A large, semi-transparent circular overlay covers the central portion of the slide. In the background, through the circle, the modern architecture of KAUST's buildings and a reflecting pool are visible.

# Recent Advances in Optimization for Machine Learning\*

Peter Richtárik

# Optimization and Machine Learning Lab



Photo: February 2019

## Research Scientists

Laurent Condat (from Grenoble)  
Zhize Li (from Tsinghua)

## Postdocs

Mher Safaryan (from Yerevan)  
Adil Salim (from Télécom Paris)  
Xun Qian (from Hong Kong)

## PhD Students

Filip Hanzely (now Assistant Prof @ TTIC)  
Konstantin Mishchenko (from ENS Paris-Saclay)  
Alibek Sailanbayev (from Nazarbayev)  
Samuel Horváth (from Comenius)  
Elnur Gasanov (from MIPT)  
Dmitry Kovalev (from MIPT)  
Konstantin Burlachenko (from Huawei)  
Slavomír Hanzely (from Comenius)  
Lukang Sun (from Nanjing)

## MS Students

Egor Shulgin (from MIPT)  
Grigory Malinovsky (from MIPT)  
Igor Sokolov (from MIPT)

## Research Interns

Ilyas Fatkhullin (from Munich)  
Rustem Islamov (from MIPT)  
Bokun Wang (from UC Davis)  
Eduard Gorbunov (from MIPT)  
Ahmed Khaled (from Cairo)

# Papers Since 2019

2021

[160] G. Malinovsky, A. Sainalbayev and P. Richtárik  
Random reshuffling with variance reduction: new analysis and better rates

[159] A. Salim, L. Condat, D. Kovalev and P. Richtárik  
An optimal algorithm for strongly convex minimization under affine constraints

[158] Zhen Shi, N. Loizou, P. Richtárik and M. Takáč  
AI-SARAH: Adaptive and implicit stochastic recursive gradient methods

[157] D. Kovalev, E. Shulgin, P. Richtárik, A. Rogozin and A. Gasnikov  
ADOM: Accelerated decentralized optimization method for time-varying networks  
NSF-TRIPODS Workshop: Communication Efficient Distributed Optimization

[156] K. Mishchenko, B. Wang, D. Kovalev and P. Richtárik  
IntSGD: Floatless compression of stochastic gradients

[155] E. Gorbunov, K. Burlachenko, Z. Li and P. Richtárik  
MARINA: faster non-convex distributed learning with compression

[154] M. Safaryan, F. Hanzely and P. Richtárik  
Smoothness metrics beat smoothness constants: better communication compression techniques for distributed optimization

[ICLR 2021 (Workshop: Distributed and Private Machine Learning)]  
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[153] R. Islamyan, X. Qian and P. Richtárik  
Distributed second order methods with fast rates and compressed communication  
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[152] K. Mishchenko, A. Khaled and P. Richtárik  
Proximal and federated random reshuffling  
NSF-TRIPODS Workshop: Communication Efficient Distributed Optimization

2020

[151] S. Horváth, A. Klein, P. Richtárik and C. Archambeau  
Hyperparameter transfer learning with adaptive complexity  
AISTATS 2021

[150] X. Qian, H. Dong, P. Richtárik and T. Zhang  
Error compensated loopless SVRG for distributed optimization  
NeurIPS 2020 (12th Annual Workshop on Optimization for ML)

[149] X. Qian, H. Dong, P. Richtárik and T. Zhang  
Error compensated proximal SGD and RDA  
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[148] E. Gorbunov, F. Hanzely and P. Richtárik  
Local SGD: unified theory and new efficient methods  
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[147] D. Kovalev, A. Koloskova, M. Jaggi, P. Richtárik and S.U. Stich  
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[146] W. Chen, S. Horváth and P. Richtárik  
Optimal client sampling for federated learning  
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[145] E. Gorbunov, D. Kovalev, D. Makarenko and P. Richtárik  
Linearity compensating error compensated SGD  
NeurIPS 2020

[144] Alyazeed Albasonyi, M. Safaryan, L. Condat and P. Richtárik  
Optimal gradient compression for distributed and federated learning  
NeurIPS 2020 (Scalability, Privacy and Security in Federated Learning)

[143] F. Hanzely, Slavomír Hanzely, S. Horváth and P. Richtárik  
Lower bounds and optimal algorithms for personalized federated learning  
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[142] L. Condat, G. Malinovsky and P. Richtárik  
Distributed proximal splitting algorithms with rates and acceleration  
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[141] R. M. Gower, M. Schmidt, F. Bach and P. Richtárik  
Variance-reduced methods for machine learning  
Proceedings of the IEEE, 2020

