33

THE NATURE AND PROPAGATION OF LIGHT

VP33.2.1. **IDENTIFY:** We have reflection and refraction. Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: **(a)** For reflected rays, the angle of reflection is equal to the angle of incidence. So the angle of reflection is 70.0°.

(b) Apply Snell's law. $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.00)\sin 70.0° = (1.80)\sin\theta_b$. $\theta_b = 31.5°$.

EVALUATE: The light is bent toward the normal in the glass.

VP33.2.2. **IDENTIFY:** We have refraction at a flat surface, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $n = c/v$.

EXECUTE: **(a)** We want n. (1.33) sin 55.0° = $n$sin37.0°. n = 1.81.

(b) We want speed of light in the glass. $v = c/n = c/1.81 = 1.66\times10^8$ m/s.

EVALUATE: The light is bent toward the normal in the glass because $n_{\text{glass}} > n_{\text{water}}$.

VP33.2.3. **IDENTIFY:** We have refraction at a flat surface, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$, λ_{air} is almost the same as λ_{vacuum}.

EXECUTE: **(a)** We want n. $\lambda_n = \dfrac{\lambda_0}{n}$. $n = \lambda_0/\lambda_n$ = (635 nm)/(508 nm) = 1.25.

(b) We want θ_b (in the liquid). Apply $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.00)\sin 35.0° = (1.25)\sin\theta_b$. $\theta_b = 27.3°$.

(c) We want the frequency in air. $f_a = c/\lambda_a = c/(635\text{ nm}) = 4.72\times10^{14}$ Hz.

(d) We want the frequency in the liquid. The frequency does not change, so it is the same as in part (c): $f_a = 4.72\times10^{14}$ Hz.

EVALUATE: Our answer to (a) gives n = 1.25, which is reasonable because n is always greater than one.

VP33.2.4. **IDENTIFY:** We have refraction at a flat surface, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$, λ_{air} is almost the same as λ_{vacuum}.

EXECUTE: **(a)** We want θ_a (in ethanol). Apply $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.36)\sin\theta_a = (1.309)\sin 85.0°$. $\theta_a = 73.5°$.

(b) We want $\lambda_{\text{ice}}/\lambda_{\text{ethanol}}$. Use $\lambda_n = \dfrac{\lambda_0}{n}$. $\lambda_{\text{ice}}/\lambda_{\text{ethanol}} = \dfrac{\dfrac{\lambda_0}{n_i}}{\dfrac{\lambda_0}{n_e}} = \dfrac{n_e}{n_i} = \dfrac{1.36}{1.309} = 1.04$.

EVALUATE: Since $n_i < n_e$, we expect that $\theta_i > \theta_e$, which is what we have found.

VP33.5.1. **IDENTIFY:** This problem deals with polarized light, so Malus's law applies.

SET UP: $I = I_0 \cos^2 \phi$.

EXECUTE: We want the intensity. $I = I_0 \cos^2 \phi$ = (255 W/m^2) cos^2 15.0° = 238 W/m^2.

EVALUATE: The filter axis and the polarization direction of the beam are closely aligned, so most of the light should get through, which our result shows.

VP33.5.2. **IDENTIFY:** This problem involves a polarizing filter, so Malus's law applies.

SET UP: Fig. VP33.5.2 illustrates the situation. $I = I_0 \cos^2 \phi$.

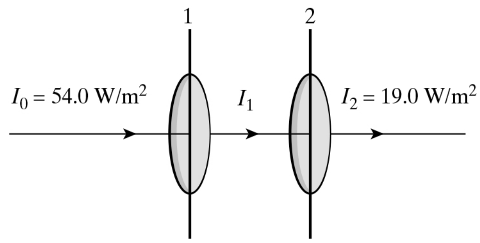


Figure VP33.5.2

EXECUTE: **(a)** We want the intensity after the first polarizer. A polarizer absorbs half the unpolarized light, so I_1 = (52.0 W/m^2)/2 = 27.0 W/m^2.

(b) We want ϕ . $I_2 = I_1 \cos^2 \phi$. 19.0 W/m^2 = (27.0 W/m^2) cos^2 ϕ . ϕ = 33.0°.

EVALUATE: The first polarizer reduced the intensity of the incident light by a greater percent than the second filter did.

VP33.5.3. **IDENTIFY:** This problem involves a polarizing filter, so Malus's law applies.

SET UP: Fig. VP33.5.3 illustrates the arrangement. $I = I_0 \cos^2 \phi$.

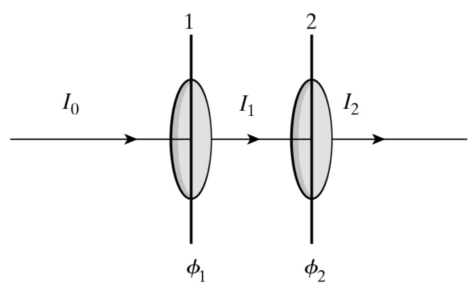


Figure VP33.5.3

EXECUTE: **(a)** We want I_1. $I_1 = I_0 \cos^2 \phi_1$ = (60.0 W/m^2) cos^2 25.0° = 49.3 W/m^2.

(b) We want I_2. I_1 is polarized at 25.0° from the y direction, so ϕ_2 = 50.0° – 25.0° = 25.0°.

$I_2 = I_1 \cos^2 \phi_2$ = (49.3 W/m^2) cos^2 25.0° = 40.5 W/m^2.

(c) We want I_2. ϕ = 50.0°. I_2 = (60.0 W/m^2) cos^2 50.0° = 25.8 W/m^2.

EVALUATE: Surprisingly we remove more of the light with just one filter, even though it is at the same angle as it was in the first case.

VP33.5.4. **IDENTIFY:** We are dealing with polarized light, so Malus's law applies.

SET UP: $I = I_0 \cos^2 \phi$. Two beams, one polarized and the other unpolarized, pass through the same filter simultaneously. We want the intensity of the emerging light.

EXECUTE: The emerging light has intensity $I = I_p + I_u$. $I_p = I_0\cos^2\phi = I_0\cos^2 30° = 0.750I_0$.

$I_u = 0.500I_0$. $I = 0.750I_0 + 0.500I_0 = 1.25I_0 = \dfrac{5}{4}I_0$.

EVALUATE: If there were no filter, the emerging intensity would be $2I_0$. With the filter, it is $1.25I_0$. So the final intensity is $1.25I_0/2.00I_0 = 0.625 = 62.5\%$ of the initial intensity. Therefore the filter has removed 37.5% of the light.

VP33.6.1. **IDENTIFY:** We are looking at polarization by reflection, so Brewster's law applies.
SET UP: $\tan\theta_p = n_b/n_a$.

EXECUTE: **(a)** We want the angle of incidence so the reflected light will be totally polarized. This is true at Brewster's angle $\tan\theta_p = n_b/n_a$, where $n_a = 1.00$ (air) and $n_b = 1.73$ (solid). This gives

$\tan\theta_p = 1.73/1.00$, so $\theta_p = 60.0°$.

(b) The reflected light is 100% polarized, but not all the light with that angle of polarization is reflected – some goes into the solid. So the transmitted light is only *partially* polarized.
EVALUATE: If water ($n = 1.33$) were on the solid, the polarizing angle would be arctan(1.73/1.33) = 52.4°.

VP33.6.2. **IDENTIFY:** This problem involves polarization by reflection, so Brewster's law applies as well as Snell's law.
SET UP: $\tan\theta_p = n_b/n_a$, $n_a\sin\theta_a = n_b\sin\theta_b$.

EXECUTE: **(a)** We want n for the glass. Since the reflected light is completely polarized, the angle of incidence must have Brewster's angle. Use $\tan\theta_p = n_b/n_a$ and solve for n.

$n = n_a\tan\theta_p = (1.00)\tan 57.0° = 1.54$.

(b) We want the angle of refraction. Use $n_a\sin\theta_a = n_b\sin\theta_b$. $(1.00)\sin 57.0° = (1.54)\sin\theta_b$. $\theta_b = 33.0°$.

EVALUATE: Since $n_{glass} > n_{air}$, the light should have been bent toward the normal in the glass, which in fact it was, so our results are reasonable.

VP33.6.3. **IDENTIFY:** This problem involves polarization by reflection, so Brewster's law applies.
SET UP: $\tan\theta_p = n_b/n_a$. We want the angle of incidence so no light reflects from the glass.

EXECUTE: **(a)** The incident light is already polarized perpendicular to the plane of incidence, so there is *no angle* at which no light reflects.
(b) At Brewster's angle, all the reflected light is polarized perpendicular to the plane of incidence. But for this light, none of it is polarized that way. So at Brewster's angle, none of it reflects. $\tan\theta_p = n_b/n_a$ = 1.66/1.00 = 1.66, so $\theta_p = 58.9°$.

EVALUATE: In part (b) all the light enters the glass.

VP33.6.4. **IDENTIFY:** This problem involves polarization by reflection, so Brewster's law applies.
SET UP: $\tan\theta_p = n_b/n_a$. At Brewster's angle, the refracted light is perpendicular to the reflected light.

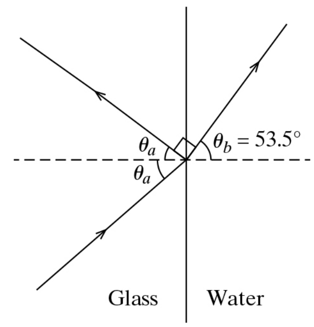


Figure VP33.6.4

EXECUTE: **(a)** We want θ_p. For 100% of the reflected light to be polarized, the angle of incidence must be Brewster's angle. The reflected and refracted rays are perpendicular, so $\theta_b + 90° + \theta_a = 180°$, which gives $\theta_a = 180° - 90° - \theta_b = 90° - 53.5° = 36.5°$.

EVALUATE: $n_{glass} > n_{water}$ so the light should be bent away from the normal in the water, which agrees with our result since θ_b (53.5°) > θ_a (36.5°).

33.1. **IDENTIFY:** For reflection, $\theta_r = \theta_a$.

SET UP: The desired path of the ray is sketched in Figure 33.1.

EXECUTE: $\tan\phi = \dfrac{14.0 \text{ cm}}{11.5 \text{ cm}}$, so $\phi = 50.6°$. $\theta_r = 90° - \phi = 39.4°$ and $\theta_r = \theta_a = 39.4°$.

EVALUATE: The angle of incidence is measured from the normal to the surface.

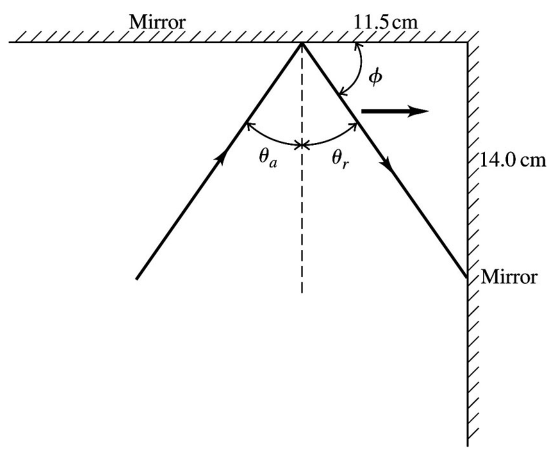


Figure 33.1

33.2. **IDENTIFY:** The speed and the wavelength of the light will be affected by the vitreous humor, but not the frequency.

