Circuits
-
First-order circuits

Spring 2022

First order circuits

Definition

Circuits that contain a single capacitor or inductor (and resistors, sources and switches).

It can contain multiple capacitors or inductors as long as we can combine them into a single equivalent capacitor/inductor.

Analysis

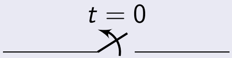
We will use KCL and KVL for the analysis of such circuits.

2 kinds of response

- **Natural response**: when the circuit does not contain any source (also called source-free)
- **Forced response**: when the circuit contains a source

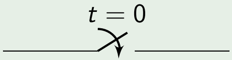
Switches

Opening switch



- $t < 0$: equivalent to a short-circuit
- $t > 0$: equivalent to an open-circuit

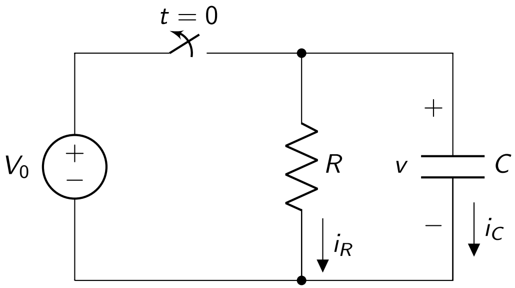
Closing switch



- $t < 0$: equivalent to an open-circuit
- $t > 0$: equivalent to a short-circuit

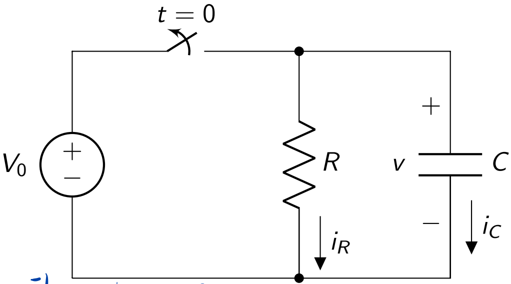
RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



$W(t=0^-) = \frac{1}{2} C V_0^2$

$t < 0$

For $t < 0$, we suppose that the switch has been closed for a *long* time

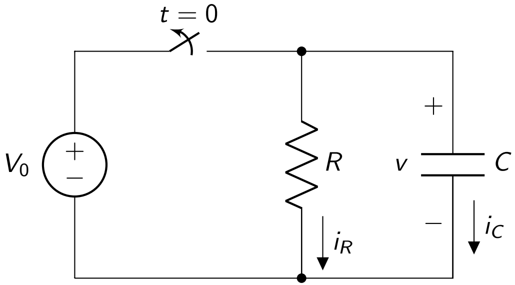
- Capacitor voltage $v(t = 0^-)$? V_0
- Capacitor current $i_C(t = 0^-)$? 0
- Stored energy in the capacitor $w(t = 0^-)$?

the response is very stable now

the energy's already stored

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



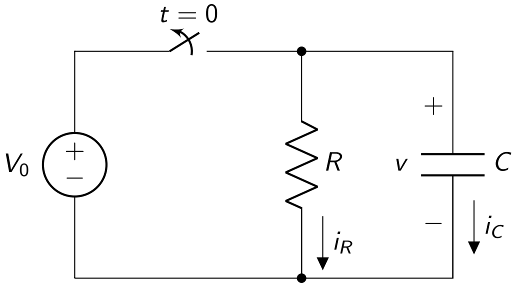
$t < 0$

For $t < 0$, we suppose that the switch has been closed for a *long* time

- Capacitor voltage $v(t = 0^-)$? $v(t = 0^-) = V_0$
- Capacitor current $i_C(t = 0^-)$?
- Stored energy in the capacitor $w(t = 0^-)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



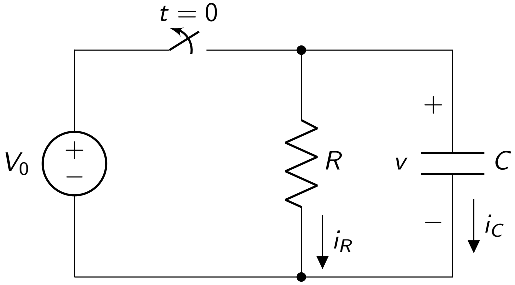
$t < 0$

For $t < 0$, we suppose that the switch has been closed for a *long* time

- Capacitor voltage $v(t = 0^-)$? $v(t = 0^-) = V_0$
- Capacitor current $i_C(t = 0^-)$? $i_C(t = 0^-) = 0$
- Stored energy in the capacitor $w(t = 0^-)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



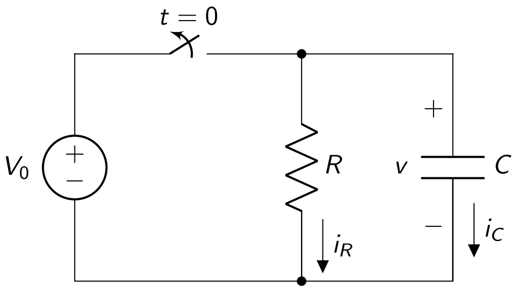
$t < 0$

For $t < 0$, we suppose that the switch has been closed for a *long* time

- Capacitor voltage $v(t = 0^-)$? $v(t = 0^-) = V_0$
- Capacitor current $i_C(t = 0^-)$? $i_C(t = 0^-) = 0$
- Stored energy in the capacitor $w(t = 0^-)$? $w(t = 0^-) = \frac{1}{2} C \cdot V_0^2$

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



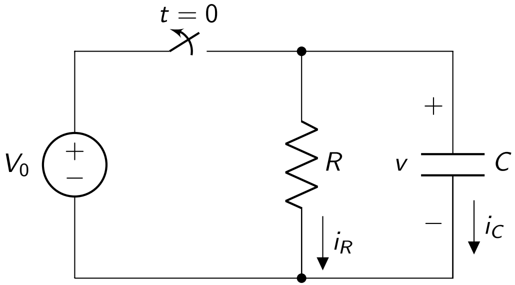
$t > 0$

The switch has just been opened ($t = 0^+$)

- Capacitor voltage $v(t = 0^+)$? V_0
- Capacitor current $i_C(t = 0^+)$?
- Stored energy in the capacitor $w(t = 0^+)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



$t > 0$

The switch has just been opened $(t = 0^+)$

- Capacitor voltage $v(t = 0^+)$? $v(t = 0^+) = V_0$
- Capacitor current $i_C(t = 0^+)$?
- Stored energy in the capacitor $w(t = 0^+)$?

RC circuits (Natural response)

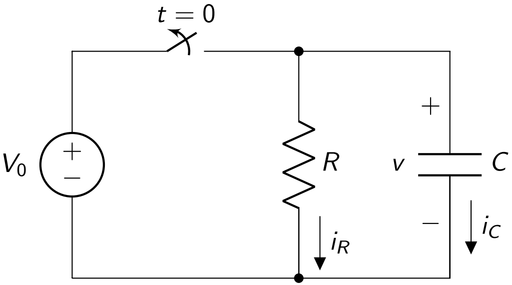
The switch, initially closed, opens at $t = 0$

$i_R = -i_C$

$V + i_C R = 0$

$i_C = -\frac{V_0}{R}$

$= -\frac{V_0}{R}$



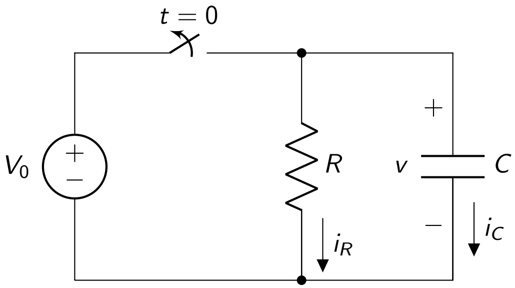
$t > 0$

The switch has just been opened ($t = 0^+$)

- Capacitor voltage $v(t = 0^+)$? $v(t = 0^+) = V_0$
- Capacitor current $i_C(t = 0^+)$? $i_C(t = 0^+) = -\frac{V_0}{R}$
- Stored energy in the capacitor $w(t = 0^+)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



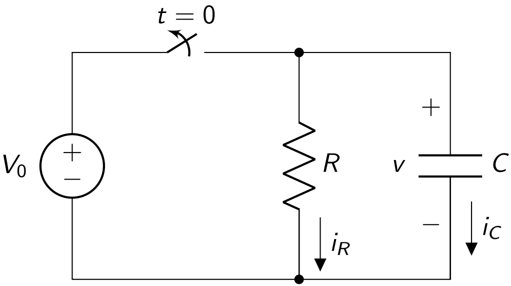
$t > 0$

The switch has just been opened $(t = 0^+)$

- Capacitor voltage $v(t = 0^+)$? $v(t = 0^+) = V_0$
- Capacitor current $i_C(t = 0^+)$? $i_C(t = 0^+) = -\frac{V_0}{R}$
- Stored energy in the capacitor $w(t = 0^+)$? $w(t = 0^+) = \frac{1}{2} C \cdot V_0^2$

