2

MOTION ALONG A STRAIGHT LINE

VP2.5.1. **IDENTIFY:** The bus and the car leave the same point at the same time. The bus has constant velocity, but the car starts from rest with constant acceleration. So the constant-acceleration formulas apply. We want to know how long it takes for the car to catch up to the bus and how far they both travel during that time.

SET UP: When they meet, x is the same for both of them and they have traveled for the same time. The formulas $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$ both apply.

EXECUTE: (a) When the car and bus meet, they have traveled the same distance in the same time. We apply the formula $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to each of them, with the origin at their starting point, which makes $x_0 = 0$ for both of them. The bus has no acceleration and the car has no initial velocity. The equation reduces to $\frac{1}{2}a_{car}t^2 = v_{bus}t \quad \rightarrow \quad t = 2v_{bus}/a_{car}$.

$t = 2(18 \text{ m/s})/(8.0 \text{ m/s}^2) = 4.5$ s.

(b) The bus has zero acceleration, so $v_x = v_{0x} + a_x t$ reduces to $x_{bus} = v_{bus}t$

$x_{bus} = (18 \text{ m/s})(4.5 \text{ s}) = 81$ m.

EVALUATE: To check, use the car's motion to find the distance.

$x_{car} = \frac{1}{2}a_{car}t^2 = \frac{1}{2}(8.0 \text{ m/s}^2)(4.5 \text{ s})^2 = 81$ m, which agrees with our result in part (b).

VP2.5.2. **IDENTIFY:** This is very similar to VP2.5.1 and VP2.5.2. The motorcycle and the SUV leave the same point at the same time. The motorcycle has a constant velocity, but the SUV has an initial velocity and a constant acceleration. So the constant-acceleration formulas apply.

SET UP: When they meet, x is the same for both of them and they have traveled for the same time. The formulas $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$ both apply.

EXECUTE: (a) When the SUV and motorcycle meet, they have traveled the same distance in the same time. We apply the formula $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to each of them, with the origin at their starting point, which makes $x_0 = 0$ for both of them. The motorcycle has no acceleration and the SUV has an initial velocity and an acceleration. The acceleration is *opposite* to the velocity of the SUV. If we take the x-axis to be in the direction of motion, a_{SUV} is negative. The equation reduces to

$v_{motorcycle}t = v_{SUV}t + \frac{1}{2}a_{SUV}t^2$. We want the time. Putting in the numbers gives

$(20.0 \text{ m/s})t = (30.0 \text{ m/s})t + \frac{1}{2}(-1.80 \text{ m/s}^2)t^2$

$t = 0$ s and $t = 11.1$ s. The $t = 0$ s solution is when they both of them leave the same point, and the $t =$ 11.1 s is the time when the motorcycle passes the SUV.

(b) Both have traveled the same distance when they meet. For the motorcycle this gives $x_{\text{motorcycle}} = v_{\text{motorcycle}}t = (20.0 \text{ m/s})(11.1 \text{ s}) = 222$ m.

(c) The equation $v_x = v_{0x} + a_x t$ gives the speed of the SUV when they meet in 11.1 s.

$v_x = 30.0 \text{ m/s} + (-1.80 \text{ m/s}^2)(11.1 \text{ s}) = 10.0$ s.

EVALUATE: Use $x = v_{\text{av}}t$ to find the distance the SUV has traveled in 11.1 s. For constant acceleration, the average velocity is $v_{\text{av}} = (v_1 + v_2)/2$, which gives us

$x = [(30.0 \text{ m/s} + 10.0 \text{ m/s})/2](11.1 \text{ s}) = 222$ m, which agrees with our previous result.

VP2.5.4. **IDENTIFY:** The truck and car have constant (but different) accelerations and the car has an initial velocity but the truck starts from rest. They leave from the same place at the same time and the truck eventually passes the car. The constant-acceleration equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ applies.

SET UP: (a) The truck and car have traveled the same distance in the same time when the truck reaches the car to pass. They start at the same place so x_0 is the same for both and the truck has no initial velocity.

EXECUTE: Use the equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ for each of them, which gives

$$\frac{1}{2}a_{\text{T}}t^2 = v_{\text{C}}t - \frac{1}{2}a_{\text{C}}t^2 \quad \to \quad t = \frac{2v_{\text{C}}}{a_{\text{T}} + a_{\text{C}}}.$$

(b) Looking at the truck gives $x_{\text{T}} = \frac{1}{2}a_{\text{T}}t^2 = \frac{1}{2}a_{\text{T}}\left(\frac{2v_{\text{C}}}{a_{\text{T}} + a_{\text{C}}}\right)^2 = \frac{2a_{\text{T}}v_{\text{C}}^2}{(a_{\text{T}} + a_{\text{C}})^2}$.

EVALUATE: We can calculate the distance the car travels using $x_{\text{C}} = v_{\text{C}}t - \frac{1}{2}a_{\text{C}}t^2$ and the value of t we found in part (a). Doing this and simplifying the result gives the same answer as in part (b).

VP2.7.1. **IDENTIFY:** The ball is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y = v_{0y} + a_y t$ apply to the motion of the ball. We know that $a_y = 9.80 \text{ m/s}^2$ downward and $v_{0y} = 12.0$ m/s upward.

EXECUTE: (a) At time $t = 0.300$ s, the vertical coordinate of the ball is given by $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, where $y_0 = 0$ at the location of the hand.

$y = 0 + (12.0 \text{ m/s})(0.300 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.300 \text{ s})^2 = 3.16$ m. Since y is positive, the ball is above the hand. The vertical velocity is given by $v_y = v_{0y} + a_y t$.

$v_y = 12.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.300 \text{ s}) = 9.06$ m/s. Since v_y is positive, the ball is moving upward.

(b) At $t = 2.60$ s, $y = (12.0 \text{ m/s})(2.60 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(2.60 \text{ s})^2 = -1.92$ m. Since y is negative, the ball is now below the hand. The ball must be moving downward since it is now below the hand.

EVALUATE: Check with v_y: $v_y = v_{0y} + a_y t = 12.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.60 \text{ s}) = -13.5$ m/s. Since v_y is negative, the ball is moving downward, as we saw above.

VP2.7.2. **IDENTIFY:** The stone is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply to the motion of the stone. We know that $a_y = 9.80\ \text{m/s}^2$ downward, $v_{0y} = 8.00\ \text{m/s}$ downward, and $y_0 = 0$.

EXECUTE: (a) The equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

$y = 0 + (–8.00\ \text{m/s})(1.50\ \text{s}) + \frac{1}{2}(–9.80\ \text{m/s}^2)(1.50\ \text{s})^2 = –23.0\ \text{m}$. The minus sign means that the stone is below your hand. The velocity of the stone is given by

$v_y = v_{0y} + a_y t = –8.00\ \text{m/s} + (–9.80\ \text{m/s}^2)(1.50\ \text{s}) = –22.7\ \text{m/s}$. The minus sign tells us it is moving downward.

(b) We know the stone's position and acceleration and want its velocity. The equation

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y^2 = (–8.00\ \text{m/s})^2 + 2(–9.80\ \text{m/s}^2)(–8.00\ \text{m})$

$v_y = \pm 14.9\ \text{m/s}$. The stone must be moving downward, so $v_y = –14.9\ \text{m/s}$.

EVALUATE: When the stone returned to the level of your hand, its speed was the same as its initial speed of 8.00 m/s. But as the stone has continued to accelerate downward since then, its speed must be greater than its initial speed, which is what we found.

VP2.7.3. **IDENTIFY:** The football is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. We know that $a_y = 9.80\ \text{m/s}^2$ downward, $v_y = 0.500\ \text{m/s}$ upward when $y = 4.00\ \text{m}$, and $y_0 = 0$.

EXECUTE: (a) We know the speed, acceleration, and position of the ball and want its initial speed, so we use the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find its initial speed v_{0y}.

$(0.500\ \text{m/s})^2 = v_{0y}^2 + 2(–9.80\ \text{m/s}^2)(4.00\ \text{m}) \rightarrow \quad v_{0y} = 8.87\ \text{m/s}$.

(b) Use the result from (a) in the equation $v_y = v_{0y} + a_y t$ to find the time.

$0.500\ \text{m/s} = 8.87\ \text{m/s} + (–9.80\ \text{m/s}^2)t \quad \rightarrow \quad t = 0.854\ \text{s}$.

EVALUATE: Calculate y using the time from (b) and compare it with the given value of 4.00 m.

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + (8.87\ \text{m/s})(0.854\ \text{s}) + \frac{1}{2}(–9.80\ \text{m/s}^2)(0.854\ \text{s})^2 = 4.00\ \text{m}$, which agrees with the given value.

VP2.7.4. **IDENTIFY:** The tennis ball is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: When the ball is at its highest point, its vertical velocity is zero. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies.

EXECUTE: (a) At the highest point, $v_y = 0$. Use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find v_{0y}.

$0 = v_{0y}^2 + 2(–g)(H) \quad \rightarrow \quad v_{0y} = \sqrt{2gH}$

(b) We now know H, v_{0y} and want v_y. The same equation gives

$v_y^2 = v_{0y}^2 + 2(-g)(H/2) = 2gH – 2gH/2 = 2gH/2$

$$v_y = \sqrt{\frac{2gH}{2}} = \frac{\sqrt{2gH}}{\sqrt{2}} = \frac{v_0}{\sqrt{2}}.$$

(c) We want $y - y_0$, we know v_0, a, and v. The same equation gives

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\left(\frac{v_0}{2}\right)^2 = v_0^2 + 2(-g)(y - y_0)$$

$$y - y_0 = \frac{3v_0^2}{8g} = \frac{3(2gH)}{8g} = \frac{3H}{4}.$$

EVALUATE: When the ball is half way to the top, $v \neq v_0/2$ because the motion equations involve the *squares* of quanties such as v^2 and t^2.

VP2.8.1. **IDENTIFY:** The rock is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: The equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ applies.

EXECUTE: (a) With the origin at the hand and the y-axis positive upward, $y = 4.00$ m and $y_0 = 0$. We want the time at which this occurs. The equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

4.00 m = (12.0 m/s)t + $\frac{1}{2}$(–9.80 m/s^2)t^2. Solving this quadratic equation for t gives two answers: t = 0.398 s and t = 2.05 s.

(b) Use the same procedure as in (a) except that y = –4.00 m. This gives

–4.00 m = (12.0 m/s)t + $\frac{1}{2}$(–9.80 m/s^2)t^2. This quadratic equation has two solutions, t = 2.75 s and t = –0.297 s. The negative answer is not physical, so t = 2.75 s.

EVALUATE: The ball is at 4.00 m above your hand twice, when it is going up and when it is going down, so we get two answers. It is at 4.00 m below the hand only once, when it is going down, so we have just one answer.

VP2.8.2. **IDENTIFY:** The ball is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertical with the origin at the hand, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. We know that a_y = 9.80 m/s^2 downward and v_y is initially 9.00 m/s downward. Since all the quantities are downward, it is convenient to call the +y-axis downward.

EXECUTE: First find v_y when y = 5.00 m. Then use this result to find the time for the ball to reach this height.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ = (9.00 m/s)^2 + 2(9.80 m/s^2)(5.00 m)

v_y = 13.38 m/s

Now use $v_y = v_{0y} + a_y t$ to find the time t.

13.38 m/s = 9.00 m/s + (9.80 m/s^2)t → t = 0.447 s.

EVALUATE: Check using the equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ when t = 0.447 s.

y = (9.00 m/s)(0.447 s) + $\frac{1}{2}$(9.80 m/s^2)(0.447 s)^2 = 5.00 m, which agrees with the given value.

VP2.8.3. **IDENTIFY:** The apple is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertically upward with the origin at the hand, the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. We know that a_y = 9.80 m/s^2 downward and v_y is initially 5.50 m/s upward.

EXECUTE: (a) Using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

1.30 m = (5.50 m/s)t + $\frac{1}{2}$(–9.80 m/s^2)(0.447 s)^2

Solving using the quadratic formula gives t = 0.338 s and t = 0.784 s. The apple passes through this point twice: going up at 0.338 s and going down at 0784 s.

(b) Use the same approach as in (a).

1.80 m = (5.50 m/s)t + $\frac{1}{2}$(–9.80 m/s^2)t^2

This equation has no real solutions, so the apple *never* reaches a height of 1.80 m.

EVALUATE: The highest point the apple reaches is when v_y = 0. Use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find the maximum height.

0 = (5.50 m/s)^2 + 2(–9.80 m/s^2)($y - y_0$)

$y - y_0$ = 1.54 m, which is *less than* 1.80 m. This is why we had no solutions to the quadratic equation in part (b).

VP2.8.4. **IDENTIFY:** The orange is in freefall so its acceleration is g downward and the constant-acceleration equations apply.

SET UP: Calling the y-axis vertically upward with the origin at the hand, the formula $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ applies. We know that $a_y = g$ downward and v_y is initially v_0 upward.

EXECUTE: We want the time when $y = \frac{v_0^2}{2g}$, so we use $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$.

$\frac{v_0^2}{2g} = v_0 t - \frac{1}{2}gt^2$. Solving this quadratic equation gives $t = v_0/g$. There is only one solution, so the orange reaches this height only once.

(b) Use the same approach as in part (a). $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives

$\frac{3v_0^2}{8g} = v_0 t - \frac{1}{2}gt^2$. The quadratic formula gives two solutions: $t = v_0/2g$ and $t = 3v_0/2g$.

The orange is going up at the smaller solution and going down at the larger solution.

EVALUATE: We got only one solution for part (a) because $y = \frac{v_0^2}{2g}$ is the highest point the orange reaches, and that occurs only once because the orange stops there. In (b) the height $\frac{3v_0^2}{8g}$ is less than the maximum height, so the orange reaches this height twice, once going up and once going down.

2.1. **IDENTIFY:** $\Delta x = v_{\text{av-}x}\Delta t$

SET UP: We know the average velocity is 6.25 m/s.

EXECUTE: $\Delta x = v_{\text{av-}x}\Delta t = 25.0$ m

EVALUATE: In round numbers, 6 m/s × 4 s = 24 m ≈ 25 m, so the answer is reasonable.

2.2. **IDENTIFY:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$

SET UP: $13.5 \text{ days} = 1.166\times10^6 \text{ s}$. At the release point, $x = +5.150\times10^6 \text{ m}$.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{x_2 - x_1}{\Delta t} = \dfrac{-5.150\times10^6 \text{ m}}{1.166\times10^6 \text{ s}} = -4.42 \text{ m/s}$.

(b) For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

EVALUATE: The average velocity for the trip from the nest to the release point is positive.

2.3. **IDENTIFY:** Target variable is the time Δt it takes to make the trip in heavy traffic. Use Eq. (2.2) that relates the average velocity to the displacement and average time.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-}x}\Delta t$ and $\Delta t = \dfrac{\Delta x}{v_{\text{av-}x}}$.

EXECUTE: Use the information given for normal driving conditions to calculate the distance between the two cities, where the time is 1 h and 50 min, which is 110 min:

$$\Delta x = v_{\text{av-}x}\Delta t = (105 \text{ km/h})(1 \text{ h}/60 \text{ min})(110 \text{ min}) = 192.5 \text{ km}.$$

Now use $v_{\text{av-}x}$ for heavy traffic to calculate Δt; Δx is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-}x}} = \frac{192.5 \text{ km}}{70 \text{ km/h}} = 2.75 \text{ h} = 2 \text{ h and } 45 \text{ min}.$$

The additional time is (2 h and 45 min) – (1 h and 50 min) = (1 h and 105 min) – (1 h and 50 min) = 55 min.

EVALUATE: At the normal speed of 105 km/s the trip takes 110 min, but at the reduced speed of 70 km/h it takes 165 min. So decreasing your average speed by about 30% adds 55 min to the time, which is 50% of 110 min. Thus a 30% reduction in speed leads to a 50% increase in travel time. This result (perhaps surprising) occurs because the time interval is inversely proportional to the average speed, not directly proportional to it.

2.4. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled divided by the time.

SET UP: The post is 80 m west of the pillar. The total distance traveled is $200 \text{ m} + 280 \text{ m} = 480 \text{ m}$.

EXECUTE: **(a)** The eastward run takes time $\dfrac{200 \text{ m}}{5.0 \text{ m/s}} = 40.0 \text{ s}$ and the westward run takes $\dfrac{280 \text{ m}}{4.0 \text{ m/s}} = 70.0 \text{ s}$. The average speed for the entire trip is $\dfrac{480 \text{ m}}{110.0 \text{ s}} = 4.4 \text{ m/s}$.

(b) $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{-80 \text{ m}}{110.0 \text{ s}} = -0.73 \text{ m/s}$. The average velocity is directed westward.

EVALUATE: The displacement is much less than the distance traveled, and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

2.5. **IDENTIFY:** Given two displacements, we want the average velocity and the average speed.

SET UP: The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ and the average speed is just the total distance walked divided by the total time to walk this distance.

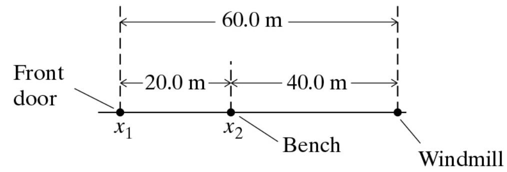


Figure 2.5

EXECUTE: (a) Let $+x$ be eastward with the origin at the front door. The trip begins at the front door and ends at the bench as shown in Fig. 2.5. Therefore $x_1 = 0.00$ m and $x_2 = 20.0$ m.
$\Delta x = x_2 - x_1 = 20.0\text{ m} - 0.00\text{ m} = 20.0\text{ m}$. The total time is $\Delta t = 28.0\text{ s} + 36.0\text{ s} = 64.0\text{ s}$. So
$v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{20.0\text{ m}}{64.0\text{ s}} = 0.313\text{ m/s}.$

(b) Average speed $= \dfrac{60.0\text{ m} + 40.0\text{ m}}{64.0\text{ s}} = 1.56\text{ m/s}$.

