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CSCI 3302

Homework 2

Introduction to Robotics
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1. You want to find a driving route that minimizes the number of turns between two locations in Boulder.
    a. Would you rather employ a sampling-based or a discrete algorithm such as Dijkstra or A* for doing this and why?

        I would rather **employ a discrete algorithm** for finding the minimal turn path between two locations. Sampling-based algorithms create possible paths by randomly adding points to a tree until some solution is found or time expires. This is useful for probabilistic complete issues, where the solution is sometimes unknown to be possible. However, sampling will most likely not find the optimal path in this case, while complete discrete algorithms, such as Dijkstra, A* or D* can find optimal paths if the solution is known to be possible.

    b. Provide a possible cost function that encodes both path length, l, and the number of turns, t.

        Say l is number of edges and t are the vertices. We can denote the Possible Cost of the absolute worst cost to be Dijkstra's worst case, since Dijkstra's is the least efficient algorithm compared to A* and D*:

        $O(|l| + |t| \log(|t|))$

        However, A* and D* both depend on what heuristics are being used to determine the time complexity. In the case for both possible worst costs:

        Time: $O(|l|)$ Space: $O(|t|)$

        However, if we are discussing about simple distance cost, a proper function would be:

        Cost = l * (t+1)

        This function takes into account the length of travel at the expense of each turn. For example, taking no turns would still cost the length of travel, l. However, as the amount of turns increases, so does the cost of travel by length l per turn t.

2. Assuming points are sampled uniformly at random in a randomized planning algorithm. Calculate the limiting behavior of the following ratio (number of points in tree)/(number of points sampled) as the number of points sampled goes to infinity. Assume the total area A_{total} (including free space and obstacles) and the area of free space A_{free} (not including obstacles) within are known.

Assuming that the number of points in the tree is denoted t, and the number points sampled are denoted s: As the limit $_{s \rightarrow \text{infinity}}$ for $\frac{t}{s}$, the ratio converges to be the equivalent to **A_{free} / A_{total}.**

3. Assuming a k-d-tree is used as a nearest-neighbor data structure to store a RRT data structure, and points are sampled uniformly at random, calculate the run-time of inserting a point into a tree of size N. Use "big-Oh" notation, e.g. O(N).

   Insertion to a N sized k-d-tree, assuming that it is uniformly balanced, is **O(log(N))**.

4. Why does the bandwidth of a Ultra-sound based distance sensor decreases significantly when increasing its dynamic range, but that of a laser range scanner does not for typical operation?

   Ultra-sound based sensors becoming increasingly more inaccurate with distance. These sensors send out waves of sound frequencies and waits for the sound to bounce off of an object and return to the sensor face in order to receive signals. The time delay between sending and receiving the sound waves is how ultrasound determines distance between itself and other objects. As the dynamically changing range increases, the delay becomes increasing longer, which is cause in bandwidth loss.
   For a laser range scanner, uses lights travelling as light speed to determine distance. This is significantly faster and more efficient than ultrasound traveling at the speed of sound, which is the reason lasers retain bandwidth at longer ranges.

5. You are designing an autonomous electric car to transport goods on campus. As you are worried about cost, you are thinking about whether to use a laser scanner or an ultra-sound sensor for detecting obstacles. As you drive rather slowly, you are required to sense up to 15 meters. The laser scanner you are considering can sense up to this range and has a bandwidth of 10Hz.
   a. Calculate the time it takes until you hear back from the US sensor when detecting an obstacle 15m away. Assume that the robot is not moving at this point. Use c = 300m/s for the speed of sound.
      Assume c is the speed of sound = 300, t is time and d is distance = 15

      $t = (2 * d) * \frac{1}{c} \Rightarrow t = (2 * 15) * \frac{1}{300} =$ **0.1 Seconds**

   b. Calculate the time it takes until you hear back from the laser scanner. Hint: you don't need the speed of light to answer this question.
      Assume f is the frequency = 10.

      $t = \frac{1}{f} \Rightarrow t = \frac{1}{10} =$ **0.1 Seconds**

6. A Global Positioning System sensor provides position estimates within a circle of approximately 3m diameter. Every now and then the satellites on the horizon change and

the center of this circle moves elsewhere, approximately staying within a 30 meter radius of the true location of the receiver.

a. Given the error data above, which value corresponds to accuracy and which to precision?
    i. **Accuracy**: The 30 meter radius from the true location of the receiver's true location, since it is focusing on the exact value.
    ii. **Precision**: The 3 meter diameter circle reading estimating the position, since all values are close to one another relatively.

b. The sensor provides 18000 readings per hour. What is its bandwidth?

    18000 readings an hour is equal to 300 reading per minute, or 5 readings a second. Using this information, we can determine a single reading occurs every 0.2 seconds.

    Assume frequency is denoted f and time is denoted as t:

$$f = \frac{1}{t} \Rightarrow f = \frac{1}{0.2} = \mathbf{5\ Hz}$$