In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns
import warnings
from scipy.stats import anderson, expon, norm, uniform, weibull_min, gumbel_l, g
from statsmodels.stats.diagnostic import anderson_statistic
from scipy.optimize import minimize, brentq, LinearConstraint
from sklearn.mixture import GaussianMixture
```

In [ ]:

```
data = pd.read_csv('CFPREC_data1.csv')
data['diff'] = data['CFP'] - data['TFM']
data.head()
```

Out[ ]:

|  | Item | characteristic | CFP | TFM | diff |
|---|---|---|---|---|---|
| 0 | 4021.0 | Length | 1516.845 | 1517.221 | -0.376 |
| 1 | 4027.0 | Length | 2116.209 | 2116.412 | -0.203 |
| 2 | 4025.0 | Length | 1362.367 | 1362.914 | -0.547 |
| 3 | 13023.0 | Length | 1077.254 | 1077.332 | -0.078 |
| 4 | 4028.0 | Length | 2709.086 | 2709.821 | -0.735 |

In [ ]:

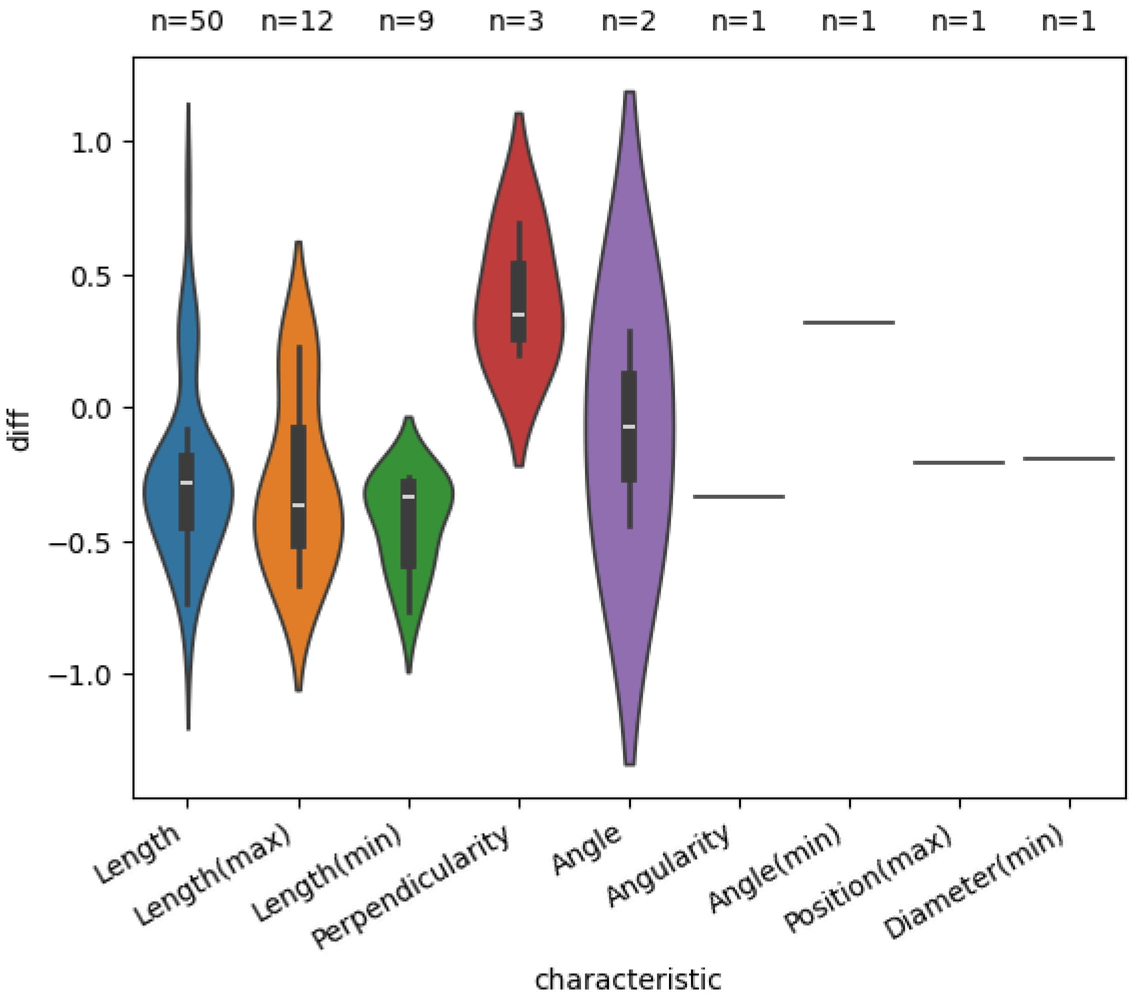
```
characteristic_summary = data['characteristic'].value_counts().reset_index()
characteristic_summary.columns = ['Characteristic', 'Count']
characteristic_summary
```

Out[ ]:

|  | Characteristic | Count |
|---|---|---|
| 0 | Length | 50 |
| 1 | Length(max) | 12 |
| 2 | Length(min) | 9 |
| 3 | Perpendicularity | 3 |
| 4 | Angle | 2 |
| 5 | Angularity | 1 |
| 6 | Angle(min) | 1 |
| 7 | Position(max) | 1 |
| 8 | Diameter(min) | 1 |

In [ ]:

```
warnings.filterwarnings('ignore')
sns.violinplot(x=data['characteristic'], y=data['diff'], hue=data['characteristi
ax = plt.gca()
ax.set_xticklabels(ax.get_xticklabels(), rotation=30, ha='right')
h = ax.get_ylim()[1]
for i, row in characteristic_summary.iterrows():
    plt.annotate(f"n={row['Count']}", (i, h), xytext=(0, 10), textcoords='offset
plt.show()
```



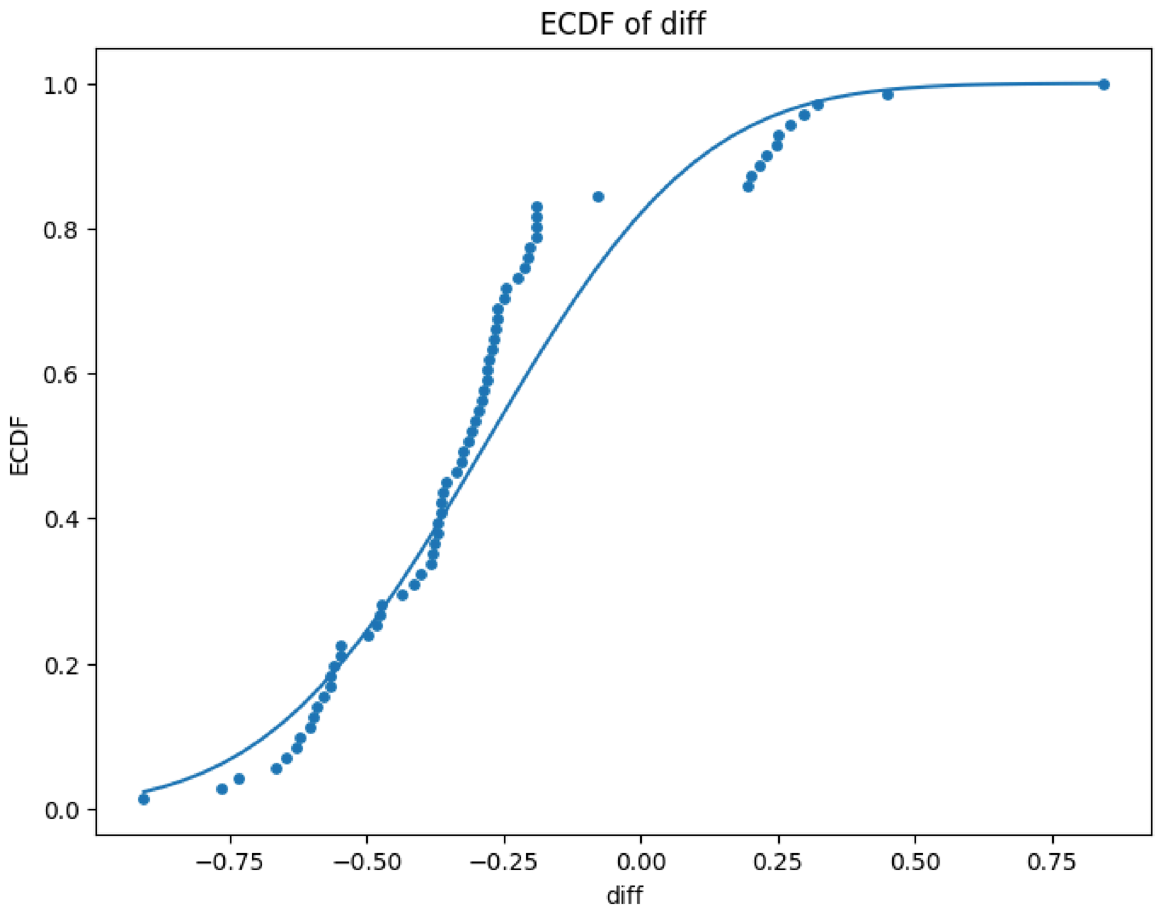
Length、Length(max)、Length(min) 樣本數比較多，然後看起來有兩個峰

In [ ]:

```
x = data.loc[data['characteristic'].isin(['Length', 'Length(max)', 'Length(min)'
# ecdf
x = np.sort(x)
y = np.arange(1, len(x)+1) / len(x)
dist = norm

def LL(theta, data): # Log Likelihood
    return -np.sum(np.log(dist.pdf(data-theta[0], scale=theta[1])))
res = minimize(LL, (0, 0.1), args = (x), method='nelder-mead',
            options={'xatol': 1e-6, 'disp': False})
# print(res.x)

fig, ax = plt.subplots(1, 1, figsize=(8, 6))
ax.scatter(x, y, s=15)
ax.set_xlabel('diff')
ax.set_ylabel('ECDF')
ax.set_title('ECDF of diff')
ax.plot(np.linspace(x[0], x[-1], 50), \
        dist.cdf(np.linspace(x[0], x[-1], 50), loc=res.x[0], scale=res.x[1]), la
fig.show()
```



從 empirical cdf 看的出來不像常態，而且有兩群。

嘗試用 mixturenormal 分成兩群結果只是分成大於 0 跟小於 0，跟前面的觀察吻合。

看起來比 0 大的比較像 noncentral exponential；比 0 小的像 normal (可能也沒那麼像)。代表 error 可能有兩種來源: exponential 跟 normal。

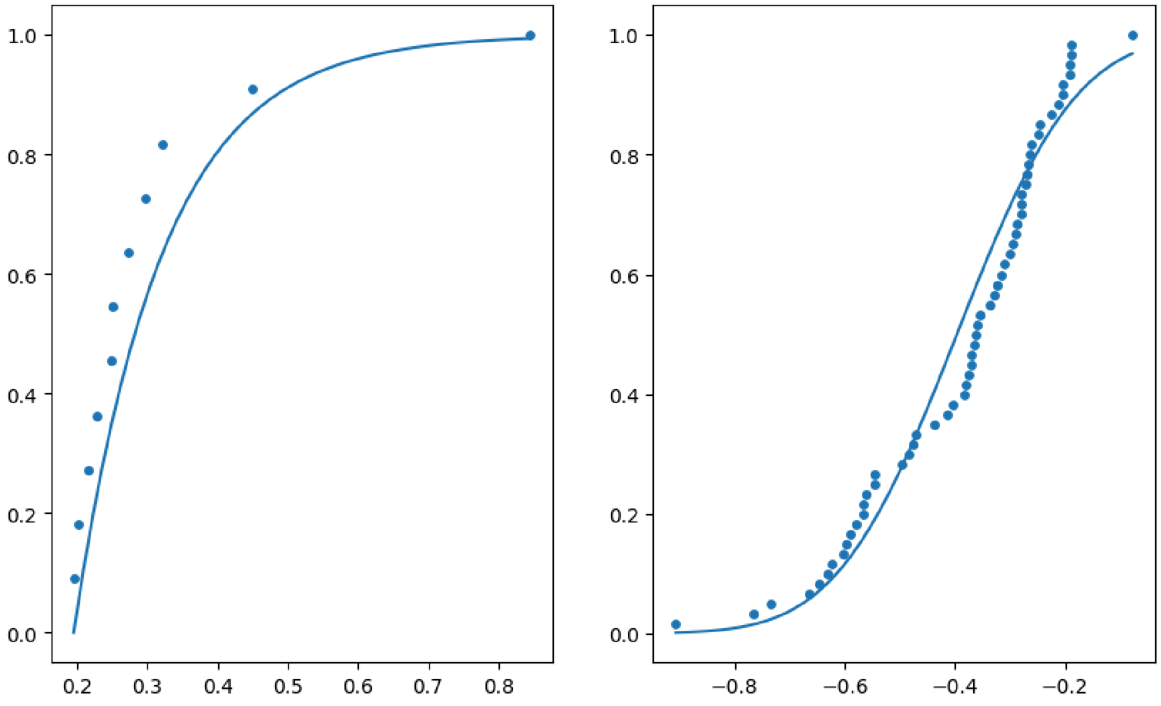
In [ ]:

