

THERMAL PROPERTIES OF MATTER

VP18.4.1. IDENTIFY: An ideal gas inside a tire undergoes a temperature change at constant volume, so the ideal gas law applies.

SET UP: Use $pV = nRT$, with $T_1 = 30.0^\circ\text{C} = 303\text{ K}$ and $T_2 = 10.0^\circ\text{C} = 283\text{ K}$.

EXECUTE: (a) We want the new pressure p_2 . At fixed volume, p/T is constant, so $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ which gives

$$p_2 = p_1 \frac{T_2}{T_1} = (5.00 \times 10^5 \text{ Pa}) \left(\frac{283 \text{ K}}{303 \text{ K}} \right) = 4.67 \times 10^5 \text{ Pa}.$$

(b) We want the number n of moles in the tire. Solve $pV = nRT$ for n , giving $n = pV/RT$. Using $p = 5.00 \times 10^5 \text{ Pa}$, $T = 303 \text{ K}$, and $V = 1.40 \times 10^{-3} \text{ m}^3$ gives $n = 0.278 \text{ mol}$.

EVALUATE: The temperature T must *always* be in Kelvins when using the ideal gas law.

VP18.4.2. IDENTIFY: As the balloon rises, the temperature and pressure decrease and the volume increases. The ideal gas law applies.

SET UP: Use $pV = nRT$. Since n remains constant, pV/T is constant.

At sea level: $T_1 = 15.0^\circ\text{C} = 288 \text{ K}$, $p_1 = 1.01 \times 10^5 \text{ Pa}$, $V_1 = 13.0 \text{ m}^3$.

At 32.0 km: $T_2 = -44.5^\circ\text{C} = 228.5 \text{ K}$, $p_2 = 868 \text{ Pa}$, $V_2 = ?$

EXECUTE: (a) We want the volume at 32.0 km altitude. Since pV/T is constant, we have $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$,

so $V_2 = V_1 \frac{T_2}{T_1} \frac{p_1}{p_2}$. Putting in the numbers shown above, we get $V_2 = 1.20 \times 10^3 \text{ m}^3$.

(b) We want the ratio of the radii R_2/R_1 of the balloons. Using the volume of a sphere gives us

$$\frac{V_2}{V_1} = \frac{\frac{4}{3}\pi R_2^3}{\frac{4}{3}\pi R_1^3} = \left(\frac{R_2}{R_1} \right)^3, \text{ so } \frac{R_2}{R_1} = \left(\frac{V_2}{V_1} \right)^{1/3} = \left(\frac{1.20 \times 10^3 \text{ m}^3}{13.0 \text{ m}^3} \right)^{1/3} = 4.52, \text{ so the radius is 4.52 times greater at}$$

32.0 km than it is at sea level.

EVALUATE: The volume at 32.0 km is $1200/13 = 92$ times as great as at sea level.

VP18.4.3. IDENTIFY: The pressure and temperature of Pluto's atmosphere are much lower than they are in Earth's atmosphere. The ideal gas law applies.

SET UP: Use $pV = nRT$, where $T = 42 \text{ K}$ and $p = 1.0 \text{ Pa}$ on Pluto. The molar mass of N_2 is 0.0028 kg/mol .

EXECUTE: (a) We want the molar density of Pluto's atmosphere in mol/m^3 . Using $pV = nRT$ gives $n/V = p/RT$. Using $p = 1.0 \text{ Pa}$ and $T = 42 \text{ K}$, we get $n/V = 2.9 \times 10^{-3} \text{ mol/m}^3$.

(b) We want the mass density of Pluto's atmosphere in kg/m^3 . Convert the molar density to mass density using the fact that the molar density of N_2 is 0.028 kg/mol . This gives

$$(2.9 \times 10^{-3} \text{ mol/m}) (0.028 \text{ kg/mol}) = 8.0 \times 10^{-5} \text{ kg/m}^3.$$

EVALUATE: Compare our results on Pluto to those on Earth. $\frac{n/V_P}{n/V_E} = \frac{2.9 \times 10^{-3} \text{ mol/m}^3}{42 \text{ mol/m}^3} = 6.8 \times 10^{-5}$.

$$\frac{m/V_P}{m/V_E} = \frac{8.0 \times 10^{-5} \text{ kg/m}^3}{1.2 \text{ kg/m}^3} = 6.7 \times 10^{-5}. \text{ The ratios are nearly the same, and both are very small due to}$$

Pluto's extremely low pressure.

VP18.4.4. IDENTIFY: We are investigating the effects of changes in an ideal gas using the ideal gas law.

SET UP: $pV = nRT$, and for constant n we have pV/T is constant. $\rho = m/V$ so $V = m/\rho$. We are given that $\rho_2 = \rho_1(p_2/p_1)^{3/5}$.

EXECUTE: (a) We want T_2 . $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$, so $T_2 = T_1 \frac{V_2}{V_1} \frac{p_2}{p_1} = T_1 \frac{p_2}{p_1} \frac{m/\rho_1}{m/\rho_2} = T_1 \frac{p_2}{p_1} \frac{\rho_1}{\rho_2}$. Using

$$\rho_2 = \rho_1(p_2/p_1)^{3/5} \text{ gives } T_2 = T_1 \frac{p_2}{p_1} \frac{\rho_1}{\rho_1 \left(\frac{p_2}{p_1}\right)^{3/5}} = T_1 \left(\frac{p_2}{p_1}\right) \left(\frac{p_1}{p_2}\right)^{3/5} = T_1 \left(\frac{p_2}{p_1}\right)^{2/5}.$$

(b) We want ρ_2/ρ_1 and T_2/T_1 when $p_2 = 0.500p_1$. Using $\rho_2 = \rho_1(p_2/p_1)^{3/5}$ we get

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{3/5} = \left(\frac{0.500p_1}{p_1}\right)^{3/5} = 0.660.$$

Using our result from part (a) gives $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{2/5} = \left(\frac{0.500p_1}{p_1}\right)^{2/5} = 0.758$.

(c) We want ρ_2/ρ_1 and T_2/T_1 when $p_2 = 2.00p_1$. Using $\rho_2 = \rho_1(p_2/p_1)^{3/5}$ we get

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{3/5} = \left(\frac{2.00p_1}{p_1}\right)^{3/5} = 1.52.$$

Using our result from part (a) gives $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{2/5} = \left(\frac{2.00p_1}{p_1}\right)^{2/5} = 1.32$.

EVALUATE: In part (b) our result says that decreasing p decreases ρ , and in part (c) it says that increasing p increases ρ . These are reasonable results.

VP18.7.1. IDENTIFY: We are comparing molecular speeds of different molecules.

SET UP and EXECUTE: Use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$.

$$\text{N}_2 \text{ molecule at } 20.0^\circ\text{C: } v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3R(293 \text{ K})}{28.0 \text{ g/mol}}}.$$

$$\text{He atom with the same } v_{\text{rms:}} v_{\text{rms}} = \sqrt{\frac{3RT}{4.00 \text{ g/mol}}}$$

$$\text{Equate the speeds and solve for } T: \sqrt{\frac{3RT}{4.00 \text{ g/mol}}} = \sqrt{\frac{3R(293 \text{ K})}{28.0 \text{ g/mol}}}, T = 41.86 \text{ K} = -231^\circ\text{C}.$$

EVALUATE: It was not necessary to use R or convert g to kg since they divided out. It is best to save numerical calculations until the end to minimize tedious work.

VP18.7.2. IDENTIFY: We are looking at atomic kinetic energy and speed.

SET UP: $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$, $K_{\text{tr}} = \frac{3}{2}kT$. We want the average kinetic energy of an atom and the rms speed.

EXECUTE: (a) $K_{\text{av}} = \frac{3}{2}kT = \frac{3}{2}(1.381 \times 10^{-23} \text{ J/K})(3000 \text{ K}) = 6.2 \times 10^{-20} \text{ J}$.

(b) Use $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$ with $T = 3000 \text{ K}$ and $m = 1.67 \times 10^{-27} \text{ kg}$, giving $v_{\text{rms}} = 8600 \text{ m/s}$.

EVALUATE: The speed in part (b) is about 19,000 mph!

VP18.7.3. IDENTIFY: We want to find out how many molecules are in a typical room and how much kinetic energy they have.

SET UP: Use $pV = nRT$, $K_{\text{tr}} = \frac{3}{2}kT$, $K_{\text{tot}} = NK_{\text{tr}}$, and $N = nN_A$. The target variables are N and K_{tr} .

EXECUTE: (a) Solve $pV = nRT$ to get $n = pV/RT$. Using $V = (5.0 \text{ m})(5.0 \text{ m})(2.4 \text{ m}) = 612 \text{ m}^3$, $p = 1.01 \times 10^5 \text{ Pa}$, and $T = 293 \text{ K}$ gives $n = 2488 \text{ mol}$. The number of molecules in 2488 mol is $N = nN_A = (2488 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 1.5 \times 10^{27} \text{ molecules}$.

(b) $K_{\text{tot}} = NK_{\text{tr}} = N \frac{3}{2}kT$. Using $T = 293 \text{ K}$ and $N = 1.5 \times 10^{27} \text{ molecules}$ gives $K_{\text{tot}} = 9.1 \times 10^6 \text{ J}$.

(c) We want the speed of the car so its kinetic energy would be $9.1 \times 10^6 \text{ J}$. $K_{\text{car}} = K_{\text{tot}}$ gives

$$\frac{1}{2}mv^2 = K_{\text{tot}}, \text{ so } v = \sqrt{\frac{2K_{\text{tot}}}{m}} = \sqrt{\frac{2(9.1 \times 10^6 \text{ J})}{1500 \text{ kg}}} = 110 \text{ m/s}.$$

EVALUATE: The speed in part (c) is about 250 mph or 400 km/h.

VP18.7.4. IDENTIFY: We want to compare the average, average of squares, and rms for the first 10 integers.

SET UP and EXECUTE: (a) The average is $n_{\text{av}} = \frac{1+2+3+\dots+10}{10} = \frac{55}{10} = 5.5$.

(b) The average of the squares is $(n^2)_{\text{av}} = \frac{1^2+2^2+3^2+\dots+10^2}{10} = \frac{385}{10} = 38.5$.

(c) The rms value is $n_{\text{rms}} = \sqrt{(n^2)_{\text{av}}} = \sqrt{38.5} = 6.20$.

EVALUATE: Note that the rms value is *not* the same as the average value.

VP18.8.1. IDENTIFY: We are comparing the mean free path for molecules on Mars with those on Earth.

SET UP: We use $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$, with $T_{\text{M}} = -63^\circ\text{C} = 210 \text{ K}$ and $T_{\text{E}} = 15^\circ\text{C} = 288 \text{ K}$, $p_{\text{M}} = 6.0 \times 10^2 \text{ Pa}$.

The molecular radius is $2.0 \times 10^{-10} \text{ m}$.

EXECUTE: (a) We want the mean free path on Mars. Using $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$ with values given above

we get $\lambda_{\text{M}} = 6.8 \times 10^{-6} \text{ m}$.

(b) We want to compare the mean free path on Mars to that on Earth by finding $\lambda_{\text{M}} / \lambda_{\text{E}}$. Taking the

ratio gives $\frac{\lambda_{\text{M}}}{\lambda_{\text{E}}} = \frac{T_{\text{M}} / p_{\text{M}}}{T_{\text{E}} / p_{\text{E}}} = \left(\frac{T_{\text{M}}}{T_{\text{E}}}\right)\left(\frac{p_{\text{E}}}{p_{\text{M}}}\right) = \left(\frac{210 \text{ K}}{288 \text{ K}}\right)\left(\frac{1.01 \times 10^5 \text{ Pa}}{600 \text{ Pa}}\right) = 120$. The mean free path on

Mars is 120 times greater than it is on Earth.

EVALUATE: The low pressure on Mars means that molecules are far apart compared to those on Earth, so they can travel much farther on Mars before running into each other.

VP18.8.2. IDENTIFY: We are investigating the mean free path of molecules.

SET UP: Use $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$ and $pV = nRT$.

EXECUTE: (a) We want to find the pressure so that the mean free path is 1.00 m. Solve $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$

for p , giving $p = \frac{kT}{4\pi\sqrt{2}r^2\lambda}$. Using $T = 293$ K, $r = 2.0 \times 10^{-10}$ m, and $\lambda = 1.00$ m, we get $p =$

$$5.7 \times 10^{-3} \text{ Pa.}$$

(b) Our target variable is the number of moles n inside the box. Solving $pV = nRT$ for n gives $n = pV/RT$. Using the pressure from part (a) and the same values for the other variables gives

$$n = 2.3 \times 10^{-6} \text{ mol.}$$

EVALUATE: Even at this low pressure, the number of air molecules in the box is $N = nN_A = (2.3 \times 10^{-6} \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 1.4 \times 10^{18}$ molecules.

VP18.8.3. IDENTIFY: We want to find the mean free path and mean free time for helium in a cylindrical tank.

SET UP: We use $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$, $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$, and $pV = nRT$. $v_{\text{rms}}t_{\text{mean}} = \lambda$. We know that $T =$

27.0°C = 300.0 K, $V = 5.00 \times 10^{-2} \text{ m}^3$, $n = 400$ mol, and $r = 3.1 \times 10^{-11}$ m, so $\lambda = 1.2 \times 10^{-8}$ m.

EXECUTE: (a) We want the mean free path. First use $pV = nRT$ to find p , so $p = nRT/V$. Using this

result gives $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p} = \frac{kT}{4\pi\sqrt{2}r^2\left(\frac{nRT}{V}\right)} = \frac{kV}{4\pi\sqrt{2}r^2nR}$. Using the given values for V , n , and r

gives $\lambda = 1.2 \times 10^{-8}$ m.

(b) We want the mean free time. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $t_{\text{mean}} = \lambda / v_{\text{rms}}$. Combining these equations gives

$$t_{\text{mean}} = \frac{\lambda}{v_{\text{rms}}} = \frac{\lambda}{\sqrt{\frac{3RT}{M}}} = \lambda \sqrt{\frac{M}{3RT}}. \text{ Using } M = 4.00 \times 10^{-3} \text{ kg/mol, } \lambda = 1.2 \times 10^{-8} \text{ m, and } T = 300 \text{ K, we}$$

get $t_{\text{mean}} = 8.9 \times 10^{-12}$ s.

EVALUATE: The number of collisions per second would be $1/t_{\text{mean}} = 1.1 \times 10^{11}$ collision/s.

VP18.8.4. IDENTIFY: We want to find the mean free time for gas molecules.

SET UP: We use $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$, $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$, $v_{\text{rms}}t_{\text{mean}} = \lambda$.

EXECUTE: (a) Combining the above equations gives $t_{\text{mean}} = \frac{\lambda}{v_{\text{rms}}} = \frac{kT}{4\pi\sqrt{2}r^2p} \cdot \frac{1}{\sqrt{\frac{3kT}{m}}}$, which simplifies to

$$t_{\text{mean}} = \frac{1}{4\pi r^2 p} \sqrt{\frac{mkT}{6}}.$$

(b) From part (a), we see that $t_{\text{mean}} \propto \frac{\sqrt{T}}{r^2 p}$. From this result we can see the following.

Doubling p would cut t_{mean} in half.

Doubling T would increase t_{mean} by a factor of $\sqrt{2}$.

Doubling r would decrease t_{mean} by a factor of $\frac{1}{2^2} = \frac{1}{4}$.

Therefore doubling r would have the greatest effect on the mean free time.

EVALUATE: If we doubled all of these quantities at once, the effect would reduce t_{mean} by a factor of

$$\frac{\sqrt{2}}{8} \approx 0.177.$$

18.1. IDENTIFY: (a) We are asked about a single state of the system.

SET UP: Use $m_{\text{total}} = nM$ to calculate the number of moles and then apply the ideal-gas equation.

EXECUTE: $n = \frac{m_{\text{tot}}}{M} = \frac{4.86 \times 10^{-4} \text{ kg}}{4.00 \times 10^{-3} \text{ kg/mol}} = 0.122 \text{ mol}.$

(b) $pV = nRT$ implies $p = nRT/V$. T must be in kelvins, so $T = (18 + 273) \text{ K} = 291 \text{ K}.$

$$p = \frac{(0.122 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(291 \text{ K})}{20.0 \times 10^{-3} \text{ m}^3} = 1.47 \times 10^4 \text{ Pa}.$$

$$p = (1.47 \times 10^4 \text{ Pa})(1.00 \text{ atm}/1.013 \times 10^5 \text{ Pa}) = 0.145 \text{ atm}.$$

EVALUATE: The tank contains about 1/10 mole of He at around standard temperature, so a pressure around 1/10 atmosphere is reasonable.

18.2. IDENTIFY: $pV = nRT$.

SET UP: $T_1 = 41.0^\circ\text{C} = 314 \text{ K}$. $R = 0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K}.$

EXECUTE: n and R are constant so $\frac{pV}{T} = nR$ is constant. $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}.$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) \left(\frac{V_2}{V_1} \right) = (314 \text{ K})(2)(2) = 1.256 \times 10^3 \text{ K} = 983^\circ\text{C}.$$

(b) $n = \frac{pV}{RT} = \frac{(0.180 \text{ atm})(3.20 \text{ L})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(314 \text{ K})} = 0.02235 \text{ mol}.$

$$m_{\text{tot}} = nM = (0.02235 \text{ mol})(4.00 \text{ g/mol}) = 0.0894 \text{ g}.$$

EVALUATE: T is directly proportional to p and to V , so when p and V are each doubled the Kelvin temperature increases by a factor of 4.

18.3. IDENTIFY: $pV = nRT$.

SET UP: T is constant.

EXECUTE: nRT is constant so $p_1 V_1 = p_2 V_2$.

