Big Data Optimization at SAS

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Outline

About SAS

The company

- Leader in business analytics software and services
- About $3 billion worldwide revenue
- Largest private software company in the world
- World's Best Multinational Workplace in 2012
- More than 11,000 employees, 400 offices and 600 alliances
- SAS customers or their affiliates represent over 90% of the top 100 FORTUNE 500 companies

The software

- Originally created for basic statistics by professors at NCSU
- Extended tremendously over the decades
- Covers all aspects of analytics and business intelligence

SAS/OR Offerings

Optimization modelling and solvers

Algebraic Modelling Language with all the usual solvers
(LP, QP, MILP, NLP, CP, Scheduling, Decomposition, . . .)

Other tools

Graph and network algorithms
Discrete event simulation + the rest of SAS

Solutions

Marketing Optimization, Service Parts Optimization,
Revenue Optimization, Size Optimization, . . .

Services

Technical Support, Training, Professional Services, Consulting

Platforms

Windows, Linux, Solaris x64/SPARC, HP–UX, AIX, z/OS

What is Big Data?

Nathan Brixius (in a recent blog post)

A big data analytics application is simply an analytics application where

- the required data does not fit on a single machine and
- needs to be considered in full to produce a result.

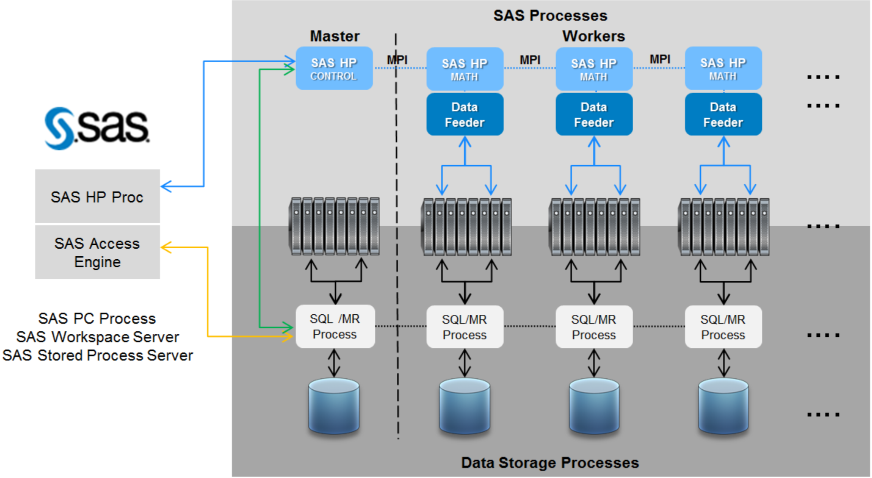
SAS

Big data is relative; it applies whenever an organization's need to handle, store and analyze data exceeds its current capacity.

Related concepts/tools

- large-scale optimization
- distributed optimization
- parallel optimization

High Performance Analytics at SAS



Parallel Implementations and Determinism

Non-determinism

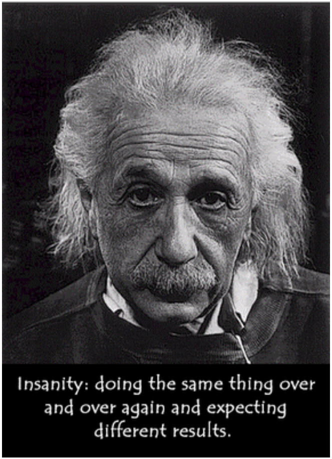
a ***dirty word*** in the commercial world

Sources of non-determinism

- adding columns in a different order
- aggregating results in a different order
- arbitrary random number seeds
- different machines in the pool
- time limits

Workarounds

- operations in a fixed order
- deterministic criteria (nodes, iterations)
- *deterministic ticks* (see Xpress/Cplex)



Determinism comes with a performance penalty.

