8

MOMENTUM, IMPULSE, AND COLLISIONS

VP8.6.1. **IDENTIFY:** This problem involves a one-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$. Call the x-axis positive horizontally to the right.

EXECUTE: **(a)** After the gliders move free of the spring, their total momentum is conserved. Before the gliders were released, they were at rest so their total momentum was zero. This means that their total momentum will be zero later also. So $P_{1x} = P_{2x}$.

$m_A v_{Ax} + m_B v_{Bx} = 0$

$(0.125\text{ kg})(0.600\text{ m/s}) + (0.375\text{ kg})v_{Bx} \quad \rightarrow \quad v_{Bx} = -0.200\text{ m/s}$, to the left.

(b) Figure VP8.6.1 shows before and after sketches of the collision. Use momentum conservation.

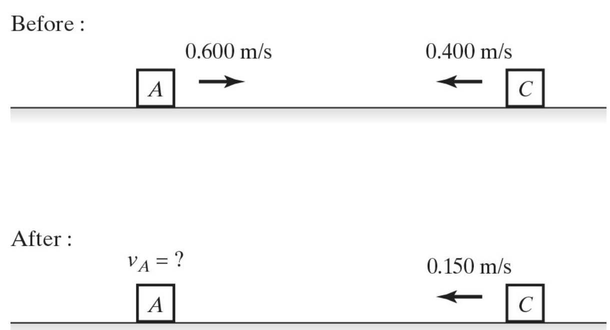

Figure VP8.6.1

$$m_A v_{A1x} + m_C v_{C1x} = m_A v_{A2x} + m_C v_{C2x}$$

$(0.125\text{ kg})(0.600\text{ m/s}) + (0.750\text{ kg})(-0.400\text{ m/s}) = (0.125\text{ kg})\, v_{A2x} + (0.750\text{ kg})(-0.150\text{ m/s})$

$v_{A2x} = -0.900\text{ m/s}$, to the left.

EVALUATE: Since momentum is a vector, we need to pay close attention to the *signs* of its components.

VP8.6.2. **IDENTIFY:** This problem involves a one-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$. Call the x-axis positive horizontally to the right. Let B stand for Buffy and M for Madeleine. Fig. VP8.6.2 shows before and after sketches.

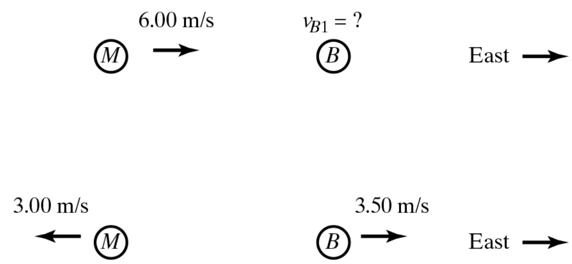

Figure VP8.6.2

EXECUTE: **(a)** $P_{1x} = P_{2x}$ $\rightarrow$ $m_M v_{M1x} + m_B v_{B1x} = m_M v_{M2x} + m_B v_{B2x}$

(65.0 kg)(6.00 m/s) + (55.0 kg) v_{B1x} = (65.0 kg)(–3.00 m/s) + (55.0 kg)(3.50 m/s)

v_{B1x} = –7.14 m/s. The minus sign means it is to the west.

(b) $\Delta v_{Mx} = v_{M2x} - v_{M1x}$ = –3.00 m/s – 6.00 m/s = –9.00 m/s, to the west.

$\Delta v_{Bx} = v_{B2x} - v_{B1x}$ = 3.50 m/s – (–7.14 m/s) = 10.6 m/s, to the east. Buffy has the greater magnitude velocity change.

EVALUATE: It is reasonable that the magnitude of light-weight Buffy's velocity change is greater than that of the heavier Madeleine. Both experience the same magnitude change in *momentum*, but the smaller-mass Buffy needs a larger velocity change than Madeleine so their momentum changes can be equal in magnitude.

VP8.6.3. **IDENTIFY:** This problem involves a two-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and likewise for y-axis. Call A the 2.40-kg stone and B the 4.00-kg stone. Fig. VP8.6.3 shows before and after sketches.

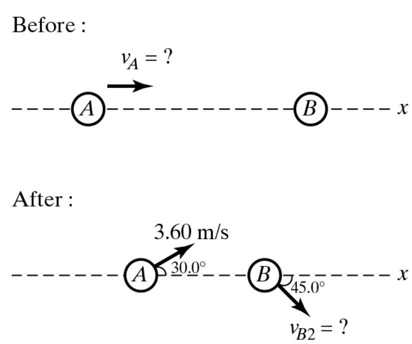

Figure VP8.6.3

EXECUTE: **(a)** $m_A v_{A1y}$ = (2.40 kg)(3.60 m/s) sin 30.0° = 4.32 kg · m/s . The initial y-component of the momentum is zero, so the final y-component must be zero. So p_{B2y} = –4.32 kg · m/s .

(b) $p_{B2y} = m_B v_{B2y} \sin 45.0°$ $\rightarrow$ –4.32 kg · m/s = (4.00 kg) v_{B2y} sin 45.0°

v_{B2y} = –1.53 m/s, so its speed is 1.53 m/s.

(c) $P_x = p_{Ax} + p_{Bx}$

P_x = (2.40 kg)(3.60 m/s) cos 30.0° + (4.00 kg)(1.53 m/s) cos 45.0° = 11.8 kg · m/s .

(d) Initially A has all the x-momentum. Since the x-component of the momentum is conserved (2.40 kg)v_{Ax}= P_x = 11.8 kg · m/s $\rightarrow$ v_{Ax} = 4.92 m/s.

EVALUATE: In a two-dimensional collision, the x-components of the momentum must *always* be treated separately from the y-components.

VP8.6.4. **IDENTIFY:** This problem involves a two-dimensional collision, so we use momentum conservation. The total momentum before the collision must equal the total momentum after the collision.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and likewise for y-axis. Call P the hockey puck and S the stone. Figure VP8.6.4 shows before and after sketches.

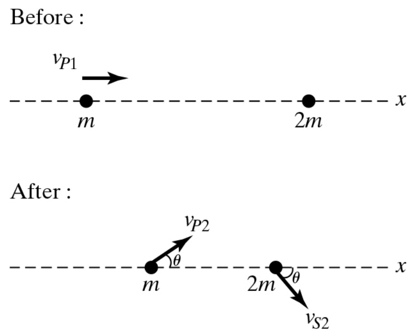

Figure VP8.6.4

EXECUTE: **(a)** The puck and stone must have equal-magnitude y-components of their momentum. $mv_{P2}\sin\theta = (2m)v_{S2} \quad \sin\theta \quad \rightarrow \quad v_{S2}/v_{P2} = ½$.

(b) The x-components of the momentum give $mv_{P1} = mv_{P2}\cos\theta + (2m)v_{S2}\cos\theta$. From part (a) we have $v_{P2} = 2v_{S2}$. Combining these two equations gives $v_{S2} = \dfrac{v_{P1}}{4\cos\theta}$ and $v_{P2} = \dfrac{v_{P1}}{2\cos\theta}$.

EVALUATE: In a two-dimensional collision, the x-components of the momentum must *always* be treated separately from the y-components.

VP8.9.1. **IDENTIFY:** This problem involves a one-dimensional collision in which the colliding objects stick together. This makes it a completely *inelastic* collision. Momentum is conserved.

SET UP: $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$. $K = \dfrac{1}{2}mv^2$.

EXECUTE: **(a)** $K_1 = \dfrac{1}{2}mv_1^2 = \dfrac{1}{2}(1.00\text{ kg})\ v_1^2 = 32.0\text{ J} \quad \rightarrow \quad v_1 = 8.00\text{ m/s}$.

Momentum conservation gives $m_1v_1 = (m_1 + m_2)v_2 \quad \rightarrow \quad (1.00\text{ kg})(8.00\text{ m/s}) = (5.00\text{ kg})v_2$, so $v_2 = 1.60$ m/s.

The lost kinetic energy is

$K_1 - K_2 = K_1 - \dfrac{1}{2}(m_1 + m_2)\, v_2^2 = 32.0\text{ J} - \dfrac{1}{2}(5.00\text{ kg})(1.60\text{ m/s})^2 = 25.6\text{ J}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(4.00\text{ kg})v^2 = 32\text{ J} \quad \rightarrow \quad v = 4.00\text{ m/s}$.

Momentum conservation gives $m_1v_1 = (m_1 + m_2)v_2 \quad \rightarrow \quad (4.00\text{ kg})(4.00\text{ m/s}) = (5.00\text{ kg})v_2$

$v_2 = 3.20$ m/s. The loss of kinetic energy is $K_1 - K_2 = K_1 - \dfrac{1}{2}(m_1 + m_2)v^2$

$K_1 - K_2 = 32.0\text{ J} - \dfrac{1}{2}(5.00\text{ kg})(3.20\text{ m/s})^2 = 6.4\text{ J}$.

(c) More kinetic energy is lost if a light object collides with a stationary heavy object, which is case (i).

EVALUATE: Careful! Even though momentum is always conserved during a collision, kinetic energy may or may not be conserved. The amount of energy lost depends on the nature of the collision and the relative masses of the colliding objects.

VP8.9.2. **IDENTIFY:** During the collision, momentum is conserved. After the collision, energy is conserved. We must break this problem up into two parts.

SET UP: During the collision, we use $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and momentum conservation. After the collision we use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where $W_{\text{tot}} = 0$. Take the x-axis to be horizontal in the direction in which the cheese is originally moving. Figure VP8.9.2 shows the given information.

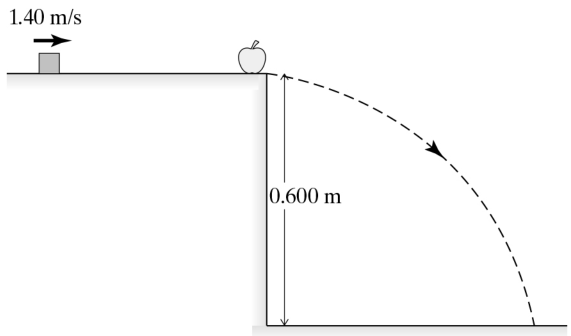

Figure VP8.9.2

EXECUTE: (a) During the collision: $p_{\text{cheese}} = p_{\text{apple + cheese}}$, so $m_c v_c = (m_c + m_a)v$
(0.500 kg)(1.40 m/s) = (0.700 kg)v $\rightarrow$ v = 1.00 m/s. Only momentum is conserved because no net external forces to the apple-cheese system due to the collision. The kinetic energy is *not* conserved because the objects stick together.

(b) Energy conservation after the collision gives $K_1 + U_1 = K_2 + U_2$. Call $y = 0$ at the floor level.
$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv_2^2$, which gives $v_2 = \sqrt{v_1^2 + 2gh} = \sqrt{(1.00 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(0.600 \text{ m})} = 3.57$ m/s. Only the total mechanical energy is conserved during the fall. Gravity is an external force acting on the falling object, so its momentum is *not* conserved.

EVALUATE: We cannot do this type of problem in a single step. Some of the initial mechanical energy of the cheese is lost during the collision, and we have no way of knowing how much without examining the collision.

VP8.9.3. **IDENTIFY:** This is a collision, but we do not know if it's elastic or inelastic from the given information. Momentum is conserved during the collision regardless of whether it is elastic or inelastic.

SET UP: During the collision, we use $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and momentum conservation. Let C refer to the coffee can and M refer to the macaroni.

EXECUTE: **(a)** The momentum before the collision is equal to the momentum after the collision. Using $m_C v_{C1} = m_C v_{C2} + m_M v_{M2}$ gives (2.40 kg)(1.50 m/s) = (2.40 kg)(0.825 m/s) + (1.20 kg)v_{M2}, so v_{M2} = 1.35 m/s in the $+x$ direction.

(b) $K_{C1} = \frac{1}{2}m_C v_{C1}^2 = \frac{1}{2}(2.40 \text{ kg})(1.50 \text{ m/s})^2 = 2.70$ J.

$K_{C2} = \frac{1}{2}m_C v_{C2}^2 = \frac{1}{2}(2.40 \text{ kg})(0.825 \text{ m/s})^2 = 0.817$ J.

$K_{M2} = \frac{1}{2}m_M v_{M2}^2 = \frac{1}{2}(1.20 \text{ kg})(1.35 \text{ m/s})^2 = 1.09$ J.

(c) $K_1 = K_{C1}$ = 2.70 J
$K_2 = K_{C2} + K_{M2}$ = 0.817 J + 1.09 J = 1.91 J
$K_2 < K_1$, so the collision is *inelastic*.

EVALUATE: This collision is inelastic, but it is not *perfectly* elastic because the objects do not stick together.

VP8.9.4. **IDENTIFY:** This is a two-dimensional collision in which the objects stick together, so it is perfectly inelastic. Momentum is conserved during the collision regardless of whether it is elastic or inelastic. We treat the *x*-components of the momentum separately from the *y*-components.

SET UP: During the collision, we use $p_x = mv_x$ and $P_x = p_{1x} + p_{2x} + \ldots$ and likewise for P_y. Call the *x*-axis positive toward the east and *y*-axis positive toward the north. Let θ be the angle with the +*x*-axis at which the blocks move together after the collision, and let V be their common speed. Figure VP8.9.4 shows before and after sketches.

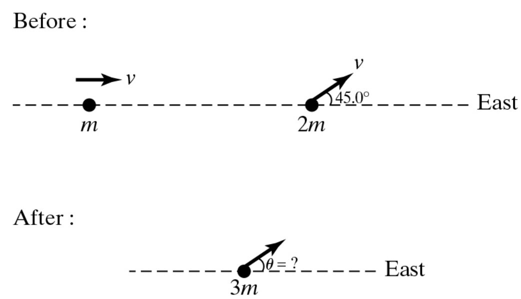

Figure VP8.9.4

EXECUTE: Before the collision the *x*-components of the momentum are mv and $(2m)v\cos 45.0°$ and the *y*-component is $(2m)v \sin 45.0°$. After the collision the *x*-component is $(3m)V\cos\theta$ and the *y*-component is $(3m)V\sin\theta$. These must also be the components after the collision because momentum is conserved.

Easterly components: $mv + 2mv\cos 45.0° = 3mV\cos\theta \quad\rightarrow\quad v(1 + 2\cos 45.0°) = 3V\cos\theta$

Northerly components: $2mv\sin 45.0° = 3mV\sin\theta \quad\rightarrow\quad 2v\sin 45.0° = 3V\sin\theta$

Dividing these two equations gives $\dfrac{3V\sin\theta}{3V\cos\theta} = \dfrac{2v\sin 45.0°}{v(1+2\cos 45.0°)}$ which simplifies to

$$\tan\theta = \frac{2\sin 45.0°}{1+2\cos 45.0°} \quad\rightarrow\quad \theta = 30.4° \text{ north of east.}$$

EVALUATE: The momentum is conserved because the collision generates no external forces on the system of colliding objects. Kinetic energy is not conserved since the objects stick together.

VP8.14.1. **IDENTIFY:** We want to find the location of the center of mass of a system of particles.

SET UP: The *x*-coordinate of the center of mass is $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ and likewise for the *y*-coordinate.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{(0.500\text{ kg})(0) + (1.25\text{ kg})(0.150\text{ m}) + (0.750\text{ kg})(0.200\text{ m})}{2.50\text{ kg}} = 0.135\text{ m}.$

$y_{\text{cm}} = \dfrac{(0.500\text{ kg})(0) + (1.25\text{ kg})(0.200\text{ m}) + (0.750\text{ kg})(-0.800\text{ m})}{2.50\text{ kg}} = -0.140\text{ m}.$

(b) Calling d the distance, we can calculate the distance between the center of mass and a given particle. For the 0.500-kg particle, we have

$d_{0.500} = \sqrt{(x_{\text{cm}} - x_{0.5})^2 + (y_{\text{cm}} - y_{0.5})^2} = \sqrt{(0.135\text{ m} - 0)^2 + (-0.140\text{ m} - 0)^2} = 0.1945$ m. Likewise for the 1.25-kg particle we find $d_{1.25} = 0.3403$ m. The result is that the center of mass is closest to the 0.500-kg particle.

EVALUATE: A plot on graph paper indicates that the center of mass is closest to the 0.500-kg particle, as we just calculated.

VP8.14.2. **IDENTIFY:** We know the location of the center of mass of a system of particles and the location of two of them. We want to find the location of the third particle. The particles all lie on the x-axis.

SET UP: The x-coordinate of the center of mass is $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$.

EXECUTE: Putting the known x-coordinates into the center of mass formula gives

$$-0.200\text{ m} = \frac{(3.00\text{ kg})(0) + (2.00\text{ kg})(1.50\text{ m}) + (1.20\text{ kg})x}{6.20\text{ kg}} \quad \rightarrow \quad x = -3.53\text{ m}.$$

EVALUATE: Since x_{cm} is negative, the 1.20-kg object must lie on the $-x$ side of the origin and farther from the origin than the center of mass, which it does. So our result is reasonable.

VP8.14.3. **IDENTIFY:** The force of the spring is internal to the two-glider system, so the center of mass of that system does not move.

SET UP: Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. Take the origin as the original position of the center of mass of the gliders, which makes $x_{cm} = 0$.

EXECUTE: $$0 = \frac{(0.125\text{ kg})(-0.960\text{ m}) + (0.500\text{ kg})x}{0.625\text{ kg}} \quad \rightarrow \quad x = 0.240\text{ m}.$$

EVALUATE: Since B is more massive than A,it should have moved a shorter distance since both of them felt the same force from the spring. This agrees with our result.

VP8.14.4. **IDENTIFY:** The objects exert forces on each other, but no external forces act on the system of three objects. Therefore the center of mass of this system does not move.

SET UP: The x-coordinate of the center of mass is $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$ and likewise for the y-coordinate. Using the initial conditions, first find the location of the center of mass of the system. Then find the center of mass (which is still the same) after the objects have moved and use it to find the position x of the third object.

EXECUTE: Initially: $x_{cm} = \dfrac{m(-L) + m(0) + m(L)}{3m} = 0$ and $y_{cm} = \dfrac{m(0) + m(L) + m(0)}{3m} = \dfrac{L}{3}$. So the center of mass is at $(0, L/3)$ and does not change location.

For the new arrangement: $x_{cm} = \dfrac{m(-L/3) + m(L/2) + mx}{3m} = 0 \quad \rightarrow \quad x = -\dfrac{L}{6}$.

$y_{cm} = \dfrac{m(L/4) + m(-L) + my}{3m} = \dfrac{L}{3} \quad \rightarrow \quad y = \dfrac{7L}{4}$.

EVALUATE: As a check, use the coordinates of the third mass $(-L/6, 7L/4)$ to calculate the location of the center of mass of the new arrangement. The result should come out $(0, L/3)$.

8.1. **IDENTIFY** and **SET UP:** $p = mv$. $K = \tfrac{1}{2}mv^2$.

EXECUTE: (a) $p = (10{,}000\text{ kg})(12.0\text{ m/s}) = 1.20\times10^5\text{ kg}\cdot\text{m/s}$

(b) (i) $v = \dfrac{p}{m} = \dfrac{1.20\times10^5\text{ kg}\cdot\text{m/s}}{2000\text{ kg}} = 60.0\text{ m/s}$. (ii) $\tfrac{1}{2}m_Tv_T^2 = \tfrac{1}{2}m_{SUV}v_{SUV}^2$, so

$$v_{SUV} = \sqrt{\frac{m_T}{m_{SUV}}}v_T = \sqrt{\frac{10{,}000\text{ kg}}{2000\text{ kg}}}(12.0\text{ m/s}) = 26.8\text{ m/s}$$

EVALUATE: The SUV must have less speed to have the same kinetic energy as the truck than to have the same momentum as the truck.

8.2. IDENTIFY: Each momentum component is the mass times the corresponding velocity component.
SET UP: Let $+x$ be along the horizontal motion of the shotput. Let $+y$ be vertically upward.
$v_x = v\cos\theta$, $v_y = v\sin\theta$.

EXECUTE: The horizontal component of the initial momentum is

$$p_x = mv_x = mv\cos\theta = (7.30\text{ kg})(15.0\text{ m/s})\cos 40.0° = 83.9\text{ kg}\cdot\text{m/s}.$$

The vertical component of the initial momentum is

$$p_y = mv_y = mv\sin\theta = (7.30\text{ kg})(15.0\text{ m/s})\sin 40.0° = 70.4\text{ kg}\cdot\text{m/s}.$$

EVALUATE: The initial momentum is directed at $40.0°$ above the horizontal.

8.3. IDENTIFY and SET UP: We use $p = mv$ and add the respective components.
EXECUTE: **(a)** $P_x = p_{Ax} + p_{Cx} = 0 + (10.0\text{ kg})(-3.0\text{ m/s}) = -30\text{ kg}\cdot\text{m/s}$

$P_y = p_{Ay} + p_{Cy} = (5.0\text{ kg})(-11.0\text{ m/s}) + 0 = -55\text{ kg}\cdot\text{m/s}$

(b) $P_x = p_{Bx} + p_{Cx} = (6.0\text{ kg})(10.0\text{ m/s}\cos 60°) + (10.0\text{ kg})(-3.0\text{ m/s}) = 0$

$P_y = p_{By} + p_{Cy} = (6.0\text{ kg})(10.0\text{ m/s}\sin 60°) + 0 = 52\text{ kg}\cdot\text{m/s}$

(c) $P_x = p_{Ax} + p_{Bx} + p_{Cx} = 0 + (6.0\text{ kg})(10.0\text{ m/s}\cos 60°) + (10.0\text{ kg})(-3.0\text{ m/s}) = 0$

$P_y = p_{Ay} + p_{By} + p_{Cy} = (5.0\text{ kg})(-11.0\text{ m/s}) + (6.0\text{ kg})(10.0\text{ m/s}\sin 60°) + 0 = -3.0\text{ kg}\cdot\text{m/s}$

EVALUATE: A has no x-component of momentum so P_x is the same in (b) and (c). C has no y-component of momentum so P_y in (c) is the sum of P_y in (a) and (b).

8.4. IDENTIFY: For each object $\vec{p} = m\vec{v}$ and the net momentum of the system is $\vec{P} = \vec{p}_A + \vec{p}_B$. The momentum vectors are added by adding components. The magnitude and direction of the net momentum is calculated from its x- and y-components.
SET UP: Let object A be the pickup and object B be the sedan. $v_{Ax} = -14.0\text{ m/s}$, $v_{Ay} = 0$. $v_{Bx} = 0$, $v_{By} = +23.0\text{ m/s}$.

