6

WORK AND KINETIC ENERGY

VP6.2.1. **IDENTIFY:** This problem requires the calculation of work knowing the components of the force and displacement vectors.

SET UP: $W = \vec{F} \cdot \vec{S}$, which in terms of components is $W = F_x s_x + F_y s_y$.

EXECUTE: **(a)** $W = F_x s_x + F_y s_y$ = (126 N)(3.00 m) + (168 N)(–6.50 m) = –714 J

(b) The football player's force is equal and opposite to that of the opponent, but his displacement is the same, so the work he does is the negative of the work done by his opponent. W = –(–714 J) = +714 J.

EVALUATE: As we see in this problem, negative work definitely has physical meaning.

VP6.2.2. **IDENTIFY:** This problem requires a calculation using work. We know the work, force, and displacement and want to find the angle between the force and displacement vectors.

SET UP: To find the angle, we write work in the form $W = \vec{F} \cdot \vec{S} = Fs \cos \phi$.

EXECUTE: $W = Fs \cos \phi \quad \rightarrow \quad 1.47 \times 10^3$ J = (215 N)(8.40 m) $\cos \phi \quad \rightarrow \quad \phi$ = 35.5°.

EVALUATE: The angle ϕ is less then 90°, so the work should be positive, which it is.

VP6.2.3. **IDENTIFY:** This problem requires the calculation of work for three forces knowing the magnitudes of the forces and displacements and the angles between them. Work is a scalar quantity, so the total work is the algebraic sum of the individual works.

SET UP: Use $W = Fs \cos \phi$ in each case. Then find the sum of the works.

EXECUTE: **(a)** The friction force is along the surface of the ramp but in the opposite direction from the displacement, so ϕ = 180°. Thus

$W_f = f_k s \cos \phi$ ° = (30.0 N)(3.00 m) cos 180° = –90.0 J.

(b) Gravity acts downward and the displacement is along the ramp surface, so ϕ = 62.0°. Thus W_g = mgs cos 62.0° = (15.0 kg)(9.80 m/s^2)(3.00 m) cos 62.0° = 207 J.

(c) The normal force is perpendicular to the displacment, so ϕ = 90° and $\cos \phi$ = 0, so the normal force does no work.

(d) $W_{tot} = W_f + W_g + W_n$ = –90.0 J + 207 J + 0 =117 J.

EVALUATE: The work is positive because the gravitational force along the ramp is greater than the friction, so the net force does positive work and the displacement is down the ramp.

VP6.2.4. **IDENTIFY:** We want to calculate work using the components of the force and displacement.

SET UP: Since we know the components, we use $W = F_x s_x + F_y s_y$.

EXECUTE: **(a)** $W_1 = F_0(2d) = 2F_0 d$, $W_2 = (-3F_0)d = -3F_0 d$, $W_3 = (-4F_0)(2d) + Gd = -8F_0 d + Gd$.

(b) $W_{tot} = 2F_0 d + (-3F_0 d) + (-8F_0 d + Gd) = 0 \qquad \rightarrow \qquad G = 9F_0$.

EVALUATE: The equation $W = Fs \cos \phi$ is still correct, but it would not allow us to easily calculate the work for these forces.

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VP6.4.1. **IDENTIFY:** This problem requires the work-energy theorem. As the cylinder falls, work is done on it by gravity and friction. This work changes its kinetic energy.

SET UP: $W_{tot} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Figure VP6.4.1 illustrates the arrangement.

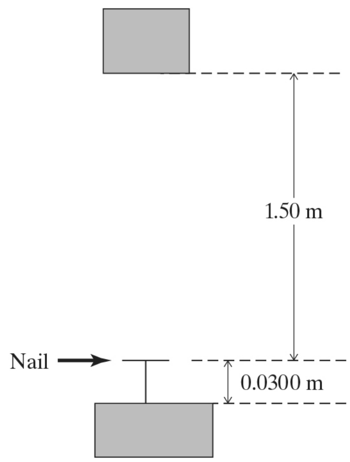

Figure VP6.4.1

EXECUTE: **(a)** Call point 1 where the cylinder is first released, and point 2 where it is just about to hit the nail. $W_g = mgs$ and $W_f = -fs$, so $W_{tot} = mgs - fs$. $K_1 = 0$ and $K_2 = \frac{1}{2}mv^2$. The work-energy theorem gives $mgs - fs = \frac{1}{2}mv^2$. Using $s = 1.50$ m, $f = 16.0$ N, $m = 20.0$ kg gives $v = 5.20$ m/s.

(b) Now we want to know about the force the cylinder exerts on the nail while hammering it in. Therefore we choose point 1 to be the instant just as the cylinder strikes the nail (that was point 2 in part (a)) and point 2 to be the instant when the nail has first stopped. As the cylinder pushes in the nail, it exerts a force F_N on the nail while pushing 0.0300 m into the block of wood. Likewise the other forces act for the same distance.

$W_{tot} = W_g + W_f + W_N = mgs - fs - F_Ns$, $K_1 = \frac{1}{2}mv^2$ (using $v = 5.20$ m/s), $K_2 = 0$. The work-energy theorem gives $mgs - fs - f_Ns = 0 - \frac{1}{2}mv^2$. Putting in the numbers, with $s = 0.0300$ m, gives $F_N = 9180$ N.

EVALUATE: The force in (a) is around a thousand pounds. From experience, we know that it takes a large force to hammer in a nail.

VP6.4.2. **IDENTIFY:** This problem requires the work-energy theorem. Gravity and the tension in the rope do work on the crate.

SET UP: $W_{tot} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Call point 1 when you just increase the tension to 175 N and point 2 just after you have lifted it the additional 1.25 m.

EXECUTE: **(a)** $W_T = Ts = (175\text{ N})(1.25\text{ m}) = 219$ J.

(b) $W_g = mgs\cos\phi = (14.5\text{ kg})(9.80\text{ m/s}^2)(1.25\text{ m})\cos 180° = -178$ J

(c) $F_{net} = T - mg$, so $W_{tot} = F_{net}s = (T - mg)s = W_T + W_g = 219\text{ J} - 178\text{ J} = 41$ J.

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(d) $W_{tot} = K_2 - K_1 = K_2 - \frac{1}{2}mv_1^2$. Solving for K_2 gives $K_2 = W_{tot} + \frac{1}{2}mv_1^2$, so we get

$K_2 = 41\text{ J} + \frac{1}{2}(14.5\text{ kg})(0.500\text{ m/s})^2 = 43\text{ J} = \frac{1}{2}mv_2^2 = \frac{1}{2}(14.5\text{ kg})\,v_2^2 \rightarrow v_2 = 2.4\text{ m/s}.$

EVALUATE: The tension does positive work and therefore increases the crate's kinetic energy.

VP6.4.3. **IDENTIFY:** This problem requires the work-energy theorem. Gravity and the thrust of the engine do work on the helicopter.

SET UP: $W_{tot} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Call point 1 when the pilot just increases the thrust and point 2 just as the speed has decreased to 0.450 m/s.

EXECUTE: **(a)** $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1400\text{ kg})(3.00\text{ m/s})^2 = 6300\text{ J}.$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(1400\text{ kg})(0.450\text{ m/s})^2 = 142\text{ J}.$

(b) $W_{tot} = K_2 - K_1 = 142\text{ J} - 6300\text{ J} = -6160\text{ J}$

$W_g = mg\Delta y = (1400\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 27{,}400\text{ J}$

$W_{tot} = W_{thrust} + W_g \rightarrow -6160\text{ J} = W_{thrust} + 27{,}400\text{ J} \rightarrow W_{thrust} = -3.36\times10^4\text{ J}.$

(c) $W_{thrust} = F_{thrust}s\cos\phi \rightarrow -3.36\times10^4\text{ J} = F_{thrust}(2.00\text{ m})\cos180° \rightarrow F_{thrust} = 1.68\times10^4\text{ N}.$

EVALUATE: The magnitude of the thrust force is around ten times as great as the force of gravity, so the net work is negative in order to slow down the helicopter.

VP6.4.4. **IDENTIFY:** This problem requires the work-energy theorem.

SET UP: $W_{tot} = \Delta K = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$. Call point 1 the point where the block is released and point 2 the bottom of the ramp. Let d be the distance it moves along the surface of the ramp. We also use $\sum F_y = 0$.

EXECUTE: **(a)** See Fig. VP6.4.4. $W_g = mgd\cos\phi = mgd\sin\theta$.

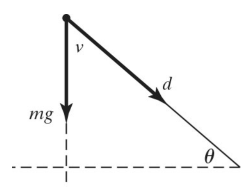

Figure VP6.4.4

(b) $W_f = f_k d\cos180° = -f_k d = -\mu_k n$. Using $\sum F_y = 0$, where the y-axis is perpendicular to the face of the ramp, gives $n = mg\cos\theta$, so $W_f = -\mu_k mgd\cos\theta$.

(c) $W_{tot} = K_2 - K_1 = W_g + W_f + W_n$. $W_n = 0$ and $K_1 = 0$, so we get

$mgd\sin\theta - \mu_k mgd\cos\theta = \frac{1}{2}mv_2^2 \rightarrow v_2 = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$.

EVALUATE: To check our result, let $\mu_k = 0$ for a frictionless surface. In that case, our result reduces to $v = \sqrt{2gd\sin\theta} = \sqrt{2gh}$. From our study of free-fall, we know that an object falling a distance h from rest has speed $v = \sqrt{2gh}$, which agrees with our result in this special case.

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VP6.8.1. **IDENTIFY:** This problem requires the work-energy theorem and involves the work done by a spring.

SET UP: $W_{\text{tot}} = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$, and the work done to compress a spring is $W = \frac{1}{2}kx^2$. Call point 1 the point just before the glider hits the spring and point 2 to be at maximum compression of the spring.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$. The final kinetic energy is zero and the work done by the spring on the glider is $-\frac{1}{2}kd^2$, so we get $-\frac{1}{2}kd^2 = -\frac{1}{2}mv_1^2$, which gives

$(30.0\text{ N/m})d^2 = (0.150\text{ kg})(1.25\text{ m/s})^2 \quad \rightarrow \quad d = 0.0884\text{ m} = 8.84\text{ cm}$.

(b) Both friction and the spring oppose the glider's motion and therefore do negative work on the glider. The work done by the spring is still $-\frac{1}{2}kd^2$, but d is now different. The work done by kinetic friction is $W_f = -f_k d = -\mu_k mgd$. So the work-energy theorem gives

$-\frac{1}{2}kd^2 - \mu_k mgd = 0 - \frac{1}{2}mv_1^2$. Putting in the numbers and simplifying gives

$(30.0\text{ N/m})d^2 + 2(0.320)(0.150\text{ kg})(9.80\text{ m/s}^2)d - (0.150\text{ kg})(1.25\text{ m/s})^2 = 0$. The quadratic formula gives two solutions (one positive and one negative), but we need the positive root, which is $d = 0.0741\text{ m} = 7.41\text{ cm}$.

EVALUATE: With no friction, the glider traveled 8.84 cm, but with friction it traveled only 7.41 cm. This is reasonable since both friction and the spring slowed down the glider in the case with friction.

VP6.8.2. **IDENTIFY:** This problem requires the work-energy theorem and involves the work done by a spring.

SET UP: $W_{\text{tot}} = K_2 - K_1$, where $K = \frac{1}{2}mv^2$. The work is $W = Fs\cos\phi$, and the work done to compress a spring is $W = \frac{1}{2}kx^2$. Call point 1 the point just before the glider hits the spring and point 2 to be at maximum compression of the spring.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$. The final kinetic energy is zero and the work done by the spring on the glider is $-\frac{1}{2}kd^2$. The work done by kinetic friction is $W_f = -f_k d = -\mu_k mgd$. So the work-energy theorem gives so the work-energy theorem gives $-\frac{1}{2}kd^2 - \mu_k mgd = 0 - \frac{1}{2}mv_1^2$. Solving for μ_k gives

$$\mu_k = \frac{mv_1^2 - kd^2}{2mgd}.$$

EVALUATE: Checking units shows that $\frac{mv_1^2 - kd^2}{2mgd}$ is dimensionless, which is correct for a coefficient of friction.

VP6.8.3. **IDENTIFY:** This problem involves the work done on the bob of a swinging pendulum.

SET UP: $W = Fs\cos\phi$. Let L be the length of the string and θ the angle it makes with the vertical.

EXECUTE: **(a)** $W_g = mg\Delta y = mgL(1-\cos\theta)$

$W_g = (0.500\text{ kg})(9.80\text{ m/s}^2)(0.750\text{ m})(1 - \cos 35.0°) = 0.665\text{ J}$.

(b) $W_g = mg\Delta y$ where Δy is negative, so $W_g = -0.665\text{ J}$.

(c) $W_{\text{tot}} = W_{AB} + W_{BC} = 0.665\text{ J} - 0.665\text{ J} = 0$.

EVALUATE: The total work is zero because gravity does positive work as the bob is moving downward and negative work as it is moving upward.

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VP6.8.4. **IDENTIFY:** This problem requires application of the work-energy theorem to a pendulum.

SET UP: $W_{tot} = K_2 - K_1$, where $K = \frac{1}{2}mv^2$, and work is $W = Fs\cos\phi$. Call point 1 the highest point in its swing point 2 the lowest point.

EXECUTE: The tension in the silk strand does zero work because the tension is perpendicular to the path of the spider, so only gravity does work. $K_1 = 0$ and $K_2 = \frac{1}{2}mv^2$. Therefore

$mgL(1 - \cos\theta) = \frac{1}{2}mv^2 \qquad \rightarrow \qquad v = \sqrt{2gL(1-\cos\theta)}$.

EVALUATE: Using our result, we see that as θ approaches 90°, $\cos\theta$ approaches zero, so v gets larger and larger. This is reasonable because the spider starts swinging from a higher and higher elevation.

6.1. **IDENTIFY** and **SET UP:** For parts (a) through (d), identify the appropriate value of ϕ and use the relation $W = F_P s = (F\cos\phi)s$. In part (e), apply the relation $W_{net} = W_{student} + W_{grav} + W_n + W_f$.

EXECUTE: **(a)** Since you are applying a horizontal force, $\phi = 0°$. Thus,

$W_{student} = (2.40\text{ N})(\cos 0°)(1.50\text{ m}) = 3.60\text{ J}$.

(b) The friction force acts in the horizontal direction, opposite to the motion, so $\phi = 180°$.

$W_f = (F_f\cos\phi)s = (0.600\text{ N})(\cos 180°)(1.50\text{ m}) = -0.900\text{ J}$.

(c) Since the normal force acts upward and perpendicular to the tabletop, $\phi = 90°$.

$W_n = (n\cos\phi)s = (ns)(\cos 90°) = 0.0\text{ J}$.

(d) Since gravity acts downward and perpendicular to the tabletop, $\phi = 270°$.

$W_{grav} = (mg\cos\phi)s = (mgs)(\cos 270°) = 0.0\text{ J}$.

(e) $W_{net} = W_{student} + W_{grav} + W_n + W_f = 3.60\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 0.900\text{ J} = 2.70\text{ J}$.

EVALUATE: Whenever a force acts perpendicular to the direction of motion, its contribution to the net work is zero.