EVALUATE: The average speed is much greater than the average velocity because the total distance walked is much greater than the magnitude of the displacement vector.

2.6. IDENTIFY: The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use $x(t)$ to find x for each t.

SET UP: $x(0) = 0$, $x(2.00\text{ s}) = 5.60\text{ m}$, and $x(4.00\text{ s}) = 20.8\text{ m}$

EXECUTE: (a) $v_{\text{av-}x} = \dfrac{5.60\text{ m} - 0}{2.00\text{ s}} = +2.80\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 0}{4.00\text{ s}} = +5.20\text{ m/s}$

(c) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 5.60\text{ m}}{2.00\text{ s}} = +7.60\text{ m/s}$

EVALUATE: The average velocity depends on the time interval being considered.

2.7. (a) IDENTIFY: Calculate the average velocity using $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$
$t = 10.0$ s: $x = (2.40\text{ m/s}^2)(10.0\text{ s})^2 - (0.120\text{ m/s}^3)(10.0\text{ s})^3 = 240\text{ m} - 120\text{ m} = 120\text{ m}.$
Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120\text{ m}}{10.0\text{ s}} = 12.0\text{ m/s}.$

(b) IDENTIFY: Use $v_x = \dfrac{dx}{dt}$ to calculate $v_x(t)$ and evaluate this expression at each specified t.

SET UP: $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2$.

EXECUTE: (i) $t = 0$: $v_x = 0$
(ii) $t = 5.0$ s: $v_x = 2(2.40\text{ m/s}^2)(5.0\text{ s}) - 3(0.120\text{ m/s}^3)(5.0\text{ s})^2 = 24.0\text{ m/s} - 9.0\text{ m/s} = 15.0\text{ m/s}.$
(iii) $t = 10.0$ s: $v_x = 2(2.40\text{ m/s}^2)(10.0\text{ s}) - 3(0.120\text{ m/s}^3)(10.0\text{ s})^2 = 48.0\text{ m/s} - 36.0\text{ m/s} = 12.0\text{ m/s}.$
(c) IDENTIFY: Find the value of t when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$
$v_x = 0$ at $t = 0$.
$v_x = 0$ next when $2bt - 3ct^2 = 0$

Execute: $2b = 3ct$ so $t = \dfrac{2b}{3c} = \dfrac{2(2.40\ \text{m/s}^2)}{3(0.120\ \text{m/s}^3)} = 13.3\ \text{s}$

Evaluate: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

2.8. **Identify:** We know the position $x(t)$ of the bird as a function of time and want to find its instantaneous velocity at a particular time.

Set Up: The instantaneous velocity is $v_x(t) = \dfrac{dx}{dt} = \dfrac{d\left[28.0\ \text{m} + (12.4\ \text{m/s})t - (0.0450\ \text{m/s}^3)t^3\right]}{dt}$.

Execute: $v_x(t) = \dfrac{dx}{dt} = 12.4\ \text{m/s} - (0.135\ \text{m/s}^3)t^2$. Evaluating this at $t = 8.0$ s gives $v_x = 3.76$ m/s.

Evaluate: The acceleration is not constant in this case.

2.9. **Identify:** The average velocity is given by $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. We can find the displacement Δt for each constant velocity time interval. The average speed is the distance traveled divided by the time.

Set Up: For $t = 0$ to $t = 2.0$ s, $v_x = 2.0$ m/s. For $t = 2.0$ s to $t = 3.0$ s, $v_x = 3.0$ m/s. In part (b), $v_x = -3.0$ m/s for $t = 2.0$ s to $t = 3.0$ s. When the velocity is constant, $\Delta x = v_x \Delta t$.

Execute: **(a)** For $t = 0$ to $t = 2.0$ s, $\Delta x = (2.0\ \text{m/s})(2.0\ \text{s}) = 4.0$ m. For $t = 2.0$ s to $t = 3.0$ s, $\Delta x = (3.0\ \text{m/s})(1.0\ \text{s}) = 3.0$ m. For the first 3.0 s, $\Delta x = 4.0\ \text{m} + 3.0\ \text{m} = 7.0$ m. The distance traveled is also 7.0 m. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{7.0\ \text{m}}{3.0\ \text{s}} = 2.33$ m/s. The average speed is also 2.33 m/s.

(b) For $t = 2.0$ s to 3.0 s, $\Delta x = (-3.0\ \text{m/s})(1.0\ \text{s}) = -3.0$ m. For the first 3.0 s, $\Delta x = 4.0\ \text{m} + (-3.0\ \text{m}) = +1.0$ m. The ball travels 4.0 m in the $+x$-direction and then 3.0 m in the $-x$-direction, so the distance traveled is still 7.0 m. $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.0\ \text{m}}{3.0\ \text{s}} = 0.33$ m/s. The average speed is $\dfrac{7.00\ \text{m}}{3.00\ \text{s}} = 2.33$ m/s.

Evaluate: When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

2.10. **Identify** and **Set Up:** The instantaneous velocity is the slope of the tangent to the x versus t graph.

Execute: **(a)** The velocity is zero where the graph is horizontal; point IV.

(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.

(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.

(d) The slope is positive and increasing at point II.

(e) The slope is positive and decreasing at point III.

Evaluate: The sign of the velocity indicates its direction.

2.11. **Identify:** Find the instantaneous velocity of a car using a graph of its position as a function of time.

Set Up: The instantaneous velocity at any point is the slope of the x versus t graph at that point. Estimate the slope from the graph.

Execute: A: $v_x = 6.7$ m/s; B: $v_x = 6.7$ m/s; C: $v_x = 0$; D: $v_x = -40.0$ m/s; E: $v_x = -40.0$ m/s; F: $v_x = -40.0$ m/s; G: $v_x = 0$.

Evaluate: The sign of v_x shows the direction the car is moving. v_x is constant when x versus t is a straight line.

2.12. **Identify:** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the v_x versus t graph.

Set Up: 60 km/h = 16.7 m/s

EXECUTE: **(a)** (i) $a_{\text{av-}x} = \dfrac{16.7\text{ m/s} - 0}{10\text{ s}} = 1.7\text{ m/s}^2$. (ii) $a_{\text{av-}x} = \dfrac{0 - 16.7\text{ m/s}}{10\text{ s}} = -1.7\text{ m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$. (iv) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

(b) At $t = 20$ s, v_x is constant and $a_x = 0$. At $t = 35$ s, the graph of v_x versus *t* is a straight line and $a_x = a_{\text{av-}x} = -1.7\text{ m/s}^2$.

EVALUATE: When $a_{\text{av-}x}$ and v_x have the same sign the speed is increasing. When they have opposite signs, the speed is decreasing.

2.13. **IDENTIFY** and **SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \dfrac{dx}{dt} = 2.00\text{ cm/s} - (0.125\text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0$ cm, $v_x = 2.00$ cm/s, $a_x = -0.125\text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for *t*: $t = 16.0$ s.

(c) Set $x = 50.0$ cm and solve for *t*. This gives $t = 0$ and $t = 32.0$ s. The turtle returns to the starting point after 32.0 s.

(d) The turtle is 10.0 cm from starting point when $x = 60.0$ cm or $x = 40.0$ cm.

Set $x = 60.0$ cm and solve for *t*: $t = 6.20$ s and $t = 25.8$ s.

At $t = 6.20$ s, $v_x = +1.23$ cm/s.

At $t = 25.8$ s, $v_x = -1.23$ cm/s.

Set $x = 40.0$ cm and solve for *t*: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).

At $t = 36.4$ s, $v_x = -2.55$ cm/s.

(e) The graphs are sketched in Figure 2.13.

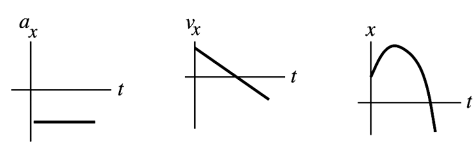


Figure 2.13

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

2.14. **IDENTIFY:** We know the velocity $v(t)$ of the car as a function of time and want to find its acceleration at the instant that its velocity is 12.0 m/s.

SET UP: We know that $v_x(t) = (0.860\text{ m/s}^3)t^2$ and that $a_x(t) = \dfrac{dv_x}{dt} = \dfrac{d\left[(0.860\text{ m/s}^3)t^2\right]}{dt}$.

EXECUTE: $a_x(t) = \dfrac{dv_x}{dt} = (1.72\text{ m/s}^3)t$. When $v_x = 12.0$ m/s, $(0.860\text{ m/s}^3)t^2 = 12.0$ m/s, which gives $t = 3.735$ s. At this time, $a_x = 6.42\text{ m/s}^2$.

EVALUATE: The acceleration of this car is not constant.

2.15. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find v_x at each t. The instantaneous acceleration is $a_x = \frac{dv_x}{dt}$.

SET UP: $v_x(0) = 3.00\text{ m/s}$ and $v_x(5.00\text{ s}) = 5.50\text{ m/s}$.

EXECUTE: **(a)** $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{5.50\text{ m/s} - 3.00\text{ m/s}}{5.00\text{ s}} = 0.500\text{ m/s}^2$

(b) $a_x = \frac{dv_x}{dt} = (0.100\text{ m/s}^3)(2t) = (0.200\text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00\text{ s}$, $a_x = 1.00\text{ m/s}^2$.

(c) Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.15.

EVALUATE: $a_x(t)$ is the slope of $v_x(t)$ and increases as t increases. The average acceleration for $t = 0$ to $t = 5.00\text{ s}$ equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50\text{ s}$, since $a_x(t)$ is a linear function of t.

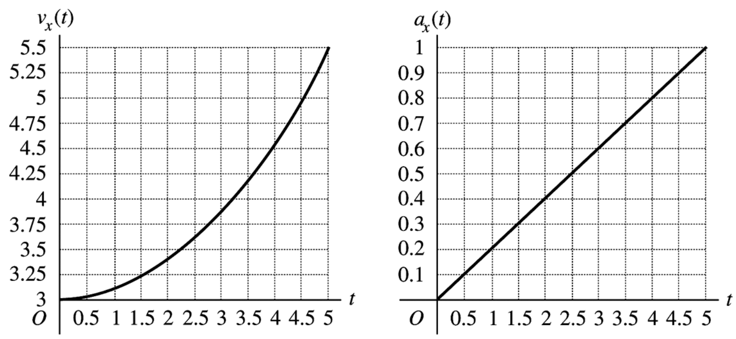


Figure 2.15

2.16. **IDENTIFY:** Use $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t}$, with $\Delta t = 10\text{ s}$ in all cases.

SET UP: v_x is negative if the motion is to the left.

EXECUTE: **(a)** $[(5.0\text{ m/s}) - (15.0\text{ m/s})]/(10\text{ s}) = -1.0\text{ m/s}^2$

(b) $[(-15.0\text{ m/s}) - (-5.0\text{ m/s})]/(10\text{ s}) = -1.0\text{ m/s}^2$

(c) $[(-15.0\text{ m/s}) - (+15.0\text{ m/s})]/(10\text{ s}) = -3.0\text{ m/s}^2$

EVALUATE: In all cases, the negative acceleration indicates an acceleration to the left.

2.17. **IDENTIFY:** $v_x(t) = \frac{dx}{dt}$ and $a_x(t) = \frac{dv_x}{dt}$

SET UP: $\frac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

EXECUTE: **(a)** $v_x(t) = (9.60\text{ m/s}^2)t - (0.600\text{ m/s}^6)t^5$ and $a_x(t) = 9.60\text{ m/s}^2 - (3.00\text{ m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00\text{ s}$. At $t = 0$, $x = 2.17\text{ m}$ and $a_x = 9.60\text{ m/s}^2$. At $t = 2.00\text{ s}$, $x = 15.0\text{ m}$ and $a_x = -38.4\text{ m/s}^2$.

(b) The graphs are given in Figure 2.17.

EVALUATE: For the entire time interval from $t = 0$ to $t = 2.00\text{ s}$, the velocity v_x is positive and x increases. While a_x is also positive the speed increases and while a_x is negative the speed decreases.

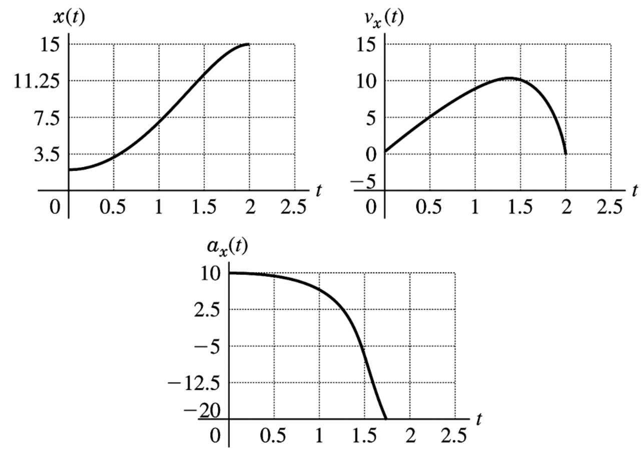


Figure 2.17

2.18. **IDENTIFY:** We have motion with constant acceleration, so the constant-acceleration equations apply. We want to determine the acceleration of a car on the entrance ramp of a highway.

SET UP: Estimate: 300 ft on the entrance ramp. We know the initial and final velocities and the distance traveled, and we want to find the acceleration. So $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ applies.

EXECUTE: Convert units: $30\text{ mph}\left(\dfrac{1.466\text{ ft/s}}{1\text{ mph}}\right) = 44\text{ ft/s}$; $70\text{ mph}\left(\dfrac{1.466\text{ ft/s}}{1\text{ mph}}\right) = 103\text{ ft/s}.$

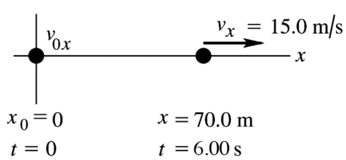
Now use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$: $(103\text{ ft/s})^2 = (44\text{ ft/s})^2 + 2a_x(300\text{ ft})$ $\rightarrow$ $a_x = 14\text{ ft/s}^2$.

In SI units, $a_x = 14\text{ ft/s}^2\left(\dfrac{0.3048\text{ m/s}^2}{1\text{ ft/s}^2}\right) = 4.4\text{ m/s}^2$.

EVALUATE: Compare with acceleration due to gravity: $a/g = (14\text{ ft/s}^2)/(32\text{ ft/s}^2) = 0.45$, so this acceleration is around 45% of g. This seems rather large for an ordinary car. Calculate the time to reach the 70 mph speed, using $v_x = v_{0x} + a_x t$: $103\text{ ft/s} = 44\text{ ft/s} + (14\text{ ft/s}^2)t$ $\rightarrow$ $t = 4.2$ s. This is too small to be reasonable for most cars. My estimate of the length of the on-ramp must be too small.

2.19. **IDENTIFY:** Use the constant acceleration equations to find v_{0x} and a_x.

(a) SET UP: The situation is sketched in Figure 2.19.



$x - x_0 = 70.0$ m

$t = 6.00$ s

$v_x = 15.0$ m/s

$v_{0x} = ?$

Figure 2.19

EXECUTE: Use $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$, so $v_{0x} = \dfrac{2(x - x_0)}{t} - v_x = \dfrac{2(70.0\text{ m})}{6.00\text{ s}} - 15.0\text{ m/s} = 8.33\text{ m/s}.$

(b) Use $v_x = v_{0x} + a_x t$, so $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{15.0\text{ m/s} - 5.0\text{ m/s}}{6.00\text{ s}} = 1.11\text{ m/s}^2.$

EVALUATE: The average velocity is $(70.0\text{ m})/(6.00\text{ s}) = 11.7$ m/s. The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x} < v_x$ and $a_x > 0$.

2.20. IDENTIFY: For constant acceleration, the standard kinematics equations apply.
SET UP: Assume the ball moves in the $+x$ direction.
EXECUTE: **(a)** $v_x = 73.14\text{ m/s}$, $v_{0x} = 0$ and $t = 30.0\text{ ms}$. $v_x = v_{0x} + a_x t$ gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{73.14\text{ m/s} - 0}{30.0\times 10^{-3}\text{ s}} = 2440\text{ m/s}^2.$$

(b) $x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t = \left(\dfrac{0 + 73.14\text{ m/s}}{2}\right)(30.0\times 10^{-3}\text{ s}) = 1.10\text{ m}$.

EVALUATE: We could also use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to calculate $x - x_0$:

$x - x_0 = \frac{1}{2}(2440\text{ m/s}^2)(30.0\times 10^{-3}\text{ s})^2 = 1.10\text{ m}$, which agrees with our previous result. The acceleration of the ball is very large.

2.21. IDENTIFY: For constant acceleration, the standard kinematics equations apply.
SET UP: Assume the ball starts from rest and moves in the $+x$-direction.
EXECUTE: **(a)** $x - x_0 = 1.50\text{ m}$, $v_x = 45.0\text{ m/s}$ and $v_{0x} = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x-x_0)} = \frac{(45.0\text{ m/s})^2}{2(1.50\text{ m})} = 675\text{ m/s}^2.$$

(b) $x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t$ gives $t = \dfrac{2(x-x_0)}{v_{0x}+v_x} = \dfrac{2(1.50\text{ m})}{45.0\text{ m/s}} = 0.0667\text{ s}$

EVALUATE: We could also use $v_x = v_{0x} + a_x t$ to find $t = \dfrac{v_x}{a_x} = \dfrac{45.0\text{ m/s}}{675\text{ m/s}^2} = 0.0667\text{ s}$ which agrees with our previous result. The acceleration of the ball is very large.

