University of Colorado
Department of Computer Science

Numerical Computation CSCI 3656

Spring 2016

Problem Set 9

Issued: 15 March 2016

Due: 29 March 2016

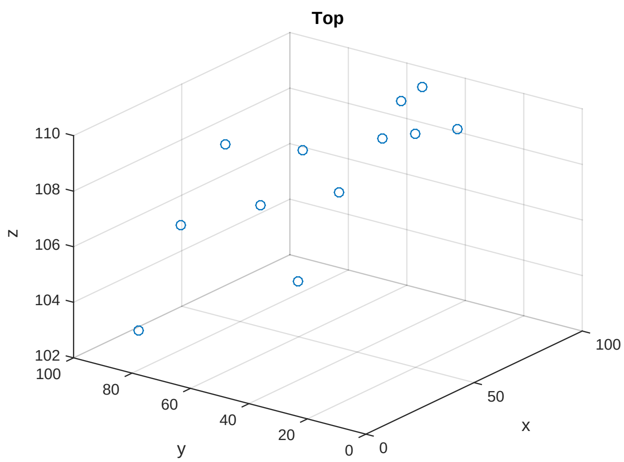
The purpose of the first three problems in this assignment is to explore how to deal with sparse, real-world data in interpolation methods. You will need your interpolation code from previous problem sets in order to do this assignment, and you will need to do some creative thinking. Do *not* start this one the night before it's due!

1. *[5 pts]* Here are (x, y, z) coordinates for 22 points on the Worthington glacier near Valdez, Alaska — twelve on the top surface and ten on the bottom:

top surface	bottom surface
(33.44 87.93 105.88)	(15.59 35.07 12.88)
(8.81 84.07 103.11)	(38.57 37.17 13.33)
(15.62 34.83 105.98)	(61.10 67.15 17.31)
(40.16 38.71 108.13)	(58.97 92.05 19.09)
(61.45 67.07 108.12)	(36.98 63.24 16.51)
(58.81 91.44 107.72)	(64.45 42.66 20.01)
(36.97 63.29 107.14)	(89.18 46.85 27.71)
(64.71 42.38 109.07)	(66.87 18.48 14.24)
(89.11 46.49 109.93)	(65.90 31.93 21.0)
(67.24 18.32 109.99)	(76.55 44.51 22.0)
(65.90 31.93 109.51)	
(76.55 44.51 109.91)	

Plot the points on the top surface (just the *points*, not the whole surface!) using your favorite 3D plotting tool. Repeat for the bottom set. Notice how changing the perspective affects your ability to make any sense of the surface from the points. If your language/package supports it, try connecting the points in a "wireframe" plot. Turn in one copy of the best version of each of the two surfaces, and please write down what language/command you used.

See the scatter plot of the top surface below. This was generated using the `scatter3` function in MATLAB.



2. *[20 pts]* Your task in this problem is to perform a 3D surface interpolation of these data. Fitting a smooth surface to these data is nontrivial; each triple of numbers in the table above required laser/GPS surveying (and, for the bottom points, it also entailed drilling a hole completely through the glacier!), so the points are unevenly spaced and very sparse — too sparse for most surface interpolation functions to deal with. You will have to improvise, creatively, in order to solve this problem well.

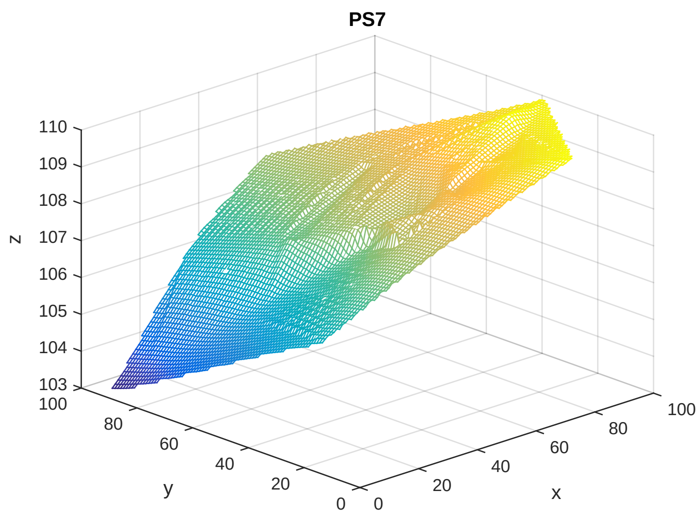
   Using whatever *smooth* surface interpolation function is available in your favorite programming environment (e.g., Matlab's `griddata` or Mathematica's `Interpolation` command), fit a surface to the *top* points. The surface may pass through the points or not, as you wish. **NOTE!** because the glacier data are sparse and irregularly spaced, you will not be able to blindly toss them into these garden-variety tools and expect to get a reasonable solution. Simply plugging these points into `griddata` will garner very little credit, and plugging them into `Interpolation` won't work at all. Rather, you will have to figure out what the glacier surface looks like *between* and *beyond* the existing data points — using some sensible interpolation and extrapolation methods based on your code from previous problem sets in this course — and use that information to add some new (made-up, but hopefully consistent) points to the data set. Depending on the method you use, you may have to improvise at the edges to get surfaces that end smoothly — e.g., adding extra copies of the end points.

