On 5th Generation of Local Training Methods in Federated Learning

Peter Richtárik

Scientific Computing and Machine Learning Workshop (SCML)

KAUST

November 14-18, 2022

Similar talk given recently @ 2022 Google Workshop on Federated Learning and Analytics (Nov 9-10, 2022)

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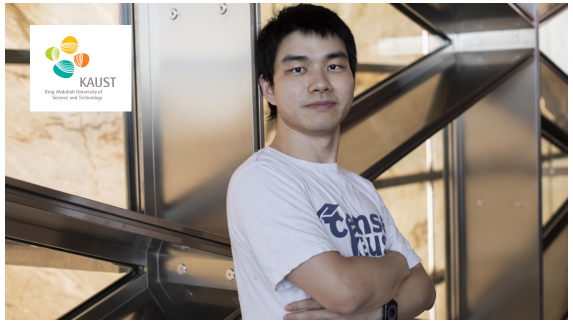
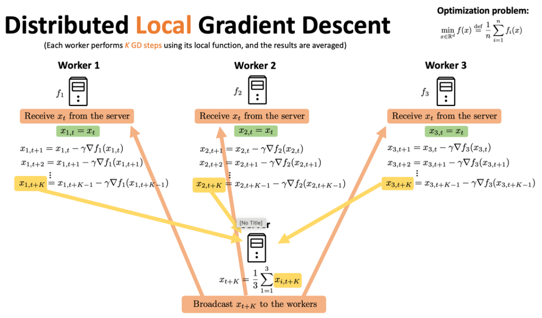


Kai Yi

Outline of the Talk

local updates
new global model
local updates
local data
local data
local data
local data
local data
local data
local data
local data
learnt model: next-word prediction
Subject
Thank you for the feedback tab

1. Local Training
2. Brief History of Local Training
3. 5th Generation of Local Training Methods
4. ProxSkip
5. GradSkip



Distributed Local Gradient Descent

(Each worker performs K GD steps using its local function, and the results are averaged)

Optimization problem:

$$\min_{x\in\mathbb{R}^d} f(x) \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$

Worker 1

f_1

Receive x_t from the server

$x_{1,t} = x_t$

$x_{1,t+1} = x_{1,t} - \gamma\nabla f_1(x_{1,t})$

$x_{1,t+2} = x_{1,t+1} - \gamma\nabla f_1(x_{1,t+1})$

$\vdots$

$x_{1,t+K} = x_{1,t+K-1} - \gamma\nabla f_1(x_{1,t+K-1})$

Worker 2

f_2

Receive x_t from the server

$x_{2,t} = x_t$

$x_{2,t+1} = x_{2,t} - \gamma\nabla f_2(x_{2,t})$

$x_{2,t+2} = x_{2,t+1} - \gamma\nabla f_2(x_{2,t+1})$

$\vdots$

$x_{2,t+K} = x_{2,t+K-1} - \gamma\nabla f_2(x_{2,t+K-1})$

Worker 3

f_3

Receive x_t from the server

$x_{3,t} = x_t$

$x_{3,t+1} = x_{3,t} - \gamma\nabla f_3(x_{3,t})$

$x_{3,t+2} = x_{3,t+1} - \gamma\nabla f_3(x_{3,t+1})$

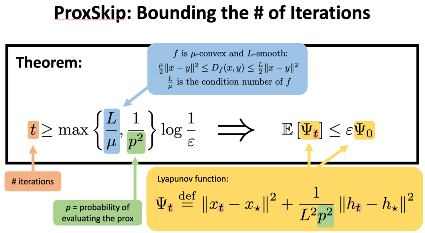
$\vdots$

$x_{3,t+K} = x_{3,t+K-1} - \gamma\nabla f_3(x_{3,t+K-1})$

[No Title] r

$$x_{t+K} = \frac{1}{3}\sum_{1=1}^{3} x_{i,t+K}$$

Broadcast x_{t+K} to the workers



ProxSkip: Bounding the # of Iterations

Theorem:

f is μ-convex and L-smooth:

$\frac{\mu}{2}\|x-y\|^2 \le D_f(x,y) \le \frac{L}{2}\|x-y\|^2$

$\frac{L}{\mu}$ is the condition number of f

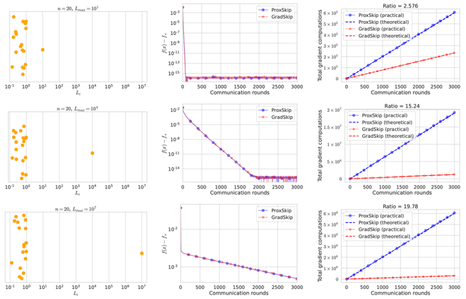
$$t \ge \max\left\{\frac{L}{\mu}, \frac{1}{p^2}\right\}\log\frac{1}{\varepsilon} \quad\Longrightarrow\quad \mathbb{E}\left[\Psi_t\right] \le \varepsilon\Psi_0$$

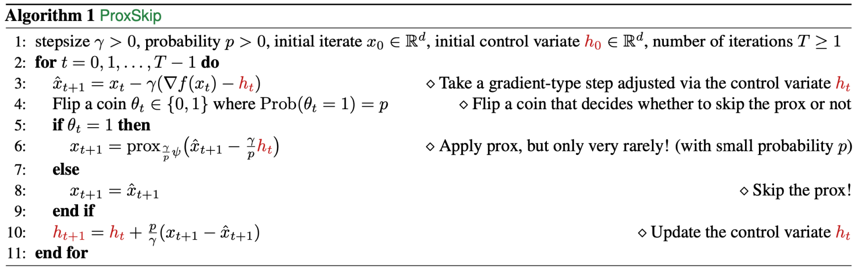
iterations

p = probability of evaluating the prox

Lyapunov function:

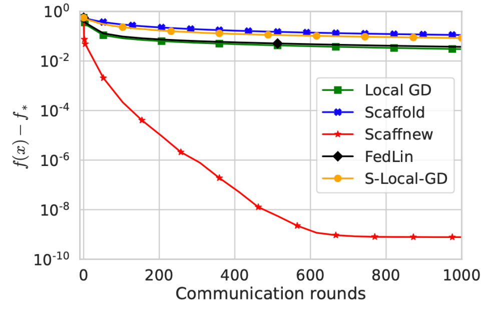
$$\Psi_t \overset{\text{def}}{=} \|x_t - x_\star\|^2 + \frac{1}{L^2p^2}\|h_t - h_\star\|^2$$





Algorithm 1 ProxSkip

```
stepsize $\gamma > 0$, probability $p > 0$, initial iterate $x_0 \in \mathbb{R}^d$, initial control variate $h_0 \in \mathbb{R}^d$, number of iterations $T \ge 1$
for $t = 0, 1, \dots, T-1$ do
    $\hat{x}_{t+1} = x_t - \gamma(\nabla f(x_t) - h_t)$        ⋄ Take a gradient-type step adjusted via the control variate $h_t$
    Flip a coin $\theta_t \in \{0,1\}$ where $\text{Prob}(\theta_t = 1) = p$        ⋄ Flip a coin that decides whether to skip the prox or not
    if $\theta_t = 1$ then
        $x_{t+1} = \text{prox}_{\frac{\gamma}{p}\psi}\left(\hat{x}_{t+1} - \frac{\gamma}{p}h_t\right)$        ⋄ Apply prox, but only very rarely! (with small probability $p$)
    else
        $x_{t+1} = \hat{x}_{t+1}$        ⋄ Skip the prox!
    end if
    $h_{t+1} = h_t + \frac{p}{\gamma}(x_{t+1} - \hat{x}_{t+1})$        ⋄ Update the control variate $h_t$
end for
```



(c) theoretical hyper-parameters

Part 1
Local Training

Optimization Formulation of Federated Learning

$$\min_{x \in \mathbb{R}^d} f(x) \overset{\text{def}}{=} \frac{1}{n} \sum_{i=1}^{n} f_i(x)$$

# devices / machines

# model parameters / features

Loss on local data $\mathcal{D}_i$ stored on device i

$$f_i(x) = \mathbb{E}_{\xi \sim \mathcal{D}_i} f_{i,\xi}(x)$$

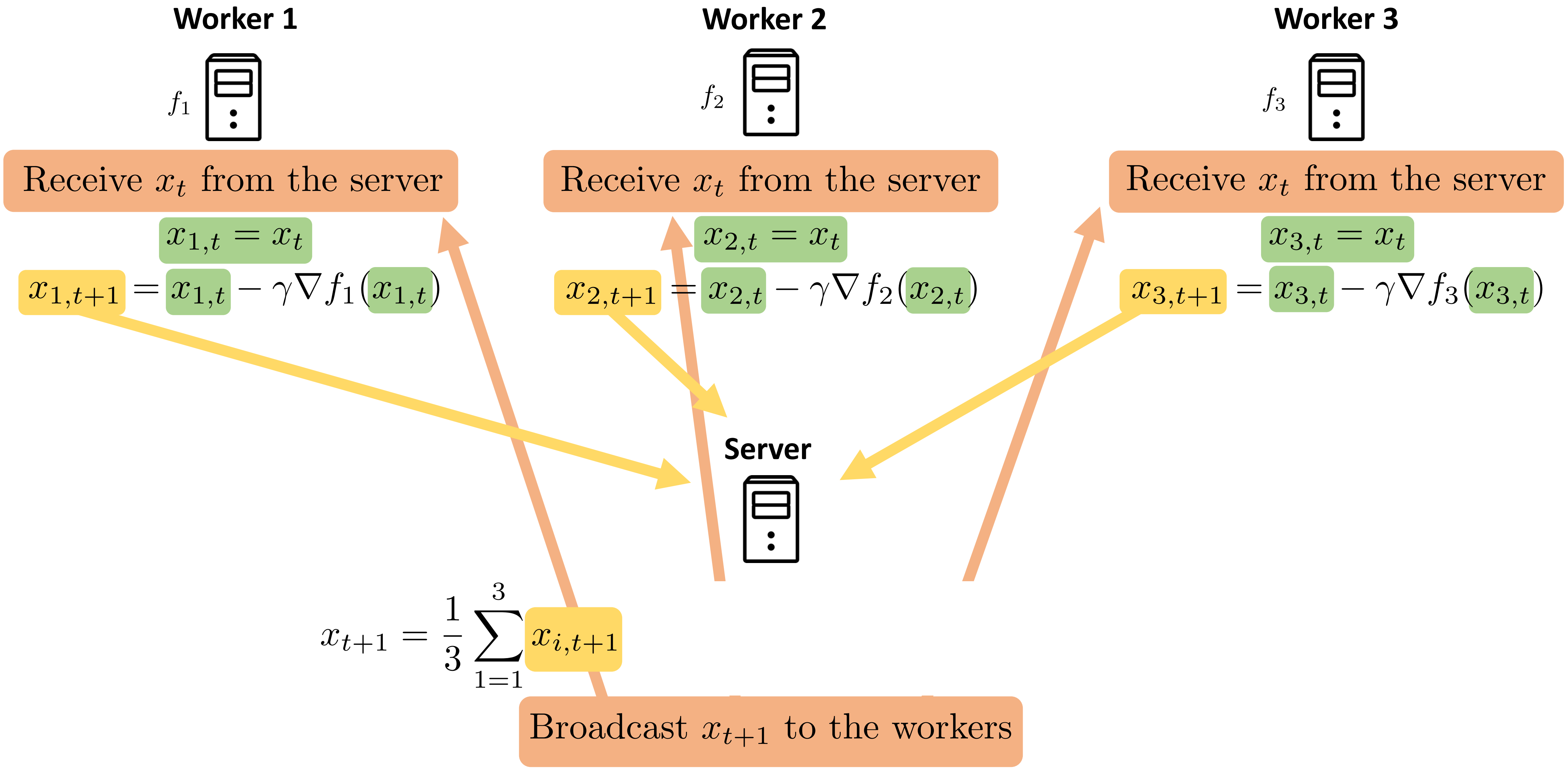
The datasets $\mathcal{D}_1, \ldots, \mathcal{D}_n$ can be arbitrarily heterogeneous

