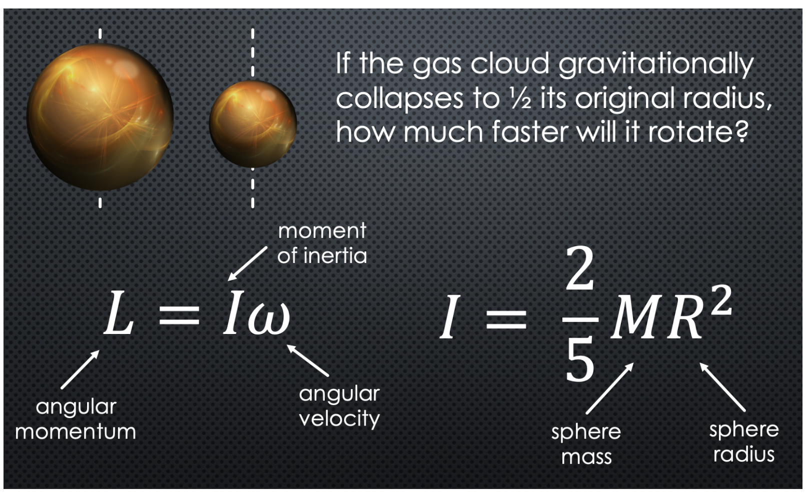
Designing Science Fiction Planets GEOPHYS 30N, EPS 30N
Homework #1
Due: Tuesday, October 10, 11:59 PM.

Students are encouraged to work together on assignments but turn in your own work/writeup! **Please** come to office hours or schedule an appointment with me (smtikoo@stanford.edu) if you are having trouble. You've got this! ☺

Total = 20 points

Part 1 (3 points total): Molecular cloud collapse/angular momentum conservation

Imagine you had a perfectly spherical molecular cloud (pre-solar nebula) that started collapsing. If the cloud collapses to half of its original radius, how many times faster will the cloud spin than its original angular velocity? Use the equations below and the law of conservation of angular momentum to obtain your solution (Remember, L should be the same for both the initial state and after the slow down).



Part 2 (7 points total): Planetary formation and internal melting

Use the gravitational potential energy formula from lecture:

$$E_G = -\frac{16}{15} G\pi^2\rho^2 R^5$$

G = Newton's gravitational constant
= 6.67×10^{-11} N•m²/kg²

ρ = planet density
(Earth's density = 5500 kg/m³)

R = radius of the planet
(Earth radius = 6371 km)

$$1\ \mathrm{N} = 1\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}.$$

(N = Newton, a unit for force)

Use the formula to calculate:

a. Gravitational potential energy E_G for the Earth, assuming R = 6378 km and ρ = 5000 kg m^{-3}, and for an asteroid assuming R = 100 km and ρ = 2500 kg m^{-3} (3 points).

b. If all this gravitational energy is converted into heat (i.e., $E_G = -\Delta Q$), and $\Delta Q = M{*}C_p{*}\Delta T$, where M = mass, C_p = specific heat of rock = 0.84 kJ kg^{-1} K^{-1}, calculate the change in temperature ΔT experienced for both the Earth (from part a) and the asteroid (from part b) (3 points).

c. From this, do you expect the Earth and the asteroid to undergo large-scale melting and differentiation/core formation from gravitational energy alone or do you need a boost from radiogenic heating (1 point)?

Pro tip: Remember to convert units properly to base SI units before proceeding! mass = kg, distance = m, time = s!!!!

Part 2: Temperature conditions (10 points total)

Here we are going to explore temperature conditions in different parts of solar systems. For the purposes of this assignment, we are going to assume that a given planet behaves as a "blackbody", which means that it is a hypothetical physical body that absorbs all incoming electromagnetic radiation. It's called a "blackbody" because it would absorb all the colors of visible light.

$F_{\odot}$= the solar constant, which is the solar flux at 1AU (the Earth-Sun distance).
FYI: $\odot$ means Sun. $r_{\odot}$ means distance of a planet from the Sun, and $r_{\odot\mathrm{AU}}$ means distance of a planet from the Sun in AU.

$F_{\odot} = \left(\frac{L_{\odot}}{4\pi r_{\odot}^2}\right) = 1366\ \mathrm{W\,m^{-2}}$ for Earth at 1 AU. $L_{\odot}$ is the Sun's luminosity (amount of light emitted) at present. Note the "solar flux" means energy transmitted by the Sun per unit time over the exposed surface area of the planet.

The solar flux at some other distance (in units of AU) may be written as:

$$\frac{F_{\odot}}{r_{\odot\mathrm{AU}}^2}$$

For example, in the denominator, $r_{\odot\mathrm{AU}}^2$ at Mars would be $(1.5)^2 = 2.25$ because Mars is 1.5 AU from the Sun.

The sunlit hemisphere of a planet orbiting around the Sun receives this amount of incoming radiation, where *R* is the radius of the planet:

$$P_{\mathrm{in}} = (1 - A_{\mathrm{b}})\left(\frac{L_{\odot}}{4\pi r_{\odot\mathrm{AU}}^2}\right)\pi R^2$$

A_b is called the Bond albedo (or the reflectivity of the planet). This value ranges between 0 and 1 where zero is not reflective at all and 1 is completely reflective.

A rapidly rotating body like a planet re-radiates the below amount of energy from its whole surface:

$$P_{\mathrm{out}} = 4\pi R^2 \epsilon \sigma T^4$$

In this expression, ϵ is the emissivity of the planet (just use 0.9 in all cases) and σ is the Stefan-Boltzmann constant = 5.67×10^{-8} W m^{-2} K^{-1}. *T* is the temperature of the body.

1. The equilibrium temperature of a body (T_{eq}) is the temperature at which the incoming radiation is equivalent to the outgoing radiation. Set $P_{in} = P_{out}$ to create an expression for T_{eq} in terms of $r^2_{\odot AU}$. (2 points)

2. Calculate the following (3 points)

   A) Solve for the equilibrium temperature of the Earth. The Earth's Bond albedo is 0.3.
   B) Solve for the equilibrium temperature of Mars. Mars' Bond albedo is 0.25.
   C) Solve for the equilibrium temperature of Venus. Venus' Bond albedo is 0.76.

   Hint: It is difficult to do these calculations on a handheld calculator without input errors so I recommend using a software like Matlab (if you have access) or using the online calculator at www.wolframalpha.com where you can make sure that what you type in was read correctly by the code.

3. Look up the range of typical surface temperatures for Earth, Mars, and Venus. How do the equilibrium temperatures for each of these planets compare to their mean surface temperatures? If these planets were perfect blackbodies, would have habitable surface temperature ranges? (3 points)

4. What factors in real life/properties of these planets might contribute to the mismatch between the equilibrium temperatures and the actual surface temperatures on these planets? (2 points)