26

DIRECT-CURRENT CIRCUITS

VP26.2.1. **IDENTIFY:** We have resistors in series and parallel.

SET UP: Parallel: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$, series: $R_{eq} = R_1 + R_2 + \cdots$. We want the equivalent resistance in each combination.

EXECUTE: **(a)** Series: $R_{eq} = 1.00\ \Omega + 2.00\ \Omega + 4.00\ \Omega = 7.00\ \Omega$.

(b) Parallel: $1/R_{eq} = 1/(1.00\ \Omega) + 1/(2.00\ \Omega) + 1/(4.00\ \Omega)$. $R_{eq} = 0.571\ \Omega$.

(c) Series/parallel combination: R_2 and R_3 in parallel. $1/R_p = 1/(2.00\ \Omega) + 1/(4.00\ \Omega)$. $R_p = 1.33\ \Omega$. $R_{eq} = R_1 + R_p = 1.00\ \Omega + 1.33\ \Omega = 2.33\ \Omega$.

(d) Series/parallel combination: R_2 and R_3 in series: $R_s = 2.00\ \Omega + 4.00\ \Omega = 6.00\ \Omega$. The parallel combination: $1/R_{eq} = 1/R_1 + 1/R_s = 1/(1.00\ \Omega) + 1/(6.00\ \Omega)$. $R_{eq} = 0.857\ \Omega$.

EVALUATE: Note that for a parallel combination the equivalent resistance is *less than* the *smallest* resistance. For a series combination, R_{eq} is *greater than* the *largest* resistance.

VP26.2.2. **IDENTIFY:** We have resistors in series and parallel and a battery.

SET UP: Parallel: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$, series: $R_{eq} = R_1 + R_2 + \cdots$. We want the currents.

EXECUTE: **(a)** We want the current through the battery. First find R_{eq} for the circuit. R_2 and R_3 are in series, so $R_s = 11.0\ \Omega$. This combination is in parallel with R_1 so $1/R_{eq} = 1/R_1 + 1/R_s$. This gives $1/R_{eq} = 1/(4.00\ \Omega) + 1/(11.0\ \Omega)$. $R_{eq} = 2.933\ \Omega$. = (24.0 V)/(2.933 Ω) = 8.18 A.

(b) We want the current through R_1. $I_1 = \mathcal{E}/R_1$ = (24.0 V)/(4.00 Ω) = 6.00 A.

(c) We want the current through R_2. $I_2 = \mathcal{E}/(R_2 + R_3)$ = (24.0 V)/(11.0 Ω) = 2.18 A.

(d) We want the current through R_3. The resistors are in series so $I_3 = I_2 = 2.18$ A.

EVALUATE: Check: $I_1 + I_2 = 6.00\text{ A} + 2.18\text{ A} = 8.18$ A, which is the current from the battery as it should be.

VP26.2.3. **IDENTIFY:** We have three resistors in parallel across a battery. We want the power in each circuit element.

SET UP: $P = I^2R = V^2/R = IV$. $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$ The potential difference is 12.0 V across each resistor.

EXECUTE: **(a)** We want $P_{battery}$. First get the equivalent resistance to find the current the battery puts out. $1/R_{eq} = 1/(7.00\ \Omega) + 1/(8.00\ \Omega) + 1/(9.00\ \Omega)$. $R_{eq} = 2.64\ \Omega$. Now find I. $I = \mathcal{E}/R_{eq}$ = (12.0 V)/(2.64 Ω) = 4.548 A. $P_{battery} = I\mathcal{E}$ = (4.548 A)(12.0 V) = 54.6 W.

(b) We want the power dissipated in R_1. $P_1 = \mathcal{E}^2/R_1 = (12.0\text{ V})^2/(7.00\ \Omega) = 20.6$ W.

(c) $P_2 = (24.0\text{ V})^2/(8.00\ \Omega) = 18.0$ W.

(d) $P_3 = (12.0\text{ V})^2/(9.00\ \Omega) = 16.0$ W.

EVALUATE: The total power dissipated in the resistors is 20.6 W + 18.0 W + 16.0 W = 54.6 W, which is the power output of the battery. This agrees with energy conservation.

VP26.2.4. **IDENTIFY:** We have a series/parallel resistor combination across a battery. We want the power in each circuit element.

SET UP: $P = I^2R = V^2/R = IV$. $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \cdots$. $V = RI$.

EXECUTE: **(a)** We want P_{battery}. First get the equivalent resistance of the circuit. For the parallel part: $1/R_p = 1/(6.00\ \Omega) + 1/(7.00\ \Omega)$. $R_p = 3.231\ \Omega$. $R_{\text{eq}} = 5.00\ \Omega + 3.231\ \Omega = 8.23\ \Omega$. $I = \mathcal{E}/R_{\text{eq}} = (9.00\ \text{V})/(8.23\ \Omega) = 1.0935\ \text{A}$. $P_{\text{battery}} = I\mathcal{E} = (1.0935\ \text{A})(9.00\ \text{V}) = 9.84\ \text{W}$.

(b) We want P_1. $P_1 = I_1^2R_1 = I^2R_1 = (1.0935\ \text{A})^2(5.00\ \Omega) = 5.98\ \text{W}$.

(c) We want P_2. $V_2 = \mathcal{E} - V_1 = \mathcal{E} - R_1I_1 = 9.00\ \text{V} - (5.00\ \Omega)(1.0935\ \text{A}) = 3.5325\ \text{V}$. $P_2 = V_2^2/R_2 = (3.5325\ \text{V})^2/(6.00\ \Omega) = 2.08\ \text{W}$.

(d) $P_3 = (3.5325\ \text{V})^2/(7.00\ \Omega) = 1.78\ \text{W}$.

EVALUATE: Check: $P_1 + P_2 + P_3 = 5.98\ \text{W} + 2.08\ \text{W} + 1.78\ \text{W} = 9.84\ \text{W}$, which is the battery power, as it should be by energy conservation.

VP26.7.1. **IDENTIFY:** We have a circuit with two batteries. Kirchhoff's rules apply.

SET UP: $P = IV = I^2R$.

EXECUTE: **(a)** We want the current. Make a loop using the same path as in Fig. 26.10(a) in the text. This gives: $12\ \text{V} - (2\ \Omega)I - (3\ \Omega)I - (4\ \Omega)I + 4\ \text{V} - (7\ \Omega)I = 0$. $I = 1\ \text{A}$.

(b) $V_{ab} = V_a - V_b = (7\ \Omega)(1\ \text{A}) - 4\ \text{V} + (4\ \Omega)(1\ \text{A}) = +7\ \text{V}$.

(c) We want the power. $P_4 = I\mathcal{E}_4 = (1\ \text{A})(4\ \text{V}) = 4\ \text{W}$. $P_{12} = (1\ \text{A})(12\ \text{V}) = 12\ \text{W}$.

EVALUATE: Check: Power dissipated in the resistors is $I^2R = (1\ \text{A})^2(16\ \Omega) = 16\ \text{W}$. The power output of the batteries is 4W + 12 W = 16 W. They agree, as they should by energy conservation.

VP26.7.2. **IDENTIFY:** This problem requires Kirchhoff's rules.

SET UP: Refer to Fig. 26.6(a) in the text.

EXECUTE: **(a)** We want $V_{ab} = V_a - V_b$. $I_R = I_1 + I_2 = 1.55\ \text{A}$. Go from b to a through R. This gives $V_{ab} = (5.00\ \Omega)(1.55\ \text{A}) = +7.75\ \text{V}$.

(b) We want r_1. $V_{ac} = V_{ab} = 7.75\ \text{V}$. Go from $c \to a$: $8.00\ \text{V} - r_1(0.200\ \text{A}) = 7.75\ \text{V}$. $r_1 = 1.25\ \Omega$.

(c) We want r_2. $V_{ab} = 9.00\ \text{V} - r_2(1.35\ \text{A}) = 7.75\ \text{V}$. $r_2 = 0.926\ \Omega$.

EVALUATE: Check for r_1: Make a loop from $c \to a \to b \to c$ through R, giving $+8.00\ \text{V} - r_1(0.200\ \text{A}) - (5.00\ \Omega)(1.55\ \text{A}) = 0$, which gives $r_1 = 1.25\ \Omega$, in agreement with our result.

VP26.7.3. **IDENTIFY:** We need to use Kirchhoff's rules and want to find the power in each resistor.

SET UP: Refer to Fig. 26.12 in the text and the equations in Example 26.6. We first must find the currents, as in the example in the text. Using the same loops, we find that Equations (1), (3), and $(1')$ are the same as in the example. Equation (2) becomes: $-I_2(1\ \Omega) - (I_2 + I_3)(3\ \Omega) + 13\ \text{V} = 0$. Equation $(2')$ becomes: $13\ \text{V} = I_1(4\ \Omega) + I_3(7\ \Omega)$. Solve these equations as was shown in the example. The results are: $I_1 = 5.778\ \text{A}$, $I_2 = 4.333\ \text{A}$, and $I_3 = -1.445\ \text{A}$. Now we can find the powers.

EXECUTE: **(a)** $P_{ca} = I_1^2(1\ \Omega) = (5.778\ \text{A})^2(1\ \Omega) = 33.4\ \text{W}$.

(b) $P_{cb} = I_2^2(1\ \Omega) = (4.333\ \text{A})^2(1\ \Omega) = 18.8\ \text{W}$.

(c) $P_{ab} = I_3^2(1\ \Omega) = (-1.445\ \text{A})^2(1\ \Omega) = 2.09\ \text{W}$.

(d) $P_{ad} = (I_1 - I_3)^2(1\ \Omega) = [5.778\ \text{A} - (-1.445\ \text{A})](1\ \Omega) = 2.09\ \text{W}$.

(e) $P_{bd} = (I_2 + I_3)^2(3\ \Omega) = (4.333\ \text{A} - 1.445\ \text{A})^2(3\ \Omega) = 25.0\ \text{W}$.

EVALUATE: Check: $P_R = 33.4\ \text{W} + 18.8\ \text{W} + 2.09\ \text{W} + 52.2\ \text{W} + 25.0\ \text{W} = 131\ \text{W}$. The power the battery produces is $P_{\text{battery}} = I\mathcal{E} = (I_1 + I_2)\mathcal{E} = (5.778\ \text{A} + 4.333\ \text{A})(13\ \text{V}) = 131\ \text{W}$. Our results are consistent with the conservation of energy.

VP26.7.4. **IDENTIFY:** Kirchhoff's rules apply to this circuit. We want the currents.

SET UP: Refer to Fig. VP26.7.4. Use the same approach as shown in Fig. 26.9(a) in the text.

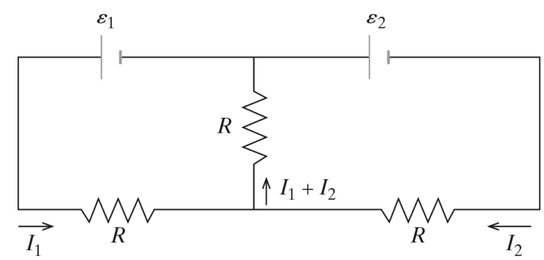


Figure VP26.7.4

EXECUTE: Take a counterclockwise loop in the left half of the circuit, which gives $\mathcal{E}_1 - RI_1 - (I_1 + I_2)R = 0$. This simplifies to s $\mathcal{E}_1 = 2RI_1 + RI_2$ (Eq. 1)

Take a counterclockwise loop in the right half of the circuit, which gives $\mathcal{E}_2 + R(I_1 + I_2) + I_2R = 0$. This simplifies to $\mathcal{E}_2 = -RI_1 - 2RI_2$ (Eq. 2)

The junction rule gives $I_3 = I_1 + I_2$ (Eq. 3)

Solving these three equations by substitution (or any other method) gives the following results:

(a) $I_1 = \dfrac{2\mathcal{E}_1 + \mathcal{E}_2}{3R}$.

(b) $I_2 = -\dfrac{\mathcal{E}_1 + 2\mathcal{E}_2}{3R}$.

(c) $I_3 = -\dfrac{\mathcal{E}_1 - \mathcal{E}_2}{3R}$.

EVALUATE: It is not possible to solve a circuit like this with simple series/parallel reduction.

VP26.13.1. **IDENTIFY:** This circuit is a charging capacitor in an *R-C* circuit.

SET UP: $q = Q_f(1 - e^{-t/RC})$, $i = I_0 e^{-t/RC}$, $\tau = RC$.

EXECUTE: **(a)** We want the final charge. The current has stopped, so $\mathcal{E} = V_C = Q_f/C$. Therefore $Q_f = C\mathcal{E} = (3.20\ \mu F)(9.00\ V) = 28.8\ \mu C$.

(b) We want the initial current. Initially $q_C = 0$, so $\mathcal{E} = V_R = RI_0$. $I_0 = \mathcal{E}/R = (9.00\ V)/(10.0\ M\Omega) = 0.900\ \mu A$.

(c) We want the time constant. $\tau = RC = (10.0\ M\Omega)(3.20\ \mu F) = 32.0\ s$.

(d) We want q/Q_f at $t = 18.0$ s. Using $q = Q_f(1 - e^{-t/RC})$ gives $q/Q_f = 1 - e^{-(18.0\ s)/(32.0\ s)} = 0.430$.

(e) We want i/I_0 at 18.0 s. Using $i = I_0 e^{-t/RC}$ gives $i/I_0 = e^{-(18.0\ s)/(32.0\ s)} = 0.570$.

EVALUATE: As the current in the circuit decreases the charge on the capacitor increases. Both of them follow a form of exponential change.

VP26.13.2. **IDENTIFY:** This circuit is a discharging capacitor in an *R-C* circuit.

SET UP: $i = I_0 e^{-t/RC}$, $q = Q_0 e^{-t/RC}$

EXECUTE: **(a)** We want the time when $q = 1.20\ \mu C$. Solve $q = Q_0 e^{-t/RC}$ for t by taking logarithms, giving $t = -RC \ln(q/Q_0) = -(4.00\ M\Omega)(2.20\ \mu C) \ln(1.20/4.20) = 11.0\ s$.

(b) We want the current at $t = 11.0$ s. $i = I_0 e^{-t/RC}$. From (a), $e^{-t/RC} = q/Q_0 = 1.20/4.20 = 0.2857$. Find I_0:

$$I_0 = \frac{V_0}{R} = \frac{Q_0/C}{R} = \frac{4.20\ \mu C}{(4.00\ M\Omega)(2.20\ \mu F)} = 4.77\times10^{-7}\ A.\ \ i = (4.77\times10^{-7}\ A)(0.2857) = 1.36\times10^{-7}\ A.$$

EVALUATE: For a discharging capacitor, both the current and the charge on the capacitor plates decrease exponentially.

VP26.13.3. **IDENTIFY:** We have a discharging capacitor.

SET UP: $i = I_0 e^{-t/RC}$, $q = Q_0 e^{-t/RC}$

EXECUTE: **(a)** We want the resistance. Solving $q = Q_0 e^{-t/RC}$ for R gives

$$R = -\frac{t}{C\ln(q/Q_0)} = -\frac{17.0\text{ s}}{(8.00\ \mu\text{F})\ln(1.10/5.50)} = 1.32\text{ M}\Omega.$$

(b) We want I_0. $I_0 = \frac{V_0}{R} = \frac{Q_0/C}{R} = \frac{5.50\ \mu\text{C}}{(1.32\text{ M}\Omega)(8.00\ \mu F)} = 5.21\times10^{-7}\text{ A}.$

(c) We want the current at $t = 17.0$ s. $i = I_0 e^{-t/RC} = 1.04\times10^{-7}\text{ A}$ using the given and calculated values for t, R, C and I_0.

EVALUATE: At $t = 17.0$ s, check V_C and V_R. $V_C = q/C = (1.10\ \mu\text{C})/(8.00\ \mu\text{F}) = 0.1375$ V. $V_R = Ri = (1.32\text{ M}\Omega)(1.04\times10^{-7}\text{ A}) = 0.1374$ V. They agree, as they should. The tiny difference is due to rounding.

VP26.13.4. **IDENTIFY:** This problem involves a capacitor that charges and then discharges.

SET UP and **EXECUTE:** **(a)** We want the charge at 51.0 s. We have a charging capacitor so $q = C\mathcal{E}(1 - e^{-t/RC})$. Using $t = 51.0$ s, $R = 5.00$ MΩ, and the other given values, we get $q = 61.2\ \mu$C.

(b) We want q 70.0 s after closing the switch. We have a discharging capacitor with $Q_0 = 61.2\ \mu$C. Using $q = Q_0 e^{-t/RC}$ with $t = 70.0$ s, $R = 6.00$ MΩ and the other given values, we get $q = 9.89\ \mu$C.

EVALUATE: The charging time constant is different from the discharging time constant because the resistance is different in the two cases.

26.1. **IDENTIFY:** The newly-formed wire is a combination of series and parallel resistors.

SET UP: Each of the three linear segments has resistance $R/3$. The circle is two $R/6$ resistors in parallel.

EXECUTE: The resistance of the circle is $R/12$ since it consists of two $R/6$ resistors in parallel. The equivalent resistance is two $R/3$ resistors in series with an $R/12$ resistor, giving $R_{\text{equiv}} = R/3 + R/3 + R/12 = 3R/4.$

EVALUATE: The equivalent resistance of the original wire has been reduced because the circle's resistance is less than it was as a linear wire.

26.2. **IDENTIFY:** It may appear that the meter measures X directly. But note that X is in parallel with three other resistors, so the meter measures the equivalent parallel resistance between ab.

SET UP: We use the formula for resistors in parallel.

EXECUTE: $1/(2.00\ \Omega) = 1/X + 1/(15.0\ \Omega) + 1/(5.0\ \Omega) + 1/(10.0\ \Omega)$, so $X = 7.5\ \Omega$.

EVALUATE: X is *greater* than the equivalent parallel resistance of $2.00\ \Omega$.

26.3. **IDENTIFY:** The emf of the battery remains constant, but changing the resistance across it changes its power output.

SET UP: The power consumption in a resistor is $P = \frac{V^2}{R}$.

EXECUTE: With just R_1, $P_1 = \frac{V^2}{R_1}$ and $V = \sqrt{P_1 R_1} = \sqrt{(36.0\text{ W})(25.0\ \Omega)} = 30.0\text{ V}$ is the battery voltage. With R_2 added, $R_{\text{tot}} = 40.0\ \Omega$. $P = \frac{V^2}{R_{\text{tot}}} = \frac{(30.0\text{ V})^2}{40.0\ \Omega} = 22.5\text{ W}.$

EVALUATE: The two resistors in series dissipate electrical energy at a smaller rate than R_1 alone.

26.4. **IDENTIFY:** For resistors in parallel the voltages are the same and equal to the voltage across the equivalent resistance.

SET UP: $V = IR$. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

EXECUTE: **(a)** $R_{\text{eq}} = \left(\dfrac{1}{42\ \Omega} + \dfrac{1}{20\ \Omega}\right)^{-1} = 13.548\ \Omega$, which rounds to 13 Ω.

(b) $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{240\ \text{V}}{13.548\ \Omega} = 17.7\ \text{A}$, which rounds to 18 A.

(c) $I_{42\Omega} = \dfrac{V}{R} = \dfrac{240\ \text{V}}{42\ \Omega} = 5.7\ \text{A}$; $I_{20\Omega} = \dfrac{V}{R} = \dfrac{240\ \text{V}}{20\ \Omega} = 12\ \text{A}$.

EVALUATE: More current flows through the resistor that has the smaller R.

26.5. **IDENTIFY:** The equivalent resistance will vary for the different connections because the series-parallel combinations vary, and hence the current will vary.

SET UP: First calculate the equivalent resistance using the series-parallel formulas, then use Ohm's law $(V = RI)$ to find the current.

EXECUTE: **(a)** $1/R = 1/(15.0\ \Omega) + 1/(30.0\ \Omega)$ gives $R = 10.0\ \Omega$. $I = V/R = (35.0\ \text{V})/(10.0\ \Omega) = 3.50\ \text{A}$.

(b) $1/R = 1/(10.0\ \Omega) + 1/(35.0\ \Omega)$ gives $R = 7.78\ \Omega$. $I = (35.0\ \text{V})/(7.78\ \Omega) = 4.50\ \text{A}$.

(c) $1/R = 1/(20.0\ \Omega) + 1/(25.0\ \Omega)$ gives $R = 11.11\ \Omega$, so $I = (35.0\ \text{V})/(11.11\ \Omega) = 3.15\ \text{A}$.

(d) From part (b), the resistance of the triangle alone is 7.78 Ω. Adding the 3.00-Ω internal resistance of the battery gives an equivalent resistance for the circuit of 10.78 Ω. Therefore the current is $I = (35.0\text{V})/(10.78\ \Omega) = 3.25\ \text{A}$.

EVALUATE: It makes a big difference how the triangle is connected to the battery.

26.6. **IDENTIFY:** The potential drop is the same across the resistors in parallel, and the current into the parallel combination is the same as the current through the 45.0-Ω resistor.

(a) SET UP: Apply Ohm's law in the parallel branch to find the current through the 45.0-Ω resistor. Then apply Ohm's law to the 45.0-Ω resistor to find the potential drop across it.