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



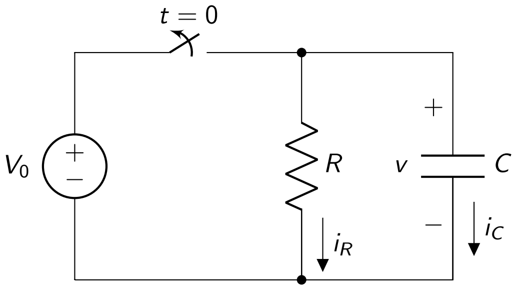
$t \to \infty$

The switch has been opened for a *long* time

- Capacitor voltage $v(t \to \infty)$?
- Capacitor current $i_C(t \to \infty)$?
- Stored energy in the capacitor $w(t \to \infty)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



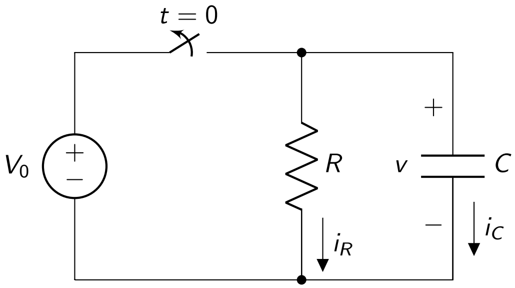
$t \to \infty$

The switch has been opened for a *long* time

- Capacitor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Capacitor current $i_C(t \to \infty)$?
- Stored energy in the capacitor $w(t \to \infty)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



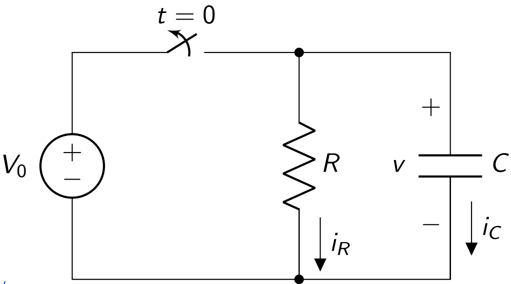
$t \to \infty$

The switch has been opened for a *long* time

- Capacitor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Capacitor current $i_C(t \to \infty)$? $i_C(t \to \infty) = 0$
- Stored energy in the capacitor $w(t \to \infty)$?

RC circuits (Natural response)

The switch, initially closed, opens at $t = 0$



All the electrons have met their protons

$t \to \infty$

The switch has been opened for a *long* time

- Capacitor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Capacitor current $i_C(t \to \infty)$? $i_C(t \to \infty) = 0$
- Stored energy in the capacitor $w(t \to \infty)$? $w(t \to \infty) = 0$

RC circuits (Natural response)

Determination of $v(t)$

For $t \geq 0$, we can write:

- $i_C(t) + i_R(t) = 0$
- $C\dfrac{dv(t)}{dt} + \dfrac{v(t)}{R} = 0$
- $\dfrac{dv(t)}{dt} + \dfrac{v(t)}{\tau} = 0$

Time constant τ

The unit of the product RC is proportional to **time** (usually in s, or ms, or μs...).

This product RC is called the **time constant** τ with $\tau = RC$.

RC circuits (Natural response)

Determination of $v(t)$

For $t \geq 0$, we have: $$\frac{dv(t)}{dt} + \frac{v(t)}{\tau} = 0$$

Solution (for $t \geq 0$)

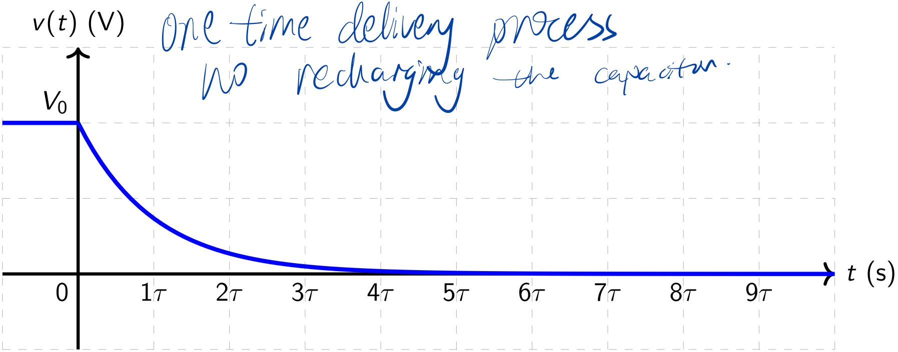
- $\dfrac{dv(t)}{dt} = -\dfrac{v(t)}{\tau}$
- $\dfrac{1}{v(t)} \cdot \dfrac{dv(t)}{dt} = -\dfrac{1}{\tau}$
- $\displaystyle\int_0^t \frac{1}{v(t')} \cdot \frac{dv(t')}{dt'} dt' = -\frac{1}{\tau} \int_0^t dt'$
- $\displaystyle\int_{v(0)=V_0}^{v(t)} \frac{1}{v'} \cdot dv' = -\frac{t}{\tau} \quad \Longrightarrow \quad \ln\left(\frac{v(t)}{V_0}\right) = -\frac{t}{\tau}$

RC circuits (Natural response)

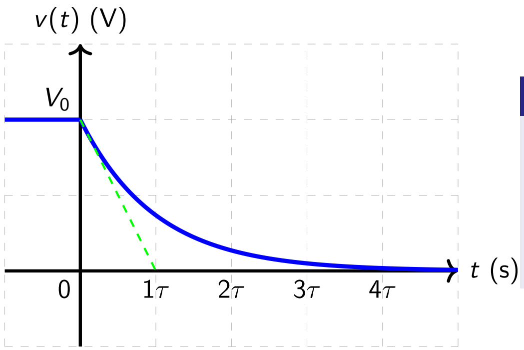
RC circuit natural response

$$v(t) = \begin{cases} V_s, & \text{if } t \le 0 \\ V_0 e^{-\frac{t}{\tau}}, & \text{if } t \ge 0 \end{cases}$$

One time delivery process
No recharging the capacitor.



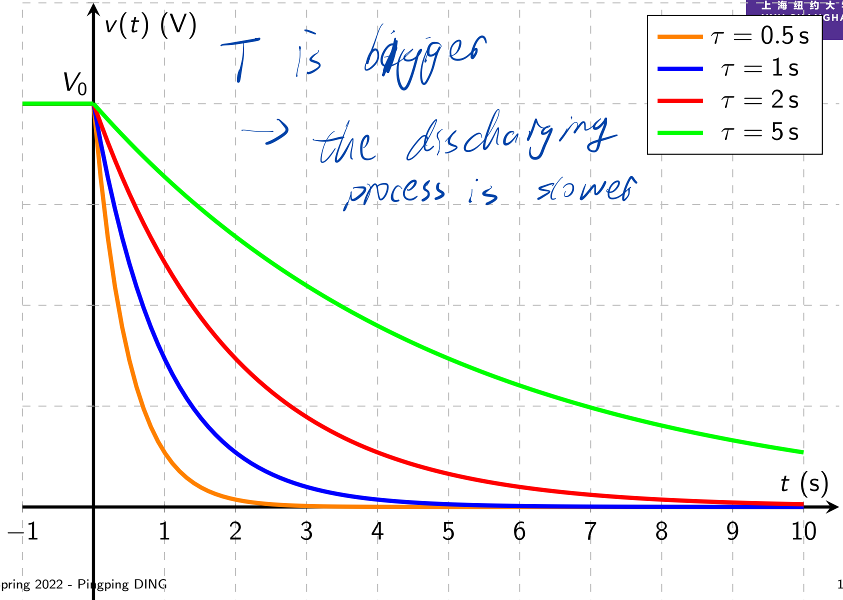
RC circuits (Natural response)



