A natural stabilized column generation method †

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Column Generation

Motivation

Exploit the advantages of a **primal-dual interior point method** to naturally stabilize the **column generation method**.

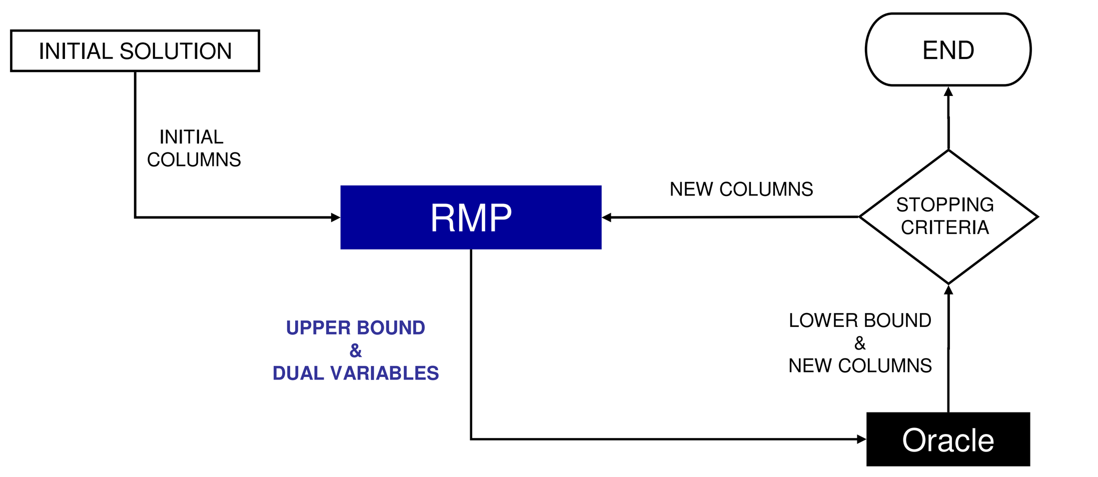

Fig. 1: Column generation scheme

Master Problem

$$z^* := \min \sum_{j \in N} c_j \lambda_j, \quad \text{s.t.} \sum_{j \in N} a_j \lambda_j = b, \quad \lambda_j \geq 0, \quad \forall j \in N.$$

Restricted Master Problem

$$z_{RMP} := \min \sum_{j \in \bar{N}} c_j \lambda_j, \quad \text{s.t.} \sum_{j \in \bar{N}} a_j \lambda_j = b, \quad \lambda_j \geq 0, \quad \forall j \in \bar{N}.$$

The RMP works with a subset $\bar{N}$ of MP columns N.

Therefore, $z^* \leq z_{RMP}$.

Oracle

$$z_{SP} := \min\{0; c_j - \bar{u}^T a_j \mid a_j \in \mathcal{A}\}.$$

Bounds

$$z_{RMP} + \kappa z_{SP} \leq z^* \leq z_{RMP}.$$

where $\bar{u}$ is the vector of duals and κ is a known parameter.

Issues

- Heading-in and tailing-off effects.

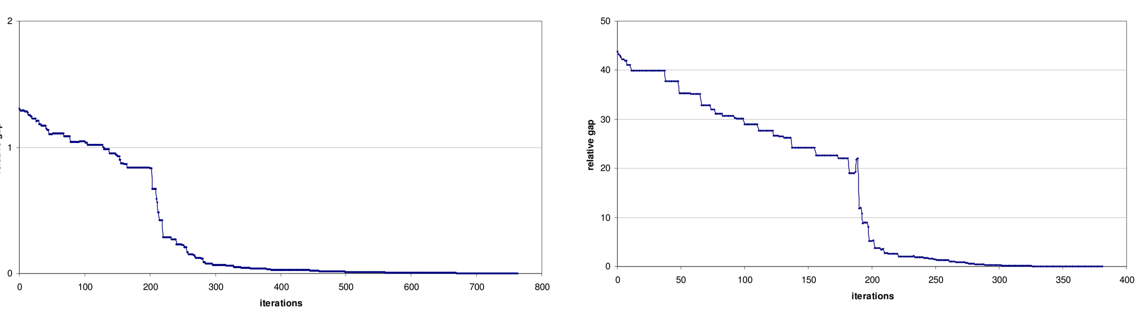

Fig. 2: Unstable behaviour of the standard column generation (CSP and UFLP)

