# 24

## CAPACITANCE AND DIELECTRICS

**VP24.4.1.** **IDENTIFY:** We are dealing with a parallel-plate capacitor.

**SET UP** and **EXECUTE:** **(a)** We want the capacitance. $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\epsilon_0 (2.75\ \text{m}^2)}{0.00350\ \text{m}} = 6.95$ nF.

**(b)** We want the potential difference between the plates. $V = Ed = (\sigma/\epsilon_0)\, d$. Using the given values gives $V = 5.54$ kV.

**EVALUATE:** $E = \sigma/\epsilon_0 = 1.58$ MV/m between the plates, so we get a large potential difference of over 5000 V.

**VP24.4.2.** **IDENTIFY:** We are dealing with a parallel-plate capacitor.

**SET UP** and **EXECUTE:** **(a)** We want the plate spacing. Solving $C = \dfrac{\epsilon_0 A}{d}$ for $d$ gives $d = \dfrac{\epsilon_0 A}{C}$. Using the given numbers we get $d = 4.80$ mm.

**(b)** We want the charge on the plates and the potential difference between them when $V = 3.00$ kV. $Q = CV = (8.30\ \text{nF})(3.00\ \text{kV}) = 24.9\ \mu\text{C}$. $E = V/d = (3.00\ \text{kV})/(4.80\ \text{mm}) = 625$ kV/m.

**EVALUATE:** Use $E = \sigma/\epsilon_0 = \dfrac{Q/A}{\epsilon_0}$, which gives the same answer as in (b).

**VP24.4.3.** **IDENTIFY** and **SET UP:** We have a spherical capacitor. $C = 4\pi\epsilon_0 \left(\dfrac{r_a r_b}{r_b - r_a}\right)$.

**EXECUTE:** **(a)** We want the outer radius of the inner shell, which is $r_a$. Solve $C = 4\pi\epsilon_0 \left(\dfrac{r_a r_b}{r_b - r_a}\right)$ for $r_a$ giving $r_a = \dfrac{r_b C}{C + 4\pi\epsilon_0\, r_b}$. The given numbers yield $r_a = 8.33$ cm.

**(b)** We want the charge when $V = 355$ V. $Q = CV = (125\ \text{pF})(355\ \text{V}) = 44.4$ nC.

**(c)** We want the surface charge density at both surfaces. $\sigma = \dfrac{Q}{A} = \dfrac{Q}{4\pi r^2}$.

Outer surface: $\sigma_b = \dfrac{Q}{4\pi r_b^2} = \dfrac{-44.4\ \text{nC}}{4\pi(0.0900\ \text{m})^2} = -4.36\times10^{-7}\ \text{C/m}^2$.

Inner surface: $\sigma_a = \dfrac{Q}{4\pi r_a^2} = \dfrac{+44.4\ \text{nC}}{4\pi(0.0833\ \text{m})^2} = +5.09\times10^{-7}\ \text{C/m}^2$.

**EVALUATE:** The inner and outer surfaces carry the same magnitude charge, but the inner surface has a smaller area so its charge density should be greater than that of the outer surface. This agrees with our result.

**VP24.4.4.** **IDENTIFY** and **SET UP:** We have a cylindrical capacitor. From Example 24.4, $C/L = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$.

**EXECUTE:** **(a)** We want $r_b/r_a$. Solving $C/L = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for $r_b/r_a$ and putting in the given numbers we have $\ln(r_b/r_a) = \dfrac{2\pi\epsilon_0}{C/L} = 0.80524$. $r_b/r_a = e^{0.80524} = 2.24$.

**(b)** We want the potential difference. $C = \dfrac{Q}{V}$, so $C/L = \dfrac{Q/L}{V}$, giving

$V = \dfrac{Q/L}{C/L} = \dfrac{8.62 \text{ nC/m}}{69.0 \text{ pF/m}} = 125 \text{ V}$. The inner conductor is positive, so the electric field between the cylinders does work on a positive charge in going from $a$ to $b$, so the inner conductor is at a higher potential then the outer one.

**EVALUATE:** A kilometer of this cylinder would have a capacitance of only 69 nF.

**VP24.9.1.** **IDENTIFY:** We have capacitors in series and parallel.

**SET UP:** In series: $1/C_{\text{eq}} = 1/C_1 + 1/C_2 + \cdots$, in parallel: $C_{\text{eq}} = C_1 + C_2 + \text{L}$. We want the equivalent capacitance.

**EXECUTE:** **(a)** $1/C_{\text{eq}} = 1/(1.00\ \mu\text{F}) + 1/(2.50\ \mu\text{F}) + 1/(5.00\ \mu\text{F})$. $C_{\text{eq}} = 0.625\ \mu\text{F}$.

**(b)** $C_{\text{eq}} = 1.00\ \mu\text{F} + 2.50\ \mu\text{F} + 5.00\ \mu\text{F} = 8.50\ \mu\text{F}$.

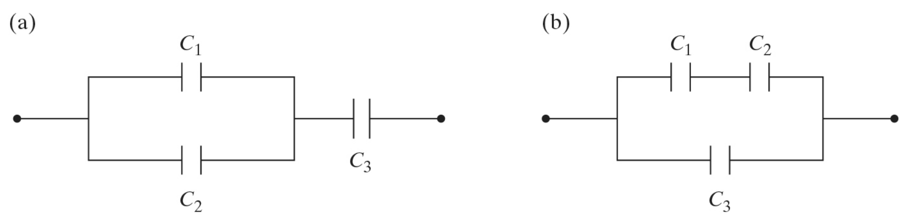



**Figure VP24.9.1**

**(c)** Figure VP24.9.1(a) shows the combination. The parallel combination is given by $C_p = 1.00\ \mu\text{F} + 2.50\ \mu\text{F} = 3.50\ \mu\text{F}$. The series combination is given by $1/C_S = 1/(3.50\ \mu\text{F}) + 1/(5.00\ \mu\text{F})$. This gives $C_S = 2.06\ \mu\text{F}$.

**(d)** Fig. VP24.9.1(b) shows the combination. For the series combination, $1/C_S = 1/(1.00\ \mu\text{F}) + 1/(2.50\ \mu\text{F})$. $C_S = 0.715\ \mu\text{F}$. The equivalent capacitance is $C_{\text{eq}} = 0.714\ \mu\text{F} + 5.00\ \mu\text{F} = 5.71\ \mu\text{F}$.

**EVALUATE:** Notice that for capacitors in series, the equivalent capacitance is less than the smallest capacitance, but when they are in parallel the equivalent capacitance is larger than the largest capacitance.

**VP24.9.2.** **IDENTIFY:** We are dealing with the energy stored in capacitors in combination.

**SET UP:** $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2$. We want to find the stored energy.

**EXECUTE:** **(a)** $U_1 = \dfrac{Q^2}{2C_1} =$ 0.0675 J using the given numbers. Similarly $U_2 = 0.0169$ J. We see that $C_1$ stores more energy than $C_2$.

**(b)** $U_1 = \dfrac{1}{2}C_1V^2 = 0.0117$ J using the given numbers. Similarly $U_2 = 0.0469$ J. $C_2$ has more energy than $C_1$.

**EVALUATE:** Note that a larger capacitor does *not necessarily* store more energy than a small capacitor. It depends on how they are connected.

**VP24.9.3.** **IDENTIFY:** We are dealing with the energy stored in capacitors in combination.

**SET UP:** $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2$. $1/C_{\text{eq}} = 1/C_1 + 1/C_2 = \dfrac{C_1 + C_2}{C_1 C_2}$. We want the stored energy.

**EXECUTE:** **(a)** Series: $U_S = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\left(\dfrac{C_1C_2}{C_1 + C_2}\right)V^2 = 0.0115$ J using the given numbers.

Parallel: $U_p = \dfrac{1}{2}(C_1 + C_2)V^2 =$ 0.0578 J using the given numbers. The parallel combination stores more energy than the series combination.

**(b)** Series: $U_S = \dfrac{Q^2}{2C_{\text{eq}}} = \dfrac{Q^2}{2}\left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right) = 0.0733$ J using the given numbers.

Parallel: $U_p = \dfrac{Q^2}{2C_{\text{eq}}} = \dfrac{Q^2}{2(C_1 + C_2)} = 0.0145$ J using the given numbers. The series combination stores more energy.

**EVALUATE:** In part (a) the parallel combination stored more energy, but in (b) the series store more. The result depends on how the capacitors are connected in the circuit.

**VP24.9.4.** **IDENTIFY:** This problem involves a parallel-plate capacitor.

**SET UP** and **EXECUTE:** **(a)** We want the surface charge density on the plates. Using $\sigma = Q/A$ and the given numbers, we get $\sigma$ = 5.10 $\mu$C/m$^2$.f

**(b)** We want the electric field. $E = \sigma/\epsilon_0$ = 576 kV/m.

**(c)** We want the energy density. $u = \dfrac{1}{2}\epsilon_0 E^2$ = 1.47 J/m$^3$ using the result from (b).

**(d)** We want the total energy. $U = u(\text{volume}) = uAd$ = (1.47 J/m$^3$)(2.45 m$^2$)(0.00140 m) = 5.04 mJ.

**EVALUATE:** As a check, use $U = \dfrac{Q^2}{2C}$ and $C = \epsilon_0 A/d$ to get $U = \dfrac{Q^2 d}{2\epsilon_0 A}$ = 5.04 mJ, which agrees with our result in (d).

**VP24.11.1.** **IDENTIFY:** We are dealing with a capacitor containing dielectric.

**SET UP** and **EXECUTE:** **(a)** We want $C$ without dielectric. $C = \dfrac{\epsilon_0 A}{d}$ = 1.36 nF using the given numbers.

**(b)** We want the charge. $Q = CV$ = (1.36 nF)(4.00 kV) = 5.43 $\mu$C.

**(c)** We want the dielectric constant. $V = V_0/K$, so $K = V_0/V$ = (4.00 kV)/(2.50 kV) = 1.60.

**(d)** We want the capacitance. $C = KC_0$ = (1.60)(1.36 nF) = 2.17 nF.

**(e)** We want the induced charge. $Q_{\neg i} = Q(1 - 1/K)$ = (5.43 $\mu$C)(1 – 1/1.60) = 2.04 $\mu$C.

**EVALUATE:** The dielectric increases the capacitance. The presence of the induced charge decreases the potential difference between the plates. This charge produces and electric field opposite to the original field.

**VP24.11.2.** **IDENTIFY:** We are dealing with a capacitor containing dielectric.

**SET UP** and **EXECUTE:** **(a)** We want the capacitance. $C = \epsilon_0 A/d$ = 2.36 nF using the given numbers.

**(b)** We want the charge. $Q = swCV$ = (2.36 nF)(3.50 kV) = 8.26 $\mu$C.

**(c)** We want $C$ with the dielectric. $C = KC_0$ = (2.50)(2.36 nF) = 5.90 nF.

**(d)** We want the charge. $Q = CV$ = (5.90 nF)(3.50 kV) = 20.7 $\mu$C.

**(e)** We want the induced charge. With the battery connected, $V$ remains the same, so $E = V/d$ also remains the same. Thus there must be more charge on the plates than for an empty capacitor to make up for the electric field due to the induced charge of the dielectric. Using $E = \sigma/\epsilon_0$ gives

$\dfrac{\sigma_{\text{plates}}}{\epsilon_0} = \dfrac{\sigma_0}{\epsilon_0} + \dfrac{|\sigma_{\text{induced}}|}{\epsilon_0}$. Which gives $|Q_{\text{induced}}| = Q_{\text{plates}} - Q_0$ = 20.7 $\mu$C – 8.26 $\mu$C = 12.4 $\mu$C.

**EVALUATE:** If the battery were not left connected while the dielectric was inserted, $V$ would *not* remain constant but the charge $Q$ on the plates would stay constant as in problem VP24.11.1.

**VP24.11.3.** **IDENTIFY:** We are dealing with the energy stored in a capacitor with dielectric.

**SET UP** and **EXECUTE:** We want the energy stored before and after the dielectric is inserted.

**(a)** $U_0 = \frac{1}{2}CV_0^2 = 17.6$ J using the given numbers.

**(b)** $K = V_0/V$ and $C = KC_0$, so $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)V^2 = \frac{1}{2}\left(\frac{V_0}{V}C_0\right)V^2 = \frac{1}{2}C_0V_0V$. Using $C_0 = 4.50\ \mu$F, $V_0 = 2.80$ kV, and $V = 1.20$ kV gives $U = 7.56$ J.

**EVALUATE:** The dielectric increased the capacitance but decreased the stored energy because $V$ decreased.

**VP24.11.4.** **IDENTIFY:** We are dealing with the energy stored in a capacitor with dielectric.

**SET UP** and **EXECUTE:** We want the energy stored before and after the dielectric is inserted.

**(a)** $U_0 = \frac{1}{2}CV_0^2 = 6.30$ J using the given numbers.

**(b)** $U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)V^2 = K\left(\frac{1}{2}C_0V^2\right) = KU_0 = (2.85)(6.30\text{ J}) = 18.0$ J.

**EVALUATE:** With the battery connected as the dielectric is inserted, $V$ stays the same and $C$ increases, so $U$ increases by a factor of $K$.

**24.1.** **IDENTIFY:** The capacitance depends on the geometry (area and plate separation) of the plates.

**SET UP:** For a parallel-plate capacitor, $V_{ab} = Ed$, $E = \frac{Q}{\epsilon_0 A}$, and $C = \frac{Q}{V_{ab}}$.

**EXECUTE:** **(a)** $V_{ab} = Ed = (4.00\times10^6\text{ V/m})(2.50\times10^{-3}\text{ m}) = 1.00\times10^4\text{ V}$.

**(b)** Solving for the area gives

$$A = \frac{Q}{E\epsilon_0} = \frac{80.0\times10^{-9}\text{ C}}{(4.00\times10^6\text{ V/m})[8.854\times10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)]} = 2.26\times10^{-3}\text{ m}^2 = 22.6\text{ cm}^2.$$

**(c)** $C = \frac{Q}{V_{ab}} = \frac{80.0\times10^{-9}\text{ C}}{1.00\times10^4\text{ V}} = 8.00\times10^{-12}\text{ F} = 8.00\text{ pF}$.

**EVALUATE:** The capacitance is reasonable for laboratory capacitors, but the area is rather large.

**24.2.** **IDENTIFY** and **SET UP:** $C = \frac{\epsilon_0 A}{d}$, $C = \frac{Q}{V}$ and $V = Ed$.

**EXECUTE:** **(a)** $C = \epsilon_0 \frac{A}{d} = \epsilon_0 \frac{0.000982\text{ m}^2}{0.00328\text{ m}} = 2.65\text{ pF}$.

**(b)** $V = \frac{Q}{C} = \frac{4.35\times10^{-8}\text{ C}}{2.65\times10^{-12}\text{ F}} = 16.4\text{ kV}$.

**(c)** $E = \frac{V}{d} = \frac{16.4\times10^3\text{ V}}{0.00328\text{ m}} = 5.00\times10^6\text{ V/m}$.

**EVALUATE:** The electric field is uniform between the plates, at points that aren't close to the edges.

**24.3.** **IDENTIFY** and **SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Section 24.1.

**EXECUTE:** **(a)** $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6}\ \text{C}}{245\times10^{-12}\ \text{F}} = 604\ \text{V}.$

**(b)** $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(245\times10^{-12}\ \text{F})(0.328\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3}\ \text{m}^2 = 90.8\ \text{cm}^2.$

**(c)** $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604\ \text{V}}{0.328\times10^{-3}\ \text{m}} = 1.84\times10^6\ \text{V/m}.$

**(d)** $E = \dfrac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84\times10^6\ \text{V/m})(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5}\ \text{C/m}^2.$

**EVALUATE:** We could also calculate $\sigma$ directly as $Q/A$. $\sigma = \dfrac{Q}{A} = \dfrac{0.148\times10^{-6}\ \text{C}}{9.08\times10^{-3}\ \text{m}^2} = 1.63\times10^{-5}\ \text{C/m}^2,$ which checks.