6.2. **IDENTIFY:** In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: In part **(a)**, when the cable pulls horizontally $\phi = 0°$ and when it pulls at $35.0°$ above the horizontal $\phi = 35.0°$. In part **(b)**, if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at $35.0°$ above the horizontal it pulls on the truck at $35.0°$ below the horizontal and ϕ 145.0°. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

EXECUTE: **(a)** When the cable is horizontal, $W = (1350\text{ N})(5.00\times10^3\text{ m})\cos 0° = 6.75\times10^6\text{ J}$. When the cable is $35.0°$ above the horizontal, $W = (1350\text{ N})(5.00\times10^3\text{ m})\cos 35.0° = 5.53\times10^6\text{ J}$.

(b) $\cos 180° = -\cos 0°$ and $\cos 145.0° = -\cos 35.0°$, so the answers are -6.75×10^6 J and -5.53×10^6 J.

(c) Since $\cos\phi = \cos 90° = 0$, $W = 0$ in both cases.

EVALUATE: If the car and truck are taken together as the system, the tension in the cable does no net work.

6.3. **IDENTIFY:** Each force can be used in the relation $W = F_{\parallel}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{net} = W_{worker} + W_{grav} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{worker} = f_k = \mu_k n$.

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{worker} = f_k = \mu_k n = \mu_k mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

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(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}} \cos\phi)s = [(74\text{ N})(\cos 0°)](4.5\text{ m}) = +333\text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_k \cos\phi)s = [(74\text{ N})(\cos 180°)](4.5\text{ m}) = -333\text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0$ J.

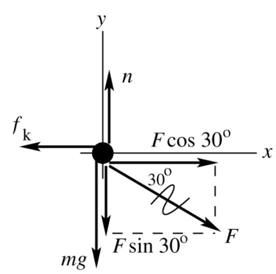
(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 333\text{ J} = 0.0\text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.4. **IDENTIFY:** The forces are constant so we can use $W = Fs\cos\phi$ to calculate the work. Constant speed implies $a = 0$. We must use $\Sigma\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

(a) SET UP: The free-body diagram for the crate is given in Figure 6.4.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

Figure 6.4

$\Sigma F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2\text{ N}$$

(b) $W_F = (F\cos\phi)s = (99.2\text{ N})(\cos 30°)(4.5\text{ m}) = 387\text{ J}$

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

(c) We have an expression for f_k from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0\text{ kg})(9.80\text{ m/s}^2) + (99.2\text{ N})(\sin 30°)] = 85.9\text{ N}$$

$\phi = 180°$ since f_k is opposite to the displacement. Thus

$W_f = (f_k \cos\phi)s = (85.9\text{ N})(\cos 180°)(4.5\text{ m}) = -387$ J.

(d) The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0$. The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0$.

(e) $W_{\text{tot}} = W_F + W_f + W_n + W_w = +387\text{ J} + (-387\text{ J}) = 0$

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EVALUATE: Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work, and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{\text{net}} = 0$ since $a = 0$ and $W_{\text{tot}} = 0,$ which agrees with the sum calculated in part (e).

6.5. **IDENTIFY:** The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi.$

SET UP: The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°.$ $w = mg = 735$ N.

EXECUTE: **(a)** $W = (735\text{ N})(2.75\text{ m})\cos 150.0° = -1750$ J.

(b) No, the gravity force is independent of the motion of the painter.

EVALUATE: Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

6.6. **IDENTIFY** and **SET UP:** $W_F = (F\cos\phi)s,$ since the forces are constant. We can calculate the total work by adding the work done by each force. The forces are sketched in Figure 6.6.

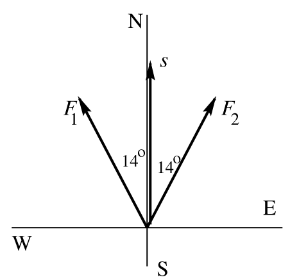

Figure 6.6

EXECUTE: $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times10^6\text{ N})(0.75\times10^3\text{ m})\cos 14°$

$W_1 = 1.31\times10^9$ J

$W_2 = F_2 s\cos\phi_2 = W_1$

$W_{\text{tot}} = W_1 + W_2 = 2(1.31\times10^9\text{ J}) = 2.62\times10^9$ J

EVALUATE: Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

6.7. **IDENTIFY:** All forces are constant and each block moves in a straight line, so $W = Fs\cos\phi.$ The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

SET UP: Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension T in the string is $T = 12.0$ N. Since the 20.0 N block moves to the right at constant speed, the friction force f_k on it is to the left and $f_k = T = 12.0$ N.

EXECUTE: **(a)** (i) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00$ J. (ii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00$ J.

(b) (i) $\phi = 90°$ and $W = 0.$ (ii) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00$ J. (iii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00$ J. (iv) $\phi = 90°$ and $W = 0.$

(c) $W_{\text{tot}} = 0$ for each block.

EVALUATE: For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.8. **IDENTIFY:** Apply $W = Fs\cos\phi$.

SET UP: $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{i}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{j}} = 1$ and $\hat{\boldsymbol{i}}\cdot\hat{\boldsymbol{j}} = \hat{\boldsymbol{j}}\cdot\hat{\boldsymbol{i}} = 0$

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EXECUTE: The work you do is $\vec{F}\cdot\vec{s} = \left[(30\text{ N})\hat{i} - (40\text{ N})\hat{j}\right]\cdot\left[(-9.0\text{ m})\hat{i} - (3.0\text{ m})\hat{j}\right]$

$\vec{F}\cdot\vec{s} = (30\text{ N})(-9.0\text{ m}) + (-40\text{ N})(-3.0\text{ m}) = -270\text{ N}\cdot\text{m} + 120\text{ N}\cdot\text{m} = -150\text{ J}.$

EVALUATE: The x-component of $\vec{F}$ does negative work and the y-component of $\vec{F}$ does positive work. The total work done by $\vec{F}$ is the sum of the work done by each of its components.

6.9. **IDENTIFY:** We want to compare the work done by friction over two different paths.
SET UP: Work is $W = Fs\cos\phi$. Call A the starting point and C the ending point for the first trip, and call B the end of the first segment of the second trip. On the first trip, $W_{AC} = -4.8$ J. We want to find the work done on the second trip, which is $W_{ABC} = W_{AB} + W_{BC}$. Call f_k the friction force, which is the same on all parts of the trips.
EXECUTE: $W_{AC} = -f_k s_{AC} = -f_k(0.500\text{ m})$. Likewise $W_{ABC} = W_{AB} + W_{BC}$ which gives $W_{ABC} = -f_k(0.300\text{ m}) - f_k(0.400\text{ m}) = -f_k(0.700\text{ m})$. Comparing the work in the two paths gives $\dfrac{W_{ABC}}{W_{AC}} = \dfrac{(-0.700\text{ m})f_k}{(-0.500\text{ m})f_k} = 1.40$. So $W_{ABC} = 1.40W_{AC} = (1.40)(-4.8\text{ J}) = -6.7\text{ J}$.
EVALUATE: We found that friction does more work during the ABC path than during the AC path. This is reasonable because the force is the same but the distance is greater during the ABC path. Notice that the work done by friction between two points depends on the *path* taken between those points.

6.10. **IDENTIFY** and **SET UP:** Use $W = F_p s = (F\cos\phi)s$ to calculate the work done in each of parts (a) through (c). In part (d), the net work consists of the contributions due to all three forces, or $w_{\text{net}} = w_{\text{grav}} + w_n + w_f$.

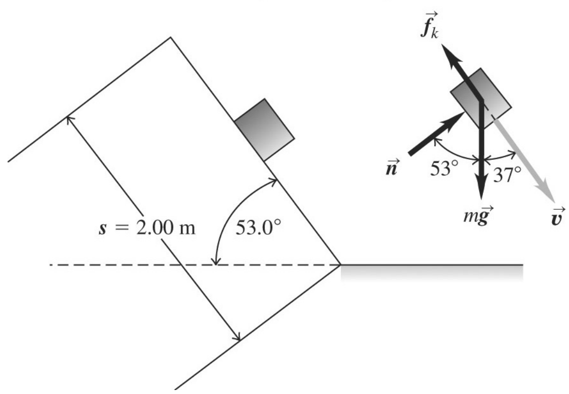

Figure 6.10

EXECUTE: **(a)** As the package slides, work is done by the frictional force which acts at $\phi = 180°$ to the displacement. The normal force is $mg\cos 53.0°$. Thus for $\mu_k = 0.40$,

$W_f = F_p s = (f_k\cos\phi)s = (\mu_k n\cos\phi)s = [\mu_k(mg\cos 53.0°)](\cos 180°)s.$

$W_f = (0.40)[(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 53.0°)](\cos 180°)(2.00\text{ m}) = -57\text{ J}.$

(b) Work is done by the component of the gravitational force parallel to the displacement. $\phi = 90° - 53° = 37°$ and the work of gravity is

$W_{\text{grav}} = (mg\cos\phi)s = [(12.0\text{ kg})(9.80\text{ m/s}^2)(\cos 37.0°)](2.00\text{ m}) = +188\text{ J}.$

(c) $W_n = 0$ since the normal force is perpendicular to the displacement.

(d) The net work done on the package is $W_{\text{net}} = W_{\text{grav}} + W_n + W_f = 188\text{ J} + 0.0\text{ J} - 57\text{ J} = 131\text{ J}.$

EVALUATE: The net work is positive because gravity does more positive work than the magnitude of the negative work done by friction.

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6.11. **IDENTIFY:** We need to calculate work and use the work-energy theorem.

SET UP: $W = Fs\cos\phi$, $W_{tot} = K_2 - K_1$, $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** Using $W = Fs\cos\phi$, we see that to double the work for the same force and displacement, the angle $\cos\phi$ must double. Thus $\cos\phi = 2\cos 60° = 1.00$, so $\phi = 0°$. We should pull horizontally.

(b) The block starts from rest so $K_1 = 0$. Using the work-energy theorem tells us that in this case $W_1 = \frac{1}{2}mv^2$. To double v, the work must be $W_2 = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4K_1 = 4W_1$. Since only the force F varies, F must increase by a factor of 4 to double the speed.

EVALUATE: To double the speed does not take twice as much work; it takes 4 times as much.

6.12. **IDENTIFY:** Since the speed is constant, the acceleration and the net force on the monitor are zero.

SET UP: Use the fact that the net force on the monitor is zero to develop expressions for the friction force, f_k, and the normal force, n. Then use $W = F_P s = (F\cos\phi)s$ to calculate W.

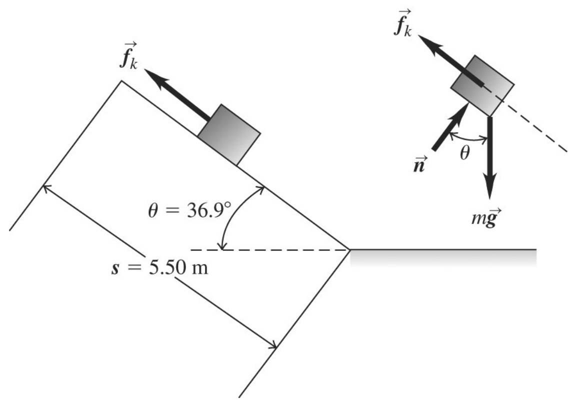

Figure 6.12

EXECUTE: **(a)** Summing forces along the incline, $\Sigma F = ma = 0 = f_k - mg\sin\theta$, giving $f_k = mg\sin\theta$, directed up the incline. Substituting gives $W_f = (f_k\cos\phi)s = [(mg\sin\theta)\cos\phi]s$.

$W_f = [(10.0 \text{ kg})(9.80 \text{ m/s}^2)(\sin 36.9°)](\cos 0°)(5.50 \text{ m}) = +324 \text{ J}.$

(b) The gravity force is downward and the displacement is directed up the incline so $\phi = 126.9°$.

$W_{grav} = (10.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 126.9°)(5.50 \text{ m}) = -324 \text{ J}.$

(c) The normal force, n, is perpendicular to the displacement and thus does zero work.

EVALUATE: Friction does positive work and gravity does negative work. The net work done is zero.

6.13. **IDENTIFY:** We want the work done by a known force acting through a known displacement.

SET UP: $W = Fs\cos\phi$

EXECUTE: $W = (48.0 \text{ N})(12.0 \text{ m})\cos(173°) = -572$ J.

EVALUATE: The force has a component opposite to the displacement, so it does negative work.

6.14. **IDENTIFY:** We want to find the work done by a known force acting through a known displacement.

SET UP: $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$. We know the components of $\vec{F}$ but need to find the components of the displacement $\vec{s}$.

EXECUTE: Using the magnitude and direction of $\vec{s}$, its components are

$x = (48.0 \text{ m})\cos 240.0° = -24.0$ m and $y = (48.0 \text{ m})\sin 240.0° = -41.57$ m. Therefore,

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$\vec{s} = (-24.0\text{ m})\hat{i} + (-41.57\text{ m})\hat{j}$. The definition of work gives

$W = \vec{F} \cdot \vec{s} = (-68.0\text{ N})(-24.0\text{ m}) + (36.0\text{ N})(-41.57\text{ m}) = +1632\text{ J} - 1497\text{ J} = +135\text{ J}.$

EVALUATE: The mass of the car is not needed since it is the given force that is doing the work.

6.15. **IDENTIFY:** We want the work done by the force, and we know the force and the displacement in terms of their components.

SET UP: We can use either $W = \vec{F} \cdot \vec{s} = F_x s_x + F_y s_y$ or $W = Fs\cos\phi$, depending on what we know.

EXECUTE: (a) We know the magnitudes of the two given vectors and the angle between them, so $W = Fs\cos\phi$ = (30.0 N)(5.00 m)(cos37°) = 120 J.

(b) As in (a), we have $W = Fs\cos\phi$ = (30.0 N)(6.00 m)(cos127°) = –108 J.

(c) We know the components of both vectors, so we use $W = \vec{F} \cdot \vec{s} = F_x s_x + F_y s_y$.

$W = \vec{F} \cdot \vec{s} = F_x s_x + F_y s_y$ = (30.0 N)(cos37°)(–2.00 m) + (30.00 N)(sin37°)(4.00 m) = 24.3 J.

EVALUATE: We could check parts (a) and (b) using the method from part (c).

6.16. **IDENTIFY:** The book changes its speed and hence its kinetic energy, so work must have been done on it.

SET UP: Use the work-kinetic energy theorem $W_{\text{net}} = K_f - K_i$, with $K = \frac{1}{2}mv^2$. In part (a) use K_i and K_f to calculate W. In parts (b) and (c) use K_i and W to calculate K_f.

EXECUTE: **(a)** Substituting the notation $i = A$ and $f = B$,

$W_{\text{net}} = K_B - K_A = \frac{1}{2}(1.50\text{ kg})[(1.25\text{ m/s})^2 - (3.21\text{ m/s})^2] = -6.56\text{ J}.$

(b) Noting $i = B$ and $f = C$,

$K_C = K_B + W_{\text{net}} = \frac{1}{2}(1.50\text{ kg})(1.25\text{ m/s})^2 - 0.750\text{ J} = +0.422\text{ J}.$ $K_C = \frac{1}{2}mv_C^2$ so

$v_C = \sqrt{2K_C/m} = 0.750\text{ m/s}.$

(c) Similarly, $K_C = \frac{1}{2}(1.50\text{ kg})(1.25\text{ m/s})^2 + 0.750\text{ J} = 1.922\text{ J}$ and $v_C = 1.60\text{ m/s}.$

EVALUATE: Negative W_{net} corresponds to a decrease in kinetic energy (slowing down) and positive W_{net} corresponds to an increase in kinetic energy (speeding up).

