25

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

VP25.3.1. **IDENTIFY:** We are looking at resistivity, resistance, and the electric field in a wire.

SET UP: $R = \dfrac{\rho L}{A}$, $V = EL$, $V = RI$.

EXECUTE: **(a)** We want the electric field. $E = V/L = (1.45\text{ V})/(35.0\text{ m}) = 0.0414\text{ V/m}$.

(b) The resistance is the target variable. $R = V/I = (1.45\text{ V})/(1.20\text{ A}) = 1.21\ \Omega$.

(c) We want the cross-sectional area of the wire. Solve $R = \dfrac{\rho L}{A}$ for A, giving $A = \dfrac{\rho L}{R}$. Using the given numbers we get $A = 7.97\times10^{-7}\text{ m}^2$.

EVALUATE: Copper has a resistivity of 1.72×10^{-8} ž·m, so if this wire were made of copper, its resistance would be $[(1.72)/(2.75)](1.21\ \Omega) = 0.756\ \Omega$, which is roughly 60% that of the aluminum wire.

VP25.3.2. **IDENTIFY:** We are looking at resistivity and the resistance of a wire.

SET UP: $R = \dfrac{\rho L}{A}$, $V = EL$, $V = RI$.

EXECUTE: **(a)** We want the resistance. Combine $V = EL$ and $V = RI$ to get $R = EL/I = 0.596\ \Omega$ using the given values.

(b) Resistivity is the target variable. Solve $R = \dfrac{\rho L}{A}$ for ρ, giving $\rho = \dfrac{RA}{L} = 2.44\times10^{-8}$ ž·m using the given or calculated values for R, A, and L.

EVALUATE: According to Table 25.1, this resistivity is the same as that of gold. These wires must be really special!

VP25.3.3. **IDENTIFY:** We want to compare the electrical characteristics of two wires.

SET UP: Wire 1: radius r and length L. Wire 2: radius $2r$, length $2L$. The potential difference is the same across both wires.

EXECUTE: **(a)** We want R_2/R_1. Use $R = \dfrac{\rho L}{A}$ and take the ratio of the resistances, which gives

$$\frac{R_2}{R_1} = \frac{\rho L_2 / A_2}{\rho L_1 / A_1} = \frac{L_2}{L_1}\frac{A_1}{A_2} = \frac{L_2}{L_1}\left(\frac{r_1}{r_2}\right)^2 = \frac{2L}{L}\left(\frac{r}{2r}\right)^2 = \frac{1}{2}.$$

(b) We want I_2/I_1. Use $I = V/R$ and take the ratio. $\dfrac{I_2}{I_1} = \dfrac{V / R_2}{V / R_1} = \dfrac{R_1}{R_2} = 2.$

(c) We want J_2/J_1. Use $J = I/A$ and take the ratio. $\dfrac{J_2}{J_1} = \dfrac{I_2 / (\pi r_2^2)}{I_1 / (\pi r_1^2)} = \dfrac{I_2}{I_1}\left(\dfrac{r_1}{r_2}\right)^2 = 2\left(\dfrac{r}{2r}\right)^2 = \dfrac{1}{2}.$

(d) We want E_2/E_1. Use $E = V/L$ and take the ratio. $\dfrac{E_2}{E_1} = \dfrac{V/L_2}{V/L_1} = \dfrac{L_1}{L_2} = \dfrac{L}{2L} = \dfrac{1}{2}$.

EVALUATE: As should now be clear, the *geometry* of a wire affects its electrical characteristics.

VP25.3.4. **IDENTIFY:** We are investigating the effect of temperatures on the resistance of a wire.
SET UP: $\rho(T) = \rho_0\left[1+\alpha(T-T_0)\right]$ so $R(T) = R_0\left[1+\alpha(T-T_0)\right]$. $V = RI$.

EXECUTE: **(a)** We want to find R at 20.0°C and 80.0°C.
At 20.0°C: $R = V/I$ = (1.50 V)/(2.40 A) = 0.625 Ω.
At 80.0°C: $R = V/I$ = (1.50 V)/(2.00 A) = 0.750 Ω.
(b) The target variable is α. Call T_0 = 20.0°C and T = 80.0°C and use $R(T) = R_0\left[1+\alpha(T-T_0)\right]$.
0.750 Ω = (0.625 Ω)$\left[1+\alpha(80.0°\text{C}-20.0°\text{C})\right]$. α = 0.00333 (C°)$^{-1}$.

EVALUATE: As we see, resistance can vary considerably as the temperature changes.

VP25.5.1. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and EXECUTE: **(a)** We want the current. $I = \dfrac{\mathcal{E}}{R} = \dfrac{9.00\ \text{V}}{15.3\ \Omega + 1.10\ \Omega} = 0.549$ A.
(b) We want V across the resistor. $V = RI$ = (15.3 Ω)(0.549 A) = 8.40 V.
(c) We want the terminal voltage. $\mathcal{E}_{\text{terminal}} = \mathcal{E} - rI =$ 9.00 V – (1.10 Ω)(0.549 A) = 8.40 V.

EVALUATE: The terminal voltage (8.40 V) is less than the internal emf (9.00 V) because there is a potential drop across the internal resistance.

VP25.5.2. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and EXECUTE: **(a)** We want the potential difference across the resistor.
$V = RI$ = (25.0 Ω)(0.480 A) = 12.0 V.
(b) We want the internal resistance. $\mathcal{E} = (R+r)I \quad\rightarrow\quad r = \dfrac{\mathcal{E}}{I} - R = \dfrac{12.6\ \text{V}}{0.480\ \text{A}} - 25.0\ \Omega = 1.3\ \Omega$.

EVALUATE: An internal resistance of 1.3 Ω is large enough to affect a circuit containing only small resistors.

VP25.5.3. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and EXECUTE: **(a)** We want the current. $I = \dfrac{\mathcal{E}_{\text{terminal}}}{R}$ = (1.30 V)/(20.0 Ω) = 0.0650 A.
(b) We want the internal resistance. $\mathcal{E} - \mathcal{E}_{\text{terminal}} = rI$. 1.50 V – 1.30 V = (0.0650 A)r. r = 3.1 Ω.

EVALUATE: An internal resistance of 3.1 Ω is large enough to considerably affect a circuit containing only small resistors.

VP25.5.4. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and EXECUTE: **(a)** We want the resistance. $\mathcal{E} = rI + RI$ so $R = \dfrac{\mathcal{E} - rI}{I}$. This gives

$R = \dfrac{1.50\ \text{V} - (0.160\ \Omega)(2.00\ \text{A})}{2.00\ \text{A}} =$ 0.590 Ω.
(b) We want the internal resistance. $\mathcal{E}_{\text{terminal}} = \mathcal{E} - rI$ = 1.50 V – (0.160 Ω)(2.00 A) = 1.18 V.

EVALUATE: A resistance smaller than 0.590 Ω would draw a current greater than 2.00 A which would run down the battery.

VP25.9.1. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.
SET UP and EXECUTE: **(a)** We want the current. $\mathcal{E} = rI + RI$. $I = \dfrac{\mathcal{E}}{R+r} = \dfrac{24.0\ \text{V}}{18.0\ \Omega + 1.30\ \Omega} = 1.24$ A.

(b) We want the rate of energy conversion in the battery, which is the power. $P = I\mathcal{E}$ which gives P = (1.24 A)(24.0 V) = 29.8 W.

(c) We want the rate of energy dissipation in the battery. This takes place in the internal resistance. $P_{\text{int}} = I^2 r$ = (1.24 A)^2(1.30 Ω) = 2.01 W.

(d) We want the net power output of the battery. $P_{\text{net}} = P - P_{\text{int}}$ = 29.8 W – 2.01 W = 27.8 W.

(e) We want the power in the 18.0 ohm resistor. $P_{18} = I^2 R$ = (1.24 A)^2(18.0 Ω) = 27.8 W.

EVALUATE: Note that the net power output of the battery is equal to the power dissipated in the 18.0 ohm resistor. This result is consistent with the conservation of energy.

VP25.9.2. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and **EXECUTE:** **(a)** We want the current. Solve $P = I^2R$ for I. $I = \sqrt{P/R}$ = 0.738 W using the given values.

(b) We want the net power output of the battery. This is the power dissipated in the resistor, which is given as P = 6.54 W.

(c) Target variable is the terminal voltage. $\mathcal{E}_{\text{terminal}} = RI$ = (0.738 A)(12.0 Ω) = 8.86 V.

(d) Target variable is the internal resistance. $\mathcal{E} = (R + r)I.\, r = \dfrac{\mathcal{E}}{I} - R.$ Using the numbers gives

$r = \dfrac{9.00\text{ V}}{0.738\text{ A}} - 12.0\ \Omega = 0.191\ \Omega,$ which rounds to 0.2 Ω.

EVALUATE: The battery gives 6.54 W of power to the 12.0 Ω resistor, but the internal resistance dissipates energy at a rate of I^2r = (0.738 A)^2(0.191 Ω) = 0.10 W. This power is small compared to the power in the 12.0 Ω resistor.

VP25.9.3. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and **EXECUTE:** **(a)** We want the terminal voltage. $\mathcal{E}_{\text{terminal}} = RI$ = (16.0 Ω)(0.720 A) = 11.5 V.

(b) We want the internal resistance. $\mathcal{E} = (R + r)I.\, r = \dfrac{\mathcal{E}}{I} - R.$ $r = \dfrac{12.0\text{ V}}{0.720\text{ A}} - 16.0\ \Omega = 0.667\ \Omega,$ which rounds to 0.7 Ω.

(c) The target variable is the power in the resistor. $P = I^2R$ = (0.720 A)^2(16.0 Ω) = 8.29 W.

(d) We want the power generated in the battery. $P = I\mathcal{E} =$ (0.720 A)(12.0 V) = 8.64 W.

(e) We want the rate of energy dissipation within the battery. This power is due to the internal resistance, so $P = I^2r$ = (0.720 A)^2(0.667 Ω) = 0.346 W, which rounds to 0.3 W.

EVALUATE: The internal resistance is much smaller than the external resistance so most of the power is dissipated in the external resistor.

VP25.9.4. **IDENTIFY:** We are dealing with internal resistance of a battery in a circuit.

SET UP and **EXECUTE:** **(a)** We want I_2/I_1.

<u>First circuit</u>: $\mathcal{E} = (R_1 + r)I_1.\, I_1 = \dfrac{\mathcal{E}}{R_1 + r} = \dfrac{\mathcal{E}}{R + r}.$

<u>Second circuit</u>: $\mathcal{E} = (R_2 + r)I_2.\, I_2 = \dfrac{\mathcal{E}}{R_2 + r} = \dfrac{\mathcal{E}}{2R + r}.$

$$\frac{I_2}{I_1} = \frac{\dfrac{\mathcal{E}}{2R + r}}{\dfrac{\mathcal{E}}{R + r}} = \frac{R + r}{2R + r}.$$

(b) We want P_2/P_1.

<u>First circuit</u>: $P_1 = I_1^2 R_1 = \left(\dfrac{\mathcal{E}}{R + r}\right)^2 R.$

<u>Second circuit</u>: $P_2 = I_2^2 R_2 = \left(\dfrac{\mathcal{E}}{2R + r}\right)^2 2R.$

$$\frac{P_2}{P_1}=\frac{\left(\frac{\mathcal{E}}{2R+r}\right)^2 2R}{\left(\frac{\mathcal{E}}{R+r}\right)^2 R}=2\left(\frac{R+r}{2R+r}\right)^2.$$

(c) We want to compare the currents and the power dissipation. $\frac{I_2}{I_1}=\frac{R+r}{2R+r}$. Since $2R+r>R+r$, $\frac{R+r}{2R+r}$ < 1, so the current is greater in R_1. $\frac{P_2}{P_1}=2\left(\frac{R+r}{2R+r}\right)^2$. Since $R > r$, $0<r\le R$. If $r = R$, $\frac{P_2}{P_1}=2\left(\frac{2R}{3R}\right)^2=\frac{8}{9}$, so $P_1 > P_2$. If $r = 0$, $\frac{P_2}{P_1}=2\left(\frac{R}{2R}\right)^2=\frac{1}{2}$, so $P_1 > P_2$. In all cases, $P_1 > P_2$, so the power dissipation is greatest in R_1.

EVALUATE: Our result tells us that increase the resistance decreases the current (which is reasonable) and decreases the power dissipation (which is also reasonable).

25.1. **IDENTIFY** and **SET UP:** The lightning is a current that lasts for a brief time. $I=\frac{\Delta Q}{\Delta t}$.

EXECUTE: $\Delta Q=I\Delta t=(25{,}000\text{ A})(40\times10^{-6}\text{ s})=1.0\text{ C}$.

EVALUATE: Even though it lasts for only 40 μs, the lightning carries a huge amount of charge since it is an enormous current.

25.2. **IDENTIFY:** $I=Q/t$. Use $I=n|q|v_\mathrm{d}A$ to calculate the drift velocity v_d.

SET UP: $n=5.8\times10^{28}\text{ m}^{-3}$. $|q|=1.60\times10^{-19}$ C.

EXECUTE: **(a)** $I=\frac{Q}{t}=\frac{420\text{ C}}{80(60\text{ s})}=8.75\times10^{-2}\text{ A}$.

(b) $I=n|q|v_\mathrm{d}A$. This gives $v_\mathrm{d}=\frac{I}{n|q|A}=\frac{8.75\times10^{-2}\text{ A}}{(5.8\times10^{28})(1.60\times10^{-19}\text{ C})(\pi(1.3\times10^{-3}\text{ m})^2)}=1.78\times10^{-6}$ m/s.

EVALUATE: v_d is smaller than in Example 25.1, because I is smaller in this problem.

25.3. **IDENTIFY:** $I=Q/t$. $J=I/A$. $J=n|q|v_\mathrm{d}$.

SET UP: $A=(\pi/4)D^2$, with $D=2.05\times10^{-3}$ m. The charge of an electron has magnitude $+e=1.60\times10^{-19}$ C.