Big Data Optimization at SAS

Quadratic programming

Support vector machine

Linear programming

Quantile regression

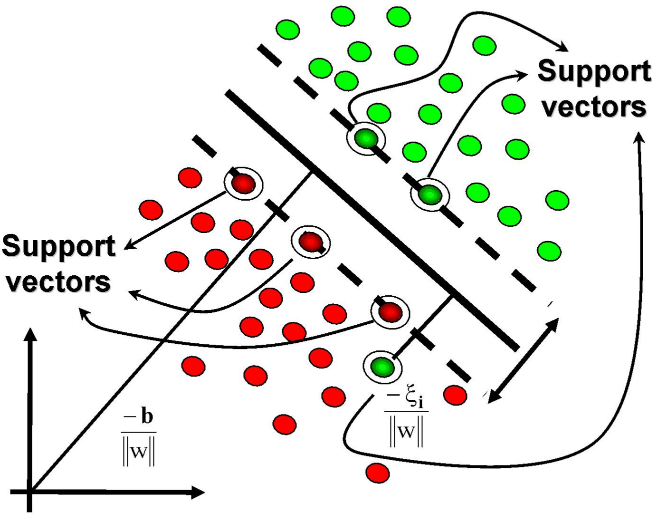
Mixed-integer linear programming

Marketing optimization

Derivative-free optimization

Local search optimization

Support Vector Machines



http://www.cac.science.ru.nl/people/ustun/SVM.JPG

Linear SVM Problem

Primal formulation

$$\begin{aligned} \underset{w,z,\beta}{\text{minimize}} \quad & \frac{1}{2}\|w\|_2^2 + \tau e^T z \\ \text{subject to} \quad & Yw - \beta d \geq e - z \\ & z \geq 0. \end{aligned}$$

Dual formulation

$$\begin{aligned} \underset{v}{\text{minimize}} \quad & -e^T v + \frac{1}{2} v^T Y Y^T v \\ \text{subject to} \quad & d^T v = 0, \\ & 0 \leq v \leq \tau e. \end{aligned}$$

Using Primal-Dual Interior-Point Approach

Dominant cost per iteration is forming/solving Newton system

Many more observations than columns/features

$$(I + Y^T \Omega_k^{-1} Y - v_k v_k^T) \Delta w = -r_w$$

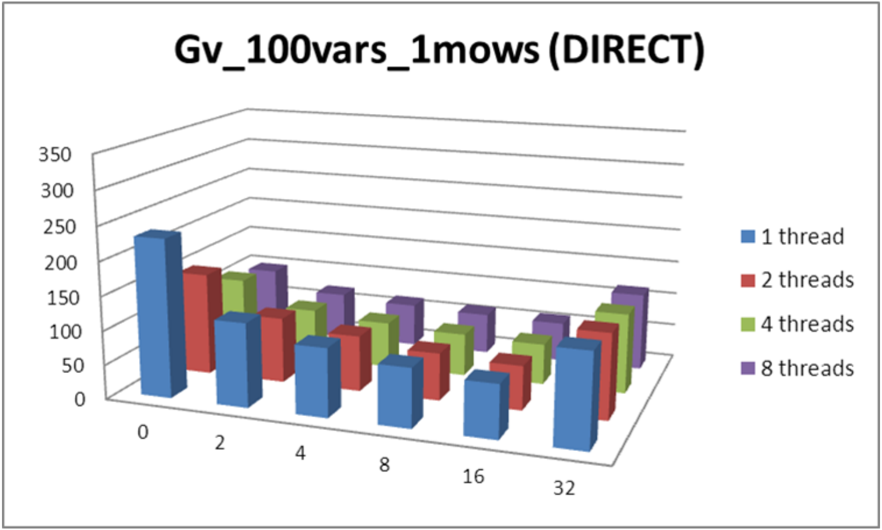
$Y^T \Omega_k^{-1} Y$ must be formed every iteration

Many more columns/features than observations

$$\begin{pmatrix} YY^T + \Omega_k & d \\ d^T & 0 \end{pmatrix} \begin{pmatrix} \Delta v \\ -\Delta \beta \end{pmatrix} = - \begin{pmatrix} \rho_\beta \\ r_\Omega \end{pmatrix}$$