6.17. **IDENTIFY:** Find the kinetic energy of the cheetah knowing its mass and speed.

SET UP: Use $K = \frac{1}{2}mv^2$ to relate v and K.

EXECUTE: **(a)** $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(70\text{ kg})(32\text{ m/s})^2 = 3.6 \times 10^4\text{ J}.$

(b) K is proportional to v^2, so K increases by a factor of 4 when v doubles.

EVALUATE: A running person, even with a mass of 70 kg, would have only 1/100 of the cheetah's kinetic energy since a person's top speed is only about 1/10 that of the cheetah.

6.18. **IDENTIFY:** Apply the work-energy theorem to the ball.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \dfrac{1}{2}mv^2$.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1 = \dfrac{1}{2}mv_2^2 - 0 = \dfrac{1}{2}(0.145\text{ kg})(30.0\text{ m/s})^2 = 65.3\text{ J}.$

(b) $W_{\text{tot}} = \dfrac{1}{2}mv_2^2 - \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}(0.145\text{ kg})(-30.0\text{ m/s})^2 - \dfrac{1}{2}(0.145\text{ kg})(20.0\text{ m/s})^2 = 36.3\text{ J}.$

(c) The work was greater in part (a) than in part (b) because the change in kinetic energy was greater.

EVALUATE: In part (a) the bat does all positive work on the ball because the force and displacement are in the same direction. In part (b) the bat does negative work to stop the ball and then positive work to increase its speed in the opposite direction. So the *total* work the bat does is less than in part (a).

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6.19. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

SET UP: $v = 12\text{ km/s} = 1.2\times10^4\text{ m/s}$. A 1.0 megaton bomb releases $4.184\times10^{15}\text{ J}$ of energy.

EXECUTE: **(a)** $K = \frac{1}{2}(1.4\times10^8\text{ kg})(1.2\times10^4\text{ m/s})^2 = 1.0\times10^{16}\text{ J}$.

(b) $\dfrac{1.0\times10^{16}\text{ J}}{4.184\times10^{15}\text{ J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

EVALUATE: Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

6.20. **IDENTIFY:** Only gravity does work on the watermelon, so $W_{\text{tot}} = W_{\text{grav}}$. $W_{\text{tot}} = \Delta K$ and $K = \frac{1}{2}mv^2$.

SET UP: Since the watermelon is dropped from rest, $K_1 = 0$.

EXECUTE: **(a)** $W_{\text{grav}} = mgs = (4.80\text{ kg})(9.80\text{ m/s}^2)(18.0\text{ m}) = 847\text{ J}$.

(b) (i) $W_{\text{tot}} = K_2 - K_1$ so $K_2 = 847\text{ J}$. **(ii)** $v = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(847\text{ J})}{4.80\text{ kg}}} = 18.8\text{ m/s}$.

(c) The work done by gravity would be the same. Air resistance would do negative work and W_{tot} would be less than W_{grav}. The answer in (a) would be unchanged and both answers in (b) would decrease.

EVALUATE: The gravity force is downward and the displacement is downward, so gravity does positive work.

6.21. **IDENTIFY:** We need to calculate work.

SET UP: $W = Fs\cos\phi$. Since d and ϕ are the same in both cases, but F is different.

EXECUTE: If the box travels a distance d in time t starting from rest, $x = x_0 + v_{0x}t + \dfrac{1}{2}a_xt^2$ tells us that $d = \dfrac{1}{2}a_1t^2$. If it travels the same distance in half the time, we have $d = \dfrac{1}{2}a_2\left(\dfrac{t}{2}\right)^2 = \dfrac{1}{2}a_2\dfrac{t^2}{4} = \dfrac{1}{2}\left(\dfrac{a_2}{4}\right)t^2$.

Comparing these two equations for d shows that $a_2/4 = a_1$ so $a_2 = 4a_1$. $\sum F_x = ma_x$ tells us that the force F is 4 times as great, and so the work $W = Fs\cos\phi$ done by the force is 4 times as great. Therefore the work is $4W_1$.

EVALUATE: We must be careful of using ratios when quantities are squared.

6.22. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. In each case calculate W_{tot} from what we know about the force and the displacement.

SET UP: The gravity force is mg, downward. The friction force is $f_\text{k} = \mu_\text{k}n = \mu_\text{k}mg$ and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

EXECUTE: **(a)** $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{\text{tot}} = W_f = -\mu_\text{k}mgs$. $-\mu_\text{k}mgs = -\frac{1}{2}mv_1^2$.

$$s = \frac{v_1^2}{2\mu_\text{k}g} = \frac{(5.00\text{ m/s})^2}{2(0.220)(9.80\text{ m/s}^2)} = 5.80\text{ m}.$$

(b) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$. $W_{\text{tot}} = W_f = -\mu_\text{k}mgs$. $K_2 = W_{\text{tot}} + K_1$. $\frac{1}{2}mv_2^2 = -\mu_\text{k}mgs + \frac{1}{2}mv_1^2$.

$$v_2 = \sqrt{v_1^2 - 2\mu_\text{k}gs} = \sqrt{(5.00\text{ m/s})^2 - 2(0.220)(9.80\text{ m/s}^2)(2.90\text{ m})} = 3.53\text{ m/s}.$$

(c) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = 0$. $W_{\text{grav}} = -mgy_2$, where y_2 is the vertical height. $-mgy_2 = -\frac{1}{2}mv_1^2$ and

$$y_2 = \frac{v_1^2}{2g} = \frac{(12.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 7.35\text{ m}.$$

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EVALUATE: In parts (a) and (b), friction does negative work and the kinetic energy is reduced. In part (c), gravity does negative work and the speed decreases. The vertical height in part (c) is independent of the slope angle of the hill.

6.23. IDENTIFY and SET UP: Apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the box. Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for K_1 and from that obtain the required initial speed.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\ \ K_2 = 0$

Work is done by gravity and friction, so $W_{\text{tot}} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where s is the distance the box travels along the incline.

$W_f = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem gives $-mgh - \mu_k mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.

Solving for v_0 then gives $v_0 = \sqrt{2gh(1 + \mu_k/\tan\alpha)}$.

EVALUATE: The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

6.24. IDENTIFY: From the work-energy relation, $W = W_{\text{grav}} = \Delta K_{\text{rock}}$.

SET UP: As the rock rises, the gravitational force, $F = mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, s, the work is negative. Let h be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_{\text{grav}} = K_2 - K_1$ we obtain $-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$. Dividing by m and solving for v_1, $v_1 = \sqrt{v_2^2 + 2gh}$. Substituting $h = 15.0$ m and $v_2 = 25.0$ m/s,

$$v_1 = \sqrt{(25.0\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(15.0\text{ m})} = 30.3\text{ m/s}$$

(b) Solve the same work-energy relation for h. At the maximum height $v_2 = 0$.

$-mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ and $h = \dfrac{v_1^2 - v_2^2}{2g} = \dfrac{(30.3\text{ m/s})^2 - (0.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 46.8\text{ m}.$

EVALUATE: Note that the weight of the rock was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, m. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.25. IDENTIFY: Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\phi = 0°$

EXECUTE: Use $W = Fs\cos\phi$, $W_{\text{tot}} = K_2 - K_1$, and $K = \dfrac{1}{2}mv^2$ and solve for F, giving

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(12.0\text{ kg})\left[(6.00\text{ m/s})^2 - (4.00\text{ m/s})^2\right]}{(2.50\text{ m})} = 48.0\text{ N}$$

EVALUATE: The force is in the direction of the displacement, so the force does positive work and the kinetic energy of the object increases.

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6.26. IDENTIFY and SET UP: Use the work-energy theorem to calculate the work done by the foot on the ball. Then use $W = Fs\cos\phi$ to find the distance over which this force acts.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420\text{ kg})(2.00\text{ m/s})^2 = 0.84\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420\text{ kg})(6.00\text{ m/s})^2 = 7.56\text{ J}$

$W_{\text{tot}} = K_2 - K_1 = 7.56\text{ J} - 0.84\text{ J} = 6.72\text{ J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work. $W_F = (F\cos\phi)s$ gives that $s = \dfrac{W}{F\cos\phi} = \dfrac{6.72\text{ J}}{(40.0\text{ N})(\cos 0)} = 0.168\text{ m}.$

EVALUATE: The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

6.27. IDENTIFY: Apply $W_{\text{tot}} = \Delta K.$

SET UP: $v_1 = 0,\; v_2 = v.\; f_k = \mu_k mg$ and f_k does negative work. The force $F = 36.0\text{ N}$ is in the direction of the motion and does positive work.

EXECUTE: **(a)** If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0\text{ N})(1.20\text{ m})}{(4.30\text{ kg})}} = 4.48\text{ m/s}.$$

(b) The net work is $Fs - f_k s = (F - \mu_k mg)s,$ so

$$v = \sqrt{\frac{2(F - \mu_k mg)s}{m}} = \sqrt{\frac{2(36.0\text{ N} - (0.30)(4.30\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m})}{(4.30\text{ kg})}} = 3.61\text{ m/s}$$

EVALUATE: The total work done is larger in the absence of friction and the final speed is larger in that case.

6.28. IDENTIFY: Apply $W_{\text{tot}} = K_2 - K_1.$

SET UP: $K_1 = 0.$ The normal force does no work. The work W done by gravity is $W = mgh,$ where $h = L\sin\theta$ is the vertical distance the block has dropped when it has traveled a distance L down the incline and θ is the angle the plane makes with the horizontal.

EXECUTE: The work-energy theorem gives $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta}.$ Using the given numbers, $v = \sqrt{2(9.80\text{ m/s}^2)(1.35\text{ m})\sin 36.9°} = 3.99\text{ m/s}.$

6.29. IDENTIFY: We use the work-energy theorem.

SET UP: $W_{\text{tot}} = K_2 - K_1,\; K = \dfrac{1}{2}mv^2$.

EXECUTE: **(a)** $K_A = \dfrac{1}{2}m_A v_A^2 = 27\text{ J}$ and $K_B = \dfrac{1}{2}\left(\dfrac{m_A}{4}\right)v_B^2 = 27\text{ J}.$ Equate both expressions, which gives $\dfrac{1}{2}m_A v_A^2 = = \dfrac{1}{2}\left(\dfrac{m_A}{4}\right)v_B^2.$ Solving for v_B we have $v_B = 2v_A.$ So B is moving faster and its speed is twice that of A.

(b) Apply the work-energy theorem to A. $-18\text{ J} = K_2 - 27\text{ J}$, so $K_2 = 9\text{ J}$. Take the ratio of its initial and final kinetic energy, giving $\dfrac{K_2}{K_1} = \dfrac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \dfrac{9\text{ J}}{27\text{ J}} = \dfrac{1}{3} = \dfrac{v_2^2}{v_1^2}.$ Thus $v_2 = \dfrac{v_1}{\sqrt{3}}.$ We will get the same result

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for B because both objects have the same initial kinetic energy (27 J) and the same amount of work (–18 J) is done on them.

EVALUATE: The result to part (a) is reasonable since a lighter object must move faster to have the same kinetic energy as a heavier object. Part (b) is reasonable because the kinetic energy decreases since the work done on the objects is negative.

6.30. **IDENTIFY:** We know (or can calculate) the change in the kinetic energy of the crate and want to find the work needed to cause this change, so the work-energy theorem applies.

SET UP: $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$.

EXECUTE: $W_{\text{tot}} = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}(30.0 \text{ kg})(5.62 \text{ m/s})^2 - \frac{1}{2}(30.0 \text{ kg})(3.90 \text{ m/s})^2$.

$W_{\text{tot}} = 473.8 \text{ J} - 228.2 \text{ J} = 246 \text{ J}$.

EVALUATE: Kinetic energy is a scalar and does not depend on direction, so only the initial and final speeds are relevant.

6.31. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. Only friction does work.

SET UP: $W_{\text{tot}} = W_{f_{\text{k}}} = -\mu_{\text{k}}mgs$. $K_2 = 0$ (car stops). $K_1 = \frac{1}{2}mv_0^2$.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_{\text{k}}mgs = -\frac{1}{2}mv_0^2$. $s = \dfrac{v_0^2}{2\mu_{\text{k}}g}$.

(b) (i) $\mu_{\text{k}b} = 2\mu_{\text{k}a}$. $s\mu_{\text{k}} = \dfrac{v_0^2}{2g} = \text{constant}$ so $s_a\mu_{\text{k}a} = s_b\mu_{\text{k}b}$. $s_b = \left(\dfrac{\mu_{\text{k}a}}{\mu_{\text{k}b}}\right)s_a = s_a/2$. The minimum stopping distance would be halved. (ii) $v_{0b} = 2v_{0a}$. $\dfrac{s}{v_0^2} = \dfrac{1}{2\mu_{\text{k}}g} = \text{constant}$, so $\dfrac{s_a}{v_{0a}^2} = \dfrac{s_b}{v_{0b}^2}$.

$s_b = s_a\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = 4s_a$. The stopping distance would become 4 times as great. (iii) $v_{0b} = 2v_{0a}$,

$\mu_{\text{k}b} = 2\mu_{\text{k}a}$. $\dfrac{s\mu_{\text{k}}}{v_0^2} = \dfrac{1}{2g} = \text{constant}$, so $\dfrac{s_a\mu_{\text{k}a}}{v_{0a}^2} = \dfrac{s_b\mu_{\text{k}b}}{v_{0b}^2}$. $s_b = s_a\left(\dfrac{\mu_{\text{k}a}}{\mu_{\text{k}b}}\right)\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = s_a\left(\dfrac{1}{2}\right)(2)^2 = 2s_a$. The stopping distance would double.

EVALUATE: The stopping distance is directly proportional to the square of the initial speed and indirectly proportional to the coefficient of kinetic friction.

6.32. **IDENTIFY:** The work that must be done to move the end of a spring from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. The force required to hold the end of the spring at displacement x is $F_x = kx$.

SET UP: When the spring is at its unstretched length, $x = 0$. When the spring is stretched, $x > 0$, and when the spring is compressed, $x < 0$.

EXECUTE: **(a)** $x_1 = 0$ and $W = \frac{1}{2}kx_2^2$. $k = \dfrac{2W}{x_2^2} = \dfrac{2(12.0 \text{ J})}{(0.0300 \text{ m})^2} = 2.67\times10^4 \text{ N/m}$.

(b) $F_x = kx = (2.67\times10^4 \text{ N/m})(0.0300 \text{ m}) = 801 \text{ N}$.

(c) $x_1 = 0$, $x_2 = -0.0400 \text{ m}$. $W = \frac{1}{2}(2.67\times10^4 \text{ N/m})(-0.0400 \text{ m})^2 = 21.4 \text{ J}$.

$F_x = kx = (2.67\times10^4 \text{ N/m})(0.0400 \text{ m}) = 1070 \text{ N}$.

EVALUATE: When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

6.33. **IDENTIFY:** The springs obey Hooke's law and balance the downward force of gravity.

SET UP: Use coordinates with $+y$ upward. Label the masses 1, 2, and 3, with 1 the top mass and 3 the bottom mass, and call the amounts the springs are stretched x_1, x_2, and x_3. Each spring force is kx.

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EXECUTE: **(a)** The three free-body diagrams are shown in Figure 6.33.

