Probabilistic Foundations of Artificial Intelligence
Problem Set 1

Sep 30, 2016

1. Conditional Probabilities

For each statement below, either prove it is true, or give a counterexample showing it is false. In the following, we assume that all events have non-zero probability.

(a) If $P(a|b,c) = P(b|a,c)$, then $P(a|c) = P(b|c)$

(b) If $P(a|b,c) = P(a)$, then $P(b|c) = P(b)$

(c) If $P(a|b) = P(a)$, then $P(a|b,c) = P(a|c)$

2. Finding the fake coin

Suppose you are given a bag containing n unbiased coins. You are also told that $n-1$ of these coins are normal, that is, they have a head on one side and a tail in the other. The remaining one is fake and has heads on both sides.

(a) Suppose you pick a coin from the bag uniformly at random, you flip it, and get a head. Given this result, what is the probability that the coin you picked is the fake one? (Note that we ask for a conditional probability.)

(b) Suppose you continue flipping the same coin for a total of k times and you get k heads. Given this result, what is the probability that you picked the fake coin?

(c) Now, suppose you devise the following method to determine if the coin is fake or not. You flip it k times, after which you conclude that it is the fake one if all k flips have resulted in heads, else you conclude that it is normal. What is the probability that using this method you arrive at a wrong conclusion? (Note that this time we ask for an unconditional probability.)

3. Naive Bayes

Suppose you have a bag of three biased coins a, b, and c, with probabilities of coming up heads of 0.2, 0.6, and 0.8 respectively. You draw a coin uniformly at random from the bag and flip it three times to generate the sequence of outcomes X_1, X_2, X_3.

(a) Draw the Bayesian Network corresponding to this setup and specify the necessary Conditional Probability Tables (CPTs).

(b) Calculate which coin was most likely to have been drawn from the bag, if two of the observed outcomes were heads and the other was a tail.