YY^T constant for all iterations

SVM: Parallel Performance



SVM: Features and Plans

Features

- frequency/weight term
- iterative (PCG) or direct (Cholesky, threaded) method to solve the Newton system
- balance threads to avoid cache misses
- balance number of compute nodes to limit communication

In progress

- nonlinear SVM
- build a distributed QP solver

Available soon in SAS/OR

Quantile Regression

Goal

Approximate the median or some other quantile of the response variable of a number of observations

$$\begin{aligned} \min \ \tau u^+ + (1-\tau)u^- \\ A(\beta^+ - \beta^-) + u^+ + u^- &= b \\ \beta^+, \beta^-, u^+, u^- &\geq 0 \end{aligned}$$

A: observations in rows, fully dense

b: response variable

τ: quantile level

Problem size

Up to 10^8 observations each of dimension 10^4

Quantile Regression

Features

- distributed IPM, similar to the SVM case
- Newton system solved directly or iteratively
- different preconditioner

Plans

- categorical variables – sparse observations
- nonlinear quantile regression
- build a distributed dense LP/IPM solver

Available soon in SAS/OR

Marketing Optimization

Problem

Assign ads/offers to customers based on budget, policy, user preferences, history and other kinds of constraints.

Formulation

Typical sizes: millions of customers, hundreds of offers
Formulated as a MILP with millions of binary variables

Solution

Special decomposition
Subproblems are solved distributed on the grid
Subgradient algorithm for the master

Available as a SAS solution.

Marketing Optimization

Typical data (telecommunications)

- 15 million customers
- 910 communications
- 14 aggregate constraints
- 19 rolling contact policies (per day, per week, per month)
- 90 million offers in the contact history

Performance

- Used to take 10 hours on a single machine with regular MO
- Solved in 2 minutes on an EMC Greenplum appliance (32 nodes, 24 threads, 48GB ram)
- Allows for scenario analysis

Local Search Optimization

Algorithm

- GA-guided pattern search
- Continuous, discrete and categorical variables
- Up to about 100 variables

Implementation

- Classical: each worker evaluates the function at a given point
- Big data: each worker computes part of the function value from its own data, then these are aggregated
- Function value cache
- Leading to simulation-based optimization

Available as part of SAS/OR

Parallel Optimization

Not Big Data, but uses the same infrastructure

- Decomposition
- Multistart NLP
- Option tuner for MILP

Decomposition — Outline

Algorithm

- Dantzig-Wolfe Decomposition embedded in B&B
  - » a specific variant of *column generation*
- Relax A and subproblem B becomes tractable (even separable)
- Find convex combinations of extreme points of subproblems that satisfy the continuous relaxation of the master constraints
- Iterate between master A (reformulated space) and B

Blocks

From user, network or auto

Available in SAS/OR

$$\left(\begin{array}{cccc} A_1 & A_2 & \cdots & A_{|K|} \\ \hline B_1 & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_{|K|} \end{array}\right)$$

Decomposition — Parallel Implementation

Implementation

- Shared (Threaded) and/or Distributed Memory (Gridded)
- Subproblems use a standard queue

Areas of parallelism

- Branch & Bound
- ✓ Heuristics (non-blocking price-and-branch)
- ✓ Subproblem solves (across subproblems)
- ✓ Master solve (IPM or concurrent)
- ✓ Subproblem solves (for each subproblem)

Factors affecting parallel performance

- percentage of time in subprob vs master (modeling)
- load balance — aggregate subproblems
- enforce balance with time limits? (non-deterministic)
- MPI overhead — jobs must be *significant*

Other parallel optimization applications

Network tools

- Graph centrality, community detection
- Social network analysis for fraud detection
- Marketing analysis for telecommunications

Multistart NLP (Global optimization)

- Standard NLP solvers started from different points
- Function evalutions and solvers are distributed

Option tuner for MILP

- Find the best option setting for a set of MILP problems
- Continuous, discrete and categorical options are all included

Future Plans

Extend the list of HP enabled procedures

- driven by customers' needs
- distributed LP/QP
- distributed graph algorithms
- parallel MILP
- parallel solves in OPTMODEL
- simulation-based optimization

Thank you for your attention.

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