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right) = (0.355 \text{ atm}) \left(\frac{0.110 \text{ m}^3}{0.390 \text{ m}^3} \right) = 0.100 \text{ atm}.$$

EVALUATE: For T constant, p decreases as V increases.

18.4. IDENTIFY: $pV = nRT$.

SET UP: $T_1 = 20.0^\circ\text{C} = 293 \text{ K}.$

EXECUTE: (a) n , R and V are constant. $\frac{p}{T} = \frac{nR}{V} = \text{constant}.$ $\frac{p_1}{T_1} = \frac{p_2}{T_2}.$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) = (293 \text{ K}) \left(\frac{1.00 \text{ atm}}{3.00 \text{ atm}} \right) = 97.7 \text{ K} = -175^\circ\text{C}.$$

(b) $p_2 = 1.00 \text{ atm}$, $V_2 = 3.00 \text{ L}$. $p_3 = 3.00 \text{ atm}$. n , R and T are constant so $pV = nRT = \text{constant}.$

$$p_2 V_2 = p_3 V_3.$$

$$V_3 = V_2 \left(\frac{p_2}{p_3} \right) = (3.00 \text{ L}) \left(\frac{1.00 \text{ atm}}{3.00 \text{ atm}} \right) = 1.00 \text{ L}.$$

EVALUATE: The final volume is one-third the initial volume. The initial and final pressures are the same, but the final kelvin temperature is one-third the initial kelvin temperature.

- 18.5. IDENTIFY:** This problem compares the ideal gas law to the van der Waals equation for carbon dioxide gas.

SET UP and EXECUTE: The ideal gas law is $pV = nRT$ and the van der Waals equation is

$$\left(p + \frac{an^2}{V^2}\right)(V - nb) = nRT. \text{ We have } n = 1 \text{ mol, } V = 4.48 \times 10^{-4} \text{ m}^3, \text{ and } T = 273.0 \text{ K.}$$

(a) Using these numbers, $p = nRT/V = 5.7 \times 10^6 \text{ Pa}$.

(b) Using the van der Waals equation and solving for p gives $p = \frac{nRT}{V - nb} - \frac{an^2}{V^2}$. Using the same values

as for part (a), and in addition for CO_2 $a = 0.364 \text{ J} \cdot \text{m}^3/\text{mol}^2$ and $b = 4.27 \times 10^{-5} \text{ m}^3/\text{mol}$, we get $p = 3.79 \times 10^6 \text{ Pa}$.

EVALUATE: The percent difference is $\frac{\Delta p}{p} = \frac{3.79 - 5.07}{5.07} = -0.252 = -25.2\%$, which is not very good agreement.

- 18.6. IDENTIFY:** $pV = nRT$ and the mass of the gas is $m_{\text{tot}} = nM$.

SET UP: The temperature is $T = 22.0^\circ\text{C} = 295.15 \text{ K}$. The average molar mass of air is

$M = 28.8 \times 10^{-3} \text{ kg/mol}$. For helium $M = 4.00 \times 10^{-3} \text{ kg/mol}$.

EXECUTE: (a) $m_{\text{tot}} = nM = \frac{pV}{RT}M = \frac{(1.00 \text{ atm})(0.900 \text{ L})(28.8 \times 10^{-3} \text{ kg/mol})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(295.15 \text{ K})} = 1.07 \times 10^{-3} \text{ kg}$.

(b) $m_{\text{tot}} = nM = \frac{pV}{RT}M = \frac{(1.00 \text{ atm})(0.900 \text{ L})(4.00 \times 10^{-3} \text{ kg/mol})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(295.15 \text{ K})} = 1.49 \times 10^{-4} \text{ kg}$.

EVALUATE: $n = \frac{N}{N_A} = \frac{pV}{RT}$ says that in each case the balloon contains the same number of molecules.

The mass is greater for air since the mass of one molecule is greater than for helium.

- 18.7. IDENTIFY:** We are asked to compare two states. Use the ideal gas law to obtain T_2 in terms of T_1 and ratios of pressures and volumes of the gas in the two states.

SET UP: $pV = nRT$ and n, R constant implies $pV/T = nR = \text{constant}$ and $p_1V_1/T_1 = p_2V_2/T_2$.

EXECUTE: $T_1 = (27 + 273)\text{K} = 300 \text{ K}$

$$p_1 = 1.01 \times 10^5 \text{ Pa}$$

$p_2 = 2.72 \times 10^6 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 2.82 \times 10^6 \text{ Pa}$ (in the ideal gas equation the pressures must be absolute, not gauge, pressures)

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) \left(\frac{V_2}{V_1} \right) = 300 \text{ K} \left(\frac{2.82 \times 10^6 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right) \left(\frac{46.2 \text{ cm}^3}{499 \text{ cm}^3} \right) = 776 \text{ K}$$

$$T_2 = (776 - 273)^\circ\text{C} = 503^\circ\text{C}.$$

EVALUATE: The units cancel in the V_2/V_1 volume ratio, so it was not necessary to convert the volumes in cm^3 to m^3 . It was essential, however, to use T in kelvins.

- 18.8. IDENTIFY:** $pV = nRT$ and $m = nM$.

SET UP: We must use absolute pressure in $pV = nRT$. $p_1 = 4.01 \times 10^5 \text{ Pa}$, $p_2 = 2.81 \times 10^5 \text{ Pa}$.

$$T_1 = 310 \text{ K}, T_2 = 295 \text{ K}.$$

EXECUTE: (a) $n_1 = \frac{p_1V_1}{RT_1} = \frac{(4.01 \times 10^5 \text{ Pa})(0.075 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(310 \text{ K})} = 11.7 \text{ mol}$.

$$m = nM = (11.7 \text{ mol})(32.0 \text{ g/mol}) = 374 \text{ g}.$$

$$(b) n_2 = \frac{p_2 V_2}{RT_2} = \frac{(2.81 \times 10^5 \text{ Pa})(0.075 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(295 \text{ K})} = 8.59 \text{ mol. } m = 275 \text{ g.}$$

The mass that has leaked out is $374 \text{ g} - 275 \text{ g} = 99 \text{ g}$.

EVALUATE: In the ideal gas law we must use absolute pressure, expressed in Pa, and T must be in kelvins.

18.9. IDENTIFY: $pV = nRT$.

SET UP: $T_1 = 300 \text{ K}$, $T_2 = 430 \text{ K}$.

EXECUTE: (a) n , R are constant so $\frac{pV}{T} = nR = \text{constant}$. $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$.

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right) \left(\frac{T_2}{T_1} \right) = (7.50 \times 10^3 \text{ Pa}) \left(\frac{0.750 \text{ m}^3}{0.410 \text{ m}^3} \right) \left(\frac{430 \text{ K}}{300 \text{ K}} \right) = 1.97 \times 10^4 \text{ Pa.}$$

EVALUATE: Since the temperature increased while the volume decreased, the pressure must have increased. In $pV = nRT$, T must be in kelvins, even if we use a ratio of temperatures.

18.10. IDENTIFY: Use the ideal-gas equation to calculate the number of moles, n . The mass m_{total} of the gas is $m_{\text{total}} = nM$.

SET UP: The volume of the cylinder is $V = \pi r^2 l$, where $r = 0.450 \text{ m}$ and $l = 1.50 \text{ m}$.

$T = 22.0^\circ\text{C} = 293.15 \text{ K}$. $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$. $M = 32.0 \times 10^{-3} \text{ kg/mol}$. $R = 8.314 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) $pV = nRT$ gives

$$n = \frac{pV}{RT} = \frac{(21.0 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})\pi(0.450 \text{ m})^2(1.50 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K})} = 827 \text{ mol.}$$

(b) $m_{\text{total}} = (827 \text{ mol})(32.0 \times 10^{-3} \text{ kg/mol}) = 26.5 \text{ kg}$

EVALUATE: In the ideal-gas law, T must be in kelvins. Since we used R in units of $\text{J/mol} \cdot \text{K}$ we had to express p in units of Pa and V in units of m^3 .

18.11. IDENTIFY: We are asked to compare two states. Use the ideal-gas law to obtain V_1 in terms of V_2 and the ratio of the temperatures in the two states.

SET UP: $pV = nRT$ and n , R , p are constant so $V/T = nR/p = \text{constant}$ and $V_1/T_1 = V_2/T_2$.

EXECUTE: $T_1 = (19 + 273) \text{ K} = 292 \text{ K}$ (T must be in kelvins)

$$V_2 = V_1(T_2/T_1) = (0.600 \text{ L})(77.3 \text{ K}/292 \text{ K}) = 0.159 \text{ L.}$$

EVALUATE: p is constant so the ideal-gas equation says that a decrease in T means a decrease in V .

18.12. IDENTIFY: The ideal gas law applies. We should be able to identify the gas from its molecular weight.

SET UP: $pV = nRT$, $\rho = m/V = (m/\text{mole})n/V$. $R = 8.314 \text{ J/mol} \cdot \text{K}$.

EXECUTE: $\rho = m/V = (m/\text{mole})n/V$, so $m/\text{mole} = \frac{\rho V}{n} = \rho \frac{RT}{p}$, using $pV = nRT$.

$$\rho = 1.33 \times 10^{-3} \text{ kg/m}^3, p = 101.3 \text{ Pa}, T = 293.15 \text{ K.}$$

Therefore $m/\text{mole} = (1.33 \times 10^{-3} \text{ kg/m}^3)(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})/(101.3 \text{ Pa}) = 0.0320 \text{ kg/mol} = 32.0 \text{ g/mol}$. The gas is probably diatomic oxygen, O_2 .

EVALUATE: There are few gases having small molecular weights, so we are quite certain that O_2 is the correct gas.

18.13. IDENTIFY: We know the volume of the gas at STP on the earth and want to find the volume it would occupy on Venus where the pressure and temperature are much greater.

SET UP: STP is $T = 273 \text{ K}$ and $p = 1 \text{ atm}$. Set up a ratio using $pV = nRT$ with nR constant.

$$T_V = 1003 + 273 = 1276 \text{ K.}$$

EXECUTE: $pV = nRT$ gives $\frac{pV}{T} = nR = \text{constant}$, so $\frac{p_E V_E}{T_E} = \frac{p_V V_V}{T_V}$.

$$V_V = V_E \left(\frac{p_E}{p_V} \right) \left(\frac{T_V}{T_E} \right) = V \left(\frac{1 \text{ atm}}{92 \text{ atm}} \right) \left(\frac{1276 \text{ K}}{273 \text{ K}} \right) = 0.0508V.$$

EVALUATE: Even though the temperature on Venus is higher than it is on earth, the pressure there is much greater than on earth, so the volume of the gas on Venus is only about 5% what it is on earth.

18.14. IDENTIFY: $pV = nRT$.

SET UP: $T_1 = 277 \text{ K}$. $T_2 = 296 \text{ K}$. Assume the number of moles of gas in the bubble remains constant.

EXECUTE: (a) n , R are constant so $\frac{pV}{T} = nR = \text{constant}$. $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ and

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2} \right) \left(\frac{T_2}{T_1} \right) = \left(\frac{3.50 \text{ atm}}{1.00 \text{ atm}} \right) \left(\frac{296 \text{ K}}{277 \text{ K}} \right) = 3.74.$$

(b) This increase in volume of air in the lungs would be dangerous.

EVALUATE: The large decrease in pressure results in a large increase in volume.

18.15. IDENTIFY: We are looking at the gauge pressure in a gas, so we apply the ideal gas law.

SET UP: $pV = nRT$, $p_{\text{tot}} = p_{\text{atm}} + p_{\text{gauge}}$. The pressure we need to use is the *total* pressure since that is the pressure the gas exerts in the container, so $p_{\text{tot}} = 0.876 \text{ atm} + 1.000 \text{ atm} = 1.876 \text{ atm}$.

EXECUTE: Using the numbers given in the problem, $n = pV/RT$. Putting in the numbers gives

$$n = \frac{(1.876 \text{ atm})(5.43 \text{ L})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(295.2 \text{ K})} = 0.421 \text{ mol}.$$

EVALUATE: We use the gauge pressure because that is the actual pressure in the tank. A pressure gauge reads the gauge pressure, but that is not the tank pressure since it is only the pressure above one atmosphere.

18.16. IDENTIFY: $F = pA$ and $pV = nRT$.

SET UP: For a cube, $V/A = L$.

EXECUTE: (a) The force of any side of the cube is $F = pA = (nRT/V)A = (nRT)/L$, since the ratio of area to volume is $A/V = 1/L$. For $T = 20.0^\circ\text{C} = 293.15 \text{ K}$, so

$$F = \frac{nRT}{L} = \frac{(3 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})}{0.300 \text{ m}} = 2.44 \times 10^4 \text{ N}.$$

(b) For $T = 100.00^\circ\text{C} = 373.15 \text{ K}$, so

$$F = \frac{nRT}{L} = \frac{(3 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(373.15 \text{ K})}{0.300 \text{ m}} = 3.10 \times 10^4 \text{ N}.$$

EVALUATE: When the temperature increases while the volume is kept constant, the pressure increases and therefore the force increases. The force increases by the ratio T_2/T_1 of Kelvin temperatures.

18.17. IDENTIFY: We know the volume, pressure, and temperature of the gas and want to find its mass and density.

SET UP: $V = 3.00 \times 10^{-3} \text{ m}^3$. $T = 295 \text{ K}$. $p = 2.03 \times 10^{-8} \text{ Pa}$. The ideal gas law, $pV = nRT$, applies.

EXECUTE: (a) $pV = nRT$ gives

$$n = \frac{pV}{RT} = \frac{(2.03 \times 10^{-8} \text{ Pa})(3.00 \times 10^{-3} \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(295 \text{ K})} = 2.48 \times 10^{-14} \text{ mol}.$$

The mass of this amount of gas is

$$m = nM = (2.48 \times 10^{-14} \text{ mol})(28.0 \times 10^{-3} \text{ kg/mol}) = 6.95 \times 10^{-16} \text{ kg}.$$

$$(b) \rho = \frac{m}{V} = \frac{6.95 \times 10^{-16} \text{ kg}}{3.00 \times 10^{-3} \text{ m}^3} = 2.32 \times 10^{-13} \text{ kg/m}^3.$$

EVALUATE: The density at this level of vacuum is 13 orders of magnitude less than the density of air at STP, which is 1.20 kg/m^3 .

18.18. IDENTIFY: Use $\rho = pM / RT$ and solve for p .

SET UP: $\rho = pM / RT$ and $p = RT\rho / M$

$$T = (-56.5 + 273.15) \text{ K} = 216.6 \text{ K}$$

For air $M = 28.8 \times 10^{-3} \text{ kg/mol}$ (Example 18.3).

$$\text{EXECUTE: } p = \frac{(8.3145 \text{ J/mol} \cdot \text{K})(216.6 \text{ K})(0.364 \text{ kg/m}^3)}{28.8 \times 10^{-3} \text{ kg/mol}} = 2.28 \times 10^4 \text{ Pa.}$$

EVALUATE: The pressure is about one-fifth the pressure at sea-level.

18.19. IDENTIFY: $n = \frac{m}{M} = \frac{N}{N_A}$.

SET UP: $N_A = 6.022 \times 10^{23}$ molecules/mol. For water, $M = 18 \times 10^{-3} \text{ kg/mol}$.

$$\text{EXECUTE: } n = \frac{m}{M} = \frac{1.00 \text{ kg}}{18 \times 10^{-3} \text{ kg/mol}} = 55.6 \text{ mol.}$$

$$N = nN_A = (55.6 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 3.35 \times 10^{25} \text{ molecules.}$$

EVALUATE: Note that we converted M to kg/mol .

18.20. IDENTIFY: The molar mass is $M = N_A m$, where m is the mass of one molecule.

SET UP: $N_A = 6.02 \times 10^{23}$ molecules/mol.

$$\text{EXECUTE: } M = N_A m = (6.02 \times 10^{23} \text{ molecules/mol})(1.41 \times 10^{-21} \text{ kg/molecule}) = 849 \text{ kg/mol.}$$

EVALUATE: For a carbon atom, $M = 12 \times 10^{-3} \text{ kg/mol}$. If this molecule is mostly carbon, so the average mass of its atoms is the mass of carbon, the molecule would contain

$$\frac{849 \text{ kg/mol}}{12 \times 10^{-3} \text{ kg/mol}} = 71,000 \text{ atoms.}$$

18.21. IDENTIFY: Use $pV = nRT$ to calculate the number of moles and then the number of molecules would be $N = nN_A$.

SET UP: $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$. $1.00 \text{ cm}^3 = 1.00 \times 10^{-6} \text{ m}^3$. $N_A = 6.022 \times 10^{23}$ molecules/mol.

$$\text{EXECUTE: (a) } n = \frac{pV}{RT} = \frac{(9.00 \times 10^{-14} \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(1.00 \times 10^{-6} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300.0 \text{ K})} = 3.655 \times 10^{-18} \text{ mol.}$$

$$N = nN_A = (3.655 \times 10^{-18} \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 2.20 \times 10^6 \text{ molecules.}$$

$$\text{(b) } N = \frac{pVN_A}{RT} \text{ so } \frac{N}{p} = \frac{VN_A}{RT} = \text{constant and } \frac{N_1}{p_1} = \frac{N_2}{p_2}.$$

$$N_2 = N_1 \left(\frac{p_2}{p_1} \right) = (2.20 \times 10^6 \text{ molecules}) \left(\frac{1.00 \text{ atm}}{9.00 \times 10^{-14} \text{ atm}} \right) = 2.44 \times 10^{19} \text{ molecules.}$$

EVALUATE: The number of molecules in a given volume is directly proportional to the pressure. Even at the very low pressure in part (a) the number of molecules in 1.00 cm^3 is very large.

