1. $E[y] = E[a + b'x + e] = E[a] + E[b'x] + E[e] = a + b'x + \mu_e$

$var[y] = E[(y - E[y])^2] = E[(a + x'b + e - (a + x'b + \mu_e))^2] =$

$E[(e - \mu_e)^2] = \sigma_e^2$

2. $E[y] = E[a_1 x_1 - a_2 x_2] = a_1 E[x_1] - a_2 E[x_2] = a_1 \cdot \mu_1 - a_2 \cdot \mu_2$

$Var(y) = Var[a_1 x_1 - a_2 x_2] = E[(a_1 x_1 - a_2 x_2 - (a_1 \mu_1 - a_2 \mu_2))^2]$

$= E[(a_1(x_1 - \mu_1) - a_2(x_2 - \mu_2))^2] =$

$E[a_1^2 (x_1 - \mu_1)^2 + a_2^2 (x_2 - \mu_2)^2 - 2a_1 a_2 (x_1 - \mu_1)(x_2 - \mu_2)] =$

$a_1^2 E[(x_1 - \mu_1)^2] + a_2^2 E[(x_2 - \mu_2)^2] - 2a_1 a_2 E[(x_1 - \mu_1)(x_2 - \mu_2)] =$

$a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2 - 2a_1 a_2 \sigma_{12}$

3. $E[y] = E[a_1 x_1 + (1 - a_1) x_2] = a_1 E[x_1] + (1 - a_1) E[x_2] = a_1 \mu + (1 - a_1)\mu = \mu$

$Var[y] = var[a_1 x_1 + (1 - a_1) x_2] = a_1^2 \sigma^2 + (1 - a_1)^2 \sigma^2 + 2a_1(1 - a_1) cov[x_1, x_2]$

$= (2a_1^2 - 2a_1 + 1)\sigma^2 + (2a_1 - 2a_1^2) e \sigma^2$

$= ((2a_1^2 - 2a_1)(1 - e) + 1)\sigma^2$

$cov[x_1, x_2] = \sigma_1 e \sigma_2 = e \sigma^2$

4. $E[y] = E\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] = \frac{1}{n}\sum_{i=1}^{n} E[x_i] = \frac{1}{n}\sum_{i=1}^{n} \mu = \mu$

a) $var[y] = var\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] = \frac{1}{n^2}\sum_{i=1}^{n} var[x_i] + \frac{2}{n^2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} cov[x_i, x_j]$

$= \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 = \frac{1}{n}\sigma^2$

b) $cov[y, x_i] = cov\left[\frac{1}{n}\sum_{j=1}^{n} x_j, x_i\right] = E\left[\left(\frac{1}{n}\sum_{j=1}^{n} x_j - \mu\right)(x_i - \mu)\right]$

$= E\left[\left(\frac{1}{n}\sum_{j=1}^{n}(x_j - \mu)\right)(x_i - \mu)\right] = E\left[\frac{1}{n}\sum_{j=1}^{n}((x_j - \mu)(x_i - \mu))\right]$

$= \frac{1}{n}\sum_{j=1}^{n} E[(x_j - \mu)(x_i - \mu)] = \frac{1}{n}E[(x_i - \mu)^2] + \frac{1}{n}\sum_{\substack{j=1 \\ j \neq i}}^{n} E[(x_j - \mu)(x_i - \mu)]$

$= \frac{1}{n}\sigma^2$

$corr[y, x_i] = \frac{cov[y, x_i]}{\sqrt{var[y]\, var[x_i]}}$

$= \frac{\sigma^2/n}{\sqrt{\sigma^2/n \cdot \sigma^2}}$

$= \frac{\sigma^2/n}{\sigma^2/\sqrt{n}}$

$= 1/\sqrt{n}$

5. $E[y] = E[h'x] = h'E[x] = \frac{1}{n}\iota_n'\iota_n\mu = \mu$

(a) $\text{var}[y] = E[(y-E(y))'(y-E(y))] = E[(h'x-\mu)(h'x-\mu)']$

$= E[h'(x-\mu)(x-\mu)'h] = h'E[(x-\mu)(x-\mu)']h$

$= \frac{1}{n^2}\iota_n'\sigma^2 I_n \iota_n = \frac{1}{n}\sigma^2$

the symbol μ marked in yellow (4x) denotes the (n×1) vector $\mu\iota_n$ with all its elements equal to μ

(b) $E(z) = E[Hx] = HE[x] = H\iota_n\mu = \begin{pmatrix} I\iota_n \\ h'\iota_n \end{pmatrix} = \mu\iota_{n+1}$

$\text{var}[z] = E[(z-E[z])(z-E[z])'] = E(H(x-E(x))(H(x-E(x)))')$

$= HE[(x-E[x])(x-E[x])']H' = H\sigma^2 IH' = \sigma^2 HH'$

$= \sigma^2 \begin{pmatrix} I \\ h' \end{pmatrix} \begin{pmatrix} I' & h \end{pmatrix} = \sigma^2 \begin{pmatrix} II' & Ih \\ h'I & h'h \end{pmatrix}$

$= \sigma^2 \begin{pmatrix} II' & Ih \\ h'I & h'h \end{pmatrix} = \sigma^2 \begin{pmatrix} I & \frac{1}{n}\iota_n \\ \frac{1}{n}\iota_n' & \frac{1}{n^2}\iota_n'\iota_n \end{pmatrix}$

$= \frac{1}{n}\sigma^2 \begin{pmatrix} nI & \iota_n \\ \iota_n' & 1 \end{pmatrix}$