Distributed Gradient Descent

(Each worker performs 1 GD step using its local function, and the results are averaged)

Optimization problem:

$$\min_{x\in\mathbb{R}^d} f(x) \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$



Worker 1

f_1

Receive x_t from the server

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$$x_{1,t+1} = x_{1,t} - \gamma\nabla f_1(x_{1,t})$$

Worker 2

f_2

Receive x_t from the server

$$x_{2,t} = x_t$$

$$x_{2,t+1} = x_{2,t} - \gamma\nabla f_2(x_{2,t})$$

Worker 3

f_3

Receive x_t from the server

$$x_{3,t} = x_t$$

$$x_{3,t+1} = x_{3,t} - \gamma\nabla f_3(x_{3,t})$$

Server

$$x_{t+1} = \frac{1}{3}\sum_{1=1}^{3} x_{i,t+1}$$

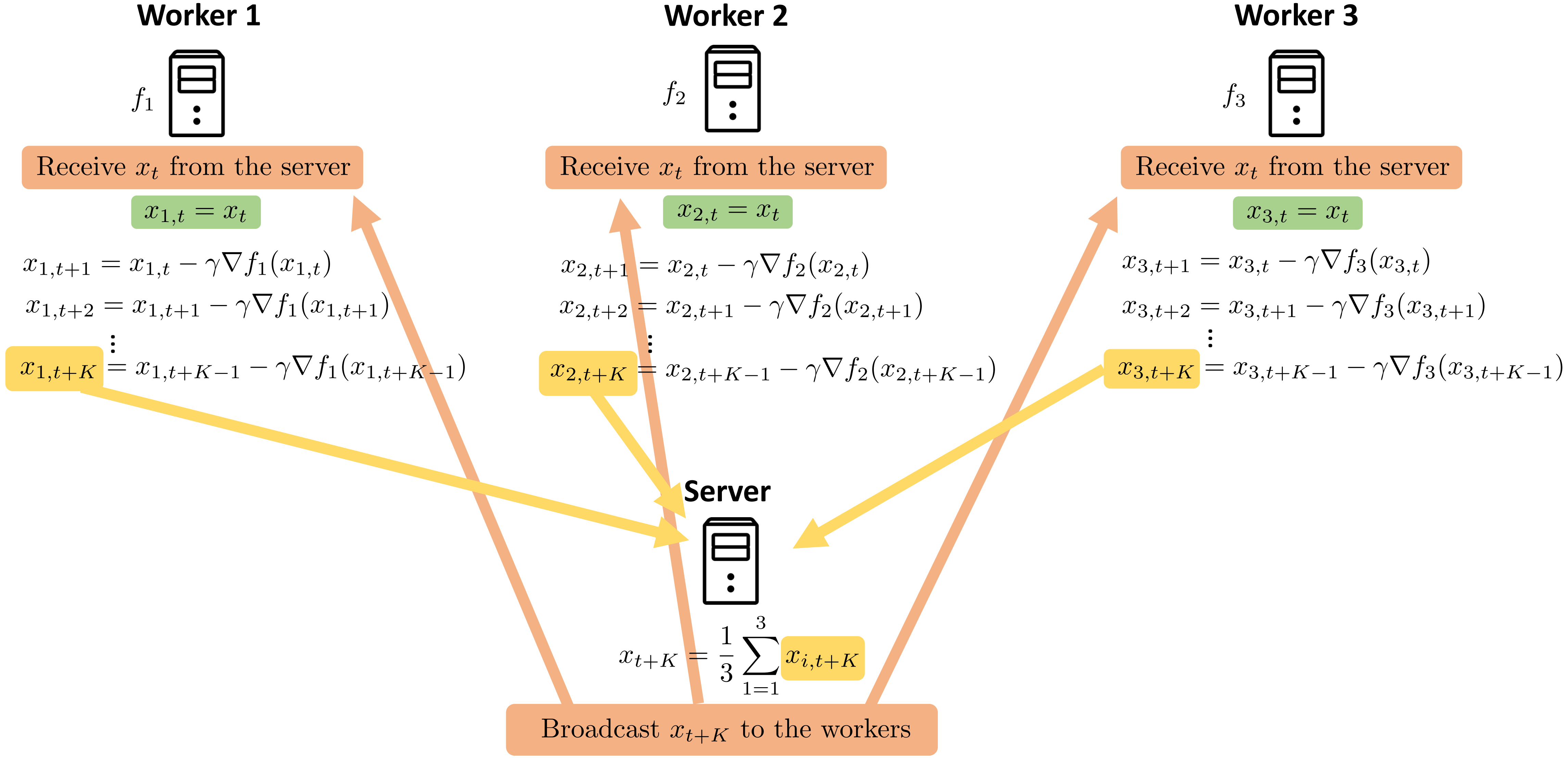
Broadcast x_{t+1} to the workers

Distributed Local Gradient Descent

(Each worker performs *K* GD steps using its local function, and the results are averaged)

Optimization problem:

$$\min_{x\in\mathbb{R}^d} f(x) \overset{\text{def}}{=} \frac{1}{n}\sum_{i=1}^{n} f_i(x)$$



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$\vdots$

$x_{3,t+K} = x_{3,t+K-1} - \gamma\nabla f_3(x_{3,t+K-1})$

Server

$$x_{t+K} = \frac{1}{3}\sum_{1=1}^{3} x_{i,t+K}$$

Broadcast x_{t+K} to the workers

Part 2
Brief History of Local Training

Grigory Malinovsky, Kai Yi and P.R.
Variance reduced ProxSkip: algorithm, theory and application to federated learning
NeurIPS 2022

Brief History of Local Training Methods

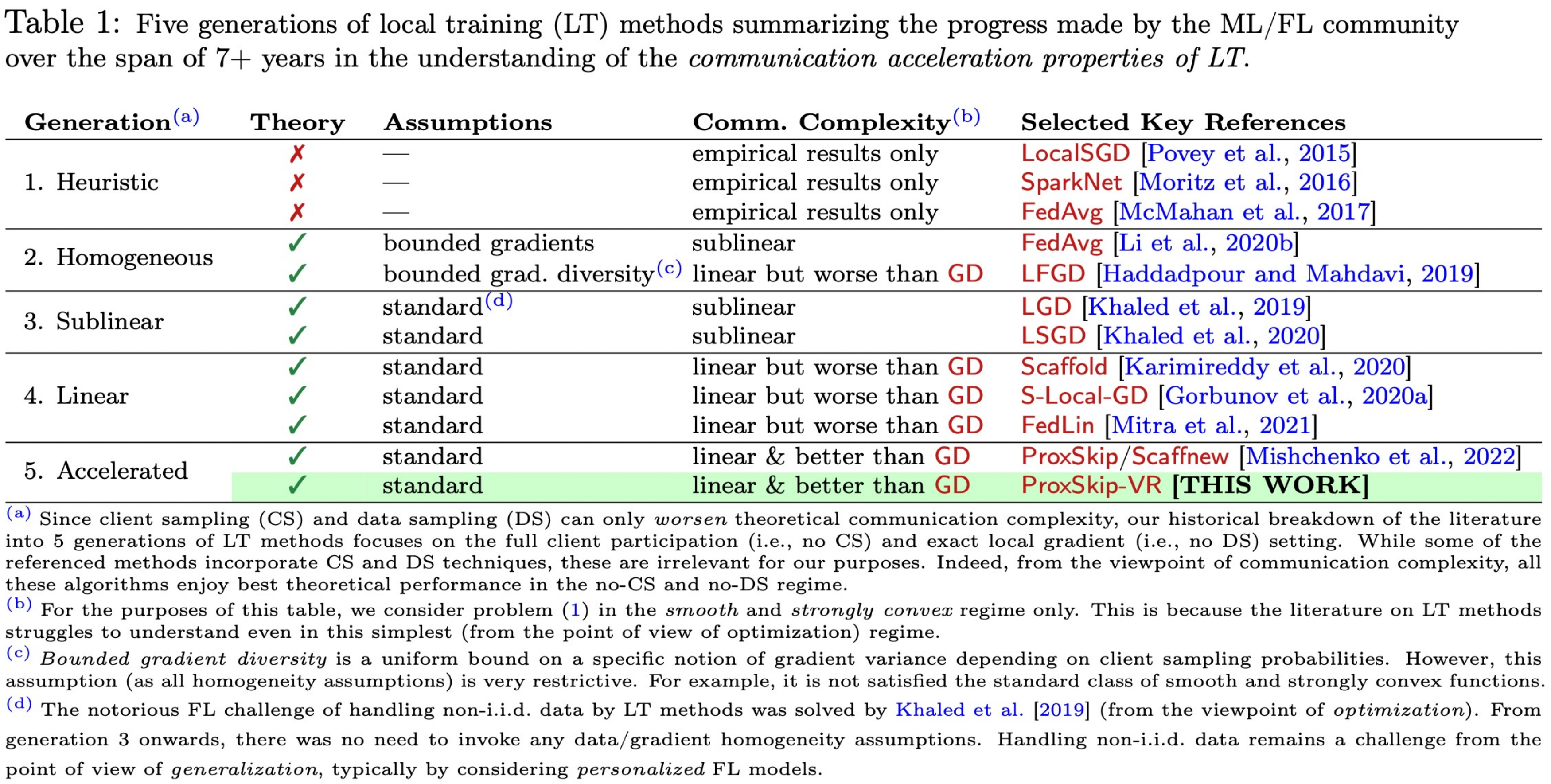


Table 1: Five generations of local training (LT) methods summarizing the progress made by the ML/FL community over the span of 7+ years in the understanding of the *communication acceleration properties of LT*.

Generation[(a)]	Theory	Assumptions	Comm. Complexity[(b)]	Selected Key References
1. Heuristic	✗	—	empirical results only	LocalSGD [Povey et al., 2015]
	✗	—	empirical results only	SparkNet [Moritz et al., 2016]
	✗	—	empirical results only	FedAvg [McMahan et al., 2017]
2. Homogeneous	✓	bounded gradients	sublinear	FedAvg [Li et al., 2020b]
	✓	bounded grad. diversity[(c)]	linear but worse than GD	LFGD [Haddadpour and Mahdavi, 2019]
3. Sublinear	✓	standard[(d)]	sublinear	LGD [Khaled et al., 2019]
	✓	standard	sublinear	LSGD [Khaled et al., 2020]
4. Linear	✓	standard	linear but worse than GD	Scaffold [Karimireddy et al., 2020]
	✓	standard	linear but worse than GD	S-Local-GD [Gorbunov et al., 2020a]
	✓	standard	linear but worse than GD	FedLin [Mitra et al., 2021]
5. Accelerated	✓	standard	linear & better than GD	ProxSkip/Scaffnew [Mishchenko et al., 2022]
	✓	standard	linear & better than GD	ProxSkip-VR **[THIS WORK]**

[(a)] Since client sampling (CS) and data sampling (DS) can only *worsen* theoretical communication complexity, our historical breakdown of the literature into 5 generations of LT methods focuses on the full client participation (i.e., no CS) and exact local gradient (i.e., no DS) setting. While some of the referenced methods incorporate CS and DS techniques, these are irrelevant for our purposes. Indeed, from the viewpoint of communication complexity, all these algorithms enjoy best theoretical performance in the no-CS and no-DS regime.

[(b)] For the purposes of this table, we consider problem (1) in the *smooth* and *strongly convex* regime only. This is because the literature on LT methods struggles to understand even in this simplest (from the point of view of optimization) regime.

[(c)] *Bounded gradient diversity* is a uniform bound on a specific notion of gradient variance depending on client sampling probabilities. However, this assumption (as all homogeneity assumptions) is very restrictive. For example, it is not satisfied the standard class of smooth and strongly convex functions.

[(d)] The notorious FL challenge of handling non-i.i.d. data by LT methods was solved by Khaled et al. [2019] (from the viewpoint of *optimization*). From generation 3 onwards, there was no need to invoke any data/gradient homogeneity assumptions. Handling non-i.i.d. data remains a challenge from the point of view of *generalization*, typically by considering *personalized* FL models.