18.22. IDENTIFY: We are asked about a single state of the system.

SET UP: Use the ideal-gas law. Write n in terms of the number of molecules N .

(a) EXECUTE: $pV = nRT$, $n = N / N_A$ so $pV = (N / N_A)RT$

$$p = \left(\frac{N}{V} \right) \left(\frac{R}{N_A} \right) T$$

$$p = \left(\frac{80 \text{ molecules}}{1 \times 10^{-6} \text{ m}^3} \right) \left(\frac{8.3145 \text{ J/mol} \cdot \text{K}}{6.022 \times 10^{23} \text{ molecules/mol}} \right) (7500 \text{ K}) = 8.28 \times 10^{-12} \text{ Pa}$$

$p = 8.2 \times 10^{-17} \text{ atm}$. This is much lower than the laboratory pressure of $9 \times 10^{-14} \text{ atm}$ in Exercise 18.21.

(b) EVALUATE: The Lagoon Nebula is a very rarefied low pressure gas. The gas would exert *very* little force on an object passing through it.

- 18.23. IDENTIFY:** Use $pV = nRT$ and $n = \frac{N}{N_A}$ with $N = 1$ to calculate the volume V occupied by 1

molecule. The length l of the side of the cube with volume V is given by $V = l^3$.

SET UP: $T = 27^\circ\text{C} = 300 \text{ K}$. $p = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$. $R = 8.314 \text{ J/mol} \cdot \text{K}$.

$N_A = 6.022 \times 10^{23} \text{ molecules/mol}$.

The diameter of a typical molecule is about 10^{-10} m . $0.3 \text{ nm} = 0.3 \times 10^{-9} \text{ m}$.

EXECUTE: (a) $pV = nRT$ and $n = \frac{N}{N_A}$ gives

$$V = \frac{NRT}{N_A p} = \frac{(1.00)(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{(6.022 \times 10^{23} \text{ molecules/mol})(1.013 \times 10^5 \text{ Pa})} = 4.09 \times 10^{-26} \text{ m}^3. \quad l = V^{1/3} = 3.45 \times 10^{-9} \text{ m}.$$

(b) The distance in part (a) is about 10 times the diameter of a typical molecule.

(c) The spacing is about 10 times the spacing of atoms in solids.

EVALUATE: There is space between molecules in a gas whereas in a solid the atoms are closely packed together.

- 18.24. IDENTIFY:** We are dealing with an ideal monatomic gas. We are removing gas from a tank and doubling the pressure and rms speed at the same time.

SET UP and EXECUTE: We want to know how many moles we need to remove from the tank. The volume stays the same (rigid walls), but n , p , and T can change. We want $p_2 = 2p_1$ and $v_{\text{rms},2} = 2v_{\text{rms},1}$.

We know that $v_{\text{rms}} = \sqrt{\frac{2kT}{m}}$ and $pV = nRT$. We want to know about the number of moles, so for the

original and final states, the ideal gas law gives $n = \frac{p_1 V}{RT_1}$ and $n_2 = \frac{p_2 V}{RT_2}$. Squaring $v_{\text{rms}} = \sqrt{\frac{2kT}{m}}$ gives

$v_{\text{rms}}^2 = \frac{2kT}{m}$. This tells us that if we square v_{rms} , the temperature increases by a factor of 4, which means

that $T_2 = 4T_1$. Putting this and $p_2 = 2p_1$ into the ideal gas law gives $n_2 = \frac{p_2 V}{RT_2} = \frac{2p_1 V}{R(4T_1)} = \frac{1}{2} \frac{p_1 V}{RT_1} = \frac{1}{2} n$.

Therefore we take out $\frac{1}{2}n$ moles.

EVALUATE: Just taking out half the moles would cut the pressure in half, and only doubling the rms speed of the molecules would increase the temperature a factor of 4. Doing both together increases the pressure by a factor of 2.

- 18.25. IDENTIFY:** The ideal gas law applies. The translational kinetic energy of a gas depends on its absolute temperature.

SET UP: $pV = nRT$, $K_{\text{tr}} = 3/2 nRT$, $K = \frac{1}{2} mv^2$.

EXECUTE: (a) From $pV = nRT$, we have $n = pV/RT$. Putting this into $K_{\text{tr}} = 3/2 nRT$, we have

$$K_{\text{tr}} = 3/2 (pV/RT)(RT) = 3/2 pV = (3/2)(1.013 \times 10^5 \text{ Pa})(8.00 \text{ m})(12.00 \text{ m})(4.00 \text{ m}) = 5.83 \times 10^7 \text{ J}.$$

(b) $K = \frac{1}{2} mv^2$: $\frac{1}{2}(2000 \text{ kg})v^2 = 5.83 \times 10^7 \text{ J}$, gives $v = 242 \text{ m/s}$.

EVALUATE: No automobile can travel this fast! Obviously the molecules in the room have a great deal of kinetic energy because there are so many of them.

18.26. IDENTIFY: $K_{\text{av}} = \frac{3}{2}kT$. $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$.

SET UP: $M_{\text{Ne}} = 20.180 \text{ g/mol}$, $M_{\text{Kr}} = 83.80 \text{ g/mol}$, and $M_{\text{Rn}} = 222 \text{ g/mol}$.

EXECUTE: (a) $K_{\text{av}} = \frac{3}{2}kT$ depends only on the temperature so it is the same for each species of atom in the mixture.

(b) $\frac{v_{\text{rms,Ne}}}{v_{\text{rms,Kr}}} = \sqrt{\frac{M_{\text{Kr}}}{M_{\text{Ne}}}} = \sqrt{\frac{83.80 \text{ g/mol}}{20.18 \text{ g/mol}}} = 2.04$. $\frac{v_{\text{rms,Ne}}}{v_{\text{rms,Rn}}} = \sqrt{\frac{M_{\text{Rn}}}{M_{\text{Ne}}}} = \sqrt{\frac{222 \text{ g/mol}}{20.18 \text{ g/mol}}} = 3.32$.

$\frac{v_{\text{rms,Kr}}}{v_{\text{rms,Rn}}} = \sqrt{\frac{M_{\text{Rn}}}{M_{\text{Kr}}}} = \sqrt{\frac{222 \text{ g/mol}}{83.80 \text{ g/mol}}} = 1.63$.

EVALUATE: The average kinetic energies are the same. The gas atoms with smaller mass have larger v_{rms} .

18.27. IDENTIFY: We make several measurements of the pressure and temperature of a gas. Using a graph of the pressure versus the temperature, we want to determine the number N of gas molecules in the container.

SET UP: We use the ideal gas law to find a relationship between p and T_C so we can interpret the graph. The gas volume is $V = 80.0 \text{ cm}^3 = 0.0800 \text{ L}$. We use $pV = nRT$ and $N = nN_A$. The target variable is N .

EXECUTE: $pV = nRT = nR(T_C + 273)$, so $p = \frac{nR(T_C + 273)}{V} = \frac{nR}{V}T_C + \frac{273nR}{V}$. Therefore a graph of p versus T_C should be a straight line having slope equal to nR/V . Putting this in terms of N gives slope = $\frac{nR}{V} = \frac{(N/N_A)R}{V}$, so $N = \frac{N_A V (\text{slope})}{R}$. Putting in the appropriate numbers gives

$$N = \frac{(6.022 \times 10^{23} \text{ molec/mol})(0.0800 \text{ L})(1.10 \text{ atm/C}^\circ)}{0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K}} = 6.46 \times 10^{23} \text{ molecules.}$$

EVALUATE: The number of moles is $\frac{6.46 \times 10^{23}}{6.022 \times 10^{23}} = 1.07 \text{ mol}$.

18.28. IDENTIFY: We can relate the temperature to the rms speed and the temperature to the pressure using the ideal gas law. The target variable is the pressure.

SET UP: $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ and $pV = nRT$, where $n = m/M$.

EXECUTE: Use v_{rms} to calculate T : $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ so

$$T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{(28.014 \times 10^{-3} \text{ kg/mol})(182 \text{ m/s})^2}{3(8.314 \text{ J/mol} \cdot \text{K})} = 37.20 \text{ K. The ideal gas law gives } p = \frac{nRT}{V}.$$

$$n = \frac{m}{M} = \frac{0.226 \times 10^{-3} \text{ kg}}{28.014 \times 10^{-3} \text{ kg/mol}} = 8.067 \times 10^{-3} \text{ mol. Solving for } p \text{ gives}$$

$$p = \frac{(8.067 \times 10^{-3} \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(37.20 \text{ K})}{1.64 \times 10^{-3} \text{ m}^3} = 1.52 \times 10^3 \text{ Pa.}$$

EVALUATE: This pressure is around 1.5% of atmospheric pressure, which is not unreasonable since we have only around 1% of a mole of gas.

18.29. IDENTIFY: $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$

SET UP: The mass of a deuteron is $m = m_p + m_n = 1.673 \times 10^{-27} \text{ kg} + 1.675 \times 10^{-27} \text{ kg} = 3.35 \times 10^{-27} \text{ kg}$.
 $c = 3.00 \times 10^8 \text{ m/s}$. $k = 1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$.

EXECUTE: (a) $v_{\text{rms}} = \sqrt{\frac{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(300 \times 10^6 \text{ K})}{3.35 \times 10^{-27} \text{ kg}}} = 1.93 \times 10^6 \text{ m/s}$. $\frac{v_{\text{rms}}}{c} = 6.43 \times 10^{-3}$.

(b) $T = \left(\frac{m}{3k}\right)(v_{\text{rms}})^2 = \left(\frac{3.35 \times 10^{-27} \text{ kg}}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})}\right)(3.0 \times 10^7 \text{ m/s})^2 = 7.3 \times 10^{10} \text{ K}$.

EVALUATE: Even at very high temperatures and for this light nucleus, v_{rms} is a small fraction of the speed of light.

18.30. IDENTIFY: $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$, where T is in kelvins. $pV = nRT$ gives $\frac{n}{V} = \frac{p}{RT}$.

SET UP: $R = 8.314 \text{ J/mol} \cdot \text{K}$. $M = 44.0 \times 10^{-3} \text{ kg/mol}$.

EXECUTE: (a) For $T = 0.0^\circ\text{C} = 273.15 \text{ K}$, $v_{\text{rms}} = \sqrt{\frac{3(8.314 \text{ J/mol} \cdot \text{K})(273.15 \text{ K})}{44.0 \times 10^{-3} \text{ kg/mol}}} = 393 \text{ m/s}$. For

$T = -100.0^\circ\text{C} = 173 \text{ K}$, $v_{\text{rms}} = 313 \text{ m/s}$. The range of speeds is 393 m/s to 313 m/s.

(b) For $T = 273.15 \text{ K}$, $\frac{n}{V} = \frac{650 \text{ Pa}}{(8.314 \text{ J/mol} \cdot \text{K})(273.15 \text{ K})} = 0.286 \text{ mol/m}^3$. For $T = 173.15 \text{ K}$,

$\frac{n}{V} = 0.452 \text{ mol/m}^3$. The range of densities is 0.286 mol/m^3 to 0.452 mol/m^3 .

EVALUATE: When the temperature decreases the rms speed decreases and the density increases.

18.31. IDENTIFY and SET UP: Apply the analysis of Section 18.3.

EXECUTE: (a) $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(300 \text{ K}) = 6.21 \times 10^{-21} \text{ J}$.

(b) We need the mass m of one molecule:

$m = \frac{M}{N_A} = \frac{32.0 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} = 5.314 \times 10^{-26} \text{ kg/molecule}$.

Then $\frac{1}{2}m(v^2)_{\text{av}} = 6.21 \times 10^{-21} \text{ J}$ (from part (a)) gives

$(v^2)_{\text{av}} = \frac{2(6.21 \times 10^{-21} \text{ J})}{m} = \frac{2(6.21 \times 10^{-21} \text{ J})}{5.314 \times 10^{-26} \text{ kg}} = 2.34 \times 10^5 \text{ m}^2/\text{s}^2$.

(c) $v_{\text{rms}} = \sqrt{(v^2)_{\text{rms}}} = \sqrt{2.34 \times 10^5 \text{ m}^2/\text{s}^2} = 484 \text{ m/s}$.

(d) $p = mv_{\text{rms}} = (5.314 \times 10^{-26} \text{ kg})(484 \text{ m/s}) = 2.57 \times 10^{-23} \text{ kg} \cdot \text{m/s}$.

(e) Time between collisions with one wall is $t = \frac{0.20 \text{ m}}{v_{\text{rms}}} = \frac{0.20 \text{ m}}{484 \text{ m/s}} = 4.13 \times 10^{-4} \text{ s}$.

In a collision \vec{v} changes direction, so $\Delta p = 2mv_{\text{rms}} = 2(2.57 \times 10^{-23} \text{ kg} \cdot \text{m/s}) = 5.14 \times 10^{-23} \text{ kg} \cdot \text{m/s}$

$F = \frac{dp}{dt}$ so $F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{5.14 \times 10^{-23} \text{ kg} \cdot \text{m/s}}{4.13 \times 10^{-4} \text{ s}} = 1.24 \times 10^{-19} \text{ N}$.

(f) pressure = $F / A = 1.24 \times 10^{-19} \text{ N} / (0.10 \text{ m})^2 = 1.24 \times 10^{-17} \text{ Pa}$ (due to one molecule).

(g) pressure = $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$.

Number of molecules needed is $1.013 \times 10^5 \text{ Pa} / (1.24 \times 10^{-17} \text{ Pa/molecule}) = 8.17 \times 10^{21}$ molecules.

(h) $pV = NkT$ (Eq. 18.18), so $N = \frac{pV}{kT} = \frac{(1.013 \times 10^5 \text{ Pa})(0.10 \text{ m})^3}{(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(300 \text{ K})} = 2.45 \times 10^{22}$ molecules.

(i) From the factor of $\frac{1}{3}$ in $(v_x^2)_{\text{av}} = \frac{1}{3}(v^2)_{\text{av}}$.

EVALUATE: This exercise shows that the pressure exerted by a gas arises from collisions of the molecules of the gas with the walls.

18.32. IDENTIFY: Apply $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p}$ and calculate λ .

SET UP: $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$, so $p = 3.55 \times 10^{-8} \text{ Pa}$. $r = 2.0 \times 10^{-10} \text{ m}$ and $k = 1.38 \times 10^{-23} \text{ J/K}$.

EXECUTE: $\lambda = \frac{kT}{4\pi\sqrt{2}r^2p} = \frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{4\pi\sqrt{2}(2.0 \times 10^{-10} \text{ m})^2(3.55 \times 10^{-8} \text{ Pa})} = 1.6 \times 10^5 \text{ m}$

EVALUATE: At this very low pressure the mean free path is very large. If $v = 484 \text{ m/s}$, as in Example 18.8, then $t_{\text{mean}} = \frac{\lambda}{v} = 330 \text{ s}$. Collisions are infrequent.

18.33. IDENTIFY and SET UP: Use equal v_{rms} to relate T and M for the two gases. $v_{\text{rms}} = \sqrt{3RT/M}$, so $v_{\text{rms}}^2/3R = T/M$, where T must be in kelvins. Same v_{rms} so same T/M for the two gases and $T_{\text{N}_2}/M_{\text{N}_2} = T_{\text{H}_2}/M_{\text{H}_2}$.

EXECUTE: $T_{\text{N}_2} = T_{\text{H}_2} \left(\frac{M_{\text{N}_2}}{M_{\text{H}_2}} \right) = [(20 + 273) \text{ K}] \left(\frac{28.014 \text{ g/mol}}{2.016 \text{ g/mol}} \right) = 4.071 \times 10^3 \text{ K}$

$T_{\text{N}_2} = (4071 - 273)^\circ\text{C} = 3800^\circ\text{C}$.

EVALUATE: A N_2 molecule has more mass so N_2 gas must be at a higher temperature to have the same v_{rms} .

18.34. IDENTIFY: $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$.

SET UP: $k = 1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$.

EXECUTE: (a) $v_{\text{rms}} = \sqrt{\frac{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(300 \text{ K})}{3.00 \times 10^{-16} \text{ kg}}} = 6.44 \times 10^{-3} \text{ m/s} = 6.44 \text{ mm/s}$

EVALUATE: (b) No. The rms speed depends on the average kinetic energy of the particles. At this T , H_2 molecules would have larger v_{rms} than the typical air molecules but would have the same average kinetic energy and the average kinetic energy of the smoke particles would be the same.

18.35. IDENTIFY: We add heat energy to a gas in a sealed rigid container and want to find the new rms speed of the molecules due to this added energy.

SET UP: We know that at constant volume $Q = nC_V\Delta T$ where $C_V = \frac{3}{2}R$ for a monatomic gas, and

that $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$. First use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ to get the original temperature. Then use $Q = nC_V\Delta T$ to get

ΔT , and finally use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ again to get the new v_{rms} .

EXECUTE: Get the original temperature T_1 . Square $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ to get $T_1 = \frac{Mv_{\text{rms}}^2}{3R}$. Using the

numbers gives $T_1 = \frac{(0.00400 \text{ kg/mol})(900 \text{ m/s})^2}{3(8.314 \text{ J/mol} \cdot \text{K})} = 129.9 \text{ K}$. Now get the new temperature T_2 . Using

$Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$ gives $2400 \text{ J} = (3.00 \text{ mol})(3/2)(8.314 \text{ J/mol} \cdot \text{K}) \Delta T$, so we have $\Delta T =$

64.15 K . Thus $T_2 = T_1 + \Delta T = 129.9 \text{ K} + 64.15 \text{ K} = 194.05 \text{ K}$. Finally use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ to calculate the new rms speed using $T_2 = 194.05 \text{ K}$ and $M = 0.00400 \text{ kg/mol}$. The result is $v_{\text{rms}} = 1100 \text{ m/s}$.

