Quantitative Text Analysis

Meeting 7

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Naïve Bayes Classifier

- Probabilistic classifier
- Simple
- Fast
- Good Accuracy

Bayes Theorem

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

Bayes Theorem

Likelihood of observing the evidence B given that event A is true

Prior probability of event A, representing our initial belief or knowledge about A before considering the evidence

Posterior probability of event A given the evidence B

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

Probability of observing the evidence B

Bayes Theorem

Example

- Probability of having a chocolate banana allergy after a positive test
- P(Allergy): The prior probability of having a banana allergy. Let's assume it is 0.1, meaning 10% of the population has a banana allergy.
- P(No Allergy): The complement of P(Allergy), representing the prior probability of not having a banana allergy (0.9 in this case).
- P(Positive|Allergy): The probability of testing positive for a banana allergy given that the person actually has an allergy. Let's say it is 0.95, meaning the test correctly identifies 95% of the people with a banana allergy.
- P(Negative|No Allergy): The probability of testing negative for a banana allergy given that the person does not have an allergy. Let's assume it is 0.90, meaning the test correctly identifies 90% of the people without a banana allergy.

Bayes Theorem

Example

$$P(Allergy \mid Positive) = \frac{P(Positive \mid Allergy) \times P(Allergy)}{P(Positive)}$$

Bayes Theorem

Example

.95 .1

$$P(Allergy \mid Positive) = \frac{P(Positive \mid Allergy) \times P(Allergy)}{P(Positive)}$$

?

Bayes Theorem

Example

$$P(Allergy \mid Positive) = \frac{\overset{.95}{P(Positive \mid Allergy)} \times \overset{.1}{P(Allergy)}}{\underset{?}{P(Positive)}}$$

$$P(Positive) = (P(Positive \,|\, Allergy) \times P(Allergy)) + (P(Positive \,|\, NoAllergy) \times P(NoAllergy))$$

Bayes Theorem

Example

$$P(Allergy \mid Positive) = \frac{\overset{.95}{P(Positive \mid Allergy)} \times \overset{.1}{P(Allergy)}}{P(Positive)}$$

?

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$$P(Positive) = (0.95 \times 0.1) + (0.10 \times 0.9) = 0.095 + 0.09 = 0.185$$

Bayes Theorem

Example

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$$P(Allergy \,|\, Positive) = \frac{0.95 \times 0.1}{0.185} \approx 0.5135$$

Bayes Theorem

Example

.95 .1

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?

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Posterior Probability

Questions so far?

Bayes Theorem

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Prior probability of event A, representing our initial belief or knowledge about A before considering the evidence

Posterior probability of event A given the evidence B

$$P(A \mid B) = \frac{P(B \mid A) \times P(A)}{P(B)}$$

Probability of observing the evidence B

Bayes Theorem

$$P(A|B) \propto P(B|A) \times P(A)$$

Naïve Bayes Classifier

$$P(C_k | x_1, x_2, ..., x_n)$$

Naïve Bayes Classifier

Features

$$P(C_k|x_1, x_2, ..., x_n)$$

Class

Naïve Bayes Classifier

Features

$$P(C_k|x_1, x_2, ..., x_n)$$

Class

Features are assumed to be independent. Hence, "***Naïve***"

Naïve Bayes Classifier

$$P(C_k|\mathbf{x}) = \frac{P(C_k) \times P(\mathbf{x}|C_k)}{P(\mathbf{x})}$$

Naïve Bayes Classifier

$$P(C_k|\mathbf{x}) \propto P(C_k) \times P(\mathbf{x}|C_k)$$

Naïve Bayes Classifier

$$\begin{aligned} p(C_k \mid x_1, \dots, x_n) &\propto p(C_k, x_1, \dots, x_n) \\ &\propto p(C_k)\, p(x_1 \mid C_k)\, p(x_2 \mid C_k)\, p(x_3 \mid C_k)\, \cdots \\ &\propto p(C_k) \prod_{i=1}^{n} p(x_i \mid C_k)\,, \end{aligned}$$

Decision Rule

$$\hat{y} = argmax\ p(C_k) \prod_{i=1}^{n} p(x_i|C_k)$$

Naïve Bayes Classifier

- Implemented in many stats/ML packages

Support Vector Machine

- Comes from computer science
- Very good
- Rather difficult math

- Considered one of the best of-the-shelf classification algorithms

Hyperplane

- n-1 dimensional plane that separates the n-dimensional space

Hyperplane

- n-1 dimensional plane that separates the n-dimensional space
- 2-dimensional hyperplane:

$$\beta_0 + \beta_1 X_1 = 0$$

Hyperplane

- n-1 dimensional plane that separates the n-dimensional space
- 2-dimensional hyperplane:
- line equation

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Hyperplane

- n-1 dimensional plane that separates the n-dimensional space
- 2-dimensional hyperplane:
- line equation

