Exercise: PageRank Algorithm

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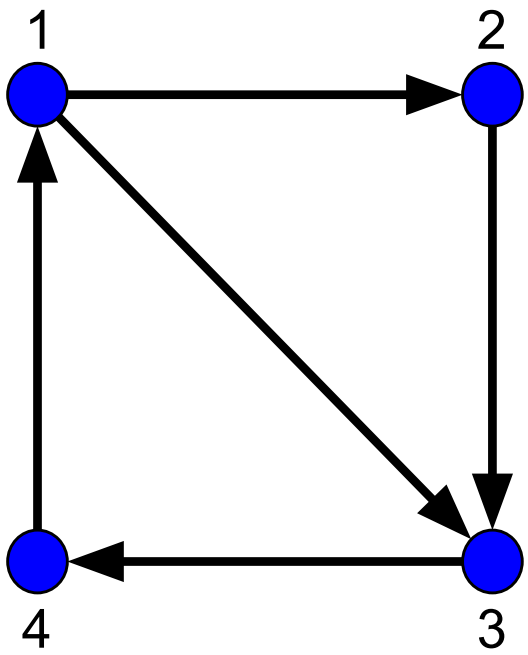
Simple Web Graph and its Matrix

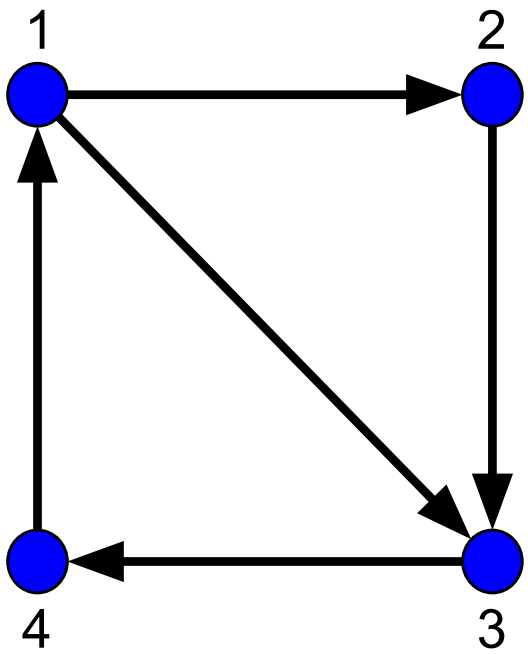


Simple Web graph G
(no random jumps)

	1	2	3	4
1				
2				
3				
4				

Its matrix M_G

Simple Web Graph and its Matrix

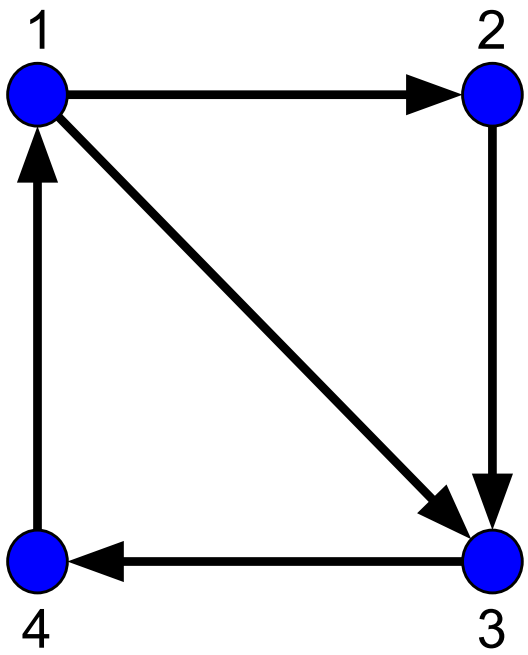


Simple Web graph G
(no random jumps)

	1	2	3	4
1	0			
2	$\frac{1}{2}$			
3	$\frac{1}{2}$			
4	0			

Its matrix M_G

Simple Web Graph and its Matrix

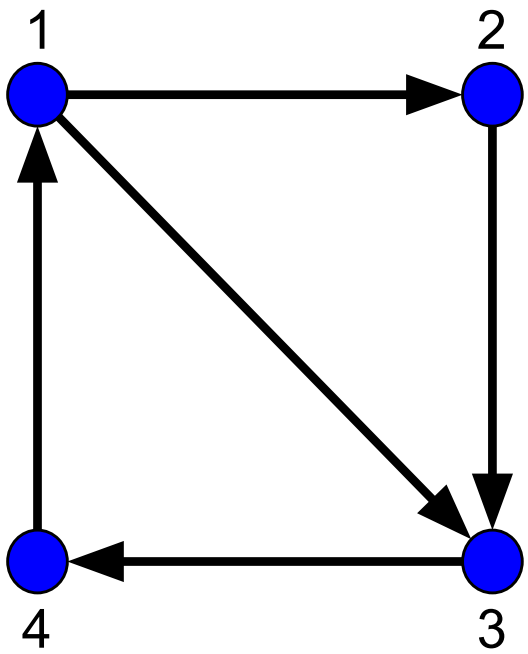


Simple Web graph G
(no random jumps)