[140] X. Qian, P. Richtárik and T. Zhang  
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[139] A. S. Berahas, M. Jahani, P. Richtárik and M. Takáč  
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[136] A. Khaled, O. Sebbouh, N. Loizou, R. M. Gower and P. Richtárik  
Unified analysis of stochastic gradient methods for composite convex and smooth optimization

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ICLR 2021

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[130] A. Salim, L. Condat, K. Mishchenko and P. Richtárik  
Duality, split, randomized: fast nonsmooth optimization algorithms  
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[128] G. Malinovsky, D. Kovalev, E. Gasanov, L. Condat and P. Richtárik  
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[127] A. Beznosikov, S. Horváth, P. Richtárik and M. Safaryan  
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[125] D. Kovalev, R. M. Gower, P. Richtárik and A. Rogozin  
Fast linear convergence of randomized BFGS  
NeurIPS 2020 (Fed, Learning for Data Privacy and Confidentiality)

[124] F. Hanzely, Nikita Doikov, P. Richtárik and Yurii Nesterov  
Stochastic subspace cubic Newton method  
ICML 2020

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Adaptivity of stochastic gradient methods for nonconvex optimization  
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[120] F. Hanzely, D. Kovalev and P. Richtárik  
Variable-reduced coordinate descent with acceleration: new methods with a surprising application to finite-sum problems  
KDD 2020

[119] A. Khaled and P. Richtárik  
Better theory for SGD in the nonconvex world  
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[112] S. Chraibi, A. Khaled, D. Kovalev, A. Salim, P. Richtárik and M. Takáč  
Distributed fixed point methods with compressed iterates  
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[111] S. Horváth, C.-Y. Ho, L. Horváth, A.N. Sahu, M. Canini and P. Richtárik  
IMTl: Natural compression for distributed deep learning  
SOSP 2019 (Workshop on AI Systems)

[110] D. Kovalev, K. Mishchenko and P. Richtárik  
Stochastic Newton and cubic Newton methods with simple local quadratic rates  
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A unified theory of SGD: variance reduction, sampling, quantization and coordinate descent  
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MISO is making a comeback with better proofs and rates  
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SGD: general analysis and improved rates  
ICML 2019

[96] F. Hanzely and P. Richtárik  
Don't jump through hoops and remove those loops: SVRG and Katyusha are better without the outer loop  
ALT 2020

[95] F. Hanzely, C.-Y. Ho and P. Richtárik  
SAGA with arbitrary sampling  
ICML 2019

[94] S. Horváth, D. Kovalev, K. Mishchenko, P. Richtárik and S. Stich  
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[93] E. H. Bergou, E. Gorbunov and P. Richtárik  
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**NeurIPS: Neural Inf. Process. Systems**

**ICML: Int. Conf. on Machine Learning**

**AISTATS: Artificial Intellig. & Statistics**

**ICLR: Int. Conf. on Learning Represent.**

**JMLR: J. Machine Learning Research**

**ALT: Algorithmic Learning Theory**

**UAI: Uncertainty in AI**

**AAAI: Conference on AI**

**SIAM, IEEE and IMA Journals**

**NSDI: USENIX Symp. on Networked Systems Design and Implementation**

**SOSP: Symp. Operating Syst. Principles**

# 17 Spotlight (5 min) Talks

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[160] G. Malinovsky, A. Sainanbayev and P. Richtárik  
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Smoothness matrices beat smoothness constants: better communication compression techniques for distributed optimization  
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[153] R. Islamov, X. Qian and P. Richtárik  
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[145] E. Gorbunov, D. Kovalev, D. Makarenko and P. Richtárik  
Linearly converging error compensated SGD  
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[144] Alyazeed Albayoni, M. Safanyan, L. Condat and P. Richtárik  
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Better theory for SGD in the nonconvex world  
NeurIPS 2020

[118] A. Khaled, K. Mishchenko and P. Richtárik  
Tighter theory for local SGD on identical and heterogeneous data  
AISTATS 2020

[117] S. Chraibi, A. Khaled, D. Kovalev, A. Salim, P. Richtárik and M. Takáč  
Distributed fixed point methods with compressed iterates  
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Better communication complexity for local SGD  
NeurIPS 2019 (Fed. Learning for Data Privacy and Confidentiality)

[113] A. Khaled and P. Richtárik  
Gradient descent with compressed iterates  
NeurIPS 2019 (Fed. Learning for Data Privacy and Confidentiality)

