Bayesian Text Analysis
Comptext 2025

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What Is Bayesian Text Analysis?

- Applying Bayesian inference to model text data with explicit probability distributions
- Treats model parameters (e.g., topic weights, sentiment mixtures) as random variables
- Combines prior knowledge (lexicons, hierarchical structure) with observed word counts
- Full posterior distributions $\rightarrow$ credible intervals & uncertainty quantification

Motivation

- Uncertainty quantification in text analytics (full posterior distribution)
- Natural incorporation of statistical models
- Incorporation of prior knowledge
- Smoothing (with priors)
- Hierarchical models

Bayes' Theorem

$$P(\theta \mid D) = \frac{P(D \mid \theta)\, P(\theta)}{P(D)}$$

- Prior $P(\theta)$ prior distribution of the parameters
- Likelihood $P(D \mid \theta)$ represents the probability of observing the data given the parameters θ
- Posterior $P(\theta \mid D)$ Posterior distribution of the model parameters θ given the data D
- Marginal Likelihood $P(D)$ Probability of observing the data

Bayes' Theorem

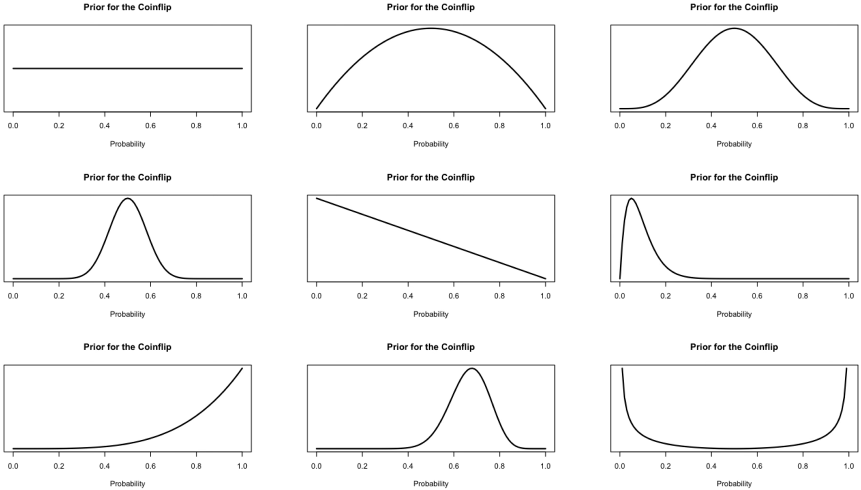
$$P(\theta \mid D) = \frac{P(D \mid \theta)\, P(\theta)}{P(D)}$$

- Prior $P(\theta)$ encodes domain knowledge
- Likelihood $P(D \mid \theta)$ measures data fit
- Posterior $P(\theta \mid D)$ captures uncertainty
- Posterior $P(D)$ normalisation constant

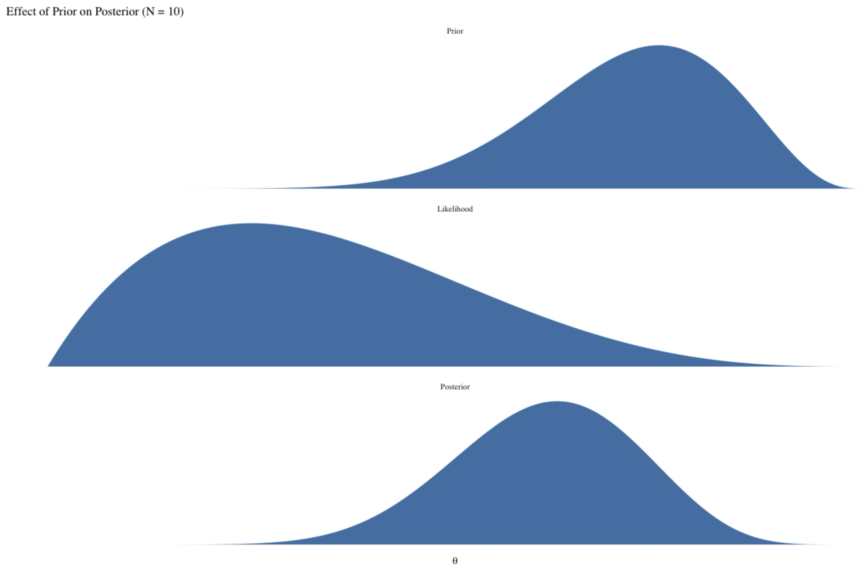
Bayesian Inference (priors)