	1	2	3	4
1	0	0		
2	$\frac{1}{2}$	0		
3	$\frac{1}{2}$	1		
4	0	0		

Its matrix M_G

Simple Web Graph and its Matrix

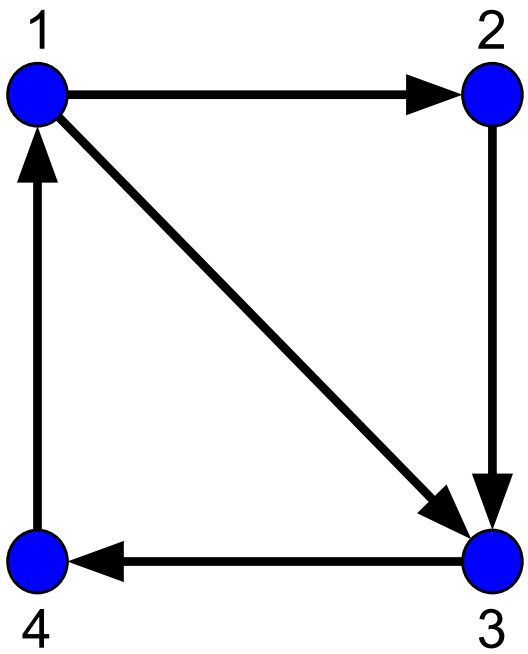


Simple Web graph G
(no random jumps)

	1	2	3	4
1	0	0	0	
2	$\frac{1}{2}$	0	0	
3	$\frac{1}{2}$	1	0	
4	0	0	1	

Its matrix M_G

Simple Web Graph and its Matrix



Simple Web graph G
(no random jumps)

	1	2	3	4
1	0	0	0	1
2	$\frac{1}{2}$	0	0	0
3	$\frac{1}{2}$	1	0	0
4	0	0	1	0

Its matrix M_G

System of Linear Equations

Let x_1, x_2, x_3, x_4 be the PageRank scores (importance) of pages $1, 2, 3, 4$, respectively. By multiplying M_G with vector $\mathbf{x} = x_1, x_2, x_3, x_4$

System of Linear Equations

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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & \frac{1}{2} & 0 & 0 & 0 \\ 3 & \frac{1}{2} & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

System of Linear Equations

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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} & 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & \frac{1}{2} & 0 & 0 & 0 \\ 3 & \frac{1}{2} & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

we obtain the following system of equations:

$$\begin{cases} x_1 = x_4 \\ x_2 = \frac{x_1}{2} \\ x_3 = \frac{x_1}{2} + x_2 \\ x_4 = x_3 \end{cases}$$

System of Linear Equations

We add the constraint $x_1 + x_2 + x_3 + x_4 = 1$ and we obtain:

$$\begin{cases} x_1 = x_4 \\ x_2 = \frac{x_1}{2} \\ x_3 = \frac{x_1}{2} + x_2 \\ x_4 = x_3 \\ x_1 + x_2 + x_3 + x_4 = 1 \end{cases}$$

By solving the system, we obtain:

$$\begin{aligned} x_1 &= \frac{2}{7} \\ x_2 &= \frac{1}{7} \\ x_3 &= \frac{2}{7} \\ x_4 &= \frac{2}{7} \end{aligned}$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{matrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{matrix}$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{array}{ll} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} \end{array}$$

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$$\begin{array}{ll} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} \end{array}$$

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We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{array}{cccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} \end{array}$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{array}{ccccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \end{array}$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{array}{cccccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} \end{array}$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{matrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} \end{matrix} \quad \ldots$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{matrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} & \frac{1}{4} & \cdots \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} & \end{matrix} \quad \begin{matrix} \frac{2}{7} \\ \frac{1}{7} \\ \frac{2}{7} \\ \frac{2}{7} \end{matrix}$$

PageRank Algorithm

We start with $r^0 = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ and we iteratively compute at each step $k \geq 1$, $r^k = M \cdot r^{k-1}$. We obtain:

$$\begin{array}{cccccccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \\ \frac{1}{4} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{3}{16} & \frac{1}{8} & \frac{1}{8} & \\ \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} & \frac{1}{4} & \cdots \\ \frac{1}{4} & \frac{1}{4} & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} & \frac{5}{16} & \frac{5}{16} & \end{array} \quad \begin{array}{c} \\ \frac{2}{7} \\ \frac{1}{7} \\ \frac{2}{7} \\ \frac{2}{7} \end{array}$$

At the exam, you will be asked to compute only 1-2 iterations.