$$\beta_0 + \beta_1 X_1 = 0$$

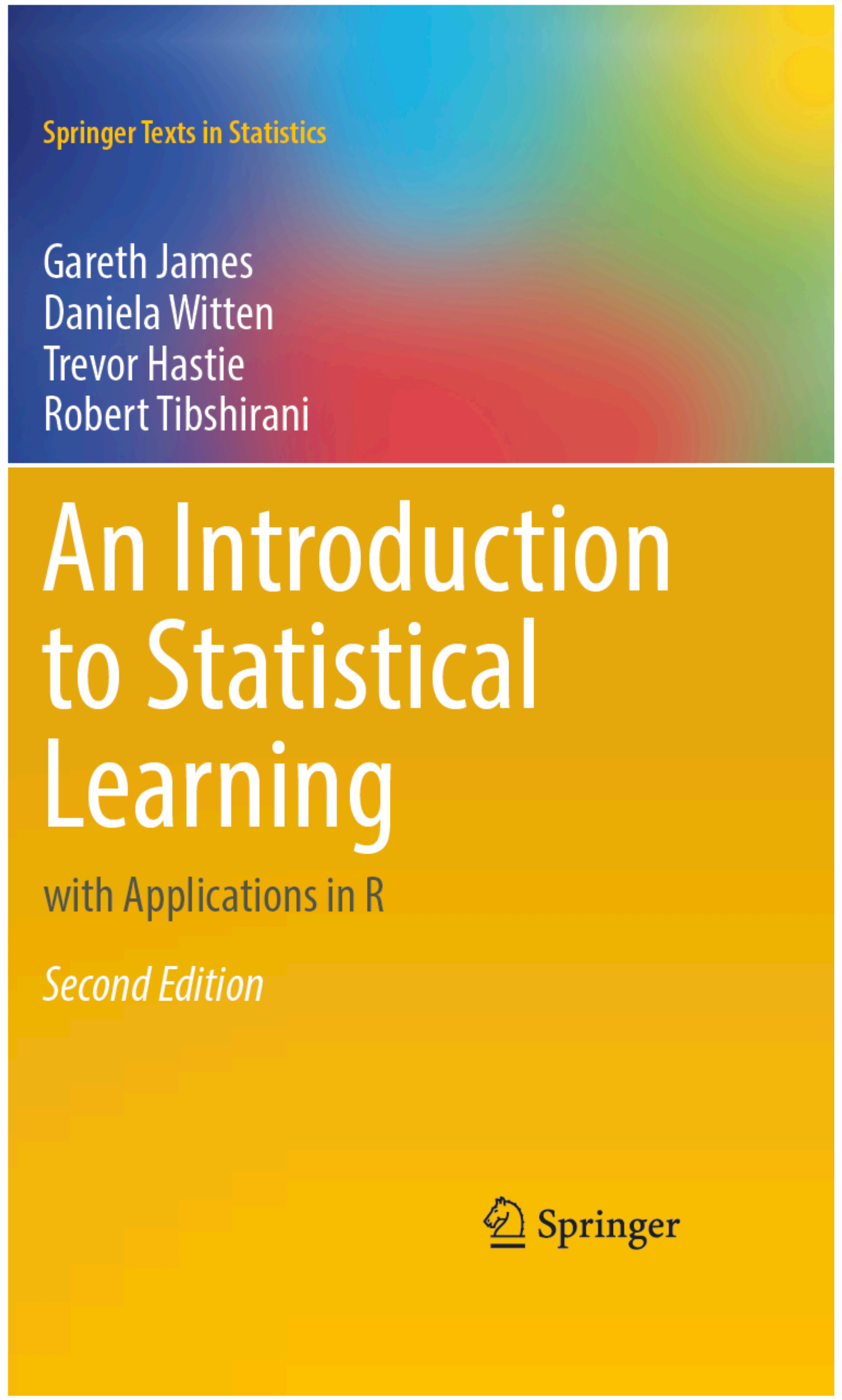
$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p = 0$$

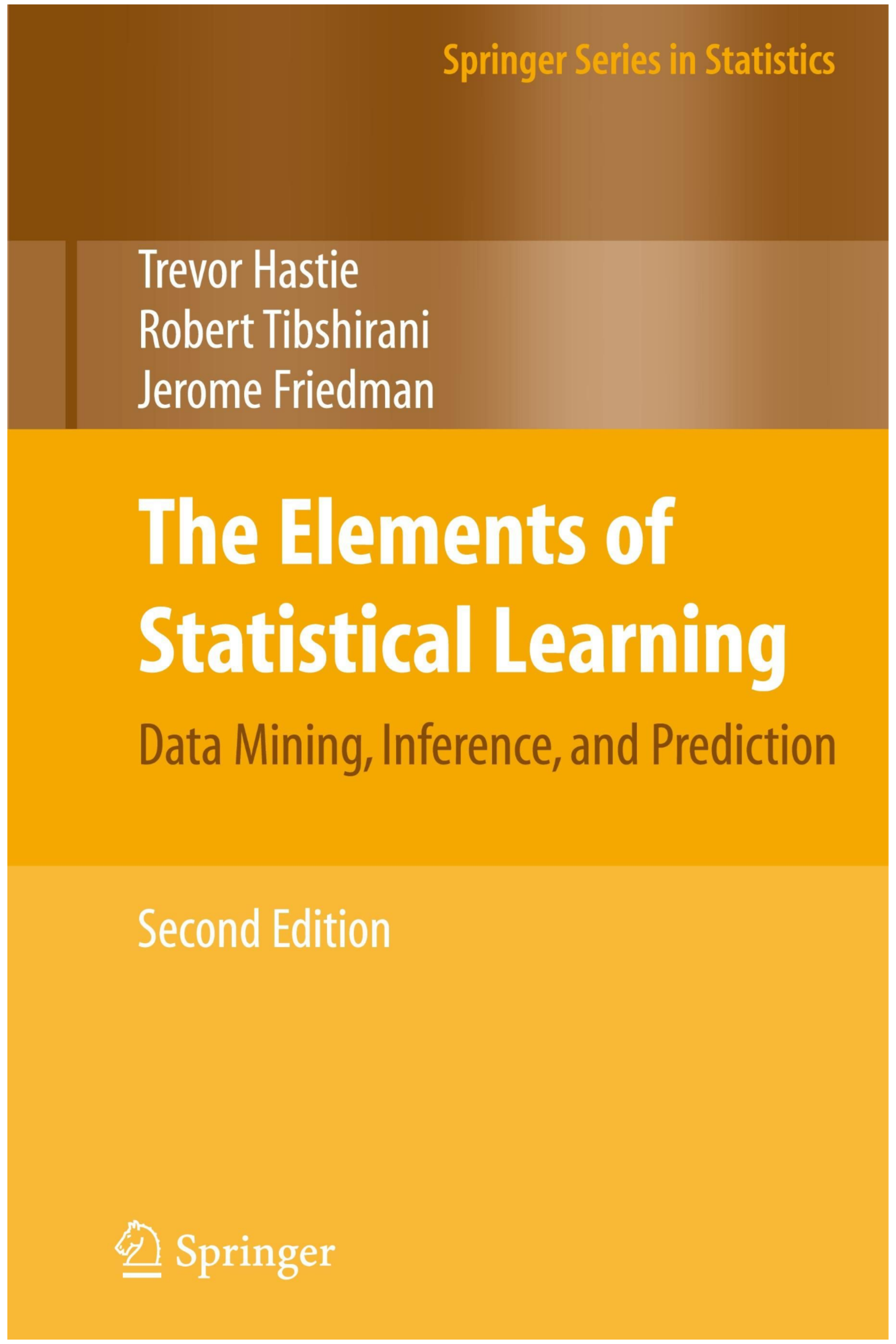
Classification

$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p > 0.$$

$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p < 0,$$

Following images from:





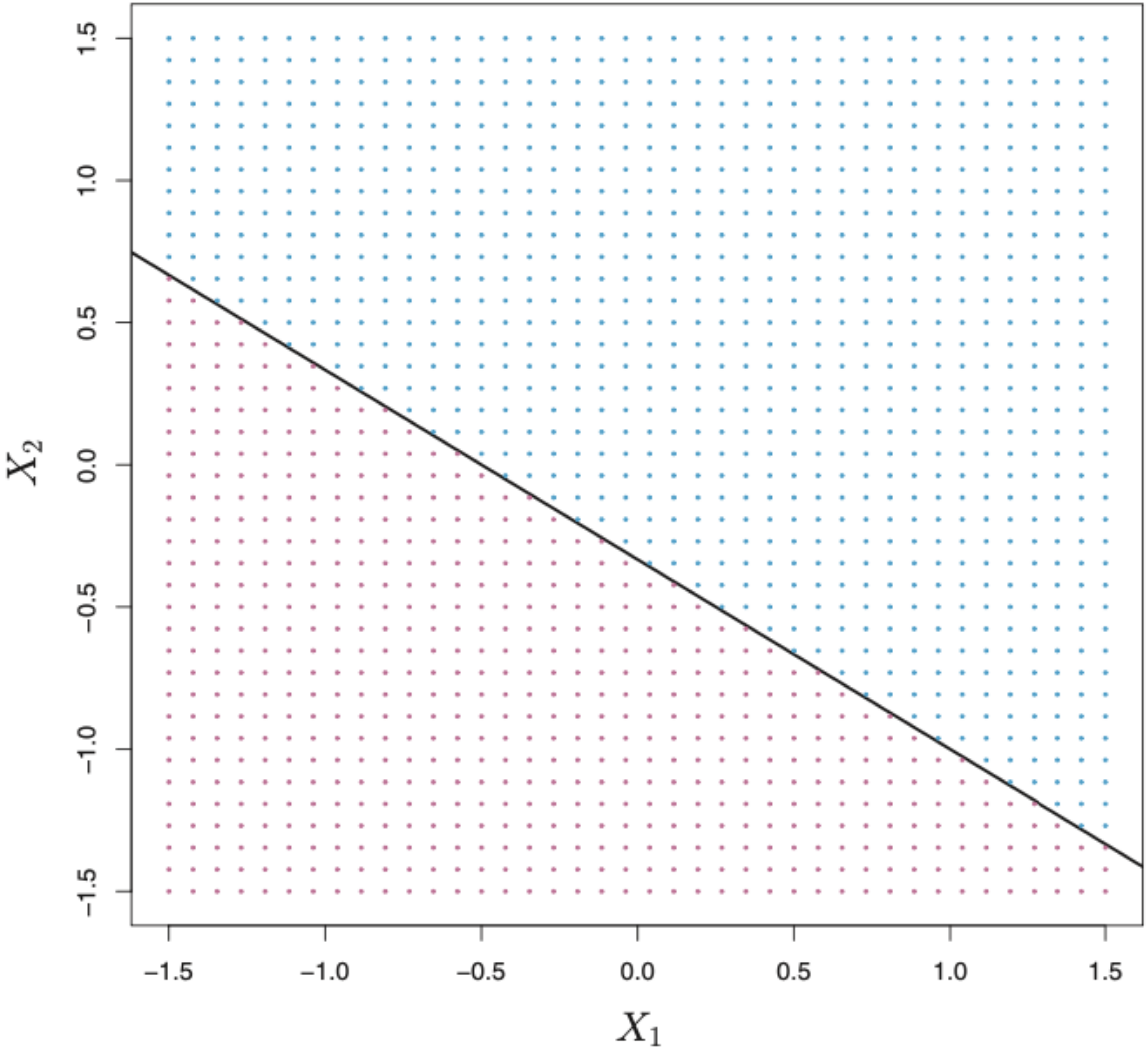
Springer Texts in Statistics

Gareth James · Daniela Witten · Trevor Hastie · Robert Tibshirani · Jonathan Taylor

