Econometrics: Methods and Applications

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Peer-graded Assignment: Test Exercise 5

Goals and skills being used:

- Get experience with the interpretation of parameters of the logit model
- Get experience with the interpretation of the effect of dummy variables

Questions

Consider again the application in lecture 5.5, where we have analysed response to a direct mailing using the following logit specification

$$\Pr[\text{resp}_i = 1] = \frac{\exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 \text{male}_i + \beta_2 \text{active}_i + \beta_3 \text{age}_i + \beta_4 (\text{age}_i/10)^2)}$$

For i = 1…925. The maximum likelihood estimates of parameters are given by

Variable	Coefficient	Std. Error	t-value	p-value
Intercept	-2.488	0.890	-2.796	0.005
Male	0.954	0.158	6.029	0.000
Active	0.914	0.185	4.945	0.000
Age	0.070	0.036	1.964	0.050
$(\text{Age}/10)^2$	-0.069	0.034	-2.015	0.044

(a) The marginal effect of activity status is defined as

$$\frac{\partial \Pr[\text{resp}_i = 1]}{\partial \text{active}_i} = \Pr[\text{resp}_i = 1] \Pr[\text{resp}_i = 0] \beta_2.$$

We could use this result to construct an activity status elasticity

$$\frac{\partial \Pr[\text{resp}_i = 1]}{\partial \text{active}_i} \frac{\text{active}_i}{\Pr[\text{resp}_i = 1]} = \Pr[\text{resp}_i = 0] \text{active}_i \beta_2.$$

Use these results to compute the elasticity effect of active status for a 50-year-old active male customer. Do the same for a 50-year-old inactive male customer.

<u>Answer:</u>

$$\Pr[resp_i = 1] = \frac{e^{(\beta_0 + \beta_1 male + \beta_2\, active + \beta_3\, age + \beta_4 \left(\frac{age}{10}\right)^2)}}{1 + e^{(\beta_0 + \beta_1 male + \beta_2\, active + \beta_3\, age + \beta_4 \left(\frac{age}{10}\right)^2)}}$$

$\Pr[resp_i = 0] = 1 - \Pr[resp_i = 1$

$$\Leftrightarrow \Pr[resp_i = 0] = 1 - \frac{e^{(\beta_0 + \beta_1 male + \beta_2\, active + \beta_3\, age + \beta_4 \left(\frac{age}{10}\right)^2)}}{1 + e^{(\beta_0 + \beta_1 male + \beta_2\, active + \beta_3\, age + \beta_4 \left(\frac{age}{10}\right)^2)}}$$

$$\Leftrightarrow \Pr[resp_i = 0] = \frac{1}{1 + e^{(\beta_0 + \beta_1 male + \beta_2\, active + \beta_3\, age + \beta_4 \left(\frac{age}{10}\right)^2)}}$$

Activity status elasticity for a 50-year-old active male customer

elasticity $= \Pr[resp_i = 0] * active_i * \beta_2$ and $\beta_2 = 0.914$ and $active_i = 1$

$$\text{elasticity} = \frac{1}{1 + e^{(-2.488 + 0.954*1 + 0.914*1 + 0.070*50 + (-0.069*(\frac{50}{10})^2)}} * 1 * 0.914$$

$\Leftrightarrow$ elasticity $= \dfrac{1}{1+ e^{(-2.488+0.954*1+0.914*1+ 0.070*50+(-0.069*(\frac{50}{10})^2)}}$ * 1 * 0.914

$\Leftrightarrow$ elasticity $= \dfrac{1}{1+ e^{(1.155)}}$ * 1 * 0.914

$\Leftrightarrow$ elasticity $= \dfrac{1}{1+ e^{1.155}}$ * 1 * 0.914

$\Leftrightarrow$ elasticity $= \dfrac{1}{1+3.1740}$ * 1 * 0.914

$\Leftrightarrow$ elasticity $=$ 0.24 * 1 * 0.914

$\Leftrightarrow$ elasticity $=$ 0.219

Activity status elasticity for a 50-year-old inactive male customer

elasticity $=$ Pr $[resp_i = 0]$ * $active_i$ * β_2 and $\beta_2 = 0.914$ and $active_i = 0$

elasticity $= \dfrac{1}{1+ e^{(-2.488+0.954*1+0.914*0+ 0.070*50+(-0.069*(\frac{50}{10})^2)}}$ * 0 * 0.914

elasticity $=$ 0 Multiplication with 0 leads to meaningless result!

(b) The activity status variable is only a dummy variable and hence it can take only two values. It is therefore better to define the elasticity as

$$\frac{\Pr[\text{resp}_i = 1|\text{active}_i = 1] - \Pr[\text{resp}_i = 1|\text{active}_i = 0]}{\Pr[\text{resp}_i = 1|\text{active}_i = 0]}.$$

Show that you can simplify the expression for the elasticity as

$$(\exp(\beta_2) - 1)\Pr[\text{resp}_i = 0|\text{active}_i = 1].$$

(c) Use the formula in (b) to compute the activity elasticity of 50 years old male active customer.

$$\text{elasticity} = \ (e^{\beta_2}\text{-1}) * \frac{1}{1+ e^{\,(\beta_0+\beta_1 male+ \beta_2\, active+ \beta_3\, age+\beta_4\left(\frac{age}{10}\right)^2)}}$$

and $\beta_2 = 0.914$ and $active_i$ = 1 and age = 50

$$\Leftrightarrow \text{elasticity} = \ (e^{0.914}\text{-1}) * \frac{1}{1+ e^{\,(-2.488+0.954*1+0.914*1+\ 0.070*50+(-0.069*(\frac{50}{10})^2)}}$$

$\Leftrightarrow$ elasticity = 1.494 * 0.24

$\Leftrightarrow$ elasticity = 0.36