Circuits
-
Superposition & Thévenin/Norton equivalence

NYU 上海 SHANGHAI 纽约大学

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Superposition

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Linear system:

having one independent source at one time.

Linear circuits

We will analyze linear circuits.

One of the most interesting consequence of linearity is **superposition**

Superposition

The response of a circuit having more than one independent source can be obtained by **adding the responses** caused by the **independent sources acting alone**

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

$$f(a\,x_1) = a\,f(x_1)$$

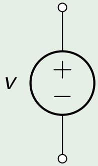
$$i = \frac{V}{R} = \frac{V_1 + V_2}{R} = \frac{V_1}{R} + \frac{V_2}{R}$$

Superposition

Turning off sources

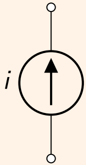
Since the response of the circuit can be obtained by superposition, **all the independent sources but one have to be turned off** at a time

Voltage source turned off (zeroed out)



When turned off $\Longrightarrow$

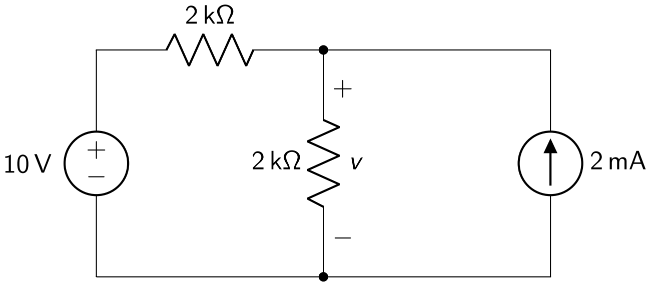
Current source turned off (zeroed out)



When turned off $\Longrightarrow$

Superposition

Determine the voltage v



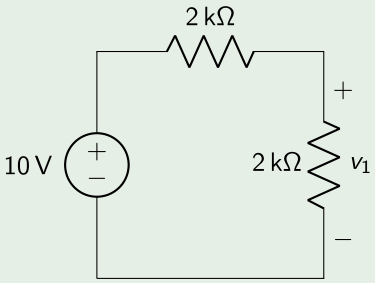
Nodal analysis (for example):

- $2\,\text{mA} + \dfrac{10\,\text{V} - v}{2\,\text{k}\Omega} = \dfrac{v}{2\,\text{k}\Omega}$
- $v = 7\,\text{V}$

Superposition

When there're too many independent sources, superposition

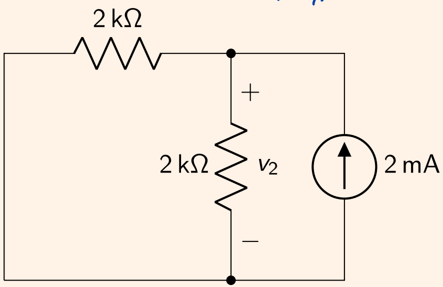
Analysis 1



- $v_1 = 5\,\text{V}$

Analysis 2

is not helpful.



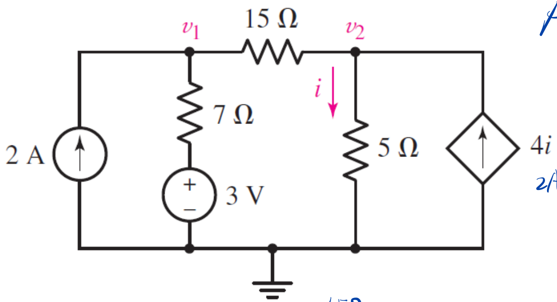
- $v_2 = 2\,\text{V}$

Superposition

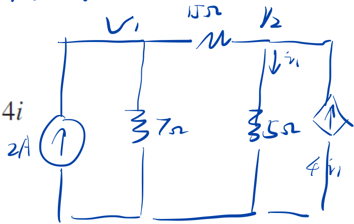
- $v = v_1 + v_2 = 7\,\text{V}$

Examples

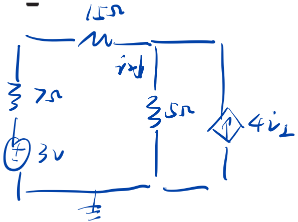
Determine the voltage across each current source.



