Power computations for interaction effects

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In this paper we provide a tool for planning a sample for a simple design. The reader who is planning his study and needs to know the optimal sample size for his study has two options. First, he may study the tables with recommended sample sizes given in this paper and select the table, which equals or is close to the design he actually intends to use. Second, he may use the R-functions that we have developed to run a simulation with the parameters of his choice. These functions are freely available.

Here we focus on conditional versus unconditional effects of a predictor on a dependent variable. When the effect of a predictor is conditional of the value of another variable this is called moderation.

The moderation model

In a moderation model we assume that the effect of the predictor *x* on the dependent variable *y* is conditional on another variable *z*, which is called the moderator. In the statistical model (Hayes, 2013) this implies that there is an interaction term between *x* and *z* in the model (usually the product between both variables: *xz*). The basic moderation model thus consists of three predictors of *y*: *x*, *z* and *xz*.

Assume the following simple moderation model:

$$y_i = b_0 + b_1 x_i + b_2 z_i + b_3 x_i z_i + \epsilon, \qquad [1]$$

with *y* the dependent variable measured on subject *i*, *x* the predictor and *z* the moderator, *xz* the interaction term. The *b*'s are regression coefficients. Variables *x* and *z* are distributed as $N(0,1)$ and the error term ε as $N(0, \sigma_\varepsilon)$. The correlation between *x* and *z* is *r*. The error term ε is uncorrelated with both *x* and *z*.

Simulation study 1: moderation in the one-level model

We simulate *x*, *z* and *ε* from a multivariate normal distribution using the R function *mvrnorm*(). In the appendix it is explained how the parameter values are connected with each other. The correlation between *x* and *z* has one of three values, representing no correlation (r = 0), small correlation (r = .30), moderate correlation (r = .50), and high (r = .80). The interaction term *xz* is constructed by multiplying *x* and *z*. The *b* coefficients are chosen as the square roots from respectively, 0.5, 0.3, and 0.2.

The *y* is then computed as the weighted linear combination of these three variables (see formula 1), the intercept is assumed to be zero. Finally, the variance of ε is taken as one of three values, representing small (1), medium (3), and large levels (9) of random error. This error term is also added to *y* and is chosen such that it corresponds with *R* squared values of respectively .50, .25, and .10.

Using the formula's given in the appendix the effect sizes and *R* squares in all conditions are given in table 1. With uncorrelated predictors the table shows that the effect sizes (beta's) of the three parameters are respectively, 0.50, 0.39, 0.32 in the small error condition, 0.35, 0.27, 0.22 in the medium error condition, and 0.22, 0.17, and 0,14 in the large error condition. For correlated predictoirs, see other cells in table 1.

Table 1. Conditions and effect sizes used for simulation in single level moderation model.

var(e)	*r*(x,z)	Beta1	Beta2	Beta3	R^2
1	0.00	0.50	0.39	0.32	0.50
1	0.30	0.47	0.37	0.32	0.56
1	0.50	0.45	0.35	0.32	0.59
1	0.80	0.43	0.33	0.34	0.64
3	0.00	0.35	0.27	0.22	0.25
3	0.30	0.34	0.27	0.23	0.29
3	0.50	0.34	0.26	0.24	0.33
3	0.80	0.32	0.25	0.26	0.37
9	0.00	0.22	0.17	0.14	0.12
9	0.30	0.22	0.17	0.15	0.13
9	0.50	0.22	0.17	0.15	0.14
9	0.80	0.22	0.17	0.17	0.16

Because the relevant effect size for moderation (Beta 3) only slightly changes as the correlation changes, we report only one value of rho (r = .30) in subsequent analyses. The effect sizes for the interaction term seem to have a realistic and relevant range. Values smaller than .15 are not likely to have much practical relevance. Larger effect size values than 0.35 do not occur very often, and if they do occur it will not be problematic to detect them.