[112] A. Khaled, K. Mishchenko and P. Richtárik  
First analysis of local GD on heterogeneous data  
NeurIPS 2019 (Fed. Learning for Data Privacy and Confidentiality)

[111] J. Xiong, P. Richtárik and W. Heidrich  
Stochastic convolutional sparse coding  
Int. Symposium on Vision, Modeling and Visualization 2019

[110] X. Qian, Z. Qiu and P. Richtárik  
L-SVRG and L-Katyusha with arbitrary sampling  
Journal of Machine Learning Research, 2021

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AISTATS 2020

[102] F. Hanzely and P. Richtárik  
One method to rule them all: variance reduction for data, parameters and many new methods

[101] E. Gorbunov, F. Hanzely and P. Richtárik  
A unified theory of SGD: variance reduction, sampling, quantization and coordinate descent  
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Natural compression for distributed deep learning  
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RSN: Randomized Subspace Newton  
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[96] N. Loizou and P. Richtárik  
Convergence analysis of inexact randomized iterative methods  
SIAM Journal on Scientific Computing, 2020

[95] A. Sapio, M. Canini, C.-Y. Ho, J. Nelson, P. Kalnis, C. Kim, A. Krishnamurthy, M. Moshref, D. R. K. Ports and P. Richtárik  
Scaling distributed machine learning with in-network aggregation  
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ICML 2019

+ Samuel Horváth:  
Federated Learning under Heterogeneous Clients

+ Adil Salim:  
Complexity Analysis of Stein Variational Gradient Descent

# 17 Spotlight (5 min) Talks



Konstantin Burlachenko  
CS PhD Student



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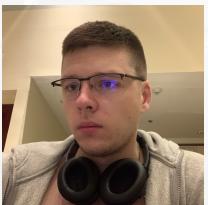
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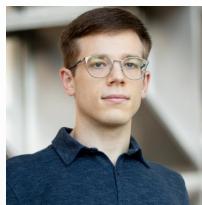
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## Random Reshuffling: Simple Analysis with Vast Improvements

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### Abstract

Random Reshuffling (RR) is an algorithm for minimizing finite-sum functions that utilizes iterative gradient steps in conjunction with random permutations. Often compared with the existing Stochastic Gradient Descent (SGD), RR is usually faster in practice and enjoys significant popularity in convex and non-convex optimization. The convergence rate of RR has attracted substantial attention recently and, for strongly convex and smooth functions, it was shown to converge faster than SGD if (1) the size of the data is large, (2) the gradients are bounded, and (3) the number of epochs is large. We remove these 3 assumptions, improve the dependence on the condition number from  $\kappa^2$  to  $\kappa$  (resp. from  $\kappa$  to  $\sqrt{\kappa}$ ) and, in addition, show that RR has a different type of variance. We argue through theory and experiments that the new variance type is an additional justification for the superior performance of RR.

### 1 Introduction

We study the finite-sum minimization problem

$$\min_{x \in \mathbb{R}^d} \left[ f(x) = \frac{1}{n} \sum_{i=1}^n f_i(x) \right], \quad (1)$$

where each  $f_i : \mathbb{R}^d \rightarrow \mathbb{R}$  is differentiable and smooth, and are particularly interested in the big data machine learning setting where the number of functions  $n$  is large. Thanks to their scalability and low memory requirements, first-order methods are especially popular in this setting (Bottou et al., 2018). Shuffling the data is a common technique used in a lot of machine learning and data mining communities and are often used in combination with various practical heuristics. Explaining these heuristics may lead to further development of stable and efficient training algorithms. In this work, we aim at better and sharper theoretical explanation of one intriguingly simple but notoriously elusive heuristic: *data permutation/shuffling*.

#### 1.1 Data permutation

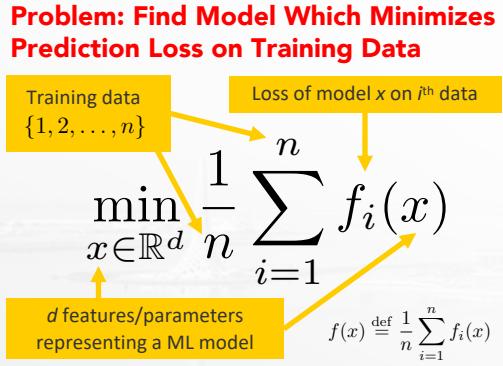
In particular, the goal of our paper is to obtain deeper theoretical understanding of methods for solving (1) which rely on random or deterministic *permutation/shuffling* of the data  $\{1, 2, \dots, n\}$  and

34th Conference on Neural Information Processing Systems (NeurIPS 2020), Vancouver, Canada.