EXECUTE: The potential drop across the 25.0-Ω resistor is $V_{25} = (25.0\ \Omega)(1.25\ \text{A}) = 31.25\ \text{V}$. The potential drop across each of the parallel branches is 31.25 V. For the 15.0-Ω resistor: $I_{15} = (31.25\text{V})/(15.0\ \Omega) = 2.083\ \text{A}$. The resistance of the 10.0-Ω + 15.0-Ω combination is 25.0 Ω, so the current through it must be the same as the current through the upper 25.0-Ω resistor: $I_{10+15} = 1.25\ \text{A}$. The sum of currents in the parallel branch will be the current through the 45.0-Ω resistor.

$$I_{\text{Total}} = 1.25\ \text{A} + 2.083\ \text{A} + 1.25\ \text{A} = 4.58\ \text{A}.$$

Apply Ohm's law to the 45.0-Ω resistor: $V_{45} = (4.58\ \text{A})(45.0\ \Omega) = 206\ \text{V}$.

(b) SET UP: First find the equivalent resistance of the circuit and then apply Ohm's law to it.

EXECUTE: The resistance of the parallel branch is $1/R = 1/(25.0\ \Omega) + 1/(15.0\ \Omega) + 1/(25.0\ \Omega)$, so $R = 6.82\ \Omega$. The equivalent resistance of the circuit is $6.82\ \Omega + 45.0\ \Omega + 35.00\ \Omega = 86.82\ \Omega$. Ohm's law gives $V_{\text{Bat}} = (86.62\ \Omega)(4.58\ \text{A}) = 398\ \text{V}$.

EVALUATE: The emf of the battery is the sum of the potential drops across each of the three segments (parallel branch and two series resistors).

26.7. **IDENTIFY:** First do as much series-parallel reduction as possible.

SET UP: The 45.0-Ω and 15.0-Ω resistors are in parallel, so first reduce them to a single equivalent resistance. Then find the equivalent series resistance of the circuit.

EXECUTE: $1/R_{\text{p}} = 1/(45.0\ \Omega) + 1/(15.0\ \Omega)$ and $R_{\text{p}} = 11.25\ \Omega$. The total equivalent resistance is $18.0\ \Omega + 11.25\ \Omega + 3.26\ \Omega = 32.5\ \Omega$. Ohm's law gives $I = (25.0\ \text{V})/(32.5\ \Omega) = 0.769\ \text{A}$.

EVALUATE: The circuit appears complicated until we realize that the 45.0-Ω and 15.0-Ω resistors are in parallel.

26.8. **IDENTIFY:** We are measuring an unknown resistance using Ohm's law.
SET UP and **EXECUTE:** We want R_1. For resistors in parallel $1/R_{\text{eq}} = 1/R_1 + 1/R_2 + \cdots$ The graph plots I versus V, so we need to find a relationship between those variables. Ohm's law gives $I = \dfrac{V}{R_{\text{eq}}} = V\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$. So a graph of I versus V should be a straight line having slope equal to $\dfrac{1}{R_1} + \dfrac{1}{R_2}$. Solve for R_1: $R_1 = \dfrac{1}{\text{slope} - 1/R_2} = \dfrac{1}{0.208\ \Omega^{-1} - 1/(8.00\ \Omega)} = 12.0\ \Omega.$
EVALUATE: This procedure might be useful if R_1 were not accessible to direct measurement, or if one lacked a working ohmmeter.

26.9. **IDENTIFY:** We have a combination of resistors in a circuit.
SET UP and **EXECUTE:** **(a)** We want the current. The potential difference is the same across all the resistors in parallel, so $I = \dfrac{\mathcal{E}}{R}$ for each of the resistors since they are equal.

(b) We want the current. $R_{\text{eq}} = R_1 + R_2 + \cdots = 6R$. $I = \dfrac{\mathcal{E}}{6R}$. The same current flows through all of the series resistors.

(c) Parallel: $P_{\text{p}} = \dfrac{\mathcal{E}^2}{R}$ in each resistor.

Series: $P_{\text{s}} = I^2R = \left(\dfrac{\mathcal{E}}{6R}\right)^2 R = \dfrac{\mathcal{E}^2}{36R}$.

The power is greatest in the parallel connection.

EVALUATE: An alternate approach in (c): $P_{\text{s}} = \dfrac{V_R^2}{R} = \dfrac{(\mathcal{E}/6)^2}{R} = \dfrac{\mathcal{E}^2}{36R}$, which agrees with our result.

26.10. **IDENTIFY:** The current, and hence the power, depends on the potential difference across the resistor.
SET UP: $P = V^2/R$.
EXECUTE: **(a)** $V = \sqrt{PR} = \sqrt{(5.0\ \text{W})(15{,}000\ \Omega)} = 274\ \text{V}$.
(b) $P = V^2/R = (120\ \text{V})^2/(9{,}000\ \Omega) = 1.6\ \text{W}$.
(c) SET UP: If the larger resistor generates 2.00 W, the smaller one will generate less and hence will be safe. Therefore the maximum power in the larger resistor must be 2.00 W. Use $P = I^2R$ to find the maximum current through the series combination and use Ohm's law to find the potential difference across the combination.
EXECUTE: $P = I^2R$ gives $I = \sqrt{P/R} = \sqrt{(2.00\ \text{W})/(150\ \Omega)} = 0.115\ \text{A}$. The same current flows through both resistors, and their equivalent resistance is $250\ \Omega$. Ohm's law gives $V = IR = (0.115\ \text{A})(250\ \Omega) = 28.8\ \text{V}$. Therefore $P_{150} = 2.00\ \text{W}$ and $P_{100} = I^2R = (0.115\ \text{A})^2(100\ \Omega) = 1.32\ \text{W}$.
EVALUATE: If the resistors in a series combination all have the same power rating, it is the *largest* resistance that limits the amount of current.

26.11. **IDENTIFY** and **SET UP:** Ohm's law applies to the resistors, the potential drop across resistors in parallel is the same for each of them, and at a junction the currents in must equal the currents out.
EXECUTE: **(a)** $V_2 = I_2R_2 = (4.00\ \text{A})(6.00\ \Omega) = 24.0\ \text{V}$. $V_1 = V_2 = 24.0\ \text{V}$.

$I_1 = \dfrac{V_1}{R_1} = \dfrac{24.0\text{ V}}{3.00\ \Omega} = 8.00\text{ A}.$ $I_3 = I_1 + I_2 = 4.00\text{ A} + 8.00\text{ A} = 12.0\text{ A}.$

(b) $V_3 = I_3 R_3 = (12.0\text{ A})(5.00\ \Omega) = 60.0\text{ V}.$ $\mathcal{E} = V_1 + V_3 = 24.0\text{ V} + 60.0\text{ V} = 84.0\text{ V}.$

EVALUATE: Series/parallel reduction was not necessary in this case.

26.12. **IDENTIFY** and **SET UP:** Ohm's law applies to the resistors, and at a junction the currents in must equal the currents out.

EXECUTE: $V_1 = I_1 R_1 = (1.50\text{ A})(5.00\ \Omega) = 7.50\text{ V}.$ $V_2 = 7.50\text{ V}.$ $I_1 + I_2 = I_3$ so

$I_2 = I_3 - I_1 = 4.50\text{ A} - 1.50\text{ A} = 3.00\text{ A}.$ $R_2 = \dfrac{V_2}{I_2} = \dfrac{7.50\text{ V}}{3.00\text{ A}} = 2.50\ \Omega.$

$V_3 = \mathcal{E} - V_1 = 35.0\text{ V} - 7.50\text{ V} = 27.5\text{ V}.$ $R_3 = \dfrac{V_3}{I_3} = \dfrac{27.5\text{ V}}{4.50\text{ A}} = 6.11\ \Omega.$

EVALUATE: Series/parallel reduction was not necessary in this case.

26.13. **IDENTIFY:** For resistors in parallel, the voltages are the same and the currents add. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ so

$R_{\text{eq}} = \dfrac{R_1 R_2}{R_1 + R_2},$ For resistors in series, the currents are the same and the voltages add. $R_{\text{eq}} = R_1 + R_2.$

SET UP: The rules for combining resistors in series and parallel lead to the sequences of equivalent circuits shown in Figure 26.13.

EXECUTE: In Figure 26.13c, $I = \dfrac{60.0\text{ V}}{5.00\ \Omega} = 12.0\text{ A}.$ This is the current through each of the resistors in Figure 26.13b. $V_{12} = IR_{12} = (12.0\text{ A})(2.00\ \Omega) = 24.0\text{ V}.$ $V_{34} = IR_{34} = (12.0\text{ A})(3.00\ \Omega) = 36.0\text{ V}.$ Note that $V_{12} + V_{34} = 60.0\text{ V}.$ V_{12} is the voltage across R_1 and across R_2, so $I_1 = \dfrac{V_{12}}{R_1} = \dfrac{24.0\text{ V}}{3.00\ \Omega} = 8.00\text{ A}$ and V_{34} is the voltage across R_3 and across R_4, so $I_3 = \dfrac{V_{34}}{R_3} = \dfrac{36.0\text{ V}}{12.0\ \Omega} = 3.00\text{ A}$ and

$I_4 = \dfrac{V_{34}}{R_4} = \dfrac{36.0\text{ V}}{4.00\ \Omega} = 9.00\text{ A}.$

EVALUATE: Note that $I_1 + I_2 = I_3 + I_4.$

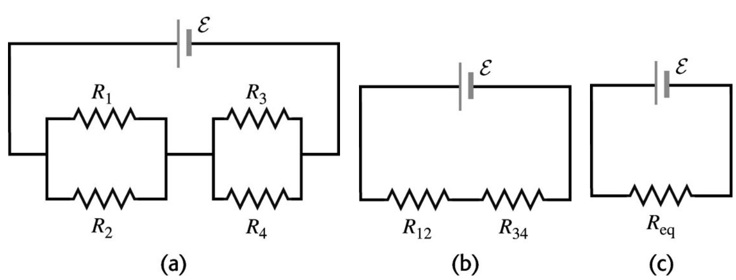


Figure 26.13

26.14. **IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.14a.

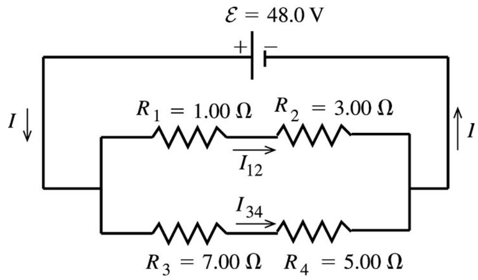


Figure 26.14a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00\ \Omega.$ R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0\ \Omega.$

The circuit is equivalent to the circuit sketched in Figure 26.14b.

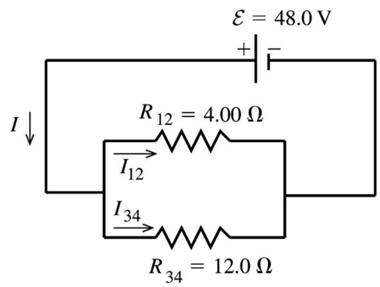


Figure 26.14b

R_{12} and R_{34} in parallel are equivalent to R_{eq} given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{12}} + \dfrac{1}{R_{34}} = \dfrac{R_{12}+R_{34}}{R_{12}R_{34}}.$

$R_{eq} = \dfrac{R_{12}R_{34}}{R_{12}+R_{34}}.$ $R_{eq} = \dfrac{(4.00\ \Omega)(12.0\ \Omega)}{4.00\ \Omega + 12.0\ \Omega} = 3.00\ \Omega.$

The voltage across each bred belowanch of the parallel combination is $\mathcal{E}$, so $\mathcal{E} - I_{12}R_{12} = 0.$

$$I_{12} = \frac{\mathcal{E}}{R_{12}} = \frac{48.0\ \text{V}}{4.00\ \Omega} = 12.0\ \text{A}.$$

$$\mathcal{E} - I_{34}R_{34} = 0 \ \text{ so } \ I_{34} = \frac{\mathcal{E}}{R_{34}} = \frac{48.0\ \text{V}}{12.0\ \Omega} = 4.0\ \text{A}.$$

The current is 12.0 A through the $1.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors, and it is 4.0 A through the $7.00\text{-}\Omega$ and $5.00\text{-}\Omega$ resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0\ \text{A} + 4.0\ \text{A} = 16.0\ \text{A},$ and this is equal to $\mathcal{E}/R_{eq} = 48.0\ \text{V}/3.00\ \Omega = 16.0\ \text{A}.$

26.15. **IDENTIFY:** In both circuits, with and without R_4, replace series and parallel combinations of resistors by their equivalents. Calculate the currents and voltages in the equivalent circuit and infer from this the currents and voltages in the original circuit. Use $P = I^2R$ to calculate the power dissipated in each bulb.

(a) SET UP: The circuit is sketched in Figure 26.15a.

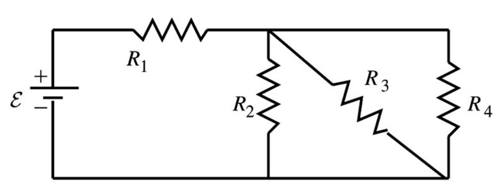


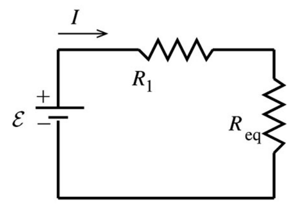
Figure 26.15a

EXECUTE: R_2, R_3, and R_4 are in parallel, so their equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}.$$

$\dfrac{1}{R_{\text{eq}}} = \dfrac{3}{4.50\ \Omega}$ and $R_{\text{eq}} = 1.50\ \Omega.$

The equivalent circuit is drawn in Figure 26.15b.



$\mathcal{E} - I(R_1 + R_{\text{eq}}) = 0.$

$I = \dfrac{\mathcal{E}}{R_1 + R_{\text{eq}}}.$

Figure 26.15b

$I = \dfrac{9.00\ \text{V}}{4.50\ \Omega + 1.50\ \Omega} = 1.50\ \text{A}$ and $I_1 = 1.50\ \text{A}.$

Then $V_1 = I_1 R_1 = (1.50\ \text{A})(4.50\ \Omega) = 6.75\ \text{V}.$

$I_{\text{eq}} = 1.50\ \text{A},\ V_{\text{eq}} = I_{\text{eq}} R_{\text{eq}} = (1.50\ \text{A})(1.50\ \Omega) = 2.25\ \text{V}.$

For resistors in parallel the voltages are equal and are the same as the voltage across the equivalent resistor, so $V_2 = V_3 = V_4 = 2.25\ \text{V}.$

$I_2 = \dfrac{V_2}{R_2} = \dfrac{2.25\ \text{V}}{4.50\ \Omega} = 0.500\ \text{A},\ I_3 = \dfrac{V_3}{R_3} = 0.500\ \text{A},\ I_4 = \dfrac{V_4}{R_4} = 0.500\ \text{A}.$

EVALUATE: Note that $I_2 + I_3 + I_4 = 1.50\ \text{A},$ which is I_{eq}. For resistors in parallel the currents add and their sum is the current through the equivalent resistor.

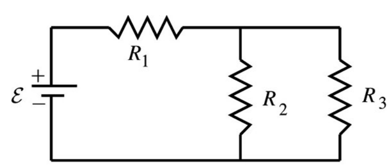
(b) SET UP: $P = I^2 R.$

EXECUTE: $P_1 = (1.50\ \text{A})^2 (4.50\ \Omega) = 10.1\ \text{W}.$

$P_2 = P_3 = P_4 = (0.500\ \text{A})^2 (4.50\ \Omega) = 1.125\ \text{W},$ which rounds to 1.12 W. R_1 glows brightest.

EVALUATE: Note that $P_2 + P_3 + P_4 = 3.37\ \text{W}$. This equals $P_{\text{eq}} = I_{\text{eq}}^2 R_{\text{eq}} = (1.50\ \text{A})^2 (1.50\ \Omega) = 3.37\ \text{W},$ the power dissipated in the equivalent resistor.

(c) SET UP: With R_4 removed the circuit becomes the circuit in Figure 26.15c.

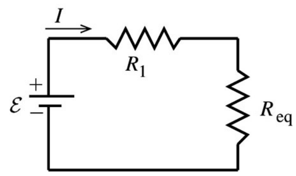


EXECUTE: R_2 and R_3 are in parallel and their equivalent resistance R_{eq} is given by

$\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{2}{4.50\ \Omega}$ and $R_{\text{eq}} = 2.25\ \Omega.$

Figure 26.15c

The equivalent circuit is shown in Figure 26.15d.



$$\mathcal{E} - I(R_1 + R_{eq}) = 0.$$
$$I = \frac{\mathcal{E}}{R_1 + R_{eq}}.$$
$$I = \frac{9.00\text{ V}}{4.50\ \Omega + 2.25\ \Omega} = 1.333\text{ A}.$$

Figure 26.15d

$I_1 = 1.33$ A, $V_1 = I_1 R_1 = (1.333\text{ A})(4.50\ \Omega) = 6.00$ V.

$I_{eq} = 1.33$ A, $V_{eq} = I_{eq} R_{eq} = (1.333\text{ A})(2.25\ \Omega) = 3.00$ V and $V_2 = V_3 = 3.00$ V.

$I_2 = \frac{V_2}{R_2} = \frac{3.00\text{ V}}{4.50\ \Omega} = 0.667$ A, $I_3 = \frac{V_3}{R_3} = 0.667$ A.

(d) SET UP: $P = I^2 R.$

EXECUTE: $P_1 = (1.333\text{ A})^2(4.50\ \Omega) = 8.00$ W.

$P_2 = P_3 = (0.667\text{ A})^2(4.50\ \Omega) = 2.00$ W.

EVALUATE: (e) When R_4 is removed, P_1 decreases and P_2 and P_3 increase. Bulb R_1 glows less brightly and bulbs R_2 and R_3 glow more brightly. When R_4 is removed the equivalent resistance of the circuit increases and the current through R_1 decreases. But in the parallel combination this current divides into two equal currents rather than three, so the currents through R_2 and R_3 increase. Can also see this by noting that with R_4 removed and less current through R_1 the voltage drop across R_1 is less so the voltage drop across R_2 and across R_3 must become larger.

26.16. IDENTIFY: Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: From Ohm's law, the voltage drop across the $6.00\text{-}\Omega$ resistor is $V = IR = (4.00\text{ A})(6.00\ \Omega) = 24.0$ V. The voltage drop across the $8.00\text{-}\Omega$ resistor is the same, since these two resistors are wired in parallel. The current through the $8.00\text{-}\Omega$ resistor is then $I = V/R = 24.0\text{ V}/8.00\ \Omega = 3.00$ A. The current through the $25.0\text{-}\Omega$ resistor is the sum of the current through these two resistors: 7.00 A. The voltage drop across the $25.0\text{-}\Omega$ resistor is $V = IR = (7.00\text{ A})(25.0\ \Omega) = 175$ V, and total voltage drop across the top branch of the circuit is $175\text{ V} + 24.0\text{ V} = 199$ V, which is also the voltage drop across the $20.0\text{-}\Omega$ resistor. The current through the $20.0\text{-}\Omega$ resistor is then $I = V/R = 199\text{ V}/20\ \Omega = 9.95$ A.

EVALUATE: The total current through the battery is $7.00\text{ A} + 9.95\text{ A} = 16.95$ A. Note that we did not need to calculate the emf of the battery.

26.17. IDENTIFY: Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: The current through the $2.00\text{-}\Omega$ resistor is 6.00 A. Current through the $1.00\text{-}\Omega$ resistor also is 6.00 A and the voltage is 6.00 V. Voltage across the $6.00\text{-}\Omega$ resistor is $12.0\text{ V} + 6.0\text{ V} = 18.0$ V. Current through the $6.00\text{-}\Omega$ resistor is $(18.0\text{ V})/(6.00\ \Omega) = 3.00$ A. The battery emf is 18.0 V.

EVALUATE: The current through the battery is $6.00 \text{ A} + 3.00 \text{ A} = 9.00 \text{ A}$. The equivalent resistor of the resistor network is 2.00 Ω, and this equals $(18.0 \text{ V})/(9.00 \text{ A})$.

26.18. **IDENTIFY:** Ohm's law applies to each resistor. In one case, the resistors are connected in series, and in the other case they are in parallel.

SET UP: $V = RI$, $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$ (in parallel), $R_{\text{eq}} = R_1 + R_2 + \ldots$ (in series). Figure 26.18 shows the equivalent circuit when S is open and when S is closed.

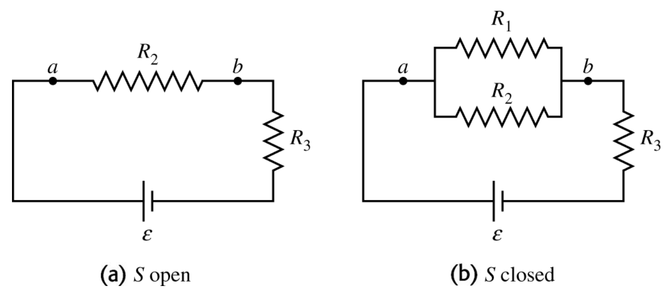


Figure 26.18

EXECUTE: **(a)** <u>S open</u>: We use the circuit in Figure 26.18a. R_2 and R_3 are in series. Ohm's law gives $\mathcal{E} = (R_2 + R_3)I$.

$I = \mathcal{E}/(R_2 + R_3) = (36.0 \; V)/(9.00 \; \Omega) = 4.00 \text{ A}$.

$V_{ab} = R_2 I = (6.00 \; \Omega)(4.00 \text{ A}) = 24.0 \text{ V}$.

