

Optimal Asset Allocation with MS-Excel

Final Assignment

Course Title: Financial Modeling and Business Decisions
Course Code: BUS 150 W
Instructor: Iddo Hadar
Quarter: Summer 2017
Grade Option: Letter Grade

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0 Proposal for an Analytical Asset Allocation Model

Customer/Decision Maker	Members of the Senior Management of a US Pension Fund (Asset Allocation Committee)
Decision	Strategic Asset Allocation Decision of Portfolio weights for various stock markets, taken quarterly. Prepare a list of recommended in- and outflows for the sub-funds (e.g. S&P, NASDAQ) for the the Asset Allocation Committee and Rebalance Pension Fund Master-Portfolio shortly thereafter.
Goal/Objective/Target	Invest the Penion Fund's Master-Portfolio in an optimal asset allocation mix with the highest possible Sharpe ratio available from the investable universe (max. Sharpe Ratio = $E(\text{Excess Return_PF}) / E(\text{Vol}_\text{PF})$)
Key Relationships/Drivers	Stock market returns are stochastic variables and stock returns follow a log normal distribution (LN)
Key Assumptions	Estimate Portfolio $\mu = E(\text{Return_PF})$ = Geometric Average Log Returns of weighted stock constituents (Assumption: All cash flows are reinvested). Estimate Portfolio $\delta = E(\text{STD})$ = Past STD of that portfolio. Correlation $\rho = E(\text{Corr})$ between two or more stocks is equal to past correlations.
Key Constraints	Sum of weights = 100 (i.e. no leverage). Weight of USD (incl. HKD) min. 40%.
Model User(s)	Senior Analyst or Portfolio Manager at a Pension Fund
Source for Raw Data	Yahoo Finance (Stock Market Indices in local currency since Jan 1988 - now, weekly) and FXTOP for currencies (since Jan-1988 - now, monthly averages)

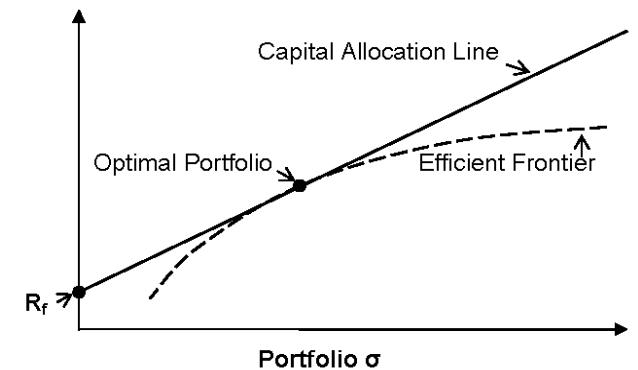
Financial Model

Modern Portfolio Theory (MPT)
 (Harry Markowitz, Merton H. Miller, William F. Sharpe, Nobel Price in Economics 1990)

Capital Allocation Line formula

$$r_p = r_f + \sigma_p \left(\frac{r_m - r_f}{\sigma_m} \right)$$

Expected
Return



Capital Allocation Line \approx Capital Market Line

1

Domestic Asset Allocations

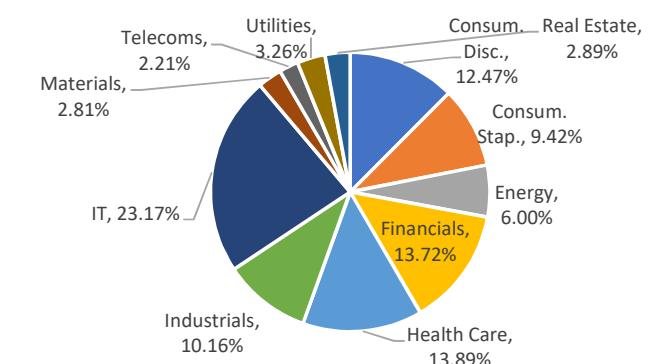
The S&P 500 index is traditional „Default Option“ for US asset allocation



Problems:

High concentration of „large cap“ issuers and strong industry concentration (Information Technology).

STANDARD & POOR'S

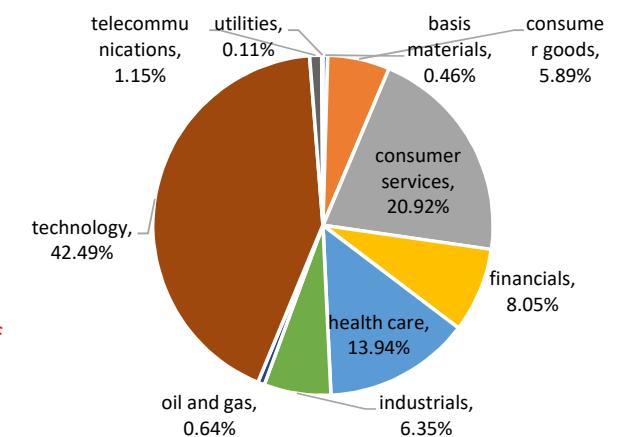


The Standard & Poor's 500, often abbreviated as the S&P 500, or just "the S&P", is an American stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. The S&P 500 index components and their weightings are determined by S&P Dow Jones Indices. It differs from other U.S. stock market indices, such as the Dow Jones Industrial Average or the Nasdaq Composite index, because of its diverse constituency and weighting methodology. It is one of the most commonly followed equity indices, and many consider it one of the best representations of the U.S. stock market, and a bellwether for the U.S. economy. The National Bureau of Economic Research has classified common stocks as a leading indicator of business cycles.

NASDAQ is the traditional „admixture“ to US asset allocation



NASDAQ®



The NASDAQ Composite is a stock market index of the common stocks and similar securities (e.g. ADRs, tracking stocks, limited partnership interests) listed on the NASDAQ stock market.

Along with the Dow Jones Average and S&P 500 it is one of the three most-followed indices in US stock markets. The composition of the NASDAQ Composite is **heavily weighted towards information technology companies**.