# Part I

# RANDOM RESHUFFLING: SIMPLE ANALYSIS WITH VAST IMPROVEMENTS

# Random Reshuffling: Simple Analysis With Vast Improvements



**Theorem (Strongly Convex Case)**

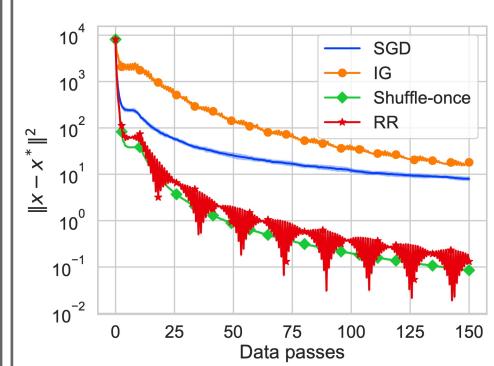
Model trained after  $t$  data passes → learning rate  $\gamma \leq \frac{1}{L}$  → A new notion: "shuffling variance"

$$\mathbb{E} [\|x^{tn} - x^*\|^2] \leq (1 - \gamma\mu)^{tn} \|x^0 - x^*\|^2 + \frac{2\gamma\sigma_{\text{Shuffle}}^2}{\mu}$$

solution → Strong convexity parameter

- Dramatically new proof technique
- Better dependence on  $n$  and condition number
- New notion: shuffling variance
- Variant Shuffle-Once: tightly matches lower bound of Safran and Shamir (2020)

Rajput et al (2020): "Current theoretical bounds [for Shuffle-Once] are insufficient to explain this phenomenon, and a new theoretical breakthrough may be required to tackle it."



## Algorithm: How to Choose the Next Training Data Point to Learn From?

1. Sampling **With** Replacement, aka **Stochastic Gradient Descent (SGD)**

$$x^{k+1} = x^k - \gamma \nabla f_{i^k}(x^k)$$

Example for  $n = 5$ :  $\{i^1, i^2, i^3, i^4, i^5\} = \{3, 2, 2, 1, 3\}$

- unbiased gradient estimator  $\mathbb{E} [\nabla f_{i^k}(x^k) | x^k] = \nabla f(x^k)$
- thousands of papers since 1950s well understood

2. Sampling **Without** Replacement, aka **Random Reshuffling (RR)**

$$x^{k+1} = x^k - \gamma \nabla f_{\pi^k}(x^k)$$

Example for  $n = 5$ :  $\{\pi^1, \pi^2, \pi^3, \pi^4, \pi^5\} = \{4, 3, 1, 2, 5\}$

- biased gradient estimator  $\mathbb{E} [\nabla f_{\pi^k}(x^k) | x^k] \neq \nabla f(x^k)$
- a handful of papers only!
- not understood
- default in deep learning software

**Our Theoretical Rates Significantly Improve on SOTA (in strongly convex, convex and also nonconvex regimes)**

Assumptions	$\mu$ -Strongly Convex	Non-Strongly Convex	Non-Convex	Citation
N.L. <sup>(1)</sup> ✓	$\kappa^2 n + \frac{\kappa n \sigma_x}{\mu \sqrt{\varepsilon}}$	—	—	Ying et al. (2019)
U.V. <sup>(2)</sup> ✗	$\kappa^2 n + \frac{\kappa \sqrt{n} G}{\mu \sqrt{\varepsilon}}$	$\frac{LD^2}{\varepsilon} + \frac{G^2 D^2}{\varepsilon^2}$ <sup>(3)</sup>	—	Nagaraj et al. (2019)
✗ ✗	—	—	$\frac{Ln}{\varepsilon^2} + \frac{LnG}{\varepsilon^3}$	Nguyen et al. (2020)
✓ ✓	$\frac{\kappa^2 n}{\sqrt{\mu \varepsilon}} + \frac{\kappa^2 n \sigma_x}{\mu \sqrt{\varepsilon}}$ <sup>(4)</sup>	—	—	Nguyen et al. (2020)
✗ ✗	$\frac{\kappa \alpha}{\varepsilon^2} + \frac{\kappa \sqrt{n} G \alpha^{3/2}}{\mu \sqrt{\varepsilon}}$ <sup>(5)</sup>	—	—	Ahn and Sra (2020)
✓ ✓	$\kappa n + \frac{\sqrt{n}}{\sqrt{\mu \varepsilon}} + \frac{\kappa \sqrt{n} G_0}{\mu \sqrt{\varepsilon}}$ <sup>(6)</sup>	—	—	Ahn et al. (2020)
→		$\frac{\kappa + \frac{\sqrt{\kappa n} \sigma_x}{\mu \sqrt{\varepsilon}}}{\kappa n + \frac{\sqrt{\kappa n} \sigma_x}{\mu \sqrt{\varepsilon}}}$ <sup>(7)</sup>	$\frac{Ln}{\varepsilon} + \frac{\sqrt{Ln} \sigma_x}{\varepsilon^{3/2}}$	$\frac{Ln}{\varepsilon^2} + \frac{L \sqrt{n(B+\sqrt{A})}}{\varepsilon^3}$
This work				