Stabilized CG

- **Artificial**: boxstep method[1], interior point stabilization[2], bundle method[3].
- **Natural**: analytic centre cutting plane method[4], primal-dual cutting plane method[5].

Primal-Dual Column Generation Method (PDCGM)

Features

- Allows suboptimal solutions
- Provides a reliable lower bound
- Delivers well-centred dual prices
- Adjusts the optimality tolerance dynamically
- Can warmstart

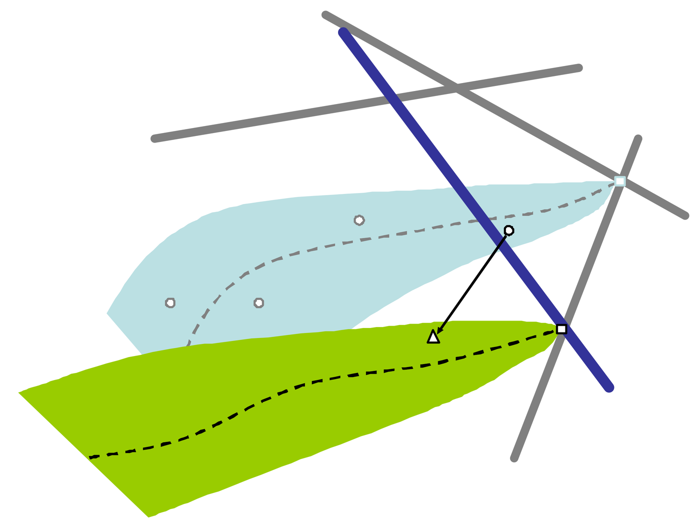

Fig. 3: Warmstarting strategy

Algorithm

1. Input: Initial RMP; parameters $\kappa, \varepsilon_{\max} > 0, D > 1, \delta > 0$.

2. Set $LB = -\infty, UB = \infty, gap = \infty, \varepsilon = 0.5$;

3. while $(gap \geq \delta)$ do

- find a well-centred ε-optimal solution $(\tilde{\lambda}, \tilde{u})$ of the RMP;
- $UB = \min\{UB, \tilde{z}_{RMP}\}$;
- call the *oracle* with the query point $\tilde{u}$ and solve $\tilde{z}_{SP}$;
- $LB = \max\{LB, \kappa \tilde{z}_{SP} + b^T \tilde{u}\}$;
- $gap = (UB - LB) / (1 + |UB|)$;
- $\varepsilon = \min\{\varepsilon_{\max}, gap / D\}$;
- if $(\tilde{z}_{SP} < 0)$ then add the new columns to the RMP;

4. end (while)

Numerical Results

Solve the linear-relaxed reformulations of three well-known combinatorial optimization problems.

Strategies

- **Standard column generation (SCG)**
  - Simplex (CPLEX 12.0)
  - Optimal vertex strategy
- **Analytic centre cutting plane method (ACCPM)**
  - Weighted projective algorithm (OBOE)
  - Central prices (analytic centre of localization set)
- **Primal-dual column generation method (PDCGM)**
  - Primal-dual interior point method (HOPDM)
  - Close-to-optimality strategy (μ-centre)