**24.4.** **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

**SET UP:** When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab} = 12.0\ \text{V}$.

**EXECUTE:** **(a)** 12.0 V.

**(b)** (i) When $d$ is doubled, $C$ is halved. $V_{ab} = \dfrac{Q}{C}$ and $Q$ is constant, so $V$ doubles. $V = 24.0\ \text{V}$.

(ii) When $r$ is doubled, $A$ increases by a factor of 4. $V$ decreases by a factor of 4 and $V = 3.0\ \text{V}$.

**EVALUATE:** The electric field between the plates is $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{\epsilon_0 A}$. $V_{ab} = Ed$. When $d$ is doubled $E$ is unchanged and $V$ doubles. When $A$ is increased by a factor of 4, $E$ decreases by a factor of 4 so $V$ decreases by a factor of 4.

**24.5.** **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

**SET UP:** When the capacitor is connected to the battery, $V_{ab} = 12.0\ \text{V}$.

**EXECUTE:** **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\ \text{F})(12.0\ \text{V}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}.$

**(b)** When $d$ is doubled $C$ is halved, so $Q$ is halved. $Q = 60\ \mu\text{C}$.

**(c)** If $r$ is doubled, $A$ increases by a factor of 4. $C$ increases by a factor of 4 and $Q$ increases by a factor of 4. $Q = 480\ \mu\text{C}$.

**EVALUATE:** When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, $\sigma$. To produce the same $\sigma$, more charge is required when the area increases.

**24.6.** **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $V_{ab} = Ed$. $C = \dfrac{\epsilon_0 A}{d}$.

**SET UP:** We want $E = 1.00\times10^4\ \text{N/C}$ when $V = 100\ \text{V}$.

**EXECUTE:** **(a)** $d = \dfrac{V_{ab}}{E} = \dfrac{1.00\times 10^2\ \text{V}}{1.00\times 10^4\ \text{N/C}} = 1.00\times 10^{-2}\ \text{m} = 1.00\ \text{cm}.$

$A = \dfrac{Cd}{\epsilon_0} = \dfrac{(5.00\times 10^{-12}\ \text{F})(1.00\times 10^{-2}\ \text{m})}{8.854\times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = 5.65\times 10^{-3}\ \text{m}^2.$ $A = \pi r^2$ so

$r = \sqrt{\dfrac{A}{\pi}} = 4.24\times 10^{-2}\ \text{m} = 4.24\ \text{cm}.$

**(b)** $Q = CV_{ab} = (5.00\times 10^{-12}\ \text{F})(1.00\times 10^2\ \text{V}) = 5.00\times 10^{-10}\ \text{C} = 500\ \text{pC}.$

**EVALUATE:** $C = \dfrac{\epsilon_0 A}{d}$. We could have a larger $d$, along with a larger $A$, and still achieve the required $C$ without exceeding the maximum allowed $E$.

**24.7.** **IDENTIFY:** The energy stored in a capacitor depends on its capacitance, which in turn depends on its geometry.

**SET UP:** $C = Q/V$ for any capacitor, and $C = \dfrac{\epsilon_0 A}{d}$ for a parallel-plate capacitor.

**EXECUTE:** **(a)** $C = \dfrac{Q}{V} = \dfrac{2.40\times 10^{-10}\ \text{C}}{42.0\ \text{V}} = 5.714\times 10^{-12}\ \text{F}.$ Using $C = \dfrac{\epsilon_0 A}{d}$ gives

$d = \dfrac{\epsilon_0 A}{C} = \dfrac{[8.854\times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)](6.80\times 10^{-4}\ \text{m}^2)}{5.714\times 10^{-12}\ \text{F}} = 1.05\ \text{mm}.$

**(b)** $d = 2.10\times 10^{-3}\ \text{m}.$ $C = \dfrac{\epsilon_0 A}{d} = \dfrac{5.714\times 10^{-12}\ \text{F}}{2} = 2.857\times 10^{-12}\ \text{F}.$ $V = \dfrac{Q}{C}$, so

$V = 2(42.0\ \text{V}) = 84.0\ \text{V}.$

**EVALUATE:** Doubling the plate separation halves the capacitance, so twice the potential difference is required to keep the same charge on the plates.

**24.8.** **IDENTIFY:** Capacitance depends on the geometry of the object.

**(a) SET UP:** The capacitance of a cylindrical capacitor is $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$. Solving for $r_b$ gives

$r_b = r_a e^{2\pi\epsilon_0 L/C}$.

**EXECUTE:** Substituting in the numbers for the exponent gives

$$\frac{2\pi(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.120\ \text{m})}{3.67\times 10^{-11}\ \text{F}} = 0.182.$$

Now use this value to calculate $r_b$: $r_b = r_a e^{0.182} = (0.250\ \text{cm})e^{0.182} = 0.300\ \text{cm}.$

**(b) SET UP:** For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the numbers gives $\lambda = Q/L = CV/L.$

**EXECUTE:** Numerically we get

$$\lambda = \frac{CV}{L} = \frac{(3.67\times 10^{-11}\ \text{F})(125\ \text{V})}{0.120\ \text{m}} = 3.82\times 10^{-8}\ \text{C/m} = 38.2\ \text{nC/m}.$$

**EVALUATE:** The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just 0.50 mm. These cylinders would have to be carefully constructed.

**24.9.** **IDENTIFY:** Apply the results of Example 24.4. $C = Q/V$.

**SET UP:** $r_a = 0.50\ \text{mm}$, $r_b = 5.00\ \text{mm}$.

**EXECUTE:** **(a)** $C = \dfrac{L2\pi\epsilon_0}{\ln(r_b/r_a)} = \dfrac{(0.180\ \text{m})2\pi\epsilon_0}{\ln(5.00/0.50)} = 4.35\times 10^{-12}\ \text{F}.$

**(b)** $V = Q/C = (10.0\times10^{-12}\ \text{C})/(4.35\times10^{-12}\ \text{F}) = 2.30\ \text{V}.$

**EVALUATE:** $\dfrac{C}{L} = 24.2$ pF. This value is similar to those in Example 24.4. The capacitance is determined entirely by the dimensions of the cylinders.

**24.10.** **IDENTIFY** and **SET UP:** Use $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ which was derived in Example 24.4. Then use $Q = CV$ to calculate $Q$.

**EXECUTE:** **(a)** Using $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ gives

$$\frac{C}{L} = \frac{2\pi(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{\ln[(3.5\ \text{mm})/(2.2\ \text{mm})]} = 1.2\times10^{-10}\ \text{F/m} = 120\ \text{pF/m}.$$

**(b)** $C = (1.20\times10^{-10}\ \text{F/m})(2.8\ \text{m}) = 3.355\times10^{-10}\ \text{F}.$

$Q = CV = (3.355\times10^{-10}\ \text{F})(350\times10^{-3}\ \text{V}) = 1.2\times10^{-10}\ \text{C} = 120\ \text{pC}.$

The conductor at higher potential has the positive charge, so there is +120 pC on the inner conductor and −120 pC on the outer conductor.

**EVALUATE:** $C$ depends only on the dimensions of the capacitor. $Q$ and $V$ are proportional.

**24.11.** **IDENTIFY:** We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

**(a) SET UP:** By the definition of capacitance, $C = Q/V$.

**EXECUTE:** $C = \dfrac{Q}{V} = \dfrac{3.30\times10^{-9}\ \text{C}}{2.20\times10^{2}\ \text{V}} = 1.50\times10^{-11}\ \text{F} = 15.0\ \text{pF}.$

**(b) SET UP:** The capacitance of a spherical capacitor is $C = 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b - r_a}$.

**EXECUTE:** Solve for $r_a$ and evaluate using $C = 15.0$ pF and $r_b = 4.00$ cm, giving $r_a = 3.09$ cm.

**(c) SET UP:** We can treat the inner sphere as a point charge located at its center and use Coulomb's law, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$.

**EXECUTE:** $E = \dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.30\times10^{-9}\ \text{C})}{(0.0309\ \text{m})^2} = 3.12\times10^{4}\ \text{N/C}.$

**EVALUATE:** Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

**24.12.** **IDENTIFY:** Apply the results of Example 24.3. $C = Q/V$.

**SET UP:** $r_a = 15.0$ cm. Solve for $r_b$.

**EXECUTE:** **(a)** For two concentric spherical shells, the capacitance is $C = \dfrac{1}{k}\left(\dfrac{r_a r_b}{r_b - r_a}\right)$.

$kCr_b - kCr_a = r_a r_b$ and $r_b = \dfrac{kCr_a}{kC - r_a} = \dfrac{k(116\times10^{-12}\ \text{F})(0.150\ \text{m})}{k(116\times10^{-12}\ \text{F}) - 0.150\ \text{m}} = 0.175\ \text{m} = 17.5\ \text{cm}.$

**(b)** $V = 220$ V and $Q = CV = (116\times10^{-12}\ \text{F})(220\ \text{V}) = 2.55\times10^{-8}\ \text{C} = 25.5\ \text{nC}.$

**EVALUATE:** A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33\ \text{m}^2$ and $d = r_b - r_a = 2.5\times10^{-2}$ m has $C = \dfrac{\epsilon_0 A}{d} = 117$ pF, in excellent agreement with the value of $C$ for the spherical capacitor.

**24.13.** **IDENTIFY:** This problem involves dielectrics and capacitors in series.

**SET UP:** First sketch the circuit as in Fig. 24.13. The graph plots $V_2$ versus $V$, so we need to find a relationship between these quantities so we can interpret the slope. We want to find $C_1$.

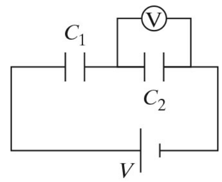



**Figure 24.13**

**EXECUTE:** For capacitors in series $Q_1 = Q_2 = Q = C_{eq}V$. $V_2 = Q/C_2 = C_{eq}V/C_2$. $C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$, so

$V_2 = \left(\dfrac{C_1C_2}{C_1+C_2}\right)\dfrac{V}{C_2} = \left(\dfrac{C_1}{C_1+C_2}\right)V$. Therefore the slope of the graph is $\dfrac{C_1}{C_1+C_2}$ = slope, which gives

$C_1 = \dfrac{C_2(\text{slope})}{1-\text{slope}} = \dfrac{(3.00\,\mu\text{F})(0.650)}{1-0.650} = 5.57\ \mu\text{F}$.

**EVALUATE:** This seems an odd way to measure $C_1$, but potentials are easy to measure using simple voltmeters, so these measurements could easily be made.

**24.14.** **IDENTIFY:** Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

**SET UP:** For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$
For capacitors in parallel the voltages are the same and the charges add; $C_{eq} = C_1 + C_2 + \ldots$ $C = \dfrac{Q}{V}$.

**EXECUTE:** **(a)** The equivalent capacitance of the $5.0\ \mu\text{F}$ and $8.0\ \mu\text{F}$ capacitors in parallel is $13.0\ \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.14. The equivalent capacitance of these three capacitors in series is $3.47\ \mu\text{F}$.

**(b)** $Q_{tot} = C_{tot}V = (3.47\ \mu\text{F})(50.0\ \text{V}) = 174\ \mu\text{C}$.

**(c)** $Q_{tot}$ is the same as $Q$ for each of the capacitors in the series combination shown in Figure 24.22, so $Q$ for each of the capacitors is $174\ \mu\text{C}$.

**EVALUATE:** The voltages across each capacitor in Figure 24.14 are $V_{10} = \dfrac{Q_{tot}}{C_{10}} = 17.4\ \text{V}$,

$V_{13} = \dfrac{Q_{tot}}{C_{13}} = 13.4\ \text{V}$, and $V_9 = \dfrac{Q_{tot}}{C_9} = 19.3\ \text{V}$. $V_{10} + V_{13} + V_9 = 17.4\ \text{V} + 13.4\ \text{V} + 19.3\ \text{V} = 50.1\ \text{V}$. The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

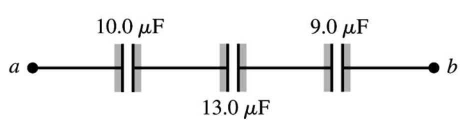



**Figure 24.14**

**24.15.** **IDENTIFY:** For capacitors in series the voltage across the combination equals the sum of the voltages in the individual capacitors. For capacitors in parallel the voltage across the combination is the same as the voltage across each individual capacitor.

**SET UP** and **EXECUTE:** **(a)** Connect the capacitors in series so their voltages will add.

**(b)** $V = V_1 + V_2 + V_3 + \ldots = NV_1,$ where $N$ is the number of capacitors in the series combination, since the capacitors are identical. $N = \dfrac{V}{V_1} = \dfrac{500\text{ V}}{0.10\text{ V}} = 5000.$

**EVALUATE:** It requires many small cells to produce a large voltage surge.

**24.16.** **IDENTIFY:** The capacitors between $b$ and $c$ are in parallel. This combination is in series with the 15 pF capacitor.

**SET UP:** Let $C_1 = 15\text{ pF},\ C_2 = 9.0\text{ pF}$ and $C_3 = 11\text{ pF}.$

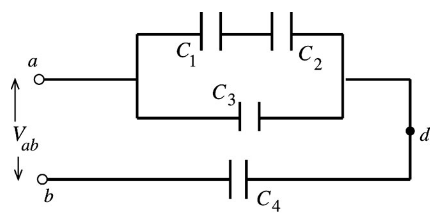
**EXECUTE:** **(a)** For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \ldots$ so $C_{23} = C_2 + C_3 = 20\text{ pF}.$

**(b)** $C_1 = 15\text{ pF}$ is in series with $C_{23} = 20\text{ pF}.$ For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$ so $\dfrac{1}{C_{123}} = \dfrac{1}{C_1} + \dfrac{1}{C_{23}}$ and $C_{123} = \dfrac{C_1 C_{23}}{C_1 + C_{23}} = \dfrac{(15\text{ pF})(20\text{ pF})}{15\text{ pF} + 20\text{ pF}} = 8.6\text{ pF}.$

**EVALUATE:** For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

**24.17.** **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for $Q$ and $V$ for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

**SET UP:** Do parts (a) and (b) together. The capacitor network is drawn in Figure 24.17a.

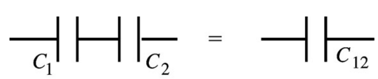



$C_1 = C_2 = C_3 = C_4 = 4.00\ \mu\text{F}.$

$V_{ab} = 28.0\text{ V}.$

**Figure 24.17a**

**EXECUTE:** Simplify the circuit by replacing the capacitor combinations by their equivalents: $C_1$ and $C_2$ are in series and are equivalent to $C_{12}$ (Figure 24.17b).

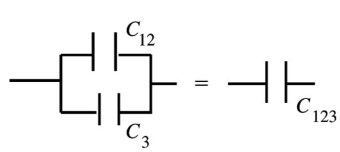



$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

**Figure 24.17b**

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00\times10^{-6}\text{ F})(4.00\times10^{-6}\text{ F})}{4.00\times10^{-6}\text{ F} + 4.00\times10^{-6}\text{ F}} = 2.00\times10^{-6}\text{ F}.$$

$C_{12}$ and $C_3$ are in parallel and are equivalent to $C_{123}$ (Figure 24.17c).



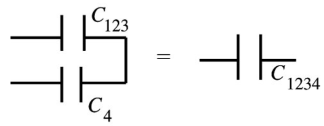

$C_{123} = C_{12} + C_3.$

$C_{123} = 2.00\times10^{-6}\text{ F} + 4.00\times10^{-6}\text{ F}.$

$C_{123} = 6.00\times10^{-6}\text{ F}.$

**Figure 24.17c**

$C_{123}$ and $C_4$ are in series and are equivalent to $C_{1234}$ (Figure 24.17d).

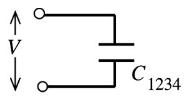



$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4}.$$

**Figure 24.17d**

$$C_{1234} = \frac{C_{123}C_4}{C_{123}+C_4} = \frac{(6.00\times10^{-6}\ \text{F})(4.00\times10^{-6}\ \text{F})}{6.00\times10^{-6}\ \text{F}+4.00\times10^{-6}\ \text{F}} = 2.40\times10^{-6}\ \text{F}.$$

The circuit is equivalent to the circuit shown in Figure 24.17e.

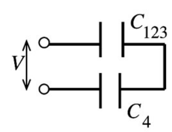



$V_{1234} = V = 28.0$ V.