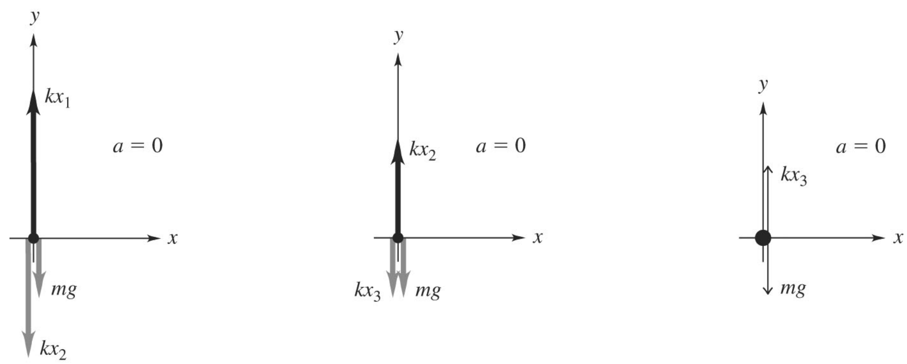

Figure 6.33

(b) Balancing forces on each of the masses and using $F = kx$ gives $kx_3 = mg$ so

$x_3 = \dfrac{mg}{k} = \dfrac{(8.50\text{ kg})(9.80\text{ m/s}^2)}{7.80\times10^3\text{ N/m}} = 1.068\text{ cm}.$ $kx_2 = mg + kx_3 = 2mg$ so $x_2 = 2\left(\dfrac{mg}{k}\right) = 2.136\text{ cm}.$

$kx_1 = mg + kx_2 = 3mg$ so $x_3 = 3\left(\dfrac{mg}{k}\right) = 3.204\text{ cm}.$ Adding the original lengths to the distance stretched, the lengths of the springs, starting from the bottom one, are 13.1 cm, 14.1 cm, and 15.2 cm.

N**EVALUATE:** The top spring stretches most because it supports the most weight, while the bottom spring stretches least because it supports the least weight.

6.34. **IDENTIFY:** The magnitude of the work can be found by finding the area under the graph.

SET UP: The area under each triangle is 1/2 base$\times$height. $F_x > 0$, so the work done is positive when x increases during the displacement.

EXECUTE: **(a)** $1/2\ (8\text{ m})(10\text{ N}) = 40\text{ J}.$

(b) $1/2\ (4\text{ m})(10\text{ N}) = 20\text{ J}.$

(c) $1/2\ (12\text{ m})(10\text{ N}) = 60\text{ J}.$

EVALUATE: The sum of the answers to parts (a) and (b) equals the answer to part (c).

6.35. **IDENTIFY:** Use the work-energy theorem and the results of Problem 6.36.

SET UP: For $x = 0$ to $x = 8.0$ m, $W_{\text{tot}} = 40$ J. For $x = 0$ to $x = 12.0$ m, $W_{\text{tot}} = 60$ J.

EXECUTE: **(a)** $v = \sqrt{\dfrac{(2)(40\text{ J})}{10\text{ kg}}} = 2.83\text{ m/s}$

(b) $v = \sqrt{\dfrac{(2)(60\text{ J})}{10\text{ kg}}} = 3.46\text{ m/s}.$

EVALUATE: $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

6.36. **IDENTIFY:** The spring obeys Hooke's law.

SET UP: Solve $F = kx$ for x to determine the length of stretch and use $W = +\frac{1}{2}kx^2$ to assess the corresponding work.

EXECUTE: $x = \dfrac{F}{k} = \dfrac{15.0\text{ N}}{300.0\text{ N/m}} = 0.0500\text{ m}.$ The new length will be $0.240\text{ m} + 0.0500\text{ m} = 0.290\text{ m}.$

The corresponding work done is $W = \dfrac{1}{2}(300.0\text{ N/m})(0.0500\text{ m})^2 = 0.375\text{ J}.$

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EVALUATE: In $F = kx$, F is always the force applied to one end of the spring, thus we did not need to double the 15.0 N force. Consider a free-body diagram of a spring at rest; forces of equal magnitude and opposite direction are always applied to both ends of every section of the spring examined.

6.37. IDENTIFY: Apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the box.

SET UP: Let point 1 be just before the box reaches the end of the spring and let point 2 be where the spring has maximum compression and the box has momentarily come to rest.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2, \;\; K_2 = 0$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx_2^2,$ where x_2 is the amount the spring is compressed.

$-\frac{1}{2}kx_2^2 = -\frac{1}{2}mv_0^2$ and $x_2 = v_0\sqrt{m/k} = (3.0 \text{ m/s})\sqrt{(6.0 \text{ kg})/(7500 \text{ N/m})} = 8.5 \text{ cm}$

EVALUATE: The compression of the spring increases when either v_0 or m increases and decreases when k increases (stiffer spring).

6.38. IDENTIFY: The force applied to the springs is $F_x = kx$. The work done on a spring to move its end from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. Use the information that is given to calculate k.

SET UP: When the springs are compressed 0.200 m from their uncompressed length, $x_1 = 0$ and $x_2 = -0.200$ m. When the platform is moved 0.200 m farther, x_2 becomes -0.400 m.

EXECUTE: **(a)** $k = \dfrac{2W}{x_2^2 - x_1^2} = \dfrac{2(80.0 \text{ J})}{(0.200 \text{ m})^2 - 0} = 4000 \text{ N/m}.$ $F_x = kx = (4000 \text{ N/m})(-0.200 \text{ m}) = -800 \text{ N}.$

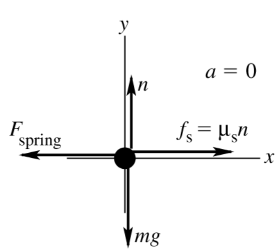
The magnitude of force that is required is 800 N.

(b) To compress the springs from $x_1 = 0$ to $x_2 = -0.400$ m, the work required is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 = \frac{1}{2}(4000 \text{ N/m})(-0.400 \text{ m})^2 = 320 \text{ J}.$ The additional work required is $320 \text{ J} - 80 \text{ J} = 240 \text{ J}.$ For $x = -0.400$ m, $F_x = kx = -1600$ N. The magnitude of force required is 1600 N.

EVALUATE: More work is required to move the end of the spring from $x = -0.200$ m to $x = -0.400$ m than to move it from $x = 0$ to $x = -0.200$ m, even though the displacement of the platform is the same in each case. The magnitude of the force increases as the compression of the spring increases.

6.39. IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to calculate the μ_s required for the static friction force to equal the spring force.

SET UP: **(a)** The free-body diagram for the glider is given in Figure 6.39.



EXECUTE: $\Sigma F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s mg$

Figure 6.39

$\Sigma F_x = ma_x$

$f_s - F_{\text{spring}} = 0$

$\mu_s mg - kd = 0$

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$$\mu_s = \frac{kd}{mg} = \frac{(20.0\text{ N/m})(0.086\text{ m})}{(0.100\text{ kg})(9.80\text{ m/s}^2)} = 1.76$$

(b) IDENTIFY and **SET UP:** Apply $\Sigma \vec{F} = m\vec{a}$ to find the maximum amount the spring can be compressed and still have the spring force balanced by friction. Then use $W_{\text{tot}} = K_2 - K_1$ to find the initial speed that results in this compression of the spring when the glider stops.

EXECUTE: $\mu_s mg = kd$

$$d = \frac{\mu_s mg}{k} = \frac{(0.60)(0.100\text{ kg})(9.80\text{ m/s}^2)}{20.0\text{ N/m}} = 0.0294\text{ m}$$

Now apply the work-energy theorem to the motion of the glider:

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (instantaneously stops)

$W_{\text{tot}} = W_{\text{spring}} + W_{\text{fric}} = -\frac{1}{2}kd^2 - \mu_k mgd$ (as in Example 6.7)

$W_{\text{tot}} = -\frac{1}{2}(20.0\text{ N/m})(0.0294\text{ m})^2 - 0.47(0.100\text{ kg})(9.80\text{ m/s}^2)(0.0294\text{ m}) = -0.02218\text{ J}$

Then $W_{\text{tot}} = K_2 - K_1$ gives $-0.02218\text{ J} = -\frac{1}{2}mv_1^2$.

$$v_1 = \sqrt{\frac{2(0.02218\text{ J})}{0.100\text{ kg}}} = 0.67\text{ m/s}.$$

EVALUATE: In Example 6.7 an initial speed of 1.50 m/s compresses the spring 0.086 m and in part (a) of this problem we found that the glider doesn't stay at rest. In part (b) we found that a smaller displacement of 0.0294 m when the glider stops is required if it is to stay at rest. And we calculate a smaller initial speed (0.67 m/s) to produce this smaller displacement.

6.40. **IDENTIFY:** For the spring, $W = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $x_1 = -0.025$ m and $x_2 = 0$.

EXECUTE: **(a)** $W = \frac{1}{2}kx_1^2 = \frac{1}{2}(200\text{ N/m})(-0.025\text{ m})^2 = 0.0625\text{ J}$, which rounds to 0.063 J.

(b) The work-energy theorem gives $v_2 = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(0.0625\text{ J})}{(4.0\text{ kg})}} = 0.18\text{ m/s}$.

EVALUATE: The block moves in the direction of the spring force, the spring does positive work and the kinetic energy of the block increases.

6.41. **IDENTIFY** and **SET UP:** The magnitude of the work done by F_x equals the area under the F_x versus x curve. The work is positive when F_x and the displacement are in the same direction; it is negative when they are in opposite directions.

EXECUTE: **(a)** F_x is positive and the displacement Δx is positive, so $W > 0$.

$W = \frac{1}{2}(2.0\text{ N})(2.0\text{ m}) + (2.0\text{ N})(1.0\text{ m}) = +4.0\text{ J}$

(b) During this displacement $F_x = 0$, so $W = 0$.

(c) F_x is negative, Δx is positive, so $W < 0$. $W = -\frac{1}{2}(1.0\text{ N})(2.0\text{ m}) = -1.0\text{ J}$

(d) The work is the sum of the answers to parts (a), (b), and (c), so $W = 4.0\text{ J} + 0 - 1.0\text{ J} = +3.0\text{ J}$.

(e) The work done for $x = 7.0$ m to $x = 3.0$ m is $+1.0$ J. This work is positive since the displacement and the force are both in the $-x$-direction. The magnitude of the work done for $x = 3.0$ m to $x = 2.0$ m is 2.0 J, the area under F_x versus x. This work is negative since the displacement is in the $-x$-direction and the force is in the $+x$-direction. Thus $W = +1.0\text{ J} - 2.0\text{ J} = -1.0\text{ J}$.

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EVALUATE: The work done when the car moves from $x = 2.0$ m to $x = 0$ is $-\frac{1}{2}(2.0\text{ N})(2.0\text{ m}) = -2.0$ J. Adding this to the work for $x = 7.0$ m to $x = 2.0$ m gives a total of $W = -3.0$ J for $x = 7.0$ m to $x = 0$. The work for $x = 7.0$ m to $x = 0$ is the negative of the work for $x = 0$ to $x = 7.0$ m.

6.42. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $K_1 = 0$. From Exercise 6.41, the work for $x = 0$ to $x = 3.0$ m is 4.0 J. W for $x = 0$ to $x = 4.0$ m is also 4.0 J. For $x = 0$ to $x = 7.0$ m, $W = 3.0$ J.

EXECUTE: **(a)** $K = 4.0$ J, so $v = \sqrt{2K/m} = \sqrt{2(4.0\text{ J})/(2.0\text{ kg})} = 2.00$ m/s.

(b) No work is done between $x = 3.0$ m and $x = 4.0$ m, so the speed is the same, 2.00 m/s.

(c) $K = 3.0$ J, so $v = \sqrt{2K/m} = \sqrt{2(3.0\text{ J})/(2.0\text{ kg})} = 1.73$ m/s.

EVALUATE: In each case the work done by F is positive and the car gains kinetic energy.

6.43. **IDENTIFY** and **SET UP:** Apply the work-energy theorem. Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200$ m for part (b). Use $W = \dfrac{1}{2}kx^2$ for the work done by the spring and calculate K_2. Then $K_2 = \frac{1}{2}mv_2^2$ gives v_2.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \frac{1}{2}mv_2^2$

The only force doing work is the spring force. $W = \dfrac{1}{2}kx^2$ gives the work done *on* the spring to move its end from x_1 to x_2. The force the spring exerts on an object attached to it is $F = -kx$, so the work the spring does is

$W_{\text{spr}} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Here $x_1 = -0.375$ m and $x_2 = 0$. Thus

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - 0 = 281$ J.

$K_2 = K_1 + W_{\text{tot}} = 0 + 281\text{ J} = 281$ J.

Then $K_2 = \frac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281\text{ J})}{70.0\text{ kg}}} = 2.83$ m/s.

(b) $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Now $x_2 = -0.200$ m, so

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - \frac{1}{2}(4000\text{ N/m})(-0.200\text{ m})^2 = 281\text{ J} - 80\text{ J} = 201$ J

Thus $K_2 = 0 + 201\text{ J} = 201$ J and $K_2 = \frac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201\text{ J})}{70.0\text{ kg}}} = 2.40$ m/s.

EVALUATE: The spring does positive work and the sled gains speed as it returns to $x = 0$. More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

6.44. **IDENTIFY** and **SET UP:** Apply the work-energy theorem to the glider. Work is done by the spring and by gravity. Take point 1 to be where the glider is released. In part (a) point 2 is where the glider has traveled 1.80 m and $K_2 = 0$. There are two points shown in Figure 6.44a. In part (b) point 2 is where the glider has traveled 0.80 m.

EXECUTE: (a) $W_{\text{tot}} = K_2 - K_1 = 0$. Solve for x_1, the amount the spring is initially compressed.

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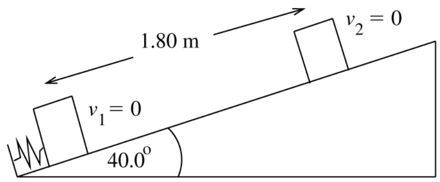


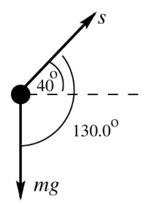
$W_{tot} = W_{spr} + W_w = 0$

So $W_{spr} = -W_w$

(The spring does positive work on the glider since the spring force is directed up the incline, the same as the direction of the displacement.)

Figure 6.44a

The directions of the displacement and of the gravity force are shown in Figure 6.44b.



$W_w = (w\cos\phi)s = (mg\cos 130.0°)s$

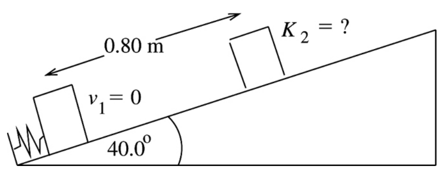
$W_w = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(1.80\text{ m}) = -1.020\text{ J}$ (The component of w parallel to the incline is directed down the incline, opposite to the displacement, so gravity does negative work.)

Figure 6.44b

$W_{spr} = -W_w = +1.020\text{ J}$

$W_{spr} = \frac{1}{2}kx_1^2$ so $x_1 = \sqrt{\frac{2W_{spr}}{k}} = \sqrt{\frac{2(1.020\text{ J})}{640\text{ N/m}}} = 0.0565\text{ m}$

(b) The spring was compressed only 0.0565 m so at this point in the motion the glider is no longer in contact with the spring. Points 1 and 2 are shown in Figure 6.44c.