EXECUTE: **(a)** $P_x = p_{Ax} + p_{Bx} = m_A v_{Ax} + m_B v_{Bx} = (2500\text{ kg})(-14.0\text{ m/s}) + 0 = -3.50\times 10^4\text{ kg}\cdot\text{m/s}$

$$P_y = p_{Ay} + p_{By} = m_A v_{Ay} + m_B v_{By} = (1500\text{ kg})(+23.0\text{ m/s}) = +3.45\times 10^4\text{ kg}\cdot\text{m/s}$$

(b) $P = \sqrt{P_x^2 + P_y^2} = 4.91\times 10^4\text{ kg}\cdot\text{m/s}$. From Figure 8.4, $\tan\theta = \left|\dfrac{P_x}{P_y}\right| = \dfrac{3.50\times 10^4\text{ kg}\cdot\text{m/s}}{3.45\times 10^4\text{ kg}\cdot\text{m/s}}$ and

$\theta = 45.4°$. The net momentum has magnitude $4.91\times 10^4\text{ kg}\cdot\text{m/s}$ and is directed at $45.4°$ west of north.

EVALUATE: The momenta of the two objects must be added as vectors. The momentum of one object is west and the other is north. The momenta of the two objects are nearly equal in magnitude, so the net momentum is directed approximately midway between west and north.

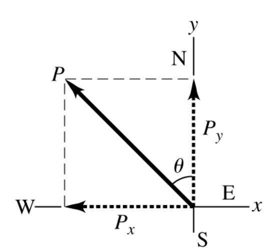

Figure 8.4

8.5. **IDENTIFY:** For each object, $\vec{p} = m\vec{v}$ and $K = \frac{1}{2}mv^2$. The total momentum is the vector sum of the momenta of each object. The total kinetic energy is the scalar sum of the kinetic energies of each object.
SET UP: Let object A be the 110 kg lineman and object B the 125 kg lineman. Let $+x$ be to the right, so $v_{Ax} = +2.75$ m/s and $v_{Bx} = -2.60$ m/s.

EXECUTE: **(a)** $P_x = m_A v_{Ax} + m_B v_{Bx} = (110 \text{ kg})(2.75 \text{ m/s}) + (125 \text{ kg})(-2.60 \text{ m/s}) = -22.5 \text{ kg} \cdot \text{m/s}$. The net momentum has magnitude $22.5 \text{ kg} \cdot \text{m/s}$ and is directed to the left.

(b) $K = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(110 \text{ kg})(2.75 \text{ m/s})^2 + \frac{1}{2}(125 \text{ kg})(2.60 \text{ m/s})^2 = 838 \text{ J}$

EVALUATE: The kinetic energy of an object is a scalar and is never negative. It depends only on the magnitude of the velocity of the object, not on its direction. The momentum of an object is a vector and has both magnitude and direction. When two objects are in motion, their total kinetic energy is greater than the kinetic energy of either one. But if they are moving in opposite directions, the net momentum of the system has a smaller magnitude than the magnitude of the momentum of either object.

8.6. **IDENTIFY:** We know the contact time of the ball with the racket, the change in velocity of the ball, and the mass of the ball. From this information we can use the fact that the impulse is equal to the change in momentum to find the force exerted on the ball by the racket.
SET UP: $J_x = \Delta p_x$ and $J_x = F_x \Delta t$. In part (a), take the $+x$-direction to be along the final direction of motion of the ball. The initial speed of the ball is zero. In part (b), take the $+x$-direction to be in the direction the ball is traveling before it is hit by the opponent's racket.

EXECUTE: **(a)** $J_x = mv_{2x} - mv_{1x} = (57 \times 10^{-3} \text{ kg})(73 \text{ m/s} - 0) = 4.16 \text{ kg} \cdot \text{m/s}$. Using $J_x = F_x \Delta t$ gives

$$F_x = \frac{J_x}{\Delta t} = \frac{4.16 \text{ kg} \cdot \text{m/s}}{30.0 \times 10^{-3} \text{ s}} = 140 \text{ N}.$$

(b) $J_x = mv_{2x} - mv_{1x} = (57 \times 10^{-3} \text{ kg})(-55 \text{ m/s} - 73 \text{ m/s}) = -7.30 \text{ kg} \cdot \text{m/s}$.

$$F_x = \frac{J_x}{\Delta t} = \frac{-7.30 \text{ kg} \cdot \text{m/s}}{30.0 \times 10^{-3} \text{ s}} = -240 \text{ N}.$$

EVALUATE: The signs of J_x and F_x show their direction. $140 \text{ N} = 31 \text{ lb}$. This very attainable force has a large effect on the light ball. 140 N is 250 times the weight of the ball.

8.7. **IDENTIFY:** The average force on an object and the object's change in momentum are related by $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t}$. The weight of the ball is $w = mg$.

SET UP: Let $+x$ be in the direction of the final velocity of the ball, so $v_{1x} = 0$ and $v_{2x} = 25.0$ m/s.

EXECUTE: $(F_{\text{av}})_x(t_2 - t_1) = mv_{2x} - mv_{1x}$ gives $(F_{\text{av}})_x = \dfrac{mv_{2x} - mv_{1x}}{t_2 - t_1} = \dfrac{(0.0450 \text{ kg})(25.0 \text{ m/s})}{2.00 \times 10^{-3} \text{ s}} = 562 \text{ N}.$

$w = (0.0450 \text{ kg})(9.80 \text{ m/s}^2) = 0.441 \text{ N}$. The force exerted by the club is much greater than the weight of the ball, so the effect of the weight of the ball during the time of contact is not significant.
EVALUATE: Forces exerted during collisions typically are very large but act for a short time.

8.8. **IDENTIFY:** The change in momentum, the impulse, and the average force are related by $J_x = \Delta p_x$ and $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t}$.

SET UP: Let the direction in which the batted ball is traveling be the $+x$-direction, so $v_{1x} = -45.0$ m/s and $v_{2x} = 55.0$ m/s.

EXECUTE: **(a)** $\Delta p_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x}) = (0.145 \text{ kg})[55.0 \text{ m/s} - (-45.0 \text{ m/s})] = 14.5 \text{ kg} \cdot \text{m/s}$.
$J_x = \Delta p_x$, so $J_x = 14.5 \text{ kg} \cdot \text{m/s}$. Both the change in momentum and the impulse have magnitude $14.5 \text{ kg} \cdot \text{m/s}$.

(b) $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t} = \dfrac{14.5\ \text{kg}\cdot\text{m/s}}{2.00\times10^{-3}\ \text{s}} = 7250\ \text{N}.$

EVALUATE: The force is in the direction of the momentum change.

8.9. **IDENTIFY:** Use $J_x = p_{2x} - p_{1x}$. We know the initial momentum and the impulse so can solve for the final momentum and then the final velocity.

SET UP: Take the x-axis to be toward the right, so $v_{1x} = +3.00$ m/s. Use $J_x = F_x \Delta t$ to calculate the impulse, since the force is constant.

EXECUTE: **(a)** $J_x = p_{2x} - p_{1x}$

$$J_x = F_x(t_2 - t_1) = (+25.0\ \text{N})(0.050\ \text{s}) = +1.25\ \text{kg}\cdot\text{m/s}$$

Thus $p_{2x} = J_x + p_{1x} = +1.25\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = +1.73\ \text{kg}\cdot\text{m/s}$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{1.73\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = +10.8\ \text{m/s (to the right)}$$

(b) $J_x = F_x(t_2 - t_1) = (-12.0\ \text{N})(0.050\ \text{s}) = -0.600\ \text{kg}\cdot\text{m/s}$ (negative since force is to left)

$$p_{2x} = J_x + p_{1x} = -0.600\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = -0.120\ \text{kg}\cdot\text{m/s}$$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{-0.120\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = -0.75\ \text{m/s (to the left)}$$

EVALUATE: In part (a) the impulse and initial momentum are in the same direction and v_x increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

8.10. **IDENTIFY:** Apply $J_x = \Delta p_x = mv_{2x} - mv_{1x}$ and $J_y = \Delta p_y = mv_{2y} - mv_{1y}$ to relate the change in momentum to the components of the average force on it.

SET UP: Let $+x$ be to the right and $+y$ be upward.

EXECUTE: $J_x = \Delta p_x = mv_{2x} - mv_{1x} = (0.145\ \text{kg})[-(52.0\ \text{m/s})\cos 30° - 40.0\ \text{m/s}] = -12.33\ \text{kg}\cdot\text{m/s}.$

$J_y = \Delta p_y = mv_{2y} - mv_{1y} = (0.145\ \text{kg})[(52.0\ \text{m/s})\sin 30° - 0] = 3.770\ \text{kg}\cdot\text{m/s}.$

The horizontal component is $12.33\ \text{kg}\cdot\text{m/s}$, to the left and the vertical component is $3.770\ \text{kg}\cdot\text{m/s}$, upward.

$F_{\text{av-}x} = \dfrac{J_x}{\Delta t} = \dfrac{-12.33\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = -7050\ \text{N}.$ $F_{\text{av-}y} = \dfrac{J_y}{\Delta t} = \dfrac{3.770\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = 2150\ \text{N}.$

The horizontal component is 7050 N, to the left, and the vertical component is 2150 N, upward.

EVALUATE: The ball gains momentum to the left and upward and the force components are in these directions.

8.11. **IDENTIFY:** The force is not constant so $\vec{J} = \int_{t_1}^{t_2} \vec{F}dt$. The impulse is related to the change in velocity by $J_x = m(v_{2x} - v_{1x})$.

SET UP: Only the x-component of the force is nonzero, so $J_x = \int_{t_1}^{t_2} F_x dt$ is the only nonzero component of $\vec{J}$. $J_x = m(v_{2x} - v_{1x})$. $t_1 = 2.00$ s, $t_2 = 3.50$ s.

EXECUTE: **(a)** $A = \dfrac{F_x}{t^2} = \dfrac{781.25\ \text{N}}{(1.25\ \text{s})^2} = 500\ \text{N/s}^2.$

(b) $J_x = \int_{t_1}^{t_2} At^2 dt = \frac{1}{3}A(t_2^3 - t_1^3) = \frac{1}{3}(500\ \text{N/s}^2)([3.50\ \text{s}]^3 - [2.00\ \text{s}]^3) = 5.81\times10^3\ \text{N}\cdot\text{s}.$

(c) $\Delta v_x = v_{2x} - v_{1x} = \dfrac{J_x}{m} = \dfrac{5.81\times10^3\ \text{N}\cdot\text{s}}{2150\ \text{kg}} = 2.70\ \text{m/s}.$ The x-component of the velocity of the rocket increases by 2.70 m/s.

EVALUATE: The change in velocity is in the same direction as the impulse, which in turn is in the direction of the net force. In this problem the net force equals the force applied by the engine, since that is the only force on the rocket.

8.12. IDENTIFY: This problem requires the graphical interpretation of impulse and momentum.
SET UP: The impulse is the area under the curve on a graph of force versus time. Impulse is equal to the change in momentum: $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. Fig. 8.12 shows a graph of F versus t.

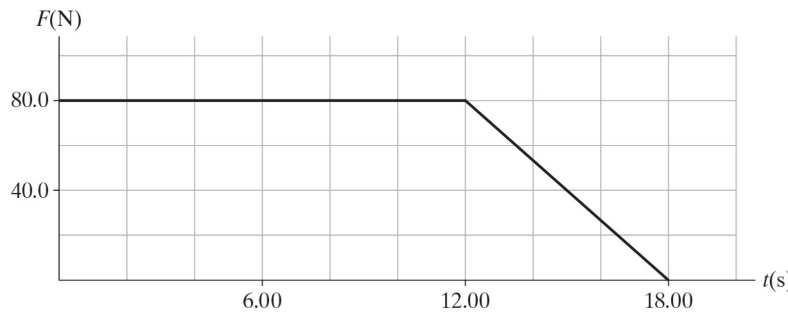

Figure 8.12

EXECUTE: To compute the area under the F-t graph, break it into two figures, a rectangle and a triangle. Using the numbers on the graph, we have $J_x = (80.0\text{ N})(12.0\text{ s}) + \frac{1}{2}(80.0\text{ N})(6.0\text{ s}) = 1200$ N·s. $J_x = p_{2x} - p_{1x} = mv - 0 \quad \rightarrow \quad (80.0\text{ kg})v = 1200\text{ N}\cdot\text{s} \quad \rightarrow \quad v = 15.0\text{ m/s}$.
EVALUATE: The longer a force acts, the more it changes the momentum of an object.

8.13. IDENTIFY: The force is constant during the 1.0 ms interval that it acts, so $\vec{J} = \vec{F}\Delta t$.
$\vec{J} = \vec{p}_2 - \vec{p}_1 = m(\vec{v}_2 - \vec{v}_1)$.
SET UP: Let $+x$ be to the right, so $v_{1x} = +5.00$ m/s. Only the x-component of $\vec{J}$ is nonzero, and $J_x = m(v_{2x} - v_{1x})$.
EXECUTE: **(a)** The magnitude of the impulse is $J = F\Delta t = (2.50\times10^3\text{ N})(1.00\times10^{-3}\text{ s}) = 2.50\text{ N}\cdot\text{s}$. The direction of the impulse is the direction of the force.
(b) (i) $v_{2x} = \dfrac{J_x}{m} + v_{1x}$. $J_x = +2.50\text{ N}\cdot\text{s}$. $v_{2x} = \dfrac{+2.50\text{ N}\cdot\text{s}}{2.00\text{ kg}} + 5.00\text{ m/s} = 6.25\text{ m/s}$. The stone's velocity has magnitude 6.25 m/s and is directed to the right. (ii) Now $J_x = -2.50\text{ N}\cdot\text{s}$ and
$v_{2x} = \dfrac{-2.50\text{ N}\cdot\text{s}}{2.00\text{ kg}} + 5.00\text{ m/s} = 3.75\text{ m/s}$. The stone's velocity has magnitude 3.75 m/s and is directed to the right.
EVALUATE: When the force and initial velocity are in the same direction the speed increases, and when they are in opposite directions the speed decreases.

8.14. IDENTIFY: We know the force acting on a box as a function of time and its initial momentum and want to find its momentum at a later time. The target variable is the final momentum.
SET UP: Use $\int_{t_1}^{t_2} \vec{F}(t)dt = \vec{p}_2 - \vec{p}_1$ to find $\vec{p}_2$ since we know $\vec{p}_1$ and $\vec{F}(t)$.

EXECUTE: $\vec{p}_1 = (-3.00 \text{ kg}\cdot\text{m/s})\hat{i} + (4.00 \text{ kg}\cdot\text{m/s})\hat{j}$ at $t_1 = 0$, and $t_2 = 2.00$ s. Work with the components of the force and momentum. $\int_{t_1}^{t_2} F_x(t)dt = (0.280 \text{ N/s})\int_{t_1}^{t_2} t\,dt = (0.140 \text{ N/s})t_2^2 = 0.560 \text{ N}\cdot\text{s}$

$p_{2x} = p_{1x} + 0.560 \text{ N}\cdot\text{s} = -3.00 \text{ kg}\cdot\text{m/s} + 0.560 \text{ N}\cdot\text{s} = -2.44 \text{ kg}\cdot\text{m/s}.$

$\int_{t_1}^{t_2} F_y(t)dt = (-0.450 \text{ N/s}^2)\int_{t_1}^{t_2} t^2 dt = (-0.150 \text{ N/s}^2)t_2^3 = -1.20 \text{ N}\cdot\text{s}.$

$p_{2y} = p_{1y} + (-1.20 \text{ N}\cdot\text{s}) = 4.00 \text{ kg}\cdot\text{m/s} + (-1.20 \text{ N}\cdot\text{s}) = +2.80 \text{ kg}\cdot\text{m/s}.$ So

$\vec{p}_2 = (-2.44 \text{ kg}\cdot\text{m/s})\hat{i} + (2.80 \text{ kg}\cdot\text{m/s})\hat{j}$

EVALUATE: Since the given force has *x*- and *y*-components, it changes both components of the box's momentum.

8.15. **IDENTIFY:** This problem requires the use of momentum, impulse, and kinetic energy.

SET UP: $p_x = mv_x$, $J_x = p_{2x} - p_{1x}$, $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $p_x = mv_x = (40.0 \text{ kg})(20.0 \text{ m/s}) = 800 \text{ kg}\cdot\text{m/s}$.

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}(40.0 \text{ kg})(20.0 \text{ m/s})^2 = 8.00\times 10^3$ J.

(c) $J_x = p_{2x} - p_{1x} = 0 - p_{1x}$, so $Ft = -p_1$ gives $F(5.00 \text{ s}) = -800 \text{ kg}\cdot\text{m/s}$. $F = -160$ N. The minus sign tells us that the force is opposite to her velocity.

EVALUATE: Unlike kinetic energy, momentum has components that can be negative as well as positive.

8.16. **IDENTIFY:** Apply conservation of momentum to the system of the astronaut and tool.

SET UP: Let *A* be the astronaut and *B* be the tool. Let $+x$ be the direction in which she throws the tool, so $v_{B2x} = +3.20$ m/s. Assume she is initially at rest, so $v_{A1x} = v_{B1x} = 0$. Solve for v_{A2x}.

EXECUTE: $P_{1x} = P_{2x}$. $P_{1x} = m_A v_{A1x} + m_B v_{B1x} = 0$. $P_{2x} = m_A v_{A2x} + m_B v_{B2x} = 0$ and

$v_{A2x} = -\frac{m_B v_{A2x}}{m_A} = -\frac{(2.25 \text{ kg})(3.20 \text{ m/s})}{68.5 \text{ kg}} = -0.105$ m/s. Her speed is 0.105 m/s and she moves opposite to the direction in which she throws the tool.

EVALUATE: Her mass is much larger than that of the tool, so to have the same magnitude of momentum as the tool her speed is much less.

8.17. **IDENTIFY:** Since the rifle is loosely held there is no net external force on the system consisting of the rifle, bullet, and propellant gases and the momentum of this system is conserved. Before the rifle is fired everything in the system is at rest and the initial momentum of the system is zero.

SET UP: Let $+x$ be in the direction of the bullet's motion. The bullet has speed $601 \text{ m/s} - 1.85 \text{ m/s} = 599 \text{ m/s}$ relative to the earth. $P_{2x} = p_{rx} + p_{bx} + p_{gx}$, the momenta of the rifle, bullet, and gases. $v_{rx} = -1.85$ m/s and $v_{bx} = +599$ m/s.

EXECUTE: $P_{2x} = P_{1x} = 0$. $p_{rx} + p_{bx} + p_{gx} = 0$.

$p_{gx} = -p_{rx} - p_{bx} = -(2.80 \text{ kg})(-1.85 \text{ m/s}) - (0.00720 \text{ kg})(599 \text{ m/s})$ and

$p_{gx} = +5.18 \text{ kg}\cdot\text{m/s} - 4.31 \text{ kg}\cdot\text{m/s} = 0.87 \text{ kg}\cdot\text{m/s}$. The propellant gases have momentum $0.87 \text{ kg}\cdot\text{m/s}$, in the same direction as the bullet is traveling.

EVALUATE: The magnitude of the momentum of the recoiling rifle equals the magnitude of the momentum of the bullet plus that of the gases as both exit the muzzle.

8.18. **IDENTIFY:** The total momentum of the two skaters is conserved, but not their kinetic energy.

SET UP: There is no horizontal external force so, $P_{i,x} = P_{f,x}$, $p = mv$, $K = ½\, mv^2$.

EXECUTE: **(a)** $P_{i,x} = P_{f,x}$. The skaters are initially at rest so $P_{i,x} = 0$. $0 = m_A(v_{A,f})_x + m_B(v_{B,f})_x$

$(v_{A,f})_x = -\dfrac{m_B(v_{B,f})_x}{m_A} = -\dfrac{(74.0\text{ kg})(1.50\text{ m/s})}{63.8\text{ kg}} = -1.74\text{ m/s}$. The lighter skater travels to the left at 1.74 m/s.

(b) $K_i = 0$. $K_f = \frac{1}{2}m_A v_{A,f}{}^2 + \frac{1}{2}m_B v_{B,f}{}^2 = \frac{1}{2}(63.8\text{ kg})(1.74\text{ m/s})^2 + \frac{1}{2}(74.0\text{ kg})(1.50\text{ m/s})^2 = 180\text{ J}$.

EVALUATE: The kinetic energy of the system was produced by the work the two skaters do on each other.

8.19. **IDENTIFY:** Since drag effects are neglected, there is no net external force on the system of squid plus expelled water, and the total momentum of the system is conserved. Since the squid is initially at rest, with the water in its cavity, the initial momentum of the system is zero. For each object, $K = \frac{1}{2}mv^2$.

SET UP: Let A be the squid and B be the water it expels, so $m_A = 6.50\text{ kg} - 1.75\text{ kg} = 4.75\text{ kg}$. Let $+x$ be the direction in which the water is expelled. $v_{A2x} = -2.50\text{ m/s}$. Solve for v_{B2x}.

EXECUTE: **(a)** $P_{1x} = 0$. $P_{2x} = P_{1x}$, so $0 = m_A v_{A2x} + m_B v_{B2x}$.

$v_{B2x} = -\dfrac{m_A v_{A2x}}{m_B} = -\dfrac{(4.75\text{ kg})(-2.50\text{ m/s})}{1.75\text{ kg}} = +6.79\text{ m/s}$.

(b) $K_2 = K_{A2} + K_{B2} = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(4.75\text{ kg})(2.50\text{ m/s})^2 + \frac{1}{2}(1.75\text{ kg})(6.79\text{ m/s})^2 = 55.2\text{ J}$. The initial kinetic energy is zero, so the kinetic energy produced is $K_2 = 55.2\text{ J}$.

EVALUATE: The two objects end up with momenta that are equal in magnitude and opposite in direction, so the total momentum of the system remains zero. The kinetic energy is created by the work done by the squid as it expels the water.

8.20. **IDENTIFY:** Apply conservation of momentum to the system of you and the ball. In part (a) both objects have the same final velocity.

SET UP: Let $+x$ be in the direction the ball is traveling initially. $m_A = 0.600\text{ kg}$ (ball). $m_B = 70.0\text{ kg}$ (you).

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $(0.600\text{ kg})(10.0\text{ m/s}) = (0.600\text{ kg} + 70.0\text{ kg})v_2$ so $v_2 = 0.0850\text{ m/s}$.

(b) $P_{1x} = P_{2x}$ gives $(0.600\text{ kg})(10.0\text{ m/s}) = (0.600\text{ kg})(-8.00\text{ m/s}) + (70.0\text{ kg})v_{B2}$ so $v_{B2} = 0.154\text{ m/s}$.

EVALUATE: When the ball bounces off it has a greater change in momentum and you acquire a greater final speed.

8.21. **IDENTIFY:** Apply conservation of momentum to the system of the two pucks.

