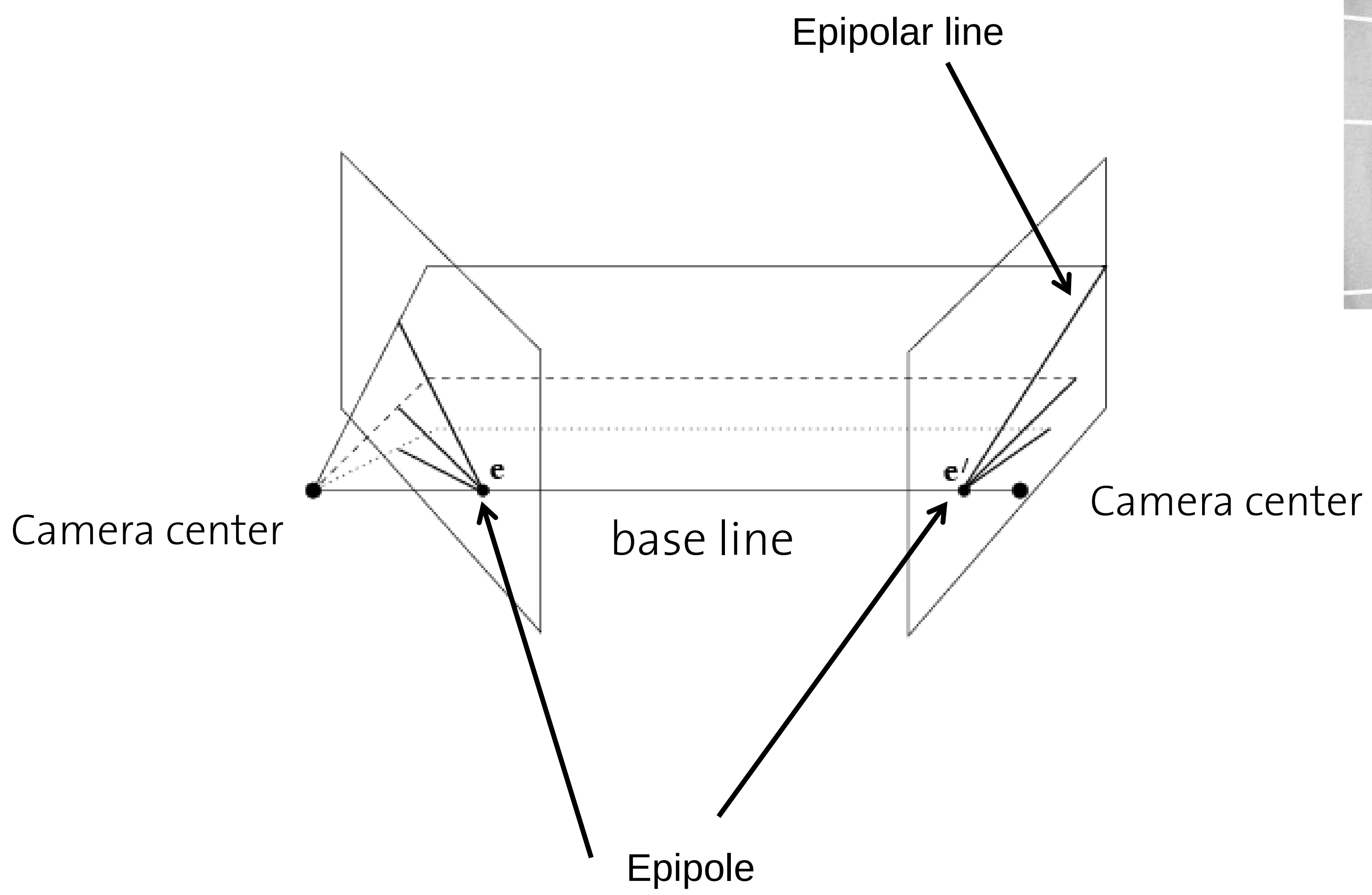
Computer Vision and Geometry Lab

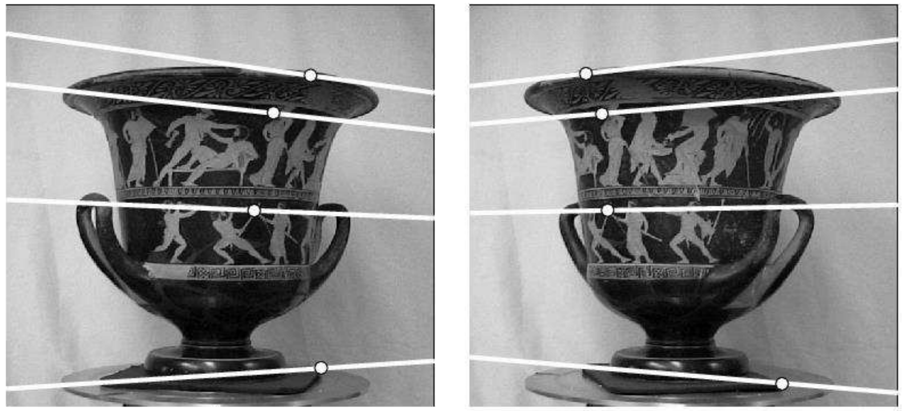
Computer Vision

Exercise Session 4

Fundamental matrix

- Epipolar constraint $x'^T Fx = 0$





Fundamental matrix

F is the unique 3x3 rank 2 matrix that satisfies $x'^T F x = 0$ for all $x \leftrightarrow x'$

(i) **Transpose:** if F is fundamental matrix for (P,P'), then F^T is fundamental matrix for (P',P)

(ii) **Epipolar lines:** $l' = Fx$ & $l = F^T x'$