EXECUTE: **(a)** $Q=It=(5.00\text{ A})(1.00\text{ s})=5.00\text{ C}$. The number of electrons is $\frac{Q}{e}=3.12\times10^{19}$.

(b) $J=\frac{I}{(\pi/4)D^2}=\frac{5.00\text{ A}}{(\pi/4)(2.05\times10^{-3}\text{ m})^2}=1.51\times10^6\text{ A/m}^2$.

(c) $v_\mathrm{d}=\frac{J}{n|q|}=\frac{1.51\times10^6\text{ A/m}^2}{(8.5\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})}=1.11\times10^{-4}\text{ m/s}=0.111\text{ mm/s}$.

EVALUATE: **(d)** If I is the same, $J=I/A$ would decrease and v_d would decrease. The number of electrons passing through the light bulb in 1.00 s would not change.

25.4. **(a) IDENTIFY:** By definition, $J=I/A$ and radius is one-half the diameter.

SET UP: Solve for the current: $I=JA=J\pi(D/2)^2$

EXECUTE: $I = (3.20\times 10^6\ \text{A/m}^2)(\pi)[(0.00102\ \text{m})/2]^2 = 2.61\ \text{A}.$

EVALUATE: This is a realistic current.

(b) IDENTIFY: The current density is $J = n|q|v_\text{d}$.

SET UP: Solve for the drift velocity: $v_\text{d} = J/n|q|$

EXECUTE: We use the value of n for copper, giving

$v_\text{d} = (3.20\times 10^6\ \text{A/m}^2)/[(8.5\times 10^{28}/\text{m}^3)(1.60\times 10^{-19}\ \text{C})] = 2.4\times 10^{-4}\ \text{m/s} = 0.24\ \text{mm/s}.$

EVALUATE: This is a typical drift velocity for ordinary currents and wires.

25.5. **IDENTIFY:** This problem is about the drift speed in a current-carrying wire.

SET UP: $\rho = E/J$ and $J = nqv_\text{d}$.

EXECUTE: **(a)** The drift velocity is the target variable. Combine $\rho = E/J$ and $J = nqv_\text{d}$. $J = E/\rho =$ nqv_d gives $v_\text{d} = \dfrac{E}{nq\rho} = \dfrac{E}{ne\rho}$. Using the given numbers gives v_d = 0.26 mm/s.

(b) We want the potential difference. $V = Ed$ = (0.0600 N/C)(0.200 m) = 0.0120 N/C.

EVALUATE: Note how small v_d, E, and V are for metals.

25.6. **IDENTIFY:** The resistance depends on the length, cross-sectional area, and material of the wires.

SET UP: $R = \dfrac{\rho L}{A}$, $A = \pi r^2 = d^2/4$. The resistivities come from Table 25.1.

EXECUTE: **(a)** Combining $R = \dfrac{\rho L}{A}$ and $A = \pi d^2/4$, gives $R = \dfrac{\rho L}{\frac{\pi}{4}d^2} = \dfrac{4\rho L}{\pi d^2}$. Solving for L gives

$L = \dfrac{R\pi d^2}{4\rho}$. Using this formula gives the length of each type of metal.

Gold: $L = \dfrac{(1.00\,\Omega)\pi(1.00\times 10^{-3}\ \text{m})^2}{4(2.44\times 10^{-8}\ \Omega\cdot\text{m})} = 32.2\ \text{m}.$

Copper: Using $\rho = 1.72\times 10^{-8}\ \Omega\cdot\text{m}$ we get L = 45.7 m.

Aluminum: Using $\rho = 2.75\times 10^{-8}\ \Omega\cdot\text{m}$, we get L = 28.6 m.

(b) The mass of the gold is the product of its mass density and its volume, so

$m = (\textit{density})(\pi d^2/4)L = (1.93\times 10^4\ \text{kg/m}^3)\pi(1.00\times 10^{-3}\ \text{m})^2(32.2\ \text{m})/4 = 0.488\ \text{kg} = 488\ g.$

If gold is currently worth \$40 per gram, the cost of the gold wire would be (\$40/g)(488 g) = \$19,500. At this price, you wouldn't want to wire your house with gold wires!

EVALUATE: The resistivities of the three metals are all fairly close to each other, so it is reasonable to expect that the lengths of the wires would also be fairly close to each other, which is just what we find.

25.7. **IDENTIFY** and **SET UP:** Apply $I = \dfrac{dQ}{dt}$ to find the charge dQ in time dt. Integrate to find the total charge in the whole time interval.

EXECUTE: **(a)** $dQ = I\,dt$.

$Q = \int_0^{8.0\text{s}} (55\ \text{A} - (0.65\ \text{A/s}^2)t^2)dt = \left[(55\ \text{A})t - (0.217\ \text{A/s}^2)t^3\right]_0^{8.0\ \text{s}}.$

$Q = (55\ \text{A})(8.0\ \text{s}) - (0.217\ \text{A/s}^2)(8.0\ \text{s})^3 = 330\ \text{C}.$

(b) $I = \dfrac{Q}{t} = \dfrac{330\ \text{C}}{8.0\ \text{s}} = 41\ \text{A}.$

EVALUATE: The current decreases from 55 A to 13.4 A during the interval. The decrease is not linear and the average current is not equal to $(55\text{A} + 13.4\ \text{A})/2$.

25.8. **IDENTIFY:** $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$.

EXECUTE: **(a)** $Q_{\text{total}} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92\times10^{16} + 2.68\times10^{16})(1.60\times10^{-19}\text{ C}) = 0.0106\text{ C}$. Then $I = \dfrac{Q_{\text{total}}}{t} = \dfrac{0.0106\text{ C}}{1.00\text{ s}} = 0.0106\text{ A} = 10.6\text{ mA}$.

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.9. **IDENTIFY** and **SET UP:** The number of ions that enter gives the charge that enters the axon in the specified time. $I = \dfrac{\Delta Q}{\Delta t}$.

EXECUTE: $\Delta Q = (5.6\times10^{11}\text{ ions})(1.60\times10^{-19}\text{ C/ion}) = 9.0\times10^{-8}\text{ C}$. $I = \dfrac{\Delta Q}{\Delta t} = \dfrac{9.0\times10^{-8}\text{ C}}{10\times10^{-3}\text{ s}} = 9.0\ \mu\text{A}$.

EVALUATE: This current is much smaller than household currents but are comparable to many currents in electronic equipment.

25.10. **IDENTIFY:** This problem deals with free-electron density.

SET UP and **EXECUTE:** First find the number of silver atoms per cubic meter, then use that to get the number of free electrons per cubic meter.

$$n = \left(10.5\times10^3\,\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1\text{ mol}}{108\text{ g}}\right)\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\left(\frac{6.022\times10^{23}\text{atoms}}{1\text{ mol}}\right)\left(\frac{1\text{ free electron}}{1\text{ atom}}\right) = 5.85\times10^{28}\,\frac{\text{el}}{\text{m}^3}.$$ Now

compare this result to the value in Example 25.1. $n_{\text{Ag}}/n_{\text{Cu}} = 5.85/8.5 = 0.69$, so n_{Ag} is $0.69n_{\text{Cu}}$.

EVALUATE: Copper has more free electrons per cubic meter than silver does even though silver is denser than copper.

25.11. **IDENTIFY:** We want to find the resistivity of the metal.

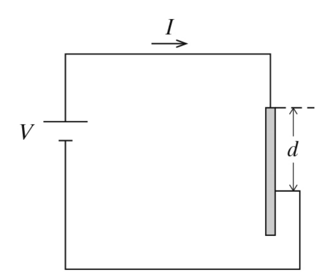


Figure 25.11

SET UP and **EXECUTE:** First sketch the circuit as in Fig. 25.11. Combine $I = V/R$ and $R = \dfrac{\rho d}{A}$ to relate I to $1/d$. $I = \dfrac{V}{R} = \dfrac{V}{\rho d / A} = \left(\dfrac{VA}{\rho}\right)\dfrac{1}{d}$. A graph of I versus $1/d$ should be a straight line having slope equal to VA/ρ. Thus $\rho = \dfrac{VA}{\text{slope}} = \dfrac{(12.0\text{ V})\pi(0.800\text{ mm})^2}{600\text{ A}\cdot\text{m}} = 4.02\times10^{-8}\ \Omega\cdot\text{m}$.

EVALUATE: From Table 25.1 we see that this resistivity is between that of aluminum and tungsten, so our result is physically reasonable.

25.12. **(a) IDENTIFY:** Start with the definition of resistivity and solve for E.

SET UP: $E = \rho J = \rho I/\pi r^2$.

EXECUTE: $E = (1.72\times10^{-8}\ \Omega\cdot\text{m})(4.50\ \text{A})/[\pi(0.001025\ \text{m})^2] = 2.345\times10^{-2}\ \text{V/m}$, which rounds to 0.0235 V/m.

EVALUATE: The field is quite weak, since the potential would drop only a volt in 43 m of wire.

(b) IDENTIFY: Take the ratio of the field in silver to the field in copper.

SET UP: Take the ratio and solve for the field in silver: $E_\text{S} = E_\text{C}(\rho_\text{S}/\rho_\text{C})$.

EXECUTE: $E_\text{S} = (0.02345\ \text{V/m})[(1.47)/(1.72)] = 2.00\times10^{-2}\ \text{V/m}$.

EVALUATE: Since silver is a better conductor than copper, the field in silver is smaller than the field in copper.

25.13. **IDENTIFY:** First use Ohm's law to find the resistance at 20.0°C; then calculate the resistivity from the resistance. Finally use the dependence of resistance on temperature to calculate the temperature coefficient of resistance.

SET UP: Ohm's law is $R = V/I$, $R = \rho L/A$, $R = R_0[1+\alpha(T-T_0)]$, and the radius is one-half the diameter.

EXECUTE: **(a)** At 20.0°C, $R = V/I = (15.0\ \text{V})/(18.5\ \text{A}) = 0.811\ \Omega$. Using $R = \rho L/A$ and solving for ρ gives $\rho = RA/L = R\pi(D/2)^2/L = (0.811\ \Omega)\pi[(0.00500\ \text{m})/2]^2/(1.50\ \text{m}) = 1.06\times10^{-5}\ \Omega\cdot\text{m}$.

(b) At 92.0°C, $R = V/I = (15.0\ \text{V})/(17.2\ \text{A}) = 0.872\ \Omega$. Using $R = R_0[1+\alpha(T-T_0)]$ with T_0 taken as 20.0°C, we have $0.872\ \Omega = (0.811\ \Omega)[1+\alpha(92.0°\text{C} - 20.0°\text{C})]$. This gives $\alpha = 0.00105\ (\text{C}°)^{-1}$.

EVALUATE: The results are typical of ordinary metals.

25.14. **IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_\text{d}A$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \dfrac{I}{A} = \dfrac{3.6\ \text{A}}{(2.3\times10^{-3}\ \text{m})^2} = 6.81\times10^5\ \text{A/m}^2$.

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\ \text{A/m}^2) = 0.012\ \text{V/m}$.

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_\text{d}} = \frac{ln|q|A}{I} = \frac{(4.0\ \text{m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\ \text{C})(2.3\times10^{-3}\ \text{m})^2}{3.6\ \text{A}} = 8.0\times10^4\ \text{s}.$$

$t = 1333\ \text{min} \approx 22\ \text{hrs!}$

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.15. **IDENTIFY:** Knowing the resistivity of a metal, its geometry and the current through it, we can use Ohm's law to find the potential difference across it.

SET UP: $V = IR$. For copper, Table 25.1 gives that $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ and for silver, $\rho = 1.47\times10^{-8}\ \Omega\cdot\text{m}$. $R = \dfrac{\rho L}{A}$.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.00\ \text{m})}{\pi(0.814\times10^{-3}\ \text{m})^2} = 1.65\times10^{-2}\ \Omega$.

$V = (12.5\times10^{-3}\ \text{A})(1.65\times10^{-2}\ \Omega) = 2.06\times10^{-4}\ \text{V}$.

(b) $V = \dfrac{I\rho L}{A}$. $\dfrac{V}{\rho} = \dfrac{IL}{A} = \text{constant}$, so $\dfrac{V_s}{\rho_s} = \dfrac{V_c}{\rho_c}$.

$$V_s = V_c\left(\frac{\rho_s}{\rho_c}\right) = (2.06\times10^{-4}\ \text{V})\left(\frac{1.47\times10^{-8}\ \Omega\cdot\text{m}}{1.72\times10^{-8}\ \Omega\cdot\text{m}}\right) = 1.76\times10^{-4}\ \text{V}.$$

EVALUATE: The potential difference across a 2-m length of wire is less than 0.2 mV, so normally we do not need to worry about these potential drops in laboratory circuits.

25.16. **IDENTIFY:** The resistivity of the wire should identify what the material is.
SET UP: $R = \rho L/A$ and the radius of the wire is half its diameter.
EXECUTE: Solve for ρ and substitute the numerical values.

$$\rho = AR/L = \pi(D/2)^2 R/L = \frac{\pi([0.00205\ \text{m}]/2)^2(0.0290\ \Omega)}{6.50\ \text{m}} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$$

EVALUATE: This result is the same as the resistivity of silver, which implies that the material is silver.

25.17. **IDENTIFY:** We want to compare the electric field in two metals.

SET UP and **EXECUTE:** $\rho = E/J$ and $J = I/A$. $E = \rho J = \rho\left(\dfrac{I}{A}\right) = \dfrac{\rho I}{\pi r^2}$. The current I is the same for both wires, and we get the resistivities from Table 25.1.

$$\frac{E_{\text{Cu}}}{E_{\text{Ag}}} = \frac{\rho_{\text{Cu}} I/\pi r_{\text{Cu}}^2}{\rho_{\text{Ag}} I/\pi r_{\text{Ag}}^2} = \left(\frac{\rho_{\text{Cu}}}{\rho_{\text{Ag}}}\right)\left(\frac{r_{\text{Ag}}}{r_{\text{Cu}}}\right)^2 = \left(\frac{1.72}{1.47}\right)\left(\frac{0.500}{0.800}\right)^2 = 0.457.$$

EVALUATE: The field in copper is about half the field in silver.