$W_{tot} = K_2 - K_1$

$K_2 = K_1 + W_{tot}$

$K_1 = 0$

Figure 6.44c

$W_{tot} = W_{spr} + W_w$

From part (a), $W_{spr} = 1.020\text{ J}$ and

$W_w = (mg\cos 130.0°)s = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(0.80\text{ m}) = -0.454\text{ J}$

Then $K_2 = W_{spr} + W_w = +1.020\text{ J} - 0.454\text{ J} = +0.57\text{ J}.$

EVALUATE: The kinetic energy in part (b) is positive, as it must be. In part (a), $x_2 = 0$ since the spring force is no longer applied past this point. In computing the work done by gravity we use the full 0.80 m the glider moves.

6.45. **IDENTIFY:** The force does work on the box, which gives it kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the box.

SET UP: $W_{tot} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ and $W_{tot} = \int_{x_1}^{x_2} F(x)dx.$

EXECUTE: $W_{tot} = \int_{x_1}^{x_2} F(x)dx = \int_0^{14.0\text{ m}} [18.0\text{ N} - (0.530\text{ N/m})x]dx$

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$W_{\text{tot}} = (18.0\text{ N})(14.0\text{ m}) - (0.265\text{ N/m})(14.0\text{ m})^2 = 252.0\text{ J} - 51.94\text{ J} = 200.1\text{ J}$. The initial kinetic energy is zero, so $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2$. Solving for v_{f} gives $v_{\text{f}} = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(200.1\text{ J})}{6.00\text{ kg}}} = 8.17\text{ m/s}$.

EVALUATE: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of *x*, not of *t*. The work-energy theorem provides a much simpler method.

6.46. **IDENTIFY:** The force acts through a distance over time, so it does work on the crate and hence supplies power to it. The force exerted by the worker is variable but the acceleration of the cart is constant.
SET UP: Use $P = Fv$ to find the power, and we can use $v = v_0 + at$ to find the instantaneous velocity.
EXECUTE: First find the instantaneous force and velocity: $F = (5.40\text{ N/s})(5.00\text{ s}) = 27.0\text{ N}$ and $v = v_0 + at = (2.80\text{ m/s}^2)(5.00\text{ s}) = 14.0\text{ m/s}$. Now find the power: $P = (27.0\text{ N})(14.0\text{ m/s}) = 378\text{ W}$.
EVALUATE: The instantaneous power will increase as the worker pushes harder and harder.

6.47. **IDENTIFY:** Apply the relation between energy and power.
SET UP: Use $P = \frac{W}{\Delta t}$ to solve for *W*, the energy the bulb uses. Then set this value equal to $\frac{1}{2}mv^2$ and solve for the speed.
EXECUTE: $W = P\Delta t = (100\text{ W})(3600\text{ s}) = 3.6\times10^5\text{ J}$

$K = 3.6\times10^5\text{ J}$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.6\times10^5\text{ J})}{70\text{ kg}}} = 100\text{ m/s}$

EVALUATE: Olympic runners achieve speeds up to approximately 10 m/s, or roughly one-tenth the result calculated.

6.48. **IDENTIFY:** Knowing the rate at which energy is consumed, we want to find out the total energy used.
SET UP: Find the elapsed time Δt in each case by dividing the distance by the speed, $\Delta t = d/v$. Then calculate the energy as $W = P\Delta t$.
EXECUTE: Running: $\Delta t = (5.0\text{ km})/(10\text{ km/h}) = 0.50\text{ h} = 1.8\times10^3\text{ s}$. The energy used is $W = (700\text{ W})(1.8\times10^3\text{ s}) = 1.3\times10^6\text{ J}$.

Walking: $\Delta t = \frac{5.0\text{ km}}{3.0\text{ km/h}}\left(\frac{3600\text{ s}}{\text{h}}\right) = 6.0\times10^3\text{ s}$. The energy used is

$W = (290\text{ W})(6.0\times10^3\text{ s}) = 1.7\times10^6\text{ J}$.

EVALUATE: The less intense exercise lasts longer and therefore burns up more energy than the intense exercise.

6.49. **IDENTIFY:** We want to calculate power.
SET UP: Estimate: 4 bags/min, so 20 bags in 5.0 min. $P_{\text{av}} = W/t$.
EXECUTE: $W = wh = (30\text{ lb})(4.0\text{ ft}) = 120\text{ ft}\cdot\text{lb}$ per bag. The total work is $(120\text{ ft}\cdot\text{lb})(20\text{ bags}) = 2400\text{ ft}\cdot\text{lb}$. The time is 5.0 min =300 s. So the power is

$P_{\text{av}} = \frac{W}{t} = \frac{2400\text{ ft}\cdot\text{lb}}{300\text{ s}} = 8.0\text{ ft}\cdot\text{lb/s}$. Convert to horsepower: $(8.0\text{ ft}\cdot\text{lb/s})\left(\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}}\right) = 1.5\times10^{-2}\text{ hp}$.

Convert to watts: $\left(1.5\times10^{-2}\text{ hp}\right)\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 11\text{ W}$.

EVALUATE: This college student puts a lot less power than the marathon runner in Example 6.10 in the text!

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6.50. **IDENTIFY:** The thermal energy is produced as a result of the force of friction, $F = \mu_k mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{av}$.

SET UP: $v_{av} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00 \text{ m/s} + 0}{2}\right) = 4.00 \text{ m/s}$

EXECUTE: $P = Fv_{av} = [(0.200)(20.0 \text{ kg})(9.80 \text{ m/s}^2)](4.00 \text{ m/s}) = 157 \text{ W}$

EVALUATE: The power could also be determined as the rate of change of kinetic energy, $\Delta K / t$, where the time is calculated from $v_f = v_i + at$ and a is calculated from a force balance, $\Sigma F = ma = \mu_k mg$.

6.51. **IDENTIFY:** We need to use power.
SET UP: Estimates: student weight is 150 lb which is about 667 N. $P_{av} = W/t$.
EXECUTE: Convert units: 50 ft = 15.2 m. The work is $W = wy = (667 \text{ N})(15.2 \text{ m}) = 10{,}000$ J. Solve $P_{av} = W/t$ for t: $t = W/P_{av} = (10{,}000 \text{ J})/(500 \text{ W}) = 20$ s.
EVALUATE: 20 s seems like a reasonable time to climb three flights of stairs.

6.52. **IDENTIFY** and **SET UP:** Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.
EXECUTE: The rate at which work is being done against gravity is
$P = Fv = mgv = (700 \text{ kg})(9.80 \text{ m/s}^2)(2.5 \text{ m/s}) = 17.15 \text{ kW}$.

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is $17.15 \text{ kW}/75 \text{ kW} = 0.23$.
EVALUATE: The power we calculate for making the airplane climb is considerably less than the power output of the engine.

6.53. **IDENTIFY:** $P_{av} = \dfrac{\Delta W}{\Delta t}$. The work you do in lifting mass m a height h is mgh.

SET UP: $1 \text{ hp} = 746 \text{ W}$

EXECUTE: **(a)** The number per minute would be the average power divided by the work (mgh) required to lift one box, $\dfrac{(0.50 \text{ hp})(746 \text{ W/hp})}{(30 \text{ kg})(9.80 \text{ m/s}^2)(0.90 \text{ m})} = 1.41/\text{s}$, or $84.6/\text{min}$.

(b) Similarly, $\dfrac{(100 \text{ W})}{(30 \text{ kg})(9.80 \text{ m/s}^2)(0.90 \text{ m})} = 0.378/\text{s}$, or $22.7/\text{min}$.

EVALUATE: A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

6.54. **IDENTIFY** and **SET UP:** Use $P_{av} = \dfrac{\Delta W}{\Delta t}$ to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.
EXECUTE: Find the total mass that can be lifted:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{av}t}{gh}$$

$$P_{av} = (40 \text{ hp})\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 2.984 \times 10^4 \text{ W}$$

$$m = \frac{P_{av}t}{gh} = \frac{(2.984 \times 10^4 \text{ W})(16.0 \text{ s})}{(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 2.436 \times 10^3 \text{ kg}$$

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This is the total mass of elevator plus passengers. The mass of the passengers is $2.436\times10^3\ \text{kg} - 600\ \text{kg} = 1.836\times10^3\ \text{kg}$. The number of passengers is $\dfrac{1.836\times10^3\ \text{kg}}{65.0\ \text{kg}} = 28.2$. 28 passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

6.55. **IDENTIFY:** To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers, $F_{\text{rope}} = Nmg\sin\alpha$.

SET UP: $P = F_{\parallel}v = F_{\text{rope}}v$

EXECUTE: $P_{\text{rope}} = F_{\text{rope}}v = [+Nmg(\cos\phi)]v$.

$$P_{\text{rope}} = [(50\ \text{riders})(70.0\ \text{kg})(9.80\ \text{m/s}^2)(\cos 75.0)]\left[(12.0\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.60\ \text{km/h}}\right)\right].$$

$P_{\text{rope}} = 2.96\times10^4\ \text{W} = 29.6\ \text{kW}$.

EVALUATE: Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

6.56. **IDENTIFY:** We want to find the power supplied by a known force acting on a crate at a known velocity.

SET UP: We know the vector components, so we use $P = \vec{F}\cdot\vec{v} = F_xv_x + F_yv_y$

EXECUTE: $P = F_xv_x + F_yv_y = (-8.00\ \text{N})(3.20\ \text{m/s}) + (3.00\ \text{N})(2.20\ \text{m/s}) = -19.0\ \text{W}$.

EVALUATE: The power is negative because the x-component of the force is opposite to the x-component of the velocity and hence opposes the motion of the crate.

6.57. **IDENTIFY:** Relate power, work, and time.

SET UP: Work done in each stroke is $W = Fs$ and $P_{\text{av}} = W/t$.

EXECUTE: 100 strokes per second means $P_{\text{av}} = 100Fs/t$ with $t = 1.00$ s, $F = 2mg$ and $s = 0.010$ m. $P_{\text{av}} = 0.20$ W.

EVALUATE: For a 70-kg person to apply a force of twice his weight through a distance of 0.50 m for 100 times per second, the average power output would be 7.0×10^4 W. This power output is very far beyond the capability of a person.

6.58. **IDENTIFY:** The force has only an x-component and the motion is along the x-direction, so $W = \int_{x_1}^{x_2} F_x dx$.

SET UP: $x_1 = 0$ and $x_2 = 6.9$ m.

EXECUTE: The work you do with your changing force is

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2}(-20.0\ \text{N})dx - \int_{x_1}^{x_2}(3.0\ \text{N/m})xdx = (-20.0\ \text{N})x\Big|_{x_1}^{x_2} - (3.0\ \text{N/m})(x^2/2)\Big|_{x_1}^{x_2}$$

$W = -138\ \text{N}\cdot\text{m} - 71.4\ \text{N}\cdot\text{m} = -209\ \text{J}$.

EVALUATE: The work is negative because the cow continues to move forward (in the $+x$-direction) as you vainly attempt to push her backward.

6.59. **IDENTIFY** and **SET UP:** Since the forces are constant, $W_F = (F\cos\phi)s$ can be used to calculate the work done by each force. The forces on the suitcase are shown in Figure 6.59a.

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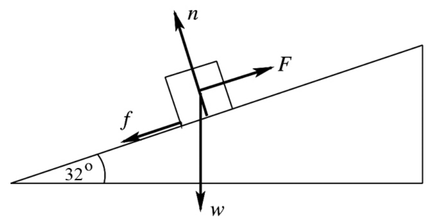


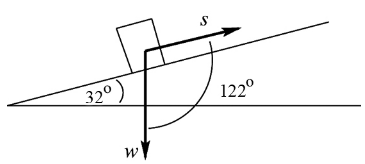
Figure 6.59a

In part (f), the work-energy theorem is used to relate the total work to the initial and final kinetic energy.

EXECUTE: **(a)** $W_F = (F\cos\phi)s$

Both $\vec{F}$ and $\vec{s}$ are parallel to the incline and in the same direction, so $\phi = 90°$ and $W_F = Fs = (160\text{ N})(3.80\text{ m}) = 608\text{ J}.$

(b) The directions of the displacement and of the gravity force are shown in Figure 6.59b.



$W_w = (w\cos\phi)s$

$\phi = 122°$, so

$W_w = (196\text{ N})(\cos 122°)(3.80\text{ m})$

$W_w = -395\text{ J}$

Figure 6.59b

Alternatively, the component of w parallel to the incline is $w\sin 32°$. This component is down the incline so its angle with $\vec{s}$ is $\phi = 180°$. $W_{w\sin 32°} = (196\text{ N}\sin 32°)(\cos 180°)(3.80\text{ m}) = -395\text{ J}$. The other component of w, $w\cos 32°$, is perpendicular to $\vec{s}$ and hence does no work. Thus $W_w = W_{w\sin 25°} = -315\text{ J}$, which agrees with the above.

(c) The normal force is perpendicular to the displacement $(\phi = 90°)$, so $W_n = 0$.

(d) $n = w\cos 32°$ so $f_\text{k} = \mu_\text{k} n = \mu_\text{k} w\cos 32° = (0.30)(196\text{ N})\cos 32° = 49.87\text{ N}$

$W_f = (f_\text{k}\cos\phi)x = (49.87\text{ N})(\cos 180°)(3.80\text{ m}) = -189\text{ J}.$

(e) $W_\text{tot} = W_F + W_w + W_n + W_f = +608\text{ J} - 395\text{ J} + 0 - 189\text{ J} = 24\text{ J}.$

(f) $W_\text{tot} = K_2 - K_1$, $K_1 = 0$, so $K_2 = W_\text{tot}$

$\frac{1}{2}mv_2^2 = W_\text{tot}$ so $v_2 = \sqrt{\dfrac{2W_\text{tot}}{m}} = \sqrt{\dfrac{2(24\text{ J})}{20.0\text{ kg}}} = 1.5\text{ m/s}.$

EVALUATE: The total work done is positive and the kinetic energy of the suitcase increases as it moves up the incline.

6.60. **IDENTIFY:** We need to use projectile motion and the work-energy theorem.

SET UP: $W_\text{tot} = K_2 - K_1$, $K = \dfrac{1}{2}mv^2$. First use projectile motion to find the initial velocity, and then use the work-energy theorem to find the work.

EXECUTE: **(a)** When the can returns to the ground, $y = 0$, so $y = (v_0\sin\alpha_0)t - \dfrac{1}{2}gt^2$ gives us $0 = v_0\sin\alpha_0 T - \dfrac{1}{2}gT^2$, so $T = \dfrac{2v_0\sin\alpha_0}{g}$, which is the time in the air, and $v_0 = \dfrac{gT}{2\sin\alpha_0}$. Now use $W_\text{tot} = K_2 - K_1$ with $K_1 = 0$ to find the work. $W = \dfrac{1}{2}Mv_0^2 = \dfrac{1}{2}M\left(\dfrac{gT}{2\sin\alpha_0}\right)^2 = \dfrac{M}{8}\left(\dfrac{gT}{\sin}\alpha_0\right)^2.$

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(b) If it is in the air twice as long, T is doubled. Using our result from part (a), we see that T is *squared*, so $(2T)^2$ would become $4T$. Therefore the work would become $W = \frac{M}{8}\left(\frac{g2T}{\sin}\alpha_0\right)^2 = \frac{M}{2}\left(\frac{gT}{\sin}\alpha_0\right)^2$, which is 4 times as much as in part (a).