SET UP: $n = \dfrac{c}{v}$. $v = f\lambda$. $\lambda = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** $\lambda_v = \dfrac{\lambda_{0,v}}{n} = \dfrac{380 \text{ nm}}{1.34} = 284 \text{ nm}$. $\lambda_r = \dfrac{\lambda_{0,r}}{n} = \dfrac{750 \text{ nm}}{1.34} = 560 \text{ nm}$. The range is 284 nm to 560 nm.

(b) Calculate the frequency in air, where $v = c = 3.00\times10^8$ m/s.

$f_r = \dfrac{c}{\lambda_r} = \dfrac{3.00\times10^8 \text{ m/s}}{750\times10^{-9} \text{ m}} = 4.00\times10^{14}$ Hz. $f_v = \dfrac{c}{\lambda_v} = \dfrac{3.00\times10^8 \text{ m/s}}{380\times10^{-9} \text{ m}} = 7.89\times10^{14}$ Hz. The range is 4.00×10^{14} Hz to 7.89×10^{14} Hz.

(c) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.34} = 2.24\times10^8$ m/s.

EVALUATE: The frequency range in air is the same as in the vitreous humor.

33.3. **IDENTIFY** and **SET UP:** Use $n = \dfrac{c}{v}$ and $\lambda = \dfrac{\lambda_0}{n}$ to calculate v and λ in the liquid.

EXECUTE: **(a)** $n = \dfrac{c}{v}$ so $v = \dfrac{c}{n} = \dfrac{2.998\times10^8 \text{ m/s}}{1.47} = 2.04\times10^8$ m/s.

(b) $\lambda = \dfrac{\lambda_0}{n} = \dfrac{650 \text{ nm}}{1.47} = 442$ nm.

EVALUATE: Light is slower in the liquid than in vacuum. By $v = f\lambda$, when v is smaller, λ is smaller.

33.4. **IDENTIFY:** In air, $c = f\lambda_0$. In glass, $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $\lambda_0 = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{5.80\times10^{14} \text{ Hz}} = 517$ nm.

(b) $\lambda = \dfrac{\lambda_0}{n} = \dfrac{517 \text{ nm}}{1.52} = 340$ nm.

EVALUATE: In glass the light travels slower than in vacuum and the wavelength is smaller.

33.5. **IDENTIFY:** This problem involves refraction at a glass-liquid boundary. Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $n = c/v$, $\lambda_n = \dfrac{\lambda_0}{n}$. In the liquid we want the speed, wavelength, and frequency of the light. Let subscript g denote glass and L denote the liquid.

EXECUTE: Find n_g: $n_g = c/v = c/(1.85\times10^8 \text{ m/s}) = 1.622$.

Find n_L: Use $n_a \sin\theta_a = n_b \sin\theta_b$. $(1.622)\sin 38.0° = n_L \sin 44.7°$. $n_L = 1.419$.

$v_L = c/n_L = c/(1.419) = 2.11\times10^8$ m/s. $\lambda_L = \dfrac{\lambda_0}{n_L}$. $\lambda_g = \dfrac{\lambda_0}{n_g}$, so $\lambda_0 = n_g\lambda_g$. Combining gives

$$\lambda_L = \frac{\lambda_g n_g}{n_L} = \frac{(365 \text{ nm})(1.622)}{1.419} = 417 \text{ nm}.$$

The frequency does not change, so $f_L = f_g = 5.07\times10^{14}$ Hz.

EVALUATE: The light was bent away from the normal in the liquid, so it should be true that $n_L < n_g$. We found that $n_g = 1.622$ and $n_L = 1.419$, so our result is reasonable.

33.6. **IDENTIFY:** $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: From Table 33.1, $n_{\text{water}} = 1.333$ and $n_{\text{benzene}} = 1.501$.

EXECUTE: $\lambda_{\text{water}} n_{\text{water}} = \lambda_{\text{benzene}} n_{\text{benzene}} = \lambda_0$. $\lambda_{\text{benzene}} = \lambda_{\text{water}}\left(\dfrac{n_{\text{water}}}{n_{\text{benzene}}}\right) = (526 \text{ nm})\left(\dfrac{1.333}{1.501}\right) = 467$ nm.

EVALUATE: λ is smallest in benzene, since n is largest for benzene.

33.7. IDENTIFY: Apply the law of reflection and Snell's law to calculate θ_r and θ_b. The angles in these equations are measured with respect to the normal, not the surface.

SET UP: The incident, reflected and refracted rays are shown in Figure 33.7. The law of reflection is $\theta_r = \theta_a$, and Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$.

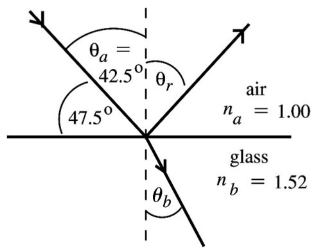


Figure 33.7

EXECUTE: (a) $\theta_r = \theta_a = 42.5°$ The reflected ray makes an angle of $90.0° - \theta_r = 47.5°$ with the surface of the glass.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$, where the angles are measured from the normal to the interface.

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.00)(\sin 42.5°)}{1.66} = 0.4070.$$

$\theta_b = 24.0°.$

The refracted ray makes an angle of $90.0° - \theta_b = 66.0°$ with the surface of the glass.

EVALUATE: The light is bent toward the normal when the light enters the material of larger refractive index.

33.8. IDENTIFY: The time delay occurs because the beam going through the transparent material travels slower than the beam in air.

SET UP: $v = \dfrac{c}{n}$ in the material, but $v = c$ in air.

EXECUTE: The time for the beam traveling in air to reach the detector is

$t = \dfrac{d}{c} = \dfrac{2.50 \text{ m}}{3.00\times 10^8 \text{ m/s}} = 8.33\times 10^{-9}$ s. The light traveling in the block takes time

$t = 8.33\times 10^{-9} \text{ s} + 6.25\times 10^{-9} \text{ s} = 1.46\times 10^{-8}$ s. The speed of light in the block is

$v = \dfrac{d}{t} = \dfrac{2.50 \text{ m}}{1.46\times 10^{-8} \text{ s}} = 1.71\times 10^8$ m/s. The refractive index of the block is $n = \dfrac{c}{v} = \dfrac{3.00\times 10^8 \text{ m/s}}{1.71\times 10^8 \text{ m/s}} = 1.75.$

EVALUATE: $n > 1$, as it must be, and 1.75 is a reasonable index of refraction for a transparent material such as plastic.

33.9. IDENTIFY and SET UP: Use Snell's law to find the index of refraction of the plastic and then use $n = \dfrac{c}{v}$ to calculate the speed v of light in the plastic.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$.

$$n_b = n_a\left(\frac{\sin\theta_a}{\sin\theta_b}\right) = 1.00\left(\frac{\sin 62.7°}{\sin 48.1°}\right) = 1.194.$$

$n = \dfrac{c}{v}$ so $v = \dfrac{c}{n} = (3.00\times 10^8 \text{ m/s})/1.194 = 2.51\times 10^8$ m/s.

EVALUATE: Light is slower in plastic than in air. When the light goes from air into the plastic it is bent toward the normal.

33.10. **IDENTIFY:** Apply Snell's law at both interfaces.

SET UP: The path of the ray is sketched in Figure 33.10. Table 33.1 gives $n = 1.329$ for the methanol.

EXECUTE: **(a)** At the air-glass interface $(1.00)\sin 41.3° = n_{\text{glass}} \sin\alpha$. At the glass-methanol interface $n_{\text{glass}} \sin\alpha = (1.329)\sin\theta$. Combining these two equations gives $\sin 41.3° = 1.329\sin\theta$ and $\theta = 29.8°$.

(b) The same figure applies apply as for part (a), except $\theta = 20.2°$. $(1.00)\sin 41.3° = n\sin 20.2°$ and $n = 1.91$.

EVALUATE: The angle α is $25.2°$. The index of refraction of methanol is less than that of the glass and the ray is bent away from the normal at the glass $\rightarrow$ methanol interface. The unknown liquid has an index of refraction greater than that of the glass, so the ray is bent toward the normal at the glass $\rightarrow$ liquid interface.

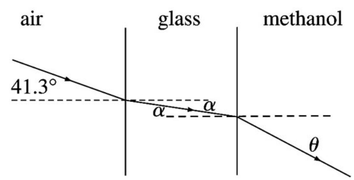


Figure 33.10

33.11. **IDENTIFY:** The figure shows the angle of incidence and angle of refraction for light going from the water into material *X*. Snell's law applies at the air-water and water-*X* boundaries.

SET UP: Snell's law says $n_a \sin\theta_a = n_b \sin\theta_b$. Apply Snell's law to the refraction from material *X* into the water and then from the water into the air.

EXECUTE: **(a)** Material *X* to water: $n_a = n_X$, $n_b = n_{\text{w}} = 1.333$. $\theta_a = 25°$ and $\theta_b = 48°$.

$$n_a = n_b\left(\frac{\sin\theta_b}{\sin\theta_a}\right) = (1.333)\left(\frac{\sin 48°}{\sin 25°}\right) = 2.34.$$

(b) Water to air: As Figure 33.11 shows, $\theta_a = 48°$. $n_a = 1.333$ and $n_b = 1.00$.

$$\sin\theta_b = \left(\frac{n_a}{n_b}\right)\sin\theta_a = (1.333)\sin 48° = 82°.$$

EVALUATE: $n > 1$ for material *X*, as it must be.

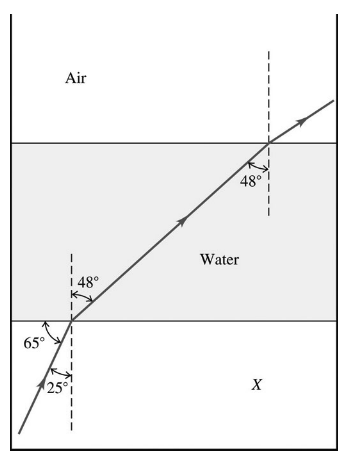


Figure 33.11

33.12. IDENTIFY: Apply Snell's law to the refraction at each interface.

SET UP: $n_{\text{air}} = 1.00$. $n_{\text{water}} = 1.333$.

EXECUTE: (a) $\theta_{\text{water}} = \arcsin\left(\dfrac{n_{\text{air}}}{n_{\text{water}}}\sin\theta_{\text{air}}\right) = \arcsin\left(\dfrac{1.00}{1.333}\sin 35.0^\circ\right) = 25.5^\circ$.

EVALUATE: (b) This calculation has no dependence on the glass because we can omit that step in the chain: $n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{glass}} \sin\theta_{\text{glass}} = n_{\text{water}} \sin\theta_{\text{water}}$.

33.13. IDENTIFY: This problem involves refraction at a boundary, so Snell's law applies.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$. Let the subscript *s* denote the solid and *g* denote the glass. We want θ_s.

EXECUTE: Snell's law gives $n_g \sin\theta_g = n_s \sin\theta_s$, so see that we need n_g and n_s. $\lambda_g = \dfrac{\lambda_0}{n_g}$ and $\lambda_s = \dfrac{\lambda_0}{n_s}$.