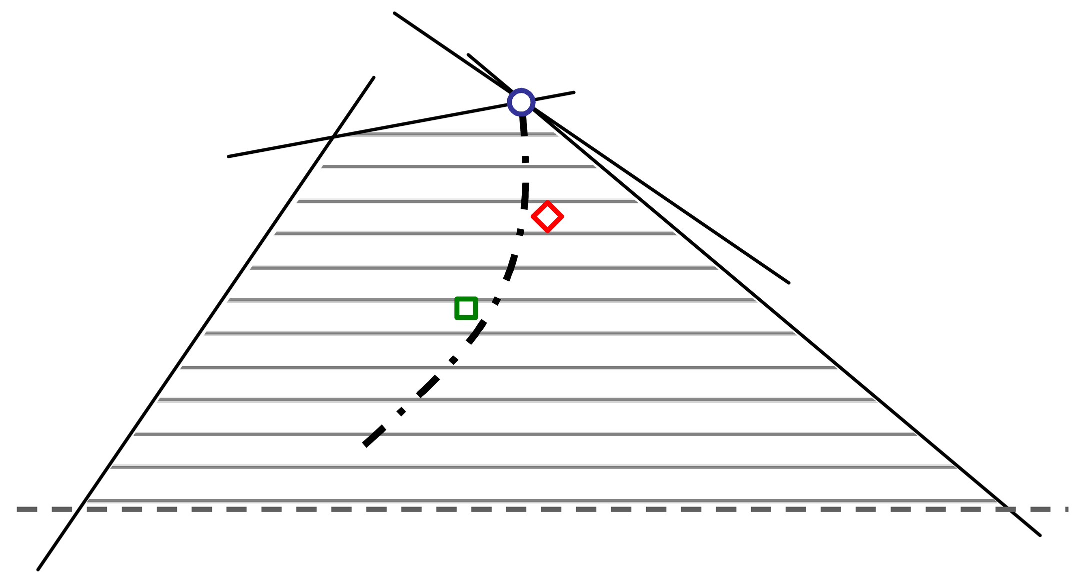

Fig. 4: Strategies in the dual space (RMP)

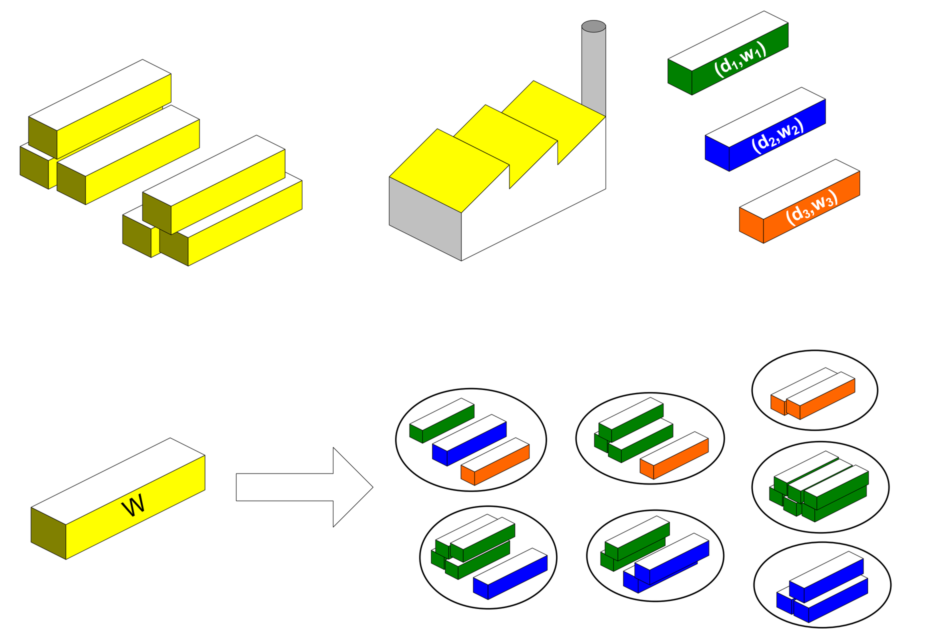

Fig. 5: Cutting stock problem (CSP)