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Hongyan Bao  
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Xiangliang Zhang  
(KAUST Associate Professor)

## PAGE: A Simple and Optimal Probabilistic Gradient Estimator for Nonconvex Optimization

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Abstract

In this paper, we propose a novel stochastic gradient estimator – Probabilistic Gradient Estimator (PAGE) – for nonconvex optimization. PAGE is easy to implement as it is designed via a small adjustment to vanilla SGD in each iteration. PAGE uses the vanilla minibatch SGD update with probability  $p$  or renounces the previous gradient with a small adjustment, at a much lower computational cost, with probability  $1-p$ . We prove that PAGE is optimal in the sense that it matches the lower bound for nonconvex problems, which are of independent interest. Moreover, we prove matching upper bounds both in the finite-sum and online regimes, which establish that PAGE is an optimal method. Besides, we show that PAGE is robust to noise and can be used in conjunction with other methods, such as SGD, to easily switch to a faster linear convergence rate. Finally, we conduct several deep learning experiments (e.g., LeNet, VGG, ResNet) on real datasets in PyTorch, and the results demonstrate that PAGE not only converges much faster than SGD in training but also achieves the higher test accuracy, validating our theoretical results and confirming the practical superiority of PAGE.

### 1 Introduction

#### 1.1 The problem

Motivated by this development, we consider the general optimization problem

$$\min_{x \in \mathbb{R}^d} f(x), \quad (1)$$

Nonconvex optimization is ubiquitous across many domains of machine learning, including robust regression, low rank matrix recovery, sparse recovery and supervised learning [14]. Driven by the applied success of deep neural networks [22], and the critical place nonconvex optimization plays in training them, research in nonconvex optimization has been undergoing a renaissance [9, 10, 47, 7, 26, 29].

$$f(x) := \frac{1}{n} \sum_{i=1}^n f_i(x), \quad (2)$$

where the functions  $f_i$  are also differentiable and possibly nonconvex. Form (2) captures the standard empirical risk minimization problems in machine learning [41]. Moreover, if the number of data samples  $n$  is very large or even infinite, e.g., in the online/streaming case, then  $f(x)$  usually is modeled via the online form

$$f(x) := \mathbb{E}_{\zeta \sim P}[F(x, \zeta)], \quad (3)$$

1

## Part II

# PAGE: A SIMPLE AND OPTIMAL PROBABILISTIC GRADIENT ESTIMATOR FOR NONCONVEX OPTIMIZATION

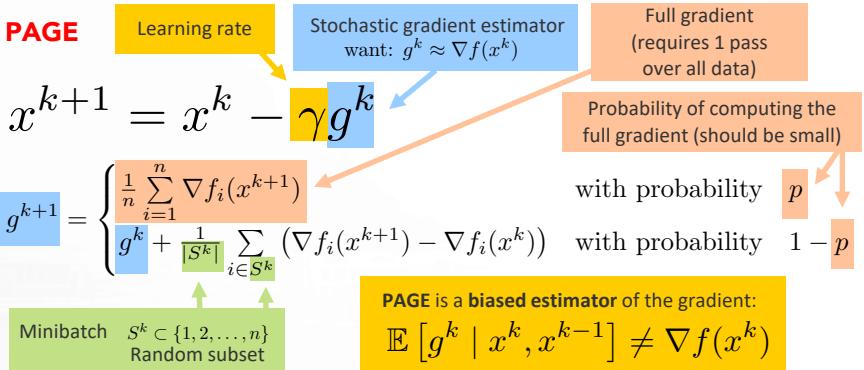
# PAGE: A Simple and Optimal Probabilistic Gradient Estimator for Nonconvex Optimization