$\dfrac{\lambda_g}{\lambda_s} = \dfrac{447\text{ nm}}{315\text{ nm}} = 1.419 = \dfrac{\lambda_0/n_g}{\lambda_0/n_s} = \dfrac{n_s}{n_g}$, so $n_g = 1.419n_g$. Thus $n_g \sin 62.0^\circ = 1.419 n_g \sin\theta_s$, so $\theta_s = 38.5^\circ$.

EVALUATE: $n_s > n_g$, so the light should be bent toward the normal, which agrees with our result.

33.14. IDENTIFY: The wavelength of the light depends on the index of refraction of the material through which it is traveling, and Snell's law applies at the water-glass interface.

SET UP: $\lambda_0 = \lambda n$ so $\lambda_{\text{w}} n_{\text{w}} = \lambda_{\text{gl}} n_{\text{gl}}$. Snell's law gives $n_{\text{gl}} \sin\theta_{\text{gl}} = n_{\text{w}} \sin\theta_{\text{w}}$.

EXECUTE: $n_{\text{gl}} = n_{\text{w}}\left(\dfrac{\lambda_{\text{w}}}{\lambda_{\text{gl}}}\right) = (1.333)\left(\dfrac{726\text{ nm}}{544\text{ nm}}\right) = 1.779$. Now apply $n_{\text{gl}} \sin\theta_{\text{gl}} = n_{\text{w}} \sin\theta_{\text{w}}$.

$\sin\theta_{\text{gl}} = \left(\dfrac{n_{\text{w}}}{n_{\text{gl}}}\right)\sin\theta_{\text{w}} = \left(\dfrac{1.333}{1.779}\right)\sin 56.0^\circ = 0.6212$. $\theta_{\text{gl}} = 38.4^\circ$.

EVALUATE: $\theta_{\text{gl}} < \theta_{\text{w}}$ because $n_{\text{gl}} > n_{\text{w}}$.

33.15. IDENTIFY: The critical angle for total internal reflection is θ_a that gives $\theta_b = 90^\circ$ in Snell's law.

SET UP: In Figure 33.15 the angle of incidence θ_a is related to angle β by $\theta_a + \beta = 90^\circ$.

EXECUTE: (a) Calculate θ_a that gives $\theta_b = 90^\circ$. $n_a = 1.60$, $n_b = 1.00$ so $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$(1.60)\sin\theta_a = (1.00)\sin 90^\circ$. $\sin\theta_a = \dfrac{1.00}{1.60}$ and $\theta_a = 38.7^\circ$. $\beta = 90^\circ - \theta_a = 51.3^\circ$.

(b) $n_a = 1.60$, $n_b = 1.333$. $(1.60)\sin\theta_a = (1.333)\sin 90^\circ$. $\sin\theta_a = \dfrac{1.333}{1.60}$ and $\theta_a = 56.4^\circ$.

$\beta = 90^\circ - \theta_a = 33.6^\circ$.

EVALUATE: The critical angle increases when the ratio $\dfrac{n_a}{n_b}$ decreases.

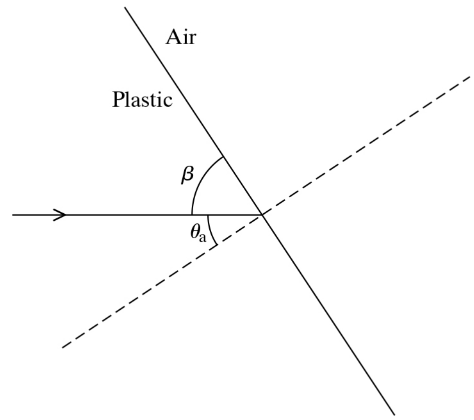


Figure 33.15

33.16. **IDENTIFY:** No light will enter the water if total internal reflection occurs at the glass-water boundary. Snell's law applies at the boundary.

SET UP: Find n_g, the refractive index of the glass. Then apply Snell's law at the boundary.

$n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: $n_g \sin 36.2° = n_w \sin 49.8°$. $n_g = (1.333)\left(\dfrac{\sin 49.8°}{\sin 36.2°}\right) = 1.724$. Now find θ_{crit} for the glass to water refraction. $n_g \sin\theta_{crit} = n_w \sin 90.0°$. $\sin\theta_{crit} = \dfrac{1.333}{1.724}$ and $\theta_{crit} = 50.6°$.

EVALUATE: For $\theta > 50.6°$ at the glass-water boundary, no light is refracted into the water.

33.17. **IDENTIFY:** Use the critical angle to find the index of refraction of the liquid.

SET UP: Total internal reflection requires that the light be incident on the material with the larger n, in this case the liquid. Apply $n_a \sin\theta_a = n_b \sin\theta_b$ with a = liquid and b = air, so $n_a = n_{liq}$ and $n_b = 1.0$.

EXECUTE: $\theta_a = \theta_{crit}$ when $\theta_b = 90°$, so $n_{liq} \sin\theta_{crit} = (1.0)\sin 90°$.

$$n_{liq} = \frac{1}{\sin\theta_{crit}} = \frac{1}{\sin 42.5°} = 1.48.$$

(a) $n_a \sin\theta_a = n_b \sin\theta_b$ (a = liquid, b = air).

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.48)\sin 35.0°}{1.0} = 0.8489 \text{ and } \theta_b = 58.1°.$$

(b) Now $n_a \sin\theta_a = n_b \sin\theta_b$ with a = air, b = liquid.

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.0)\sin 35.0°}{1.48} = 0.3876 \text{ and } \theta_b = 22.8°.$$

EVALUATE: Light traveling from liquid to air is bent away from the normal. Light traveling from air to liquid is bent toward the normal.

33.18. **IDENTIFY:** Since the refractive index of the glass is greater than that of air or water, total internal reflection will occur at the cube surface if the angle of incidence is greater than or equal to the critical angle.

SET UP: At the critical angle θ_{crit}, Snell's law gives $n_{glass} \sin\theta_{crit} = n_{air} \sin 90°$ and likewise for water.

EXECUTE: **(a)** At the critical angle θ_{crit}, $n_{glass} \sin\theta_{crit} = n_{air} \sin 90°$.

$1.62 \sin\theta_{crit} = (1.00)(1)$ and $\theta_{crit} = 38.1°$.

(b) Using the same procedure as in part (a), we have $1.62 \sin\theta_{crit} = 1.333 \sin 90°$ and $\theta_{crit} = 55.4°$.

EVALUATE: Since the refractive index of water is closer to the refractive index of glass than the refractive index of air is, the critical angle for glass-to-water is greater than for glass-to-air.

33.19. **IDENTIFY** and **SET UP:** For glass $\rightarrow$ water, $\theta_{\text{crit}} = 48.7°$. Apply Snell's law with $\theta_a = \theta_{\text{crit}}$ to calculate the index of refraction n_a of the glass.

EXECUTE: $n_a \sin\theta_{\text{crit}} = n_b \sin 90°$, so $n_a = \dfrac{n_b}{\sin\theta_{\text{crit}}} = \dfrac{1.333}{\sin 48.7°} = 1.77$

EVALUATE: For total internal reflection to occur the light must be incident in the material of larger refractive index. Our results give $n_{\text{glass}} > n_{\text{water}}$, in agreement with this.

33.20. **IDENTIFY:** The largest angle of incidence for which any light refracts into the air is the critical angle for water $\rightarrow$ air.

SET UP: Figure 33.20 shows a ray incident at the critical angle and therefore at the edge of the circle of light. The radius of this circle is r and $d = 10.0$ m is the distance from the ring to the surface of the water.

EXECUTE: From the figure, $r = d\tan\theta_{\text{crit}}$. θ_{crit} is calculated from $n_a \sin\theta_a = n_b \sin\theta_b$ with $n_a = 1.333$, $\theta_a = \theta_{\text{crit}}$, $n_b = 1.00$, and $\theta_b = 90°$. $\sin\theta_{\text{crit}} = \dfrac{(1.00)\sin 90°}{1.333}$ and $\theta_{\text{crit}} = 48.6°$.

$r = (10.0\text{ m})\tan 48.6° = 11.3\text{ m}.$

$A = \pi r^2 = \pi(11.3\text{ m})^2 = 401\text{ m}^2.$

EVALUATE: When the incident angle in the water is larger than the critical angle, no light refracts into the air.

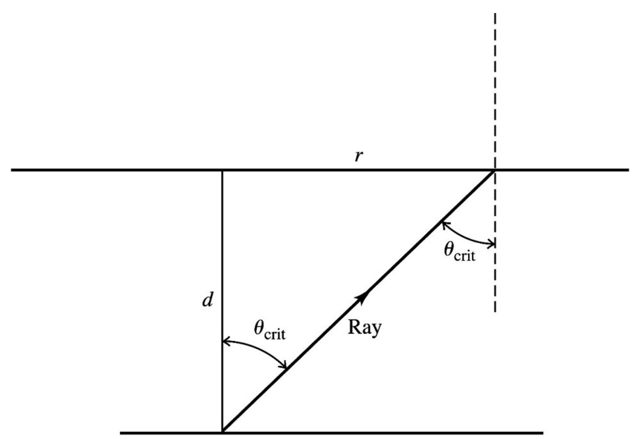


Figure 33.20

33.21. **IDENTIFY:** If no light refracts out of the glass at the glass to air interface, then the incident angle at that interface is θ_{crit}.

SET UP: The ray has an angle of incidence of $0°$ at the first surface of the glass, so enters the glass without being bent, as shown in Figure 33.21. The figure shows that $\alpha + \theta_{\text{crit}} = 90°$.

EXECUTE: **(a)** For the glass-air interface $\theta_a = \theta_{\text{crit}}$, $n_a = 1.52$, $n_b = 1.00$, and $\theta_b = 90°$.

$n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_{\text{crit}} = \dfrac{(1.00)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 41.1°$. $\alpha = 90° - \theta_{\text{crit}} = 48.9°$.

(b) Now the second interface is glass $\rightarrow$ water and $n_b = 1.333$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives

$\sin\theta_{\text{crit}} = \dfrac{(1.333)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 61.3°$. $\alpha = 90° - \theta_{\text{crit}} = 28.7°$.

EVALUATE: The critical angle increases when the air is replaced by water.

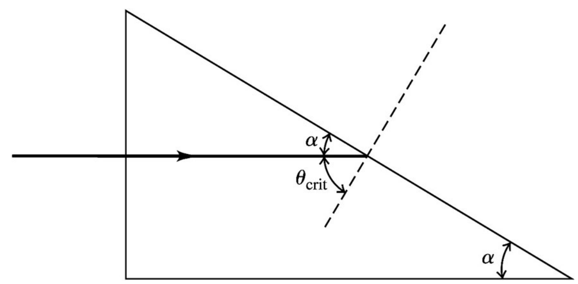


Figure 33.21

33.22. **IDENTIFY:** This problem involves Snell's law and total internal reflection.
SET UP: We want the wavelength in the plastic. Let subscript p denote the plastic and g denote the glass. $n_a \sin\theta_a = n_b \sin\theta_b$, $\lambda_n = \dfrac{\lambda_0}{n}$.

EXECUTE: At the critical angle $n_g \sin 48.6° = n_p \sin 90° = n_p$. $\lambda_p = \dfrac{\lambda_0}{n_p} = \dfrac{\lambda_0}{\lambda_g \sin 48.6°}$. $\lambda_g = \dfrac{\lambda_0}{n_g}$, so $\lambda_0 = n_g \lambda_g$. Combining gives $\lambda_p = \dfrac{n_g \lambda_g}{n_g \sin 48.6°} = \dfrac{\lambda_g}{\sin 48.6°} = \dfrac{350 \text{ nm}}{\sin 48.6°} = 467$ nm.