SET UP: Let $+x$ be to the right.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ says $(0.250\text{ kg})v_{A1} = (0.250\text{ kg})(-0.120\text{ m/s}) + (0.350\text{ kg})(0.650\text{ m/s})$ and $v_{A1} = 0.790\text{ m/s}$.

(b) $K_1 = \frac{1}{2}(0.250\text{ kg})(0.790\text{ m/s})^2 = 0.0780\text{ J}$.

$K_2 = \frac{1}{2}(0.250\text{ kg})(0.120\text{ m/s})^2 + \frac{1}{2}(0.350\text{ kg})(0.650\text{ m/s})^2 = 0.0757\text{ J}$ and $\Delta K = K_2 - K_1 = -0.0023\text{ J}$.

EVALUATE: The total momentum of the system is conserved but the total kinetic energy decreases.

8.22. **IDENTIFY:** Since road friction is neglected, there is no net external force on the system of the two cars and the total momentum of the system is conserved. For each object, $K = \frac{1}{2}mv^2$.

SET UP: Let A be the 1750 kg car and B be the 1450 kg car. Let $+x$ be to the right, so $v_{A1x} = +1.50\text{ m/s}$, $v_{B1x} = -1.10\text{ m/s}$, and $v_{A2x} = +0.250\text{ m/s}$. Solve for v_{B2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$.

$$v_{B2x} = \frac{(1750\text{ kg})(1.50\text{ m/s}) + (1450\text{ kg})(-1.10\text{ m/s}) - (1750\text{ kg})(0.250\text{ m/s})}{1450\text{ kg}} = 0.409\text{ m/s}.$$

After the collision the lighter car is moving to the right with a speed of 0.409 m/s.

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(1750\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(1.10\text{ m/s})^2 = 2846\text{ J}.$

$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1750\text{ kg})(0.250\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(0.409\text{ m/s})^2 = 176\text{ J}.$

The change in kinetic energy is $\Delta K = K_2 - K_1 = 176\text{ J} - 2846\text{ J} = -2670\text{ J}.$

EVALUATE: The total momentum of the system is constant because there is no net external force during the collision. The kinetic energy of the system decreases because of negative work done by the forces the cars exert on each other during the collision.

8.23. **IDENTIFY:** The momentum and the mechanical energy of the system are both conserved. The mechanical energy consists of the kinetic energy of the masses and the elastic potential energy of the spring. The potential energy stored in the spring is transformed into the kinetic energy of the two masses.

SET UP: Let the system be the two masses and the spring. The system is sketched in Figure 8.23, in its initial and final situations. Use coordinates where $+x$ is to the right. Call the masses A and B.

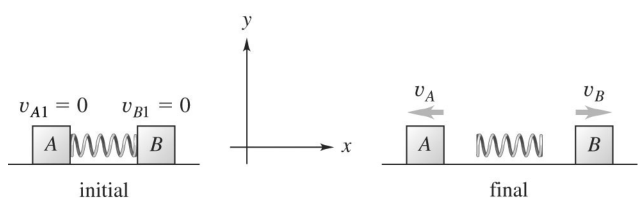

Figure 8.23

EXECUTE: $P_{1x} = P_{2x}$ so $0 = (0.900\text{ kg})(-v_A) + (0.900\text{ kg})(v_B)$ and, since the masses are equal, $v_A = v_B$. Energy conservation says the potential energy originally stored in the spring is all converted into kinetic energy of the masses, so $\frac{1}{2}kx_1^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$. Since $v_A = v_B$, this equation gives

$$v_A = x_1\sqrt{\frac{k}{2m}} = (0.200\text{ m})\sqrt{\frac{175\text{ N/m}}{2(0.900\text{ kg})}} = 1.97\text{ m/s}.$$

EVALUATE: If the objects have different masses they will end up with different speeds. The lighter one will have the greater speed, since they end up with equal magnitudes of momentum.

8.24. **IDENTIFY:** In part (a) no horizontal force implies P_x is constant. In part (b) use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the potential energy initially in the spring.

SET UP: Initially both blocks are at rest.

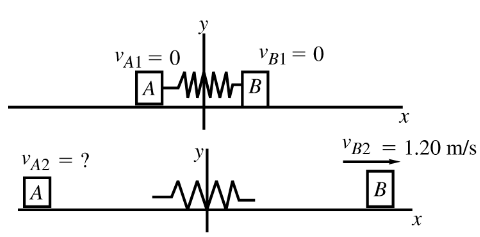

Figure 8.24

EXECUTE: **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right) v_{B2x} = -\left(\frac{3.00\text{ kg}}{1.00\text{ kg}}\right)(+1.20\text{ m/s}) = -3.60\text{ m/s}$$

Block A has a final speed of 3.60 m/s, and moves off in the opposite direction to B.

(b) Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2 = \tfrac{1}{2}(1.00\text{ kg})(3.60\text{ m/s})^2 + \tfrac{1}{2}(3.00\text{ kg})(1.20\text{ m/s})^2 = 8.64\text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64$ J.

EVALUATE: The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

8.25. **IDENTIFY:** Since friction at the pond surface is neglected, there is no net external horizontal force, and the horizontal component of the momentum of the system of hunter plus bullet is conserved. Both objects are initially at rest, so the initial momentum of the system is zero. Gravity and the normal force exerted by the ice together produce a net vertical force while the rifle is firing, so the vertical component of momentum is not conserved.

SET UP: Let object A be the hunter and object B be the bullet. Let $+x$ be the direction of the horizontal component of velocity of the bullet. Solve for v_{A2x}.

EXECUTE: **(a)** $v_{B2x} = +965$ m/s. $P_{1x} = P_{2x} = 0$. $0 = m_A v_{A2x} + m_B v_{B2x}$ and

$$v_{A2x} = -\frac{m_B}{m_A} v_{B2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(965\text{ m/s}) = -0.0559\text{ m/s}.$$

(b) $v_{B2x} = v_{B2}\cos\theta = (965\text{ m/s})\cos 56.0° = 540$ m/s. $v_{A2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(540\text{ m/s}) = -0.0313$ m/s.

EVALUATE: The mass of the bullet is much less than the mass of the hunter, so the final mass of the hunter plus gun is still 72.5 kg, to three significant figures. Since the hunter has much larger mass, his final speed is much less than the speed of the bullet.

8.26. **IDENTIFY:** Assume the nucleus is initially at rest. $K = \tfrac{1}{2}mv^2$.

SET UP: Let $+x$ be to the right. $v_{A2x} = -v_A$ and $v_{B2x} = +v_B$.

EXECUTE: **(a)** $P_{2x} = P_{1x} = 0$ gives $m_A v_{A2x} + m_B v_{B2x} = 0$. $v_B = \left(\frac{m_A}{m_B}\right) v_A$.

(b) $\dfrac{K_A}{K_B} = \dfrac{\tfrac{1}{2} m_A v_A^2}{\tfrac{1}{2} m_B v_B^2} = \dfrac{m_A v_A^2}{m_B (m_A v_A / m_B)^2} = \dfrac{m_B}{m_A}$.

EVALUATE: The lighter fragment has the greater kinetic energy.

8.27. **IDENTIFY:** Each horizontal component of momentum is conserved. $K = \tfrac{1}{2}mv^2$.

SET UP: Let $+x$ be the direction of Rebecca's initial velocity and let the $+y$ axis make an angle of $36.9°$ with respect to the direction of her final velocity. $v_{\text{D}1x} = v_{\text{D}1y} = 0$. $v_{\text{R}1x} = 13.0$ m/s; $v_{\text{R}1y} = 0$.

$v_{R2x} = (8.00 \text{ m/s})\cos 53.1° = 4.80 \text{ m/s}$; $v_{R2y} = (8.00 \text{ m/s})\sin 53.1° = 6.40 \text{ m/s}$. Solve for v_{D2x} and v_{D2y}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_R v_{R1x} = m_R v_{R2x} + m_D v_{D2x}$.

$$v_{D2x} = \frac{m_R(v_{R1x} - v_{R2x})}{m_D} = \frac{(45.0 \text{ kg})(13.0 \text{ m/s} - 4.80 \text{ m/s})}{65.0 \text{ kg}} = 5.68 \text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $0 = m_R v_{R2y} + m_D v_{D2y}$. $v_{D2y} = -\dfrac{m_R}{m_D} v_{R2y} = -\left(\dfrac{45.0 \text{ kg}}{65.0 \text{ kg}}\right)(6.40 \text{ m/s}) = -4.43 \text{ m/s}.$

The directions of $\vec{v}_{R1}$, $\vec{v}_{R2}$ and $\vec{v}_{D2}$ are sketched in Figure 8.27. $\tan\theta = \left|\dfrac{v_{D2y}}{v_{D2x}}\right| = \dfrac{4.43 \text{ m/s}}{5.68 \text{ m/s}}$ and $\theta = 38.0°$. $v_D = \sqrt{v_{D2x}^2 + v_{D2y}^2} = 7.20 \text{ m/s}$.

(b) $K_1 = \frac{1}{2} m_R v_{R1}^2 = \frac{1}{2}(45.0 \text{ kg})(13.0 \text{ m/s})^2 = 3.80 \times 10^3 \text{ J}$.

$K_2 = \frac{1}{2} m_R v_{R2}^2 + \frac{1}{2} m_D v_{D2}^2 = \frac{1}{2}(45.0 \text{ kg})(8.00 \text{ m/s})^2 + \frac{1}{2}(65.0 \text{ kg})(7.20 \text{ m/s})^2 = 3.12 \times 10^3 \text{ J}$.

$\Delta K = K_2 - K_1 = -680 \text{ J}$.

EVALUATE: Each component of momentum is separately conserved. The kinetic energy of the system decreases.

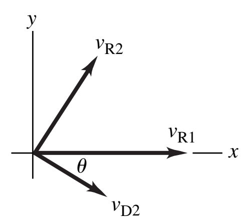

Figure 8.27

8.28. **IDENTIFY** and **SET UP:** Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so P_x is constant. Let object A be you and object B be the rock.

EXECUTE: $0 = -m_A v_A + m_B v_B \cos 35.0°$ gives $v_A = \dfrac{m_B v_B \cos 35.0°}{m_A} = 0.421 \text{ m/s}$.

EVALUATE: P_y is not conserved because there is a net external force in the vertical direction; as you throw the rock the normal force exerted on you by the ice is larger than the total weight of the system.

8.29. **IDENTIFY:** In the absence of a horizontal force, we know that momentum is conserved.
SET UP: $p = mv$. Let $+x$ be the direction you are moving. Before you catch it, the flour sack has no momentum along the x-axis. The total mass of you and your skateboard is 60 kg. You, the skateboard, and the flour sack are all moving with the same velocity, after the catch.
EXECUTE: **(a)** Since $P_{i,x} = P_{f,x}$, we have $(60 \text{ kg})(4.5 \text{ m/s}) = (62.5 \text{ kg})v_{f,x}$. Solving for the final velocity we obtain $v_{f,x} = 4.3 \text{ m/s}$.

(b) To bring the flour sack up to your speed, you must exert a horizontal force on it. Consequently, it exerts an equal and opposite force on you, which slows you down.
(c) Since you exert a vertical force on the flour sack, your horizontal speed does not change and remains at 4.3 m/s. Since the flour sack is only accelerated in the vertical direction, its horizontal velocity-component remains at 4.3 m/s as well.
EVALUATE: Unless you or the flour sack are deflected by an outside force, you will need to be ready to catch the flour sack as it returns to your arms!

8.30. **IDENTIFY:** The x- and y-components of the momentum of the system of the two asteroids are separately conserved.

SET UP: The before and after diagrams are given in Figure 8.30 and the choice of coordinates is indicated. Each asteroid has mass m.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $mv_{A1} = mv_{A2}\cos 30.0° + mv_{B2}\cos 45.0°$.

$40.0 \text{ m/s} = 0.866v_{A2} + 0.707v_{B2}$ and $0.707v_{B2} = 40.0 \text{ m/s} - 0.866v_{A2}$.

$P_{2y} = P_{2y}$ gives $0 = mv_{A2}\sin 30.0° - mv_{B2}\sin 45.0°$ and $0.500v_{A2} = 0.707v_{B2}$.

Combining these two equations gives $0.500v_{A2} = 40.0 \text{ m/s} - 0.866v_{A2}$ and $v_{A2} = 29.3 \text{ m/s}$. Then

$$v_{B2} = \left(\frac{0.500}{0.707}\right)(29.3 \text{ m/s}) = 20.7 \text{ m/s}.$$

(b) $K_1 = \frac{1}{2}mv_{A1}^2$. $K_2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2$. $\dfrac{K_2}{K_1} = \dfrac{v_{A2}^2 + v_{B2}^2}{v_{A1}^2} = \dfrac{(29.3 \text{ m/s})^2 + (20.7 \text{ m/s})^2}{(40.0 \text{ m/s})^2} = 0.804.$

$$\frac{\Delta K}{K_1} = \frac{K_2 - K_1}{K_1} = \frac{K_2}{K_1} - 1 = -0.196.$$

19.6% of the original kinetic energy is dissipated during the collision.

EVALUATE: We could use any directions we wish for the x- and y-coordinate directions, but the particular choice we have made is especially convenient.

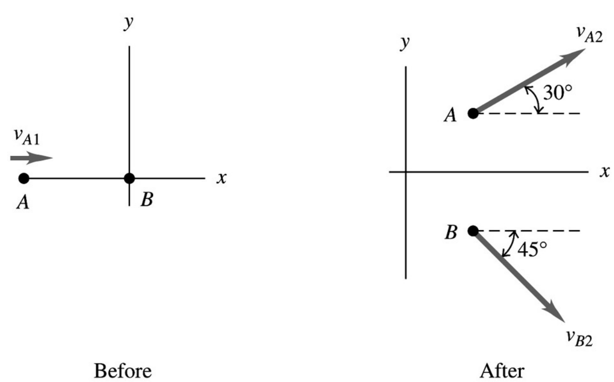

Figure 8.30

8.31. **IDENTIFY:** Momentum is conserved during the collision.

SET UP: $p_x = mv_x$. Call the $+x$-axis pointing northward. We want the speed v of the hockey players after they collide and become intertwined.

EXECUTE: The momentum before the collision is equal to the momentum after the collision. $m_1v_1 + m_2v_2 = (m_1 + m_2)v \rightarrow$ (70 kg)(5.5 m/s) + (110 kg)(–4.0 m/s) = (180 kg)v $\rightarrow$ $v = -0.31$ m/s. The minus sign tells that they are traveling toward the south.

EVALUATE: Even though the heavier player was traveling slower than the lighter player, his larger mass gave him greater momentum than that of the faster lighter player.

8.32. **IDENTIFY:** There is no net external force on the system of the two skaters and the momentum of the system is conserved.

SET UP: Let object A be the skater with mass 70.0 kg and object B be the skater with mass 65.0 kg. Let $+x$ be to the right, so $v_{A1x} = +4.00$ m/s and $v_{B1x} = -2.50$ m/s. After the collision, the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.

EXECUTE: $P_{1x} = P_{2x}$. $m_Av_{A1x} + m_Bv_{B1x} = (m_A + m_B)v_{2x}$.

$v_{2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \dfrac{(70.0\text{ kg})(4.00\text{ m/s}) + (65.0\text{ kg})(-2.50\text{ m/s})}{70.0\text{ kg} + 65.0\text{ kg}} = 0.870\text{ m/s}.$ The two skaters move to the right at 0.870 m/s.

EVALUATE: There is a large decrease in kinetic energy.

8.33. **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of two fish and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

SET UP: Let object A be the 15.0 kg fish and B be the 4.50 kg fish. Let $+x$ be the direction the large fish is moving initially, so $v_{A1x} = 1.10\text{ m/s}$ and $v_{B1x} = 0$. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(15.0\text{ kg})(1.10\text{ m/s}) + 0}{15.0\text{ kg} + 4.50\text{ kg}} = 0.846\text{ m/s}.$$

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(15.0\text{ kg})(1.10\text{ m/s})^2 = 9.08\text{ J}.$

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(19.5\text{ kg})(0.846\text{ m/s})^2 = 6.98\text{ J}.$

$\Delta K = K_2 - K_1 = \mathbf{2}2.10\text{ J}.$ 2.10 J of mechanical energy is dissipated.

EVALUATE: The total kinetic energy always decreases in a collision where the two objects become combined.

8.34. **IDENTIFY:** This problem requires use of conservation of momentum and conservation of energy. We need to break it into two parts: the collision and the motion after the collision.

SET UP: During the collision momentum is conserved. After the collision energy is conserved. We use $p_x = mv_x$, $K = \dfrac{1}{2}mv^2$, $U_g = mgh$, and $U_1 + K_1 = U_2 + K_2$. We want to find the maximum height the apple will reach after being hit by the dart.

EXECUTE: <u>Collision</u>: Use momentum conservation to find the speed v of the apple-dart system just after the collision. $\dfrac{M}{4}v_0 = \left(M + \dfrac{M}{4}\right)v$, so $v = \dfrac{v_0}{5}$.

<u>After collision</u>: Use energy conservation with the initial speed $v = \dfrac{v_0}{5}$ and the final speed zero.

$U_1 + K_1 = U_2 + K_2$ gives $\dfrac{1}{2}\left(M + \dfrac{M}{4}\right)\left(\dfrac{v_0}{5}\right)^2 = \left(M + \dfrac{M}{4}\right)gh \quad \rightarrow \quad h = \dfrac{v_0^2}{50g}.$

EVALUATE: The momentum is conserved but the kinetic energy is *not* conserved. After the collision the mechanical energy is conserved but the momentum is *not* conserved.

8.35. **IDENTIFY:** This problem involves a collision, so momentum is conserved.

SET UP: We use $p_x = mv_x$ and $K = \dfrac{1}{2}mv^2$. The total momentum P_x before the collision is equal to the total momentum after the collision, where $P_x = p_{1x} + p_{2x}$. Call the $+x$-axis eastward. We want to find the decrease in kinetic energy during the collision. Start by making a before-and-after sketch of the collision, as shown in Fig. 8.35.

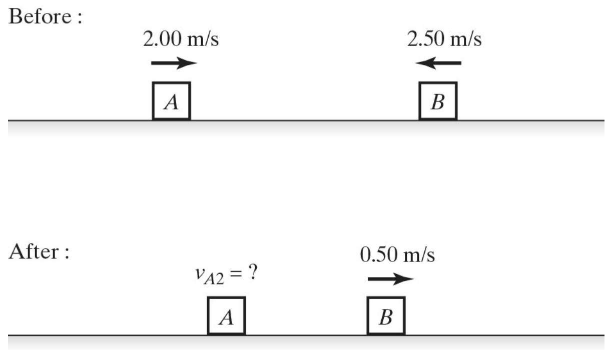

Figure 8.35

EXECUTE: We first need to find the velocity of A after the collision. Using $P_{1x} = P_{2x}$ gives $p_{A1} + p_{B1} = p_{A2} + p_{B2}$, so $m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$. Putting in the numbers gives us (4.00 kg)(2.00 m/s) – (6.00 kg)(2.50 m/s) = (4.00 kg)v_{A2} + (6.00 kg)(0.50 m/s) , so $v_{A2} = -2.50$ m/s.

Now find the decrease in kinetic energy, which is the initial value minus the final value. Using $v_{A2} = -2.50$ m/s and the quantities given in the problem, we get

$$\text{Decrease} = K_1 - K_2 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 - \left(\frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2\right) = 13.5\text{ J}.$$

EVALUATE: We see that the momentum is conserved during this collision but the kinetic energy is not conserved. The final kinetic energy is 13.5 J less than the initial kinetic energy.

8.36. **IDENTIFY:** The forces the two vehicles exert on each other during the collision are much larger than the horizontal forces exerted by the road, and it is a good approximation to assume momentum conservation.

SET UP: Let $+x$ be eastward. After the collision two vehicles move with a common velocity $\vec{v}_2$.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x} = (m_{\text{SC}} + m_{\text{T}})v_{2x}$.

$$v_{2x} = \frac{m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x}}{m_{\text{SC}} + m_{\text{T}}} = \frac{(1050\text{ kg})(-15.0\text{ m/s}) + (6320\text{ kg})(+10.0\text{ m/s})}{1050\text{ kg} + 6320\text{ kg}} = 6.44\text{ m/s}.$$

The final velocity is 6.44 m/s, eastward.

(b) $P_{1x} = P_{2x} = 0$ gives $m_{\text{SC}}v_{\text{SC}x} + m_{\text{T}}v_{\text{T}x} = 0$. $v_{\text{T}x} = -\left(\frac{m_{\text{SC}}}{m_{\text{T}}}\right)v_{\text{SC}x} = -\left(\frac{1050\text{ kg}}{6320\text{ kg}}\right)(-15.0\text{ m/s}) = 2.50\text{ m/s}.$

The truck would need to have initial speed 2.50 m/s.

(c) part (a):

$$\Delta K = \tfrac{1}{2}(7370\text{ kg})(6.44\text{ m/s})^2 - \tfrac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \tfrac{1}{2}(6320\text{ kg})(10.0\text{ m/s})^2 = -2.81\times10^5\text{ J}$$

part (b): $\Delta K = 0 - \tfrac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \tfrac{1}{2}(6320\text{ kg})(2.50\text{ m/s})^2 = -1.38\times10^5$ J. The change in kinetic energy has the greater magnitude in part (a).

EVALUATE: In part (a) the eastward momentum of the truck has a greater magnitude than the westward momentum of the car and the wreckage moves eastward after the collision. In part (b) the two vehicles have equal magnitudes of momentum, the total momentum of the system is zero and the wreckage is at rest after the collision.

8.37. **IDENTIFY:** The forces the two players exert on each other during the collision are much larger than the horizontal forces exerted by the slippery ground and it is a good approximation to assume momentum conservation. Each component of momentum is separately conserved.

Set Up: Let $+x$ be east and $+y$ be north. After the collision the two players have velocity $\vec{v}_2$. Let the linebacker be object *A* and the halfback be object *B*, so $v_{A1x} = 0$, $v_{A1y} = 8.8$ m/s, $v_{B1x} = 7.2$ m/s and $v_{B1y} = 0$. Solve for v_{2x} and v_{2y}.

Execute: $P_{1x} = P_{2x}$ gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(85\text{ kg})(7.2\text{ m/s})}{110\text{ kg} + 85\text{ kg}} = 3.14\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{2y} = \frac{m_A v_{A1y} + m_B v_{B1y}}{m_A + m_B} = \frac{(110\text{ kg})(8.8\text{ m/s})}{110\text{ kg} + 85\text{ kg}} = 4.96\text{ m/s}.$$

$v = \sqrt{v_{2x}^2 + v_{2y}^2} = 5.9$ m/s.

$$\tan\theta = \frac{v_{2y}}{v_{2x}} = \frac{4.96\text{ m/s}}{3.14\text{ m/s}} \text{ and } \theta = 58°.$$

The players move with a speed of 5.9 m/s and in a direction $58°$ north of east.