- We need to assign priors for the model to work
- Priors are our beliefs about the event *prior* to seeing the data
- We can be very certain or very uncertain (or everything in between)

Priors (coinflip)



Effect of Prior on Posterior



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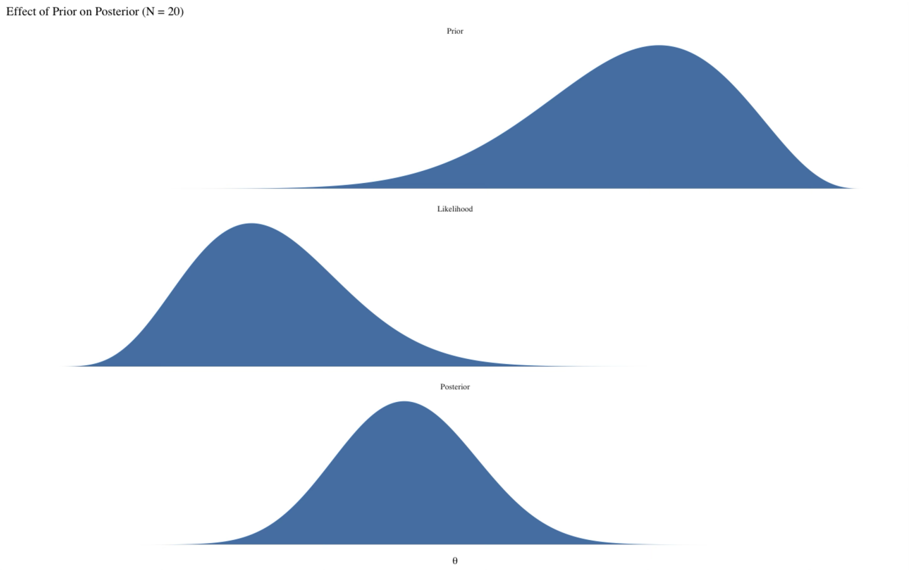

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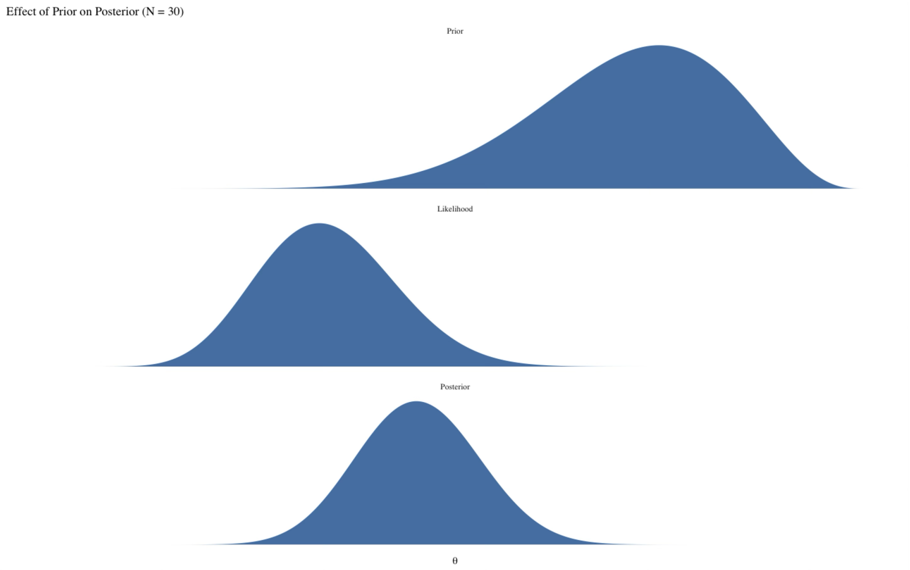