EVALUATE: Total internal reflection occurs only for light going from a high-n material to a low-n material, such as glass to water or water to air.

33.23. **IDENTIFY:** The index of refraction depends on the wavelength of light, so the light from the red and violet ends of the spectrum will be bent through different angles as it passes into the glass. Snell's law applies at the surface.
SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. From the graph in Figure 33.17 in the textbook, for $\lambda = 400$ nm (the violet end of the visible spectrum), $n = 1.67$ and for $\lambda = 700$ nm (the red end of the visible spectrum), $n = 1.62$. The path of a ray with a single wavelength is sketched in Figure 33.23.

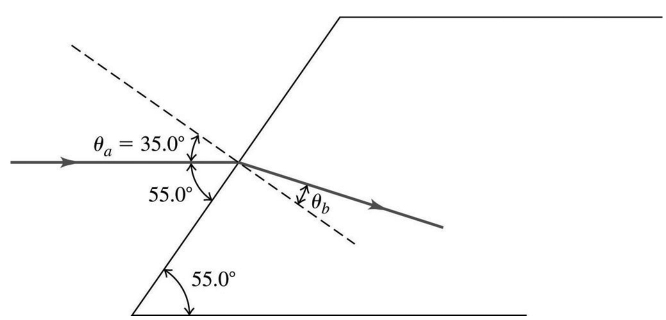


Figure 33.23

EXECUTE: For $\lambda = 400$ nm, $\sin\theta_b = \dfrac{n_a}{n_b}\sin\theta_a = \dfrac{1.00}{1.67}\sin 35.0°$, so $\theta_b = 20.1°$. For $\lambda = 700$ nm, $\sin\theta_b = \dfrac{1.00}{1.62}\sin 35.0°$, so $\theta_b = 20.7°$. $\Delta\theta$ is about $0.6°$.

EVALUATE: This angle is small, but the separation of the beams could be fairly large if the light travels through a fairly large slab.

33.24. IDENTIFY: The red and violet light will be bent through different angles in the glass because they have slightly different indexes of refraction. Use Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$.

SET UP: Apply Snell's law twice: the first time use the index of refraction for red light $(n = 2.41)$ and the second time use the index of refraction for violet light $(n = 2.46)$. Assume that the index of refraction for air is $n = 1.00$.

EXECUTE: For red light Snell's law gives $(1.00)\sin 53.5^\circ = (2.41)\sin\theta_{\text{red}}$. Solving this equation we find $\theta_{\text{red}} = 19.48^\circ$. For violet light Snell's law gives $(1.00)\sin 53.5^\circ = (2.46)\sin\theta_{\text{violet}}$. Solving this equation we find $\theta_{\text{violet}} = 19.07^\circ$. From these two values we can calculate the angle between the two initially coincident rays: $\Delta\theta = \theta_{\text{red}} - \theta_{\text{violet}} = 19.48^\circ - 19.07^\circ = 0.41^\circ$.

EVALUATE: Violet light is refracted more than red light since it has the larger index of refraction. Although the angular separation between the red and the blue rays is small, it is easily noticeable under the right circumstances.

33.25. IDENTIFY: Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$. $v = \dfrac{c}{n}$.

SET UP: $a = \text{air}$, $b = \text{glass}$.

EXECUTE: **(a)** red: $n_b = \dfrac{n_a \sin\theta_a}{\sin\theta_b} = \dfrac{(1.00)\sin 57.0^\circ}{\sin 38.1^\circ} = 1.36$. violet: $n_b = \dfrac{(1.00)\sin 57.0^\circ}{\sin 36.7^\circ} = 1.40$.

(b) red: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.36} = 2.21\times10^8 \text{ m/s}$; violet: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.40} = 2.14\times10^8 \text{ m/s}$.

EVALUATE: n is larger for the violet light and therefore this light is bent more toward the normal, and the violet light has a smaller speed in the glass than the red light.

33.26. IDENTIFY: This problem involves birefringence and a quarter-wave plate.

SET UP: For calcite, n = 1.658 and 1.486 for λ = 589 nm in vacuum, $v = c/n$, and $\lambda = cT$. For this light, what should be the minimum thickness of the quarter-wave plate?

EXECUTE: To cancel, the difference in time between the two waves must be ¼ of the period T. The two waves differ because there are two indices of refraction. $t_1 = d/v_1$ and $t_2 = d/v_2$.

$\Delta t = t_2 - t_1 = \dfrac{d}{v_2} - \dfrac{d}{v_1} = \dfrac{1}{4}T$. Using $v = c/n$ and $\lambda = cT$ gives $\dfrac{d}{c/n_2} - \dfrac{d}{c/n_1} = \dfrac{1}{4}T = \dfrac{1}{4}\left(\dfrac{\lambda}{c}\right)$. Simplifying and solving for d gives $d = \dfrac{\lambda}{4(n_2 - n_2)} = \dfrac{589 \text{ nm}}{4(1.658 - 1.486)} = 856 \text{ nm}$.

EVALUATE: This is the minimum thickness. If the time difference is $T + T/4$, $2T + T/4$, … , cancellation will also occur.

33.27. IDENTIFY: The first polarizer filters out half the incident light. The fraction filtered out by the second polarizer depends on the angle between the axes of the two filters.

SET UP: Use Malus's law: $I = I_0 \cos^2\phi$.

EXECUTE: After the first filter, $I = \frac{1}{2}I_0$. After the second filter, $I = (\frac{1}{2}I_0)\cos^2\phi$, which gives $I = \left(\frac{1}{2}I_0\right)\cos^2 30.0^\circ = 0.375 I_0$.

EVALUATE: The only variable that affects the answer is the angle between the axes of the two polarizers.

33.28. **IDENTIFY:** This problem involves polarization by reflection and Snell's law.
SET UP: $\tan\theta_{\text{p}} = n_b/n_a$, $n_a \sin\theta_a = n_b \sin\theta_b$. We want to find polarizing angles. For light traveling from a to b, the polarizing angle is given by $\tan\theta_{\text{p}} = n_b/n_a$. At this angle, 100% of the reflected light is polarized. Use the critical angle to relate the indices of refraction. Let subscript L denote the liquid and s the solid.
EXECUTE: **(a)** $n_s \sin 38.7° = 0.6252 n_s = n_L \sin 90° = n_L$. Now use Brewster's law.
$\tan\theta_{\text{p}} = n_L/n_s = 0.6252 n_s/n_s = 0.6252$. $\theta_{\text{p}} = 32.0°$.
(b) If we reverse the rays, we get $\tan\theta_{\text{p}} = n_s/n_L = n_s/(0.6252 n_s) = 1.599$. $\theta_{\text{p}} = 58.0°$.
EVALUATE: Of course we would not have total internal reflection for the rays in part (b) since that only occurs for light going from high-n to low-n materials. However polarization would still occur in the reflected beam.

33.29. **IDENTIFY:** When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity $I_{\max}$ is incident on a polarizer, the transmitted intensity is $I = I_{\max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.
SET UP: For the second polarizer $\phi = 60°$. For the third polarizer, $\phi = 90° - 60° = 30°$.
EXECUTE: **(a)** At point A the intensity is $I_0/2$ and the light is polarized along the vertical direction. At point B the intensity is $(I_0/2)(\cos 60°)^2 = 0.125 I_0$, and the light is polarized along the axis of the second polarizer. At point C the intensity is $(0.125 I_0)(\cos 30°)^2 = 0.0938 I_0$.
(b) Now for the last filter $\phi = 90°$ and $I = 0$.
EVALUATE: Adding the middle filter increases the transmitted intensity.

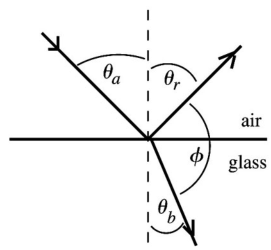
33.30. **IDENTIFY:** Set $I = I_0/10$, where I is the intensity of light passed by the second polarizer.
SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity $I_{\max}$ is incident on a polarizer, the transmitted intensity is $I = I_{\max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.
EXECUTE: **(a)** After the first filter $I = \dfrac{I_0}{2}$ and the light is polarized along the vertical direction. After the second filter we want $I = \dfrac{I_0}{10}$, so $\dfrac{I_0}{10} = \left(\dfrac{I_0}{2}\right)(\cos\phi)^2$. $\cos\phi = \sqrt{2/10}$ and $\phi = 63.4°$.
(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter
$\dfrac{I_0}{10} = I_0(\cos\phi)^2$. $\cos\phi = \sqrt{1/10}$ and $\phi = 71.6°$.
EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.

33.31. **IDENTIFY** and **SET UP:** Reflected beam completely linearly polarized implies that the angle of incidence equals the polarizing angle, so $\theta_{\text{p}} = 54.5°$. Use Brewster's law, $\tan\theta_{\text{p}} = \dfrac{n_b}{n_a}$, to calculate the refractive index of the glass. Then use Snell's law to calculate the angle of refraction. See Figure 33.29.
EXECUTE: **(a)** $\tan\theta_{\text{p}} = \dfrac{n_b}{n_a}$ gives $n_{\text{glass}} = n_{\text{air}}\tan\theta_{\text{p}} = (1.00)\tan 54.5° = 1.40$.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$.

$\sin\theta_b = \dfrac{n_a \sin\theta_a}{n_b} = \dfrac{(1.00)\sin 54.5°}{1.40} = 0.5815$ and $\theta_b = 35.5°$.

EVALUATE:



Note: $\phi = 180.0° - \theta_r - \theta_b$ and $\theta_r = \theta_a$. Thus $\phi = 180.0° - 54.5° - 35.5° = 90.0°$; the reflected ray and the refracted ray are perpendicular to each other. This agrees with Figure 33.28 in the textbook.

Figure 33.29

33.32. **IDENTIFY:** The reflected light is completely polarized when the angle of incidence equals the polarizing angle θ_p, where $\tan\theta_p = \dfrac{n_b}{n_a}$.

SET UP: $n_b = 1.66$.

EXECUTE: **(a)** $n_a = 1.00$. $\tan\theta_p = \dfrac{1.66}{1.00}$ and $\theta_p = 58.9°$.

(b) $n_a = 1.333$. $\tan\theta_p = \dfrac{1.66}{1.333}$ and $\theta_p = 51.2°$.

EVALUATE: The polarizing angle depends on the refractive indicies of both materials at the interface.

33.33. **IDENTIFY:** When unpolarized light of intensity I_0 is incident on a polarizing filter, the transmitted light has intensity $\frac{1}{2}I_0$ and is polarized along the filter axis. When polarized light of intensity I_0 is incident on a polarizing filter the transmitted light has intensity $I_0 \cos^2\phi$.

SET UP: For the second filter, $\phi = 62.0° - 25.0° = 37.0°$.

EXECUTE: After the first filter the intensity is $\frac{1}{2}I_0 = 10.0\ \text{W/cm}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0\cos^2\phi$, where $I_0 = 10.0\ \text{W/cm}^2$ and $\phi = 37.0°$. This gives $I = 6.38\ \text{W/cm}^2$.

EVALUATE: The transmitted intensity depends on the angle between the axes of the two filters.