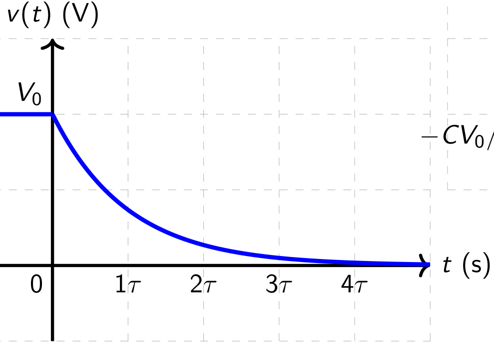
Exponential values

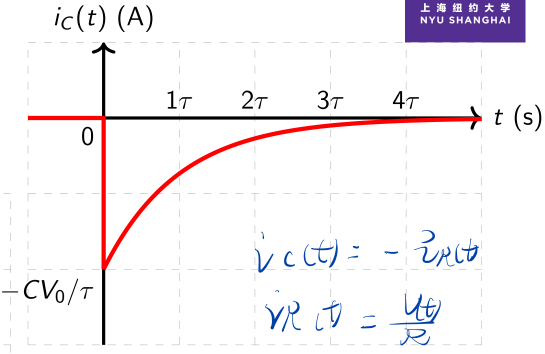
t	$v(t)$
τ	$0.37V_s$
2τ	$0.14V_s$
3τ	$0.05V_s$
5τ	$0.01V_s$

RC circuits (Natural response)



RC circuits (Natural response)





$i_C(t) = -i_R(t)$

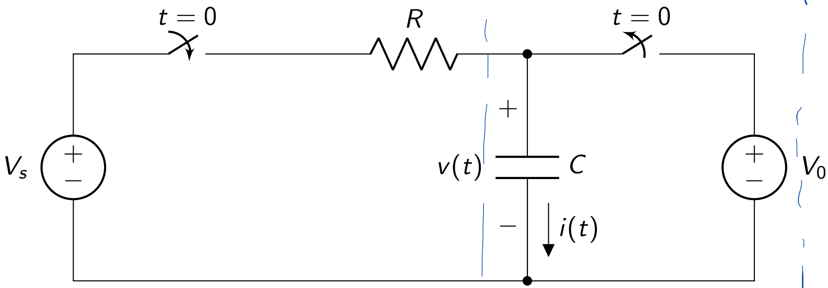
$i_R(t) = \frac{v(t)}{R}$

$i_C(t) = -\frac{v(t)}{R}$

RC circuits (Forced response)

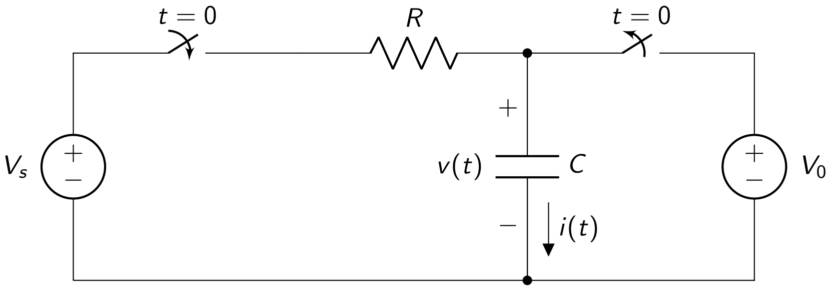
$t = 0^+$

$t = 0^-$



$v(t = 0^-) = V_0$

RC circuits (Forced response)

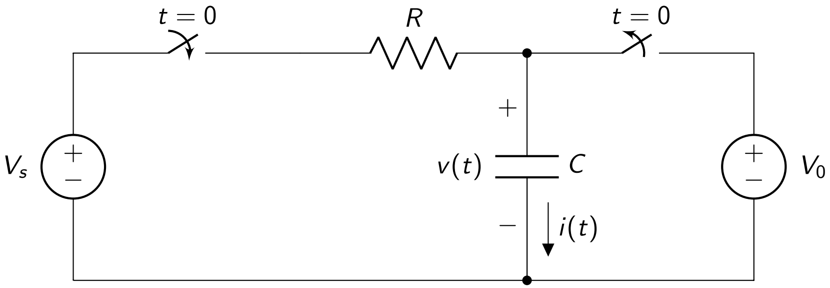


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Capacitor voltage $v(t = 0^-)$?
- Capacitor current $i(t = 0^-)$?
- Stored energy in the capacitor $w(t = 0^-)$?

RC circuits (Forced response)

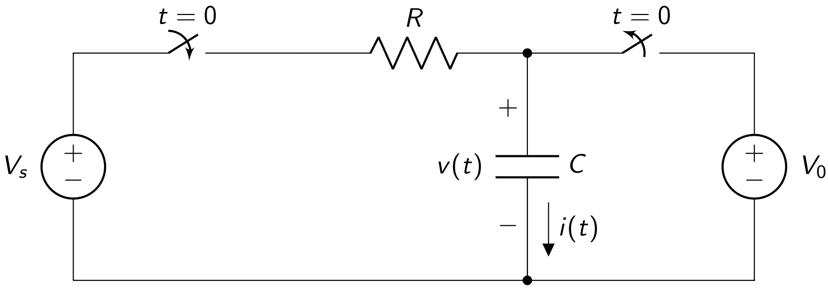


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Capacitor voltage $v(t = 0^-)$? $v(t = 0^-) = V_0$
- Capacitor current $i(t = 0^-)$?
- Stored energy in the capacitor $w(t = 0^-)$?

RC circuits (Forced response)

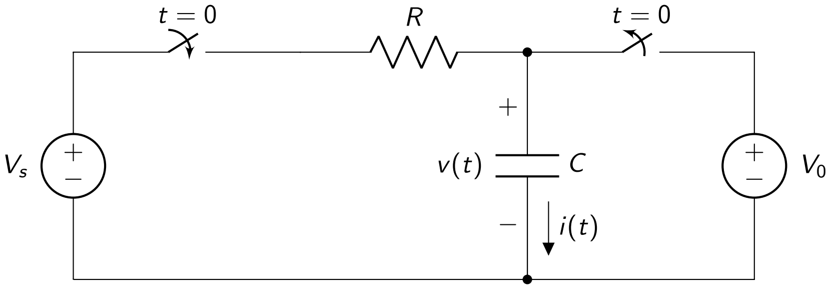


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Capacitor voltage $v(t = 0^-)$? $v(t = 0^-) = V_0$
- Capacitor current $i(t = 0^-)$? $i(t = 0^-) = 0$
- Stored energy in the capacitor $w(t = 0^-)$?

RC circuits (Forced response)

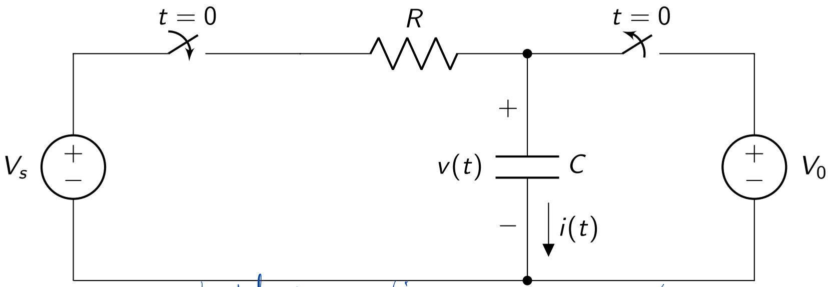


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Capacitor voltage $v(t=0^-)$? $v(t=0^-) = V_0$
- Capacitor current $i(t=0^-)$? $i(t=0^-) = 0$
- Stored energy in the capacitor $w(t=0^-)$? $w(t=0^-) = \frac{1}{2}C \cdot V_0^2$

RC circuits (Forced response)

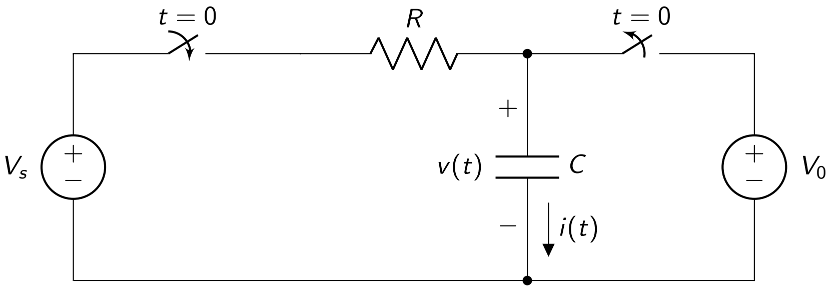


should be continuous all the time.

$t > 0$

- Capacitor voltage $v(t = 0^+)$?
- Capacitor current $i(t = 0^+)$?
- Stored energy in the capacitor $w(t = 0^+)$?