EVALUATE: Since $v_{\text{rms}} \propto \sqrt{T}$, when our fractional increase in T is $\frac{194.05}{129.9} = 1.494$, the fractional increase in v_{rms} is $\sqrt{1.494} = 1.222$. Thus we would expect that $v_2 = 1.222v_1 = (1.222)(900 \text{ m/s}) = 1100 \text{ m/s}$, which is exactly what we found.

- 18.36. IDENTIFY:** We add 6000 J of heat energy to a monatomic gas in a rigid container and want to find the new pressure.

SET UP: We know that $Q = nC_V\Delta T$ where $C_V = \frac{3}{2}R$ for a monatomic gas, and $pV = nRT$. Our target variable is the final pressure, and we know that the volume remains constant.

EXECUTE: Use $Q = nC_V\Delta T = 6000 \text{ J}$ to find ΔT and from this get the new temperature.

$$\Delta T = \frac{Q}{nC_V} = \frac{Q}{n\left(\frac{3}{2}R\right)} = \frac{6000 \text{ J}}{(4.00 \text{ mol})(3/2)(8.314 \text{ J/mol} \cdot \text{K})} = 120.3 \text{ K}, \text{ so } T_2 = 300 \text{ K} + 120 \text{ K} = 420 \text{ K}.$$

Since V and n are constant, $pV = nRT$ gives $T/p = \text{constant}$, so

$$p_2 = p_1 \frac{T_2}{T_1} = (6.00 \times 10^4 \text{ Pa}) \frac{420 \text{ K}}{300 \text{ K}} = 8.41 \times 10^4 \text{ Pa}.$$

EVALUATE: The temperature ratio is $\frac{420}{300} = 1.40$ and the pressure ratio is $\frac{8.41}{6.00} = 1.40$, so they agree.

- 18.37. IDENTIFY:** Use $dQ = nC_V dT$ applied to a finite temperature change.

SET UP: $C_V = 5R/2$ for a diatomic ideal gas and $C_V = 3R/2$ for a monatomic ideal gas.

EXECUTE: (a) $Q = nC_V\Delta T = n\left(\frac{5}{2}R\right)\Delta T$. $Q = (1.80 \text{ mol})\left(\frac{5}{2}\right)(8.314 \text{ J/mol} \cdot \text{K})(50.0 \text{ K}) = 1870 \text{ J}$.

(b) $Q = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$. $Q = (1.80 \text{ mol})\left(\frac{3}{2}\right)(8.314 \text{ J/mol} \cdot \text{K})(50.0 \text{ K}) = 1120 \text{ J}$.

EVALUATE: More heat is required for the diatomic gas; not all the heat that goes into the gas appears as translational kinetic energy, some goes into energy of the internal motion of the molecules (rotations).

- 18.38. IDENTIFY:** The heat Q added is related to the temperature increase ΔT by $Q = nC_V\Delta T$.

SET UP: For ideal H_2 (a diatomic gas), $C_{V,\text{H}_2} = 5/2R$, and for ideal Ne (a monatomic gas),

$$C_{V,\text{Ne}} = 3/2R.$$

EXECUTE: $C_V\Delta T = \frac{Q}{n} = \text{constant}$, so $C_{V,\text{H}_2}\Delta T_{\text{H}_2} = C_{V,\text{Ne}}\Delta T_{\text{Ne}}$.

$$\Delta T_{\text{Ne}} = \left(\frac{C_{V,\text{H}_2}}{C_{V,\text{Ne}}}\right)\Delta T_{\text{H}_2} = \left(\frac{5/2R}{3/2R}\right)(2.50 \text{ C}^\circ) = 4.17 \text{ C}^\circ = 4.17 \text{ K}.$$

EVALUATE: The same amount of heat causes a smaller temperature increase for H_2 since some of the energy input goes into the internal degrees of freedom.

- 18.39. IDENTIFY:** $C = Mc$, where C is the molar heat capacity and c is the specific heat capacity.

$$pV = nRT = \frac{m}{M}RT.$$

SET UP: $M_{\text{N}_2} = 2(14.007 \text{ g/mol}) = 28.014 \times 10^{-3} \text{ kg/mol}$. For water, $c_w = 4190 \text{ J/kg} \cdot \text{K}$. For N_2 ,

$$C_V = 20.76 \text{ J/mol} \cdot \text{K}.$$

EXECUTE: (a) $c_{N_2} = \frac{C}{M} = \frac{20.76 \text{ J/mol} \cdot \text{K}}{28.014 \times 10^{-3} \text{ kg/mol}} = 741 \text{ J/kg} \cdot \text{K}$. $\frac{c_w}{c_{N_2}} = 5.65$; c_w is over five times

larger.

(b) To warm the water, $Q = mc_w \Delta T = (1.00 \text{ kg})(4190 \text{ J/mol} \cdot \text{K})(10.0 \text{ K}) = 4.19 \times 10^4 \text{ J}$. For air,

$$m = \frac{Q}{c_{N_2} \Delta T} = \frac{4.19 \times 10^4 \text{ J}}{(741 \text{ J/kg} \cdot \text{K})(10.0 \text{ K})} = 5.65 \text{ kg}.$$

$$V = \frac{mRT}{Mp} = \frac{(5.65 \text{ kg})(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})}{(28.014 \times 10^{-3} \text{ kg/mol})(1.013 \times 10^5 \text{ Pa})} = 4.85 \text{ m}^3 = 4850 \text{ L}.$$

EVALUATE: c is smaller for N_2 , so less heat is needed for 1.0 kg of N_2 than for 1.0 kg of water.

18.40. (a) IDENTIFY and SET UP: $\frac{1}{2}R$ contribution to C_V for each degree of freedom. The molar heat capacity C is related to the specific heat capacity c by $C = Mc$.

EXECUTE: $C_V = 6\left(\frac{1}{2}R\right) = 3R = 3(8.3145 \text{ J/mol} \cdot \text{K}) = 24.9 \text{ J/mol} \cdot \text{K}$. The specific heat capacity is

$$c_V = C_V / M = (24.9 \text{ J/mol} \cdot \text{K}) / (18.0 \times 10^{-3} \text{ kg/mol}) = 1380 \text{ J/kg} \cdot \text{K}.$$

(b) For water vapor the specific heat capacity is $c = 2000 \text{ J/kg} \cdot \text{K}$. The molar heat capacity is

$$C = Mc = (18.0 \times 10^{-3} \text{ kg/mol})(2000 \text{ J/kg} \cdot \text{K}) = 36.0 \text{ J/mol} \cdot \text{K}.$$

EVALUATE: The difference is $36.0 \text{ J/mol} \cdot \text{K} - 24.9 \text{ J/mol} \cdot \text{K} = 11.1 \text{ J/mol} \cdot \text{K}$, which is about $2.7\left(\frac{1}{2}R\right)$; the vibrational degrees of freedom make a significant contribution.

18.41. IDENTIFY: Apply $v_{mp} = \sqrt{2kT/m}$, $v_{av} = \sqrt{8kT/\pi m}$, and $v_{rms} = \sqrt{3kT/m}$.

SET UP: Note that $\frac{k}{m} = \frac{R/N_A}{M/N_A} = \frac{R}{M}$. $M = 44.0 \times 10^{-3} \text{ kg/mol}$.

EXECUTE: (a) $v_{mp} = \sqrt{2(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K}) / (44.0 \times 10^{-3} \text{ kg/mol})} = 3.37 \times 10^2 \text{ m/s}$.

(b) $v_{av} = \sqrt{8(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K}) / (\pi(44.0 \times 10^{-3} \text{ kg/mol}))} = 3.80 \times 10^2 \text{ m/s}$.

(c) $v_{rms} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K}) / (44.0 \times 10^{-3} \text{ kg/mol})} = 4.12 \times 10^2 \text{ m/s}$.

EVALUATE: The average speed is greater than the most probable speed and the rms speed is greater than the average speed.

18.42. IDENTIFY: Table 18.2 gives the value of v/v_{rms} for which 94.7% of the molecules have a smaller

value of v/v_{rms} . $v_{rms} = \sqrt{\frac{3RT}{M}}$.

SET UP: For N_2 , $M = 28.0 \times 10^{-3} \text{ kg/mol}$. $v/v_{rms} = 1.60$.

EXECUTE: $v_{rms} = \frac{v}{1.60} = \sqrt{\frac{3RT}{M}}$, so the temperature is

$$T = \frac{Mv^2}{3(1.60)^2 R} = \frac{(28.0 \times 10^{-3} \text{ kg/mol})}{3(1.60)^2 (8.3145 \text{ J/mol} \cdot \text{K})} v^2 = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2) v^2.$$

(a) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(1500 \text{ m/s})^2 = 987 \text{ K}$.

(b) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(1000 \text{ m/s})^2 = 438 \text{ K}$.

(c) $T = (4.385 \times 10^{-4} \text{ K} \cdot \text{s}^2/\text{m}^2)(500 \text{ m/s})^2 = 110 \text{ K}$.

EVALUATE: As T decreases the distribution of molecular speeds shifts to lower values.

- 18.43. IDENTIFY:** We want to compare the speed of sound in a gas to the rms speed and average speed of its molecules.

SET UP: $v = \sqrt{\frac{\gamma RT}{M}}$ (for sound), $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$, and $v_{\text{av}} = \sqrt{\frac{8kT}{\pi m}}$. Using $k/m = R/M$ gives

$$v_{\text{av}} = \sqrt{\frac{8RT}{\pi M}}.$$

EXECUTE: (a) Compare v_{rms} to v : $\frac{v_{\text{rms}}}{v} = \frac{\sqrt{3RT/M}}{\sqrt{\gamma RT/M}} = \sqrt{3/\gamma} = \sqrt{3/1.67} = 1.34.$

(b) Compare v_{av} to v : $\frac{v_{\text{av}}}{v} = \frac{\sqrt{8RT/\pi M}}{\sqrt{\gamma RT/M}} = \sqrt{\frac{8}{\pi\gamma}} = \sqrt{\frac{8}{\pi(1.67)}} = 1.23.$

EVALUATE: The average speed is closer to the speed of sound, but both are greater than it.

- 18.44. IDENTIFY:** This problem concerns the distribution of molecular speeds.

SET UP and EXECUTE: We want to find the number of atoms within 20% of the rms speed. This means

that the limits on v are $v = v_{\text{rms}} \pm 0.20v_{\text{rms}} = \begin{cases} v_{\text{rms}} + 0.20v_{\text{rms}} = 1.20v_{\text{rms}} \\ v_{\text{rms}} - 0.20v_{\text{rms}} = 0.80v_{\text{rms}} \end{cases}$. Table 18.2 tells us that if we

have N atoms (or molecules), $0.771N$ have speeds less than $1.20v_{\text{rms}}$ and $0.411N$ have speeds less than $0.80v_{\text{rms}}$. Therefore the number with speeds between $0.80v_{\text{rms}}$ and $1.20v_{\text{rms}}$ is

$0.771N - 0.411N = 0.360N$. In 0.0345 mol of gas, the number of molecules is $(0.0345 \text{ mol})N_A =$

$(0.0345 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 2.0776 \times 10^{22}$ molecules. In this gas the number of

molecules with speeds between $0.80v_{\text{rms}}$ and $1.20v_{\text{rms}}$ is $(0.360)(2.0776 \times 10^{22}) = 7.48 \times 10^{21}$.

EVALUATE: Careful! We do not want the number of molecules that have a speed of 20% the rms speed. We want the number *within* 20% of the rms speed.

- 18.45. IDENTIFY:** Refer to the phase diagram in Figure 18.24 in the textbook.

SET UP: For water the triple-point pressure is 610 Pa and the critical-point pressure is 2.212×10^7 Pa.

EXECUTE: (a) To observe a solid to liquid (melting) phase transition the pressure must be greater than the triple-point pressure, so $p_1 = 610$ Pa. For $p < p_1$ the solid to vapor (sublimation) phase transition is observed.

(b) No liquid to vapor (boiling) phase transition is observed if the pressure is greater than the critical-point pressure. $p_2 = 2.212 \times 10^7$ Pa. For $p_1 < p < p_2$ the sequence of phase transitions is solid to liquid and then liquid to vapor.

EVALUATE: Normal atmospheric pressure is approximately 1.0×10^5 Pa, so the solid to liquid to vapor sequence of phase transitions is normally observed when the material is water.

- 18.46. IDENTIFY:** Apply the definition of relative humidity given in the problem. $pV = nRT = \frac{m_{\text{tot}}}{M}RT$.

SET UP: $M = 18.0 \times 10^{-3}$ kg/mol.

EXECUTE: (a) The pressure due to water vapor is $(0.60)(2.34 \times 10^3 \text{ Pa}) = 1.40 \times 10^3$ Pa.

(b) $m_{\text{tot}} = \frac{MpV}{RT} = \frac{(18.0 \times 10^{-3} \text{ kg/mol})(1.40 \times 10^3 \text{ Pa})(1.00 \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} = 10 \text{ g}.$

EVALUATE: The vapor pressure of water vapor at this temperature is much less than the total atmospheric pressure of 1.0×10^5 Pa.

18.47. IDENTIFY: We are investigating characteristics of the gas in a party balloon.

SET UP: Estimate: The sound intensity level is 95 dB. The sound lasts 100 ms = 0.10 s. We use

$$I = P / A = P / 4\pi r^2 \text{ and } \beta = (10 \text{ dB}) \log \frac{I}{I_0}.$$

EXECUTE: (a) $95 \text{ dB} = (10 \text{ dB}) \log \frac{I}{I_0}$, so $I = 10^{9.5} I_0 = 10^{9.5} 10^{-12} \text{ W/m}^2 = 3.2 \times 10^{-3} \text{ W/m}^2$. The

intensity is $I = P/A$, so $P_{\text{av}} = IA = (3.2 \times 10^{-3} \text{ W/m}^2) (4\pi)(2.0 \text{ m})^2 = 0.16 \text{ W}$. The energy E in this sound is $E = P_{\text{av}} t = (0.16 \text{ W})(0.10 \text{ s}) = 0.016 \text{ J}$.

(b) $T = E/A = E/(4\pi r^2) = (0.016 \text{ J})/[4\pi(0.075 \text{ m})^2] = 0.23 \text{ J/m}^2$.

(c) $p_{\text{gauge}} = T/R = (0.23 \text{ J/m}^2)/(0.075 \text{ m}) = 3.0 \text{ Pa}$.

EVALUATE: These results are only a rough approximation for the gauge pressure.

18.48. IDENTIFY: The ideal gas law will tell us the number of moles of gas in the room, which we can use to find the number of molecules.

SET UP: $pV = nRT$, $N = nN_A$, and $m = nM$.

EXECUTE: (a) $T = 27.0^\circ\text{C} + 273 = 300 \text{ K}$. $p = 1.013 \times 10^5 \text{ Pa}$.

$$n = \frac{pV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(216 \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 8773 \text{ mol}.$$

$N = nN_A = (8773 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 5.28 \times 10^{27} \text{ molecules}$.

(b) $V = (216 \text{ m}^3)(1 \text{ cm}^3/10^{-6} \text{ m}^3) = 2.16 \times 10^8 \text{ cm}^3$. The particle density is

$$\frac{5.28 \times 10^{27} \text{ molecules}}{2.16 \times 10^8 \text{ cm}^3} = 2.45 \times 10^{19} \text{ molecules/cm}^3.$$

(c) $m = nM = (8773 \text{ mol})(28.014 \times 10^{-3} \text{ kg/mol}) = 246 \text{ kg}$.

EVALUATE: A cubic centimeter of air (about the size of a sugar cube) contains around 10^{19} molecules, and the air in the room weighs about 500 lb!

18.49. IDENTIFY: We can model the atmosphere as a fluid of constant density, so the pressure depends on the depth in the fluid, as we saw in Section 12.2.

SET UP: The pressure difference between two points in a fluid is $\Delta p = \rho gh$, where h is the difference in height of two points.

EXECUTE: (a) $\Delta p = \rho gh = (1.2 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1000 \text{ m}) = 1.18 \times 10^4 \text{ Pa}$.

(b) At the bottom of the mountain, $p = 1.013 \times 10^5 \text{ Pa}$. At the top, $p = 8.95 \times 10^4 \text{ Pa}$.

$$pV = nRT = \text{constant} \text{ so } p_b V_b = p_t V_t \text{ and } V_t = V_b \left(\frac{p_b}{p_t} \right) = (0.50 \text{ L}) \left(\frac{1.013 \times 10^5 \text{ Pa}}{8.95 \times 10^4 \text{ Pa}} \right) = 0.566 \text{ L}.$$

EVALUATE: The pressure variation with altitude is affected by changes in air density and temperature and we have neglected those effects. The pressure decreases with altitude and the volume increases. You may have noticed this effect: bags of potato chips “puff up” when taken to the top of a mountain.

18.50. IDENTIFY: As the pressure on the bubble changes, its volume will change. As we saw in Section 12.2, the pressure in a fluid depends on the depth.

SET UP: The pressure at depth h in a fluid is $p = p_0 + \rho gh$, where p_0 is the pressure at the surface.

$p_0 = p_{\text{air}} = 1.013 \times 10^5 \text{ Pa}$. The density of water is $\rho = 1000 \text{ kg/m}^3$.

EXECUTE: $p_1 = p_0 + \rho gh = 1.013 \times 10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(25 \text{ m}) = 3.463 \times 10^5 \text{ Pa}$.

$p_2 = p_{\text{air}} = 1.013 \times 10^5 \text{ Pa}$. $V_1 = 1.0 \text{ mm}^3$. n , R and T are constant so $pV = nRT = \text{constant}$. $p_1 V_1 = p_2 V_2$ and $V_2 = V_1 \left(\frac{p_1}{p_2} \right) = (1.0 \text{ mm}^3) \left(\frac{3.463 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) = 3.4 \text{ mm}^3$.

