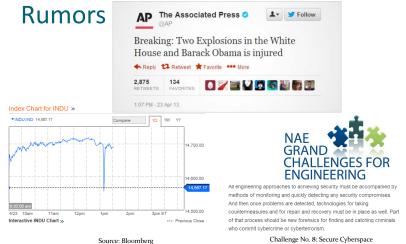
A Probabilistic Approach to Rumor Source Detection and Graph-based Message Passing Algorithms

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MOTIVATION



Challenge No. 8: Secure Cyberspace



WHO IS THE CULPRIT?

■ Spread of computer virus



■ Tweeting and Retweeting in Twitter Network



- A dynamical process, e.g., rumor, virus, spreads on a network
- SI (Susceptible-Infectious) model [Kermack & McKendrick'1927]
- Observe prior suspect set and infected nodes
- Can we find the spreading source accurately and reliably?



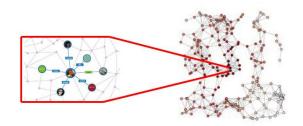
RUMOR SOURCE ESTIMATOR

Maximum likelihood (ML) estimator

$$\hat{v} \in \arg\max_{v \in G_N} P(G_N \mid v, T),$$

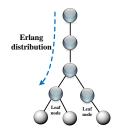
where $P(G_N | v, T)$ is the probability of observing G_N at time T supposing that v is the rumor source.

⇒ Most likely source is at the "center" of the network!



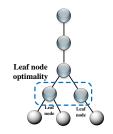


ML ESTIMATOR FOR TREE GRAPHS



$$P\left(v_{i} \bigcap_{v_{l} \in \text{child}(v_{i})}^{L} \overline{v}_{l} \middle| v_{j}\right)$$

$$= \int_{0}^{T} \frac{t^{K_{ij}-1}e^{-t}}{(K_{ij}-1)!} e^{-(T-t)(d_{i}-1)} dt$$

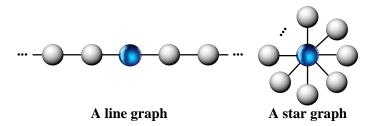


$$\hat{v} \in \arg\max_{v \in G_N} P(G_N \mid v, T)$$

$$= \arg\max_{v \in G_N} \prod_{v_i \in \text{leaf}(G_N \mid v)}^{L} P\left(v_i \bigcap_{v_l \in \text{child}(v_i)}^{L} \bar{v}_l \mid v\right)$$



SPECIAL OPTIMALITY CONDITION



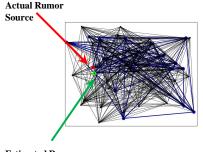
Proposition 1: Leaf nodes are never the estimated rumor source when N > 2.

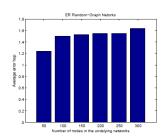
Proposition 2: The estimated rumor source of 2-degree regular tree is the node(s) in the middle of the line.

Proposition 3: The estimated rumor source of a star graph is the internal node.

DETECTION IN COMPLEX NETWORKS

Erdos-Renyi random graph





Estimated Rumor Source





Thank you!

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