<u>S closed</u>: We use the circuit in Figure 26.18b. R_1 and R_2 are in parallel, and this combination is in series with R_3. For the parallel branch

$\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$ = 1/(4.00 Ω) + 1/(6.00 Ω), which gives $R_{\text{eq}} = 2.40 \; \Omega$. The equivalent resistance R of the circuit is 2.40 Ω + 3.00 Ω = 5.40 Ω. The current is $I = \mathcal{E}/R$ = (36.0 V)/(5.40 Ω) = 6.667 A. Therefore $V_{ab} = IR_{\text{eq}}$ = (6.667 A)(2.40 Ω) = 16.0 V.

(b) <u>S open</u>: From part (a), we know that I_2 = 4.00 A through R_2. Since S is open, no current can flow through R_1, so $I_1 = 0$, $I_2 = I_3$ = 4.00 A.

<u>S closed</u>: $I_1 = V_{ab}/R_1$ = (16.0 V)/(4.00 Ω) = 4.00 A. $I_2 = V_{ab}/R_2$ = (16.0 V)/(6.00 Ω) = 2.67 A.

$I_3 = I_1 + I_2$ = 4.00 A + 2.67 A = 6.67 A.

I_1 increased from 0 to 4.00 A.

I_2 decreased from 4.00 A to 2.67 A.

I_3 increased from 4.00 A to 6.67 A.

EVALUATE: With S closed, $V_{ab} + V_3$ = 16.0 V + (3.00 Ω)(6.67 A) = 36.0 V, which is equal to $\mathcal{E}$, as it should be.

26.19. **IDENTIFY** and **SET UP:** Replace series and parallel combinations of resistors by their equivalents until the circuit is reduced to a single loop. Use the loop equation to find the current through the 20.0-Ω resistor. Set $P = I^2 R$ for the 20.0-Ω resistor equal to the rate Q/t at which heat goes into the water and set $Q = mc\Delta T$.

EXECUTE: Replace the network by the equivalent resistor, as shown in Figure 26.19.

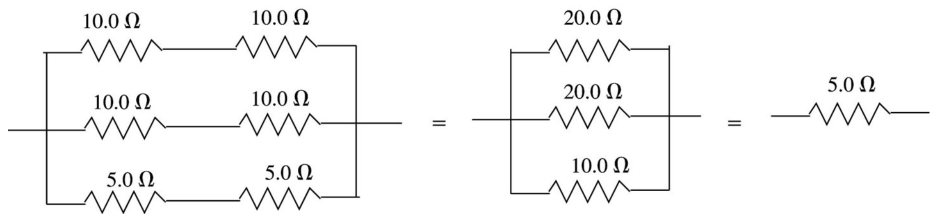


Figure 26.19

$30.0\text{ V} - I(20.0\ \Omega + 5.0\ \Omega + 5.0\ \Omega) = 0;\ I = 1.00\text{ A}.$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2R = 20.0\text{ W}.$

$Q = Pt$ and $Q = mc\Delta T$ gives $t = \dfrac{mc\Delta T}{P} = \dfrac{(0.100\text{ kg})(4190\text{ J/kg}\cdot\text{K})(48.0\text{ C}°)}{20.0\text{ W}} = 1.01 \times 10^3\text{ s}.$

EVALUATE: The battery is supplying heat at the rate $P = \mathcal{E}I = 30.0\text{ W}.$ In the series circuit, more energy is dissipated in the larger resistor $(20.0\ \Omega)$ than in the smaller ones $(5.00\ \Omega).$

26.20. IDENTIFY: $P = I^2R$ determines R_1. $R_1, R_2,$ and the $10.0\text{-}\Omega$ resistor are all in parallel so have the same voltage. Apply the junction rule to find the current through R_2.

SET UP: $P = I^2R$ for a resistor and $P = \mathcal{E}I$ for an emf. The emf inputs electrical energy into the circuit and electrical energy is removed in the resistors.

EXECUTE: (a) $P_1 = I_1^2R_1$. $15.0\text{ W} = (2.00\text{ A})^2R_1$ so $R_1 = 3.75\ \Omega.$ R_1 and $10.0\ \Omega$ are in parallel, so $(10.0\ \Omega)I_{10} = (3.75\ \Omega)(2.00\text{ A})$ so $I_{10} = 0.750\text{ A}.$ So $I_2 = 3.50\text{ A} - I_1 - I_{10} =$ 3.50 A – 2.00 A – 0.750 A = 0.750 A. R_1 and R_2 are in parallel, so $(0.750\text{ A})R_2 = (2.00\text{ A})(3.75\ \Omega)$ which gives $R_2 = 10.0\ \Omega.$

(b) $\mathcal{E} = V_1 = (2.00\text{ A})(3.75\ \Omega) = 7.50\text{ V}.$

(c) From part (a), $I_2 = 0.750\text{ A}, I_{10} = 0.750\text{ A}.$

(d) $P_1 = 15.0\text{ W}$ (given). $P_2 = I_2^2R_2 = (0.750\text{ A})^2(10.0\ \Omega) = 5.625\text{ W},$ which rounds to 5.63 W.

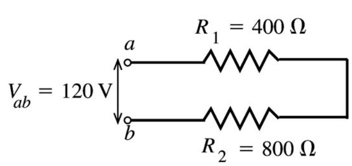
$P_{10} = I_{10}^2R_{10} = (0.750\text{ A})^2(10.0\ \Omega) = 5.625\text{ W}.$ The total rate at which the resistors remove electrical energy is $P_{\text{Resist}} = 15.0\text{ W} + 5.625\text{ W} + 5.625\text{ W} = 26.25\text{ W},$ which rounds to 26.3 W.

The total rate at which the battery inputs electrical energy is $P_{\text{Battery}} = I\mathcal{E} = (3.50\text{ A})(7.50\text{ V}) =$ 26.3 W. Therefore $P_{\text{Resist}} = P_{\text{Battery}}$, which agrees with conservation of energy.

EVALUATE: The three resistors are in parallel, so the voltage for each is the battery voltage, 7.50 V. The currents in the three resistors add to give the current in the battery.

26.21. IDENTIFY: For resistors in series, the voltages add and the current is the same. For resistors in parallel, the voltages are the same and the currents add. $P = I^2R.$

(a) SET UP: The circuit is sketched in Figure 26.21a.



For resistors in series the current is the same through each.

Figure 26.21a

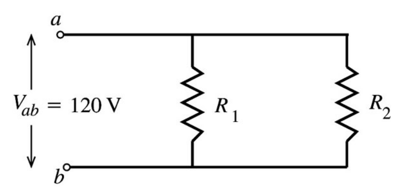
EXECUTE: $R_{eq} = R_1 + R_2 = 1200\ \Omega$. $I = \frac{V}{R_{eq}} = \frac{120\text{ V}}{1200\ \Omega} = 0.100\text{ A}$. This is the current drawn from the line.

(b) $P_1 = I_1^2 R_1 = (0.100\text{ A})^2(400\ \Omega) = 4.0\text{ W}$.

$P_2 = I_2^2 R_2 = (0.100\text{ A})^2(800\ \Omega) = 8.0\text{ W}$.

(c) $P_{out} = P_1 + P_2 = 12.0\text{ W}$, the total power dissipated in both bulbs. Note that $P_{in} = V_{ab}I = (120\text{ V})(0.100\text{ A}) = 12.0\text{ W}$, the power delivered by the potential source, equals P_{out}.

(d) SET UP: The circuit is sketched in Figure 26.21b.



For resistors in parallel the voltage across each resistor is the same.

Figure 26.21b

EXECUTE: $I_1 = \frac{V_1}{R_1} = \frac{120\text{ V}}{400\ \Omega} = 0.300\text{ A}, I_2 = \frac{V_2}{R_2} = \frac{120\text{ V}}{800\ \Omega} = 0.150\text{ A}$.

EVALUATE: Note that each current is larger than the current when the resistors are connected in series.

EXECUTE: **(e)** $P_1 = I_1^2 R_1 = (0.300\text{ A})^2(400\ \Omega) = 36.0\text{ W}$.

$P_2 = I_2^2 R_2 = (0.150\text{ A})^2(800\ \Omega) = 18.0\text{ W}$.

(f) $P_{out} = P_1 + P_2 = 54.0\text{ W}$.

EVALUATE: Note that the total current drawn from the line is $I = I_1 + I_2 = 0.450\text{ A}$. The power input from the line is $P_{in} = V_{ab}I = (120\text{ V})(0.450\text{ A}) = 54.0\text{ W}$, which equals the total power dissipated by the bulbs.

(g) The bulb that is dissipating the most power glows most brightly. For the series connection the currents are the same and by $P = I^2R$ the bulb with the larger *R* has the larger *P*; the $800\text{-}\Omega$ bulb glows more brightly. For the parallel combination the voltages are the same and by $P = V^2/R$ the bulb with the smaller *R* has the larger *P*; the $400\text{-}\Omega$ bulb glows more brightly.

(h) The total power output P_{out} equals $P_{in} = V_{ab}I$, so P_{out} is larger for the parallel connection where the current drawn from the line is larger (because the equivalent resistance is smaller.)

26.22. **IDENTIFY:** This circuit cannot be reduced using series/parallel combinations, so we apply Kirchhoff's rules. The target variables are the currents in each segment.

SET UP: Assume the unknown currents have the directions shown in Figure 26.22. We have used the junction rule to write the current through the 10.0 V battery as $I_1 + I_2$. There are two unknowns, I_1 and I_2, so we will need two equations. Three possible circuit loops are shown in the figure.

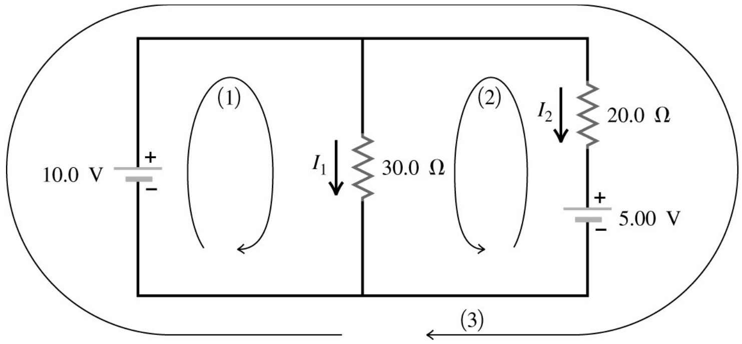


Figure 26.22

EXECUTE: **(a)** Apply the loop rule to loop (1), going around the loop in the direction shown: $+10.0\text{ V}-(30.0\ \Omega)I_1=0$ and $I_1=0.333\text{ A}$.

(b) Apply the loop rule to loop (3): $+10.0\text{ V}-(20.0\ \Omega)I_2-5.00\text{ V}=0$ and $I_2=0.250\text{ A}$.

(c) $I_1+I_2=0.333\text{ A}+0.250\text{ A}=0.583\text{ A}$.

EVALUATE: For loop (2) we get
$+5.00\text{ V}+I_2(20.0\ \Omega)-I_1(30.0\ \Omega)=5.00\text{ V}+(0.250\text{ A})(20.0\ \Omega)-(0.333\text{ A})(30.0\ \Omega)=$
$5.00\text{ V}+5.00\text{ V}-10.0\text{ V}=0$, so that with the currents we have calculated the loop rule is satisfied for this third loop.

26.23. **IDENTIFY:** Apply Kirchhoff's junction rule at point *a* to find the current through *R*. Apply Kirchhoff's loop rule to loops (1) and (2) shown in Figure 26.23a to calculate *R* and $\mathcal{E}$. Travel around each loop in the direction shown.

SET UP:

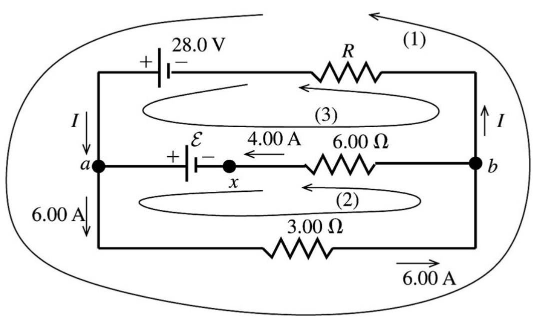


Figure 26.23a

EXECUTE: **(a)** Apply Kirchhoff's junction rule to point *a*: $\sum I=0$ so $I+4.00\text{ A}-6.00\text{ A}=0$
$I=2.00\text{ A}$ (in the direction shown in the diagram).

(b) Apply Kirchhoff's loop rule to loop (1): $-(6.00\text{ A})(3.00\ \Omega)-(2.00\text{ A})R+28.0\text{ V}=0$

$-18.0\text{ V}-(2.00\ \Omega)R+28.0\text{ V}=0$.

$$R=\frac{28.0\text{ V}-18.0\text{ V}}{2.00\text{ A}}=5.00\ \Omega.$$

(c) Apply Kirchhoff's loop rule to loop (2): $-(6.00\text{ A})(3.00\ \Omega)-(4.00\text{ A})(6.00\ \Omega)+\varepsilon=0$.

$\mathcal{E}=18.0\text{ V}+24.0\text{ V}=42.0\text{ V}$.

EVALUATE: We can check that the loop rule is satisfied for loop (3), as a check of our work:
$28.0\text{ V}-\mathcal{E}+(4.00\text{ A})(6.00\ \Omega)-(2.00\text{ A})R=0$.

$28.0\text{ V}-42.0\text{ V}+24.0\text{ V}-(2.00\text{ A})(5.00\ \Omega)=0$.

$52.0\text{ V} = 42.0\text{ V} + 10.0\text{ V}.$

$52.0\text{ V} = 52.0\text{ V},$ so the loop rule is satisfied for this loop.

(d) IDENTIFY: If the circuit is broken at point x there can be no current in the $6.00\text{-}\Omega$ resistor. There is now only a single current path and we can apply the loop rule to this path.

SET UP: The circuit is sketched in Figure 26.23b.

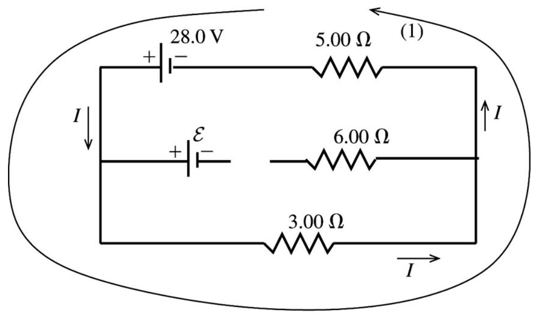


Figure 26.23b

EXECUTE: $+28.0\text{ V} - (3.00\,\Omega)I - (5.00\,\Omega)I = 0.$

$$I = \frac{28.0\text{ V}}{8.00\,\Omega} = 3.50\text{ A}.$$

EVALUATE: Breaking the circuit at x removes the 42.0-V emf from the circuit and the current through the $3.00\text{-}\Omega$ resistor is reduced.

26.24. IDENTIFY: Apply Kirchhoff's loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.24. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{V} - (1.00\text{ A})(1.00\,\Omega) + (1.00\text{ A})(4.00\,\Omega) + (1.00\text{ A})(1.00\,\Omega) - \mathcal{E}_1 - (1.00\text{ A})(6.00\,\Omega) = 0.$

$\mathcal{E}_1 = 20.0\text{ V} - 1.00\text{ V} + 4.00\text{ V} + 1.00\text{ V} - 6.00\text{ V} = 18.0\text{ V}.$ The loop rule applied to loop (2) gives:

$+20.0\text{ V} - (1.00\text{ A})(1.00\,\Omega) - (2.00\text{ A})(1.00\,\Omega) - \mathcal{E}_2 - (2.00\text{ A})(2.00\,\Omega) - (1.00\text{ A})(6.00\,\Omega) = 0.$

$\mathcal{E}_2 = 20.0\text{ V} - 1.00\text{ V} - 2.00\text{ V} - 4.00\text{ V} - 6.00\text{ V} = 7.0\text{ V}.$ Going from b to a along the lower branch,

$V_b + (2.00\text{ A})(2.00\,\Omega) + 7.0\text{ V} + (2.00\text{ A})(1.00\,\Omega) = V_a \cdot V_b - V_a = -13.0\text{ V};$ point b is at 13.0 V lower potential than point a.

EVALUATE: We can also calculate $V_b - V_a$ by going from b to a along the upper branch of the circuit. $V_b - (1.00\text{ A})(6.00\,\Omega) + 20.0\text{ V} - (1.00\text{ A})(1.00\,\Omega) = V_a$ and $V_b - V_a = -13.0\text{ V}.$ This agrees with $V_b - V_a$ calculated along a different path between b and a.

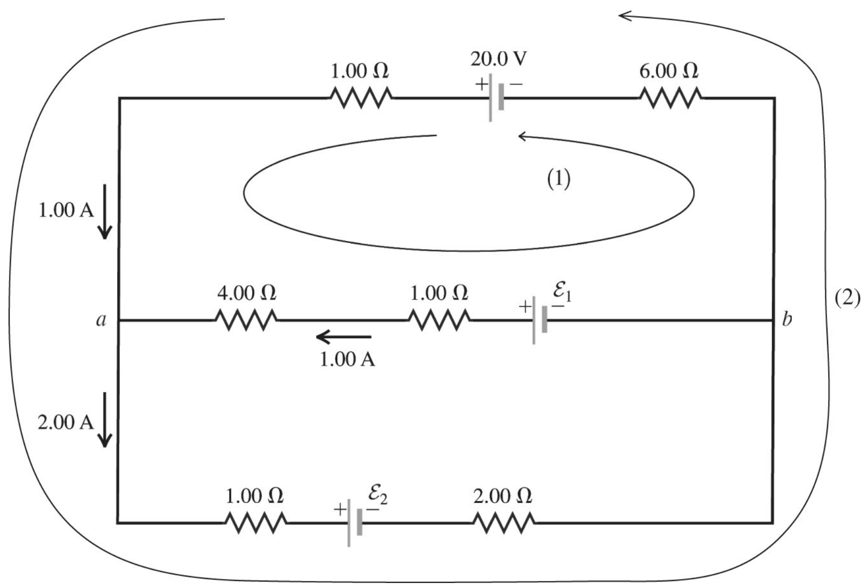


Figure 26.24

26.25. **IDENTIFY:** Apply Kirchhoff's junction rule at points *a*, *b*, *c*, and *d* to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1, \mathcal{E}_2$, and *R*.

SET UP: The circuit is sketched in Figure 26.25.

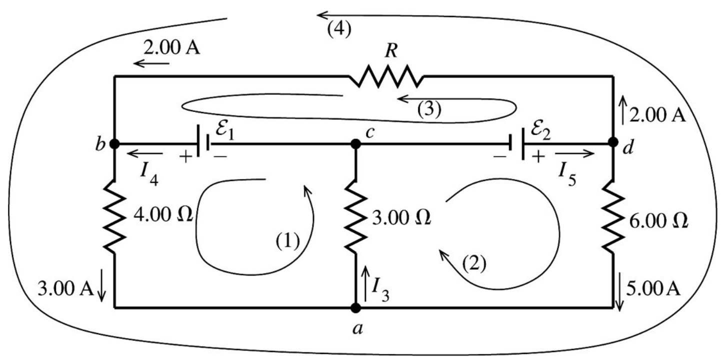


Figure 26.25

(a) EXECUTE: Apply the junction rule to point *a*: $3.00 \text{ A} + 5.00 \text{ A} - I_3 = 0.$

$I_3 = 8.00 \text{ A}.$

Apply the junction rule to point *b*: $2.00 \text{ A} + I_4 - 3.00 \text{ A} = 0.$

$I_4 = 1.00 \text{ A}.$

Apply the junction rule to point *c*: $I_3 - I_4 - I_5 = 0.$

$I_5 = I_3 - I_4 = 8.00 \text{ A} - 1.00 \text{ A} = 7.00 \text{ A}.$

EVALUATE: As a check, apply the junction rule to point *d*: $I_5 - 2.00 \text{ A} - 5.00 \text{ A} = 0.$

$I_5 = 7.00 \text{ A}.$

(b) EXECUTE: Apply the loop rule to loop (1): $\mathcal{E}_1 - (3.00 \text{ A})(4.00\ \Omega) - I_3(3.00\ \Omega) = 0.$

$\mathcal{E}_1 = 12.0 \text{ V} + (8.00 \text{ A})(3.00\ \Omega) = 36.0 \text{ V}.$

Apply the loop rule to loop (2): $\mathcal{E}_2 - (5.00 \text{ A})(6.00\ \Omega) - I_3(3.00\ \Omega) = 0.$

$\mathcal{E}_2 = 30.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 54.0\text{ V}.$

(c) EXECUTE: Apply the loop rule to loop (3): $-(2.00\text{ A})R - \mathcal{E}_1 + \mathcal{E}_2 = 0.$

$R = \dfrac{\mathcal{E}_2 - \mathcal{E}_1}{2.00\text{ A}} = \dfrac{54.0\text{ V} - 36.0\text{ V}}{2.00\text{ A}} = 9.00\ \Omega.$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:
$-(2.00\text{ A})R - (3.00\text{ A})(4.00\ \Omega) + (5.00\text{ A})(6.00\ \Omega) = 0.$

$-(2.00\text{ A})(9.00\ \Omega) - 12.0\text{ V} + 30.0\text{ V} = 0.$

$-18.0\text{ V} + 18.0\text{ V} = 0.$

26.26. **IDENTIFY:** Use Kirchhoff's rules to find the currents.
SET UP: Since the 10.0-V battery has the larger voltage, assume I_1 is to the left through the 10-V battery, I_2 is to the right through the 5-V battery, and I_3 is to the right through the 10-Ω resistor. Go around each loop in the counterclockwise direction.
EXECUTE: **(a)** Upper loop: $10.0\text{ V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\text{ V} = 0.$ This gives $5.0\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0,$ and $\Rightarrow I_1 + I_2 = 1.00\text{ A}.$

Lower loop: $5.00\text{ V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0.$ This gives

$5.00\text{ V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0,$ and $I_2 - 2I_3 = -1.00\text{ A}.$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$I_1 = 0.800\text{ A}, I_2 = 0.200\text{ A}, I_3 = 0.600\text{ A}.$

(b) $V_{ab} = -(0.200\text{ A})(4.00\ \Omega) - (0.800\text{ A})(3.00\ \Omega) = -3.20\text{ V}.$

EVALUATE: Traveling from *b* to *a* through the 4.00-Ω and 3.00-Ω resistors you pass through the resistors in the direction of the current and the potential decreases. Therefore point *b* is at higher potential than point *a*.

