Numerical Computation CSCI 3656
Spring 2016
Problem Set 4

Issued: 4 February 2016

Due: 11 February 2016

1. Write a program that:

   - takes as input
     - a 3×3 matrix $\mathbf{A}$
     - a three-element vector $\vec{b}$
   - uses Gaussian elimination — *without pivoting* — to find the solution $\vec{x}$ to the matrix equation $\mathbf{A}\vec{x} = \vec{b}$
   - prints out the augmented matrix at every step

   For this problem, please write the code in python, C, C++, Lisp, Java, etc. That is, do this *from scratch*, without using any linear algebra libraries or calling upon any built-in linear algebra commands from packages like Maple, Matlab, IDL, Mathematica, etc. (You may use those programming environments if you wish, but do not use their built-in linear algebra stuff. One of the points of this problem is to get you thinking about what data structures, loops, etc., you should use for matrix problems.)

   For this assignment, we would like to see both a transcript of your interaction with the program **and a copy of your code.** And do remember that you need to let us know if you worked with other class members on that code.

   (a) *[25 pts]* Use your program to solve the following set of equations and turn in a transcript of the session:

   $$
   \begin{aligned}
   -11x_1 + 3x_2 + 3x_3 &= 18 \\
   -2x_1 - 5x_2 + x_3 &= -3 \\
   x_1 - 10x_2 + 8x_3 &= 5
   \end{aligned}
   $$

   (b) *[5 pts]* Use your program to solve the following set of equations and turn in a transcript of the session:

$$0.16x_1 + 0.85x_2 + 0.34x_3 = 0.27$$
$$-0.25x_1 - 1.5x_2 + 0.50x_3 = -0.21$$
$$1.03x_1 + 5.46x_2 + 1.77x_3 = 1.75$$

(c) *[5 pts]* Use your program to solve the following set of equations and turn in a transcript of the session:

$$-2x_1 + x_2 - 4x_3 = -5$$
$$7x_1 - 3x_2 + x_3 = -2$$
$$3x_1 - x_2 - 7x_3 = 1$$

2. *[15 pts]* Compute the condition number of the three matrices in problem 1 using the $||.||_\infty$ norm. What do those numbers tell you about those matrices? Does that line up with what happened when you ran your Gaussian elimination code on them?

3. *[15 pts]* Solve the system in problem 1(b) *by hand* using Gaussian elimination *with partial pivoting*. (You're welcome to modify your code from problem 1 to do pivoting in order to check your answers, but it's worth knowing how to do these kinds of problems by hand — e.g., for the hour exams.) Do you get the same answer with and without pivoting? Why or why not?

4. *[15 pts]* Write down the PA=LU factorization of the system in problem 1(b) and use those factors to solve this system:

$$0.16x_1 + 0.85x_2 + 0.34x_3 = 0.27$$
$$-0.25x_1 - 1.5x_2 + 0.50x_3 = -0.21$$
$$1.03x_1 + 5.46x_2 + 1.77x_3 = 1.75$$

Do this *by hand*, in the most efficient way possible. Please show your work, and be neat enough that we can follow what you're doing. To finish off this problem, please explain why LU factorization is a good idea—i.e., what is the advantage of doing it? In what situation, for example, is that a Good Thing?

5. *[5 pts]* What is the difference between Jacobi and Gauss-Seidel, in terms of the way they work? Which one converges faster? Why?