$Q_{1234} = C_{1234}V = (2.40\times10^{-6}\ \text{F})(28.0\ \text{V}) = 67.2\ \mu\text{C}.$

**Figure 24.17e**

Now build back up the original circuit, step by step. $C_{1234}$ represents $C_{123}$ and $C_4$ in series (Figure 24.17f).




$Q_{123} = Q_4 = Q_{1234} = 67.2\ \mu\text{C}$
(charge same for capacitors in series).

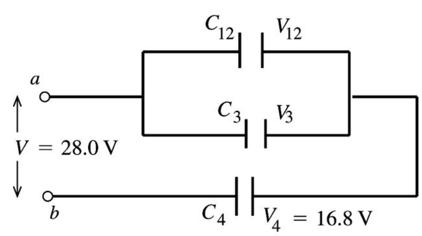
**Figure 24.17f**

Then $V_{123} = \dfrac{Q_{123}}{C_{123}} = \dfrac{67.2\ \mu\text{C}}{6.00\ \mu\text{F}} = 11.2\ \text{V}$ .

$$V_4 = \frac{Q_4}{C_4} = \frac{67.2\ \mu\text{C}}{4.00\ \mu\text{F}} = 16.8\ \text{V}.$$

Note that $V_4 + V_{123} = 16.8\ \text{V} + 11.2\ \text{V} = 28.0\ \text{V}$, as it should.

Next consider the circuit as written in Figure 24.17g.




$V_3 = V_{12} = 28.0\ \text{V} - V_4.$

$V_3 = 11.2$ V.

$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(11.2\ \text{V}).$

$Q_3 = 44.8\ \mu\text{C}.$

$Q_{12} = C_{12}V_{12} = (2.00\ \mu\text{F})(11.2\ \text{V}).$

$Q_{12} = 22.4\ \mu\text{C}.$

**Figure 24.17g**

Finally, consider the original circuit, as shown in Figure 24.17h.

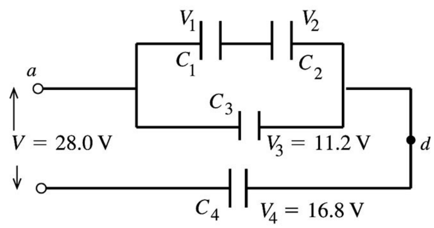



Figure 24.17h

$Q_1 = Q_2 = Q_{12} = 22.4\ \mu\text{C}$

(charge same for capacitors in series).

$V_1 = \dfrac{Q_1}{C_1} = \dfrac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6$ V.

$V_2 = \dfrac{Q_2}{C_2} = \dfrac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6$ V.

Note that $V_1 + V_2 = 11.2$ V, which equals $V_3$ as it should.

Summary: $Q_1 = 22.4\ \mu\text{C}$, $V_1 = 5.6$ V.

$Q_2 = 22.4\ \mu\text{C}$, $V_2 = 5.6$ V.

$Q_3 = 44.8\ \mu\text{C}$, $V_3 = 11.2$ V.

$Q_4 = 67.2\ \mu\text{C}$, $V_4 = 16.8$ V.

**(c)** $V_{ad} = V_3 = 11.2$ V.

**EVALUATE:** $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2, Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

**24.18.** **IDENTIFY:** The two capacitors are in series. The equivalent capacitance is given by $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

**SET UP:** For capacitors in series the charges are the same and the potentials add to give the potential across the network.

**EXECUTE:** **(a)** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{(3.00\times10^{-6}\ \text{F})} + \dfrac{1}{(5.00\times10^{-6}\ \text{F})}$, so $C_{\text{eq}} = 1.875\times10^{-6}$ F. Then

$Q = VC_{\text{eq}} = (64.0\ \text{V})(1.875\times10^{-6}\ \text{F}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$. Each capacitor has a charge of $1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$.

**(b)** $V_1 = Q/C_1 = (1.20\times10^{-4}\ \text{C})/(3.0\times10^{-6}\ \text{F}) = 40.0$ V.

$V_2 = Q/C_2 = (1.20\times10^{-4}\ \text{C})/(5.0\times10^{-6}\ \text{F}) = 24.0$ V.

**EVALUATE:** $V_1 + V_2 = 64.0$ V, which is equal to the applied potential $V_{ab}$. The capacitor with the smaller *C* has the larger *V*.

**24.19.** **IDENTIFY:** The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage $V_{ab}$.

**SET UP:** Do parts (a) and (b) together. The network is sketched in Figure 24.19.

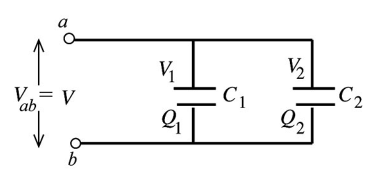



Figure 24.19

**EXECUTE:** $V_1 = V_2 = V$.

$V_1 = 52.0$ V.

$V_2 = 52.0$ V.

$C = Q/V$ so $Q = CV$.

$Q_1 = C_1V_1 = (3.00\ \mu\text{F})(52.0\ \text{V}) = 156\ \mu\text{C}$. $Q_2 = C_2V_2 = (5.00\ \mu\text{F})(52.0\ \text{V}) = 260\ \mu\text{C}$.

**EVALUATE:** To produce the same potential difference, the capacitor with the larger $C$ has the larger $Q$.

**24.20.** **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

**SET UP:** $C_1$ and $C_2$ are in parallel and $C_3$ is in series with the parallel combination of $C_1$ and $C_2$.

**EXECUTE:** **(a)** $C_1$ and $C_2$ are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{30.0\times10^{-6}\ \text{C}}{3.00\times10^{-6}\ \text{F}} = 10.0\ \text{V}$. Therefore, $Q_1 = V_1C_1 = (10.0\ \text{V})(6.00\times10^{-6}\ \text{F}) = 60.0\times10^{-6}\ \text{C}$.

Since $C_3$ is in series with the parallel combination of $C_1$ and $C_2$, its charge must be equal to their combined charge: $Q_3 = 30.0\times10^{-6}\ \text{C} + 60.0\times10^{-6}\ \text{C} = 90.0\times10^{-6}\ \text{C}$.

**(b)** The total capacitance is found from $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times10^{-6}\ \text{F}} + \dfrac{1}{5.00\times10^{-6}\ \text{F}}$ and

$C_{\text{eq}} = 3.21\ \mu\text{F}$. $V_{ab} = \dfrac{Q_{\text{tot}}}{C_{\text{eq}}} = \dfrac{90.0\times10^{-6}\ \text{C}}{3.21\times10^{-6}\ \text{F}} = 28.0\ \text{V}$.

**EVALUATE:** $V_3 = \dfrac{Q_3}{C_3} = \dfrac{90.0\times10^{-6}\ \text{C}}{5.00\times10^{-6}\ \text{F}} = 18.0\ \text{V}$. $V_{ab} = V_1 + V_3 = 10.0\ \text{V} + 18.0\ \text{V} = 28.0\ \text{V}$, as we just found.

**24.21.** **IDENTIFY:** Three of the capacitors are in series, and this combination is in parallel with the other two capacitors.

**SET UP:** For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$. For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \ldots$. $C = \dfrac{Q}{V}$.

**EXECUTE:** **(a)** The equivalent capacitance of the 18.0 nF, 30.0 nF and 10.0 nF capacitors in series is 5.29 nF. When these capacitors are replaced by their equivalent we get the network sketched in Figure 24.21. The equivalent capacitance of these three capacitors in parallel is 19.3 nF, and this is the equivalent capacitance of the original network.

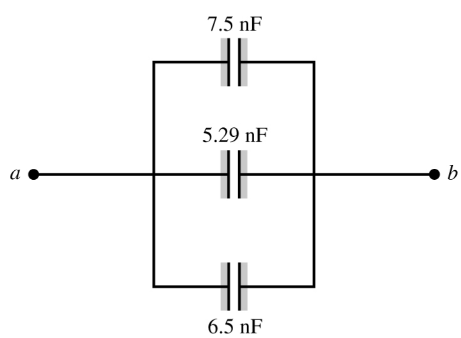



**Figure 24.21**

**(b)** $Q_{\text{tot}} = C_{\text{eq}}V = (19.3 \text{ nF})(25 \text{ V}) = 482 \text{ nC}.$

**(c)** The potential across each capacitor in the parallel network of Figure 24.21 is 25 V.
$Q_{6.5} = C_{6.5}V_{6.5} = (6.5 \text{ nF})(25 \text{ V}) = 162 \text{ nC}.$

**(d)** 25 V.

**EVALUATE:** As with most circuits, we must go through a series of steps to simplify it as we solve for the unknowns.

**24.22.** **IDENTIFY:** Apply $u = \frac{1}{2}\epsilon_0 E^2$.

**SET UP:** Example 24.3 shows that $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that $\dfrac{Q}{4\pi\epsilon_0} = \left(\dfrac{r_a r_b}{r_b - r_a}\right)V_{ab}$.

**EXECUTE:** $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{(0.125 \text{ m})(0.148 \text{ m})}{0.148 \text{ m} - 0.125 \text{ m}}\right)\dfrac{120 \text{ V}}{r^2} = \dfrac{96.5 \text{ V}\cdot\text{m}}{r^2}.$

**(a)** For $r = 0.126$ m, $E = 6.08\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64\times10^{-4} \text{ J/m}^3$.

**(b)** For $r = 0.147$ m, $E = 4.47\times10^3$ V/m. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85\times10^{-5} \text{ J/m}^3$.

**EVALUATE:** **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. *E* decreases as *r* increases, so *u* decreases also.

**24.23.** **IDENTIFY** and **SET UP:** The energy density is given by $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for *E*.

**EXECUTE:** Calculate *E*: $E = \dfrac{V}{d} = \dfrac{400 \text{ V}}{5.00\times10^{-3} \text{ m}} = 8.00\times10^4 \text{ V/m}.$

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}(8.854\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^4 \text{ V/m})^2 = 0.0283 \text{ J/m}^3$.

**EVALUATE:** *E* is smaller than the value in Example 24.8 by about a factor of 6 so *u* is smaller by about a factor of $6^2 = 36$.

**24.24.** **IDENTIFY:** Apply $C = Q/V$. $C = \dfrac{\epsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

**SET UP:** $U = \dfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}.$

**EXECUTE:** **(a)** $V = Q/C = (3.90\ \mu\text{C})/(920\times10^{-12} \text{ F}) = 4240 \text{ V} = 4.24 \text{ kV}.$

**(b)** $C = \dfrac{\epsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 8480 V = 8.48 kV.

**(c)** $U_i = \dfrac{Q^2}{2C} = \dfrac{(3.90\times10^{-6} \text{ C})^2}{2(920\times10^{-12} \text{ F})} = 8.27\times10^{-3} \text{ J} = 8.27 \text{ mJ}.$ If the separation is doubled while *Q* stays the same, the capacitance halves, and the energy stored doubles to $2U_i$. The amount of work done to move the plates equals the difference in energy stored in the capacitor, so
$\Delta U = U_f - U_i = 2U_i - U_i = U_i = 8.27 \text{ mJ}.$

**EVALUATE:** The oppositely charged plates attract each other so positive work must be done by an external force to pull them farther apart.

**24.25.** **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2}QV$.

**SET UP:** $d = 1.50\times10^{-3}$ m. $1\ \mu\text{C} = 10^{-6}$ C

**EXECUTE:** **(a)** $C = \dfrac{0.0180\times10^{-6}\ \text{C}}{200\ \text{V}} = 9.00\times10^{-11}\ \text{F} = 90.0\ \text{pF}.$

**(b)** $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(9.00\times10^{-11}\ \text{F})(1.50\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)} = 0.0152\ \text{m}^2.$

**(c)** $V = Ed = (3.0\times10^{6}\ \text{V/m})(1.50\times10^{-3}\ \text{m}) = 4.5\times10^{3}\ \text{V} = 4.5\ \text{kV}.$

**(d)** Energy $= \frac{1}{2}QV = \frac{1}{2}(0.0180\times10^{-6}\ \text{C})(200\ \text{V}) = 1.80\times10^{-6}\ \text{J} = 1.80\ \mu\text{J}.$

**EVALUATE:** We could also calculate the stored energy as $\dfrac{Q^2}{2C} = \dfrac{(0.0180\times10^{-6}\ \text{C})^2}{2(9.00\times10^{-11}\ \text{F})} = 1.80\ \mu\text{J}.$

**24.26.** **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. The stored energy can be expressed either as $\dfrac{Q^2}{2C}$ or as $\dfrac{CV^2}{2}$, whichever is more convenient for the calculation.

**SET UP:** Since $d$ is halved, $C$ doubles.

**EXECUTE:** **(a)** If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

**(b)** If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when $V$ is held constant throughout.

**EVALUATE:** When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

**24.27.** **IDENTIFY:** Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express $U$, $Q$, and $E$ in terms of the capacitor voltage.

**SET UP:** Let the applied voltage be $V$. Let each capacitor have capacitance $C$. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage $V$.

**EXECUTE:** **(a)** Series: The voltage across each capacitor is $V/2$. The total energy stored is $U_s = 2(\frac{1}{2}C(V/2)^2) = \frac{1}{4}CV^2$.

Parallel: The voltage across each capacitor is $V$. The total energy stored is

$U_p = 2(\frac{1}{2}CV^2) = CV^2 \quad \rightarrow \quad U_p = 4U_s$.

**(b)** $Q = CV$ for a single capacitor with voltage $V$. $Q_s = 2[C(V/2)] = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$.

**(c)** $E = V/d$ for a capacitor with voltage $V$. $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$.

**EVALUATE:** The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

**24.28. IDENTIFY:** The two capacitors are in series. $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$, $C = \frac{Q}{V}$, and $U = \frac{1}{2}CV^2$.

**SET UP:** For capacitors in series the voltages add and the charges are the same.

**EXECUTE:** **(a)** $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$ so $C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(150\text{ nF})(120\text{ nF})}{150\text{ nF} + 120\text{ nF}} = 66.7\text{ nF}$.

$Q = CV = (66.7\text{ nF})(48\text{ V}) = 3.2 \times 10^{-6}\text{ C} = 3.2\ \mu\text{C}$.

**(b)** $Q = 3.2\ \mu\text{C}$ for each capacitor.

**(c)** $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(66.7 \times 10^{-9}\text{ F})(48\text{ V})^2 = 77\ \mu\text{J}$.

**(d)** We know $C$ and $Q$ for each capacitor so rewrite $U$ in terms of these quantities.

$U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$.

150 nF: $U = \frac{(3.2 \times 10^{-6}\text{ C})^2}{2(150 \times 10^{-9}\text{ F})} = 34\ \mu\text{J}$.

120 nF: $U = \frac{(3.2 \times 10^{-6}\text{ C})^2}{2(120 \times 10^{-9}\text{ F})} = 43\ \mu\text{J}$.

Note that $34\ \mu\text{J} + 43\ \mu\text{J} = 77\ \mu\text{J}$, the total stored energy calculated in part (c).

**(e)** 150 nF: $V = \frac{Q}{C} = \frac{3.2 \times 10^{-6}\text{ C}}{150 \times 10^{-9}\text{ F}} = 21\text{ V}$.

120 nF: $V = \frac{Q}{C} = \frac{3.2 \times 10^{-6}\text{ C}}{120 \times 10^{-9}\text{ F}} = 27\text{ V}$.

Note that these two voltages sum to 48 V, the voltage applied across the network.

**EVALUATE:** Since $Q$ is the same, the capacitor with smaller $C$ stores more energy $\left(U = Q^2/2C\right)$ and has a larger voltage $(V = Q/C)$.

**24.29. IDENTIFY:** The two capacitors are in parallel. $C_{\text{eq}} = C_1 + C_2$. $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

**SET UP:** For capacitors in parallel, the voltages are the same and the charges add.

**EXECUTE:** **(a)** $C_{\text{eq}} = C_1 + C_2 = 35\text{ nF} + 75\text{ nF} = 110\text{ nF}$. $Q_{\text{tot}} = C_{\text{eq}}V = (110 \times 10^{-9}\text{ F})(220\text{ V}) = 24.2\ \mu\text{C}$

**(b)** $V = 220\text{ V}$ for each capacitor.

35 nF: $Q_{35} = C_{35}V = (35 \times 10^{-9}\text{ F})(220\text{ V}) = 7.7\ \mu\text{C}$; 75 nF:

$Q_{75} = C_{75}V = (75 \times 10^{-9}\text{ F})(220\text{ V}) = 16.5\ \mu\text{C}$. Note that $Q_{35} + Q_{75} = Q_{\text{tot}}$.

