University of Colorado
Department of Computer Science

Numerical Computation CSCI 3656

Spring 2015

Exam 2

1. *[10 pts]* Consider the following chunk of Matlab code, which calculates a set of cubic Bezier curves through an arbitrary number of points. The percent sign (%) is the comment character in Matlab; the ellipsis (...) is the way to split a long line of code across two lines in the file.

```
function Bezier(x,y)
figure
hold on
 n = size(x);
 if round((n(2) - 1)/3) < (n(2) - 1)/3
 else
    sprintf('Complain')
    return
 end
 u = 0:0.01:1;
counter = 0;
while counter < (n(2) - 1)/3
xu = (1-u).^3*x(counter * 3 + 1) + 3*(1-u).^2.*u*x(counter * 3 + 2) ...
    + 3*(1-u).*u.^2*x(counter * 3 + 3) + u.^3*x(counter * 3 + 4);
yu = (1-u).^3*y(counter * 3 + 1) + 3*(1-u).^2.*u*y(counter * 3 + 2) ...
    + 3*(1-u).*u.^2*y(counter * 3 + 3) + u.^3*y(counter * 3 + 4);
plot(xu,yu);
counter = counter + 1;
end
end
```

   There's a bug in that code—not in the formula lines in the middle of the while loop, though; those are correct (albeit ugly, but that's because the formula itself is ugly). Please find and circle that bug. What should the chunk of code that you circled really be if this is to work right?

2. *[4 pts]* How many different polynomials of degree two can be fit to three (non-colinear) points?

3. *[4 pts]* How many different polynomials of degree three can be fit to three (non-colinear) points?

4. *[6 pts]* List three ways to find a *global* interpolating polynomial $P_{n-1}(x)$ of degree $n-1$ that passes **through** n points.

5. *[5 pts]* List a *geometric* difference between Bezier curves and B-splines.

6. *[5 pts]* Bezier curves and cubic natural splines are both patchwork (non-global) interpolating functions. Identify one difference between the geometry with which the curves meet at the patch boundaries in these two methods.

7. *[5 pts]* Give an example of a commercial use of either Bezier curves or B-splines.

8. *[7 pts]* Give an example of a problem that the "Newton's divided differences" method solves. (Note: there is no need to *solve* the problem; just describe it to us.)

9. *[13 pts]* Use least squares to fit a power law to these three points: $(x, y) =$(1, 0.2), (3, 6.7), and (4, 16.9). Here are the normal equations: $A^T A\bar{x} = A^T b$.

10. *[10 pts]* What is the RMSE of your fit in the previous problem **measured in the log space**? What about the RMSE in the original (x, y) space?

11. *[10 pts]* Explain what the Runge phenomenon is (perhaps with a drawing) and describe any bad effects that it can have on numerical methods.

12. *[13 pts]* Using 3D interpolation on the values for $f(x, y)$ given in the table below, figure out $f(2, 4)$.

$f(x,y)$	$y=2$	$y=3$	$y=5$
$x=1$	1	3	-2
$x=3$	-1	1	4
$x=5$	3	4	-2

13. *[8 pts]* How might you parallelize the steps in your solution to the previous problem?