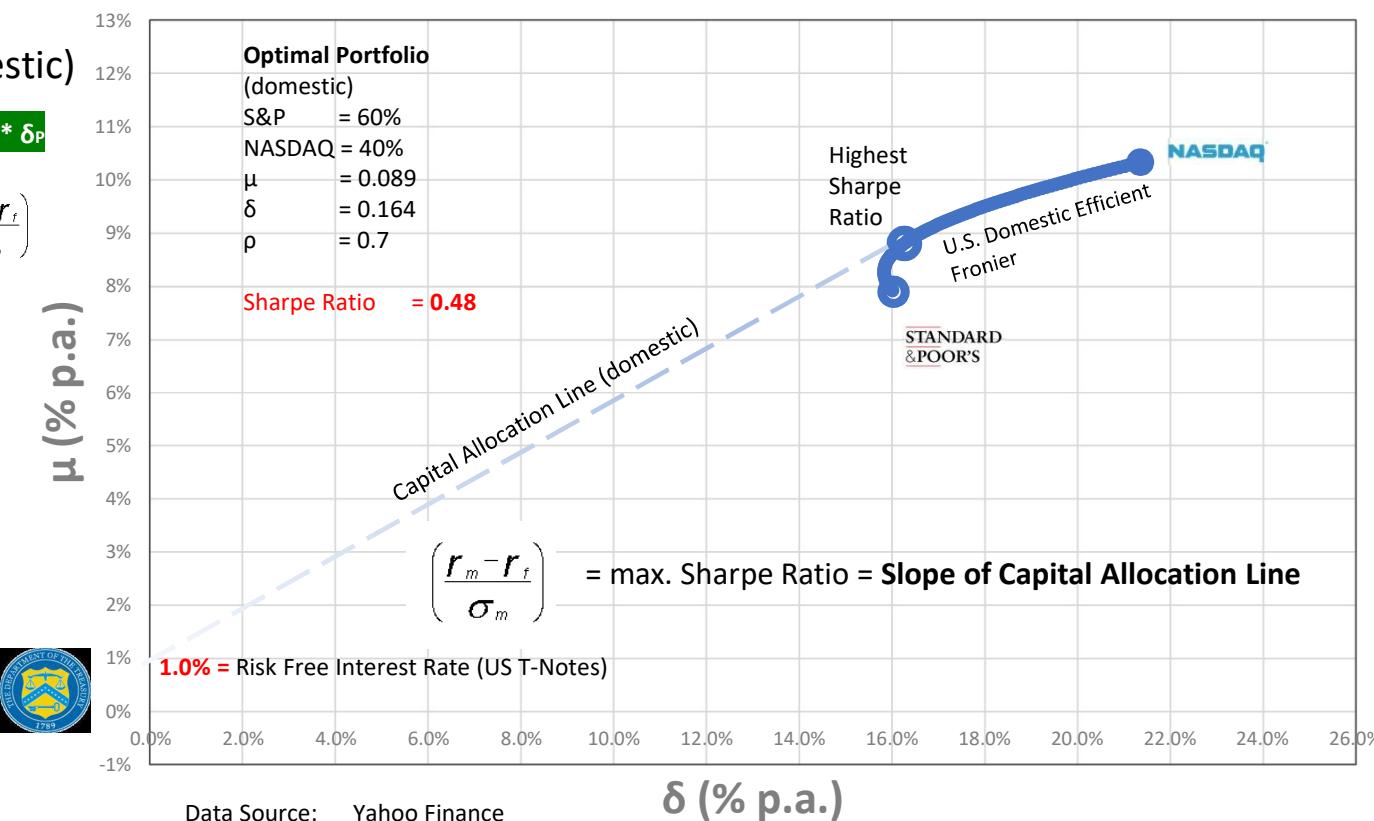


Domestic Mean-variance optimization Risk-Return Expectation (in USD) 1988-2017 (weekly)

CAL (domestic)

$$\mu = 0.01 + 0.48 * \delta_p$$

$$r_p = r_f + \sigma_p \left(\frac{r_m - r_f}{\sigma_m} \right)$$



Financial Model

Modern Portfolio Theory (MPT)
(Harry Markowitz, Merton H. Miller, William F. Sharpe, Nobel Price in Economics 1990)

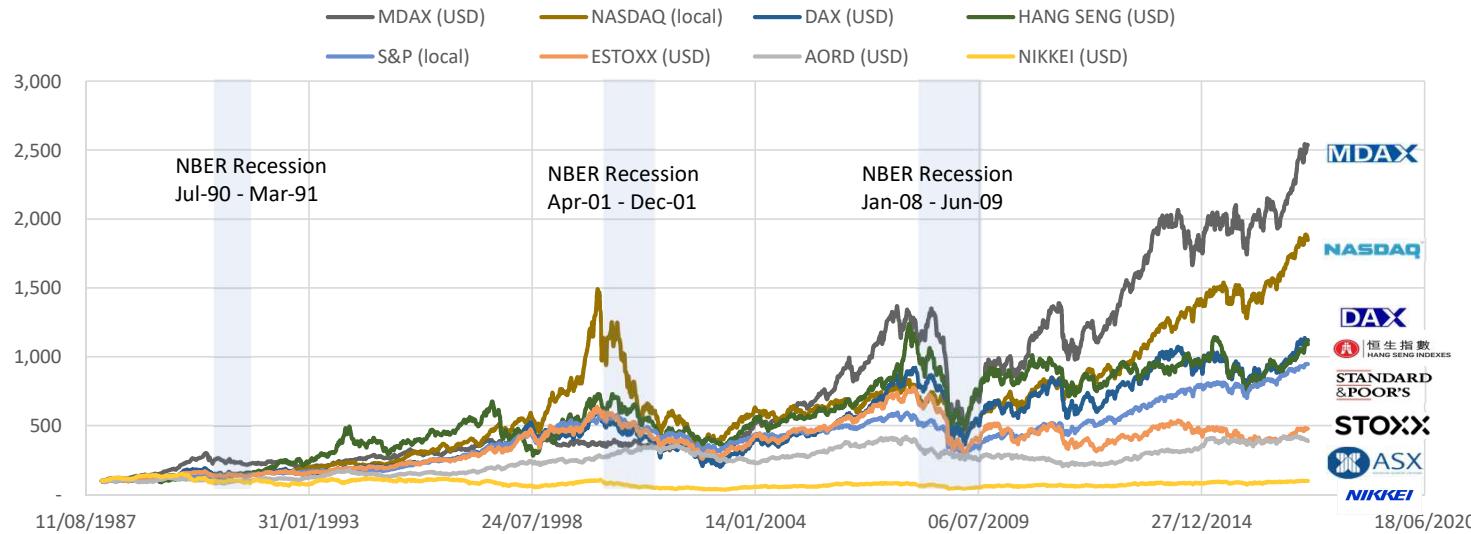
MPT assumptions:

- (1) Investors make rational decisions – i.e., after considering all relevant data investors make unbiased buy and sell decisions.
- (2) Investors want to be compensated for risk – i.e., increased risk must be compensated with higher returns.
- (3) Investors assess risk/return characteristics of their portfolio as a whole.
- (4) Investors minimize risk for a given expected return and maximize expected return for a given level of risk – i.e., investors assemble efficient portfolios.
- (5) Systematic risk assumption is rewarded – non-systematic risk assumption is not rewarded.

The desired outcome of applying MPT to portfolio selection is achieving maximum return while minimizing risk by way of diversification. The MPT implication for assembling a portfolio is an asset should not be evaluated based on its individual risk/return characteristics; rather its addition to a portfolio should be evaluated based on how it impacts the portfolio's overall risk/return characteristics.

<http://financialplanningbodyofknowledge.com>

② Screening International stock markets & data collection



World's Top 10 Countries by Market Cap				
Rank	Market	Mkt Cap (US\$ trillion) October 2016	Mkt Cap (US\$ trillion) October 2003	% Change
1	U.S.	23.8	12.70	87%
2	China	6.6	0.42	1479%
3	Japan	5.2	3.10	68%
4	Hong Kong	4.1	0.83	394%
5	U.K.	3.0	2.20	36%
6	Canada	1.9	0.74	157%
7	France	1.9	1.30	46%
8	Germany	1.8	0.94	91%
9	India	1.7	0.23	639%
10	Switzerland	1.4	0.66	112%
Top 10		51.4	23.12	122%
World		65.6	28.10	133%

Source: Bloomberg www.truewealthpublishing.asia

Geometric Mean annual	E (S&P)	E (NASDAQ)	E (HANG SENG)	E (DAX)	E (MDAX)	E (ESTOXX)	E (NIKKEI)	E (AORD)
	7.9%	10.4%	8.4%	8.5%	11.5%	5.4%	0.0%	4.7%
Standard Deviation annual	Vola (S&P)	Vola (NASDAQ)	Vola (HANG SENG)	Vola (DAX)	Vola (MDAX)	Vola (Euronext)	Vola (NIKKEI)	Vola (AORD)
	16.0%	21.5%	23.8%	23.3%	19.9%	21.7%	22.9%	15.5%
Sharpe Ratio	SharpeRatio (S&P)	SharpeRatio (NASDAQ)	SharpeRatio (HANG SENG)	SharpeRatio (DAX)	SharpeRatio (MDAX)	SharpeRatio (ESTOXX)	SharpeRatio (NIKKEI)	SharpeRatio (AORD)
(Risk Free Rate = 1%)	0.43	0.44	0.31	0.32	0.53	0.20	-0.04	0.24
Log Normal Total Return last 30 years	In TR (S&P)	In TR (NASDAQ)	In TR (Hang Seng)	In TR (DAX)	In TR (MDAX)	In TR (ESTOXX)	In TR (NIKKEI)	In TR (AORD)
	224.9%	291.7%	239.1%	241.8%	323.4%	156.6%	-0.3%	135.8%
STD of Log Normal Returns last 30 years	STD_30yr (S&P)	STD_30yr (NASDAQ)	STD_30yr (HANG SENG)	STD_30yr (DAX)	STD_30yr (MDAX)	STD_30yr (ESTOXX)	STD_30yr (NIKKEI)	STD_30yr (AORD)
	87.3%	116.9%	129.4%	126.5%	108.3%	118.3%	124.4%	84.6%
min. TR (95% Confidence) p.a.	MinReturn (S&P)	MinReturn (NASDAQ)	MinReturn (Hang Seng)	MinReturn (DAX)	MinReturn (MDAX)	MinReturn (ESTOXX)	MinReturn (NIKKEI)	MinReturn (AORD)
	2.0%	2.4%	0.8%	1.0%	3.1%	-1.6%	#NUM!	-0.1%