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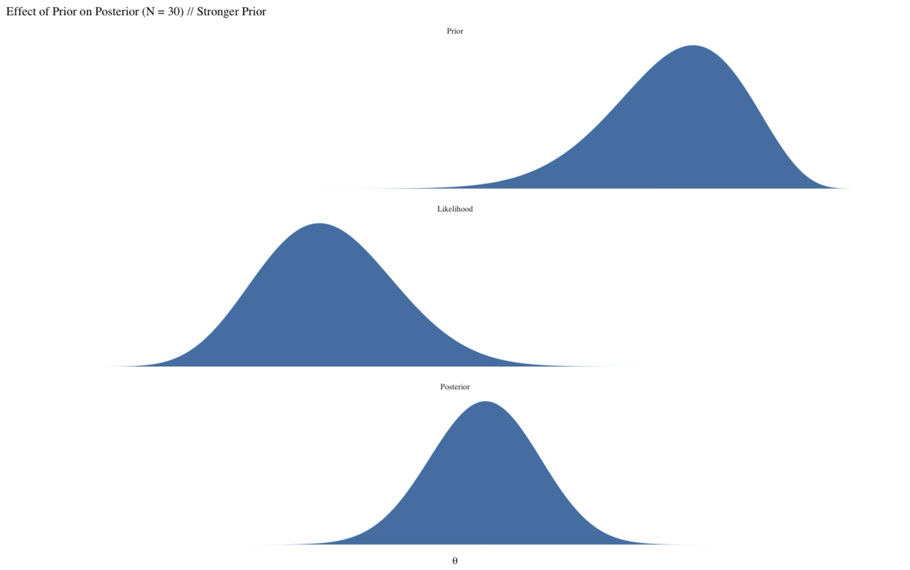

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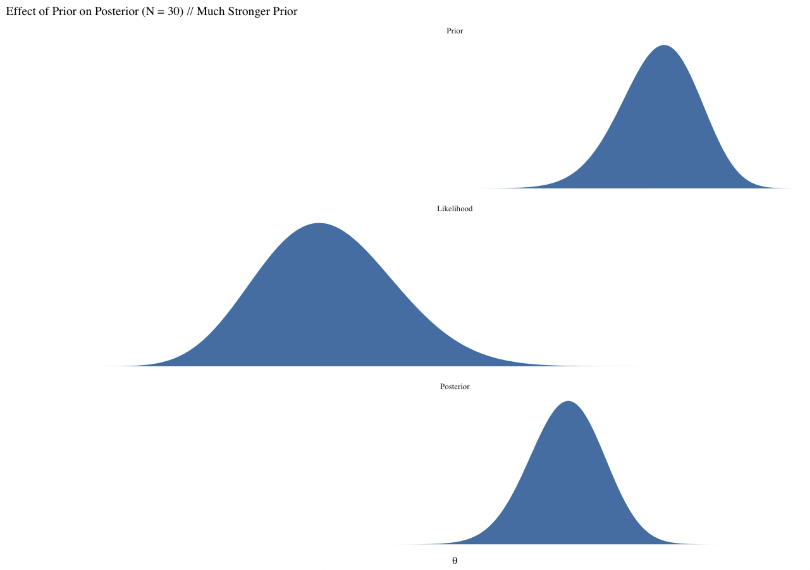