**(c)** $U_{\text{tot}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(110 \times 10^{-9}\text{ F})(220\text{ V})^2 = 2.66\text{ mJ}$.

**(d)** 35 nF: $U_{35} = \frac{1}{2}C_{35}V^2 = \frac{1}{2}(35 \times 10^{-9}\text{ F})(220\text{ V})^2 = 0.85\text{ mJ}$;

75 nF: $U_{75} = \frac{1}{2}C_{75}V^2 = \frac{1}{2}(75 \times 10^{-9}\text{ F})(220\text{ V})^2 = 1.81\text{ mJ}$. Since $V$ is the same the capacitor with larger $C$ stores more energy.

**(e)** 220 V for each capacitor.

**EVALUATE:** The capacitor with the larger $C$ has the larger $Q$.

**24.30.** **IDENTIFY:** This problem involves dielectrics and capacitors in parallel.

**SET UP:** First sketch the circuit as in Fig. 24.30. $C_{eq} = C_1 + C_2$, $C = KC_0$, and $Q = CV$. We want the charge in both cases.

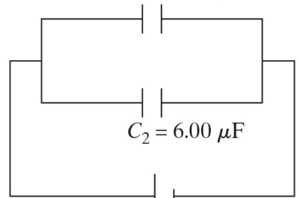



**Figure 24.30**

**EXECUTE:** **(a)** $Q = C_{eq}V = (C_1 + C_2)V = (10.0\ \mu\text{F})(36.0\ \text{V}) = 360\ \mu\text{C}$.

**(b)** $C_1$ is now $KC_0 = (5.00)(4.00\ \mu\text{F}) = 20.0\ \mu\text{F}$. $Q = C_{eq}V = (26.0\ \mu\text{F})(36.0\ \text{V}) = 936\ \mu\text{C}$.

**EVALUATE:** The total charge increases due to the insertion of the dielectric. The dielectric increases the equivalent capacitance which increases the stored charge.

**24.31.** **IDENTIFY:** $C = KC_0$. $U = \frac{1}{2}CV^2$.

**SET UP:** $C_0 = 12.5\ \mu\text{F}$ is the value of the capacitance without the dielectric present.

**EXECUTE:** **(a)** With the dielectric, $C = (3.75)(12.5\ \mu\text{F}) = 46.9\ \mu\text{F}$.

Before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5\times10^{-6}\ \text{F})(24.0\ \text{V})^2 = 3.60\ \text{mJ}$.

After: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9\times10^{-6}\ \text{F})(24.0\ \text{V})^2 = 13.5\ \text{mJ}$.

**(b)** $\Delta U = 13.5\ \text{mJ} - 3.6\ \text{mJ} = 9.9\ \text{mJ}$. The energy increased.

**EVALUATE:** The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

**24.32.** **IDENTIFY:** $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

**SET UP:** $V = Ed$ holds both with and without the dielectric.

**EXECUTE:** **(a)** $V = Ed = (3.00\times10^4\ \text{V/m})(1.50\times10^{-3}\ \text{m}) = 45.0\ \text{V}$.

$Q = C_0V = (8.00\times10^{-12}\ \text{F})(45.0\ \text{V}) = 3.60\times10^{-10}\ \text{C} = 360\ \text{pC}$.

**(b)** With the dielectric, $C = KC_0 = (2.70)(8.00\ \text{pF}) = 21.6\ \text{pF}$. $V$ is still 45.0 V, so

$Q = CV = (21.6\times10^{-12}\ \text{F})(45.0\ \text{V}) = 9.72\times10^{-10}\ \text{C} = 972\ \text{pC}$.

**EVALUATE:** The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. $Q$ increases by a factor of $K$.

**24.33.** **IDENTIFY** and **SET UP:** $Q$ is constant so we can apply Eq. (24.14). The charge density on each surface of the dielectric is given by $\sigma_i = \sigma(1 - 1/K)$.

**EXECUTE:** $E = \dfrac{E_0}{K}$ so $K = \dfrac{E_0}{E} = \dfrac{3.20\times10^5\ \text{V/m}}{2.50\times10^5\ \text{V/m}} = 1.28$.

**(a)** $\sigma_i = \sigma(1 - 1/K)$.

$\sigma = \epsilon_0 E_0 = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.20\times10^5\ \text{N/C}) = 2.833\times10^{-6}\ \text{C/m}^2$.

$\sigma_i = (2.833\times10^{-6}\ \text{C/m}^2)(1 - 1/1.28) = 6.20\times10^{-7}\ \text{C/m}^2$.

**(b)** As calculated above, $K = 1.28$.

**EVALUATE:** The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

**24.34. IDENTIFY:** We are dealing with the energy in a capacitor with dielectric.

**SET UP:** $1/C_{eq} = 1/C_1 + 1/C_2$. $U = \frac{1}{2}CV^2$. In this case, $C_1 = C_2 = C$. We want the ratio of $U/U_0$.

**EXECUTE:** Without dielectric: $1/C_{eq} = 1/C + 1/C = 2/C$, so $C_{eq} = C/2$.

$$U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\left(\frac{C}{2}\right)V^2 = \frac{CV^2}{4}.$$

With dielectric: $1/C_{eq} = 1/KC + 1/C = KC/(1 + K)$. $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}\left(\frac{KC}{1+K}\right)V^2 = \frac{CV^2}{4}$.

$$\frac{U}{U_0} = \frac{\frac{1}{2}\left(\frac{KC}{1+K}\right)V^2}{\frac{CV^2}{4}} = \frac{2K}{1+K}.$$

**EVALUATE:** $\frac{U}{U_0} = \frac{2K}{1+K}$. $K \geq 1$, so the smallest this ratio can be is 1 and the largest it can be is 2.

Therefore $U \geq U_0$, so the dielectric *increases* the stored energy.

**24.35. IDENTIFY:** This problem involves capacitors in series, with and without dielectric.

**SET UP:** $1/C_{eq} = 1/C_1 + 1/C_2$. $C = KC_0$. $Q = CV$. The charge is the same on capacitors in series. We want the charge on $C_1$ (the capacitor with dielectric).

**EXECUTE:** Without dielectric: $1/C_{eq} = 1/C + 1/C = 2/C$, so $C_{eq} = C/2$. $Q_0 = C_{eq}V = CV/2$.

With dielectric: $1/C_{eq} = 1/KC + 1/C$, so $C_{eq} = CK/(1 + K)$. $Q = CV_{eq} = \left(\frac{CK}{1+K}\right)V$. $Q_0 = CV/2$, so $V = 2Q_0/C$, which gives $Q = \left(\frac{CK}{1+K}\right)\left(\frac{2Q_0}{C}\right) = \frac{2KQ_0}{1+K}$.

**EVALUATE:** $K \geq 1$, so $Q_{min} = Q_0$ and $Q_{max} = 2Q_0$. Therefore $Q$ *increases*.

**24.36. IDENTIFY:** We are dealing with a capacitors in series and in parallel with dielectric.

**SET UP:** $E = V/d$, series: $1/C_{eq} = 1/C_1 + 1/C_2$, parallel: $C_{eq} = C_1 + C_2$. $Q = CV$. We want the ratio $E_2/E_{02}$.

**EXECUTE: (a)** Without dielectric: $1/C_{eq} = 1/(3.00\ \mu\text{F}) + 1/(6.00\ \mu\text{F})$. $C_{eq} = 2.00\ \mu\text{F}$. For the combination $Q = C_{eq}V = (2.00\ \mu\text{F})V$, which is the same charge on each capacitor. For $C_2$, $V_2 = Q_2/C_2 = Q/C_2 = \frac{(2.00\mu\text{F})V}{6.00\mu\text{F}} = \frac{V}{3}$. $E_{02} = V_2/d = V/3d$.

With dielectric: $C_1 = KC_0 = (4)(3.00\ \mu\text{F}) = 12.0\ \mu\text{F}$. $1/C_{eq} = 1/(12.0\ \mu\text{F}) + 1/(6.00\ \mu\text{F})$, which gives $C_{eq} = 4.00\ \mu\text{F}$. $Q = C_{eq}V = (4.00\ \mu\text{F})V$. For $C_2$, $V_2 = Q/C_2 = \frac{(4.00\mu\text{F})V}{6.00\mu\text{F}} = \frac{2V}{3}$. $E_2 = \frac{V_2}{d} = \frac{2V/3}{d}$.

$\frac{E_2}{E_{02}} = \frac{2V/3d}{V/3d} = 2.00$. The field in $C_2$ has *increased*.

**(b)** Without dielectric: $V$ is the same for both capacitors, so $E_{02} = V/d$.

With dielectric: $V$ is unchanged, so $E_2 = V/d$. $E_2/E_{02} = 1$. The dielectric in $C_1$ does not affect the electric field in $C_2$.

**EVALUATE:** When the capacitors were in series, the capacitor in one affected the field in the other one. But when they were in parallel, the dielectric had no effect on the other capacitor.

**24.37.** **IDENTIFY** and **SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C = K\varepsilon_0 A/d$. Minimum $A$ means smallest possible $d$. $d$ is limited by the requirement that $E$ be less than $1.60\times 10^7$ V/m when $V$ is as large as 5500 V.

**EXECUTE:** $V = Ed$ so $d = \dfrac{V}{E} = \dfrac{5500\text{ V}}{1.60\times 10^7\text{ V/m}} = 3.44\times 10^{-4}$ m.

Then $A = \dfrac{Cd}{K\epsilon_0} = \dfrac{(1.25\times 10^{-9}\text{ F})(3.44\times 10^{-4}\text{ m})}{(3.60)(8.854\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)} = 0.0135\text{ m}^2$.

**EVALUATE:** The relation $V = Ed$ applies with or without a dielectric present. $A$ would have to be larger if there were no dielectric.

**24.38.** **IDENTIFY:** We can model the cell wall as a large sheet carrying equal but opposite charges, which makes it equivalent to a parallel-plate capacitor.

**SET UP:** With air between the layers, $E_0 = \dfrac{Q}{\epsilon_0 A} = \dfrac{\sigma}{\epsilon_0}$ and $V_0 = E_0 d$. The energy density in the electric field is $u = \frac{1}{2}\epsilon_0 E^2$. The volume of a shell of thickness $t$ and average radius $R$ is $4\pi R^2 t$. The volume of a solid sphere of radius $R$ is $\frac{4}{3}\pi R^3$. With the dielectric present, $E = \dfrac{E_0}{K}$ and $V = \dfrac{V_0}{K}$.

**EXECUTE:** **(a)** $E_0 = \dfrac{\sigma}{\epsilon_0} = \dfrac{0.50\times 10^{-3}\text{ C/m}^2}{8.854\times 10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.6\times 10^7$ V/m.

**(b)** $V_0 = E_0 d = (5.6\times 10^7\text{ V/m})(5.0\times 10^{-9}\text{ m}) = 0.28$ V. The outer wall of the cell is at higher potential, since it has positive charge.

**(c)** For the cell, $V_{\text{cell}} = \frac{4}{3}\pi R^3$, which gives $R = \left(\dfrac{3V_{\text{cell}}}{4\pi}\right)^{1/3} = \left(\dfrac{3(10^{-16}\text{ m}^3)}{4\pi}\right)^{1/3} = 2.9\times 10^{-6}$ m. The volume of the cell wall is $V_{\text{wall}} = 4\pi R^2 t = 4\pi(2.9\times 10^{-6}\text{ m})^2(5.0\times 10^{-9}\text{ m}) = 5.3\times 10^{-19}\text{ m}^3$. The energy density in the cell wall is $u_0 = \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2}[8.854\times 10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)](5.6\times 10^7\text{ V/m})^2 = 1.39\times 10^4\text{ J/m}^3$. The total electric-field energy in the cell wall is $(1.39\times 10^4\text{ J/m}^3)(5.3\times 10^{-19}\text{ m}^3) = 7\times 10^{-15}$ J.

**(d)** $E = \dfrac{E_0}{K} = \dfrac{5.6\times 10^7\text{ V/m}}{5.4} = 1.0\times 10^7$ V/m and $V = \dfrac{V_0}{K} = \dfrac{0.28\text{ V}}{5.4} = 0.052$ V.

**EVALUATE:** To a first approximation, many biological structures can be modeled as basic circuit elements.

**24.39.** **IDENTIFY:** $C = Q/V$. $C = KC_0$. $V = Ed$.

**SET UP:** Table 24.1 gives $K = 3.1$ for mylar.

**EXECUTE:** **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times 10^{-7}\text{ F})(12\text{ V}) = 6.3\times 10^{-6}$ C.

**(b)** $\sigma_{\text{i}} = \sigma(1 - 1/K)$ so $Q_{\text{i}} = Q(1 - 1/K) = (9.3\times 10^{-6}\text{ C})(1 - 1/3.1) = 6.3\times 10^{-6}$ C.

**(c)** The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

**EVALUATE:** $E = V/d$ and $V$ is constant so $E$ doesn't change when the dielectric is inserted.

**24.40.** **IDENTIFY** and **SET UP:** The energy density is due to the electric field in the dielectric. $u = \frac{1}{2}\epsilon E^2$, where $\epsilon = K\epsilon_0$. $V = Ed$. In this case, $E = 0.800E_{\text{m}}$.

**EXECUTE:** **(a)** Using $u = \frac{1}{2}\epsilon E^2$ with $\epsilon = K\epsilon_0$, we have

$u = (1/2)(2.6)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)[(0.800)(2.0\times10^7\ \text{V/m})]^2 = 2945\ \text{J/m}^3$, which rounds to 2900 J/m$^3$.

**(b)** First get the plate separation $d$: $V = Ed$ gives

$d = V/E = (500\ V)/[(0.800)(2.0\times10^7\ \text{V/m})] = 3.125\times10^{-5}$ m.

The stored energy is $U = u\times\text{volume} = uAd$, so

$A = U/ud = (0.200\times10^{-3}\ \text{J})/[(2945\ \text{J/m}^3)(3.125\times10^{-5}\text{m})] = 2.2\times10^{-3}\ \text{m}^2 = 22\ \text{cm}^2$.

**EVALUATE:** If this capacitor has square plates, their dimensions would be $x = (22\ \text{cm}^2)^{1/2} = 4.7$ cm on each side. This is considerably larger than ordinary laboratory capacitors used in circuits.

**24.41.** **(a) IDENTIFY** and **SET UP:** Since the capacitor remains connected to the power supply the potential difference doesn't change when the dielectric is inserted. Use $U = \frac{1}{2}CV^2$ to calculate $V$ and combine it with $K = C/C_0$ to obtain a relation between the stored energies and the dielectric constant and use this to calculate $K$.

**EXECUTE:** Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\dfrac{2U_0}{C_0}} = \sqrt{\dfrac{2(1.85\times10^{-5}\ \text{J})}{360\times10^{-9}\ \text{F}}} = 10.1$ V.

**(b)** $K = C/C_0$.

$U_0 = \frac{1}{2}C_0V^2,\ U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$.

$$K = \frac{U}{U_0} = \frac{1.85\times10^{-5}\ \text{J} + 2.32\times10^{-5}\ \text{J}}{1.85\times10^{-5}\ \text{J}} = 2.25.$$

**EVALUATE:** $K$ increases the capacitance and then from $U = \frac{1}{2}CV^2$, with $V$ constant an increase in $C$ gives an increase in $U$.

