Erasmus School of Economics

MOOC Econometrics

Training Exercise 1.4

Questions

Consider the situation where the x-variable is observed with measurement error, which is rather common for complex macroeconomic variables like national income.

Let x^* be the true, unobserved economic variable, and let the data generating process (DGP) be given by $y_i = \alpha + \beta x_i^* + \varepsilon_i^*$, where x_i^* and ε_i^* are uncorrelated.

The observed x-values are $x_i = x_i^* + v_i$, with measurement errors v_i that are uncorrelated with x_i^* and ε_i^*. The signal-to-noise ratio is defined as $SN = \sigma_*^2/\sigma_v^2$, where σ_*^2 is the variance of x^* and σ_v^2 that of v.

The estimated regression model is $y_i = \alpha + \beta x_i + \varepsilon_i$, and we consider the least squares estimator b of β.

(a) Do you think that the value of b depends on the variance of the measurement errors? Why?

(b) Show that $b = \beta + \frac{\sum_{i=1}^n (x_i - \bar{x})(\varepsilon_i - \bar{\varepsilon})}{\sum_{i=1}^n (x_i - \bar{x})^2}$.

(c) Show that $\varepsilon_i = \varepsilon_i^* - \beta v_i$.

(d) Show that the covariance between x_i and ε_i is equal to $-\beta\sigma_v^2$.

(e) Show that for large sample size n we get $b - \beta \approx \frac{-\beta\sigma_v^2}{\sigma_*^2 + \sigma_v^2}$.

(f) Compute the approximate bias $(b - \beta)$ for $\beta = 1$ in the cases $SN = 1$, $SN = 3$, and $SN = 10$.