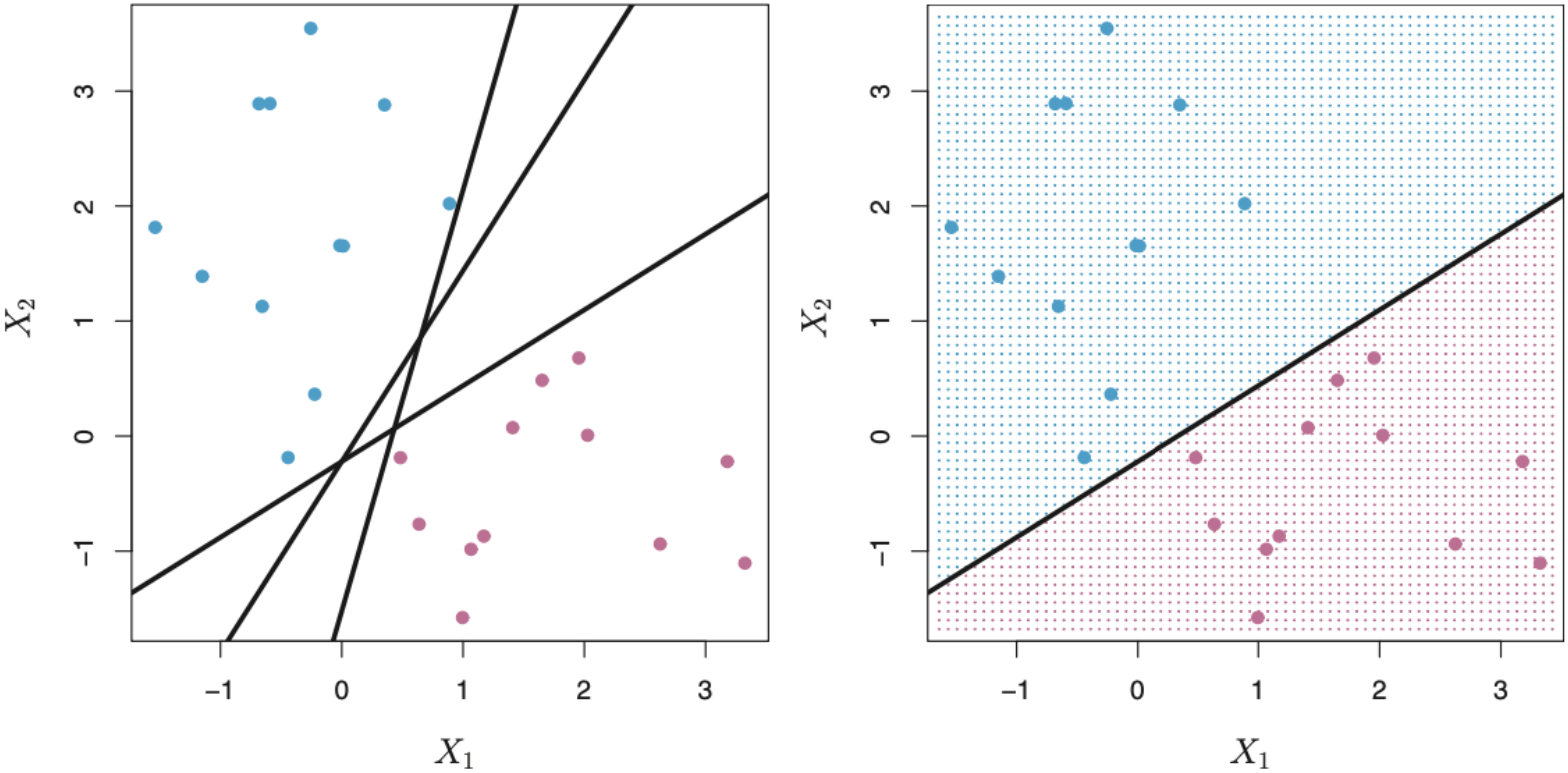
An Introduction to Statistical Learning

with Applications in Python

Springer

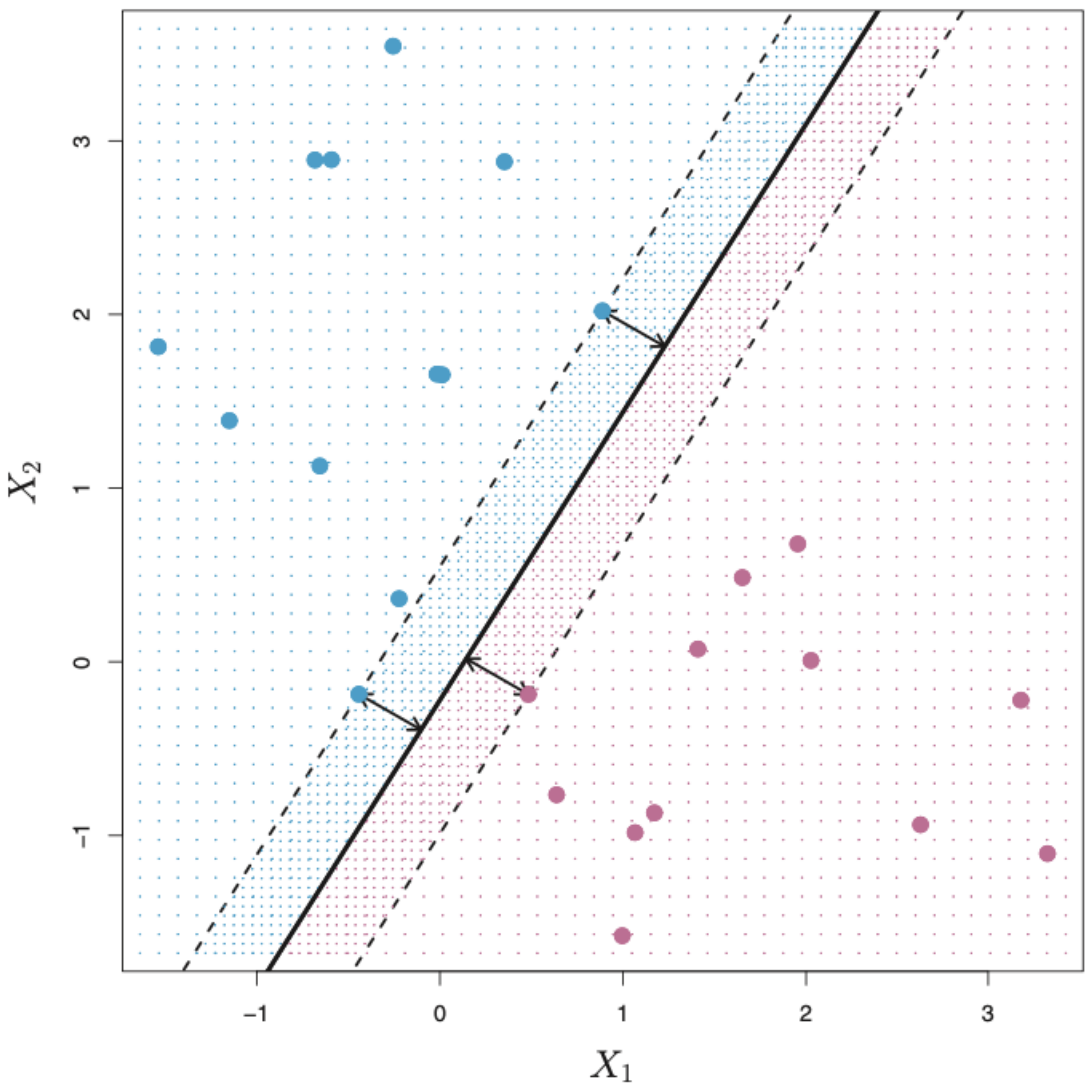


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
X_2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X_1