**24.42.** **IDENTIFY:** $C = KC_0$. $C = Q/V$. $V = Ed$.

**SET UP:** Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

**EXECUTE:** **(a)** The capacitance changes by a factor of $K$ when the dielectric is inserted. Since $V$ is unchanged (the battery is still connected), $\dfrac{C_{\text{after}}}{C_{\text{before}}} = \dfrac{Q_{\text{after}}}{Q_{\text{before}}} = \dfrac{45.0\ \text{pC}}{25.0\ \text{pC}} = K = 1.80.$

**(b)** The area of the plates is $\pi r^2 = \pi(0.0300\ \text{m})^2 = 2.827\times10^{-3}\ \text{m}^2$ and the separation between them is thus $d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)}{12.5\times10^{-12}\ \text{F}} = 2.00\times10^{-3}$ m. Before the dielectric is inserted, $C = \dfrac{\epsilon_0 A}{d} = \dfrac{Q}{V}$ and $V = \dfrac{Qd}{\epsilon_0 A} = \dfrac{(25.0\times10^{-12}\ \text{C})(2.00\times10^{-3}\ \text{m})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 2.00$ V. The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

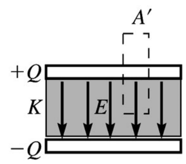
**(c)** Before the dielectric is inserted, $E = \dfrac{Q}{\epsilon_0 A} = \dfrac{25.0\times10^{-12}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)} = 1000$ N/C.

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

**EVALUATE:** $E = \dfrac{V}{d} = \dfrac{2.00\ \text{V}}{2.00\times10^{-3}\ \text{m}} = 1000$ N/C, whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need $d$ to calculate $E$ because $V$ is the potential difference between points separated by distance $d$.

**24.43.** **IDENTIFY:** Apply $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\epsilon_0}$ to calculate $E$. $V = Ed$ and $C = Q/V$ apply whether there is a dielectric between the plates or not.

**(a) SET UP:** Apply $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\epsilon_0}$ to the dashed surface in Figure 24.43.




**EXECUTE:** $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\epsilon_0}$.

$\oint K\vec{E}\cdot d\vec{A} = KEA'$.

since $E = 0$ outside the plates

$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'$.

**Figure 24.43**

Thus $KEA' = \dfrac{(Q/A)A'}{\epsilon_0}$ and $E = \dfrac{Q}{\epsilon_0 AK}$.

**SET UP** and **EXECUTE:** **(b)** $V = Ed = \dfrac{Qd}{\epsilon_0 AK}$.

**(c)** $C = \dfrac{Q}{V} = \dfrac{Q}{Qd/\epsilon_0 AK} = K\dfrac{\epsilon_0 A}{d} = KC_0$.

**EVALUATE:** Our result shows that $K = C/C_0$, which is Eq. (24.12).

**24.44.** **IDENTIFY:** Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of $K$ and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

**(a) SET UP:** The capacitance of a parallel-plate capacitor is $C = K\epsilon_0 A/d$ and the charge on its plates is $Q = CV$.

**EXECUTE:** First find the capacitance:

$$C = \frac{K\epsilon_0 A}{d} = \frac{(2.1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 4.18\times10^{-10}\ \text{F}.$$

Now find the charge on the plates: $Q = CV = (4.18\times10^{-10}\ \text{F})(12.0\ \text{V}) = 5.02\times10^{-9}\ \text{C}$.

**(b) SET UP:** Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\epsilon_0$.

**EXECUTE:** Solving for $E$ gives

$$E = \frac{Q_{\text{free}}}{KA\epsilon_0} = \frac{5.02\times10^{-9}\ \text{C}}{(2.1)(0.0225\ \text{m}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.20\times10^4\ \text{N/C}.$$

**(c) SET UP:** Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \epsilon_0 A/d$ and Gauss's law now gives $EA = Q/\epsilon_0$.

**EXECUTE:** First find the capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 1.99\times10^{-10}\ \text{F}.$$

The potential difference is $V = \dfrac{Q}{C} = \dfrac{5.02\times10^{-9}\ \text{C}}{1.99\times10^{-10}\ \text{F}} = 25.2\ \text{V}$. From Gauss's law, the electric field is

$$E = \frac{Q}{\epsilon_0 A} = \frac{5.02\times10^{-9}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)} = 2.52\times10^4\ \text{N/C}.$$

**EVALUATE:** The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

**24.45. IDENTIFY:** $P = E/t$, where $E$ is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

**SET UP:** $E = 0.95U$.

**EXECUTE:** **(a)** The power output is $2.70\times10^5$ W, and 95% of the original energy is converted, so $E = Pt = (2.70\times10^5\ \text{W})(1.48\times10^{-3}\ \text{s}) = 400\ \text{J}$. $U = \dfrac{400\ \text{J}}{0.95} = 421\ \text{J}$.

**(b)** $U = \frac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421\ \text{J})}{(125\ \text{V})^2} = 0.054\ \text{F}$.

**EVALUATE:** For a given $V$, the stored energy increases linearly with $C$.

**24.46. IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2}CV^2$. With the battery disconnected, $Q$ is constant. When the separation $d$ is doubled, $C$ is halved.

**EXECUTE:** **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\epsilon_0 (0.12\ \text{m})^2}{3.7\times10^{-3}\ \text{m}} = 3.446\times10^{-11}\ \text{F}$, which rounds to 34 pF.

**(b)** $Q = CV = (3.446\times10^{-11}\ \text{F})(12\ \text{V}) = 4.135\times10^{-10}\ \text{C}$, which rounds to 410 pC.

**(c)** $E = V/d = (12\ \text{V})/(3.7\times10^{-3}\ \text{m}) = 3200\ \text{V/m}$.

**(d)** $U = \frac{1}{2}CV^2 = \frac{1}{2}(3.446\times10^{-11}\ \text{F})(12\ \text{V})^2 = 2.48\times10^{-9}\ \text{J}$, which rounds to 2.5 nJ.

**(e)** If the battery is disconnected, so the charge remains constant, and the plates are pulled farther apart to 0.0074 m, then the calculations above can be carried out just as before, and we find:

(a) $C = 1.7\times10^{-11}\ \text{F} = 17\ \text{pF}$.

(b) $Q = 4.1\times10^{-10}\ \text{C} = 410\ \text{pC}$.

(c) $E = 3200\ \text{V/m}$.

(d) $U = \dfrac{Q^2}{2C} = \dfrac{(4.1\times10^{-10}\ \text{C})^2}{2(1.7\times10^{-11}\ \text{F})} = 5.0\times10^{-9}\ \text{J} = 5.0\ \text{nJ}$.

**EVALUATE:** $Q$ is unchanged. $E = \dfrac{Q}{\epsilon_0 A}$ so $E$ is therefore unchanged. $U$ doubles because $C$ is halved with $Q$ unchanged. The additional stored energy comes from the work done by the force that pulled the plates apart.

**24.47. IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$.

**SET UP:** $A = 4.2\times10^{-5}\ \text{m}^2$. The original separation between the plates is $d = 0.700\times10^{-3}\ \text{m}$. $d'$ is the separation between the plates at the new value of $C$.

**EXECUTE:** $C_0 = \dfrac{A\epsilon_0}{d} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\epsilon_0}{7.00\times10^{-4}\ \text{m}} = 5.31\times10^{-13}\ \text{F}$. The new value of $C$ is $C = C_0 + 0.25\ \text{pF} = 7.81\times10^{-13}\ \text{F}$. But $C = \dfrac{A\epsilon_0}{d'}$, so $d' = \dfrac{A\epsilon_0}{C} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\epsilon_0}{7.81\times10^{-13}\ \text{F}} = 4.76\times10^{-4}\ \text{m}$.

Therefore the key must be depressed by a distance of $7.00\times10^{-4}\ \text{m} - 4.76\times10^{-4}\ \text{m} = 0.224\ \text{mm}$.

**EVALUATE:** When the key is depressed, $d$ decreases and $C$ increases.

**24.48. IDENTIFY:** $C = KC_0 = K\epsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

**SET UP:** $A = 1.0\text{ cm}^2 = 1.0\times10^{-4}\text{ m}^2$.

**EXECUTE:** **(a)** $C = (10)\dfrac{(8.85\times10^{-12}\text{ F/m})(1.0\times10^{-4}\text{ m}^2)}{7.5\times10^{-9}\text{ m}} = 1.18\ \mu\text{F per cm}^2$.

**(b)** $E = \dfrac{V}{d} = \dfrac{85\text{ mV}}{7.5\times10^{-9}\text{ m}} = 1.13\times10^{7}\text{ V/m}$.

**EVALUATE:** The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

**24.49.** **IDENTIFY:** Some of the charge from the original capacitor flows onto the uncharged capacitor until the potential differences across the two capacitors are the same.

**SET UP:** $C = \dfrac{Q}{V_{ab}}$. Let $C_1 = 20.0\ \mu\text{F}$ and $C_2 = 10.0\ \mu\text{F}$. The energy stored in a capacitor is $\frac{1}{2}QV_{ab} = \frac{1}{2}CV_{ab}^2 = \dfrac{Q^2}{2C}$.

**EXECUTE:** **(a)** The initial charge on the $20.0\ \mu\text{F}$ capacitor is

$Q = C_1(800\text{ V}) = (20.0\times10^{-6}\text{ F})(800\text{ V}) = 0.0160\text{ C}$.

**(b)** In the final circuit, charge $Q$ is distributed between the two capacitors and $Q_1 + Q_2 = Q$. The final circuit contains only the two capacitors, so the voltage across each is the same, $V_1 = V_2$. $V = \dfrac{Q}{C}$ so $V_1 = V_2$ gives $\dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$. $Q_1 = \dfrac{C_1}{C_2}Q_2 = 2Q_2$. Using this in $Q_1 + Q_2 = 0.0160\text{ C}$ gives $3Q_2 = 0.0160\text{ C}$ and $Q_2 = 5.33\times10^{-3}\text{ C}$. $Q = 2Q_2 = 1.066\times10^{-2}\text{ C}$. $V_1 = \dfrac{Q_1}{C_1} = \dfrac{1.066\times10^{-2}\text{ C}}{20.0\times10^{-6}\text{ F}} = 533\text{ V}$. $V_2 = \dfrac{Q_2}{C_2} = \dfrac{5.33\times10^{23}\text{ C}}{10.0\times10^{26}\text{ F}} = 533\text{ V}$. The potential differences across the capacitors are the same, as they should be.

**(c)** $\text{Energy} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = \frac{1}{2}(C_1 + C_2)V^2$ gives

$\text{Energy} = \frac{1}{2}(20.0\times10^{-6}\text{ F} + 10.0\times10^{-6}\text{ F})(533\text{ V})^2 = 4.26\text{ J}$.

**(d)** The $20.0\ \mu\text{F}$ capacitor initially has $\text{energy} = \frac{1}{2}C_1V^2 = \frac{1}{2}(20.0\times10^{-6}\text{ F})(800\text{ V})^2 = 6.40\text{ J}$. The decrease in stored energy that occurs when the capacitors are connected is $6.40\text{ J} - 4.26\text{ J} = 2.14\text{ J}$.

**EVALUATE:** The decrease in stored energy is because of conversion of electrical energy to other forms during the motion of the charge when it becomes distributed between the two capacitors. Thermal energy is generated by the current in the wires and energy is emitted in electromagnetic waves.

**24.50.** **IDENTIFY:** We model a car as a spherical capacitor.

**SET UP:** $C = 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b - r_a}$.

**EXECUTE:** **(a)** Using $C = 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b - r_a}$ and let $r_b \to \infty$. $C \to 4\pi\epsilon_0 \dfrac{r_a r_b}{r_b} = 4\pi\epsilon_0 r_a$.

Using a charged sphere, $C = Q/V = \dfrac{Q}{\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r_a}} = 4\pi\epsilon_0 r_a$. Our results agree.

**(b)** Estimate: Care is about 4 paces long $\approx$ 4.0 m long, so $r \approx 2.0$ m.

**(c)** $C = 4\pi\epsilon_0 r_a = 4\pi\epsilon_0 (2.0\text{ m})$ 220 pF.

**(d)** $Q = CV = (220\text{ pF})(100\text{ MV}) = 22\text{ mC}$.

**EVALUATE:** This is quite a large charge, which why lightning strikes can be fatal!

**24.51.** **IDENTIFY:** Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \frac{1}{2}CV^2$.

**SET UP:** For capacitors in series the voltages add and the charges are the same; $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$ For capacitors in parallel the voltages are the same and the charges add; $C_{eq} = C_1 + C_2 + \ldots$ $C = \frac{Q}{V}$. $U = \frac{1}{2}CV^2$.

**EXECUTE:** **(a)** Find $C_{eq}$ for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.51.

$U_{tot} = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(2.19\times10^{-6}\text{ F})(12.0\text{ V})^2 = 1.58\times10^{-4}\text{ J} = 158\ \mu\text{J}$.

**(b)** From Figure 24.51c, $Q_{tot} = C_{eq}V = (2.19\times10^{-6}\text{ F})(12.0\text{ V}) = 2.63\times10^{-5}\text{ C}$. From Figure 24.51b,

$Q_{4.8} = 2.63\times10^{-5}\text{ C}$. $V_{4.8} = \frac{Q_{4.8}}{C_{4.8}} = \frac{2.63\times10^{-5}\text{ C}}{4.80\times10^{-6}\text{ F}} = 5.48\text{ V}$.

$U_{4.8} = \frac{1}{2}CV^2 = \frac{1}{2}(4.80\times10^{-6}\text{ F})(5.48\text{ V})^2 = 7.21\times10^{-5}\text{ J} = 72.1\ \mu\text{J}$.

This one capacitor stores nearly half the total stored energy.

**EVALUATE:** $U = \frac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest *C* stores the greatest amount of energy.

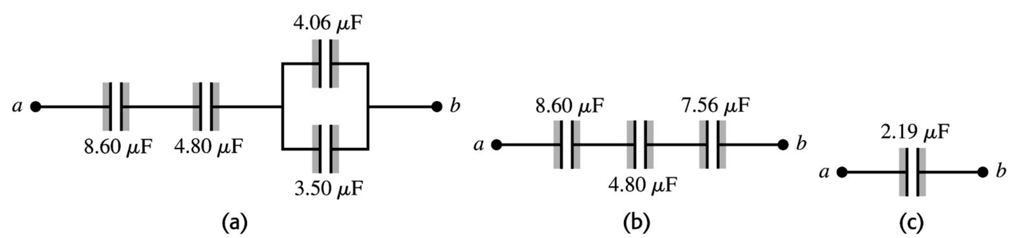



**Figure 24.51**

**24.52.** **IDENTIFY** and **SET UP:** The charge *Q* is the same on capacitors in series, and the potential *V* is the same for capacitors in parallel. $C_1$ is in series with $C_2$, and that combination is in parallel with $C_3$. The $C_1$-$C_2$-$C_3$ combination is in series with $C_4$. $V = Q/C$.

**EXECUTE:** **(a)** Since $C_1$ and $C_2$ are in series, and that combination is in parallel with $C_3$, the potential difference across the $C_1$-$C_2$ combination is the same as the potential difference across $C_3$, which is 40.0 V. Also, $Q_1 = Q_2 = Q$.

$V_1 + V_2 = 40.0$ V.

$Q/C_1 + Q/C_2 = 40.0$ V.

$Q/(6.00\ \mu\text{F}) + Q/(3.00\ \mu\text{F}) = 40.0$ V.

$Q = 80.0\ \mu\text{C}$.

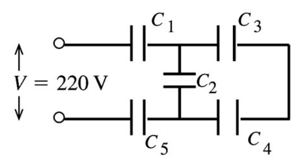
Therefore

$V_1 = Q/C_1 = (80.0\ \mu\text{C})/(6.00\ \mu\text{F}) = 13.3$ V.

$V_2 = Q/C_2 = (80.0\ \mu\text{C})/(3.00\ \mu\text{F}) = 26.7$ V.

**(b)** First get the charge $Q_4$ on $C_4$. We know that $Q_1 = Q_{-2} = Q = 80.0\ \mu\text{C}$. We also have
$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(40.0\ \text{V}) = 160\ \mu\text{C}$.
$Q_4 = Q + Q_3 = 80.0\ \mu\text{C} + 160\ \mu\text{C} = 240\ \mu\text{C}$.
$V_4 = Q_4/C_4 = (240\ \mu\text{C})/(8.00\ \mu\text{F}) = 30.0\ \text{V}$.
**(c)** $V_{ab} = V_3 + V_4 = 40.0\ \text{V} + 30.0\ \text{V} = 70.0\ \text{V}$.
**EVALUATE:** $C_3$ and $C_4$ are *not* in parallel, so $V_3 \neq V_4$.