33.34. **IDENTIFY:** Use the transmitted intensity when all three polarizers are present to solve for the incident intensity I_0. Then repeat the calculation with only the first and third polarizers.

SET UP: For unpolarized light incident on a filter, $I = \frac{1}{2}I_0$ and the light is linearly polarized along the filter axis. For polarized light incident on a filter, $I = I_{max}(\cos\phi)^2$, where I_{max} is the intensity of the incident light, and the emerging light is linearly polarized along the filter axis.

EXECUTE: With all three polarizers, if the incident intensity is I_0 the transmitted intensity is

$I = \frac{1}{2}I_0\cos^2 23.0°\cos^2(62.0° - 23.0°) = 0.2559I_0$. $I_0 = \dfrac{I}{0.2559} = \dfrac{55.0\ \text{W/cm}^2}{0.2559} = 215\ \text{W/cm}^2$. With only the first and third polarizers, $I = \frac{1}{2}I_0\cos^2 62.0° = 0.110I_0 = (0.110)(215\ \text{W/cm}^2) = 23.7\ \text{W/cm}^2$.

EVALUATE: The transmitted intensity is greater when all three filters are present.

33.35. **IDENTIFY:** The shorter the wavelength of light, the more it is scattered. The intensity is inversely proportional to the fourth power of the wavelength.

SET UP: The intensity of the scattered light is proportional to $1/\lambda^4$; we can write it as $I = (\text{constant})/\lambda^4$.

EXECUTE: **(a)** Since I is proportional to $1/\lambda^4$, we have $I = (\text{constant})/\lambda^4$. Taking the ratio of the intensity of the red light to that of the green light gives

$$\frac{I_{\text{R}}}{I} = \frac{(\text{constant})/\lambda_{\text{R}}^4}{(\text{constant})/\lambda_{\text{G}}^4} = \left(\frac{\lambda_{\text{G}}}{\lambda_{\text{R}}}\right)^4 = \left(\frac{532 \text{ nm}}{685 \text{ nm}}\right)^4 = 0.364, \text{ so } I_{\text{R}} = 0.364I.$$

(b) Following the same procedure as in part (a) gives $\dfrac{I_{\text{V}}}{I} = \left(\dfrac{\lambda_{\text{G}}}{\lambda_{\text{V}}}\right)^4 = \left(\dfrac{532 \text{ nm}}{415 \text{ nm}}\right)^4 = 2.70, \text{ so } I_{\text{V}} = 2.70I.$

EVALUATE: In the scattered light, the intensity of the short-wavelength violet light is about 7 times as great as that of the red light, so this scattered light will have a blue-violet color.

33.36. **IDENTIFY:** We use measurements to determine the index of refraction for water. Snell's law applies. $n_a \sin\theta_a = n_b \sin\theta_b$

SET UP and **EXECUTE:** **(a)** Estimate: $s = 8.0$ mm.

(b) Estimate: $d = 25$ mm.

(c) Using Fig. P33.36 in the textbook, we have $\tan\theta_a = \dfrac{d}{H}$, $\tan\theta_b = \dfrac{s+d}{H}$.

(d) Apply Snell's law. $n_a \sin\theta_a = (1.00)\sin\theta_b$. $\sin\theta_a \approx \tan\theta_a$. $n\dfrac{d}{H} = \dfrac{s+d}{H}$. $n = \dfrac{s+d}{d} = 1 + \dfrac{s}{d}$.

(e) $n = 1 + \dfrac{8.0 \text{ mm}}{25 \text{ mm}} = 1.3.$

EVALUATE: This result is very close to the accepted value of 1.33, which is surprising given the rather crude estimates involved.

33.37. **IDENTIFY:** Snell's law applies to the sound waves in the heart.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. If θ_a is the critical angle then $\theta_b = 90°$. For air, $n_{\text{air}} = 1.00$. For heart muscle, $n_{\text{mus}} = \dfrac{344 \text{ m/s}}{1480 \text{ m/s}} = 0.2324$.

EXECUTE: **(a)** $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.00)\sin(9.73°) = (0.2324)\sin\theta_b$. $\sin\theta_b = \dfrac{\sin(9.73°)}{0.2324}$ so $\theta_b = 46.7°$.

(b) $(1.00)\sin\theta_{\text{crit}} = (0.2324)\sin 90°$ gives $\theta_{\text{crit}} = 13.4°$.

EVALUATE: To interpret a sonogram, it should be important to know the true direction of travel of the sound waves within muscle. This would require knowledge of the refractive index of the muscle.

33.38. **IDENTIFY:** Use the change in transit time to find the speed v of light in the slab, and then apply $n = \dfrac{c}{v}$ and $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam.

EXECUTE: $\dfrac{0.840\text{ m}}{c/n} - \dfrac{0.840\text{ m}}{c} = (n-1)\dfrac{0.840\text{ m}}{c} = 4.2\text{ ns}.$ We can now solve for the index of refraction: $n = \dfrac{(4.2\times10^{-9}\text{ s})(3.00\times10^{8}\text{ m/s})}{0.840\text{ m}} + 1 = 2.50.$ The wavelength inside of the glass is $\lambda = \dfrac{490\text{ nm}}{2.50} = 196\text{ nm}.$

EVALUATE: Light travels slower in the slab than in air and the wavelength is shorter.

33.39. **IDENTIFY:** The angle of incidence at A is to be the critical angle. Apply Snell's law at the air to glass refraction at the top of the block.

SET UP: The ray is sketched in Figure 33.39.

EXECUTE: For glass $\to$ air at point A, Snell's law gives $(1.38)\sin\theta_{\text{crit}} = (1.00)\sin 90°$ and $\theta_{\text{crit}} = 46.4°$. $\theta_b = 90° - \theta_{\text{crit}} = 43.6°$. Snell's law applied to the refraction from air to glass at the top of the block gives $(1.00)\sin\theta_a = (1.38)\sin(43.6°)$ and $\theta_a = 72.1°$.

EVALUATE: If θ_a is larger than $72.1°$ then the angle of incidence at point A is less than the initial critical angle and total internal reflection doesn't occur.

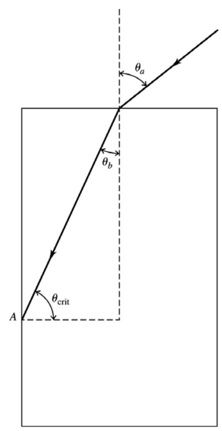


Figure 33.39

33.40. **IDENTIFY:** As the light crosses the glass-air interface along AB, it is refracted and obeys Snell's law.

SET UP: Snell's law is $n_a \sin\theta_{\text{a}} = n_b \sin\theta_{\text{b}}$ and $n = 1.000$ for air. At point B the angle of the prism is $30.0°$.

EXECUTE: Apply Snell's law at AB. The prism angle at A is 60.0°, so for the upper ray, the angle of refraction at AB is $60.0° + 12.0° = 72.0°$. Using this value gives $n_1 \sin 60.0° = \sin 72.0°$ and $n_1 = 1.10$. For the lower ray, the angle of refraction at AB is $60.0° + 12.0° + 8.50° = 80.5°$, giving $n_2 \sin 60.0° = \sin 80.5°$ and $n_2 = 1.14$.

EVALUATE: The lower ray is deflected more than the upper ray because that wavelength has a slightly greater index of refraction than the upper ray.

33.41. **IDENTIFY:** For total internal reflection, the angle of incidence must be at least as large as the critical angle.

SET UP: The angle of incidence for the glass-oil interface must be the critical angle, so $\theta_b = 90°$. $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.52)\sin 57.2° = n_{\text{oil}} \sin 90°$. $n_{\text{oil}} = (1.52)\sin 57.2° = 1.28$.

EVALUATE: $n_{\text{oil}} > 1$, which it must be, and 1.28 is a reasonable value for an oil.

33.42. **IDENTIFY:** Because the prism is a right-angle prism, the normals at point A and at surface BC are perpendicular to each other (see Figure 33.42). Therefore the angle of incidence at A is 50.0°, and this is the critical angle at that surface. Apply Snell's law at A and at surface BC. For light incident at the critical angle, the angle of refraction is 90°.

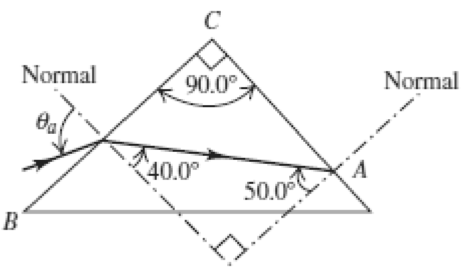


Figure 33.42

SET UP: Apply Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$. Use n = 1.00 for air, and let n be the index of refraction of the glass.
EXECUTE: Apply Snell's law at point A.
$n \sin(50.0°) = (1.00) \sin(90°) = 1.00$.
$n = 1.305$.
Now apply Snell's law at surface BC.
$(1.00) \sin\theta = (1.305) \sin(40.0°)$.
$\theta = 57.0°$.
EVALUATE: The critical angle at A would not be 50.0° if the prism were not a right-angle prism.

33.43. **IDENTIFY:** Apply $\lambda = \dfrac{\lambda_0}{n}$. The number of wavelengths in a distance d of a material is $\dfrac{d}{\lambda}$ where λ is the wavelength in the material.
SET UP: The distance in glass is $d_{\text{glass}} = 0.00250 \text{ m}$. The distance in air is $d_{\text{air}} = 0.0180 \text{ m} - 0.00250 \text{ m} = 0.0155 \text{ m}$.
EXECUTE: number of wavelengths = number in air + number in glass.

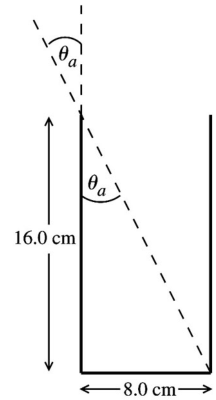
$$\text{number of wavelengths} = \frac{d_{\text{air}}}{\lambda} + \frac{d_{\text{glass}}}{\lambda} n = \frac{0.0155 \text{ m}}{5.40\times10^{-7} \text{ m}} + \frac{0.00250 \text{ m}}{5.40\times10^{-7} \text{ m}}(1.40) = 3.52\times10^4.$$

EVALUATE: Without the glass plate the number of wavelengths between the source and screen is $\dfrac{0.0180 \text{ m}}{5.40\times10^{-3} \text{ m}} = 3.33\times10^4$. The wavelength is shorter in the glass so there are more wavelengths in a distance in glass than there are in the same distance in air.

33.44. **IDENTIFY:** Apply Snell's law to the refraction of the light as it passes from water into air.
SET UP: $\theta_a = \arctan\left(\dfrac{1.5 \text{ m}}{1.2 \text{ m}}\right) = 51°$. $n_a = 1.00$. $n_b = 1.333$.
EXECUTE: $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.00}{1.333}\sin 51°\right) = 36°$. Therefore, the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is $x = (4.0 \text{ m})\tan\theta_b = (4.0 \text{ m})\tan 36° = 2.9 \text{ m}$. $x_{\text{total}} = 1.5 \text{ m} + x = 1.5 \text{ m} + 2.9 \text{ m} = 4.4 \text{ m}$.
EVALUATE: The light ray from the flashlight is bent toward the normal when it refracts into the water.