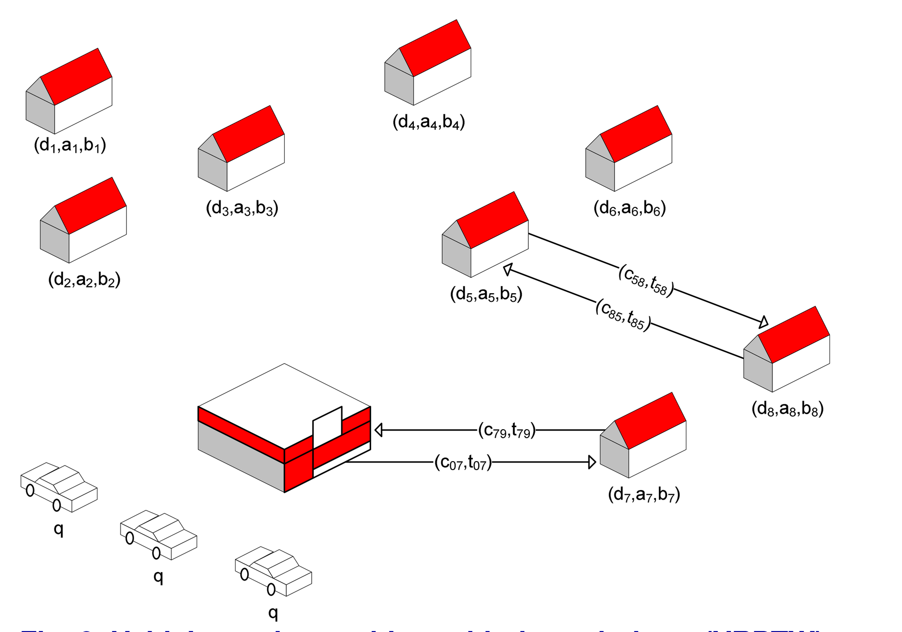

Fig. 6: Vehicle routing problem with time windows (VRPTW)

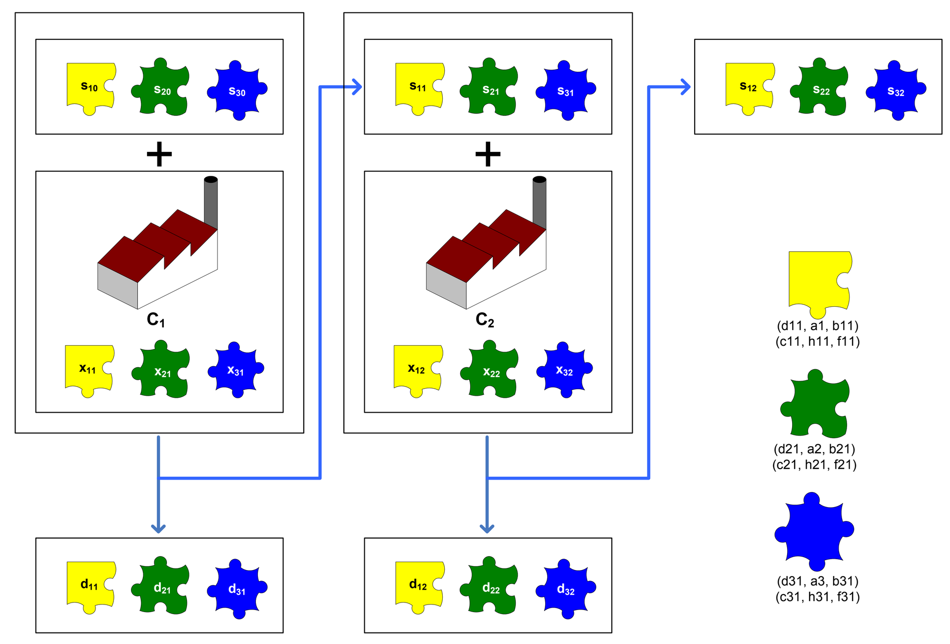

Fig. 7: Capacitated lot sizing problem with setup times (CLSPST)