EVALUATE: This is a large change and would have serious effects.

- 18.51. IDENTIFY:** The buoyant force on the balloon must be equal to the weight of the load plus the weight of the gas.

SET UP: The buoyant force is $F_B = \rho_{\text{air}} V g$. A lift of 290 kg means $\frac{F_B}{g} - m_{\text{hot}} = 290 \text{ kg}$, where m_{hot} is the mass of hot air in the balloon. $m = \rho V$.

EXECUTE: $m_{\text{hot}} = \rho_{\text{hot}} V$. $\frac{F_B}{g} - m_{\text{hot}} = 290 \text{ kg}$ gives $(\rho_{\text{air}} - \rho_{\text{hot}})V = 290 \text{ kg}$.

Solving for ρ_{hot} gives $\rho_{\text{hot}} = \rho_{\text{air}} - \frac{290 \text{ kg}}{V} = 1.23 \text{ kg/m}^3 - \frac{290 \text{ kg}}{500.0 \text{ m}^3} = 0.65 \text{ kg/m}^3$. $\rho_{\text{hot}} = \frac{pM}{RT_{\text{hot}}}$.

$\rho_{\text{air}} = \frac{pM}{RT_{\text{air}}}$. $\rho_{\text{hot}} T_{\text{hot}} = \rho_{\text{air}} T_{\text{air}}$ so

$T_{\text{hot}} = T_{\text{air}} \left(\frac{\rho_{\text{air}}}{\rho_{\text{hot}}} \right) = (288 \text{ K}) \left(\frac{1.23 \text{ kg/m}^3}{0.65 \text{ kg/m}^3} \right) = 545 \text{ K} = 272^\circ\text{C}$.

EVALUATE: This temperature is well above normal air temperatures, so the air in the balloon would need considerable heating.

- 18.52. IDENTIFY:** The ideal gas law applies. The gas pressure supports the disk and whatever is on it.

SET UP: For constant temperature, $pV = nRT$ gives $p_1 V_1 = p_2 V_2$. Call A the area of the disk, m the mass of the disk, M the mass of the lead brick, and h_2 the final height of the disk. $p = F_{\perp}/A$.

EXECUTE: $p_1 = mg/A$, $p_2 = (m + M)g/A$. For constant temperature, $p_1 V_1 = p_2 V_2$, which gives $p_2 = p_1 (V_1/V_2) = (mg/A)(Ah/Ah_2) = (m + M)g/A$. Solving for h_2 gives $h_2 = mh/(m + M)$, which gives $h_2 = (3.00 \text{ kg})(4.00 \text{ m})/(12.0 \text{ kg}) = 1.00 \text{ m}$.

EVALUATE: Since $p \rightarrow 4p$, $V \rightarrow \frac{1}{4}V$, so it is reasonable that $h \rightarrow \frac{1}{4}h$, as we found.

- 18.53. IDENTIFY:** We are asked to compare two states. Use the ideal-gas law to obtain m_2 in terms of m_1 and the ratio of pressures in the two states. Apply $pV = \frac{m_{\text{total}}}{M}RT$ to the initial state to calculate m_1 .

SET UP: $pV = nRT$ can be written $pV = (m/M)RT$

T , V , M , R are all constant, so $p/m = RT/MV = \text{constant}$.

So $p_1/m_1 = p_2/m_2$, where m is the mass of the gas in the tank.

EXECUTE: $p_1 = 1.30 \times 10^6 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 1.40 \times 10^6 \text{ Pa}$

$p_2 = 3.40 \times 10^5 \text{ Pa} + 1.01 \times 10^5 \text{ Pa} = 4.41 \times 10^5 \text{ Pa}$

$m_1 = p_1 VM/RT$; $V = hA = h\pi r^2 = (1.00 \text{ m})\pi(0.060 \text{ m})^2 = 0.01131 \text{ m}^3$

$m_1 = \frac{(1.40 \times 10^6 \text{ Pa})(0.01131 \text{ m}^3)(44.1 \times 10^{-3} \text{ kg/mol})}{(8.3145 \text{ J/mol} \cdot \text{K})(22.0 + 273.15 \text{ K})} = 0.2845 \text{ kg}$

Then $m_2 = m_1 \left(\frac{p_2}{p_1} \right) = (0.2845 \text{ kg}) \left(\frac{4.41 \times 10^5 \text{ Pa}}{1.40 \times 10^6 \text{ Pa}} \right) = 0.0896 \text{ kg}$.

m_2 is the mass that remains in the tank. The mass that has been used is

$m_1 - m_2 = 0.2845 \text{ kg} - 0.0896 \text{ kg} = 0.195 \text{ kg}$.

EVALUATE: Note that we have to use absolute pressures. The absolute pressure decreases by a factor of approximately 3 and the mass of gas in the tank decreases by a factor of approximately 3.

- 18.54. IDENTIFY:** Apply $pV = nRT$ to the air inside the diving bell. The pressure p at depth y below the surface of the water is $p = p_{\text{atm}} + \rho gy$.

SET UP: $p = 1.013 \times 10^5$ Pa. $T = 300.15$ K at the surface and $T' = 280.15$ K at the depth of 13.0 m.

EXECUTE: (a) The height h' of the air column in the diving bell at this depth will be proportional to the volume, and hence inversely proportional to the pressure and proportional to the Kelvin temperature:

$$h' = h \frac{p}{p'} \frac{T'}{T} = h \frac{p_{\text{atm}}}{p_{\text{atm}} + \rho gy} \frac{T'}{T}.$$

$$h' = (2.30 \text{ m}) \frac{(1.013 \times 10^5 \text{ Pa})}{(1.013 \times 10^5 \text{ Pa}) + (1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(73.0 \text{ m})} \left(\frac{280.15 \text{ K}}{300.15 \text{ K}} \right) = 0.26 \text{ m}.$$

The height of the water inside the diving bell is $h - h' = 2.04$ m.

(b) The necessary gauge pressure is the term ρgy from the above calculation, $p_{\text{gauge}} = 7.37 \times 10^5$ Pa.

EVALUATE: The gauge pressure required in part (b) is about 7 atm.

- 18.55. IDENTIFY:** We are investigating the movement of air above a campfire. The air is heated by the fire and expands so the number of molecules in a given volume decreases.

SET UP and EXECUTE: The mass of a volume of air is $m = nM_{\text{air}}$, where M is the molar mass and n is the number of moles. If m_{out} is the mass of gas out of the heated region and m_{in} is the mass within that region, the upward force F_{up} on a parcel of air above the fire can be roughly approximated by $F_{\text{up}} = (m_{\text{out}} - m_{\text{in}})g$. We can use $pV = nRT$ and $\sum F_y = ma_y$.

(a) We want the acceleration a of the air parcel as a function of $T_{\text{out}}/T_{\text{in}}$. Using $m = nM_{\text{air}}$ we have

$$F_{\text{up}} = (m_{\text{out}} - m_{\text{in}})g = (n_{\text{out}}M_{\text{air}} - n_{\text{in}}M_{\text{air}})g = M_{\text{air}}(n_{\text{out}} - n_{\text{in}})g. \text{ Now solve for } n \text{ in } pV = nRT \text{ to get } n = pV/RT \text{ and use this in the equation for } F_{\text{up}}, \text{ which gives } F_{\text{up}} = \frac{M_{\text{air}}g}{R} \left(\frac{p_{\text{out}}V_{\text{out}}}{T_{\text{out}}} - \frac{p_{\text{in}}V_{\text{in}}}{T_{\text{in}}} \right).$$

As stated in the problem, the pressure above the fire is the same as the ambient pressure, so $p_0 = p_1 = p_{\text{amb}}$. This gives

$$F_{\text{up}} = \frac{M_{\text{air}}gp_{\text{amb}}}{R} \left(\frac{V_{\text{out}}}{T_{\text{out}}} - \frac{V_{\text{in}}}{T_{\text{in}}} \right). \text{ Now apply } \sum F_y = ma_y \text{ where } m \text{ is the mass of air in the air parcel over}$$

the fire. This mass is $m = n_{\text{in}}M_{\text{air}}$, so $F_{\text{up}} = \frac{M_{\text{air}}gp_{\text{amb}}}{R} \left(\frac{V_{\text{out}}}{T_{\text{out}}} - \frac{V_{\text{in}}}{T_{\text{in}}} \right) = n_{\text{in}}M_{\text{air}}a$. Substituting for n_{in} gives

$$\frac{M_{\text{air}}gp_{\text{amb}}}{R} \left(\frac{V_{\text{out}}}{T_{\text{out}}} - \frac{V_{\text{in}}}{T_{\text{in}}} \right) = \left(\frac{p_{\text{in}}V_{\text{in}}}{RT_{\text{in}}} \right) M_{\text{air}}a. \text{ We are told that } p_{\text{in}} = p_{\text{amb}} \text{ and the masses are for similar}$$

volumes of air, so $V_{\text{in}} = V_{\text{out}}$, this equation simplifies to $F_{\text{up}} = g \left(\frac{1}{T_0} - \frac{1}{T_1} \right) = \left(\frac{1}{T_{\text{in}}} \right) a$. Solving for a we

$$\text{get } a = \left(\frac{T_{\text{in}}}{T_{\text{out}}} - 1 \right) g.$$

(b) Solving for T_{in} as a function of a gives $T_{\text{in}} = T_{\text{out}} \left(\frac{a}{g} + 1 \right)$.

(c) Estimate: $a \approx 1.5g$.

(d) $T_{\text{out}} = 15^\circ\text{C} = 288 \text{ K}$, so $T_{\text{in}} = (288 \text{ K}) \left(\frac{1.5g}{g} + 1 \right) = 720 \text{ K} \blacklozenge 450^\circ\text{C}$.

EVALUATE: This would be a roaring fire at 840°F !

18.56. IDENTIFY: For constant temperature, the variation of pressure with altitude is calculated in Example

$$18.4 \text{ to be } p = p_0 e^{-Mgy/RT}. \quad v_{\text{rms}} = \sqrt{\frac{3RT}{M}}.$$

SET UP: $g_{\text{earth}} = 9.80 \text{ m/s}^2$. $T = 460^\circ\text{C} = 733 \text{ K}$. $M = 44.0 \text{ g/mol} = 44.0 \times 10^{-3} \text{ kg/mol}$.

EXECUTE: (a) $\frac{Mgy}{RT} = \frac{(44.0 \times 10^{-3} \text{ kg/mol})(0.894)(9.80 \text{ m/s}^2)(1.00 \times 10^3 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(733 \text{ K})} = 0.06326$.

$p = p_0 e^{-Mgy/RT} = (92 \text{ atm})e^{-0.06326} = 86 \text{ atm}$. The pressure is 86 earth-atmospheres, or 0.94 Venus-atmospheres.

(b) $v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \text{ J/mol} \cdot \text{K})(733 \text{ K})}{44.0 \times 10^{-3} \text{ kg/mol}}} = 645 \text{ m/s}$. v_{rms} has this value both at the surface and at an altitude of 1.00 km.

EVALUATE: v_{rms} depends only on T and the molar mass of the gas. For Venus compared to earth, the surface temperature, in kelvins, is nearly a factor of three larger and the molecular mass of the gas in the atmosphere is only about 50% larger, so v_{rms} for the Venus atmosphere is larger than it is for the earth's atmosphere.

18.57. IDENTIFY: Air and water can be pumped in and out of a diving bell.

SET UP and EXECUTE: The average density can be changed by pumping water in and out of the bell.

We use $\rho = m/V$, $p = p_0 + \rho gh$, and $pV = nRT$.

(a) We want to know what volume of seawater to add to the bell so it is neutrally buoyant. The average density of the bell and all its contents must be equal to the density of seawater. Call m_b the mass of the bell, m_d the mass of the diver, and m_w the mass of the added water. Using $\rho_{\text{bell}} = \rho_{\text{seawater}}$ gives

$$\frac{m_b + m_d + m_w}{V_{\text{bell}}} = \rho_{\text{seawater}}. \text{ Using } V_{\text{bell}} = \pi r^2 L \text{ and solving for } m_w \text{ gives } m_w = \pi r^2 L \rho_{\text{seawater}} - m_b - m_d.$$

Using $\rho_{\text{seawater}} = 1025 \text{ kg/m}^3$, $r = 0.750 \text{ m}$, $L = 2.50 \text{ m}$, $m_b = 4350 \text{ kg}$, and $m_d = 80.0 \text{ kg}$, we get $m_w =$

$$98.31 \text{ kg}. \text{ The volume of this water is } V = m/\rho \text{ so } V = \frac{98.31 \text{ kg}}{1025 \text{ kg/m}^3} = 0.0959 \text{ m}^3 = 95.9 \text{ L}.$$

(b) We want to know the rate at which we should release air from the tank to maintain a constant pressure as the bell descends at a steady 1.0 m/s, so we want dn/dt . Combining $p = p_0 + \rho gh$ and $pV = nRT$ gives $p_0 + \rho gh = (RT/V)n$. Taking the time derivative of both sides of this equation gives

$$\rho g \frac{dy}{dt} = \left(\frac{RT}{V} \right) \frac{dn}{dt}. \text{ Solving for } dn/dt \text{ gives } \frac{dn}{dt} = \frac{\rho g V}{RT} \frac{dy}{dt} = \frac{\rho g V v}{RT}. \text{ } V \text{ is the volume of air in the bell}$$

which is $V = V_b - V_w = \pi r^2 L - V_w = \pi(0.750 \text{ m})^2(2.50 \text{ m}) - 0.0959 \text{ m}^3 = 4.322 \text{ m}^3$. Using this value for V plus $\rho = 1025 \text{ kg/m}^3$, $T = 293 \text{ K}$, $v = 1.00 \text{ m/s}$, $R = 8.314 \text{ J/mol} \cdot \text{K}$, and $L = 2.50 \text{ m}$, we get $dn/dt = 17.8 \text{ mol/s}$.

(c) The tank contains 600 ft³ of gas, which is $1.6992 \times 10^4 \text{ L}$. It was loaded at standard conditions of 1.0 atm at $0^\circ\text{C} = 273 \text{ K}$, so $n = pV/RT$. Putting in the numbers gives $n = 758.5 \text{ mol}$. The tank releases gas at a rate of 17.8 mol/s, so $(17.8 \text{ mol/s})t = 758.5 \text{ mol}$, which gives $t = 42.6 \text{ s}$. During this time the bell is descending at a steady 1.00 m/s, so the distance it travels is $(1.00 \text{ m/s})(42.6 \text{ s}) = 42.6 \text{ m}$.

EVALUATE: At 42.6 m the water pressure would be $p = p_0 + \rho gh = p_0 + (1025 \text{ kg/m}^3)(g)(42.6 \text{ m}) = 5.3 \text{ atm}$, so the diving bell should not have any leaks!

18.58. IDENTIFY: In part (a), apply $pV = nRT$ to the ethane in the flask. The volume is constant once the

stopcock is in place. In part (b) apply $pV = \frac{m_{\text{tot}}}{M} RT$ to the ethane at its final temperature and pressure.

SET UP: $1.50 \text{ L} = 1.50 \times 10^{-3} \text{ m}^3$. $M = 30.1 \times 10^{-3} \text{ kg/mol}$. Neglect the thermal expansion of the flask.

EXECUTE: (a) $p_2 = p_1(T_2/T_1) = (1.013 \times 10^5 \text{ Pa})(300 \text{ K}/550 \text{ K}) = 5.525 \times 10^4 \text{ Pa}$, which rounds to $5.53 \times 10^4 \text{ Pa}$.

$$(b) m_{\text{tot}} = \left(\frac{p_2 V}{RT_2} \right) M = \left(\frac{(5.525 \times 10^4 \text{ Pa})(1.50 \times 10^{-3} \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} \right) (30.1 \times 10^{-3} \text{ kg/mol}) = 1.00 \text{ g}.$$

EVALUATE: We could also calculate m_{tot} with $p = 1.013 \times 10^5 \text{ Pa}$ and $T = 550 \text{ K}$, and we would obtain the same result. Originally, before the system was warmed, the mass of ethane in the flask was

$$m = (1.00 \text{ g}) \left(\frac{1.013 \times 10^5 \text{ Pa}}{5.525 \times 10^4 \text{ Pa}} \right) = 1.83 \text{ g}.$$

- 18.59. (a) IDENTIFY:** Consider the gas in one cylinder. Calculate the volume to which this volume of gas expands when the pressure is decreased from $(1.20 \times 10^6 \text{ Pa} + 1.01 \times 10^5 \text{ Pa}) = 1.30 \times 10^6 \text{ Pa}$ to $1.01 \times 10^5 \text{ Pa}$. Apply the ideal-gas law to the two states of the system to obtain an expression for V_2 in terms of V_1 and the ratio of the pressures in the two states.

SET UP: $pV = nRT$

n, R, T constant implies $pV = nRT = \text{constant}$, so $p_1 V_1 = p_2 V_2$.

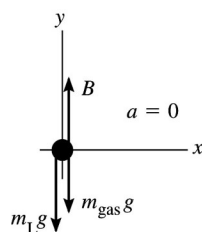
$$\text{EXECUTE: } V_2 = V_1(p_1 / p_2) = (1.90 \text{ m}^3) \left(\frac{1.30 \times 10^6 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right) = 24.46 \text{ m}^3$$

The number of cylinders required to fill a 750 m^3 balloon is $750 \text{ m}^3 / 24.46 \text{ m}^3 = 30.7$ cylinders.

EVALUATE: The ratio of the volume of the balloon to the volume of a cylinder is about 400. Fewer cylinders than this are required because of the large factor by which the gas is compressed in the cylinders.

