Erasmus
School of
Economics

MOOC Econometrics

Lecture 5.2 on Binary Choice: Representation

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Introduction

Let y_i be a binary variable with value 0 or 1 and assume

$$y_i \sim \mathit{Bernoulli}(\pi),$$

such that

$$\pi = \Pr[y_i = 1] \text{ with } 0 < \pi < 1$$

and hence

$$\Pr[y_i = 0] = 1 - \pi.$$

Individual-specific probabilities:

$$\Pr[y_i = 1] = \pi_i$$

Logit model

Individual-specific probabilities:

$$\Pr[y_i = 1] = \pi_i,$$

where the value of π_i depends on the explanatory variable x_i.

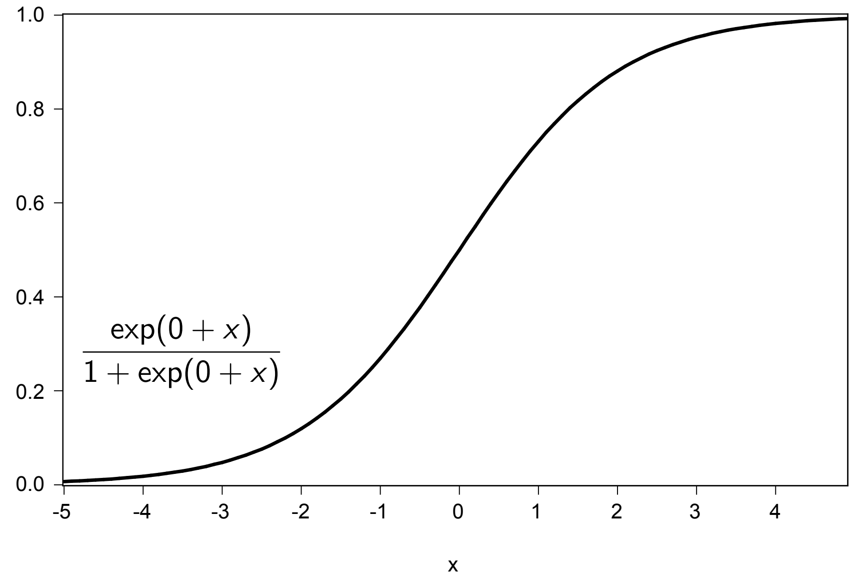
Logit model:

$$\Pr[y_i = 1] = \frac{\exp(\beta_1 + \beta_2 x_i)}{1 + \exp(\beta_1 + \beta_2 x_i)}$$

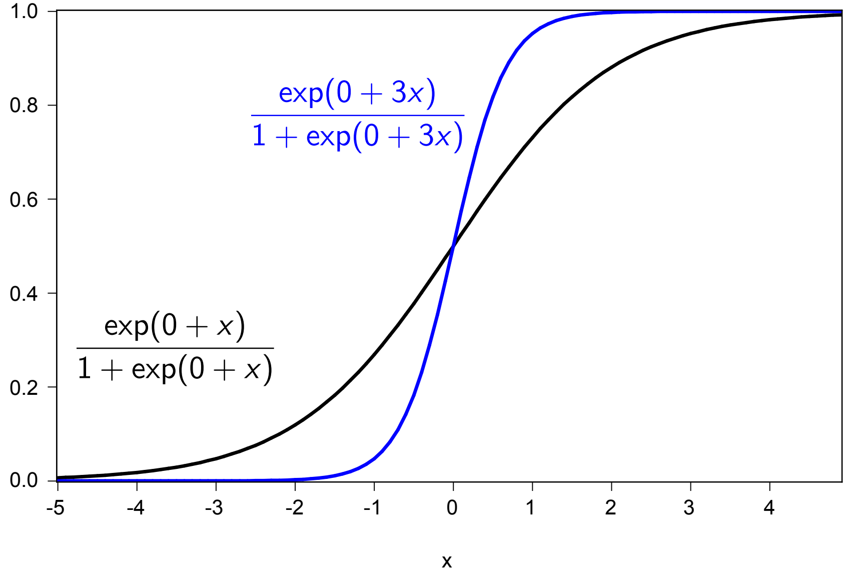
and

$$\Pr[y_i = 0] = 1 - \frac{\exp(\beta_1 + \beta_2 x_i)}{1 + \exp(\beta_1 + \beta_2 x_i)} = \frac{1}{1 + \exp(\beta_1 + \beta_2 x_i)}$$