26.27. **IDENTIFY:** Apply the junction rule to reduce the number of unknown currents. Apply the loop rule to two loops to obtain two equations for the unknown currents I_1 and I_2.
(a) SET UP: The circuit is sketched in Figure 26.27.

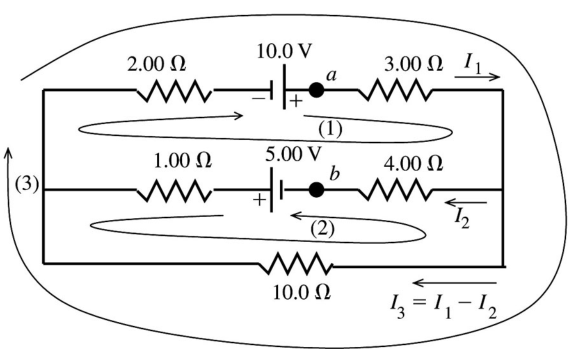


Figure 26.27

Let I_1 be the current in the 3.00-Ω resistor and I_2 be the current in the 4.00-Ω resistor and assume that these currents are in the directions shown. Then the current in the 10.0-Ω resistor is $I_3 = I_1 - I_2$, in the direction shown, where we have used Kirchhoff's junction rule to relate I_3 to I_1 and I_2. If we get a negative answer for any of these currents we know the current is actually in the opposite direction to what we have assumed. Three loops and directions to travel around the loops are shown in the circiut diagram in Figure 26.27. Apply Kirchhoff's loop rule to each loop.
EXECUTE: Loop (1):
$+10.0\text{ V} - I_1(3.00\ \Omega) - I_2(4.00\ \Omega) + 5.00\text{ V} - I_2(1.00\ \Omega) - I_1(2.00\ \Omega) = 0.$

$15.00\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0.$

$3.00\text{ A} - I_1 - I_2 = 0.$

Loop (2):

$+5.00\text{ V} - I_2(1.00\ \Omega) + (I_1 - I_2)10.0\ \Omega - I_2(4.00\ \Omega) = 0.$

$5.00\text{ V} + (10.0\ \Omega)I_1 - (15.0\ \Omega)I_2 = 0.$

$1.00\text{ A} + 2.00I_1 - 3.00I_2 = 0.$

The first equation says $I_2 = 3.00\text{ A} - I_1.$

Use this in the second equation: $1.00\text{ A} + 2.00I_1 - 9.00\text{ A} + 3.00I_1 = 0.$

$5.00I_1 = 8.00\text{ A}, I_1 = 1.60\text{ A}.$

Then $I_2 = 3.00\text{ A} - I_1 = 3.00\text{ A} - 1.60\text{ A} = 1.40\text{ A}.$

$I_3 = I_1 - I_2 = 1.60\text{ A} - 1.40\text{ A} = 0.20\text{ A}.$

EVALUATE: Loop (3) can be used as a check.

$+10.0\text{ V} - (1.60\text{ A})(3.00\ \Omega) - (0.20\text{ A})(10.00\ \Omega) - (1.60\text{ A})(2.00\ \Omega) = 0.$

$10.0\text{ V} = 4.8\text{ V} + 2.0\text{ V} + 3.2\text{ V}.$

$10.0\text{ V} = 10.0\text{ V}.$

We find that with our calculated currents the loop rule is satisfied for loop (3). Also, all the currents came out to be positive, so the current directions in the circuit diagram are correct.

(b) IDENTIFY and **SET UP:** To find $V_{ab} = V_a - V_b$ start at point *b* and travel to point *a*. Many different routes can be taken from *b* to *a* and all must yield the same result for V_{ab}.

EXECUTE: Travel through the $4.00\text{-}\Omega$ resistor and then through the $3.00\text{-}\Omega$ resistor:

$V_b + I_2(4.00\ \Omega) + I_1(3.00\ \Omega) = V_a.$

$V_a - V_b = (1.40\text{ A})(4.00\ \Omega) + (1.60\text{ A})(3.00\ \Omega) = 5.60\text{ V} + 4.8\text{ V} = 10.4\text{ V}$ (point *a* is at higher potential than point *b*).

EVALUATE: Alternatively, travel through the 5.00-V emf, the $1.00\text{-}\Omega$ resistor, the $2.00\text{-}\Omega$ resistor, and the 10.0-V emf.

$V_b + 5.00\text{ V} - I_2(1.00\ \Omega) - I_1(2.00\ \Omega) + 10.0\text{ V} = V_a.$

$V_a - V_b = 15.0\text{ V} - (1.40\text{ A})(1.00\ \Omega) - (1.60\text{ A})(2.00\ \Omega) = 15.0\text{ V} - 1.40\text{ V} - 3.20\text{ V} = 10.4\text{ V},$ the same as before.

26.28. IDENTIFY: Use Kirchhoff's rules to find the currents.

SET UP: Since the 15.0-V battery has the largest voltage, assume I_1 is to the right through the 10.0-V battery, I_2 is to the left through the 15.0-V battery, and I_3 is to the right through the $10.00\text{-}\Omega$ resistor. Go around each loop in the counterclockwise direction.

EXECUTE: (a) Upper loop: $10.0\text{ V} + (2.00\ \Omega + 3.00\ \Omega)I_1 + (1.00\ \Omega + 4.00\ \Omega)I_2 - 15.00\text{ V} = 0.$

$-5.00\text{ V} + (5.00\ \Omega)I_1 + (5.00\ \Omega)I_2 = 0,$ so $I_1 + I_2 = +1.00\text{A}.$

Lower loop: $15.00\text{ V} - (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0.$

$15.00\text{ V} - (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0,$ so $I_2 + 2I_3 = 3.00\text{ A}.$

Along with $I_2 = I_1 + I_3,$ we can solve for the three currents and find

$I_1 = 0.00\text{ A}, I_2 = +1.00\text{ A (to the left)}, I_3 = +1.00\text{ A (to the right)}.$

(b) $V_{ab} = I_2(4.00\ \Omega) + I_1(3.00\ \Omega) = (1.00\text{ A})(4.00\ \Omega) + (0.00\text{ A})(3.00\ \Omega) = 4.00\text{ V}.$

EVALUATE: Traveling from *b* to *a* through the $4.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors you pass through each resistor opposite to the direction of the current and the potential increases; point *a* is at higher potential than point *b*.

26.29. **(a) IDENTIFY:** With the switch open, the circuit can be solved using series-parallel reduction.
SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.
EXECUTE: The $30.0\text{-}\Omega$ and $50.0\text{-}\Omega$ resistors are in series, and hence have the same current. Using Ohm's law $I_{50} = (15.0\text{ V})/(50.0\ \Omega) = 0.300\text{ A} = I_{30}$. The potential drop across the $75.0\text{-}\Omega$ resistor is the same as the potential drop across the $80.0\text{-}\Omega$ series combination. We can use this fact to find the current through the $75.0\text{-}\Omega$ resistor using Ohm's law: $V_{75} = V_{80} = (0.300\text{ A})(80.0\ \Omega) = 24.0\text{ V}$ and $I_{75} = (24.0\text{ V})/(75.0\ \Omega) = 0.320\text{ A}$.

The current through the unknown battery is the sum of the two currents we just found: $I_{\text{Total}} = 0.300\text{ A} + 0.320\text{ A} = 0.620\text{ A}$.

The equivalent resistance of the resistors in parallel is $1/R_{\text{p}} = 1/(75.0\ \Omega) + 1/(80.0\ \Omega)$. This gives $R_{\text{p}} = 38.7\ \Omega$. The equivalent resistance "seen" by the battery is $R_{\text{equiv}} = 20.0\ \Omega + 38.7\ \Omega = 58.7\ \Omega$. Applying Ohm's law to the battery gives $\mathcal{E} = R_{\text{equiv}} I_{\text{Total}} = (58.7\ \Omega)(0.620\text{ A}) = 36.4\text{ V}$.

(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the $50.0\text{-}\Omega$ resistor.
SET UP: Take a loop around the right part of the circuit.
EXECUTE: Ohm's law gives $I = (25.0\text{ V})/(50.0\ \Omega) = 0.500\text{ A}$.
EVALUATE: The current through the $50.0\text{-}\Omega$ resistor, and the rest of the circuit, depends on whether or not the switch is open.

26.30. **IDENTIFY:** We need to use Kirchhoff's rules.
SET UP: Take a loop around the outside of the circuit, apply the junction rule at the upper junction, and then take a loop around the right side of the circuit.
EXECUTE: The outside loop gives $75.0\text{ V} - (12.0\ \Omega)(1.50\text{ A}) - (48.0\ \Omega)I_{48} = 0$, so $I_{48} = 1.188\text{ A}$. At a junction we have $1.50\text{A} = I_{\mathcal{E}} + 1.188\text{ A}$, and $I_{\mathcal{E}} = 0.313\text{ A}$. A loop around the right part of the circuit gives $\mathcal{E} - (48\ \Omega)(1.188\text{ A}) + (15.0\ \Omega)(0.313\text{ A})$. $\mathcal{E} = 52.3\text{ V}$, with the polarity shown in the figure in the problem.
EVALUATE: The unknown battery has a smaller emf than the known one, so the current through it goes against its polarity.

26.31. **(a) IDENTIFY:** With the switch open, we have a series circuit with two batteries.
SET UP: Take a loop to find the current, then use Ohm's law to find the potential difference between *a* and *b*.
EXECUTE: Taking the loop: $I = (40.0\text{ V})/(175\ \Omega) = 0.229\text{ A}$. The potential difference between *a* and *b* is $V_b - V_a = +15.0\text{ V} - (75.0\ \Omega)(0.229\text{ A}) = -2.14\text{ V}$.
EVALUATE: The minus sign means that *a* is at a higher potential than *b*.
(b) IDENTIFY: With the switch closed, the ammeter part of the circuit divides the original circuit into two circuits. We can apply Kirchhoff's rules to both parts.
SET UP: Take loops around the left and right parts of the circuit, and then look at the current at the junction.
EXECUTE: The left-hand loop gives $I_{100} = (25.0\text{ V})/(100.0\ \Omega) = 0.250\text{ A}$. The right-hand loop gives $I_{75} = (15.0\text{ V})/(75.0\ \Omega) = 0.200\text{ A}$. At the junction just above the switch we have $I_{100} = 0.250\text{ A}$ (in) and $I_{75} = 0.200\text{ A}$ (out), so $I_{\text{A}} = 0.250\text{ A} - 0.200\text{ A} = 0.050\text{ A}$, downward. The voltmeter reads zero because the potential difference across it is zero with the switch closed.
EVALUATE: The ideal ammeter acts like a short circuit, making *a* and *b* at the same potential. Hence the voltmeter reads zero.

26.32. **IDENTIFY:** We first reduce the parallel combination of the $20.0\text{-}\Omega$ resistors and then apply Kirchhoff's rules.

SET UP: $P = I^2R$ so the power consumption of the $6.0\text{-}\Omega$ resistor allows us to calculate the current through it. Unknown currents $I_1, I_2,$ and I_3 are shown in Figure 26.32. The junction rule says that $I_1 = I_2 + I_3$. In Figure 26.34 the two $20.0\text{-}\Omega$ resistors in parallel have been replaced by their equivalent $(10.0\ \Omega)$.

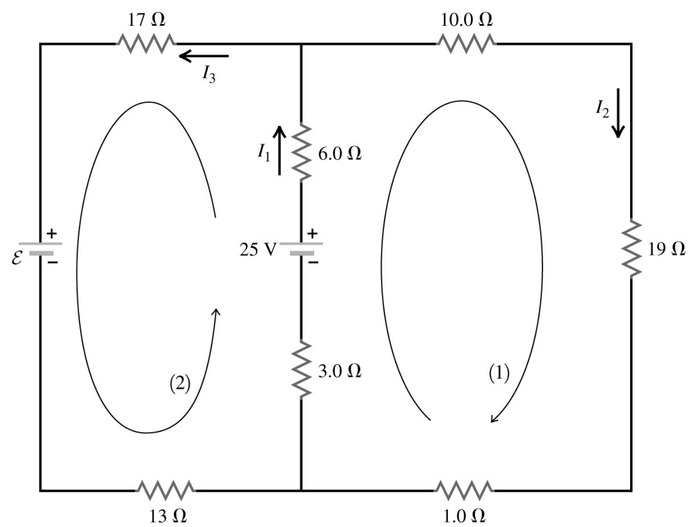


Figure 26.32

EXECUTE: **(a)** $P = I^2R$ gives $I_1 = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{24\ \text{J/s}}{6.0\ \Omega}} = 2.0\ \text{A}$. The loop rule applied to loop (1) gives:

$-(2.0\ \text{A})(3.0\ \Omega) - (2.0\ \text{A})(6.0\ \Omega) + 25\ \text{V} - I_2(10.0\ \Omega + 19.0\ \Omega + 1.0\ \Omega) = 0.$

$I_2 = \dfrac{25\ \text{V} - 18\ \text{V}}{30.0\ \Omega} = 0.233\ \text{A}.$

(b) $I_3 = I_1 - I_2 = 2.0\ \text{A} - 0.233\ \text{A} = 1.77\ \text{A}$. The loop rule applied to loop (2) gives:

$-(2.0\ \text{A})(3.0\ \Omega + 6.0\ \Omega) + 25\ \text{V} - (1.77\ \text{A})(17\ \Omega) - \mathcal{E} - (1.77\ \text{A})(13\ \Omega) = 0.$

$\mathcal{E} = 25\ \text{V} - 18\ \text{V} - 53.1\ \text{V} = -46.1\ \text{V}$. The emf is 46.1 V.

EVALUATE: Because of the minus sign for the emf, the polarity of the battery is opposite to what is shown in the figure in the problem; the + terminal is adjacent to the $13\text{-}\Omega$ resistor.

26.33. **IDENTIFY:** To construct an ammeter, add a shunt resistor in parallel with the galvanometer coil. To construct a voltmeter, add a resistor in series with the galvanometer coil.

SET UP: The full-scale deflection current is $500\ \mu\text{A}$ and the coil resistance is $25.0\ \Omega$.

EXECUTE: **(a)** For a 20-mA ammeter, the two resistances are in parallel and the voltages across each are the same. $V_\text{c} = V_\text{s}$ gives $I_\text{c}R_\text{c} = I_\text{s}R_\text{s}$. $(500 \times 10^{-6}\ \text{A})(25.0\ \Omega) = (20 \times 10^{-3}\ \text{A} - 500 \times 10^{-6}\ \text{A})R_\text{s}$ and $R_\text{s} = 0.641\ \Omega$.

(b) For a 500-mV voltmeter, the resistances are in series and the current is the same through each:

$V_{ab} = I(R_\text{c} + R_\text{s})$ and $R_\text{s} = \dfrac{V_{ab}}{I} - R_\text{c} = \dfrac{500 \times 10^{-3}\ \text{V}}{500 \times 10^{-6}\ \text{A}} - 25.0\ \Omega = 975\ \Omega.$

EVALUATE: The equivalent resistance of the voltmeter is $R_{eq} = R_s + R_c = 1000\ \Omega$. The equivalent resistance of the ammeter is given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{sh}} + \dfrac{1}{R_c}$ and $R_{eq} = 0.625\ \Omega$. The voltmeter is a high-resistance device and the ammeter is a low-resistance device.

26.34. **IDENTIFY:** The galvanometer is represented in the circuit as a resistance R_G. Use the junction rule to relate the current through the galvanometer and the current through the shunt resistor. The voltage drop across each parallel path is the same; use this to write an equation for the resistance *R*.

SET UP: The circuit is sketched in Figure 26.34.

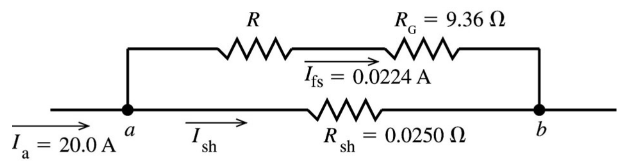


Figure 26.34

We want that $I_a = 20.0\text{ A}$ in the external circuit to produce $I_{fs} = 0.0224\text{ A}$ through the galvanometer coil.

EXECUTE: Applying the junction rule to point *a* gives $I_a - I_{fs} - I_{sh} = 0$.

$I_{sh} = I_a - I_{fs} = 20.0\text{ A} - 0.0224\text{ A} = 19.98\text{ A}$.

The potential difference V_{ab} between points *a* and *b* must be the same for both paths between these two points: $I_{fs}(R + R_G) = I_{sh}R_{sh}$.

$$R = \frac{I_{sh}R_{sh}}{I_{fs}} - R_G = \frac{(19.98\text{ A})(0.0250\ \Omega)}{0.0224\text{ A}} - 9.36\ \Omega = 22.30\ \Omega - 9.36\ \Omega = 12.9\ \Omega.$$

EVALUATE: $R_{sh} << R + R_G$; most of the current goes through the shunt. Adding *R* decreases the fraction of the current that goes through R_G.

26.35. **IDENTIFY:** The meter introduces resistance into the circuit, which affects the current through the $5.00\text{-k}\Omega$ resistor and hence the potential drop across it.

SET UP: Use Ohm's law to find the current through the $5.00\text{-k}\Omega$ resistor and then the potential drop across it.

EXECUTE: **(a)** The parallel resistance with the voltmeter is $3.33\text{ k}\Omega$, so the total equivalent resistance across the battery is $9.33\text{ k}\Omega$, giving $I = (50.0\text{ V})/(9.33\text{ k}\Omega) = 5.36\text{ mA}$. Ohm's law gives the potential drop across the $5.00\text{-k}\Omega$ resistor: $V_{5\text{ k}\Omega} = (3.33\text{ k}\Omega)(5.36\text{ mA}) = 17.9\text{ V}$.

(b) The current in the circuit is now $I = (50.0\text{ V})/(11.0\text{ k}\Omega) = 4.55\text{ mA}$.

$V_{5\text{ k}\Omega} = (5.00\text{ k}\Omega)(4.55\text{ mA}) = 22.7\text{ V}$.

(c) % error $= (22.7\text{ V} - 17.9\text{ V})/(22.7\text{ V}) = 0.214 = 21.4\%$. (We carried extra decimal places for accuracy since we had to subtract our answers.)

EVALUATE: The presence of the meter made a very large percent error in the reading of the "true" potential across the resistor.

26.36. **IDENTIFY:** We are measuring a capacitor in a circuit. We measure the time $T_{1/2}$ for the voltage to decrease to $V_0/2$ and then graph $T_{1/2}$ versus *R*.

SET UP: We need to find a relationship between $T_{1/2}$ and *R* to interpret the graph.

EXECUTE: For discharging $V = V_0 e^{-t/RC}$. $V_{1/2} = \frac{1}{2}V_0 = V_0 e^{-T_{1/2}/RC}$. Solve for $T_{1/2}$ by taking logarithms.

$T_{1/2} = (C\ln 2)R$, so a graph of $T_{1/2}$ versus R should be a straight line having slope equal to C ln 2. C = (slope)/(ln 2) = (5.00 μF)/(ln 2) = 7.21 μF.

EVALUATE: Note that $T_{1/2}$ is *not* the time constant.

26.37. IDENTIFY: The capacitor discharges exponentially through the voltmeter. Since the potential difference across the capacitor is directly proportional to the charge on the plates, the voltage across the plates decreases exponentially with the same time constant as the charge.

SET UP: The reading of the voltmeter obeys the equation $V = V_{0e}^{-t/RC}$, where RC is the time constant.

EXECUTE: **(a)** Solving for C and evaluating the result when $t = 4.00\text{ s}$ gives

$$C = \frac{t}{R\ln(V/V_0)} = \frac{4.00\text{ s}}{(3.40\times10^6\ \Omega)\ln\left(\dfrac{12.0\text{ V}}{3.00\text{ V}}\right)} = 8.49\times10^{-7}\text{ F}.$$

(b) $\tau = RC = (3.40\times10^6\ \Omega)(8.49\times10^{-7}\text{ F}) = 2.89\text{ s}.$

EVALUATE: In most laboratory circuits, time constants are much shorter than this one.

26.38. IDENTIFY: When S is closed, charge starts to flow and charge the capacitor until the potential difference across the capacitor is equal to the emf of the battery.

SET UP: $V_R = RI$, $V_C = \mathcal{E}(1 - e^{-t/RC})$, and $U_C = Q^2/2C$.

EXECUTE: **(a)** Kirchhoff's loop rule gives $V_C + V_R = \mathcal{E}$,

so $I = (\mathcal{E} - V_C)/R$ = (36.0 V – 8.00 V)/(120 Ω) = 0.2333 A, which rounds to 0.233 A.