Grigory Malinovsky, Kai Yi and P.R.

Variance Reduced ProxSkip: Algorithm, Theory and Application to Federated Learning

NeurIPS 2022

Brief History of Local Training Methods

Generation 1: Heuristic

“No theory”

10/2014

PDF

Daniel Povey, Xiaohui Zhang, and Sanjeev Khudanpur
Parallel Training of DNNs with Natural Gradient and Parameter Averaging
ICLR Workshops 2015

11/2015

PDF

Philipp Moritz, Robert Nishihara, Ion Stoica, Michael I. Jordan
SparkNet: Training Deep Networks in Spark
ICLR 2015

02/2016

PDF

H. Brendan_McMahan, Eider Moore, Daniel Ramage, Seth Hampson, Blaise Agüera y Arcas
Communication-Efficient Learning of Deep Networks from Decentralized Data
AISTATS 2017

Accepted as a workshop contribution at ICLR 2015

PARALLEL TRAINING OF DNNS WITH NATURAL GRADIENT AND PARAMETER AVERAGING

Daniel Povey, Xiaohui Zhang & Sanjeev Khudanpur
Center for Language and Speech Processing & Human Language Technology Center of Excellence,
The Johns Hopkins University, Baltimore, MD 21218, USA
{dpovey@gmail.com}, {xiaohui,khudanpur@jhu.edu}

arXiv:1410.7455v8 [cs.NE] 22 Jun 2015

ABSTRACT

We describe the neural-network training framework used in the Kaldi speech recognition toolkit, which is geared towards training DNNs with large amounts of training data using multiple GPU-equipped or multi-core machines. In order to be as hardware-agnostic as possible, we needed a way to use multiple machines without generating excessive network traffic. Our method is to average the neural network parameters periodically (typically every minute or two), and redistribute the averaged parameters to the machines for further training. Each machine sees different data. By itself, this method does not work very well. However, we have another method, an approximate and efficient implementation of Natural Gradient for Stochastic Gradient Descent (NG-SGD), which seems to allow our periodic-averaging method to work well, as well as substantially improving the convergence of SGD on a single machine.

1 INTRODUCTION

Parallel training of neural networks generally makes use of some combination of model parallelism and data parallelism (Dean et al., 2012), and the normal approach to data parallelism involves communication of model parameters for each minibatch. Here we describe our neural-net training framework which uses a different version of data parallelism: we have multiple SGD processes on separate machines, and only infrequently (every minute or so) average the model parameters and redistribute them to the individual machines. This is very effective for us for large-scale training of systems for speech recognition– but in our case it only works well when combined with an efficient implementation of natural gradient stochastic gradient descent (NG-SGD) that we have developed. We don't attempt in this paper to develop a framework that explains why parameter averaging should work well despite non-convexity of DNNs, or why NG-SGD is so helpful. The point of this paper is to describe our methods and to establish that empirically they work well. The significance of this work is that we show that it is possible to get a linear speedup when increasing the number of GPUs, without requiring frequent data transfer (however, this only holds up to about 4 or 8 GPUs).

In Section 2 we describe our problem setting, which is Deep Neural Networks (DNNs) applied to speech recognition– although our ideas are more general than this. In Section 3 we introduce the parallel training method. In Section 4 we describe the general ideas behind our natural gradient method, although most of the technical details have been relegated to appendices. In this paper we don't give any proofs, but we do discuss in Section 5 what we think we can and can't be proven about our methods. Section 6 has experiments on the convergence of SGD with and without natural gradient and parallelism. We conclude in Section 7.

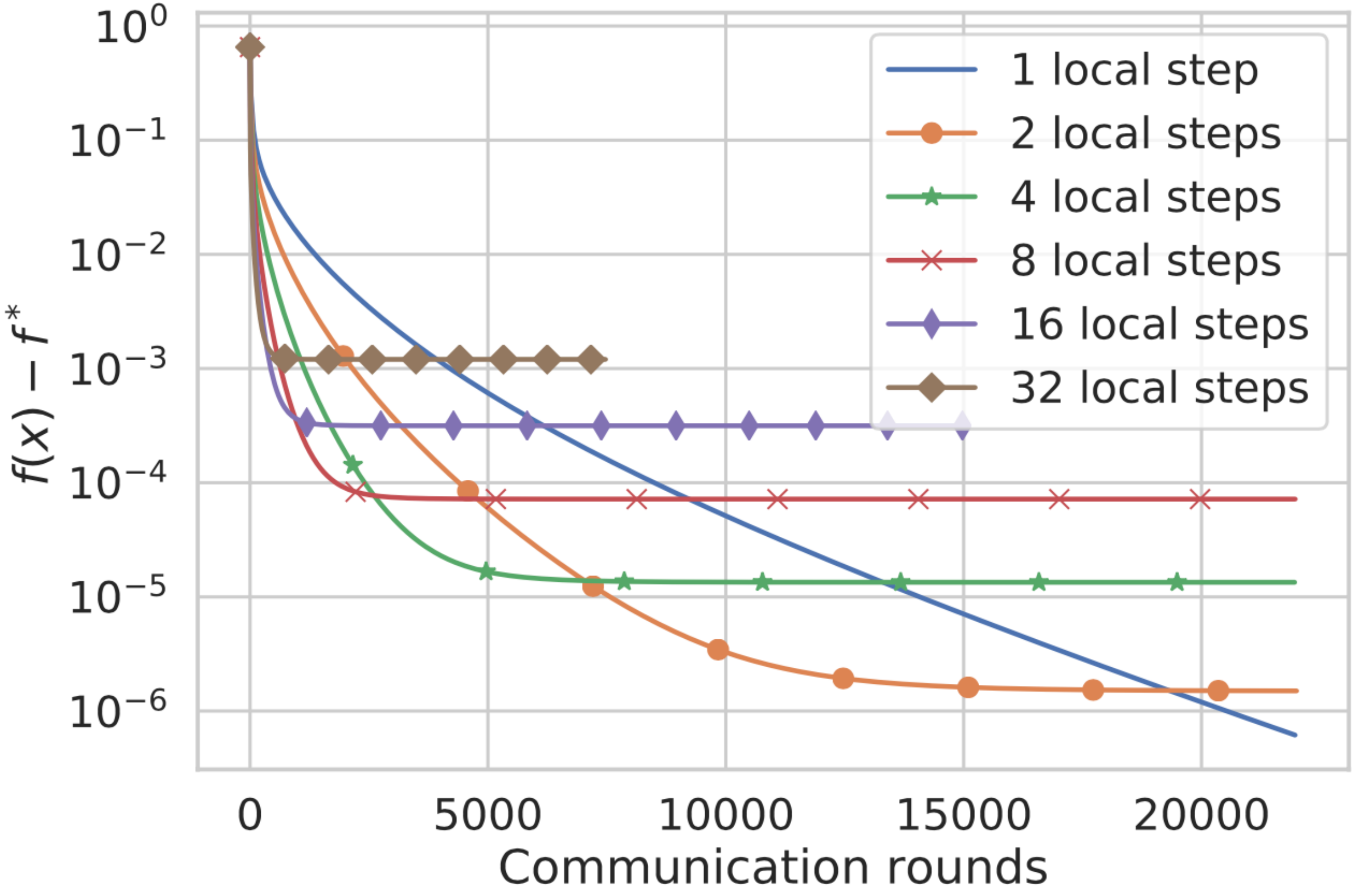
There are two versions of our NG-SGD method: a “simple” version and an “online” one. Technical details for these are in Appendices A and B respectively. Appendix C has background information on our DNN implementation.

2 PROBLEM SETTING

When training DNNs for speech recognition, the immediate problem is that of classifying vectors $\mathbf{x} \in \mathbb{R}^D$ as corresponding to discrete labels $y \in \mathcal{Y}$. The dimension D is typically several hundred,

1

Brief History of Local Training Methods
Generation 3: Heuristic



L2-regularized logistic regression
LibSVM `mushrooms` dataset

Brief History of Local Training Methods
Generation 2: Homogeneous

"Theory requires data to be similar/homogeneous across the clients"

07/2019

Xiang Li, Kaixuan Huang, Wenhao Yang, Shusen Wang and Zhihua Zhang
On the Convergence of FedAvg on Non-IID Data
ICLR 2020

Bounded gradients:

$$\|\nabla f_i(x)\| \leq B \quad \forall x \in \mathbb{R}^d \quad \forall i \in \{1, 2, \ldots, n\}$$

10/2019

Farzin Haddadpour and Mehrdad Mahdavi
On the Convergence of Local Descent Methods in Federated Learning
arXiv:1910.14425, 2019

Bounded gradient diversity (aka strong growth):

$$\frac{1}{n}\sum_{i=1}^{n} \|\nabla f_i(x)\|^2 \leq C \|\nabla f(x)\|^2 \quad \forall x \in \mathbb{R}^d$$

Brief History of Local Training Methods
Generation 3: Sublinear

"Heterogeneous data is allowed, but the rate is worse than GD"

10/2019 PDF

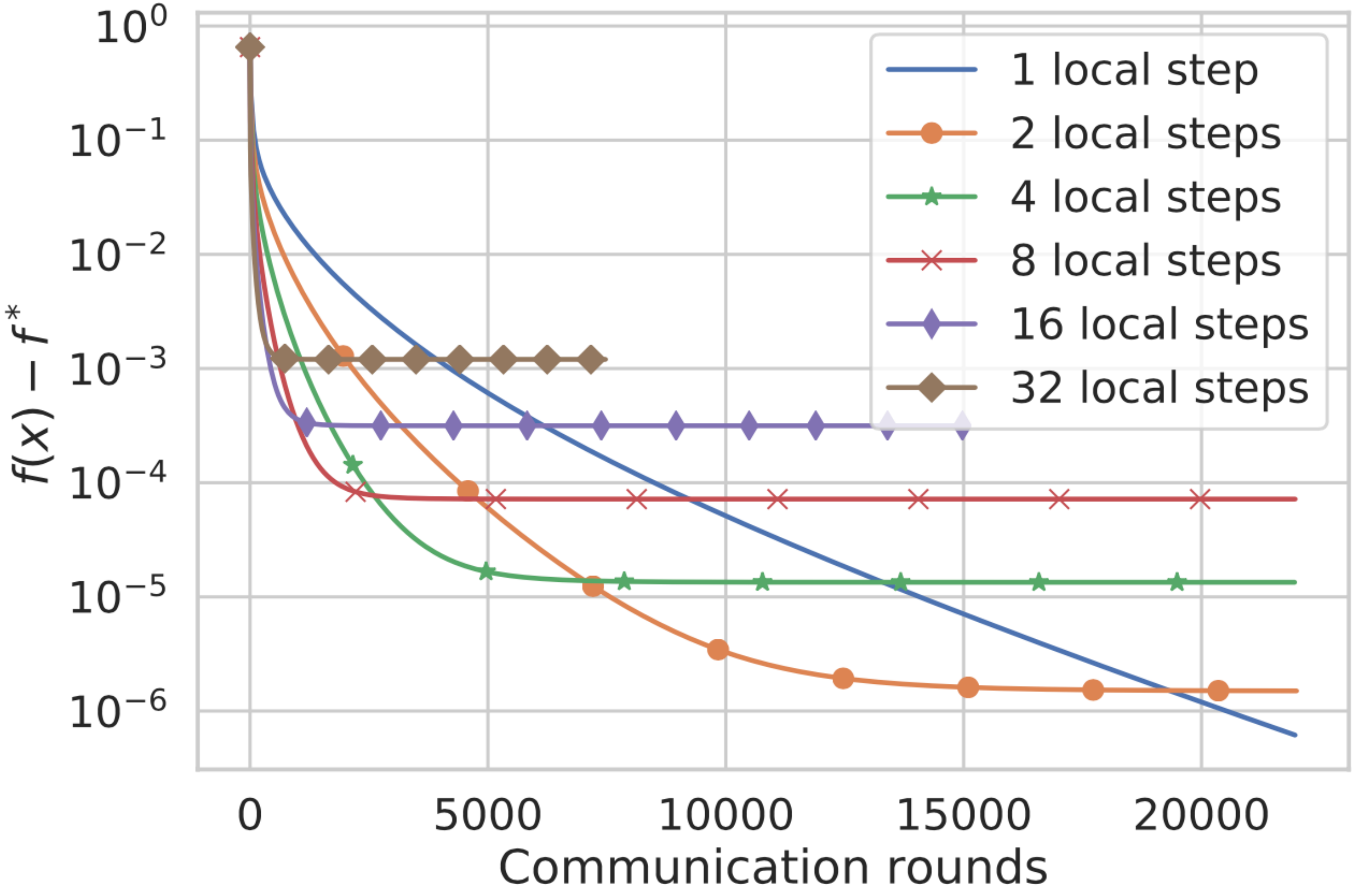
Ahmed Khaled, Konstantin Mishchenko and P.R.
First Analysis of Local GD on Heterogeneous Data
NeurIPS 2019 Workshop on Federated Learning for Data Privacy and Confidentiality, 2019