Evaluate: Each component of momentum is separately conserved.

8.38. **Identify:** The momentum is conserved during the collision. Since the motions involved are in two dimensions, we must consider the components separately.

Set Up: Use coordinates where $+x$ is east and $+y$ is south. The system of two cars before and after the collision is sketched in Figure 8.38. Neglect friction from the road during the collision. The enmeshed cars have a total mass of $2000\text{ kg} + 1500\text{ kg} = 3500\text{ kg}$. Momentum conservation tells us that $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

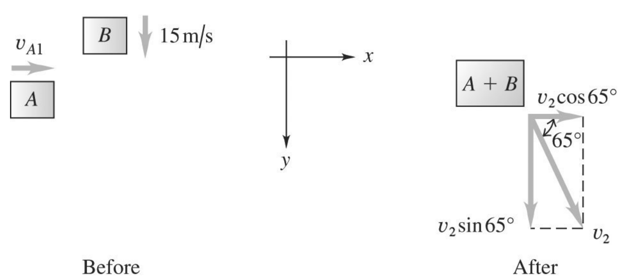

Figure 8.38

Execute: There are no external horizontal forces during the collision, so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

(a) $P_{1x} = P_{2x}$ gives $(1500\text{ kg})(15\text{ m/s}) = (3500\text{ kg})v_2\sin 65°$ and $v_2 = 7.1$ m/s.

(b) $P_{1y} = P_{2y}$ gives $(2000\text{ kg})v_{A1} = (3500\text{ kg})v_2\cos 65°$. And then using $v_2 = 7.1$ m/s, we have $v_{A1} = 5.2$ m/s.

Evaluate: Momentum is a vector so we must treat each component separately.

8.39. **Identify:** The collision generates only internal forces to the Jack-Jill system, so momentum is conserved.

Set Up: Call the *x*-axis Jack's initial direction (eastward), and the *y*-axis perpendicular to that (northward). The initial *y*-component of the momentum is zero. Call *v* Jill's speed just after the collision and call θ the angle her velocity makes with the $+x$-axis.

Execute: In the *x*-direction: (55.0 kg)(8.00 m/s) = (55.0 kg)(5.00 m/s)(cos34.0°) + (48.0 kg)$v\cos\theta$.
In the *y*-direction: (55.0 kg)(5.00 m/s)(sin34.0°) = (48.0 kg)$v\sin\theta$.
Separating $v\sin\theta$ and $v\cos\theta$ and dividing gives

$\tan\theta$ = (5.00 m/s)(sin34.0°)/[8.00 m/s – (5.00 m/s)(cos34.0°)] = 0.72532, so θ = 36.0° south of east. Using the y-direction momentum equation gives
v = (55.0 kg)(5.00 m/s)(sin34.0°)/[(48.0 kg)(sin36.0°) = 5.46 m/s.

EVALUATE: Jill has a bit less mass than Jack, so the angle her momentum makes with the +x-axis (36.0°) has to be a bit larger than Jack's (34.0°) for their y-component momenta to be equal in magnitude.

8.40. **IDENTIFY:** The collision forces are large so gravity can be neglected during the collision. Therefore, the horizontal and vertical components of the momentum of the system of the two birds are conserved.

SET UP: The system before and after the collision is sketched in Figure 8.40. Use the coordinates shown.

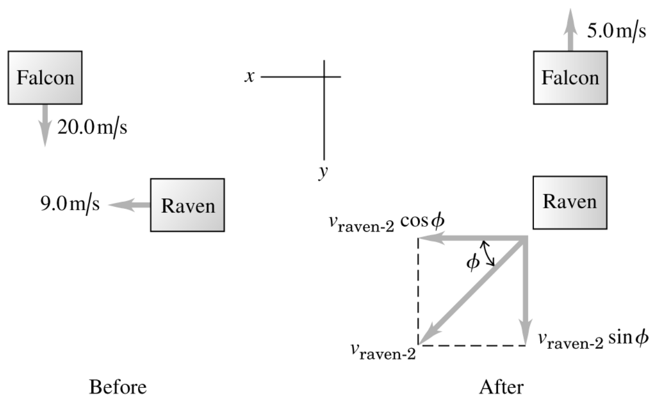

Figure 8.40

EXECUTE: **(a)** There is no external force on the system so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

$P_{1x} = P_{2x}$ gives $(1.5\text{ kg})(9.0\text{ m/s}) = (1.5\text{ kg})v_{\text{raven-2}}\cos\phi$ and $v_{\text{raven-2}}\cos\phi = 9.0\text{ m/s}$.

$P_{1y} = P_{2y}$ gives $(0.600\text{ kg})(20.0\text{ m/s}) = (0.600\text{ kg})(-5.0\text{ m/s}) + (1.5\text{ kg})v_{\text{raven-2}}\sin\phi$ and $v_{\text{raven-2}}\sin\phi = 10.0\text{ m/s}$.

Combining these two equations gives $\tan\phi = \dfrac{10.0\text{ m/s}}{9.0\text{ m/s}}$ and $\phi = 48°$.

(b) $v_{\text{raven-2}} = 13.5$ m/s

EVALUATE: Due to its large initial speed the lighter falcon was able to produce a large change in the raven's direction of motion.

8.41. **IDENTIFY:** Since friction forces from the road are ignored, the x- and y-components of momentum are conserved.

SET UP: Let object A be the subcompact and object B be the truck. After the collision the two objects move together with velocity $\vec{v}_2$. Use the x- and y-coordinates given in the problem. $v_{A1y} = v_{B1x} = 0$.

$v_{2x} = (16.0\text{ m/s})\sin 24.0° = 6.5\text{ m/s}$; $v_{2y} = (16.0\text{ m/s})\cos 24.0° = 14.6\text{ m/s}$.

EXECUTE: $P_{1x} = P_{2x}$ gives $m_A v_{A1x} = (m_A + m_B)v_{2x}$.

$$v_{A1x} = \left(\frac{m_A + m_B}{m_A}\right)v_{2x} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{950\text{ kg}}\right)(6.5\text{ m/s}) = 19.5\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_B v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{B1y} = \left(\frac{m_A + m_B}{m_B}\right)v_{2y} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{1900\text{ kg}}\right)(14.6\text{ m/s}) = 21.9\text{ m/s}.$$

Before the collision the subcompact car has speed 19.5 m/s and the truck has speed 21.9 m/s.

EVALUATE: Each component of momentum is independently conserved.

8.42. **IDENTIFY:** Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

SET UP: Conservation of momentum applied to the collision between the bullet and the block: Let object A be the bullet and object B be the block. Let v_A be the speed of the bullet before the collision and let V be the speed of the block with the bullet inside just after the collision.

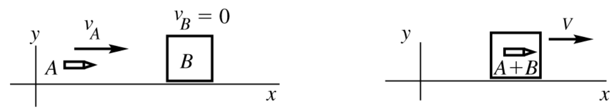

Figure 8.42a

P_x is constant gives $m_A v_A = (m_A + m_B)V$.

Conservation of energy applied to the motion of the block after the collision:

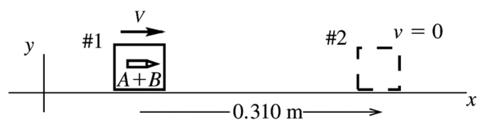

Figure 8.42b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $+y$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80\text{ m/s}^2)(0.310\text{ m})} = 1.1\text{ m/s}$

Use this result in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00\times10^{-3}\text{ kg} + 1.20\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)(1.1\text{ m/s}) = 266\text{ m/s, which rounds to 270 m/s.}$$

EVALUATE: When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

8.43. **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. After the collision the kinetic energy of the combined object is converted to gravitational potential energy.

SET UP: Immediately after the collision the combined object has speed V. Let h be the vertical height through which the pendulum rises.

EXECUTE: **(a)** Conservation of momentum applied to the collision gives

$(12.0\times10^{-3}\text{ kg})(380\text{ m/s}) = (6.00\text{ kg} + 12.0\times10^{-3}\text{ kg})V$ and $V = 0.758\text{ m/s}$.

Conservation of energy applied to the motion after the collision gives $\frac{1}{2}m_{\text{tot}}V^2 = m_{\text{tot}}gh$ and

$$h = \frac{V^2}{2g} = \frac{(0.758\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0293\text{ m} = 2.93\text{ cm}.$$

(b) $K = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(12.0\times10^{-3}\text{ kg})(380\text{ m/s})^2 = 866\text{ J}.$

(c) $K = \frac{1}{2}m_{tot}V^2 = \frac{1}{2}(6.00\text{ kg} + 12.0\times10^{-3}\text{ kg})(0.758\text{ m/s})^2 = 1.73\text{ J}.$

EVALUATE: Most of the initial kinetic energy of the bullet is dissipated in the collision.

8.44. **IDENTIFY:** During the collision, momentum is conserved. After the collision, mechanical energy is conserved.

SET UP: The collision occurs over a short time interval and the block moves very little during the collision, so the spring force during the collision can be neglected. Use coordinates where $+x$ is to the right. During the collision, momentum conservation gives $P_{1x} = P_{2x}$. After the collision, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$.

EXECUTE: *Collision*: There is no external horizontal force during the collision and $P_{1x} = P_{2x}$, so $(3.00\text{ kg})(8.00\text{ m/s}) = (15.0\text{ kg})v_{\text{block, 2}} - (3.00\text{ kg})(2.00\text{ m/s})$ and $v_{\text{block, 2}} = 2.00\text{ m/s}$.

Motion after the collision: When the spring has been compressed the maximum amount, all the initial kinetic energy of the block has been converted into potential energy $\frac{1}{2}kx^2$ that is stored in the compressed spring. Conservation of energy gives $\frac{1}{2}(15.0\text{ kg})(2.00\text{ m/s})^2 = \frac{1}{2}(500.0\text{ kg})x^2$, so $x = 0.346\text{ m}$.

EVALUATE: We cannot say that the momentum was converted to potential energy, because momentum and energy are different types of quantities.

8.45. **IDENTIFY:** The missile gives momentum to the ornament causing it to swing in a circular arc and thereby be accelerated toward the center of the circle.

SET UP: After the collision the ornament moves in an arc of a circle and has acceleration $a_{\text{rad}} = \frac{v^2}{r}$.

During the collision, momentum is conserved, so $P_{1x} = P_{2x}$. The free-body diagram for the ornament plus missile is given in Figure 8.45. Take $+y$ to be upward, since that is the direction of the acceleration. Take the $+x$-direction to be the initial direction of motion of the missile.

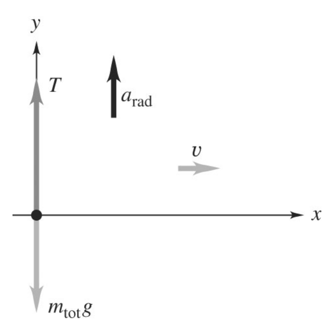

Figure 8.45

EXECUTE: Apply conservation of momentum to the collision. Using $P_{1x} = P_{2x}$, we get $(0.200\text{ kg})(12.0\text{ m/s}) = (1.00\text{ kg})V$, which gives $V = 2.40\text{ m/s}$, the speed of the ornament immediately after the collision. Then $\Sigma F_y = ma_y$ gives $T - m_{\text{tot}}g = m_{\text{tot}}\frac{v^2}{r}$. Solving for T gives

$$T = m_{\text{tot}}\left(g + \frac{v^2}{r}\right) = (1.00\text{ kg})\left(9.80\text{ m/s}^2 + \frac{(2.40\text{ m/s})^2}{1.50\text{ m}}\right) = 13.6\text{ N}.$$

EVALUATE: We cannot use energy conservation during the collision because it is an inelastic collision (the objects stick together).

8.46. IDENTIFY: No net external horizontal force so P_x is conserved. Elastic collision so $K_1 = K_2$ and can use $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$.

SET UP:

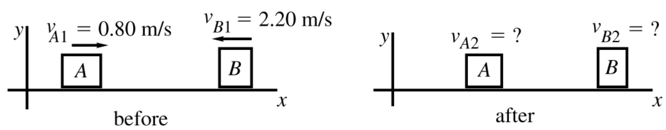

Figure 8.46

EXECUTE: From conservation of x-component of momentum:

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$m_A v_{A1} - m_B v_{B1} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.150\text{ kg})(0.80\text{ m/s}) - (0.300\text{ kg})(2.20\text{ m/s}) = (0.150\text{ kg})v_{A2x} + (0.300\text{ kg})v_{B2x}$$

$$-3.60\text{ m/s} = v_{\text{A}2x} + 2v_{B2x}$$

From the relative velocity equation for an elastic collision Eq. 8.27:

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) = -(-2.20\text{ m/s} - 0.80\text{ m/s}) = +3.00\text{ m/s}$$

$$3.00\text{ m/s} = -v_{\text{A}2x} + v_{B2x}$$

Adding the two equations gives $-0.60\text{ m/s} = 3v_{B2x}$ and $v_{B2x} = -0.20\text{ m/s}$. Then $v_{A2x} = v_{B2x} - 3.00\text{ m/s} = -3.20\text{ m/s}$.

The 0.150 kg glider (A) is moving to the left at 3.20 m/s and the 0.300 kg glider (B) is moving to the left at 0.20 m/s.

EVALUATE: We can use our v_{A2x} and v_{B2x} to show that P_x is constant and $K_1 = K_2$.

8.47. IDENTIFY: When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Since the collision is elastic, $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$ give the final velocity of each block after the collision.

SET UP: Let $+x$ be the direction of the initial motion of A.

EXECUTE: (a) Momentum conservation gives $(2.00\text{ kg})(2.00\text{ m/s}) = (8.00\text{ kg})V$ so $V = 0.500\text{ m/s}$.

Both blocks are moving at 0.500 m/s, in the direction of the initial motion of block A. Conservation of energy says the initial kinetic energy of A equals the total kinetic energy at maximum compression plus the potential energy U_b stored in the bumpers: $\frac{1}{2}(2.00\text{ kg})(2.00\text{ m/s})^2 = U_\text{b} + \frac{1}{2}(8.00\text{ kg})(0.500\text{ m/s})^2$ so $U_\text{b} = 3.00\text{ J}$.

(b) $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\dfrac{2.00\text{ kg} - 6.0\text{ kg}}{8.00\text{ kg}}\right)(2.00\text{ m/s}) = -1.00\text{ m/s}$. Block A is moving in the $-x$-direction at 1.00 m/s.

$v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x} = \dfrac{2(2.00\text{ kg})}{8.00\text{ kg}}(2.00\text{ m/s}) = +1.00\text{ m/s}$. Block B is moving in the $+x$-direction at 1.00 m/s.

EVALUATE: When the spring is compressed the maximum amount, the system must still be moving in order to conserve momentum.

8.48. **IDENTIFY:** Since the collision is elastic, both momentum conservation and equation $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ apply.

SET UP: Let object A be the 30.0 g marble and let object B be the 10.0 g marble. Let $+x$ be to the right.

EXECUTE: **(a)** Conservation of momentum gives
$(0.0300\text{ kg})(0.200\text{ m/s}) + (0.0100\text{ kg})(-0.400\text{ m/s}) = (0.0300\text{ kg})v_{A2x} + (0.0100\text{ kg})v_{B2x}$.
$3v_{A2x} + v_{B2x} = 0.200\text{ m/s}$. $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ says
$v_{B2x} - v_{A2x} = -(-0.400\text{ m/s} - 0.200\text{ m/s}) = +0.600\text{ m/s}$. Solving this pair of equations gives
$v_{A2x} = -0.100\text{ m/s}$ and $v_{B2x} = +0.500\text{ m/s}$. The 30.0 g marble is moving to the left at 0.100 m/s and the 10.0 g marble is moving to the right at 0.500 m/s.

(b) For marble A, $\Delta P_{Ax} = m_A v_{A2x} - m_A v_{A1x} = (0.0300\text{ kg})(-0.100\text{ m/s} - 0.200\text{ m/s}) = -0.00900\text{ kg}\cdot\text{m/s}$.
For marble B, $\Delta P_{Bx} = m_B v_{B2x} - m_B v_{B1x} = (0.0100\text{ kg})(0.500\text{ m/s} - [-0.400\text{ m/s}]) = +0.00900\text{ kg}\cdot\text{m/s}$.
The changes in momentum have the same magnitude and opposite sign.

(c) For marble A, $\Delta K_A = \frac{1}{2}m_A v_{A2}^2 - \frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}(0.0300\text{ kg})([0.100\text{ m/s}]^2 - [0.200\text{ m/s}]^2) = -4.5\times10^{-4}\text{ J}$.
For marble B, $\Delta K_B = \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(0.0100\text{ kg})([0.500\text{ m/s}]^2 - [0.400\text{ m/s}]^2) = +4.5\times10^{-4}\text{ J}$.
The changes in kinetic energy have the same magnitude and opposite sign.

EVALUATE: The results of parts (b) and (c) show that momentum and kinetic energy are conserved in the collision.

8.49. **IDENTIFY:** Equation $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ applyies, with object A being the neutron.

SET UP: Let $+x$ be the direction of the initial momentum of the neutron. The mass of a neutron is $m_{\text{n}} = 1.0\text{ u}$.

EXECUTE: **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \dfrac{1.0\text{ u} - 2.0\text{ u}}{1.0\text{ u} + 2.0\text{ u}}v_{A1x} = -v_{A1x}/3.0$. The speed of the neutron after the collision is one-third its initial speed.

(b) $K_2 = \frac{1}{2}m_{\text{n}}v_{\text{n}}^2 = \frac{1}{2}m_{\text{n}}(v_{A1}/3.0)^2 = \dfrac{1}{9.0}K_1$.

(c) After n collisions, $v_{A2} = \left(\dfrac{1}{3.0}\right)^n v_{A1}$. $\left(\dfrac{1}{3.0}\right)^n = \dfrac{1}{59{,}000}$, so $3.0^n = 59{,}000$. $n\log 3.0 = \log 59{,}000$ and $n = 10$.

EVALUATE: Since the collision is elastic, in each collision the kinetic energy lost by the neutron equals the kinetic energy gained by the deuteron.

8.50. **IDENTIFY:** Elastic collision. Solve for mass and speed of target nucleus.

SET UP: **(a)** Let A be the proton and B be the target nucleus. The collision is elastic, all velocities lie along a line, and B is at rest before the collision. Hence the results of equations $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$ apply.

EXECUTE: $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x}$: $m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax})$, where v_x is the velocity component of *A* before the collision and v_{Ax} is the velocity component of *A* after the collision. Here, $v_x = 1.50\times10^7$ m/s (take direction of incident beam to be positive) and $v_{Ax} = -1.20\times10^7$ m/s (negative since traveling in direction opposite to incident beam).

$$m_B = m_A\left(\frac{v_x - v_{Ax}}{v_x + v_{Ax}}\right) = m\left(\frac{1.50\times10^7\ \text{m/s} + 1.20\times10^7\ \text{m/s}}{1.50\times10^7\ \text{m/s} - 1.20\times10^7\ \text{m/s}}\right) = m\left(\frac{2.70}{0.30}\right) = 9.00m.$$

(b) $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_{A1x}$: $v_{Bx} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_x = \left(\dfrac{2m}{m + 9.00m}\right)(1.50\times10^7\ \text{m/s}) = 3.00\times10^6\ \text{m/s}.$

EVALUATE: Can use our calculated v_{Bx} and m_B to show that P_x is constant and that $K_1 = K_2$.

8.51. IDENTIFY: In any collision, momentum is conserved. But in this one, kinetic energy is conserved because it is an *elastic* collision.

SET UP: We use $p_x = mv_x$ and $K = \dfrac{1}{2}mv^2$. The total momentum is $P_x = p_{1x} + p_{2x}$. Call the +*x*-axis the original direction of object *A*. Start by making a before-and-after sketch of the collision, as shown in Fig. 8.51. Call v_0 the original speed of *A*. We want to find v_A after the collision.

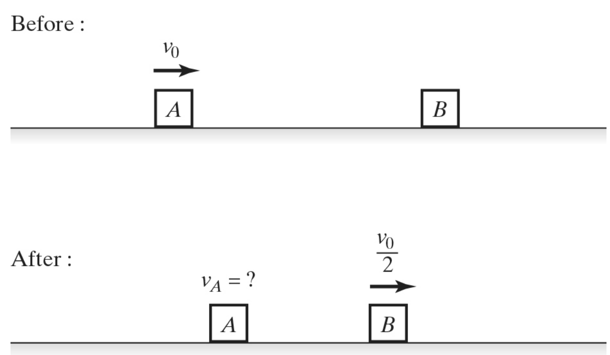

Figure 8.51

EXECUTE: **(a)** Which has greater mass, *A* or *B*? Apply momentum conservation and energy conservation to the collision. We know that $v_B = v_0/2$ after the collision. See the figure for the quantities used.

<u>Momentum conservation</u>: $m_A v_0 = m_A v_A + m_B \dfrac{v_0}{2}$ (Eq. 1)

<u>Energy conservation</u>: $\dfrac{1}{2}m_A v_0^2 = \dfrac{1}{2}m_A v_A^2 + \dfrac{1}{2}m_B\left(\dfrac{v_0}{2}\right)^2$ (Eq. 2)

Defining $R = m_B/m_A$, Eq. 1 becomes $v_A = v_0\left(1 - \dfrac{R}{2}\right)$. Using this result and simplifying, Eq. 2 becomes

$1 = \left(1 - \dfrac{R}{2}\right)^2 + \dfrac{R}{4}$. Squaring and solving for *R* gives *R* =3.

(b) Therefore $m_B/m_A = 3$, so *B* has 3 times the mass of *A*.

(c) From our result in part (a), $v_A = v_0\left(1 - \dfrac{R}{2}\right) = v_0\left(1 - \dfrac{3}{2}\right) = -\dfrac{v_0}{2} = -\dfrac{6.0\ \text{m/s}}{2} = -3.0\ \text{m/s}.$ The minus sign tells us that *A* is moving opposite to its original direction.

EVALUATE: Use our results to calculate the kinetic energy before and after the collision. $K_1 = \frac{1}{2}m_A v_0^2$ and $K_2 = \frac{1}{2}m_A\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}(3m_A)\left(\frac{v_0}{2}\right)^2 = \frac{1}{2}m_A v_0^2 = K_1$. This agrees with the fact that it is an elastic collision.

8.52. IDENTIFY: Calculate x_{cm}.

SET UP: Apply $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ with the sun as mass 1 and Jupiter as mass 2. Take the origin at the sun and let Jupiter lie on the positive x-axis.