(iii) **Epipoles:** on all epipolar lines, thus $e'^T F x = 0, \forall x$ $\Rightarrow e'^T F = 0$, similarly $Fe = 0$

(iv) **F** has 7 d.o.f. , i.e. 3x3-1(homogeneous)-1(rank2)

Eight-point algorithm

- Epipolar constraint $x'^T Fx = 0$

$$x = (x, y, 1)^T \qquad x' = (x', y', 1)^T$$

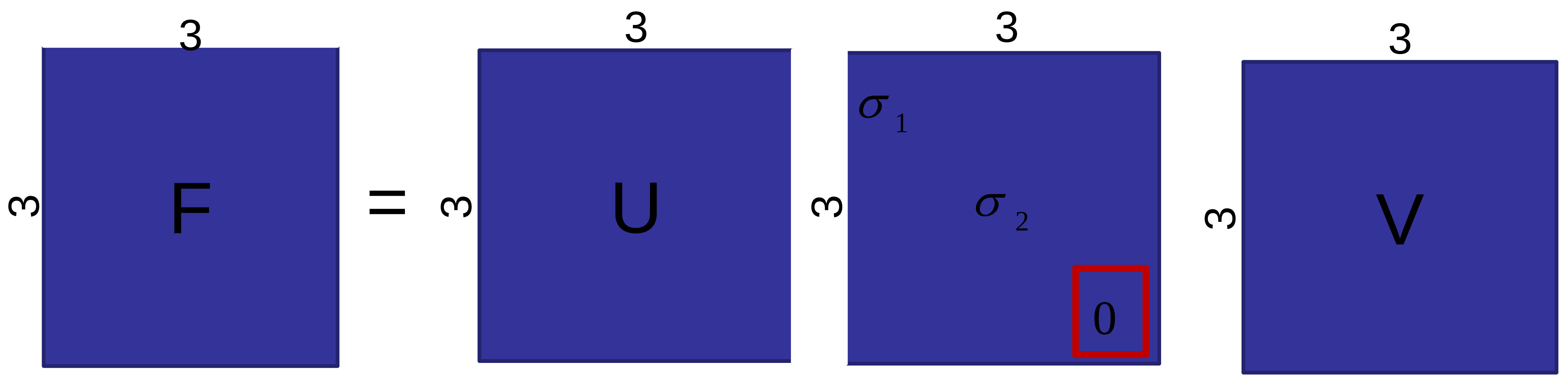
$$(x' \quad y' \quad 1)\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0 \;\Rightarrow\; (x'_1 x_1 \quad x'_1 y_1 \quad x'_1 \quad y'_1 x_1 \quad y'_1 y_1 \quad y'_1 \quad x_1 \quad y_1 \quad 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