**Problem: Train a ML Model on 1 Machine Using Minimal # of Data Samples**

$$\min_{x \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^n f_i(x)$$

Training data  $\{1, 2, \dots, n\}$       Loss of model  $x$  on  $i^{\text{th}}$  data  
 $f(x) \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^n f_i(x)$   
 $d$  features/parameters representing a ML model



## Comparison to Existing Results

Table 1: Gradient complexity for finding  $\tilde{x}$  satisfying  $\mathbb{E}\|\nabla f(\tilde{x})\| \leq \epsilon$  in nonconvex problems

Problem	Assumption	Algorithm or Lower Bound	Gradient complexity
Finite-sum (2)	Asp. 2	GD [34]	$O(n)$
Finite-sum (2)	Asp. 2	SVRG [3, 40], SCSG [24], SVRG+ [27]	$O(n + \frac{n^{1/2}}{\epsilon})$
Finite-sum (2)	Asp. 2	SNVRG [47], Geom-SARAH [13]	$\tilde{O}(n + \frac{\sqrt{n}}{\epsilon^2})$
Finite-sum (2)	Asp. 2	SPIDER [7], SpiderBoost [49], SARAH [37], SSGD [26]	$O(n + \frac{\sqrt{n}}{\epsilon^2})$
Finite-sum (2)	Asp. 2	PAGE (this paper)	$O(n + \frac{\sqrt{n}}{\epsilon^2})$
Finite-sum (2)	Asp. 2	Lower bound [7]	$\Omega(\frac{\sqrt{n}}{\epsilon})$ if $n > O(\frac{1}{\epsilon})$
Finite-sum (2)	Asp. 2	Lower bound (this paper)	$\Omega(n + \frac{\sqrt{n}}{\epsilon^2})$
Finite-sum (2)	Asp. 2 and 3	PAGE (this paper)	$O((n + \sqrt{n}\log \frac{1}{\epsilon}))$
Online (3) <sup>2</sup>	Asp. 1 and 2	SGD [10, 16, 29]	$O(\frac{1}{\epsilon^2})$
Online (3)	Asp. 1 and 2	SCSG [24], SVRG+ [27]	$O(b + \frac{\sqrt{b^2 - b}}{\epsilon^2})$
Online (3)	Asp. 1 and 2	SNVRG [47], Geom-SARAH [13]	$\tilde{O}(b + \frac{\sqrt{b}}{\epsilon^2})$
Online (3)	Asp. 1 and 2	SPIDER [7], SpiderBoost [49], SARAH [37], SSGD [26]	$O(b + \frac{\sqrt{b}}{\epsilon^2})$
Online (3)	Asp. 1 and 2	PAGE (this paper)	$O(b + \frac{\sqrt{b}}{\epsilon^2})$
Online (3)	Asp. 1 and 2	Lower bound (this paper)	$\Omega(b + \frac{\sqrt{b}}{\epsilon})$
Online (3)	Asp. 1, 2 and 3	PAGE (this paper)	$O(b + \sqrt{b}\log \frac{1}{\epsilon})$

## Assumptions:

- 1  $f_i$  can be nonconvex
- 2  $f$  is lower bounded
- 3  $f_i$  is “smooth”

$$\|\nabla f_i(x) - \nabla f_i(y)\| \leq L_i \|x - y\| \quad \forall x, y \in \mathbb{R}^d$$

**Goal: Find random vector  $\hat{x}$  such that**

$$\mathbb{E} [\|\nabla f(\hat{x})\|^2] \leq \epsilon^2$$

## Theorem

PAGE solves the problem using

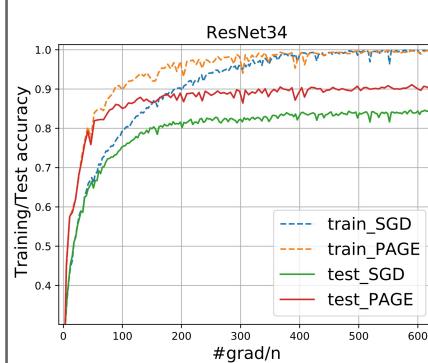
$$\mathcal{O}\left(n + \frac{\sqrt{n}}{\epsilon^2}\right) \text{ data samples}$$

with either of these two parameter choices:

A       $|S^k| = 1$  and  $p = \frac{1}{1+n}$

B       $|S^k| = \sqrt{n}$  and  $p = \frac{1}{1+\sqrt{n}}$

We prove that PAGE is “optimal” (= in a precise mathematical sense, this is the best gradient-type method for solving smooth nonconvex problems)