(b) From $V_C = \mathcal{E}(1 - e^{-t/RC})$, we get $e^{-t/RC} = 1 - V_C/\mathcal{E}$. Taking logs gives $-t/RC = \ln(1 - V_C/\mathcal{E})$. Solving for t gives t = –(120 Ω)(5.00 μF) ln[1 – (8.00 V)/(36.0 V)] = 151 μs.

(c) $U_C = Q^2/2C$, so $P_C = dU_C/dt = (Q/C)\,dQ/dt = V_C I$ = (8.00 V)(0.2333 A) = 1.87 W.

EVALUATE: $P_C + P_R = P_C + I^2R$ = 1.87 W + (0.2333 A)^2(120 Ω) = 8.40 W. $P_{\mathcal{E}} = I\mathcal{E}$ = (0.2333 A)(36.0 V) = 8.40 W. These results for the power agree, as they should by conservation of energy.

26.39. IDENTIFY: An uncharged capacitor is placed into a circuit. Apply the loop rule at each time.

SET UP: The voltage across a capacitor is $V_C = q/C$.

EXECUTE: **(a)** At the instant the circuit is completed, there is no voltage across the capacitor, since it has no charge stored.

(b) Since s the full battery voltage appears across the resistor $V_R = \mathcal{E} = 245$ V.

(c) There is no charge on the capacitor.

(d) The current through the resistor is $i = \dfrac{\mathcal{E}}{R_{\text{total}}} = \dfrac{245\text{ V}}{7500\ \Omega} = 0.0327\text{ A}$ = 32.7 mA.

(e) After a long time has passed the full battery voltage is across the capacitor and $i = 0$. The voltage across the capacitor balances the emf: $V_C = 245$ V. The voltage across the resistor is zero. The capacitor's charge is $q = CV_C = (4.60\times10^{-6}\text{ F})(245\text{ V}) = 1.13\times10^{-3}$ C. The current in the circuit is zero.

EVALUATE: The current in the circuit starts at 0.0327 A and decays to zero. The charge on the capacitor starts at zero and rises to $q = 1.13\times10^{-3}$ C.

26.40. IDENTIFY: Once the switch S is closed, current starts to flow and charge the capacitor.

SET UP: $P = IV$, $V_R = RI$, $U_C = Q^2/2C$, $Q = C\mathcal{E}(1 - e^{-t/RC})$, $(1 - e^{-t/RC})$, and $I = (\mathcal{E}/R)\,e^{-t/RC}$.

EXECUTE: **(a)** $\mathcal{E} = V_R + V_C = IR + Q/C$ = (3.00 A)(12.0 Ω) + (40.0 μC)/(5.00 μF) = 44.0 V.

(b) The current is $I = (\mathcal{E}/R)\,e^{-t/RC}$. The current is 3.00 A when Q = 40.0 μC, so

3.00 A = [(44.0 V)/(12.0 Ω)]$e^{-t/RC}$. Taking logs and solving for t gives

$-t/RC$ = ln(36.0/44.0).

$t = -(12.0\ \Omega)(5.00\ \mu\text{F})\ \ln(36.0/44.0) = 12.0\ \mu\text{s}$.

(c) (i) The power in the capacitor is $P_C = dU/dt = d(Q^2/2C)/dt = (Q/C)\ dQ/dt = QI/C$, so $P_C = (40.0\ \mu C)(3.00\ A)/(5.00\ \mu F) = 24.0$ W.

(ii) $P_{\mathcal{E}} = I\mathcal{E}$ = (3.00 A)(44.0 V) = 132 W.

EVALUATE: In (c), when $I = 3.00$ A, $P_R = I^2R = (3.00\ A)^2(12.0\ \Omega) = 108\ W$. Therefore $P_R + P_C = 108$ W + 24.0 W = 132 W, which is equal to $P_{\mathcal{E}}$, as it should be by energy conservation. In (b), we can use the equation $Q = C\mathcal{E}(1-e^{-t/RC})$ to calculate Q when $t = 12.0\ \mu$s; it should be 40.0 μC. We have $Q = (44.0\ \text{V})(5.00\ \mu\text{F})(1-e^{-(12.0\ \mu s)/[(12.0\ \Omega)(5.00\ \mu F)]}) = 40.0\ \mu\text{C}$, as expected.

26.41. IDENTIFY: The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q, V must obey the same exponential equation as Q, $V = V_0 e^{-t/RC}$. The current is $I = (V_0/R)\ e^{-t/RC}$.

EXECUTE: **(a)** Solve for time when the potential across each capacitor is 10.0 V:

$t = -RC\ \ln(V/V_0) = -(80.0\ \Omega)(35.0\ \mu\text{F})\ \ln(10/45) = 4210\ \mu\text{s} = 4.21$ ms.

(b) $I = (V_0/R)\ e^{-t/RC}$. Using the above values, with $V_0 = 45.0$ V, gives $I = 0.125$ A.

EVALUATE: Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.42. IDENTIFY: For a charging capacitor $q(t) = C\mathcal{E}(1-e^{-t/\tau})$ and $i(t) = \dfrac{\mathcal{E}}{R}e^{-t/\tau}$.

SET UP: The time constant is $RC = (0.895\times10^6\ \Omega)\ (12.4\times10^{-6}\ \text{F}) = 11.1\ \text{s}$.

EXECUTE: **(a)** At $t = 0$ s: $q = C\mathcal{E}(1-e^{-t/RC}) = 0$.

At $t = 5$ s: $q = C\mathcal{E}(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1-e^{-(5.0\ \text{s})/(11.1\ \text{s})}) = 2.70\times10^{-4}$ C.

At $t = 10$ s: $q = C\mathcal{E}(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1-e^{-(10.0\ \text{s})/(11.1\ \text{s})}) = 4.42\times10^{-4}$ C.

At $t = 20$ s: $q = C\mathcal{E}(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1-e^{-(20.0\ \text{s})/(11.1\ \text{s})}) = 6.21\times10^{-4}$ C.

At $t = 100$ s: $q = C\mathcal{E}(1-e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1-e^{-(100\ \text{s})/(11.1\ \text{s})}) = 7.44\times10^{-4}$ C.

(b) The current at time t is given by: $i = \dfrac{\mathcal{E}}{R}e^{-t/RC}$.

At $t = 0$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-0/11.1} = 6.70\times10^{-5}$ A.

At $t = 5$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-5/11.1} = 4.27\times10^{-5}$ A.

At $t = 10$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-10/11.1} = 2.72\times10^{-5}$ A.

At $t = 20$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-20/11.1} = 1.11\times10^{-5}$ A.

At $t = 100$ s: $i = \dfrac{60.0\ \text{V}}{8.95\times10^5\ \Omega}e^{-100/11.1} = 8.20\times10^{-9}$ A.

(c) The graphs of $q(t)$ and $i(t)$ are given in Figure 26.42a and b.

EVALUATE: The charge on the capacitor increases in time as the current decreases.

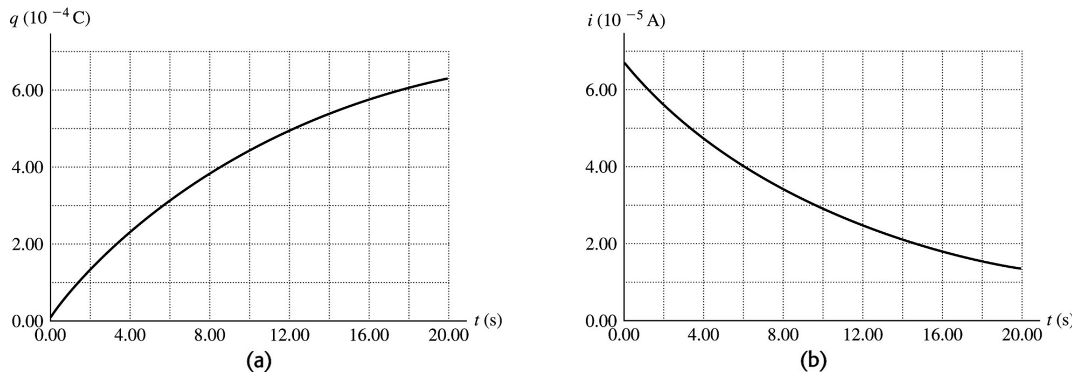


Figure 26.42

26.43. **IDENTIFY** and **SET UP:** Apply Kirchhoff's loop rule. The voltage across the resistor depends on the current through it and the voltage across the capacitor depends on the charge on its plates.
EXECUTE: $\mathcal{E} - V_R - V_C = 0.$

$\mathcal{E} = 120$ V, $V_R = IR = (0.900\text{ A})(80.0\ \Omega) = 72$ V, so $V_C = 48$ V.

$Q = CV = (4.00\times10^{-6}\text{ F})(48\text{ V}) = 192\ \mu\text{C}.$

EVALUATE: The initial charge is zero and the final charge is $C\mathcal{E} = 480\ \mu\text{C}$. Since current is flowing at the instant considered in the problem the capacitor is still being charged and its charge has not reached its final value.

26.44. **IDENTIFY:** We have two capacitors charging through a resistor.
SET UP: We want the time constant if the capacitors are in series and if they are in parallel. $\tau = RC$. $1/C_{\text{eq}} = 1/C_1 + 1/C_2 + \cdots$ (series) and $C_{\text{eq}} = C_1 + C_2 + \cdots$ (parallel).

EXECUTE: **(a)** Series: $1/C_{\text{eq}} = 1/C_1 + 1/C_2 + \cdots = 1/C + 1/C = 2/C$. $C_{\text{eq}} = C/2$. $\tau = RC_{\text{eq}} = RC/2$.

(b) Parallel: $C_{\text{eq}} = C_1 + C_2 + \cdots = C + C = 2C$. $\tau = RC_{\text{eq}} = R(2C) = 2RC$.

(c) The series connection has the shorter time constant so the current decreases faster. This means that V_{-R} also decreases faster since $V_R = IR$. So the answer is *series*.
EVALUATE: There is a factor of 4 difference in the time constants. The series circuit changes much faster than the parallel circuit.

26.45. **IDENTIFY:** The stored energy is proportional to the square of the charge on the capacitor, so it will obey an exponential equation, but not the same equation as the charge.
SET UP: The energy stored in the capacitor is $U = Q^2/2C$ and the charge on the plates is $Q_0 e^{-t/RC}$.

The current is $I = I_0 e^{-t/RC}$.

EXECUTE: $U = Q^2/2C = (Q_0 e^{-t/RC})^2/2\text{C} = U_0 e^{-2t/RC}$. When the capacitor has lost 80% of its stored energy, the energy is 20% of the initial energy, which is $U_0/5. U_0/5 = U_0\ e^{-2t/RC}$ gives $t = (RC/2)\ \ln 5 = (25.0\ \Omega)(4.62\text{ pF})(\ln 5)/2 = 92.9$ ps.

At this time, the current is $I = I_0\ e^{-t/RC} = (Q_0/RC)\ e^{-t/RC}$, so

$$I = (3.5\text{ nC})/[(25.0\ \Omega)(4.62\text{ pF})]\ \text{e}^{-(92.9\text{ ps})/[(25.0\ \Omega)(4.62\text{ pF})]} = 13.6\text{ A}.$$

EVALUATE: When the energy is reduced by 80%, neither the current nor the charge are reduced by that percent.

26.46. **IDENTIFY:** The charge is increasing while the current is decreasing. Both obey exponential equations, but they are not the same equation.

SET UP: The charge obeys the equation $Q = Q_{\text{max}}(1-e^{-t/RC})$, but the equation for the current is $I = I_{\text{max}}e^{-t/RC}$.

EXECUTE: When the charge has reached $\frac{1}{4}$ of its maximum value, we have $Q_{\text{max}}/4 = Q_{\text{max}}(1-e^{-t/RC})$, which says that the exponential term has the value $e^{-t/RC} = \frac{3}{4}$. The current at this time is $I = I_{\text{max}}e^{-t/RC} = I_{\text{max}}(3/4) = (3/4)[(10.0\text{ V})/(12.0\ \Omega)] = 0.625\text{ A}$.

EVALUATE: Notice that the current will be $\frac{3}{4}$, not $\frac{1}{4}$, of its maximum value when the charge is $\frac{1}{4}$ of its maximum. Although current and charge both obey exponential equations, the equations have different forms for a charging capacitor.

26.47. **IDENTIFY:** In both cases, simplify the complicated circuit by eliminating the appropriate circuit elements. The potential across an uncharged capacitor is initially zero, so it behaves like a short circuit. A fully charged capacitor allows no current to flow through it.

(a) SET UP: Just after closing the switch, the uncharged capacitors all behave like short circuits, so any resistors in parallel with them are eliminated from the circuit.

EXECUTE: The equivalent circuit consists of $50\ \Omega$ and $25\ \Omega$ in parallel, with this combination in series with $75\ \Omega$, $15\ \Omega$, and the 100-V battery. The equivalent resistance is $90\ \Omega + 16.7\ \Omega = 106.7\ \Omega$, which gives $I = (100\text{ V})/(106.7\ \Omega) = 0.937\text{ A}$.

(b) SET UP: Long after closing the switch, the capacitors are essentially charged up and behave like open circuits since no charge can flow through them. They effectively eliminate any resistors in series with them since no current can flow through these resistors.

EXECUTE: The equivalent circuit consists of resistances of $75\ \Omega$, $15\ \Omega$, and three 25-Ω resistors, all in series with the 100-V battery, for a total resistance of $165\ \Omega$. Therefore $I = (100\text{V})/(165\ \Omega) = 0.606\text{ A}$.

EVALUATE: The initial and final behavior of the circuit can be calculated quite easily using simple series-parallel circuit analysis. Intermediate times would require much more difficult calculations!

26.48. **IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0e^{-t/RC})^2/2C = U_0e^{-2t/RC}$.

EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0e^{-t/RC}$. This gives $t = RC\ \ln 2 = (225\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.871\text{ ms}$, which rounds to 1.87 ms.

(b) The energy is reduced by half: $U_0/2 = U_0e^{-2t/RC}$. This gives $t = (RC\ \ln 2)/2 = (1.871\text{ ms})/2 = 0.936\text{ ms}$.

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

26.49. **IDENTIFY:** When the capacitor is fully charged the voltage V across the capacitor equals the battery emf and $Q = CV$. For a charging capacitor, $q = Q(1-e^{-t/RC})$.

SET UP: $\ln e^x = x$.

EXECUTE: **(a)** $Q = CV = (5.90\times10^{-6}\text{ F})(28.0\text{ V}) = 1.65\times10^{-4}\text{ C} = 165\ \mu\text{C}$.

(b) $q = Q(1-e^{-t/RC})$, so $e^{-t/RC} = 1-\dfrac{q}{Q}$ and $R = \dfrac{-t}{C\ln(1-q/Q)}$. After

$$t = 3\times10^{-3}\text{ s}: R = \frac{-3\times10^{-3}\text{ s}}{(5.90\times10^{-6}\text{ F})(\ln(1-110/165))} = 463\ \Omega.$$

(c) If the charge is to be 99% of final value: $\frac{q}{Q} = (1 - e^{-t/RC})$ gives

$t = -RC\ln(1 - q/Q) = -(463\ \Omega)(5.90\times10^{-6}\ \text{F})\ln(0.01) = 0.0126\ \text{s} = 12.6$ ms.

EVALUATE: The time constant is $\tau = RC = 2.73$ ms. The time in part (b) is a bit more than one time constant and the time in part (c) is about 4.6 time constants.

26.50. **IDENTIFY:** We have a capacitor discharging through a resistor.

SET UP: $i = I_0 e^{-t/RC}$ and $q = Q_0 e^{-t/RC}$. When the current is 0.180 A, we want the charge and time.

EXECUTE: **(a)** $V_C = V_R$, so $q/C = RI$. $q = RIC = (185\ \Omega)(0.180\ \text{A})(6.00\ \mu\text{F}) = 200\ \mu\text{C}$.

(b) $i = I_0 e^{-t/RC}$, where $I_0 = V_0/R = (50.0\ \text{V})/(185\ \Omega) = 0.2703$ A. Use logarithms to solve for t.

$t = -RC\ln(i/I_0) = -(185\ \Omega)(6.00\ \mu\text{F})\ln(0.180/0.2703) = 451\ \mu\text{s}$.

EVALUATE: Check: At the time in (b), q should be 200 μC. Putting the numbers into $q = Q_0 e^{-t/RC}$ gives 200 μC, so our result checks.

26.51. **IDENTIFY** and **SET UP:** The heater and hair dryer are in parallel so the voltage across each is 120 V and the current through the fuse is the sum of the currents through each appliance. As the power consumed by the dryer increases, the current through it increases. The maximum power setting is the highest one for which the current through the fuse is less than 20 A.

EXECUTE: Find the current through the heater. $P = VI$ so $I = P/V = (1500\ \text{W})/(120\ \text{V}) = 12.5$ A. The maximum total current allowed is 20 A, so the current through the dryer must be less than $20\ \text{A} - 12.5\ \text{A} = 7.5$ A. The power dissipated by the dryer if the current has this value is $P = VI = (120\ \text{V})(7.5\ \text{A}) = 900$ W. For P at this value or larger the circuit breaker trips.

EVALUATE: $P = V^2/R$ and for the dryer V is a constant 120 V. The higher power settings correspond to a smaller resistance R and larger current through the device.

26.52. **IDENTIFY:** $P = VI = I^2 R$

SET UP: Problem 25.76 says that for 12-gauge wire the maximum safe current is 25 A.

EXECUTE: **(a)** $I = \frac{P}{V} = \frac{4100\ \text{W}}{240\ \text{V}} = 17.1$ A. So we need at least 14-gauge wire (good up to 18 A). 12-gauge is also ok (good up to 25 A).

(b) $P = \frac{V^2}{R}$ and $R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{4100\ \text{W}} = 14\ \Omega$.

(c) At 11¢ per kWh, for 1 hour the cost is $(11¢/\text{kWh})(1\ \text{h})(4.1\ \text{kW}) = 45¢$.

EVALUATE: The cost to operate the device is proportional to its power consumption.

26.53. **IDENTIFY:** We have a capacitor charging through a resistor.

SET UP: $i = I_0 e^{-t/RC}$, $U_C = \frac{1}{2}CV^2$, $P_\mathcal{E} = i\mathcal{E}$, and $P = i^2R$. Our target variable is the energy.

EXECUTE: **(a)** We want the energy in the capacitor. When fully charged, $i = 0$ so $V_C = \mathcal{E}$. Thus $U_C = \frac{1}{2}C\mathcal{E}^2$.

(b) $P_\mathcal{E} = i\mathcal{E}$. Since i is variable, we need to integrate to find the energy.

$U_\mathcal{E} = \int_0^\infty i\mathcal{E}dt = \int_0^\infty I_0 e^{-t/RC}dt = I_0\mathcal{E}(-RC)e^{-t/RC}\Big|_0^\infty = I_0\mathcal{E}RC$. $I_0 = \frac{\mathcal{E}}{R}$ so $U_\mathcal{E} = C\mathcal{E}^2$.

(c) $P_R = i^2R$, so $U_R = \int_0^\infty i^2 R dt = \int_0^\infty \left(I_0 e^{-t/RC}\right)^2 R dt = I_0^2 R\int_0^\infty e^{-2t/RC}dt = \left(I_0^2 R\right)\left(\frac{RC}{2}\right)$. $I_0 = \frac{\mathcal{E}}{R}$ so $U_R = \frac{1}{2}C\mathcal{E}^2$.

(d) $\dfrac{U_C}{U_{\mathcal{E}}} = \dfrac{\frac{1}{2}C\mathcal{E}^2}{C\mathcal{E}^2} = \dfrac{1}{2}$. $\dfrac{U_R}{U_{\mathcal{E}}} = \dfrac{\frac{1}{2}C\mathcal{E}^2}{C\mathcal{E}^2} = \dfrac{1}{2}$. Half the energy is stored in the capacitor and half is dissipated in the resistor.

EVALUATE: The result in (d) is compatible with energy conservation.

26.54. **IDENTIFY:** We need to do series/parallel reduction to solve this circuit.

SET UP: $P = \dfrac{\mathcal{E}^2}{R}$, where R is the equivalent resistance of the network. For resistors in series, $R_{\text{eq}} = R_1 + R_2$, and for resistors in parallel $1/R_{\text{P}} = 1/R_1 + 1/R_2$.

EXECUTE: $R = \dfrac{\mathcal{E}^2}{P} = \dfrac{(48.0\text{ V})^2}{295\text{ W}} = 7.810\ \Omega$. $R_{12} = R_1 + R_2 = 8.00\ \Omega$. $R = R_{123} + R_4$.

$R_{123} = R - R_4 = 7.810\ \Omega - 3.00\ \Omega = 4.810\ \Omega$. $\dfrac{1}{R_{12}} + \dfrac{1}{R_3} = \dfrac{1}{R_{123}}$. $\dfrac{1}{R_3} = \dfrac{1}{R_{123}} - \dfrac{1}{R_{12}} = \dfrac{R_{12} - R_{123}}{R_{123}R_{12}}$.

$R_3 = \dfrac{R_{123}R_{12}}{R_{12} - R_{123}} = \dfrac{(4.810\ \Omega)(8.00\ \Omega)}{8.00\ \Omega - 4.810\ \Omega} = 12.1\ \Omega.$

EVALUATE: The resistance R_3 is greater than R, since the equivalent parallel resistance is less than any of the resistors in parallel.