- (b) IDENTIFY:** The upward force on the balloon is given by Archimedes's principle: $B = \text{weight of air displaced by balloon} = \rho_{\text{air}} V g$. Apply Newton's second law to the balloon and solve for the weight of the load that can be supported. Use the ideal-gas equation to find the mass of the gas in the balloon.

SET UP: The free-body diagram for the balloon is given in Figure 18.59.



m_{gas} is the mass of the gas that is inside the balloon; m_L is the mass of the load that is supported by the balloon.

EXECUTE: $\sum F_y = ma_y$

$$B - m_L g - m_{\text{gas}} g = 0$$

Figure 18.59

$$\rho_{\text{air}} V g - m_L g - m_{\text{gas}} g = 0$$

$$m_L = \rho_{\text{air}} V - m_{\text{gas}}$$

Calculate m_{gas} , the mass of hydrogen that occupies 750 m^3 at 15°C and $p = 1.01 \times 10^5 \text{ Pa}$.

$$pV = nRT = (m_{\text{gas}} / M) RT$$

$$\text{gives } m_{\text{gas}} = pVM / RT = \frac{(1.01 \times 10^5 \text{ Pa})(750 \text{ m}^3)(2.02 \times 10^{-3} \text{ kg/mol})}{(8.3145 \text{ J/mol} \cdot \text{K})(288 \text{ K})} = 63.9 \text{ kg}.$$

Then $m_L = (1.23 \text{ kg/m}^3)(750 \text{ m}^3) - 63.9 \text{ kg} = 859 \text{ kg}$, and the weight that can be supported is

$$w_L = m_L g = (859 \text{ kg})(9.80 \text{ m/s}^2) = 8420 \text{ N}.$$

$$(c) m_L = \rho_{\text{air}} V - m_{\text{gas}}$$

$$m_{\text{gas}} = pVM / RT = (63.9 \text{ kg})((4.00 \text{ g/mol}) / (2.02 \text{ g/mol})) = 126.5 \text{ kg} \quad (\text{using the results of part (b)}).$$

$$\text{Then } m_L = (1.23 \text{ kg/m}^3)(750 \text{ m}^3) - 126.5 \text{ kg} = 796 \text{ kg}.$$

$$w_L = m_L g = (796 \text{ kg})(9.80 \text{ m/s}^2) = 7800 \text{ N}.$$

EVALUATE: A greater weight can be supported when hydrogen is used because its density is less.

- 18.60. IDENTIFY:** The upward force exerted by the gas on the piston must equal the piston's weight. Use $pV = nRT$ to calculate the volume of the gas, and from this the height of the column of gas in the cylinder.

SET UP: $F = pA = p\pi r^2$, with $r = 0.100 \text{ m}$ and $p = 0.300 \text{ atm} = 3.039 \times 10^4 \text{ Pa}$. For the cylinder, $V = \pi r^2 h$.

$$\text{EXECUTE: (a) } p\pi r^2 = mg \text{ and } m = \frac{p\pi r^2}{g} = \frac{(3.039 \times 10^4 \text{ Pa})\pi(0.100 \text{ m})^2}{9.80 \text{ m/s}^2} = 97.4 \text{ kg}.$$

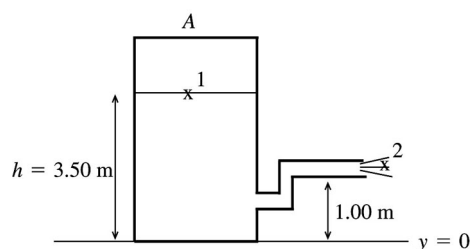
$$(b) V = \pi r^2 h \text{ and } V = nRT/p. \text{ Combining these equations gives } h = nRT/\pi r^2 p, \text{ which gives}$$

$$h = \frac{(1.80 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})}{\pi(0.100 \text{ m})^2(3.039 \times 10^4 \text{ Pa})} = 4.60 \text{ m}.$$

EVALUATE: The calculation assumes a vacuum ($p = 0$) in the tank above the piston.

- 18.61. IDENTIFY:** Apply Bernoulli's equation to relate the efflux speed of water out the hose to the height of water in the tank and the pressure of the air above the water in the tank. Use the ideal-gas equation to relate the volume of the air in the tank to the pressure of the air.

SET UP: Points 1 and 2 are shown in Figure 18.61.



$$p_1 = 4.20 \times 10^5 \text{ Pa}$$

$$p_2 = p_{\text{air}} = 1.00 \times 10^5 \text{ Pa}$$

$$\text{large tank implies } v_1 \approx 0$$

Figure 18.61

$$\text{EXECUTE: (a) } p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

$$\frac{1}{2} \rho v_2^2 = p_1 - p_2 + \rho g (y_1 - y_2)$$

$$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$$

$$v_2 = 26.2 \text{ m/s}.$$

$$(b) \underline{h = 3.00 \text{ m}}$$

The volume of the air in the tank increases so its pressure decreases. $pV = nRT = \text{constant}$, so

$$pV = p_0 V_0 \quad (p_0 \text{ is the pressure for } h_0 = 3.50 \text{ m and } p \text{ is the pressure for } h = 3.00 \text{ m})$$

$$p(4.00 \text{ m} - h)A = p_0(4.00 \text{ m} - h_0)A$$

$$p = p_0 \left(\frac{4.00 \text{ m} - h_0}{4.00 \text{ m} - h} \right) = (4.20 \times 10^5 \text{ Pa}) \left(\frac{4.00 \text{ m} - 3.50 \text{ m}}{4.00 \text{ m} - 3.00 \text{ m}} \right) = 2.10 \times 10^5 \text{ Pa}.$$

Repeat the calculation of part (a), but now $p_1 = 2.10 \times 10^5 \text{ Pa}$ and $y_1 = 3.00 \text{ m}$.

$$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$$

$$v_2 = 16.1 \text{ m/s}$$

$$h = 2.00 \text{ m}$$

$$p = p_0 \left(\frac{4.00 \text{ m} - h_0}{4.00 \text{ m} - h} \right) = (4.20 \times 10^5 \text{ Pa}) \left(\frac{4.00 \text{ m} - 3.50 \text{ m}}{4.00 \text{ m} - 2.00 \text{ m}} \right) = 1.05 \times 10^5 \text{ Pa}$$

$$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$$

$$v_2 = 5.44 \text{ m/s.}$$

$$(c) \quad v_2 = 0 \text{ means } (2/\rho)(p_1 - p_2) + 2g(y_1 - y_2) = 0$$

$$p_1 - p_2 = -\rho g(y_1 - y_2)$$

$$y_1 - y_2 = h - 1.00 \text{ m}$$

$$p = p_0 \left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h} \right) = (4.20 \times 10^5 \text{ Pa}) \left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h} \right). \text{ This is } p_1, \text{ so}$$

$$(4.20 \times 10^5 \text{ Pa}) \left(\frac{0.50 \text{ m}}{4.00 \text{ m} - h} \right) - 1.00 \times 10^5 \text{ Pa} = (9.80 \text{ m/s}^2)(1000 \text{ kg/m}^3)(1.00 \text{ m} - h)$$

$$(210 / (4.00 - h)) - 100 = 9.80 - 9.80h, \text{ with } h \text{ in meters.}$$

$$210 = (4.00 - h)(109.8 - 9.80h)$$

$$9.80h^2 - 149h + 229.2 = 0 \text{ and } h^2 - 15.20h + 23.39 = 0$$

$$\text{quadratic formula: } h = \frac{1}{2} \left(15.20 \pm \sqrt{(15.20)^2 - 4(23.39)} \right) = (7.60 \pm 5.86) \text{ m}$$

h must be less than 4.00 m, so the only acceptable value is $h = 7.60 \text{ m} - 5.86 \text{ m} = 1.74 \text{ m}$.

EVALUATE: The flow stops when $p + \rho g(y_1 - y_2)$ equals air pressure. For $h = 1.74 \text{ m}$,

$p = 9.3 \times 10^4 \text{ Pa}$ and $\rho g(y_1 - y_2) = 0.7 \times 10^4 \text{ Pa}$, so $p + \rho g(y_1 - y_2) = 1.0 \times 10^5 \text{ Pa}$, which is air pressure.

18.62. IDENTIFY: The vertical forces on the plastic sphere must balance. Archimedes's principle and the ideal gas law both apply.

SET UP: $pV = nRT$, $\rho = m/V$. Call V the volume of the sphere, m its mass, and ρ its volume. Let ρ_a be the density of the air and F the tension in the thread. The buoyant force B is equal to the weight of the air displaced by the sphere, so $B = \rho_a Vg$.

EXECUTE: (a) Balancing vertical forces gives $F + B = mg$. $B = \rho_a Vg$ and $V = m/\rho$.

For the air: $\rho_a = (\text{mass of } n \text{ moles})/(\text{volume of } n \text{ moles}) = Mn/V$, where M is the molar mass of air, which is $28.8 \text{ g/mol} = 0.0288 \text{ kg/mol}$. From $pV = nRT$ we have $n/V = p/RT$, so the density of air is $\rho_a = M(n/V) = Mp/RT$.

$$\rho_a = (0.0288 \text{ kg/mol})(1.013 \times 10^5 \text{ Pa}) / [(8.314 \text{ J/mol} \cdot \text{K})(298.15 \text{ K})] = 1.177 \text{ kg/m}^3.$$

$$\text{Therefore } B = \rho_a Vg = \rho_a (m/\rho)g$$

$$B = (1.177 \text{ kg/m}^3)(0.00900 \text{ kg})(9.80 \text{ m/s}^2)/(4.00 \text{ kg/m}^3) = 0.02595 \text{ N.}$$

$$F = mg - B = (0.00900 \text{ kg})(9.80 \text{ m/s}^2) - 0.02595 \text{ N} = 0.0622 \text{ N.}$$

(b) If the buoyant force increases, the tension decreases, so $\Delta F = -\Delta B = -\frac{mg}{\rho} \Delta \rho_a$. This gives us

$$\Delta F = -\frac{mg}{\rho} (\rho_{a,2} - \rho_{a,1}). \text{ Using } \rho_a = pM/RT, \text{ this equation becomes}$$

$$\Delta F = -\frac{mgpM}{\rho R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right) = \frac{mgpM}{\rho R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right). \text{ Putting in the numbers gives}$$

$$\Delta F = \frac{(0.00900 \text{ kg})(9.80 \text{ m/s}^2)(1.013 \times 10^5 \text{ Pa})(0.0288 \text{ kg/mol})}{(4.00 \text{ kg/m}^3)(8.314 \text{ J/mol} \cdot \text{K})} \left(\frac{1}{278.15 \text{ K}} - \frac{1}{308.15 \text{ K}} \right)$$

$\Delta F = 2.71 \times 10^{-3} \text{ N} = 2.71 \text{ mN}$. The positive sign tells us that the tension increases.

EVALUATE: As the air temperature increased, its density decreased, so the buoyant force it exerted decreased. So it is reasonable that the tension should increase.

- 18.63. IDENTIFY and SET UP:** Apply $m_{\text{total}} = nM$ to find n and then use Avogadro's number to find the number of molecules.

EXECUTE: Calculate the number of water molecules N .

$$\text{Number of moles: } n = \frac{m_{\text{tot}}}{M} = \frac{50 \text{ kg}}{18.0 \times 10^{-3} \text{ kg/mol}} = 2.778 \times 10^3 \text{ mol}$$

$$N = nN_A = (2.778 \times 10^3 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 1.7 \times 10^{27} \text{ molecules}$$

Each water molecule has three atoms, so the number of atoms is $3(1.7 \times 10^{27}) = 5.1 \times 10^{27}$ atoms.

EVALUATE: We could also use the masses in Example 18.5 to find the mass m of one H_2O molecule:

$$m = 2.99 \times 10^{-26} \text{ kg. Then } N = m_{\text{tot}} / m = 1.7 \times 10^{27} \text{ molecules, which checks.}$$

- 18.64. IDENTIFY:** Use the ideal gas law to find the number of moles of air taken in with each breath and from this calculate the number of oxygen molecules taken in. Then find the pressure at an elevation of 2000 m and repeat the calculation.

SET UP: The number of molecules in a mole is $N_A = 6.022 \times 10^{23}$ molecules/mol.

$R = 0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$. Example 18.4 shows that the pressure variation with altitude y , when constant temperature is assumed, is $p = p_0 e^{-Mgy/RT}$. For air, $M = 28.8 \times 10^{-3} \text{ kg/mol}$.

EXECUTE: (a) $pV = nRT$ gives $n = \frac{pV}{RT} = \frac{(1.00 \text{ atm})(0.50 \text{ L})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(293.15 \text{ K})} = 0.0208 \text{ mol}$.

$$N = (0.210)nN_A = (0.210)(0.0208 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 2.63 \times 10^{21} \text{ molecules.}$$

(b) $\frac{Mgy}{RT} = \frac{(28.8 \times 10^{-3} \text{ kg/mol})(9.80 \text{ m/s}^2)(2000 \text{ m})}{(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} = 0.2316$.

$$p = p_0 e^{-Mgy/RT} = (1.00 \text{ atm})e^{-0.2316} = 0.793 \text{ atm.}$$

N is proportional to n , which is in turn proportional to p , so

$$N = \left(\frac{0.793 \text{ atm}}{1.00 \text{ atm}} \right) (2.63 \times 10^{21} \text{ molecules}) = 2.09 \times 10^{21} \text{ molecules.}$$

(c) Less O_2 is taken in with each breath at the higher altitude, so the person must take more breaths per minute.

EVALUATE: A given volume of gas contains fewer molecules when the pressure is lowered and the temperature is kept constant.

- 18.65. IDENTIFY:** The mass of one molecule is the molar mass, M , divided by the number of molecules in a mole, N_A . The average translational kinetic energy of a single molecule is $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$. Use $pV = NkT$ to calculate N , the number of molecules.

SET UP: $k = 1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K}$. $M = 28.0 \times 10^{-3} \text{ kg/mol}$. $T = 295.15 \text{ K}$. The volume of the balloon is $V = \frac{4}{3}\pi(0.250 \text{ m})^3 = 0.0654 \text{ m}^3$. $p = 1.25 \text{ atm} = 1.27 \times 10^5 \text{ Pa}$.

EXECUTE: (a) $m = \frac{M}{N_A} = \frac{28.0 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} = 4.65 \times 10^{-26} \text{ kg}$.

(b) $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT = \frac{3}{2}(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(295.15 \text{ K}) = 6.11 \times 10^{-21} \text{ J}$.

(c) $N = \frac{pV}{kT} = \frac{(1.27 \times 10^5 \text{ Pa})(0.0654 \text{ m}^3)}{(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})(295.15 \text{ K})} = 2.04 \times 10^{24} \text{ molecules.}$

(d) The total average translational kinetic energy is

$$N\left(\frac{1}{2}m(v^2)_{\text{av}}\right) = (2.04 \times 10^{24} \text{ molecules})(6.11 \times 10^{-21} \text{ J/molecule}) = 1.25 \times 10^4 \text{ J}.$$

EVALUATE: The number of moles is $n = \frac{N}{N_A} = \frac{2.04 \times 10^{24} \text{ molecules}}{6.022 \times 10^{23} \text{ molecules/mol}} = 3.39 \text{ mol}.$

$K_{\text{tr}} = \frac{3}{2}nRT = \frac{3}{2}(3.39 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K}) = 1.25 \times 10^4 \text{ J},$ which agrees with our results in part (d).

18.66. IDENTIFY: $pV = nRT = \frac{N}{N_A}RT.$ Deviations will be noticeable when the volume V of a molecule is on the order of 1% of the volume of gas that contains one molecule.

SET UP: The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3.$

EXECUTE: The volume of gas per molecule is $\frac{RT}{N_A p},$ and the volume of a molecule is about

$$V_0 = \frac{4}{3}\pi(2.0 \times 10^{-10} \text{ m})^3 = 3.4 \times 10^{-29} \text{ m}^3. \text{ Denoting the ratio of these volumes as}$$

$$f, p = f \frac{RT}{N_A V_0} = f \frac{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{(6.022 \times 10^{23} \text{ molecules/mol})(3.4 \times 10^{-29} \text{ m}^3)} = (1.2 \times 10^8 \text{ Pa})f.$$

“Noticeable deviations” is a subjective term, but f on the order of 1.0% gives a pressure of $10^6 \text{ Pa}.$

EVALUATE: The forces between molecules also cause deviations from ideal-gas behavior.

18.67. IDENTIFY and SET UP: At equilibrium $F(r) = 0.$ The work done to increase the separation from r_2 to ∞ is $U(\infty) - U(r_2).$

(a) **EXECUTE:** $U(r) = U_0[(R_0/r)^{12} - 2(R_0/r)^6]$

From Chapter 14: $F(r) = 12(U_0/R_0)[(R_0/r)^{13} - (R_0/r)^7].$ The graphs are given in Figure 18.67.

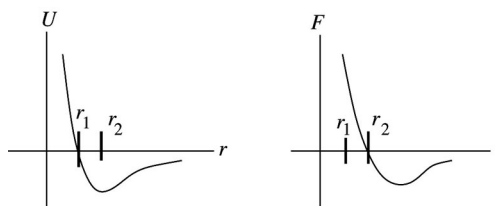


Figure 18.67

(b) Equilibrium requires $F = 0;$ occurs at point $r_2.$ r_2 is where U is a minimum (stable equilibrium).