Analysis 1



Analysis 2



Then use Nodal Analysis

Superposition

Summary

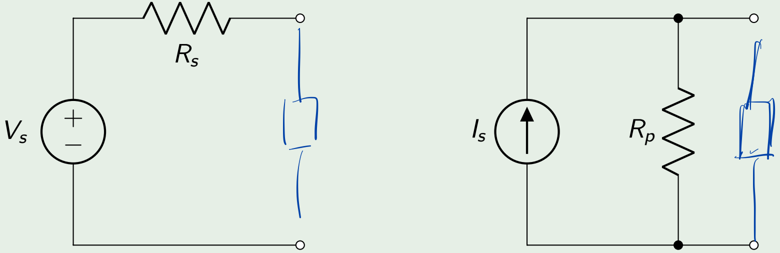
- Superposition relies on linearity
- Analyze as many circuits as there are **independent sources**
- Dependent sources are never zeroed out
- More circuits to analyze, sometimes leading to more equations...
- But the different circuits are also simpler to analyze, with simpler equations

Source transformation

Concept

A voltage source with a resistor in series is **equivalent** to a current source with a resistor in parallel

Equivalence



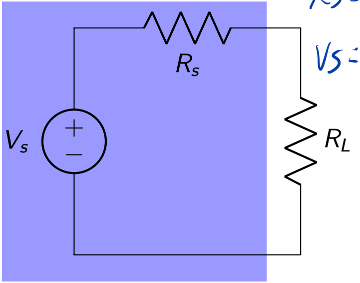
Validity

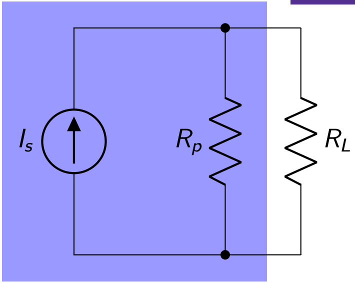
The equivalence holds if both circuits have the same **same voltage-current relationship** at their terminals

Source transformation

$R_s = R_p$

$V_s = R_p I_s$





Double check the polarity. check the current direction in the load

Terminal

- $V = \dfrac{R_L}{R_s + R_L} V_s$
- $I = \dfrac{V_s}{R_s + R_L}$

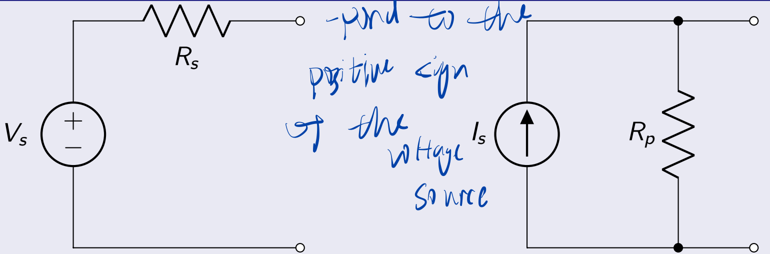
Terminal

- $V = \dfrac{R_L}{R_p + R_L} R_p I_s$
- $I = \dfrac{R_p I_s}{R_p + R_L}$

Source transformation

the head of the current source should correspond to the

Equivalence



positive sign of the voltage source

Equivalent when:

- $R_s = R_p = R$
- $V_s = RI_s$

what's the role of the polarity of the voltage source & the direction of the current source

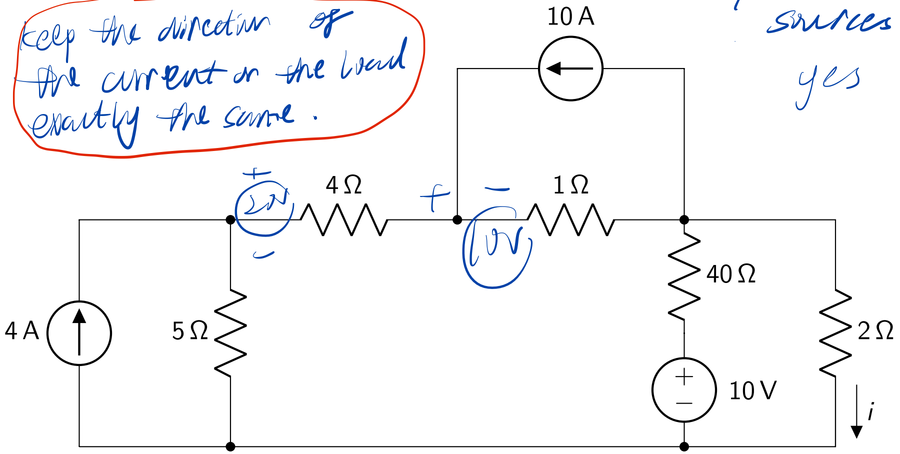
Source transformation

Can we apply source transformation to dependent sources?

yes

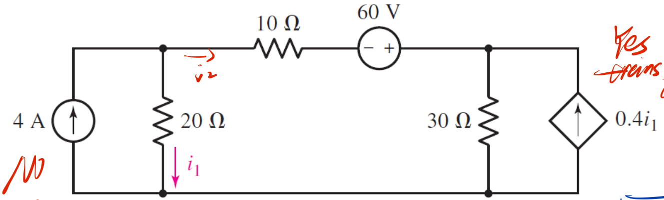
Apply multiple source transformation to determine i

Keep the direction of the current on the load exactly the same.