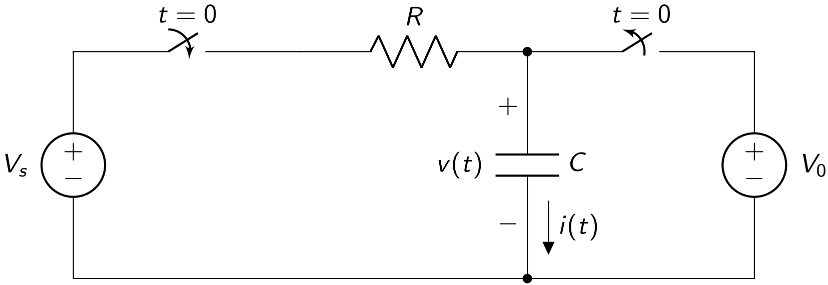
RC circuits (Forced response)



$t > 0$

- Capacitor voltage $v(t=0^+)$? $v(t=0^+) = V_0$
- Capacitor current $i(t=0^+)$? $-V_s + R\,i(t) + V_0 = 0$
- Stored energy in the capacitor $w(t=0^+)$? $i(t) = \frac{V_s - V_0}{R}$

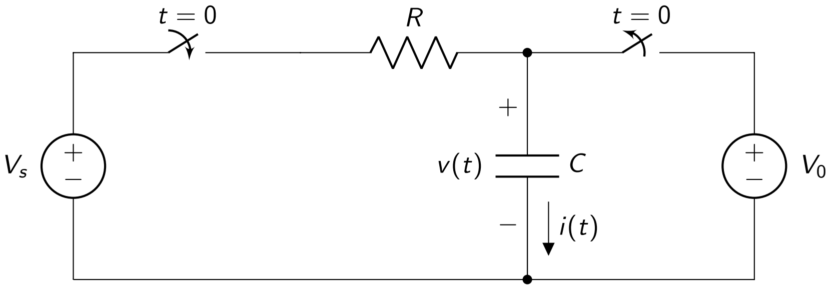
RC circuits (Forced response)



$t > 0$

- Capacitor voltage $v(t = 0^+)$? $v(t = 0^+) = V_0$
- Capacitor current $i(t = 0^+)$? $i(t = 0^+) = \frac{V_s - V_0}{R}$
- Stored energy in the capacitor $w(t = 0^+)$?

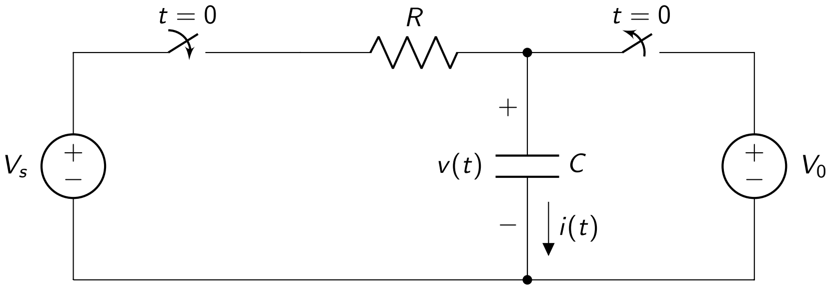
RC circuits (Forced response)



$t > 0$

- Capacitor voltage $v(t=0^+)$? $v(t=0^+) = V_0$
- Capacitor current $i(t=0^+)$? $i(t=0^+) = \frac{V_s - V_0}{R}$
- Stored energy in the capacitor $w(t=0^+)$? $w(t=0^+) = \frac{1}{2} C \cdot V_0^2$

RC circuits (Forced response)

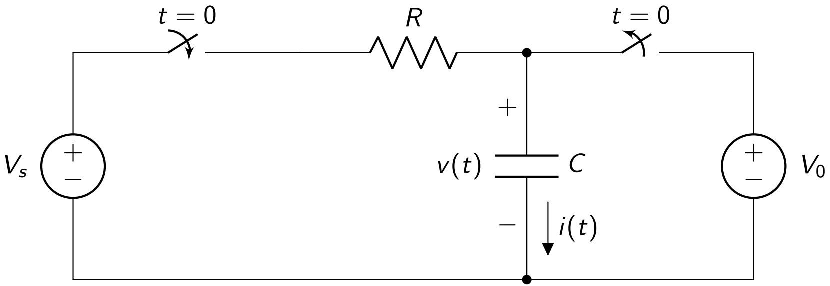


very stable ; capacitor as open circuit now

$t \to \infty$

- Capacitor voltage $v(t \to \infty)$? Vs
- Capacitor current $i(t \to \infty)$? 0
- Stored energy in the capacitor $w(t \to \infty)$?

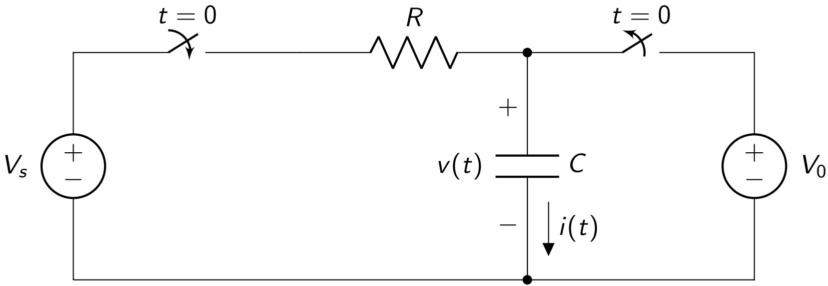
RC circuits (Forced response)



$t \to \infty$

- Capacitor voltage $v(t \to \infty)$? $v(t \to \infty) = V_s$
- Capacitor current $i(t \to \infty)$?
- Stored energy in the capacitor $w(t \to \infty)$?

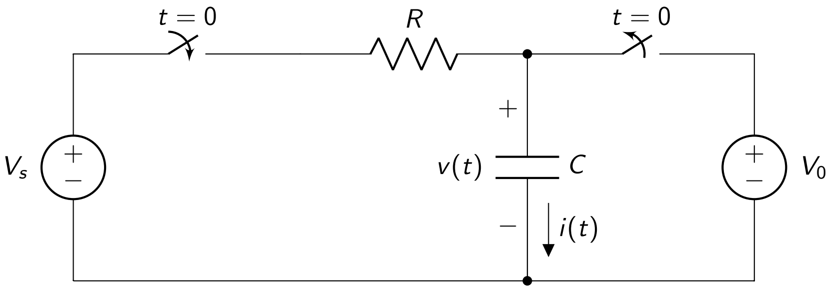
RC circuits (Forced response)



$t \to \infty$

- Capacitor voltage $v(t \to \infty)$? $v(t \to \infty) = V_s$
- Capacitor current $i(t \to \infty)$? $i(t \to \infty) = 0$
- Stored energy in the capacitor $w(t \to \infty)$?

RC circuits (Forced response)



$t \to \infty$

- Capacitor voltage $v(t \to \infty)$? $v(t \to \infty) = V_s$
- Capacitor current $i(t \to \infty)$? $i(t \to \infty) = 0$
- Stored energy in the capacitor $w(t \to \infty)$? $w(t \to \infty) = \frac{1}{2}C \cdot V_s^2$

RC circuits (Forced response)

natural response.

Determination of $v(t)$

For $t \geq 0$, we can write:

$$\frac{dv(t)}{dt} + \frac{V(t)}{\tau} = 0$$

- $i(t) = C\dfrac{dv(t)}{dt}$
- $V_s = R \cdot i(t) + v(t) \qquad \Longrightarrow \qquad \tau\dfrac{dv(t)}{dt} + v(t) = V_s$

Solution

Solution similar to natural response.

By setting $v_2(t) = v(t) - V_s$, we get: $\tau\dfrac{dv_2(t)}{dt} + v_2(t) = 0$

RC circuits (Forced response)

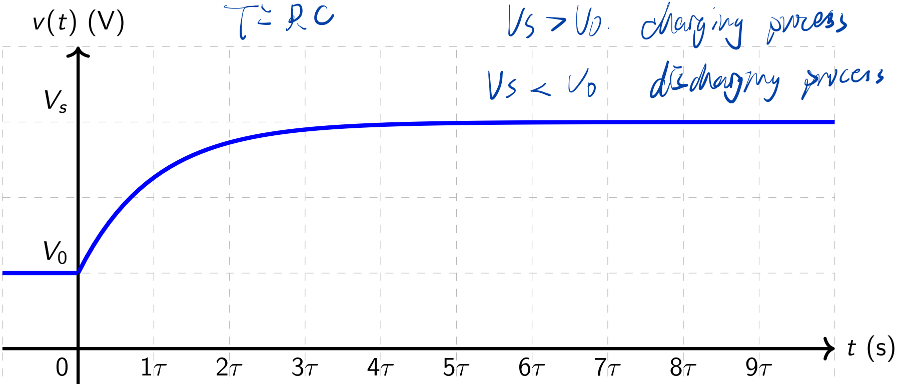
RC circuit forced response

$$v(t) = \begin{cases} V_0, & \text{if } t \leq 0 \\ (V_0 - V_s)e^{-\frac{t}{\tau}} + V_s, & \text{if } t \geq 0 \end{cases}$$

