1. $\frac{\partial f}{\partial b}(b) = b + b = 2b$

$f(b) = \sum_{i=1}^{p} b_i^2 \qquad \frac{\partial f}{\partial b_i}(b) = 2b_i$

$\frac{\partial^2 f}{\partial b_j \partial b_i}(b) = \begin{cases} 2 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \Rightarrow \frac{\partial^2 f}{\partial b' \partial b}(b) = 2I$ $(p \times p)$ identity matrix

2. A $(p \times p)$ diagonal matrix $\quad x'Ax = \sum_{i=1}^{p} a_{ii} x_i^2 \quad x_i^2 > 0$ for $x_i \neq 0$

$x'Ax = \begin{cases} > 0 & \text{if } a_{ii} > 0 \text{ for all } i \\ < 0 & \text{if } a_{ii} < 0 \text{ for all } i \end{cases}$

3. $f(b) = (y - Xb)'(y - Xb) = y'y - 2y'Xb + b'X'Xb$

$\frac{\partial f}{\partial b}(b) = -(2y'X)' + (X'X + (X'X))b = -2X'y + 2X'Xb$

$\frac{\partial (a'b)}{\partial b}(b) = (a')' = a \qquad X'X$ symmetric

$\frac{\partial f}{\partial b}(b^*) = -2X'y + 2X'Xb^* = 0$

$X'Xb^* = X'y$ → exists as $X'X$ has full rank

$b^* = (X'X)^{-1}X'y$

$\frac{\partial^2 f}{\partial b' \partial b}(b) = 2X'X \Rightarrow$ positive definite $\Rightarrow b^*$ is a minimum