(c) $U = 0$ implies $(R_0/r)^{12} - 2(R_0/r)^6 = 0$

$$(r_1/R_0)^6 = 1/2 \text{ and } r_1 = R_0/(2)^{1/6}.$$

$$F = 0 \text{ implies } (R_0/r)^{13} - (R_0/r)^7 = 0$$

$$(r_2/R_0)^6 = 1 \text{ and } r_2 = R_0.$$

$$\text{Then } r_1/r_2 = (R_0/2^{1/6})/R_0 = 2^{-1/6}.$$

(d) $W_{\text{other}} = \Delta U.$

$$\text{At } r \rightarrow \infty, U = 0, \text{ so } W = -U(R_0) = -U_0[(R_0/R_0)^{12} - 2(R_0/R_0)^6] = +U_0.$$

EVALUATE: The answer to part (d), $U_0,$ is the depth of the potential well shown in the graph of $U(r).$

18.68. IDENTIFY: Use $pV = nRT$ to calculate the number of moles, n . Then $K_{tr} = \frac{3}{2}nRT$. The mass of the gas, m_{tot} , is given by $m_{tot} = nM$.

SET UP: $5.00 \text{ L} = 5.00 \times 10^{-3} \text{ m}^3$.

EXECUTE: (a) $n = \frac{pV}{RT} = \frac{(1.01 \times 10^5 \text{ Pa})(5.00 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 0.2025 \text{ moles}$

$K_{tr} = \frac{3}{2}(0.2025 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(300 \text{ K}) = 758 \text{ J}$.

(b) $m_{tot} = nM = (0.2025 \text{ mol})(2.016 \times 10^{-3} \text{ kg/mol}) = 4.08 \times 10^{-4} \text{ kg}$. The kinetic energy due to the speed of the jet is $K = \frac{1}{2}mv^2 = \frac{1}{2}(4.08 \times 10^{-4} \text{ kg})(300.0 \text{ m/s})^2 = 18.4 \text{ J}$. The total kinetic energy is

$K_{tot} = K + K_{tr} = 18.4 \text{ J} + 758 \text{ J} = 776 \text{ J}$. The percentage increase is

$$\frac{K}{K_{tot}} \times 100\% = \frac{18.4 \text{ J}}{776 \text{ J}} \times 100\% = 2.37\%.$$

(c) No. The temperature is associated with the random translational motion, and that hasn't changed.

EVALUATE: The equation $pV = \frac{2}{3}K_{tr}$ gives $K_{tr} = \frac{3}{2}pV = \frac{3}{2}(1.01 \times 10^5 \text{ Pa})(5.00 \times 10^{-3} \text{ m}^3) = 758 \text{ J}$,

which agrees with our result in part (a). $v_{rms} = \sqrt{\frac{3RT}{M}} = 1.93 \times 10^3 \text{ m/s}$. v_{rms} is a lot larger than the speed of the jet, so the percentage increase in the total kinetic energy, calculated in part (b), is small.

18.69. IDENTIFY: The equipartition principle says that each atom has an average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. There is an equal average potential energy.

SET UP: The atoms in a three-dimensional solid have three degrees of freedom and the atoms in a two-dimensional solid have two degrees of freedom.

EXECUTE: (a) In the same manner that $C_V = 3R$ was obtained, the heat capacity of the two-dimensional solid would be $2R = 16.6 \text{ J/mol} \cdot \text{K}$.

(b) The heat capacity would behave qualitatively like those in Figure 18.21 in the textbook, and the heat capacity would decrease with decreasing temperature.

EVALUATE: At very low temperatures the equipartition theorem doesn't apply. Most of the atoms remain in their lowest energy states because the next higher energy level is not accessible.

18.70. IDENTIFY: $v_{rms} = \sqrt{\frac{3kT}{m}}$.

SET UP: $M = 1.99 \times 10^{30} \text{ kg}$, $R = 6.96 \times 10^8 \text{ m}$ and $G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

EXECUTE: (a) $v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(5800 \text{ K})}{(1.67 \times 10^{-27} \text{ kg})}} = 1.20 \times 10^4 \text{ m/s}$.

(b) $v_{escape} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}{(6.96 \times 10^8 \text{ m})}} = 6.18 \times 10^5 \text{ m/s}$.

EVALUATE: (c) The escape speed is about 50 times the rms speed, and either Figure 18.23 in the textbook, the Maxwell-Boltzmann distribution, or Table 18.2 will indicate that there is a negligibly small fraction of molecules with the escape speed.

18.71. (a) IDENTIFY and SET UP: Apply conservation of energy $K_1 + U_1 + W_{other} = K_2 + U_2$, where

$U = -Gmm_p/r$. Let point 1 be at the surface of the planet, where the projectile is launched, and let point 2 be far from the earth. Just barely escapes says $v_2 = 0$.

EXECUTE: Only gravity does work says $W_{other} = 0$.

$U_1 = -Gmm_p/R_p$; $r_2 \rightarrow \infty$ so $U_2 = 0$; $v_2 = 0$ so $K_2 = 0$.

The conservation of energy equation becomes $K_1 - Gmm_p / R_p = 0$ and $K_1 = Gmm_p / R_p$.

But $g = Gm_p / R_p^2$ so $Gm_p / R_p = R_p g$ and $K_1 = mgR_p$, as was to be shown.

EVALUATE: The greater gR_p is, the more initial kinetic energy is required for escape.

(b) IDENTIFY and SET UP: Set K_1 from part (a) equal to the average kinetic energy of a molecule as given by the equation $\frac{1}{2}m(v^2)_{av} = \frac{3}{2}kT$. $\frac{1}{2}m(v^2)_{av} = mgR_p$ (from part (a)). But also, $\frac{1}{2}m(v^2)_{av} = \frac{3}{2}kT$, so $mgR_p = \frac{3}{2}kT$.

EXECUTE: $T = \frac{2mgR_p}{3k}$

Nitrogen:

$$m_{N_2} = (28.0 \times 10^{-3} \text{ kg/mol}) / (6.022 \times 10^{23} \text{ molecules/mol}) = 4.65 \times 10^{-26} \text{ kg/molecule}$$

$$T = \frac{2mgR_p}{3k} = \frac{2(4.65 \times 10^{-26} \text{ kg/molecule})(9.80 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 1.40 \times 10^5 \text{ K}$$

Hydrogen:

$$m_{H_2} = (2.02 \times 10^{-3} \text{ kg/mol}) / (6.022 \times 10^{23} \text{ molecules/mol}) = 3.354 \times 10^{-27} \text{ kg/molecule}$$

$$T = \frac{2mgR_p}{3k} = \frac{2(3.354 \times 10^{-27} \text{ kg/molecule})(9.80 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 1.01 \times 10^4 \text{ K}$$

(c) $T = \frac{2mgR_p}{3k}$

Nitrogen:

$$T = \frac{2(4.65 \times 10^{-26} \text{ kg/molecule})(1.63 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 6370 \text{ K}$$

Hydrogen:

$$T = \frac{2(3.354 \times 10^{-27} \text{ kg/molecule})(1.63 \text{ m/s}^2)(1.74 \times 10^6 \text{ m})}{3(1.381 \times 10^{-23} \text{ J/molecule} \cdot \text{K})} = 459 \text{ K}$$

(d) EVALUATE: The “escape temperatures” are much less for the moon than for the earth. For the moon a larger fraction of the molecules at a given temperature will have speeds in the Maxwell-Boltzmann distribution larger than the escape speed. After a long time most of the molecules will have escaped from the moon.

18.72. IDENTIFY: The ideal gas law applies. The rms speed depends on the Kelvin temperature of the gas.

SET UP: $pV = nRT$, $v_{rms} = \sqrt{3kT/m}$. For constant volume, the ideal gas law gives $p_2/T_2 = p_1/T_1$. All temperatures must be in kelvins.

EXECUTE: Solving for p_2 gives $p_2 = p_1(T_2/T_1)$. Solving $v_{rms} = \sqrt{3kT/m}$ for T gives

$$T = \frac{m}{3k} v_{rms}^2. \text{ Applying this result to the pressure equation gives}$$

$$p_2 = p_1 \frac{(m/3k)v_{rms,2}^2}{(m/3k)v_{rms,1}^2} = (2.00 \text{ atm}) \left(\frac{276 \text{ m/s}}{176 \text{ m/s}} \right)^2 = 4.92 \text{ atm. The change in pressure is}$$

$$4.92 \text{ atm} - 2.00 \text{ atm} = 2.92 \text{ atm.}$$

EVALUATE: An increase in speed of $(100 \text{ m/s})/(176 \text{ m/s}) = 57\%$ produced an increase in pressure of $(2.92 \text{ atm})/(2.00 \text{ atm}) = 146\%$. The large increase in pressure for a much smaller increase in molecular speed is due to the fact that the temperature, and hence the pressure, depends on the *square* of the molecular speed.

18.73. IDENTIFY and SET UP: Evaluate the integral as specified in the problem.

EXECUTE: $\int_0^\infty v^2 f(v) dv = 4\pi(m/2\pi kT)^{3/2} \int_0^\infty v^4 e^{-mv^2/2kT} dv.$

The integral formula with $n=2$ gives $\int_0^\infty v^4 e^{-av^2} dv = (3/8a^2)\sqrt{\pi/a}.$

Apply with $a = m/2kT,$

$$\int_0^\infty v^2 f(v) dv = 4\pi(m/2\pi kT)^{3/2} (3/8)(2kT/m)^2 \sqrt{2\pi kT/m} = (3/2)(2kT/m) = 3kT/m.$$

EVALUATE: $\frac{1}{2}m(v^2)_{av} = \frac{3}{2}kT$ says $\frac{1}{2}m(v^2)_{av} = 3kT/2$, so $(v^2)_{av} = 3kT/m$, in agreement with our calculation.

18.74. IDENTIFY: The equipartition principle says that each molecule has average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. $I = 2m(L/2)^2$, where L is the distance between the two atoms in the molecule.

$$K_{rot} = \frac{1}{2}I\omega^2. \omega_{rms} = \sqrt{(\omega^2)_{av}}.$$

SET UP: The mass of one atom is $m = M/N_A = (16.0 \times 10^{-3} \text{ kg/mol})/(6.022 \times 10^{23} \text{ molecules/mol}) = 2.66 \times 10^{-26} \text{ kg}.$

EXECUTE: (a) The two degrees of freedom associated with the rotation for a diatomic molecule account for two-fifths of the total kinetic energy, so $K_{rot} = nRT = (1.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K}) = 2.49 \times 10^3 \text{ J}.$

(b) $I = 2m(L/2)^2 = 2 \left(\frac{16.0 \times 10^{-3} \text{ kg/mol}}{6.022 \times 10^{23} \text{ molecules/mol}} \right) (6.05 \times 10^{-11} \text{ m})^2 = 1.94 \times 10^{-46} \text{ kg} \cdot \text{m}^2.$

(c) Since the result in part (b) is for one mole, the rotational kinetic energy for one atom is K_{rot}/N_A

and $\omega_{rms} = \sqrt{\frac{2K_{rot}/N_A}{I}} = \sqrt{\frac{2(2.49 \times 10^3 \text{ J})}{(1.94 \times 10^{-46} \text{ kg} \cdot \text{m}^2)(6.022 \times 10^{23} \text{ molecules/mol})}} = 6.52 \times 10^{12} \text{ rad/s}.$ This

is much larger than the typical value for a piece of rotating machinery.

EVALUATE: The average rotational period, $T = \frac{2\pi \text{ rad}}{\omega_{rms}}$, for molecules is very short.

18.75. IDENTIFY: $f(v)dv$ is the probability that a particle has a speed between v and $v+dv$. The equation for the Maxwell-Boltzmann distribution gives $f(v)$. v_{mp} is given by $v_{mp} = \sqrt{2kT/m}.$

SET UP: For O_2 , the mass of one molecule is $m = M/N_A = 5.32 \times 10^{-26} \text{ kg}.$

EXECUTE: (a) $f(v)dv$ is the fraction of the particles that have speed in the range from v to $v+dv$. The number of particles with speeds between v and $v+dv$ is therefore $dN = Nf(v)dv$ and

$$\Delta N = N \int_v^{v+\Delta v} f(v)dv.$$

(b) Setting $v = v_{mp} = \sqrt{\frac{2kT}{m}}$ in $f(v)$ gives $f(v_{mp}) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \left(\frac{2kT}{m} \right) e^{-1} = \frac{4}{e\sqrt{\pi}v_{mp}}.$ For oxygen

gas at 300 K, $v_{mp} = 3.95 \times 10^2 \text{ m/s}$ and $f(v)\Delta v = 0.0421.$

(c) Increasing v by a factor of 7 changes f by a factor of $7^2 e^{-48}$, and $f(v)\Delta v = 2.94 \times 10^{-21}.$

(d) Multiplying the temperature by a factor of 2 increases the most probable speed by a factor of $\sqrt{2}$, and the answers are decreased by $\sqrt{2}$: 0.0297 and $2.08 \times 10^{-21}.$

(e) Similarly, when the temperature is one-half what it was in parts (b) and (c), the fractions increase by $\sqrt{2}$ to 0.0595 and $4.15 \times 10^{-21}.$

EVALUATE: (f) At lower temperatures, the distribution is more sharply peaked about the maximum (the most probable speed), as is shown in Figure 18.23a in the textbook.

18.76. IDENTIFY: Follow the procedure specified in the problem.

SET UP: If $v^2 = x$, then $dx = 2v dv$.

EXECUTE: $\int_0^\infty v f(v) dv = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 e^{-mv^2/2kT} dv$. Making the suggested change of variable,

$v^2 = x$. $2v dv = dx$, $v^3 dv = (1/2)x dx$, and the integral

becomes $\int_0^\infty v f(v) dv = 2\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty x e^{-mx/2kT} dx = 2\pi \left(\frac{m}{2\pi kT} \right)^{3/2} \left(\frac{2kT}{m} \right)^2 = \frac{2}{\sqrt{\pi}} \sqrt{\frac{2kT}{m}} = \sqrt{\frac{8kT}{\pi m}}$

which is the equation $v_{av} = \sqrt{8kT/\pi m}$.

EVALUATE: The integral $\int_0^\infty v f(v) dv$ is the definition of v_{av} .

18.77. IDENTIFY: At equilibrium the net upward force of the gas on the piston equals the weight of the piston. When the piston moves upward the gas expands, the pressure of the gas drops and there is a net downward force on the piston. For simple harmonic motion the net force has the form $F_y = -ky$, for a

displacement y from equilibrium, and $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$.

SET UP: $pV = nRT$. T is constant.

EXECUTE: (a) The difference between the pressure, inside and outside the cylinder, multiplied by the area of the piston, must be the weight of the piston. The pressure in the trapped gas is

$$p_0 + \frac{mg}{A} = p_0 + \frac{mg}{\pi r^2}.$$

(b) When the piston is a distance $h + y$ above the cylinder, the pressure in the trapped gas is

$\left(p_0 + \frac{mg}{\pi r^2} \right) \left(\frac{h}{h+y} \right)$ and for values of y small compared to h , $\frac{h}{h+y} = \left(1 + \frac{y}{h} \right)^{-1} \sim 1 - \frac{y}{h}$. The net force,

taking the positive direction to be upward, is then

$$F_y = \left[\left(p_0 + \frac{mg}{\pi r^2} \right) \left(1 - \frac{y}{h} \right) - p_0 \right] (\pi r^2) - mg = - \left(\frac{y}{h} \right) (p_0 \pi r^2 + mg).$$

This form shows that for positive h , the net force is down; the trapped gas is at a lower pressure than the equilibrium pressure, and so the net force tends to restore the piston to equilibrium.

(c) The angular frequency of small oscillations would be given by

$$\omega^2 = \frac{(p_0 \pi r^2 + mg)/h}{m} = \frac{g}{h} \left(1 + \frac{p_0 \pi r^2}{mg} \right). \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g}{h} \left(1 + \frac{p_0 \pi r^2}{mg} \right)}.$$

If the displacements are not small, the motion is not simple harmonic. This can be seen by considering what happens if $y \sim -h$; the gas is compressed to a very small volume, and the force due to the pressure of the gas would become unboundedly large for a finite displacement, which is not characteristic of simple harmonic motion. If $y \gg h$ (but not so large that the piston leaves the cylinder), the force due to the pressure of the gas becomes small, and the restoring force due to the atmosphere and the weight would tend toward a constant, and this is not characteristic of simple harmonic motion.

EVALUATE: The assumption of small oscillations was made when $\frac{h}{h+y}$ was replaced by $1 - y/h$; this is accurate only when y/h is small.

18.78. IDENTIFY and SET UP: For an ideal gas, its pressure approaches zero as its temperature approaches absolute zero. For volume expansion, $\Delta V = V_0 \beta \Delta T$.

EXECUTE: (a) Graph the pressure p versus the temperature T . This graph is shown in Figure 18.78.

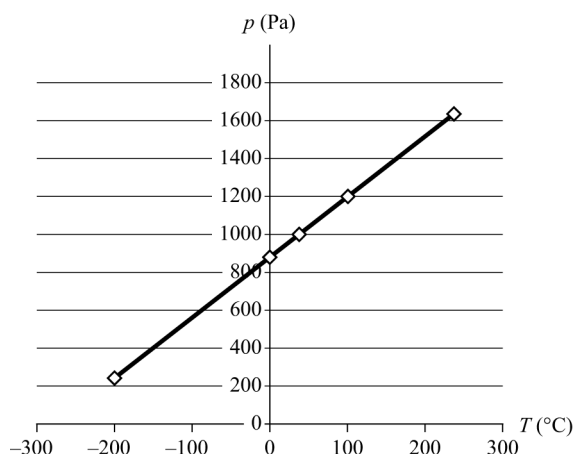


Figure 18.78

The best-fit equation for the graph is $p = (3.2289 \text{ Pa/}^\circ\text{C})T + 888.81 \text{ Pa}$. The temperature when $p = 0$ is given by $T = -\frac{888.81 \text{ Pa}}{3.2289 \text{ Pa/}^\circ\text{C}} = -275^\circ\text{C}$, so this temperature is our determined value for absolute zero.

