Erasmus
School of
Economics

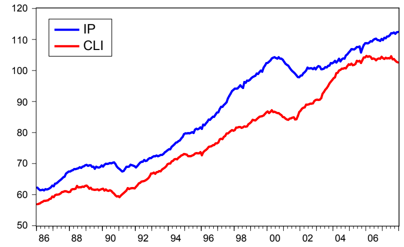
MOOC Econometrics

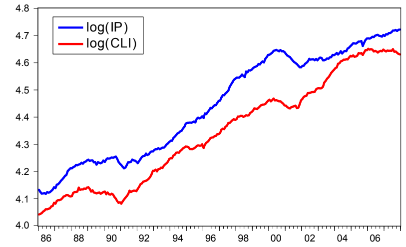
Lecture 6.5 on Time Series: Application

Dick van Dijk, Philip Hans Franses, Christiaan Heij

Erasmus University Rotterdam

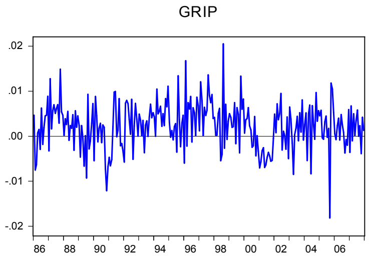
Industrial Production and Composite Leading Index

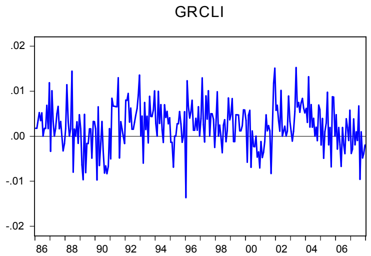




- IP: Industrial production USA (monthly data 1986 - 2007, $n = 264$)
- CLI: Composite Leading Index USA (Conference Board)
- Goal: Forecast IP one quarter (three months) ahead

Monthly growth rates: GRIP and GRCLI



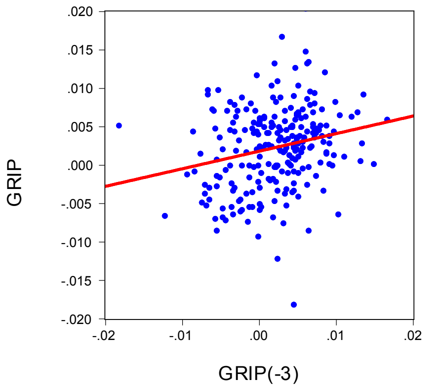


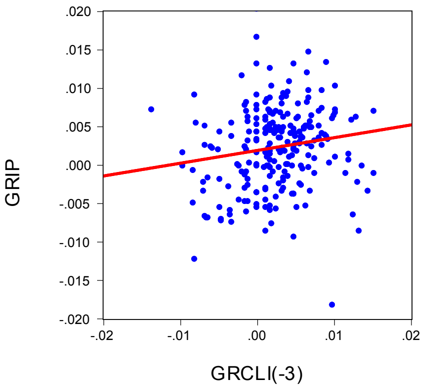
- Monthly growth rates: GRIP $= \Delta\log(\text{IP})$, GRCLI $= \Delta\log(\text{CLI})$
- Estimation sample: 1986 - 2005 ($n = 240$)
- Hold-out forecast sample: 2006 - 2007 ($n = 24$)

Tests on stationarity

- Let y_t denote log(IP) or log(CLI): trend
  ADF: $\Delta y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{j=1}^{3} \gamma_j \Delta y_{t-j} + \varepsilon_t$
  $t_{\hat{\rho}} = -1.6$ for log(IP), $t_{\hat{\rho}} = -1.8$ for log(CLI) $\rightarrow$ not stationary
- Let y_t denote GRIP $= \Delta\log(\text{IP})$ or GRCLI $= \Delta\log(\text{CLI})$: no trend
  ADF: $\Delta y_t = \alpha + \rho y_{t-1} + \sum_{j=1}^{3} \gamma_j \Delta y_{t-j} + \varepsilon_t$
  $t_{\hat{\rho}} = -5.2$ for GRIP, $t_{\hat{\rho}} = -5.6$ for GRCLI $\rightarrow$ stationary
- Engle-Granger test on cointegration:
  Step 1: OLS: $\log(\text{IP}_t) = 0.08 + 1.01\log(\text{CLI}_t) + e_t$
  Step 2: ADF: $\Delta e_t = 0.00 + 0.00t - 0.01e_{t-1} + 0.04\Delta e_{t-1} + \text{res}_t$
  t-value e_{t-1} is $-0.6 > -3.8$ $\rightarrow$ not cointegrated