33.45. **IDENTIFY:** Use Snell's law to determine the effect of the liquid on the direction of travel of the light as it enters the liquid.

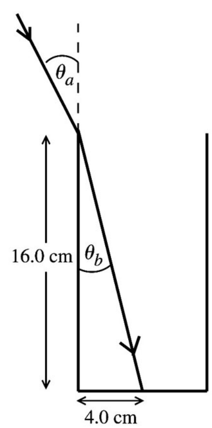
SET UP: Use geometry to find the angles of incidence and refraction. Before the liquid is poured in, the ray along your line of sight has the path shown in Figure 33.45a.



$$\tan\theta_a = \frac{8.0 \text{ cm}}{16.0 \text{ cm}} = 0.500.$$
$$\theta_a = 26.57°.$$

Figure 33.45a

After the liquid is poured in, θ_a is the same and the refracted ray passes through the center of the bottom of the glass, as shown in Figure 33.45b.



$$\tan\theta_b = \frac{4.0 \text{ cm}}{16.0 \text{ cm}} = 0.250.$$
$$\theta_b = 14.04°.$$

Figure 33.45b

EXECUTE: Use Snell's law to find n_b, the refractive index of the liquid:

$$n_a \sin\theta_a = n_b \sin\theta_b.$$
$$n_b = \frac{n_a \sin\theta_a}{\sin\theta_b} = \frac{(1.00)(\sin 26.57°)}{\sin 14.04°} = 1.84.$$

EVALUATE: When the light goes from air to liquid (larger refractive index) it is bent toward the normal.

33.46. **IDENTIFY:** Apply Snell's law. For light incident at the critical angle, the angle of refraction is 90°.

SET UP: Apply $n_a \sin\theta_a = n_b \sin\theta_b$ and use $n = 1.00$ for air.

EXECUTE: **(a)** Apply Snell's law at the interface between the cladding and the core. At that surface, the angle of incidence is the critical angle.

$n_1 \sin\theta_{\text{crit}} = n_2 \sin(90°) = n_2$.

$1.465 \sin\theta_{\text{crit}} = 1.450.$

$\theta_{\text{crit}} = 81.8°.$

(b) Apply Snell's law at the flat end of the cable and then at the core-cladding interface. Call θ the angle of refraction at the flat end, and α the angle of incidence at the core-cladding interface. Because the flat end is perpendicular to the surface at the core-cladding interface, $\sin\alpha = \cos\theta$.
(See Figure 33.46.)

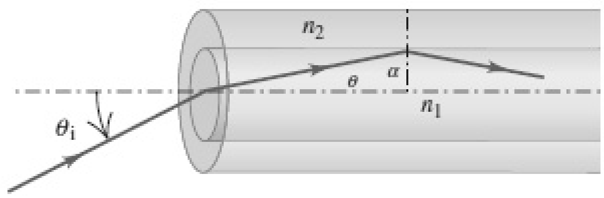


Figure 33.46

<u>At the flat end of the cable</u>: $(1.00)\sin\theta_i = n_1 \sin\theta \quad\rightarrow\quad \sin\theta = \dfrac{\sin\theta_i}{n_1}.$

<u>At the core-cladding interface</u>: $n_1 \sin\alpha = n_2 \sin(90°) = n_2 \quad\rightarrow\quad n_1\cos\theta = n_2 \rightarrow \cos\theta = n_2/n_1.$

Using the fact that $\sin^2\theta + \cos^2\theta = 1$, we get $\left(\dfrac{\sin\theta_i}{n_1}\right)^2 + \left(\dfrac{n_2}{n_1}\right)^2 = 1.$ Solving for $\sin\theta_i$ gives

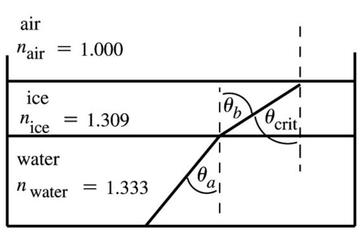
$\sin\theta_i = \sqrt{n_1^2 - n_2^2}.$

(c) Using the formula we just derived gives $\sin\theta_i = \sqrt{1.465^2 - 1.450^2} = 0.20911,$ so $\theta_i = 12.1°.$

Evaluate: If $n_2 > n_1$, the square root in (b) is not a real number, so there is no solution for θ_i. This is reasonable since total internal reflection will not occur unless $n_2 < n_1$.

33.47. Identify: Apply Snell's law to the water $\rightarrow$ ice and ice $\rightarrow$ air interfaces.

(a) Set Up: Consider the ray shown in Figure 33.47.



We want to find the incident angle θ_a at the water-ice interface that causes the incident angle at the ice-air interface to be the critical angle.

Figure 33.47

Execute: ice-air interface: $n_{\text{ice}} \sin\theta_{\text{crit}} = 1.0 \sin 90°.$

$n_{\text{ice}} \sin\theta_{\text{crit}} = 1.0$ so $\sin\theta_{\text{crit}} = \dfrac{1}{n_{\text{ice}}}.$

But from the diagram we see that $\theta_b = \theta_{\text{crit}}$, so $\sin\theta_b = \dfrac{1}{n_{\text{ice}}}.$

water-ice interface: $n_{\text{w}} \sin\theta_a = n_{\text{ice}} \sin\theta_b.$

But $\sin\theta_b = \dfrac{1}{n_{\text{ice}}}$ so $n_w \sin\theta_a = 1.0.$ $\sin\theta_a = \dfrac{1}{n_{\text{w}}} = \dfrac{1}{1.333} = 0.7502$ and $\theta_a = 48.6°.$

EVALUATE: **(b)** The angle calculated in part (a) is the critical angle for a water-air interface; the answer would be the same if the ice layer wasn't there!

33.48. **IDENTIFY:** This problem is about polarizing filters, so Malus's law applies.

SET UP: $I = I_0 \cos^2\phi$. We are looking for the intensity after light has pass through N polarizers, each with its polarizing axis turned slightly from the one before it.

EXECUTE: **(a)** Each time turn the polarizers are turned through an angle $\phi = 90°/N$ from each other.

After the first polarizer: $I = I_0 \cos^2(90°/N)$.

After the second polarizer: $I = \left[I_0 \cos^2(90°/N)\right]\cos^2(90°/N) = I_0 \cos^4(90°/N)$.

After the third polarizer: $I = \left[I_0 \cos^4(90°/N)\right]\cos^2(90°/N) = I_0 \cos^6(90°/N)$.

As we can see, the emerging pattern is $I = I_0 \cos^{2N}(90°/N)$.

(b) We want the minimum N so that the light is rotated by 90° yet retains more than 90° of its intensity. Use $I = I_0 \cos^{2N}(90°/N) = 0.90 I_0$. We want I/I_0 to be 0.90 or greater. Using a calculator, try several values to zero in on the appropriate value of N. For example:

$2N = 10$: $I/I_0 = \cos^{10}(90°/5) = 0.605$.

$2N = 30$: $I/I_0 = \cos^{30}(90°/15) = 0.85$.

$2N = 50$: $I/I_0 = \cos^{50}(90°/25) = 0.91$.

We want the minimum intensity ratio to be 0.90, so try $2N = 48$:

$2N = 48$: $I/I_0 = \cos^{48}(90°/24) = 0.90$.

$2N = 48$ gives the required intensity, so $N = 24$ polarizers.

(c) Use the same procedure as in (b). In both cases we want the *minimum N*. For $I/I_0 = 0.95$, $2N = 98$, so $N = 49$ polarizers. For $I/I_0 = 0.99$, $2N = 492$, so $N = 246$ polarizers.

EVALUATE: It takes a great number of polarizers to retain a high intensity. If the material of which the polarizers are made absorbs light due to impurities, it could be self-defeating to use so many of them unless they are extremely pure.

33.49. **IDENTIFY:** Apply Snell's law to the refraction of each ray as it emerges from the glass. The angle of incidence equals the angle $A = 25.0°$.

SET UP: The paths of the two rays are sketched in Figure 33.49.

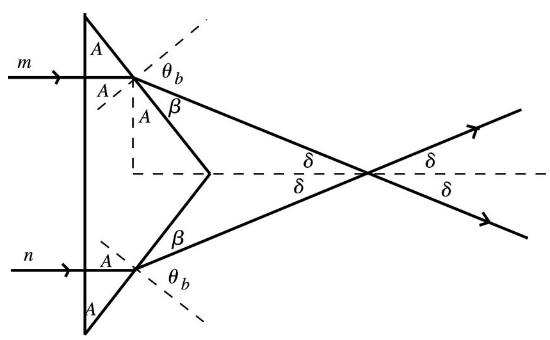


Figure 33.49

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$.

$n_{\text{glass}} \sin 25.0° = 1.00 \sin\theta_b$.

$\sin\theta_b = n_{\text{glass}} \sin 25.0°$.

$\sin\theta_b = 1.66 \sin 25.0° = 0.7015$.

$\theta_b = 44.55°$.

$\beta = 90.0° - \theta_b = 45.45°.$

Then $\delta = 90.0° - A - \beta = 90.0° - 25.0° - 45.45° = 19.55°.$ The angle between the two rays is $2\delta = 39.1°.$

EVALUATE: The light is incident normally on the front face of the prism so the light is not bent as it enters the prism.

33.50. **IDENTIFY:** The ray shown in the figure that accompanies the problem is to be incident at the critical angle.

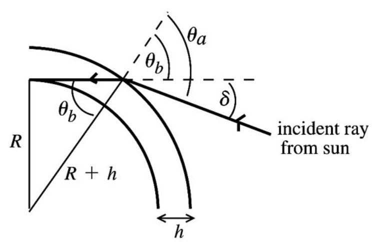
SET UP: $\theta_b = 90°.$ The incident angle for the ray in the figure is $60°.$

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $n_b = \left(\dfrac{n_a \sin\theta_a}{\sin\theta_b}\right) = \left(\dfrac{1.56 \sin 60°}{\sin 90°}\right) = 1.35.$

EVALUATE: Total internal reflection occurs only when the light is incident in the material of the greater refractive index.

33.51. **IDENTIFY:** Apply Snell's law to the refraction of the light as it enters the atmosphere.

SET UP: The path of a ray from the sun is sketched in Figure 33.51.



$\delta = \theta_a - \theta_b.$

From the diagram $\sin\theta_b = \dfrac{R}{R+h}.$

$\theta_b = \arcsin\left(\dfrac{R}{R+h}\right).$

Figure 33.51

EXECUTE: **(a)** Apply Snell's law to the refraction that occurs at the top of the atmosphere: $n_a \sin\theta_a = n_b \sin\theta_b$ (a = vacuum of space, refractive index 1.0; b = atmosphere, refractive index n).

$\sin\theta_a = n\sin\theta_b = n\left(\dfrac{R}{R+h}\right)$ so $\theta_a = \arcsin\left(\dfrac{nR}{R+h}\right).$

$\delta = \theta_a - \theta_b = \arcsin\left(\dfrac{nR}{R+h}\right) - \arcsin\left(\dfrac{R}{R+h}\right).$

(b) $\dfrac{R}{R+h} = \dfrac{6.38\times10^6\ \text{m}}{6.38\times10^6\ \text{m} + 20\times10^3\ \text{m}} = 0.99688.$

$\dfrac{nR}{R+h} = 1.0003(0.99688) = 0.99718.$

$\theta_b = \arcsin\left(\dfrac{R}{R+h}\right) = 85.47°.$

$\theta_a = \arcsin\left(\dfrac{nR}{R+h}\right) = 85.70°.$

$\delta = \theta_a - \theta_b = 85.70° - 85.47° = 0.23°.$

EVALUATE: The calculated δ is about the same as the angular radius of the sun.

