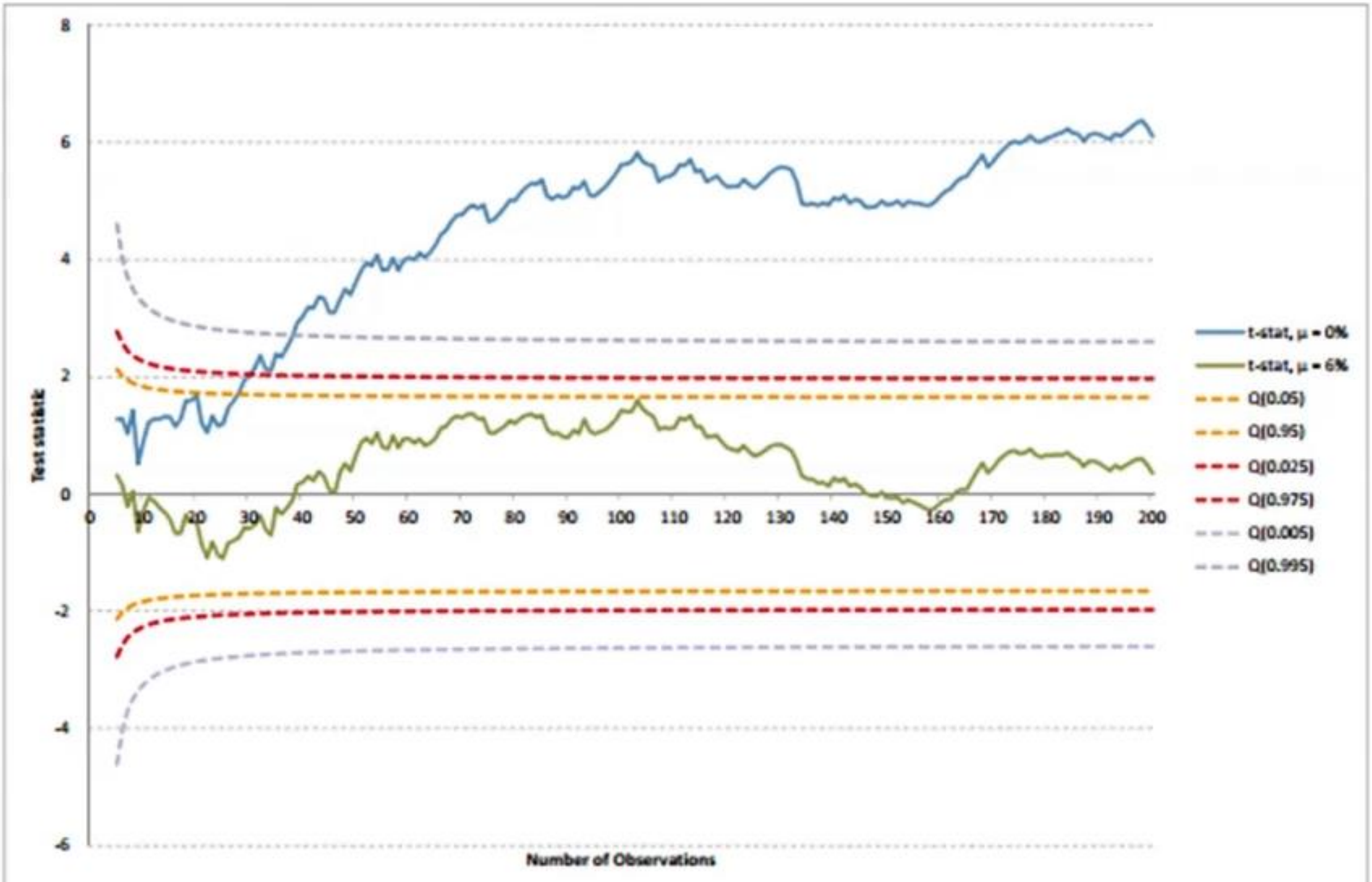
1 a) $t_i = \dfrac{m_i - \mu_0}{s_i/\sqrt{i}}, \quad m_i = \dfrac{1}{i}\sum_{j=1}^{i} y_j, \quad s_i^2 = \dfrac{1}{i-1}\sum_{j=1}^{i}(y_j - m_i)^2$