2.22. IDENTIFY: A car is slowing down with uniform acceleration, so the constant-acceleration equations apply. We want to determine the acceleration of the car as it slows down.
SET UP: Estimate: It takes 5.0 s to reduce the speed from 70 mph to 30 mph. $v_x = v_{0x} + a_x t$,

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, and $a_{x\text{-av}} = \dfrac{\Delta v_x}{\Delta t}$ apply. Call the $+x$-axis the direction in which the car is moving.

EXECUTE: **(a)** Δv_x = 30 mph – 70 mph = –40 mph = –59 ft/s, so $a_{x\text{-av}} = \dfrac{-59\text{ ft/s}}{5.0\text{ s}} = -12\text{ ft/s}^2$. The magnitude is 12 ft/s^2 and its direction is opposite to the velocity of the car.
(b) The initial velocity is 70 mph = 103 ft/s and $v_x = 0$ when the car stops, so $v_x = v_{0x} + a_x t$ gives $0 = 103$ ft/s + (–12 ft/s^2)t → $t = 8.6$ s. This is the time from first hitting the brakes. The time to stop from 30 mph is 8.6 s – 5.0 s = 3.6 s.
(c) Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ since we know everything in it except the distance traveled.
$0 = (103\text{ ft/s})^2 + 2(-12\text{ ft/s}^2)(x - x_0)$ → $x - x_0 = 440$ ft.

EVALUATE: As a check, use $x = x_0 + v_{0x}t + \dfrac{1}{2}a_x t^2$ to calculate the distance in (c). This gives

$x = 0 + (103\text{ ft/s})(8.6\text{ s}) + \dfrac{1}{2}(-12\text{ ft/s}^2)(8.6\text{ s})^2 = 440$ ft, which agrees with our answer. Compare the acceleration to g: $a_x g = (12\text{ ft/s}^2)/(32\text{ ft/s}^2) = 0.38$, so a_x is about 38% of g. This is fairly large, but if you really slam on your brakes, it might be reasonable.

2.23. IDENTIFY: Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x = -250\text{ m/s}^2$.
105 km/h = 29.17 m/s.

EXECUTE: $v_{0x} = 29.17 \text{ m/s}$. $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (29.17 \text{ m/s})^2}{2(-250 \text{ m/s}^2)} = 1.70 \text{ m}.$$

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

2.24. IDENTIFY: Apply constant acceleration equations to the motion of the car.
SET UP: Let $+x$ be the direction the car is moving.

EXECUTE: **(a)** From $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, with $v_{0x} = 0$, $a_x = \dfrac{v_x^2}{2(x - x_0)} = \dfrac{(20 \text{ m/s})^2}{2(120 \text{ m})} = 1.67 \text{ m/s}^2$.

(b) Using $x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$, we have $t = 2(x - x_0)/v_x = 2(120 \text{ m})/(20 \text{ m/s}) = 12 \text{ s}$.

(c) $(12 \text{ s})(20 \text{ m/s}) = 240 \text{ m}$.

EVALUATE: The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

2.25. IDENTIFY: If a person comes to a stop in 36 ms while slowing down with an acceleration of 60*g*, how far does he travel during this time?
SET UP: Let $+x$ be the direction the person travels. $v_x = 0$ (he stops), a_x is negative since it is opposite to the direction of the motion, and $t = 36 \text{ ms} = 3.6 \times 10^{-2} \text{ s}$. The equations $v_x = v_{0x} + a_x t$ and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ both apply since the acceleration is constant.

EXECUTE: Solving $v_x = v_{0x} + a_x t$ for v_{0x} gives $v_{0x} = -a_x t$. Then $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x = -\frac{1}{2}a_x t^2 = -\frac{1}{2}(-588 \text{ m/s}^2)(3.6 \times 10^{-2} \text{ s})^2 = 38 \text{ cm}$.

EVALUATE: Notice that we were not given the initial speed, but we could find it:

$$v_{0x} = -a_x t = -(-588 \text{ m/s}^2)(36 \times 10^{-3} \text{ s}) = 21 \text{ m/s} = 47 \text{ mph}.$$

2.26. IDENTIFY: The acceleration a_x is the slope of the graph of v_x versus *t*.
SET UP: The signs of v_x and of a_x indicate their directions.
EXECUTE: **(a)** Reading from the graph, at $t = 4.0 \text{ s}$, $v_x = 2.7 \text{ cm/s}$, to the right and at $t = 7.0 \text{ s}$, $v_x = 1.3 \text{ cm/s}$, to the left.

(b) v_x versus *t* is a straight line with slope $-\dfrac{8.0 \text{ cm/s}}{6.0 \text{ s}} = -1.3 \text{ cm/s}^2$. The acceleration is constant and equal to 1.3 cm/s^2, to the left. It has this value at all times.

(c) Since the acceleration is constant, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. Call $x_0 = 0$ the cat's position when $t = 0$.

<u>During the first 4.5 s</u>: $x = (8.0 \text{ cm/s})(4.5 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(4.5 \text{ s})^2 = 22.8 \text{ cm}$ which rounds to 23 cm.

This is the change in the cat's position, but it is also the distance the cat walks.
<u>During the first 7.5 s</u>: After 6.0 s, v_x becomes negative so the cat is walking backward. However the *distance* it is moving does *not* become negative. The position of the car at $t = 6.0$ s is $x = (8.0 \text{ cm/s})(6.0 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(6.0 \text{ s})^2 = 24.6 \text{ cm}$. At the end of 7.5 s, the position is $x = (8.0 \text{ cm/s})(7.5 \text{ s}) + \frac{1}{2}(-1.3 \text{ cm/s}^2)(7.5 \text{ s})^2 = 23.4 \text{ cm}$. Therefore from $t = 6.0$ s to $t = 7.5$ s, the cat has walked back a *distance* of 24.6 cm – 23.4 cm = 1.2 cm. During the first 7.5 s, the cat has walked a total distance of 24.6 cm + 1.2 cm = 25.8 cm which rounds to 26 cm.
(d) The graphs of a_x and *x* versus *t* are given in Figure 2.26.

EVALUATE: In part (c) we could have instead used $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t.$

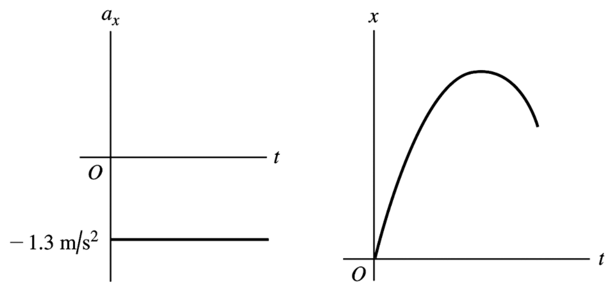


Figure 2.26

2.27. IDENTIFY: We know the initial and final velocities of the object, and the distance over which the velocity change occurs. From this we want to find the magnitude and duration of the acceleration of the object.

SET UP: The constant-acceleration kinematics formulas apply. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0),$ where $v_{0x} = 0,\ v_x = 5.0\times10^3$ m/s, and $x - x_0 = 4.0$ m.

EXECUTE: **(a)** $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(5.0\times10^3\ \text{m/s})^2}{2(4.0\ \text{m})} = 3.1\times10^6\ \text{m/s}^2 = 3.2\times10^5\ g.$$

(b) $v_x = v_{0x} + a_x t$ gives $t = \frac{v_x - v_{0x}}{a_x} = \frac{5.0\times10^3\ \text{m/s}}{3.1\times10^6\ \text{m/s}^2} = 1.6$ ms.

EVALUATE: **(c)** The calculated a is less than 450,000 g so the acceleration required doesn't rule out this hypothesis.

2.28. IDENTIFY: $v_x(t)$ is the slope of the x versus t graph. Car B moves with constant speed and zero acceleration. Car A moves with positive acceleration; assume the acceleration is constant.

SET UP: For car B, v_x is positive and $a_x = 0$. For car A, a_x is positive and v_x increases with t.

EXECUTE: **(a)** The motion diagrams for the cars are given in Figure 2.28a.

(b) The two cars have the same position at times when their x-t graphs cross. The figure in the problem shows this occurs at approximately $t = 1$ s and $t = 3$ s.

(c) The graphs of v_x versus t for each car are sketched in Figure 2.28b.

(d) The cars have the same velocity when their x-t graphs have the same slope. This occurs at approximately $t = 2$ s.

(e) Car A passes car B when x_A moves above x_B in the x-t graph. This happens at $t = 3$ s.

(f) Car B passes car A when x_B moves above x_A in the x-t graph. This happens at $t = 1$ s.

EVALUATE: When $a_x = 0$, the graph of v_x versus t is a horizontal line. When a_x is positive, the graph of v_x versus t is a straight line with positive slope.

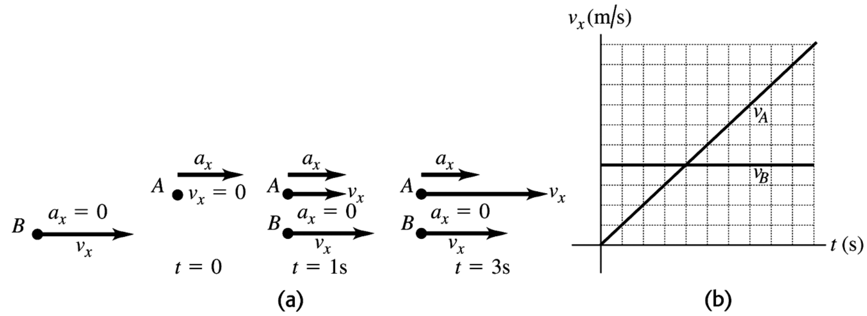


Figure 2.28

2.29. **(a) IDENTIFY** and **SET UP:** The acceleration a_x at time t is the slope of the tangent to the v_x versus t curve at time t.

EXECUTE: At $t = 3$ s, the v_x versus t curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the v_x versus t curve is a straight-line segment with slope $\dfrac{45\text{ m/s} - 20\text{ m/s}}{9\text{ s} - 5\text{ s}} = 6.3\text{ m/s}^2$.

Thus $a_x = 6.3\text{ m/s}^2$.

At $t = 11$ s the curve is again a straight-line segment, now with slope $\dfrac{-0 - 45\text{ m/s}}{13\text{ s} - 9\text{ s}} = -11.2\text{ m/s}^2$.

Thus $a_x = -11.2\text{ m/s}^2$.

EVALUATE: $a_x = 0$ when v_x is constant, $a_x > 0$ when v_x is positive and the speed is increasing, and $a_x < 0$ when v_x is positive and the speed is decreasing.

(b) IDENTIFY: Calculate the displacement during the specified time interval.

SET UP: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5$ s the acceleration is constant and equal to zero. For the time interval $t = 5$ s to $t = 9$ s the acceleration is constant and equal to 6.25 m/s^2. For the interval $t = 9$ s to $t = 13$ s the acceleration is constant and equal to -11.2 m/s^2.

EXECUTE: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$v_{0x} = 20\text{ m/s}$ $\quad a_x = 0$ $\quad t = 5\text{ s}$ $\quad x - x_0 = ?$

$x - x_0 = v_{0x}t$ ($a_x = 0$ so no $\frac{1}{2}a_xt^2$ term)

$x - x_0 = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$; this is the distance the officer travels in the first 5 seconds.

During the interval $t = 5$ s to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4-second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 4$ s. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9$ s interval.)

$v_{0x} = 20\text{ m/s}$ $\quad a_x = 6.25\text{ m/s}^2$ $\quad t = 4\text{ s}$ $\quad x_0 = 100\text{ m}$ $\quad x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$

$x - x_0 = (20\text{ m/s})(4\text{ s}) + \frac{1}{2}(6.25\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m} + 50\text{ m} = 130\text{ m}.$

Thus $x - x_0 + 130\text{ m} = 100\text{ m} + 130\text{ m} = 230\text{ m}.$

At $t = 9$ s the officer is at $x = 230$ m, so she has traveled 230 m in the first 9 seconds.

During the interval $t = 9$ s to $t = 13$ s the acceleration is again constant. The constant acceleration formulas can be applied for this 4-second interval but *not* for the whole $t = 0$ to $t = 13$ s interval. To use the equations restart our clock so this interval begins at time $t = 0$ and ends at time $t = 4$ s.

$v_{0x} = 45$ m/s (at the start of this time interval)

$a_x = -11.2 \text{ m/s}^2$ $t = 4$ s $x_0 = 230$ m $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$x - x_0 = (45 \text{ m/s})(4 \text{ s}) + \frac{1}{2}(-11.2 \text{ m/s}^2)(4 \text{ s})^2 = 180 \text{ m} - 89.6 \text{ m} = 90.4 \text{ m}.$

Thus $x = x_0 + 90.4 \text{ m} = 230 \text{ m} + 90.4 \text{ m} = 320 \text{ m}.$

At $t = 13$ s the officer is at $x = 320$ m, so she has traveled 320 m in the first 13 seconds.

EVALUATE: The velocity v_x is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval Δt is $v_{\text{av-}x} = \Delta x/\Delta t.$ For $t = 0$ to 5 s, $v_{\text{av-}x} = 20$ m/s. For $t = 0$ to 9 s, $v_{\text{av-}x} = 26$ m/s. For $t = 0$ to 13 s, $v_{\text{av-}x} = 25$ m/s. These results are consistent with the figure in the textbook.

2.30. **IDENTIFY:** For constant acceleration, the kinematics formulas apply. We can use the total displacement and final velocity to calculate the acceleration and then use the acceleration and shorter distance to find the speed.

SET UP: Take +*x* to be down the incline, so the motion is in the +*x* direction. The formula $v_x^2 = v_{0x}^2 + 2a(x - x_0)$ applies.

EXECUTE: First look at the motion over 6.80 m. We use the following numbers: $v_{0x} = 0$, $x - x_0 = 6.80$ m, and $v_x = 3.80$ /s. Solving the above equation for a_x gives $a_x = 1.062 \text{ m/s}^2$. Now look at the motion over the 3.40 m using $v_{0x} = 0$, $a_x = 1.062 \text{ m/s}^2$ and $x - x_0 = 3.40$ m. Solving the same equation, but this time for v_x, gives $v_x = 2.69$ m/s.

EVALUATE: Even though the block has traveled half way down the incline, its speed is not half of its speed at the bottom.

2.31. **IDENTIFY:** Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g$, downward. Take the origin at the ground and the positive direction to be upward.

(a) SET UP: At the maximum height $v_y = 0$.

$v_y = 0$ $y - y_0 = 0.440$ m $a_y = -9.80 \text{ m/s}^2$ $v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80 \text{ m/s}^2)(0.440 \text{ m})} = 2.94$ m/s

(b) SET UP: When the flea has returned to the ground $y - y_0 = 0$.

$y - y_0 = 0$ $v_{0y} = +2.94$ m/s $a_y = -9.80 \text{ m/s}^2$ $t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(2.94 \text{ m/s})}{-9.80 \text{ m/s}^2} = 0.600$ s.

EVALUATE: We can use $v_y = v_{0y} + a_y t$ to show that with $v_{0y} = 2.94$ m/s, $v_y = 0$ after 0.300 s.

2.32. **IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s^2. We know its initial velocity and position and its final position.

SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: **(a)** $y - y_0 = -30$ m, $v_{0y} = 22.0$ m/s, $a_y = -9.80$ m/s^2. The kinematics formulas give $v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(22.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-30 \text{ m})} = -32.74$ m/s, so the speed is 32.7 m/s.

(b) $v_y = v_{0y} + a_y t$ and $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-32.74 \text{ m/s} - 22.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 5.59$ s.

EVALUATE: The vertical velocity in part (a) is negative because the rock is moving downward, but the speed is always positive. The 5.59 s is the total time in the air.

2.33. **IDENTIFY:** The pin has a constant downward acceleration of 9.80 m/s^2 and returns to its initial position.
SET UP: We can use the kinematics formulas for constant acceleration.

EXECUTE: The kinematics formulas give $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. We know that $y - y_0 = 0$, so

$t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(8.20 \text{ m/s})}{-9.80 \text{ m/s}^2} = +1.67$ s.

EVALUATE: It takes the pin half this time to reach its highest point and the remainder of the time to return.

2.34. **IDENTIFY:** The putty has a constant downward acceleration of 9.80 m/s^2. We know the initial velocity of the putty and the distance it travels.
SET UP: We can use the kinematics formulas for constant acceleration.
EXECUTE: **(a)** $v_{0y} = 9.50$ m/s and $y - y_0 = 3.60$ m, which gives
$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(9.50 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(3.60 \text{ m})} = 4.44$ m/s

(b) $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{4.44 \text{ m/s} - 9.50 \text{ m/s}}{-9.8 \text{ m/s}^2} = 0.517$ s

EVALUATE: The putty is stopped by the ceiling, not by gravity.

2.35. **IDENTIFY:** A ball on Mars that is hit directly upward returns to the same level in 8.5 s with a constant downward acceleration of $0.379g$. How high did it go and how fast was it initially traveling upward?
SET UP: Take $+y$ upward. $v_y = 0$ at the maximum height. $a_y = -0.379g = -3.71$ m/s^2. The constant-acceleration formulas $v_y = v_{0y} + a_y t$ and $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ both apply.

EXECUTE: Consider the motion from the maximum height back to the initial level. For this motion $v_{0y} = 0$ and $t = 4.25$ s. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.71 \text{ m/s}^2)(4.25 \text{ s})^2 = -33.5$ m. The ball went 33.5 m above its original position.
(b) Consider the motion from just after it was hit to the maximum height. For this motion $v_y = 0$ and $t = 4.25$ s. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = -(-3.71 \text{ m/s}^2)(4.25 \text{ s}) = 15.8$ m/s.
(c) The graphs are sketched in Figure 2.35.