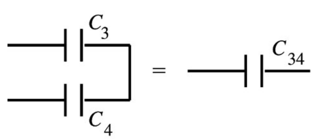
**24.53.** **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.
**SET UP:** The network is sketched in Figure 24.53a.




$C_1 = C_5 = 8.4\ \mu\text{F}$.
$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$.

**Figure 24.53a**

**EXECUTE:** Simplify the circuit by replacing the capacitor combinations by their equivalents:
$C_3$ and $C_4$ are in series and can be replaced by $C_{34}$ (Figure 24.53b):

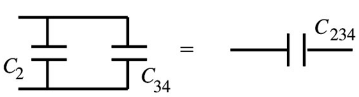



$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}.$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3C_4}.$$

**Figure 24.53b**

$$C_{34} = \frac{C_3C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}.$$

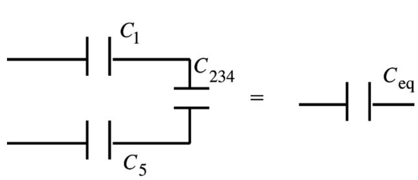
$C_2$ and $C_{34}$ are in parallel and can be replaced by their equivalent (Figure 24.53c):




$$C_{234} = C_2 + C_{34}.$$
$$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}.$$
$$C_{234} = 6.3\ \mu\text{F}.$$

**Figure 24.53c**

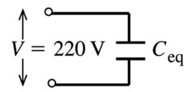
$C_1, C_5$, and $C_{234}$ are in series and can be replaced by $C_{\text{eq}}$ (Figure 24.53d):




$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}.$$
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}.$$
$$C_{\text{eq}} = 2.5\ \mu\text{F}.$$

**Figure 24.53d**

**EVALUATE:** For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
**(b) IDENTIFY** and **SET UP:** In each equivalent network apply the rules for $Q$ and $V$ for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
**EXECUTE:** The equivalent circuit is drawn in Figure 24.53e.




$Q_{\text{eq}} = C_{\text{eq}}V.$
$Q_{\text{eq}} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}.$

**Figure 24.53e**

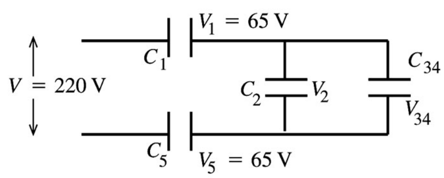
$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge).

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}.$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}.$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}.$$

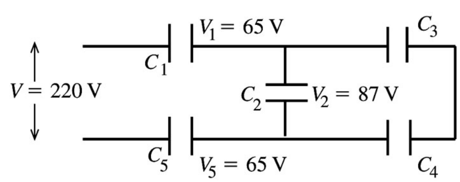
Now draw the network as in Figure 24.53f.




$V_2 = V_{34} = V_{234} = 87\ \text{V}$
capacitors in parallel have the same potential.

**Figure 24.53f**

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\ \text{V}) = 370\ \mu\text{C}.$
$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\ \text{V}) = 180\ \mu\text{C}.$
Finally, consider the original circuit (Figure 24.53g).




$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$
capacitors in series have the same charge.

**Figure 24.53g**

$$V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}.$$

$$V_4 = \frac{Q_4}{C_4} = \frac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\ \text{V}.$$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\ \text{V}.$
$Q_2 = 370\ \mu\text{C},\ V_2 = 87\ \text{V}.$
$Q_3 = 180\ \mu\text{C},\ V_3 = 43\ \text{V}.$
$Q_4 = 180\ \mu\text{C},\ V_4 = 43\ \text{V}.$

$Q_5 = 550\ \mu\text{C},\ V_5 = 65\ \text{V}.$

**EVALUATE:** $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\ \text{V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$.

**24.54.** **IDENTIFY** and **SET UP:** We want to estimate the excess charge rubbed onto our head and the resulting voltage when we comb our hair. Treat the head as a sphere and model it as a spherical capacitor.
**EXECUTE:** **(a)** Estimate: $L \approx 15$ cm = 0.15 m long.
**(b)** $m$ = (65 $\mu$g/cm)(15 cm) ≈ 975 $\mu$g.
**(c)** Estimate: $N$ = 25 hairs.
**(d)** The electric force $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$ is the force between the charges at both ends of a hair. $q_1 = q_2 = Q/N$, $r = L$, and the force is twice the weight of the hair, which is $2mg$. Therefore $2mg = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/N)^2}{L^2}$. Solve for $Q$: $Q = \sqrt{(2mg)(4\pi\epsilon_0)L^2 N}$. Using $m$ = 975 $\mu$g, $L$ = 0.15 m, and $N$ = 25 gives $Q$ = 0.35 $\mu$C. The total charge on your head is $2Q$, so $Q_{\text{head}}$ = 0.70 $\mu$C.
**(e)** Estimate: Diameter ≈ 22 cm, so $R \approx 11$ cm. $C = 4\pi\epsilon_0 R = 4\pi\epsilon_0\,(0.11\ \text{m}) = 12$ pF.
**(f)** $V = Q/C$ = (0.35 $\mu$C)/(12 pF) = 29 kV.
**EVALUATE:** This is a large potential but it is not dangerous because of the small amount of charge.

**24.55.** **IDENTIFY:** Capacitors in series carry the same charge, while capacitors in parallel have the same potential difference across them.
**SET UP:** $V_{ab} = 150\ \text{V},\ Q_1 = 150\ \mu\text{C},\ Q_3 = 450\ \mu\text{C},$ and $V = Q/C$.
**EXECUTE:** $C_1 = 3.00\ \mu\text{F}$ so $V_1 = \dfrac{Q_1}{C_1} = \dfrac{150\ \mu\text{C}}{3.00\ \mu\text{F}} = 50.0\ \text{V}$ and $V_1 = V_2 = 50.0\ \text{V}$. $V_1 + V_3 = V_{ab}$ so $V_3 = 100\ \text{V}$. $C_3 = \dfrac{Q_3}{V_3} = \dfrac{450\ \mu\text{C}}{100\ \text{V}} = 4.50\ \mu\text{F}$. $Q_1 + Q_2 = Q_3$ so $Q_2 = Q_3 - Q_1 = 450\ \mu\text{C} - 150\ \mu\text{C} = 300\ \mu\text{C}$ and $C_2 = \dfrac{Q_2}{V_2} = \dfrac{300\ \mu\text{C}}{50.0\ \text{V}} = 6.00\ \mu\text{F}$.
**EVALUATE:** Capacitors in parallel only carry the same charge if they have the same capacitance.

**24.56.** **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.
**SET UP:** With the switch open, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed, each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.
**EXECUTE:** **(a)** With the switch open $C_{\text{eq}} = \left(\left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}$.

$Q_{\text{total}} = C_{\text{eq}}V = (4.00\ \mu\text{F})(210\ \text{V}) = 8.40\times10^{-4}\ \text{C}$. By symmetry, each capacitor carries $4.20\times10^{-4}$ C. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140\ \text{V}$ and $V_{ac} = Q/C_6 = 70\ \text{V}$. $V_{cd} = V_{ad} - V_{ac} = 70\ \text{V}$.
**(b)** When the switch is closed, the points $c$ and $d$ must be at the same potential, so the equivalent capacitance is $C_{\text{eq}} = \left(\dfrac{1}{(3.00+6.00)\ \mu\text{F}} + \dfrac{1}{(3.00+6.00)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}$.

$Q_{\text{total}} = C_{\text{eq}}V = (4.50\ \mu\text{F})(210\ \text{V}) = 9.5\times10^{-4}\ \text{C}$, and each capacitor has the same potential difference of 105 V (again, by symmetry).

**(c)** Consider the $C_3 = 3.00\ \mu\text{F}$ and $C_6 = 6.00\ \mu\text{F}$ capacitors in the upper branch of the network. The only way for the net charge $Q_{\text{net}}$ on the negative plate of $C_3$ and the positive plate of $C_6$ to change is by charge to flow through the switch. With the switch open all four capacitors have the same charge and $Q_{\text{net}} = 0$. With the switch closed the charge on $C_3$ is $Q_3 = (3.00\ \mu\text{F})(105\ \text{V}) = 315\ \mu\text{C}$ and the charge on $C_6$ is $Q_6 = (6.00\ \mu\text{F})(105\ \text{V}) = 630\ \mu\text{C}$ and $Q_{\text{net}} = Q_2 - Q_1 = 315\ \mu\text{C}$. Therefore, the change in $Q_{\text{net}}$ is $315\ \mu\text{C}$ and this is the amount of charge that flowed through the switch when it was closed.

**EVALUATE:** When the switch is closed the charge must redistribute to make points *c* and *d* be at the same potential.

**24.57.** **(a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

**SET UP:** The three capacitors can be replaced by their equivalent as shown in Figure 24.57a.

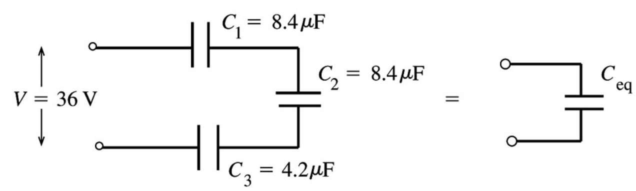



**Figure 24.57a**

**EXECUTE:** $C_3 = C_1/2$ so $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{\text{eq}} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}$.

$Q = C_{\text{eq}}V = (2.1\ \mu\text{F})(36\ \text{V}) = 76\ \mu\text{C}$.

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}$.

**EVALUATE:** The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

**(b) IDENTIFY** and **SET UP:** Use $U = \frac{1}{2}QV$. We know each $Q$ and we know that $V_1 + V_2 + V_3 = 36\ \text{V}$.

**EXECUTE:** $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$.

But also $V_1 + V_2 + V_3 = V = 36\ \text{V}$, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\ \text{V}) = 1.4 \times 10^{-3}\ \text{J}$.

**EVALUATE:** We could also use $U = Q^2/2C$ and calculate $U$ for each capacitor.

**(c) IDENTIFY:** The charges on the plates redistribute to make the potentials across each capacitor the same.

**SET UP:** The capacitors before and after they are connected are sketched in Figure 24.57b.

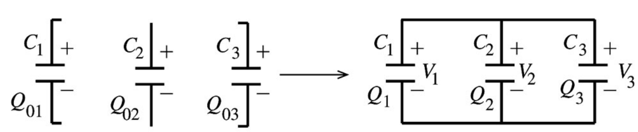



**Figure 24.57b**

**EXECUTE:** The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76\ \mu\text{C}) = 228\ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228\ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4\ \mu\text{F} / 4.2\ \mu\text{F}) = 2Q_3$.

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228\ \mu\text{C}$.
$5Q_3 = 228\ \mu\text{C}$ and $Q_3 = 45.6\ \mu\text{C}$, $Q_1 = Q_2 = 91.2\ \mu\text{C}$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11\ \text{V}$, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6\ \mu\text{C}}{4.2\ \mu\text{F}} = 11\ \text{V}$.

The voltage across each capacitor in the parallel combination is 11 V.

**(d)** $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $V_1 = V_2 = V_3$ so $U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11\ \text{V})(228\ \mu\text{C}) = 1.3\times10^{-3}\ \text{J}$.

**EVALUATE:** This is less than the original energy of $1.4\times10^{-3}$ J. The stored energy has decreased, as in Example 24.7.

**24.58. IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. $C = \dfrac{Q}{V}$. $V = Ed$. $U = \frac{1}{2}QV$.

**SET UP:** $d = 3.0\times10^3$ m. $A = \pi r^2$, with $r = 1.0\times10^3$ m.

**EXECUTE:** **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(1.0\times10^3\ \text{m})^2}{3.0\times10^3\ \text{m}} = 9.3\times10^{-9}\ \text{F}$.

**(b)** $V = \dfrac{Q}{C} = \dfrac{20\ \text{C}}{9.3\times10^{-9}\ \text{F}} = 2.2\times10^9\ \text{V}$.

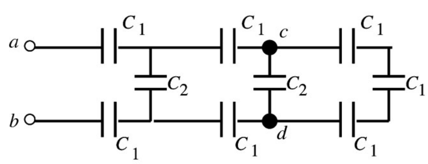
**(c)** $E = \dfrac{V}{d} = \dfrac{2.2\times10^9\ \text{V}}{3.0\times10^3\ \text{m}} = 7.3\times10^5\ \text{V/m}$.

**(d)** $U = \frac{1}{2}QV = \frac{1}{2}(20\ \text{C})(2.2\times10^9\ \text{V}) = 2.2\times10^{10}\ \text{J}$.

**EVALUATE:** Thunderclouds involve very large potential differences and large amounts of stored energy.

**24.59. IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for $Q$ and $V$ for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

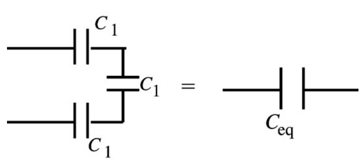
**(a) SET UP:** The network is sketched in Figure 24.59a.




$C_1 = 6.9\ \mu\text{F}$.
$C_2 = 4.6\ \mu\text{F}$.

**Figure 24.59a**

**EXECUTE:** Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.59b.

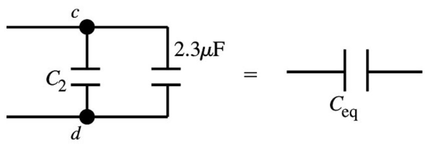



$\dfrac{1}{C_{\text{eq}}} = \dfrac{3}{C_1}$.

$C_{\text{eq}} = \dfrac{C_1}{3} = \dfrac{6.9\ \mu\text{F}}{3} = 2.3\ \mu\text{F}$.

**Figure 24.59b**

Next make the replacement shown in Figure 24.59c.

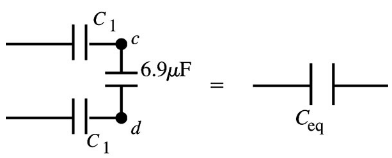



$$C_{eq} = 2.3\ \mu\text{F} + C_2.$$

$$C_{eq} = 2.3\ \mu\text{F} + 4.6\ \mu\text{F} = 6.9\ \mu\text{F}.$$

**Figure 24.59c**

Make the replacement shown in Figure 24.59d.

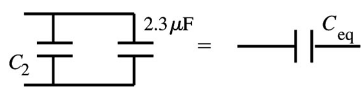



$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}.$$

$$C_{eq} = 2.3\ \mu\text{F}.$$

**Figure 24.59d**

Make the replacement shown in Figure 24.59e.

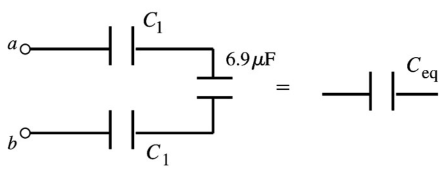



$$C_{eq} = C_2 + 2.3\ \mu\text{F} = 4.6\ \mu\text{F} + 2.3\ \mu\text{F}.$$

$$C_{eq} = 6.9\ \mu\text{F}.$$

**Figure 24.59e**

Make the replacement shown in Figure 24.59f.

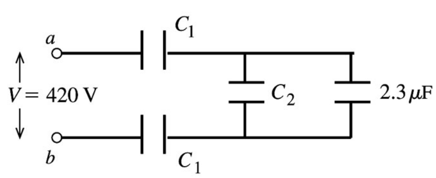



$$\frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}.$$

$$C_{eq} = 2.3\ \mu\text{F}.$$

**Figure 24.59f**

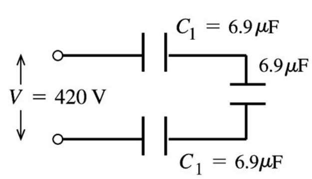
**(b) SET UP** and **EXECUTE**: Consider the network as drawn in Figure 24.59g.




From part (a) $2.3\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

**Figure 24.59g**

The equivalent network is shown in Figure 24.59h.

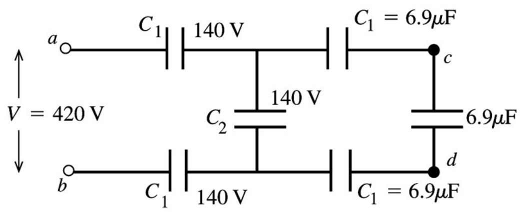



The capacitors are in series, so all three capacitors have the same $Q$.