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## MARINA: Faster Non-Convex Distributed Learning with Compression

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### Abstract

We develop and analyze MARINA: a new communication efficient method for non-convex distributed learning with compression. MARINA uses a gradient compression strategy based on the compression of gradient differences which is reminiscent of but different from the strategy employed in the DIANA method of Mischler et al (2019). Unlike DIANA, MARINA is based on a compressed gradient estimator, which is a key to its superior theoretical and practical performance. To the best of our knowledge, the communication complexity bounds we prove for MARINA are among the best known. As a first application of MARINA, we develop and analyze two variants of MARINA: VR-MARINA and PP-MARINA. The first method is designed for the case when the local loss functions owned by clients are either of a finite sum or of an expectation form, and the second method allows for partial participation of clients: a feature important in federated learning. MARINA is shown to be competitive to state-of-the-art methods in terms of the oracle/communication complexity. Finally, we provide convergence analysis of all methods for problems satisfying the Polyak-Lojasiewicz condition.

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arXiv:2102.07845v1 [cs.LG] 15 Feb 2021

## Part III

# MARINA: FASTER NON-CONVEX DISTRIBUTED LEARNING WITH (COMMUNICATION) COMPRESSION

# MARINA: Faster Non-convex Distributed Learning with (Communication) Compression



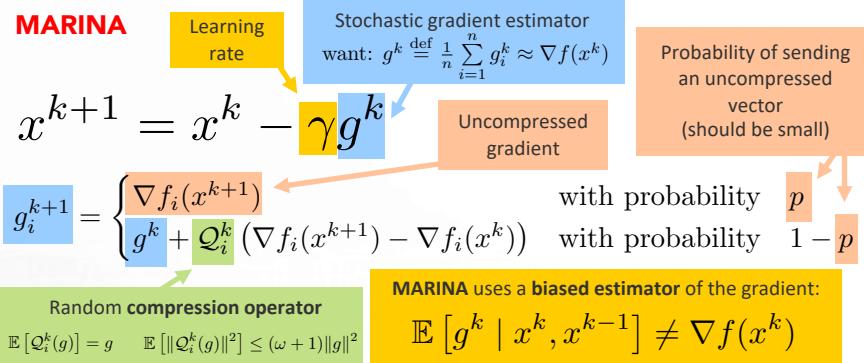
**Problem: Train a ML Model on  $n$  Machines Using Minimal # of Bits Communicated by the  $n$  Workers to the Master**

$$\min_{x \in \mathbb{R}^d} \frac{1}{n} \sum_{i=1}^n f_i(x)$$

$f(x) \stackrel{\text{def}}{=} \frac{1}{n} \sum_{i=1}^n f_i(x)$

# of machines  $\downarrow$   $n$  Loss of model  $x$  on data stored on machine  $i$

$d$  features/parameters representing a ML model



**Assumptions:**

- 1  $f_i$  can be nonconvex
- 2  $f$  is lower bounded
- 3  $f_i$  is “smooth”

$$\|\nabla f_i(x) - \nabla f_i(y)\| \leq L_i \|x - y\| \quad \forall x, y \in \mathbb{R}^d$$

**Goal: Find random vector  $\hat{x}$  such that**

$$\mathbb{E}[\|\nabla f(\hat{x})\|^2] \leq \varepsilon^2$$

**Theorem (simplified)**

If  $\mathcal{Q}_i^k$  is the Rand-1 sparsifier, and  $p = \frac{1}{d+1}$ , then MARINA solves the problem using

$$\mathcal{O}\left(\frac{1+d/\sqrt{n}}{\varepsilon^2}\right) \text{ communicated bits / machine}$$

**Previous SOTA:**  
Mishchenko et al 2019; Horváth et al 2019; Li & R. 2020

**Gradient Descent**

**DIANA**

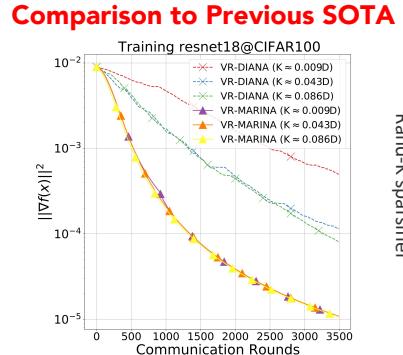
$$\mathcal{O}\left(\frac{d}{\varepsilon^2}\right)$$