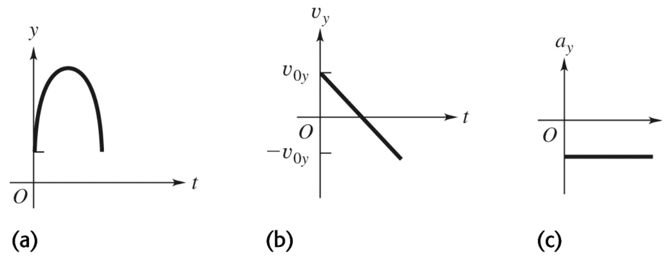


Figure 2.35

EVALUATE: The answers can be checked several ways. For example, $v_y = 0,\ v_{0y} = 15.8$ m/s, and

$a_y = -3.71\text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (15.8\text{ m/s})^2}{2(-3.71\text{ m/s}^2)} = 33.6$ m,

which agrees with the height calculated in (a).

2.36. **IDENTIFY:** A baseball is thrown upward, so its acceleration is downward and uniform. Therefore the constant-acceleration equations apply.

SET UP: Estimate: It is not easy to throw a ball straight up, so estimate 25 ft for the maximum height. Its acceleration is $g = 32.2\text{ ft/s}^2$ downward, and the formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. Call the $+y$-axis upward, with the origin at the point where the ball leaves the hand; this makes $y_0 = 0$ and $a_y = -32.2\text{ ft/s}^2$. At its maximum height, the ball stops moving, so $v_y = 0$.

EXECUTE: **(a)** First find the initial speed of the ball using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $v_y = 0$ at the maximum height. $0 = v_{0y}^2 + 2(-32.2\text{ ft/s}^2)(25\text{ ft}) \rightarrow v_{0y} = 40.1$ ft/s. Now use

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to get the time t to reach the maximum height.'

$0 = 0 + (40.1\text{ ft/s})t + \frac{1}{2}(-32.2\text{ ft/s}^2)t^2 \rightarrow t = 2.5$ s.

(b) Estimate: The ball moves about 2.5 ft while it is being thrown. We want the average accelerating during this time. The ball starts from rest and reaches a velocity of 40.1 ft/s while traveling 2.5 ft upward. So we use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.

$(40.1\text{ ft/s})^2 = 0 + 2a_y(2.5\text{ ft}) \rightarrow a_y = 320\text{ ft/s}^2 \approx 10g$.

EVALUATE: The result in (b) seems rather large, so maybe the 2.5-ft estimate was too short, or maybe the maximum height of 25 ft was too large.

2.37. **IDENTIFY:** A rock is thrown upward, so its acceleration is downward and uniform. Therefore the constant-acceleration equations apply. We want to know the rock's velocity at times 1.0 s and 3.0 s after it is thrown.

SET UP: The formula $v_y = v_{0y} + a_y t$ applies. Call the $+y$-axis upward, with the origin at the point where the rock leaves the hand; this makes $y_0 = 0$, $v_{0y} = 24.0$ m/s, and $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** At 1.0 s: $v_y = v_{0y} + a_y t = 24.0\text{ m/s} + (-9.80\text{ m/s}^2)(1.0\text{ s}) = +14.2$ m/s. The acceleration is downward. The velocity is upward but the speed is decreasing because the acceleration is downward.

(b) At 3.0 s: $v_y = v_{0y} + a_y t = 24.0\text{ m/s} + (-9.80\text{ m/s}^2)(3.0\text{ s}) = -5.40$ m/s. The acceleration is downward. The velocity is downward and the speed is increasing because the acceleration is also downward. The rock has passed its highest point and is now coming down.

EVALUATE: If only gravity acts on an object, its acceleration is always downward with a magnitude of 9.80 m/s^2.

2.38. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

SET UP: Let $+y$ be downward. $a_y = 9.80\text{ m/s}^2$

EXECUTE: **(a)**

$v_{0y} = 0,\ t = 1.90\text{ s},\ a_y = 9.80\text{ m/s}^2.\ y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(9.80\text{ m/s}^2)(1.90\text{ s})^2 = 17.7\text{ m}.$

The building is 17.7 m tall.

(b) $v_y = v_{0y} + a_y t = 0 + (9.80\text{ m/s}^2)(1.90\text{ s}) = 18.6\text{ m/s}$

(c) The graphs of a_y, v_y and y versus t are given in Figure 2.38. Take $y = 0$ at the ground.

EVALUATE: We could use either $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

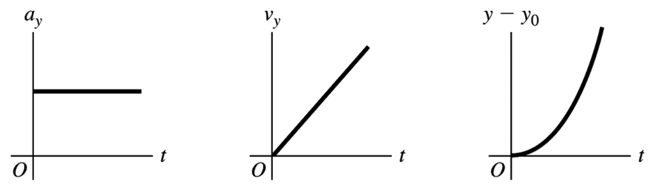


Figure 2.38

2.39. **IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

SET UP: Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80\text{ m/s}^2$. Let *d* be the distance the meterstick falls.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $d = (4.90\text{ m/s}^2)t^2$ and $t = \sqrt{\frac{d}{4.90\text{ m/s}^2}}$.

(b) $t = \sqrt{\frac{0.176\text{ m}}{4.90\text{ m/s}^2}} = 0.190\text{ s}$

EVALUATE: The reaction time is proportional to the square of the distance the stick falls.

2.40. **IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

SET UP: Let $+y$ be downward. Since the lander is in free-fall, $a_y = +1.6\text{ m/s}^2$.

EXECUTE: $v_{0y} = 0.8\text{ m/s}$, $y - y_0 = 5.0\text{ m}$, $a_y = +1.6\text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8\text{ m/s})^2 + 2(1.6\text{ m/s}^2)(5.0\text{ m})} = 4.1\text{ m/s}$.

EVALUATE: The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

2.41. **IDENTIFY:** When the only force is gravity the acceleration is 9.80 m/s^2, downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

SET UP: Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

EXECUTE: **(a)** Find the velocity when the engines cut off. $y - y_0 = 525\text{ m}$, $a_y = 2.25\text{ m/s}^2$, $v_{0y} = 0$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(2.25\text{ m/s}^2)(525\text{ m})} = 48.6\text{ m/s}$.

Now consider the motion from engine cut-off to maximum height: $y_0 = 525\text{ m}$, $v_{0y} = +48.6\text{ m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80\text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (48.6\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 121\text{ m}$ and $y = 121\text{ m} + 525\text{ m} = 646\text{ m}$.

(b) Consider the motion from engine failure until just before the rocket strikes the ground:

$y - y_0 = -525\text{ m}$, $a_y = -9.80\text{ m/s}^2$, $v_{0y} = +48.6\text{ m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(48.6\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-525\text{ m})} = -112\text{ m/s}$. Then $v_y = v_{0y} + a_y t$ gives

$t = \frac{v_y - v_{0y}}{a_y} = \frac{-112\text{ m/s} - 48.6\text{ m/s}}{-9.80\text{ m/s}^2} = 16.4\text{ s}$.

(c) Find the time from blast-off until engine failure: $y - y_0 = 525 \text{ m}$, $v_{0y} = 0$, $a_y = +2.25 \text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(525 \text{ m})}{2.25 \text{ m/s}^2}} = 21.6 \text{ s}$. The rocket strikes the launch pad $21.6 \text{ s} + 16.4 \text{ s} = 38.0 \text{ s}$ after blast-off. The acceleration a_y is $+2.25 \text{ m/s}^2$ from $t = 0$ to $t = 21.6 \text{ s}$. It is -9.80 m/s^2 from $t = 21.6 \text{ s}$ to 38.0 s. $v_y = v_{0y} + a_y t$ applies during each constant acceleration segment, so the graph of v_y versus *t* is a straight line with positive slope of 2.25 m/s^2 during the blast-off phase and with negative slope of -9.80 m/s^2 after engine failure. During each phase $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. The sign of a_y determines the curvature of $y(t)$. At $t = 38.0 \text{ s}$ the rocket has returned to $y = 0$. The graphs are sketched in Figure 2.41.

EVALUATE: In part (b) we could have found the time from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, finding v_y first allows us to avoid solving for *t* from a quadratic equation.

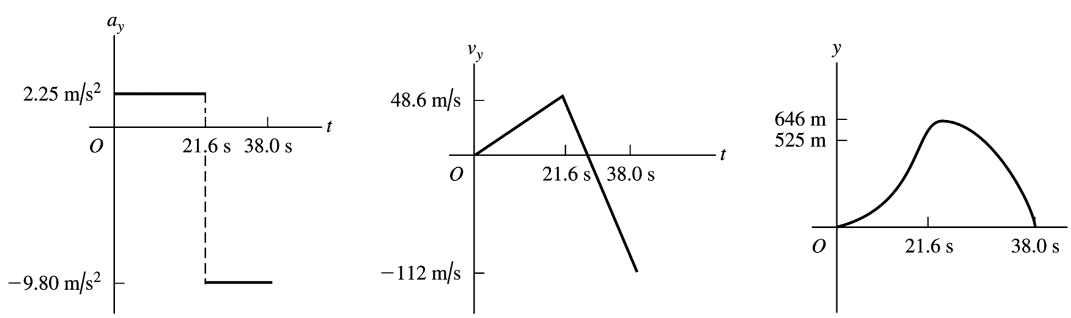


Figure 2.41

2.42. IDENTIFY: Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y = -9.80 \text{ m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00 \text{ m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0 \text{ m}$. At its maximum height the sandbag has $v_y = 0$.

EXECUTE: (a)

$t = 0.250 \text{ s}$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00 \text{ m/s})(0.250 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.250 \text{ s})^2 = 0.94 \text{ m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.250 \text{ s}) = 2.55 \text{ m/s}$.

$t = 1.00 \text{ s}$: $y - y_0 = (5.00 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 0.10 \text{ m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = -4.80 \text{ m/s}$.

(b) $y - y_0 = -40.0 \text{ m}$, $v_{0y} = 5.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-40.0 \text{ m} = (5.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. $(4.90 \text{ m/s}^2)t^2 - (5.00 \text{ m/s})t - 40.0 \text{ m} = 0$ and

$t = \dfrac{1}{9.80}\left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)}\right) \text{ s} = (0.51 \pm 2.90) \text{ s}$. *t* must be positive, so $t = 3.41 \text{ s}$.

(c) $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.41 \text{ s}) = -28.4 \text{ m/s}$

(d) $v_{0y} = 5.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (5.00 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 1.28 \text{ m}$. The maximum height is 41.3 m above the ground.

(e) The graphs of a_y, v_y, and y versus t are given in Figure 2.42. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

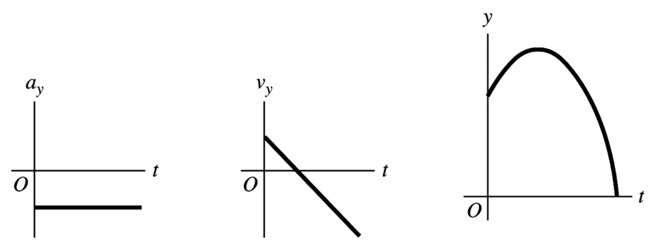


Figure 2.42

2.43. **IDENTIFY:** You throw a rock thrown upward, so its acceleration is downward and uniform. Therefore the constant-acceleration equations apply.

SET UP: We want to know how high the rock went if it returned to your hand 3.60 s after you threw it upward. With constant downward acceleration, the time it takes the rock to reach its highest point is the same as the time to fall back to your hand. Therefore it took 1.80 s to reach that highest point, and the rock's velocity was zero at the point. We can use $v_y = v_{0y} + a_y t$ to find the initial speed and then use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find how high it went.

EXECUTE: First find v_{0y} using $v_y = v_{0y} + a_y t$ with $v_y = 0$ at the highest point.

$0 = v_{0y} - gt$, so $v_{0y} = (9.80\ \text{m/s}^2)(1.80\ \text{s}) = 17.64$ m/s. Now use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find the maximum height the rock reached. $0 = (17.64\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(y - y_0) \rightarrow y - y_0 = 15.9$ m.

EVALUATE: Check by using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to calculate y at the maximum height.

$y = 0 + (17.64\ \text{m/s})(1.80\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.80\ \text{s})^2 = 15.9$ m, which agrees with our result.

2.44. **IDENTIFY:** Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude $9.80\ \text{m/s}^2$. Apply the constant acceleration equations to the motion of the egg.

SET UP: Take $+y$ to be upward. At the maximum height, $v_y = 0$.

EXECUTE: **(a)** $y - y_0 = -30.0$ m, $t = 5.00$ s, $a_y = -9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \tfrac{1}{2}a_y t = \frac{-30.0\ \text{m}}{5.00\ \text{s}} - \tfrac{1}{2}(-9.80\ \text{m/s}^2)(5.00\ \text{s}) = +18.5\ \text{m/s}.$$

(b) $v_{0y} = +18.5$ m/s, $v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (18.5\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 17.5\ \text{m}.$$

(c) At the maximum height $v_y = 0$.

(d) The acceleration is constant and equal to $9.80\ \text{m/s}^2$, downward, at all points in the motion, including at the maximum height.

(e) The graphs are sketched in Figure 2.44.

EVALUATE: The time for the egg to reach its maximum height is $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-18.5\text{ m/s}}{-9.8\text{ m/s}^2} = 1.89\text{ s}.$

The egg has returned to the level of the cornice after 3.78 s and after 5.00 s it has traveled downward from the cornice for 1.22 s.

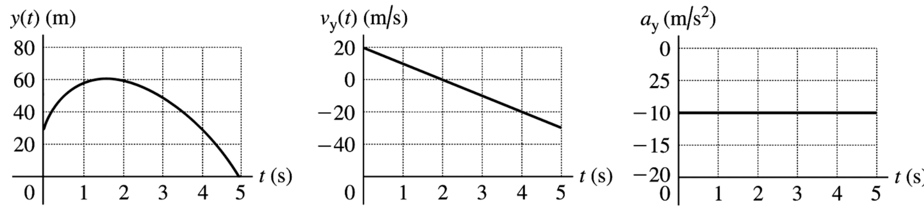


Figure 2.44

2.45. **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall, and acceleration due to gravity, and setting up a ratio involving time of fall and acceleration due to gravity.

SET UP: Let g_{En} be the acceleration due to gravity on Enceladus and let g be this quantity on earth. Let h be the common height from which the object is dropped. Let $+y$ be downward, so

$y - y_0 = h.\ v_{0y} = 0$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}gt_{\text{E}}^2$ and $h = \frac{1}{2}g_{\text{En}}t_{\text{En}}^2$. Combining these two equations gives $gt_{\text{E}}^2 = g_{\text{En}}t_{\text{En}}^2$ and $g_{\text{En}} = g\left(\dfrac{t_{\text{E}}}{t_{\text{En}}}\right)^2 = (9.80\text{ m/s}^2)\left(\dfrac{1.75\text{ s}}{18.6\text{ s}}\right)^2 = 0.0868\text{ m/s}^2.$

EVALUATE: The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.46. **IDENTIFY:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the boulder.

SET UP: Take $+y$ to be upward.

EXECUTE: **(a)** $v_{0y} = +40.0\text{ m/s},\ v_y = +20.0\text{ m/s},\ a_y = -9.80\text{ m/s}^2.\ v_y = v_{0y} + a_y t$ gives

$t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{20.0\text{ m/s} - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = +2.04\text{ s}.$

(b) $v_y = -20.0\text{ m/s}.\ t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-20.0\text{ m/s} - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = +6.12\text{ s}.$

(c) $y - y_0 = 0,\ v_{0y} = +40.0\text{ m/s},\ a_y = -9.80\text{ m/s}^2.\ y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 0$ and

$t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(40.0\text{ m/s})}{-9.80\text{ m/s}^2} = +8.16\text{ s}.$

(d) $v_y = 0,\ v_{0y} = +40.0\text{ m/s},\ a_y = -9.80\text{ m/s}^2.\ v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{0 - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = 4.08\text{ s}.$

(e) The acceleration is 9.80 m/s^2, downward, at all points in the motion.

(f) The graphs are sketched in Figure 2.46.

EVALUATE: $v_y = 0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04\text{ s} < 4.08\text{ s} < 6.12\text{ s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

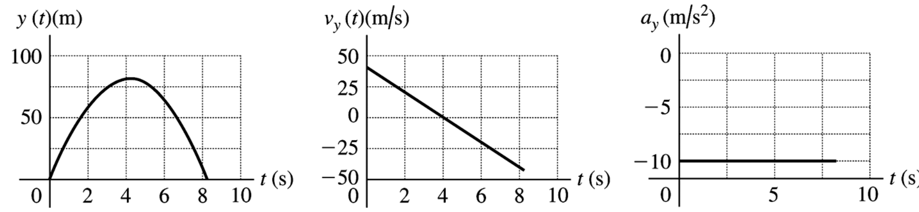


Figure 2.46

2.47. **IDENTIFY:** The rock has a constant downward acceleration of 9.80 m/s^2. The constant-acceleration kinematics formulas apply.

SET UP: The formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply. Call $+y$ upward. First find the initial velocity and then the final speed.

EXECUTE: **(a)** 6.00 s after it is thrown, the rock is back at its original height, so $y = y_0$ at that instant. Using $a_y = -9.80\text{ m/s}^2$ and $t = 6.00$ s, the equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $v_{0y} = 29.4$ m/s. When the rock reaches the water, $y - y_0 = -28.0$ m. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = -37.6$ m/s, so its speed is 37.6 m/s.

EVALUATE: The final speed is greater than the initial speed because the rock accelerated on its way down below the bridge.

