University of Colorado
Department of Computer Science

Numerical Computation CSCI 3656

Spring 2016

Problem Set 8 Solutions

1. *[10 pts]* Go to one of the desktop machines in the CSEL and invoke the Unix drawing tool `xfig`. Play with the four spline- and curve-drawing tools, whose icons look like figure eights and S curves. Try some of the examples from the sheet that I handed out in class on March 8th.

   Using your knowledge about different kinds of splines, do some experiments and form some conjectures about which ones `xfig` uses for the tool whose icon looks like a smooth figure eight and the one that looks like a figure eight with superimposed dots. With each answer, turn in a few sentences that describe your rationale for making that diagnosis.

   Suggestions:

   - try some colinear points
   - use the 'move-point' tool (next to the 'move' tool)
   - look at the slope of the spline at the endpoint

   Answers will vary. We were looking for good reasoning that is firmly grounded in the course material.

   The smooth S/figure-8 with superimposed dots creates a curve that goes through each of the specified points, and appears to be differentiable at the junctions between points. The only two possibilities here, among the interpolation methods covered in class, are cubic splines or some kind of polynomial fit. And since a polynomial fit to an arbitrary number of points would go nuts outside the fit range, that eliminates **that** possibility. So of the methods we've studied, the only possible culprit here is the cubic natural spline.

   For the smooth S/figure-8 with no superimposed points, the resulting curve does not seem to hit each of the points, so it is not a cubic natural spline or a polynomial fit. That leaves Bezier curves or B-splines. You can verify this by yanking on one of the points and noting that the change that causes is localized. Recall that both of these are built "patch wise"—from groups of n points—and that neither one is constrained to hit all the intermediate points in the patch, but that B-splines need not hit the points at the two ends of the patches. This fact isn't really useful in this task, though, because you don't know the patch size that `xfig` uses. So you

need to rely on other properties of those two kinds of splines in order to figure out which one is being used here. There are lots of properties that you could use for this. For example, the `xfig` curve appears to stay within the convex hull of the points. Both Bezier curves and B-splines do this, so that property isn't a useful discriminator. However, it also appears that the slope of the interpolated curve at the endpoints is equal to the slope of the secant line between the endpoint and the corresponding intermediate point. This is a characteristic of a Bezier curve.

The difference between the S and figure-8 tools is that `xfig` forces the curve to close in the latter—probably by adding extra control points on top of the first point you clicked in.