

Questions

1. Consider the model

$$y = Xb + e,$$

with

$$y = \begin{pmatrix} 15.1 \\ 7.9 \\ 4.5 \\ 12.8 \\ 10.5 \end{pmatrix}, \quad X = \begin{pmatrix} 1 & 25.5 & 1.23 \\ 1 & 40.8 & 1.89 \\ 1 & 30.2 & 1.55 \\ 1 & 4.3 & 1.18 \\ 1 & 10.7 & 1.68 \end{pmatrix},$$

and b a 3×1 vector, and e a 5×1 vector. Calculate the values of the vector e for

$$b = \begin{pmatrix} 23 \\ 0.1 \\ -8 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 22 \\ -0.2 \\ -7 \end{pmatrix}.$$

2. The vector e contains the unexplained part of the model. We like the unexplained part to be small, with which we mean that each element of the vector e is close to zero. You can measure the distance of an element to zero by its absolute value, or by its square. Which of the two candidates for the vector b gives the smallest unexplained part for the two distance measures?
3. Let A be a $(p \times q)$ matrix, u a $(1 \times p)$ vector, and v a $(q \times 1)$ vector. What dimensions does $d = u \cdot A \cdot v$ have? Write the result in sigma notation (i.e., with \sum).
4. Find a simplified expression without parentheses for $(A + I)^2$, with A a $(p \times p)$ matrix, and I the $(p \times p)$ identity matrix.