26.55. **IDENTIFY:** This problem requires Kirchhoff's rules. The target variables are the currents.

SET UP: Refer to Fig. 26.6(a) in the textbook and use the same loops shown there. For the currents choose I_1 downward through r_1, I_2 upward through r_2, and I_R downward through R. Do all the loops in a counterclockwise sense, as in the textbook. Now apply Kirchhoff's rules.

EXECUTE: <u>Junction rule</u>: $I_2 = I_1 + I_R$.

<u>Loop s</u>: $-r_2I_2 - \mathcal{E}_1 + RI_R = 0 \quad \rightarrow \quad -(2.00\ \Omega)I_1 - 24.0\text{ V} + (20.0\ \Omega)I_R = 0$.

<u>Loop 2</u>: $\mathcal{E}_2 - r_2I_2 - r_1I_1 - \mathcal{E}_1 = 0 \quad \rightarrow \quad 36.0\text{ V} - (2.00\ \Omega)I_2 - (2.00\ \Omega)I_1 - 24.0\text{ V} = 0$.

<u>Loop 3</u>: $+r_2I_2 - \mathcal{E}_2 + RI_R = 0 \quad \rightarrow \quad (2.00\ \Omega)I_2 - 36.0\text{ V} + (20.0\ \Omega)I_R = 0$.

Solve these equations by substitution (or any other method) and obtain the following answers.

(a) $I_{-1} = 2.29$ A.

(b) $I_2 = 3.71$ A.

(c) $I_R = 1.43$ A.

EVALUATE: Check: $I_1 + I_R = 2.29\text{ A} + 1.43\text{ A} = 3.72$ A. This agrees with our answer in (b).The slight difference is due to rounding during calculations.

26.56. **IDENTIFY:** Half the current flows through each parallel resistor and the full current flows through the third resistor, that is in series with the parallel combination. Therefore, only the series resistor will be at its maximum power.

SET UP: $P = I^2R$.

EXECUTE: The maximum allowed power is when the total current is the maximum allowed value of Then half the current flows through the parallel resistors and the maximum power is

$P_{\text{max}} = (I/2)^2R + (I/2)^2R + I^2R = \frac{3}{2}I^2R = \frac{3}{2}(4.47\text{ A})^2(2.4\ \Omega) = 72\text{ W}.$

EVALUATE: If all three resistors were in series or all three were in parallel, then the maximum power would be $3(48\text{ W}) = 144\text{ W}$. For the network in this problem, the maximum power is half this value.

26.57. **(a) IDENTIFY:** Break the circuit between points *a* and *b* means no current in the middle branch that contains the 3.00-Ω resistor and the 10.0-V battery. The circuit therefore has a single current path. Find the current, so that potential drops across the resistors can be calculated. Calculate V_{ab} by traveling from *a* to *b*, keeping track of the potential changes along the path taken.

SET UP: The circuit is sketched in Figure 26.57a.

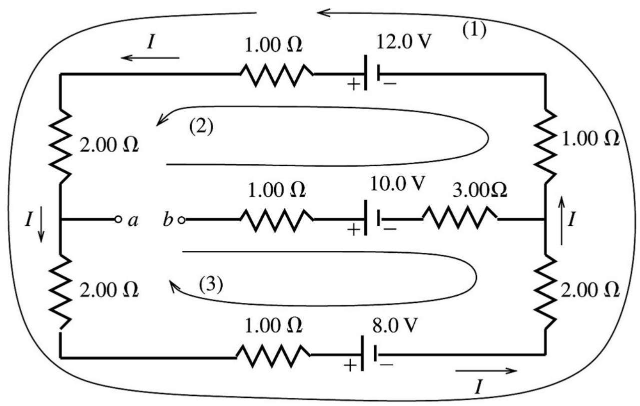


Figure 26.57a

EXECUTE: Apply Kirchhoff's loop rule to loop (1).
$+12.0\text{ V} - I(1.00\,\Omega + 2.00\,\Omega + 2.00\,\Omega + 1.00\,\Omega) - 8.0\text{ V} - I(2.00\,\Omega + 1.00\,\Omega) = 0.$

$$I = \frac{12.0\text{ V} - 8.0\text{ V}}{9.00\,\Omega} = 0.4444\text{ A}.$$

To find V_{ab} start at point *b* and travel to *a*, adding up the potential rises and drops. Travel on path (2) shown on the diagram. The 1.00-Ω and 3.00-Ω resistors in the middle branch have no current through them and hence no voltage across them. Therefore,
$V_b - 10.0\text{ V} + 12.0\text{ V} - I(1.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) = V_a$; thus
$V_a - V_b = 2.0\text{ V} - (0.4444\text{ A})(4.00\,\Omega) = +0.22\text{ V}$ (point *a* is at higher potential).

EVALUATE: As a check on this calculation we also compute V_{ab} by traveling from *b* to *a* on path (3).
$V_b - 10.0\text{ V} + 8.0\text{ V} + I(2.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) = V_a.$
$V_{ab} = -2.00\text{ V} + (0.4444\text{ A})(5.00\,\Omega) = +0.22\text{ V}$, which checks.

(b) IDENTIFY and SET UP: With points *a* and *b* connected by a wire there are three current branches, as shown in Figure 26.57b.

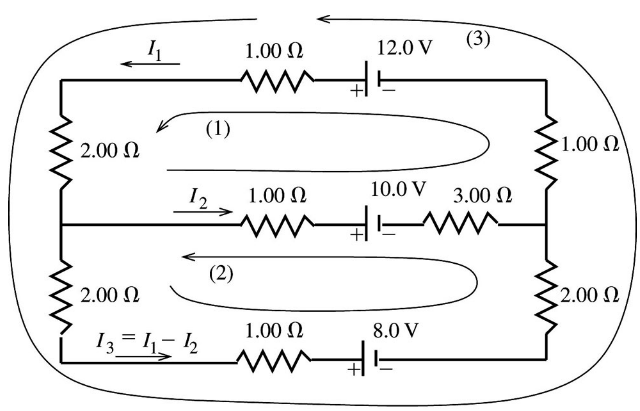


Figure 26.57b

The junction rule has been used to write the third current (in the 8.0-V battery) in terms of the other currents. Apply the loop rule to loops (1) and (2) to obtain two equations for the two unknowns I_1 and I_2.

EXECUTE: Apply the loop rule to loop (1).

$12.0\text{ V} - I_1(1.00\,\Omega) - I_1(2.00\,\Omega) - I_2(1.00\,\Omega) - 10.0\text{ V} - I_2(3.00\,\Omega) - I_1(1.00\,\Omega) = 0$

$2.0\text{ V} - I_1(4.00\,\Omega) - I_2(4.00\,\Omega) = 0$

$(2.00\,\Omega)I_1 + (2.00\,\Omega)I_2 = 1.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$-(I_1 - I_2)(2.00\,\Omega) - (I_1 - I_2)(1.00\,\Omega) - 8.0\text{ V} - (I_1 - I_2)(2.00\,\Omega) + I_2(3.00\,\Omega) + 10.0\text{ V} + I_2(1.00\,\Omega) = 0$

$2.0\text{ V} - (5.00\,\Omega)I_1 + (9.00\,\Omega)I_2 = 0$ eq. (2)

Solve eq. (1) for I_2 and use this to replace I_2 in eq. (2).

$I_2 = 0.50\text{ A} - I_1$

$2.0\text{ V} - (5.00\,\Omega)I_1 + (9.00\,\Omega)(0.50\text{ A} - I_1) = 0$

$(14.0\,\Omega)I_1 = 6.50\text{ V}$ so $I_1 = (6.50\text{ V})/(14.0\,\Omega) = 0.464\text{ A}$

$I_2 = 0.500\text{ A} - 0.464\text{ A} = 0.036\text{ A}.$

The current in the 12.0-V battery is $I_1 = 0.464\text{ A}$

EVALUATE: We can apply the loop rule to loop (3) as a check.

$+12.0\text{ V} - I_1(1.00\,\Omega + 2.00\,\Omega + 1.00\,\Omega) - (I_1 - I_2)(2.00\,\Omega + 1.00\,\Omega + 2.00\,\Omega) - 8.0\text{ V} = 4.0\text{ V} - 1.86\text{ V} - 2.14\text{ V} = 0,$ as it should.

26.58. **IDENTIFY:** Heat, which is generated in the resistor, melts the ice.

SET UP: Find the rate at which heat is generated in the $20.0\text{-}\Omega$ resistor using $P = V^2/R$. Then use the heat of fusion of ice to find the rate at which the ice melts. The heat dH to melt a mass of ice dm is $dH = L_F\ dm$, where L_F is the latent heat of fusion. The rate at which heat enters the ice, dH/dt, is the power P in the resistor, so $P = L_F\ dm/dt$. Therefore the rate of melting of the ice is $dm/dt = P/L_F$.

EXECUTE: The equivalent resistance of the parallel branch is $5.00\,\Omega$, so the total resistance in the circuit is $35.0\,\Omega$. Therefore the total current in the circuit is $I_{\text{Total}} = (45.0\text{ V})/(35.0\,\Omega) = 1.286\text{ A}$. The potential difference across the $20.0\text{-}\Omega$ resistor in the ice is the same as the potential difference across the parallel branch: $V_{\text{ice}} = I_{\text{Total}}R_p = (1.286\text{ A})(5.00\,\Omega) = 6.429\text{ V}$. The rate of heating of the ice is $P_{\text{ice}} = V_{\text{ice}}^2/R = (6.429\text{ V})^2/(20.0\,\Omega) = 2.066\text{ W}$. This power goes into to heat to melt the ice, so

$dm/dt = P/L_F = (2.066\text{ W})/(3.34\times10^5\text{ J/kg}) = 6.19\times10^{-6}\text{ kg/s} = 6.19\times10^{-3}\text{ g/s}.$

EVALUATE: The melt rate is about 6 mg/s, which is not much. It would take 1000 s to melt just 6 g of ice.

25.59. **IDENTIFY:** Apply Kirchhoff's junction rule to express the currents through the $5.00\text{-}\Omega$ and $8.00\text{-}\Omega$ resistors in terms of I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.59.

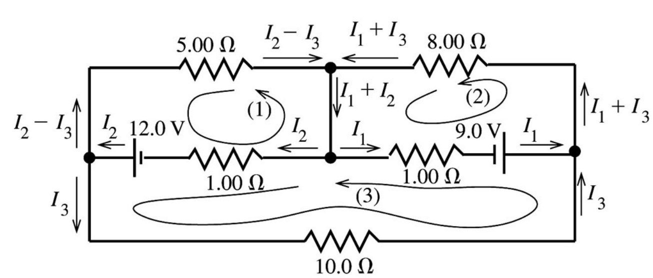


Figure 26.59

The current in each branch has been written in terms of I_1, I_2, and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2(1.00\,\Omega) + (I_2 - I_3)(5.00\,\Omega) = 0$

$I_2(6.00\,\Omega) - I_3(5.00\,\Omega) = 12.0\text{ V}$ eq. (1)

Apply the loop rule to loop (2).

$-I_1(1.00\,\Omega) + 9.00\text{ V} - (I_1 + I_3)(8.00\,\Omega) = 0$

$I_1(9.00\,\Omega) + I_3(8.00\,\Omega) = 9.00\text{ V}$ eq. (2)

Apply the loop rule to loop (3).

$-I_3(10.0\,\Omega) - 9.00\text{ V} + I_1(1.00\,\Omega) - I_2(1.00\,\Omega) + 12.0\text{ V} = 0$

$-I_1(1.00\,\Omega) + I_2(1.00\,\Omega) + I_3(10.0\,\Omega) = 3.00\text{ V}$ eq. (3)

Eq. (1) gives $I_2 = 2.00\text{ A} + \frac{5}{6}I_3$; eq. (2) gives $I_1 = 1.00\text{ A} - \frac{8}{9}I_3$.

Using these results in eq. (3) gives

$-(1.00\text{ A} - \frac{8}{9}I_3)(1.00\,\Omega) + (2.00\text{ A} + \frac{5}{6}I_3)(1.00\,\Omega) + I_3(10.0\,\Omega) = 3.00\text{ V}.$

$(\frac{16+15+180}{18})I_3 = 2.00\text{ A}$; $I_3 = \frac{18}{211}(2.00\text{ A}) = 0.171\text{ A}.$

Then $I_2 = 2.00\text{ A} + \frac{5}{6}I_3 = 2.00\text{ A} + \frac{5}{6}(0.171\text{ A}) = 2.14\text{ A}$ and

$I_1 = 1.00\text{ A} - \frac{8}{9}I_3 = 1.00\text{ A} - \frac{8}{9}(0.171\text{ A}) = 0.848\text{ A}.$

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\text{-}\Omega$, $8.00\text{-}\Omega$ and $10.0\text{-}\Omega$ resistors. Going around the loop clockwise:

$-(I_2 - I_3)(5.00\,\Omega) + (I_1 + I_3)(8.00\,\Omega) + I_3(10.0\,\Omega) = -9.85\text{ V} + 8.15\text{ V} + 1.71\text{ V}$, which does equal zero, apart from rounding.

26.60. IDENTIFY: Apply the junction rule and the loop rule to the circuit.

SET UP: Because of the polarity of each emf, the current in the $7.00\text{-}\Omega$ resistor must be in the direction shown in Figure 26.60a. Let I be the current in the 24.0-V battery.

EXECUTE: The loop rule applied to loop (1) gives: $+24.0\text{ V} - (1.80\text{ A})(7.00\,\Omega) - I(3.00\,\Omega) = 0.$

$I = 3.80\text{ A}.$ The junction rule then says that the current in the middle branch is 2.00 A, as shown in Figure 26.64b. The loop rule applied to loop (2) gives: $+\mathcal{E} - (1.80\text{ A})(7.00\,\Omega) + (2.00\text{ A})(2.00\,\Omega) = 0$ and $\mathcal{E} = 8.6\text{ V}.$

EVALUATE: We can check our results by applying the loop rule to loop (3) in Figure 26.60b:

$+24.0\text{ V} - \mathcal{E} - (2.00\text{ A})(2.00\,\Omega) - (3.80\text{ A})(3.00\,\Omega) = 0$ and $\mathcal{E} = 24.0\text{ V} - 4.0\text{ V} - 11.4\text{ V} = 8.6\text{ V}$, which agrees with our result from loop (2).

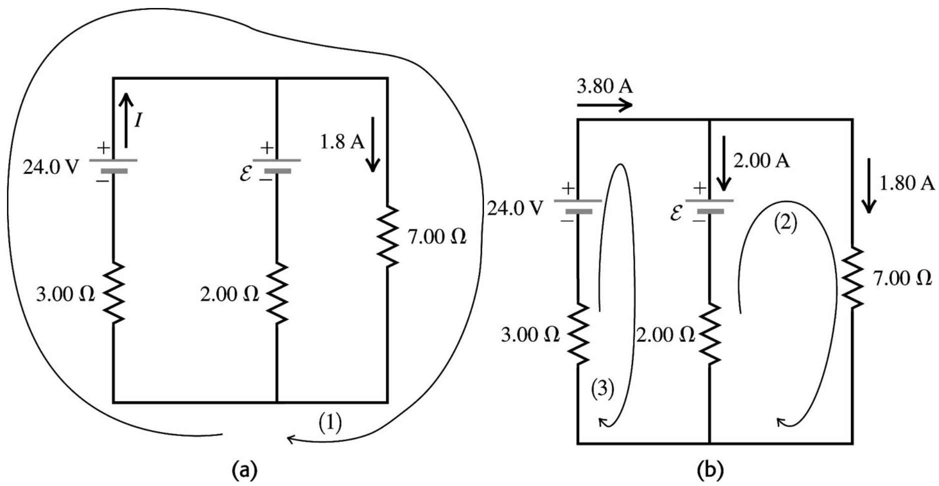


Figure 26.60

26.61. **IDENTIFY:** This problem involves resistivity and a resistor in a circuit.

SET UP and **EXECUTE:** $R = \dfrac{\rho L}{A}$.

(a) Estimate: Diameter = 0.50 mm.

(b) Solve $R = \dfrac{\rho L}{A}$ for R/L giving $\dfrac{R}{L} = \dfrac{\rho}{A} = \dfrac{\rho}{\pi r^2} = \dfrac{1.72 \times 10^{-8}\ \Omega \cdot \text{m}}{\pi (0.25\ \text{mm})^2} = 0.088\ \Omega/\text{m}.$

(c) $IR = 1.0$ V, so $(1.0\text{ A})R = 1.0$ V, which gives $R = 1.0\ \Omega$. $(R/L)L = R$, so $(0.088\ \Omega/\text{m})L = 1.0\ \Omega$. This gives $L = 11$ m.

EVALUATE: The wire must be tightly wound to be 11 m (≈35 ft) long.

26.62. **IDENTIFY:** Apply the loop and junction rules.

SET UP: Use the currents as defined on the circuit diagram in Figure 26.62 and obtain three equations to solve for the currents.

EXECUTE: **(a)** Left loop: $14 - I_1 - 2(I_1 - I_2) = 0$ and $3I_1 - 2I_2 = 14$.

Top loop: $-2(I - I_1) + I_2 + I_1 = 0$ and $-2I + 3I_1 + I_2 = 0$.

Bottom loop: $-(I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0$ and $-I + 3I_1 - 4I_2 = 0$.

Solving these equations for the currents we find: $I = I_{\text{battery}} = 10.0$ A; $I_1 = I_{R_1} = 6.0$ A; $I_2 = I_{R_3} = 2.0$ A.

So the other currents are: $I_{R_2} = I - I_1 = 4.0$ A; $I_{R_4} = I_1 - I_2 = 4.0$ A; $I_{R_5} = I - I_1 + I_2 = 6.0$ A.

(b) $R_{\text{eq}} = \dfrac{V}{I} = \dfrac{14.0\text{ V}}{10.0\text{ A}} = 1.40\ \Omega.$

EVALUATE: It isn't possible to simplify the resistor network using the rules for resistors in series and parallel. But the equivalent resistance is still defined by $V = IR_{\text{eq}}$.

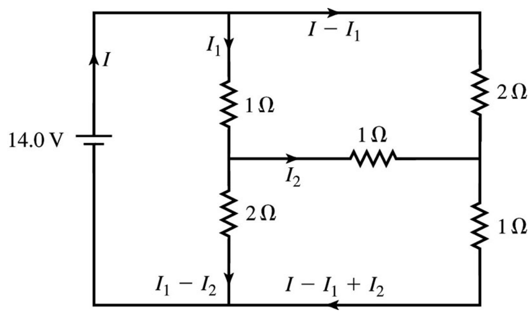


Figure 26.62

26.63. **IDENTIFY:** Simplify the resistor networks as much as possible using the rule for series and parallel combinations of resistors. Then apply Kirchhoff's laws.

SET UP: First do the series/parallel reduction. This gives the circuit in Figure 26.63. The rate at which the $10.0\text{-}\Omega$ resistor generates thermal energy is $P = I^2R$.

EXECUTE: **(a)** Apply Kirchhoff's laws and solve for $\mathcal{E}$.

$\Delta V_{\text{adefa}} = 0$: $-(20\,\Omega)(2\text{ A}) - 5\text{ V} - (20\,\Omega)I_2 = 0$.

This gives $I_2 = -2.25$ A. Then $I_1 + I_2 = 2$ A gives $I_1 = 2\text{ A} - (-2.25\text{ A}) = 4.25$ A.

$\Delta V_{\text{abcdefa}} = 0$: $(15\,\Omega)(4.25\text{ A}) + \mathcal{E} - (20\,\Omega)(-2.25\text{ A}) = 0$. This gives $\mathcal{E} = -109$ V. Since $\mathcal{E}$ is calculated to be negative, its polarity should be reversed.

(b) The parallel network that contains the $10.0\text{-}\Omega$ resistor in one branch has an equivalent resistance of $10\,\Omega$. The voltage across each branch of the parallel network is $V_{\text{par}} = RI = (10\,\Omega)(2\text{A}) = 20$ V. The current in the upper branch is $I = \dfrac{V}{R} = \dfrac{20\text{ V}}{30\,\Omega} = \dfrac{2}{3}$ A. $Pt = E$, so $I^2\,Rt = E$, where $E = 60.0$ J.

$\left(\tfrac{2}{3}\text{A}\right)^2 (10\,\Omega)t = 60$ J, and $t = 13.5$ s.

EVALUATE: For the $10.0\text{-}\Omega$ resistor, $P = I^2R = 4.44$ W. The total rate at which electrical energy is inputted to the circuit in the emf is $(5.0\text{ V})(2.0\text{ A}) + (109\text{ V})(4.25\text{ A}) = 473$ J. Only a small fraction of the energy is dissipated in the $10.0\text{-}\Omega$ resistor.

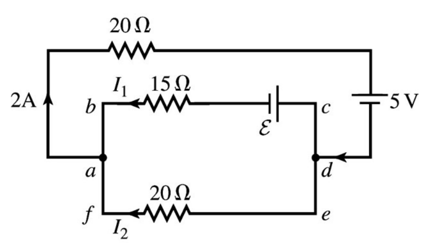


Figure 26.63

26.64. **IDENTIFY:** The resistor R_2 can vary between 3.00 Ω and 24.0 Ω. R_2 is in parallel with R_1, so as R_2 is changed it affects the current in R_1 and hence the power dissipated in R_1. Ohm's law and Kirchhoff's rules apply.

SET UP: $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots$, $V_R = IR$, $P_R = I^2R$.

