University of Colorado
Department of Computer Science

Numerical Computation CSCI 3656

Spring 2015

Exam 1

1. *[8 pts]* Consider the following chunk of Matlab code, which uses Gaussian elimination (without pivoting) and back substitution to solve $A\vec{x} = \vec{b}$. The percent sign (%) is the comment character in Matlab.

```
function x = gauss_elim(A, b)
% Find size of matrix A and vector b:
[n, n] = size(A);
[n, k] = size(b);
for i = 1:n-1
    m = -A(i,i)/A(i+n,i);
    % Following line is the "row <- row + multiplier*(other row)" step:
    A(i+1:n,:) = A(i+1:n,:) + m*A(i,:);
    % Gotta do that same thing to the b vector too:
    b(i+1:n,:) = b(i+1:n,:) + m*b(i,:);
end;
% Below here is the back substitution step
x = zeros(n,k);
x(n,:) = b(n,:)/A(n,n);
for i = n-1:-1:1
    x(i,:) = (b(i,:) - A(i,i+1:n)*x(i+1:n,:))/A(i,i);
end
```

There's a bug in the forward elimination section of that code. Please circle it. What should the chunk of code that you circled really be if this is to work right?

2. *[10 pts]* What is the $PA = LU$ factorization of this matrix? Show your work, and be neat enough that we can see what you're doing. Please specify no more than one significant figure to the right of the decimal place in the entries of the matrices. If you don't have a calculator, you may leave those entries in the form of fractions.

$$\begin{bmatrix} 3 & 1 & -2 \\ -6 & 2 & 2 \\ -1 & 3 & 2 \end{bmatrix}$$

3. *[5 pts]* Why do you need the P matrix in the $PA = LU$ factorization—what purpose does it serve?

4. *[5 pts]* If $cond(B) = 100000$ and $||\vec{b} - \vec{b'}||$ is very small, what can you say about $||\vec{x} - \vec{x'}||$, where $\vec{x}$ and $\vec{x'}$ are the solutions to $B\vec{x} = \vec{b}$ and $B\vec{x'} = \vec{b'}$, respectively?

5. *[10 pts]* If A is the matrix in the problem 2 above and $\vec{b} = (-17, 4, 1)^T$, what is the solution $\vec{x}$ to $A\vec{x} = \vec{b}$? (Please do this *in the most economical way possible.*)

6. *[8 pts]* What is the difference between Gaussian elimination and the Gauss-Jordan method, in terms of the way they work? Which one is more computationally efficient?

7. *[6 pts]* Why is it important to consider the effects of floating-point arithmetic in numerical computing problems?

8. *[8 pts]* Consider a variation of the IEEE standard in which the exponent is represented by six bits and the fractional part of the mantissa is represented by three bits. What is the bit string for the largest positive machine number that can be represented in this system (not including $\pm\infty$)? What decimal number does that represent? (If you don't have a calculator, you may leave it in base-two scientific notation.) Assume that all other rules of the IEEE standard apply, including standard normalization (e.g., no subnormals) and reserved exponent values.

9. *[8 pts]* What is the difference between the Jacobi and Gauss-Seidel methods? Which one *parallelizes* more effectively?

10. *[8 pts]* Perform two iterations of the fixed-point method on $f(x) = x^2 - 5x + 4$ starting from the guess of $x_0 = 2$. What do you predict about the rest of the iteration sequence?

11. *[5 pts]* How do you determine whether a matrix is diagonally dominant?

12. *[5 pts]* Give an example of a problem that the bisection method solves. Be sure to mention any conditions on guesses, functions, parameters, etc. (Note: there is no need to *solve* the problem; just describe it to us.)

13. *[4 pts]* List four methods that can be used to solve $A\vec{x} = \vec{b}$ problems. (We will take your first four answers, so please choose carefully.)

14. *[5 pts]* If the $n \times n$ matrix A is nonsingular, how many possible solutions are there to the set of n linear equations $A\vec{x} = \vec{b}$?

15. *[5 pts]* What is swamping?