33.52. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the angles and distances defined in the figure that accompanies the problem.

EXECUTE: **(a)** For light in air incident on a parallel-faced plate, Snell's Law yields:
$n\sin\theta_a = n'\sin\theta_b' = n'\sin\theta_b = n\sin\theta_a' \Rightarrow \sin\theta_a = \sin\theta_a' \Rightarrow \theta_a = \theta_a'.$

(b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta_n' = \theta_n$ and the chain of equations can continue.

(c) The lateral displacement of the beam can be calculated using geometry:

$d = L\sin(\theta_a - \theta_b')$ and $L = \dfrac{t}{\cos\theta_b'} \Rightarrow d = \dfrac{t\sin(\theta_a - \theta_b')}{\cos\theta_b'}$.

(d) $\theta_b' = \arcsin\left(\dfrac{n\sin\theta_a}{n'}\right) = \arcsin\left(\dfrac{\sin 66.0°}{1.80}\right) = 30.5°$ and $d = \dfrac{(2.40\text{ cm})\sin(66.0° - 30.5°)}{\cos 30.5°} = 1.62\text{ cm}.$

EVALUATE: The lateral displacement in part (d) is large, of the same order as the thickness of the plate.

33.53. **IDENTIFY:** The reflected light is totally polarized when light strikes a surface at Brewster's angle.

SET UP: At the plastic wall, Brewster's angle obeys the equation $\tan\theta_p = n_b/n_a$, and Snell's law, $n_a\sin\theta_a = n_b\sin\theta_b$, applies at the air-water surface.

EXECUTE: To be totally polarized, the reflected sunlight must have struck the wall at Brewster's angle. $\tan\theta_p = n_b/n_a = (1.61)/(1.00)$ and $\theta_p = 58.15°$.

This is the angle of incidence at the wall. A little geometry tells us that the angle of incidence at the water surface is $90.00° - 58.15° = 31.85°$. Applying Snell's law at the water surface gives $(1.00)\sin 31.85° = 1.333\sin\theta$ and $\theta = 23.3°$.

EVALUATE: We have two different principles involved here: Reflection at Brewster's angle at the wall and Snell's law at the water surface.

33.54. **IDENTIFY:** We are dealing with circularly polarized electromagnetic waves.

SET UP: $\tan\theta_p = n_b/n_a$, $n_a\sin\theta_a = n_b\sin\theta_b$

EXECUTE: **(a)** We want the angle of refraction. First find θ_p. $\tan\theta_p = n_b/n_a$ = (1.62)/(1.00) = 1.62, so θ_p = 58.31°. Now use Snell's law. $(1.00)\sin 58.31° = (1.62)\sin\theta_b$. θ_b = 31.7°.

(b) We want the reflected intensity. The reflecting surface is perpendicular to the *xz*-plane, so the component of the incident electric field parallel to that is E_y. Using Fresnel's equation gives

$\dfrac{E_{\text{refl}}}{E_{\text{inc},y}} = \dfrac{\sin(\theta_p - \theta_b)}{\sin(\theta_p + \theta_b)} = \dfrac{\sin(58.32° - 31.7°)}{\sin(58.32° + 31.7°)} = 0.448.$ $E_{\text{inc},y} = \dfrac{E_{\text{inc}}}{\sqrt{2}}$, so $E_{\text{refl}} = (0.448)\dfrac{E_{\text{inc}}}{\sqrt{2}} = 0.3168E_{\text{inc}}.$

The intensity is proportional to E^2, so $\dfrac{I_{\text{refl}}}{I_{\text{inc}}} = \dfrac{E_{\text{refl}}^2}{E_{\text{inc}}^2} = \dfrac{(0.3168E_{\text{inc}})^2}{E_{\text{inc}}^2} = 0.1004.$ Therefore we have

$I_{\text{refl}} = 0.1004I_{\text{inc}} = (0.1004)(150\text{ W/m}^2) = 15.1\text{ W/m}^2$.

(c) and **(d)** We want $I_{||}$. $I_{\text{refr}} = I_{\text{inc}} - I_{\text{refl}} = 150\text{ W/m}^2 - 15.1\text{ W/m}^2 = 134.9\text{ W/m}^2$. In the incident light, $E_x = E_y$, so the intensity is the same for both of them, which is (150 W/m^2)/2 = 75.0 W/m^2. After reflection, none of the E_x light is reflected since the reflected light is 100% polarized in the *y* direction. Therefore E_x is the same in the transmitted (refracted) light, so that intensity is 75.0 W/m^2. The remainder of the light in the refracted beam is 134.9 W/m^2 – 75.0 W/m^2 = 59.9 W/m^2. Therefore the final answers are: (c) 59.9 W/m^2 and (d) 75.0 W/m^2.

(e) We want *e*. $e = \sqrt{1 - (E_1/E_2)^2}$. I is proportional to E^2, so $e = \sqrt{1 - I_1/I_2} = \sqrt{1 - \dfrac{59.9\text{ W/m}^2}{75.0\text{ W/m}^2}} = 0.449.$

EVALUATE: In the transmitted light E_x and E_y have different amplitudes so the light is elliptically polarized.

33.55. **IDENTIFY:** Apply Snell's law in part (a). In part (b), we know from Chapter 32 that in a dielectric, $n^2 = KK_m$. In this case, we are told that K_m is very close to 1, so $n^2 \approx K$, where K is the dielectric constant of the material.

SET UP: Use $n_a\sin\theta_a = n_b\sin\theta_b$ (Snell's law) in (a) and $n^2 \approx K$ in (b). Use n = 1.00 for air. $f\lambda = c$.

EXECUTE: **(a)** For each liquid, apply $n_a \sin\theta_a = n_b \sin\theta_b$ using the data in the table with the problem. The angle of incidence is 60.0° in each case.

$\sin(60.0°) = n \sin\theta_b$, which gives $n = \dfrac{\sin(60.0°)}{\sin\theta_b}$. Apply this formula for each liquid and then use the information in Table 33.1 to identify the liquids.

<u>Liquid A</u>: $n_\text{A} = \dfrac{\sin(60.0°)}{\sin(36.4°)} = 1.46$ (carbon tetrachloride).

<u>Liquid B</u>: $n_\text{B} = \dfrac{\sin(60.0°)}{\sin(40.5°)} = 1.33$ (water).

<u>Liquid C</u>: $n_\text{C} = \dfrac{\sin(60.0°)}{\sin(32.1°)} = 1.63$ (carbon disulfide).

<u>Liquid D</u>: $n_\text{D} = \dfrac{\sin(60.0°)}{\sin(35.2°)} = 1.50$ (benzene).

(b) Use $K = n^2$ for each liquid.
$K_\text{A} = (1.46)^2 = 2.13$.
$K_\text{B} = (1.33)^2 = 1.77$.
$K_\text{C} = (1.63)^2 = 2.66$.
$K_\text{D} = (1.50)^2 = 2.25$.
(c) Use $f\lambda = c$: $f = c/\lambda$ = $(3.00\times10^8 \text{ m/s})/(589\times10^{-9} \text{ m}) = 5.09\times10^{14}$ Hz. This is the frequency in air and also in each liquid.
EVALUATE: The indexes of refraction are accurate, but the dielectric constants are less so because $n^2 \approx K$ is an approximation.

33.56. **IDENTIFY:** Apply Snell's law at the air-glass interface and also at the glass-liquid interface. For light incident at the critical angle, the angle of refraction is 90°.
SET UP: Use Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$. If θ_b is the angle of refraction at the air-glass interface and θ_c is the angle of incidence at the glass-liquid interface, then $\sin\theta_c = \cos\theta_b$. This is true because the normal at the air-glass interface and the glass-liquid interface are perpendicular to each other. We also know that θ_c is the critical angle for the glass-liquid boundary. Use 1.00 for the index of refraction of air, and call n the index of refraction for the liquid.
EXECUTE: First apply Snell's law at the air-glass boundary to find θ_b, and then use that result to find $\sin\theta_c$. Finally use Snell's law at the glass-liquid boundary to find n for the liquid.
<u>Liquid A</u>: At the air-glass boundary we have
$(1.00)\sin(52.0°) = (1.52)\sin\theta_b$, which gives $\theta_b = 31.226°$ and $\cos\theta_b = 0.85512 = \sin\theta_c$.
At the glass-liquid boundary we have
$(1.52)\sin\theta_c = n\sin(90°) = n$.
$n_\text{A} = (1.52)(0.85512) = 1.30$.
<u>Liquid B</u>: At the air-glass boundary we have
$(1.00)\sin(44.3°) = (1.52)\sin\theta_b$, so $\theta_b = 27.3538°$, so $\cos\theta_b = 0.8882 = \sin\theta_c$.
At the glass-liquid boundary we have
$n_\text{B} = (1.52)(0.8882) = 1.35$.
<u>Liquid C</u>: Air-glass boundary: $(1.00)\sin(36.3°) = (1.52)\sin\theta_b$, $\theta_b = 22.922°$, $\cos\theta_b = 0.9210 = \sin\theta_c$.
Glass-liquid boundary: $n_\text{C} = (1.52)(0.9210) = 1.40$.
EVALUATE: The indexes of refraction would be slightly different at wavelengths other than 638 nm since n depends on the wavelength of the light. All the values for n are greater than 1, which they must be.

33.57. **IDENTIFY** and **SET UP**: The polarizer passes $\frac{1}{2}$ of the intensity of the unpolarized component, independent of α. Malus's law tells us that out of the intensity I_p of the polarized component, the polarizer passes intensity $I_p \cos^2(\alpha - \theta)$, where $\alpha - \theta$ is the angle between the plane of polarization and the axis of the polarizer.

EXECUTE: **(a)** Use the angle where the transmitted intensity is maximum or minimum to find θ. See Figure 33.57.

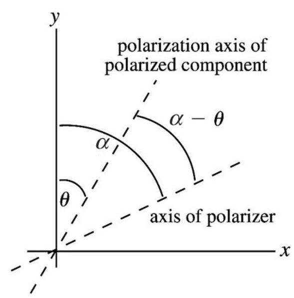


Figure 33.57

The total transmitted intensity is $I = \frac{1}{2}I_0 + I_p \cos^2(\alpha - \theta)$. This is maximum when $\theta = \alpha$, and from the graph in the problem this occurs when α is approximately 35°, so $\theta = 35°$. Alternatively, the total transmitted intensity is minimum when $\alpha - \theta = 90°$ and from the graph this occurs for $\alpha = 125°$. Thus, $\theta = \alpha - 90° = 125° - 90° = 35°$, which is in agreement with what we just found.

(b) For the equation, $I = \frac{1}{2}I_0 + I_p \cos^2(\alpha - \theta)$, we use data at two values of α to determine I_0 and I_p. It is easiest to use data where *I* is a maximum and a minimum. From the graph, we see that these extremes are 25 W/m^2 at α = 35° and 5.0 W/m^2 at $\alpha = 125°$.