Start	01 January 1988
End	04 August 2017
Years of Data	29.6
Frequency	weekly
Data points	1,545
Data type	Index values (local currency)
Data source:	Yahoo Finance
FX data (monthly)	FXTOP

3

Mean-Variance Optimization (Solver)

Portfolio Valuation (monthly rebalancing)				
	Weight (w_n)	Annual volatility (σ_n)	Expected returns	$w_n\sigma_n$
S&P500	10%	16.0%	7.9%	1.60%
NASDAQ	25%	21.5%	10.4%	5.37%
HANG SENG	5%	23.8%	8.4%	1.19%
DAX30	0%	23.3%	8.5%	0.00%
MDAX50	55%	19.9%	11.5%	10.94%
EuroSTOXX50	0%	21.7%	5.4%	0.00%
Nikkei 225	0%	22.9%	0.0%	0.00%
AORD	5%	15.5%	4.7%	0.78%
US T-Notes	0%	0.0%	1.0%	0.00%
<u>Sum</u>	<u>100.0%</u>			

	Correlation matrix							
	S&P500	NASDAQ	HANG SENG	DAX30	MDAX50	EuroSTOXX50	Nikkei 225	AORD
S&P500	1	0.7	0.5	0.6	0.6	0.6	0.5	0.5
NASDAQ	0.7	1	0.4	0.6	0.5	0.6	0.4	0.4
HANG SENG	0.5	0.4	1	0.5	0.5	0.5	0.4	0.4
DAX30	0.6	0.6	0.5	1.0	0.8	0.9	0.5	0.3
MDAX50	0.6	0.5	0.5	0.8	1.0	0.8	0.5	0.3
EuroSTOXX50	0.6	0.6	0.5	0.9	0.8	1.0	0.5	0.3
Nikkei 225	0.5	0.4	0.4	0.5	0.5	0.5	1.0	0.3
AORD	0.5	0.4	0.4	0.3	0.3	0.3	0.3	1.0

Expected returns	μ	10.4%
Annual variance		0.0279
Standard Deviation	δ	16.7%
Risk Free Rate		1%
Sharpe Ratio =		0.56

$$\text{Example of an efficient portfolio with the highest possible Sharpe Ratio in this data sample}$$

$$= [w_1 \sigma_1 \dots w_n \sigma_n] \times \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1n} \\ \rho_{21} & 1 & \dots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \dots & \dots & 1 \end{bmatrix} \times \begin{bmatrix} w_1 \sigma_1 \\ \vdots \\ w_n \sigma_n \end{bmatrix}$$

Mean-Variance Optimization Description

1. Set Risk Free Interest Rate
2. **Run Solver**
max. Expected Return by changing stock markets' weights
3. Compare graphically Solver's risk/return output with Efficient Frontier and Security Market line (Chart Asset Allocation).
4. Make sure Solver's output's Sharpe Ratio is higher than 0.5
5. run Solver again with different risk tolerance until you reach the highest Sharpe Ratio (Efficient Portfolio)

③ Create efficient and investable Model Portfolios in accordance with investment guidelines (e.g. only investment grade, no leverage, min.40% USD)

Asset Allocation Decision							
	(3) Blue Ships plus	(2) Global Broad	(1) Opportunity	S&P500	MDAX50	NASDAQ	Typical Solver-Output
δ (% p.a.)	16.1%	16.7%	18.1%	16.0%	19.9%	21.5%	16.8%
μ (% p.a.)	9.8%	10.4%	11.1%	7.9%	11.5%	10.4%	10.4%
Sharpe Ratio	0.55	0.56	0.56	0.43	0.53	0.44	0.56
S&P500	40%	10%	0%	100%			16.7%
NASDAQ	20%	25%	40%			100%	28.3%
HANG SENG	0%	5%	0%				1.7%
DAX30	0%	0%	0%				0.0%
MDAX50	40%	55%	60%		100%		51.6%
EuroSTO XX50	0%	0%					0.0%
Nikkei 225	0%	0%					0.0%
AORD	0%	5%					1.7%
Total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
USD weight	60.0%	40.0%	40.0%	100.0%	0.0%	100.0%	46.6%

Financial Model

Modern Portfolio Theory
(Harry Markowitz, Merton H. Miller,
William F. Sharpe, Nobel Price in
Economics 1990).

← Highest Sharpe Ratio (=most efficient Portfolios)

Business Decisions

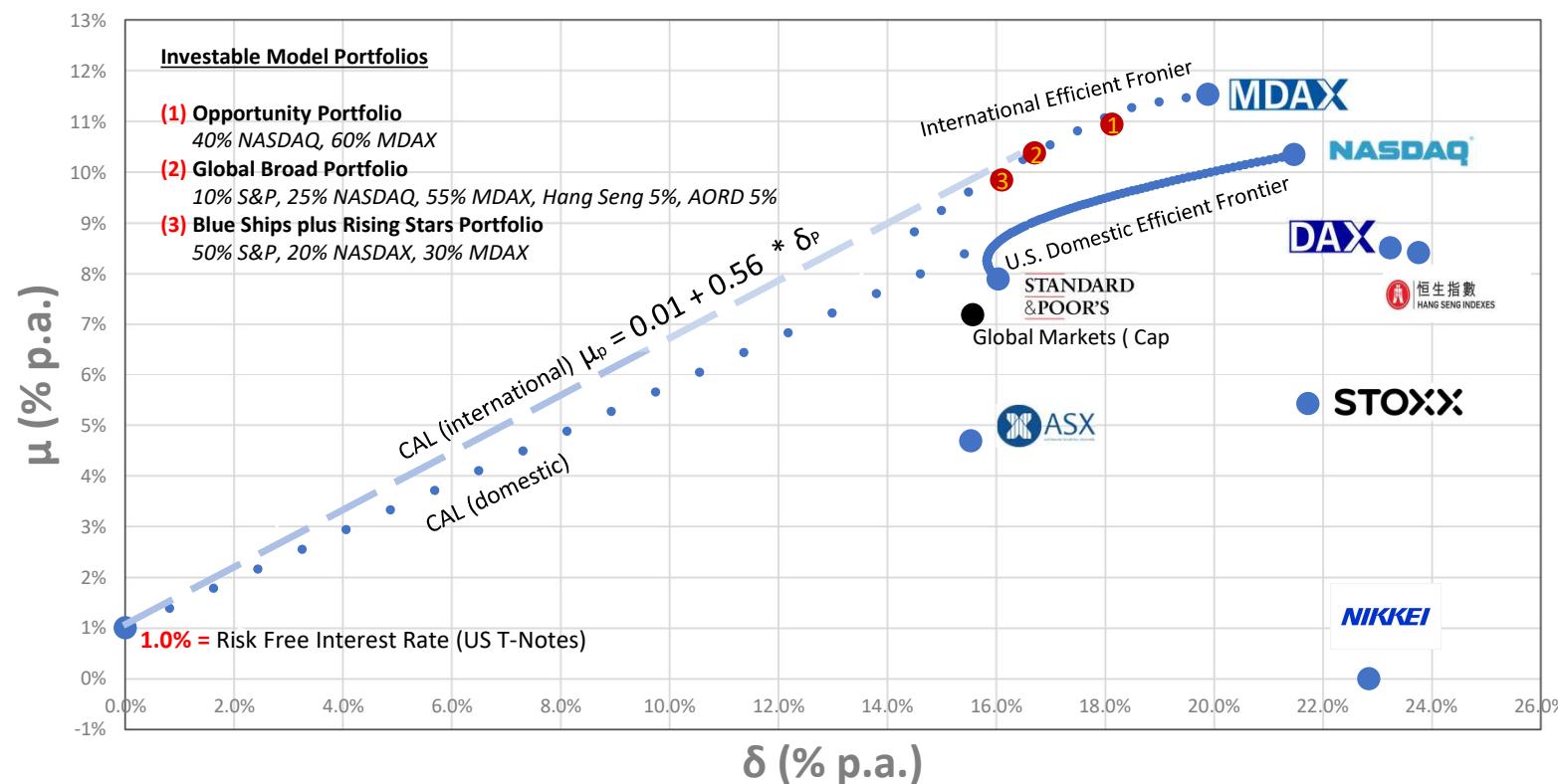
Invest only in efficient portfolios and combine them with investments in credit risk free US-Treasury Notes of an appropriate maturity to reduce the portfolio volatility if necessary (i.e. move along the „International Efficient Frontier Incl. Risk Free Investments“).