2.48. **IDENTIFY:** We want to interpret a graph of v_x versus t.

SET UP: The area under a graph of v_x versus t is equal to the change in position $x_2 - x_1$. The average velocity is $v_{x\text{-av}} = \dfrac{\Delta x}{\Delta t}$.

EXECUTE: **(a)** The area of a triangle is $\frac{1}{2}bh$, which gives $A = \frac{1}{2}(6.0\text{ s})(8.0\text{ m/s}) = 24$ m.

(b) The area under a graph of v_x versus t is equal to the change in position $x_2 - x_1$, which in this case is the distance traveled, which is 24 m in 6.0 s. Thus $v_{x\text{-av}} = \dfrac{\Delta x}{\Delta t} = \dfrac{24\text{ m}}{6.0\text{ s}} = 4.0$ m/s.

(c) $\Delta x = v_{x\text{-av}}\,\Delta t = (4.0\text{ m/s})(6.0\text{ s}) = 24$ m, which is the same as the area in (a).

EVALUATE: For constant acceleration $v_{x\text{-av}} = \dfrac{v_{x\text{-}0} + v_x}{2} = \dfrac{8.0\text{ m/s} + 0}{2} = 4.0$ m/s, which agrees with our answer in (b).

2.49. **IDENTIFY:** The acceleration is not constant, but we know how it varies with time. We can use the definitions of instantaneous velocity and position to find the rocket's position and speed.

SET UP: The basic definitions of velocity and position are $v_y(t) = v_{0y} + \int_0^t a_y dt$ and $y - y_0 = \int_0^t v_y dt$.

EXECUTE: **(a)** $v_y(t) = \int_0^t a_y dt = \int_0^t (2.80\text{ m/s}^3)t\,dt = (1.40\text{ m/s}^3)t^2$

$y - y_0 = \int_0^t v_y dt = \int_0^t (1.40\text{ m/s}^3)t^2 dt = (0.4667\text{ m/s}^3)t^3$. For $t = 10.0$ s, $y - y_0 = 467$ m.

(b) $y - y_0 = 325$ m so $(0.4667\text{ m/s}^3)t^3 = 325$ m and $t = 8.864$ s. At this time $v_y = (1.40\text{ m/s}^3)(8.864\text{ s})^2 = 110$ m/s.

EVALUATE: The time in part (b) is less than 10.0 s, so the given formulas are valid.

2.50. **IDENTIFY:** The acceleration is not constant, so we must use calculus instead of the standard kinematics formulas.

SET UP: The general calculus formulas are $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. First integrate a_x to find $v(t)$, and then integrate that to find $x(t)$.

EXECUTE: Find $v(t)$: $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -(0.0320\text{ m/s}^3)(15.0\text{ s} - t)dt$. Carrying out the integral and putting in the numbers gives $v_x(t) = 8.00$ m/s – (0.0320 m/s^3)[(15.0 s)$t - t^2/2$]. Now use this result to find $x(t)$.

$x = x_0 + \int_0^t v_x dt = x_0 + \int_0^t \left[8.00\text{ m/s} - (0.0320\text{ m/s}^3)\left((15.0\text{ s})t - \frac{t^2}{2}\right)\right]dt$, which gives

$x = x_0 + (8.00\text{ m/s})t - (0.0320\text{ m/s}^3)[(7.50\text{ s})t^2 - t^3/6)]$. Using $x_0 = -14.0$ m and $t = 10.0$ s, we get $x = 47.3$ m.

EVALUATE: The standard kinematics formulas apply only when the acceleration is constant.

2.51. **(a) IDENTIFY:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

SET UP: $v_x = v_{0x} + \int_0^t a_x\, dt$, $a_x = At - Bt^2$ with $A = 1.50\text{ m/s}^3$ and $B = 0.120\text{ m/s}^4$.

EXECUTE: $v_x = v_{0x} + \int_0^t (At - Bt^2)\, dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0$, so

$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50\text{ m/s}^3)t^2 - \frac{1}{3}(0.120\text{ m/s}^4)t^3$

$v_x = (0.75\text{ m/s}^3)t^2 - (0.040\text{ m/s}^4)t^3$

SET UP: $x - x_0 + \int_0^t v_x\, dt$

EXECUTE: $x = x_0 + \int_0^t (\frac{1}{2}At^2 - \frac{1}{3}Bt^3)\, dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0$, so

$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50\text{ m/s}^3)t^3 - \frac{1}{12}(0.120\text{ m/s}^4)t^4$

$x = (0.25\text{ m/s}^3)t^3 - (0.010\text{ m/s}^4)t^4$

EVALUATE: We can check our results by using them to verify that $v_x(t) = \frac{dx}{dt}$ and $a_x(t) = \frac{dv_x}{dt}$.

(b) IDENTIFY and **SET UP:** At time t, when v_x is a maximum, $\frac{dv_x}{dt} = 0$. (Since $a_x = \frac{dv_x}{dt}$, the maximum velocity is when $a_x = 0$. For earlier times a_x is positive so v_x is still increasing. For later times a_x is negative and v_x is decreasing.)

EXECUTE: $a_x = \frac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0$, but at this time $v_x = 0$ and not a maximum.

The other root is $t = \frac{A}{B} = \frac{1.50\text{ m/s}^3}{0.120\text{ m/s}^4} = 12.5\text{ s}$

At this time $v_x = (0.75\text{ m/s}^3)t^2 - (0.040\text{ m/s}^4)t^3$ gives

$v_x = (0.75\text{ m/s}^3)(12.5\text{ s})^2 - (0.040\text{ m/s}^4)(12.5\text{ s})^3 = 117.2\text{ m/s} - 78.1\text{ m/s} = 39.1\text{ m/s}.$

EVALUATE: For $t < 12.5$ s, $a_x > 0$ and v_x is increasing. For $t > 12.5$ s, $a_x < 0$ and v_x is decreasing.

2.52. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$. Use the values of v_x and of x at $t = 1.0$ s to evaluate v_{0x} and x_0.

SET UP: $\int t^n dt = \frac{1}{n+1} t^{n+1}$, for $n \geq 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \int_0^t \alpha t dt = v_{0x} + \frac{1}{2}\alpha t^2 = v_{0x} + (0.60 \text{ m/s}^3)t^2$. $v_x = 5.0$ m/s when $t = 1.0$ s gives $v_{0x} = 4.4$ m/s. Then, at $t = 2.0$ s, $v_x = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)(2.0 \text{ s})^2 = 6.8$ m/s.

(b) $x = x_0 + \int_0^t (v_{0x} + \frac{1}{2}\alpha t^2) dt = x_0 + v_{0x}t + \frac{1}{6}\alpha t^3$. $x = 6.0$ m at $t = 1.0$ s gives $x_0 = 1.4$ m. Then, at $t = 2.0$ s, $x = 1.4 \text{ m} + (4.4 \text{ m/s})(2.0 \text{ s}) + \frac{1}{6}(1.2 \text{ m/s}^3)(2.0 \text{ s})^3 = 11.8$ m.

(c) $x(t) = 1.4 \text{ m} + (4.4 \text{ m/s})t + (0.20 \text{ m/s}^3)t^3$. $v_x(t) = 4.4 \text{ m/s} + (0.60 \text{ m/s}^3)t^2$. $a_x(t) = (1.20 \text{ m/s}^3)t$. The graphs are sketched in Figure 2.52.

EVALUATE: We can verify that $a_x = \frac{dv_x}{dt}$ and $v_x = \frac{dx}{dt}$.

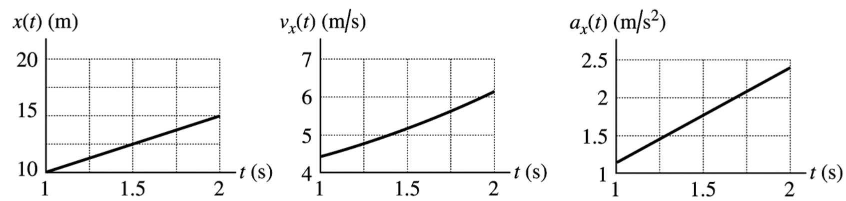


Figure 2.52

2.53. **IDENTIFY:** The sprinter's acceleration is constant for the first 2.0 s but zero after that, so it is not constant over the entire race. We need to break up the race into segments.

SET UP: When the acceleration is constant, the formula $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ applies. The average velocity is $v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$.

EXECUTE: **(a)** $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 10.0 \text{ m/s}}{2}\right)(2.0 \text{ s}) = 10.0$ m.

(b) (i) 40.0 m at 10.0 m/s so time at constant speed is 4.0 s. The total time is 6.0 s, so $v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{50.0 \text{ m}}{6.0 \text{ s}} = 8.33$ m/s.

(ii) He runs 90.0 m at 10.0 m/s so the time at constant speed is 9.0 s. The total time is 11.0 s, so $v_{\text{av-}x} = \frac{100 \text{ m}}{11.0 \text{ s}} = 9.09$ m/s.

(iii) He runs 190 m at 10.0 m/s so time at constant speed is 19.0 s. His total time is 21.0 s, so $v_{\text{av-}x} = \frac{200 \text{ m}}{21.0 \text{ s}} = 9.52$ m/s.

EVALUATE: His average velocity keeps increasing because he is running more and more of the race at his top speed.

2.54. **IDENTIFY:** We know the vertical position of the lander as a function of time and want to use this to find its velocity initially and just before it hits the lunar surface.

SET UP: By definition, $v_y(t) = \dfrac{dy}{dt}$, so we can find v_y as a function of time and then evaluate it for the desired cases.

EXECUTE: **(a)** $v_y(t) = \dfrac{dy}{dt} = -c + 2dt$. At $t = 0$, $v_y(t) = -c = -60.0$ m/s. The initial velocity is 60.0 m/s downward.

(b) $y(t) = 0$ says $b - ct + dt^2 = 0$. The quadratic formula says $t = 28.57$ s ± 7.38 s. It reaches the surface at $t = 21.19$ s. At this time, $v_y = -60.0 \text{ m/s} + 2(1.05 \text{ m/s}^2)(21.19 \text{ s}) = -15.5 \text{ m/s}$.

EVALUATE: The given formula for $y(t)$ is of the form $y = y_0 + v_{0y}t + \frac{1}{2}at^2$. For part (a), $v_{0y} = -c = -60$ m/s.

2.55. **IDENTIFY:** In time t_S the S-waves travel a distance $d = v_S t_S$ and in time t_P the P-waves travel a distance $d = v_P t_P$.

SET UP: $t_S = t_P + 33$ s

EXECUTE: $\dfrac{d}{v_S} = \dfrac{d}{v_P} + 33 \text{ s}$. $d\left(\dfrac{1}{3.5 \text{ km/s}} - \dfrac{1}{6.5 \text{ km/s}}\right) = 33$ s and $d = 250$ km.

EVALUATE: The times of travel for each wave are $t_S = 71$ s and $t_P = 38$ s.

2.56. **IDENTIFY:** A rock is thrown upward from the edge of a roof and eventually lands on the ground a distance H below the edge. Its acceleration is g downward, so the constant-acceleration equations apply.

SET UP: The time to reach the maximum height is T_{max} and the time after throwing to reach the ground is $3T_{max}$. The equations $v_y = v_{0y} + a_y t$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. Call the y-axis upward with $y = 0$ at the edge of the roof. At the highest point, $v_y = 0$ and at the ground $y = -H$. We want the initial speed V_0 in terms of H. Making a sketch helps to organize the information, as in Fig. 2.56.

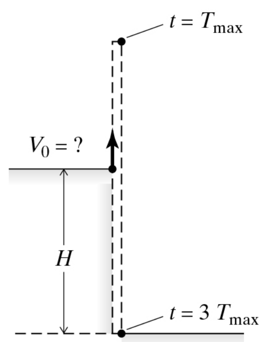


Figure 2.56

EXECUTE: At the highest point: $v_y = v_{0y} + a_y t$ gives $0 = V_0 - gT_{max}$, so $V_0 = gT_{max}$.

At the ground: After the rock was thrown, it took time T_{max} to reach the highest point and time $3T_{max}$ to reach the ground. Therefore the time to fall from the highest point to the ground was $2T_{max}$. Apply $v_y = v_{0y} + a_y t$ to the interval from the highest point to the ground to find the speed v_y when the rock reaches the ground. $v_y = 0 + (-g)(2T_{max}) = -2gT_{max}$. Now look at the entire motion from the instant the rock is thrown up until the instant it reaches the ground a distance H below the edge of the roof. We have already found that $V_0 = gT_{max}$ and $v_y = -2gT_{max}$ at ground level. Therefore $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

gives $(-2gT_{\max})^2 = V_0^2 + 2(-g)(-H)$. Using $T_{\max} = V_0/g$ gives $\left[-2g\left(\frac{V_0}{g}\right)\right]^2 = V_0^2 + 2gH$ $\rightarrow$

$$V_0 = \sqrt{\frac{2gH}{3}}.$$

EVALUATE: It might seem like a good idea to check the special case if $H = 0$ (the edge of the roof is at ground level). Our result seems to give $V_0 = 0$ in that case. But if the roof were at ground level, the time for the entire up-and-down trip would take $2T_{\max}$, not $3T_{\max}$, so our result would not apply. Checking units in the final answer would be a good idea.

2.57. IDENTIFY: The average velocity is $v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be upward.

EXECUTE: **(a)** $v_{\text{av-}x} = \frac{1000\text{ m} - 63\text{ m}}{4.75\text{ s}} = 197\text{ m/s}$

(b) $v_{\text{av-}x} = \frac{1000\text{ m} - 0}{5.90\text{ s}} = 169\text{ m/s}$

EVALUATE: For the first 1.15 s of the flight, $v_{\text{av-}x} = \frac{63\text{ m} - 0}{1.15\text{ s}} = 54.8\text{ m/s}$. When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

2.58. IDENTIFY: The block has a constant westward acceleration, so we can use the constant-acceleration equations.

SET UP: The equations $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, $v_x = v_{0x} + a_x t$, $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, and $v_{av\text{-}x} = \frac{\Delta x}{\Delta t}$ apply.

EXECUTE: **(a)** The target variable is the time for the block to return to $x = 0$. Using $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $0 = (12.0\text{ m/s})t + \frac{1}{2}(-2.00\text{ m/s}^2)t^2$. So $t = 12.0$ s.

(b) The block instantaneously stops when it reaches it maximum distance east, so $v_x = v_{0x} + a_x t$ gives 0 = 12.0 m/s + (–2.00 m/s)t, which gives $t = 6.00$ s. Using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ we have $0 = (12.0\text{ m/s})^2 + 2(-2.00\text{ m/s}^2)\Delta x$, which gives $\Delta x = 36.0$ m.

EVALUATE: To check, we can use $v_{av\text{-}x} = \frac{\Delta x}{\Delta t}$ to find Δx. This gives $\Delta x = v_{\text{av}}\Delta t = \left(\frac{0 + 12.0\text{ m/s}}{2}\right)(6.00\text{ s}) =$ 36.0 m, which agrees with our result in (b).

2.59. IDENTIFY: A block is moving with constant acceleration on an incline, so the constant-acceleration equations apply.

SET UP: All quantities are *down* the surface of the incline, so choose the x-axis along this surface and pointing downward. We first find the acceleration of the block and then use that to find its speed after sliding 16.0 m starting from rest and how long it takes to slide that distance.

EXECUTE: **(a)** First use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find a_x. $(3.00\text{ m/s})^2 = 0 + 2a_x(8.00\text{ m})$. $a_x = 0.5625$ m/s^2. Now use the same equation to find the speed when the block has moved 16.0 m.

$v_x^2 = 0 + 2(0.5625\text{ m/s}^2)(16.0\text{ m})$ $\rightarrow$ $v_x = 4.24$ m/s.

(b) Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the time to travel 16.0 m.

$16.0\text{ m} = 0 + 0 + \frac{1}{2}(0.5625\text{ m/s}^2)t^2$ $\rightarrow$ $t = 7.54$ s.

EVALUATE: From $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, with $v_{0x} = 0$, we see that $v_x^2 \propto (x - x_0)$, so $v_x \propto \sqrt{x - x_0}$. So if $(x - x_0)$ increases by a factor of 2, v_x should increase by a factor of $\sqrt{2}$. At the end of 8.00 m, the block was moving at 3.00 m/s, so at the end of 16.0 m, it should be moving at (3.00 m/s) $\sqrt{2}$ = 4.24 m/s, which agrees with our result in (a).

2.60. IDENTIFY: Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(1.60 \text{ m/s}^2)(14.0 \text{ s})^2 = 157 \text{ m}.$

At $t = 14.0$ s, the speed is $v_x = v_{0x} + a_x t = (1.60 \text{ m/s}^2)(14.0 \text{ s}) = 22.4 \text{ m/s}.$ In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4 \text{ m/s})(70.0 \text{ s}) = 1568 \text{ m}.$

For the interval during which the train is slowing down, $v_{0x} = 22.4$ m/s, $a_x = -3.50 \text{ m/s}^2$ and $v_x = 0.$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4 \text{ m/s})^2}{2(-3.50 \text{ m/s}^2)} = 72 \text{ m}.$

The total distance traveled is $157 \text{ m} + 1568 \text{ m} + 72 \text{ m} = 1800 \text{ m}.$

EVALUATE: The acceleration is not constant for the entire motion, but it does consist of constant acceleration segments, and we can use constant acceleration equations for each segment.