10/2019 PDF

Ahmed Khaled, Konstantin Mishchenko and P.R.
Tighter Theory for Local SGD on Identical and Heterogeneous Data
AISTATS 2020

Brief History of Local Training Methods

Generation 3: Sublinear



L2-regularized logistic regression
LibSVM `mushrooms` dataset

Brief History of Local Training Methods
Generation 4: Linear

“Heterogeneous data is allowed, but the rate ay best matches that of GD”

10/2019

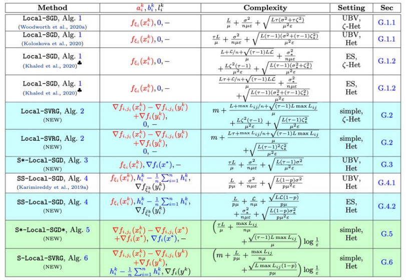
Scaffold

Sai P. Karimireddy, S. Kale, M. Mohri, S. J. Reddi, S. U. Stich, A. T. Suresh
SCAFFOLD: Stochastic Controlled Averaging for Federated Learning
ICML 2020

11/2020

S-Local-GD, Local-GD*
S-Local-SVRG

Eduard Gorbunov, Filip Hanzely and P.R.
Local SGD: Unified Theory and New Efficient Methods
AISTATS 2021



Method	a_i^k, b_i^k, l_i^k	Complexity	Setting	Sec
Local-SGD, Alg. 1 (Woodworth et al., 2020a)	$f_{\xi_i}(x_i^k), 0, -$	$\frac{L}{\mu} + \frac{\sigma^2}{n\mu\varepsilon} + \sqrt{\frac{L\tau(\sigma^2+\tau\zeta^2)}{\mu^2\varepsilon}}$	UBV, ζ-Het	G.1.1
Local-SGD, Alg. 1 (Koloskova et al., 2020)	$f_{\xi_i}(x_i^k), 0, -$	$\frac{\tau L}{\mu} + \frac{\sigma^2}{n\mu\varepsilon} + \sqrt{\frac{L(\tau-1)(\sigma^2+(\tau-1)\zeta_*^2)}{\mu^2\varepsilon}}$	UBV, Het	G.1.1
Local-SGD, Alg. 1 (Khaled et al., 2020)♣	$f_{\xi_i}(x_i^k), 0, -$	$\frac{L+\mathcal{L}/n+\sqrt{(\tau-1)L\mathcal{L}}}{\mu} + \frac{\sigma_*^2}{n\mu\varepsilon}$ $+ \frac{L\zeta^2(\tau-1)}{\mu^2\varepsilon} + \sqrt{\frac{L(\tau-1)(\sigma_*^2+\zeta_*^2)}{\mu^2\varepsilon}}$	ES, ζ-Het	G.1.2
Local-SGD, Alg. 1 (Khaled et al., 2020)♣	$f_{\xi_i}(x_i^k), 0, -$	$\frac{L\tau+\mathcal{L}/n+\sqrt{(\tau-1)L\mathcal{L}}}{\mu} + \frac{\sigma_*^2}{n\mu\varepsilon}$ $+\sqrt{\frac{L(\tau-1)(\sigma_*^2+(\tau-1)\zeta_*^2)}{\mu^2\varepsilon}}$	ES, Het	G.1.2
Local-SVRG, Alg. 2 (NEW)	$\nabla f_{i,j_i}(x_i^k) - \nabla f_{i,j_i}(y_i^k)$ $+\nabla f_i(y_i^k)$, $0, -$	$m + \frac{L+\max L_{ij}/n+\sqrt{(\tau-1)L\max L_{ij}}}{\mu}$ $+ \frac{L\zeta^2(\tau-1)}{\mu^2\varepsilon} + \sqrt{\frac{L(\tau-1)\zeta_*^2}{\mu^2\varepsilon}}$	simple, ζ-Het	G.2
Local-SVRG, Alg. 2 (NEW)	$\nabla f_{i,j_i}(x_i^k) - \nabla f_{i,j_i}(y_i^k)$ $+\nabla f_i(y_i^k)$, $0, -$	$m + \frac{L\tau+\max L_{ij}/n+\sqrt{(\tau-1)L\max L_{ij}}}{\mu}$ $+\sqrt{\frac{L(\tau-1)^2\zeta_*^2}{\mu^2\varepsilon}}$	simple, Het	G.2
S*-Local-SGD, Alg. 3 (NEW)	$f_{\xi_i}(x_i^k), \nabla f_i(x^*), -$	$\frac{\tau L}{\mu} + \frac{\sigma^2}{n\mu\varepsilon} + \sqrt{\frac{L(\tau-1)\sigma^2}{\mu^2\varepsilon}}$	UBV, Het	G.3
SS-Local-SGD, Alg. 4 (Karimireddy et al., 2019a)	$f_{\xi_i}(x_i^k), h_i^k - \frac{1}{n}\sum_{i=1}^n h_i^k$, $\nabla f_{\tilde{\xi}_i^k}(y_i^k)$	$\frac{L}{p\mu} + \frac{\sigma^2}{n\mu\varepsilon} + \sqrt{\frac{L(1-p)\sigma^2}{p\mu^2\varepsilon}}$	UBV, Het	G.4.1
SS-Local-SGD, Alg. 4 (NEW)	$f_{\xi_i}(x_i^k), h_i^k - \frac{1}{n}\sum_{i=1}^n h_i^k$, $\nabla f_{\tilde{\xi}_i^k}(y_i^k)$	$\frac{L}{p\mu} + \frac{\mathcal{L}}{n\mu} + \frac{\sqrt{L\mathcal{L}(1-p)}}{p\mu}$ $+ \frac{\sigma_*^2}{n\mu\varepsilon} + \sqrt{\frac{L(1-p)\sigma_*^2}{p\mu^2\varepsilon}}$	ES, Het	G.4.2
S*-Local-SGD*, Alg. 5 (NEW)	$\nabla f_{i,j_i}(x_i^k) - \nabla f_{i,j_i}(x^*)$ $+\nabla f_i(x^*), \nabla f_i(x^*), -$	$\left(\frac{\tau L}{\mu} + \frac{\max L_{ij}}{n\mu} + \frac{\sqrt{(\tau-1)L\max L_{ij}}}{\mu}\right)\log\frac{1}{\varepsilon}$	simple, Het	G.5
S-Local-SVRG, Alg. 6 (NEW)	$\nabla f_{i,j_i}(x_i^k) - \nabla f_{i,j_i}(y^k)$ $+\nabla f_i(y^k)$, $h_i^k - \frac{1}{n}\sum_{i=1}^n h_i^k, \nabla f_i(y^k)$	$\left(m + \frac{L}{p\mu} + \frac{\max L_{ij}}{n\mu} + \frac{\sqrt{L\max L_{ij}(1-p)}}{p\mu}\right)\log\frac{1}{\varepsilon}$	simple, Het	G.6

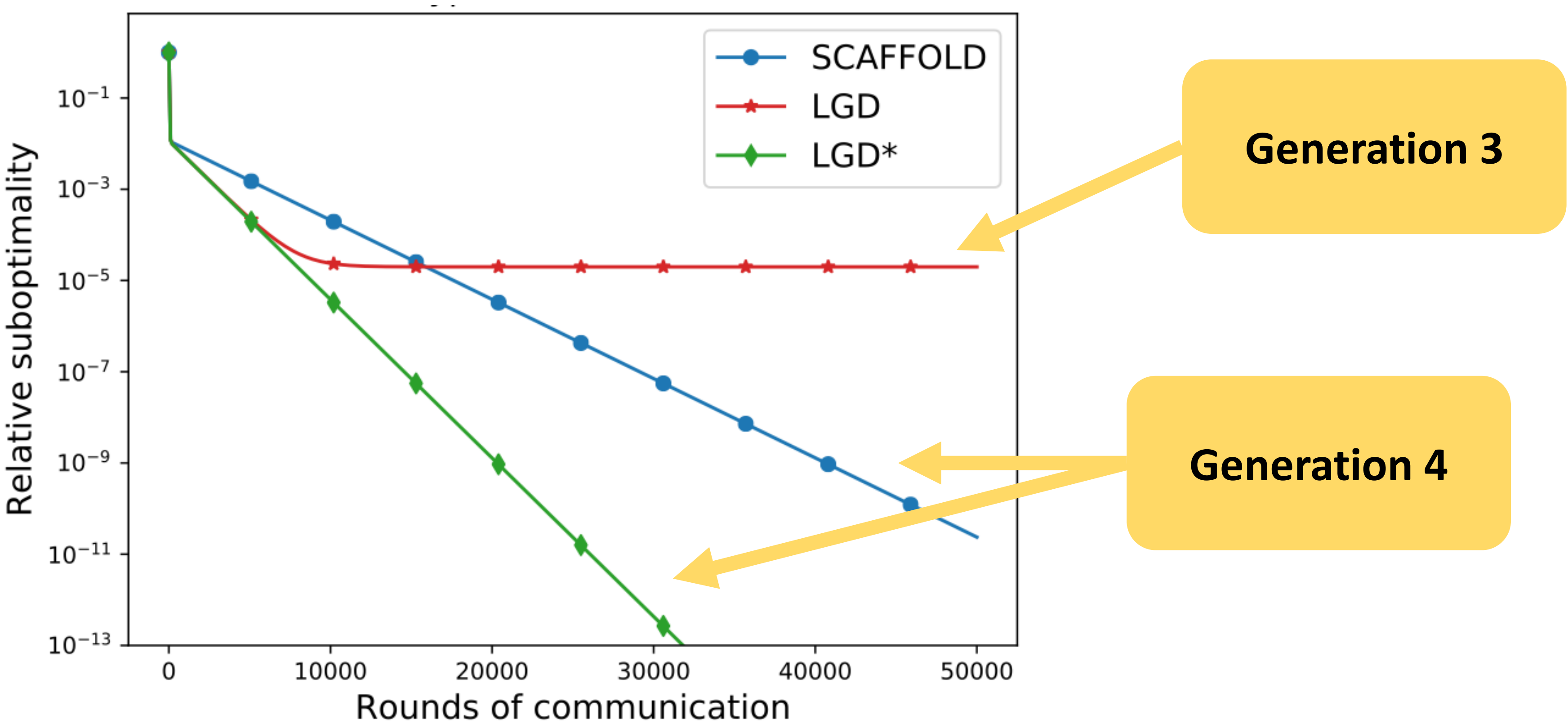
02/2021

FedLin

Aritra Mitra, Rayana Jaafar, George J. Pappas, Hamed Hassani
Linear Convergence in Federated Learning: Tackling Client Heterogeneity & Sparse Gradients
NeurIPS 2021

Brief History of Local Training Methods
Generation 4: Linear

“Heterogeneous data is allowed, but the rate ay best matches that of GD”



Generation 3

Generation 4

Part 3
5th Generation of
Local Training Methods

Brief History of Local Training Methods
Generation 5: Accelerated

“Communication complexity is better than GD for heterogeneous data”

In practice, local training significantly improves communication efficiency.