For example, 10 years before out payment starts at age 65 a defined contribution pension plan starts shifting 10% of a policy holder's assets into 10yr T-Notes. The following year the risk-free allocation is increased to 20% by buying 9yr T-Notes

Asset Allocation Mean-Variance Optimization

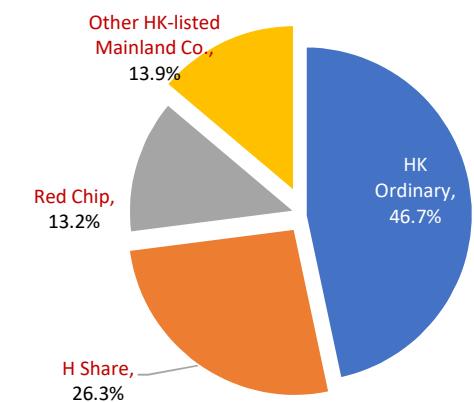
Risk-Return Expectation (in USD)

1988-2017 (weekly)



Key Outcome:
The international Capital Allocation Line $\text{CAL}_{\text{Intern.}}$ offers higher expected return for every level of expected risk than the CAL_{us}

5 Break down multiple risk factors



The Hang Seng Index (恒生指數) is a freefloat-adjusted market capitalization-weighted stock market index in Hong Kong. These 50 constituent companies represent about 58% of the capitalisation of the Hong Kong Stock Exchange.

Break down industry and country risk weightings (Pivot tables)



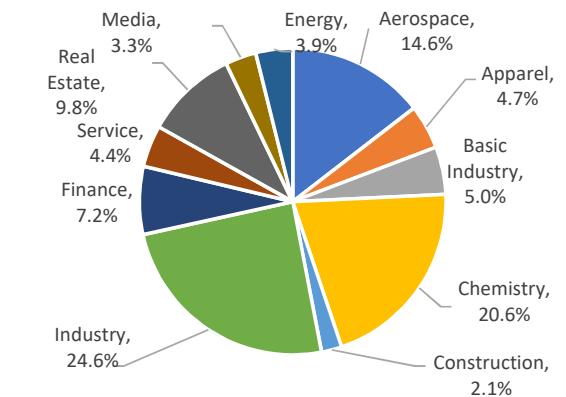
Components						
www.hsi.com.hk						
No.	Stock Code	ISIN Code	Company Name	Industry Classification	Share Type	Weighting (%)
1	700	KYG875721634	Tencent	Information Technology	Other HK-listed Mainland Co.	11.0%
2	5	GB0005405286	HSBC Holdings	Financials	HK Ordinary	10.9%
3	1299	HK0000069689	AIA	Financials	HK Ordinary	8.6%
4	939	CNE1000002H1	CCB	Financials	H Share	8.1%
5	941	HK0941009539	China Mobile	Telecommunications	Red Chip	6.0%
6	1398	CNE1000003G1	ICBC	Financials	H Share	4.7%
7	3988	CNE1000001Z5	Bank of China	Financials	H Share	3.5%
8	2318	CNE1000003X6	Ping An	Financials	H Share	3.5%
9	1	KYG217651051	CKH Holdings	Conglomerates	HK Ordinary	3.2%
10	388	HK0388045442	HKEx	Financials	HK Ordinary	3.0%
11	2628	CNE1000002L3	China Life	Financials	H Share	2.1%
12	1113	KYG2103F1019	CK Property	Properties & Construction	HK Ordinary	2.0%
13	2	HK0002007356	CLP Holdings	Utilities	HK Ordinary	1.8%
14	16	HK0016000132	SHK Ppt	Properties & Construction	HK Ordinary	1.8%
15	883	HK0883013259	CNOOC	Energy	Red Chip	1.8%
16	386	CNE1000002Q2	Sinopec Corp	Energy	H Share	1.8%
17	2388	HK2388011192	BOC Hong Kong	Financials	HK Ordinary	1.6%
18	823	HK0823032773	Link REIT	Properties & Construction	HK Ordinary	1.6%
19	11	HK0011000095	Hang Seng Bank	Financials	HK Ordinary	1.5%
20	3	HK0003000038	HK & China Gas	Utilities	HK Ordinary	1.4%
						79.9%

Sum of Weighting (%)	Column Labels			
	Red Chip	Other HK-listed Mainland Co.	HK Ordinary	H Share
Conglomerates	0.8%		3.7%	4.5%
Consumer Goods	0.5%	2.0%		2.5%
Consumer Services			3.2%	3.2%
Energy	2.1%		3.7%	5.8%
Financials			26.2%	22.6%
Industrials	0.4%			0.4%
Information Technology	0.4%	11.9%		12.3%
Properties & Construction	2.0%		8.5%	10.5%
Telecommunications	6.7%			6.7%
Utilities	0.3%		5.1%	5.4%
Grand Total	13.2%	13.9%	46.7%	26.3%
				100.0%

H-shares are shares of a company incorporated in the **Chinese mainland** that is listed on the Hong Kong Stock Exchange or other foreign exchange.

Red Chip'A company based in Mainland China that is incorporated internationally and listed on the Hong Kong Stock Exchange. Red chip stocks are expected to maintain the filing and reporting requirements of the Hong Kong exchange, which makes them a main outlet for foreign investors who wish to participate in the rapid growth of the Chinese economy. Red chips may be issued in addition to A-shares in the same companies, although only Chinese citizens can invest in A-shares

Analyse multiple Risk factors



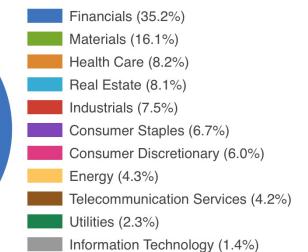
The MDAX is a stock index which lists German companies. The index is calculated by Deutsche Börse.

It includes the 50 Prime Standard shares from sectors **excluding technology** that rank immediately below the companies included in the DAX index. The company size is based on terms of order book volume and market capitalization.