At $\alpha = 125°$ the net intensity is 5.0 W/m^2, so we have

$$5.0\ \text{W/m}^2 = \tfrac{1}{2}I_0 + I_p \cos^2(125° - 35°) = \tfrac{1}{2}I_0 + I_p \cos^2(90°) = \tfrac{1}{2}I_0 \ \rightarrow\ I_0 = 10\ \text{W/m}^2.$$

At $\alpha = 35°$ the net intensity is 25 W/m^2, so we have

$$25\ \text{W/m}^2 = \tfrac{1}{2}I_0 + I_p \cos^2 0° = \tfrac{1}{2}I_0 + I_p = 5\ \text{W/m}^2 + I_p \ \rightarrow\ I_p = 20\ \text{W/m}^2.$$

EVALUATE: Now that we have $I_0, I_p,$ and θ we can verify that $I = \frac{1}{2}I_0 + I_p \cos^2(\phi - \theta)$ describes the data in the graph.

33.58. **IDENTIFY:** This problem involves refraction at a spherical surface.

SET UP: Refer to Fig. P33.58 in the textbook and follow the items requested. Snell's law applies: $n_a \sin\theta_a = n_b \sin\theta_b$.

EXECUTE: **(a)** We want $\sin\theta_a$. The hypotenuse is *R*, so $\sin\theta_a = r/R$.

(b) We want $\sin\theta_b$. From the figure we see that $\sin\theta_b = r'/R$.

(c) Apply Snell's law at the surface of the sphere, giving $n\sin\theta_a = (1.00)\sin\theta_b$. Use the results from parts (a) and (b). $n\left(\dfrac{r}{R}\right) = \dfrac{r'}{R}$, which gives $r' = nr$.

(d) The diameter is $D = 2r' = 2nr$ = (1.53)(45.0 mm) = 68.9 mm.

EVALUATE: The seed head will be magnified by a factor of 68.9/45.0 = 1.53 times.

33.59. **IDENTIFY:** This problem deals with total internal reflection and Snell's law.

SET UP: $n_a \sin\theta_a = n_b \sin\theta_b$. Follow the items requested in the parts of the problem.

EXECUTE: **(a)** We want θ. Follow the suggestion in the problem and refer to Fig. P33.59 in the textbook. $\sin\theta = \frac{bc}{ac} = \frac{R-d/2}{R+d/2} = \frac{2R-d}{2R+d}.$

(b) We want *R*. θ should be the critical angle, so $n_1 \sin\theta = n_2 \sin 90° = n_2$. Now use the result from part (a). $n_1\left(\frac{2R-d}{2R+d}\right) = n_2$. Solve for *R*, giving $R = \frac{d(n_1+n_2)}{2(n_1-n_2)}$.

(c) Using the given values, we get *R* = 2.07 cm.

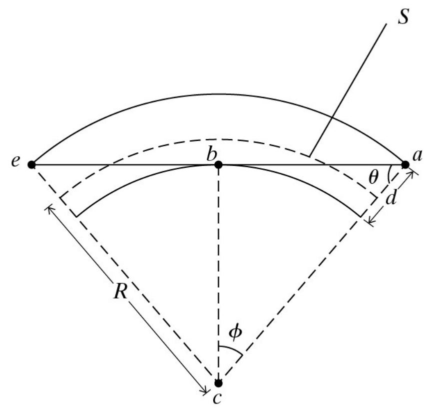


Figure 33.59

(d) The pattern shown in Fig. 33.59 keeps repeating as light goes down the cable. The dashed curve represents the path without reflections, which is a total of 1.00 km. In Fig. 33.59, the light follows the path *ae*, but without reflections it would follow the dashed curve. The angle θ is the critical angle. First find the distance *ae*. Referring to Fig. 33.59, we can see the following. *ae* = 2*ab*. $(ab)^2 + (R-d/2)^2 = (R+d/2)^2$. Solving for *ab* gives $(ab)^2 = 2Rd$, so $ab = \sqrt{2Rd}$. *ae* = 2*ab*, so $ae = 2\sqrt{2Rd} = 2\sqrt{2(0.02065 \text{ m})(50.0\ \mu\text{m})}$ = 2.8740 mm. Now get the length *S* of the dashed curved path. Since θ is the critical angle, $\sin\theta = n_2/n_1$ = 1.4440/1.4475, which gives θ = 86.015°. The length of *S* is the length of the curved path of radius *R* that subtends an angle 2ϕ. With ϕ in radians, we get $S = R(2\phi) = 2R\phi.$ $\phi = 90° - \theta = 90° - 86.015° = 3.985° = 0.06955$ rad. Therefore $S = 2R\phi =$ 2(0.02065 m)(0.06955 rad) = 2.8726 mm. So while the unreflected ray has traveled 2.8726 mm, the reflected ray has traveled 2.8740 mm. Now find the distance the reflected ray travels (call it *L*) while the unreflected ray has traveled 1.00 km. Using proportionality, we have $\frac{L}{1.00 \text{ km}} = \frac{ae}{S}$, so $L = (1.00 \text{ km})\frac{ae}{S}$. We want the *difference* in distance between these two rays, which is *L* – 1.00 km. This is $L - 1.00 \text{ km} = (1.00 \text{ km})\frac{ae}{S} - 1.00 \text{ km} = (1.00 \text{ km})\left(\frac{ae}{S} - 1\right)$. Using the results we just found for *ae* and *S* gives $L - 1.00 \text{ km} = (1.00 \text{ km})\left(\frac{2.8740 \text{ mm}}{2.8726 \text{ mm}} - 1\right) = 48.7 \text{ cm},$ which is about 50 cm. (Note that this part involves the subtraction of two numbers that are nearly equal. Therefore the answer is heavily dependent on the amount of rounding in the intermediate numerical calculations.)

(e) From part (d), we saw that the extra distance is only around 50 cm, so that makes a negligible time difference for the two rays. However there is a time difference because the ray in an air-filled cable travels at the speed of light c while one in the filled cable travels at speed c/n. Calling x the length of the cable, we have $\Delta t = t_{\text{cable}} - t_{\text{air}} = \dfrac{x}{c/n} - \dfrac{x}{c} = \dfrac{x}{c}(n-1) = \dfrac{1.00\text{ km}}{c}(1.4475-1) = 1.49\,\mu\text{s}.$

EVALUATE: Compare the times: $t_{\text{cable}}/t_{\text{air}} = (nx/c)/(x/c) = n = 1.744.$ So the time through the cable is over 1.7 times as long as through the air. The difference is small but the *fractional* difference is large.

33.60. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$. Therefore, $\sin\theta_b^C = n\sin\theta_a^C = n\sin\theta_b^A = \sin\theta_a^A = \theta_b^C = \theta_a^A.$

(b) The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi.$

(c) From Snell's law, $\sin\theta_a^A = n\sin\theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right).$

$$\Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + \pi.$$

(d) $$\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 4\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{4}{\sqrt{1-\dfrac{\sin^2\theta_1}{n^2}}}\cdot\left(\frac{\cos\theta_1}{n}\right)\cdot 4\left(1-\frac{\sin^2\theta_1}{n^2}\right) = \left(\frac{16\cos^2\theta_1}{n^2}\right).$$

$$4\cos^2\theta_1 = n^2 - 1 + \cos^2\theta_1.\ 3\cos^2\theta_1 = n^2 - 1.\ \cos^2\theta_1 = \frac{1}{3}(n^2-1).$$

(e) For violet: $\theta_1 = \arccos\left(\sqrt{\tfrac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{3}(1.342^2-1)}\right) = 58.89°.$

$\Delta_{\text{violet}} = 139.2° \Rightarrow \theta_{\text{violet}} = 40.8°.$

For red: $\theta_1 = \arccos\left(\sqrt{\tfrac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{3}(1.330^2-1)}\right) = 59.58°.$

$\Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°.$

EVALUATE: The angles we have calculated agree with the values given in Figure 33.19d in the textbook. θ_1 is larger for red than for violet, so red in the rainbow is higher above the horizon.

33.61. **IDENTIFY:** Follow similar steps to Challenge Problem 33.60.

SET UP: Refer to Figure 33.19e in the textbook.

EXECUTE: **(a)** The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi,$ where we have used the fact from the previous problem that all the internal angles are equal and the two external equals are equal. Also using the Snell's law relationship,

we have: $\theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right).\ \Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right) + 2\pi.$

(b) $$\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 6\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{6}{\sqrt{1-\dfrac{\sin\theta_2}{n^2}}}\cdot\left(\frac{\cos\theta_2}{n}\right).$$

$$n^2\left(1-\frac{\sin^2\theta_2}{n^2}\right) = (n^2 - 1 + \cos^2\theta_2) = 9\cos^2\theta_2.\ \cos^2\theta_2 = \frac{1}{8}(n^2-1).$$

(c) For violet, $\theta_2 = \arccos\left(\sqrt{\tfrac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{8}(1.342^2-1)}\right) = 71.55°$. $\Delta_{\text{violet}} = 233.2°$ and $\theta_{\text{violet}} = 53.2°$.

For red, $\theta_2 = \arccos\left(\sqrt{\tfrac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\tfrac{1}{8}(1.330^2-1)}\right) = 71.94°$. $\Delta_{\text{red}} = 230.1°$ and $\theta_{\text{red}} = 50.1°$.

EVALUATE: The angles we calculated agree with those given in Figure 33.19e in the textbook. The color that appears higher above the horizon is violet. The colors appear in reverse order in a secondary rainbow compared to a primary rainbow.

33.62. **IDENTIFY** and **SET UP:** Light polarized at 45° with the horizontal has both a horizontal component and a vertical component to its electric field.
EXECUTE: Since the light has both horizontal and vertical components, both H-type and V-type cells will be able to detect it, which makes choice (a) correct.
EVALUATE: Since the light is polarized at 45° with the horizontal, its horizontal and vertical components will be equal. So both types of cells should respond to it equally.

33.63. **IDENTIFY:** Light reflected from a glass surface is polarized to varying degrees, depending on the angle of incidence. At Brewster's angle the reflected light is 100% polarized parallel to the surface.
SET UP: Brewster's angle is given by $\tan\theta_p = n_b/n_a$. $n = 1.5$ for glass and $n = 1.0$ for air.
EXECUTE: For reflection from glass, $\tan\theta_p = n_b/n_a = (1.5)/(1.0) = 1.5$, so $\theta_p = 56°$. This is the angle with the normal to the glass. The light in this case makes an angle of 35° with the plane of the glass, so its angle of incidence is 55°, which is very close to Brewster's angle. Therefore the reflected light is almost totally polarized horizontally (since the glass is horizontal). Thus H cells will respond much more strongly to this light than will V cells, which is choice (d).
EVALUATE: The incident light is not *exactly* at Brewster's angle, so the reflected light will not be 100% horizontally polarized. Therefore the V cells will respond slightly to the reflect light.

33.64. **IDENTIFY** and **SET UP:** A polarizer reduces the intensity of unpolarized light by 50%.
EXECUTE: The first polarizer, with a vertical transmission axis, decreases the light intensity by half and leaves the transmitted light vertically polarized, so the intensity I after the first polarizer is $I = I_0/2$. The second polarizer removed none of the light, so it must have had a vertical transmission axis. Therefore the light emerging from both polarizers is vertically polarized. Thus only the V cells of the insect will detect this light, which is choice (b).
EVALUATE: If the second polarizer were rotated by 90°, no light would have emerged from the system.