$$\begin{pmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ & & & & & & & & \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{pmatrix} f = 0$$

- Use SVD to solve for F

Eight-point algorithm

- Enforce the singularity constraint on F
  - Factorize F using SVD
  - Set the third singular value of F to zero



Essential matrix (calibrated cameras)

- Essential matrix (5 d.o.f)

$$nx' E nx = 0 \qquad E = [t]_\times R \qquad [t]_\times = \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix}$$

- $nx' \leftrightarrow nx$ are the normalized image coordinates

$$nx = K^{-1} x \qquad nx' = K^{-1} x'$$

  - K^{-1} inverse camera calibration matrix

- Linear solution for E using 8 point correspondences
- Enforce the property of E that the first two singular values are equal and the third is zero
- Compare F with $K^{-T} E K^{-1}$

Essential matrix (calibrated cameras)

- Enforce the property of E that the first two singular values are equal and the third is zero.
  - Factorize E using SVD, where S is the diagonal matrix with the singular values, $S = diag\ (r, s, t)$.
  - Replace S with $dig\ ((r+s)/2, (r+s)/2, 0)$.

Essential Matrix

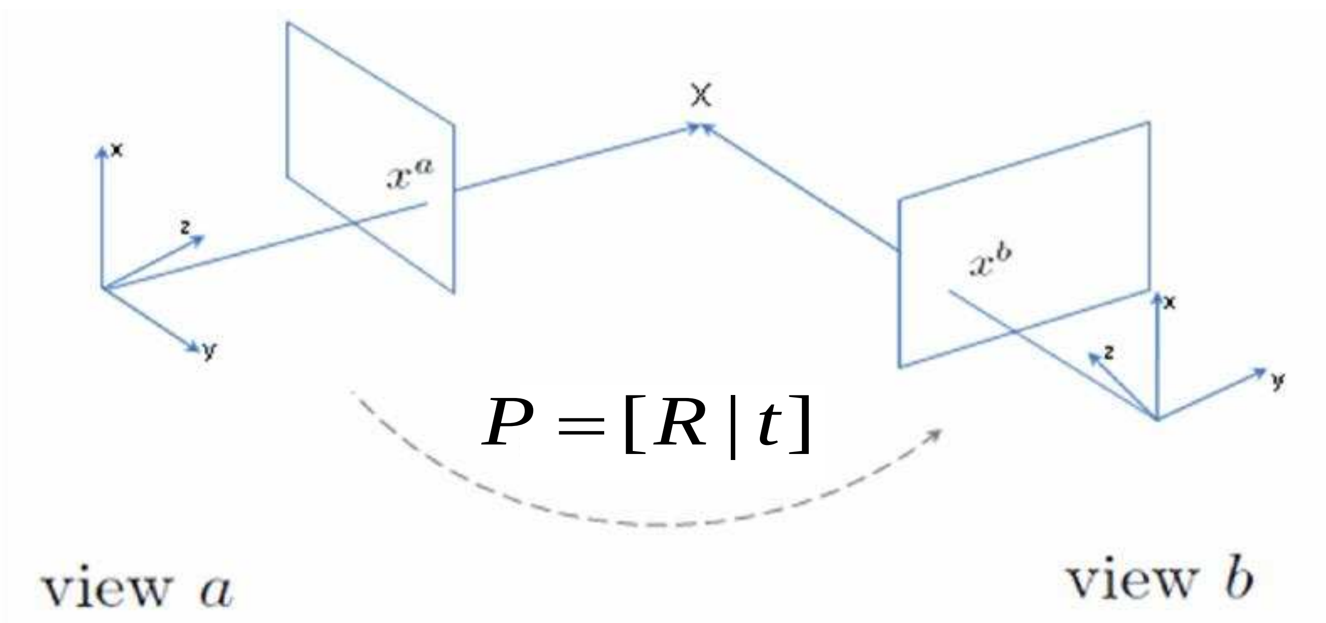
- Decompose $E = [t]_{\times} R$
  - Translation t is the left null-vector of E (ie last column of U if $E = UDV^{T}$)
    - The length of t is unknown and can be set to 1
  - Rotation matrix is obtained by decomposing

