Erasmus School of Economics

MOOC Econometrics

Training Exercise 5.3

Questions

Consider a logit model with only an intercept parameter such that

$$\Pr[y_i = 1] = \frac{\exp(\beta_1)}{1 + \exp(\beta_1)},$$

for $i = 1, \ldots, n$. To estimate the parameter β_1 we use the maximum likelihood method. As shown in lecture 5.3 the first-order condition for maximum likelihood can be written as

$$\frac{1}{n}\sum_{i=1}^{n} \frac{\exp(b_1)}{1 + \exp(b_1)} = \frac{1}{n}\sum_{i=1}^{n} y_i,$$

where b_1 is the maximum likelihood estimator (MLE).

(a) Use the first-order condition given above to show that the maximum likelihood estimator of β_1 equals

$$b_1 = \log\left(\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n}(1 - y_i)}\right).$$

(b) Show that the maximum likelihood estimator b_1 implies that

$$\widehat{\Pr}[y_i = 1] = \frac{\sum_{i=1}^{n} y_i}{n}.$$

(c) Use the general formula to estimate the covariance matrix of the MLE in a logit model to show that this formula simplifies to

$$\hat{V} = \left(n\frac{\sum_{i=1}^{n} y_i}{n}(1 - \frac{\sum_{i=1}^{n} y_i)}{n})\right)^{-1}$$

in case the logit model only contains an intercept.