X_2
X_1
−1
0
1
2
3

SV Classifier



Support Vector Machine

- Non-linear version of the Support Vector Classifier
- Extension using Kernels

Support Vector Machines

$$f(x) = \beta_0 + \sum_{i \in \mathcal{S}} \alpha_i \langle x, x_i \rangle$$

$$f(x) = \beta_0 + \sum_{i \in \mathcal{S}} \alpha_i K(x, x_i)$$

Support Vector Machines

$$f(x) = \beta_0 + \sum_{i \in \mathcal{S}} \alpha_i \langle x, x_i \rangle$$

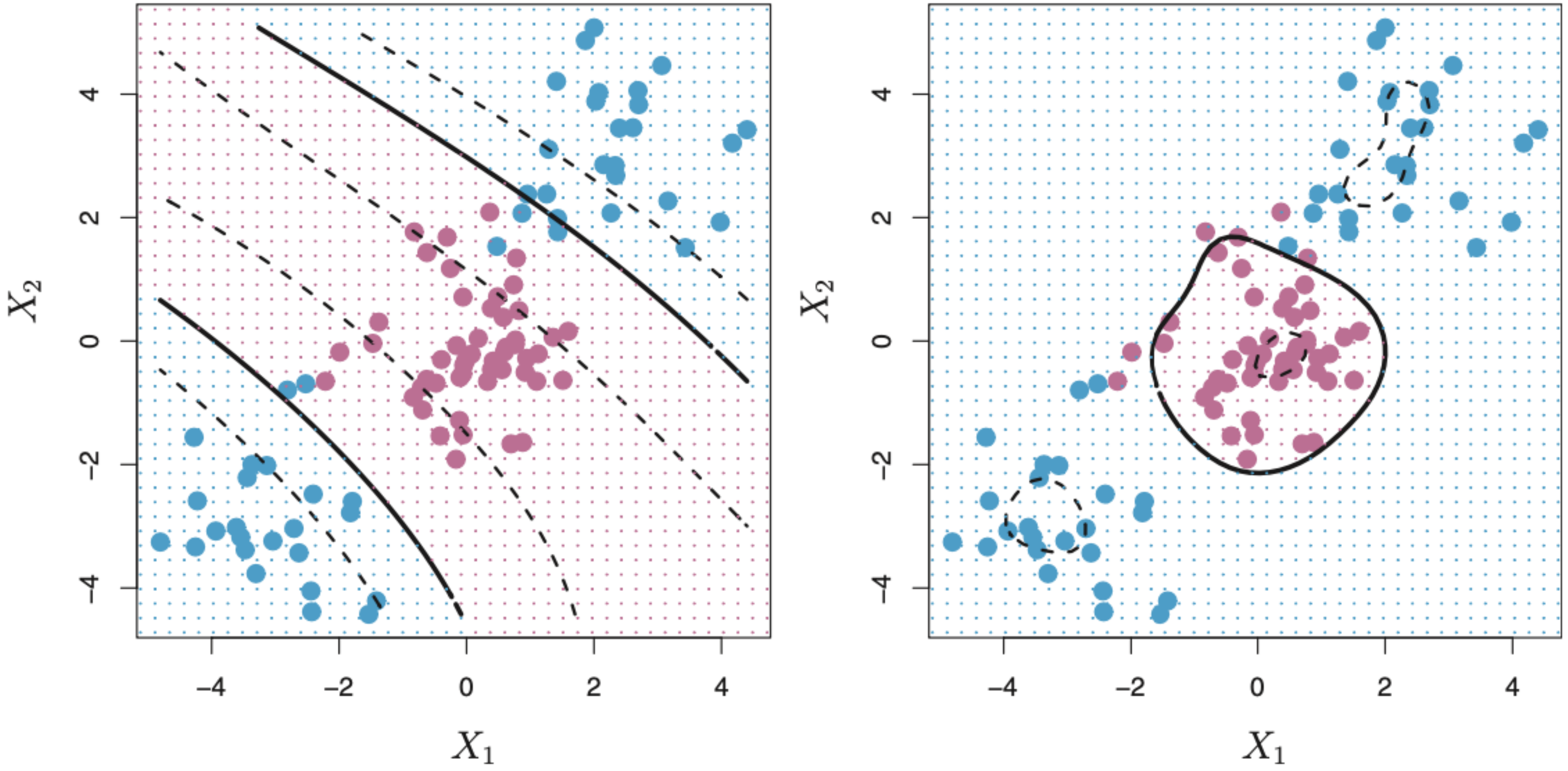
$$f(x) = \beta_0 + \sum_{i \in \mathcal{S}} \alpha_i K(x, x_i)$$

