Preconditioned alternating projection algorithms for High-Resolution Image Reconstruction with Displacement-Errors

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Joint work with Yuesheng Xu and Lixin Shen

Outline

Outline

Background

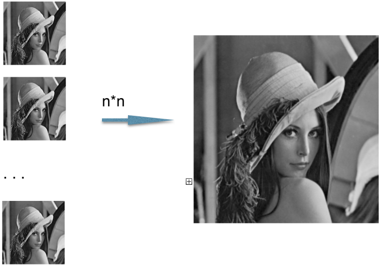


Figure: Reconstruct one HR image from an array of LR images.

Application

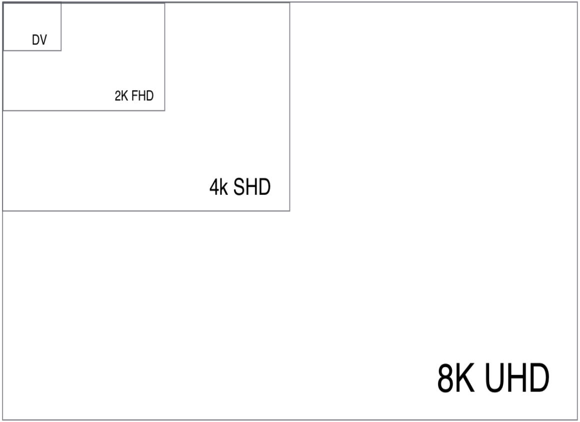


Figure: Image size of ultra high display.

Application

Table: Television display standards.

Format	Size	Resoution
DV		480×720
General laptop	14"	1366×768
MacBook with Retina display	15"	2880×1800
1080p TV		1920×1080
4K	80"	4096×2160
8K		7680×4320

Outline

Optimization Model

$$u^* = \operatorname*{argmin}_{u \in \mathbb{R}^n} \{f_1(u) + f_2(u)\}$$

- $f_1, f_2 \in \Gamma_0(\mathbb{R}^n)$
- f_2 is differentiable with a $1/\beta$ -Lipschitz continuous gradient

Optimization Model

$$u^* = \arg\min_{u \in \mathbb{R}^n} \{p_1(Bu) + p_2(Au)\} \tag{2.1}$$

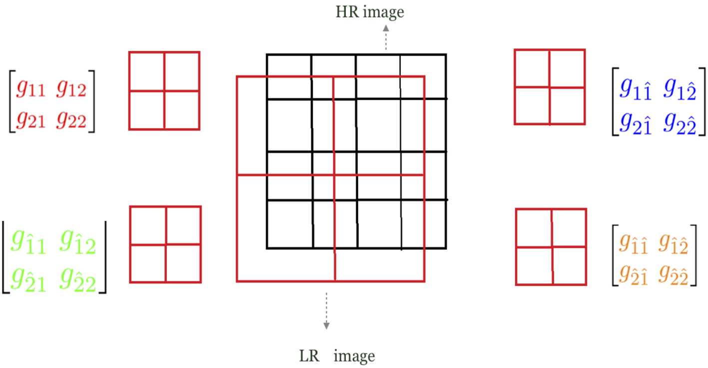
- $A \in \mathbb{R}^{m \times n}$: convolution kernel
- $B := \begin{bmatrix} I_N \bigotimes D \\ D \bigotimes I_N \end{bmatrix}$

Existing Works

- Gabay D, Mercier B. A dual algorithm for the solution of nonlinear variational problems via finite element approximation[J]. Computers & Mathematics with Applications, 1976, 2(1): 17-40.
- Chambolle A, Pock T. A first-order primal-dual algorithm for convex problems with applications to imaging[J]. Journal of Mathematical Imaging and Vision, 2011, 40(1): 120-145.

Observation Model

- **Bose and Boo model**



Observation Model

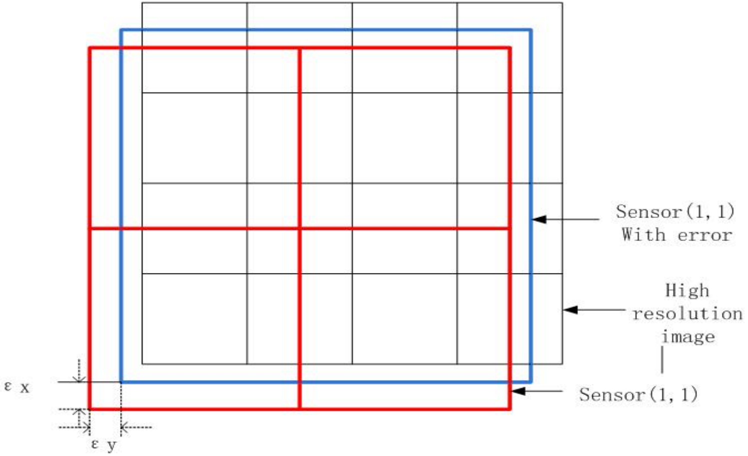
$$\begin{bmatrix} g_{11} & g_{1\hat{1}} & g_{12} & g_{1\hat{2}} \\ g_{\hat{1}1} & g_{\hat{1}\hat{1}} & g_{\hat{1}2} & g_{\hat{1}\hat{2}} \\ g_{21} & g_{2\hat{1}} & g_{22} & g_{2\hat{2}} \\ g_{\hat{2}1} & g_{\hat{2}\hat{1}} & g_{\hat{2}2} & g_{\hat{2}\hat{2}} \end{bmatrix} \qquad \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix}$$