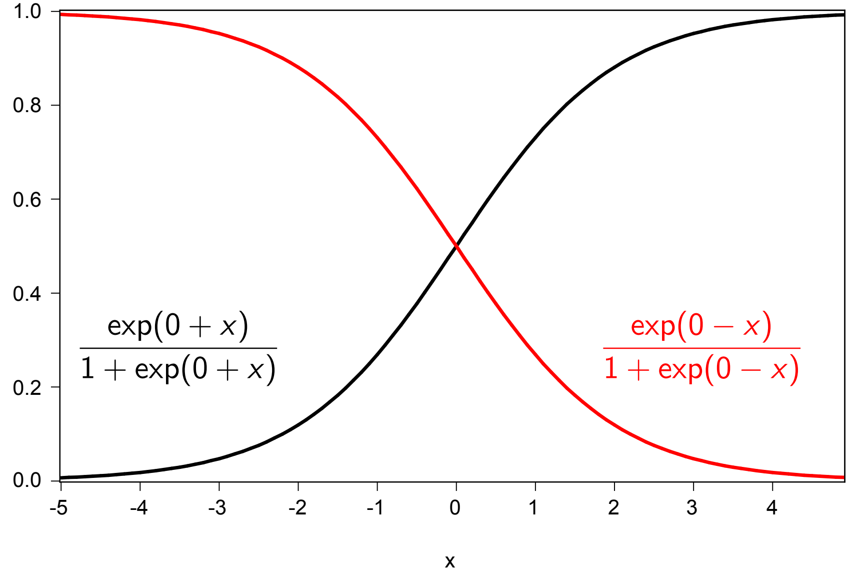
Graphical interpretation



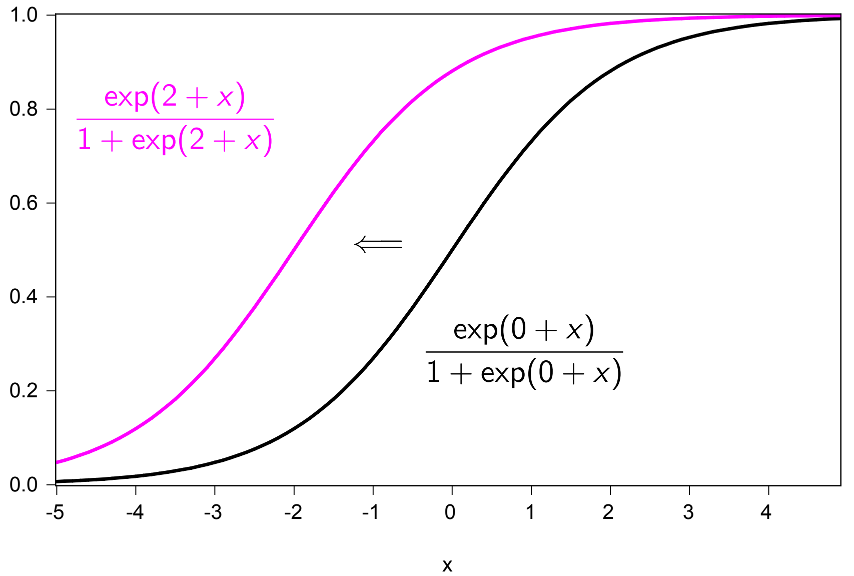
Graphical interpretation



Graphical interpretation



Graphical interpretation



Test question

Test

What happens to the location and shape of the logit function

$$\frac{\exp(\beta_1 + x)}{1 + \exp(\beta_1 + x)}$$

if you change the β_1 parameter from $\beta_1 = 0$ to $\beta_1 = -2$?

The logit function only shifts 2 units to the right.

Odds ratio

Logit model:

$$\Pr[y_i = 1] = \frac{\exp(\beta_1 + \beta_2 x_i)}{1 + \exp(\beta_1 + \beta_2 x_i)}$$
$$\Pr[y_i = 0] = \frac{1}{1 + \exp(\beta_1 + \beta_2 x_i)}$$

Odds ratio:

$$\frac{\Pr[y_i = 1]}{\Pr[y_i = 0]} = \exp(\beta_1 + \beta_2 x_i)$$

Log odds ratio:

$$\log\left(\frac{\Pr[y_i = 1]}{\Pr[y_i = 0]}\right) = \beta_1 + \beta_2 x_i$$

Marginal effect

Marginal effect:

$$\frac{d\Pr[y_i = 1]}{d\ x_i} = \Pr[y_i = 1]\Pr[y_i = 0]\beta_2$$

Change in probability that $y_i = 1$ due to change in x_i.

Average marginal effect:

$$\frac{1}{n}\sum_{i=1}^{n}\frac{d\Pr[y_i = 1]}{d\ x_i} = \left(\frac{1}{n}\sum_{i=1}^{n}\Pr[y_i = 1]\Pr[y_i = 0]\right)\beta_2$$

More explanatory variables

Logit specification with $x_{2i}, \ldots, x_{ki}$ as explanatory variables:

$$\Pr[y_i = 1] = \frac{\exp(\beta_1 + \sum_{j=2}^{k}\beta_j x_{ji})}{1 + \exp(\beta_1 + \sum_{j=2}^{k}\beta_j x_{ji})}$$

Log odds ratio:

$$\log\left(\frac{\Pr[y_i = 1]}{\Pr[y_i = 0]}\right) = \beta_1 + \sum_{j=2}^{k}\beta_j x_{ji}$$

Marginal effect:

$$\frac{\partial\Pr[y_i = 1]}{\partial x_{ji}} = \Pr[y_i = 1]\Pr[y_i = 0]\beta_j \text{ for } j = 2, \ldots, k.$$

Change in probability that $y_i = 1$ due to change in x_{ji}.

Training Exercise 5.2

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).