$$\mathcal{O}\left(\frac{1+d^{3/2}/\sqrt{n}}{\varepsilon^2}\right)$$

**Comparison to Existing Results: MARINA is SOTA**

Setup	Method	Citation	Communication Complexity	Oracle Complexity
(1)	DIANA	[Mishchenko et al., 2019] [Horváth et al., 2019] [Li and Richtárik, 2020]	$\frac{1+(1+\omega)\sqrt{\omega/n}}{\varepsilon^2}$	$\frac{1+(1+\omega)\sqrt{\omega/n}}{\varepsilon^2}$
(1)	FedCOMATE (1) FedSTEPH, $r = n$	[Haddadpour et al., 2020] [Das et al., 2020]	$\frac{1+\omega}{\varepsilon^2}$ $\frac{1+\omega}{\varepsilon^2}$ $\frac{1+\omega\sqrt{n}}{\varepsilon^2}$	$\frac{1+\omega}{\varepsilon^2}$ $\frac{n\sqrt{\omega}}{\varepsilon^2}$ $\frac{1+\omega\sqrt{n}}{\varepsilon^2}$
(1)	MARINA (Alg. 1)	Thm. 2.1 & Cor. 2.1 (NEW)	$\frac{1+\omega\sqrt{n}}{\varepsilon^2}$	$\frac{1+\omega\sqrt{n}}{\varepsilon^2}$
(1)+(5)	DIANA	[Li and Richtárik, 2020]	$\frac{1+(1+\omega)\sqrt{\omega/n} + \frac{1+\omega}{n\sqrt{4}}}{\varepsilon^2}$	$\frac{1+(1+\omega)\sqrt{\omega/n} + \frac{1+\omega}{n\sqrt{4}}}{\varepsilon^2}$
(1)+(5)	VR-DIANA	[Horváth et al., 2019]	$\frac{(m\sqrt{3}+\omega)\sqrt{1+\omega/n}}{\varepsilon^2}$	$\frac{(m\sqrt{3}+\omega)\sqrt{1+\omega/n}}{\varepsilon^2}$
(1)+(5)	VR-MARINA (Alg. 2) $b' = 1^{(2)}$	Thm. 3.1 & Cor. 3.1 (NEW)	$\frac{1+\max\{\omega, \sqrt{(1+\omega)m}\}/\sqrt{n}}{\varepsilon^2}$	$\frac{1+\max\{\omega, \sqrt{(1+\omega)m}\}/\sqrt{n}}{\varepsilon^2}$
(1)+(8)	DIANA (3) FedCOMATE (3)	[Mishchenko et al., 2019] [Li and Richtárik, 2020] [Haddadpour et al., 2020]	$\frac{1+(1+\omega)\sqrt{\omega/n} + \frac{1+\omega}{n\sqrt{4}}}{\varepsilon^2}$ $\frac{1+\omega}{\varepsilon^2}$	$\frac{1+(1+\omega)\sqrt{\omega/n} + \frac{1+\omega}{n\sqrt{4}}}{\varepsilon^2}$ $\frac{1+\omega}{\varepsilon^2}$
(1)+(8)	VR-MARINA (Alg. 2) $b' = 1$	Thm. 3.2 & Cor. 3.2 (NEW)	$\frac{1+\omega\sqrt{n}}{\varepsilon^2} + \frac{\sqrt{1+\omega}}{n\varepsilon^3}$	$\frac{1+\omega\sqrt{n}}{\varepsilon^2} + \frac{\sqrt{1+\omega}}{n\varepsilon^3}$
(1)+(8)	VR-MARINA (Alg. 2) $b' = \Theta\left(\frac{1}{n^{1/2}}\right)$	Thm. 3.2 & Cor. 3.2 (NEW)	$\frac{1+\omega\sqrt{n}}{\varepsilon^2}$	$\frac{1+\omega\sqrt{n}}{n\varepsilon^4} + \frac{1+\omega}{n\varepsilon^3}$
PP, (1)	FedSTEPH	[Das et al., 2020]	$\frac{1+\omega/n + (1+\omega)(n-r)}{r\varepsilon^4}$	$\frac{1+\omega/n}{r\varepsilon^4} + \frac{(1+\omega)(n-r)}{r\varepsilon^4}$
PP, (1)	PP-MARINA (Alg. 4)	Thm. 4.1 & Cor. 4.1 (NEW)	$\frac{1+(1+\omega)\sqrt{n/r}}{\varepsilon^2}$	$\frac{1+(1+\omega)\sqrt{n/r}}{\varepsilon^2}$

Rand-K sparsifier





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# ARTIFICIAL INTELLIGENCE INITIATIVE