Kernel function

Support Vector Machines

$$K(x_i, x_{i'}) = (1 + \sum_{j=1}^{p} x_{ij} x_{i'j})^d.$$

Polynomial Kernel

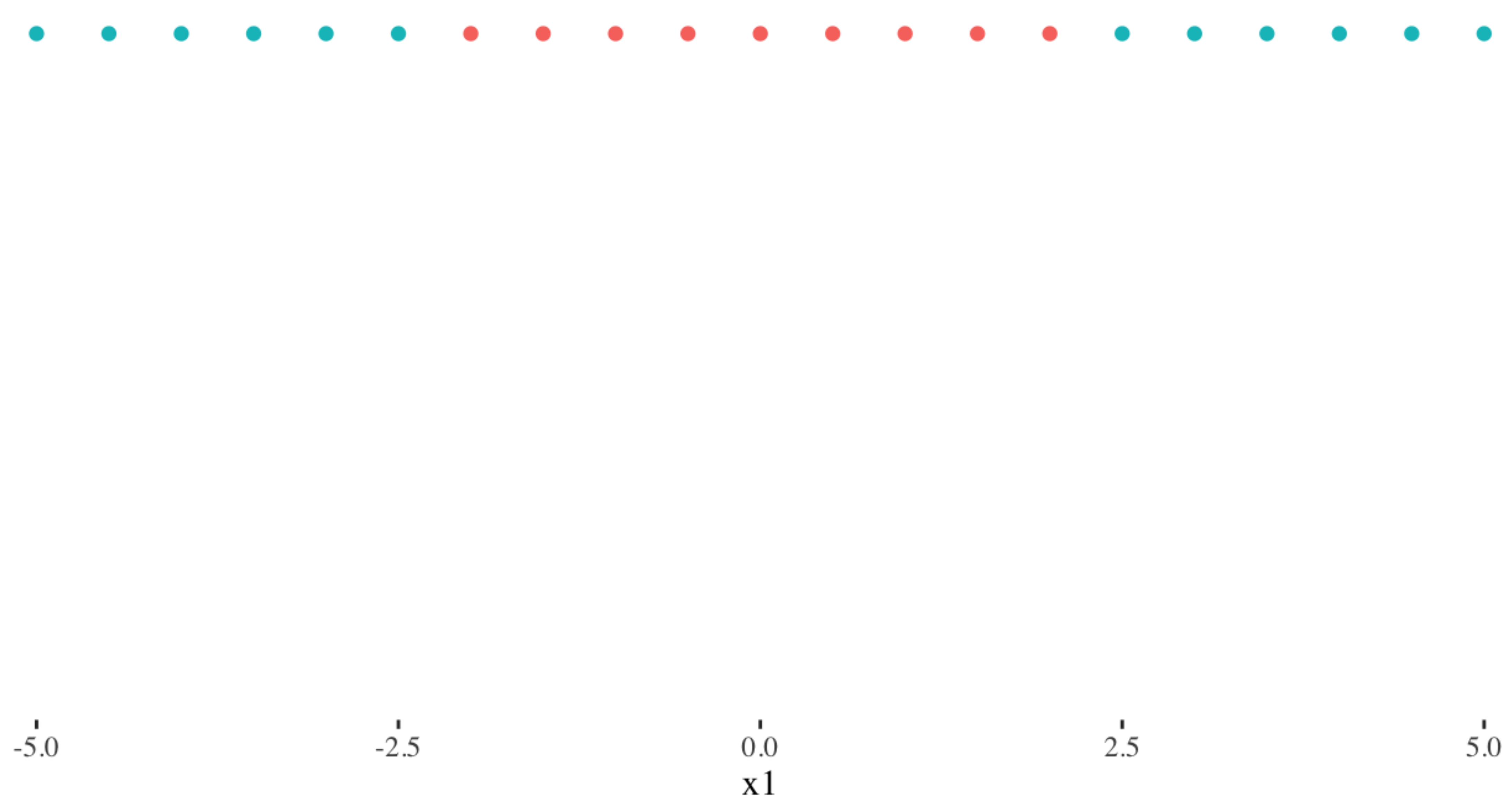


X_1
X_2
−4
−2
0
2
4
X_1
X_2

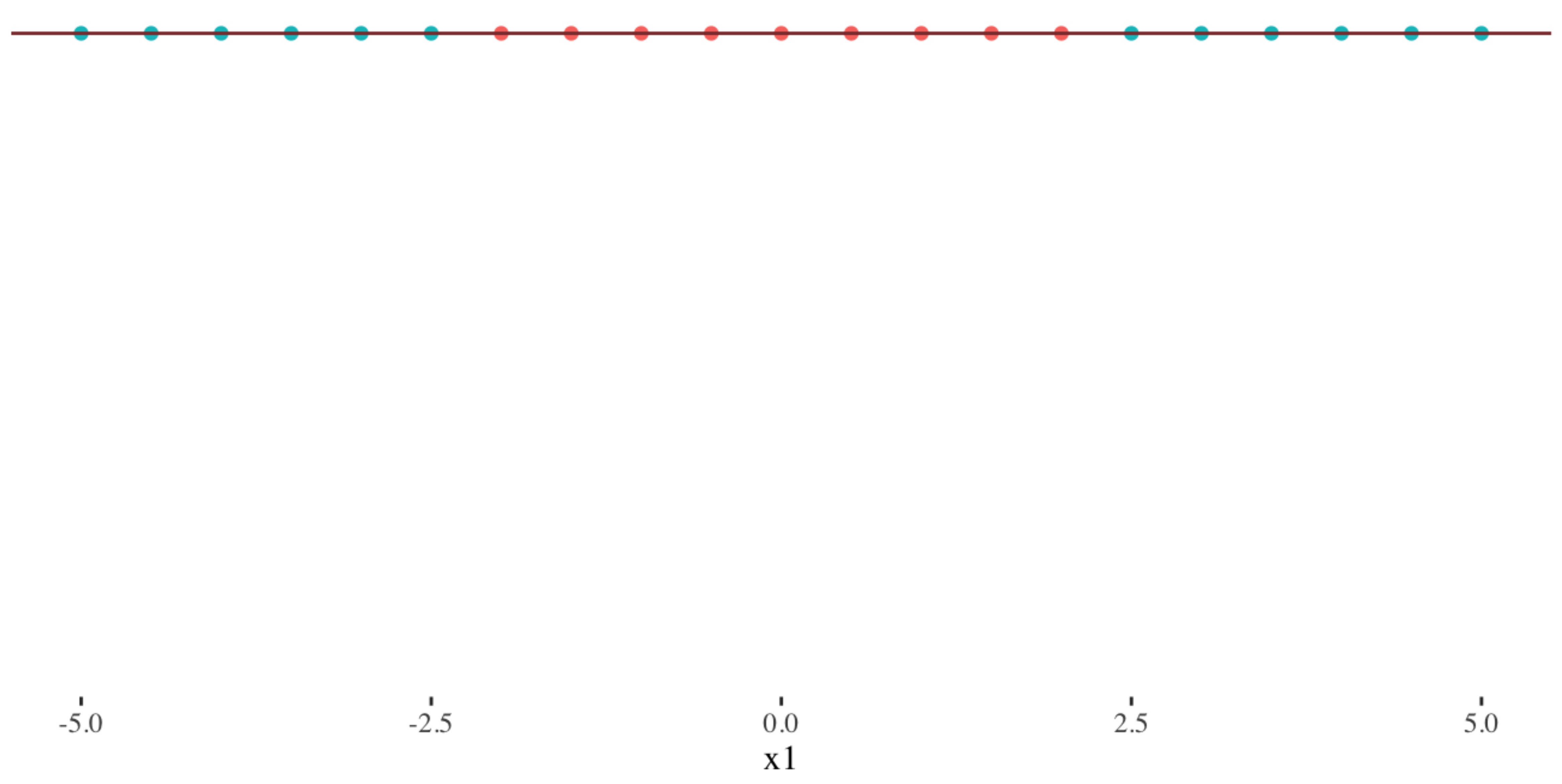