After generating data according to the specifications set above, we used the *lm*() function in R to perform the analyses. The number of replications in each condition was set 2,000. The results are shown in Figure 1 for alpha = .05 and in Figure 2 for alpha = .01.

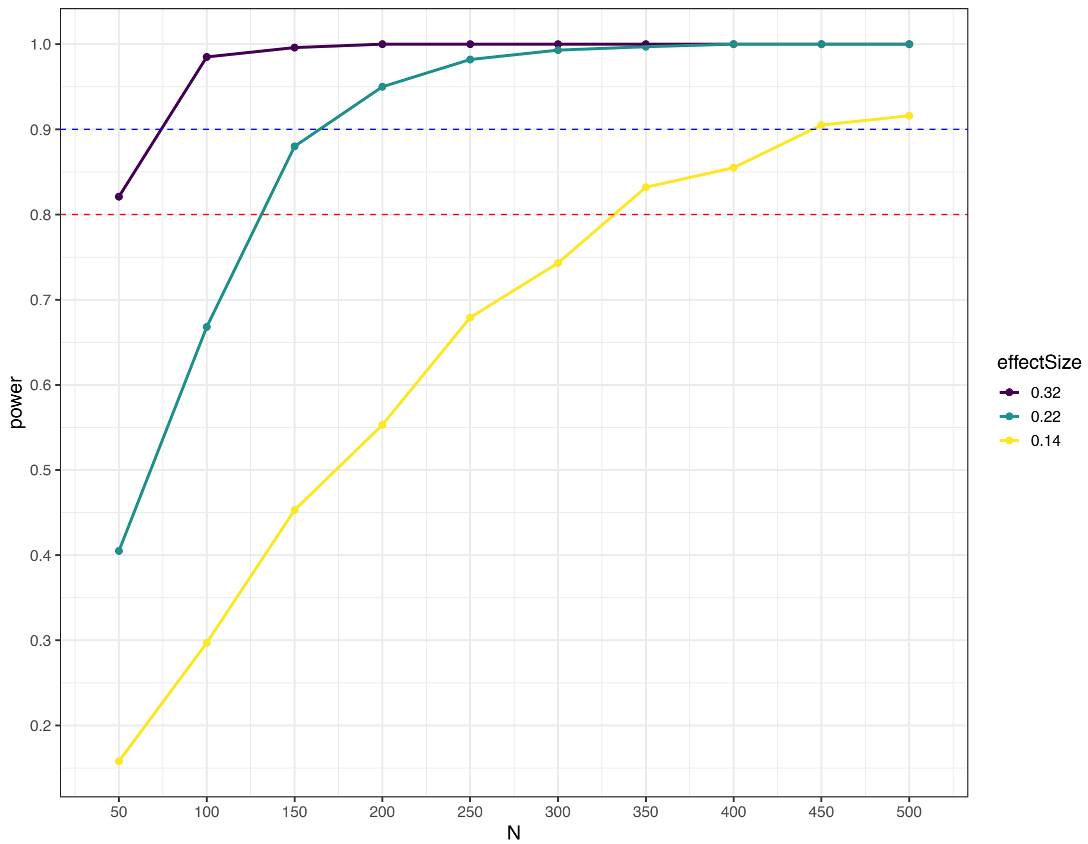

Figure 1. Power curve for the interaction effect for alpha = 0.05. The red and blue dashed lines indicate power of 80% and 90% respectively. Correlation between predictors is 0.30.

From these analyses we can conclude that with relatively large effects (> 0.30) for the interaction term a sample of N = 50 is sufficient if you accept a type I error of 5%. For N = 100 the power is even more than 90%. When the interaction corresponds with a small effect size, which is far more common, N = 350 is necessary for a power of 84%.