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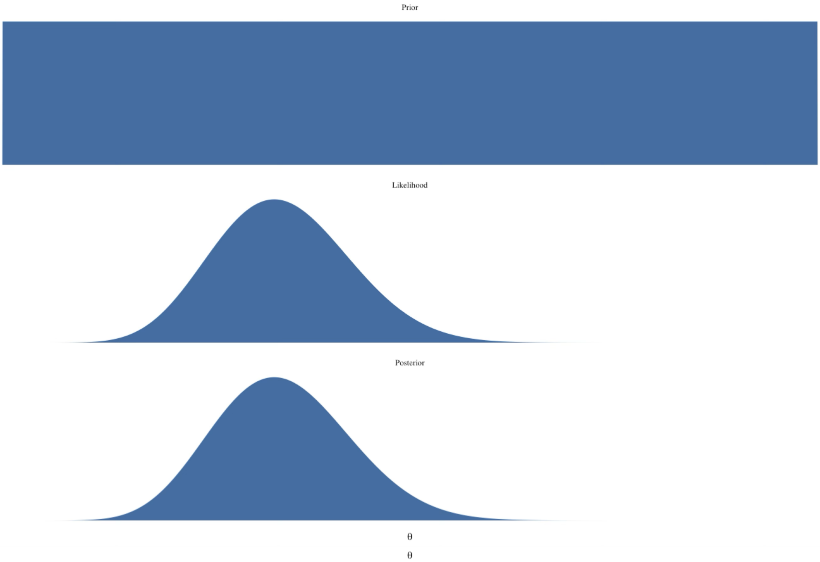

Statistical Model

$$y = \beta_0 + \beta_1 X_1 + ... + \beta_k X_k + \epsilon$$

Statistical Model: Components Explained

$$y \sim \text{Normal}(\mu,\ \sigma)$$

$$\mu = \beta_0 + \beta_1 X_1 + \cdots + \beta_k X_k$$

$$\beta_j \sim \text{Normal}(0,\ 5) \quad (j = 0, \ldots, k)$$

$$\sigma \sim \text{Exponential}(1)$$

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- **Likelihood** $y \sim Normal(\mu, \sigma)$ How the data arise given μ, σ.
- **Linear predictor** $\mu = \beta_0 + \beta_1 X_1 + \cdots + \beta_k X_k$ Maps covariates X_j to the mean response.
- **Priors on coefficients** $\beta_j \sim \mathrm{Normal}(0, 5)$ Each β_j centered at 0 with moderate spread.
- **Prior on scale** $\sigma \sim \mathrm{Exponential}(1)$ Belief that $\sigma > 0$, mean 1.

Parameter Interpretation: Bayesian vs. Frequentist

Bayesian Inference

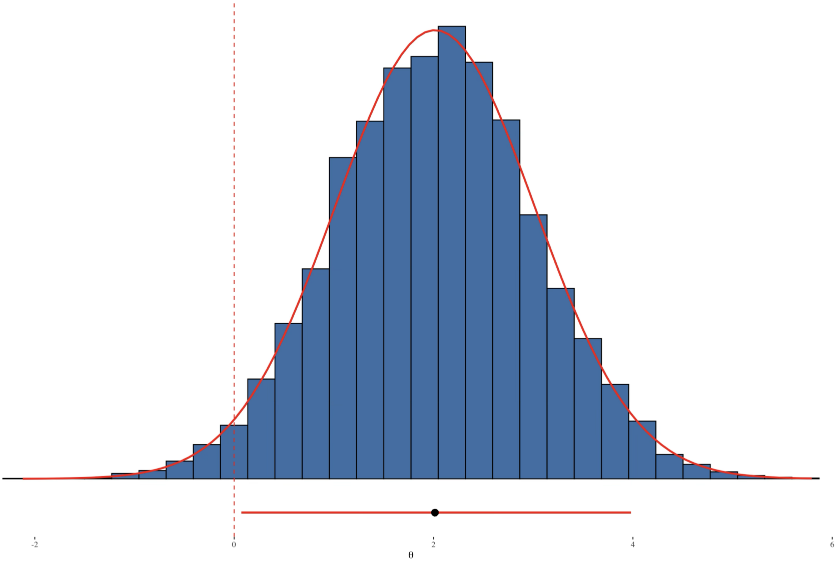
- *Parameters are random variables* with a distribution.
- Encode prior beliefs via a **prior** $P(\theta)$.
- Update beliefs with data via the **posterior** $P(\theta \mid D)$
- Uncertainty expressed by **credible intervals**: $\Pr(\theta \in C \mid D) = 0.95$
- Interpret intervals directly as "there's a 95% chance θ lies in C"

