Optimization Model for Islanding of Power Systems

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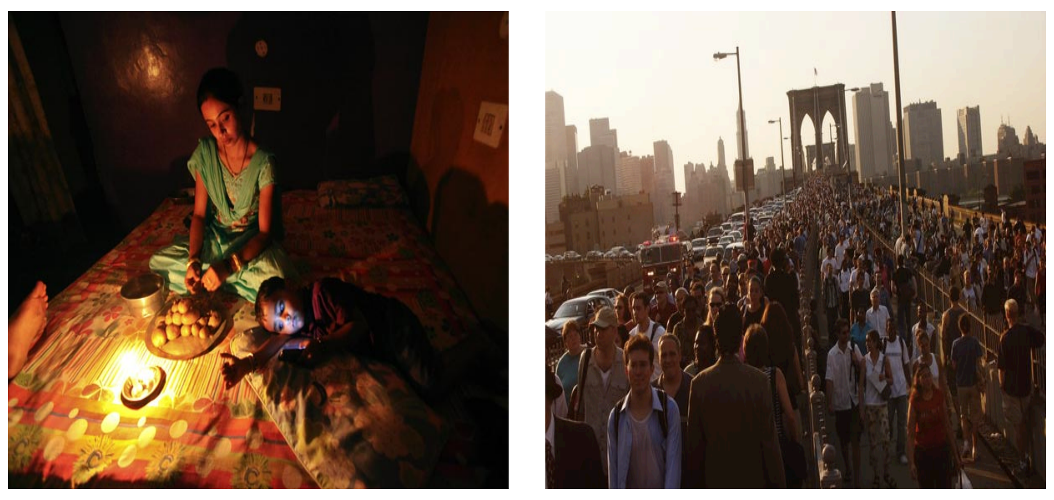
Abstract

A mixed integer linear programming (MILP) formulation is presented for intentionally forming islands in a power network. We give the motivation, formulation and numerical results of the model. Numerical results on test networks upto 300 buses show the method is computationally efficient on large networks.

Motivation

Modern power systems are designed to handle *single* outage at a time. But sometimes unpredicted events like wind storms or natural disasters happen and they cause two or more outages at a time. Since power system is not designed to handle such a situation, cascading outages occur and lead to large area blackout.



(a) Indian Blackout 2012 (b) US & Canada Blackout 2003

Figure 1: Two Major Blackouts of Recent History

Source:World's Worst Power Outages, National Geographic

In the last decade we have witnessed numerous wide area blackouts. Figure 1 shows the chaos and pain which follows after the blackout. The biggest blackout of history happened recently in India. 670 million people were affected by this blackout. This blackout was caused by tripping of a 400 kV transmission line and a substation. Cascading tripping followed and 32GW of generating capacity was taken out of the system. In 2003, wrong calculations and tripping of a high flow line caused a blackout in US and Canada. 55 million people were affected in that blackout, and it took one week to fully restore the power.

The risk of blackouts is increasing due to liberalization and integration of renewable resources in the power system. The recent liberalization of energy markets has reduced the security margin of operation. Also there is a lot of emphasis on renewable resources. The downside of renewable energy is the volatile supply of power which is causing problems. Figure 2 shows the increasing trend of blackouts.

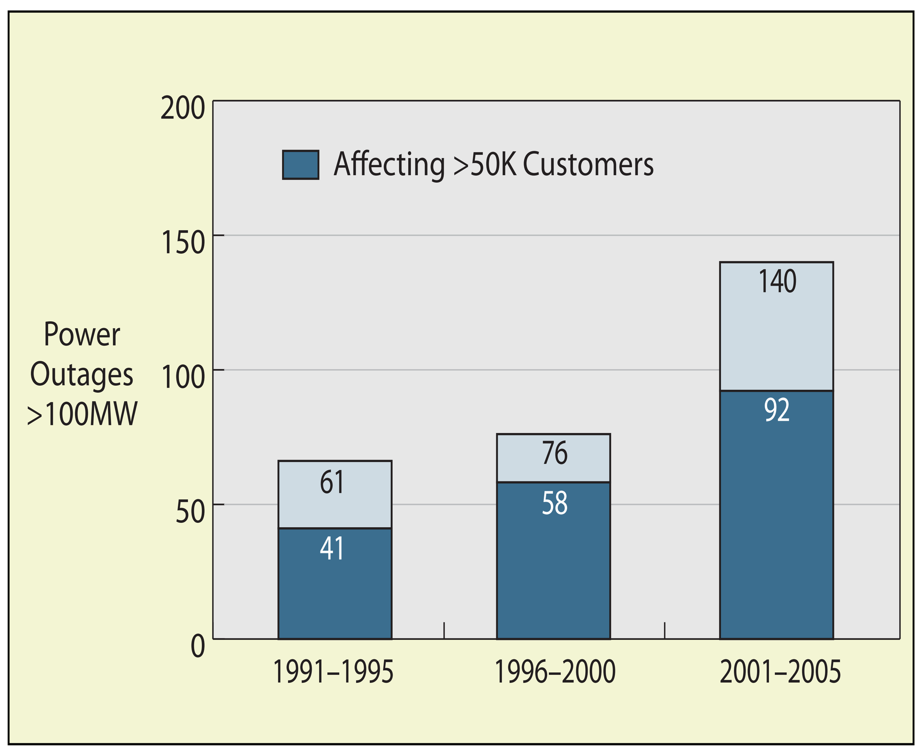


Figure 2: Increasing Risk of Blackouts

Source: IEEE Power & Energy Magazine

Our Idea of Islanding

The case studies of historic blackouts suggest that blackouts were initially triggered by a disturbance in a small part of the network. Disturbance was not curtailed to the affected area in time and eventually it spread across the network causing wide area blackout.