$\tau = RC$

$V_s > V_0$ charging process

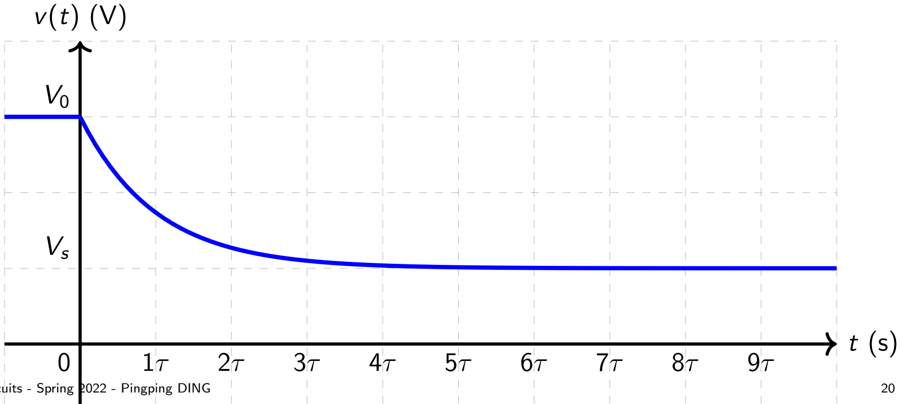
$V_s < V_0$ discharging process



RC circuits (Forced response)

RC circuit forced response

$$v(t) = \begin{cases} V_0, & \text{if } t \le 0 \\ (V_0 - V_s)e^{-\frac{t}{\tau}} + V_s, & \text{if } t \ge 0 \end{cases}$$



RC circuits

Natural Response $\begin{cases} V_0 & t \le 0 \\ V_0 e^{-\frac{t}{\tau}} & t > 0 \end{cases}$

Generic method

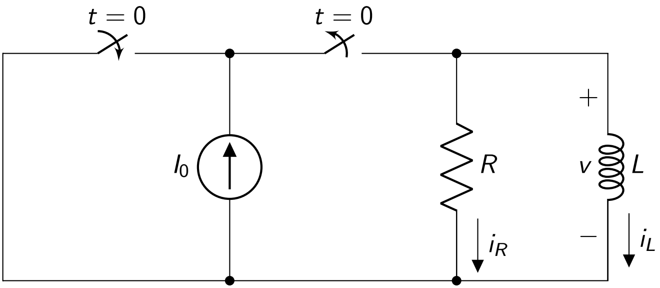
1. Zero-out all the independent sources, and determine R_{eq}, C_{eq} and the time constant $\tau = R_{eq} C_{eq}$
2. Determine the capacitor voltage $v(0^-)$ before the change ($t \le 0$) by DC analysis, meaning considering the capacitor is **equivalent to an open-circuit**
3. Find the steady-state capacitor voltage $v(t \to \infty)$ by using DC analysis, still considering the capacitor is **equivalent to an open-circuit**
4. The response (for $t \ge 0$) is then:
$v(t) = v(t \to \infty) + (v(0^-) - v(t \to \infty))e^{-t/\tau}$

Forced Response.

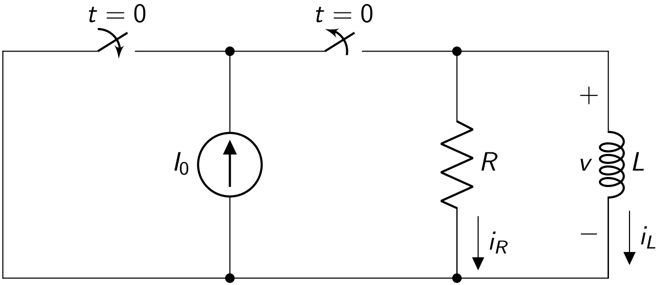
$$v(t) = \begin{cases} V_0 & t \le 0 \\ (V_0 - V_s) e^{-\frac{t}{\tau}} + V_s & t > 0 \end{cases}$$

RL circuits (Natural response)

Why chose voltage as the response?
voltage is the consequence. current is the premise.



RL circuits (Natural response)

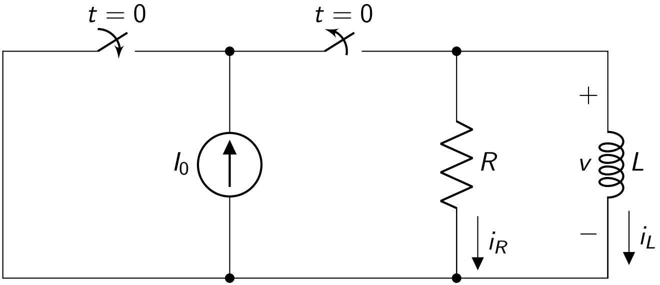


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i_L(t = 0^-)$?
- Inductor voltage $v(t = 0^-)$?
- Stored energy in the inductor $w(t = 0^-)$?

RL circuits (Natural response)

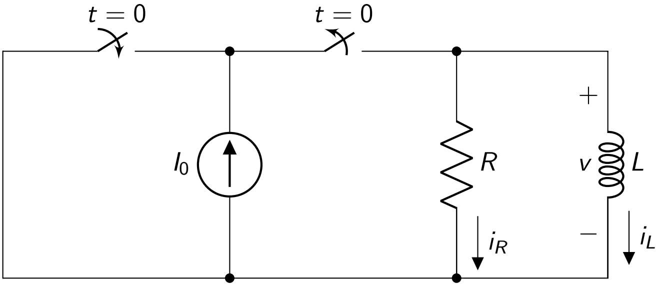


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i_L(t = 0^-)$? $i_L(t = 0^-) = I_0$
- Inductor voltage $v(t = 0^-)$?
- Stored energy in the inductor $w(t = 0^-)$?

RL circuits (Natural response)

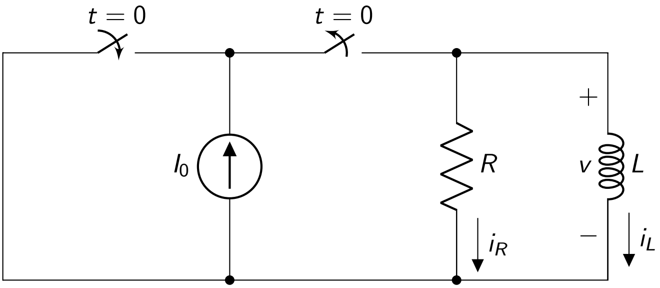


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i_L(t=0^-)$? $i_L(t=0^-) = I_0$
- Inductor voltage $v(t=0^-)$? $v(t=0^-) = 0$
- Stored energy in the inductor $w(t=0^-)$?

RL circuits (Natural response)

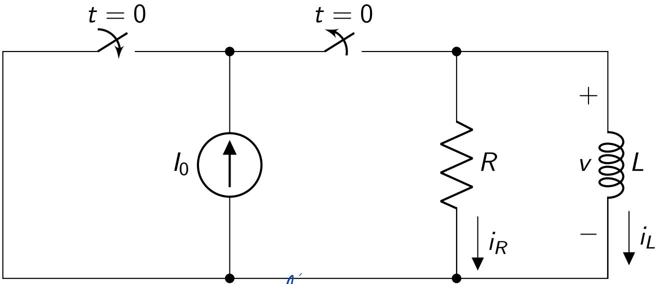


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i_L(t = 0^-)$? $i_L(t = 0^-) = I_0$
- Inductor voltage $v(t = 0^-)$? $v(t = 0^-) = 0$
- Stored energy in the inductor $w(t = 0^-)$? $w(t = 0^-) = \frac{1}{2} L \cdot I_0^2$

RL circuits (Natural response)

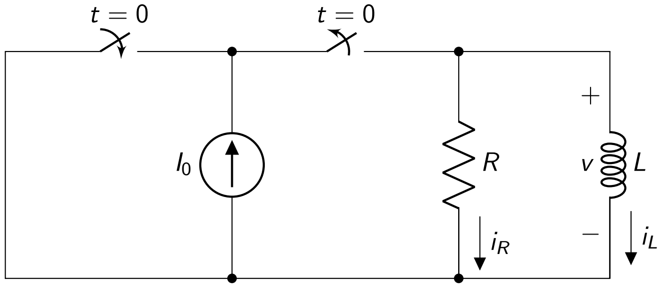


$V = L\frac{di}{dt}$. i is continuous

$t > 0$

- Inductor current $i_L(t = 0^+)$?
- Inductor voltage $v(t = 0^+)$?
- Stored energy in the inductor $w(t = 0^-)$?

