Questions

1. Let a and b be two $(p \times 1)$ vectors. Multiply out the parentheses and simplify $(a + b)'(a + b)$.

2. Let a be a $(p \times 1)$ vector. Use the definition of trace to show that the inner product $a'a$ is equal to the trace of the outer product aa'.

3. Let A be a square $(p \times p)$ matrix and c a scalar. Show that $\text{tr}(cA) = c\,\text{tr}(A)$.

4. Let A be a square invertible $(p \times p)$ matrix and $c \neq 0$ a scalar. Find the inverse of (cA).

5. Let A be a square (2×2) matrix given by $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find the value for f such that $B(f) = \dfrac{1}{f} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ gives the inverse of A. What condition should the matrix elements satisfy for the inverse to exist?

6. Simplify the expression $\iota'\iota$ and $(\iota\iota')^2$, where ι is the $(p \times 1)$ unit vector.