As a mitigation step to wide area blackouts we propose an islanding approach. We assume that there is problem in a small part of the network. Let $\mathcal{B}_0$ denotes the set of troubled buses, and let $\mathcal{B}_1$ be the set of buses which are operating normally. We want to isolate buses in $\mathcal{B}_0$ by creating atleast two disconnected components of the network $\mathcal{S}_0$ and $\mathcal{S}_1$ such that $\mathcal{B}_0 \subseteq \mathcal{S}_0$, and $\mathcal{B}_1 \subseteq \mathcal{S}_1$.

Optimization Model for Power Systems Islanding

Consider a power system network with $n^{\mathcal{B}}$ buses. Let $\mathcal{G}_b$ and $\mathcal{D}_b$ be the set of generators and demands at bus b, let $\mathcal{B}_b$ be the set of buses connected by a line to bus b, let $\mathcal{L}$ be the set of lines and let b_0 be the reference bus. Parameters P_g^- and P_g^+ are the bounds on variable p_g^{G}, the real power output of generator g; and parameter P_d^{D} is the real power consumed by load d. Variable $p_{bb'}^{\mathsf{L}}$ is the real power flowing into line bb' from bus b, and parameter $P_{bb'}^+$ is the real power line rating of the line bb'. Variable θ_b is the voltage phase angle at bus b.

Sectioning Constraints

We define binary variables γ_b for each bus b and ρ_l for each line l in the network respectively. The constraints to partition the network into two sections are as follows:

$$\rho_{bb'} \leq 1 + \gamma_b - \gamma_{b'} \quad \forall\, bb' \in \mathcal{L} \tag{1}$$
$$\rho_{bb'} \leq 1 - \gamma_b + \gamma_{b'} \quad \forall\, bb' \in \mathcal{L} \tag{2}$$
$$\gamma_b = 0 \quad \forall\ b\ \in \mathcal{B}_0 \tag{3}$$
$$\gamma_b = 1 \quad \forall\ b\ \in \mathcal{B}_1 \tag{4}$$

DC Power Flow Equations

The linear real power flow equation is given by:

$$\hat{p}_{bb'}^L = \quad -B_{bb'}(\theta_b - \theta_{b'}) \tag{8}$$

Here $\hat{p}_{bb'}^L$ is the auxiliary variable for real power flow. When the line bb' is connected, we must have $p_{bb'}^L = \hat{p}_{bb'}^L$, and when the line is disconnected then $p_{bb'}^L = 0$ and $\hat{p}_{bb'}^L$ is free. This is modelled as:

$$-\rho_{bb'} P_{bb'}^+ \leq p_{bb'}^L \leq \rho_{bb'} P_{bb'}^+, \tag{9}$$
$$-(1-\rho_{bb'})\hat{P}_{bb'}^+ \leq \hat{p}_{bb'}^L - p_{bb'}^L \leq (1-\rho_{bb'})\hat{P}_{bb'}^+ \tag{10}$$

Load Model

We define $p_d^{\mathsf{D}} = \alpha_d P_d^{\mathsf{D}}$, where α_d is the proportion of the load delivered at demand d. We define β_d as the load satisfaction probability in section 0. Probability of the load satisfaction in section 1 is unity. This is achieved by following set of equations:

$$\alpha_d = \alpha_{0d} + \alpha_{1d}, \tag{5}$$
$$0 \leq \alpha_{0d} \leq \alpha_{1d}, \tag{6}$$
$$0 \leq \alpha_{1d} \leq \gamma_b \tag{7}$$

Other Constraints

The power balance constraint at each bus of the network is given by:

$$\sum_{g \in \mathcal{G}_b} p_g^{\mathsf{G}} = \sum_{d \in \mathcal{D}_b} P_d^{\mathsf{D}} + \sum_{b' \in \mathcal{B}_b} p_{bb'}^{\mathsf{L}} \tag{11}$$

The reference bus and real power generation constraints are:

$$\theta_{b_0} = 0, \tag{12}$$
$$P_g^- \leq p_g \leq P_g^+ \tag{13}$$

Optimization Problem

The objective of our optimization model is to maximize the amount of load delivered to customers. Let M_d be the reward per unit of real power delivered at demand d. The optimization problem is as follows

$$\max \sum_{d \in \mathcal{D}} M_d P_d (\beta_d \alpha_{0d} + \alpha_{1d})$$

subject to: (1) - (13)

Numerical Results

We have tested our model on standard IEEE test cases ranging from 9 to 300 bus nodes. Here we give an example of 24 bus network and the computation times to solve big IEEE test networks.