EVALUATE: Another approach is to use $T = \frac{2v_0 \sin\alpha_0}{g}$ to see that if T is doubled, so is v_0. Therefore the kinetic energy $\frac{1}{2}mv^2$ is 4 times as large, so the work must have 4 times as much.

6.61. IDENTIFY: This problem requires use of the work-energy theorem. Both friction and gravity do work on the block as it slides down the ramp, but the normal force does no work.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $W = Fs\cos\phi$, $K = \frac{1}{2}mv^2$, $K_1 = 0$ when the block is released. Fig. 6.61 shows the information in the problem.

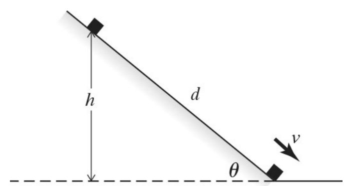

Figure 6.61

EXECUTE: **(a)** We want the work done by friction. $W_{\text{tot}} = K_2 - K_1$, where $K_1 = 0$, $K_2 = \frac{1}{2}mv^2$, $W_{\text{tot}} = W_f + W_g$. So $W_f = K_2 - W_g = \frac{1}{2}mv^2 - mgh$.

$W_f = \frac{1}{2}(5.00\text{ kg})(5.00\text{ m/s})^2 - (5.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = -35.5\text{ J}$.

(b) Now the ramp is lowered to 50.0° and the block is released from higher up but still 2.00 m above the bottom. Using $W = Fs\cos\phi$, we have $W_f = -f_k d = -\mu_k nd$. Using $\sum F_y = 0$ perpendicular to the surface of the ramp tells us that $n = mg\cos\theta$, and Fig. 6.61 shows that $d = \frac{h}{\sin\theta}$, so

$W_f = -\mu_k mg\cos\theta\left(\frac{h}{\sin\theta}\right) = -\frac{\mu_k mgh}{\tan\theta}$. From this result we see that as θ decreases, $\tan\theta$ decreases, so $1/\tan\theta$ increases. Therefore the magnitude of the work done by friction *increases* as θ decreases providing that h remains the same.

(c) Take the ratio of the works to find $W_{50°}$. $\frac{W_{50°}}{W_{60°}} = \frac{-\frac{\mu_k mgh}{\tan 50.0°}}{-\frac{\mu_k mgh}{\tan 60.0°}} = \frac{\tan 60.0°}{\tan 50.0°} = 1.45$, which gives $W_{50°} = (1.45)W_{60°} = (1.45)(-35.5\text{ J}) = -51.6\text{ J}$.

EVALUATE: As the slope angle θ decreases, the normal force increases so friction increases, and the distance d that the block slides also increases. So the magnitude of the work $W_f = f_k d$ increases, which agrees with our result.

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6.62. **IDENTIFY:** We want to calculate the work that friction does on a block as it slides down an incline.

SET UP: $W = Fs\cos\phi$

EXECUTE: In this case, F is the friction force f, s is the distance that the block slides along the surface of the incline, and ϕ is the angle between the friction force and the displacement of the block, which is 180°. $W_f = fs \cos 180° = -fs$. In terms of h, $s = h/\sin\alpha$, and $f = \mu_k n$. $\sum F_y = 0$ perpendicular to the surface of the incline tells us that $n = mg\cos\alpha$, so $f = \mu_k mg\cos\alpha$. Combining these relations gives $W_f = -\mu_k mg\cos\alpha\left(\dfrac{h}{\sin\alpha}\right) = -\dfrac{\mu_k mgh}{\tan\alpha}$.

EVALUATE: As α is decreased, $\tan\alpha$ decreases, so W_f increases in magnitude.

6.63. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each block to find the tension in the string. Each force is constant and $W = Fs\cos\phi$.

SET UP: The free-body diagram for each block is shown in Figure 6.63. $m_A = \dfrac{20.0\text{ N}}{g} = 2.04\text{ kg}$ and $m_B = \dfrac{12.0\text{ N}}{g} = 1.22\text{ kg}$.

EXECUTE: $T - f_k = m_A a$. $w_B - T = m_B a$. $w_B - f_k = (m_A + m_B)a$.

(a) $f_k = 0$. $a = \left(\dfrac{w_B}{m_A + m_B}\right)$ and $T = w_B\left(\dfrac{m_A}{m_A + m_B}\right) = w_B\left(\dfrac{w_A}{w_A + w_B}\right) = 7.50\text{ N}$.

20.0 N block: $W_{tot} = Ts = (7.50\text{ N})(0.750\text{ m}) = 5.62\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 7.50\text{ N})(0.750\text{ m}) = 3.38\text{ J}$.

(b) $f_k = \mu_k w_A = 6.50\text{ N}$. $a = \dfrac{w_B - \mu_k w_A}{m_A + m_B}$.

$$T = f_k + (w_B - \mu_k w_A)\left(\frac{m_A}{m_A + m_B}\right) = \mu_k w_A + (w_B - \mu_k w_A)\left(\frac{w_A}{w_A + w_B}\right).$$

$T = 6.50\text{ N} + (5.50\text{ N})(0.625) = 9.94\text{ N}$.

20.0 N block: $W_{tot} = (T - f_k)s = (9.94\text{ N} - 6.50\text{ N})(0.750\text{ m}) = 2.58\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 9.94\text{ N})(0.750\text{ m}) = 1.54\text{ J}$.

EVALUATE: Since the two blocks move with equal speeds, for each block $W_{tot} = K_2 - K_1$ is proportional to the mass (or weight) of that block. With friction the gain in kinetic energy is less, so the total work on each block is less.

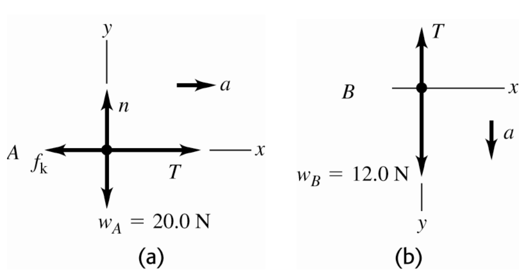

Figure 6.63

6.64. **IDENTIFY:** $W = Fs\cos\phi$ and $W_{tot} = K_2 - K_1$.

SET UP: $f_k = \mu_k n$. The normal force is $n = mg\cos\theta$, with $\theta = 24.0°$. The component of the weight parallel to the incline is $mg\sin\theta$.

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EXECUTE: **(a)** $\phi = 180°$ and

$W_f = -f_k s = -(\mu_k mg\cos\theta)s = -(0.31)(5.00\text{ kg})(9.80\text{ m/s}^2)(\cos 24.0°)(2.80\text{ m}) = -38.9\text{ J}.$

(b) $(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 24.0°)(2.80\text{ m}) = 55.8\text{ J}.$

(c) The normal force does no work.

(d) $W_{tot} = 55.8\text{ J} - 38.9\text{ J} = +16.9\text{ J}.$

(e) $K_2 = K_1 + W_{tot} = (1/2)(5.00\text{ kg})(2.20\text{ m/s})^2 + 16.9\text{ J} = 29.0\text{ J},$ and so

$v_2 = \sqrt{2(29.0\text{ J})/(5.00\text{ kg})} = 3.41\text{ m/s}.$

EVALUATE: Friction does negative work and gravity does positive work. The net work is positive and the kinetic energy of the object increases.

6.65. **IDENTIFY:** The initial kinetic energy of the head is absorbed by the neck bones during a sudden stop. Newton's second law applies to the passengers as well as to their heads.

SET UP: In part (a), the initial kinetic energy of the head is absorbed by the neck bones, so $\frac{1}{2}mv_{max}^2 = 8.0\text{ J}.$ For part (b), assume constant acceleration and use $v_f = v_i + at$ with $v_i = 0,$ to calculate a; then apply $F_{net} = ma$ to find the net accelerating force.

Solve: (a) $v_{max} = \sqrt{\dfrac{2(8.0\text{ J})}{5.0\text{ kg}}} = 1.8\text{ m/s} = 4.0\text{ mph}.$

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{1.8\text{ m/s} - 0}{10.0\times 10^{-3}\text{ s}} = 180\text{ m/s}^2 \approx 18g,$ and $F_{net} = ma = (5.0\text{ kg})(180\text{ m/s}^2) = 900\text{ N}.$

EVALUATE: The acceleration is very large, but if it lasts for only 10 ms it does not do much damage.

6.66. **IDENTIFY:** The force does work on the object, which changes its kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the object.

SET UP: $W_{tot} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ and $W_{tot} = \int_{x_1}^{x_2} F(x)dx.$

EXECUTE: $W_{tot} = \int_{x_1}^{x_2} F(x)dx = \int_0^{5.00\text{ m}} [-12.0\text{ N} + (0.300\text{ N/m}^2)x^2]dx.$

$W_{tot} = -(12.0\text{ N})(5.00\text{ m}) + (0.100\text{ N/m}^2)(5.00\text{ m})^3 = -60.0\text{ J} + 12.5\text{ J} = -47.5\text{ J}.$

$W_{tot} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -47.5\text{ J},$ so the final velocity is

$v_f = \sqrt{v_i^2 - \dfrac{2(47.5\text{ J})}{m}} = \sqrt{(6.00\text{ m/s})^2 - \dfrac{2(47.5\text{ J})}{5.00\text{ kg}}} = 4.12\text{ m/s}.$

EVALUATE: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of *x*, not of *t*. The work-energy theorem provides a much simpler method.

6.67. **IDENTIFY:** Calculate the work done by friction and apply $W_{tot} = K_2 - K_1.$ Since the friction force is not constant, use $W = \int F_x dx$ to calculate the work.

SET UP: Let *x* be the distance past *P*. Since μ_k increases linearly with *x*, $\mu_k = 0.100 + Ax.$ When $x = 12.5\text{ m},$ $\mu_k = 0.600,$ so $A = 0.500/(12.5\text{ m}) = 0.0400/\text{m}.$

EXECUTE: **(a)** $W_{tot} = \Delta K = K_2 - K_1$ gives $-\int \mu_k mg dx = 0 - \frac{1}{2}mv_1^2.$ Using the above expression for $\mu_k,$ $g\int_0^{x_2}(0.100 + Ax)dx = \frac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\frac{x_2^2}{2}\right] = \frac{1}{2}v_1^2.$

$(9.80\text{ m/s}^2)\left[(0.100)x_2 + (0.0400/\text{m})\frac{x_2^2}{2}\right] = \frac{1}{2}(4.50\text{ m/s})^2.$ Solving for x_2 gives $x_2 = 5.11\text{ m}.$

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(b) $\mu_k = 0.100 + (0.0400/\text{m})(5.11\text{ m}) = 0.304$

(c) $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgx_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \dfrac{v_1^2}{2\mu_k g} = \dfrac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3\text{ m}$.

EVALUATE: The box goes farther when the friction coefficient doesn't increase.

6.68. **IDENTIFY:** Use $W = \int F_x dx$ to calculate W.

SET UP: $x_1 = 0$. In part (a), $x_2 = 0.050$ m. In part (b), $x_2 = -0.050$ m.

EXECUTE: **(a)** $W = \int_0^{x_2} F_x dx = \int_0^{x_2} (kx - bx^2 + cx^3)\,dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$.

$W = (50.0\text{ N/m})\,x_2^2 - (233\text{ N/m}^2)\,x_2^3 + (3000\text{ N/m}^3)\,x_2^4$. When $x_2 = 0.050\text{ m}$, $W = 0.12\text{ J}$.

(b) When $x_2 = -0.050\text{ m}$, $W = 0.17\text{ J}$.

(c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

EVALUATE: When $x = 0.050$ m, $F_x = 4.75$ N. When $x = -0.050$ m, $F_x = -8.25$ N.

6.69. **IDENTIFY:** Use the work-energy theorem.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \frac{1}{2}mv^2$. When a force does work W_D on the box, its speed is V starting from rest, so $W_D = \frac{1}{2}mV^2$.

EXECUTE: **(a)** We want to find the speed v of the box when half the total work has been done on it. Using $W_{\text{tot}} = K_2 - K_1$, we have $\frac{1}{2}W_D = \frac{1}{2}mv^2$. But as we saw above, $W_D = \frac{1}{2}mV^2$. So $\frac{1}{2}\left(\frac{1}{2}mV^2\right) = \frac{1}{2}mv^2$ which gives $v = \frac{V}{\sqrt{2}}$. Since $v \approx 0.707V$, $v > V/2$.

(b) Now we want to find how much work has been done on the box to reach half of its maximum speed, which is $\frac{1}{2}V$. The work-energy theorem gives $W = \frac{1}{2}m\left(\frac{V}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}mV^2\right)$. From above, we know that $W_D = \frac{1}{2}mV^2$, so $W = \frac{1}{4}W_D$, which is *less than* $\frac{1}{2}W_D$.

EVALUATE: It takes $\frac{1}{4}$ of the total work to get the box to half its maximum velocity, and $\frac{3}{4}$ of the total work to get the box from $V/2$ to V.

6.70. **IDENTIFY:** We are dealing with Hooke's law and the work to compress a spring.

SET UP: Hooke's law: The force to stretch a spring a distance x is $F = kx$. The work to compress a spring a distance x from its equilibrium position is $W = \frac{1}{2}kx^2$.

EXECUTE: First find the force constant k. With you alone on the spring at maximum compression, $kx_1 = mg$, so $k = mg/x_1 = (530\text{ N})/(0.0180\text{ m}) = 2.94\times10^4$ N/m. The spring compression with you and the dog is given by $kx_2 = w_{\text{you}} + w_{\text{dog}}$, so $x_2 = (710\text{ N})/(2.94\times10^4\text{ N/m}) = 2.411\times10^{-2}\text{ m} = 0.02411$ m. The work to compress the spring from x_1 to x_2 is $W_{1\to2} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. Using $x_1 = 0.0180$ m, $x_2 = 0.02411$ m, and $k = 2.94\times10^4$ N/m, we get $W_{1\to2} = 3.79$ J.

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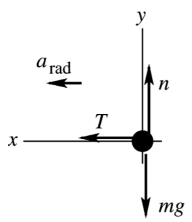
EVALUATE: The work to bring you to rest without your dog is $\frac{1}{2}kx_1^2 = \frac{1}{2}(2.944\times10^4\ \text{N/m})(0.0180\ \text{m})^2$ $= 4.77$ J. This is greater than the work to compress the spring with you and your dog because the amount of compression was less. You alone compressed the spring by 1.80 cm, but you with your dog compressed it only an additional 0.61 cm.

6.71. **IDENTIFY** and **SET UP:** Use $\Sigma\vec{F} = m\vec{a}$ to find the tension force T. The block moves in uniform circular motion and $\vec{a} = \vec{a}_{\text{rad}}$.

(a) The free-body diagram for the block is given in Figure 6.71.



EXECUTE: $\Sigma F_x = ma_x$

$$T = m\frac{v^2}{R}$$

$$T = (0.0600\ \text{kg})\frac{(0.70\ \text{m/s})^2}{0.40\ \text{m}} = 0.074\ \text{N}.$$

Figure 6.71

(b) $T = m\dfrac{v^2}{R} = (0.0600\ \text{kg})\dfrac{(2.80\ \text{m/s})^2}{0.10\ \text{m}} = 4.7\ \text{N}.$

(c) SET UP: The tension changes as the distance of the block from the hole changes. We could use $W = \int_{x_1}^{x_2} F_x\, dx$ to calculate the work. But a much simpler approach is to use $W_{\text{tot}} = K_2 - K_1$.