$$E = USV^T$$

$$R_1 = UWV^T,\ R_2 = UW^TV^T \ \text{with} \ W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For RHS coordinate system:

$$\det(R) = 1$$

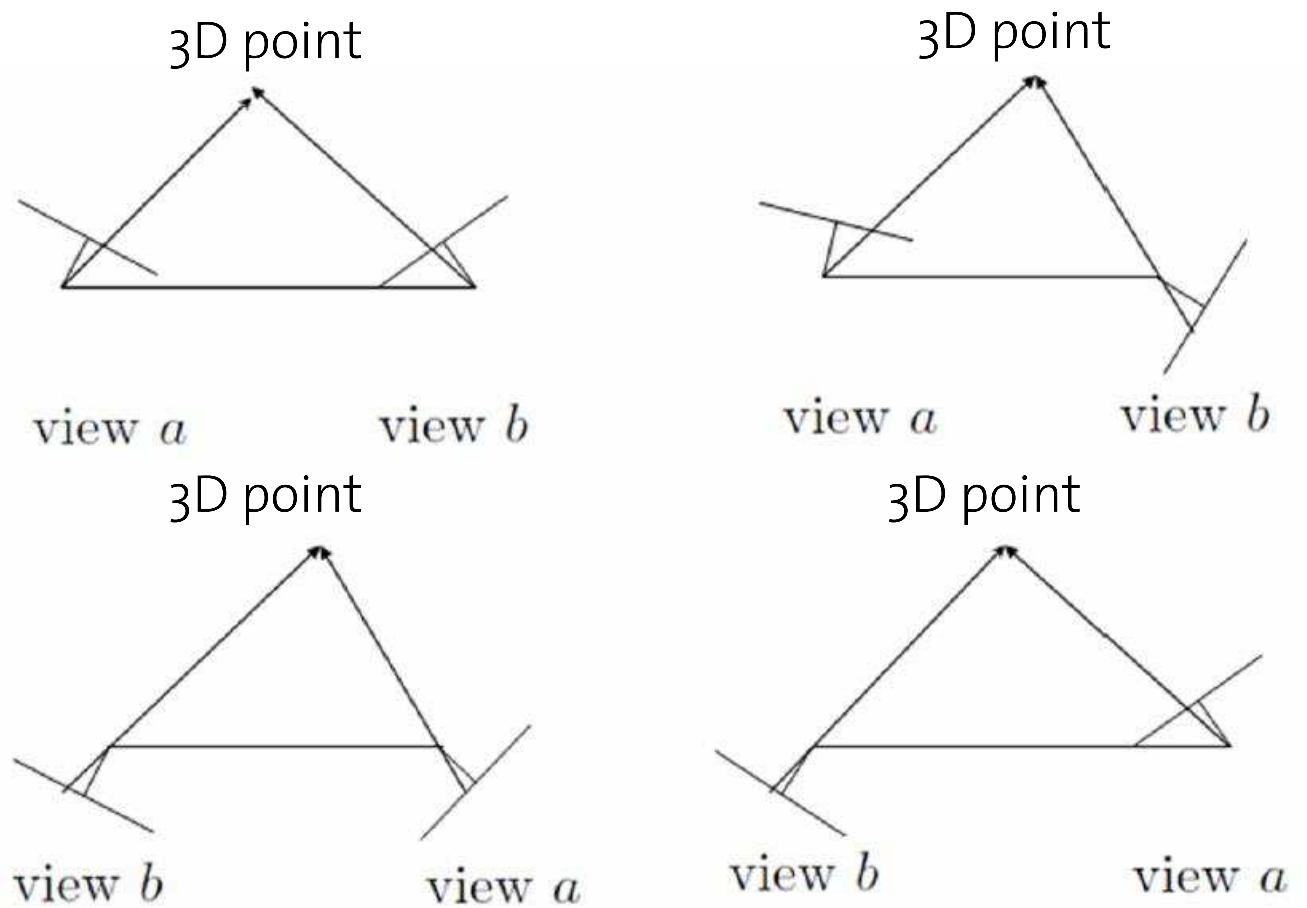


Essential Matrix

- We obtain four possible solutions

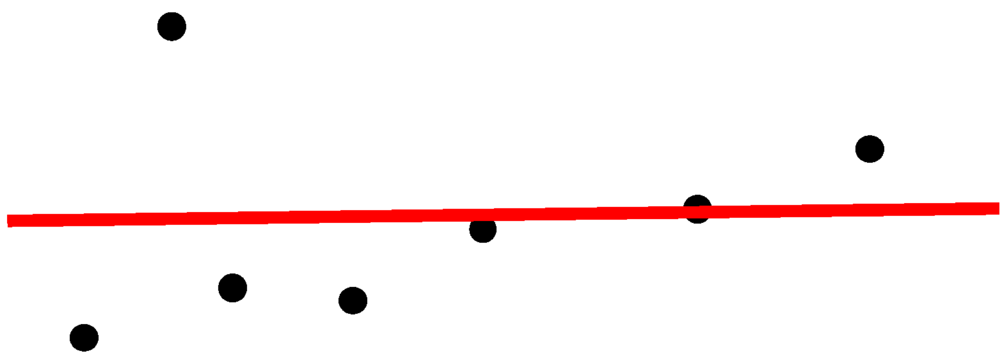