Frequentist Inference

- *Parameters are fixed* but unknown constants.
- No prior; inference derives from the **likelihood** $P(D \mid \theta)$
- Estimation via point estimators $\hat{\theta}$ (e.g. MLE).
- Uncertainty quantified by the **sampling distribution** of $\hat{\theta}$.
- **Confidence intervals**: Under repeated sampling, 95% of CIs will cover the true θ.
- Cannot say "θ has 95% probability of lying in this interval."

Confidence Interval

Posterior (Credibility Interval)



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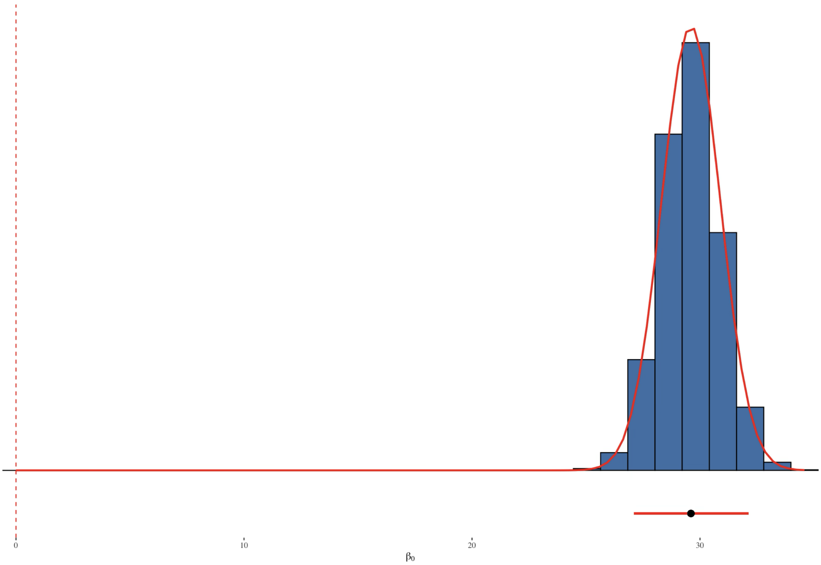

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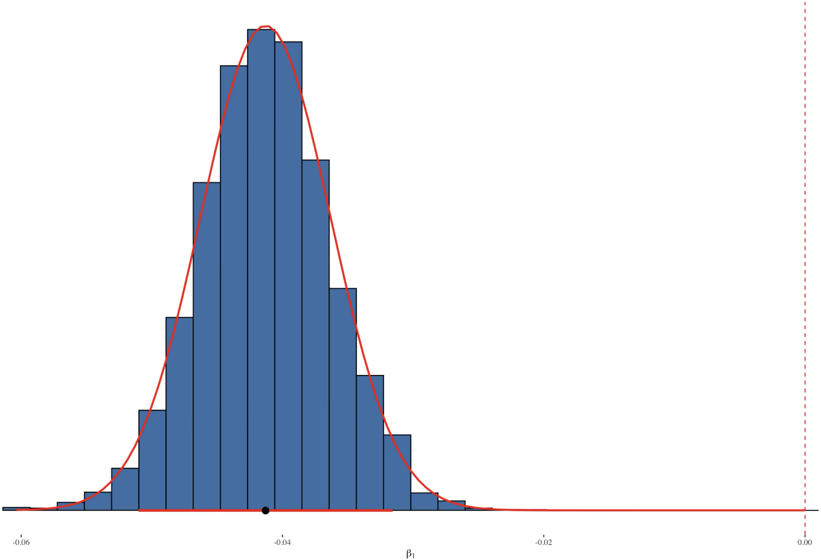

Probabilistic Programming: Overview

- **Definition:** A *probabilistic program* is code for random sampling and conditioning.
- Automates the core steps of Bayesian analysis:
  - Specifying generative models
  - Defining priors and likelihoods
  - Performing posterior inference
- "Intuitive" approach to complex hierarchical and latent-variable models.