Kernel Trick

Kernel Trick

- Actual name
- Attempt to place n-dimensional data into n+1 dimensional space



-5.0
-2.5
0.0
2.5
5.0
x1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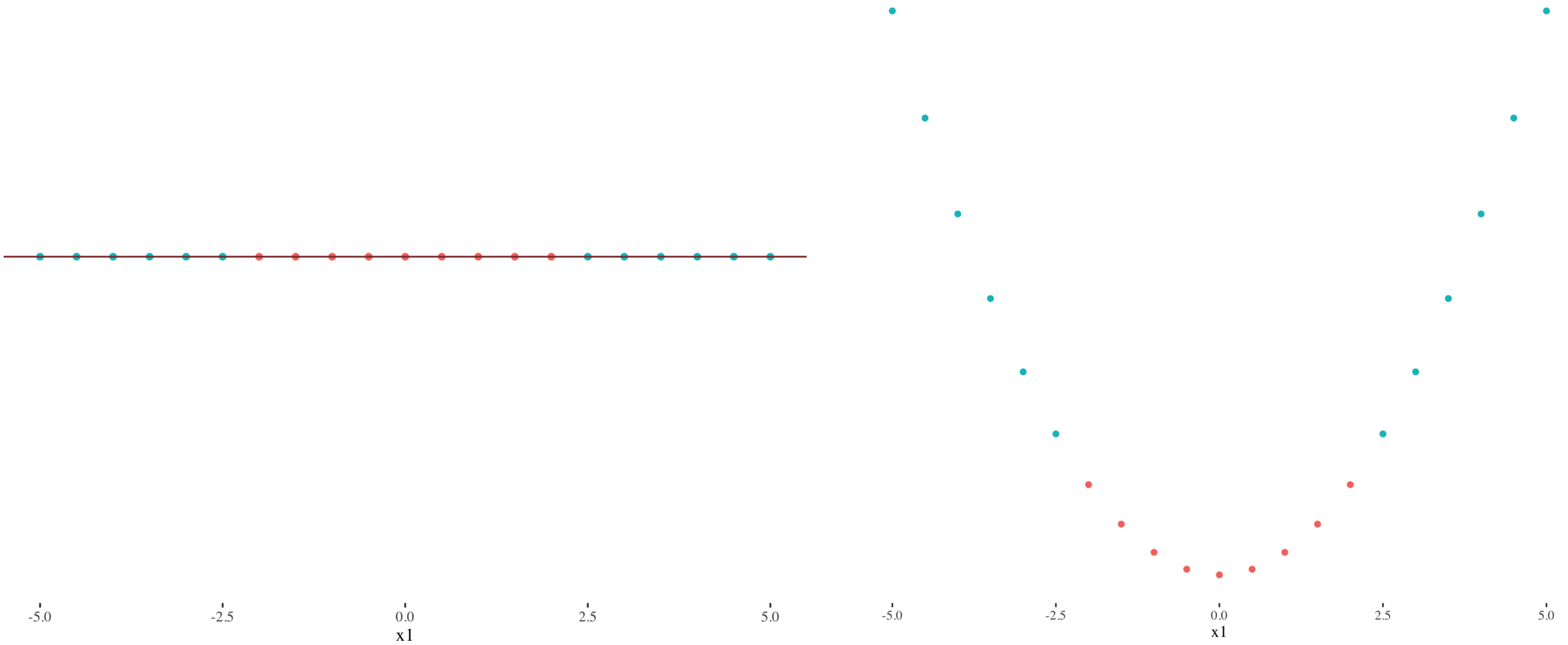