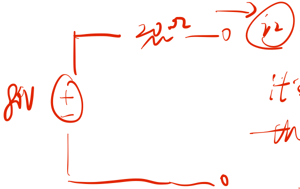
Examples

Determine the source transformation should be performed or not.



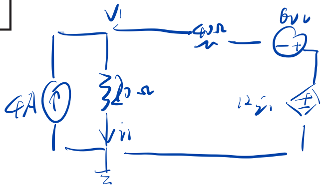
Yes ~~trans~~ allowed

No ~~trans~~



this is not i_1.

it's the current on the top branch



Thévenin equivalent circuit

Theorem

A **two-terminal linear circuit**, constituted of sources and resistors, is equivalent to a **voltage source in series with a resistor**

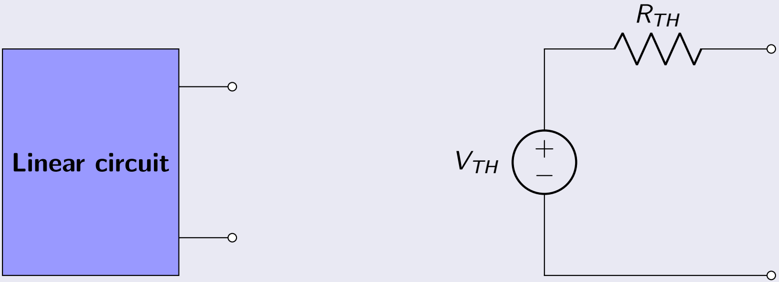
Application

Usually, it allows us to simplify the analysis of a circuit when only a few values need to be determined

for example, previous example...

Thévenin equivalent circuit

Equivalence



How to determine the Thv́enin equivalence?

Several methods:

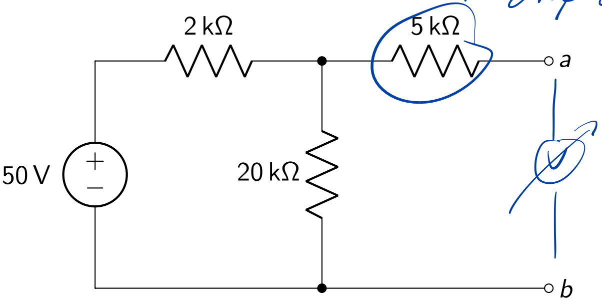
1. Evaluate the **open-circuit voltage**
2. Evaluate the **short-circuit current**
3. Evaluate the **resistor value** by turning off every independent source

Thévenin equivalent circuit

① V_{ab}. R_{eq}

② I_{ab}. R_{eq} Source Transformation

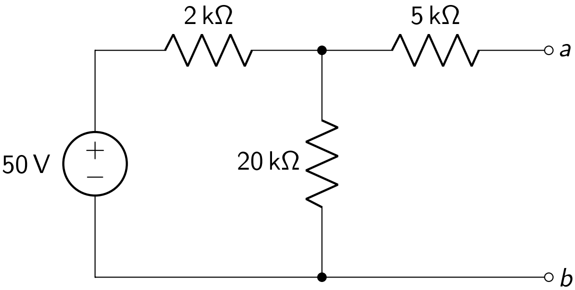
Find the Thévenin equivalence (terminal ab)



no voltage drop across the 5kΩ resistor.