(b) Solve $\Delta V = V_0 \beta \Delta T$ for the fractional change in volume:

$$\frac{\Delta V}{V} = \beta \Delta T = (3.6 \times 10^{-5} \text{ K}^{-1})(232^\circ\text{C} + 195.8^\circ\text{C}) = 1.5 \times 10^{-2} = 1.5\%.$$

EVALUATE: If the temperature range in an experiment is much less than the extremes in the table, it may be acceptable to ignore the change in volume of the cylinder. But for the range shown in the table, ignoring the volume change of the cylinder would cause a small but significant error.

- 18.79. IDENTIFY:** The measurement gives the dew point. Relative humidity is defined in Problem 18.46, and the vapor pressure table is given with the problem in the text.

SET UP: relative humidity = $\frac{\text{partial pressure of water vapor at temperature } T}{\text{vapor pressure of water at temperature } T}$. At 28.0°C the vapor

pressure of water is $3.78 \times 10^3 \text{ Pa}$.

EXECUTE: (a) The experiment shows that the dew point is 16.0°C , so the partial pressure of water vapor at 30.0°C is equal to the vapor pressure at 16.0°C , which is $1.81 \times 10^3 \text{ Pa}$.

$$\text{Thus the relative humidity} = \frac{1.81 \times 10^3 \text{ Pa}}{4.25 \times 10^3 \text{ Pa}} = 0.426 = 42.6\%.$$

(b) For a relative humidity of 35%, the partial pressure of water vapor is $(0.35)(3.78 \times 10^3 \text{ Pa}) = 1.323 \times 10^3 \text{ Pa}$. This is close to the vapor pressure at 12°C , which would be at an altitude $(30^\circ\text{C} - 12^\circ\text{C}) / (0.6^\circ\text{C}/100 \text{ m}) = 3 \text{ km}$ above the ground.

(c) For a relative humidity of 80%, the vapor pressure will be the same as the water pressure at around 24°C , corresponding to an altitude of about 1 km.

EVALUATE: The lower the dew point is compared to the air temperature, the smaller the relative humidity. Clouds form at a lower height when the relative humidity at the surface is larger.

- 18.80. IDENTIFY and SET UP:** With the multiplicity of each score denoted by n_i , the average score is

$$\left(\frac{1}{150} \right) \sum n_i x_i \text{ and the rms score is } \left[\left(\frac{1}{150} \right) \sum n_i x_i^2 \right]^{1/2}. \text{ Read the numbers from the bar graph in the problem.}$$

EXECUTE: (a) For example, $n_1 x_1 = (11)(10)$, $n_2 x_2 = (12)(20)$, $n_3 x_3 = (24)(30)$, etc. The result is 54.6.

(b) Using the same data, the result is 61.1.

EVALUATE: (c) The rms score is higher than the average score since the rms calculation gives more weight to the higher scores.

- 18.81. IDENTIFY:** We are looking at collisions between nitrogen molecules N_2 . Each molecule contains two nitrogen atoms, each of mass $m = 2.3 \times 10^{-26}$ kg and length $2r = 188$ pm $= 188 \times 10^{-12}$ m.

SET UP: For a point mass, $I = mr^2$. We also know that $K_{\text{tot}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, $L = I\omega$, and for a

point mass $L = mvr$. Angular momentum is conserved during the collision. Since it is an elastic collision, kinetic energy is also conserved.

EXECUTE: (a) We want the moment of inertia of a N_2 molecule. Calling m the mass of a single nitrogen atom, we have $I = 2(mr^2) = 2(2.3 \times 10^{-26} \text{ kg})(94.0 \times 10^{-12} \text{ m})^2 = 4.1 \times 10^{-46} \text{ kg} \cdot \text{m}^2$.

(b) and (c) We want to look at conservation of energy and angular momentum during the collision. The mass of each molecule is $2m$.

Energy conservation: $\frac{1}{2}(2m)v_1^2 + \frac{1}{2}(2m)v_1^2 = \frac{1}{2}(2m)v_2^2 + \frac{1}{2}(2m)v_2^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2$. Using $I = 2mr^2$, this equation becomes $v_1^2 = v_2^2 + r^2\omega^2$, giving $v_f = \sqrt{v_i^2 - r^2\omega^2}$.

Angular momentum conservation: Using $L = mvr$ for a translating point mass gives

$mv_1(2r) + mv_1(2r) = 2I\omega$. Using $I = 2mr^2$ and simplifying gives $4mv_1r = 4mr^2\omega$, so $\omega = v_i / r$.

Combining this result with $v_f = \sqrt{v_i^2 - r^2\omega^2}$ gives $v_f = \sqrt{v_i^2 - v_i^2} = 0$. The final results are: $\omega = v_i / r$ and $v_f = 0$.

(d) The target variable is ω when $v_i = v_{\text{rms}}$. Use $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$ with $T = 293$ K and $m =$

$2(2.3) \times 10^{-26} \text{ kg} = 4.6 \times 10^{-26} \text{ kg}$. This gives $v_{\text{rms}} = 514$ m/s. From part (c) we have $\omega = v_i / r$ so

$$\omega = \frac{514 \text{ m/s}}{94 \times 10^{-12} \text{ m}} = 5.5 \times 10^{12} \text{ rad/s}.$$

EVALUATE: The molecules stop translational motion but continue to rotate, so all the initial translational kinetic energy is transformed into rotational kinetic energy. We also should note that linear momentum must be conserved during the collision. The initial momentum is zero since the molecules have equal but opposite velocities. The final momentum is zero because they both stop, so momentum is conserved.

- 18.82. IDENTIFY:** The gas pressure in a pneumatic lift is used to support heavy loads.

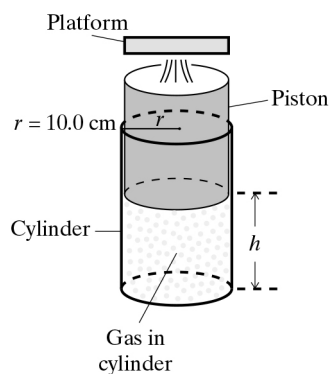


Figure 18.82

SET UP: Fig. 18.82 shows this pneumatic lift. The mass of the piston and platform together is 50.0 kg, the gas temperature is a constant $20.0^\circ\text{C} = 293\text{ K}$, and the outside pressure is 1.00 atm. We use $pV = nRT$ and $p = F/A$.

EXECUTE: (a) Our target variable is h in the figure. There is 1.00 mol of gas in the cylinder and no load on the platform. The cylinder is airtight, so the only pressure inside is due to the gas that is there. The force due to the pressure on the bottom of the piston supports the weight of the piston and the platform and it also balances the force on the top of the piston due to atmospheric pressure (remember the cylinder is airtight). Using $F = pA$, we have $F = pA = mg + p_{\text{atm}}A$, which becomes $p = \frac{mg}{A} + p_{\text{atm}}$.

Using $pV = nRT$, where $V = Ah$ is the volume of gas in the cylinder, we get $\left(\frac{mg}{A} + p_{\text{atm}}\right)Ah = nRT$.

Solving for h gives $h = \frac{nRT}{mg + p_{\text{atm}}A}$. Using $A = \pi R^2$ with $R = 0.100\text{ m}$, $p_{\text{atm}} = 1.01 \times 10^5\text{ Pa}$, $n = 1.00$

mol, $m = 50.0\text{ kg}$, and $T = 293\text{ K}$, we get $h = 0.665\text{ m}$.

(b) The change in pressure supports the added weight, and this change is due to the change in volume.

Since $p = \frac{F}{A}$, it follows that $\Delta p = \frac{\Delta F}{A}$, where ΔF is the added weight

of $(200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$. Using $pV = nRT$ we have

$$\Delta p = p_2 - p_1 = nRT \left(\frac{1}{V_2} - \frac{1}{V_1} \right) = nRT \left(\frac{1}{Ah_2} - \frac{1}{Ah_1} \right). \text{ Solving for } 1/h_2 \text{ gives}$$

$$\frac{1}{h_2} = \frac{A\Delta p}{nRT} + \frac{1}{h_1} = \frac{A \left(\frac{\Delta F}{A} \right)}{nRT} + \frac{1}{h_1} = \frac{\Delta F}{nRT} + \frac{1}{h_1}. \text{ Putting in } h_1 = 0.665\text{ m}, \Delta F = 1960\text{ N}, T = 293\text{ K}, \text{ and } n = 1.00\text{ mol, we have } h_2 = 0.4332\text{ m. The distance platform drops is } h_1 - h_2 = 0.665\text{ m} - 0.433\text{ m} = 0.232\text{ m.}$$

(c) The piston and load rise back to the original position, so they rise by a distance $\Delta h = 0.232\text{ m}$ from part (b). We want to find the amount of gas Δn that enters the cylinder. The pressure must remain the

same to support the load, so $p = \frac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}}$. Using $pV = nRT$ with constant p and T , we

have $p\Delta V = RT\Delta n$. Combining this result with $\Delta V = A\Delta h$ and our equation for p , we get

$$\left[\frac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}} \right] A\Delta h = RT\Delta n. \text{ Solving for } \Delta n \text{ using } m_{\text{piston}} + m_{\text{load}} = 250\text{ kg}, \Delta h = 0.232\text{ m}, T = 293\text{ K, and } A = \pi R^2 \text{ with } R = 0.100\text{ m, we get } \Delta n = 0.536\text{ mol.}$$

(d) We want to find Δn to raise the platform an additional 2.00 m. Everything is the same as in part (c) except that $\Delta h = 2.00\text{ m}$ instead of 0.232 m. To save a lot of arithmetic, take the ratio of Δn for the

two cases and divide out all the common factors. This leaves $\frac{\Delta n_2}{\Delta n_1} = \frac{\Delta h_2}{\Delta h_1}$, so

$$\Delta n_2 = \Delta n_1 \frac{\Delta h_2}{\Delta h_1} = (0.536\text{ mol}) \left(\frac{2.00\text{ m}}{0.232\text{ m}} \right) = 4.62\text{ mol.}$$

(e) We want the rate $\Delta n / \Delta t$ at which air should be introduced so that the platform rises at a steady 10.0

cm/s = 0.100 m/s. From part (c), we have $\left[\frac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}} \right] A\Delta h = RT\Delta n$. Solving for Δn

$$\text{gives } \Delta n = \left[\frac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}} \right] \frac{A\Delta h}{RT}. \text{ Divide by } \Delta t \text{ and realize that } \Delta h / \Delta t = v = 0.100\text{ m/s,}$$

giving $\Delta n = \left[\frac{(m_{\text{piston}} + m_{\text{load}})g}{A} + p_{\text{atm}} \right] \frac{Av}{RT}$. Putting in $v = 0.100$ m/s along with the same numbers as

before, we get $\Delta n / \Delta t = 0.231$ mol/s = 231 millimol/s.

EVALUATE: In most cases we've dealt with, the amount of gas has remained constant. But in this case the pressure was increased in parts (c), (d) and (e) by adding gas to the cylinder. This is how you inflate your car tires.

18.83. IDENTIFY: The equation $\lambda = \frac{V}{4\pi\sqrt{2}r^2N}$ gives the mean free path λ . In the equation

$t_{\text{mean}} = \frac{V}{4\pi\sqrt{2}r^2vN}$, use $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ in place of v . $pV = nRT = NkT$. The escape speed is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}.$$

SET UP: For atomic hydrogen, $M = 1.008 \times 10^{-3}$ kg/mol.

EXECUTE: (a) From $\lambda = \frac{V}{4\pi\sqrt{2}r^2N}$, we have

$$\lambda = [4\pi\sqrt{2}r^2(N/V)]^{-1} = [4\pi\sqrt{2}(5.0 \times 10^{-11} \text{ m})^2(50 \times 10^6 \text{ m}^{-3})]^{-1} = 4.5 \times 10^{11} \text{ m}.$$

(b) $v_{\text{rms}} = \sqrt{3RT/M} = \sqrt{3(8.3145 \text{ J/mol} \cdot \text{K})(20 \text{ K}) / (1.008 \times 10^{-3} \text{ kg/mol})} = 703 \text{ m/s}$, and the time between collisions is then $(4.5 \times 10^{11} \text{ m}) / (703 \text{ m/s}) = 6.4 \times 10^8 \text{ s}$, about 20 yr. Collisions are not very important.

(c) $p = (N/V)kT = (50 / 1.0 \times 10^{-6} \text{ m}^3)(1.381 \times 10^{-23} \text{ J/K})(20 \text{ K}) = 1.4 \times 10^{-14} \text{ Pa}$.

$$\text{(d) } v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G(Nm/V)(4\pi R^3/3)}{R}} = \sqrt{(8\pi/3)G(N/V)mR^2}$$

$$v_{\text{escape}} = \sqrt{(8\pi/3)(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(50 \times 10^6 \text{ m}^{-3})(1.67 \times 10^{-27} \text{ kg})(10 \times 9.46 \times 10^{15} \text{ m})^2}$$

$v_{\text{escape}} = 650 \text{ m/s}$. This is lower than v_{rms} and the cloud would tend to evaporate.

(e) In equilibrium (clearly not *thermal* equilibrium), the pressures will be the same; from $pV = NkT$, $kT_{\text{ISM}}(N/V)_{\text{ISM}} = kT_{\text{nebula}}(N/V)_{\text{nebula}}$ and the result follows.

(f) With the result of part (e),

$$T_{\text{ISM}} = T_{\text{nebula}} \left(\frac{(N/V)_{\text{nebula}}}{(N/V)_{\text{ISM}}} \right) = (20 \text{ K}) \left(\frac{50 \times 10^6 \text{ m}^{-3}}{(200 \times 10^{-6} \text{ m}^3)^{-1}} \right) = 2 \times 10^5 \text{ K},$$

more than three times the temperature of the sun. This indicates a high average kinetic energy, but the thinness of the ISM means that a ship would not burn up.

EVALUATE: The temperature of a gas is determined by the average kinetic energy per atom of the gas. The energy density for the gas also depends on the number of atoms per unit volume, and this is very small for the ISM.

18.84. IDENTIFY: Follow the procedure of Example 18.4, but use $T = T_0 - \alpha y$.

SET UP: $\ln(1+x) \approx x$ when x is very small.

EXECUTE: (a) $\frac{dp}{dy} = -\frac{pM}{RT}$, which in this case becomes $\frac{dp}{p} = -\frac{Mg}{R} \frac{dy}{T_0 - \alpha y}$. This integrates to

$$\ln\left(\frac{p}{p_0}\right) = \frac{Mg}{R\alpha} \ln\left(1 - \frac{\alpha y}{T_0}\right), \text{ or } p = p_0 \left(1 - \frac{\alpha y}{T_0}\right)^{Mg/R\alpha}.$$

(b) For sufficiently small α , $\ln\left(1 - \frac{\alpha y}{T_0}\right) \approx -\frac{\alpha y}{T_0}$, and this gives the expression derived in Example 18.4.

$$(c) \left(1 - \frac{(0.6 \times 10^{-2} \text{ C}^\circ/\text{m})(8863 \text{ m})}{(288 \text{ K})} \right) = 0.8154, \quad \frac{Mg}{R\alpha} = \frac{(28.8 \times 10^{-3})(9.80 \text{ m/s}^2)}{(8.3145 \text{ J/mol} \cdot \text{K})(0.6 \times 10^{-2} \text{ C}^\circ/\text{m})} = 5.6576 \text{ and}$$

$p_0(0.8154)^{5.6576} = 0.315 \text{ atm}$, which is 0.95 of the result found in Example 18.4.

EVALUATE: The pressure is calculated to decrease more rapidly with altitude when we assume that T also decreases with altitude.

18.85. IDENTIFY and SET UP: Noble gases are monatomic, but nitrogen and oxygen are diatomic, and k is proportional to C_V .

EXECUTE: For a monatomic gas, $C_V = 3/2 R$, but for a diatomic gas $C_V = 5/2 R$. A small C_V will give a small k , which means that a monatomic gas has a smaller thermal conductivity than a diatomic gas. This is true because rotational modes are not present for a monatomic gas, but they are present for a diatomic gas. This makes choice (a) the correct one.

EVALUATE: An added advantage of noble gases is that they are less reactive than other gases and therefore would not react with the window materials.

18.86. IDENTIFY and SET UP: $k \propto \frac{C_V}{r^2 \sqrt{M}}$.

EXECUTE: Take the ratio of the thermal conductivities for xenon and helium.

$$\frac{k_{\text{Xe}}}{k_{\text{He}}} = \sqrt{\frac{M_{\text{He}}}{M_{\text{Xe}}}} \left(\frac{r_{\text{He}}}{r_{\text{Xe}}} \right)^2 = \sqrt{\frac{4.0 \text{ g/mol}}{131 \text{ g/mol}}} \left(\frac{0.13 \text{ nm}}{0.22 \text{ nm}} \right)^2 = 0.061, \text{ which is choice (b).}$$

EVALUATE: Since both He and Xe are monatomic, they have the same C_V so this cancels out in the ratio.

18.87. IDENTIFY and SET UP: The rate of effusion is proportional to v_{rms} and $v_{\text{rms}} = \sqrt{3kT/m}$, so the rate $R = C v_{\text{rms}} = C \sqrt{3kT/m}$, where C is a constant.

EXECUTE: Take the ratio of the rates for helium and xenon:

$$\frac{R_{\text{He}}}{R_{\text{Xe}}} = \frac{C \sqrt{3kT/M_{\text{He}}}}{C \sqrt{3kT/M_{\text{Xe}}}} = \sqrt{\frac{M_{\text{Xe}}}{M_{\text{He}}}} = \sqrt{\frac{131}{4.0}} = 5.7 \approx 6, \text{ which makes choice (c) correct.}$$

EVALUATE: At a given temperature, helium atoms will be moving faster than xenon atoms, so they will more easily move through any small openings (leaks) in the window seal.