2.61. IDENTIFY: When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since v_x is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 10.0$ s segment, $v_{0x} = 4.00$ m/s and $v_x = 12.0$ m/s. For the $t = 10.0$ s to 12.0 s segment, $v_{0x} = 12.0$ m/s and $v_x = 0.$

EXECUTE: **(a)** For $t = 0$ to $t = 10.0$ s, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{4.00 \text{ m/s} + 12.0 \text{ m/s}}{2}\right)(10.0 \text{ s}) = 80.0 \text{ m}.$

For $t = 10.0$ s to $t = 12.0$ s, $x - x_0 = \left(\dfrac{12.0 \text{ m/s} + 0}{2}\right)(2.00 \text{ s}) = 12.0 \text{ m}.$ The total distance traveled is 92.0 m.

(b) $x - x_0 = 80.0 \text{ m} + 12.0 \text{ m} = 92.0 \text{ m}$

(c) For $t = 0$ to 10.0 s, $a_x = \dfrac{12.0 \text{ m/s} - 4.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2.$ For $t = 10.0$ s to 12.0 s,

$a_x = \dfrac{0 - 12.0 \text{ m/s}}{2.00 \text{ s}} = -6.00 \text{ m/s}^2.$ The graph of a_x versus t is given in Figure 2.61.

EVALUATE: When v_x and a_x are both positive, the speed increases. When v_x is positive and a_x is negative, the speed decreases.

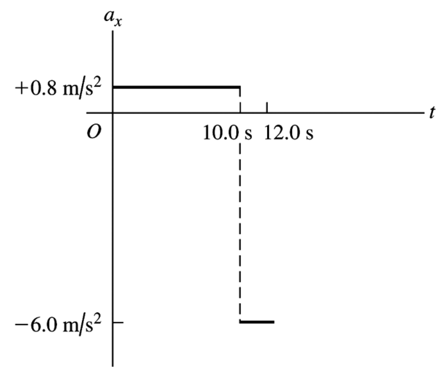


Figure 2.61

2.62. **IDENTIFY:** Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.

SET UP: Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200$ m. For the freight train $v_F = 15.0$ m/s and $a_F = 0$. For the passenger train $v_P = 25.0$ m/s and $a_P = -0.100$ m/s^2.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200 \text{ m} + v_F t$. Setting $x_P = x_F$ gives $v_P t + \frac{1}{2}a_P t^2 = 200 \text{ m} + v_F t$. $(0.0500 \text{ m/s}^2)t^2 - (10.0 \text{ m/s})t + 200 \text{ m} = 0$. The quadratic formula gives $t = \frac{1}{0.100}\left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)}\right) \text{ s} = (100 \pm 77.5) \text{ s}$. The collision occurs at $t = 100 \text{ s} - 77.5 \text{ s} = 22.5$ s. The equations that specify a collision have a physical solution (real, positive t), so a collision does occur.

(b) $x_P = (25.0 \text{ m/s})(22.5 \text{ s}) + \frac{1}{2}(-0.100 \text{ m/s}^2)(22.5 \text{ s})^2 = 537$ m. The passenger train moves 537 m before the collision. The freight train moves $(15.0 \text{ m/s})(22.5 \text{ s}) = 337$ m.

(c) The graphs of x_F and x_P versus t are sketched in Figure 2.62.

EVALUATE: The second root for the equation for t, $t = 177.5$ s is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

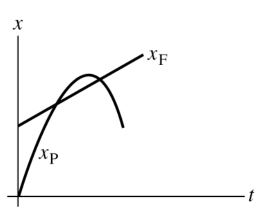


Figure 2.62

2.63. **IDENTIFY** and **SET UP:** Apply constant acceleration kinematics equations.
Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

EXECUTE: For the first 5.0 s of the motion, $v_{0x} = 0$, $t = 5.0$ s.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0 \text{ s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\text{ s}),\ t = 5.0\text{ s},\ x - x_0 = 200\text{ m}.$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $200\text{ m} = (25\text{ s}^2)a_x + (12.5\text{ s}^2)a_x$ so $a_x = 5.333\text{ m/s}^2$.

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(5.333\text{ m/s}^2)(5.0\text{ s})^2 = 67\text{ m}.$

EVALUATE: The ball is speeding up so it travels farther in the second 5.0 s interval than in the first.

2.64. **IDENTIFY:** The rock has a constant acceleration, so we can apply the constant-acceleration equations.

SET UP: We use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ and $v_x = v_{0x} + a_x t$. The force is applied starting at $t = 0$ and maintains a constant acceleration of 4.00 m/s^2 in the $-x$ direction. Our target variables are the three times when the rock is 24.0 m from where it was when the force began to be applied. We also want its velocity at those instants.

EXECUTE: When the rock is 24.0 m on the $+x$ side of the origin, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives

$24.0\text{ m} = (16.0\text{ m/s})t + \frac{1}{2}(-4.00\text{ m/s}^2)t^2.$ Using the quadratic formula we get $t = 2.00$ s and $t = 6.00$ s.

At these times $v_x = v_{0x} + a_x t$ gives $v_x = 16.0\text{ m/s} + (-4.00\text{ m/s}^2)(2.00\text{ s}) = 8.00\text{ m/s}$ and

$v_x = 16.0\text{ m/s} + (-4.00\text{ m/s}^2)(6.00\text{ s}) = -8.00\text{ m/s}.$ But the rock is also 24.0 m from the origin when it is 24.0 m on the $-x$ side. In this case we get $-24.0\text{ m} = (16.0\text{ m/s})t + \frac{1}{2}(-4.00\text{ m/s}^2)t^2.$ This quadratic equation has two roots, one of which is negative. We discard that root, leaving only $t = 9.29$ s. At this time, the velocity is $v_x = 16.0\text{ m/s} + (-4.00\text{ m/s}^2)(9.29\text{ s}) = -21.2\text{ m/s}.$

EVALUATE: To find the third time when the rock was 24.0 m from the origin, we had to think about the rock's behavior. It was not sufficient just to plug into an equation and get all three answers.

2.65. **IDENTIFY:** Apply constant acceleration equations to each object.
Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.65a.
Let d be the distance that the car initially is behind the truck, so $x_0(\text{car}) = -d$ and $x_0(\text{truck}) = 0.$ Let T be the time it takes the car to catch the truck. Thus at time T the truck has undergone a displacement $x - x_0 = 60.0\text{ m},$ so is at $x = x_0 + 60.0\text{ m} = 60.0\text{ m}.$ The car has caught the truck so at time T is also at $x = 60.0\text{ m}.$

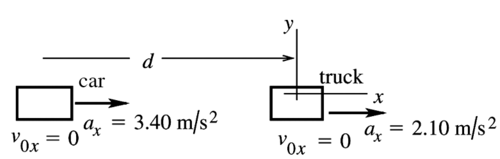


Figure 2.65a

(a) SET UP: Use the motion of the truck to calculate T:

$x - x_0 = 60.0\text{ m},\ v_{0x} = 0$ (starts from rest), $a_x = 2.10\text{ m/s}^2,\ t = T$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

Since $v_{0x} = 0,$ this gives $t = \sqrt{\dfrac{2(x - x_0)}{a_x}}$

EXECUTE: $T = \sqrt{\dfrac{2(60.0\text{ m})}{2.10\text{ m/s}^2}} = 7.56\text{ s}$

(b) SET UP: Use the motion of the car to calculate d:

$x - x_0 = 60.0\text{ m} + d,\ v_{0x} = 0,\ a_x = 3.40\text{ m/s}^2,\ t = 7.56\text{ s}$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

EXECUTE: $d + 60.0\text{ m} = \frac{1}{2}(3.40\text{ m/s}^2)(7.56\text{ s})^2$

$d = 97.16\text{ m} - 60.0\text{ m} = 37.2\text{ m}$.

(c) car: $v_x = v_{0x} + a_x t = 0 + (3.40\text{ m/s}^2)(7.56\text{ s}) = 25.7\text{ m/s}$

truck: $v_x = v_{0x} + a_x t = 0 + (2.10\text{ m/s}^2)(7.56\text{ s}) = 15.9\text{ m/s}$

(d) The graph is sketched in Figure 2.65b.

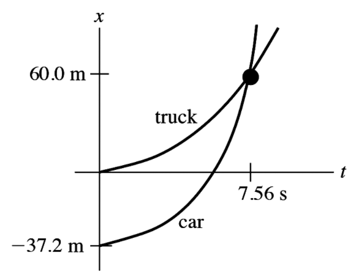


Figure 2.65b

EVALUATE: In part (c) we found that the auto was traveling faster than the truck when they came abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the x-t curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

2.66. **IDENTIFY:** The bus has a constant velocity but you have a constant acceleration, starting from rest.
SET UP: When you catch the bus, you and the bus have been traveling for the same time, but you have traveled an extra 12.0 m during that time interval. The constant-acceleration kinematics formula $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ applies.

EXECUTE: Call d the distance the bus travels after you start running and t the time until you catch the bus. For the bus we have $d = (5.00\text{ m/s})t$, and for you we have $d + 12.0\text{ m} = (1/2)(0.960\text{ m/s}^2)t^2$. Solving these two equations simultaneously, and using the positive root, gives $t = 12.43$ s and $d = 62.14$ m. The distance you must run is 12.0 m + 62.14 m = 74.1 m. Your final speed just as you reach the bus is $v_x = (0.960\text{ m/s}^2)(12.43\text{s}) = 11.9\text{ m/s}$. This might be possible for a college runner for a brief time, but it would be highly demanding!
EVALUATE: Note that when you catch the bus, you are moving much faster than it is.

2.67. **IDENTIFY:** The runner has constant acceleration in each segment of the dash, but it is not the same acceleration in the two segments. Therefore we must solve this problem in two segments.
SET UP: In the first segment, her acceleration is a constant a_x for 3.0 s, and in the next 9.0 s her speed remains constant. The total distance she runs is 100 m. A sketch as in Fig. 2.67 helps to organize the information. The constant-acceleration equations apply to each segment. We want a_x.

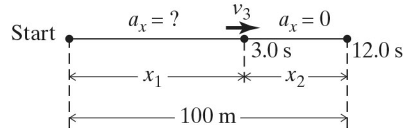


Figure 2.67

EXECUTE: Calling v_3 the velocity at the end of the first 3.0 s and using the notation shown in the figure, we look at one segment at a time. We realize that $v_0 = 0$ and v_3 is the constant speed during the second segment because she is not accelerating during that segment.

First segment: $v_x = v_{0x} + a_x t$ gives $v_3 = (3.0\text{ s})a_x$

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_1 = \frac{1}{2}a_x(3.0\text{ s})^2 = (4.5\text{ s}^2)\,a_x$.

Second segment: She has no acceleration, so $x_2 = v_3 t_2 = [(3.0\text{ s})a_x](9.0\text{ s}) = (27\text{ s}^2)a_x$.

The dash is 100 m long, so $x_1 + x_2 = 100$ m, so $(4.5\text{ s}^2)a_x + (27\text{ s}^2)a_x = 100\text{ m} \rightarrow a_x = 3.2\text{ m/s}^2$.

EVALUATE: We cannot solve this problem in a single step by averaging the accelerations because the accelerations (a_x during the first 3.0 s and zero during the last 9.0 s) do not last for the same time.

2.68. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $a_x(t) = \dfrac{dv_x}{dt}$ and $x = x_0 + \int_0^t v_x(t)dt$.

SET UP: $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$ for $n \geq 0$.

EXECUTE: **(a)** $x(t) = x_0 + \int_0^t [\alpha - \beta t^2]dt = x_0 + \alpha t - \frac{1}{3}\beta t^3$. $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$x(t) = \alpha t - \frac{1}{3}\beta t^3 = (4.00\text{ m/s})t - (0.667\text{ m/s}^3)t^3$. $a_x(t) = \dfrac{dv_x}{dt} = -2\beta t = -(4.00\text{ m/s}^3)t$.

(b) The maximum positive x is when $v_x = 0$ and $a_x < 0$. $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$t = \sqrt{\dfrac{\alpha}{\beta}} = \sqrt{\dfrac{4.00\text{ m/s}}{2.00\text{ m/s}^3}} = 1.41$ s. At this t, a_x is negative. For $t = 1.41$ s,

$x = (4.00\text{ m/s})(1.41\text{ s}) - (0.667\text{ m/s}^3)(1.41\text{ s})^3 = 3.77$ m.

EVALUATE: After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \to \infty$.

2.69. **IDENTIFY:** In this problem the acceleration is not constant, so the constant-acceleration equations do not apply. We need to go to the basic definitions of velocity and acceleration and use calculus.

SET UP: We know the object starts at $x = 0$ and its velocity is given by $v_x = \alpha t - \beta t^3$. We want to find out when it returns to the origin and what are its velocity and acceleration at that instant. We need to use the definitions $v_x = dx/dt$ and $a_x = dv_x/dt$.

EXECUTE: **(a)** To find when the object returns to the origin we need to find $x(t)$ and use it to find t when $x = 0$. Using $v_x = dx/dt$ gives $dx = v_x dt$. Using $v_x = \alpha t - \beta t^3$, we integrate to find $x(t)$.

$x - x_0 = \int v_x dt = \int\left(\alpha t - \beta t^3\right)dt = \dfrac{\alpha t^2}{2} - \dfrac{\beta t^4}{4}$. The object starts from the origin, so $x_0 = 0$ and when $t = 0$, $x = 0$. This gives $\dfrac{\alpha t^2}{2} - \dfrac{\beta t^4}{4} = 0$. Solving for t gives $t = \sqrt{\dfrac{2\alpha}{\beta}} = \sqrt{\dfrac{2(8.0\text{ m/s}^2)}{4.0\text{ m/s}^4}} = 2.0$ s.

(b) When $t = 2.0$ s, $v_x = \alpha t - \beta t^3 = (8.0\text{ m/s}^2)(2.0\text{ s}) - (4.0\text{ m/s}^4)(2.0\text{ s})^3 = -16$ m/s. The minus sign tells us the object is moving in the $-x$ direction. To find the acceleration, we $a_x = dv_x/dt$.

$a_x = \dfrac{d\left(\alpha t - \beta t^3\right)}{dt} = \alpha - 3\beta t^2 = 8.0\text{ m/s}^2 - 3(4.0\text{ m/s}^4)(2.0\text{ s})^2 = -40\text{ m/s}^2$, in the $-x$ direction.

EVALUATE: The standard constant-acceleration formulas do not apply when the acceleration is a function of time.

2.70. **IDENTIFY:** Find the distance the professor walks during the time t it takes the egg to fall to the height of his head.

SET UP: Let $+y$ be downward. The egg has $v_{0y} = 0$ and $a_y = 9.80\text{ m/s}^2$. At the height of the professor's head, the egg has $y - y_0 = 44.2\text{ m}$.

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(44.2\text{ m})}{9.80\text{ m/s}^2}} = 3.00\text{ s}$. The professor walks a distance $x - x_0 = v_{0x}t = (1.20\text{ m/s})(3.00\text{ s}) = 3.60\text{ m}$. Release the egg when your professor is 3.60 m from the point directly below you.

EVALUATE: Just before the egg lands its speed is $(9.80\text{ m/s}^2)(3.00\text{ s}) = 29.4\text{ m/s}$. It is traveling much faster than the professor.

2.71. **(a) IDENTIFY** and **SET UP:** Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$. We know $a_x(t) = \alpha + \beta t$, with $\alpha = -2.00\text{ m/s}^2$ and $\beta = 3.00\text{ m/s}^3$.

EXECUTE: $v_x = v_{0x} + \int_0^t a_x\, dt = v_{0x} + \int_0^t (\alpha + \beta t)\, dt = v_{0x} + \alpha t + \frac{1}{2}\beta t^2$

$x = x_0 + \int_0^t v_x\, dt = x_0 + \int_0^t (v_{0x} + \alpha t + \frac{1}{2}\beta t^2)\, dt = x_0 + v_{0x}t + \frac{1}{2}\alpha t^2 + \frac{1}{6}\beta t^3$

At $t = 0$, $x = x_0$.

To have $x = x_0$ at $t_1 = 4.00\text{ s}$ requires that $v_{0x}t_1 + \frac{1}{2}\alpha t_1^2 + \frac{1}{6}\beta t_1^3 = 0$.

Thus $v_{0x} = -\frac{1}{6}\beta t_1^2 - \frac{1}{2}\alpha t_1 = -\frac{1}{6}(3.00\text{ m/s}^3)(4.00\text{ s})^2 - \frac{1}{2}(-2.00\text{ m/s}^2)(4.00\text{ s}) = -4.00\text{ m/s}$.

(b) With v_{0x} as calculated in part (a) and $t = 4.00\text{ s}$,

$v_x = v_{0x} + \alpha t + \frac{1}{2}\beta t^2 = -4.00\text{ m/s} + (-2.00\text{ m/s}^2)(4.00\text{ s}) + \frac{1}{2}(3.00\text{ m/s}^3)(4.00\text{ s})^2 = +12.0\text{ m/s}$.

EVALUATE: $a_x = 0$ at $t = 0.67\text{ s}$. For $t > 0.67\text{ s}$, $a_x > 0$. At $t = 0$, the particle is moving in the $-x$-direction and is speeding up. After $t = 0.67$ s, when the acceleration is positive, the object slows down and then starts to move in the $+x$-direction with increasing speed.

2.72. **IDENTIFY:** After it is thrown upward, the rock has a constant downward acceleration g, so the constant-acceleration equations apply. It lands on the ground, a distance H below the point where it started.

SET UP: For constant acceleration, $v_{\text{av-}y} = \dfrac{v_1 + v_2}{2}$. We know $v_1 = v_0$ and can use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find v_2.

EXECUTE: **(a)** Use $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t}$. The rock starts at $y = 0$ and ends up at $y = -H$, so $\Delta y = y_2 - y_1 = -H - 0 = -H$. We need the total time from the instant the rock is thrown up until it hits the ground. Break the motion into two parts: the upward motion to the highest point and the fall from the highest point to the ground.