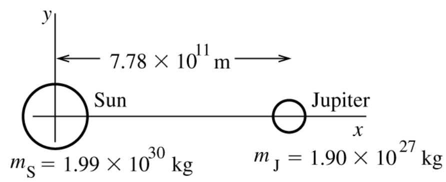

Figure 8.52

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

EXECUTE: $x_1 = 0$ and $x_2 = 7.78\times10^{11}$ m

$$x_{cm} = \frac{(1.90\times10^{27}\ \text{kg})(7.78\times10^{11}\ \text{m})}{1.99\times10^{30}\ \text{kg} + 1.90\times10^{27}\ \text{kg}} = 7.42\times10^8\ \text{m}$$

The center of mass is 7.42×10^8 m from the center of the sun and is on the line connecting the centers of the sun and Jupiter. The sun's radius is 6.96×10^8 m so the center of mass lies just outside the sun.

EVALUATE: The mass of the sun is much greater than the mass of Jupiter, so the center of mass is much closer to the sun. For each object we have considered all the mass as being at the center of mass (geometrical center) of the object.

8.53. IDENTIFY: The location of the center of mass is given by $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. The mass can be expressed in terms of the diameter. Each object can be replaced by a point mass at its center.

SET UP: Use coordinates with the origin at the center of Pluto and the $+x$-direction toward Charon, so $x_P = 0$, $x_C = 19{,}700$ km. $m = \rho V = \rho\frac{4}{3}\pi r^3 = \frac{1}{6}\rho\pi d^3$.

EXECUTE: $x_{cm} = \frac{m_Px_P + m_Cx_C}{m_P + m_C} = \left(\frac{m_C}{m_P + m_C}\right)x_C = \left(\frac{\frac{1}{6}\rho\pi d_C^3}{\frac{1}{6}\rho\pi d_P^3 + \frac{1}{6}\rho\pi d_C^3}\right)x_C = \left(\frac{d_C^3}{d_P^3 + d_C^3}\right)x_C$.

$$x_{cm} = \left(\frac{[1250\ \text{km}]^3}{[2370\ \text{km}]^3 + [1250\ \text{km}]^3}\right)(19{,}700\ \text{km}) = 2.52\times10^3\ \text{km}.$$

The center of mass of the system is 2.52×10^3 km from the center of Pluto.

EVALUATE: The center of mass is closer to Pluto because Pluto has more mass than Charon.

8.54. IDENTIFY: Apply $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$, $v_{cm,x} = \frac{m_Av_{A,x} + m_Bv_{B,x}}{m_A + m_B}$, and $P_x = Mv_{cm-x}$. There is only one component of position and velocity.

SET UP: $m_A = 1200$ kg, $m_B = 1800$ kg. $M = m_A + m_B = 3000$ kg. Let $+x$ be to the right and let the origin be at the center of mass of the station wagon.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0\text{ m}.$

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

(b) $P_x = m_A v_{A,x} + m_B v_{B,x} = (1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04 \times 10^4\text{ kg} \cdot \text{m/s}.$

(c) $v_{\text{cm},x} = \dfrac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \dfrac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{1200\text{ kg} + 1800\text{ kg}} = 16.8\text{ m/s}.$

(d) $P_x = M v_{\text{cm}-x} = (3000\text{ kg})(16.8\text{ m/s}) = 5.04 \times 10^4\text{ kg} \cdot \text{m/s},$ the same as in part (b).

EVALUATE: The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

8.55. **IDENTIFY:** This problem requires finding the location of the center of mass of a system of two objects.

SET UP: We can treat each object as a point mass located at its center of mass and use the formula $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$. We want to find the location of the center of mass of the two-mass sytstem.

EXECUTE: We'll use $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$, but first we find the dimensions of the cube. Its volume is x^3 = 0.0270 m^3, so x = 0.300 m, so its center of mass is 0.150 m above the floor. The center of mass of the sphere is 0.400 m above the top of the cube. The locations of each center of mass are $y_1 = y_{\text{cube}} = 0.150$ m and $y_2 = y_{\text{sphere}} = 0.400\text{ m} + 0.300\text{ m} = 0.700$ m. Now use the center of mass formula with the origin at the floor. $y_{\text{cm}} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \dfrac{(0.500\text{ kg})(0.150\text{ m}) + (0.800\text{ kg})(0.700\text{ m})}{1.300\text{ kg}} = 0.488$ m above the floor.

EVALUATE: The center of mass is closer to the center of the sphere than to the center of the cube since the sphere has more mass.

8.56. **IDENTIFY:** Use $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$.

SET UP: The target variable is m_1.

EXECUTE: $x_{\text{cm}} = 2.0\text{ m}$, $y_{\text{cm}} = 0$

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1(0) + (0.10\text{ kg})(8.0\text{ m})}{m_1 + (0.10\text{ kg})} = \frac{0.80\text{ kg} \cdot \text{m}}{m_1 + 0.10\text{ kg}}.$$

$$x_{\text{cm}} = 2.0\text{ m gives } 2.0\text{ m} = \frac{0.80\text{ kg} \cdot \text{m}}{m_1 + 0.10\text{ kg}}.$$

$$m_1 + 0.10\text{ kg} = \frac{0.80\text{ kg} \cdot \text{m}}{2.0\text{ m}} = 0.40\text{ kg}.$$

$$m_1 = 0.30\text{ kg}.$$

EVALUATE: The cm is closer to m_1 so its mass is larger then m_2.

(b) IDENTIFY: Use $\vec{P} = M\vec{v}_{\text{cm}}$ to calculate $\vec{P}$.

SET UP: $\vec{v}_{\text{cm}} = (5.0\text{ m/s})\,\hat{i}$.

$$\vec{P} = M\vec{v}_{\text{cm}} = (0.10\text{ kg} + 0.30\text{ kg})(5.0\text{ m/s})\,\hat{i} = (2.0\text{ kg} \cdot \text{m/s})\hat{i}.$$

(c) IDENTIFY: Use $\vec{v}_{\text{cm}} = \dfrac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$.

SET UP: $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$. The target variable is $\vec{v}_1$. Particle 2 at rest says $v_2 = 0$.

EXECUTE: $\vec{v}_1 = \left(\dfrac{m_1 + m_2}{m_1}\right)\vec{v}_{cm} = \left(\dfrac{0.30\text{ kg} + 0.10\text{ kg}}{0.30\text{ kg}}\right)(5.00\text{ m/s})\hat{i} = (6.7\text{ m/s})\hat{i}$.

EVALUATE: Using the result of part (c) we can calculate $\vec{p}_1$ and $\vec{p}_2$ and show that $\vec{P}$ as calculated in part (b) does equal $\vec{p}_1 + \vec{p}_2$.

8.57. IDENTIFY: There is no net external force on the system of James, Ramon, and the rope; the momentum of the system is conserved, and the velocity of its center of mass is constant. Initially there is no motion, and the velocity of the center of mass remains zero after Ramon has started to move.

SET UP: Let $+x$ be in the direction of Ramon's motion. Ramon has mass $m_R = 60.0$ kg and James has mass $m_J = 90.0$ kg.

EXECUTE: $v_{\text{cm-}x} = \dfrac{m_R v_{Rx} + m_J v_{Jx}}{m_R + m_J} = 0$.

$v_{Jx} = -\left(\dfrac{m_R}{m_J}\right)v_{Rx} = -\left(\dfrac{60.0\text{ kg}}{90.0\text{ kg}}\right)(1.10\text{ m/s}) = -0.733\text{ m/s}$. James' speed is 0.733 m/s.

EVALUATE: As they move, the two men have momenta that are equal in magnitude and opposite in direction, and the total momentum of the system is zero. Also, Example 8.14 shows that Ramon moves farther than James in the same time interval. This is consistent with Ramon having a greater speed.

8.58. (a) IDENTIFY and SET UP: Apply $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ and solve for m_1 and m_2.

EXECUTE: $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$m_1 + m_2 = \dfrac{m_1 y_1 + m_2 y_2}{y_{cm}} = \dfrac{m_1(0) + (0.50\text{ kg})(6.0\text{ m})}{2.4\text{ m}} = 1.25\text{ kg}$ and $m_1 = 0.75$ kg.

EVALUATE: y_{cm} is closer to m_1 since $m_1 > m_2$.

(b) IDENTIFY and SET UP: Apply $\vec{a} = d\vec{v}/dt$ for the cm motion.

EXECUTE: $\vec{a}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} = (1.5\text{ m/s}^3)t\hat{i}$.

(c) IDENTIFY and SET UP: Apply $\sum\vec{F}_{ext} = M\vec{a}_{cm}$.

EXECUTE: $\sum\vec{F}_{ext} = M\vec{a}_{cm} = (1.25\text{ kg})(1.5\text{ m/s}^3)t\hat{i}$.

At $t = 3.0$ s, $\sum\vec{F}_{ext} = (1.25\text{ kg})(1.5\text{ m/s}^3)(3.0\text{ s})\hat{i} = (5.6\text{ N})\hat{i}$.

EVALUATE: $v_{\text{cm-}x}$ is positive and increasing so $a_{\text{cm-}x}$ is positive and $\vec{F}_{ext}$ is in the $+x$-direction. There is no motion and no force component in the y-direction.

8.59. IDENTIFY: Apply $\sum\vec{F} = \dfrac{d\vec{P}}{dt}$ to the airplane.

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$. $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$

EXECUTE: $\dfrac{d\vec{P}}{dt} = [-(1.50\text{ kg}\cdot\text{m/s}^3)t]\hat{i} + (0.25\text{ kg}\cdot\text{m/s}^2)\hat{j}$. $F_x = -(1.50\text{ N/s})t$, $F_y = 0.25$ N, $F_z = 0$.

EVALUATE: There is no momentum or change in momentum in the z-direction and there is no force component in this direction.

8.60. IDENTIFY: This problem requires finding the location of the center of mass of a system of three objects.

SET UP: We can treat each object as a point mass located at its center of mass and use the formula $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ We want to find the location of the center of mass of the three-mass sytstem. The center of mass of the bottom piece is a distance $L/4$ before the edge of the table, that of the middle piece is above the edge of the table, and that of the upper piece is $L/4$ beyond the edge of the table. We treat each square of wood as a point mass located at its center of mass. Call the origin of the x-axis at the edge of the table.

EXECUTE: Using $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{m(-L/4) + m(0) + m(L/4)}{3m} = 0$, so the center of mass is directly above the edge of the table.

EVALUATE: Our result is reasonable because of the symmetry of the arrangement. We have a mass $L/4$ before the edge, an equal mass mass $L/4$ beyond the edge, and a mass at the edge, so the center of mass should be at the edge. The vertical location of the center of mass would be at the middle of the middle piece.

8.61. **IDENTIFY:** $a = -\dfrac{v_{ex}}{m}\dfrac{dm}{dt}$. Assume that dm/dt is constant over the 5.0 s interval, since m doesn't change much during that interval. The thrust is $F = -v_{ex}\dfrac{dm}{dt}$.

SET UP: Take m to have the constant value $110\text{ kg} + 70\text{ kg} = 180\text{ kg}$. dm/dt is negative since the mass of the MMU decreases as gas is ejected.

EXECUTE: **(a)** $\dfrac{dm}{dt} = -\dfrac{m}{v_{ex}}a = -\left(\dfrac{180\text{ kg}}{490\text{ m/s}}\right)(0.029\text{ m/s}^2) = -0.0106\text{ kg/s}$. In 5.0 s the mass that is ejected is $(0.0106\text{ kg/s})(5.0\text{ s}) = 0.053\text{ kg}$.

(b) $F = -v_{ex}\dfrac{dm}{dt} = -(490\text{ m/s})(-0.0106\text{ kg/s}) = 5.19\text{ N}$.

EVALUATE: The mass change in the 5.0 s is a very small fraction of the total mass m, so it is accurate to take m to be constant.

8.62. **IDENTIFY:** Use $F = -v_{ex}\dfrac{\Delta m}{\Delta t}$, applied to a finite time interval.

SET UP: $v_{ex} = 1600\text{ m/s}$

EXECUTE: **(a)** $F = -v_{ex}\dfrac{\Delta m}{\Delta t} = -(1600\text{ m/s})\dfrac{-0.0500\text{ kg}}{1.00\text{ s}} = +80.0\text{ N}$.

(b) The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the gas in a direction with a component perpendicular to the rocket's velocity and braked by ejecting it in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

EVALUATE: The thrust depends on the speed of the ejected gas relative to the rocket and on the mass of gas ejected per second.

8.63. **IDENTIFY:** This problem involves both static and kinetic friction as well as the impulse-momentum theorem and energy conservation.

SET UP: Once we start the crate moving, the friction force on it is kinetic friction. Impulse is $J_x = F_xt$, where F_x is the net force acting. The impulse-momentum theorem is $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. Energy conservation says that $U_1 + K_1 + W_{other} = U_2 + K_2$, and $W = Fs\cos\phi$. Call the push P.

EXECUTE: **(a)** Estimate: The heaviest crate weighs about 100 lb, so the maximum force need to start it moving is $f_{max} = \mu_s n = (0.500)(100\text{ lb}) = 50\text{ lb} \approx 220\text{ N}$. The mass of a 100-lb weight is about 45 kg.

(b) Now the friction force is due to kinetic friction, so we use $f_k = \mu_k n = \mu_k mg$, so the net force is $F_{net} = P - f_k = P - \mu_k mg$. The impulse-momentum theorem gives $F_{net} = P - f_k = P - \mu_k mg$, so

$$t = \frac{mv}{P - \mu_k mg} = \frac{(45\text{ kg})(8.0\text{ m/s})}{220\text{ N} - (0.300)(450\text{ N})} = 4.2\text{ s.}$$

(c) Using $U_1 + K_1 + W_{other} = U_2 + K_2$, we have $U_1 = U_2 = 0$, $K_1 = 0$, and $W_{tot} = W_P + W_f$. Using $W = Fs\cos\phi$ gives $W_{tot} = -\mu_k mgs + Ps$, so energy conservation gives $-\mu_k mgs + Ps = \frac{1}{2}mv^2$. Solving for s gives $s = \dfrac{\frac{1}{2}mv^2}{P - \mu_k mg} = s = \dfrac{\frac{1}{2}(45\text{ kg})(8.0\text{ m/s})^2}{220\text{ N} - (0.300)(450\text{ N})} = 17\text{ m.}$

EVALUATE: As a check, use $\sum F_x = ma_x$: $-\mu_k mg + P = ma_x$ gives $a_x = 1.89\text{ m/s}^2$. Then use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, which gives $x - x_0 = 17$ m, as we just found.

8.64. IDENTIFY: Use the heights to find v_{1y} and v_{2y}, the velocity of the ball just before and just after it strikes the slab. Then apply $J_y = F_y\Delta t = \Delta p_y$.

SET UP: Let $+y$ be downward.

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

$v_{1y} = +\sqrt{2(9.80\text{ m/s}^2)(2.00\text{ m})} = 6.26\text{ m/s}$. $v_{2y} = -\sqrt{2(9.80\text{ m/s}^2)(1.60\text{ m})} = -5.60\text{ m/s}$.

$$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times10^{-3}\text{ kg})(-5.60\text{ m/s} - 6.26\text{ m/s}) = -0.474\text{ kg}\cdot\text{m/s}.$$

The impulse is $0.474\text{ kg}\cdot\text{m/s}$, upward.

(b) $F_y = \dfrac{J_y}{\Delta t} = \dfrac{-0.474\text{ kg}\cdot\text{m/s}}{2.00\times10^{-3}\text{ s}} = -237\text{ N}$. The average force on the ball is 237 N, upward.

EVALUATE: The upward force, on the ball changes the direction of its momentum.

8.65. IDENTIFY: The impulse, force, and change in velocity are related by $J_x = F_x\Delta t$.

SET UP: $m = w/g = 0.0571\text{ kg}$. Since the force is constant, $\vec{F} = \vec{F}_{av}$.

EXECUTE: **(a)** $J_x = F_x\Delta t = (-380\text{ N})(3.00\times10^{-3}\text{ s}) = -1.14\text{ N}\cdot\text{s}$.

$J_y = F_y\Delta t = (110\text{ N})(3.00\times10^{-3}\text{ s}) = 0.330\text{ N}\cdot\text{s}$.

(b) $v_{2x} = \dfrac{J_x}{m} + v_{1x} = \dfrac{-1.14\text{ N}\cdot\text{s}}{0.0571\text{ kg}} + 20.0\text{ m/s} = 0.04\text{ m/s}$.

$v_{2y} = \dfrac{J_y}{m} + v_{1y} = \dfrac{0.330\text{ N}\cdot\text{s}}{0.0571\text{ kg}} + (-4.0\text{ m/s}) = +1.8\text{ m/s}$.

EVALUATE: The change in velocity $\Delta\vec{v}$ is in the same direction as the force, so $\Delta\vec{v}$ has a negative x-component and a positive y-component.

8.66. IDENTIFY: We use the impulse-momentum theorem.

SET UP: The impulse-momentum theorem is $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. For a variable force, we use $J_x = \int F_x dt$. From the information given, $F_x = (3.00\text{ N/s})t$.

EXECUTE: The impulse is $J_x = \int F_x dt = \int (3.00\text{ N/s})t\,dt = (1.50\text{ N/s})t^2$, where we have used $J_x = 0$ when $t = 0$. Now use $J_x = p_{2x} - p_{1x}$, giving $(1.50\text{ N/s})t^2 = mv - 0$. Solving for t gives

$t = \sqrt{\dfrac{mv}{1.50\text{ N/s}}} = \sqrt{\dfrac{(2.00\text{ kg})(9.00\text{ m/s})}{1.50\text{ N/s}}} = 3.46\text{ s}$. Therefore $F = (3.00\text{ N/s})(3.464\text{ s}) = 10.4$ N.

EVALUATE: We could find the impulse graphically as the area under the F versus t graph. This area is a triangle of base t and height F, where F = (3.00 N/s)t. The area of a triangle is $\frac{1}{2}bh$, so the impulse is $J_x = \frac{1}{2}Ft = \frac{1}{2}(3.00\text{ N/s})t = (1.50\text{ N/s})t^2$.

8.67. **IDENTIFY** and **SET UP:** When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity V relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Let $+x$ be the direction of the initial motion of A. The collision is elastic.

SET UP: $p = mv$, $K = ½\, mv^2$, $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ for an elastic collision.

EXECUTE: **(a)** The maximum energy stored in the spring is at maximum compression, at which time the blocks have the same velocity. Momentum conservation gives $m_A v_{A1} + m_B v_{B1} = (m_A + m_B)V$. Putting in the numbers we have (2.00 kg)(2.00 m/s) + (10.0 kg)(–0.500 m/s) = (12.0 kg)V, giving V = –0.08333 m/s. The energy U_{spring} stored in the spring is the loss of kinetic of the system. Therefore $U_{\text{spring}} = K_1 - K_2 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{V1}^2 - \frac{1}{2}(m_A + m_B)V^2$. Putting in the same set of numbers as above, and using V = –0.08333 m/s, we get U_{spring} = 5.21 J. At this time, the blocks are both moving to the left, so their velocities are each –0.0833 m/s.

(b) Momentum conservation gives $m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$. Putting in the numbers gives –1 m/s = $2v_{A2} + 10v_{B2}$. Using $v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$ we get $v_{B2x} - v_{A2x}$ = –(–0.500 m/s – 2.00 m/s) = +2.50 m/s. Solving this equation and the momentum equation simultaneously gives v_{A2x} = 2.17 m/s and v_{B2x} = 0.333 m/s.

EVALUATE: The total kinetic energy before the collision is 5.25 J, and it is the same after, which is consistent with an elastic collision.

8.68. **IDENTIFY:** Use a coordinate system attached to the ground. Take the x-axis to be east (along the tracks) and the y-axis to be north (parallel to the ground and perpendicular to the tracks). Then P_x is conserved and P_y is *not* conserved, due to the sideways force exerted by the tracks, the force that keeps the handcar on the tracks.

(a) SET UP: Let A be the 25.0 kg mass and B be the car (mass 175 kg). After the mass is thrown sideways relative to the car it still has the same eastward component of velocity, 5.00 m/s as it had before it was thrown.

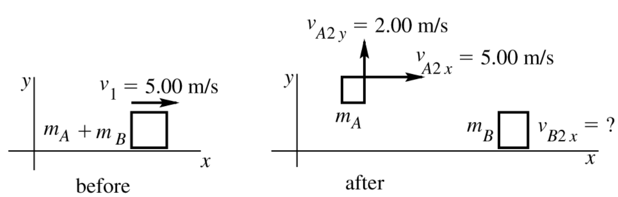

Figure 8.68a

P_x is conserved so $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$

EXECUTE: $(200\text{ kg})(5.00\text{ m/s}) = (25.0\text{ kg})(5.00\text{ m/s}) + (175\text{ kg})v_{B2x}$.

$$v_{B2x} = \frac{1000\text{ kg}\cdot\text{m/s} - 125\text{ kg}\cdot\text{m/s}}{175\text{ kg}} = 5.00\text{ m/s}.$$

The final velocity of the car is 5.00 m/s, east (unchanged).

EVALUATE: The thrower exerts a force on the mass in the y-direction and by Newton's third law the mass exerts an equal and opposite force in the $-y$-direction on the thrower and car.

(b) SET UP: We are applying $P_x = \text{constant}$ in coordinates attached to the ground, so we need the final velocity of A relative to the ground. Use the relative velocity addition equation. Then use $P_x = \text{constant}$ to find the final velocity of the car.

EXECUTE: $\vec{v}_{A/E} = \vec{v}_{A/B} + \vec{v}_{B/E}$

$v_{B/E} = +5.00\text{ m/s}$

$v_{A/B} = -5.00\text{ m/s}$ (minus since the mass is moving west relative to the car). This gives $v_{A/E} = 0$; the mass is at rest relative to the earth after it is thrown backwards from the car.

As in part (a) $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$.

Now $v_{A2x} = 0$, so $(m_A + m_B)v_1 = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A + m_B}{m_B}\right)v_1 = \left(\frac{200\text{ kg}}{175\text{ kg}}\right)(5.00\text{ m/s}) = 5.71\text{ m/s}.$$

The final velocity of the car is 5.71 m/s, east.

EVALUATE: The thrower exerts a force in the $-x$-direction so the mass exerts a force on him in the $+x$-direction, and he and the car speed up.

(c) SET UP: Let A be the 25.0 kg mass and B be the car (mass $m_B = 200\text{ kg}$).

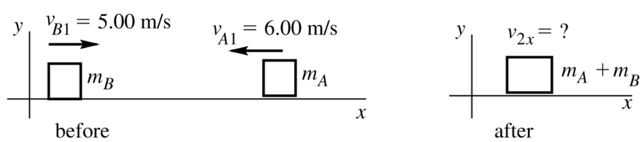

Figure 8.68b

P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

EXECUTE: $-m_A v_{A1} + m_B v_{B1} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_B v_{B1} - m_A v_{A1}}{m_A + m_B} = \frac{(200\text{ kg})(5.00\text{ m/s}) - (25.0\text{ kg})(6.00\text{ m/s})}{200\text{ kg} + 25.0\text{ kg}} = 3.78\text{ m/s}.$$

The final velocity of the car is 3.78 m/s, east.