Why Bayesian for Text Analysis?

- **Full uncertainty quantification** We get the full posterior over parameters (topic weights, embedding coefficients, etc.) instead of just point estimates.
- **Principled smoothing** Dirichlet- or Normal-priors naturally smooth sparse word counts or regression weights.
- **Prior knowledge incorporation** Encode beliefs (e.g. lexicon strengths, topic correlations) via informative priors.
- **Probabilistic Predictions** E.g., Posterior Predictive distributions

Explicit Statistical Modeling

- **Generative story** "Words $\sim$ Multinomial(topic mixture), topic mixture $\sim$ Dirichlet(α)" makes assumptions explicit.
- **Model comparison & checking** Bayes factors, WAIC/LOO, and posterior predictive checks.
- **Extensibility**

Hierarchical & Latent-Variable Benefits

- **Document-level random effects** Capture author or genre variability via hierarchical priors on topic proportions.
- **Word-level latent structure** Bayesian embedding models (e.g. neural variational LDA) recover nuanced semantic representations.
- **Cross-document coupling** Share statistical strength across documents, corpora, languages, or time via multi-level priors.
- **Uncertainty propagation** Downstream tasks (classification, prediction) inherit posterior uncertainty automatically.

Take-home, Flexible, Interpretable

- **Flexible:** Easily swap likelihoods (e.g. Poisson for counts, negative-binomial for overdispersion) or build new hierarchies
- **Interpretable:** Each parameter has a clear probabilistic role; credible intervals and posteriors speak directly to belief
- **Reproducible:** Full generative code (in Stan/Pyro/PyMC)

Laplace Smoothing vs Dirichlet Priors

Laplace Smoothing

$$\hat{\phi}_{k,j} = \frac{n_{k,j} + 1}{\sum_{j'}(n_{k,j'} + 1)}$$

Fixed κ, point estimates only.

Dirichlet Prior

$$\phi_k \sim \mathrm{Dirichlet}(\alpha)$$

Posterior: $\mathrm{Dir}(\alpha + n)$, with variance.

Latent Dirichlet Allocation: Model Specification

$$
\begin{aligned}
\theta_d &\sim \mathrm{Dirichlet}(\alpha), & d &= 1, \ldots, D \\
\phi_k &\sim \mathrm{Dirichlet}(\beta), & k &= 1, \ldots, K \\
z_{d,n} &\sim \mathrm{Categorical}(\theta_d), & n &= 1, \ldots, N_d \\
w_{d,n} &\sim \mathrm{Categorical}(\phi_{z_{d,n}}), & n &= 1, \ldots, N_d
\end{aligned}
$$

- α: concentration hyperparameter for document–topic Dirichlet prior
- β: concentration hyperparameter for topic–word Dirichlet prior
- θ_d: topic proportion vector for document d
- ϕ_k: word probability vector for topic k
- $z_{d,n}$: latent topic assignment of the nth word in document d
- $w_{d,n}$: observed nth word in document d, drawn from topic $z_{d,n}$

Naive Bayes Classifier

- For a document $d = (w_1, \ldots, w_N)$, posterior over class c:

$$P(c \mid d) \;\propto\; P(c) \prod_{i=1}^{N} P(w_i \mid c)$$

- Conditional independence: words w_i independent given class c
- Priors $P(c)$: frequency of each class in training data
- Likelihoods $P(w \mid c)$: estimated from word counts with Laplace (add-1) smoothing
- Prediction: choose $\hat{c} = \arg\max_c P(c \mid d)$
- Not *fully* Bayesian, relies on MLE estimates

Fightin' Words Methodology (Monroe et al., 2008)

- Objective: Identify words that most distinguish two corpora (A vs. B)
- Model counts with Dirichlet–Multinomial for each corpus:

$$\mathbf{n}^{(X)} \sim \mathrm{Multinomial}(N^{(X)}, \mathbf{p}^{(X)}), \quad \mathbf{p}^{(X)} \sim \mathrm{Dirichlet}(\boldsymbol{\alpha}),\ X \in \{A, B\}$$