<u>Upward motion</u>: Using $v_y = v_{0y} + a_y t$ gives $t_{\text{up}} = v_0/g$. Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives the maximum height: $0 = v_0^2 - 2gy_{\text{top}} \rightarrow y_{\text{top}} = v_0^2/2g$.

<u>Downward motion</u>: The downward motion starts from rest, and the rock falls a distance $y_{\text{top}} + H$, so $y = y_0 + v_{0y}t + \dfrac{1}{2}a_y t^2$ gives $\dfrac{v_0^2}{2g} + H = \dfrac{1}{2}g t_{\text{fall}}^2 \rightarrow t_{\text{fall}} = \sqrt{\dfrac{v_0^2}{g^2} + \dfrac{2H}{g}}$.

The total time for the entire motion is $t_{\text{tot}} = t_{\text{up}} + t_{\text{fall}} = \dfrac{v_0}{g} + \sqrt{\dfrac{v_0^2}{g^2} + \dfrac{2H}{g}}$. Now use $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t}$.

$v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t} = \dfrac{-H}{\dfrac{v_0}{g} + \sqrt{\dfrac{v_0^2}{g^2} + \dfrac{2H}{g}}} = \dfrac{-H}{v_0 + \sqrt{v_0^2 + 2gH}}$. Rationalizing the denominator and simplifying gives

$$v_{\text{av-}y} = \frac{v_0 - \sqrt{v_0^2 + 2gH}}{2}.$$

(b) When $H = 0$, $v_{\text{av-}y} = \dfrac{v_0 - \sqrt{v_0^2 + 2gH}}{2} = \dfrac{v_0 - \sqrt{v_0^2}}{2} = 0$. This result is reasonable because in this case $v_2 = -v_0$, so $v_{\text{av-}y} = \dfrac{v_1 + v_2}{2} = \dfrac{v_0 - v_0}{2} = 0$.

EVALUATE: (c) Using $v_2^2 = v_0^2 - 2g(-H)$ gives $v_2 = -\sqrt{v_0^2 + 2gH}$. We used the negative root because the rock is moving downward. Now use $v_{\text{av-}y} = \dfrac{v_1 + v_2}{2}$, which gives $v_{\text{av-}y} = \dfrac{v_0 - \sqrt{v_0^2 + 2gH}}{2}$. We get the same result as in part (a). If $H = 0$, $y_1 = y_2$, so $\Delta y = 0$, so $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t} = 0$.

2.73. IDENTIFY: The watermelon is in freefall so it has a constant downward acceleration of g. The constant-acceleration equations apply to its motion.

SET UP: The melon starts from rest at the top of a building. You observe that 1.50 s after it is 30.0 m above the ground, it hits the ground, and you want to find the height of the building. It is very helpful to make a sketch to organize the information, as in Fig. 2.73. In the figure, we see that the height h of the building is $h = y_1 + 30.0$ m, and it takes 1.50 s to travel those last 30.0 m of the fall. We know that $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t}$, and for constant acceleration it is also true that $v_{\text{av-}y} = \dfrac{v_1 + v_2}{2}$. All quantities are downward, so choose the +y-axis downward.

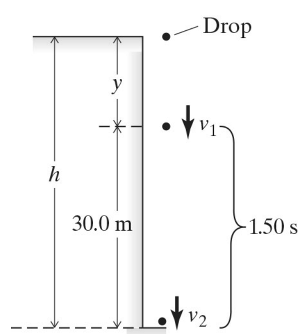


Figure 2.73

EXECUTE: We need to find y_1, but first we need to get v_1. Using $v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t}$ during the last 1.50 s of the fall, we have $v_{\text{av-}y} = \dfrac{30.0\text{ m}}{1.50\text{ s}} = 20.0$ m/s. It is also true that $v_{\text{av-}y} = \dfrac{v_1 + v_2}{2}$, so $\dfrac{v_1 + v_2}{2} = 20.0$ m/s, which gives $v_2 = 40.0\text{ m/s} - v_1$. We can also use $v_y = v_{0y} + a_y t$ to get v_2 in terms of v_1.

$v_2 = v_1 + gt = v_1 + (9.80\text{ m/s}^2)(1.50\text{ s}) = v_1 + 14.7$ m/s. Equating our two expressions for v_2 gives $v_1 + 14.7\text{ m/s} = 40.0\text{ m/s} - v_1$. Solving for v_1 gives $v_1 = 12.65$ m/s. Now use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find y_1. $(12.65\text{ m/s})^2 = 0 + 2(9.80\text{ m/s}^2)y_1 \rightarrow y_1 = 8.16$ m. The total height h of the building is $h = y_1 + 30.0\text{ m} = 8.16\text{ m} + 30.0\text{ m} = 38.2$ m.

EVALUATE: Use our results to find the time to fall the last 30.0 m; it should be 1.50 s if our answer is correct.

Time to fall 8.16 m from rest: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$: 8.16 m = 0 + $\frac{1}{2}$(9.80 m/s^2)t^2 $\rightarrow$ t = 1.29 s.

Time to fall 38.2 m from rest: 38.2 m = $\frac{1}{2}$(9.80 m/s^2)t^2 $\rightarrow$ t = 2.79 s.

Time to fall the last 30.0 m: 2.79 s – 1.29 s = 1.50 s, which agrees with the observed time.

2.74. **IDENTIFY:** The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

SET UP: Let $+y$ be downward. Throughout the motion $a_y = +9.80\text{ m/s}^2$. The constant-acceleration kinematics formulas all apply.

EXECUTE: Motion past the window:

$y - y_0 = 1.90\text{ m}$, $t = 0.380\text{ s}$, $a_y = +9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_y t = \dfrac{1.90\text{ m}}{0.380\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(0.380\text{ s}) = 3.138\text{ m/s}$. This is the velocity of the flowerpot when it is at the top of the window.

Motion from the windowsill to the top of the window: $v_{0y} = 0$, $v_y = 2.466\text{ m/s}$, $a_y = +9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(3.138\text{ m/s})^2 - 0}{2(9.80\text{ m/s}^2)} = 0.502\text{ m}$. The top of the window is 0.502 m below the windowsill.

EVALUATE: It takes the flowerpot $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{3.138\text{ m/s}}{9.80\text{ m/s}^2} = 0.320\text{ s}$ to fall from the sill to the top of the window. Our result says that from the windowsill the pot falls $0.502\text{ m} + 1.90\text{ m} = 2.4\text{ m}$ in $0.320\text{ s} + 0.380\text{ s} = 0.700\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(0.700\text{ s})^2 = 2.4\text{ m}$, which checks.

2.75. **(a) IDENTIFY:** Consider the motion from when he applies the acceleration to when the shot leaves his hand.

SET UP: Take positive y to be upward. $v_{0y} = 0$, $v_y = ?$, $a_y = 35.0\text{ m/s}^2$, $y - y_0 = 0.640\text{ m}$,

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(35.0\text{ m/s}^2)(0.640\text{ m})} = 6.69\text{ m/s}$

(b) IDENTIFY: Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0$. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20\text{ m}$, $y = ?$, $a_y = -9.80\text{ m/s}^2$ (free fall), $v_{0y} = 6.69\text{ m/s}$ (from part (a), $v_y = 0$ at maximum height), $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (6.69\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 2.29\text{ m}$, $y = 2.20\text{ m} + 2.29\text{ m} = 4.49\text{ m}$.

(c) IDENTIFY: Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20\text{ m}$, $y = 1.83\text{ m}$, $a_y = -9.80\text{ m/s}^2$, $v_{0y} = +6.69\text{ m/s}$, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $1.83\text{ m} - 2.20\text{ m} = (6.69\text{ m/s})t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2 = (6.69\text{ m/s})t - (4.90\text{ m/s}^2)t^2$,

$4.90t^2 - 6.69t - 0.37 = 0$, with t in seconds. Use the quadratic formula to solve for t:

$t = \frac{1}{9.80}\left(6.69 \pm \sqrt{(6.69)^2 - 4(4.90)(-0.37)}\right) = 0.6830 \pm 0.7362$. Since t must be positive, $t = 0.6830\text{ s} + 0.7362\text{ s} = 1.42\text{ s}$.

EVALUATE: Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so $t = (v_y - v_{0y})/a_y = -(6.69\text{ m/s})/(-9.80\text{ m/s}^2) = 0.68\text{ s}$. It also takes 0.68 s to return to 2.2 m above the ground, for a total time of 1.36 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.36 s after being thrown; the answer of 1.42 s in part (c) makes sense.

2.76. IDENTIFY: The motion of the rocket can be broken into 3 stages, each of which has constant acceleration, so in each stage we can use the standard kinematics formulas for constant acceleration. But the acceleration is not the same throughout all 3 stages.

SET UP: The formulas $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, and $v_y = v_{0y} + a_y t$ apply.

EXECUTE: **(a)** Let $+y$ be upward. At $t = 25.0\text{ s}$, $y - y_0 = 1094\text{ m}$ and $v_y = 87.5\text{ m/s}$. During the next 10.0 s the rocket travels upward an additional distance

$y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t = \left(\frac{87.5\text{ m/s} + 132.5\text{ m/s}}{2}\right)(10.0\text{ s}) = 1100\text{ m}$. The height above the launch pad when the second stage quits therefore is $1094\text{ m} + 1100\text{ m} = 2194\text{ m}$. For the free-fall motion after the second stage quits: $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (132.5\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 896\text{ m}$. The maximum height above the launch pad that the rocket reaches is $2194\text{ m} + 896\text{ m} = 3090\text{ m}$.

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-2194\text{ m} = (132.5\text{ m/s})t - (4.9\text{ m/s}^2)t^2$. From the quadratic formula the positive root is $t = 38.6\text{ s}$.

(c) $v_y = v_{0y} + a_y t = 132.5\text{ m/s} + (-9.8\text{ m/s}^2)(38.6\text{ s}) = -246\text{ m/s}$. The rocket's speed will be 246 m/s just before it hits the ground.

EVALUATE: We cannot solve this problem in a single step because the acceleration, while constant in each stage, is not constant over the entire motion. The standard kinematics equations apply to each stage but not to the motion as a whole.

2.77. IDENTIFY: Two stones are thrown up with different speeds. (a) Knowing how soon the faster one returns to the ground, how long it will take the slow one to return? (b) Knowing how high the slower stone went, how high did the faster stone go?

SET UP: Use subscripts f and s to refer to the faster and slower stones, respectively. Take $+y$ to be upward and $y_0 = 0$ for both stones. $v_{0\text{f}} = 3v_{0\text{s}}$. When a stone reaches the ground, $y = 0$. The constant-acceleration formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply.

EXECUTE: **(a)** $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = -\frac{2v_{0y}}{t}$. Since both stones have the same a_y, $\frac{v_{0\text{f}}}{t_\text{f}} = \frac{v_{0\text{s}}}{t_\text{s}}$ and $t_\text{s} = t_\text{f}\left(\frac{v_{0\text{s}}}{v_{0\text{f}}}\right) = \left(\frac{1}{3}\right)(10\text{ s}) = 3.3\text{ s}$.

(b) Since $v_y = 0$ at the maximum height, then $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = -\frac{v_{0y}^2}{2y}$. Since both have the same a_y, $\frac{v_{0\text{f}}^2}{y_\text{f}} = \frac{v_{0\text{s}}^2}{y_\text{s}}$ and $y_\text{f} = y_\text{s}\left(\frac{v_{0\text{f}}}{v_{0\text{s}}}\right)^2 = 9H$.

EVALUATE: The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

2.78. **IDENTIFY:** The rocket accelerates uniformly upward at 16.0 m/s^2 with the engines on. After the engines are off, it moves upward but accelerates downward at 9.80 m/s^2.

SET UP: The formulas $y - y_0 = v_{0_y} t = \frac{1}{2} a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply to both parts of the motion since the accelerations are both constant, but the accelerations are different in both cases. Let $+y$ be upward.

EXECUTE: With the engines on, $v_{0y} = 0$, $a_y = 16.0$ m/s^2 upward, and $t = T$ at the instant the engines just shut off. Using these quantities, we get

$y - y_0 = v_{0_y} t + \frac{1}{2} a_y t^2 = (8.00 \text{ m/s}^2)T^2$ and $v_y = v_{0y} + a_y t = (16.0 \text{ m/s}^2)T$.

With the engines off (free fall), the formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for the highest point gives

$y - y_0 = (13.06 \text{ m/s}^2)T^2$, using $v_{0y} = (16.0 \text{ m/s}^2)T$, $v_y = 0$, and $a_y = -9.80$ m/s^2.

The total height reached is 960 m, so (distance in free-fall) + (distance with engines on) = 960 m.

Therefore $(13.06 \text{ m/s}^2)\, T^2 + (8.00 \text{ m/s}^2)\, T^2 = 960$ m, which gives $T = 6.75$ s.

EVALUATE: It we put in 6.75 s for T, we see that the rocket travels considerably farther during free fall than with the engines on.

2.79. **IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s, free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2.

SET UP: Let $+y$ be upward. Let $y = 0$ at the ground.

EXECUTE: **(a)** When the engine shuts off both objects have upward velocity $v_y = v_{0y} + a_y t =$ $(5.0 \text{ m/s}^2)(10.0 \text{ s}) = 50.0 \text{ m/s}$ and are at $y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(5.0 \text{ m/s}^2)(10.0 \text{ s})^2 = 250 \text{ m}$.

For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0$ m/s, $y_0 = 250$ m, and $a_y = -9.80$ m/s^2.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y = \dfrac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \dfrac{0 - (50.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} + 250 \text{ m} = 378 \text{ m}$, which rounds to 380 m.

(b) The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0$ m/s, $a_y = -9.80$ m/s^2, and $y - y_0 = -250$ m. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-250 \text{ m} = (50.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. $(4.90 \text{ m/s}^2)t^2 - (50.0 \text{ m/s})t - 250 \text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)}\right)$ s. Only the positive solution is physical, so $t = 13.9$ s.

Powers therefore has free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 for $13.9 \text{ s} - 7.0 \text{ s} = 6.9 \text{ s}$. After 7.0 s of free-fall he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 250 \text{ m} + (50.0 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(7.0 \text{ s})^2 = 360 \text{ m}$ and has velocity $v_x = v_{0x} + a_x t = 50.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(7.0 \text{ s}) = -18.6 \text{ m/s}$. After the next 6.9 s he is at $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 360 \text{ m} + (-18.6 \text{ m/s})(6.9 \text{ s}) + \frac{1}{2}(-2.00 \text{ m/s}^2)(6.9 \text{ s})^2 = 184 \text{ m}$. Powers is 184 m above the ground when the helicopter crashes.

EVALUATE: When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from 5.0 m/s^2 upward to 9.80 m/s^2 downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

2.80. **IDENTIFY:** Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

SET UP: Let t_f be the time for the rock to fall to the ground and let t_s be the time it takes the sound to travel from the impact point back to you. $t_f + t_s = 8.00$ s. Both the rock and sound travel a distance h that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = g = 9.80 \text{ m/s}^2$.

EXECUTE: **(a)** For the falling rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}gt_f^2$. For the sound, $h = v_s t_s$. Equating these two equations for h and using the fact that $t_f + t_s = 8.00$ s, we get $\frac{1}{2}gt_f^2 = v_s t_s = v_s(8.00 \text{ s} - t_f)$. Using $v_s = 330$ m/s and $g = 9.80 \text{ m/s}^2$, we get a quadratic equation. Solving it using the quadratic formula and using the positive square root, we get $t_f = 7.225$ s. Therefore $h = \frac{1}{2}gt_f^2 = (1/2)(9.80 \text{ m/s}^2)(7.225 \text{ s})^2 = 256$ m.

(b) Ignoring sound you would calculate $d = \frac{1}{2}(9.80 \text{ m/s}^2)(8.00 \text{ s})^2 = 314$ m, which is greater than the actual distance. So you would have overestimated the height of the cliff. It actually takes the rock less time than 8.00 s to fall to the ground.

EVALUATE: Once we know h we can calculate that $t_f = 7.225$ s and $t_s = 0.775$ s. The time for the sound of impact to travel back to you is 6% of the total time and should not be neglected for best precision.

2.81. **(a) IDENTIFY:** We have nonconstant acceleration, so we must use calculus instead of the standard kinematics formulas.

SET UP: We know the acceleration as a function of time is $a_x(t) = -Ct$, so we can integrate to find the velocity and then the x-coordinate of the object. We know that $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x(t)dt$.

EXECUTE: **(a)** We have information about the velocity, so we need to find that by integrating the acceleration. $v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t -Ctdt = v_{0x} - \frac{1}{2}Ct^2$. Using the facts that the initial velocity is 20.0 m/s and $v_x = 0$ when $t = 8.00$ s, we have $0 = 20.0 \text{ m/s} - C(8.00 \text{ s})^2/2$, which gives $C = 0.625 \text{ m/s}^3$.

(b) We need the change in position during the first 8.00 s. Using $x(t) = x_0 + \int_0^t v_x(t)dt$ gives

$$x - x_0 = \int_0^t \left(-\tfrac{1}{2}Ct^2 + (20.0 \text{ m/s})\right)dt = -Ct^3/6 + (20.0 \text{ m/s})t$$

Putting in $C = 0.625 \text{ m/s}^3$ and $t = 8.00$ s gives an answer of 107 m.

EVALUATE: The standard kinematics formulas are of no use in this problem since the acceleration varies with time.

2.82. **IDENTIFY:** Both objects are in free-fall and move with constant acceleration 9.80 m/s^2, downward. The two balls collide when they are at the same height at the same time.

SET UP: Let $+y$ be upward, so $a_y = -9.80 \text{ m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball A be the one thrown straight up and ball B be the one dropped from rest at height H. $y_{0A} = 0$, $y_{0B} = H$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applied to each ball gives $y_A = v_0 t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$. $y_A = y_B$ gives $v_0 t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$ and $t = \dfrac{H}{v_0}$.