EXECUTE: The only force doing work on the block is the tension in the cord, so $W_{\text{tot}} = W_T$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0600\ \text{kg})(0.70\ \text{m/s})^2 = 0.01470\ \text{J}$, $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0600\ \text{kg})(2.80\ \text{m/s})^2 = 0.2352\ \text{J}$, so $W_{\text{tot}} = K_2 - K_1 = 0.2352\ \text{J} - 0.01470\ \text{J} = 0.22\ \text{J}$. This is the amount of work done by the person who pulled the cord.

EVALUATE: The block moves inward, in the direction of the tension, so T does positive work and the kinetic energy increases.

6.72. **IDENTIFY:** Use $W = \int F_x dx$ to find the work done by F. Then apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\int \dfrac{dx}{x^2} = -\dfrac{1}{x}$.

EXECUTE: $W = \int_{x_1}^{x_2} \dfrac{\alpha}{x^2}\, dx = \alpha\left(\dfrac{1}{x_1} - \dfrac{1}{x_2}\right).$

$W = (2.12\times10^{-26}\ \text{N}\cdot\text{m}^2)\left[(0.200\ \text{m}^{-1}) - (1.25\times10^9\ \text{m}^{-1})\right] = -2.65\times10^{-17}\ \text{J}.$

Note that x_1 is so large compared to x_2 that the term $1/x_1$ is negligible. Then, using the work-energy theorem and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times10^5\ \text{m/s})^2 + \frac{2(-2.65\times10^{-17}\ \text{J})}{(1.67\times10^{-27}\ \text{kg})}} = 2.41\times10^5\ \text{m/s}.$$

(b) With $K_2 = 0, W = -K_1$. Using $W = -\dfrac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times10^{-26}\ \text{N}\cdot\text{m}^2)}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^5\ \text{m/s})^2} = 2.82\times10^{-10}\ \text{m}.$$

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(c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is 3.00×10^5 m/s.

EVALUATE: As the proton moves toward the uranium nucleus the repulsive force does negative work and the kinetic energy of the proton decreases. As the proton moves away from the uranium nucleus the repulsive force does positive work and the kinetic energy of the proton increases.

6.73. **IDENTIFY:** The negative work done by the spring equals the change in kinetic energy of the car.

SET UP: The work done by a spring when it is compressed a distance x from equilibrium is $-\dfrac{1}{2}kx^2$. $K_2 = 0$.

EXECUTE: $-\frac{1}{2}kx^2 = K_2 - K_1$ gives $\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$ and

$k = (mv_1^2)/x^2 = [(1200\text{ kg})(0.65\text{ m/s})^2]/(0.090\text{ m})^2 = 6.3\times10^4$ N/m.

EVALUATE: When the spring is compressed, the spring force is directed opposite to the displacement of the object and the work done by the spring is negative.

6.74. **IDENTIFY** and **SET UP:** Use the work-energy theorem $W_{\text{tot}} = K_2 - K_1$. You do positive work and gravity does negative work. Let point 1 be at the base of the bridge and point 2 be at the top of the bridge.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 1000\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(80.0\text{ kg})(1.50\text{ m/s})^2 = 90\text{ J}$

$W_{\text{tot}} = 90\text{ J} - 1000\text{ J} = -910\text{ J}$

(b) Neglecting friction, work is done by you (with the force you apply to the pedals) and by gravity: $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$. The gravity force is $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$, downward. The displacement is 5.20 m, upward. Thus $\phi = 180°$ and

$$W_{\text{gravity}} = (F\cos\phi)s = (784\text{ N})(5.20\text{ m})\cos180° = -4077\text{ J}$$

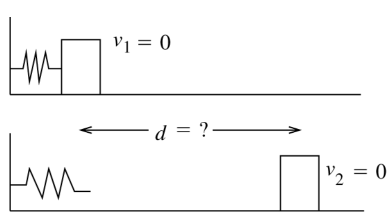
Then $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$ gives

$$W_{\text{you}} = W_{\text{tot}} - W_{\text{gravity}} = -910\text{ J} - (-4077\text{ J}) = +3170\text{ J}$$

EVALUATE: The total work done is negative and you lose kinetic energy.

6.75. **IDENTIFY** and **SET UP:** Use the work-energy theorem $W_{\text{tot}} = K_2 - K_1$. Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.75.

EXECUTE:



$W_{\text{tot}} = K_2 - K_1$

$K_1 = 0,\ \ K_2 = 0$

$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$

Figure 6.75

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250$ m (the spring force is in direction of motion of block so it does positive work).

$W_{\text{fric}} = -\mu_k mgd$

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Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$d = \dfrac{kx_1^2}{2\mu_k mg} = \dfrac{(250\text{ N/m})\,(0.250\text{ m})^2}{2(0.30)\,(2.50\text{ kg})\,(9.80\text{ m/s}^2)} = 1.1\text{ m},$ measured from the point where the block was released.

EVALUATE: The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero, and the textbook starts and ends with zero kinetic energy.

6.76. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: Let x_0 be the initial distance the spring is compressed. The work done by the spring is $\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$, where x is the final distance the spring is compressed.

EXECUTE: **(a)** Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}}(0.060\text{ m}) = 6.93\text{ m/s}.$$

(b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where f is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2f}{m}x_0} = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}(0.060\text{ m})^2 - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m})} = 4.90\text{ m/s}.$$

(c) The greatest speed occurs when the acceleration (and the net force) are zero. Let x be the amount the spring is still compressed, so the distance the ball has moved is $x_0 - x$. $kx = f$, $x = \dfrac{f}{k} = \dfrac{6.00\text{ N}}{400\text{ N/m}} =$ 0.0150 m.

The ball is 0.0150 m from the end of the barrel, or 0.0450 m from its initial position.

To find the speed, the net work is $W_{\text{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is

$$v_{\max} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}.$$

$$v_{\max} = \sqrt{\frac{400\text{ N/m}}{(0.0300\text{ kg})}\left[(0.060\text{ m})^2 - (0.0150\text{ m})^2\right] - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m} - 0.0150\text{ m})} = 5.20\text{ m/s}$$

EVALUATE: The maximum speed with friction present (part (c)) is larger than the result of part (b) but smaller than the result of part (a).

6.77. **IDENTIFY:** A constant horizontal force pushes a block against a spring on a rough floor. The work-energy theorem and Newton's second law both apply.

SET UP: In part (a), we apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the block. $f_k = \mu_k n$ and $W_{\text{spring}} = -\frac{1}{2}kx^2$. In part (b), we apply Newton's second law to the block.

EXECUTE: **(a)** $W_F + W_{\text{spring}} + W_f = K_2 - K_1$. $Fx - \frac{1}{2}kx^2 - \mu_k mgx = \frac{1}{2}mv^2 - 0$. Putting in the numbers from the problem gives (82.0 N)(0.800 m) – (130.0 N/m)(0.800 m)^2/2 – (0.400)(4.00 kg)(9.80 m/s^2)(0.800 m) = (4.00 kg)v^2/2, v = 2.39 m/s.

(b) Looking at quantities parallel to the floor, with the positive direction toward the wall, Newton's second law gives $F - f_k - F_{\text{spring}} = ma$.

$F - \mu_k mg - kx = ma$: 82.0 N – (0.400)(4.00 kg)(9.80 m/s^2) – (130.0 N/m)(0.800 m) = (4.00 kg)a

a = –9.42 m/s^2. The minus sign means that the acceleration is away from the wall.

EVALUATE: The force you apply is toward the wall but the block is accelerating away from the wall.

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6.78. **IDENTIFY:** A constant horizontal force pushes a frictionless block of ice against a spring on the floor. The work-energy theorem and Newton's second law both apply.

SET UP: In part (a), we apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the ice. $W_{\text{spring}} = -½\, kx^2$. In part (b), we apply Newton's second law to the ice.

EXECUTE: **(a)** $W_F + W_{\text{spring}} = K_2 - K_1$. $Fx - ½\, kx^2 = ½\, mv^2 - 0$. Putting in the numbers from the problem gives (54.0 N)(0.400 m) – (76.0 N/m)(0.400 m)^2/2 = (2.00 kg)v^2/2, v = 3.94 m/s.

(b) Looking at quantities parallel to the floor, with the positive direction away from the post, Newton's second law gives $F - F_{\text{spring}} = ma$, so $F - kx = ma$.

54.0 N – (76.0 N/m)(0.400 m) = (2.00 kg)a, which gives a = 11.8 m/s^2. The acceleration is positive, so the block is accelerating away from the post.

EVALUATE: The given force must be greater than the spring force since the ice is accelerating away from the post.

6.79. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the blocks.

SET UP: If X is the distance the spring is compressed, the work done by the spring is $-\frac{1}{2}kX^2$. At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy.

EXECUTE: **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv_0^2$ and $X = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$

(b) Solving for v_0 in terms of a known X, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\text{ N/m}}{5.00\text{ kg}}}(0.150\text{ m}) = 1.50\text{ m/s}.$

EVALUATE: The negative work done by the spring removes the kinetic energy of the block.

6.80. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$. $W = Fs\cos\phi$.

SET UP: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion.

EXECUTE: The total work done is W_{tot} =
$\left[(600\text{ N})\cos 30.0° + (85.0\text{ kg})(9.80\text{ m/s}^2)\cos 120.0°\right](2.50\text{ m}) = 257.8\text{ J},$ and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{\text{tot}}}{m}} = \sqrt{(2.00\text{ m/s})^2 + \frac{2(257.8\text{ J})}{(85.0\text{ kg})}} = 3.17\text{ m/s}.$

EVALUATE: The component of gravity down the incline is $mg\sin 30° = 417\text{ N}$ and the component of the push up the incline is $(600\text{ N})\cos 30° = 520\text{ N}$. The force component up the incline is greater than the force component down the incline; the net work done is positive and the speed increases.

6.81. **IDENTIFY** and **SET UP:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system consisting of both blocks. Since they are connected by the cord, both blocks have the same speed at every point in the motion. Also, when the 6.00-kg block has moved downward 1.50 m, the 8.00-kg block has moved 1.50 m to the right. The target variable, μ_k, will be a factor in the work done by friction. The forces on each block are shown in Figure 6.81.

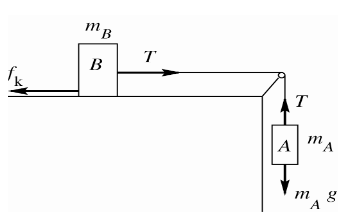

Figure 6.81

EXECUTE: $K_1 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_1^2 = \frac{1}{2}(m_A + m_B)v_1^2$

$K_2 = 0$

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The tension T in the rope does positive work on block B and the same magnitude of negative work on block A, so T does no net work on the system. Gravity does work $W_{mg} = m_A gd$ on block A, where $d = 2.00$ m. (Block B moves horizontally, so no work is done on it by gravity.) Friction does work $W_{\text{fric}} = -\mu_k m_B gd$ on block B. Thus $W_{\text{tot}} = W_{mg} + W_{\text{fric}} = m_A gd - \mu_k m_B gd$. Then $W_{\text{tot}} = K_2 - K_1$ gives $m_A gd - \mu_k m_B gd = -\frac{1}{2}(m_A + m_B)v_1^2$ and

$$\mu_k = \frac{m_A}{m_B} + \frac{\frac{1}{2}(m_A + m_B)v_1^2}{m_B gd} = \frac{6.00\text{ kg}}{8.00\text{ kg}} + \frac{(6.00\text{ kg} + 8.00\text{ kg})(0.900\text{ m/s})^2}{2(8.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})} = 0.786$$

EVALUATE: The weight of block A does positive work and the friction force on block B does negative work, so the net work is positive and the kinetic energy of the blocks increases as block A descends. Note that K_1 includes the kinetic energy of both blocks. We could have applied the work-energy theorem to block A alone, but then W_{tot} includes the work done on block A by the tension force.

6.82. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).
SET UP: Let h be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)gh and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))(9.80\text{ m/s}^2)(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\frac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

6.83. **IDENTIFY:** Apply the work-energy theorem $W_{\text{tot}} = K_2 - K_1$ to the skater.

SET UP: Let point 1 be just before she reaches the rough patch and let point 2 be where she exits from the patch. Work is done by friction. We don't know the skater's mass so can't calculate either friction or the initial kinetic energy. Leave her mass m as a variable and expect that it will divide out of the final equation.
EXECUTE: $f_k = 0.25mg$ so $W_f = W_{\text{tot}} = -(0.25mg)s$, where s is the length of the rough patch.
$W_{\text{tot}} = K_2 - K_1$
$K_1 = \frac{1}{2}mv_0^2$, $K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.55v_0)^2 = 0.3025\left(\frac{1}{2}mv_0^2\right)$
The work-energy relation gives $-(0.25mg)s = (0.3025 - 1)\frac{1}{2}mv_0^2$.
The mass divides out, and solving gives $s = 1.3$ m.
EVALUATE: Friction does negative work and this reduces her kinetic energy.

6.84. **IDENTIFY** and **SET UP:** $W = Pt$
EXECUTE: **(a)** The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.
(b) The steady output of the athlete is $(500\text{ W})/(70\text{ kg}) = 7\text{ W/kg}$, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend $1400\text{ W}/70\text{ kg} = 20\text{ W/kg}$ for short periods of time, no human-powered aircraft could stay aloft for very long.
EVALUATE: Movies of early attempts at human-powered flight bear out our results.

6.85. **IDENTIFY:** To lift a mass m a height h requires work $W = mgh$. To accelerate mass m from rest to speed v requires $W = K_2 - K_1 = \frac{1}{2}mv^2$. $P_{\text{av}} = \frac{\Delta W}{\Delta t}$.

SET UP: $t = 60$ s

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EXECUTE: **(a)** $(800\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ m}) = 1.10\times10^5\text{ J}$.

(b) $(1/2)(800\text{ kg})(18.0\text{ m/s}^2) = 1.30\times10^5\text{ J}$.

(c) $\dfrac{1.10\times10^5\text{ J} + 1.30\times10^5\text{ J}}{60\text{ s}} = 3.99\text{ kW}$.

EVALUATE: Approximately the same amount of work is required to lift the water against gravity as to accelerate it to its final speed.

6.86. **IDENTIFY:** This problem requires the work-energy theorem. Gravity and the force $\vec{F}$ do work on the steel ball, the tension in the rope does none since it is perpendicular to the displacement.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \dfrac{1}{2}mv^2$, $W = Fs\cos\phi$. The work done by gravity is $-mgL(1-\cos\alpha)$ and $K_1 = 0$. The work done by $\vec{F}$ is Fs, where s is the arc length subtended by α. So $s = L\alpha$, but α must be in radians. Converting gives $\alpha = 37.0° = 0.6458$ rad.

EXECUTE: Applying $W_{\text{tot}} = K_2 - K_1$ we have $W_F + W_g = \dfrac{1}{2}mv^2$, which gives

$FL\alpha(\text{rad}) - mgL(1-\cos\alpha) = \dfrac{1}{2}mv^2$. Putting in the numbers gives us

$(0.760\text{ N})(0.600\text{ m})(0.6458\text{ rad}) - (0.200\text{ kg})(9.80\text{ m/s}^2)(0.600\text{ m})(1 - \cos 37.0°) = \dfrac{1}{2}(0.200\text{ kg})v^2$

which gives $v = 0.759$ m/s.

