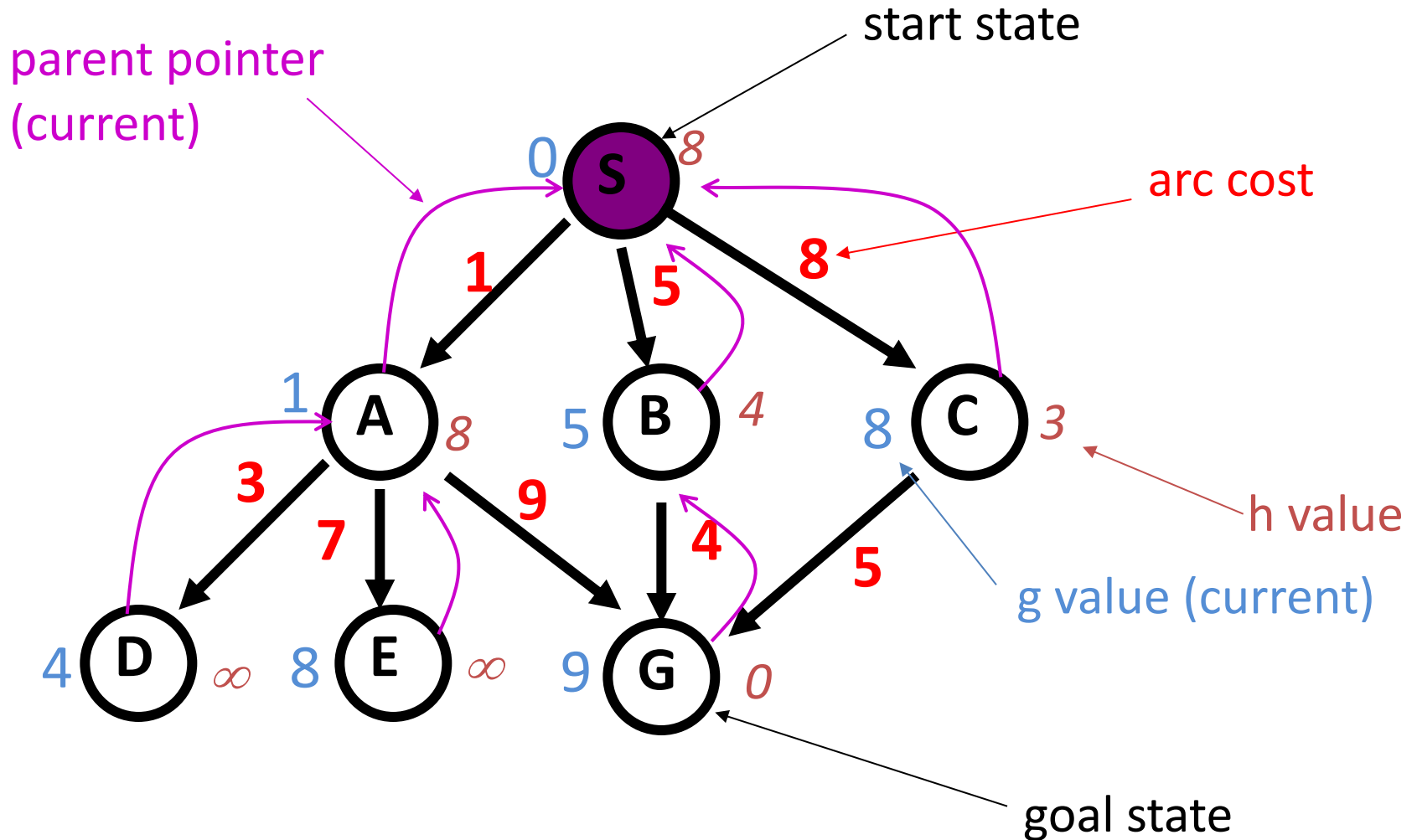
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UCS	Yes	Yes	Number of nodes with $g(n) \leq C^*$	
DFS	No	No	$O(b^m)$	$O(bm)$
IDS	Yes	If all step costs are equal	$O(b^d)$	$O(bd)$
Greedy	No	No	Worst case: $O(b^m)$ Best case: $O(bd)$	
A*	Yes	Yes	Number of nodes with $g(n)+h(n) \leq C^*$	