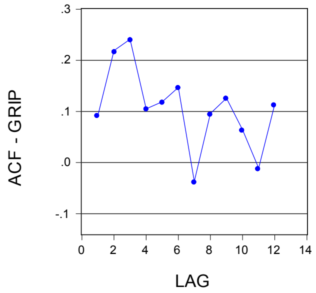
Forecast IP growth rate 3 months ahead

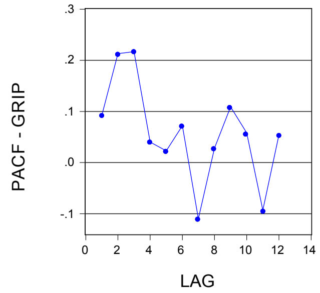




- Forecast GRIP_t with information $\{\text{GRIP}_{t-j}, \text{GRCLI}_{t-j}, j = 3, 4, \ldots\}$.
- Two models: AR for GRIP, and ADL in terms of GRIP and GRCLI.

AR model for GRIP





- $2/\sqrt{n} = 2/\sqrt{240} = 0.13 \rightarrow \text{AR}(3)$
- GRIP_{t-1} and GRIP_{t-2} may not be used
  $\rightarrow$ Start with lags 3-12 and reduce (down-testing).

AR model for GRIP

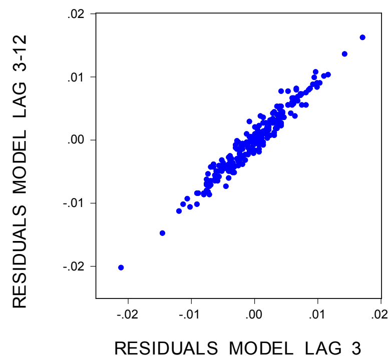
- $\text{GRIP}_t = \alpha + \sum_{j=3}^{L} \beta_j \text{GRIP}_{t-j} + \varepsilon_t$
- $L = 12$: lags 4-12 individually not significant.
- $L = 12$ has $R^2 = 0.0988$, and $L = 3$ gives $R^2 = 0.0519$

Test

Test if model with lags 3-12 can be simplified to one with lag 3 only.
Note: The relevant 5% critical value is 1.9.

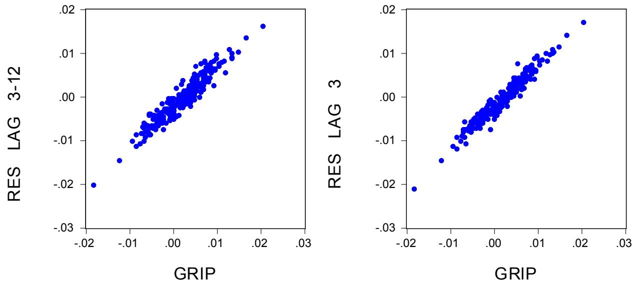
- F-test with $n = 240$, $k = 11$, and $g = 9$.
- $F = \frac{(0.0988-0.0519)/9}{(1-0.0988)/229} = 1.3 < 1.9$.
- Yes, use lag 3 only.

AR model for GRIP



- Both models nearly identical residuals.
- Also nearly identical diagnostics:
  $\rightarrow$ p-value Breusch-Godfrey (6 lags): $p_{12} = 0.03$, $p_3 = 0.03$
  $\rightarrow$ p-value Jarque-Bera: $p_{12} = 0.03$, $p_3 = 0.01$

AR model for GRIP



- Four outliers GRIP cause four associated large residuals.