EVALUATE: The mass has negative p_x so reduces the total P_x of the system and the car slows down.

8.69. IDENTIFY: The x- and y-components of the momentum of the system are conserved.

SET UP: After the collision the combined object with mass $m_{\text{tot}} = 0.100\text{ kg}$ moves with velocity $\vec{v}_2$. Solve for v_{Cx} and v_{Cy}.

EXECUTE: (a) $P_{1x} = P_{2x}$ gives $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{\text{tot}} v_{2x}$.

$$v_{Cx} = -\frac{m_A v_{Ax} + m_B v_{Bx} - m_{\text{tot}} v_{2x}}{m_C}$$

$$v_{Cx} = -\frac{(0.020\text{ kg})(-1.50\text{ m/s}) + (0.030\text{ kg})(-0.50\text{ m/s})\cos 60^\circ - (0.100\text{ kg})(0.50\text{ m/s})}{0.050\text{ kg}}.$$

$$v_{Cx} = 1.75\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{Ay} + m_B v_{By} + m_C v_{Cy} = m_{\text{tot}} v_{2y}$.

$$v_{Cy} = -\frac{m_A v_{Ay} + m_B v_{By} - m_{\text{tot}} v_{2y}}{m_C} = -\frac{(0.030\text{ kg})(-0.50\text{ m/s})\sin 60^\circ}{0.050\text{ kg}} = +0.260\text{ m/s}.$$

(b) $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 1.77$ m/s. $\Delta K = K_2 - K_1$.

$\Delta K = \frac{1}{2}(0.100\text{ kg})(0.50\text{ m/s})^2 - [\frac{1}{2}(0.020\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(0.030\text{ kg})(0.50\text{ m/s})^2 + \frac{1}{2}(0.050\text{ kg})(1.77\text{ m/s})^2]$
$\Delta K = -0.092$ J.

EVALUATE: Since there is no horizontal external force the vector momentum of the system is conserved. The forces the spheres exert on each other do negative work during the collision and this reduces the kinetic energy of the system.

8.70. **IDENTIFY:** We need to use the impulse-momentum theorem.
SET UP: The impulse-momentum theorem is $J_x = p_{2x} - p_{1x}$, where $p_x = mv_x$. The impulse is equal to the area under a graph of F versus t.
EXECUTE: **(a)** Figure 8.70 shows the graph of F versus t.

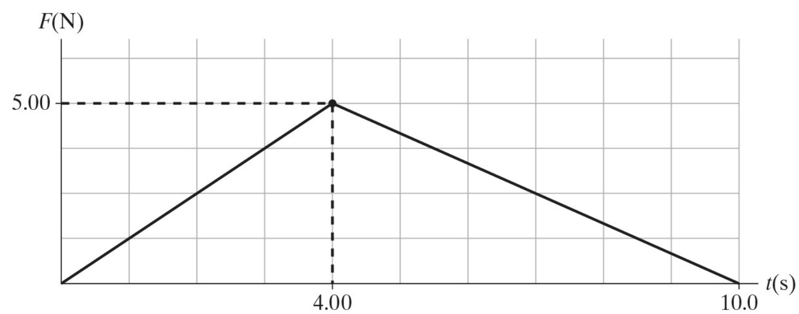

Figure 8.70

(b) Using the area of a triangle, the area under the graph is $\frac{1}{2}(5.00\text{ N})(10.0\text{ s}) = 25.0\text{ N}\cdot\text{s}$. Now use $J_x = p_{2x} - p_{1x}$: $25.0\text{ N}\cdot\text{s} = (3.00\text{ kg})v$, so $v = 8.33$ m/s.

EVALUATE: Using the area under the curve to find the J_x is much easier than integrating $\int F_x dt$ because the force changes at 4.00 s, so the integral would have to be done in two parts.

8.71. **IDENTIFY:** Momentum is conserved during the collision, and the wood (with the clay attached) is in free fall as it falls since only gravity acts on it.
SET UP: Apply conservation of momentum to the collision to find the velocity V of the combined object just after the collision. After the collision, the wood's downward acceleration is g and it has no horizontal acceleration, so we can use the standard kinematics equations: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

EXECUTE: Momentum conservation gives $(0.500\text{ kg})(24.0\text{ m/s}) = (8.50\text{ kg})V$, so $V = 1.412$ m/s. Consider the projectile motion after the collision: $a_y = +9.8\text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = +2.20$ m, and t is unknown. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(2.20\text{ m})}{9.8\text{ m/s}^2}} = 0.6701$ s. The horizontal acceleration is zero so $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.412\text{ m/s})(0.6701\text{ s}) = 0.946$ m.

EVALUATE: The momentum is *not* conserved after the collision because an external force (gravity) acts on the system. Mechanical energy is *not* conserved during the collision because the clay and block stick together, making it an inelastic collision.

8.72. **IDENTIFY:** An inelastic collision (the objects stick together) occurs during which momentum is conserved, followed by a swing during which mechanical energy is conserved. The target variable is the initial speed of the bullet.

SET UP: Newton's second law, $\Sigma\vec{F} = m\vec{a}$, will relate the tension in the cord to the speed of the block during the swing. Mechanical energy is conserved after the collision, and momentum is conserved during the collision.

EXECUTE: First find the speed v of the block, at a height of 0.800 m. The mass of the combined object is 0.812 kg. $\cos\theta = \dfrac{0.8\text{ m}}{1.6\text{ m}} = 0.50$ so $\theta = 60.0^\circ$ is the angle the cord makes with the vertical. At this position, Newton's second law gives $T - mg\cos\theta = m\dfrac{v^2}{R}$, where we have taken force components toward the center of the circle. Solving for v gives

$v = \sqrt{\dfrac{R}{m}(T - mg\cos\theta)} = \sqrt{\dfrac{1.6\text{ m}}{0.812\text{ kg}}(4.80\text{ N} - 3.979\text{ N})} = 1.272\text{ m/s}.$ Now apply conservation of energy to find the velocity V of the combined object just after the collision: $\dfrac{1}{2}mV^2 = mgh + \dfrac{1}{2}mv^2$. Solving for V gives $V = \sqrt{2gh + v^2} = \sqrt{2(9.8\text{ m/s}^2)(0.8\text{ m}) + (1.272\text{ m/s})^2} = 4.159\text{ m/s}.$ Now apply conservation of momentum to the collision: $(0.012\text{ kg})v_0 = (0.812\text{ kg})(4.159\text{ m/s}),$ which gives $v_0 = 281\text{ m/s}.$

EVALUATE: We cannot solve this problem in a single step because different conservation laws apply to the collision and the swing.

8.73. **IDENTIFY:** During the collision, momentum is conserved, but after the collision mechanical energy is conserved. We cannot solve this problem in a single step because the collision and the motion after the collision involve different conservation laws.

SET UP: Use coordinates where $+x$ is to the right and $+y$ is upward. Momentum is conserved during the collision, so $P_{1x} = P_{2x}$. Energy is conserved after the collision, so $K_1 = U_2$, where $K = \frac{1}{2}mv^2$ and $U = mgh.$

EXECUTE: *Collision*: There is no external horizontal force during the collision so $P_{1x} = P_{2x}$. This gives $(5.00\text{ kg})(12.0\text{ m/s}) = (10.0\text{ kg})v_2$ and $v_2 = 6.0\text{ m/s}.$

Motion after the collision: Only gravity does work and the initial kinetic energy of the combined chunks is converted entirely to gravitational potential energy when the chunk reaches its maximum height h above the valley floor. Conservation of energy gives $\frac{1}{2}m_{\text{tot}}v^2 = m_{\text{tot}}gh$ and

$$h = \frac{v^2}{2g} = \frac{(6.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 1.8\text{ m}.$$

EVALUATE: After the collision the energy of the system is $\frac{1}{2}m_{\text{tot}}v^2 = \frac{1}{2}(10.0\text{ kg})(6.0\text{ m/s})^2 = 180\text{ J}$ when it is all kinetic energy and the energy is $m_{\text{tot}}gh = (10.0\text{ kg})(9.8\text{ m/s}^2)(1.8\text{ m}) = 180\text{ J}$ when it is all gravitational potential energy. Mechanical energy is conserved during the motion after the collision. But before the collision the total energy of the system is $\frac{1}{2}(5.0\text{ kg})(12.0\text{ m/s})^2 = 360\text{ J}$; 50% of the mechanical energy is dissipated during the inelastic collision of the two chunks.

8.74. **IDENTIFY:** Momentum is conserved during the collision. After that we use energy conservation for B.

SET UP: $P_1 = P_2$ during the collision. For B, $K_1 + U_1 = K_2 + U_2$ after the collision.

EXECUTE: For the collision, $P_1 = P_2$: (2.00 kg)(8.00 m/s) = (2.00 kg)(–2.00 m/s) + (4.00 kg)v_B, which gives v_B = 5.00 m/s. Now look at B after the collision and apply $K_1 + U_1 = K_2 + U_2$.

$K_1 + U_1 = K_2 + 0$: $\frac{1}{2} mv_B^2 + mgh = \frac{1}{2} mv^2$

$v^2 = (5.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.60\text{ m})$, which gives v = 8.72 m/s.

EVALUATE: We cannot do this problem in a single step because we have two different conservation laws involved: momentum during the collision and energy after the collision. The energy is not conserved during the collision, and the momentum of B is not conserved after the collision.

8.75. IDENTIFY: We use momentum conservation during the collision of the carts. Half the initial kinetic energy is lost during the collision.

SET UP: Use $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$ and $K = \frac{1}{2}mv^2$. Call the $+x$-axis the original direction that cart A is moving. Fig. 8.75 shows a before and after sketch. The carts have equal masses, the final kinetic energy is one-half the initial kinetic energy, and we want to find the speed of each cart after the collision.

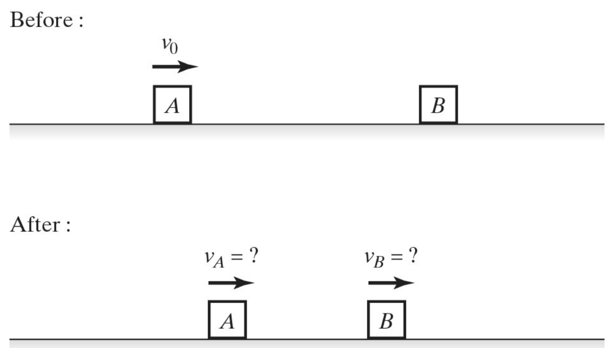

Figure 8.75

EXECUTE: Using the notation in the figure, momentum conservation gives us $mv_0 = mv_A + mv_B$, so $v_0 = v_A + v_B$. (Eq. 1)

Half the kinetic energy is lost, so $K_2 = \frac{1}{2}K_1 = \frac{1}{2}\left(\frac{1}{2}mv_0^2\right) = \frac{1}{4}mv_0^2$, which simplifies to

$v_A^2 + v_B^2 = \frac{1}{2}v_0^2$. (Eq. 2)

Solve Eq. 1 and Eq. 2 together, which gives $v_B = \frac{v_0}{2}$. Now find v_A using Eq. 1:

$v_A = v_0 - v_B = v_0 - \frac{v_0}{2} = \frac{v_0}{2}$. Returning to the notation stated in the problem, we find that each cart has speed $\frac{v_A}{2}$ after the collision.

EVALUATE: To check, we calculate the kinetic energy before and after the collision. $K_1 = \frac{1}{2}mv_0^2$ and $K_2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}m\left(\frac{v_0^2}{4} + \frac{v_0^2}{4}\right) = \frac{1}{2}\left(\frac{1}{2}mv_0^2\right) = \frac{1}{2}K_1$. Our result checks.

8.76. IDENTIFY: During the inelastic collision, momentum is conserved but not mechanical energy. After the collision, momentum is not conserved and the kinetic energy of the cars is dissipated by nonconservative friction.

SET UP: Treat the collision and motion after the collision as separate events. Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. The friction force on the combined cars is $\mu_k(m_A + m_B)g$.

EXECUTE: *Motion after the collision*: The kinetic energy of the combined cars immediately after the collision is taken away by the negative work done by friction: $\frac{1}{2}(m_A + m_B)V^2 = \mu_k(m_A + m_B)gd$, where $d = 7.15$ m. This gives $V = \sqrt{2\mu_k gd} = 9.54$ m/s.

Collision: Momentum conservation gives $m_A v_A = (m_A + m_B)V$, which gives

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{1500 \text{ kg} + 1900 \text{ kg}}{1500 \text{ kg}}\right)(9.54 \text{ m/s}) = 21.6 \text{ m/s}.$$

(b) $v_A = 21.6 \text{ m/s} = 48 \text{ mph}$, which is 13 mph greater than the speed limit.

EVALUATE: We cannot solve this problem in a single step because the collision and the motion after the collision involve different principles (momentum conservation and energy conservation).

8.77. **IDENTIFY:** During the inelastic collision, momentum is conserved (in two dimensions), but after the collision we must use energy principles.

SET UP: The friction force is $\mu_k m_{tot} g$. Use energy considerations to find the velocity of the combined object immediately after the collision. Apply conservation of momentum to the collision. Use coordinates where $+x$ is west and $+y$ is south. For momentum conservation, we have $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

EXECUTE: *Motion after collision*: The negative work done by friction takes away all the kinetic energy that the combined object has just after the collision. Calling ϕ the angle south of west at which the enmeshed cars slid, we have $\tan\phi = \dfrac{6.43 \text{ m}}{5.39 \text{ m}}$ and $\phi = 50.0°$. The wreckage slides 8.39 m in a direction 50.0° south of west. Energy conservation gives $\frac{1}{2}m_{tot}V^2 = \mu_k m_{tot} gd$, so

$V = \sqrt{2\mu_k gd} = \sqrt{2(0.75)(9.80 \text{ m/s}^2)(8.39 \text{ m})} = 11.1$ m/s. The velocity components are

$V_x = V\cos\phi = 7.13$ m/s; $V_y = V\sin\phi = 8.50$ m/s.

Collision: $P_{1x} = P_{2x}$ gives $(2200 \text{ kg})v_{SUV} = (1500 \text{ kg} + 2200 \text{ kg})V_x$ and $v_{SUV} = 12$ m/s. $P_{1y} = P_{2y}$ gives $(1500 \text{ kg})v_{sedan} = (1500 \text{ kg} + 2200 \text{ kg})V_y$ and $v_{sedan} = 21$ m/s.

EVALUATE: We cannot solve this problem in a single step because the collision and the motion after the collision involve different principles (momentum conservation and energy conservation).

8.78. **IDENTIFY:** Find k for the spring from the forces when the frame hangs at rest, use constant acceleration equations to find the speed of the putty just before it strikes the frame, apply conservation of momentum to the collision between the putty and the frame, and then apply conservation of energy to the motion of the frame after the collision.

SET UP: Use the free-body diagram in Figure 8.78a for the frame when it hangs at rest on the end of the spring to find the force constant k of the spring. Let s be the amount the spring is stretched.

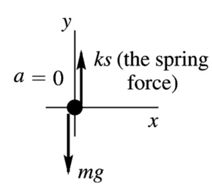

Figure 8.78a

EXECUTE: $\Sigma F_y = ma_y$ gives $-mg + ks = 0$. $k = \dfrac{mg}{s} = \dfrac{(0.150 \text{ kg})(9.80 \text{ m/s}^2)}{0.0400 \text{ m}} = 36.75$ N/m.

SET UP: Next find the speed of the putty when it reaches the frame. The putty falls with acceleration $a = g$, downward (see Figure 8.78b).

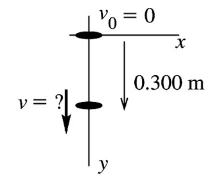

Figure 8.78b

$v_0 = 0,\ \ y - y_0 = 0.300\text{ m},\ \ a = +9.80\text{ m/s}^2,$ and we want to find v. The constant-acceleration $v^2 = v_0^2 + 2a(y - y_0)$ applies to this motion.

EXECUTE: $v = \sqrt{2a(y - y_0)} = \sqrt{2(9.80\text{ m/s}^2)(0.300\text{ m})} = 2.425\text{ m/s}.$

SET UP: Apply conservation of momentum to the collision between the putty (A) and the frame (B). See Figure 8.78c.

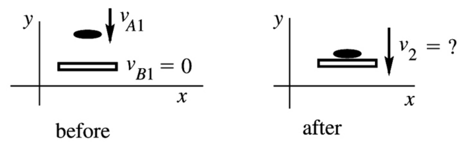

Figure 8.78c

P_y is conserved, so $-m_A v_{A1} = -(m_A + m_B)v_2.$

EXECUTE: $v_2 = \left(\dfrac{m_A}{m_A + m_B}\right)v_{A1} = \left(\dfrac{0.200\text{ kg}}{0.350\text{ kg}}\right)(2.425\text{ m/s}) = 1.386\text{ m/s}.$

SET UP: Apply conservation of energy to the motion of the frame on the end of the spring after the collision. Let point 1 be just after the putty strikes and point 2 be when the frame has its maximum downward displacement. Let d be the amount the frame moves downward (see Figure 8.78d).

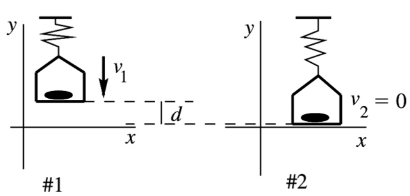

Figure 8.78d

When the frame is at position 1 the spring is stretched a distance $x_1 = 0.0400\text{ m}.$ When the frame is at position 2 the spring is stretched a distance $x_2 = 0.040\text{ m} + d.$ Use coordinates with the y-direction upward and $y = 0$ at the lowest point reached by the frame, so that $y_1 = d$ and $y_2 = 0.$ Work is done on the frame by gravity and by the spring force, so $W_{\text{other}} = 0,$ and $U = U_{\text{el}} + U_{\text{gravity}}.$

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2.\ \ W_{\text{other}} = 0.$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.350\text{ kg})(1.386\text{ m/s})^2 = 0.3362\text{ J}.$

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}} = \frac{1}{2}kx_1^2 + mgy_1 = \frac{1}{2}(36.75\text{ N/m})(0.0400\text{ m})^2 + (0.350\text{ kg})(9.80\text{ m/s}^2)d.$

$U_1 = 0.02940\text{ J} + (3.43\text{ N})d.\ \ U_2 = U_{2,\text{el}} + U_{2,\text{grav}} = \frac{1}{2}kx_2^2 + mgy_2 = \frac{1}{2}(36.75\text{ N/m})(0.0400\text{ m} + d)^2.$

$U_2 = 0.02940\text{ J} + (1.47\text{ N})d + (18.375\text{ N/m})d^2.$ Thus

$0.3362\text{ J} + 0.02940\text{ J} + (3.43\text{ N})d = 0.02940\text{ J} + (1.47\text{ N})d + (18.375\text{ N/m})d^2.$

$(18.375\text{ N/m})d^2 - (1.96\text{ N})d - 0.3362\text{ J} = 0.$ Using the quadratic formula, with the positive solution, we get $d = 0.199$ m.

EVALUATE: The collision is inelastic and mechanical energy is lost. Thus the decrease in gravitational potential energy is *not* equal to the increase in potential energy stored in the spring.

8.79. **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision.

SET UP: Let $+x$ be to the right. The total mass is $m = m_{\text{bullet}} + m_{\text{block}} = 1.00$ kg. The spring has force constant $k = \dfrac{|F|}{|x|} = \dfrac{0.750\text{ N}}{0.250\times10^{-2}\text{ m}} = 300$ N/m. Let V be the velocity of the block just after impact.

EXECUTE: **(a)** Conservation of energy for the motion after the collision gives $K_1 = U_{\text{el2}}$. $\frac{1}{2}mV^2 = \frac{1}{2}kx^2$ and

$$V = x\sqrt{\frac{k}{m}} = (0.150\text{ m})\sqrt{\frac{300\text{ N/m}}{1.00\text{ kg}}} = 2.60\text{ m/s}.$$

(b) Conservation of momentum applied to the collision gives $m_{\text{bullet}}v_1 = mV$.

$$v_1 = \frac{mV}{m_{\text{bullet}}} = \frac{(1.00\text{ kg})(2.60\text{ m/s})}{8.00\times10^{-3}\text{ kg}} = 325\text{ m/s}.$$

EVALUATE: The initial kinetic energy of the bullet is 422 J. The energy stored in the spring at maximum compression is 3.38 J. Most of the initial mechanical energy of the bullet is dissipated in the collision.

8.80. **IDENTIFY:** The horizontal components of momentum of the system of bullet plus stone are conserved. The collision is elastic if $K_1 = K_2$.

SET UP: Let A be the bullet and B be the stone.

(a)

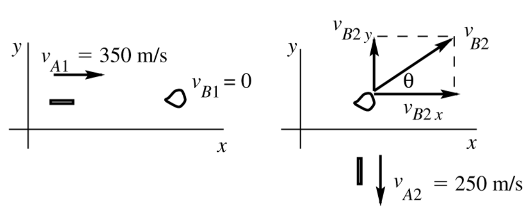

Figure 8.80

EXECUTE: P_x is conserved so $m_Av_{A1x} + m_Bv_{B1x} = m_Av_{A2x} + m_Bv_{B2x}$.

$m_Av_{A1} = m_Bv_{B2x}$.

$$v_{B2x} = \left(\frac{m_A}{m_B}\right)v_{A1} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(350\text{ m/s}) = 21.0\text{ m/s}$$

P_y is conserved so $m_Av_{A1y} + m_Bv_{B1y} = m_Av_{A2y} + m_Bv_{B2y}$.

$0 = -m_Av_{A2} + m_Bv_{B2y}$.

$$v_{B2y} = \left(\frac{m_A}{m_B}\right)v_{A2} = \left(\frac{6.00\times10^{-3}\text{ kg}}{0.100\text{ kg}}\right)(250\text{ m/s}) = 15.0\text{ m/s}.$$

$$v_{B2} = \sqrt{v_{B2x}^2 + v_{B2y}^2} = \sqrt{(21.0\text{ m/s})^2 + (15.0\text{ m/s})^2} = 25.8\text{ m/s}.$$

$$\tan\theta = \frac{v_{B2y}}{v_{B2x}} = \frac{15.0\text{ m/s}}{21.0\text{ m/s}} = 0.7143;\quad \theta = 35.5^\circ\ \text{(defined in the sketch)}.$$

(b) To answer this question compare K_1 and K_2 for the system:

$$K_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}(6.00\times10^{-3}\ \text{kg})(350\ \text{m/s})^2 = 368\ \text{J}.$$

$$K_2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2 = \tfrac{1}{2}(6.00\times10^{-3}\ \text{kg})(250\ \text{m/s})^2 + \tfrac{1}{2}(0.100\ \text{kg})(25.8\ \text{m/s})^2 = 221\ \text{J}.$$

$$\Delta K = K_2 - K_1 = 221\ \text{J} - 368\ \text{J} = -147\ \text{J}.$$

EVALUATE: The kinetic energy of the system decreases by 147 J as a result of the collision; the collision is *not* elastic. Momentum is conserved because $\Sigma \boldsymbol{F}_{\text{ext},x} = 0$ and $\Sigma \boldsymbol{F}_{\text{ext},y} = 0$. But there are internal forces between the bullet and the stone. These forces do negative work that reduces K.