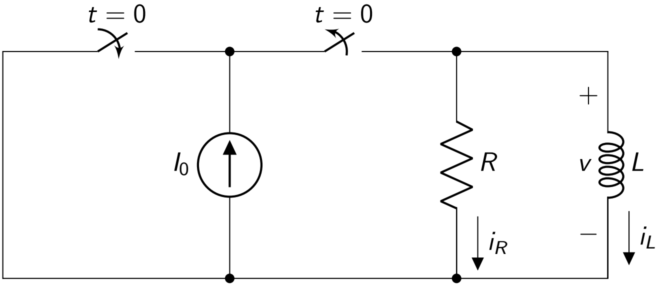
RL circuits (Natural response)



$t > 0$

- Inductor current $i_L(t = 0^+)$? $i_L(t = 0^+) = I_0$
- Inductor voltage $v(t = 0^+)$?
- Stored energy in the inductor $w(t = 0^-)$?

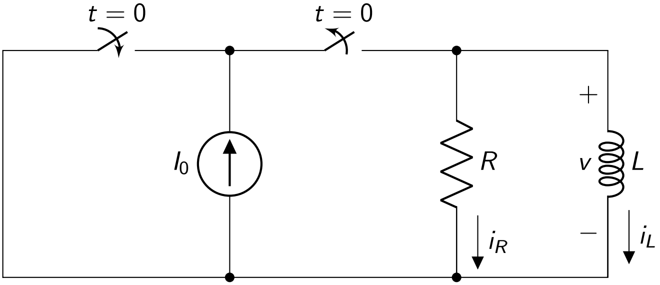
RL circuits (Natural response)



$t > 0$

- Inductor current $i_L(t = 0^+)$? $i_L(t = 0^+) = I_0$
- Inductor voltage $v(t = 0^+)$? $v(t = 0^+) = -RI_0$
- Stored energy in the inductor $w(t = 0^-)$?

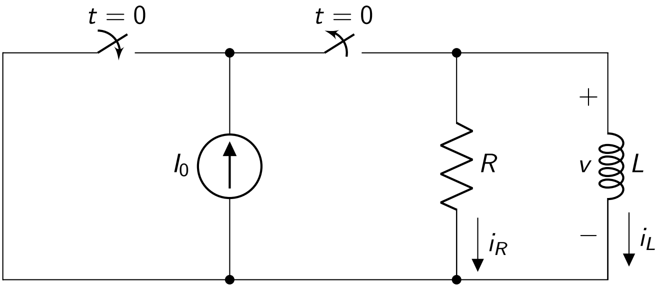
RL circuits (Natural response)



$t > 0$

- Inductor current $i_L(t=0^+)$? $i_L(t=0^+) = I_0$
- Inductor voltage $v(t=0^+)$? $v(t=0^+) = -RI_0$
- Stored energy in the inductor $w(t=0^-)$? $w(t=0^+) = \frac{1}{2}L \cdot I_0^2$

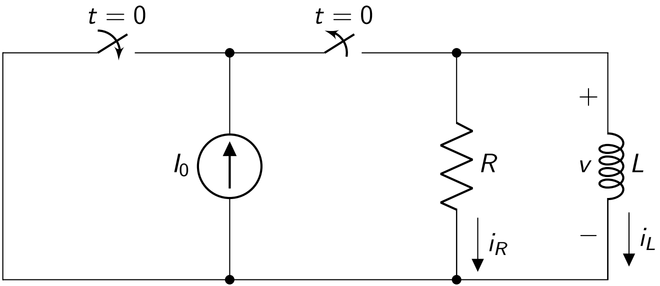
RL circuits (Natural response)



$t \to \infty$

- Inductor current $i_L(t \to \infty)$?
- Inductor voltage $v(t \to \infty)$?
- Stored energy in the inductor $w(t \to \infty)$?

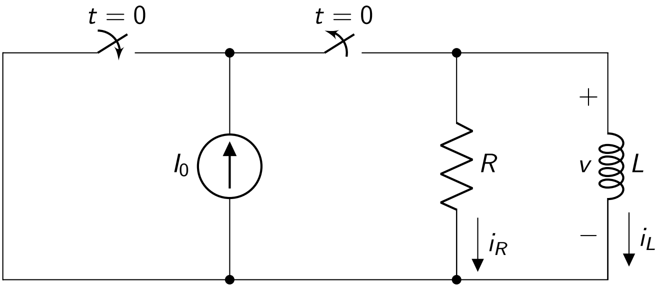
RL circuits (Natural response)



$t \to \infty$

- Inductor current $i_L(t \to \infty)$? $i_L(t \to \infty) = 0$
- Inductor voltage $v(t \to \infty)$?
- Stored energy in the inductor $w(t \to \infty)$?

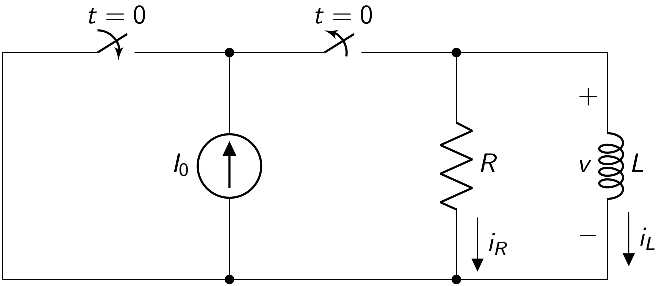
RL circuits (Natural response)



$t \to \infty$

- Inductor current $i_L(t \to \infty)$? $i_L(t \to \infty) = 0$
- Inductor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Stored energy in the inductor $w(t \to \infty)$?

RL circuits (Natural response)



$t \to \infty$

- Inductor current $i_L(t \to \infty)$? $i_L(t \to \infty) = 0$
- Inductor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Stored energy in the inductor $w(t \to \infty)$? $w(t \to \infty) = 0$

RL circuits (Natural response)

Determination of $i_L(t)$

For $t \geq 0$, we can write:

- $i_L(t) + i_R(t) = 0$
- $v(t) = L\dfrac{di_L(t)}{dt} = Ri_R(t)$
- $L\dfrac{di_L(t)}{dt} + Ri_L(t) = 0$
- $\tau\dfrac{di_L(t)}{dt} + i_L(t) = 0$

My derivation:

$\tau \frac{di}{dt} + i = 0$

$\int_{i_0}^{i(t')} \frac{di}{-i} = \int_0^{t'} \frac{1}{\tau} dt$

$\ln \frac{i(t')}{I_0} = -\frac{t'}{\tau}$

$\frac{i(t')}{I_0} = e^{-\frac{t'}{\tau}}$

Time constant τ

The unit of the ratio $\frac{L}{R}$ is proportional to **time** (usually in s, or ms, or µs. . .).

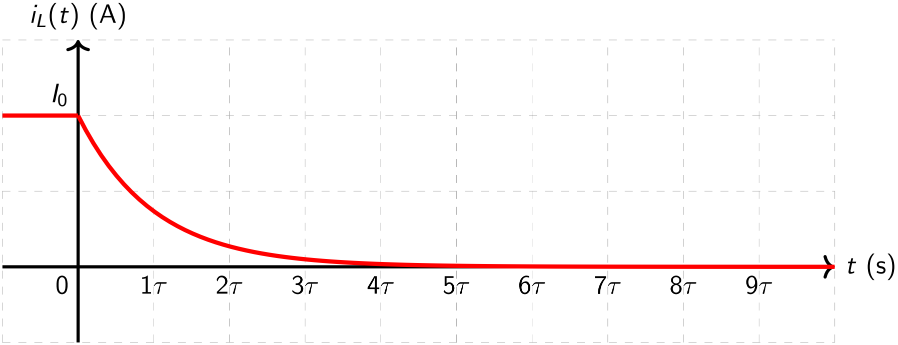
$\tau = \frac{L}{R}$ $\tau = RC$

This ratio $\frac{L}{R}$ is called the **time constant** τ with $\tau = \frac{L}{R}$.