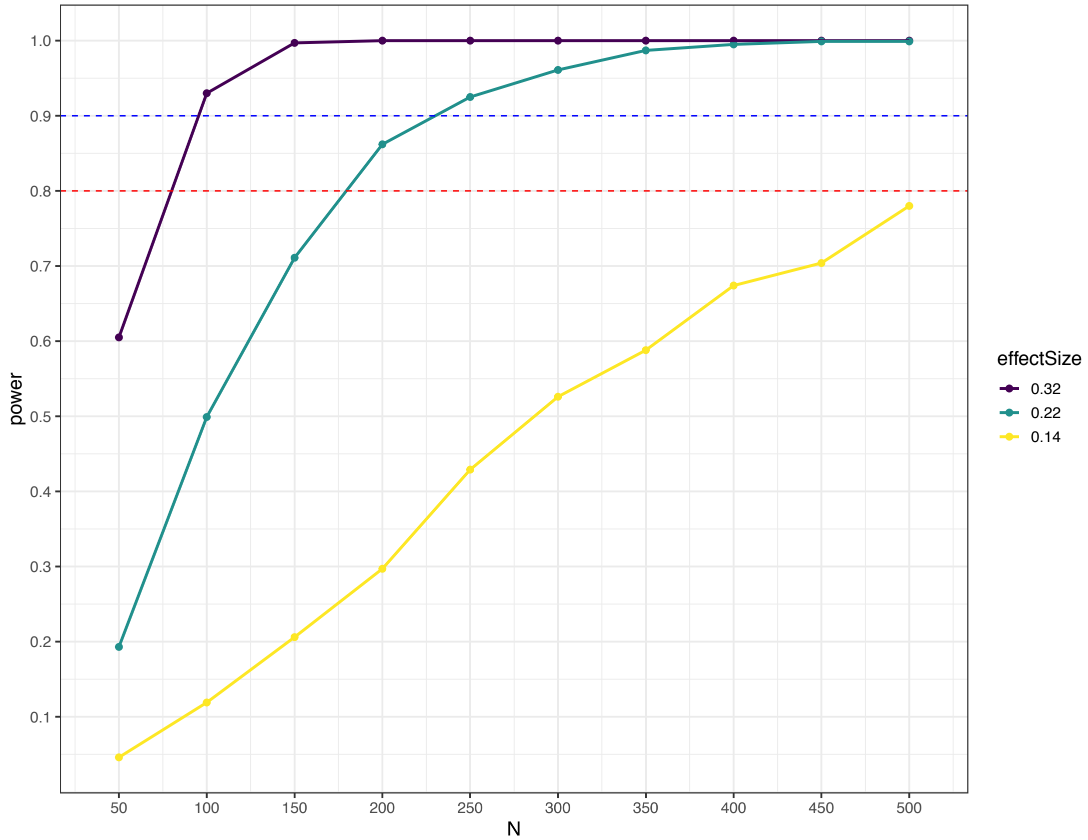

Figure 2. Power curve for the interaction effect alpha = 0.01. The red and blue dashed lines indicate power of 80% and 90% respectively. Correlation between predictors is 0.30.

From these analyses we can conclude that with relatively large effects (> 0.30) for the interaction term a sample of N = 100 if you are accept a type I error of 1%. For N = 100 the power is even more than 90%. When the interaction corresponds with a small effect size, N > 500 is necessary for a power of 80%.

Attrition and missing values

After computing an estimate for the sample size, this estimate will almost invariable underestimate the sample size required for the study, unless you carefully address the following issues. First, participants often drop out of studies, a phenomenon called attrition in longitudinal studies. Longer and more intensive studies are likely to have higher attrition rates. Participants may also have more missing data than in cross-sectional studies, because of the intensive character of the study. Second, participants may exhibit more variation (i.e. be more different) than expected, which directly inflates the error variance and therefore the effective sample size. Third, participants sometimes provide data that is not useable (e.g. errors or unrealistic values), in which case they have to be excluded for some or all analyses. In fact, mistakes can be made at all levels during the data gathering process, which causes loss of data. Because this influences the actual required sample size, it is important to be aware of these issues. If no other guidelines are available, adding 20% to the raw estimate seems reasonable.