However, there is no theoretical result explaining this!

Is the situation hopeless, or can we show/prove that local training helps?

Key Property of 5th Generation Local Training Methods

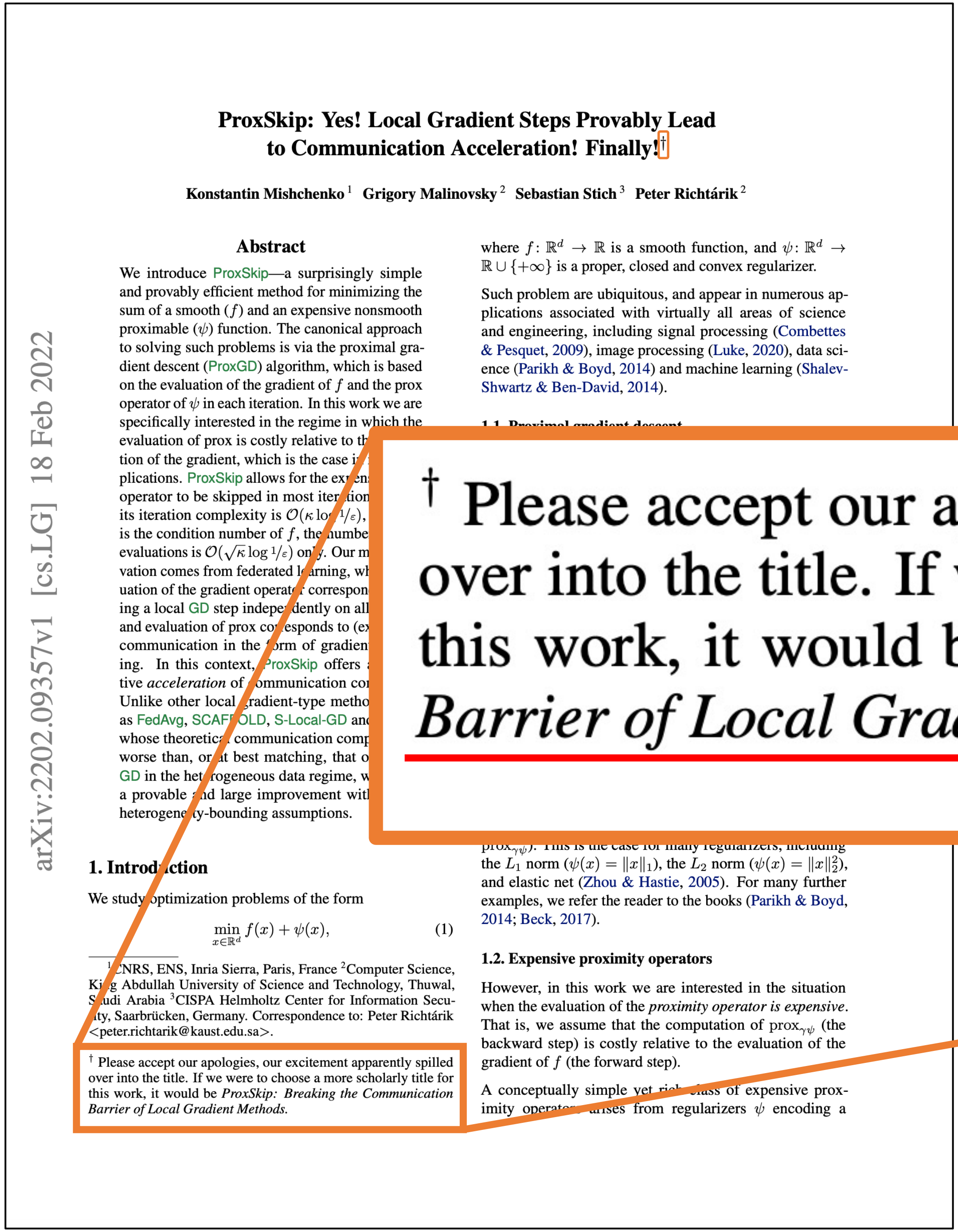
Communication complexity of 4th generation local training methods

$$\mathcal{O}\left(\frac{L}{\mu}\log\frac{1}{\varepsilon}\right)$$

Communication complexity of 5th generation local training methods

$$\mathcal{O}\left(\sqrt{\frac{L}{\mu}}\log\frac{1}{\varepsilon}\right)$$

The Beginning



ProxSkip: Yes! Local Gradient Steps Provably Lead to Communication Acceleration! Finally![†]

Konstantin Mishchenko[1] Grigory Malinovsky[2] Sebastian Stich[3] Peter Richtárik[2]

Abstract

We introduce ProxSkip—a surprisingly simple and provably efficient method for minimizing the sum of a smooth (f) and an expensive nonsmooth proximable (ψ) function. The canonical approach to solving such problems is via the proximal gradient descent (ProxGD) algorithm, which is based on the evaluation of the gradient of f and the prox operator of ψ in each iteration. In this work we are specifically interested in the regime in which the evaluation of prox is costly relative to th
tion of the gradient, which is the case i
plications. ProxSkip allows for the expens
operator to be skipped in most iteration
its iteration complexity is $\mathcal{O}(\kappa \log 1/\varepsilon)$,
is the condition number of f, the numbe
evaluations is $\mathcal{O}(\sqrt{\kappa} \log 1/\varepsilon)$ only. Our m
vation comes from federated learning, wh
uation of the gradient operator correspon
ing a local GD step independently on all
and evaluation of prox corresponds to (ex
communication in the form of gradien
ing. In this context, ProxSkip offers
tive *acceleration* of communication co
Unlike other local gradient-type metho
as FedAvg, SCAFFOLD, S-Local-GD and
whose theoretical communication comp
worse than, or at best matching, that o
GD in the heterogeneous data regime, w
a provable and large improvement wit
heterogeneity-bounding assumptions.

1. Introduction

We study optimization problems of the form

$$\min_{x\in\mathbb{R}^d} f(x)+\psi(x), \qquad (1)$$

where $f\colon \mathbb{R}^d \to \mathbb{R}$ is a smooth function, and $\psi\colon \mathbb{R}^d \to \mathbb{R}\cup\{+\infty\}$ is a proper, closed and convex regularizer.

Such problem are ubiquitous, and appear in numerous applications associated with virtually all areas of science and engineering, including signal processing (Combettes & Pesquet, 2009), image processing (Luke, 2020), data science (Parikh & Boyd, 2014) and machine learning (Shalev-Shwartz & Ben-David, 2014).

1.1. Proximal gradient descent

$\mathrm{prox}_{\gamma\psi}$). This is the case for many regularizers, including the L_1 norm ($\psi(x) = \|x\|_1$), the L_2 norm ($\psi(x) = \|x\|_2^2$), and elastic net (Zhou & Hastie, 2005). For many further examples, we refer the reader to the books (Parikh & Boyd, 2014; Beck, 2017).

1.2. Expensive proximity operators

However, in this work we are interested in the situation when the evaluation of the *proximity operator is expensive*. That is, we assume that the computation of $\mathrm{prox}_{\gamma\psi}$ (the backward step) is costly relative to the evaluation of the gradient of f (the forward step).

A conceptually simple yet rich class of expensive proximity operators arises from regularizers ψ encoding a

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[†] Please accept our apologies, our excitement apparently spilled over into the title. If we were to choose a more scholarly title for this work, it would be *ProxSkip: Breaking the Communication Barrier of Local Gradient Methods.*

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ICML
International Conference
On Machine Learning

PDF

Konstantin Mishchenko, Grigory Malinovsky, Sebastian Stich and P.R.
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ICML 2022

Brief History of Local Training Methods

Generation 5: Accelerated

“Communication complexity is better than GD for heterogeneous data”

02/2022
ProxSkip

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07/2022
APDA; APDA-Inexact

PDF

Abdurakhmon Sadiev, Dmitry Kovalev and P.R.
Communication Acceleration of Local Gradient Methods via an Accelerated Primal-Dual Algorithm with Inexact Prox
NeurIPS 2022

07/2022
ProxSkip-LSVRG

PDF

Grigory Malinovsky, Kai Yi and P.R.
Variance Reduced ProxSkip: Algorithm, Theory and Application to Federated Learning
NeurIPS 2022

07/2022
RandProx

PDF

Laurent Condat and P.R.
RandProx: Primal-Dual Optimization Algorithms with Randomized Proximal Updates
arXiv:2207.12891, 2022

Brief History of Local Training Methods

Generation 5: Accelerated

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10/2022
GradSkip

Artavazd Maranjyan, Mher Safaryan and P.R.
GradSkip: Communication-Accelerated Local Gradient Methods with Better Computational Complexity
arXiv:2210.16402, 2022

10/2022
Compressed-Scaffnew

Laurent Condat, Ivan Agarský and P.R.
Provably Doubly Accelerated Federated Learning: The First Theoretically Successful Combination of Local Training and Compressed Communication
arXiv:2210.13277, 2022

10/2022
5GCS

Michal Grudzien, Grigory Malinovsky and P.R.
Can 5th Generation Local Training Methods Support Client Sampling? Yes!
preprint, 2022

Brief History of Local Training Methods
Generation 5: Accelerated

	Comm. Acceleration	Local Optimizer	# Local Training Steps	Total Complexity (Comm. + Compute)	Client Sampling?	Comm. Compression?	Supports Decentralized Setup?	Key Insight
ProxSkip 2/22, ICML 22	✓ $\mathcal{O}\left(\sqrt{\frac{L}{\mu}}\log\frac{1}{\varepsilon}\right)$	GD	$\sqrt{\frac{L}{\mu}}$	=	✗	✗	✓	First 5th generation local training method
APDA-Inexact 7/22, NeurIPS 22	✓ $\mathcal{O}\left(\sqrt{\frac{L}{\mu}}\log\frac{1}{\varepsilon}\right)$	any	better	better	✗	✗	✓	Can use more powerful local solvers which take fewer local GD-type steps
VR-ProxSkip 7/22, NeurIPS 22	✓ worse	VR-SGD	worse	better	✗	✗	✗	Running variance reduced SGD locally can lead to better total complexity than ProxSkip
RandProx 7/22	✓ $\mathcal{O}\left(\sqrt{\frac{L}{\mu}}\log\frac{1}{\varepsilon}\right)$	GD	$\sqrt{\frac{L}{\mu}}$	=	✗	✗	✓	ProxSkip = VR mechanism for compressing the prox
GradSkip 10/22	✓ $\mathcal{O}\left(\sqrt{\frac{L}{\mu}}\log\frac{1}{\varepsilon}\right)$	GD	better	better	✗	✗	✗	Workers containing less imortant data can do fewer local training steps!
Compressed Scaffnew 10/22	✓ worse	GD	worse	better	✗	✓	✗	Can compress uplink, leads to better overal communication complexity than ProxSkip.
5GCS 10/22	✓ worse	any	$\sqrt{\frac{L}{\mu}}$	worse	✓	✗	✗	Can do client sampling

Part 4
ProxSkip: Local Training Provably Leads to Communication Acceleration

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Federated Learning: ProxSkip vs Baselines

