

## Questions

Several types of economic behavior lead to relations with time lags because it takes some time before adjustments to changed conditions take place. As an illustration, consider a company producing consumer goods. Let  $y_t$  denote the production volume in month  $t$ , and let  $x_t$  be the total demand volume in that month. Suppose that the optimal production volume  $y_t^*$  is equal to  $y_t^* = \gamma + \delta x_t$ . As an example, if the company has a market share of 25%, then  $\delta = 0.25$  could make sense for this company. If demand changes, then it may take some time for the company to increase production, because it will need to arrange for extra capital and labor. The partial adjustment (PA) model postulates that

$$y_t = y_{t-1} + \lambda(y_{t-1}^* - y_{t-1}) + \varepsilon_t,$$

where  $0 \leq \lambda \leq 1$ . If the company is pro-active, it can decide to base its production on the expected demand volume  $x_t^*$  and to produce the corresponding volume  $y_t = \gamma + \delta x_t^*$ . The adaptive expectations (AE) model postulates that the expectations are partly adjusted to the previously observed demand volume by means of

$$x_t^* = x_{t-1}^* + \lambda(x_{t-1} - x_{t-1}^*) + \varepsilon_t,$$

where  $0 \leq \lambda \leq 1$ . It is assumed that the  $\varepsilon_t$  in the PA and AE models is white noise.

- Write the partial adjustment model in terms of only the observed variables  $y$  and  $x$ , by eliminating  $y_t^*$ . What is the type of the resulting model?
- Write the adaptive expectations model in terms of only the observed variables  $y$  and  $x$ , by eliminating  $x_t^*$ . What is the type of the resulting model?
- What is the condition for stability of the two models in parts (a) and (b)? Provide an economic interpretation of this condition.
- Consider the AE model based on the last two observed sales volumes, where  $x_t^* = x_{t-1}^* + \lambda_1(x_{t-1} - x_{t-1}^*) + \lambda_2(x_{t-2} - x_{t-2}^*) + \varepsilon_t$ . Write this model in terms of only the observed variables  $y$  and  $x$ . What is the type of the resulting model?