25.18. **IDENTIFY:** The geometry of the wire is changed, so its resistance will also change.

SET UP: $R = \dfrac{\rho L}{A}$. $L_{\text{new}} = 3L$. The volume of the wire remains the same when it is stretched.

EXECUTE: Volume $= LA$ so $LA = L_{\text{new}} A_{\text{new}}$. $A_{\text{new}} = \dfrac{L}{L_{\text{new}}} A = \dfrac{A}{3}$.

$$R_{\text{new}} = \frac{\rho L_{\text{new}}}{A_{\text{new}}} = \frac{\rho(3L)}{A/3} = 9\frac{\rho L}{A} = 9R.$$

EVALUATE: When the length increases the resistance increases and when the area decreases the resistance increases.

25.19. **IDENTIFY:** $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$.

EXECUTE: $R = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(24.0\ \text{m})}{\pi(1.025\times10^{-3}\ \text{m})^2} = 0.125\ \Omega.$

EVALUATE: The resistance is proportional to the length of the piece of wire.

25.20. **IDENTIFY:** $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi d^2/4}$.

SET UP: For aluminum, $\rho_{\text{al}} = 2.75\times10^{-8}\ \Omega\cdot\text{m}$. For copper, $\rho_{\text{c}} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\dfrac{\rho}{d^2} = \dfrac{R\pi}{4L} =$ constant, so $\dfrac{\rho_{\text{al}}}{d_{\text{al}}^2} = \dfrac{\rho_{\text{c}}}{d_{\text{c}}^2}$.

$$d_{\text{c}} = d_{\text{al}}\sqrt{\frac{\rho_{\text{c}}}{\rho_{\text{al}}}} = (2.14\ \text{mm})\sqrt{\frac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.75\times10^{-8}\ \Omega\cdot\text{m}}} = 1.69\ \text{mm}.$$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

25.21. **IDENTIFY** and **SET UP:** The equation $\rho = E/J$ relates the electric field that is given to the current density. $V = EL$ gives the potential difference across a length L of wire and $V = IR$ allows us to calculate R.

EXECUTE: **(a)** $\rho = E/J$ so $J = E/\rho$.

From Table 25.1 the resistivity for gold is $2.44\times10^{-8}\ \Omega\cdot\text{m}$.

$$J = \frac{E}{\rho} = \frac{0.49\ \text{V/m}}{2.44\times10^{-8}\ \Omega\cdot\text{m}} = 2.008\times10^{7}\ \text{A/m}^2.$$

$I = JA = J\pi r^2 = (2.008\times10^{7}\ \text{A/m}^2)\pi(0.42\times10^{-3}\ \text{m})^2 = 11\ \text{A}.$

(b) $V = EL = (0.49\ \text{V/m})(6.4\ \text{m}) = 3.1\ \text{V}.$

(c) We can use Ohm's law: $V = IR$.

$$R = \frac{V}{I} = \frac{3.1\ \text{V}}{11\ \text{A}} = 0.28\ \Omega.$$

EVALUATE: We can also calculate R from the resistivity and the dimensions of the wire:

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{(2.44\times10^{-8}\ \Omega\cdot\text{m})(6.4\ \text{m})}{\pi(0.42\times10^{-3}\ \text{m})^2} = 0.28\ \Omega,\ \text{which checks.}$$

25.22. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ and $V = IR$.

SET UP: $A = \pi r^2$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{VA}{IL} = \dfrac{(4.50\ \text{V})\pi(6.54\times10^{-4}\ \text{m})^2}{(17.6\ \text{A})(2.50\ \text{m})} = 1.37\times10^{-7}\ \Omega\cdot\text{m}.$

EVALUATE: Our result for ρ shows that the wire is made of a metal with resistivity greater than that of good metallic conductors such as copper and aluminum.

25.23. **IDENTIFY:** Apply $R = R_0[1+\alpha(T-T_0)]$ to calculate the resistance at the second temperature.

(a) SET UP: $\alpha = 0.0004\ (\text{C}^\circ)^{-1}$ (Table 25.2). Let T_0 be 0.0°C and T be 11.5°C.

EXECUTE: $R_0 = \dfrac{R}{1+\alpha(T-T_0)} = \dfrac{100.0\ \Omega}{1+(0.0004\ (\text{C}^\circ)^{-1}(11.5\ \text{C}^\circ))} = 99.54\ \Omega.$

(b) SET UP: $\alpha = -0.0005\ (\text{C}^\circ)^{-1}$ (Table 25.2). Let $T_0 = 0.0°\text{C}$ and $T = 25.8°\text{C}$.

EXECUTE: $R = R_0[1+\alpha(T-T_0)] = 0.0160\ \Omega[1+(-0.0005\ (\text{C}^\circ)^{-1})(25.8\ \text{C}^\circ)] = 0.0158\ \Omega.$

EVALUATE: Nichrome, like most metallic conductors, has a positive α and its resistance increases with temperature. For carbon, α is negative and its resistance decreases as T increases.

25.24. **IDENTIFY:** $R_T = R_0[1+\alpha(T-T_0)]$.

SET UP: $R_0 = 217.3\ \Omega$. $R_T = 215.8\ \Omega$. For carbon, $\alpha = -0.00050(\text{C}^\circ)^{-1}$.

EXECUTE: $T - T_0 = \dfrac{(R_T/R_0)-1}{\alpha} = \dfrac{(215.8\ \Omega/217.3\ \Omega)-1}{-0.00050\ (\text{C}^\circ)^{-1}} = 13.8\ \text{C}^\circ.$ $T = 13.8\ \text{C}^\circ + 4.0°\text{C} = 17.8°\text{C}.$

EVALUATE: For carbon, α is negative so R decreases as T increases.

25.25. **IDENTIFY:** Use $R = \dfrac{\rho L}{A}$ to calculate R and then apply $V = IR$. $P = VI$ and energy $= Pt$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$, where $r = 0.050$ m.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times 10^{-8}\ \Omega\cdot\text{m})(100\times 10^{3}\,\text{m})}{\pi(0.050\ \text{m})^2} = 0.219\ \Omega.$

$V = IR = (125\ \text{A})(0.219\ \Omega) = 27.4\ \text{V}.$

(b) $P = VI = (27.4\ \text{V})(125\ \text{A}) = 3422\ \text{W} = 3422\ \text{J/s}$ and energy $= Pt = (3422\ \text{J/s})(3600\ \text{s}) = 1.23\times 10^{7}\ \text{J}.$

EVALUATE: The rate of electrical energy loss in the cable is large, over 3 kW.

25.26. **IDENTIFY:** When current passes through a battery in the direction from the $-$ terminal toward the + terminal, the terminal voltage V_{ab} of the battery is $V_{ab} = \mathcal{E} - Ir$. Also, $V_{ab} = IR$, the potential across the circuit resistor.

SET UP: $\mathcal{E} = 24.0\ \text{V}$. $I = 4.00\ \text{A}$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} - Ir$ gives $r = \dfrac{\mathcal{E} - V_{ab}}{I} = \dfrac{24.0\ \text{V} - 21.2\ \text{V}}{4.00\ \text{A}} = 0.700\ \Omega.$

(b) $V_{ab} - IR = 0$ so $R = \dfrac{V_{ab}}{I} = \dfrac{21.2\ \text{V}}{4.00\ \text{A}} = 5.30\ \Omega.$

EVALUATE: The voltage drop across the internal resistance of the battery causes the terminal voltage of the battery to be less than its emf. The total resistance in the circuit is $R + r = 6.00\ \Omega.$

$I = \dfrac{24.0\ \text{V}}{6.00\ \Omega} = 4.00\ \text{A},$ which agrees with the value specified in the problem.

25.27. **IDENTIFY:** The terminal voltage of the battery is $V_{ab} = \mathcal{E} - Ir$. The voltmeter reads the potential difference between its terminals.

SET UP: An ideal voltmeter has infinite resistance.

EXECUTE: **(a)** Since an ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\ \Omega$ resistor.

(b) $V_{ab} = \mathcal{E} = 5.0\ \text{V}$; Since there is no current there is no voltage lost over the internal resistance.

(c) The voltmeter reading is therefore 5.0 V since with no current flowing there is no voltage drop across either resistor.

EVALUATE: This not the proper way to connect a voltmeter. If we wish to measure the terminal voltage of the battery in a circuit that does not include the voltmeter, then connect the voltmeter across the terminals of the battery.

25.28. **IDENTIFY:** The *idealized* ammeter has no resistance so there is no potential drop across it. Therefore it acts like a short circuit across the terminals of the battery and removes the $4.00\text{-}\Omega$ resistor from the circuit. Thus the only resistance in the circuit is the $2.00\text{-}\Omega$ internal resistance of the battery.

SET UP: Use Ohm's law: $I = \mathcal{E}/r$.

EXECUTE: **(a)** $I = (10.0\ \text{V})/(2.00\ \Omega) = 5.00\ \text{A}.$

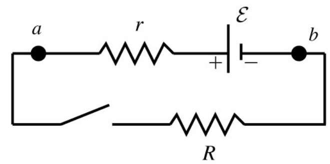
(b) The zero-resistance ammeter is in parallel with the $4.00\text{-}\Omega$ resistor, so all the current goes through the ammeter. If no current goes through the $4.00\text{-}\Omega$ resistor, the potential drop across it must be zero.

(c) The terminal voltage is zero since there is no potential drop across the ammeter.

EVALUATE: An ammeter should *never* be connected this way because it would seriously alter the circuit!

25.29. **IDENTIFY:** The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

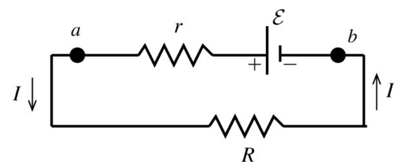
SET UP: open circuit: $I = 0$. The circuit is sketched in Figure 25.29a.



Execute: $V_{ab} = \mathcal{E} = 3.08\text{ V}$.

Figure 25.29a

Set Up: <u>switch closed:</u> The circuit is sketched in Figure 25.29b.



Execute: $V_{ab} = \mathcal{E} - Ir = 2.97\text{ V}$.

$$r = \frac{\mathcal{E} - 2.97\text{ V}}{I}.$$

$$r = \frac{3.08\text{ V} - 2.97\text{ V}}{1.65\text{ A}} = 0.067\ \Omega.$$

Figure 25.29b

And $V_{ab} = IR$ so $R = \dfrac{V_{ab}}{I} = \dfrac{2.97\text{ V}}{1.65\text{ A}} = 1.80\ \Omega$.

Evaluate: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage V is less than its emf.

25.30. **Identify:** The sum of the potential changes around the circuit loop is zero. Potential decreases by IR when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to $+$ terminal.

Set Up: The current is counterclockwise, because the 16-V battery determines the direction of current flow.

Execute: $+16.0\text{ V} - 8.0\text{ V} - I(1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega) = 0$.

$$I = \frac{16.0\text{ V} - 8.0\text{ V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 0.47\text{ A}.$$

(b) $V_b + 16.0\text{ V} - I(1.6\ \Omega) = V_a$, so $V_a - V_b = V_{ab} = 16.0\text{ V} - (1.6\ \Omega)(0.47\text{ A}) = 15.2\text{ V}$.

(c) $V_c + 8.0\text{ V} + I(1.4\ \Omega + 5.0\ \Omega) = V_a$ so $V_{ac} = (5.0\ \Omega)(0.47\text{ A}) + (1.4\ \Omega)(0.47\text{ A}) + 8.0\text{ V} = 11.0\text{ V}$.

(d) The graph is sketched in Figure 25.30.

Evaluate: $V_{cb} = (0.47\text{ A})(9.0\ \Omega) = 4.2\text{ V}$. The potential at point b is 15.2 V below the potential at point a and the potential at point c is 11.0 V below the potential at point a, so the potential of point c is $15.2\text{ V} - 11.0\text{ V} = 4.2\text{ V}$ above the potential of point b.

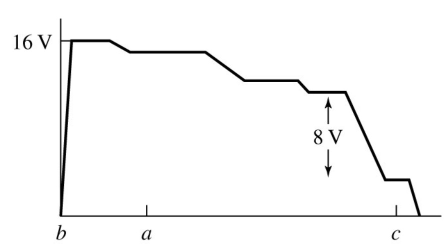


Figure 25.30

25.31. **(a) IDENTIFY** and **SET UP:** Assume that the current is clockwise. The circuit is sketched in Figure 25.31a.

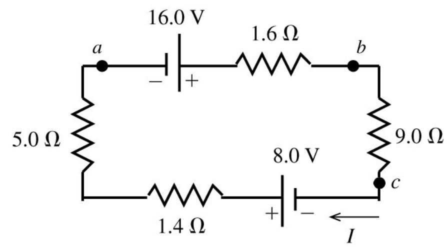


Figure 25.31a

Add up the potential rises and drops as travel clockwise around the circuit.

EXECUTE: $16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) + 8.0\text{ V} - I(1.4\ \Omega) - I(5.0\ \Omega) = 0.$

$$I = \frac{16.0\text{ V} + 8.0\text{ V}}{9.0\ \Omega + 1.4\ \Omega + 5.0\ \Omega + 1.6\ \Omega} = \frac{24.0\text{ V}}{17.0\ \Omega} = 1.41\text{ A, clockwise.}$$

EVALUATE: The 16.0-V battery and the 8.0-V battery both drive the current in the same direction.