EVALUATE: Note that we used $\alpha = 0.6458$ rad for the arc length but $\alpha = 37.0°$ for $\cos\alpha$. We could have used $\alpha = 0.6458$ rad in $\cos\alpha$, but we would have had to put our calculator in the radian mode for angles, and most people would forget to do this!

6.87. **IDENTIFY:** This problem requires the work-energy theorem. Gravity does work on the system, the tension in the rope does work on both of the blocks, and friction does work on the 8.00-kg block.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $K = \dfrac{1}{2}mv^2$, $W = Fs\cos\phi$, and $K_1 = 0$. The blocks move together and therefore have the same speed.

EXECUTE: **(a)** $W_g = mgs = (6.00\text{ kg})(9.80\text{ m/s}^2)(0.800\text{ m}) = 47.0$ J.

$W_T = -Ts = -(37.0\text{ N})(0.800\text{ m}) = -29.6$ J.

$W_{\text{tot}} = K_2 - K_1$, where $W_{\text{tot}} = 47.0\text{ J} - 29.6\text{ J} = 17.4$ J. So $17.4\text{ J} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(6.00\text{ kg})v^2$, which gives $v = 2.41$ m/s.

(b) $W_{\text{tot}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(8.00\text{ kg})(2.41\text{ m/s})^2 = 23.2$ J.

$W_T = Ts = (37.0\text{ N})(0.800\text{ m}) = 29.6$ J.

$W_{\text{tot}} = W_T + W_f \rightarrow \quad 23.2\text{ J} = 29.6\text{ J} + W_f \quad \rightarrow \quad W_f = -6.4$ J.

(c) $W_{\text{tot}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(14.0\text{ kg})(2.41\text{ m/s})^2 = 40.6$ J.

$W_g = 47.0$ J (from part (a)).

$W_f = -6.4$ J (from part (b)).

$W_{\text{tot}} = 0$ since the tension is internal. Another way to see this is that the tension does positive work on the 8.00-kg block and an equal amount of negative work on the 6.00-kg block, so its total work is zero.

EVALUATE: In part (c), the total work on the system is 47.0 J – 6.4 J = 40.6 J, which agrees from our result using the work-energy theorem.

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6.88. **IDENTIFY:** $W = \int_{x_1}^{x_2} F_x dx,$ and F_x depends on both x and y.

SET UP: In each case, use the value of y that applies to the specified path. $\int x dx = \frac{1}{2}x^2$. $\int x^2 dx = \frac{1}{3}x^3$.

EXECUTE: **(a)** Along this path, y is constant, with the value $y = 3.00$ m.

$$W = \alpha y \int_{x_1}^{x_2} x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\frac{(2.00\ \text{m})^2}{2} = 15.0\ \text{J},\ \text{since}\ x_1 = 0\ \text{and}\ x_2 = 2.00\ \text{m}.$$

(b) Since the force has no y-component, no work is done moving in the y-direction.

(c) Along this path, y varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W = \int_{x_1}^{x_2} F dx = 1.5\alpha \int_{x_1}^{x_2} x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(2.00\ \text{m})^3}{3} = 10.0\ \text{J}.$$

EVALUATE: The force depends on the position of the object along its path.

6.89. **IDENTIFY** and **SET UP:** For part (a) calculate m from the volume of blood pumped by the heart in one day. For part (b) use W calculated in part (a) in $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$.

EXECUTE: (a) The work to life the blood is $W = mgh$. We need the mass of blood lifted; we are given the volume $V = (7500\ \text{L})\left(\dfrac{1\times10^{-3}\ \text{m}^3}{1\ \text{L}}\right) = 7.50\ \text{m}^3$.

$$m = \text{density} \times \text{volume} = (1.05\times10^3\ \text{kg/m}^3)(7.50\ \text{m}^3) = 7.875\times10^3\ \text{kg}$$

Then $W = mgh = (7.875\times10^3\ \text{kg})(9.80\ \text{m/s}^2)(1.63\ \text{m}) = 1.26\times10^5\ \text{J}$.

(b) $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{1.26\times10^5\ \text{J}}{(24\ \text{h})(3600\ \text{s/h})} = 1.46\ \text{W}.$

EVALUATE: Compared to light bulbs or common electrical devices, the power output of the heart is rather small.

6.90. **IDENTIFY:** We know information about the force exerted by a stretched rubber band and want to know if it obeys Hooke's law.

SET UP: Hooke's law is $F = kx$. The graph fits the equation $F = 33.55x^{0.4871}$, with F in newtons and x in meters.

EXECUTE: **(a)** For Hooke's law, a graph of F versus x is a straight line through the origin. This graph is not a straight line, so the rubber band does not obey Hooke's law.

(b) $k_{\text{eff}} = \dfrac{dF}{dx} = \dfrac{d}{dx}(33.55x^{0.4871}) = 16.34x^{-0.5129}$. Because of the negative exponent for x, as x increases, k_{eff} decreases.

(c) The definition of work gives $W = \int_a^b F_x dx = \int_0^{0.0400\ \text{m}} 0.3355x^{0.4871}dx = (33.55/1.4871)\ 0.0400^{1.4871}$

$W = 0.188$ J. From 0.0400 m to 0.0800 m, we follow the same procedure but with different limits of integration. The result is $W = (33.55/1.4871)\ (0.0800^{1.4871} - 0.0400^{1.4871}) = 0.339$ J.

(d) $W = K_2 - K_1 = \frac{1}{2} mv^2 - 0$, which gives 0.339 J = (0.300 kg)v^2/2, $v = 1.50$ m/s.

EVALUATE: The rubber band does not obey Hooke's law, but it does obey the work-energy theorem.

6.91. **IDENTIFY:** We know a spring obeys Hooke's law, and we want to use observations of the motion of a block attached to this spring to determine its force constant and the coefficient of friction between the block and the surface on which it is sliding. The work-energy theorem applies.

SET UP: $W_{\text{tot}} = K_2 - K_1$, $W_{\text{spring}} = \frac{1}{2} kx^2$.

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EXECUTE: (a) The spring force is initially greater than friction, so the block accelerates forward. But eventually the spring force decreases enough so that it is less than the force of friction, and the block then slows down (decelerates).

(b) The spring is initially compressed a distance x_0, and after the block has moved a distance d, the spring is compressed a distance $x = x_0 - d$. Therefore the work done by the spring is

$W_{\text{spring}} = \frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - d)^2$. The work done by friction is $W_f = -\mu_k mgd$.

The work-energy theorem gives $W_{\text{spring}} + W_f = K_2 - K_1 = ½\, mv^2$. Using our previous results, we get $\frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - d)^2 - \mu_k mgd = \frac{1}{2}mv^2$. Solving for v^2 gives $v^2 = -\frac{k}{m}d^2 + 2d\left(\frac{k}{m}x_0 - \mu_k g\right)$, where $x_0 =$ 0.400 m.

(c) Figure 6.91 shows the resulting graph of v^2 versus d. Using a graphing program and a quadratic fit gives $v^2 = -39.96d^2 + 16.31d$. The maximum speed occurs when $dv^2/dd = 0$, which gives $(-39.96)(2d) + 16.31 = 0$, so $d = 0.204$ m. For this value of d, we have $v^2 = (-39.96)(0.204\text{ m})^2 + (16.31)(0.204\text{ m})$, giving $v = 1.29$ m/s.

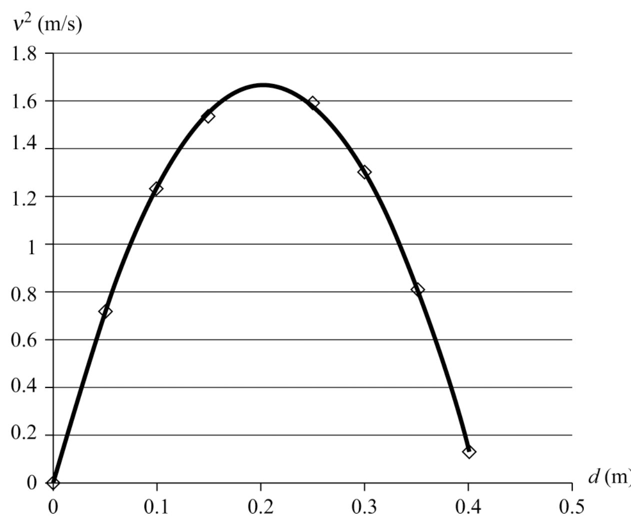

Figure 6.91

(d) From our work in (b) and (c), we know that $-k/m$ is the coefficient of d^2, so $-k/m = -39.96$, which gives $k = (39.96)(0.300\text{ kg}) = 12.0$ N/m. We also know that $2(kx_0/m - \mu_k g)$ is the coefficient of d. Solving for μ_k and putting in the numbers gives $\mu_k = 0.800$.

EVALUATE: The graphing program makes analysis of complicated behavior relatively easy.

6.92. IDENTIFY: The power output of the runners is the work they do in running from the basement to the top floor divided by the time it takes to make this run.

SET UP: $P = W/t$ and $W = mgh$.

EXECUTE: (a) For each runner, $P = mgh/t$. We must read the time of each runner from the figure shown with the problem. For example, for Tatiana we have $P = (50.2\text{ kg})(9.80\text{ m/s}^2)(16.0\text{ m})/32\text{ s} = 246.0$ W, which we must round to 2 significant figures because we cannot read the times any more accurate than that using the figure in the text. Carrying out these calculations for all the runners, we get the following results.

Tatiana: 250 W, Bill: 210 W, Ricardo: 290 W, Melanie: 170 W. Ricardo had the greatest power output, and Melanie had the least.

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(b) Solving $P = mgh/t$ for t gives $t = mgh/P$ = (62.3 kg)(9.80 m/s^2)(16.0 m)/(746 W) = 13.1 s, where we have used the fact that 1 hp = 746 W.

EVALUATE: Even though Tatiana had the shortest time, her power output was less than Ricardo's because she weighs less than he does.

6.93. **IDENTIFY:** In part (a) follow the steps outlined in the problem. For parts (b), (c), and (d) apply the work-energy theorem.

SET UP: $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v(l/L)$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time). The mass of a piece of length dl is $dm = (M/L)dl$, and so $dK = \dfrac{1}{2}(dm)u^2 = \dfrac{1}{2}\dfrac{Mv^2}{L^3}l^2 dl$, and

$$K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}.$$

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)x} = \sqrt{(3200\text{ N/m})/(0.053\text{ kg})}\,(2.50\times10^{-2}\text{ m}) = 6.1\text{ m/s}.$

(c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for v,

$$v = \sqrt{\frac{k}{m + M/3}}\,x = \sqrt{\frac{(3200\text{ N/m})}{(0.053\text{ kg}) + (0.243\text{ kg})/3}}(2.50\times10^{-2}\text{m}) = 3.9\text{ m/s}.$$

(d) Algebraically, $\dfrac{1}{2}mv^2 = \dfrac{(1/2)kx^2}{(1 + M/3m)} = 0.40\text{ J}$ and $\dfrac{1}{6}Mv^2 = \dfrac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\text{ J}.$

EVALUATE: For this ball and spring, $\dfrac{K_{\text{ball}}}{K_{\text{spring}}} = \dfrac{3m}{M} = 3\left(\dfrac{0.053\text{ kg}}{0.243\text{ kg}}\right) = 0.65.$ The percentage of the final kinetic energy that ends up with each object depends on the ratio of the masses of the two objects. As expected, when the mass of the spring is a small fraction of the mass of the ball, the fraction of the kinetic energy that ends up in the spring is small.

6.94. **IDENTIFY:** In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. Write W_0 in terms of the range R and speed v and in terms of the time of flight T and v.

SET UP: In both cases assume v is constant, so $W_0 = RF$ and $R = vT$.

EXECUTE: In terms of the range R and the constant speed v, $W_0 = RF = R\left(\alpha v^2 + \dfrac{\beta}{v^2}\right).$

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \dfrac{\beta}{v}\right).$

(a) Rather than solve for R as a function of v, differentiate the first of these relations with respect to v, setting $\dfrac{dW_0}{dv} = 0$ to obtain $\dfrac{dR}{dv}F + R\dfrac{dF}{dv} = 0.$ For the maximum range, $\dfrac{dR}{dv} = 0,$ so $\dfrac{dF}{dv} = 0.$ Performing the differentiation, $\dfrac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0,$ which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\text{ N}\cdot\text{m}^2/\text{s}^2}{0.30\text{ N}\cdot\text{s}^2/\text{m}^2}\right)^{1/4} = 32.9\text{ m/s} = 118\text{ km/h}.$$

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(b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0;$ performing the differentiation,

$$3\alpha v^2 - \beta/v^2 = 0. \quad v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5 \text{ N}\cdot\text{m}^2/\text{s}^2}{3(0.30 \text{ N}\cdot\text{s}^2/\text{m}^2)}\right)^{1/4} = 25 \text{ m/s} = 90 \text{ km/h}.$$

EVALUATE: When $v = (\beta/\alpha)^{1/4}$, F_{air} has its minimum value $F_{\text{air}} = 2\sqrt{\alpha\beta}$. For this v, $R_1 = (0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1 = (0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v = (\beta/3\alpha)^{1/4}$, $F_{\text{air}} = 2.3\sqrt{\alpha\beta}$. For this v, $R_2 = (0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2 = (0.57)\alpha^{-1/4}\beta^{-3/4}$. $R_1 > R_2$ and $T_2 > T_1$, as they should be.

6.95. **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.
SET UP: $P = W/t$, so $W = Pt$.
EXECUTE: When biking, the person travels 3 times as fast as when walking, so the bike trip takes 1/3 the time. Since $W = Pt$ and the power is the same, the energy when biking will be 1/3 of the energy when walking, which makes choice (a) the correct one.
EVALUATE: Walking is obviously a better way to burn calories than biking.

6.96. **IDENTIFY:** When walking on a grade, metabolic power is required for walking horizontally as well as the vertical climb.
SET UP: $P = W/t$, $W = mgh$.
EXECUTE: $P_{\text{tot}} = P_{\text{horiz}} + P_{\text{vert}} = P_{\text{horiz}} + mgh/t = P_{\text{horiz}} + mg(v_{\text{vert}})$. The slope is a 5% grade, so $v_{\text{vert}} = 0.05v_{\text{horiz}}$. Therefore $P_{\text{tot}} = 300 \text{ W} + (70 \text{ kg})(9.80 \text{ m/s}^2)(0.05)(1.4 \text{ m/s}) = 348 \text{ W} \approx 350 \text{ W}$, which makes choice (c) correct.
EVALUATE: Even a small grade of only 5% makes a difference of about 17% in power output.

6.97. **IDENTIFY:** Using 300 W of metabolic power, the person travels 3 times as fast when biking than when walking.
SET UP: $K = \frac{1}{2} mv^2$.
EXECUTE: The speed when biking is 3 times the speed when walking. Since the kinetic energy is proportional to the square of the speed, the kinetic energy will be $3^2 = 9$ times as great when biking, making choice (d) correct.
EVALUATE: Even a small increase in speed gives a considerable increase in kinetic energy due to the factor of v^2 in the kinetic energy.

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