Small and medium size instances

class	inst	SCG		ACCPM		PDCGM	
		ite	time	ite	time	ite	time
CSP	262	671.0	51.7	552.1	65.6	**440.3**	**17.5**
VRPTW	87	392.4	163.8	187.1	57.8	**121.3**	**45.1**

Table 1: Average results adding one column at a time

class	k	SCG		ACCPM		PDCGM	
		ite	time	ite	time	ite	time
CSP	10	182.3	25.5	290.0	65.4	**120.2**	**7.3**
	50	91.0	20.1	316.4	161.2	**74.1**	**10.0**
	100	68.8	24.6	353.6	319.1	**63.0**	**15.1**
VRPTW	10	93.7	40.2	128.7	32.8	**44.2**	**14.8**
	50	45.2	19.7	125.8	31.3	**31.0**	**9.7**
	100	34.5	15.2	126.0	31.3	**25.9**	**7.8**
	200	26.9	12.1	126.9	31.9	**23.2**	**6.9**
	300	23.3	10.7	127.7	32.2	**22.6**	**7.0**

Table 2: Average results adding k-best columns at a time

Large size instances

CLSPST

r	SCG		ACCPM		PDCGM	
	ite	time	ite	time	ite	time
5	27.5	4.7	22.5	3.2	**11.5**	**1.6**
10	32.0	62.7	29.5	49.5	**15.6**	**21.0**
15	38.4	308.8	36.4	274.3	**20.0**	**106.2**
20	45.5	975.8	42.4	941.0	**25.9**	**358.4**

Table 3: Average results on 11 instances adding one column at a time per subproblem

CSP

name	m	SCG		ACCPM		PDCGM	
		ite	time	ite	time	ite	time
U09498	1005	548	12946.8	762	21253.6	**293**	**5678.0**
U09513	975	518	9903.8	779	19362.0	**267**	**4276.7**
U09528	945	541	7797.8	740	15919.8	**276**	**4923.6**
U09543	915	506	5585.0	723	13448.9	**263**	**3723.7**

Table 4: Results adding 100 columns at a time

VRPTW

name	n	SCG		ACCPM		PDCGM	
		ite	time	ite	time	ite	time
RC1_6_1	600	258	18972.3	923	56683.3	**150**	**8844.3**
C1_4_1	400	137	551.9	272	908.6	**53**	**185.7**
C1_2_1	200	85	40.9	169	81.6	**29**	**15.2**

Table 5: Results adding 300 columns at a time

Conclusions

- PDCGM is a reliable column generation method.
- It offers improvements over other methods in the literature.
- The larger the instance, the better the relative performance of the method.
- It can be seen as a general-purpose technique due to the lack of parameter setting.

References

[1] R. E. Marsten, W. W. Hogan, J. W. Blankenship, “The boxstep method for large-scale optimization”, Operations Research 23 (3) (1975) 389–405.

[2] L.-M. Rousseau, M. Gendreau, D. Feillet, “Interior point stabilization for column generation”, Operations Research Letters 35 (5) (2007) 660–668.

[3] O. Briant, C. Lemaréchal, P. Meurdesoif, S. Michel, N. Perrot, F. Vanderbeck, “Comparison of bundle and classical column generation”, Mathematical Programming 113 (2008) 299–344.

[4] J. L. Goffin, A. Haurie and J. P. Vial, "Decomposition and nondifferentiable optimization with the projective algorithm", Management Science, 38(2)(1992), 284-302.

[5] J. E. Mitchell and B. Borchers, "Solving real-world linear ordering problems using a primal-dual interior point cutting plane method", Annals of Operations Research, 62 (1996), 253-276.

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