Example search space

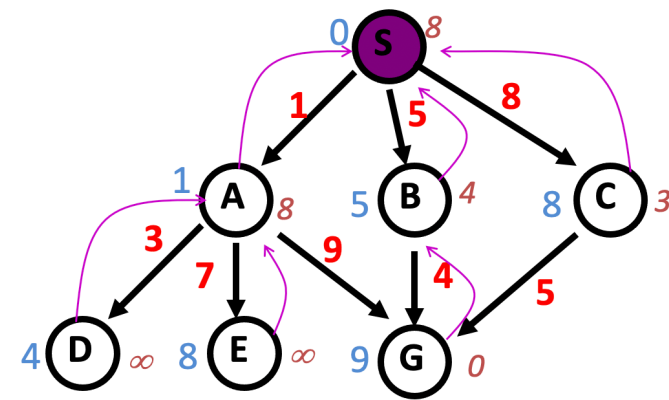




Example search space



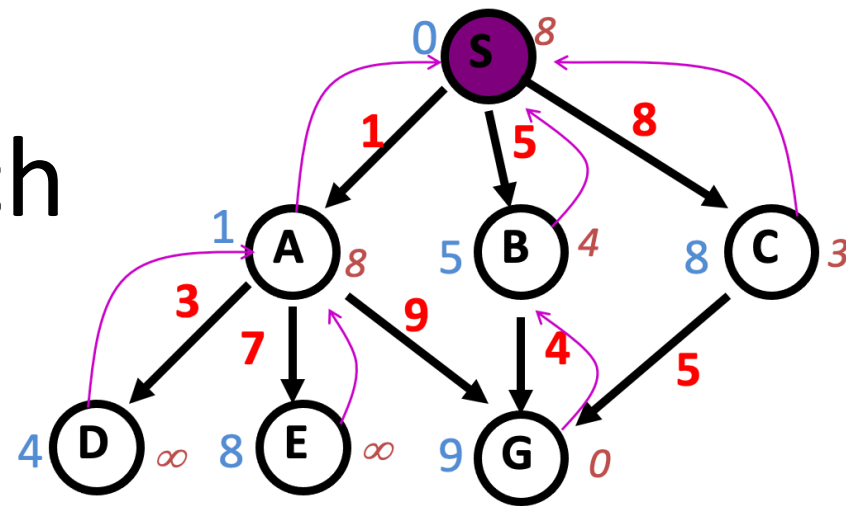
Example



n	g(n)	h(n)	f(n)	h*(n)
S	0	8	8	9
A	1	8	9	9
B	5	4	9	4
C	8	3	11	5
D	4	inf	inf	inf
E	8	inf	inf	inf
G	9	0	9	0

- $h^*(n)$ is (hypothetical) perfect heuristic (an oracle)
- Since $h(n) \leq h^*(n)$ for all n , h is admissible (optimal)
- Optimal path = $S \ B \ G$ with cost 9

Greedy search



$$f(n) = h(n)$$

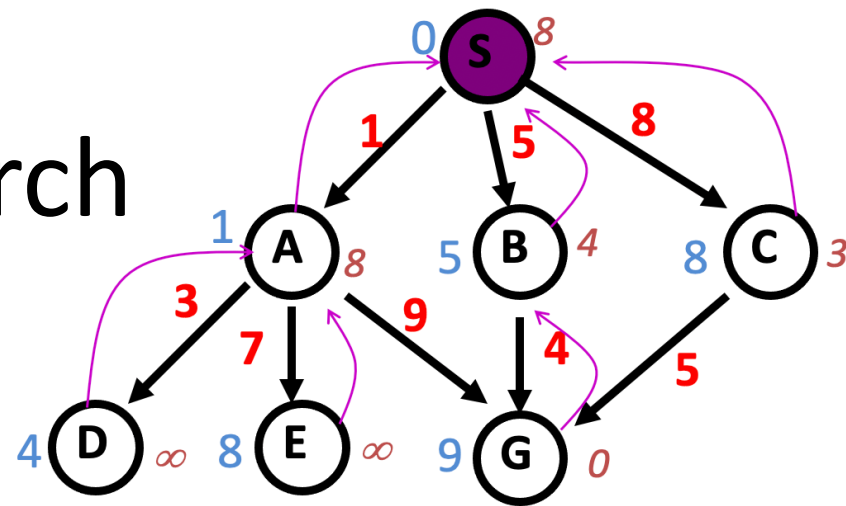
node expanded

nodes list

	{ S (8) }
S	{ C (3) B (4) A (8) }
C	{ G (0) B (4) A (8) }
G	{ B (4) A (8) }

- Solution path found is S C G, 3 nodes expanded.
- See how fast the search is!! But it is NOT optimal.

