Erasmus School of Economics

# MOOC Econometrics

## Training Exercise 2.4.1

### Questions

By solving the questions of this exercise, you provide a proof of the Gauss-Markov theorem. We use the following notation.

- The OLS estimator is $b = A_0 y$, where $A_0 = (X'X)^{-1}X'$.
- Let $\hat{\beta} = Ay$ be linear unbiased, with $A$ $(k \times n)$ matrix.
- Define the difference matrix $D = A - A_0$.

(a) Prove the following three results:

(i) $\text{var}(\hat{\beta}) = \sigma^2 AA'$.

(ii) $\hat{\beta}$ unbiased implies $AX = I$ and $DX = 0$.

(iii) Part (ii) implies $AA' = DD' + (X'X)^{-1}$.

(b) Prove that part (a-iii) implies $\text{var}(\hat{\beta}) = \text{var}(b) + \sigma^2 DD'$.

(c) Prove that part (b) implies $\text{var}(\hat{\beta}) - \text{var}(b)$ is positive semidefinite (Gauss-Markov).

(d) Prove that $\text{var}(\hat{\beta}_j) \geq \text{var}(b_j)$ for every $j = 1, \ldots, k$.