MOOC Econometrics

Training Exercise 2.3

Questions

In economic and business applications, the variables $(x_{1i}, \ldots, x_{ki})$ usually do not have natural measurement units. Personal income, for example, can be measured in units or thousands of local currency or US dollars, and per month or per year. A change of measurement scale of the j-th variable corresponds to a transformation $\tilde{x}_{ji} = a_j x_{ji}$ (with a_j fixed for $i = 1, \ldots, n$). Let $A = \text{diag}(a_1, \ldots, a_k)$ and let $\tilde{X} = XA$. We even allow for non-diagonal A and define $\tilde{X} = XA$ where A is any invertible ($k \times k$) matrix. As before, let $\hat{y} = Xb$ be the predicted values of y.

(a) Prove that $\hat{y}$, e, s^2, and R^2 do not depend on A (that is, are invariant under linear transformations).

(b) Prove that $\tilde{b} = A^{-1}b$ and provide an intuitive interpretation.