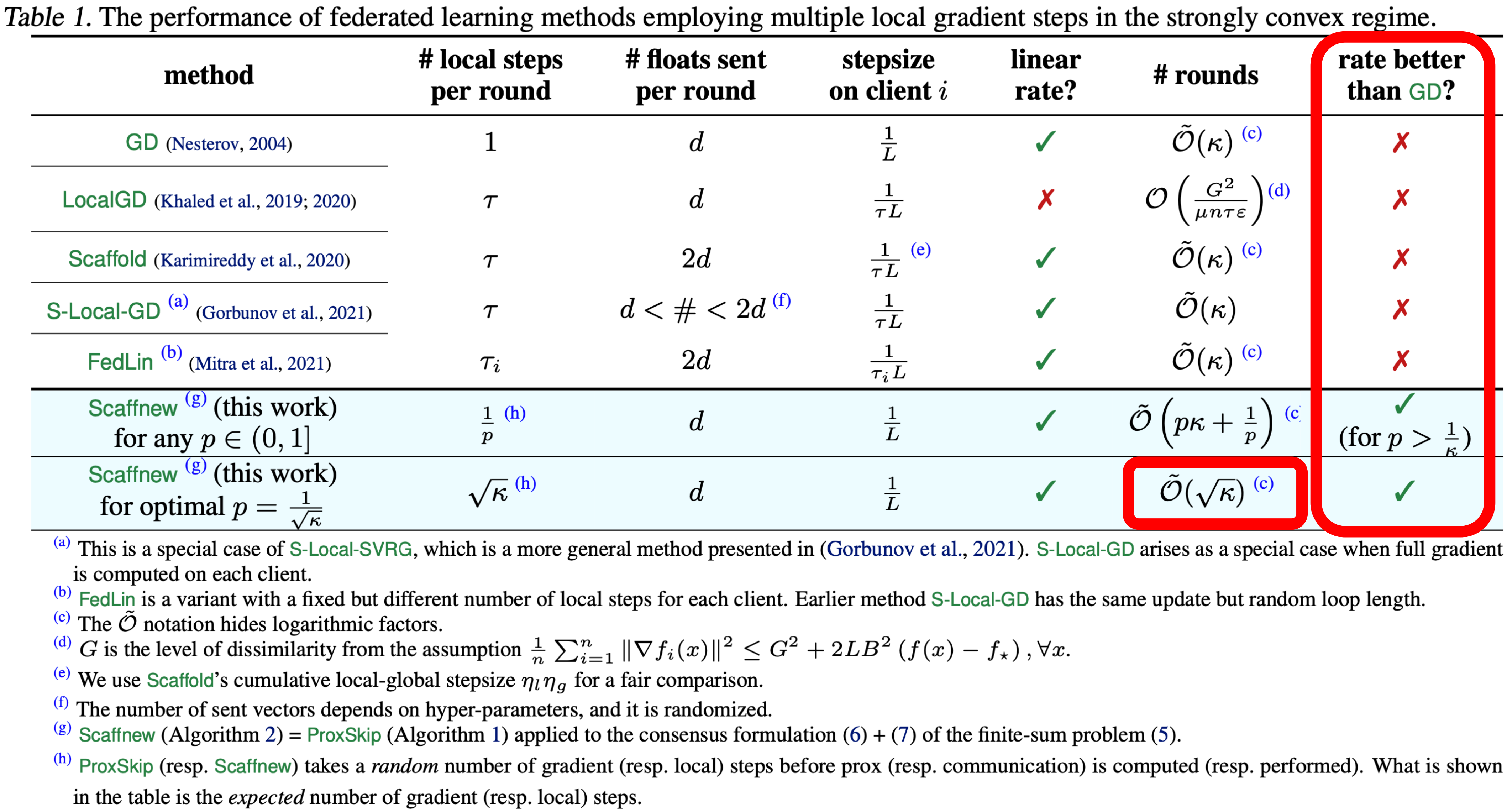


Table 1. The performance of federated learning methods employing multiple local gradient steps in the strongly convex regime.

method	**# local steps per round**	**# floats sent per round**	**stepsize on client i**	**linear rate?**	**# rounds**	**rate better than GD?**
GD (Nesterov, 2004)	1	d	$\frac{1}{L}$	✓	$\tilde{\mathcal{O}}(\kappa)$ [c]	✗
LocalGD (Khaled et al., 2019; 2020)	τ	d	$\frac{1}{\tau L}$	✗	$\mathcal{O}\left(\frac{G^2}{\mu n \tau \varepsilon}\right)$ [d]	✗
Scaffold (Karimireddy et al., 2020)	τ	$2d$	$\frac{1}{\tau L}$ [e]	✓	$\tilde{\mathcal{O}}(\kappa)$ [c]	✗
S-Local-GD [a] (Gorbunov et al., 2021)	τ	$d < \# < 2d$ [f]	$\frac{1}{\tau L}$	✓	$\tilde{\mathcal{O}}(\kappa)$	✗
FedLin [b] (Mitra et al., 2021)	τ_i	$2d$	$\frac{1}{\tau_i L}$	✓	$\tilde{\mathcal{O}}(\kappa)$ [c]	✗
Scaffnew [g] (this work) for any $p \in (0,1]$	$\frac{1}{p}$ [h]	d	$\frac{1}{L}$	✓	$\tilde{\mathcal{O}}\left(p\kappa + \frac{1}{p}\right)$ [c	✓ (for $p > \frac{1}{\kappa}$)
Scaffnew [g] (this work) for optimal $p = \frac{1}{\sqrt{\kappa}}$	$\sqrt{\kappa}$ [h]	d	$\frac{1}{L}$	✓	$\boxed{\tilde{\mathcal{O}}(\sqrt{\kappa})}$ [c]	✓

[a] This is a special case of S-Local-SVRG, which is a more general method presented in (Gorbunov et al., 2021). S-Local-GD arises as a special case when full gradient is computed on each client.

[b] FedLin is a variant with a fixed but different number of local steps for each client. Earlier method S-Local-GD has the same update but random loop length.

[c] The $\tilde{\mathcal{O}}$ notation hides logarithmic factors.

[d] G is the level of dissimilarity from the assumption $\frac{1}{n}\sum_{i=1}^{n}\|\nabla f_i(x)\|^2 \le G^2 + 2LB^2\left(f(x) - f_\star\right), \forall x$.

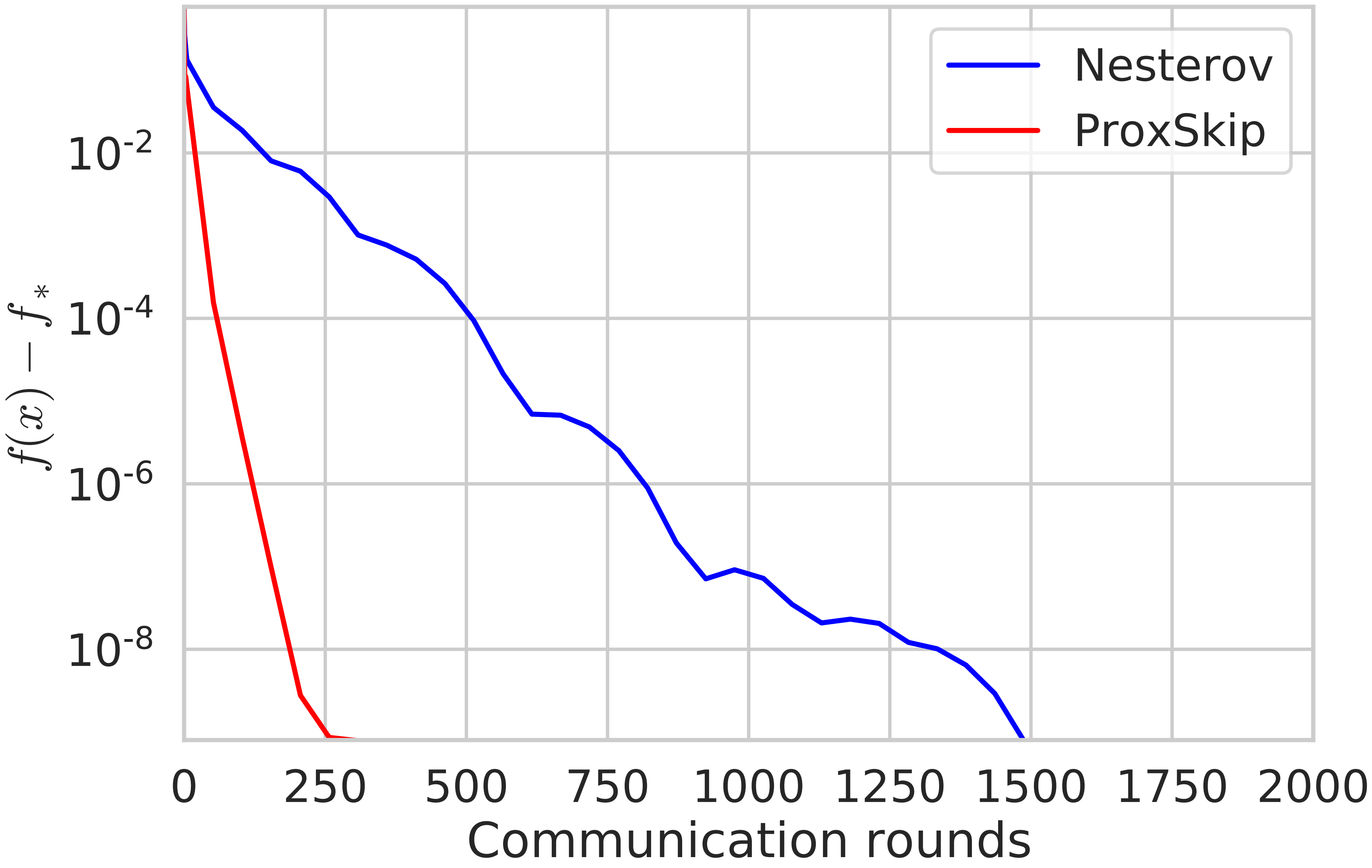
[e] We use Scaffold's cumulative local-global stepsize $\eta_l \eta_g$ for a fair comparison.

[f] The number of sent vectors depends on hyper-parameters, and it is randomized.

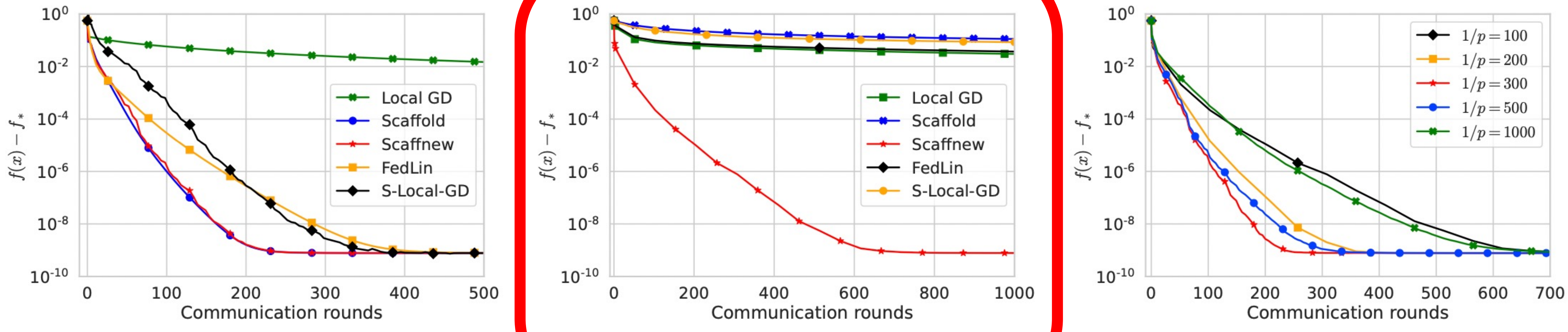
[g] Scaffnew (Algorithm 2) = ProxSkip (Algorithm 1) applied to the consensus formulation (6) + (7) of the finite-sum problem (5).

[h] ProxSkip (resp. Scaffnew) takes a *random* number of gradient (resp. local) steps before prox (resp. communication) is computed (resp. performed). What is shown in the table is the *expected* number of gradient (resp. local) steps.

ProxSkip vs Nesterov



ProxSkip + Deterministic Gradients



(a) tuned hyper-parameters

(b) theoretical hyper-parameters

(c) different options of p

Figure 1. **Deterministic Case**. Comparison of Scaffnew to other local update methods that tackle data-heterogeneity and to LocalGD. In (a) we compare communication rounds with optimally tuned hyper-parameters. In (b), we compare communication rounds with the algorithm parameters set to the best theoretical stepsizes used in the convergence proofs. In (c), we compare communication rounds with the algorithm stepsize set to the best theoretical stepsize and different options of parameter p.