Analyse industry Risk/Return factors for stocks in indices



Row Labels	Sum of Index Weight	Column Labels
Aerospace & Defense	1%	Advertising Spending Business Climate Civil Aerospace Spending Civil Aerospace Spending & Military Spending Commodity Prices Consumer Spending Discretionary Consumer Spending Global luxury spending Government Subsidies Interest Rates Oil Price Potash Price Public Health Insurance Spending Steel Price Grand Total
Alternative Energy		15%
Apparel		
Automotive	2%	
Basic Industry	1%	
Capital Goods	9%	
Chemical Industry		
Construction		
Consumer Services		
Energy		
Financial		
Furniture		
Industrial		
Media	1%	1%
Pharma		
Real Estate		
Grand Total	1%	12% 1% 15% 1% 18% 6% 4% 19% 15% 2% 3% 1% 100%

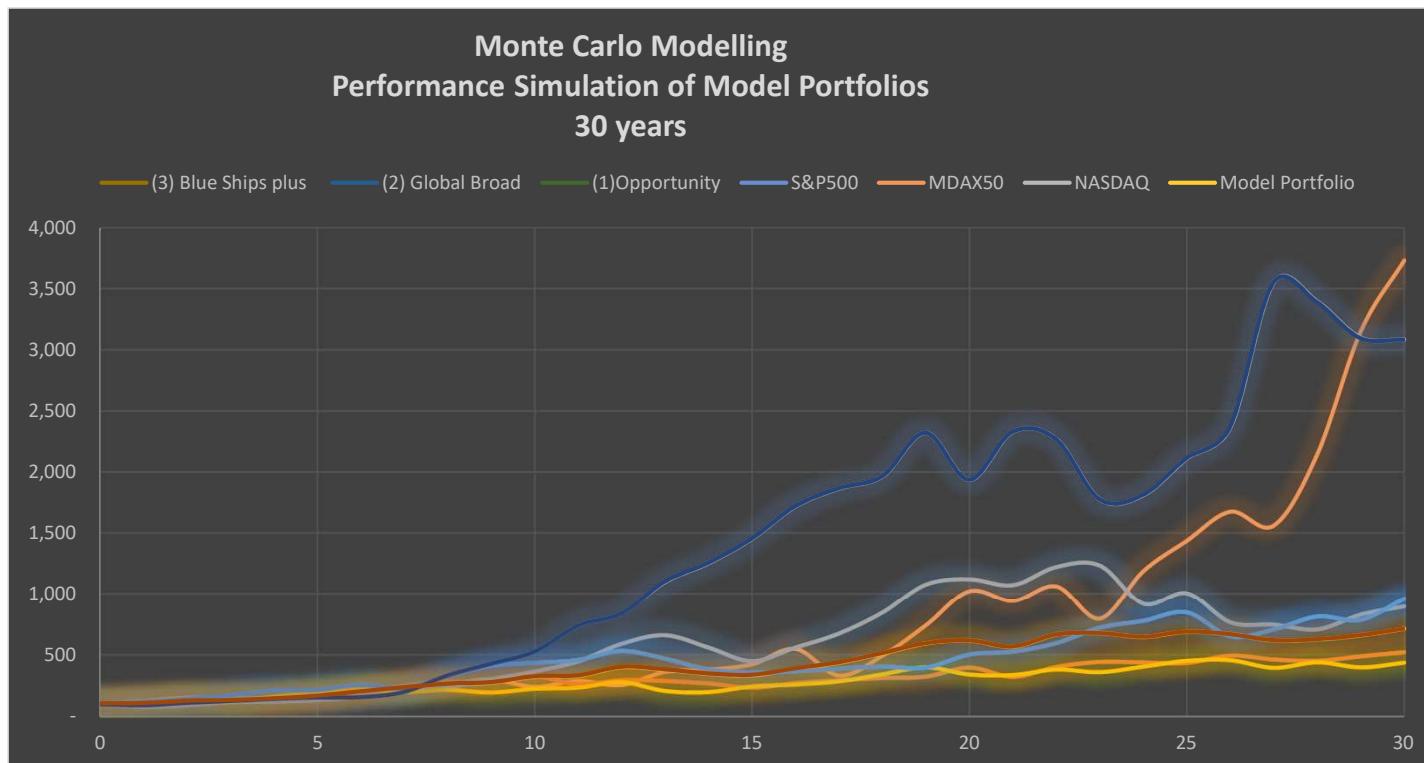


Established in January 1980, the All Ordinaries (colloquially known as the "All Ords"; also known as the All Ordinaries Index, AOI) is the oldest index of shares in Australia. It is made up of the share prices for 500 of the largest companies listed on the Australian Securities Exchange (ASX).

The market capitalization of the companies included in the All Ords index amounts to over 95% of the value of all shares listed on the ASX. The 3-letter exchange ticker in Australia for the All Ordinaries is "XAO".

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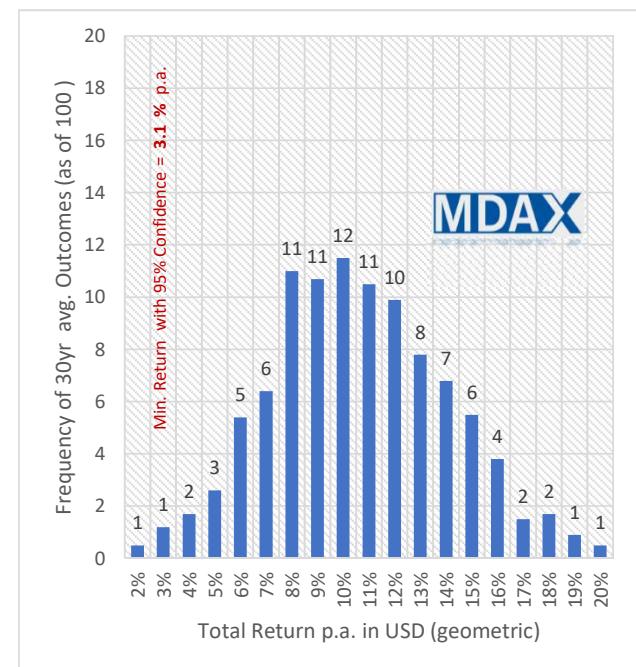
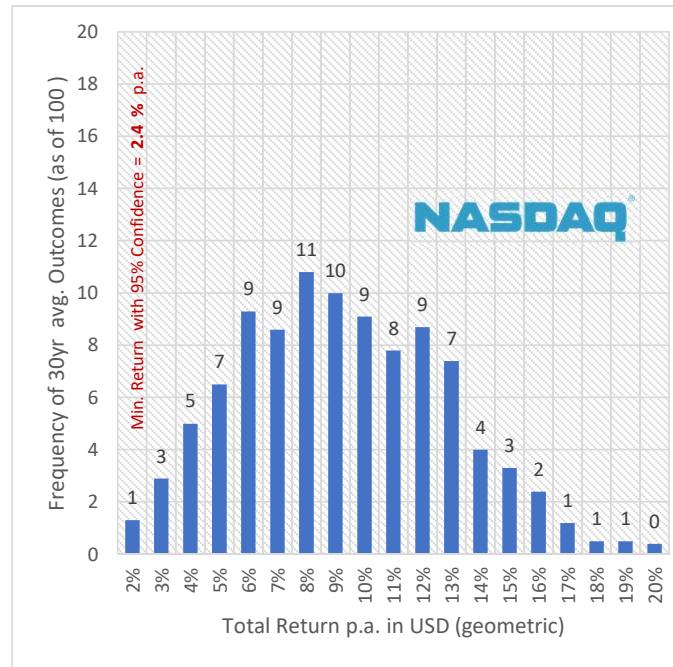
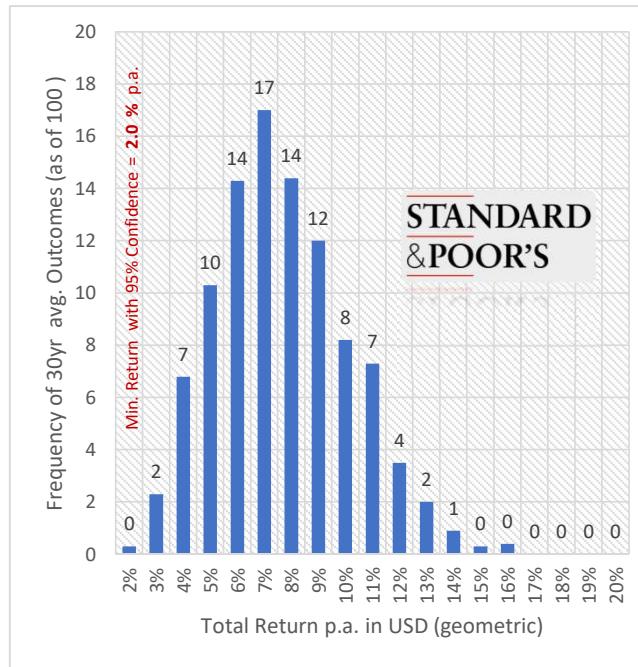
Monte Carlo Simulations