**Figure 24.59h**

But here all three have the same $C$, so by $V = Q/C$ all three must have the same $V$. The three voltages must add to 420 V, so each capacitor has $V = 140$ V. The $6.9\ \mu$F to the right is the equivalent of $C_2$ and the $2.3\ \mu$F capacitor in parallel, so $V_2 = 140$ V. (Capacitors in parallel have the same potential difference.) Hence $Q_1 = C_1V_1 = (6.9\ \mu\text{F})(140\ \text{V}) = 9.7\times10^{-4}$ C and
$Q_2 = C_2V_2 = (4.6\ \mu\text{F})(140\ \text{V}) = 6.4\times10^{-4}$ C.

**(c)** From the potentials deduced in part (b) we have the situation shown in Figure 24.59i.




From part (a) $6.9\ \mu$F is the equivalent capacitance of the rest of the network.

**Figure 24.59i**

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by $V = Q/C$ they each have the same potential difference. Their potentials must sum to $140$ V, so the potential across each is 47 V and $V_{cd} = 47$ V.

**EVALUATE:** In each capacitor network the rules for combining $V$ for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140\ \text{V}) - 2(47\ \text{V}) = V_{cd}$.

**24.60.** **IDENTIFY:** This situation is analogous to having two capacitors $C_1$ in series, each with separation $\frac{1}{2}(d-a)$.

**SET UP:** For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

**EXECUTE:** **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\epsilon_0 A}{(d-a)/2} = \dfrac{\epsilon_0 A}{d-a}$.

**(b)** $C = \dfrac{\epsilon_0 A}{d-a} = \dfrac{\epsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$.

**EVALUATE:** **(c)** As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance $y$ parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large $Q$ on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large $C$.

**24.61.** **IDENTIFY:** Capacitors in series carry the same charge, but capacitors in parallel have the same potential difference across them.

**SET UP:** $V_{ab} = 48.0$ V. $C = Q/V$ and $U = \dfrac{1}{2}CV^2$. For capacitors in parallel, $C = C_1 + C_2$, and for capacitors in series, $1/C = 1/C_1 + 1/C_2$.

**EXECUTE:** Using $U = \dfrac{1}{2}CV^2$ gives $C = \dfrac{2U}{V^2} = \dfrac{2(2.90\times10^{-3}\ \text{J})}{(48.0\ \text{V})^2} = 2.517\times10^{-6}$ F, which is the equivalent capacitance of the network. The equivalent capacitance for $C_1$ and $C_2$ in series is $C_{12} = \dfrac{1}{2}(4.00\ \mu\text{F}) = 2.00\ \mu$F. If $C_{123}$ is the equivalent capacitance for $C_{12}$ and $C_3$ in parallel, then

$\frac{1}{C_{123}} + \frac{1}{C_4} = \frac{1}{C}$. Solving for $C_{123}$ gives

$\frac{1}{C_{123}} = \frac{1}{C} - \frac{1}{C_4} = \frac{1}{2.517\times 10^{-6}\ \text{F}} - \frac{1}{8.00\times 10^{-6}\ \text{F}} = 2.722\times 10^{5}\ \text{F}^{-1}$, so $C_{123} = 3.673\times 10^{-6}$ F.

$C_{12} + C_3 = C_{123}$. $C_3 = C_{123} - C_{12} = 3.673\ \mu\text{F} - 2.00\ \mu\text{F} = 1.67\ \mu\text{F}$.

**EVALUATE:** As with most circuits, it is necessary to solve them in a series of steps rather than using a single step.

**24.62. IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \frac{Q^2}{2C}$.

**SET UP:** For this charge distribution, $E = 0$ for $r < r_a$, $E = \frac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

**EXECUTE: (a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \frac{\lambda^2}{8\pi^2\epsilon_0 r^2}$.

**(b)** $U = \int u\,dV = 2\pi L\int u r\,dr = \frac{L\lambda^2}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{dr}{r}$ and $\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

**(c)** $U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a) = \frac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

**EVALUATE:** We could have used the results of part (b) and $U = \frac{Q^2}{2C}$ to calculate $C/L$ and would obtain the same result as in Example 24.4.

**24.63. IDENTIFY:** We are dealing with a parallel-plate capacitor.
**SET UP:** $E = E_0/K$ with dielectric, where $E_0 = \sigma/\epsilon_0$.

**EXECUTE: (a)** We want the compressive force applied to the dielectric. $F_{\text{on}\,+Q} = QE_{-Q} =$

$Q\left(\frac{\sigma}{2\epsilon_0 K}\right) = Q\left(\frac{Q/A}{2\epsilon_0 K}\right) = \frac{Q^2}{2A\epsilon_0 K}$, and the force on $-Q$ is the same.

**(b)** We want the squeezing coefficient $s$. $d = d_0 - sQ^2$, so $\Delta d = d_0 - d = sQ^2$. Young's modulus is $Y = \frac{F_\perp}{A}\frac{\ell_0}{\Delta\ell}$. Applied here, $\Delta\ell = \Delta d = sQ^2$ and $\ell_0 = d_0$. Solving for $s$: $s = \frac{F_\perp d_0}{AQ^2 Y}$. Using $F$ from part (a) gives $s = \left(\frac{Q^2}{2A\epsilon_0 K}\right)\frac{d_0}{AQ^2 Y} = \frac{d_0}{2A^2\epsilon_0 YK}$.

**(c)** Using $A$ = 1.00 cm$^2$, $d_0$ = 0.400 mm, $K$ = 3.00, and $Y$ = 0.0100 GPa gives $s = 7.53\times 10^{7}\ \text{m/C}^2$.

**(d)** We want the applied voltage. $V = Ed = \frac{E_0}{K}\left(d_0 - sQ^2\right) = \frac{\sigma/\epsilon_0}{K}\left(d_0 - sQ^2\right) = \frac{Q}{A\epsilon_0 K}\left(d_0 - sQ^2\right)$.

Using $Q$ = 0.700 $\mu$C and the other given numbers, we get $V$ = 95.7 kV.
**(e)** We want the voltage if we double the charge. The same calculation but using $Q$ = 1.40 $\mu$C leads to $V$ = 133 kV.
**EVALUATE:** Doubling the charge did not double the potential.

**24.64.** **IDENTIFY:** The capacitor is equivalent to two capacitors in parallel, as shown in Figure 24.64.

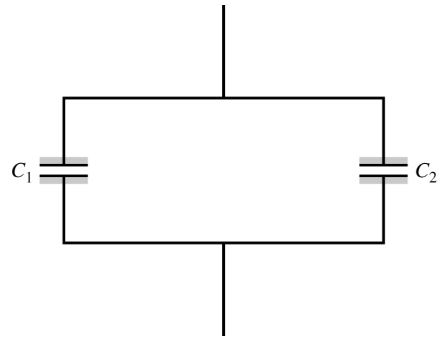



**Figure 24.64**

**SET UP:** Each of these two capacitors have plates that are 12.0 cm by 6.0 cm. For a parallel-plate capacitor with dielectric filling the volume between the plates, $C = K\epsilon_0 \dfrac{A}{d}$. For two capacitors in parallel, $C = C_1 + C_2$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

**EXECUTE:** **(a)** $C = C_1 + C_2$.

$$C_2 = \epsilon_0 \frac{A}{d} = \frac{(8.854\times10^{-12}\ \text{F/m})(0.120\ \text{m})(0.060\ \text{m})}{4.50\times10^{-3}\ \text{m}} = 1.42\times10^{-11}\ \text{F}.$$

$C_1 = KC_2 = (3.40)(1.42\times10^{-11}\ \text{F}) = 4.83\times10^{-11}\ \text{F}$. $C = C_1 + C_2 = 6.25\times10^{-11}\ \text{F} = 62.5\ \text{pF}$.

**(b)** $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.25\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 1.01\times10^{-8}\ \text{J}$.

**(c)** Now $C_1 = C_2$ and $C = 2(1.42\times10^{-11}\ \text{F}) = 2.84\times10^{-11}\ \text{F}$.

$U = \frac{1}{2}CV^2 = \frac{1}{2}(2.84\times10^{-11}\ \text{F})(18.0\ \text{V})^2 = 4.60\times10^{-9}\ \text{J}$.

**EVALUATE:** The plexiglass increases the capacitance and that increases the energy stored for the same voltage across the capacitor.

**24.65.** **IDENTIFY:** We are looking at the force between oppositely charged parallel plates.

**SET UP:** The plates are oppositely charged so they attract. The tension in the cable is equal to the force with which the plates attract each other.

**EXECUTE:** **(a)** We want the tension in the cable. The force on the upper plate is $F_{\text{upper}} = QE_{\text{lower}}$. The field between the plates is $\sigma/\epsilon_0$, half of which is due to each plate, so $E_{\text{lower}} = \sigma/2\epsilon_0$. Using $\sigma = Q/A$ we get $F_{\text{electric}} = \dfrac{Q^2}{2\epsilon_0 A}$. For a parallel-plate capacitor with plates a distance $z$ apart, $C = \epsilon_0 A/z$, where $A = \pi r^2$. $Q = CV = \dfrac{\epsilon_0 A}{z}V$, so $F_{\text{electric}} = \left(\dfrac{\epsilon_0 AV}{z}\right)\dfrac{1}{2\epsilon_0 A} = \dfrac{\pi\epsilon_0 r^2V^2}{2z^2}$, which is the tension in the cable.

**(b)** We want the work. $\displaystyle\int_d^{2d} F dz = \int_d^{2d} \frac{\pi\epsilon_0 r^2V^2}{2z^2}dz = \frac{\pi\epsilon_0 r^2V^2}{2}\left(-\frac{1}{z}\right)\Bigg|_d^{2d} = \frac{\pi\epsilon_0 r^2V^2}{4d}$.

**(c)** We want the initial energy stored in the electric field of the capacitor.

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)V^2 = \frac{\pi\epsilon_0 r^2V^2}{2d}.$$

**(d)** We want the energy stored in the electric field of the capacitor after raising the plate.

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{2d}\right)V^2 = \frac{\pi\epsilon_0 r^2V^2}{4d}.$$

**EVALUATE:** **(e)** Subtract the two energies, giving $\Delta U = -\dfrac{\pi\epsilon_0 r^2V^2}{4d}$. The work done in separating the plates is equal to the *magnitude* of the energy change in the plates. This does *not* mean that the work done is *equal* to the change in the energy stored in the plates. The work done on the plates is positive but the plates *lose* energy. The plates are connected to the battery, so the potential difference across them remains constant as they are separated. Therefore change is forced off of the plates through the battery, which does work on the battery. We have neglected gravity because it was much weaker than the electric force.

**24.66.** **IDENTIFY:** The system is equivalent to two capacitors in parallel. One of the capacitors has plate separation *d*, plate area $w(L-h)$ and air between the plates. The other has the same plate separation *d*, plate area *wh* and dielectric constant *K*.

**SET UP:** Define $K_{\text{eff}}$ by $C_{\text{eq}} = \dfrac{K_{\text{eff}}\epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

**EXECUTE:** **(a)** The capacitors are in parallel, so $C = \dfrac{\epsilon_0 w(L-h)}{d} + \dfrac{K\epsilon_0 wh}{d} = \dfrac{\epsilon_0 wL}{d}\left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$.

This gives $K_{\text{eff}} = \left(1 + \dfrac{Kh}{L} - \dfrac{h}{L}\right)$.

**(b)** For gasoline, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 1.48$; $\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 1.71$.

**(c)** For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 17$; $\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 25$.

**(d)** This kind of fuel tank sensor will work best for methanol since it has the greater range of $K_{\text{eff}}$ values.

**EVALUATE:** When $h = 0$, $K_{\text{eff}} = 1$. When $h = L$, $K_{\text{eff}} = K$.

**24.67.** **IDENTIFY** and **SET UP:** For two capacitors in series, $\dfrac{1}{C_1} + \dfrac{1}{C_2} = \dfrac{1}{C_{\text{eq}}}$, which gives $C_{\text{eq}} = \dfrac{C_1C_2}{C_1 + C_2}$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$. $C = Q/V$. The stored energy can be written as $U = \frac{1}{2}CV^2$ or $U = \dfrac{Q^2}{2C}$.

**EXECUTE:** **(a)** When connected in series, the stored energy is 0.0400 J, so

a = $\frac{1}{2}\left(\dfrac{C_1C_2}{C_1 + C_2}\right)V^2 = \frac{1}{2}\left(\dfrac{C_1C_2}{C_1 + C_2}\right)(200.0\text{ V})^2 = 0.0400\text{ J}$, which gives

$$\frac{C_1C_2}{C_1 + C_2} = 2.00\ \mu\text{F}.$$

When connected in parallel, the stored energy is 0.180 J, so

$U = \frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(C_1 + C_2)(200.0\text{ V})^2 = 0.180\text{ J}.$

$C_1 + C_2 = 9.00\ \mu\text{F}$.

Solving the two equations for $C_1$ and $C_2$ gives $C_1 = 6.00\ \mu\text{F}$ and $C_2 = 3.00\ \mu\text{F}$.

**(b)** When the capacitors are in series, both have the same charge. The stored energy is $U = \dfrac{Q^2}{2C}$, so the capacitor with the *smaller* capacitance stores more energy, which is $C_2$.
**(c)** When the capacitors are in parallel, the potential across them is the same. The stored energy is $U = \frac{1}{2}CV^2$, so the capacitor with the *larger* capacitance stores the most energy, which is $C_1$.

**EVALUATE:** When the two capacitors are connected in parallel, they can store considerably more energy than when in series.

**24.68.** **IDENTIFY** and **SET UP:** The presence of the dielectric affects the charge and energy in the capacitor for a given potential difference. $V = Ed$, $Q = CV$, $K = C/C_0$, $U = \frac{1}{2}CV^2$. We use the values for $K$ and $E_m$ from Table 24.2. In this case, $E = 0.500E_m$ and $d = 2.50$ mm = 0.00250 m.

**EXECUTE:** **(a)** Using $U = \frac{1}{2}CV^2$, $C = KC_0$, $V = Ed$, and $E = 0.500E_m$, the stored energy is

$U = \frac{1}{2}CV^2 = \frac{1}{2}KC_0(Ed)^2 = \frac{1}{2}KC_0(0.500E_m d)^2$.

For polycarbonate, $K = 2.8$ and $E_m = 3\times10^7$ V/m. Therefore the stored energy is

$U = (1/2)[(2.8)(6.00\times10^{-12}\text{ F})][(0.500)(3\times10^7\text{ V/m})(0.00250\text{ m})]^2 = 1.18\times10^{-2}$ J, which rounds to 12 mJ. Using similar calculations for the other materials, the results for $U$ are:
12 mJ (polycarbonate)
56 mJ (polyester)
51 mJ (polypropylene)
4.9 mJ (polystyrene)
2.2 mJ (pyrex)
**(b)** $Q = CV = KC_0(Ed) = KC_0(0.500E_m)d$.
For polycarbonate we have
$Q = (2.8)(6.00\times10^{-12}\text{ F})(0.500)(3\times10^7\text{ V/m})(0.00250\text{ m}) = 6.3\times10^{-7}\text{ C} = 0.63\ \mu\text{C}$.
Similar calculations for the other materials yield:
0.63 $\mu$C (polycarbonate)
1.5 $\mu$C (polyester)
1.2 $\mu$C (polypropylene)
0.39 $\mu$C (polystyrene)
0.35 $\mu$C (pyrex)
**(c)** $V = Ed = 0.500E_m d$. For polycarbonate this gives
$V = (0.500)(3\times10^7\text{ V/m})(0.00250\text{ m}) = 3.8\times10^4\text{ V} = 38\text{ kV}$.
Similar calculations for the other materials yield:
38 kV (polycarbonate)
75 kV (polyester)
88 kV (polypropylene)
25 kV (polystyrene)
13 kV (pyrex)

**EVALUATE:** **(d)** Polyester is best for maximum energy storage and maximum charge, but polypropylene is best for maximum voltage. No single material is best for all three categories. As so often occurs, the choice of materials is a trade-off.

**24.69.** **IDENTIFY** and **SET UP:** For a parallel-plate capacitor, $C=\dfrac{\epsilon_0 A}{d}$. The stored energy can be expressed as $U=\tfrac{1}{2}CV^2$ or $U=\dfrac{Q^2}{2C}$.