$$P_1 = R_1[I_{3\times3}|t],\ P_2 = R_1[I_{3\times3}|-t],\ P_3 = R_2[I_{3\times3}|t],\ P_4 = R_2[I_{3\times3}|-t]$$

  - Correct solution -> triangulated points in front of both cameras

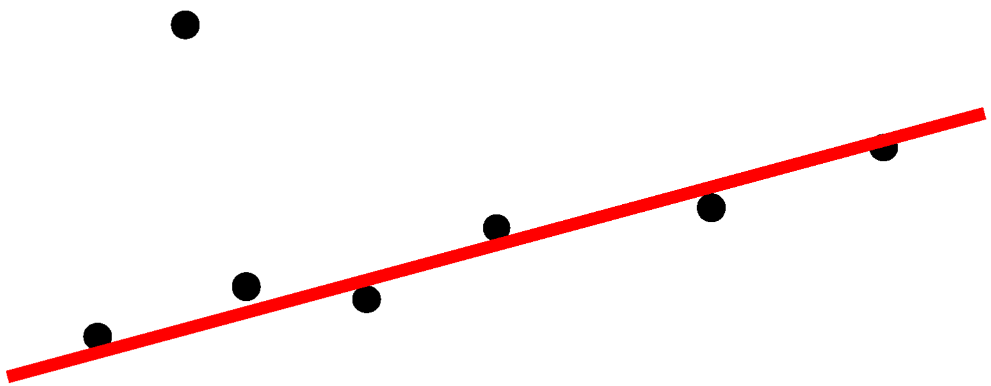


RANSAC

- Least squares solution is dramatically effected by outliers:



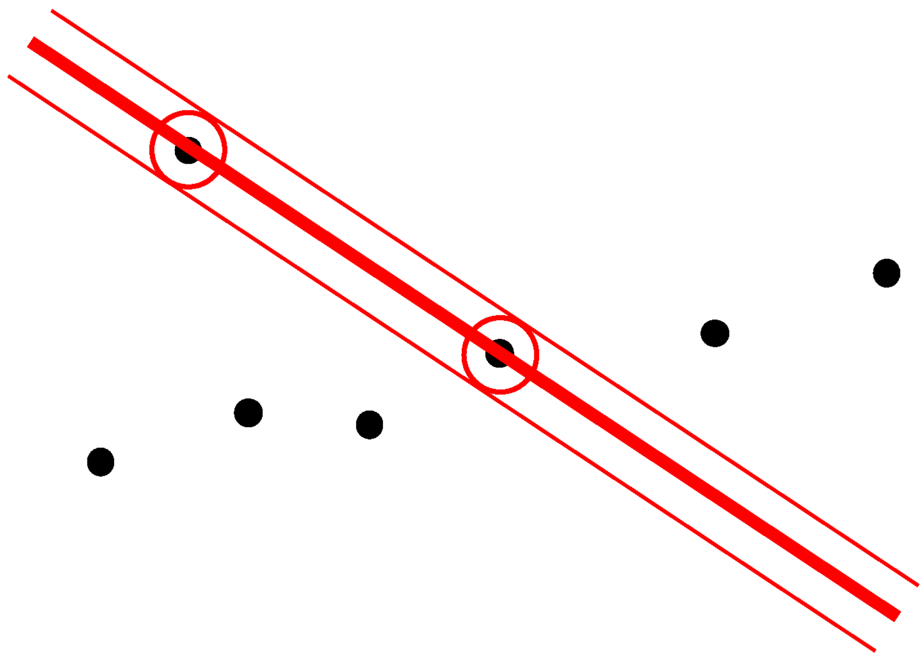
What we want to have:



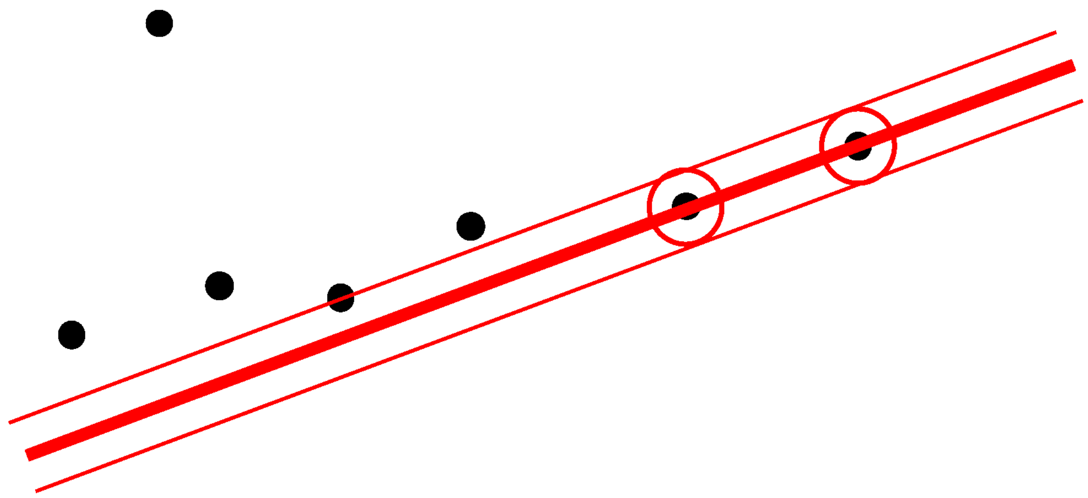
RANSAC

- Algorithm
  1. Guess N points that you hope are inliers.
  2. Compute the solution.
  3. Check how many other points fit within some threshold, i.e. are inliers.
  4. Repeat 1-3 until you're sure the solution has been found.
  5. Take the solution that has the most inliers, and compute least-squares solution from inliers.