WINSTON, Wayne (2016)
»Microsoft Excel Data
Analysis and Business
Modeling«, 5th edition,
Chapter 74

30-year Monte Carlo Simulation (1000-points)

Histogram of Average 30yr performance



<u>S&P 500</u>			
Cyclical Gain			
	Period-end	Best	Worst
Average	973%	1046%	91%
Annualized	7.9%	8.1%	-0.3%
Most Extreme		8198%	25%

<u>NASDAQ</u>			
Cyclical Gain			
	Period-end	Best	Worst
Average	1842%	2023%	86%
Annualized	10.2%	10.5%	-0.5%
Most Extreme		30841%	9%

<u>MDAX</u>			
Cyclical Gain			
	Period-end	Best	Worst
Average	2438%	2623%	90%
Annualized	11.2%	11.5%	-0.3%
Most Extreme		37579%	14%

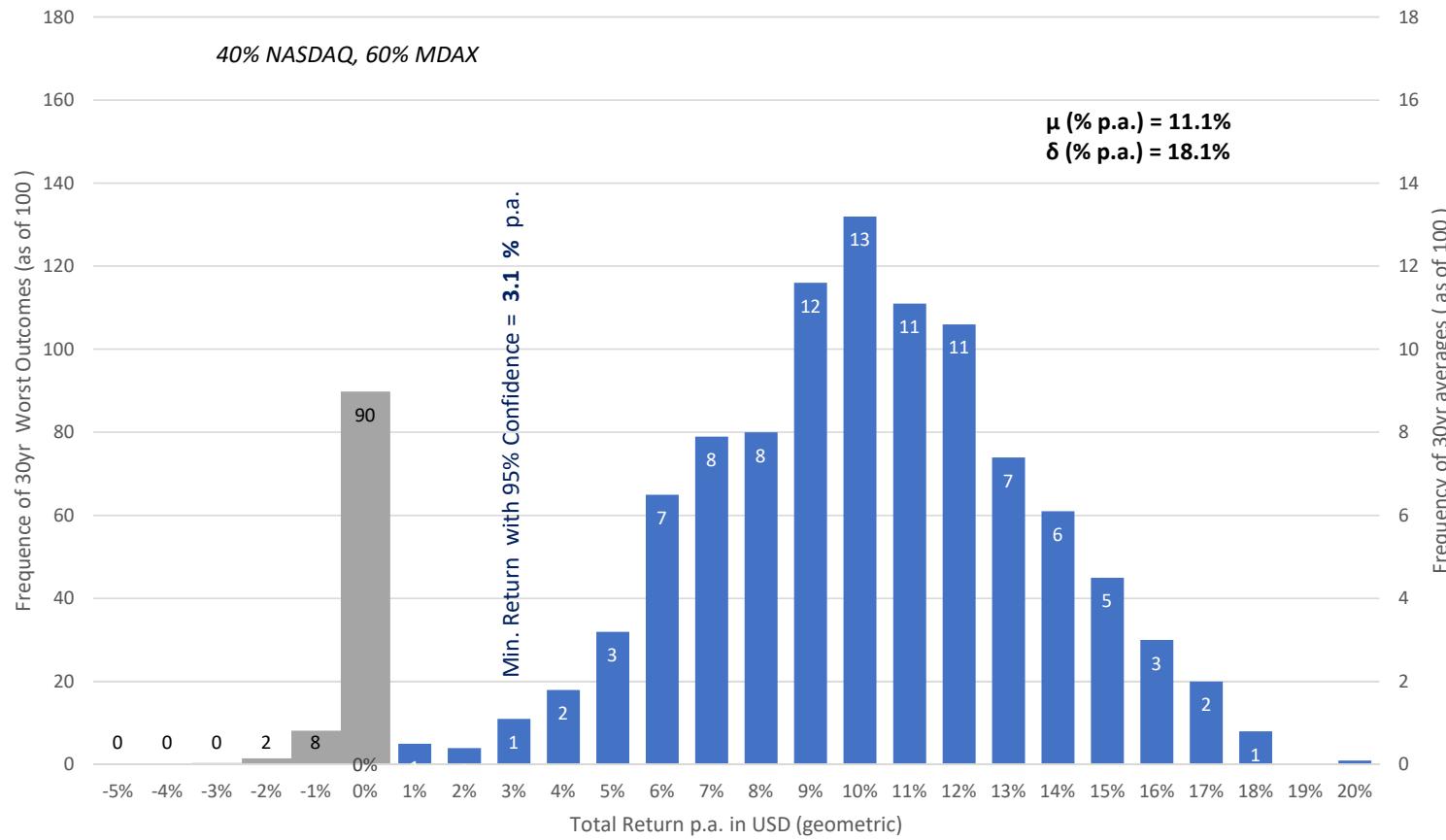


(1) Opportunity Portfolio

30-year Monte Carlo Simulation (1000-points)



Histogram of Worst 30yr performance (left) and of Average 30yr performance (right)