- Define log-odds difference for each word:

$$\delta_j = \log \frac{p_j^{(A)}}{1 - p_j^{(A)}} - \log \frac{p_j^{(B)}}{1 - p_j^{(B)}}.$$

- Approximate posterior of δ_j as Gaussian via Beta marginals
- Rank words by standardized effect: $z_j = \hat{\delta}_j / \mathrm{sd}(\delta_j)$

Fighting Words vs. Raw Log-Odds

- **Raw Log-Odds:**

$$\log \frac{n_j^{(A)}/N^{(A)}}{n_j^{(B)}/N^{(B)}}$$

    - No smoothing $\rightarrow$ infinite or undefined for zero counts
    - Lacks uncertainty quantification
- **Fighting Words:**
    - Bayesian smoothing via Dirichlet prior
    - Provides posterior mean and variance of δ_j
    - Enables credible intervals and z-scores
- **Key difference:** principled shrinkage + uncertainty vs. unstable point estimates

Example: Unsupervised Sentiment Model

Building Sentiment-Informed Priors

- For each vocabulary term v, extract its AFINN score s_v (zero if absent).
- Define two Dirichlet concentration vectors:
  - $\alpha_v^+ = 1 + \kappa \cdot \max(s_v, 0)$, boosting words with positive scores.
  - $\alpha_v^- = 1 + \kappa \cdot \max(-s_v, 0)$, boosting words with negative scores.
- The scaling constant κ (e.g. 15) controls how strongly lexicon scores influence the prior.
- These priors encourage our "positive" word distribution to favor lexicon-positive words, and similarly for "negative."

Mixture-of-Multinomials Sentiment Model

- We posit two latent word distributions: ϕ_{neg} and ϕ_{pos}, each drawn from their Dirichlet priors.
- A global mixing probability $\pi \sim \mathrm{Beta}(1, 1)$ governs the overall chance a document is "positive."
- Each document's counts C_d arise by first flipping a latent component (neg/pos) then drawing words from the corresponding multinomial.
- This captures the idea that each document is either predominantly negative or positive in tone, with uncertainty.

Example: Semi-Supervised Model

- Goal: leverage both labeled and unlabeled data in a single generative model

Model Priors

- $\pi \sim \mathrm{Beta}(1, 1)$: prior on the probability a document is "positive."
- Two sentiment-specific word distributions:

$$\phi_{\text{neg}} \sim \mathrm{Dirichlet}(\alpha^{-}), \quad \phi_{\text{pos}} \sim \mathrm{Dirichlet}(\alpha^{+}).$$

- Dirichlet priors $\alpha^{\pm}$ are constructed from AFINN scores to bias word probabilities.
- These priors encode our lexicon-informed beliefs before seeing any documents.

Labeled-Data Likelihood

- For each labeled document d with $y_d = 0$ (negative):

$$C_d \sim \mathrm{Multinomial}\big(n_d,\ \phi_{\text{neg}}\big).$$

- For each labeled document with $y_d = 1$ (positive):

$$C_d \sim \mathrm{Multinomial}\big(n_d,\ \phi_{\text{pos}}\big).$$

- We "fix" the latent component to the observed label, effectively conditioning on true sentiment.
- This uses our labeled subset to anchor the two word distributions.

Unlabeled-Data Mixture Likelihood

- For each unlabeled document d, we don't know its sentiment component.
- We model its counts as a mixture:

$$C_d \sim (1-\pi)\,\mathrm{Mult}(n_d, \phi_{\text{neg}}) \;+\; \pi\,\mathrm{Mult}(n_d, \phi_{\text{pos}}).$$

- The mixture weight π links labeled and unlabeled parts of the model.
- This lets unlabeled documents inform both the global sentiment frequency and word distributions.

Drawbacks of Bayesian Framework for Text Analysis

- Computational cost
- Sensitivity to priors
- Scalability
- Implementation complexity

Questions?