  High growth: Feb 1996 (1.7%) and Aug 1998 (2.1%)

  Large negative growth: Nov 1990 (-1.2%) and Sep 2005 (-1.8%)

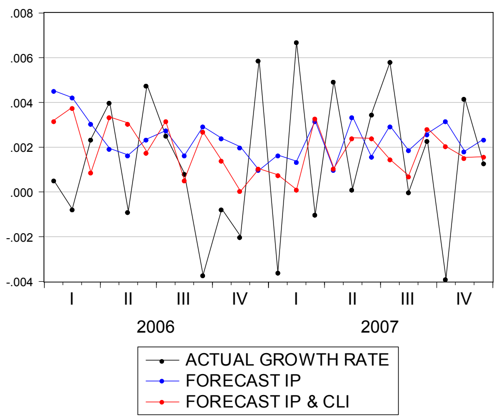
- Our forecast model: $\text{GRIP}_t = 0.0018 + 0.2288\text{GRIP}_{t-3} + e_t$
  ($t_b = 3.6$, $R^2 = 0.052$)

ADL model for GRIP

- Does Composite Leading Index help to predict GRIP 3 months ahead?
- If CLI is 'leading', by how many months?
- ADL: $\text{GRIP}_t = \alpha + \sum_{j=3}^{p} \beta_j \text{GRIP}_{t-j} + \sum_{j=3}^{r} \gamma_j \text{GRCLI}_{t-j} + \varepsilon_t$
- Start with $p = r = 6$ and reduce (down-testing).
- Model: $\text{GRIP}_t = 0.001 + 0.193\text{GRIP}_{t-3} + 0.219\text{GRCLI}_{t-6} + e_t$
  ($t_{b3} = 3.1$, $t_{b6} = 3.2$, $R^2 = 0.092$) $\rightarrow$ CLI leads IP by 6 months
- p-values : Breusch-Godfrey (6 lags): 0.36, no serial correlation
  Jarque-Bera 0.04 (same 4 outliers as before)

Out-of-sample forecast of monthly growth rate IP

- AR (lag 3) and ADL (lags 3 and 6) estimated from data 1986-2005.
- Forecast monthly GRIP for Jan 2006 - Dec 2007 ($n = 24$) and the annual growth rates of IP for the years 2006 and 2007.



Test question

Test

Monthly growth rate of y_t is $g_t^m = \Delta \log(y_t)$, and annual growth rate is $g_t^y = \log(y_t) - \log(y_{t-12})$.
Show that the annual growth rate is simply obtained by adding monthly growth rates over the previous 12 months.

Answer:

- $$g_t^y = \log(y_t) - \log(y_{t-12})$$
  $$= (\log(y_t) - \log(y_{t-1})) + (\log(y_{t-1}) - \log(y_{t-2})) + \ldots$$
  $$+ \ldots + (\log(y_{t-11}) - \log(y_{t-12}))$$
  $$= g_t^m + g_{t-1}^m + \ldots + g_{t-11}^m.$$

Out-of-sample forecast of monthly growth rate IP

- Monthly growth rate IP much fluctuation, not easy to predict.
- Evaluation criteria: RMSE and MAE (see Lecture 3)
  SUM: sum of forecast errors $\sum_{t=1}^{24}(y_t - \widehat{y}_t)$
- Table shows forecast errors for the 24 months in 2006 and 2007.
- CLI improves the monthly IP growth forecast for 3-months ahead.

Model (lags)	AR(3-12)	AR(3)	ADL(AR 3, CLI 6)
RMSE ($\times$100)	0.369	0.367	0.350
MAE ($\times$100)	0.322	0.315	0.290
SUM ($\times$100)	5.240	5.731	4.518

Out-of-sample forecast of annual growth rate IP

- Table shows actual anunal IP growth rate (in %) and forecasts.
- CLI improves annual IP growth forecast considerably.
- Such long-term forecasts are important for firms and investors.

	Actual	Forecast		
		AR(3-12)	AR(3)	ADL(AR 3, CLI 6)
2006	1.288	2.859	3.042	2.492
2007	2.037	2.382	2.689	2.025
2006 and 2007	3.325	5.240	5.731	4.518

TRAINING EXERCISE 6.5

- Train yourself by making the training exercise (see the website).
- After making this exercise, check your answers by studying the webcast solution (also available on the website).