L2-regularized logistic regression:

$$f(x) = \frac{1}{n}\sum_{i=1}^{n} \log\left(1 + \exp\left(-b_i a_i^\top x\right)\right) + \frac{\lambda}{2}\|x\|^2$$

$$a_i \in \mathbb{R}^d,\ b_i \in \{-1, +1\},\ \lambda = L/10^4$$

`w8a` **dataset from LIBSVM library (Chang & Lin, 2011)**

Consensus Reformulation

Original problem:
optimization in $\mathbb{R}^d$

$$\min_{x\in\mathbb{R}^d}\left\{f(x)\overset{\text{def}}{=}\frac{1}{n}\sum_{i=1}^{n}f_i(x)\right\}$$

$$\Updownarrow$$

Consensus reformulation:
optimization in $\mathbb{R}^{nd}$

$$\min_{x_1,\ldots,x_n\in\mathbb{R}^d}\left\{\frac{1}{n}\sum_{i=1}^{n}f_i\left(x_i\right)+\psi\left(x_1,\ldots,x_n\right)\right\}$$

Bad: non-differentiable

Good: Indicator function of a nonempty closed convex set

$$\psi\left(x_1,\ldots,x_n\right)\overset{\text{def}}{=}\begin{cases}0, & \text{if } x_1=\cdots=x_n,\\ +\infty, & \text{otherwise.}\end{cases}$$

Consensus Reformulation

Original problem:
optimization in $\mathbb{R}^d$

$$\min_{x\in\mathbb{R}^d}\left\{f(x)\overset{\text{def}}{=}\frac{1}{n}\sum_{i=1}^{n}f_i(x)\right\}$$

$$\Updownarrow$$

Consensus reformulation:
optimization in $\mathbb{R}^{nd}$

$$\min_{x_1,\dots,x_n\in\mathbb{R}^d}\left\{\frac{1}{n}\sum_{i=1}^{n}f_i(x_i)+\psi(x_1,\dots,x_n)\right\}$$

Bad: non-differentiable

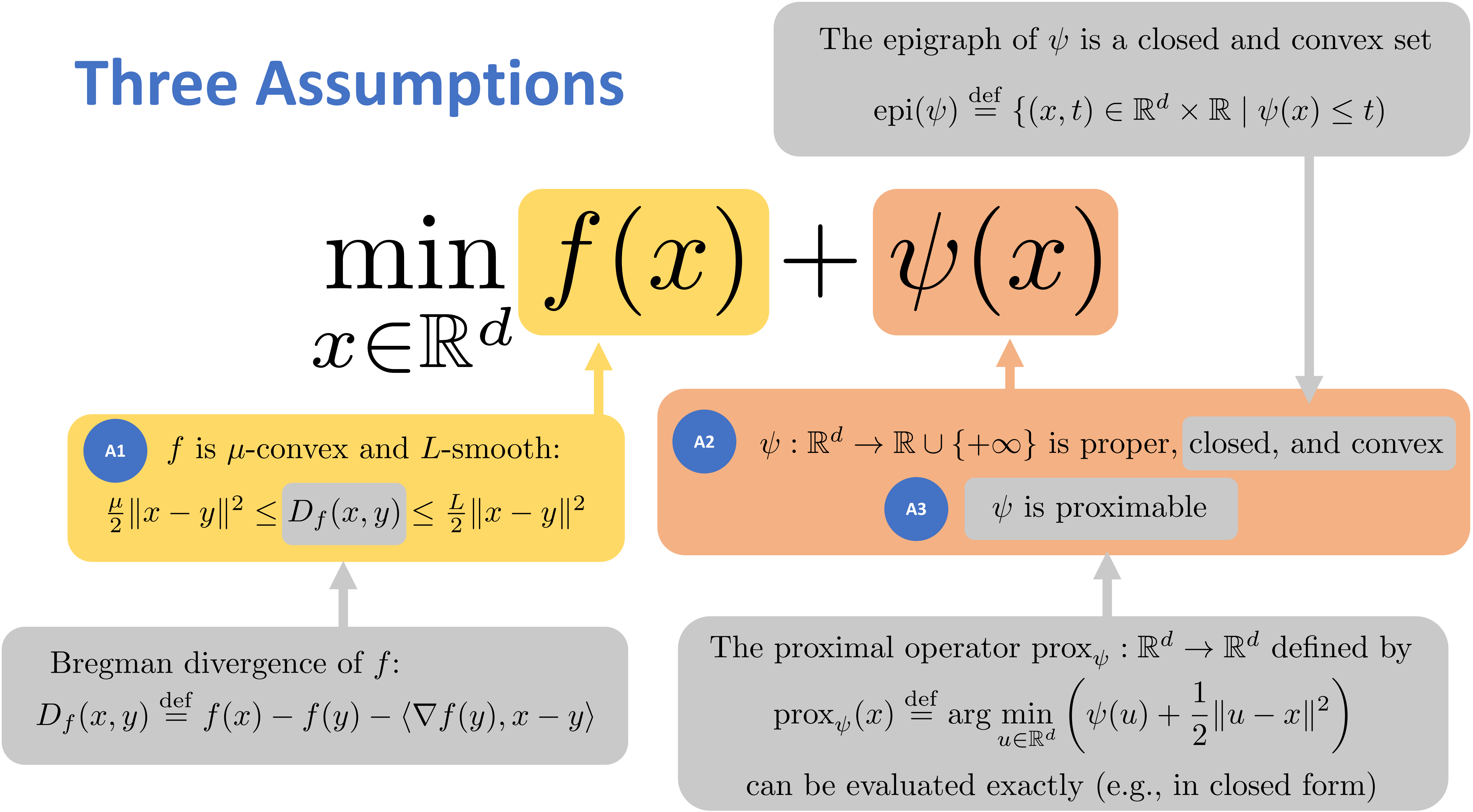
Good: proper closed convex

$$\psi(x_1,\dots,x_n):\mathbb{R}^{nd}\to\mathbb{R}\cup\{+\infty\}$$

is a proper closed convex function

$\mathrm{epi}(\psi)\overset{\text{def}}{=}\{(x,t)\mid\psi(x)\le t\}$ The epigraph of ψ is a closed and convex set

Three Assumptions



The epigraph of ψ is a closed and convex set

$$\mathrm{epi}(\psi) \overset{\text{def}}{=} \{(x,t) \in \mathbb{R}^d \times \mathbb{R} \mid \psi(x) \le t\}$$

$$\min_{x \in \mathbb{R}^d} f(x) + \psi(x)$$

A1 f is μ-convex and L-smooth:

$$\frac{\mu}{2}\|x-y\|^2 \le D_f(x,y) \le \frac{L}{2}\|x-y\|^2$$

A2 $\psi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ is proper, closed, and convex

A3 ψ is proximable

Bregman divergence of f:

$$D_f(x,y) \overset{\text{def}}{=} f(x) - f(y) - \langle \nabla f(y), x-y \rangle$$

The proximal operator $\mathrm{prox}_\psi : \mathbb{R}^d \to \mathbb{R}^d$ defined by

$$\mathrm{prox}_\psi(x) \overset{\text{def}}{=} \arg\min_{u \in \mathbb{R}^d} \left(\psi(u) + \frac{1}{2}\|u-x\|^2 \right)$$

can be evaluated exactly (e.g., in closed form)

Key Method: Proximal Gradient Descent

proximal operator:

$$\mathrm{prox}_{\psi}(x) \overset{\text{def}}{=} \arg\min_{u\in\mathbb{R}^d}\left(\psi(u)+\frac{1}{2}\|u-x\|^2\right)$$

stepsize

$$\underbrace{x_t - \boxed{\gamma}\nabla f(x_t)}$$

gradient operator

$$x \mapsto x - \gamma\nabla f(x)$$

Proximal Gradient Descent: Theory

f is μ-convex and L-smooth:

$$\frac{\mu}{2}\|x-y\|^2 \leq D_f(x,y) \leq \frac{L}{2}\|x-y\|^2$$

$\frac{L}{\mu}$ is the condition number of f

Theorem:

$$t \geq \frac{L}{\mu}\log\frac{1}{\varepsilon} \quad \Longrightarrow \quad \|x_t - x_\star\|^2 \leq \varepsilon\|x_0 - x_\star\|^2$$

(for stepsize $\gamma = \frac{1}{L}$)

# iterations

Error tolerance

$$x_\star \overset{\text{def}}{=} \arg\min_{x\in\mathbb{R}^d} f(x) + \psi(x)$$

ProxSkip: Bird’s Eye View

$$\min_{x\in\mathbb{R}^d} f(x)+\psi(x)$$

1

$$\hat{x}_{t+1} = x_t - \gamma\left(\nabla f(x_t) - h_t\right)$$

2a

with probability $1-p$ do

$1-p \approx 1$

$$x_{t+1} = \hat{x}_{t+1} \qquad h_{t+1} = h_t$$

2b

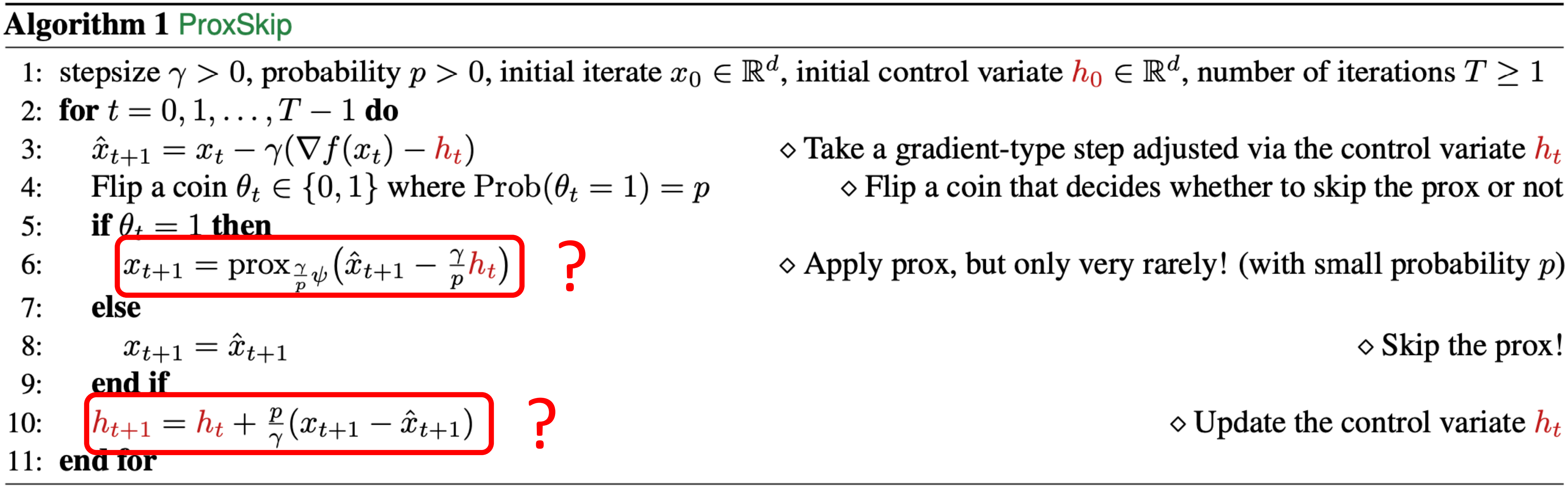
with probability p do

$p \approx 0$

evaluate $\mathrm{prox}_{\frac{\gamma}{p}\psi}(?)$

$$x_{t+1} = \ ? \qquad h_{t+1} = \ ?$$

ProxSkip: The Algorithm (Detailed View)