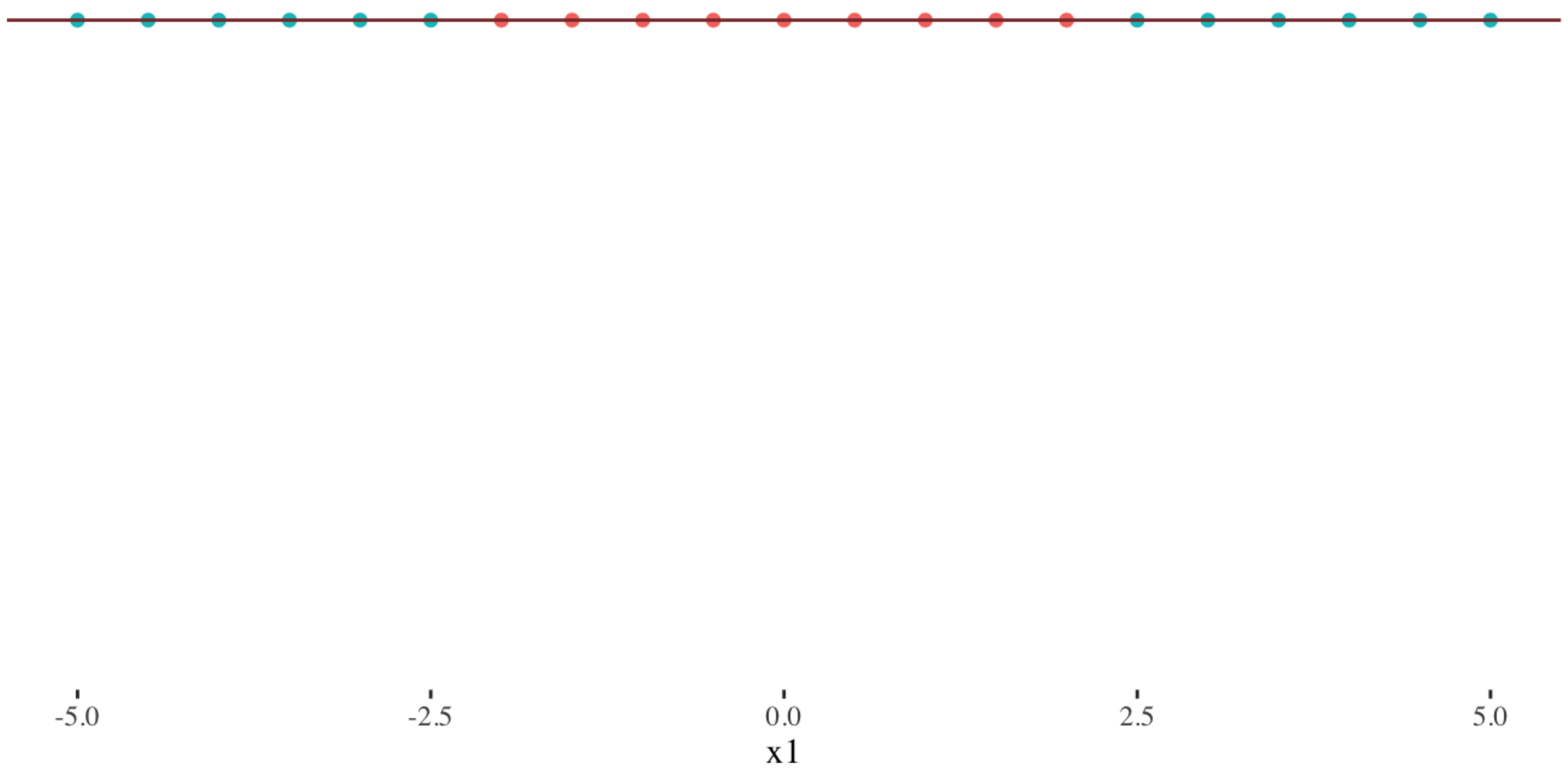
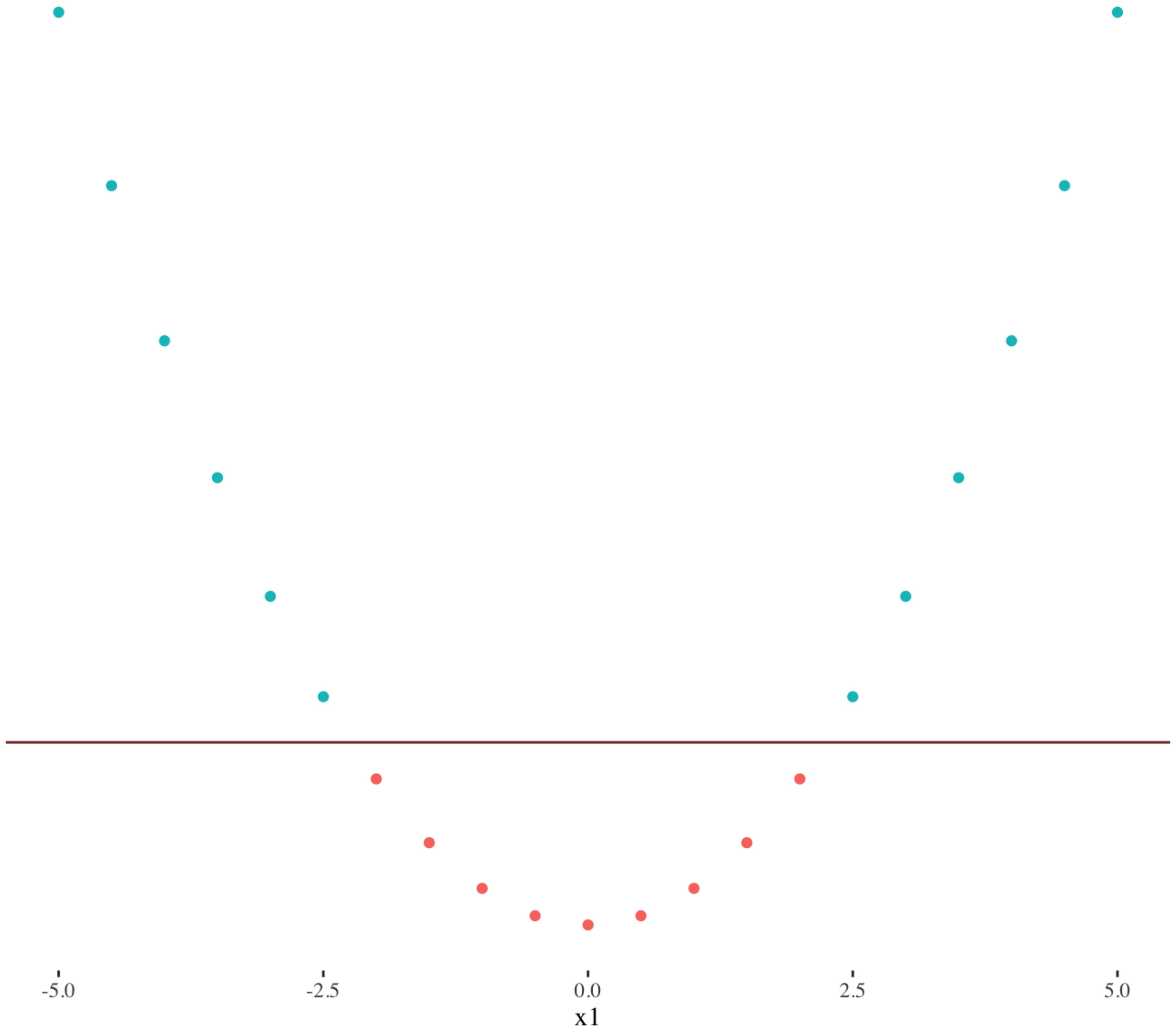
-2.5
0.0
2.5
5.0
x1



-5.0
-2.5
0.0
2.5
5.0
x1
-5.0
-2.5
0.0
2.5
5.0
x1



-5.0
-2.5
0.0
2.5
5.0
x1



-5.0
-2.5
0.0
2.5
5.0
x1