Thévenin equivalent circuit

1. Evaluate the **open-circuit voltage**

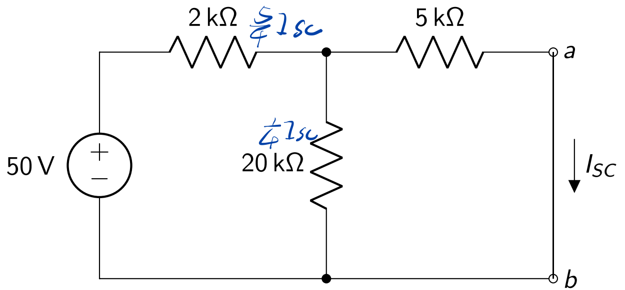


(no current in 5 kΩ resistor)

$$V_{OC} = \frac{20}{22} 50\,\text{V} = 45.45\,\text{V}$$

Thévenin equivalent circuit

2. Evaluate the **short-circuit current**



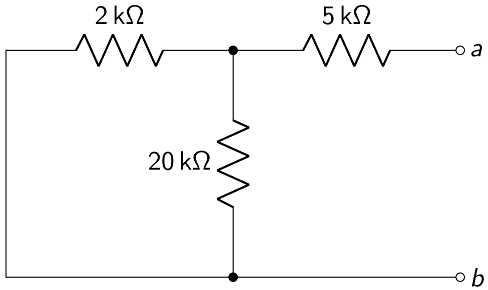
(current $I_{SC}/4$ in the 20 kΩ resistor)

$$50\,\text{V} = \left(\frac{5}{4} 2\,\text{k}\Omega + 5\,\text{k}\Omega\right) I_{SC} \quad \Longrightarrow \quad I_{SC} = 6.67\,\text{mA}$$

Thévenin equivalent circuit

all of them at the same time

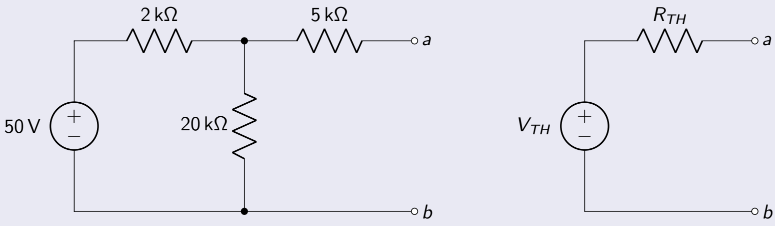
3. Evaluate the **resistor value** by turning off every independent source



$$R_{eq} = 5\,\text{k}\Omega + \frac{2\,\text{k}\Omega \cdot 20\,\text{k}\Omega}{22\,\text{k}\Omega} = 6.82\,\text{k}\Omega$$

Thévenin equivalent circuit

Equivalence



Values

Actually, only 2 experiments are necessary:

- $V_{TH} = V_{OC} = 45.45\,\text{V}$
- $R_{TH} = R_{eq} = 6.82\,\text{k}\Omega$
- $V_{TH} = R_{TH} \cdot I_{SC}$ Source Transformation

Thévenin equivalent circuit

Presence of dependent sources

It seems that determining the **open-circuit voltage** and the **equivalent resistance** (when independent sources are turned off) is the most straightforward technique to find Thévenin equivalence

But, what happens when we have dependent sources?

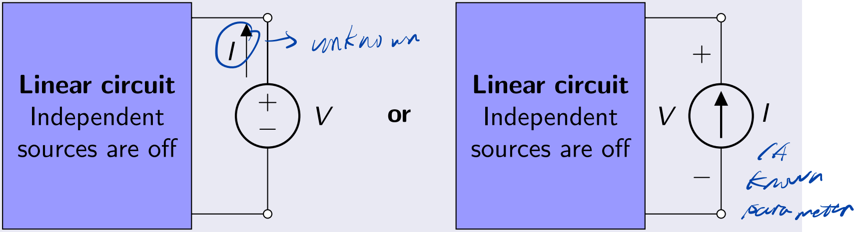
Answer: connect a source to the terminals

- connect a voltage source (or a current source) to the terminals
- the source value can be **any value**
- $R_{eq} = \dfrac{V}{I}$

Thévenin equivalent circuit

independent sources are zeroed out

Determining R_{eq} with dependent sources



R_{eq} with dependent sources

Beware!