$\mathbf{G}$ $\qquad$ $\mathbf{U}$

$$\vec{\mathrm{G}} = \mathrm{A}\vec{\mathrm{U}}$$

Observation Model

- **Sensor movement**



displacement error

$$G = (A_{\mathsf{estimated}} + A_{\mathsf{displacement}})U + \eta$$

Outline

Relationship of sub-differential and proximity

$$y \in \partial f(x) \Leftrightarrow x = \mathrm{prox}_f(x+y)$$

Characterization

Theorem

u is a solution of problem (2.1), if and only if their exists a vector $v \in \mathbb{R}^m$, such that the pair $(v, u) \in \mathbb{R}^m \times \mathbb{R}^n$ satisfies the coupled fixed point equations:

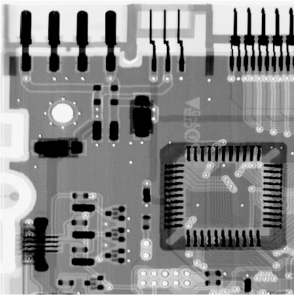
$$v = (I - \mathrm{prox}_{\lambda p_2})(Bu + v)$$

$$u = u - S(A^T \nabla p_1(Au - g) + \frac{1}{\lambda} B^T v)$$

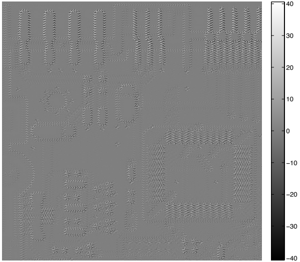
Picard Iteration

$$\begin{cases} v_{k+1} = (I - \mathrm{Prox}_{\lambda\| \ \|_1})(v_k + Bu_k), \\ u_{k+1} = u_k - S(A^T \nabla p_1(Au_k - g) + \frac{1}{\lambda} B^T(2v_{k+1} - v_k)). \end{cases} \tag{3.1}$$

Motivation



(a)



(b)

Figure: Original image and displacement error with $\varepsilon_{\max} = 0.99$

Operators

- Moreau envelope

$$\mathrm{env}_{\tau f}(x) := \min_{u \in \mathbb{R}^n} \{f(u) + \frac{1}{2\tau}\|u - x\|_2^2, x \in \mathbb{R}^n\}$$

- Differentiable

$$\nabla \mathrm{env}_{\tau f}(x) = \frac{(I - \mathrm{prox}_{\tau f})(x)}{\tau}$$

with a $1/\beta$ Lipschitz continuous gradient for some $\beta > 0$.

Operators

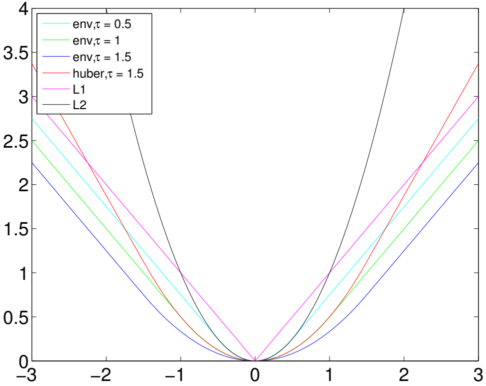


Figure: The figure of L1 norm, L2 nom and Moreau envelop

Optimization model

- New model: $\mathrm{env}_{\ell^1}/\mathrm{TV}$ model

$$f^* = \operatorname*{argmin}_{f\in\mathbb{R}^n}\{\mu\|f\|_{\mathrm{TV}} + \mathrm{env}_{\tau\|\ \|_1}(Af-g), x\in\mathbb{R}^n\}$$

Theorem

The system of the fixed-point equations is equivalent to

$$V = \mathrm{prox}_{\Phi}(EV + G(V))$$

- $V := [v, u]$
- $p_3 = 0$
- $\Phi(V) := \frac{1}{\lambda} p_2^*(y) + p_3(f)$
- $\mathrm{prox}_{\Phi}(V) := (\mathrm{prox}_{\frac{1}{\lambda}p_2^*}(v), \mathrm{prox}_{p_3}(u))$
- $E = \begin{bmatrix} I & B \\ -\lambda B^T & I \end{bmatrix}, G(V) = \begin{bmatrix} 0 \\ -\nabla p_1(u) \end{bmatrix}$

Problem

- prox operator is a firmly non-expansive operator,
- However $\|E\|_2 > 1$.

So, the Picard iteration

$$V^{k+1} = \mathrm{prox}_{\Phi}(EV^k + G(V^k))$$

may not convergent.

Splitting

Let $\quad v, w \in \mathbb{R}^{n+m},\ Q_M : v \to w,$

$$w = \operatorname{prox}_\Phi((E - R^{-1}M)w + R^{-1}Mv + G(v))$$

- $R := \begin{bmatrix} I & 0 \\ 0 & \lambda I \end{bmatrix}$

Splitting

Let $\quad v, w \in \mathbb{R}^{n+m},\ Q_M : v \to w,$

$$w = \text{prox}_\Phi((E - R^{-1}M)w + R^{-1}Mv + G(v))$$

- $R := \begin{bmatrix} I & 0 \\ 0 & \lambda I \end{bmatrix}$

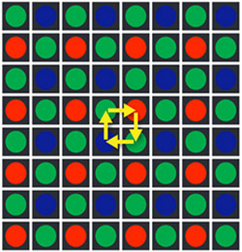
Algorithm

Given v^1, the vector v^k for $k > 1$ is calculated as

$$v^{k+1} = \text{prox}_\Phi((E - R^{-1}M)v^{k+1} + R^{-1}Mv^k + G(v^k))$$

Outline

Color Image



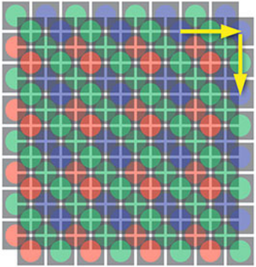


Figure: Bose and Boo model for color image

Thank You!