```
x = (data.loc[data['characteristic'].isin(['Length', 'Length(max)', 'Length(min)
gm = GaussianMixture(n_components=2, random_state=0).fit(x.reshape(-1, 1))
# print(gm.means_)
group = gm.fit_predict(x.reshape(-1, 1))

fig, ax = plt.subplots(1, 2, figsize=(10, 6))
# for i, g in enumerate(np.unique(group)):
#     x_g = np.sort(x[group==g])
#     ax[i].scatter(x_g, np.arange(1, np.sum(group==g)+1) / np.sum(group==g), s=
#     ax[i].plot(np.linspace(x_g[0], x_g[-1], 50), \
#                norm.cdf(np.linspace(x_g[0], x_g[-1], 50), gm.means_[i], np.sqr
x_g = np.sort(x[group==1])
def LL(theta, data): # Log likelihood
    return -np.sum(np.log(expon.pdf(data, loc = theta[0], scale=theta[1])))
res = minimize(LL, (0, 0.1), args = (x_g), method='nelder-mead',
               options={'xatol': 1e-6, 'disp': False})
print(res.x)
ax[0].scatter(x_g, np.arange(1, len(x_g)+1) / len(x_g), s=15)
ax[0].plot(np.linspace(x_g[0], x_g[-1], 50), \
               expon.cdf(np.linspace(x_g[0], x_g[-1], 50), loc = res.x[0], scale =
x_g = np.sort(x[group==0])
ax[1].scatter(x_g, np.arange(1, len(x_g)+1) / len(x_g), s=15)
ax[1].plot(np.linspace(x_g[0], x_g[-1], 50), \
                norm.cdf(np.linspace(x_g[0], x_g[-1], 50), gm.means_[0], np.sqrt(gm.

fig.show()
```

```
[0.195      0.12545464]
```



```
In [ ]: # 看這兩群的數字
        print(np.sort(x[group==1]))
        print(np.sort(x[group==0]))
```

```
[0.195 0.201 0.216 0.229 0.248 0.25  0.272 0.298 0.321 0.45  0.845]
[-0.907 -0.765 -0.735 -0.665 -0.648 -0.63  -0.624 -0.603 -0.598 -0.59
 -0.58  -0.567 -0.566 -0.561 -0.547 -0.547 -0.497 -0.484 -0.476 -0.472
 -0.437 -0.415 -0.403 -0.382 -0.381 -0.376 -0.371 -0.37  -0.364 -0.363
 -0.36  -0.355 -0.335 -0.328 -0.323 -0.315 -0.309 -0.301 -0.295 -0.29
 -0.286 -0.28  -0.28  -0.278 -0.271 -0.268 -0.265 -0.263 -0.262 -0.249
 -0.246 -0.226 -0.212 -0.205 -0.203 -0.19  -0.19  -0.189 -0.189 -0.078]
```

檢定 exponential dist. 的平均是不是 0

```
In [ ]: def LL0(theta, data): # Log likelihood
            return -np.sum(np.log(expon.pdf(data, loc = theta[0], scale=theta[1])))
        res = minimize(LL, (-0.1, 0.1), args = (x[group==1]), method='SLSQP',
                        options={'xatol': 1e-6, 'disp': False}, constraints=LinearConstraint
        LL0_value = res.fun
        def LL1(theta, data): # Log likelihood
            return -np.sum(np.log(expon.pdf(data, loc = theta[0], scale=theta[1])))
        res = minimize(LL, (0, 0.1), args = (x[group==1]), method='Nelder-Mead',
                        options={'xatol': 1e-6, 'disp': False})
        LL1_value = res.fun
        print(2*(LL0_value - LL1_value))
        print('p-value={}'.format(chi2.sf(2*(LL0_value - LL1_value), 1)))
```

```
42.63153264814073
p-value=6.608541074381676e-11
```

結論為不是 0。

檢定 normal dist. 的平均是不是 0

In [ ]:

```
x_bar = np.mean(x[group==0])
s = np.std(x[group==0])
t = (x_bar - 0) / (s / np.sqrt(len(x[group==0])))
print(t)
```

```
-18.075245727069653
```

t-value 一看就知道平均不是 0。