(b) IDENTIFY and **SET UP:** Start at point *a* and travel through the battery to point *b*, keeping track of the potential changes. At point *b* the potential is V_b.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) = V_b.$

$V_a - V_b = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega).$

$V_{ab} = -16.0\text{ V} + 2.3\text{ V} = -13.7\text{ V}$ (point *a* is at lower potential; it is the negative terminal). Therefore, $V_{ba} = 13.7\text{V}.$

EVALUATE: Could also go counterclockwise from *a* to *b:*

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} + (1.41\text{ A})(9.0\ \Omega) = V_b.$

$V_{ab} = -13.7\text{ V},$ which checks.

(c) IDENTIFY and **SET UP:** Start at point *a* and travel through the battery to point *c*, keeping track of the potential changes.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) = V_c.$

$V_a - V_c = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega + 9.0\ \Omega).$

$V_{ac} = -16.0\text{ V} + 15.0\text{ V} = -1.0\text{ V}$ (point *a* is at lower potential than point *c*).

EVALUATE: Could also go counterclockwise from *a* to *c*:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} = V_c.$

$V_{ac} = -1.0\text{ V},$ which checks.

(d) Call the potential zero at point *a*. Travel clockwise around the circuit. The graph is sketched in Figure 25.31b.

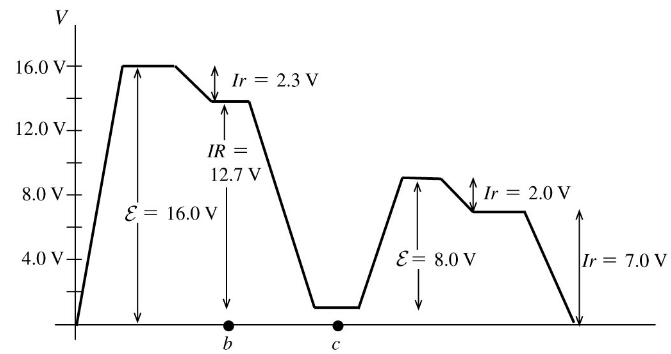


Figure 25.31b

25.32. **IDENTIFY:** This problem involves the internal resistance of a battery.

SET UP: $\mathcal{E} = (R + r)I, I = V_R / R$. The internal resistance is the target variable.

EXECUTE: $I = V_R/R = (27.0\text{ V})/(9.00\ \Omega) = 3.00\text{ A}$. Solve for r: $r = \dfrac{\mathcal{E}}{I} - R$.

$$r = \frac{30.0\text{ V}}{3.00\text{ A}} - 9.00\ \Omega = 1.0\ \Omega.$$

EVALUATE: If there were no internal resistance the current would be $(30.0\text{ V})/(9.00\ \Omega) = 3.33\text{ A}$ and V_R would be 30.0 V. The internal resistance makes a significant difference in the circuit.

25.33. **IDENTIFY:** This problem involves the internal resistance of a battery.

SET UP: $\mathcal{E} = (R + r)I,\ P = I^2R$. The target variable is the external resistance R.

EXECUTE: $P = I^2R = \left(\dfrac{\mathcal{E}}{R + r}\right)^2 R$. $\mathcal{E}^2R = P(R^2 + 2rR + r^2)$. Putting in the given values and solving this quadratic equation for R gives $R = 0.429\ \Omega$ and $R = 21.0\ \Omega$.

EVALUATE: To check, apply $P = I^2R = \left(\dfrac{\mathcal{E}}{R + r}\right)^2 R$ for both resistances.

$$P = I^2R = \left(\frac{24.0\text{ V}}{3.429\ \Omega}\right)^2 (0.429\ \Omega) = 21\text{ W} \text{ and } P = \left(\frac{24.0\text{ V}}{24.0\ \Omega}\right)^2 (21.0\ \Omega) = 21\text{ W}.$$ Both answers check.

25.34. **IDENTIFY** and **SET UP:** The resistance is the same in both cases, and $P = V^2/R$.

EXECUTE: **(a)** Solving $P = V^2/R$ for R, gives $R = V^2/P$. Since the resistance is the same in both cases, we have $\dfrac{V_1^2}{P_1} = \dfrac{V_2^2}{P_2}$. Solving for P_2 gives $P_2 = P_1(V_2/V_1)^2 = (0.0625\text{ W})[(12.5\text{ V})/(1.50\text{ V})]^2 = 4.41$ W.

(b) Solving for V_2 gives $V_2 = V_1\sqrt{\dfrac{P_2}{P_1}} = (1.50\text{ V})\sqrt{\dfrac{5.00\text{ W}}{0.0625\text{ W}}} = 13.4\text{ V}$.

EVALUATE: These calculations are correct assuming that the resistor obeys Ohm's law throughout the range of currents involved.

25.35. **IDENTIFY:** The bulbs are each connected across a 120-V potential difference.

SET UP: Use $P = V^2/R$ to solve for R and Ohm's law $(I = V/R)$ to find the current.

EXECUTE: **(a)** $R = V^2/P = (120\text{ V})^2/(100\text{ W}) = 144\ \Omega$.

(b) $R = V^2/P = (120\text{ V})^2/(60\text{ W}) = 240\ \Omega$.

(c) For the 100-W bulb: $I = V/R = (120\text{ V})/(144\ \Omega) = 0.833\text{ A}$.

For the 60-W bulb: $I = (120\text{ V})/(240\ \Omega) = 0.500\text{ A}$.

EVALUATE: The 60-W bulb has *more* resistance than the 100-W bulb, so it draws less current.

25.36. **IDENTIFY:** Across 120 V, a 75-W bulb dissipates 75 W. Use this fact to find its resistance, and then find the power the bulb dissipates across 220 V.

SET UP: $P = V^2/R$, so $R = V^2/P$.

EXECUTE: Across 120 V: $R = (120\text{ V})^2/(75\text{ W}) = 192\ \Omega$. Across a 220-V line, its power will be $P = V^2/R = (220\text{ V})^2/(192\ \Omega) = 252\text{ W}$.

EVALUATE: The bulb dissipates much more power across 220 V, so it would likely blow out at the higher voltage. An alternative solution to the problem is to take the ratio of the powers.

$\dfrac{P_{220}}{P_{120}} = \dfrac{V_{220}^2/R}{V_{120}^2/R} = \left(\dfrac{V_{220}}{V_{120}}\right)^2 = \left(\dfrac{220}{120}\right)^2$. This gives $P_{220} = (75\text{ W})\left(\dfrac{220}{120}\right)^2 = 252\text{ W}$.

25.37. **IDENTIFY:** A "100-W" European bulb dissipates 100 W when used across 220 V.

(a) SET UP: Take the ratio of the power in the U.S. to the power in Europe, as in the alternative method for Problem 25.36, using $P = V^2/R$.

EXECUTE: $\dfrac{P_{\text{US}}}{P_{\text{E}}} = \dfrac{V_{\text{US}}^2/R}{V_{\text{E}}^2/R} = \left(\dfrac{V_{\text{US}}}{V_{\text{E}}}\right)^2 = \left(\dfrac{120\text{ V}}{220\text{ V}}\right)^2$. This gives $P_{\text{US}} = (100\text{ W})\left(\dfrac{120\text{ V}}{220\text{ V}}\right)^2 = 29.8\text{ W}$.

(b) SET UP: Use $P = IV$ to find the current.

EXECUTE: $I = P/V = (29.8\text{ W})/(120\text{ V}) = 0.248\text{ A}$.

EVALUATE: The bulb draws considerably less power in the U.S., so it would be much dimmer than in Europe.

25.38. **IDENTIFY:** $P = VI$. Energy $= Pt$.

SET UP: $P = (9.0\text{ V})(0.13\text{ A}) = 1.17\text{ W}$.

EXECUTE: Energy $= (1.17\text{ W})(30\text{ min})(60\text{ s/min}) = 2100\text{ J}$.

EVALUATE: The energy consumed is proportional to the voltage, to the current and to the time.

25.39. **IDENTIFY:** Calculate the current in the circuit. The power output of a battery is its terminal voltage times the current through it. The power dissipated in a resistor is I^2R.

SET UP: The sum of the potential changes around the circuit is zero.

EXECUTE: **(a)** $I = \dfrac{8.0\text{ V}}{17\ \Omega} = 0.47\text{ A}$. Then $P_{5\Omega} = I^2R = (0.47\text{ A})^2(5.0\ \Omega) = 1.1\text{ W}$ and $P_{9\Omega} = I^2R = (0.47\text{ A})^2(9.0\ \Omega) = 2.0\text{ W}$, so the total is 3.1 W.

(b) $P_{16\text{V}} = \mathcal{E}I - I^2r = (16\text{ V})(0.47\text{ A}) - (0.47\text{ A})^2(1.6\ \Omega) = 7.2\text{ W}$.

(c) $P_{8\text{V}} = \mathcal{E}I + Ir^2 = (8.0\text{ V})(0.47\text{ A}) + (0.47\text{ A})^2(1.4\ \Omega) = 4.1\text{ W}$.

EVALUATE: **(d)** (b) = (a) + (c). The rate at which the 16.0-V battery delivers electrical energy to the circuit equals the rate at which it is consumed in the 8.0-V battery and the 5.0-Ω and 9.0-Ω resistors.

25.40. **IDENTIFY:** Knowing the current and potential difference, we can find the power.

SET UP: $P = VI$ and energy is the product of power and time.

EXECUTE: $P = (500\text{ V})(80\times10^{-3}\text{ A}) = 40\text{ W}$.

Energy $= Pt = (40\text{ W})(10\times10^{-3}\text{ s}) = 0.40\text{ J}$.

EVALUATE: The energy delivered depends not only on the voltage and current but also on the length of the pulse. The pulse is short but the voltage is large.

25.41. **IDENTIFY:** We know the current, voltage and time the current lasts, so we can calculate the power and the energy delivered.

SET UP: Power is energy per unit time. The power delivered by a voltage source is $P = V_{ab} I$.

EXECUTE: **(a)** $P = (25\text{ V})(12\text{ A}) = 300\text{ W}$.

(b) $\text{Energy} = Pt = (300\text{ W})(3.0\times10^{-3}\text{ s}) = 0.90\text{ J}$.

EVALUATE: The energy is not very great, but it is delivered in a short time (3 ms) so the power is large, which produces a short shock.

25.42. **IDENTIFY** and **SET UP:** The average power delivered by the battery can be calculated in two different ways: $P = \dfrac{\text{energy}}{\text{time}}$ or $P = VI$. The time is 5.25 h, which in seconds is $5.25\text{ h} = (5.25\text{ h})(3600\text{ s/h}) = 1.89\times10^4\text{ s}$.

EXECUTE: The average power delivered by the battery is $P = \dfrac{\text{energy}}{\text{time}} = \dfrac{3.15\times10^4\text{ J}}{1.89\times10^4\text{ s}} = 1.6667\text{ W}$. Thus, the current must be $I = \dfrac{P}{V} = \dfrac{1.6667\text{ W}}{3.70\text{ V}} = 0.450\text{ A}$.

EVALUATE: The energy stored in the battery can be expressed in joules or watt-hours. The energy is equal to Pt, so we can express the stored energy as either $3.15\times10^4\text{ J}$ or (1.6667 W)(5.25 h) = $8.75\text{ W}\cdot\text{h}$.

25.43. **(a) IDENTIFY** and **SET UP:** $P = VI$ and $\text{energy} = (\text{power})\times(\text{time})$.

EXECUTE: $P = VI = (12\text{ V})(60\text{ A}) = 720\text{ W}$.

The battery can provide this for 1.0 h, so the energy the battery has stored is $U = Pt = (720\text{ W})(3600\text{ s}) = 2.6\times10^6\text{ J}$.

(b) IDENTIFY and **SET UP:** For gasoline the heat of combustion is $L_c = 46\times10^6\text{ J/kg}$. Solve for the mass m required to supply the energy calculated in part (a) and use density $\rho = m/V$ to calculate V.

EXECUTE: The mass of gasoline that supplies $2.6\times10^6\text{ J}$ is $m = \dfrac{2.6\times10^6\text{ J}}{46\times10^6\text{ J/kg}} = 0.0565\text{ kg}$.

The volume of this mass of gasoline is

$$V = \frac{m}{\rho} = \frac{0.0565\text{ kg}}{900\text{ kg/m}^3} = 6.3\times10^{-5}\text{ m}^3\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) = 0.063\text{ L}.$$

(c) IDENTIFY and **SET UP:** $\text{Energy} = (\text{power})\times(\text{time})$; the energy is that calculated in part (a).

EXECUTE: $U = Pt,\ t = \dfrac{U}{P} = \dfrac{2.6\times10^6\text{ J}}{450\text{ W}} = 5800\text{ s} = 97\text{ min} = 1.6\text{ h}$.

EVALUATE: The battery discharges at a rate of 720 W (for 1.0 h) and is charged at a rate of 450 W (for 1.6 h), so it takes longer to charge than to discharge.

25.44. **IDENTIFY:** This problem involves the internal resistance of a battery.

SET UP: $\mathcal{E} = (R + r)I$, $P = I^2R$. The target variable is the emf of the battery.

EXECUTE: Solve $P = I^2R$ for I: $I = \sqrt{P/R} = \sqrt{\dfrac{96.0\text{ J/s}}{12.0\ \Omega}} = 2.8284\text{ A}$. Now find the emf:

$\mathcal{E} = (R + r)I =$ (14.0 Ω)(2.8284 A) = 39.6 V.

EVALUATE: The power the battery produces is $P = I\mathcal{E}$ = (2.8284 A)(39.6 V) = 112 W. So the power lost in the internal resistance is 112 W – 96 W = 16 W. We can also get this using $P = I^2R$ = (2.8284 A)^2(2.00 Ω) = 16 W.

25.45. **IDENTIFY:** Some of the power generated by the internal emf of the battery is dissipated across the battery's internal resistance, so it is not available to the bulb.

SET UP: Use $P = I^2R$ and take the ratio of the power dissipated in the internal resistance r to the total power.