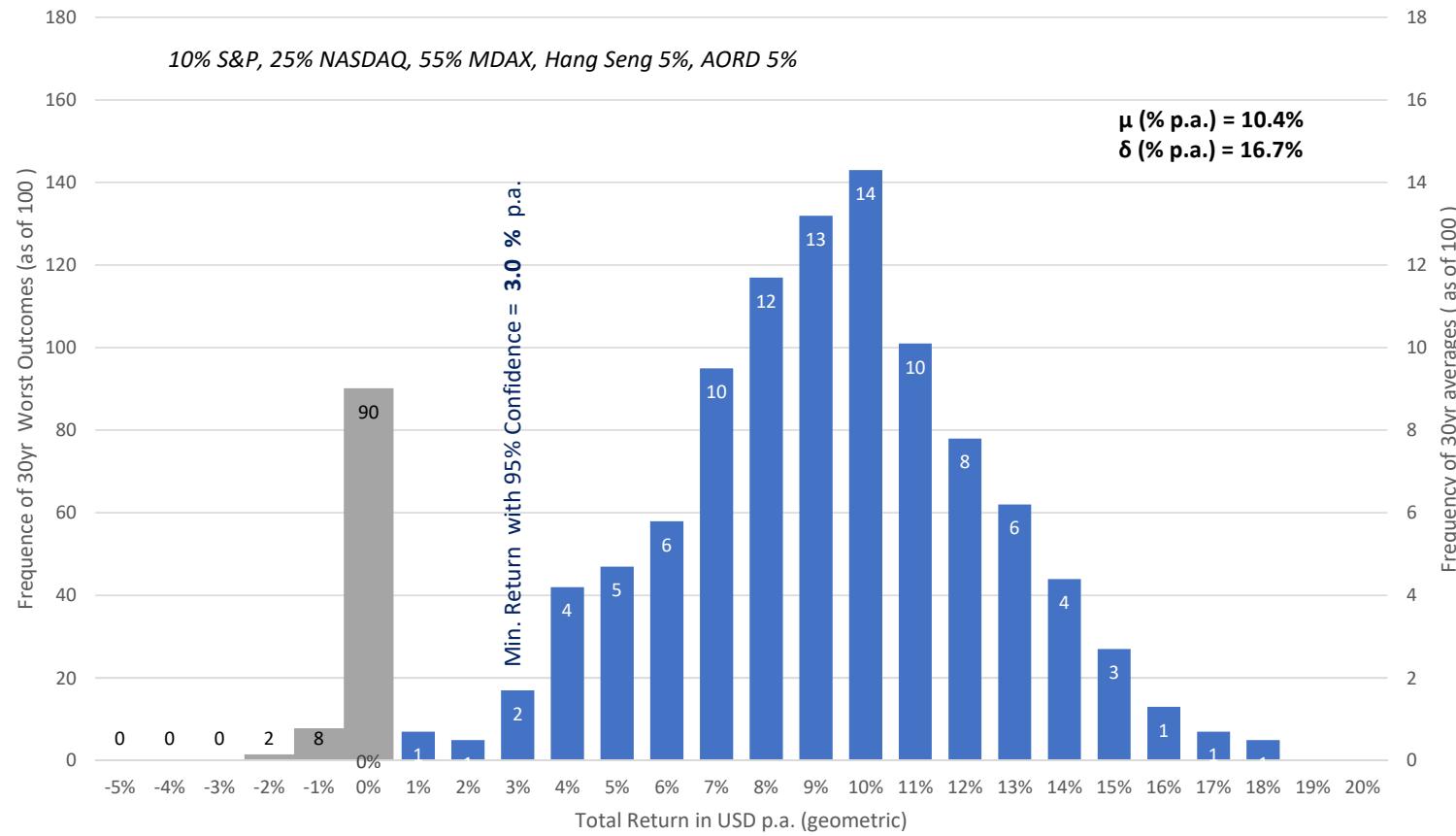


(2) Global Broad Portfolio



30-year Monte Carlo Simulation (1000-points)

Histogram of Worst 30yr performance (left) and of Average 30yr performance (right)



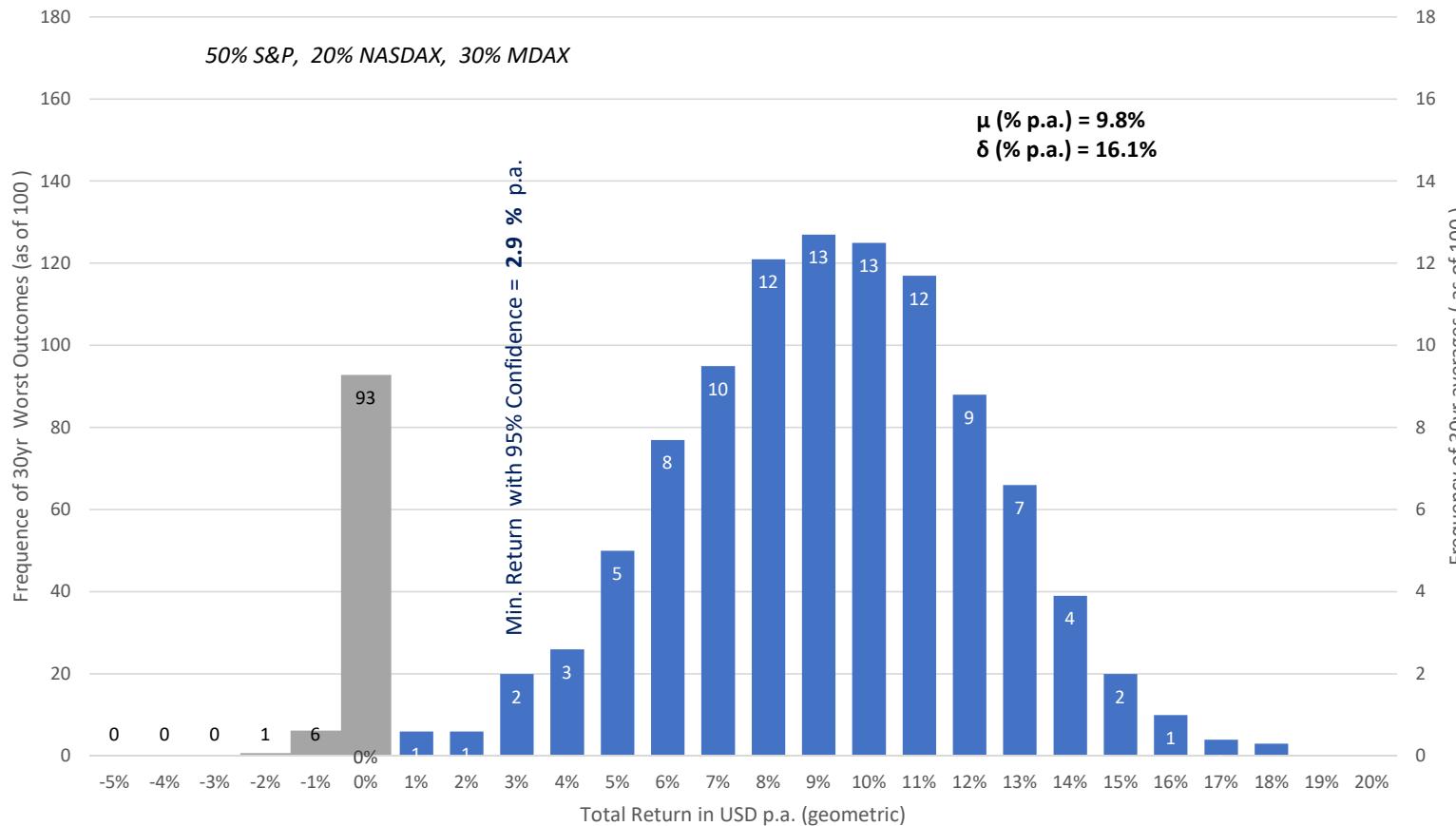


(3) Blue Ships + Rising Stars Portfolio

30-year Monte Carlo Simulation (1000-points)



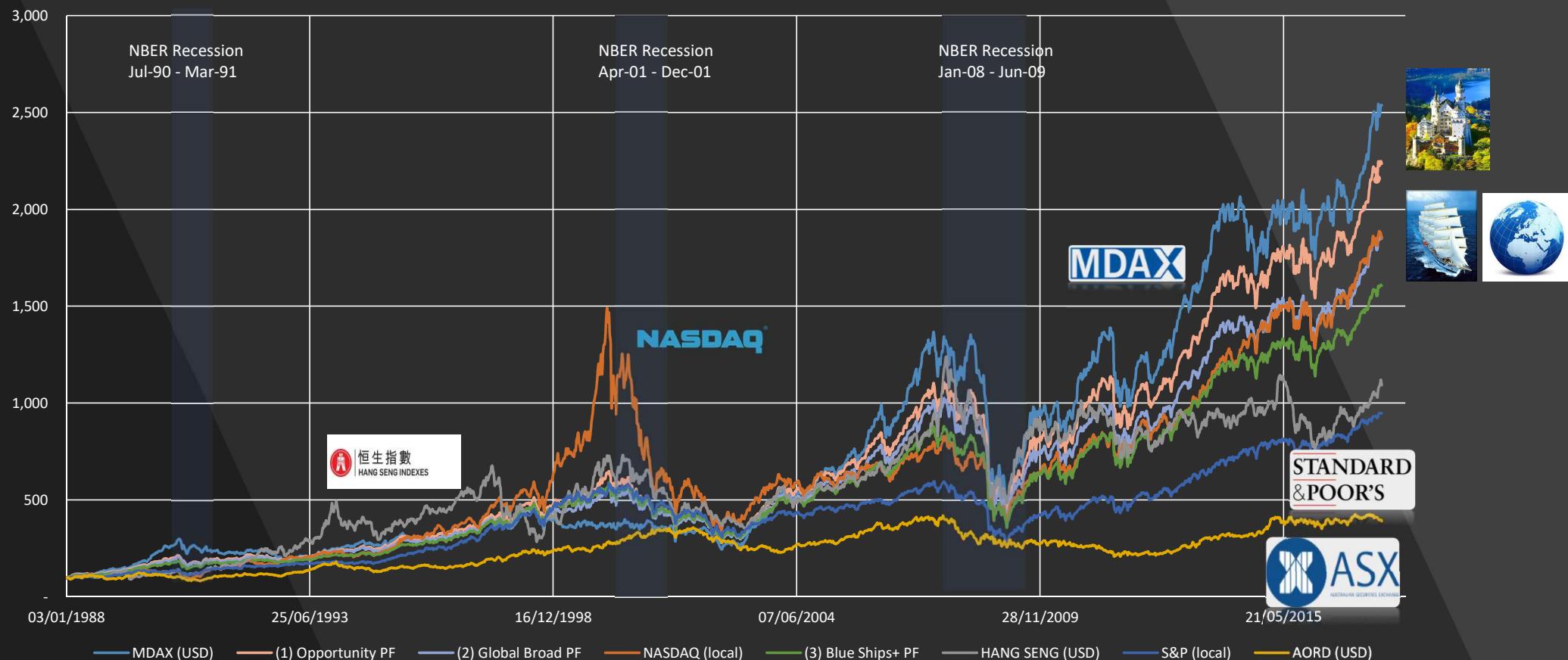
Histogram of Worst 30yr performance (left) and of Average 30yr performance (right)