Appendix

Effect sizes

One of the crucial parameters in computing power analyses is the expected effect size. We assume that x, z, and ε are normally distributed and standardized with mean 0 and variance equal to 1. The effects sizes of the three effects are the standardized coefficients (*beta*), defined as:

$$\beta_1 = b_1 \frac{\sigma_x}{\sigma_y} \, and \; \beta_2 = b_2 \frac{\sigma_z}{\sigma_y} \; and \; \beta_3 = b_3 \frac{\sigma_{xz}}{\sigma_y}.$$

If x, z and xz are uncorrelated, the variance of y is:

$$\sigma_y^2 = b_1^2\sigma_x^2 + b_2^2\sigma_z^2 + b_3^2\sigma_{xz}^2 + \sigma_\epsilon^2 = b_1^2 + b_2^2 + b_3^2 + \sigma_\epsilon^2 , \qquad [2]$$

because the variance of xz is:

$$\sigma_{xz}^2 = \sigma_x^2\sigma_z^2 = 1.$$

We choose b_1, b_2 and b_3 such that their squares sum to 1. For example:
b_1 = *sqrt*(.5), b_2 = *sqrt*(.3) and b_3 = *sqrt*(.2). If the variance of ε is also 1, it follows that the σ_y^2 = 1 + σ_ϵ^2 = 2. The expected *R* squared of this model is 0.5, which is computed by:

$$R^2 = \frac{\sigma_y^2 - \sigma_e^2}{\sigma_y^2}. \qquad [3]$$

The three effects sizes in this example then become:

$$\beta_1 = \frac{\sqrt{.5}}{\sqrt{2}} = 0.500; \;\; \beta_2 = \frac{\sqrt{.3}}{\sqrt{2}} = 0.387; \;\; \beta_3 = \frac{\sqrt{.2}}{\sqrt{2}} = 0.316.$$

By changing the variance of the error the effect sizes can be manipulated.

Correlated predictors

When *x* and *z* are correlated the variance of the interaction term *xz* becomes:

$$\sigma_{xz}^2 = \sigma_x^2\sigma_z^2 + cov(x^2, z^2) - cov(x,z)^2 = 1 + 2r^2 - r^2 = 1 + r^2. \qquad [4]$$

Here *r* is the correlation between *x* and *z*. Assuming the interaction term is independent from *x* and *z*, the variance of *y* then becomes:

$$\sigma_y^2 = b_1^2\sigma_x^2 + b_2^2\sigma_z^2 + b_3^2\sigma_{xz}^2 + 2b_1b_2r\sigma_x\sigma_z + \sigma_\epsilon^2 =$$

$$b_1^2 + b_2^2 + (1 + r^2)b_3^2 + 2b_1b_2r + \sigma_\epsilon^2. \qquad [5]$$

The effect sizes of *x* and *z* are computed with this variance term. For instance for *x*:

$$\beta_1 = b_1\frac{\sigma_x}{\sigma_y} = b_1\frac{1}{\sqrt{b_1^2 + b_2^2 + (1 + r^2)b_3^2 + 2b_1b_2r + \sigma_\epsilon^2}}$$

The effect size of the interaction term becomes:

$$\beta_3 = b_3\frac{\sigma_{xz}}{\sigma_y} = b_3\frac{\sqrt{1 + r^2.}}{\sqrt{b_1^2 + b_2^2 + (1 + r^2)b_3^2 + 2b_1b_2r + \sigma_\epsilon^2}}$$

The R^2 of the model with correlated predictors can be obtained as before by [3]. Using these expressions we can construct expected effected sizes in a simulation study.

Literature

Hayes, A. (2013). *Introduction to mediation, moderation, and conditional process analysis*. *New York, NY: Guilford*. http://doi.org/978-1-60918-230-4.