A* search

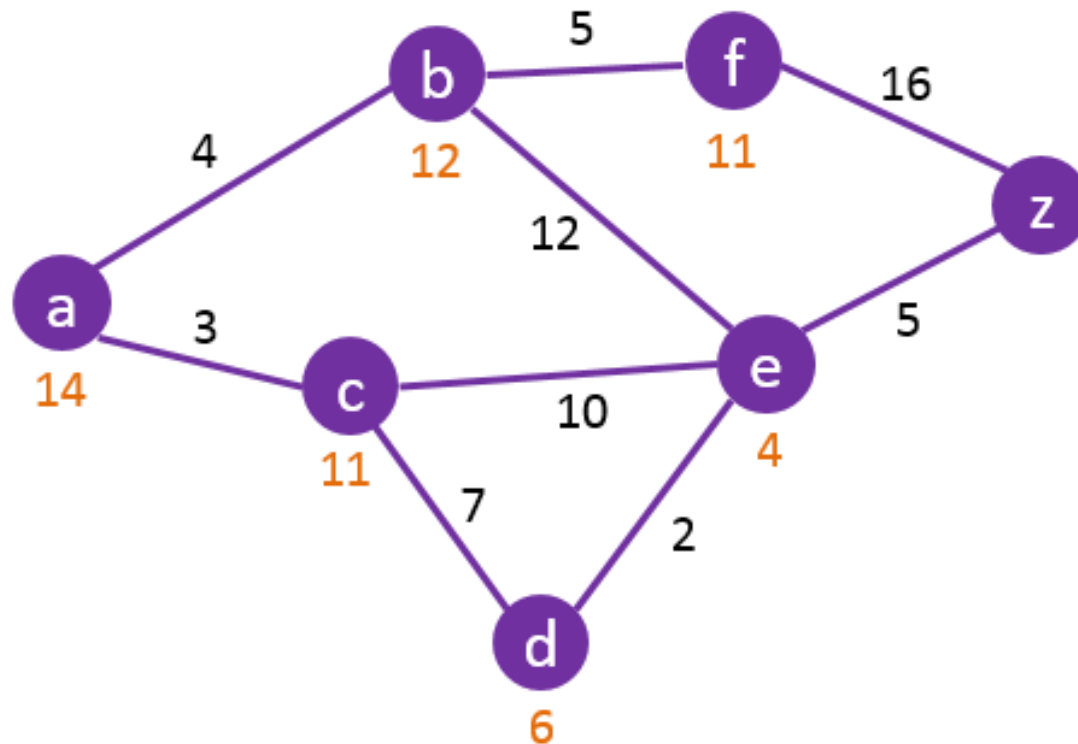


$$f(n) = g(n) + h(n)$$

node exp.	nodes list
	{ S(8) }
S	{ A(9) B(9) C(11) }
A	{ B(9) G(10) C(11) D(inf) E(inf) }
B	{ G(9) G(10) C(11) D(inf) E(inf) }
G	{ C(11) D(inf) E(inf) }

- Solution path found is S B G, 4 nodes expanded..
- Still pretty fast. And optimal, too.

Exercise (if there is time, else on your own)



A* Search Algorithm

What is the shortest path to travel from A to Z?

Numbers in orange are the heuristic values, distances in a straight line (as the crow flies) from a node to node Z.

Summary: Informed search

- **Greedy best-first search** uses minimal estimated cost $h(n)$ to goal state as measure; reduces search time, but is neither complete nor optimal
- **A* search** combines uniform-cost search & greedy best-first search: $f(n) = g(n) + h(n)$. Handles state repetitions & $h(n)$ never overestimates
 - A* is complete & optimal, but space complexity high
 - Time complexity depends on quality of heuristic function
 - IDA* and SMA* reduce the memory requirements of A*