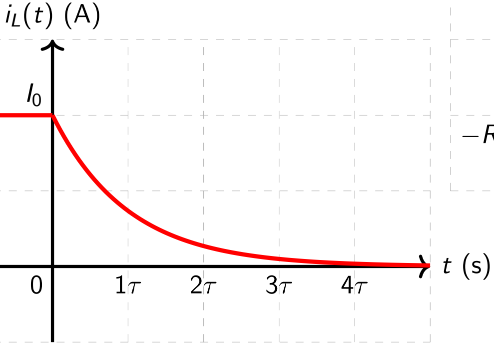
RL circuits (Natural response)

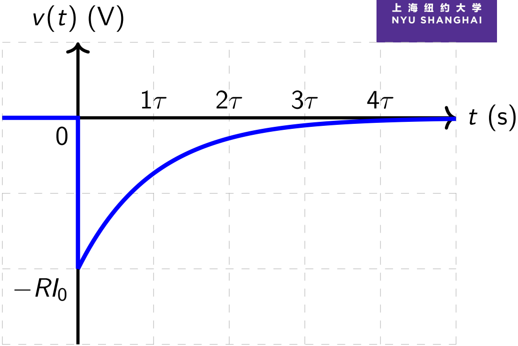
RL circuit natural response

$$i_L(t) = \begin{cases} I_0, & \text{if } t \le 0 \\ I_0 e^{-\frac{t}{\tau}}, & \text{if } t \ge 0 \end{cases}$$

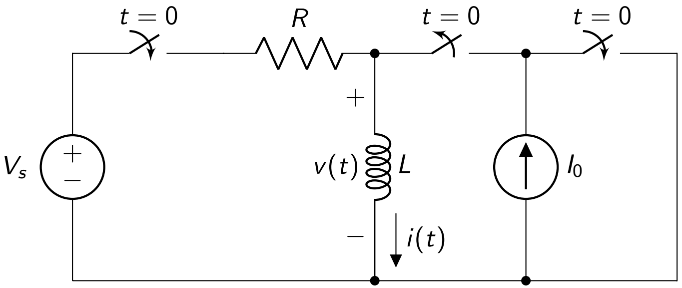


RL circuits (Natural response)

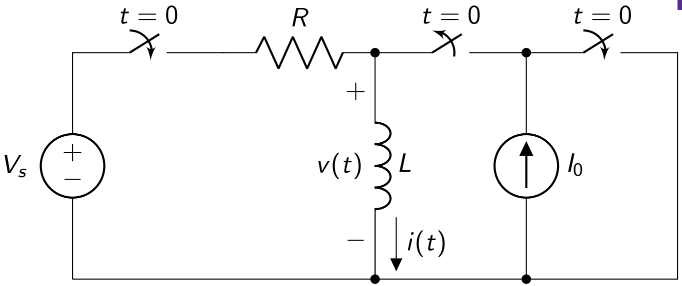




RL circuits (Forced response)



RL circuits (Forced response)

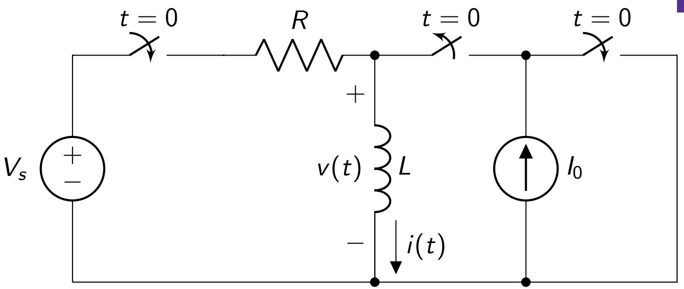


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i(t = 0^-)$?
- Inductor voltage $v(t = 0^-)$?
- Stored energy in the inductor $w(t = 0^-)$?

RL circuits (Forced response)

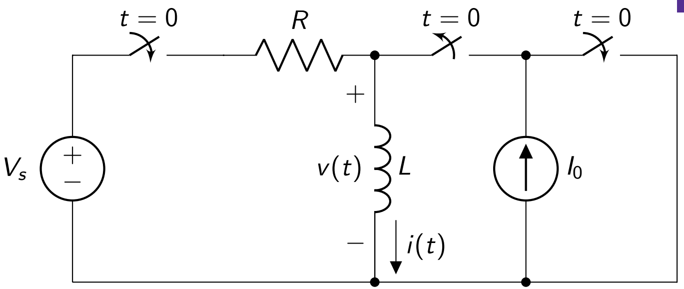


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i(t = 0^-)$? $i(t = 0^-) = I_0$
- Inductor voltage $v(t = 0^-)$?
- Stored energy in the inductor $w(t = 0^-)$?

RL circuits (Forced response)

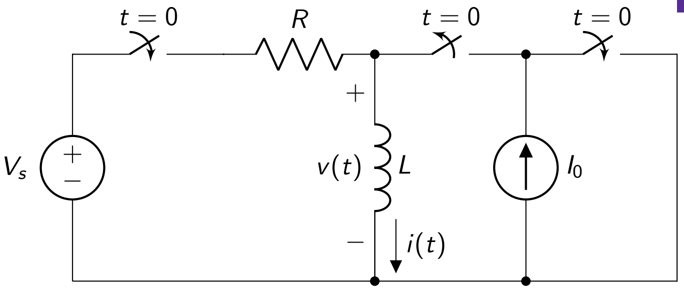


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i(t = 0^-)$? $i(t = 0^-) = I_0$
- Inductor voltage $v(t = 0^-)$? $v(t = 0^-) = 0$
- Stored energy in the inductor $w(t = 0^-)$?

RL circuits (Forced response)

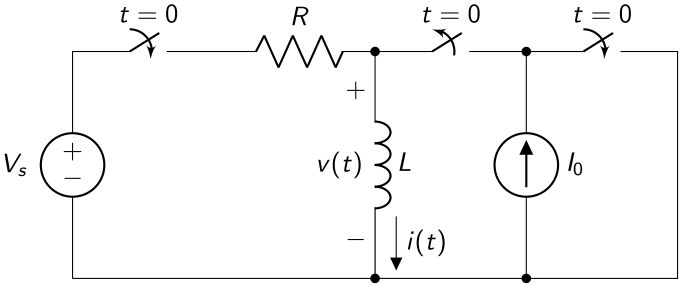


$t < 0$

For $t < 0$, we suppose that the circuit was in that state for a *long* time

- Inductor current $i(t = 0^-)$? $i(t = 0^-) = I_0$
- Inductor voltage $v(t = 0^-)$? $v(t = 0^-) = 0$
- Stored energy in the inductor $w(t = 0^-)$? $w(t = 0^-) = \frac{1}{2} L \cdot I_0^2$

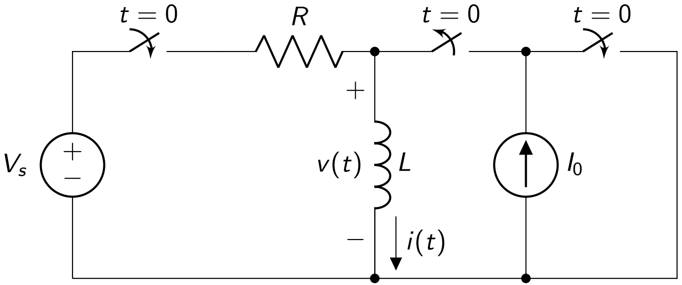
RL circuits (Forced response)



$t > 0$

- Inductor current $i(t = 0^+)$?
- Inductor voltage $v(t = 0^+)$?
- Stored energy in the inductor $w(t = 0^+)$?

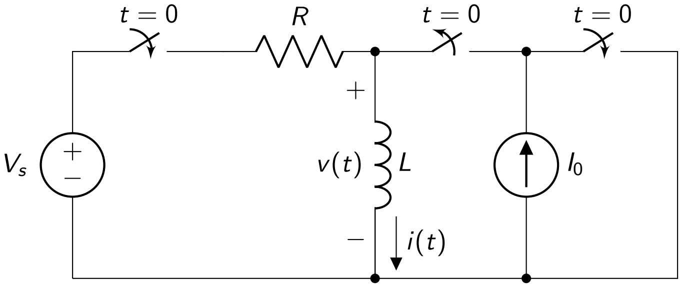
RL circuits (Forced response)



$t>0$

- Inductor current $i(t=0^+)$? $i(t=0^+)=I_0$
- Inductor voltage $v(t=0^+)$?
- Stored energy in the inductor $w(t=0^+)$?

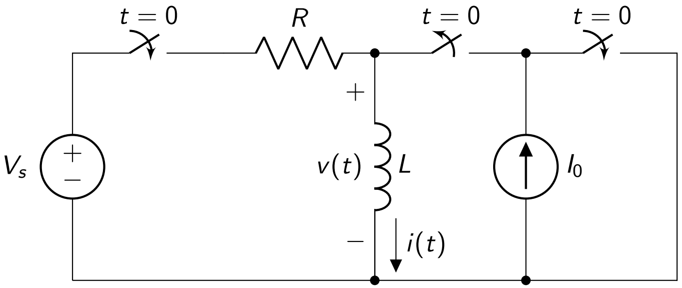
RL circuits (Forced response)



$t > 0$

- Inductor current $i(t=0^+)$? $i(t=0^+) = I_0$
- Inductor voltage $v(t=0^+)$? $v(t=0^+) = V_s - RI_0$
- Stored energy in the inductor $w(t=0^+)$?