**EXECUTE:** **(a)** If the battery remains connected, $V$ remains constant, so it is useful to write the energy in terms of $V$ and $C$:

$$U=\tfrac{1}{2}CV^2 = \tfrac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)V^2=\frac{\epsilon_0 AV^2}{2}\cdot\frac{1}{d}.$$

If the battery is disconnected, $Q$ remains constant, so it is useful to write the energy in terms of $Q$ and $C$:

$$U=\frac{Q^2}{2C} = \frac{Q^2}{2\left(\frac{\epsilon_0 A}{d}\right)}=\left(\frac{Q^2}{2\epsilon_0 A}\right)d.$$

The graph shows a linear relationship between $U$ and $1/d$, so it must represent the case where the battery remains connected to the capacitor.

**(b)** In a graph of $U$ versus $1/d$ for the equation $U=\dfrac{\epsilon_0 AV^2}{2}\cdot\dfrac{1}{d}$, the slope should be equal to $\dfrac{\epsilon_0 AV^2}{2}$.

Choosing points on the graph in the problem, the slope is $\dfrac{(73-18)\times10^{-9}\text{ J}}{20.0\text{ cm}^{-1}-5.0\text{ cm}^{-1}}=3.67\times10^{-11}\text{ J}\cdot\text{m}.$

Solving for $A$ gives

$A = 2(\text{slope})/\epsilon_0 V^2=2(3.67\times10^{-11}\text{ J}\cdot\text{m})/(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(24.0\text{ V})^2]=0.014\text{ m}^2=144\text{ cm}^2.$

**(c)** With the battery connected: $U=\dfrac{\epsilon_0 AV^2}{2}\cdot\dfrac{1}{d}$, so as we increase $d$ from 0.0500 cm to 0.400 cm, the energy *decreases* since $V$ remains constant.

With the battery disconnected: $U=\left(\dfrac{Q^2}{2\epsilon_0 A}\right)d$, so as we increase $d$, the energy *increases* since $Q$ does not change. Therefore there is more energy stored with the battery *disconnected* as $d$ is increased.

**EVALUATE:** If this capacitor were square, its plates would be $12\text{ cm}\times12\text{ cm}$. This is a reasonable size for a piece of apparatus for use in a laboratory and could easily be manufactured.

**24.70.** **IDENTIFY:** Two coaxial conducting shells with dielectric in the space between them form a cylindrical capacitor. We assume that there are no appreciable effects due to the part of the dielectric that is not within the region between the cylinders.

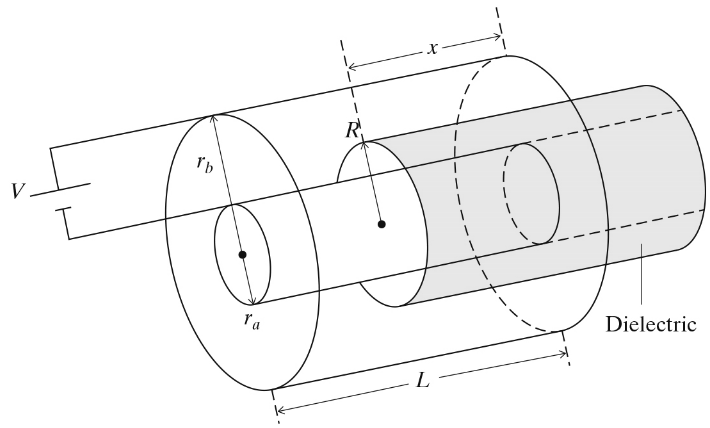



**Figure 24.70a**

**SET UP:** Refer to Fig. 24.70a. We can think of this combination as two capacitors in parallel. One capacitor is the section without dielectric and the other is the section with dielectric. They are in parallel because they share the same positive side and the same negative side. The section containing dielectric can be viewed as two cylindrical capacitors of length $x$ in series, as shown in Fig. 24.70b. If $R$ is the radius of the dielectric section, that part has an inner radius $r_a$ and outer radius $R$. The part without dielectric has inner radius $R$ and outer radius $r_b$. For a cylindrical capacitor of length $\ell$ with inner radius $r_a$ and outer radius $r_b$, $C = \dfrac{2\pi\epsilon_0 \ell}{\ln(r_b/r_a)}$. We want the capacitance of this device.

**EXECUTE: (a)** The section containing no dielectric: Use $C = \dfrac{2\pi\epsilon_0 \ell}{\ln(r_b/r_a)}$ with $\ell = L - x$. Call this capacitance $C_1$ so $C_1 = \dfrac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)}$.

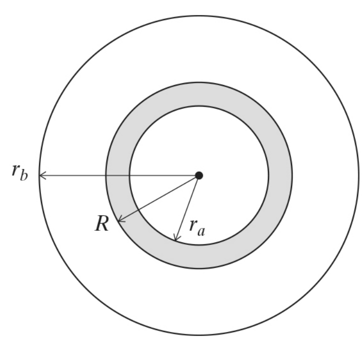



**Figure 24.70b**

The section partially filled with dielectric: Refer to Fig. 24.70b. Call the inner part (radii $r_a$ and $R$) $C_{aR}$. Applying $C = \dfrac{2\pi\epsilon_0 \ell}{\ln(r_b/r_a)}$ with $\ell = x$ and allowing for the dielectric, we have $C_{aR} = \dfrac{K 2\pi\epsilon_0 x}{\ln(R/r_a)}$. Call the outer part (radii $R$ and $r_b$) $C_{Rb}$. Applying the same formula gives $C_{Rb} = \dfrac{2\pi\epsilon_0 x}{\ln(r_b/R)}$. Call $C_2$ the equivalent capacitance of $C_{aR}$ and $C_{Rb}$. Since they are in series, their equivalent capacitance is given by $\dfrac{1}{C_2} = \dfrac{1}{C_{aR}} + \dfrac{1}{C_{Rb}} = \dfrac{\ln(R/r_a)}{K 2\pi\epsilon_0 x} + \dfrac{\ln(r_b/R)}{2\pi\epsilon_0 x}$. This gives $C_2 = \dfrac{2\pi\epsilon_0 x}{\dfrac{\ln(R/r_a)}{K} + \ln(r_b/R)}$. Since $C_1$ and $C_2$ are in

series, the equivalent capacitance of this device is $C = C_1 + C_2$, which gives
$$C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)} + \frac{2\pi\epsilon_0 x}{\frac{\ln(R/r_a)}{K} + \ln(r_b/R)}.$$

**(b)** We want the equivalent capacitance when $x = 0$. Using our result from part (a) gives $C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)}$. We recognize this as an ordinary air-filled cylindrical capacitor of length $L-x$.

Using $L = 10.0$ cm, $r_a = 1.00$ cm, $r_b = 4.00$ cm, and $K = 3.21$, we get $C = 4.01$ pF.

**(c)** We want $C$ when $x = L$. Using the result from (a) with $K = 3.21$ gives $C = 8.83$ pF.

**(d)** We want $x$. First find $C$. $C = Q/V = (6.00\text{ nC})/(1.00\text{ kV}) = 6.00$ pF. Use our result from (a) and solve for $x$: $C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)} + \frac{2\pi\epsilon_0 x}{\frac{\ln(R/r_a)}{K} + \ln(r_b/R)} = 6.00$ pF. The result is $x = 4.12$ cm.

**EVALUATE:** A device like this one could be used to make a variable capacitor that one could easily vary as desired simply by sliding the dielectric in and out.

**24.71. IDENTIFY:** This problem involves a capacitor with dielectric inside.

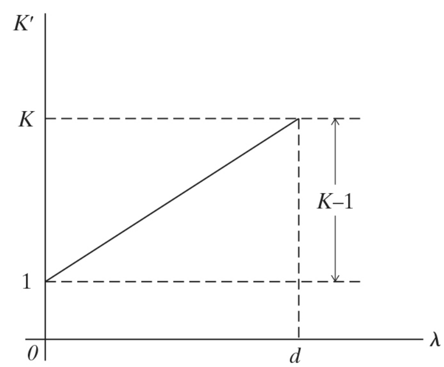



**Figure 24.71a**

**SET UP:** The dielectric constant $K'$ is not uniform within the capacitor. We first need to find an equation for $K'$. At the first plate, the $K' = 1$, at the second plate $K' = K$, and it varies linearly. Call $x$ the distance of any point from the first plate. Using this information, sketch the graph of $K'$ versus $x$, as shown in Fig. 24.71a. Using the slope-intercept form, we see that the slope of this line is $(K-1)/d$ and the $x$-intercept is 1. The equation is $K' = \left(\frac{K-1}{d}\right)x + 1$.

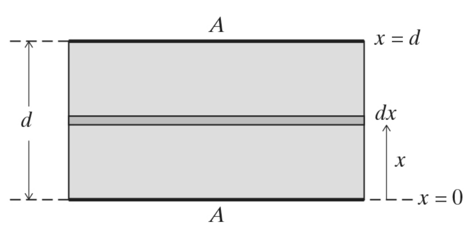



**Figure 24.71b**

**EXECUTE:** (a) We want to find the capacitance of this device. Since $K'$ depends only on the distance $x$ from the first plate, break the dielectric into thin slabs of area $A$ and thickness $dx$ as shown in Fig.

24.71b. Using the equation $C=\frac{\epsilon_0 AK}{d}$, the capacitance of a single slab is $dC=\frac{\epsilon_0 AK}{dx}$. These slabs are all in series with each other, so we apply the equation $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots$ But in this case each capacitance to be added is infinitesimal, so we must integrate to get the sum. Doing so gives $\frac{1}{C}=\int\frac{1}{dC}=\int_0^d\frac{dx}{K'\epsilon_0 A}=\frac{1}{\epsilon_0 A}\int_0^d\frac{dx}{1+(K-1)x/d}=\frac{d}{\epsilon_0 A(K-1)}\ln\left[1+(K-1)x/d\right]\Big|_0^d$. Evaluating at the two limits and rearranging gives $C=\frac{\epsilon_0 A(K-1)}{d\ln K}$.

**(b)** We want to find $C$ when $K = 1$. Putting $K = 1$ into our result gives $\frac{0}{0}$, which is indeterminate. So to evaluate the limit, we need to use L'Hopital's rule, which gives $C=\lim\limits_{K\to 1}\left[\frac{\epsilon_0 A(K-1)}{d\ln K}\right]=\lim\limits_{K\to 1}\left[\frac{\epsilon_0 A}{d}\frac{d(K-1)/dK}{d(\ln K)/dK}\right]=\left(\frac{\epsilon_0 A}{d}\right)\left(\frac{1}{1/K}\right)=\frac{K\epsilon_0 A}{d}$. We recognize this result as the capacitance of a parallel-plate capacitor filled with uniform material having dielectric constant $K$.

**EVALUATE:** It would be plausible to construct a capacitor like this by filling the gap between the plates with thin layers of material have a progressively larger value of $K$.

**24.72.** **IDENTIFY:** The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area $Lx$ and dielectric filling the space between the plates.

**SET UP:** $C=\frac{K\epsilon_0 A}{d}$ for a parallel-plate capacitor with plate area $A$.

**EXECUTE:** **(a)** $C=\frac{\epsilon_0}{D}\left[(L-x)L+xKL\right]=\frac{\epsilon_0 L}{D}\left[L+(K-1)x\right]$.

**(b)** $dU=\frac{1}{2}(dC)V^2$, where $C=C_0+\frac{\epsilon_0 L}{D}(-dx+dxK)$, with $C_0=\frac{\epsilon_0 L}{D}\left[L+(K-1)x\right]$. This gives $dU=\frac{1}{2}\left(\frac{\epsilon_0 Ldx}{D}(K-1)\right)V^2=\frac{(K-1)\epsilon_0 V^2 L}{2D}dx.$

**(c)** If the charge is kept constant on the plates, then $Q=\frac{\epsilon_0 LV}{D}\left[L+(K-1)x\right]$ and $U=\frac{1}{2}CV^2=\frac{1}{2}C_0V^2\left(\frac{C}{C_0}\right)$. $U\approx\frac{C_0V^2}{2}\left(1-\frac{\epsilon_0 L}{DC_0}(K-1)dx\right)$ and $\Delta U=U-U_0=-\frac{(K-1)\epsilon_0 V^2 L}{2D}dx.$

**(d)** Since $dU=-Fdx=-\frac{(K-1)\epsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion $dx$, meaning that the slab feels a force pushing it out.

**EVALUATE:** **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for $dU$ in part (c) to $dU=-Fdx$ gives $F=\frac{(K-1)\epsilon_0 V^2 L}{2D}$.

**24.73.** **IDENTIFY** and **SET UP:** The potential difference is $V$ = 30 mV – (–70 mV) = 100 mV, and $Q = CV$.

**EXECUTE:** $Q = CV$ gives $Q/\text{cm}^2 = (C/\text{cm}^2)V = (1\ \mu\text{F/cm}^2)(100\ \text{mV})(1\ \text{mol}/10^5\ \text{C}) = 10^{-12}\ \text{mol/cm}^2$, which is choice (c).

**EVALUATE:** This charge produces a potential difference of 100 mV = 0.1 V, which is certainly measurable using ordinary laboratory meters.

**24.74.** **IDENTIFY** and **SET UP:** The change in concentration of $Na^+$ ions is equal to the added charge divided by the volume of the spherical egg. The original concentration of ions is given as 30 mmol/L. We use the answer from Problem 24.73 to get the added charge.

**EXECUTE:** The added charge is ($10^{-12}$ mol/cm$^2$)(surface area of egg) = ($10^{-12}$ mol/cm$^2$)($4\pi R^2$), and the original volume of the egg is $(4\pi/3)R^3$. Therefore the change in concentration is

$$(10^{-12}\text{ mol/cm}^2)(4\pi R^2)/\left[(4\pi/3)R^3\right] = 3(10^{-12}\text{ mol/cm}^2)/R = 3(10^{-12}\text{ mol/cm}^2)/(100\times10^{-4}\text{cm}) =$$

$$3\times10^{10}\text{ mol/cm}^3 = 3\times10^{-5}\text{ mmol/L.}$$

The fractional change in the concentrations is $(3\times10^{-5}\text{ mmol/L})/(30\text{ mmol/L}) = 10^{-5}$, which is 1 part in $10^5$. Therefore choice (b) is correct.

**EVALUATE:** As a percent, this change is $10^{-3}\%$ = 0.001%, which is quite small yet certainly important for the organism.

**24.75.** **IDENTIFY** and **SET UP:** The calcium $Ca^{2+}$ ions carry twice the charge of the $Na^+$ ions.

**EXECUTE:** The charge to produce the given voltage change would be the same as with $Na^+$, so we would need only half as many $Ca^{2+}$ ions to accomplish this. Thus choice (a) is correct.

**EVALUATE:** $Ca^{+2}$ ions are nearly twice as heavy as $Na^+$ ions, so they may not move as readily as the sodium ions.

**24.76.** **IDENTIFY** and **SET UP:** The energy is needed to change the potential from 30 mV to –70 mV. $U = \frac{1}{2}CV^2$. The capacitance is (1 $\mu$F/cm$^2$)(surface area of egg).

**EXECUTE:** For a spherical egg, the surface area is $4\pi R^2$, so the capacitance is

$$C = (1\ \mu\text{F/cm}^2)(4\pi R^2) = (1\ \mu\text{F/cm}^2)(4\pi)(100\times10^{-4}\text{ cm})^2 = 1.26\times10^{-9}\text{ F.}$$

The change in stored energy is

$$\Delta U = \tfrac{1}{2}CV_2^2 - \tfrac{1}{2}CV_1^2 = \tfrac{1}{2}C(V_2^2 - V_1^2).$$

$\Delta U = (1/2)(1.26\times10^{-9}\text{ F})[(-70\times10^{-3}\text{ V})^2 - (30\times10^{-3}\text{ V})^2] = 2.5\times10^{-12}\text{ J} = 2.5\text{ pJ} \approx 3\text{ pJ}$, which makes choice (d) the correct one.

**EVALUATE:** The actual energy required would probably be greater than 2.5 pJ, depending on the process by which the charging is accomplished, but our value is the minimum energy needed.