EXECUTE: $\dfrac{P_r}{P_{\text{Total}}} = \dfrac{I^2 r}{I^2(r+R)} = \dfrac{r}{r+R} = \dfrac{3.5\ \Omega}{28.5\ \Omega} = 0.123 = 12.3\%.$

EVALUATE: About 88% of the power of the battery goes to the bulb. The rest appears as heat in the internal resistance.

25.46. **IDENTIFY:** The power delivered to the bulb is I^2R. Energy $= Pt$.

SET UP: The circuit is sketched in Figure 25.46. r_{total} is the combined internal resistance of both batteries.

EXECUTE: **(a)** $r_{\text{total}} = 0$. The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - I(17\ \Omega) = 0$. $I = 0.1765\text{ A}$. $P = I^2R = (0.1765\text{ A})^2(17\ \Omega) = 0.530\text{ W}$. This is also $(3.0\text{ V})(0.1765\text{ A})$.

(b) Energy $= (0.530\text{ W})(5.0\text{ h})(3600\text{ s/h}) = 9540\text{ J}$.

(c) $P = \dfrac{0.530\text{ W}}{2} = 0.265\text{ W}$. $P = I^2R$ so $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{0.265\text{ W}}{17\ \Omega}} = 0.125\text{ A}$.

The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - IR - Ir_{\text{total}} = 0$.

$r_{\text{total}} = \dfrac{3.0\text{ V} - (0.125\text{ A})(17\ \Omega)}{0.125\text{ A}} = 7.0\ \Omega.$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

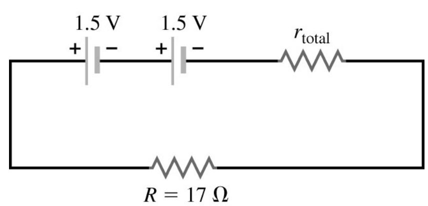


Figure 25.46

25.47. **IDENTIFY:** Solve for the current I in the circuit. Apply $P = VI = I^2R$ to the specified circuit elements to find the rates of energy conversion.

SET UP: The circuit is sketched in Figure 25.47.

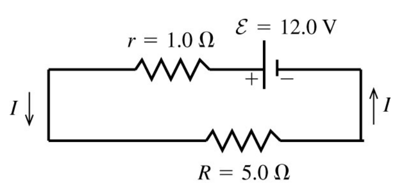


Figure 25.47

EXECUTE: Compute I:

$\mathcal{E} - Ir - IR = 0.$

$I = \dfrac{\mathcal{E}}{r+R} = \dfrac{12.0\text{ V}}{1.0\ \Omega + 5.0\ \Omega} = 2.00\text{ A}.$

(a) The rate of conversion of chemical energy to electrical energy in the emf of the battery is $P = \mathcal{E}I = (12.0\text{ V})(2.00\text{ A}) = 24.0\text{ W}$.

(b) The rate of dissipation of electrical energy in the internal resistance of the battery is $P = I^2 r = (2.00\text{ A})^2(1.0\,\Omega) = 4.0\text{ W}$.

(c) The rate of dissipation of electrical energy in the external resistor R is $P = I^2 R = (2.00\text{ A})^2(5.0\,\Omega) = 20.0\text{ W}$.

EVALUATE: The rate of production of electrical energy in the circuit is 24.0 W. The total rate of consumption of electrical energy in the circuit is $4.00\text{ W} + 20.0\text{ W} = 24.0\text{ W}$. Equal rates of production and consumption of electrical energy are required by energy conservation.

25.48. **IDENTIFY:** $P = I^2 R = \dfrac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120\text{ V}$. Energy $= Pt$.

EXECUTE: **(a)** $P = \dfrac{V^2}{R}$ so $R = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{540\text{ W}} = 26.7\,\Omega$.

(b) $P = VI$ so $I = \dfrac{P}{V} = \dfrac{540\text{ W}}{120\text{ V}} = 4.50\text{ A}$.

(c) Assuming that R remains $26.7\,\Omega$, $P = \dfrac{V^2}{R} = \dfrac{(110\text{ V})^2}{26.7\,\Omega} = 453\text{ W}$. P is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than $26.7\,\Omega$ and the power consumed will be greater than the value calculated in part (c).

25.49. **IDENTIFY:** The resistivity is $\rho = \dfrac{m}{ne^2\tau}$.

SET UP: For silicon, $\rho = 2300\,\Omega\cdot\text{m}$.

EXECUTE: **(a)** $\tau = \dfrac{m}{ne^2\rho} = \dfrac{9.11\times10^{-31}\text{ kg}}{(1.0\times10^{16}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})^2(2300\,\Omega\cdot\text{m})} = 1.55\times10^{-12}\text{ s}$.

EVALUATE: **(b)** The number of free electrons in copper $(8.5\times10^{28}\text{ m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\text{ m}^{-3})$. A smaller density of current carriers means a higher resistivity.

25.50. **IDENTIFY:** We are investigating a cell phone.

SET UP and **EXECUTE:** **(a)** Charge capacity = 2600 mAh, energy = 9.88 Wh, and potential rating = 3.8 V.

(b) 2600 mAh = (2.6 A)(3600 s) = 9360 C.

(c) (9.88 Wh)(9.88 J/s)(3600 s) = 35.6 kJ.

(d) qV = (9360 C)(3.8 V) = 35.6 kJ. They are equivalent.

(e) $C = Q/V$ = (9360 C)/(3.8 V) = 2460 F.

(f) $U = mc\Delta T \;\rightarrow\; \Delta T = U/mc$. m = 1.0 kg. $\Delta T = \dfrac{35.6\text{ kJ}}{(1.0\text{ kg})(4190\text{ J/kg}\cdot\text{K})} = 8.5\text{ C}°$.

EVALUATE: A capacitance of 2460 F would be very large!

25.51. **(a) IDENTIFY** and **SET UP:** Use $V_{ab} = \mathcal{E} - rI$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{(0.104\ \Omega)\pi(1.25\times10^{-3}\text{ m})^2}{14.0\text{ m}} = 3.65\times10^{-8}\,\Omega\cdot\text{m}$.

EVALUATE: This value is similar to that for good metallic conductors in Table 25.1.

(b) IDENTIFY and **SET UP:** Use $V = EL$ to calculate E and then Ohm's law gives I.

EXECUTE: $V = EL = (1.28\text{ V/m})(14.0\text{ m}) = 17.9\text{ V}$.

$I = \dfrac{V}{R} = \dfrac{17.9\text{ V}}{0.104\ \Omega} = 172\text{ A}.$

EVALUATE: We could do the calculation another way:

$E = \rho J$ so $J = \dfrac{E}{\rho} = \dfrac{1.28\text{ V/m}}{3.65\times10^{-8}\ \Omega\cdot\text{m}} = 3.51\times10^{7}\text{ A/m}^2.$

$I = JA = (3.51\times10^{7}\text{ A/m}^2)\pi(1.25\times10^{-3}\text{ m})^2 = 172\text{ A},$ which checks.

(c) IDENTIFY and SET UP: Calculate $J = I/A$ or $J = E/\rho$ and then use Eq. (25.3) for the target variable v_d.

EXECUTE: $J = n|q|v_\text{d} = nev_\text{d}.$

$v_\text{d} = \dfrac{J}{ne} = \dfrac{3.51\times10^{7}\text{ A/m}^2}{(8.5\times10^{28}\text{ m}^{-3})(1.602\times10^{-19}\text{ C})} = 2.58\times10^{-3}\text{ m/s} = 2.58\text{ mm/s}.$

EVALUATE: Even for this very large current the drift speed is small.

25.52. **IDENTIFY:** We are investigating a hot water heater.

SET UP and EXECUTE: **(a)** Estimate: 50 gal = (50)(3.788 L) = 190 L.

(b) $Q = mc\Delta T$ = (190 kg)(4190 J/kg · K)(25 C°) = 20 MJ.

(c) $P = Q/t$ = (20 MJ)/[(1.5)(3600 s)] = 3.7 kW.

(d) $I = P/V$ = (3.7 kW)/(220 V) = 17 A.

(e) $R = V/I$ = (220 V)/(17 A) = 13 Ω.

EVALUATE: Since $P = V^2/R$, for a given voltage we need a small resistance to have a large power.

25.53. **IDENTIFY and SET UP:** With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\mathcal{E} = 12.6\text{ V}.$

With a wire of resistance R connected to the battery current I flows and $\mathcal{E} - Ir - IR = 0,$ where r is the internal resistance of the battery. Apply this equation to each piece of wire to get two equations in the two unknowns.

EXECUTE: Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1.$

$\mathcal{E} - I_1 r - I_1 R_1 = 0;\quad 12.6\text{ V} - (7.00\text{ A})r - (7.00\text{ A})R_1 = 0.$

$\mathcal{E} - I_2 r - I_2(2R_2) = 0;\quad 12.6\text{ V} - (4.20\text{ A})r - (4.20\text{ A})(2R_1) = 0.$

Solving these two equations in two unknowns gives $R_1 = 1.20\ \Omega.$ This is the resistance of 20.0 m, so the resistance of one meter is $[1.20\ \Omega/(20.0\text{ m})](1.00\text{ m}) = 0.060\ \Omega.$

EVALUATE: We can also solve for r and we get $r = 0.600\ \Omega.$ When measuring small resistances, the internal resistance of the battery has a large effect.

25.54. **IDENTIFY:** As the resistance R varies, the current in the circuit also varies, which causes the potential drop across the internal resistance of the battery to vary. The largest current will occur when $R = 0,$ and the smallest current will occur when $R \to \infty.$ The largest terminal voltage will occur when the current is zero ($R \to \infty$) and the smallest terminal voltage will be when the current is a maximum ($R = 0$).

SET UP: If $\mathcal{E}$ is the internal emf of the battery and r is its internal resistance, then $V_{ab} = \mathcal{E} - rI.$

EXECUTE: **(a)** As $R \to \infty,$ $I \to 0,$ so $V_{ab} \to \mathcal{E} = 15.0\text{ V},$ which is the largest reading of the voltmeter. When $R = 0,$ the current is largest at $(15.0\text{ V})/(4.00\ \Omega) = 3.75\text{ A},$ so the smallest terminal voltage is $V_{ab} = \mathcal{E} - rI = 15.0\text{ V} -\ (4.00\ \Omega)(3.75\text{ A}) = 0.$

(b) From part (a), the maximum current is 3.75 A when $R = 0,$ and the minimum current is 0.00 A when $R \to \infty.$

(c) The graphs are sketched in the Figure 25.54.

EVALUATE: Increasing the resistance R increases the terminal voltage, but at the same time it decreases the current in the circuit.

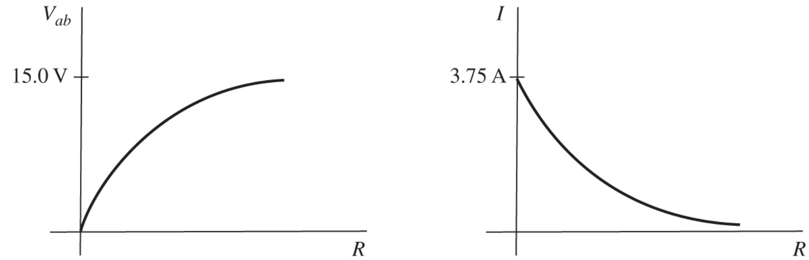


Figure 25.54

25.55. **IDENTIFY:** Conservation of charge requires that the current be the same in both sections of the wire. $E = \rho J = \dfrac{\rho I}{A}$. For each section, $V = IR = JAR = \left(\dfrac{EA}{\rho}\right)\left(\dfrac{\rho L}{A}\right) = EL$. The voltages across each section add.

SET UP: $A = (\pi/4)D^2$, where D is the diameter.

EXECUTE: **(a)** The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

(b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(1.6\times10^{-3}\ \text{m})^2} = 2.14\times10^{-5}\ \text{V/m}.$

(c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(0.80\times10^{-3}\ \text{m})^2} = 8.55\times10^{-5}\ \text{V/m}.$ This is $4E_{1.6\text{mm}}$.

(d)
$V = E_{1.6\text{mm}}L_{1.6\ \text{mm}} + E_{0.8\ \text{mm}}L_{0.8\ \text{mm}}.\ V = (2.14\times10^{-5}\ \text{V/m})(1.20\ \text{m}) + (8.55\times10^{-5}\ \text{V/m})(1.80\ \text{m}) = 1.80\times10^{-4}\ \text{V}.$

EVALUATE: The currents are the same but the current density is larger in the thinner section and the electric field is larger there.

25.56. **IDENTIFY** and **SET UP:** The voltage is the same at both temperatures since the same battery is used. The power is $P = V^2/R$ and $R = R_0(1+\alpha\Delta T)$.

EXECUTE: Since the voltage is the same, we have $V^2 = P_{80}R_{80} = P_{150}R_{150}$. Therefore $P_{80}R_0[1+\alpha(T_{80}-T_0)] = P_{150}R_0[1+\alpha(T_{150}-T_0)]$. Solving for P_{150} and putting in the numbers gives

$$P_{150} = P_{80}\frac{1+\alpha(T_{80}-T_0)}{1+\alpha(T_{150}-T_0)} = (480\ \text{W})\frac{1+(0.0045\ \text{K}^{-1})(80^\circ\text{C}-20^\circ\text{C})}{1+(0.0045\ \text{K}^{-1})(150^\circ\text{C}-20^\circ\text{C})} = 385\ \text{W}.$$

EVALUATE: This result assumes that α is the same at all the temperatures.

25.57. **IDENTIFY:** Knowing the current and the time for which it lasts, plus the resistance of the body, we can calculate the energy delivered.

SET UP: Electric energy is deposited in his body at the rate $P = I^2R$. Heat energy Q produces a temperature change ΔT according to $Q = mc\Delta T$, where $c = 4190\ \text{J/kg}\cdot\text{C}^\circ$.