8.81. IDENTIFY: Apply conservation of momentum to the collision between the two people. Apply conservation of energy to the motion of the stuntman before the collision and to the entwined people after the collision.

SET UP: For the motion of the stuntman, $y_1 - y_2 = 5.0$ m. Let v_S be the magnitude of his horizontal velocity just before the collision. Let V be the speed of the entwined people just after the collision. Let d be the distance they slide along the floor.

EXECUTE: **(a)** Motion before the collision: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$ and $\tfrac{1}{2}mv_S^2 = mg(y_1 - y_2)$.

$v_S = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\ \text{m/s}^2)(5.0\ \text{m})} = 9.90$ m/s.

Collision: $m_S v_S = m_{\text{tot}}V$. $V = \dfrac{m_S}{m_{\text{tot}}}v_S = \left(\dfrac{80.0\ \text{kg}}{150.0\ \text{kg}}\right)(9.90\ \text{m/s}) = 5.28$ m/s.

(b) Motion after the collision: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $\tfrac{1}{2}m_{\text{tot}}V^2 - \mu_k m_{\text{tot}} g d = 0$.

$$d = \frac{V^2}{2\mu_k g} = \frac{(5.28\ \text{m/s})^2}{2(0.250)(9.80\ \text{m/s}^2)} = 5.7\ \text{m}.$$

EVALUATE: Mechanical energy is dissipated in the inelastic collision, so the kinetic energy just after the collision is less than the initial potential energy of the stuntman.

8.82. IDENTIFY: Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

SET UP: Let v be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass m.

EXECUTE: Conservation of energy says $\tfrac{1}{2}mv^2 = mgR$; $v = \sqrt{2gR}$.

SET UP: This is speed v_1 for the collision. Let v_2 be the speed of the combined object just after the collision.

EXECUTE: Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so

$v_2 = v_1/2 = \sqrt{gR/2}$.

SET UP: Apply conservation of energy to the motion of the combined object after the collision. Let y_3 be the final height above the bottom of the bowl.

EXECUTE: $\tfrac{1}{2}(2m)v_2^2 = (2m)gy_3$.

$$y_3 = \frac{v_2^2}{2g} = \frac{1}{2g}\left(\frac{gR}{2}\right) = R/4.$$

EVALUATE: Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

8.83. IDENTIFY: This collision is elastic, so kinetic energy and momentum are both conserved.

Set Up: Use $v_A = \frac{m_A - m_B}{m_A + m_B} v_0$ (Eq. 8.24) and $v_B = \frac{2m_A}{m_A + m_B} v_0$ (Eq. 8.25), as well as $K = \frac{1}{2}mv^2$. Call the $+x$-axis the original direction that object A is moving. Where we let $v_{Ai} = v_0$ and we have simplified the notation of Eq. 8.24 and Eq. 8.25 somewhat. We have $m_A = \alpha m_B$.

Execute: **(a)** Using Eq. 8.25 gives the final kinetic energy of B.

$K_{B,f} = \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_B\left(\frac{2m_A v_0}{m_A + m_B}\right)^2 = \frac{1}{2}m_B\left(\frac{2m_A}{m_A + m_B}\right)^2 v_0^2$. This is equal to the initial kinetic energy of A, which is $K_{A,i} = \frac{1}{2}m_A v_0^2 = \frac{1}{2}\alpha m_B v_0^2$. Equating the two kinetic energies gives

$\frac{1}{2}\alpha m_B v_0^2 = \frac{1}{2}m_B\left(\frac{2m_A}{m_A + m_B}\right)^2 v_0^2$, which simplifies to $\alpha = \left(\frac{2m_A}{m_A + m_B}\right)^2$. Using $m_A = \alpha m_B$ gives

$\alpha = \left(\frac{2\alpha m_B}{\alpha m_B + m_B}\right)^2 = \left(\frac{2\alpha}{\alpha + 1}\right)^2$. Solving for α gives $\alpha = 1$. This means that the masses are equal.

(b) In this case, after the collision $K_A = K_B$, so $\frac{1}{2}m_A v_A^2 = \frac{1}{2}m_B v_B^2$. Using $m_A = \alpha m_B$ and simplifying gives $\alpha v_A^2 = v_B^2$. Now use Eq. 24 and Eq. 25 in the last equation, which gives

$\alpha\left(\frac{m_A - m_B}{m_A + m_B} v_0\right)^2 = \left(\frac{2m_A v_0}{m_A + m_B}\right)^2$. Using $m_A = \alpha m_B$ and simplifying gives $\alpha(\alpha - 1)^2 = 4\alpha^2$. The resulting quadratic equation has solutions $\alpha = 3 + 2\sqrt{2} \approx 5.83$ and $\alpha = 3 - 2\sqrt{2} \approx 0.172$.

Evaluate: When $\alpha = 1$, $m_A = m_B$. Object A stops and object B moves ahead with the same speed that A had. Object A has lost all of its momentum and kinetic energy and object B has gained it all, so both momentum and kinetic energy are conserved.

8.84. **Identify:** Apply conservation of energy to the motion before and after the collision. Apply conservation of momentum to the collision.

Set Up: First consider the motion after the collision. The combined object has mass $m_{\text{tot}} = 25.0$ kg. Apply $\Sigma\vec{F} = m\vec{a}$ to the object at the top of the circular loop, where the object has speed v_3. The acceleration is $a_{\text{rad}} = v_3^2/R$, downward.

Execute: $T + mg = m\frac{v_3^2}{R}$.

The minimum speed v_3 for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg}$, where $R = 2.80$ m.

Set Up: Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at point 2. Only gravity does work, so $K_2 + U_2 = K_3 + U_3$

Execute: $\frac{1}{2}m_{\text{tot}}v_2^2 = \frac{1}{2}m_{\text{tot}}v_3^2 + m_{\text{tot}}g(2R)$.

Use $v_3 = \sqrt{Rg}$ and solve for v_2: $v_2 = \sqrt{5gR} = 11.71$ m/s.

Set Up: Now apply conservation of momentum to the collision between the dart and the sphere. Let v_1 be the speed of the dart before the collision.

Execute: $(5.00\text{ kg})v_1 = (25.0\text{ kg})(11.71\text{ m/s})$, which gives $v_1 = 58.6$ m/s.

Evaluate: The collision is inelastic and mechanical energy is removed from the system by the negative work done by the forces between the dart and the sphere.

8.85. **Identify:** Apply conservation of momentum to the collision between the bullet and the block and apply conservation of energy to the motion of the block after the collision.

(a) SET UP: For the collision between the bullet and the block, let object A be the bullet and object B be the block. Apply momentum conservation to find the speed v_{B2} of the block just after the collision (see Figure 8.85a).

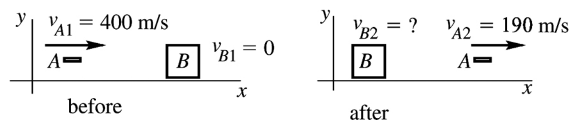

Figure 8.85a

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $m_A v_{A1} = m_A v_{A2} + m_B v_{B2x}$.

$$v_{B2x} = \frac{m_A(v_{A1} - v_{A2})}{m_B} = \frac{4.00\times 10^{-3}\ \text{kg}(400\ \text{m/s} - 190\ \text{m/s})}{0.800\ \text{kg}} = 1.05\ \text{m/s}.$$

SET UP: For the motion of the block after the collision, let point 1 in the motion be just after the collision, where the block has the speed 1.05 m/s calculated above, and let point 2 be where the block has come to rest (see Figure 8.85b).

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

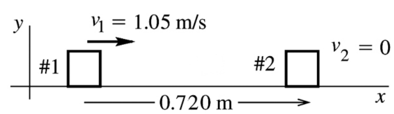

Figure 8.85b

EXECUTE: Work is done on the block by friction, so $W_{\text{other}} = W_f$.

$W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$, where $s = 0.720$ m. $U_1 = 0$, $U_2 = 0$, $K_1 = \frac{1}{2}mv_1^2$, $K_2 = 0$ (the block has come to rest). Thus $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$. Therefore

$$\mu_k = \frac{v_1^2}{2gs} = \frac{(1.05\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(0.720\ \text{m})} = 0.0781.$$

(b) For the bullet, $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(4.00\times 10^{-3}\ \text{kg})(400\ \text{m/s})^2 = 320\ \text{J}$ and

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(4.00\times 10^{-3}\ \text{kg})(190\ \text{m/s})^2 = 72.2\ \text{J}$. $\Delta K = K_2 - K_1 = 72.2\ \text{J} - 320\ \text{J} = -248\ \text{J}$. The kinetic energy of the bullet decreases by 248 J.

(c) Immediately after the collision the speed of the block is 1.05 m/s, so its kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.800\ \text{kg})(1.05\ \text{m/s})^2 = 0.441\ \text{J}$.

EVALUATE: The collision is highly inelastic. The bullet loses 248 J of kinetic energy but only 0.441 J is gained by the block. But momentum is conserved in the collision. All the momentum lost by the bullet is gained by the block.

8.86. IDENTIFY: Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

SET UP: Let $+x$ be to the right. Let the bullet be A and the block be B. Let V be the velocity of the block just after the collision.

EXECUTE: *Motion of block after the collision*: $K_1 = U_{\text{grav2}}$. $\frac{1}{2}m_B V^2 = m_B gh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(0.38\times 10^{-2}\ \text{m})} = 0.273\ \text{m/s}.$$

Collision: $v_{B2} = 0.273$ m/s. $P_{1x} = P_{2x}$ gives $m_A v_{A1} = m_A v_{A2} + m_B v_{B2}$.

$$v_{A2} = \frac{m_A v_{A1} - m_B v_{B2}}{m_A} = \frac{(5.00\times10^{-3}\text{ kg})(450\text{ m/s}) - (1.00\text{ kg})(0.273\text{ m/s})}{5.00\times10^{-3}\text{ kg}} = 395\text{ m/s}.$$

EVALUATE: We assume the block moves very little during the time it takes the bullet to pass through it.

8.87. **IDENTIFY:** Apply conservation of energy to the motion of the package before the collision and apply conservation of the horizontal component of momentum to the collision.

(a) SET UP: Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take $y = 0$ at point 2, so $y_1 = 4.00$ m. Only gravity does work, so

$$K_1 + U_1 = K_2 + U_2.$$

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{v_1^2 + 2gy_1} = 9.35$ m/s.

(b) SET UP: In the collision between the package and the cart, momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let *A* be the package and *B* be the cart.

EXECUTE: P_x is constant gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$v_{B1x} = -5.00$ m/s.

$v_{A1x} = (3.00\text{ m/s})\cos 37.0°$. (The horizontal velocity of the package is constant during its free fall.)

Solving for v_{2x} gives $v_{2x} = -3.29$ m/s. The cart is moving to the left at 3.29 m/s after the package lands in it.

EVALUATE: The cart is slowed by its collision with the package, whose horizontal component of momentum is in the opposite direction to the motion of the cart.

8.88. **IDENTIFY:** Apply conservation of momentum to the system of the neutron and its decay products.

SET UP: Let the proton be moving in the $+x$-direction with speed v_p after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$-direction after the decay. Let the speed of the electron be v_e.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_p v_p - m_e v_e$ and $v_e = \left(\frac{m_p}{m_e}\right)v_p$. The total kinetic energy after the decay is $K_{tot} = \frac{1}{2}m_e v_e^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}m_e\left(\frac{m_p}{m_e}\right)^2 v_p^2 + \frac{1}{2}m_p v_p^2 = \frac{1}{2}m_p v_p^2\left(1 + \frac{m_p}{m_e}\right)$.

Thus, $\frac{K_p}{K_{tot}} = \frac{1}{1 + m_p/m_e} = \frac{1}{1 + 1836} = 5.44\times10^{-4} = 0.0544\%$.

EVALUATE: Most of the released energy goes to the electron, since it is much lighter than the proton.

8.89. **IDENTIFY:** We have a collision, so we use momentum conservation. After the collision we have projectile motion. Fig. 8.89 illustrates this process.

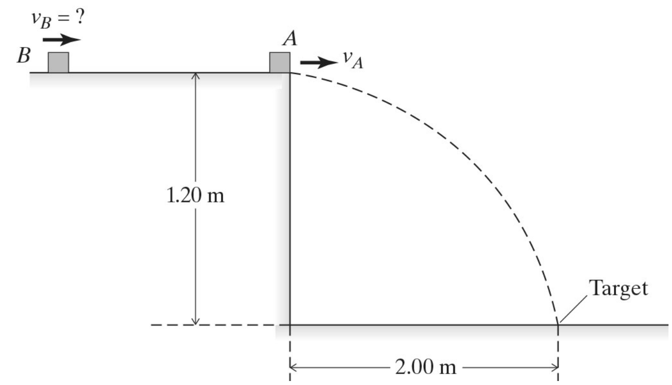

Figure 8.89

SET UP: We use projectile motion to find the speed that A needs at the top of the table to hit the target. We use momentum conservation during the collision to find the speed that B needed to give A the necessary speed. The collision is elastic, so we can use $v_A = \dfrac{m_A - m_B}{m_A + m_B} v_0$ (Eq. 8.24) and $v_B = \dfrac{2m_A}{m_A + m_B} v_0$ (Eq. 8.25). In this problem A and B are interchanged from the equations in the text because B is moving and A is initially at rest, so we must be careful. Eq. 8.24 tells us that if the masses are equal, B stops and A moves forward with the same velocity that B had.

EXECUTE: Look at the projectile motion after the collision. The constant-acceleration equations apply. The block needs to travel 2.00 m at constant horizontal velocity v_A in the same time that it falls 1.20 m starting from rest vertically. The vertical motion gives $y = \dfrac{1}{2}gt^2$, so $t = \sqrt{2y/g} = \sqrt{\dfrac{2(1.20\text{ m})}{9.80\text{ m/s}^2}} = 0.4949$ s. Horizontally $x = v_A t = 2.00$ m, so $v_A = (2.00\text{ m})/(0.4949\text{ s}) = 4.04\text{ m/s} = v_B$.

EVALUATE: If the collision were inelastic, B would have had a velocity after the collision.

8.90. **IDENTIFY:** Since there is no friction, the horizontal component of momentum of the system of Jonathan, Jane, and the sleigh is conserved.

SET UP: Let $+x$ be to the right. $w_A = 800$ N, $w_B = 600$ N and $w_C = 1000$ N.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$.

$$v_{C2x} = -\frac{m_A v_{A2x} + m_B v_{B2x}}{m_C} = -\frac{w_A v_{A2x} + w_B v_{B2x}}{w_C}.$$

$$v_{C2x} = -\frac{(800\text{ N})[-(5.00\text{ m/s})\cos 30.0°] + (600\text{ N})[+(7.00\text{ m/s})\cos 36.9°]}{1000\text{ N}} = 0.105\text{ m/s}.$$

The sleigh's velocity is 0.105 m/s, to the right.

EVALUATE: The vertical component of the momentum of the system consisting of the two people and the sleigh is not conserved, because of the net force exerted on the sleigh by the ice while they jump.

8.91. **IDENTIFY:** No net external force acts on the Burt-Ernie-log system, so the center of mass of the system does not move.

SET UP: $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$.

EXECUTE: Use coordinates where the origin is at Burt's end of the log and where $+x$ is toward Ernie, which makes $x_1 = 0$ for Burt initially. The initial coordinate of the center of mass is

$x_{\text{cm},1} = \dfrac{(20.0 \text{ kg})(1.5 \text{ m}) + (40.0 \text{ kg})(3.0 \text{ m})}{90.0 \text{ kg}}$. Let d be the distance the log moves toward Ernie's original position. The final location of the center of mass is

$x_{\text{cm},2} = \dfrac{(30.0 \text{ kg})d + (1.5 \text{ kg} + d)(20.0 \text{ kg}) + (40.0 \text{ kg})d}{90.0 \text{ kg}}$. The center of mass does not move, so

$x_{\text{cm},1} = x_{\text{cm},2}$, which gives

$(20.0 \text{ kg})(1.5 \text{ m}) + (40.0 \text{ kg})(3.0 \text{ m}) = (30.0 \text{ kg})d + (20.0 \text{ kg})(1.5 \text{ m} + d) + (40.0 \text{ kg})d$. Solving for d gives $d = 1.33$ m.

EVALUATE: Burt, Ernie, and the log all move, but the center of mass of the system does not move.

8.92. **IDENTIFY:** There is no net horizontal external force so v_{cm} is constant.

SET UP: Let $+x$ be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_{\text{A}} = 45.0$ kg, $m_B = 60.0$ kg. The center of mass of the canoe is at its center.

EXECUTE: Initially, $v_{\text{cm}} = 0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}} = \dfrac{m_A x_{A1} + m_B x_{B1}}{m_A + m_B}$.

After she walks, $x_{\text{cm2}} = \dfrac{m_A x_{A2} + m_B x_{B2}}{m_A + m_B}$. $x_{\text{cm1}} = x_{\text{cm2}}$ gives $m_A x_{A1} + m_B x_{B1} = m_A x_{A2} + m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2} = x_{B2} + 1.50$ m.

$$(45.0 \text{ kg})(1.00 \text{ m}) + (60.0 \text{ kg})(2.50 \text{ m}) = (45.0 \text{ kg})(x_{B2} + 1.50 \text{ m}) + (60.0 \text{ kg})x_{B2}.$$

$(105.0 \text{ kg})x_{B2} = 127.5 \text{ kg} \cdot \text{m}$ and $x_{B2} = 1.21$ m. $x_{B2} - x_{B1} = 1.21 \text{ m} - 2.50 \text{ m} = -1.29$ m. The canoe moves 1.29 m to the left.

EVALUATE: When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00 \text{ m} - 1.29 \text{ m} = 1.71 \text{ m}$ to the right relative to the water. Note that this distance is $(60.0 \text{ kg} / 45.0 \text{ kg})(1.29 \text{ m})$.

8.93. **IDENTIFY:** This process involves a swing, a collision, and another swing. Energy is conserved during the two swings and momentum is conserved during the collision.

SET UP: We must break this problem into three parts: the first swing, the collision, and the second swing. We cannot solve it in a single step. We want to find h_{max}, the maximum height the combined spheres reach during the second swing after the collision.

EXECUTE: Swing of B: Energy conservation gives $mgH = \frac{1}{2} m v_B^2$, so $v_B = \sqrt{2gH}$.

Collision: Momentum conservation gives $m v_B = (3m) v_{AB}$, so $v_{AB} = \dfrac{v_B}{3} = \dfrac{1}{3}\sqrt{2gH}$.

Swing of $A + B$: Energy conservation gives $\frac{1}{2}(3m) v_{AB}^2 = 3mgh_{\text{max}}$. Using v_{AB}, this becomes

$\dfrac{1}{2}\left(\dfrac{1}{3}\sqrt{2gH}\right)^2 = gh_{\text{max}}$. Solving for h_{max} gives $h_{\text{max}} = \dfrac{H}{9}$.

EVALUATE: It is reasonable that we get $h_{\text{max}} < H$ because mechanical energy is lost during the inelastic collision.

8.94. **IDENTIFY:** The explosion produces only internal forces for the fragments, so the momentum of the two-fragment system is conserved. Therefore the explosion does not affect the motion of the center of mass of this system.

SET UP: The center of mass follows a parabolic path just as a single particle would. Its horizontal range is $R = \dfrac{v_0^2 \sin(2\alpha)}{g}$. The center of mass of a two-particle system is $x_{\text{cm}} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$.

EXECUTE: **(a)** The range formula gives $R = (18.0\text{ m/s})^2(\sin 102°)/(9.80\text{ m/s}^2) = 32.34$ m, which rounds to 32.3 m.

(b) The center of mass is 32.3 m from the firing point and one fragment lands at $x_2 = 26.0$ m. Using the center of mass formula, with the origin at the firing point and calling m the mass of each fragment, we have 32.34 m = $[m(26.0\text{ m}) + mx_2]/(2m)$, which gives $x_2 = 38.68$ m, which rounds to 38.7 m.

EVALUATE: Since the fragments have equal masses, their center of mass should be midway between them. So it should be at (26.0 m + 38.68 m)/2 = 32.3 m, which it is.

8.95. **IDENTIFY:** The collision is inelastic since the blocks stick together. Momentum is conserved during the collision and energy is conserved before and after the collision.

SET UP: Hooke's law: $F = kx$. The elastic energy stored in a spring is $U_{\text{spr}} = \dfrac{1}{2}kx^2$. Energy conservation gives $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ and momentum is $p_x = mv_x$.

EXECUTE: **(a)** First find the distance the spring is compressed using Hooke's law. $mg = kx$, so

$x = \dfrac{mg}{k} = \dfrac{(0.500\text{ kg})(9.80\text{ m/s}^2)}{80.0\text{ N/m}} = 0.06125$ m. The energy stored with this compression is

$U_{\text{spr}} = \dfrac{1}{2}kx^2 = \dfrac{1}{2}(80.0\text{ N})(0.06125\text{ m})^2 = 0.150$ J.

(b) After the collision, the two-block system has kinetic energy, which can be transferred to the spring. As the spring compresses, gravity also does work on the masses. Fig. 8.95 shows the system just after the collision.

