Erasmus School of Economics

MOOC Econometrics

Training Exercise 3.2

Questions

(a) Consider again the case from the lecture, with

DGP:	$y = X_1\beta_1 + X_2\beta_2 + \varepsilon$	$\rightarrow b_1$ and b_2
Model:	$y = X_1\beta_1 + \tilde{\varepsilon}$	$\rightarrow b_R$

Prove $Var(b_R) = Var(b_1) - PVar(b_2)P'$. Use $b_R = b_1 + Pb_2$ and $Cov(b_2, b_R) = 0$ (you do not need to prove this).

(b) One way to get more insight into the bias-efficiency trade-off (also referred to as the bias-variance trade-off) is to combine bias and efficiency in the Mean Squared Error (MSE). The mean squared error is defined as

$$MSE(b) = E((b - \beta)(b - \beta)'),$$

with b a certain estimator of the unknown parameter β. Prove $E((b-\beta)(b-\beta)') = Var(b)+E(b-\beta)E(b-\beta)'$. Hints: First write the MSE out in four terms. Then, as an intermediate step, show $Var(b) = E(bb)-E(b)E(b)'$. Finally, add and subtract $E(b)E(b)'$ from the MSE expression and rewrite to get the desired result.

(c) Again in the context of the case from the lecture (see (a)), show that the mean squared error of b_1 minus the mean squared error of b_R is equal to

$$MSE(b_1) - MSE(b_R) = P(Var(b_2) - \beta_2\beta_2')P'.$$

(d) When would the restricted estimator be better, when basing this decision on the mean squared error?