EXECUTE: **(a)** $P = I^2R = (25{,}000\ \text{A})^2(1.0\ \text{k}\Omega) = 6.25\times10^{11}\ \text{W}$. The energy deposited is $Pt = (6.15\times10^{11}\ \text{W})(40\times10^{-6}\ \text{s}) = 2.5\times10^7\ \text{J}$. Find ΔT when $Q = 2.5\times10^7\ \text{J}$.

$$\Delta T = \frac{Q}{mc} = \frac{2.5\times10^7\ \text{J}}{(75\ \text{kg})(4190\ \text{J/kg}\cdot\text{C}^\circ)} = 80\ \text{C}^\circ.$$

(b) An increase of only $63\ \text{C}^\circ$ brings the water in the body to the boiling point; part of the person's body will be vaporized.

EVALUATE: Even this approximate calculation shows that being hit by lightning is very dangerous.

25.58. IDENTIFY: The current in the circuit depends on R and on the internal resistance of the battery, as well as the emf of the battery. It is only the current in R that dissipates energy in the resistor R.

SET UP: $I = \dfrac{\mathcal{E}}{R+r}$, where $\mathcal{E}$ is the emf of the battery, and $P = I^2 R$.

EXECUTE: $P = I^2 R = \dfrac{\mathcal{E}^2}{(R+r)^2} R$, which gives $\mathcal{E}^2 R = (R^2 + 2Rr + r^2)P$.

$$R^2 + \left(2r - \frac{\mathcal{E}^2}{P}\right) R + r^2 = 0. \quad R = \frac{1}{2}\left[\left(\frac{\mathcal{E}^2}{P} - 2r\right) \pm \sqrt{\left(\frac{\mathcal{E}^2}{P} - 2r\right)^2 - 4r^2}\right].$$

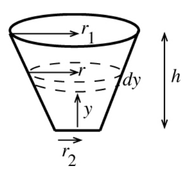
$$R = \frac{1}{2}\left[\left(\frac{(12.0\ \text{V})^2}{80.0\ \text{W}} - 2(0.40\ \Omega)\right) \pm \sqrt{\left(\frac{(12.0\ \text{V})^2}{80.0\ \text{W}} - 2(0.40\ \Omega)\right)^2 - 4(0.40\ \Omega)^2}\right].$$

$R = 0.50\ \Omega \pm 0.30\ \Omega$. $R = 0.20\ \Omega$ and $R = 0.80\ \Omega$.

EVALUATE: There are two values for R because there are two ways for the power dissipated in R to be 80 W. The power is $P = I^2 R$, so we can have a small $R(0.20\ \Omega)$ and large current, or a larger $R(0.80\ \Omega)$ and a smaller current.

25.59. (a) IDENTIFY: Apply $R = \dfrac{\rho L}{A}$ to calculate the resistance of each thin disk and then integrate over the truncated cone to find the total resistance.

SET UP:



EXECUTE: The radius of a truncated cone a distance y above the bottom is given by $r = r_2 + (y/h)(r_1 - r_2) = r_2 + y\beta$ with $\beta = (r_1 - r_2)/h$.

Figure 25.59

Consider a thin slice a distance y above the bottom. The slice has thickness dy and radius r (see Figure 25.59.) The resistance of the slice is $dR = \dfrac{\rho dy}{A} = \dfrac{\rho dy}{\pi r^2} = \dfrac{\rho dy}{\pi(r_2 + \beta y)^2}$.

The total resistance of the cone if obtained by integrating over these thin slices:

$$R = \int dR = \frac{\rho}{\pi}\int_0^h \frac{dy}{(r_2 + \beta y)^2} = \frac{\rho}{\pi}\left[-\frac{1}{\beta}(r_2 + y\beta)^{-1}\right]_0^h = -\frac{\rho}{\pi\beta}\left[\frac{1}{r_2 + h\beta} - \frac{1}{r_2}\right].$$

But $r_2 + h\beta = r_1$.

$$R = \frac{\rho}{\pi\beta}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{\rho}{\pi}\left(\frac{h}{r_1 - r_2}\right)\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{\rho h}{\pi r_1 r_2}.$$

(b) EVALUATE: Let $r_1 = r_2 = r$. Then $R = \rho h/\pi r^2 = \rho L/A$ where $A = \pi r^2$ and $L = h$. This agrees with $R = \dfrac{\rho L}{A}$.

25.60. IDENTIFY: Divide the region into thin spherical shells of radius r and thickness dr. The total resistance is the sum of the resistances of the thin shells and can be obtained by integration.

SET UP: $I = V/R$ and $J = I/4\pi r^2$, where $4\pi r^2$ is the surface area of a shell of radius r.

EXECUTE: **(a)** $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\int_a^b \dfrac{dr}{r^2} = -\dfrac{\rho}{4\pi}\dfrac{1}{r}\bigg|_a^b = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\rho}{4\pi}\left(\dfrac{b-a}{ab}\right).$

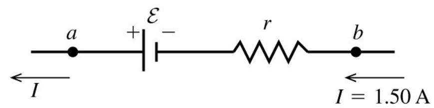
(b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)}$ and $J = \dfrac{I}{A} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab}ab}{\rho(b-a)r^2}.$

(c) If the thickness of the shells is small, then $4\pi ab \approx 4\pi a^2$ is the surface area of the conducting material. $R = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\rho(b-a)}{4\pi ab} \approx \dfrac{\rho L}{4\pi a^2} = \dfrac{\rho L}{A}$, where $L = b - a$.

EVALUATE: The current density in the material is proportional to $1/r^2$.

25.61. **IDENTIFY:** In each case write the terminal voltage in terms of $\mathcal{E}$, I, and r. Since I is known, this gives two equations in the two unknowns $\mathcal{E}$ and r.

SET UP: The battery with the 1.50-A current is sketched in Figure 25.61a.



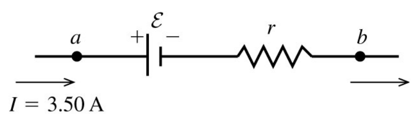
$V_{ab} = 8.40$ V.

$V_{ab} = \mathcal{E} - Ir$.

$\mathcal{E} - (1.50\text{ A})r = 8.40$ V.

Figure 25.61a

The battery with the 3.50-A current is sketched in Figure 25.61b.



$V_{ab} = 10.2$ V.

$V_{ab} = \mathcal{E} + Ir$.

$\mathcal{E} + (3.50\text{ A})r = 10.2$ V.

Figure 25.61b

EXECUTE: **(a)** Solve the first equation for $\mathcal{E}$ and use that result in the second equation:
$\mathcal{E} = 8.40\text{ V} + (1.50\text{ A})r.$

$8.40\text{ V} + (1.50\text{ A})r + (3.50\text{ A})r = 10.2\text{ V}.$

$(5.00\text{ A})r = 1.8\text{ V}$ so $r = \dfrac{1.8\text{ V}}{5.00\text{ A}} = 0.36\ \Omega.$

(b) Then $\mathcal{E} = 8.40\text{ V} + (1.50\text{ A})r = 8.40\text{ V} + (1.50\text{ A})(0.36\ \Omega) = 8.94\text{ V}.$

EVALUATE: When the current passes through the emf in the direction from $-$ to $+$, the terminal voltage is less than the emf and when it passes through from $+$ to $-$, the terminal voltage is greater than the emf.

25.62. **IDENTIFY:** Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: **(a)** $I = \dfrac{8.0\text{ V} - 4.0\text{ V}}{24.0\ \Omega} = 0.167$ A. $V_d + 8.00\text{ V} - I(0.50\ \Omega + 8.00\ \Omega) = V_a$, so

$V_{ad} = 8.00\text{ V} - (0.167\text{ A})(8.50\ \Omega) = 6.58\text{ V}.$

(b) The terminal voltage is $V_{bc} = V_b - V_c$. $V_c + 4.00\text{ V} + I(0.50\ \Omega) = V_b$ and

$V_{bc} = +4.00\text{ V} + (0.167\text{ A})(0.50\ \Omega) = +4.08\text{ V}.$

(c) Adding another battery at point d in the opposite sense to the 8.0-V battery produces a counterclockwise current with magnitude $I = \dfrac{10.3\text{ V} - 8.0\text{ V} + 4.0\text{ V}}{24.5\ \Omega} = 0.257\text{ A}.$ Then

$V_c + 4.00\text{ V} - I(0.50\ \Omega) = V_b$ and $V_{bc} = 4.00\text{ V} - (0.257\text{ A})(0.50\ \Omega) = 3.87\text{ V}.$

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.63. IDENTIFY: $R = \dfrac{\rho L}{A}.$ $V = IR.$ $P = I^2R.$

SET UP: The area of the end of a cylinder of radius r is $\pi r^2.$

EXECUTE: **(a)** $R = \dfrac{(5.0\ \Omega\cdot\text{m})(1.6\text{ m})}{\pi(0.050\text{ m})^2} = 1.0\times10^3\ \Omega.$

(b) $V = IR = (100\times10^{-3}\text{ A})(1.0\times10^3\ \Omega) = 100\text{ V}.$

(c) $P = I^2R = (100\times10^{-3}\text{ A})^2(1.0\times10^3\ \Omega) = 10\text{ W}.$

EVALUATE: The resistance between the hands when the skin is wet is about a factor of ten less than when the skin is dry (Problem 25.64).

25.64. IDENTIFY: $V = IR.$ $P = I^2R.$

SET UP: The total resistance is the resistance of the person plus the internal resistance of the power supply.

EXECUTE: **(a)** $I = \dfrac{V}{R_{\text{tot}}} = \dfrac{14\times10^3\text{ V}}{10\times10^3\ \Omega + 2000\ \Omega} = 1.17\text{ A}.$

(b) $P = I^2R = (1.17\text{ A})^2(10\times10^3\ \Omega) = 1.37\times10^4\text{ J} = 13.7\text{ kJ}.$

(c) $R_{\text{tot}} = \dfrac{V}{I} = \dfrac{14\times10^3\text{ V}}{1.00\times10^{-3}\text{ A}} = 14\times10^6\ \Omega.$ The resistance of the power supply would need to be $14\times10^6\ \Omega - 10\times10^3\ \Omega = 14\times10^6\ \Omega = 14\text{ M}\Omega.$

EVALUATE: The current through the body in part (a) is large enough to be fatal.

25.65. IDENTIFY: The cost of operating an appliance is proportional to the amount of energy consumed. The energy depends on the power the item consumes and the length of time for which it is operated.

SET UP: At a constant power, the energy is equal to Pt, and the total cost is the cost per kilowatt-hour (kWh) times the energy (in kWh).

EXECUTE: **(a)** Use the fact that $1.00\text{ k Wh} = (1000\text{ J/s})(3600\text{ s}) = 3.60\times10^6\text{ J},$ and one year contains $3.156\times10^7\text{ s}.$

$$(75\text{ J/s})\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\text{ J}}\right) = \$78.90.$$

(b) At $8\text{ h/day},$ the refrigerator runs for 1/3 of a year. Using the same procedure as above gives

$$(400\text{ J/s})\left(\frac{1}{3}\right)\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\text{ J}}\right) = \$140.27.$$

EVALUATE: Electric lights can be a substantial part of the cost of electricity in the home if they are left on for a long time!

25.66. (a) IDENTIFY: The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A$. The diameter D of the cable is twice its radius. $P = \dfrac{V^2}{R} = \dfrac{V^2}{(\rho L/A)} = \dfrac{AV^2}{\rho L} = \dfrac{\pi r^2 V^2}{\rho L}$. The electric field in the cable is equal to the potential difference across its ends divided by the length of the cable: $E = V/L$.

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(90.0\text{ W})(1.72\times10^{-8}\ \Omega\cdot\text{m})(1500\text{ m})}{\pi(220.0\text{ V})^2}} = 1.236\times10^{-4}\text{ m} = 0.1236\text{ mm}.$$

$D = 2r = 0.247$ mm.

(b) IDENTIFY and **SET UP:** $E = V/L$.

EXECUTE: $E = (220\text{ V})/(1500\text{ m}) = 0.147$ V/m.

EVALUATE: This would be an extremely thin (and hence fragile) cable.

25.67. **(a) IDENTIFY:** Since the resistivity is a function of the position along the length of the cylinder, we must integrate to find the resistance.

SET UP: The resistance of a cross-section of thickness dx is $dR = \rho dx/A$.

EXECUTE: Using the given function for the resistivity and integrating gives

$$R = \int \frac{\rho dx}{A} = \int_0^L \frac{(a + bx^2)dx}{\pi r^2} = \frac{aL + bL^3/3}{\pi r^2}.$$

Now get the constants a and b: $\rho(0) = a = 2.25\times10^{-8}\ \Omega\cdot\text{m}$ and $\rho(L) = a + bL^2$ gives $8.50\times10^{-8}\ \Omega\cdot\text{m} = 2.25\times10^{-8}\ \Omega\cdot\text{m} + b(1.50\text{ m})^2$ which gives $b = 2.78\times10^{-8}\ \Omega/\text{m}$. Now use the above result to find R.

$$R = \frac{(2.25\times10^{-8}\ \Omega\cdot\text{m})(1.50\text{ m}) + (2.78\times10^{-8}\ \Omega/\text{m})(1.50\text{ m})^3/3}{\pi(0.0110\text{ m})^2} = 1.71\times10^{-4}\ \Omega = 171\ \mu\Omega.$$

(b) IDENTIFY: Use the definition of resistivity to find the electric field at the midpoint of the cylinder, where $x = L/2$.

SET UP: $E = \rho J$. Evaluate the resistivity, using the given formula, for $x = L/2$.

EXECUTE: At the midpoint, $x = L/2$, giving $E = \dfrac{\rho I}{\pi r^2} = \dfrac{[a + b(L/2)^2]I}{\pi r^2}$.

$$E = \frac{[2.25\times10^{-8}\ \Omega\cdot\text{m} + (2.78\times10^{-8}\ \Omega/\text{m})(0.750\text{ m})^2](1.75\text{ A})}{\pi(0.0110\text{ m})^2} = 1.76\times10^{-4}\text{ V/m} = 176\ \mu\text{V/m}$$

(c) IDENTIFY: For the first segment, the result is the same as in part (a) except that the upper limit of the integral is $L/2$ instead of L.

