

$$1. (a+b)'(a+b) = a'a + a'b + b'a + b'b = a'a + 2a'b + b'b$$

$$b'a = \text{scalar}$$

$$2. aa' = \begin{pmatrix} a_1 a_1 & a_1 a_2 & \dots & a_1 a_p \\ a_2 a_1 & a_2 a_2 & \dots & a_2 a_p \\ \dots & \dots & \ddots & \dots \\ a_p a_1 & a_p a_2 & \dots & a_p a_p \end{pmatrix}$$

$$\text{tr}(aa') = \sum_{i=1}^p (aa')_{ii} = \sum_{i=1}^p a_i^2 = a'a$$

$$3. \text{tr}(cA) = \sum_{i=1}^p (cA)_{ii} = \sum_{i=1}^p c a_{ii} = c \sum_{i=1}^p a_{ii} = c \text{tr}(A)$$

$$4. cAB = I \text{ (} p \times p \text{) identity matrix}$$

$$cAB = I$$

$$AB = \frac{1}{c} I$$

$$A^{-1}AB = \frac{1}{c} A^{-1}I$$

$$B = \frac{1}{c} A^{-1}$$

$$5. B(p) \cdot A = I$$

$$B(p) \cdot A = \frac{1}{f} \begin{pmatrix} d-b & \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \frac{1}{f} \begin{pmatrix} da-bc & db-bd \\ -ca+ac & -cb+ad \end{pmatrix} =$$

$$\frac{1}{f} \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$f = ad - bc \quad ad \neq bc$$

$$6. l' l = \sum_{i=1}^p l_i l_i = \sum_{i=1}^p 1 = p$$

$$(l l')^2 = l l' l l' = l (l' l) l' = p l l'$$