EXECUTE: $P_1 = V_1^2/R_1$, so P_1 is largest when V_1 is largest. By Kirchhoff's loop rule,

$\mathcal{E} - V_1 - V_3 = 0$, so $V_1 = \mathcal{E} - V_3$, which means that V_1 is largest when V_3 is smallest.
$V_3 = IR_3 = \mathcal{E}/(R_{eq} + R_3)$, where R_{eq} is the equivalent resistance of the R_1-R_2 combination. Since they are in parallel, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$, which gives $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$. The smallest V_3 is for the smallest I, which occurs for the largest $R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{R_1}{\frac{R_1}{R_2} + 1}$.

As we can see, the largest R_{eq} occurs when R_2 is largest, which is $R_2 = 24.0\ \Omega$.
The equivalent parallel resistance is then
$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = (6.00\ \Omega)(24.0\ \Omega)/(6.00\ \Omega + 24.0\ \Omega) = 4.80\ \Omega$.
The current I is then
$I = \mathcal{E}/(R_{eq} + R_3) = (24.0\text{ V})/(4.80\ \Omega + 12.0\ \Omega) = 1.429\text{ A}$.
$V_3 = IR_3 = (1.429\text{ A})(12.0\ \Omega) = 17.148\text{ V}$.
The potential difference across R_1 is
$V_1 = \mathcal{E} - V_3 = 24.0\text{ V} - 17.148\text{ V} = 6.852\text{ V}$.
The power dissipated in R_1 is
$P_1 = V_1^2/R_1 = (6.852\text{ V})^2/(6.00\ \Omega) = 7.83\text{ W}$.

EVALUATE: Since all the circuit elements except for R_2 are fixed, varying R_2 affects the current in the circuit as well as the current through R_1.

26.65. **IDENTIFY** and **SET UP:** We want to estimate the cost to use some electrical appliances. The estimates and calculated results are shown in the table in part (a).
EXECUTE: **(a)** See the accompanying table.

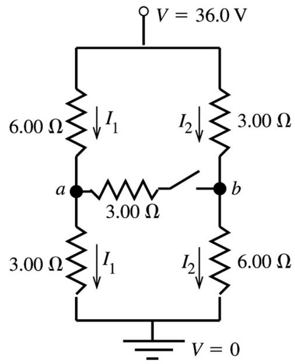
Appliance	Voltage V	Current I	Time T	Power $P = IV$	Energy $U = PT$
Refrigerator	120 V	7.2 A	24 h	864 W	21 kWh
Water heater	240 V	20 A	4 h	4800 W	19 kWh
Electric oven	240 V	16 A	1 h	3840 W	3.8 kWh
Dishwasher	120 V	10 A	2 h	1200 W	2.4 kWh
Clothes washer	120 V	10 A	2/15 h	1200 W	0.2 kWh
Clothes dryer	120 V	26 A	¼ h	3120 W	0.8 kWh
15 light bulbs	120 V	7.5 A	7 h	900 W	6.3 kWh
Stereo system	120 V	0.2	2 h	24 W	0.05 kWh

The total energy is about 54 kWh.
(b) (54 kWh/day)(30 days/mo)($0.12/kWh) = $194/month.
EVALUATE: The refrigerator and water heater are by far the most costly to use, but listening to music costs hardly anything!

26.66. **IDENTIFY:** The current through the 40.0-Ω resistor equals the current through the emf, and the current through each of the other resistors is less than or equal to this current. So, set $P_{40} = 2.00\text{ W}$, and use this to solve for the current I through the emf. If $P_{40} = 2.00\text{ W}$, then P for each of the other resistors is less than 2.00 W.
SET UP: Use the equivalent resistance for series and parallel combinations to simplify the circuit.
EXECUTE: $I^2R = P$ gives $I^2(40\ \Omega) = 2.00\text{ W}$, and $I = 0.2236\text{ A}$. Now use series/parallel reduction to simplify the circuit. The upper parallel branch is $6.38\ \Omega$ and the lower one is $25\ \Omega$. The series sum is now $126\ \Omega$. Ohm's law gives $\mathcal{E} = (126\ \Omega)(0.2236\text{ A}) = 28.2\text{ V}$.

EVALUATE: The power input from the emf is $\mathcal{E}I = 6.30$ W, so nearly one-third of the total power is dissipated in the 40.0-Ω resistor.

26.67. **(a) IDENTIFY and SET UP:** The circuit is sketched in Figure 26.67a.



With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

Figure 26.67a

The potential drop across each parallel branch is 36.0 V. Use this fact to calculate I_1 and I_2. Then travel from point *a* to point *b* and keep track of the potential rises and drops in order to calculate V_{ab}.

EXECUTE: $-I_1(6.00\,\Omega + 3.00\,\Omega) + 36.0\text{ V} = 0.$

$$I_1 = \frac{36.0\text{ V}}{6.00\ \Omega + 3.00\,\Omega} = 4.00\text{ A.}$$

$$-I_2(3.00\,\Omega + 6.00\,\Omega) + 36.0\text{ V} = 0.$$

$$I_2 = \frac{36.0\text{ V}}{3.00\ \Omega + 6.00\,\Omega} = 4.00\text{ A.}$$

To calculate $V_{ab} = V_a - V_b$ start at point *b* and travel to point *a*, adding up all the potential rises and drops along the way. We can do this by going from *b* up through the 3.00-Ω resistor:

$V_b + I_2(3.00\,\Omega) - I_1(6.00\,\Omega) = V_a.$

$V_a - V_b = (4.00\text{ A})(3.00\,\Omega) - (4.00\text{ A})(6.00\,\Omega) = 12.0\text{ V} - 24.0\text{ V} = -12.0\text{ V.}$

$V_{ab} = -12.0$ V (point *a* is 12.0 V lower in potential than point *b*).

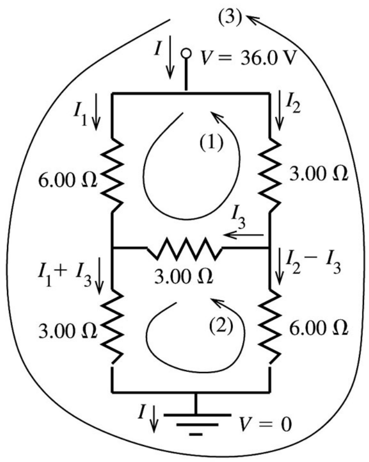
EVALUATE: Alternatively, we can go from point *b* down through the 6.00-Ω resistor.

$V_b - I_2(6.00\,\Omega) + I_1(3.00\,\Omega) = V_a.$

$V_a - V_b = -(4.00\text{ A})(6.00\,\Omega) + (4.00\text{ A})(3.00\,\Omega) = -24.0\text{ V} + 12.0\text{ V} = -12.0\text{ V}$, which checks.

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.67b. Use the junction rule to write the current in each branch in terms of three unknown currents I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3, the current through the switch. R_{eq} is calculated from $V = IR_{\text{eq}}$, where *I* is the total current that passes through the network.

SET UP:



The three unknown currents I_1, I_2, and I_3 are labeled on Figure 26.67b.

Figure 26.67b

EXECUTE: Apply the loop rule to loops (1), (2) and (3).
<u>Loop (1)</u>: $-I_1(6.00\ \Omega)+I_3(3.00\ \Omega)+I_2(3.00\ \Omega)=0$

$I_2 = 2I_1 - I_3$ eq. (1)

<u>Loop (2)</u>: $-(I_1+I_3)(3.00\ \Omega)+(I_2-I_3)(6.00\ \Omega)-I_3(3.00\ \Omega)=0$

$6I_2-12I_3-3I_1=0$ so $2I_2-4I_3-I_1=0$

Use eq (1) to replace I_2:

$4I_1-2I_3-4I_3-I_1=0$

$3I_1=6I_3$ and $I_1=2I_3$ eq. (2)

<u>Loop (3)</u>: This loop is completed through the battery (not shown), in the direction from the $-$ to the $+$ terminal.

$-I_1(6.00\ \Omega)-(I_1+I_3)(3.00\ \Omega)+36.0\ \text{V}=0$

$9I_1+3I_3=36.0\ \text{A}$ and $3I_1+I_3=12.0\ \text{A}$ eq. (3)

Use eq. (2) in eq. (3) to replace I_1:

$3(2I_3)+I_3=12.0\ \text{A}$

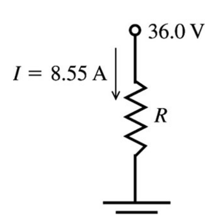
$I_3=12.0\ \text{A}/7=1.71\ \text{A}$

$I_1=2I_3=3.42\ \text{A}$

$I_2=2I_1-I_3=2(3.42\ \text{A})-1.71\ \text{A}=5.13\ \text{A}$

The current through the switch is $I_3=1.71\ \text{A}$.

(c) SET UP and **EXECUTE:** From the results in part (a) the current through the battery is $I=I_1+I_2=3.42\ \text{A}+5.13\ \text{A}=8.55\ \text{A}$. The equivalent circuit is a single resistor that produces the same current through the 36.0-V battery, as shown in Figure 26.67c.



$-IR+36.0\ \text{V}=0.$

$$R=\frac{36.0\ \text{V}}{I}=\frac{36.0\ \text{V}}{8.55\ \text{A}}=4.21\ \Omega.$$

Figure 26.67c

EVALUATE: With the switch open (part a), point b is at higher potential than point a, so when the switch is closed the current flows in the direction from b to a. With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.

26.68. IDENTIFY: $P_{\text{tot}} = \dfrac{V^2}{R_{\text{eq}}}$.

SET UP: Let R be the resistance of each resistor.

EXECUTE: When the resistors are in series, $R_{\text{eq}} = 3R$ and $P_{\text{s}} = \dfrac{V^2}{3R}$. When the resistors are in parallel, $R_{\text{eq}} = R/3$. $P_{\text{p}} = \dfrac{V^2}{R/3} = 3\dfrac{V^2}{R} = 9P_{\text{s}} = 9(45.0\text{ W}) = 405\text{ W}$.

EVALUATE: In parallel, the voltage across each resistor is the full applied voltage V. In series, the voltage across each resistor is $V/3$ and each resistor dissipates less power.

26.69. IDENTIFY: We have an R-C circuit with a discharging capacitor.

SET UP: $q = Q_0 e^{-t/RC}$, $C = \dfrac{\epsilon_0 AK}{d}$, $R = \dfrac{\rho L}{A}$, $\tau = RC$.

EXECUTE: **(a)** We want the time constant. When S is opened, we have a discharging capacitor. $\tau = RC = \left(\dfrac{\rho d}{A}\right)\left(\dfrac{\epsilon_0 AK}{d}\right) = \epsilon_0 \rho K$.

(b) We want Q_0 if $V = 5.00$ V. $Q_0 = CV = \dfrac{\epsilon_0 AKV}{d} = 1.28$ nC using the given numbers.

(c) At what time will q be $Q_0/2$? Solve $q = Q_0 e^{-t/RC} = Q_0 e^{-t/\tau}$ for t using logarithms. At this time $q = Q_0/2$. $t = \tau \ln 2 = \epsilon_0 \rho K \ln 2 = 165$ s using the given numbers.

(d) We want the current. $V_R = V_C$ so $RI = \dfrac{q}{C} = \dfrac{Q_0}{2C}$. $I = \dfrac{Q_0}{2RC} = \dfrac{Q_0}{2\epsilon_0 K\rho}$ using the result from (a).

Putting in the given numbers gives $I = 2.68$ pA.

EVALUATE: The current in (d) is very small, but the dielectric has a very large resistivity so the result is reasonable. The leakage current is the reason that capacitors in electrical devices eventually discharge if the device has been turned off for a long time.

26.70. IDENTIFY and **SET UP:** Just after the switch is closed the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through R_3 is zero. After a long time the capacitor can be replaced by a break in the circuit.

EXECUTE: **(a)** Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{6.00\ \Omega} + \dfrac{1}{3.00\ \Omega} = \dfrac{3}{6.00\ \Omega}$; $R_{\text{eq}} = 2.00\ \Omega$. In the absence of the capacitor, the total current in the circuit (the current through the $8.00\text{-}\Omega$ resistor) would be $i = \dfrac{\mathcal{E}}{R} = \dfrac{42.0\text{ V}}{8.00\ \Omega + 2.00\ \Omega} = 4.20\text{ A}$, of which 2/3, or 2.80 A, would go through the $3.00\text{-}\Omega$ resistor and 1/3, or 1.40 A, would go through the $6.00\text{-}\Omega$ resistor. Since the current through the capacitor is given by $i = \dfrac{V}{R}e^{-t/RC}$, at the instant $t = 0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The $8.00\text{-}\Omega$ and the $6.00\text{-}\Omega$ resistors are now in series, and the current through them is $i = \mathcal{E}/R = (42.0\text{ V})/(8.00\ \Omega +$

$6.00\ \Omega) = 3.00\text{ A}$. The voltage drop across both the $6.00\text{-}\Omega$ resistor and the capacitor is thus $V = iR = (3.00\text{ A})(6.00\ \Omega) = 18.0\text{ V}$. (There is no current through the $3.00\text{-}\Omega$ resistor and so no voltage drop across it.) The charge on the capacitor is $Q = CV = (4.00\times10^{-6}\text{ F})(18.0\text{ V}) = 7.2\times10^{-5}\text{ C}$.

EVALUATE: The equivalent resistance of R_2 and R_3 in parallel is less than R_3, so initially the current through R_1 is larger than its value after a long time has elapsed.

26.71. **IDENTIFY:** We have a capacitor that contains a dielectric and is in a series circuit with a resistor and a battery.

SET UP and **EXECUTE:** **(a)** We want the charge. $Q_0 = CV_0$. $C = \dfrac{\epsilon_0 A}{d} = 1.18\text{ pF}$ using the given A and d. $Q_0 = (1.18\text{ pF})(10.0\text{ V}) = 11.8\text{ pC}$.

(b) The target variable is the current. $V_C = Ed = (E_0/K)d = V_0/K$. For the complete circuit $\mathcal{E} = RI + V_C = RI + V_0/K = RI + \mathcal{E}/K$. $I = \dfrac{\mathcal{E}}{R}\left(1-\dfrac{1}{K}\right) = \dfrac{10.0\text{ V}}{10.0\ \Omega}\left(1-\dfrac{1}{12.0}\right) = 0.917\text{ A}$.

(c) We want the initial energy in the capacitor. $U_C = \dfrac{1}{2}CV_C^2 = \dfrac{1}{2}(KC_0)\left(\dfrac{V_0}{K}\right)^2 = \dfrac{U_0}{K} = 4.92\text{ pJ}$. (This result also tells us that the stored energy before the dielectric was inserted was $U_0 = (12.0)(4.92\text{ pJ}) = 59.0\text{ pJ}$.)

(d) We want the final energy in the capacitor. $U_\text{f} = \dfrac{1}{2}CV_\text{f}^2 = \dfrac{1}{2}(KC_0)\mathcal{E}^2 = KU_0$.

$\Delta U = KU_0 - U_0 = (12.0)(59.0\text{ pJ}) - 4.92\text{ pJ} = 703\text{ pJ}$.

(e) We want the total energy supplied by the battery. $U_\mathcal{E} = \int P_\mathcal{E}dt = \int i\mathcal{E}dt$. $i = I_0e^{-t/RC} = I_0e^{-t/RKC_0}$.

Therefore $U_\mathcal{E} = \int_0^\infty I_0e^{-t/RKC_0}\mathcal{E}dt = I_0KRC_0\mathcal{E} = (0.917\text{ A})(12.0)(10.0\ \Omega)(1.18\text{ pF})(10.0\text{ V}) = 1298\text{ pJ}$, which rounds to 1300 pJ.

(f) We want the energy dissipated in the resistor. $U_R = \int P_Rdt = \int i^2Rdt\int_0^\infty\left(I_0e^{-t/RKC_0}\right)^2dt = \dfrac{I_0^2R^2C_0K}{2} =$ 595 pJ using the given numbers.

EVALUATE: Check: $U_C + U_R = 703\text{ pJ} + 595\text{ pJ} = 1298\text{ pJ} = U_\mathcal{E}$, which is consistent with energy conservation.

26.72. **IDENTIFY** and **SET UP:** $P_R = i^2R$, $\mathcal{E} - iR - \dfrac{q}{C} = 0$, and $U_C = \dfrac{q^2}{2C}$.

EXECUTE: $P_R = i^2R$ so $i = \sqrt{\dfrac{P_R}{R}} = \sqrt{\dfrac{300\text{ W}}{5.00\ \Omega}} = 7.746\text{ A}$. $\mathcal{E} - iR - \dfrac{q}{C} = 0$ so

$q = C(\mathcal{E} - iR) = (6.00\times10^{-6}\text{ F})[50.0\text{ V} - (7.746\text{ A})(5.00\ \Omega)] = 6.762\times10^{-5}\text{ C}$.

$U_C = \dfrac{q^2}{2C} = \dfrac{(6.762\times10^{-5}\text{ C})^2}{2(6.00\times10^{-6}\text{ F})} = 3.81\times10^{-4}\text{ J}$.

EVALUATE: The energy stored in the capacitor can be returned to a circuit as current, but the energy dissipated in a resistor cannot.

26.73. **IDENTIFY:** Connecting the voltmeter between point b and ground gives a resistor network and we can solve for the current through each resistor. The voltmeter reading equals the potential drop across the 200-kΩ resistor.

SET UP: For two resistors in parallel, $\dfrac{1}{R_\text{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. For two resistors in series, $R_\text{eq} = R_1 + R_2$.

EXECUTE: **(a)** $R_{eq} = 100\text{ k}\Omega + \left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 140\text{ k}\Omega$. The total current is

$I = \dfrac{0.400\text{ kV}}{140\text{ k}\Omega} = 2.86\times10^{-3}$ A. The voltage across the $200\text{-k}\Omega$ resistor is

$V_{200\text{ k}\Omega} = IR = (2.86\times10^{-3}\text{ A})\left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 114.4\text{ V}.$

(b) If the resistance of the voltmeter is $5.00\times10^{6}\ \Omega$, then we carry out the same calculations as above to find $R_{eq} = 292\text{ k}\Omega$, $I = 1.37\times10^{-3}$ A and $V_{200\text{ k}\Omega} = 263$ V.

(c) If the resistance of the voltmeter is infinite, then we find $R_{eq} = 300\text{ k}\Omega$, $I = 1.33\times10^{-3}$ A and $V_{200\text{k}\Omega} = 266$ V.

EVALUATE: When a voltmeter of finite resistance is connected to a circuit, current flows through the voltmeter and the presence of the voltmeter alters the currents and voltages in the original circuit. The effect of the voltmeter on the circuit decreases as the resistance of the voltmeter increases.

26.74. **IDENTIFY** and **SET UP:** Zero current through the galvanometer means the current I_1 through *N* is also the current through *M* and the current I_2 through *P* is the same as the current through *X*. And it means that points *b* and *c* are at the same potential, so $I_1N = I_2P$.

EXECUTE: **(a)** The voltage between points *a* and *d* is $\mathcal{E}$, so $I_1 = \dfrac{\mathcal{E}}{N+M}$ and $I_2 = \dfrac{\mathcal{E}}{P+X}$. Using these expressions in $I_1N = I_2P$ gives $\dfrac{\mathcal{E}}{N+M}N = \dfrac{\mathcal{E}}{P+X}P$. $N(P+X) = P(N+M)$. $NX = PM$ and $X = MP/N$.

(b) $X = \dfrac{MP}{N} = \dfrac{(850.0\ \Omega)(33.48\ \Omega)}{15.00\ \Omega} = 1897\ \Omega$

EVALUATE: The measurement of *X* does not require that we know the value of the emf.

26.75. **IDENTIFY:** With *S* open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When *S* is closed, current *I* flows through the $6.00\text{-}\Omega$ and $3.00\text{-}\Omega$ resistors.
SET UP: With the switch closed, *a* and *b* are at the same potential and the voltage across the $6.00\text{-}\Omega$ resistor equals the voltage across the $6.00\text{-}\mu\text{F}$ capacitor and the voltage is the same across the $3.00\text{-}\mu\text{F}$ capacitor and $3.00\text{-}\Omega$ resistor.
EXECUTE: **(a)** With an open switch: $V_{ab} = \varepsilon = 18.0$ V.

(b) Point *a* is at a higher potential since it is directly connected to the positive terminal of the battery.
(c) When the switch is closed $18.0\text{ V} = I(6.00\ \Omega + 3.00\ \Omega)$. $I = 2.00$ A and $V_b = (2.00\text{ A})(3.00\ \Omega) = 6.00$ V.

(d) Initially the capacitor's charges were $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V}) = 5.40\times10^{-5}$ C and $Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V}) = 1.08\times10^{-4}$ C. After the switch is closed $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V} - 12.0\text{ V}) = 1.80\times10^{-5}$ C and $Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V} - 6.0\text{ V}) = 7.20\times10^{-5}$ C. Both capacitors lose 3.60×10^{-5} C $= 36.0\ \mu$C.

EVALUATE: The voltage across each capacitor decreases when the switch is closed, because there is then current through each resistor and therefore a potential drop across each resistor.

26.76. **IDENTIFY:** The energy stored in a capacitor is $U = q^2/2C$. The electrical power dissipated in the resistor is $P = i^2 R$.

SET UP: For a discharging capacitor, $i = -\dfrac{q}{RC}$.