SET UP: Integrating using the upper limit of $L/2$ gives $R_1 = \dfrac{a(L/2) + (b/3)(L^3/8)}{\pi r^2}$.

EXECUTE: Substituting the numbers gives

$$R_1 = \frac{(2.25\times10^{-8}\ \Omega\cdot\text{m})(0.750\text{ m}) + (2.78\times10^{-8}\ \Omega/\text{m})/3((1.50\text{ m})^3/8)}{\pi(0.0110\text{ m})^2} = 5.47\times10^{-5}\ \Omega = 54.7\ \mu\Omega.$$

The resistance R_2 of the second half is equal to the total resistance minus the resistance of the first half.

$R_2 = R - R_1 = 1.71\times10^{-4}\ \Omega - 5.47\times10^{-5}\ \Omega = 1.16\times10^{-4}\ \Omega = 116\ \mu\Omega$.

EVALUATE: The second half has a greater resistance than the first half because the resistance increases with distance along the cylinder.

25.68. **IDENTIFY:** Compact fluorescent bulbs draw much less power than incandescent bulbs and last much longer. Hence they cost less to operate.

SET UP: A kWh is power of 1 kW for a time of 1 h. $P = \dfrac{V^2}{R}$.

EXECUTE: (a) In 3.0 yr the bulbs are on for $(3.0\text{ yr})(365.24\text{ days/yr})(4.0\text{ h/day}) = 4.38\times10^3\text{ h}$.

Compact bulb: The energy used is $(23\text{ W})(4.38\times10^3\text{ h}) = 1.01\times10^5\text{ Wh} = 101\text{ kWh}$. The cost of this energy is $(\$0.080\text{/kWh})(101\text{ kWh}) = \$8.08$. One bulb will last longer than this. The bulb cost is \$11.00, so the total cost is \$19.08.

Incandescent bulb: The energy used is $(100\text{ W})(4.38\times10^3\text{ h}) = 4.38\times10^5\text{ Wh} = 438\text{ kWh}$. The cost of this energy is $(\$0.080\text{/kWh})(438\text{ kWh}) = \$35.04$. Six bulbs will be used during this time and the bulb cost will be \$4.50. The total cost will be \$39.54.

(b) The compact bulb will save $\$39.54 - \$19.08 = \$20.46$.

(c) $R = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{23\text{ W}} = 626\ \Omega.$

EVALUATE: The initial cost of the bulb is much greater for the compact fluorescent bulb but the savings soon repay the cost of the bulb. The compact bulb should last for over six years, so over a 6-year period the savings per year will be even greater. The cost of compact fluorescent bulbs has come down dramatically, so the savings today would be considerably greater than indicated here.

25.69. IDENTIFY: This problem involves capacitance, dielectrics, and resistance.

SET UP: $C = \dfrac{\epsilon_0 A}{d}$, $R = \dfrac{\rho L}{A}$, $P = I^2R$, $\rho = (s_0/s)\ \Omega\cdot\text{m}$.

EXECUTE: (a) We want the salinity s when the ammeter reads 484 mA. Using $V = (R + R_{\text{can}})I$ gives 10.0 V = (15.0 Ω + R_{can}) (0.484 A). R_{can} = 5.66 Ω. $R_{\text{can}} = \dfrac{\rho L}{A} = \left(\dfrac{s_0}{s}\right)\dfrac{L}{\pi r^2}$, so $s = \dfrac{s_0 L}{\pi r^2 R_{\text{can}}} =$

$$\frac{(6.30\text{ ppt }\Omega\cdot\text{m})(0.0300\text{ m})}{\pi(0.0500\text{ m})^2(5.66\ \Omega)} = 4.25\text{ ppt.}$$

(b) We want the charge on the left capacitor. Treat the can like an ideal parallel-plate capacitor, so $C = \dfrac{\epsilon_0 AK}{d} = \dfrac{\epsilon_0 \pi r^2 K}{L}$. $V_{\text{can}} = R_{\text{can}}I$ = (5.66 Ω)(0.484 A) = 2.739 V. Now use $Q = CV_{\text{can}}$.

$$Q = \left(\frac{\epsilon_0 \pi r^2 K}{L}\right)V_{\text{can}} = \left(\frac{\epsilon_0 \pi(0.0500\text{ m})^2(80.4)}{0.0300\text{ m}}\right)(2.739\text{ V}) = 510\text{ pC.}$$

(c) We want the power generated in the saline solution. $P = I^2R_{\text{can}} = (0.484\text{ A})^2(5.66\ \Omega) = 1.33\text{ W}$.

(d) We want the salinity level s. $P = V^2/R$, so if R and R_{can} each dissipate half the power of the battery, they must have equal resistances. Therefore $R_{\text{can}} = R = 15.0\ \Omega$. From part (a) we have

$$s = \frac{s_0 L}{\pi r^2 R_{\text{can}}} = \frac{(6.30\text{ ppt }\Omega\cdot\text{m})(0.0300\text{ m})}{\pi(0.0500\text{ m})^2(15.0\ \Omega)} = 1.60\text{ ppt.}$$

EVALUATE: The measurements needed to calculate the salinity s (V, I, and R) are easily made with simple meters, so s could easily be determined by this method.

25.70. IDENTIFY: No current flows to the capacitors when they are fully charged.

SET UP: $V_R = RI$ and $V_C = Q/C$.

EXECUTE: (a) $V_{C_1} = \dfrac{Q_1}{C_1} = \dfrac{18.0\ \mu\text{C}}{3.00\ \mu\text{F}} = 6.00\text{ V}$. $V_{C_2} = V_{C_1} = 6.00\text{ V}$.

$Q_2 = C_2V_{C_2} = (6.00\ \mu\text{F})(6.00\text{ V}) = 36.0\ \mu\text{C}$.

(b) No current flows to the capacitors when they are fully charged, so $\mathcal{E} = IR_1 + IR_2$.

$V_{R_2} = V_{C_1} = 6.00\text{ V}$. $I = \dfrac{V_{R_2}}{R_2} = \dfrac{6.00\text{ V}}{2.00\ \Omega} = 3.00\text{ A}$.

$R_1 = \dfrac{\mathcal{E} - IR_2}{I} = \dfrac{72.0\text{ V} - 6.00\text{ V}}{3.00\text{ A}} = 22.0\ \Omega.$

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

25.71. **IDENTIFY:** No current flows through the capacitor when it is fully charged.

SET UP: With the capacitor fully charged, $I = \dfrac{\mathcal{E}}{R_1 + R_2}$. $V_R = IR$ and $V_C = Q/C$.

EXECUTE: $V_C = \dfrac{Q}{C} = \dfrac{36.0\ \mu\text{C}}{9.00\ \mu\text{F}} = 4.00\ \text{V}$. $V_{R_1} = V_C = 4.00\ \text{V}$ and $I = \dfrac{V_{R_1}}{R_1} = \dfrac{4.00\ \text{V}}{6.00\ \Omega} = 0.667\ \text{A}$.

$V_{R_2} = IR_2 = (0.667\ \text{A})(4.00\ \Omega) = 2.668\ \text{V}$. $\mathcal{E} = V_{R_1} + V_{R_2} = 4.00\ \text{V} + 2.668\ \text{V} = 6.67\ \text{V}$.

EVALUATE: When a capacitor is fully charged, it acts like an open circuit and prevents any current from flowing though it.

25.72. **IDENTIFY** and **SET UP:** Ohm's law applies. The terminal voltage V_{ab} is less than the internal emf $\mathcal{E}$ due to voltage losses in the internal resistance *r* of the battery when current *I* is flowing in the circuit. $V_{ab} = \mathcal{E} - rI$.

EXECUTE: **(a)** The equation $V_{ab} = \mathcal{E} - rI$ applies to this circuit, so a graph of V_{ab} versus *I* should be a straight line with a slope equal to –*r* and a *y*-intercept equal to $\mathcal{E}$. Using points where the graph crosses grid lines, the slope is: $\text{slope} = \dfrac{22.0\ \text{V} - 30.0\ \text{V}}{7.00\ \text{A} - 3.00\ \text{A}} = -2.00\ \text{V/A}$. Therefore $r = -(-2.00\ \text{V/A}) = 2.00\ \Omega$.

The equation of the graph is $V_{ab} = \mathcal{E} - rI$, so we can solve for $\mathcal{E}$ and use a point on the graph to calculate $\mathcal{E}$. This gives

$\mathcal{E} = V_{ab} + rI = 30.0\ \text{V} + (2.00\ \Omega)(3.00\ \text{A}) = 36.0\ \text{V}$.

(b) $R = V_{ab}/I$ and $I = \dfrac{\mathcal{E} - V_{ab}}{r}$, so $R = \dfrac{V_{ab}}{\dfrac{\mathcal{E} - V_{ab}}{r}} = \dfrac{rV_{ab}}{\mathcal{E} - V_{ab}}$. Putting in the numbers gives

$R = (2.00\ \Omega)(0.800)(36.0\ \text{V})/[36.0\ \text{V} - (0.800)(36.0\ \text{V})] = 8.00\ \Omega$.

EVALUATE: For large currents, the terminal voltage can be much less than the internal emf, as shown by the graph with the problem.

25.73. **IDENTIFY:** According to Ohm's law, $R = \dfrac{V_{ab}}{I}$ = constant, and a graph of V_{ab} versus *I* will be a straight line with positive slope passing through the origin.

SET UP and **EXECUTE:** **(a)** Figure 25.73a shows the graphs of V_{ab} versus *I* and *R* versus *I* for resistor A.

Figure 25.73b shows these graphs for resistor B.

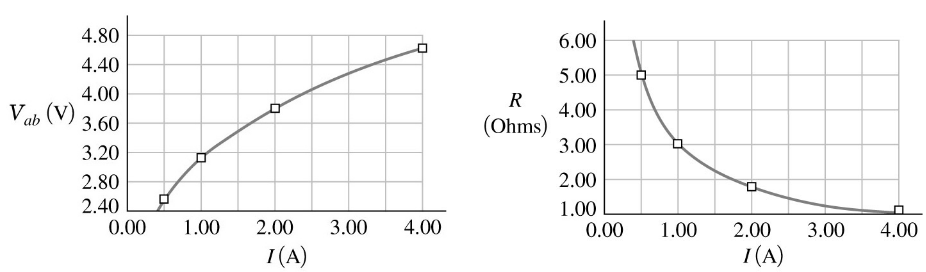


Figure 25.73a

(b) In Figure 25.73a, the graph of V_{ab} versus *I* is not a straight line so resistor A does not obey Ohm's law. In the graph of *R* versus *I*, *R* is not constant; it decreases as *I* increases.

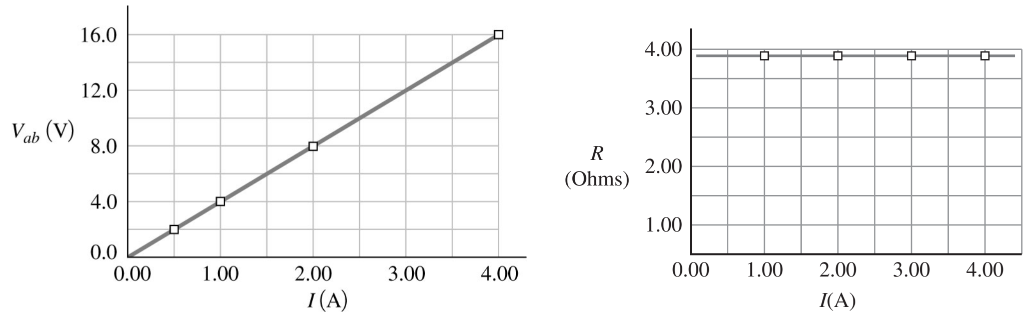


Figure 25.73b

(c) In Figure 25.73b, the graph of V_{ab} versus I is a straight line with positive slope passing through the origin, so resistor B obeys Ohm's law. The graph of R versus I is a horizontal line. This means that R is constant, which is consistent with Ohm's law.

(d) We use $P = IV$. From the graph of V_{ab} versus I in Figure 25.73a, we read that $I = 2.35$ A when $V = 4.00$ V. Therefore $P = IV = (2.35\text{ A})(4.00\text{ V}) = 9.40$ W.

(e) We use $P = V^2/R$. From the graph of R versus I in Figure 25.73b, we find that $R = 3.88\ \Omega$. Thus $P = V^2/R = (4.00\text{ V})^2/(3.88\ \Omega) = 4.12\text{ W}$.

EVALUATE: Since resistor B obeys Ohm's law $V_{ab} = RI$, R is the slope of the graph of V_{ab} versus I in Figure 25.73b. The given data points lie on the line, so we use them to calculate the slope.

slope = $R = \dfrac{15.52\text{ V} - 1.94\text{ V}}{4.00\text{ A} - 0.50\text{ A}} = 3.88\ \Omega.$ This value is the same as the one we got from the graph of R versus I in Figure 25.73b, so our results agree.

25.74. **IDENTIFY:** The power supplied to the house is $P = VI$. The rate at which electrical energy is dissipated in the wires is I^2R, where $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

(b) $P = VI$ gives $I = \dfrac{P}{V} = \dfrac{4200\text{ W}}{120\text{ V}} = 35\text{ A}$, so the 8-gauge wire is necessary, since it can carry up to 40 A.

(c) $P = I^2R = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})(42.0\text{ m})}{(\pi/4)(0.00326\text{ m})^{\,2}} = 106\text{ W}.$

(d) If 6-gauge wire is used, $P = \dfrac{I^2\rho L}{A} = \dfrac{(35\text{ A})^2\,(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42\text{ m})}{(\pi/4)\,(0.00412\text{ m})^2} = 66\text{ W}.$ The decrease in energy consumption is $\Delta E = \Delta Pt = (40\text{ W})(365\text{ days/yr})\,(12\text{ h/day}) = 175\text{ kWh/yr}$ and the savings is $(175\text{ kWh/yr})(\$0.11/\text{kWh}) = \$19.25$ per year.

