

**Notes:**

- This exercise uses the datafile TrainExer61 and requires a computer.
- The dataset TrainExer61 is available on the website.

**Questions**

The datafile TrainExer61 contains values of four series of length 250. Two of these series are uncorrelated white noise series denoted by  $\varepsilon_{xt}$  and  $\varepsilon_{yt}$ , where both variables are NID(0, 1), that is, normally and independently distributed standard normal random variables. The other two series are so-called random walks constructed from these two white noise series by  $x_t = x_{t-1} + \varepsilon_{xt}$  and  $y_t = y_{t-1} + \varepsilon_{yt}$ , with starting values  $x_1 = 0$  and  $y_1 = 0$ .

As  $\varepsilon_{xt}$  and  $\varepsilon_{ys}$  are independent for all values of  $t$  and  $s$ , the same holds true for all values of  $x_t$  and  $y_s$ . The purpose of this exercise is to experience that, nonetheless, the regression of  $y$  on  $x$  indicates a highly significant relation between  $y$  and  $x$  if evaluated by standard regression tools. This kind of result is called 'spurious regression' and is caused by the trending nature of the variables  $x$  and  $y$ . The lesson we learn is that standard regression tools are not applicable if the variables contain trends similar to those of the random walks considered here.

- Use dataset TrainExer61 to make the following graphs: the time series plot of  $x_t$  against time  $t$ , the time series plot of  $y_t$  against time  $t$ , and the scatter plot of  $y_t$  against  $x_t$ . What conclusion could you draw from these three graphs?
- To check that the series  $\varepsilon_{xt}$  and  $\varepsilon_{yt}$  are uncorrelated, regress  $\varepsilon_{yt}$  on a constant and  $\varepsilon_{xt}$ . Report the  $t$ -value and  $p$ -value of the slope coefficient.
- Extend the analysis of part (b) by regressing  $\varepsilon_{yt}$  on a constant,  $\varepsilon_{xt}$ , and three lagged values of  $\varepsilon_{yt}$  and of  $\varepsilon_{xt}$ . Perform the  $F$ -test for the joint insignificance of the seven parameters of  $\varepsilon_{xt}$  and the three lags of  $\varepsilon_{xt}$  and  $\varepsilon_{yt}$ . Report the degrees of freedom of the  $F$ -test and the numerical outcome of this test, and draw your conclusion. Note: The relevant 5% critical value is 2.0.
- Regress  $y$  on a constant and  $x$ . Report the  $t$ -value and  $p$ -value of the slope coefficient. What conclusion would you be tempted to draw if you did not know how the data were generated?
- Let  $e_t$  be the residuals of the regression of part (d). Regress  $e_t$  on a constant and the one-period lagged residual  $e_{t-1}$ . What standard assumption of regression is clearly violated for the regression in part (d)?