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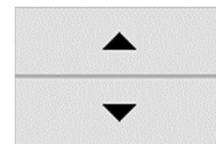
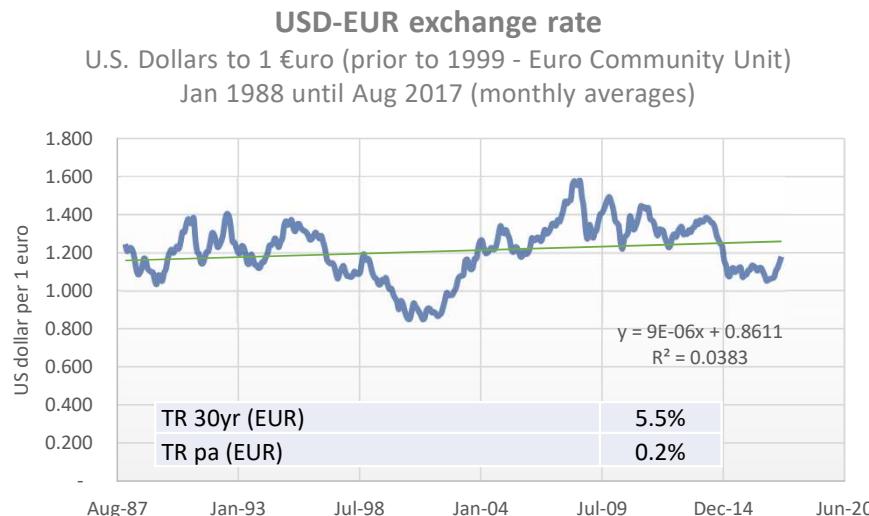
Back Testing Model Portfolios

Total Return in US\$ (last 30 years)



8

Currency Risk Modeling



Characteristics of distribution

Currenty EUR exchange rate **1.1896**

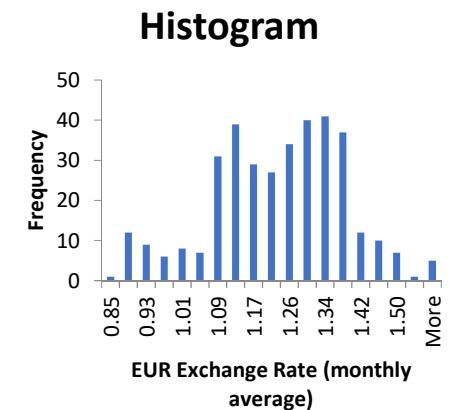
Standard Deviation **14.9%**

Simulated EUR exchange rate **1.0000**

is -1.28 standard deviations from the mean

Likelihood of 1.0000 or lower being the exchange rate? **10% Confidence**

90% chance for EUR exchange rate to stay above 1.0000 or better



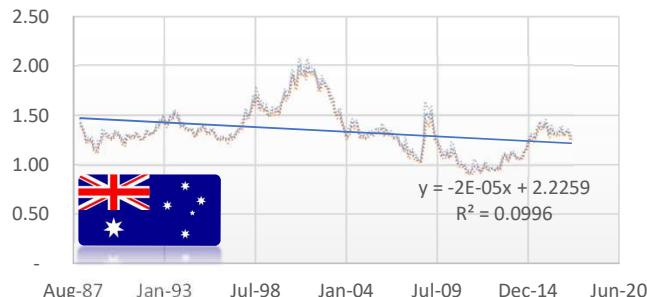
Assumption:

EUR Exchange Rate is random without a trend due to similar inflation rates in Euroland and in the USA. The Bretton Woods Quasi Gold Standard ended in early 1970's followed by a period of a high inflation rates. In the 1980's a system of independent central banks was established in both the USA and in most Western European Countries. Those independent Central Banks target money supply (e.g. German Bundesbank broad money supply M3) or inflation rates of 2% p.a. (e.g. US Federal Reserve, ECB). Therefore, EUR is volatile but mean reversing to Purchasing Power Parity (PPP) levels (currently at approx. 1.2200 according to OECD research).

JYP-USD Exchange Rate (How many JPY does 1 USD buy)



AUD-USD Exchange Rate (How many AUD does 1 USD buy)



ZAR-USD Exchange Rate (How many ZAR does 1 USD buy)



Assumption:

The JPY exchange rate is random with an appreciating trend due to lower inflation rates and lower nominal interest rates in Japan. Nevertheless, this exchange rate is highly political and levels below 100 are generally perceived to harm Japanese exports.

Economic Theory: Uncovered Interest Parity

Uncovered Interest Rate Parity Formula and Example
The formula for uncovered interest rate parity takes into account the following variables:

$E(t+k) / S(t) = \text{the expected rate of change in the exchange rate, which is simply the projected exchange rate at time } (t+k) \text{ divided by the spot rate at time } t$

$k = \text{the number of time periods into the future from time } t$

$i(c) = \text{the foreign interest rate}$

$i(d) = \text{the domestic interest rate.}$

Using these variables, the formula is:

$$(1 + i(d)) = E(t+k) / S(t) \times (1 + i(c))$$

03/09/2017

Assumption:

The AUD Exchange Rate is random with a small appreciating trend due to the "Dutch disease".

The term "**Dutch disease**" was coined by The Economist magazine in 1977. The magazine was analyzing a crisis taking place in the Netherlands following discoveries of vast natural gas deposits in the North Sea in 1959. The newfound wealth and massive exports of oil caused the Dutch guilder to rise sharply, making exports of all nonoil products less competitive on the world market. Unemployment rose from 1.1% to 5.1%, and capital investment in the country dropped.

Dutch disease became widely used in economics to describe the paradoxical situation where seemingly good news, such as the discovery of large oil reserves, turns out to have a negative impact on a country's broader economy.

Assumption:

The ZAR exchange rate is random with a strongly depreciating trend due to significantly higher inflation rates and higher nominal interest rates in South Africa. In addition, domestic political uncertainty (e.g. Banking crisis, elections) and falling commodity prices depress this exchange rate.

Economic Theory: Uncovered Interest Parity, Emerging Markets risks

Recent News on Financial Times (JULY 21, 2017) by:
Nicholas Megaw :

The government's **credit rating** has already been **downgraded to junk status by Fitch and S&P** since president Jacob Zuma sacked his respected finance minister earlier this year. Moody's also cut the government to one notch above speculative grade, and a further downgrade from would force South Africa's expulsion from the widely-tracked World Government Bond Index, a move that would likely cause a sharp outflow from South African assets.

BUS 150 W, Summer 2017, Final Assignment, Peter Schuld

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Step 9 Conclusion

- **Conclusion**

The inclusion of international stock markets into the asset allocation of US investors improves their risk/return profile. Portfolios on the International Efficient Frontier have higher Sharpe Ratios than what is available from domestic diversification (i.e. higher Sharpe Ratios than portfolios on the domestic Efficient Frontier).

- **Business Decisions**

The inclusion of the German MDAX index (Mid-Cap ex. Technology), HK Chinese HENG SENG (Exposure to Mainland China) and Sydney All Ordinary (Comodities) is most attractive.

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