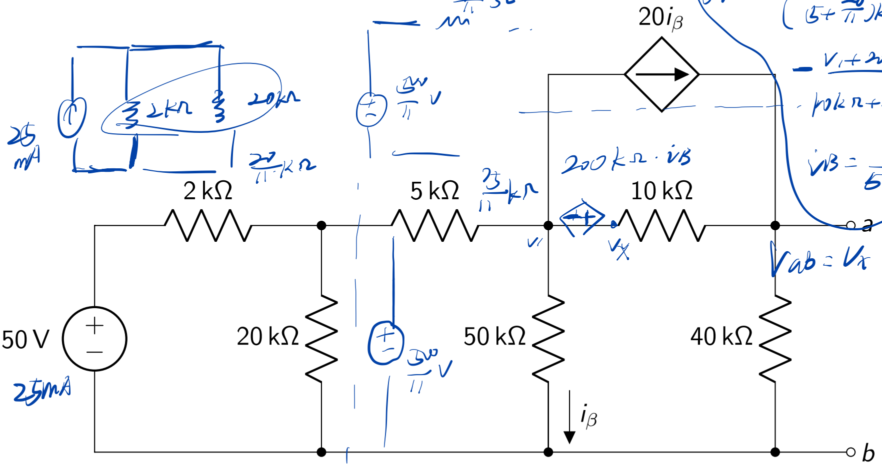
With dependent sources, it is possible to find **negative values** for R_{eq}!!!

Due to dependent sources, equivalent resistance may be a negative value

Thévenin equivalent circuit

Find the Thévenin equivalence (terminal *ab*)

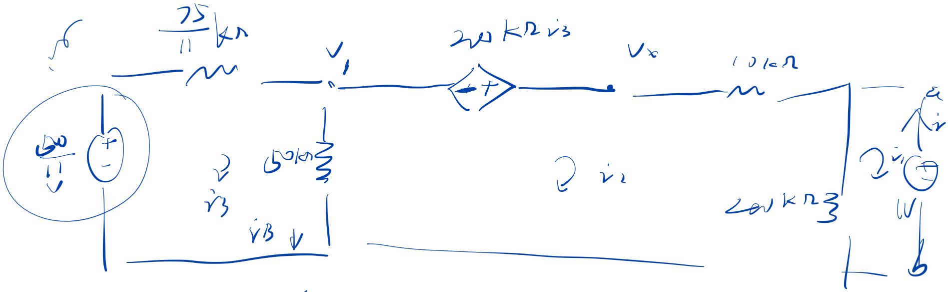
Source Transformation to simplify first.



$$\frac{V_1}{50k\Omega} = \frac{\frac{500V}{11} - V_1}{(5+\frac{20}{11})k\Omega} - \frac{V_1 + 200k\Omega\, i_B}{10k\Omega + 40k\Omega}$$

$$i_B = \frac{V_1}{50k\Omega}$$

$$V_{ab} = V_x \times \frac{40}{50}$$



$$R_{ab} = \frac{1V}{i}$$

$1V \; \cancel{+} \; 20\,k\Omega \cdot (i_1 - i_2)$

Norton equivalent circuit

Theorem

A **two-terminal linear circuit**, constituted of sources and resistors, is equivalent to a **current source in parallel with a resistor**

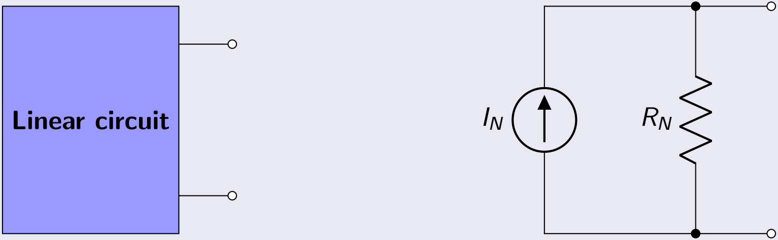
Application

Usually, it allows us to simplify the analysis of a circuit when only a few values need to be determined

for example, previous example...

Norton equivalent circuit

Equivalence



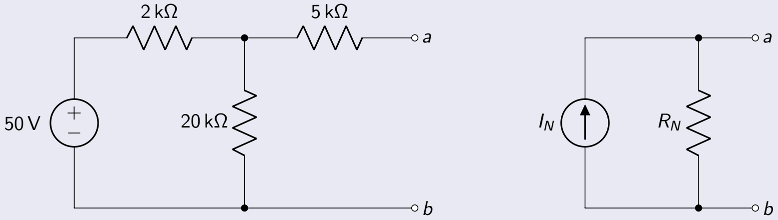
How to determine the Norton equivalence?

Several methods:

1. Evaluate the **open-circuit voltage**
2. Evaluate the **short-circuit current**
3. Evaluate the **resistor value** by turning off every independent source

Norton equivalent circuit

Equivalence



Values

Actually, only 2 experiments are necessary:

- $I_N = I_{SC} = 6.67\,\text{mA}$
- $R_N = R_{eq} = 6.82\,\text{k}\Omega$
- $I_N = \dfrac{V_{OC}}{R_N}$

Norton equivalent circuit

Find the Norton equivalence (terminal *ab*)