Plot your interpolated surface in 3D, experimenting with shading, point size, and other plotting parameters — contour versus perspective plot, various shading or coloring schemes, etc. — until it looks as good as possible. Turn in a printout of this plot, together with a one-paragraph discussion of your results and observations, including at least a few sentences on *how* and *why* you added interpolated points between those in the data set. (This description is worth half the credit for this problem.)

There is no single right answer here, and this is the hardest part of this assignment. Your creativity in working around this problem and manufacturing "good" points to add will determine how nice your surface looks. Hint: start by using some form of interpolation to add points between the existing ones. You may also use any other method you wish (e.g., neural nets); if you use a method that does not appear in the textbook, please include an explanation of how it works.

See attached surface plot of the glacier. For this solution, I adapted a cubic Bezier spline function by adding a parameterized z term analogous to the x and y terms. This was interpolating a (1D) curve that had a component in all three dimensions. The easiest approach, though, is just to use linear interpolation in three dimensions. For example, since the function varies proportionally to x in both the y and z direction, the midpoint of a line between two points is just at the value of x, y, and z exactly half way between the points of interest. This would have allowed for a quick addition of many points.

In general, with more data, there is an easier way to do this. If a plane is selected that is perpendicular to one of the coordinate axes (so that that the value coordinate is constant), any 2D interpolation technique could be used between two points in that plane. This would be a quick way to add data points in a grid-like manner. That was not feasible for this problem, because there were never multiple points in a plane perpendicular to an axis. So, I'm only telling you this in case you come across another, simpler 3D interpolation problem in the future...The final interpolation was performed using the `griddata` function in MATLAB to on an arbitrary xy grid before generating the plot using the `mesh` function.



3. *[optional for 10 pts extra credit]* Repeat problem 2 for the bottom surface. Now try *superimposing* the two surfaces, removing the hidden part of the bottom surface if your plotting tool lets you do that easily. Note that the scales in the two data sets are quite different, so when you plot them on the same axes, some surface detail may be lost.

   Note: The approach to the bottom surface is the same as that to the top but may have required that more points be added because there is a rock in the middle of it that looks like a big bump in the surface.

4. *[10 pts]* Problem 8(c) on page 198 of the textbook.

   $A =$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$b =$

$$\begin{bmatrix} 5 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$A^T A =$

$$\begin{bmatrix} 4 & 6 \\ 6 & 14 \end{bmatrix}$$

$A^T b =$

$$\begin{bmatrix} 12 \\ 12 \end{bmatrix}$$

Solving $A^T A c = A^T b$, $c =$

$$\begin{bmatrix} 4.8 \\ -1.2 \end{bmatrix}$$

For the line $y = -1.2x + 4.8$, the errors are: $0.2, -0.6, 0.6, -0.2$. The sum of squared errors is: 0.8. The RMSE is 0.4472.

5. *[10 pts]* Fit these data to a power-law model by using linearization. Find the RMSE of the fit:

t	y
1	2
1.5	3.7
2.5	7.9
3	10.4

$A =$

$$\begin{bmatrix} 1 & 0 \\ 1 & 0.4055 \\ 1 & 0.9163 \\ 1 & 1.0986 \end{bmatrix}$$

$b =$

$$\begin{bmatrix} 0.6931 \\ 1.3083 \\ 2.0669 \\ 2.3418 \end{bmatrix}$$

$A^T A =$

$$\begin{bmatrix} 4 & 2.4204 \\ 2.4204 & 2.2110 \end{bmatrix}$$

$A^T b =$

$$\begin{bmatrix} 6.4101 \\ 4.9971 \end{bmatrix}$$

Solving $A^T A[k, c_2]^T = A^T b, [k, c_2]^T =$

$$\begin{bmatrix} 0.6958 \\ 1.4984 \end{bmatrix}$$

With $c_1 = e^k$, the power law model is $y = 2.0054 * t^{1.4984}$. The errors are: $0.0054, -0.0182, 0.0154, -0.0021$. The sum of squared error is: $6.0197 * 10^{-4}$. The RMSE is 0.0123.

(This one may go away, depending on how far I get in lecture on 3/15. Please check the moodle for announcements if you missed that class.)