Algorithm 1 ProxSkip

```
stepsize $\gamma > 0$, probability $p > 0$, initial iterate $x_0 \in \mathbb{R}^d$, initial control variate $h_0 \in \mathbb{R}^d$, number of iterations $T \geq 1$
for $t = 0, 1, \ldots, T-1$ do
    $\hat{x}_{t+1} = x_t - \gamma(\nabla f(x_t) - h_t)$        ◇ Take a gradient-type step adjusted via the control variate $h_t$
    Flip a coin $\theta_t \in \{0, 1\}$ where $\mathrm{Prob}(\theta_t = 1) = p$        ◇ Flip a coin that decides whether to skip the prox or not
    if $\theta_t = 1$ then
        $\boxed{x_{t+1} = \mathrm{prox}_{\frac{\gamma}{p}\psi}\left(\hat{x}_{t+1} - \frac{\gamma}{p} h_t\right)}$  ?        ◇ Apply prox, but only very rarely! (with small probability $p$)
    else
        $x_{t+1} = \hat{x}_{t+1}$        ◇ Skip the prox!
    end if
    $\boxed{h_{t+1} = h_t + \frac{p}{\gamma}(x_{t+1} - \hat{x}_{t+1})}$  ?        ◇ Update the control variate $h_t$
end for
```

ProxSkip: Bounding the # of Iterations

Theorem:

f is μ-convex and L-smooth:

$$\frac{\mu}{2}\|x-y\|^2 \leq D_f(x,y) \leq \frac{L}{2}\|x-y\|^2$$

$\frac{L}{\mu}$ is the condition number of f

$$t \geq \max\left\{\frac{L}{\mu}, \frac{1}{p^2}\right\}\log\frac{1}{\varepsilon} \quad\Longrightarrow\quad \mathbb{E}\left[\Psi_t\right] \leq \varepsilon\Psi_0$$

# iterations

p = probability of evaluating the prox

Lyapunov function:

$$\Psi_t \overset{\text{def}}{=} \left\|x_t - x_\star\right\|^2 + \frac{1}{L^2p^2}\left\|h_t - h_\star\right\|^2$$

ProxSkip: Optimal Prox-Evaluation Probability

Since in each iteration we evaluate the prox with probability p, the expected number of prox evaluations after t iterations is:

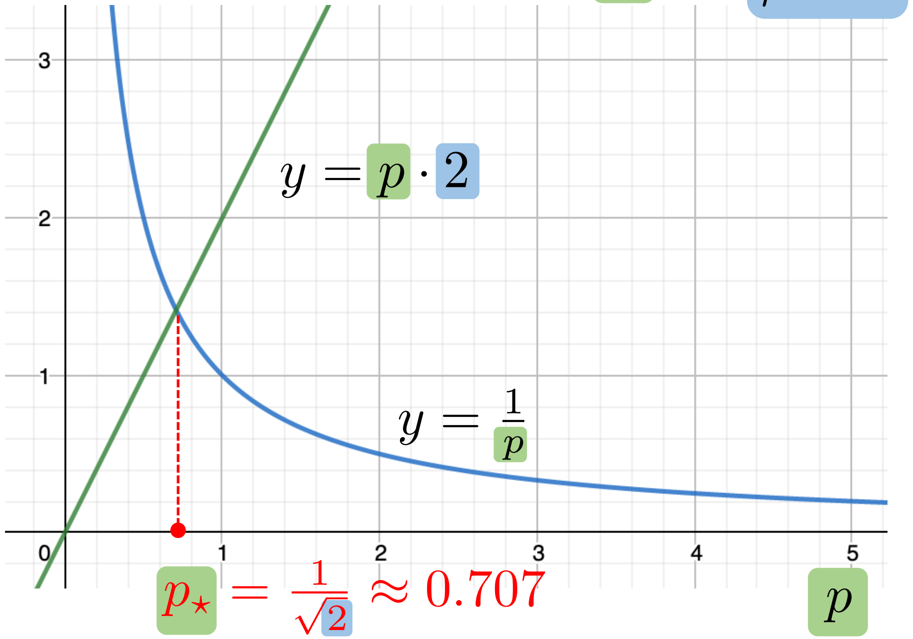
$\frac{L}{\mu}$ is the condition number of f

$$p \cdot t = p \cdot \max\left\{\frac{L}{\mu}, \frac{1}{p^2}\right\} \cdot \log\frac{1}{\varepsilon} = \max\left\{p \cdot \frac{L}{\mu}, \frac{1}{p}\right\} \cdot \log\frac{1}{\varepsilon}$$

Minimized for p satisfying $p \cdot \frac{L}{\mu} = \frac{1}{p}$

$$\Rightarrow \quad p_\star = \frac{1}{\sqrt{L/\mu}}$$

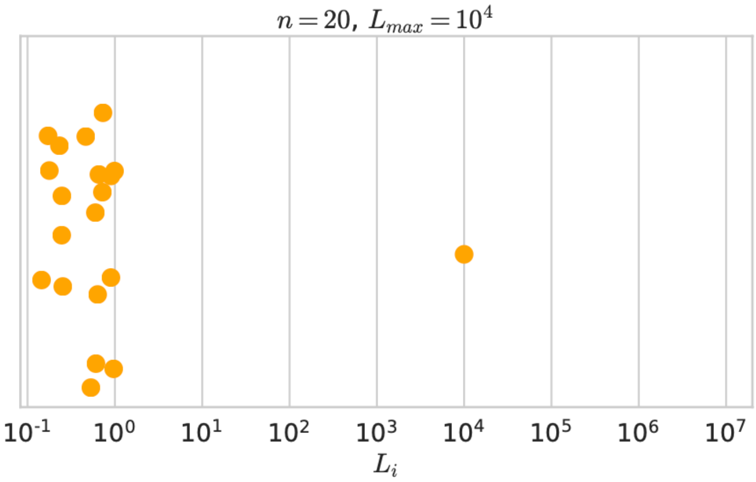
Computation of optimal $p_\star$ for $\frac{L}{\mu} = 2$

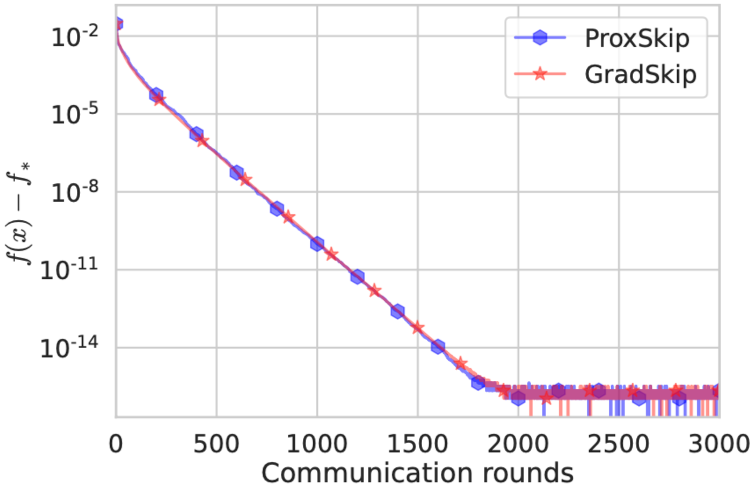


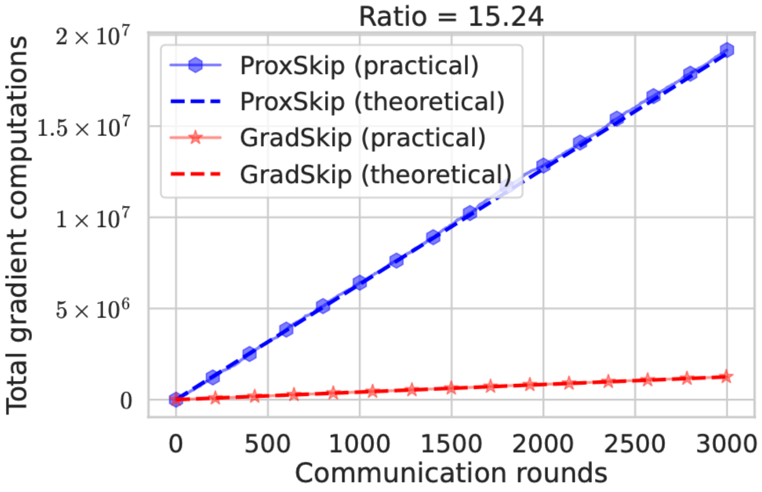
Part 5
GradSkip: Clients with Less Important Data can do Less Local Training

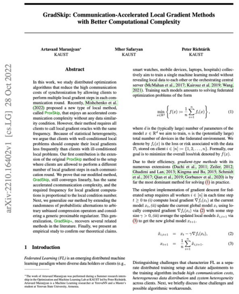
Artavazd Maranjyan, Mher Safaryan and P.R.
GradSkip: Communication-Accelerated Local Gradient Methods with Better Computational Complexity
arXiv:2210.16402, 2022

GradSkip





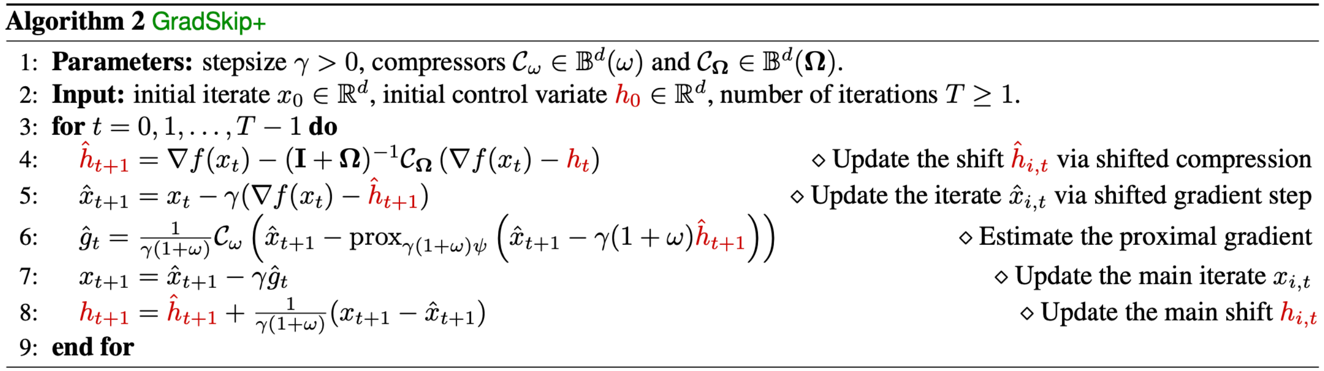




GradSkip: Communication-Accelerated Local Gradient Methods with Better Computational Complexity

Artavazd Maranjyan KAUST Mher Safaryan KAUST Peter Richtárik KAUST

Abstract



Algorithm 2 GradSkip+

```
Parameters: stepsize $\gamma > 0$, compressors $\mathcal{C}_\omega \in \mathbb{B}^d(\omega)$ and $\mathcal{C}_{\boldsymbol{\Omega}} \in \mathbb{B}^d(\boldsymbol{\Omega})$.
Input: initial iterate $x_0 \in \mathbb{R}^d$, initial control variate $h_0 \in \mathbb{R}^d$, number of iterations $T \geq 1$.
for $t = 0, 1, \ldots, T-1$ do
    $\hat{h}_{t+1} = \nabla f(x_t) - (\mathbf{I} + \boldsymbol{\Omega})^{-1} \mathcal{C}_{\boldsymbol{\Omega}} (\nabla f(x_t) - h_t)$    ◇ Update the shift $\hat{h}_{i,t}$ via shifted compression
    $\hat{x}_{t+1} = x_t - \gamma(\nabla f(x_t) - \hat{h}_{t+1})$    ◇ Update the iterate $\hat{x}_{i,t}$ via shifted gradient step
    $\hat{g}_t = \frac{1}{\gamma(1+\omega)} \mathcal{C}_\omega \left( \hat{x}_{t+1} - \mathrm{prox}_{\gamma(1+\omega)\psi} \left( \hat{x}_{t+1} - \gamma(1+\omega)\hat{h}_{t+1} \right) \right)$    ◇ Estimate the proximal gradient
    $x_{t+1} = \hat{x}_{t+1} - \gamma \hat{g}_t$    ◇ Update the main iterate $x_{i,t}$
    $h_{t+1} = \hat{h}_{t+1} + \frac{1}{\gamma(1+\omega)} (x_{t+1} - \hat{x}_{t+1})$    ◇ Update the main shift $h_{i,t}$
end for
```

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The End