(b) For ball A at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_0}{g}$. Setting this equal to the time in part (a) gives $\dfrac{H}{v_0} = \dfrac{v_0}{g}$ and $H = \dfrac{v_0^2}{g}$.

EVALUATE: In part (a), using $t = \dfrac{H}{v_0}$ in the expressions for y_A and y_B gives $y_A = y_B = H\left(1 - \dfrac{gH}{2v_0^2}\right)$. H must be less than $\dfrac{2v_0^2}{g}$ in order for the balls to collide before ball A returns to the ground. This is because it takes ball A time $t = \dfrac{2v_0}{g}$ to return to the ground and ball B falls a distance $\frac{1}{2}gt^2 = \dfrac{2v_0^2}{g}$ during this time. When $H = \dfrac{2v_0^2}{g}$ the two balls collide just as ball A reaches the ground and for H greater than this ball A reaches the ground before they collide.

2.83. IDENTIFY and SET UP: Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2$, $v_{Ax} = \dfrac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \dfrac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \dfrac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \dfrac{dv_{Bx}}{dt} = 2\gamma - 6\delta t$

(a) IDENTIFY and SET UP: The car that initially moves ahead is the one that has the larger v_{0x}.

EXECUTE: At $t = 0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car A moves ahead.

(b) IDENTIFY and SET UP: Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t = 0$, which says that they start from the same point. To find the other solutions, divide by t: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{2\delta}\left(-(\beta-\gamma) \pm \sqrt{(\beta-\gamma)^2 - 4\delta\alpha}\right) = \frac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00\ \text{s} \pm 1.73\ \text{s}$$

So $x_A = x_B$ for $t = 0$, $t = 2.27$ s and $t = 5.73$ s.

EVALUATE: Car A has constant, positive a_x. Its v_x is positive and increasing. Car B has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car B initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27$ s car B has overtaken car A and then passes it. At $t = 5.73$ s, car B is moving in the $-x$-direction as it passes car A again.

(c) IDENTIFY: The distance from A to B is $x_B - x_A$. The rate of change of this distance is $\dfrac{d(x_B - x_A)}{dt}$. If this distance is not changing, $\dfrac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between A and B is neither decreasing nor increasing at the instant when they have the same velocity.)

SET UP: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

EXECUTE: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$$

$t = 2.667\ \text{s} \pm 1.667\ s$, so $v_{Ax} = v_{Bx}$ for $t = 1.00$ s and $t = 4.33$ s.

EVALUATE: At $t = 1.00$ s, $v_{Ax} = v_{Bx} = 5.00$ m/s. At $t = 4.33$ s, $v_{Ax} = v_{Bx} = 13.0$ m/s. Now car B is slowing down while A continues to speed up, so their velocities aren't ever equal again.
(d) IDENTIFY and **SET UP:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

EXECUTE: $t = \dfrac{\gamma - \beta}{3\delta} = \dfrac{2.80 \text{ m/s}^2 - 1.20 \text{ m/s}^2}{3(0.20 \text{ m/s}^3)} = 2.67$ s.

EVALUATE: At $t = 0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t = 2.67$ s but for all times after that $a_{Bx} < a_{Ax}$.

2.84. IDENTIFY: Interpret the data on a graph to draw conclusions about the motion of a glider having constant acceleration down a frictionless air track, starting from rest at the top.
SET UP: The constant-acceleration kinematics formulas apply. Take the +x-axis along the surface of the track pointing downward.

EXECUTE: **(a)** For constant acceleration starting from rest, we have $x = \frac{1}{2}a_x t^2$. Therefore a plot of x versus t^2 should be a straight line, and the slope of that line should be $a_x/2$.
(b) To construct the graph of x versus t^2, we can use readings from the graph given in the text to construct a table of values for x and t^2, or we could use graphing software if available. The result is a graph similar to the one shown in Figure 2.84, which was obtained using software. A graph done by hand could vary slightly from this one, depending on how one reads the values on the graph in the text. The graph shown is clearly a straight line having slope 3.77 m/s^2 and x-intercept 0.0092 m. Using the slope y-intercept form of the equation of a straight line, the equation of this line is $x = 3.77t^2 + 0.0092$, where x is in meters and t in seconds.

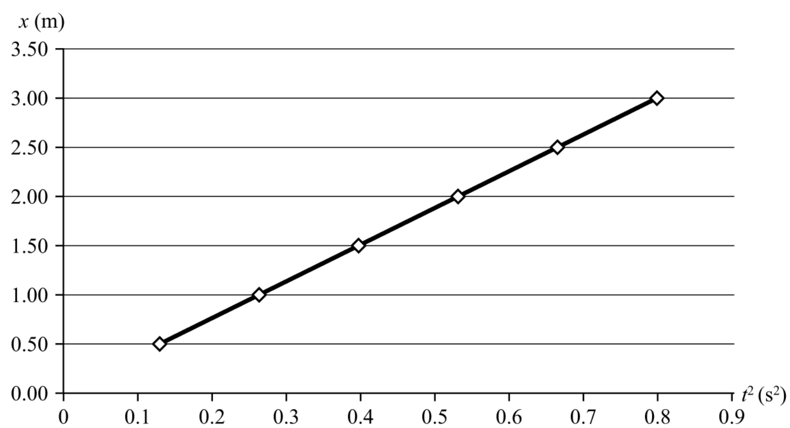


Figure 2.84

(c) The slope of the straight line in the graph is $a_x/2$, so $a_x = 2(3.77 \text{ m/s}^2) = 7.55 \text{ m/s}^2$.
(d) We know the distance traveled is 1.35 m, the acceleration is 7.55 m/s^2, and the initial velocity is zero, so we use the equation $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and solve for v_x, giving $v_x = 4.51$ m/s.

EVALUATE: For constant acceleration in part (d), the average velocity is (4.51 m/s)/2 = 2.25 m/s. With this average velocity, the time for the glider to travel 1.35 m is x/v_{av} = (1.35 m)/(2.25 m) = 0.6 s, which is approximately the value of t read from the graph in the text for x = 1.35 m.

2.85. **IDENTIFY:** A ball is dropped from rest and falls from various heights with constant acceleration. Interpret a graph of the square of its velocity just as it reaches the floor as a function of its release height.

SET UP: Let $+y$ be downward since all motion is downward. The constant-acceleration kinematics formulas apply for the ball.

EXECUTE: **(a)** The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies to the falling ball. Solving for $y - y_0$ and using $v_{0y} = 0$ and $a_y = g$, we get $y - y_0 = \dfrac{v_y^2}{2g}$. A graph of $y - y_0$ versus v_y^2 will be a straight line with slope $1/2g = 1/(19.6\ \text{m/s}^2) = 0.0510\ \text{s}^2/\text{m}$.

(b) With air resistance the acceleration is less than $9.80\ \text{m/s}^2$, so the final speed will be smaller.

(c) The graph will not be a straight line because the acceleration will vary with the speed of the ball. For a given release height, v_y with air resistance is less than without it. Alternatively, with air resistance the ball will have to fall a greater distance to achieve a given velocity than without air resistance. The graph is sketched in Figure 2.85.

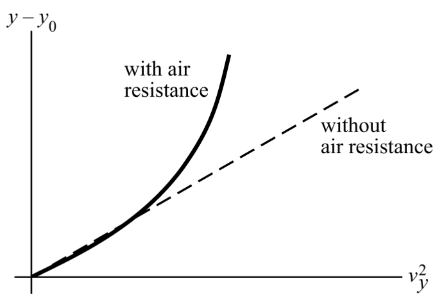


Figure 2.85

EVALUATE: Graphing $y - y_0$ versus v_y^2 for a set of data will tell us if the acceleration is constant. If the graph is a straight line, the acceleration is constant; if not, the acceleration is not constant.

2.86. **IDENTIFY:** Use data of acceleration and time for a model car to find information about its velocity and position.

SET UP: From the table of data in the text, we can see that the acceleration is not constant, so the constant-acceleration kinematics formlas do not apply. Therefore we must use calculus. The equations $v_x(t) = v_{0x} + \int_0^t a_x dt$ and $x(t) = x_0 + \int_0^t v_x dt$ apply.

EXECUTE: **(a)** Figure 2.86a shows the graph of a_x versus t. From the graph, we find that the slope of the line is $-0.5131\ \text{m/s}^3$ and the a-intercept is $6.026\ \text{m/s}^2$. Using the slope y-intercept equation of a straight line, the equation is $a(t) = -0.513\ \text{m/s}^3\ t + 6.026\ \text{m/s}^2$, where t is in seconds and a is in m/s^2.

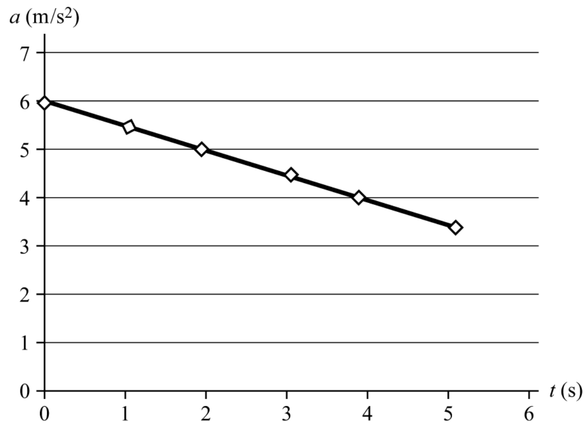


Figure 2.86a

(b) Integrate the acceleration to find the velocity, with the initial velocity equal to zero.

$v_x(t) = v_{0x} + \int_0^t a_x dt = v_{0x} + \int_0^t \left(6.026\text{ m/s}^2 - 0.513\text{ m/s}^3\ t\right)dt\ = 6.026\text{ m/s}^2\ t - 0.2565\text{ m/s}^3\ t^2.$

Figure 2.86b shows a sketch of the graph of v_x versus t.

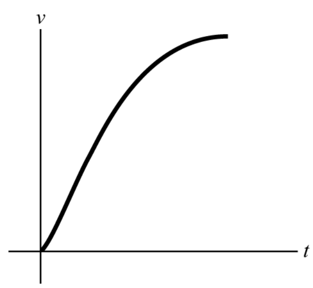


Figure 2.86b

(c) Putting $t = 5.00$ s into the equation we found in (b) gives $v_x = 23.7$ m/s.
(d) Integrate the velocity to find the change in position of the car.

$x - x_0 = \int_0^t v_x dt = \int_0^t \left[\left(6.026\text{ m/s}^2\right)t - \left(0.2565\text{ m/s}^3\right)t^2\right]dt\ = 3.013\text{ m/s}^2\ t^2 - 0.0855\text{ m/s}^3\ t^3$

At $t = 5.00$ s, this gives $x - x_0 = 64.6$ m.

EVALUATE: Since the acceleration is not constant, the standard kinematics formulas do not apply, so we must go back to basic definitions involving calculus.

2.87. IDENTIFY: Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\text{max}}/2$ on the way down equals the time spent above $y_{\text{max}}/2$ on the way up.

SET UP: Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\text{max}}/2$ when he is a distance $y_{\text{max}}/2$ above the floor.

EXECUTE: The time from the maximum height to $y_{\text{max}}/2$ above the floor is given by $y_{\text{max}}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\text{max}} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\text{max}}/2$ to the floor is $t_2 = t_{\text{tot}} - t_1$.

$$\frac{2t_1}{t_2} = \frac{2\sqrt{y_{\text{max}}/2}}{\sqrt{y_{\text{max}}} - \sqrt{y_{\text{max}}/2}} = \frac{2}{\sqrt{2} - 1} = 4.8.$$

EVALUATE: The person spends over twice as long above $y_{max}/2$ as below $y_{max}/2$. His average speed is less above $y_{max}/2$ than it is when he is below this height.

2.88. IDENTIFY: Apply constant acceleration equations to the motion of the two objects, the student and the bus.

SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: **(a)** Setting $x_1 = x_2$ and solving for the times *t* gives $t = \frac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{0.170\ \text{m/s}^2}\left(5.0\ \text{m/s} \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) = 9.55\ \text{s and } 49.3\ \text{s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

(b) The speed of the bus is $(0.170\ \text{m/s}^2)(9.55\ \text{s}) = 1.62\ \text{m/s}$.

(c) The results can be verified by noting that the *x* lines for the student and the bus intersect at two points, as shown in Figure 2.88a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $(0.170\ \text{m/s}^2)(49.3\ \text{s}) = 8.38\ \text{m/s}$.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at $3.5\ \text{m/s}$, Figure 2.88b shows that the two lines do *not* intersect.

(f) For the student to catch the bus, $v_0^2 > 2ax_0$. And so the minimum speed is $\sqrt{2(0.170\ \text{m/s}^2)(40\ \text{m/s})} = 3.688\ \text{m/s}$. She would be running for a time $\frac{3.69\ \text{m/s}}{0.170\ \text{m/s}^2} = 21.7\ \text{s}$, and covers a distance $(3.688\ \text{m/s})(21.7\ \text{s}) = 80.0\ \text{m}$. However, when the student runs at $3.688\ \text{m/s}$, the lines intersect at *one* point, at $x = 80\ \text{m}$, as shown in Figure 2.88c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{tot} - t_1$

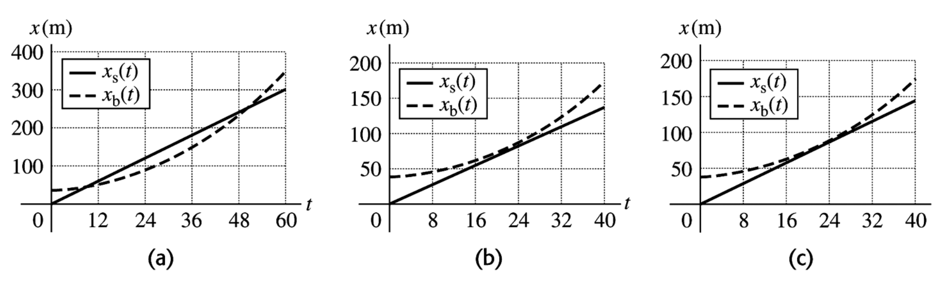


Figure 2.88

2.89. **IDENTIFY:** Apply constant acceleration equations to both objects.

SET UP: Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0 t - \frac{1}{2}gt^2$, $y_2 = h - \frac{1}{2}g(t - t_0)^2$. In this case, $t_0 = 1.00$ s.

EXECUTE: **(a)** Setting $y_1 = y_2 = 0$, expanding the binomial $(t - t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0 t = gt_0 t - \frac{1}{2}gt_0^2$. Solving for t: $t = \dfrac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \dfrac{t_0}{2}\left(\dfrac{1}{1 - v_0/(gt_0)}\right)$.

Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for h as a function of v_0 yields, after some algebra, $h = \frac{1}{2}gt_0^2 \dfrac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{(gt_0 - v_0)^2}$. Using the given value $t_0 = 1.00$ s and $g = 9.80\ \text{m/s}^2$,

$$h = 20.0\ \text{m} = (4.9\ \text{m})\left(\frac{4.9\ \text{m/s} - v_0}{9.8\ \text{m/s} - v_0}\right)^2.$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields 8.2 m/s. The graph of y versus t for each ball is given in Figure 2.89.

(b) The above expression gives for (i) 0.411 m and for (ii) 1.15 km.

(c) As v_0 approaches 9.8 m/s, the height h becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8$ m/s, the first ball can never catch the second ball.

(d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9$ m/s, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

EVALUATE: Note that the values of v_0 in parts (a) and (b) are all greater than v_{min} and less than v_{max}.

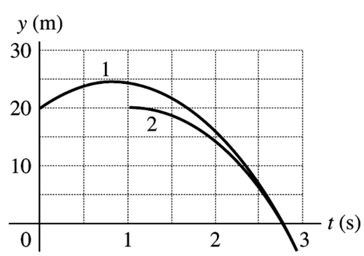


Figure 2.89

2.90. **IDENTIFY:** We know the change in velocity and the time for that change. We can use these quantities to find the average acceleration.

SET UP: The average acceleration is the change in velocity divided by the time for that change.

EXECUTE: $a_{\text{av}} = (v - v_0)/t = (0.80\ \text{m/s} - 0)/(250 \times 10^{-3}\ \text{s}) = 32\ \text{m/s}^2$, which is choice (c).

EVALUATE: This is about 1/3 the acceleration due to gravity, which is a reasonable acceleration for an organ.

2.91. **IDENTIFY:** The original area is divided into two equal areas. We want the diameter of these two areas, assuming the original and final areas are circular.

SET UP: The area A of a circle or radius r is $A = \pi r^2$ and the diameter d is $d = 2r$. $A_i = 2A_f$, and $r = d/2$, where A_f is the area of each of the two arteries.

EXECUTE: Call d the diameter of each artery. $A_i = \pi(d_a/2)^2 = 2[\pi(d/2)^2]$, which gives s $d = d_a/\sqrt{2}$, which is choice (b).

EVALUATE: The area of each artery is half the area of the aorta, but the diameters of the arteries are not half the diameter of the aorta.

2.92. **IDENTIFY:** We must interpret a graph of blood velocity during a heartbeat as a function of time.

SET UP: The instantaneous acceleration of a blood molecule is the slope of the velocity-versus-time graph.

EXECUTE: The magnitude of the acceleration is greatest when the slope of the v-t graph is steepest. That occurs at the upward sloping part of the graph, around $t = 0.10$ s, which makes choice (d) the correct one.

EVALUATE: The slope of the given graph is positive during the first 0.25 s and negative after that. Yet the velocity is positive throughout. Therefore the blood is always flowing forward, but it is increasing in speed during the first 0.25 s and slowing down after that.