University of Colorado
Department of Computer Science

Numerical Computation CSCI 3656

Spring 2015

Problem Set 1

Issued: 12 January 2016

Due: 21 January 2015 **at the beginning of class!**

Note: Email submissions of assignments will not be accepted.

1. *[10 pts]* Using your favorite computer language, play around with some simple arithmetic calculations and find one whose answer is incorrect because of the imprecision of floating-point arithmetic. Submit the line of code, the correct answer (i.e., what you'd get if you were doing that calculation with pencil & paper, with infinite-precision numbers), the answer that the calculation produced on your computer, and an explanation of what it was that the computer arithmetic system did that made the answer wrong.

2. *[6 pts]* Consider a variation of the IEEE standard in which the exponent is represented by seven bits and the fractional part of the mantissa is represented by four bits. What are the largest and smallest positive machine numbers that can be represented using this system, not including $\pm\infty$ or ± 0? Give your answers as both binary and decimal numbers. Assume that all other rules of the IEEE standard apply, including normalized numbers and reserved exponent values. Neglect subnormals (aka "graceful underflow"), which are permitted, but not *required*, by that standard.

3. *[6 pts]* Consider a variation of the IEEE standard in which the exponent is represented by e bits and the fractional part of the mantissa is represented by three bits. If the largest and smallest positive machine numbers in this system are 15.0 and 0.25, respectively, what is e? As in the previous problem, assume that all other rules of the IEEE standard apply.

4. *[6 pts]* Consider a computer that uses a really dumb arithmetic system that simply stores numbers in n-bit memory locations by dicing up the desired number range into $2^n - 1$ even-size chunks. If the range of numbers to be stored is -100000 to 100000 and $n = 5$, what is the (decimal) range of numbers that will be stored as 10000_2?

5. *[4 pts]* The thermometer at the top of the Panoramic lift at Winter Park registered 10 degrees Fahrenheit last Saturday morning, but the true temperature was 9 degrees. What were the absolute and relative errors in that measurement?

6. A bit about roots

- *[2 pts]* Give an example of a function, $f(x)$, that has no roots.
- *[2 pts]* Give an example of a function, $f(x)$, that has exactly one root.
- *[2 pts]* Give an example of a function, $f(x)$, that has an infinite number of roots.
- *[2 pts]* How many roots does $f(x) = x^2 - 4x + 6$ have? Are they real or complex?