EVALUATE: The cost of the 4200 W used by the appliances is \$2020. The savings is about 1%.

25.75. **IDENTIFY:** This problem involves resistivity.

SET UP: $E = c/r$, $R = \dfrac{\rho\ell}{A}$.

EXECUTE: **(a)** The outer conductor is at a higher potential than the inner conductor, so the electric field points inward toward the central axis.

(b) The target variable is *c*, where *E* = *c/r*. $V = \int_{r_{\text{inner}}}^{r_{\text{outer}}} E_r dr = \int_{r_{\text{inner}}}^{r_{\text{outer}}} \frac{c}{r} dr = c\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)$, which gives

$c = \dfrac{V}{\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)}.$

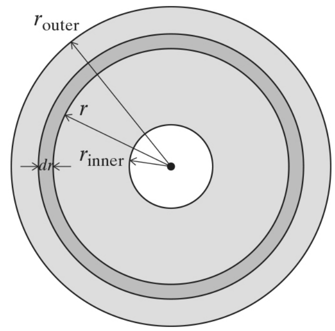


Figure 25.75

(c) We want the resistance of this device. Break the material into infinitesimal coaxial cylindrical shells of radius *r*, length *L*, and thickness *dr* as shown in Fig. 25.75. Apply $R = \dfrac{\rho\ell}{A}$ where $\ell = dr$ and *A* = $2\pi rL$. Doing so gives $R = \int_{r_{\text{inner}}}^{r_{\text{outer}}} \dfrac{\rho dr}{2\pi rL} = \dfrac{\rho}{2\pi L}\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)$.

(d) We want the resistivity so that *R* = 6.80 kΩ. Solve the result from (c) for ρ and put in the given quantities, giving $\rho = \dfrac{2\pi LR}{\ln\left(r_{\text{outer}} / r_{\text{inner}}\right)} = 616\ \Omega \cdot \text{m}.$

EVALUATE: From Table 25.1 we see that this resistivity would be about ¼ that of pure silicon but much less than for insulators and much greater than for conductors.

25.76. **IDENTIFY:** In this problem we investigate the resistance of a light bulb filament as it varies with temperature.

SET UP: $R = \dfrac{\rho L}{A}$, $R(T) = R_0\left[1 + \alpha\left(T - T_0\right)\right]$, $P = I^2R$.

EXECUTE: **(a)** The target variable is the resistance at 20.0°C. Use $R = \dfrac{\rho L}{A}$ with ρ at its 20.0°C value.

Putting in the given numbers gives R_{20} = 18.3 Ω.

(b) The target variable is the current when *V* = 120 V. First find *T* as a function of *I*. The graph of *T* versus *I* is a straight line passing through (0 A, 20.0°C) and (1.00 A, 2520°C), so its slope is 2500°C/A and its *T*-intercept is 20.0°C. Using the slope-intercept equation of a straight line, the equation is *T*(*I*) = (2500°C/A)*I* + 20.0°C. Using T_0 = 20.0°C, the resistance as a function of temperature is $R(T) = R_{20}\left[1 + \alpha\left(T - 20.0°\text{C}\right)\right]$ and a comparable equation hold for the resistivity. Now combine our equation for *T*(*I*) with *R*(*T*) to find *R* as a function of *I*. $R(I) = R_{20}\left\{1 + \alpha\left[\left(2500°\text{C/A}\right)I + 20.0°\text{C}\right]\right\}$.

Using *V* = *RI* gives $V = R_{20}\left\{1 + \alpha\left[\left(2500°\text{C/A}\right)I + 20.0°\text{C}\right]\right\}I$. Now solve for *I* when *V* = 120 V.

$120\ \text{V} = (18.3\ \Omega)\left\{1 + (0.0045 / \text{C°})\left[\left(2500°\text{C/A}\right)I + 20.0°\text{C}\right]\right\}I$. The positive solution to this quadratic equation is *I* = 716 mA which rounds to 720 mA.

(c) We want *R* when *V* is 120 V. *R* = *V*/*I* = (120 V)/(0.716 A) = 167 Ω.

(d) We want the energy dissipated during 1 minute. $U = Pt = I^2Rt$. Using the results from (b) and (c) gives $U = (0.716\text{ A})^2(167\ \Omega)(60\text{ s}) = 5140\text{ J} = 5.14\text{ kJ}$.
(e) We want the energy when $V = 60$ V. Use $P = IV$ where $V = 60$ V. Get I using the method of part (b) with $V = 60$ V, which gives $I = 0.49355$ A. $R = V/I = (60\text{ V})/(0.49355\text{ A}) = 121.6\ \Omega$. The energy is $U = Pt = I^2Rt = (0.49355\text{ A})^2(121.6\ \Omega)(60\text{ s}) = 1.78$ kJ.
EVALUATE: Our results show that temperature variation can have significant effects on circuits.

25.77. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to find the resistance of a thin slice of the rod and integrate to find the total R. $V = IR$. Also find $R(x)$, the resistance of a length x of the rod.
SET UP: $E(x) = \rho(x)J$

EXECUTE: **(a)** $dR = \dfrac{\rho dx}{A} = \dfrac{\rho_0 \exp[-x/L]\,dx}{A}$ so

$R = \dfrac{\rho_0}{A}\int_0^L \exp\left[-x/L\right]dx = \dfrac{\rho_0}{A}[-L\exp(-x/L)]_0^L = \dfrac{\rho_0 L}{A}(1-e^{-1})$ and $I = \dfrac{V_0}{R} = \dfrac{V_0 A}{\rho_0 L(1-e^{-1})}$. With an upper limit of x rather than L in the integration, $R(x) = \dfrac{\rho_0 L}{A}(1-e^{-x/L})$.

(b) $E(x) = \rho(x)J = \dfrac{I\,\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L(1-e^{-1})}$.

(c) $V = V_0 - IR(x)$. $V = V_0 - \left(\dfrac{V_0 A}{\rho_0 L[1-e^{-1}]}\right)\left(\dfrac{\rho_0 L}{A}\right)(1-e^{-x/L}) = V_0\dfrac{(e^{-x/L}-e^{-1})}{(1-e^{-1})}$.

(d) Graphs of resistivity, electric field, and potential from $x = 0$ to L are given in Figure 25.77. Each quantity is given in terms of the indicated unit.
EVALUATE: The current is the same at all points in the rod. Where the resistivity is larger the electric field must be larger, in order to produce the same current density.

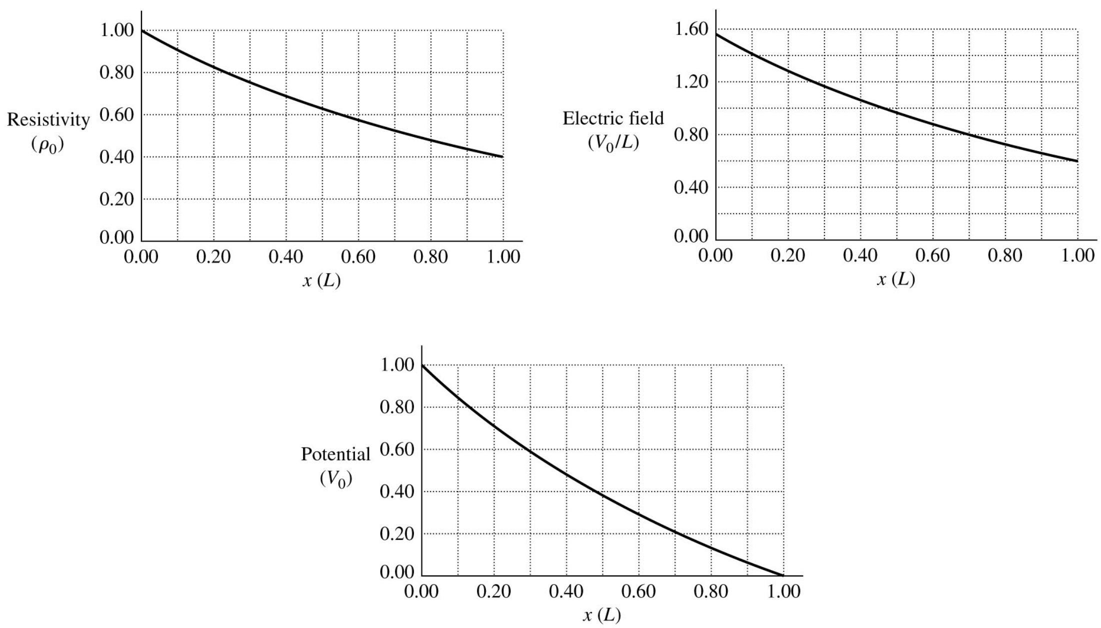


Figure 25.77

25.78. **Identify** and **Set Up:** The power output P of the source is the power delivered to the resistor R, so P is the power output of the internal emf $\mathcal{E}$ minus the power consumed by the internal resistance r. Therefore $P = \mathcal{E}I - I^2 r$. For the entire circuit, $\mathcal{E} = (R + r)I$.

Execute: **(a)** Combining $P = I^2/R$ and $\mathcal{E} = (R + r)I$ gives $P = \left(\dfrac{\mathcal{E}}{R+r}\right)^2 R = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$. From this result, we can see that as $R \to 0, \quad P \to 0$.

(b) Using the same equation as in (a), we see that as $R \to \infty, \quad P \to \dfrac{\mathcal{E}^2}{R} \to 0$.

(c) In (a) we showed that $P = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$. For maximum power, $dP/dR = 0$.

$$\frac{dP}{dR} = \mathcal{E}^2\left[-\frac{2R}{(R+r)^3} + \frac{1}{(R+r)^2}\right] = 0 \qquad \rightarrow \qquad \frac{2R}{R+r} = 1 \qquad \rightarrow \qquad R = r.$$

The maximum power is therefore

$$P_{\max} = \left.\frac{R\mathcal{E}^2}{(R+r)^2}\right|_{R=r} = \frac{r\mathcal{E}^2}{(2r)^2} = \frac{\mathcal{E}^2}{4r}.$$

(d) Use $P = \dfrac{\mathcal{E}^2 R}{(R+r)^2}$ to calculate P.

<u>For $R = 2.00\ \Omega$</u>: $P_2 = (64.0\text{ V})^2(2.00\ \Omega)/(6.00\ \Omega)^2 = 228$ W.
<u>For $R = 4.00\ \Omega$</u>: $P_4 = (64.0\text{ V})^2(4.00\ \Omega)/(8.00\ \Omega)^2 = 256$ W.
<u>For $R = 6.00\ \Omega$</u>: $P_6 = (64.0\text{ V})^2(6.00\ \Omega)/(10.0\ \Omega)^2 = 246$ W.

Evaluate: The maximum power in (d) occurred when $R = r = 4.00\ \Omega$, so it is consistent with the result from (c). The equation we found, $P_{\max} = \dfrac{\mathcal{E}^2}{4r}$, gives $P_{\max} = (64.0\text{ V})^2/[4(4.00\ \Omega)] = 256$ W, which agrees with our calculation in (d). When R is smaller than r, I is large and the $I^2 r$ losses in the battery are large. When R is larger than r, I is small and the power output $\mathcal{E}I$ of the battery emf is small.

25.79. **Identify** and **Set Up:** $R = \dfrac{\rho L}{A}$.

Execute: From the equation $R = \dfrac{\rho L}{A}$, if we double the length of a resistor and change nothing else, the resistance will double. But from the data table given in the problem, we see that doubling the length of the thread causes its resistance to do much more than double. For example, at 5 mm the resistance is $9\times 10^9\ \Omega$ and at 11 mm (approximately double) the resistance is $63\times 10^9\ \Omega$, which is much more than twice the resistance at 5 mm. Therefore as the thread stretches, its coating gets thinner, which decreases its cross-sectional area. This decreased area contributes significantly to the increase in resistance. Therefore choice (c) is correct.

Evaluate: The cross-sectional area of the coating depends on the square of the radius of the thread, so a decrease in the radius has a very large effect on the resistance.

25.80. **Identify** and **Set Up:** Use data from the table for 5 mm and 13 mm to compare the resistance. $R = \dfrac{\rho L}{A}$.

Execute: $\dfrac{R_{13}}{R_5} = \dfrac{102}{9} = \dfrac{\dfrac{\rho(13\text{ mm})}{A_5}}{\dfrac{\rho(5\text{ mm})}{A_{13}}} = \dfrac{13A_5}{5A_{13}}$. Solving for A_{13} gives

$$A_{13} = A_5\left(\frac{13}{5}\right)\left(\frac{9}{102}\right) = 0.23 \approx \frac{1}{4},$$ which is choice (b).

EVALUATE: It is reasonable that $A_{13} < A_5$ because the thread and its coating stretch out and get thinner.

25.81. **IDENTIFY** and **SET UP:** Apply Ohm's law, $V = RI$. The minimum resistance will give the maximum current. Get data from the table in the problem.

EXECUTE: $I_{max} = V/R_{min} = (9\text{ V})/(9\times10^9\ \Omega) = 1\times10^{-9}\text{ A} = 1\text{ nA}$, which is choice (d).

EVALUATE: This is a very small current, but the thread of a spider web is very thin.

25.82. **IDENTIFY** and **SET UP:** An electrically neutral conductor contains equal amounts of positive and negative charge, and these charges can move if a charged object comes near to them.

EXECUTE: If a positively charged object comes near to the web, it attracts negative charges in the web. The attraction between these negative charges in the web and the positive charges in the charged object pull the web toward the object. If a negatively charged object comes near the web, it repels negative charges in the web, leaving the web positively charged near the object. The attraction between the negatively charged object and the positive side of the web pulls the web toward the object. This is best explained by choice (d).

EVALUATE: This is similar to the principle of charging by induction. The amounts of charge are small, but the web is moved because it is extremely light.