24 Bus Example

This IEEE test network consists of 24 buses and 38 transmission lines. We assume that there is a fault on bus 9 and assigned it to set $\mathcal{B}_0$. Also we assume that $\beta_d = 0.75$.

Figure 3 shows the islanded solution. Six buses are placed in section 0, 16.5% of the load was placed in this section. 35 MW of load was shed in this islanding solution.

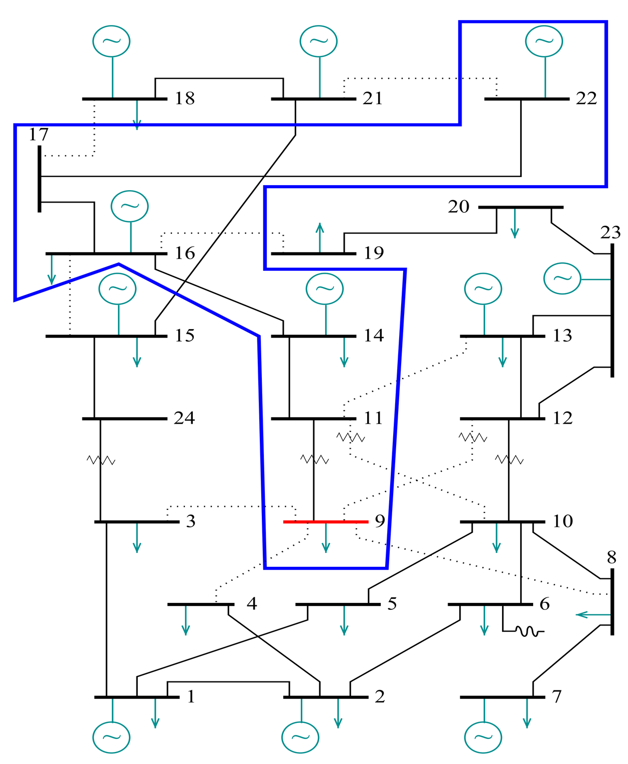


Figure 3: Islanding of 24 Bus Network

Computation Times on Big Networks

Problems were solved on a dual quad-core 64-bit Linux machine with 8GiB RAM, using AMPL 11.0 with paralled CPLEX 12.3 to solve MILP problems.

Figure 4 shows the time required to obtain feasible islanding solutions to varying proven levels of optimality. Minimum, mean and maximum times were obtained by solving each of the cases with $n^{\mathcal{B}}$ scenarios. All problems were solved to feasibility within 1 s and to 5% optimality within 5 s.

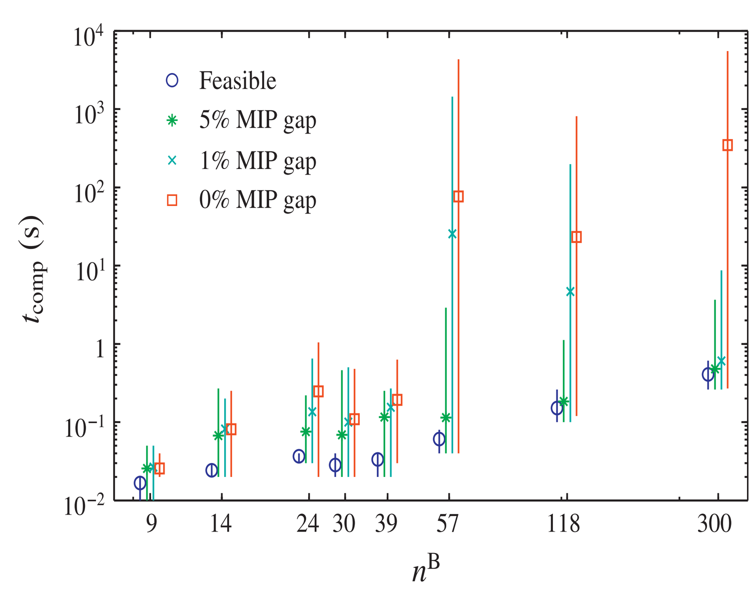


Figure 4: Mean, Max and Min times of Islanding Solutions

Conclusions

The proposed method uses MILP to determine which transmission lines to cut, loads to shed, generators to adjust in order to isolate a failure-prone region of electricity transmission network. We have demonstrated this approach through standard IEEE test cases. The ability of this method to find good islanding solutions quickly makes it very attractive for practical use.

References

[1] Trodden, P. A., Bukhsh, W. A., Grothey, A., & McKinnon, K. I. M., MILP formulation for controlled islanding of power networks, *International Journal of Electrical Power & Energy Systems*, 2013, 45(1), 501-508

[2] Trodden, P. A., Bukhsh, W. A., Grothey, A., & McKinnon, K. I. M., Optimization-based Islanding of Power Networks using Piecewise Linear AC Power FLow, submitted for publication in *IEEE Transactions on Power Systems*, April 2013