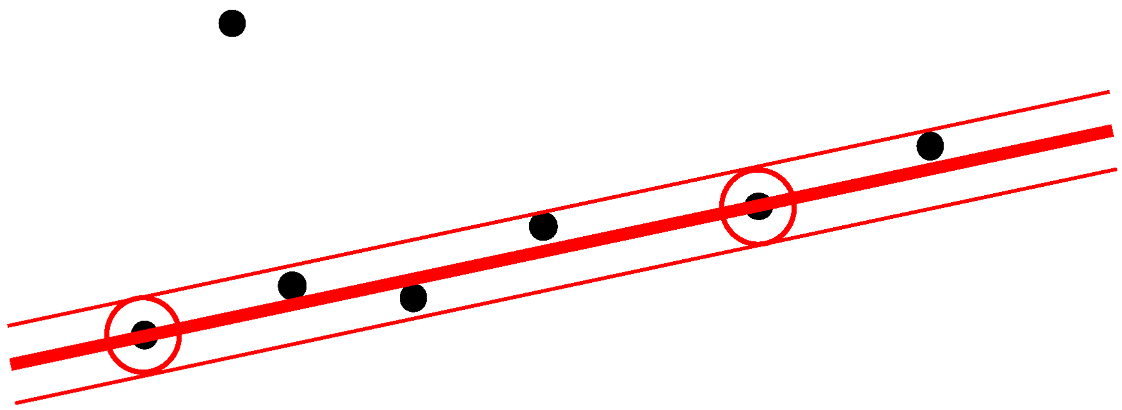
RANSAC



RANSAC



RANSAC

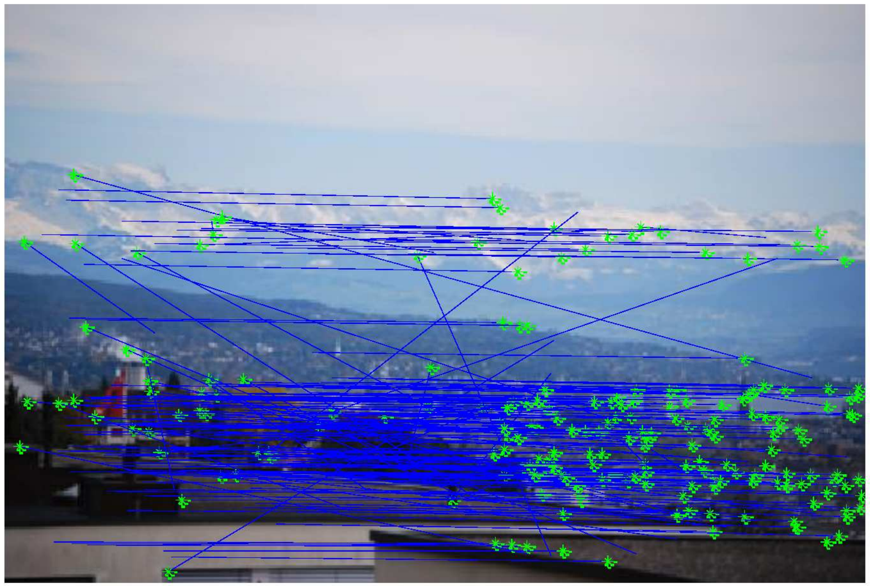


RANSAC

Detected features

Matched features

After RANSAC



RANSAC

- Probability of having found solution:

$$p = 1-(1-r^N)^M$$

  - r is inlier ratio
  - N is number of samples drawn (i.e. 8 for fundamental matrix)
  - M is number of iterations
- Adaptive RANSAC:
  - Use the largest number of inliers found so far as a lower bound on p
  - Stop iterating once the solution probability lower bound is above 0.99