$p_i = 2\,\Psi_{i-1}(-|t_i|)$

(see next page for table)

b) $c_i(\alpha) = -\Psi_{i-1}^{-1}(\alpha/2)$



	$\mu_0 = 0\%$		$\mu_0 = 6\%$			$\mu_0 = 0\%$		$\mu_0 = 6\%$	
i	t-stat	p-value	t-stat	p-value	i	t-stat	p-value	t-stat	p-value
5	1.278	0.271	0.321	0.764	18	1.594	0.129	−0.359	0.724
6	1.288	0.254	0.149	0.888	19	1.596	0.128	−0.461	0.650
7	1.035	0.341	−0.214	0.838	20	1.690	0.107	−0.479	0.638
8	1.428	0.196	0.045	0.966	21	1.211	0.240	−0.869	0.395
9	0.517	0.619	−0.639	0.541	22	1.054	0.304	−1.086	0.290
10	0.916	0.383	−0.277	0.788	23	1.336	0.195	−0.818	0.422
11	1.222	0.250	−0.059	0.954	24	1.162	0.257	−1.040	0.309
12	1.284	0.226	−0.118	0.908	25	1.200	0.242	−1.096	0.284
13	1.282	0.224	−0.234	0.819	26	1.471	0.154	−0.831	0.414
14	1.326	0.208	−0.309	0.762	27	1.594	0.123	−0.793	0.435
15	1.315	0.210	−0.432	0.673	28	1.732	0.095	−0.737	0.468
16	1.152	0.267	−0.669	0.513	29	1.948	0.062	−0.567	0.575
17	1.273	0.221	−0.664	0.516	30	2.012	0.054	−0.590	0.559