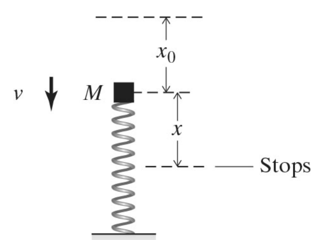

Figure 8.95

First we need to find v, the speed of the two-block system just after the collision. We do this in two steps: conservation of energy as the first block is dropped and reaches the block on the spring, followed by momentum conservation during the collision. Call v_0 the speed of the single block just before it hits the block on the spring. Energy conservation gives $mgh = \dfrac{1}{2}mv_0^2$, so $v_0 = \sqrt{2gh}$. Momentum conservation during the collision gives $mv_0 = (2m)v$, which gives

$v = \dfrac{v_0}{2} = \dfrac{\sqrt{2gh}}{2} = \dfrac{\sqrt{2(9.80\text{ m/s}^2)(4.00\text{ m})}}{2} = 19.60$ m/s. Now we use energy conservation after the collision. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ with $W_{\text{other}} = 0$. Choose point 1 to be the instant after the collision and point 2 to be when the spring has its maximum compression. At that point, the blocks stop, so $K_2 = 0$. Call x the maximum distance that the spring will compress after the collision (see Fig. 8.25). At point 1 the blocks have gravitational potential energy, which is $U_{\text{g}} = mgx$. At point 1 the system has two forms of potential, U_{g} and the elastic potential energy that is already stored in the spring. In part (a) we saw that this is 0.150 J. At point 2 the system has only elastic potential energy because the spring is now compressed a *total* distance of $x_0 + x$, where x_0 = 0.06125 m from part (a). Calling M the total mass of the system, $U_1 + K_1 = U_2 + K_2$ becomes $Mgx + U_1 + \frac{1}{2}Mv^2 = \frac{1}{2}k(x_0 + x)^2$. Expanding the square, realizing that $U_1 = \frac{1}{2}kx_0^2$, and collecting terms, this equation becomes

$-kx^2 + (2Mg - 2kx_0)x + Mv^2 = 0$. Using k = 80.0 N/m, v = 19.60 m/s, M = 1.00 kg, and x_0 = 0.06125 m, the quadratic formula gives two solutions. One of the solutions is negative, so we discard it as nonphysical. The other solution is x = 0.560 m. This is the maximum distance that the system compresses the spring *after* the collision. But the spring was already compressed by 0.06125 m before the collision, so the *total* distance that the spring is compressed at the instant the blocks stop moving is x_{total} = 0.560 m + 0.06125 m. Therefore the maximum elastic energy stored in the spring is

$U_{max} = \frac{1}{2}kx_{max}^2 = \frac{1}{2}k(x + x_0)^2 = \frac{1}{2}(80\text{ N/m})(0.560\text{ m} + 0.06125\text{ m})^2 = 15.4\text{ J}.$

(c) As shown in part (b), x = 0.560 m.

EVALUATE: We cannot overlook the gravitational potential energy in solving this problem because it could be significant compared to the elastic energy in the spring.

8.96. **IDENTIFY:** Conservation of x- and y-components of momentum applies to the collision. At the highest point of the trajectory the vertical component of the velocity of the projectile is zero.

SET UP: Let $+y$ be upward and $+x$ be horizontal and to the right. Let the two fragments be A and B, each with mass m. For the projectile before the explosion and the fragments after the explosion. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$ gives that the maximum height of the projectile is

$h = -\frac{v_{0y}^2}{2a_y} = -\frac{[(80.0\text{ m/s})\sin 60.0°]^2}{2(-9.80\text{ m/s}^2)} = 244.9\text{ m}.$ Just before the explosion the projectile is moving to the right with horizontal velocity $v_x = v_{0x} = v_0 \cos 60.0° = 40.0\text{ m/s}.$ After the explosion $v_{Ax} = 0$ since fragment A falls vertically. Conservation of momentum applied to the explosion gives $(2m)(40.0\text{ m/s}) = mv_{Bx}$ and $v_{Bx} = 80.0\text{ m/s}.$ Fragment B has zero initial vertical velocity so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives a time of fall of $t = \sqrt{-\frac{2h}{a_y}} = \sqrt{-\frac{2(244.9\text{ m})}{-9.80\text{ m/s}^2}} = 7.07\text{ s}.$ During this time the fragment travels horizontally a distance $(80.0\text{ m/s})(7.07\text{ s}) = 566\text{ m}.$ It also took the projectile 7.07 s to travel from launch to maximum height and during this time it travels a horizontal distance of $([80.0\text{ m/s}]\cos 60.0°)(7.07\text{ s}) = 283\text{ m}.$ The second fragment lands $283\text{ m} + 566\text{ m} = 849\text{ m}$ from the firing point.

(b) For the explosion, $K_1 = \frac{1}{2}(20.0\text{ kg})(40.0\text{ m/s})^2 = 1.60 \times 10^4\text{ J}.$

$K_2 = \frac{1}{2}(10.0\text{ kg})(80.0\text{ m/s})^2 = 3.20 \times 10^4\text{ J}.$ The energy released in the explosion is 1.60×10^4 J.

EVALUATE: The kinetic energy of the projectile just after it is launched is 6.40×10^4 J. We can calculate the speed of each fragment just before it strikes the ground and verify that the total kinetic energy of the fragments just before they strike the ground is $6.40 \times 10^4\text{ J} + 1.60 \times 10^4\text{ J} = 8.00 \times 10^4\text{ J}.$ Fragment A has speed 69.3 m/s just before it strikes the ground, and hence has kinetic energy 2.40×10^4 J. Fragment B has speed $\sqrt{(80.0\text{ m/s})^2 + (69.3\text{ m/s})^2} = 105.8\text{ m/s}$ just before it strikes the ground, and hence has kinetic energy 5.60×10^4 J. Also, the center of mass of the system has the same horizontal range $R = \frac{v_0^2}{g}\sin(2\alpha_0) = 565\text{ m}$ that the projectile would have had if no explosion had occurred. One fragment lands at $R/2$ so the other, equal mass fragment lands at a distance $3R/2$ from the launch point.

8.97. **IDENTIFY:** The rocket moves in projectile motion before the explosion and its fragments move in projectile motion after the explosion. Apply conservation of energy and conservation of momentum to the explosion.

(a) SET UP: Apply conservation of energy to the explosion. Just before the explosion the rocket is at its maximum height and has zero kinetic energy. Let *A* be the piece with mass 1.40 kg and *B* be the piece with mass 0.28 kg. Let v_A and v_B be the speeds of the two pieces immediately after the collision.

EXECUTE: $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = 860\text{ J}$

SET UP: Since the two fragments reach the ground at the same time, their velocities just after the explosion must be horizontal. The initial momentum of the rocket before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum: $m_A v_A = m_B v_B$.

EXECUTE: Use this to eliminate v_A in the first equation and solve for v_B:

$$\tfrac{1}{2}m_B v_B^2(1 + m_B / m_A) = 860\text{ J} \text{ and } v_B = 71.6\text{ m/s}.$$

Then $v_A = (m_B/m_A)v_B = 14.3\text{ m/s}$.

(b) SET UP: Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.

$$v_{0y} = 0,\ a_y = +9.80\text{ m/s}^2,\ y - y_0 = 80.0\text{ m},\ t = ?$$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 4.04\text{ s}$.

During this time the horizontal distance each piece moves is $x_A = v_A t = 57.8\text{ m}$ and $x_B = v_B t = 289.1\text{ m}$. They move in opposite directions, so they are $x_A + x_B = 347\text{ m}$ apart when they land.

EVALUATE: Fragment *A* has more mass so it is moving slower right after the collision, and it travels horizontally a smaller distance as it falls to the ground.

8.98. **IDENTIFY:** Apply conservation of momentum to the explosion. At the highest point of its trajectory the shell is moving horizontally. If one fragment received some upward momentum in the explosion, the other fragment would have had to receive a downward component. Since they each hit the ground at the same time, each must have zero vertical velocity immediately after the explosion.

SET UP: Let $+x$ be horizontal, along the initial direction of motion of the projectile and let $+y$ be upward. At its maximum height the projectile has $v_x = v_0\cos 55.0° = 86.0\text{ m/s}$. Let the heavier fragment be *A* and the lighter fragment be *B*. $m_A = 9.00\text{ kg}$ and $m_B = 3.00\text{ kg}$.

EXECUTE: Since fragment *A* returns to the launch point, immediately after the explosion it has $v_{Ax} = -86.0\text{ m/s}$. Conservation of momentum applied to the explosion gives $(12.0\text{ kg})(86.0\text{ m/s}) = (9.00\text{ kg})(-86.0\text{ m/s}) + (3.00\text{ kg})v_{Bx}$ and $v_{Bx} = 602\text{ m/s}$. The horizontal range of the projectile, if no explosion occurred, would be $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 2157\text{ m}$. The horizontal distance each fragment travels is proportional to its initial speed and the heavier fragment travels a horizontal distance $R/2 = 1078\text{ m}$ after the explosion, so the lighter fragment travels a horizontal distance $\left(\dfrac{602\text{ m}}{86\text{ m}}\right)(1078\text{ m}) = 7546\text{ m}$ from the point of explosion and $1078\text{ m} + 7546\text{ m} = 8624\text{ m}$ from the launch point. The energy released in the explosion is

$K_2 - K_1 = \frac{1}{2}(9.00\text{ kg})(86.0\text{ m/s})^2 + \frac{1}{2}(3.00\text{ kg})(602\text{ m/s})^2 - \frac{1}{2}(12.0\text{ kg})(86.0\text{ m/s})^2 = 5.33\times 10^5\text{ J}$.

EVALUATE: The center of mass of the system has the same horizontal range $R = 2157$ m as if the explosion didn't occur. This gives $(12.0 \text{ kg})(2157 \text{ m}) = (9.00 \text{ kg})(0) + (3.00 \text{ kg})d$ and $d = 8630$ m, where d is the distance from the launch point to where the lighter fragment lands. This agrees with our calculation.

8.99. IDENTIFY: Apply conservation of energy to the motion of the wagon before the collision. After the collision the combined object moves with constant speed on the level ground. In the collision the horizontal component of momentum is conserved.

SET UP: Let the wagon be object A and treat the two people together as object B. Let $+x$ be horizontal and to the right. Let V be the speed of the combined object after the collision.

EXECUTE: **(a)** The speed v_{A1} of the wagon just before the collision is given by conservation of energy applied to the motion of the wagon prior to the collision. $U_1 = K_2$ says

$m_A g([50 \text{ m}][\sin 6.0°]) = \frac{1}{2} m_A v_{A1}^2$. $v_{A1} = 10.12$ m/s. $P_{1x} = P_{2x}$ for the collision says $m_A v_{A1} = (m_A + m_B)V$

and $V = \left(\dfrac{300 \text{ kg}}{300 \text{ kg} + 75.0 \text{ kg} + 60.0 \text{ kg}}\right)(10.12 \text{ m/s}) = 6.98 \text{ m/s}$. In 5.0 s the wagon travels

$(6.98 \text{ m/s})(5.0 \text{ s}) = 34.9$ m, and the people will have time to jump out of the wagon before it reaches the edge of the cliff.

(b) For the wagon, $K_1 = \frac{1}{2}(300 \text{ kg})(10.12 \text{ m/s})^2 = 1.54 \times 10^4$ J. Assume that the two heroes drop from a small height, so their kinetic energy just before the wagon can be neglected compared to K_1 of the wagon. $K_2 = \frac{1}{2}(435 \text{ kg})(6.98 \text{ m/s})^2 = 1.06 \times 10^4$ J. The kinetic energy of the system decreases by $K_1 - K_2 = 4.8 \times 10^3$ J.

EVALUATE: The wagon slows down when the two heroes drop into it. The mass that is moving horizontally increases, so the speed decreases to maintain the same horizontal momentum. In the collision the vertical momentum is not conserved, because of the net external force due to the ground.

8.100. IDENTIFY: Impulse is equal to the area under the curve in a graph of force versus time.

SET UP: $J_x = \Delta p_x = F_x \Delta t$.

EXECUTE: **(a)** Impulse is the area under *F-t* curve

$J_x = [7500 \text{ N} + ½ (7500 \text{ N} + 3500 \text{ N}) + 3500 \text{ N}](1.50 \text{ s}) = 2.475 \times 10^4 \text{ N} \cdot \text{s}$.

(b) The total mass of the car and driver is $(3071 \text{ lb})(4.448 \text{ N/lb})/(9.80 \text{ m/s}^2) = 1394$ kg.

$J_x = \Delta p_x = mv_x - 0$, so $v_x = J_x/m = (2.475 \times 10^4 \text{ N} \cdot \text{s})/(1394 \text{ kg}) = 17.8$ m/s.

(c) The braking force must produce an impulse opposite to the one that accelerated the car, so $J_x = -2.475 \times 10^4 \text{ N} \cdot \text{s}$. Therefore $J_x = F_x \Delta t$ gives $\Delta t = J_x/F_x = (-24{,}750 \text{ N} \cdot \text{s})/(-5200 \text{ N}) = 4.76$ s.

(d) $W_{\text{brake}} = \Delta K = -K = -½ mv^2 = -½ (1394 \text{ kg})(17.76 \text{ m/s})^2 = -2.20 \times 10^5$ J.

(e) $W_{\text{brake}} = -B_x s$, so $s = -W_{\text{brake}}/B_x = -(2.20 \times 10^5 \text{ J})/(-5200 \text{ N}) = 42.3$ m.

EVALUATE: The result in (e) could be checked by using kinematics with an average velocity of (17.8 m/s)/2 for 4.76 s.

8.101. IDENTIFY: As the bullet strikes and embeds itself in the block, momentum is conserved. After that, we use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where W_{other} is due to kinetic friction.

SET UP: Momentum conservation during the collision gives $m_b v_b = (m_b + m)V$, where m is the mass of the block and m_b is the mass of the bullet. After the collision, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$, where M is the mass of the block plus the bullet.

EXECUTE: **(a)** From the energy equation above, we can see that the greatest compression of the spring will occur for the greatest V (since $M >> m_b$), and the greatest V will occur for the bullet with the greatest initial momentum. Using the data in the table with the problem, we get the following momenta expressed in units of grain · ft/s.

A: 1.334×10^5 grain·ft/s B: 1.181×10^5 grain·ft/s C: 2.042×10^5 grain·ft/s

D: 1.638×10^5 grain·ft/s E: 1.869×10^5 grain·ft/s

From these results, it is clear that bullet C will produce the maximum compression of the spring and bullet B will produce the least compression.

(b) For bullet C, we use $p_b = m_b v_b = (m_b + m)V$. Converting mass (in grains) and speed to SI units gives $m_b = 0.01555$ kg and $v_b = 259.38$ m/s, we have

$(0.01555 \text{ kg})(259.38 \text{ m/s}) = (0.01555 \text{ kg} + 2.00 \text{ kg})V$, so $V = 2.001$ m/s.

Now use $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$ and solve for k, giving

$k = (2.016 \text{ kg})[(2.001 \text{ m/s})^2 - 2(0.38)(9.80 \text{ m/s}^2)(0.25 \text{ m})]/(0.25 \text{ m})^2 = 69.1$ N/m, which rounds to 69 N/m.

(c) For bullet B, $m_b = 125$ grains = 0.00810 kg and $v_b = 945$ ft/s = 288.0 m/s. Momentum conservation gives

$V = (0.00810 \text{ kg})(288.0 \text{ m/s})/(2.00810 \text{ kg}) = 1.162$ m/s.

Using $\frac{1}{2}MV^2 - \mu_k Mgd = \frac{1}{2}kd^2$, the above numbers give $33.55d^2 + 7.478d - 1.356 = 0$. The quadratic formula, using the positive square root, gives $d = 0.118$ m, which rounds to 0.12 m.

EVALUATE: This method for measuring muzzle velocity involves a spring displacement of around 12 cm, which should be readily measurable.

8.102. **IDENTIFY** Momentum is conserved during the collision. After the collision, we can use energy methods.

SET UP: $p = mv$, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where W_{other} is due to kinetic friction. We need to use components of momentum. Call $+x$ eastward and $+y$ northward.

EXECUTE: **(a)** Momentum conservation gives

$p_x = [6500 \text{ lb})/g]v_D = [(9542 \text{ lb})/g]v_w\cos(39°)$

$p_y = [(3042 \text{ lb})/g](50 \text{ mph}) = [(9542 \text{ lb})/g]v_w\sin(39°)$

Solving for v_D gives $v_D = 28.9$ mph, which rounds to 29 mph.

(b) The above equations also give that the velocity of the wreckage just after impact is 25.3 mph = 37.1 ft/s. Using $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, we have $\frac{1}{2}mv_1^2 - \mu_k mgd = \frac{1}{2}mv_2^2$. Solving for v_2 gives

$v_2 = \sqrt{v_1^2 - 2\mu_k gd}$. Using $v_1 = 37.1$ ft/s, $g = 32.2$ ft/s^2 and $d = 35$ ft, we get $v_2 = 19.1$ ft/s = 13 mph.

EVALUATE: We were able to minimize unit conversions by working in British units instead of SI units since the data was given in British units.

8.103. **IDENTIFY:** From our analysis of motion with constant acceleration, if $v = at$ and a is constant, then $x - x_0 = v_0 t + \frac{1}{2}at^2$.

SET UP: Take $v_0 = 0$, $x_0 = 0$ and let $+x$ downward.

EXECUTE: **(a)** $\frac{dv}{dt} = a$, $v = at$ and $x = \frac{1}{2}at^2$. Substituting into $xg = x\frac{dv}{dt} + v^2$ gives

$\frac{1}{2}at^2 g = \frac{1}{2}at^2 a + a^2t^2 = \frac{3}{2}a^2t^2$. The nonzero solution is $a = g/3$.

(b) $x = \frac{1}{2}at^2 = \frac{1}{6}gt^2 = \frac{1}{6}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = 14.7$ m.

(c) $m = kx = (2.00 \text{ g/m})(14.7 \text{ m}) = 29.4$ g.

EVALUATE: The acceleration is less than g because the small water droplets are initially at rest, before they adhere to the falling drop. The small droplets are suspended by buoyant forces that we ignore for the raindrops.

8.104. IDENTIFY and SET UP: $dm = \rho dV$. $dV = Adx$. Since the thin rod lies along the x-axis, $y_{cm} = 0$. The mass of the rod is given by $M = \int dm$.

EXECUTE: (a) $x_{cm} = \frac{1}{M}\int_0^L xdm = \frac{\rho}{M}A\int_0^L xdx = \frac{\rho A}{M}\frac{L^2}{2}$. The volume of the rod is AL and $M = \rho AL$.

$x_{cm} = \frac{\rho AL^2}{2\rho AL} = \frac{L}{2}$. The center of mass of the uniform rod is at its geometrical center, midway between its ends.

(b) $x_{cm} = \frac{1}{M}\int_0^L xdm = \frac{1}{M}\int_0^L x\rho Adx = \frac{A\alpha}{M}\int_0^L x^2dx = \frac{A\alpha L^3}{3M}$. $M = \int dm = \int_0^L \rho Adx = \alpha A\int_0^L xdx = \frac{\alpha AL^2}{2}$.

Therefore, $x_{cm} = \left(\frac{A\alpha L^3}{3}\right)\left(\frac{2}{\alpha AL^2}\right) = \frac{2L}{3}$.

EVALUATE: When the density increases with x, the center of mass is to the right of the center of the rod.

8.105. IDENTIFY: $x_{cm} = \frac{1}{M}\int xdm$ and $y_{cm} = \frac{1}{M}\int ydm$. At the upper surface of the plate, $y^2 + x^2 = a^2$.

SET UP: To find x_{cm}, divide the plate into thin strips parallel to the y-axis, as shown in Figure 8.105a. To find y_{cm}, divide the plate into thin strips parallel to the x-axis as shown in Figure 8.105b. The plate has volume one-half that of a circular disk, so $V = \frac{1}{2}\pi a^2 t$ and $M = \frac{1}{2}\rho\pi a^2 t$.

EXECUTE: In Figure 8.105a each strip has length $y = \sqrt{a^2 - x^2}$. $x_{cm} = \frac{1}{M}\int xdm$, where

$dm = \rho tydx = \rho t\sqrt{a^2 - x^2}dx$. $x_{cm} = \frac{\rho t}{M}\int_{-a}^{a} x\sqrt{a^2 - x^2}dx = 0$, since the integrand is an odd function of x.

$x_{cm} = 0$ because of symmetry. In Figure 8.105b each strip has length $2x = 2\sqrt{a^2 - y^2}$. $y_{cm} = \frac{1}{M}\int ydm$,

where $dm = 2\rho txdy = 2\rho t\sqrt{a^2 - y^2}dy$. $y_{cm} = \frac{2\rho t}{M}\int_{-a}^{a} y\sqrt{a^2 - y^2}dy$. The integral can be evaluated using

$u = a^2 - y^2$, $du = -2ydy$. This substitution gives

$$y_{cm} = \frac{2\rho t}{M}\left(-\frac{1}{2}\right)\int_{a^2}^{0} u^{1/2}du = \frac{2\rho ta^3}{3M} = \left(\frac{2\rho ta^3}{3}\right)\left(\frac{2}{\rho\pi a^2 t}\right) = \frac{4a}{3\pi}.$$

EVALUATE: $\frac{4}{3\pi} = 0.424$. y_{cm} is less than $a/2$, as expected, since the plate becomes wider as y decreases.

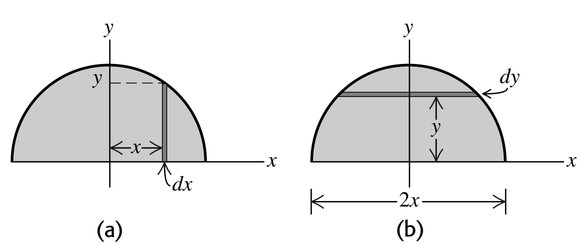

Figure 8.105

8.106. **IDENTIFY** and **SET UP:** $p = mv$.
EXECUTE: $p = mv = (0.30 \times 10^{-3}\text{ kg})(2.5\text{ m/s}) = 7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s}$, which makes choice (a) correct.

EVALUATE: This is a small amount of momentum for a speed of 2.5 m/s, but the water drop is very light.

8.107. **IDENTIFY** and **SET UP:** Momentum is conserved, $p = mv$.
EXECUTE: $(65 \times 10^{-3}\text{ kg})v_{\text{fish}} = 7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s}$, so $v_{\text{fish}} = 0.012$ m/s, which makes choice (b) correct.
EVALUATE: The fish is much ligher than the water drop and thus moves much slower.

8.108. **IDENTIFY** and **SET UP:** $J = F_{\text{av}}t = \Delta p$.
EXECUTE: $\text{F}_{\text{av}} = \Delta p/t = (7.5\times10^{-4}\text{ kg}\cdot\text{m/s})/(0.0050\text{ s}) = 0.15\text{ N}$, which is choice (d).

EVALUATE: This is a rather small force, but it acts on a very light-weight water drop, so it can give the water considerable speed.

8.109. **IDENTIFY** and **SET UP:** Momentum is conserved in the collision with the insect. $p = mv$.
EXECUTE: Using $P_1 = P_2$ gives $7.5 \times 10^{-4}\text{ kg}\cdot\text{m/s} = (m_{\text{insect}} + 3.0 \times 10^{-4}\text{ kg})(2.0\text{ m/s})$, which gives $m_{\text{insect}} = 0.075$ g, so choice (b) is correct.
EVALUATE: The insect has considerably less mass than the water drop.