

University of Colorado  
Department of Computer Science

Numerical Computation

CSCI 3656

Spring 2016

Problem Set 3

Issued:

28 January 2016

Due:

4 February 2016

1. [5 pts] Use your Newton's method code from PS2 to find a root of the function  $f(x) = x^2 - x - 2$  starting from the initial guess  $x = 1.5$ . Note the number of iterations required to get  $|f(x)| < 0.0001$ .
2. [5 pts] Use your Newton's method program to find a root of the function  $f(x) = x^3 - 3x^2 + 4$  starting from the initial guess  $x = 1.5$ . Note the number of iterations required to get  $|f(x)| < 0.0001$ .
3. [5 pts] Compare the convergence rates that you determined in the previous two problems for Newton's method on these two polynomials. Are these numbers consistent with what you know about the theoretical convergence rates of this method? Explain.
4. [5 pts] If you were to apply the bisection method to a function with a multiple root at  $x = 2$ , would the errors ( $|f(x)|$ ) converge more slowly than if that function had a *single* root at  $x = 2$ ? Explain.
5. [4 pts] What would it mean for a rootfinder to be *cubically convergent*?
6. [4 pts] What is the next-term rule?
7. Dusting off your linear algebra
  - [3 pts] Give an example of a function  $f(x)$  that is linear.
  - [3 pts] Give an example of a function  $f(x)$  that is nonlinear.
  - [3 pts] If the  $n \times n$  matrix  $A$  is nonsingular, how many possible solutions are there to the set of  $n$  linear equations  $A\vec{x} = \vec{b}$ ?
  - [3 pts] If  $A^{-1}$  exists, what is true of  $|A|$ ?

8. [10 pts] Solve this system **by hand** using Gaussian elimination. Show your work:

$$\begin{aligned}3x_1 + x_2 - 2x_3 &= -17 \\-6x_1 + 2x_2 + 2x_3 &= 4 \\-x_1 + 3x_2 + 2x_3 &= 1\end{aligned}$$