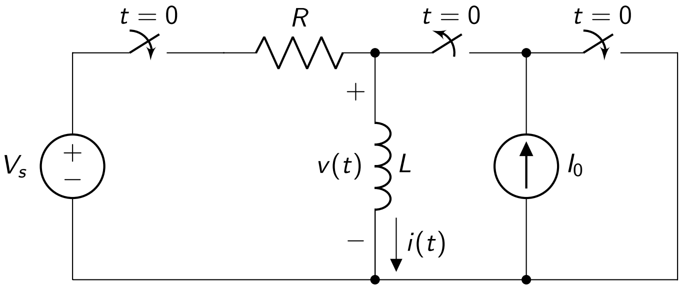
RL circuits (Forced response)



$t > 0$

- Inductor current $i(t = 0^+)$? $i(t = 0^+) = I_0$
- Inductor voltage $v(t = 0^+)$? $v(t = 0^+) = V_s - RI_0$
- Stored energy in the inductor $w(t = 0^+)$? $w(t = 0^+) = \frac{1}{2} L \cdot I_0^2$

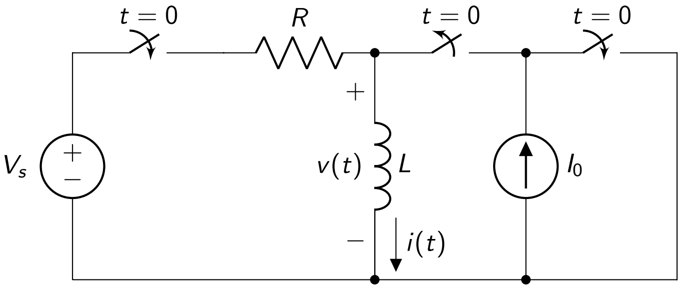
RL circuits (Forced response)



$t \to \infty$

- Inductor current $i(t \to \infty)$?
- Inductor voltage $v(t \to \infty)$?
- Stored energy in the inductor $w(t \to \infty)$?

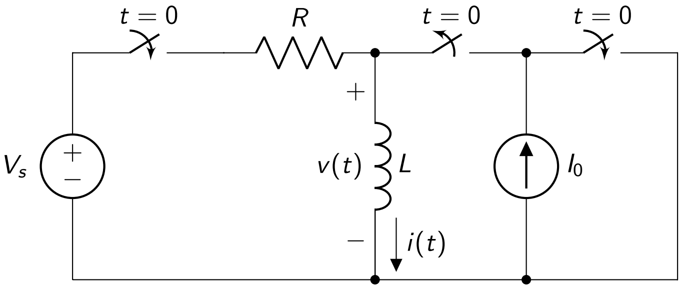
RL circuits (Forced response)



$t \to \infty$

- Inductor current $i(t \to \infty)$? $i(t \to \infty) = \frac{V_s}{R}$
- Inductor voltage $v(t \to \infty)$?
- Stored energy in the inductor $w(t \to \infty)$?

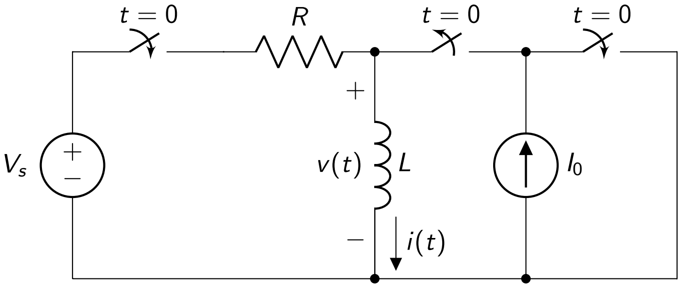
RL circuits (Forced response)



$t \to \infty$

- Inductor current $i(t \to \infty)$? $i(t \to \infty) = \frac{V_s}{R}$
- Inductor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Stored energy in the inductor $w(t \to \infty)$?

RL circuits (Forced response)



$t \to \infty$

- Inductor current $i(t \to \infty)$? $i(t \to \infty) = \frac{V_s}{R}$
- Inductor voltage $v(t \to \infty)$? $v(t \to \infty) = 0$
- Stored energy in the inductor $w(t \to \infty)$? $w(t \to \infty) = \frac{1}{2}\frac{L}{R^2}{V_s}^2$

RL circuits (Forced response)

Determination of $i(t)$

For $t \geq 0$, we can write:

- $v(t) = L\dfrac{di(t)}{dt}$
- $V_s = R \cdot i(t) + v(t) \qquad \Longrightarrow \qquad \tau\dfrac{di(t)}{dt} + i(t) = \dfrac{V_s}{R}$

Solution

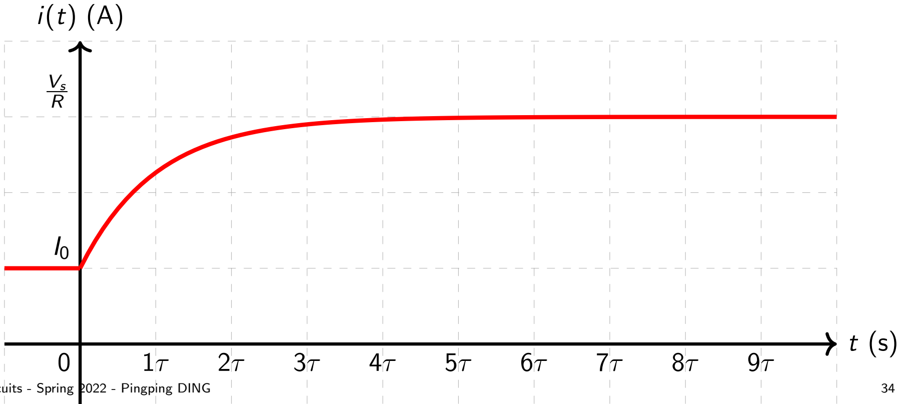
Solution similar to natural response.

By setting $i_2(t) = i(t) - \dfrac{V_s}{R}$, we get: $\tau\dfrac{di_2(t)}{dt} + i_2(t) = 0$

RL circuits (Forced response)

RL circuit forced response

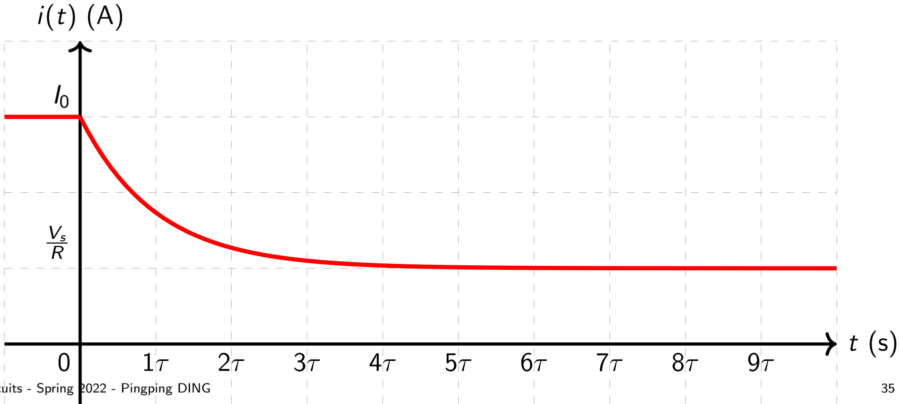
$$i(t) = \begin{cases} I_0, & \text{if } t \leq 0 \\ (I_0 - \frac{V_s}{R})e^{-\frac{t}{\tau}} + \frac{V_s}{R}, & \text{if } t \geq 0 \end{cases}$$



RL circuits (Forced response)

RL circuit forced response

$$v(t) = \begin{cases} V_0, & \text{if } t \leq 0 \\ (V_0 - V_s)e^{-\frac{t}{\tau}} + V_s, & \text{if } t \geq 0 \end{cases}$$



RL circuits

Generic method

1. Zero-out all the independent sources, and determine R_{eq}, L_{eq} and the time constant $\tau = \frac{L_{eq}}{R_{eq}}$
2. Determine the inductor current $i(0^-)$ before the change $(t \leq 0)$ by DC analysis, meaning considering the inductor is **equivalent to a short-circuit**
3. Find the steady-state inductor current $i(t \to \infty)$ by using DC analysis, still considering the inductor is **equivalent to a short-circuit**
4. The response (for $t \geq 0$) is then:
$i(t) = i(t \to \infty) + (i(0^-) - i(t \to \infty))e^{-t/\tau}$