c) $H_0: \mu = 0\%$ rejected $(n > 30)$

$H_0: \mu = 6\%$ not rejected

$H_0: \mu = 6.5\%$: t-statistics slightly below t-scastistics

of $H_0: \mu = 6\%$ => not rejected

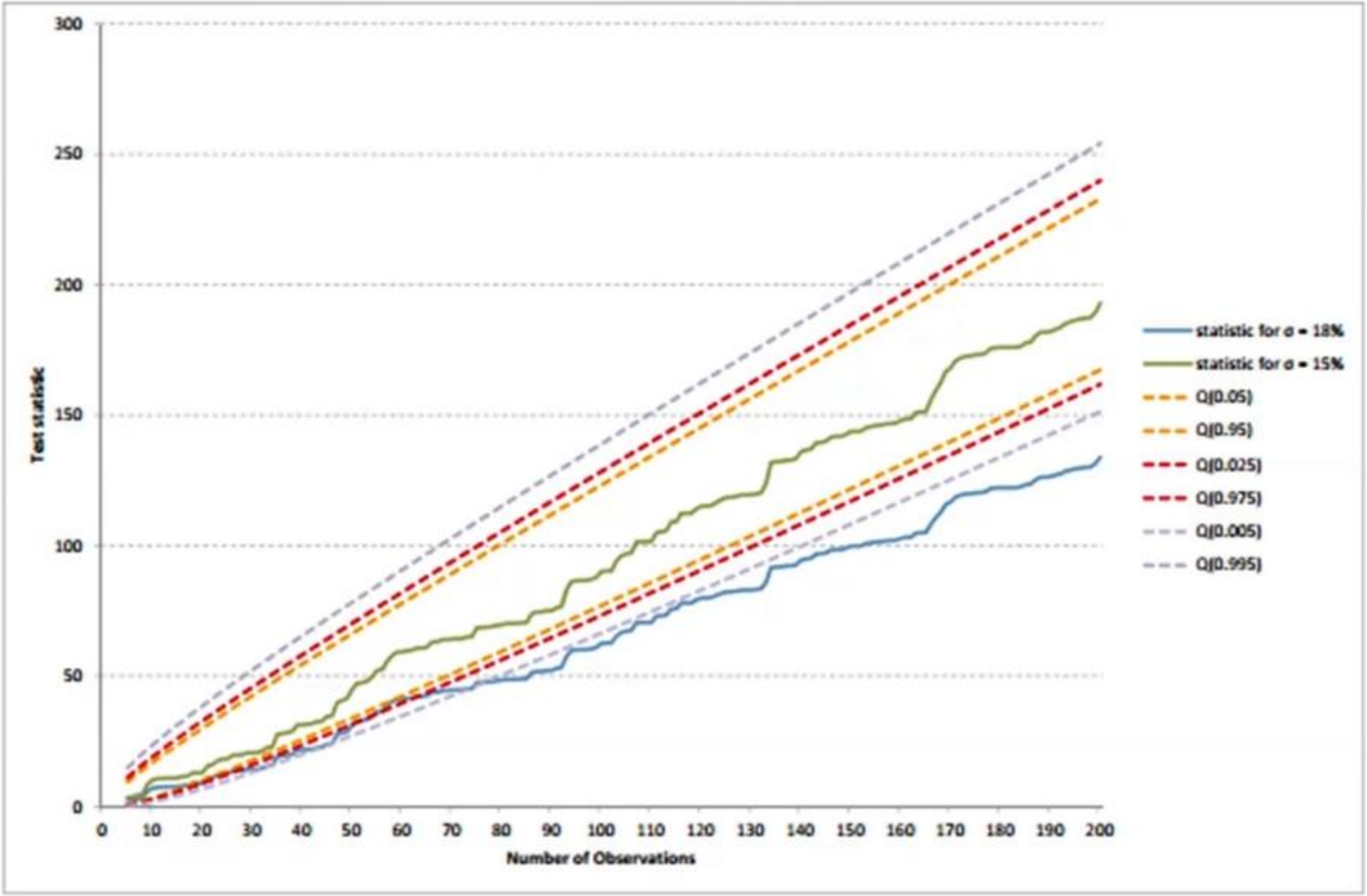
=> test never reveal the true value,

only one value for μ can be true

d) $\chi_i^2 = \frac{(i-1)s_i^2}{\sigma_0^2}$ $p_i = \begin{cases} 2G_{i-1}(\chi_i^2) & \text{for } G(\chi_i^2) \leq 1/2 \\ 2\cdot\{1 - G_{i-1}(\chi_i^2) & \text{for } G(\chi_i^2) > 1/2 \end{cases}$

	$\sigma_0 = 18\%$		$\sigma_0 = 15\%$			$\sigma_0 = 18\%$		$\sigma_0 = 15\%$	
i	χ^2-stat	p-value	χ^2-stat	p-value	i	χ^2-stat	p-value	χ^2-stat	p-value
5	2.43	0.685	3.50	0.957	18	8.92	0.114	12.84	0.506
6	2.57	0.468	3.70	0.813	19	8.98	0.080	12.93	0.408
7	2.99	0.381	4.31	0.730	20	8.98	0.052	12.93	0.316
8	3.25	0.278	4.68	0.602	21	10.79	0.097	15.54	0.510
9	5.99	0.703	8.63	0.750	22	11.20	0.083	16.13	0.476
10	7.02	0.730	10.11	0.684	23	12.12	0.090	17.45	0.524
11	7.44	0.634	10.72	0.760	24	12.65	0.082	18.21	0.508
12	7.46	0.479	10.75	0.930	25	12.65	0.057	18.21	0.415
13	7.55	0.361	10.87	0.919	26	13.63	0.064	19.62	0.467
14	7.57	0.259	10.90	0.762	27	13.69	0.046	19.71	0.390
15	7.65	0.187	11.02	0.631	28	13.79	0.034	19.85	0.326
16	8.04	0.155	11.57	0.578	29	14.27	0.030	20.55	0.313
17	8.05	0.106	11.60	0.458	30	14.27	0.020	20.55	0.250

e) $c^l(\alpha) = G_{i-1}^{-1}(\alpha/2)$, $c^r(\alpha) = G_{i-1}^{-1}(1-\alpha/2)$



f) $\sigma_0 = 18\%$ H_0 rejected at $\begin{cases} \alpha = 10\% & n > 20 \\ \alpha < 10\% & n > 70-80 \end{cases}$

$\sigma_0 = 15\%$ H_0 not rejected

g) n small -> estimation uncertainty large

-> α high

n very large -> limit probability type I error

-> α low (1% or less)

2 a) $H_0 : h'\mu = 0 \rightarrow h'\mu = (1 \;\; -1)\begin{pmatrix}\mu_1 \\ \mu_2\end{pmatrix} = \mu_1 - \mu_2 = 0$

$\Rightarrow \mu_1 = \mu_2$

b) $y \sim N(H\mu, \sigma^2 I_n) \rightarrow m \sim N$

$E[m] = T^{-1}H'E[y] = T^{-1}H'H\mu = \mu$

$var[m] = var[T^{-1}H'y] = T^{-1}H' var[y] H T^{-1} =$

$T^{-1}H'\sigma^2 I_n H T^{-1} = \sigma^2 T^{-1}H'HT^{-1} =$

$\sigma^2 T^{-1} T T^{-1} = \sigma^2 T^{-1}$

$m \sim N(\mu, \sigma^2 T^{-1})$

c) $h'm \sim N$ $\quad E[h'm] = h'E[m] = h'\mu = \mu_1 - \mu_2$

$Var[h'm] = h' Var[m] h = h'\sigma^2 T^{-1} h$

$= \sigma^2 (1 \; -1)\begin{pmatrix}\frac{1}{n_1} & 0 \\ 0 & \frac{1}{n_2}\end{pmatrix}\begin{pmatrix}1 \\ -1\end{pmatrix} = \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)$

d) $h'm \sim N\left(h'\mu, \sigma^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\right)$

$x \sim N(\mu_x, \sigma_x^2)$, $\frac{x-\mu_x}{\sigma_x} \sim N(0,1)$

$$\frac{h'm - h'\mu}{\sqrt{\sigma^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{h'(m-\mu)}{\sigma\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim N(0,1)$$

e) $$t = \frac{h'(m-\mu)}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{\dfrac{h'(m-\mu)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}}{\sqrt{\dfrac{(n-2)s^2}{\sigma^2}/(n-2)}}$$

$\frac{h'(m-\mu)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim N(0,1)$, $\frac{(n-2)s^2}{\sigma^2} \sim \chi^2(n-2)$ $\Big\}$ $t \sim t(n-2)$