EXECUTE: **(a)** $U_0 = \dfrac{Q_0^2}{2C} = \dfrac{(0.0069\text{ C})^2}{2(4.62\times10^{-6}\text{ F})} = 5.15\text{ J}.$

(b) $P_0 = I_0^2 R = \left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{(0.0069\text{ C})^2}{(850\ \Omega)(4.62\times10^{-6}\text{ F})^2} = 2620\text{ W}.$

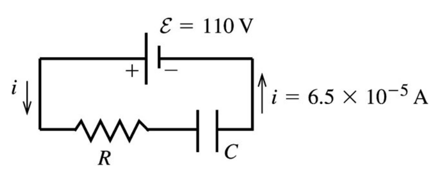
(c) Since $U = q^2/2C$, when $U \to U_0/2, q \to Q_0/\sqrt{2}$. Since $q = Q_0 e^{-t/RC}$, this means that $e^{-t/RC} = 1/\sqrt{2}$. Therefore the current is $i = i_0 e^{-t/RC} = i_0/\sqrt{2}$. Therefore

$P_R = \left(\dfrac{i_0}{\sqrt{2}}\right)^2 R = \dfrac{1}{2}\left(\dfrac{V_0}{R}\right)^2 R = \dfrac{1}{2}\left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{1}{RC}\left(\dfrac{Q_0^2}{2C}\right) = \dfrac{U_0}{RC}$. Putting in the numbers gives

$P_R = \dfrac{5.15\text{ J}}{(850\ \Omega)(4.62\ \mu\text{F})} = 1310\text{ W}.$

EVALUATE: All the energy originally stored in the capacitor is eventually dissipated as current flows through the resistor.

26.77. **IDENTIFY:** Apply the loop rule to the circuit. The initial current determines R. We can then use the time constant to calculate C.
SET UP: The circuit is sketched in Figure 26.77.



Initially, the charge of the capacitor is zero, so by $V = q/C$ the voltage across the capacitor is zero.

Figure 26.77

EXECUTE: The loop rule therefore gives $\mathcal{E} - iR = 0$ and $R = \dfrac{\mathcal{E}}{i} = \dfrac{110\text{ V}}{6.5\times10^{-5}\text{ A}} = 1.7\times10^6\ \Omega.$

The time constant is given by $\tau = RC$, so $C = \dfrac{\tau}{R} = \dfrac{5.2\text{ s}}{1.7\times10^6\ \Omega} = 3.1\ \mu\text{F}.$

EVALUATE: The resistance is large so the initial current is small and the time constant is large.

26.78. **IDENTIFY** and **SET UP:** When the switch S is closed, current begins to flow as the capacitor plates discharge. The current in the circuit is $i = (Q_0/RC)e^{-t/RC}$.

EXECUTE: **(a)** Taking logs of the equation for i gives $\ln(i) = \ln(Q_0/RC) - t/RC$. A graph of $\ln(i)$ versus t will be a straight line with slope equal to $-1/RC$.
(b) Using the points (1.50 ms, –3.0) and (3.00 ms, –4.0) on the graph in the problem, the slope is
slope = $\dfrac{-4.0-(-3.0)}{3.00\text{ ms} - 1.50\text{ ms}} = -0.667\ (\text{ms})^{-1} = -667\text{ s}^{-1}$. Therefore
$-1/RC = -667\text{ s}^{-1}$.
$C = 1/[(196\ \Omega)(667\text{ s}^{-1})] = 7.65\times10^{-6}$ F, which rounds to 7.7 μF.
Using point (1.50 ms, –3.0) on the graph, the equation of the graph gives
$-3.0 = \ln(Q_0/RC) - (1.50\text{ ms})/RC$.
Simplifying and rearranging gives
$-2.0 = \ln(Q_0/RC)$.

$Q_0 = RC\ e^{-2.0} = (196\ \Omega)(7.65\ \mu F)\ e^{-2.0} = 203\ \mu C,$ which rounds to 200 μC.

(c) Taking a loop around the circuit gives

$V_R + V_C = 0.$

$-IR + Q/C = 0.$

$Q = RCI = (196\ \Omega)(7.65\ \mu\text{F})(0.0500\ \text{A}) = 75\mu C.$

(d) From (c), we have $Q = RCI$, so $I = Q/RC = (500\mu C)/[(196\ \Omega)(7.65\ \mu\text{F})] = 0.33$ A.

EVALUATE: The accuracy of the answers depends on how well we can get information from the graph with the problem, so answers may differ slightly from those given here.

26.79. **IDENTIFY** and **SET UP:** Kirchhoff's rules apply to the circuit. Taking a loop around the circuit gives $\mathcal{E} - Ri - q/C = 0.$

EXECUTE: **(a)** Solving the loop equation for q gives $q = q = \mathcal{E}C - RCi.$ A graph of q as a function of i should be a straight line with slope equal to $-RC$ and y-intercept equal to $\mathcal{E}C.$ Figure 26.79 shows this graph.

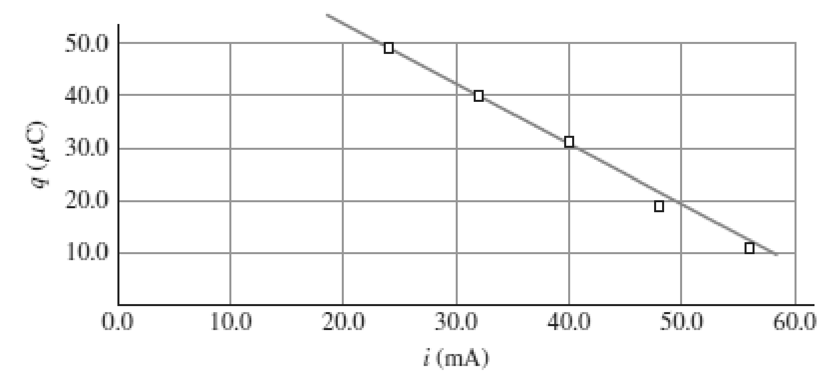


Figure 26.79

The best-fit slope of this graph is -1.233×10^{-3} C/A, and the y-intercept is 7.054×10^{-5} C.

(b) $RC = -\text{slope} = -(-1.233 \times 10^{-3}\ \text{C/A})$, which gives

$R = (-1.233 \times 10^{-3}\ \text{C/A})/(5.00 \times 10^{-6}\ \text{F}) = 246.6\ \Omega$, which rounds to 247 Ω.

The y-intercept is $\mathcal{E}C$, so

$7.054 \times 10^{-5}\ \text{C} = \mathcal{E}\ (5.00 \times 10^{-6}\ \text{F}).$

$\mathcal{E} = 15.9$ V.

(c) $V_C = \mathcal{E}(1 - e^{-t/RC}).$

$V_C/\mathcal{E} = 1 - e^{-t/RC} = (10.0\ \text{V})(15.9\ \text{V}).$

Solving for t gives

$t = (247\ \Omega)(5.00\ \mu\text{F})\ \ln(0.3714) = 1223\ \mu\text{s}$, which rounds to 1.22 ms.

(d) $V_R = \mathcal{E} - V_C = 15.9\ \text{V} - 4.00\ \text{V} = 11.9$ V.

EVALUATE: As time increases, the potential difference across the capacitor increases as it gets charged, but the potential difference across the resistor decreases as the current decreases.

26.80. **IDENTIFY** and **SET UP:** When connected in series across a 48.0-V battery, R_1 and R_2 dissipate 48.0 W of power, and when in parallel across the same battery, they dissipate a total of 256 W. $PR = I^2R = V^2/R.$

EXECUTE: **(a)** In series: $I = \mathcal{E}/(R_1 + R_2).$

$P_s = I^2(R_1 + R_2) = [\mathcal{E}/(R_1 + R_2)]2(R_1 + R_2) = \mathcal{E}^2/(R_1 + R_2).$

$48.0\ \text{W} = (48.0\ \text{V})^2/(R_1 + R_2).$

$R_1 + R_2 = 48.0\ \Omega.$

<u>In parallel</u>: $P_p = I_1^2 R_1 + I_2^2 R_2 = \frac{\mathcal{E}^2}{R_1^2} R_1 + \frac{\mathcal{E}^2}{R_2^2} R_2 = \mathcal{E}^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \mathcal{E}^2 \left(\frac{R_1 + R_2}{R_1 R_2} \right) = 256 \text{ W}.$

Therefore $(48.0 \text{ V})^2 \left(\frac{R_1 R_2}{R_1 + R_2} \right) = 256 \text{ W}$. Using $R_1 + R_2 = 48.0\ \Omega$, this becomes $R_1 R_2 = 432\ \Omega^2$.

Solving the two equations for R_1 and R_2 simultaneously, we get two sets of answers: $R_1 = 36.0\ \Omega$, $R_2 = 12.0\ \Omega$ and $R_1 = 12.0\ \Omega$, $R_2 = 36.0\ \Omega$. But we are told that that $R_1 > R_2$, so the solution to use is $R_1 = 36.0\ \Omega$, $R_2 = 12.0\ \Omega$.

(b) In series, both resistors have the same current. $P = I^2 R$, so the larger resistor, which is R_1, consumes more power.

(c) In parallel, the potential difference across both resistors is the same. $P = V^2 R$, so the smaller resistor, which is R_2, consumes more power.

EVALUATE: If we did not know which resistor was larger, we would know that one resistor was 12.0 Ω and the other was 36.0 Ω, but we would not know which one was the larger of the two.

26.81. **IDENTIFY:** This problem requires Kirchhoff's rules.

SET UP: Refer to Fig. 26.81 with the problem in the text. The given relations are:
I: $I_C + I_B = I_E$. II: $V_e = V_b - 0.60$ V. III: $I_C = \beta I_B$ ($\beta << 1$).

Use the following loops:

<u>Loop 1</u>: Clockwise through the small circuit containing V_{in} and the 100 Ω resistor.

<u>Loop 2</u>: Clockwise around the outside of the full circuit.

EXECUTE: **(a)** We want V_{out} in terms of V_{in}. If $\beta \to \infty, I_B = I_C/\beta \to 0$, so $I_C = I_E$. Apply Kirchhoff's rules.

<u>Loop 1</u>: $V_{in} - 0.60 \text{ V} - I_E(100\ \Omega) = 0$

<u>Loop 2</u>: $15 \text{ V} - I_C(1 \text{ kV}) - V_{out} = 0$

Using $I_C = I_E$ we get $I_E = (V_{in} - 0.60 \text{ V})/(100\ \Omega)$. Combining and solving gives $V_{out} = 21 \text{ V} - 10V_{in}$.

(b) We want V_{in} so V_{out} is 7.5 V. From (a): $V_{in} = (21 \text{ V} - V_{out})/10 = (21 \text{ V} - 7.5 \text{ V})/10 = 1.35$ V.

(c) The target variable is G. If V_{in} were just 15.0 V, then V_{out} would be $V_{out} = 21 \text{ V} - 10(15.0 \text{ V}) = -129$ V. But with the small $v(t)$, $V_{out} = 21 \text{ V} - 10(V_{in} + v_{in}) = 21 \text{ V} - 10(15.0 \text{ V} + v_{in})$. The coefficient of v_{in} is –10, so $G = -10$.

EVALUATE: Kirchhoff's rules apply to any type of circuit.

26.82. **IDENTIFY:** This problem involves a capacitor in an *R-C* circuit. We need to use Kirchhoff's rules.

SET UP: Refer to Fig. 26.82 with the problem in the textbook.

EXECUTE: **(a)** We want V_{out}. After a long time, the capacitor is fully charged, so $V_{out} = V_R = IR$.
$I = \mathcal{E}/5R$, so $V_{out} = R(\mathcal{E}/5R) = \mathcal{E}/5 = (15 \text{ V})/5 = 3.0$ V.

(b) We want the time constant τ_{ch} during charging, which is with S open. The resistance in the circuit is $4R$, so $\tau_{ch} = 4RC$.

(c) We want the time constant τ_d during discharging, which is with S closed. Apply Kirchhoff's rules.

The current choices are: I_1 is downward through R; I_2 is upward through C, and I_4 is downward through $4R$.

<u>Loop 1</u>: Clockwise through the small circuit with R and C: $I_1 R = q/C$.

<u>Loop 2</u>: Clockwise around the outside of the circuit: $\mathcal{E} - 4RI_4 - q/C = 0$.

<u>Junction rule</u>: $I_4 = I_1 - I_2$.

The capacitor is discharging, so $I_2 = -dq/dt$.

Combining these equations gives $\frac{dq}{dt} = \frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}$. From this result we see that $\tau_d = 4RC/5$.

(d) During the charging-discharging cycle, we want the time between successive 10.0 V output voltages across the capacitor. In one complete cycle, the potential difference across the capacitor discharges from 10.0 V to 5.0 V and then recharges from 5.0 V back to 10.0 V. The time constants in the two parts of the cycle are *not* the same.

Discharging: Using the result of part (c), solve $\frac{dq}{dt} = \frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}$. The circuit discharges from 10.0 V to 5.0 V, so the initial voltage across the capacitor is $V_{0,d}$ = 10.0 V. Separate variables and integrate.

$\int \frac{dq}{\mathcal{E}/4R - q/(4RC/5)} = \int dt$ gives $\ln\left(\frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}\right) = t + K$, where K is a constant of integration.

Putting this result into exponential form gives $K' e^{-5t/4RC} = \frac{\mathcal{E}}{4R} - \frac{q}{4RC/5}$, where K' is a constant. When $t = 0$, $q = Q_0$, which gives $K' = \frac{\mathcal{E}}{4R} - \frac{Q_0}{4RC/5}$. Using this result, $V = q/C$, and $V_{0,d} = Q_0/C$ gives

$\left(\frac{\mathcal{E}}{4R} - \frac{V_{0,d}}{4R/5}\right) e^{-5t/4RC} = \frac{\mathcal{E}}{4R} - \frac{V}{4R/5}$. Solving for V (and calling it V_d) and simplifying, we get

$V_d = \frac{\mathcal{E}}{5} + \left(V_{0,d} - \frac{\mathcal{E}}{5}\right) e^{-5t/4RC}$.

(Check: At $t = 0$, $\mathcal{E}$ = 15 V and $V_{0,d}$ = 10.0 V, which gives V_d = 10 V, as it should. For $t \to \infty$ we have V_d = 3.0 V, which agrees with our result in part (a).)

Charging: The circuit is a simple series circuit containing the battery, the capacitor, and a resistance $4R$. It charges from 5.0 V to 10.0 V, so $V_{0,ch}$ = 5.0 V. Applying Kirchhoff's loop rule gives $\frac{\mathcal{E}}{4R} - \frac{q}{4RC} = \frac{dq}{dt}$.

We solve this differential equation as we did for discharging. Separate variables and integrate, using the initial condition that $V = V_{0,ch}$ when $t = 0$. Carrying out these steps and solving for V_{ch} gives $V_{ch} = \mathcal{E} - \left(\mathcal{E} - V_{0,ch}\right) e^{-t/4RC}$. (Check: At $t = 0$, V_{ch} = 15 V – (15 V – 5.0 V) = 5.0 V, as we should get. As $t \to \infty$, $V_{ch} \to \mathcal{E}$ as it should.) Now we find the time to charge the capacitor from 5.0 V to 10.0 V and to discharge it from 10.0 V to 5.0 V.

Charging from 5.0 V to 10.0 V: Using $V_{ch} = \mathcal{E} - \left(\mathcal{E} - V_{0,ch}\right) e^{-t/4RC}$ with $\mathcal{E}$ = 15 V, $V_{0,ch}$ = 5.0 V, and V_{ch} = 10.0 V, we have 10.0 V = 15 V – (15 V – 5.0 V)$e^{-t/4RC}$. Solving for the charging time t_{ch} gives $t_{ch} = 4RC \ln 2$.

Discharging from 10.0 V to 5.0 V: Use $V_d = \frac{\mathcal{E}}{5} + \left(V_{0,d} - \frac{\mathcal{E}}{5}\right) e^{-5t/4RC}$ with $\mathcal{E}$ = 15 V, $V_{0,d}$ = 10.0 V, and V_d = 5.0 V, we have 5.0 V = 3.0 V + (10.0 V – 3.0 V)$e^{-5t/4RC}$. Solving for the discharge time t_d gives $t_d = \frac{4RC}{5}\ln(7/2)$.

The total time T for one cycle is $T = t_d + t_{ch} = \frac{4RC}{5}\ln(7/2) + 4RC\ln 2$. Simplifying gives

$T = \frac{4RC}{5}\left(\ln 7 + 4\ln 2\right)$.

(e) We want the frequency f of operation. Using $f = 1/T$ with R = 10.0 kΩ and C = 10.0 μF, we have T = 4(10.0 kΩ)(10.0 μF)(ln 7 + 4 ln 2)/5 = 0.3775 s. $f = 1/T$ = 1/(0.3775 s) = 2.65 Hz.

EVALUATE: The charging and discharging times are different because the time constants are different.

26.83. **IDENTIFY:** Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.83.

EXECUTE: $R_T = 2R_1 + \left(\dfrac{1}{R_2} + \dfrac{1}{R_T}\right)^{-1} = 2R_1 + \dfrac{R_2 R_T}{R_2 + R_T}$. $R_T{}^2 - 2R_1R_T - 2R_1R_2 = 0$.

$R_T = R_1 \pm \sqrt{R_1{}^2 + 2R_1R_2}$. $R_T > 0$, so $R_T = R_1 + \sqrt{R_1{}^2 + 2R_1R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

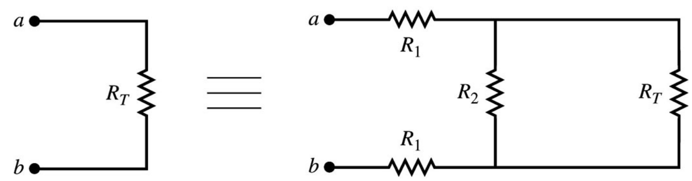


Figure 26.83

26.84. **IDENTIFY:** Assume a voltage V applied between points a and b and consider the currents that flow along each path between a and b.

SET UP: The currents are shown in Figure 26.84.

EXECUTE: Let current I enter at a and exit at b. At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is

$V = \left(\dfrac{I}{3}\right)R + \left(\dfrac{I}{6}\right)R + \left(\dfrac{I}{3}\right)R = \frac{5}{6}IR$. This must be the same as $V = IR_{\text{eq}}$, so $R_{\text{eq}} = \frac{5}{6}R$.

EVALUATE: The equivalent resistance is less than R, even though there are 12 resistors in the network.

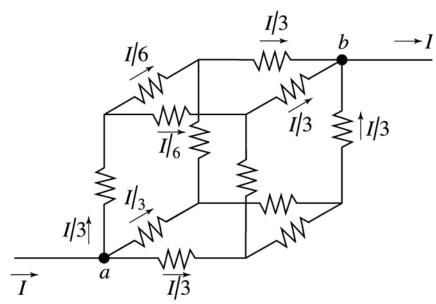


Figure 26.84

26.85. **IDENTIFY:** The network is the same as the one in Challenge Problem 26.83, and that problem shows that the equivalent resistance of the network is $R_T = \sqrt{R_1^2 + 2R_1R_2}$.

SET UP: The circuit can be redrawn as shown in Figure 26.85.

EXECUTE: **(a)** $V_{cd} = V_{ab}\dfrac{R_{\text{eq}}}{2R_1 + R_{\text{eq}}} = V_{ab}\dfrac{1}{2R_1/R_{\text{eq}} + 1}$ and $R_{\text{eq}} = \dfrac{R_2R_T}{R_2 + R_T}$. But $\beta = \dfrac{2R_1(R_T + R_2)}{R_TR_2} = \dfrac{2R_1}{R_{\text{eq}}}$,

so $V_{cd} = V_{ab}\dfrac{1}{1+\beta}$.

(b) $V_1 = \dfrac{V_0}{(1+\beta)} \Rightarrow V_2 = \dfrac{V_1}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^2} \Rightarrow V_n = \dfrac{V_{n-1}}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^n}$.

If $R_1 = R_2$, then $R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1R_1} = R_1(1+\sqrt{3})$ and $\beta = \dfrac{2(2+\sqrt{3})}{1+\sqrt{3}} = 2.73$. So, for the *n*th segment to have 1% of the original voltage, we need: $\dfrac{1}{(1+\beta)^n} = \dfrac{1}{(1+2.73)^n} \le 0.01$. This says $n = 4$, and then $V_4 = 0.005V_0$.

(c) $R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1R_2}$ gives $R_T = 6400\ \Omega + \sqrt{(6400\ \Omega)^2 + 2(6400\ \Omega)(8.0\times10^8\ \Omega)} = 3.2\times10^6\ \Omega$ and $\beta = \dfrac{2(6400\ \Omega)(3.2\times10^6\ \Omega + 8.0\times10^8\ \Omega)}{(3.2\times10^6\ \Omega)(8.0\times10^8\ \Omega)} = 4.0\times10^{-3}$.

(d) Along a length of 2.0 mm of axon, there are 2000 segments each 1.0 μm long. The voltage therefore attenuates by $V_{2000} = \dfrac{V_0}{(1+\beta)^{2000}}$, so $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+4.0\times10^{-3})^{2000}} = 3.4\times10^{-4}$.

(e) If $R_2 = 3.3\times10^{12}\ \Omega$, then $R_T = 2.1\times10^8\ \Omega$ and $\beta = 6.2\times10^{-5}$. This gives $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+6.2\times10^{-5})^{2000}} = 0.88$.

EVALUATE: As R_2 increases, β decreases and the potential difference decrease from one section to the next is less.

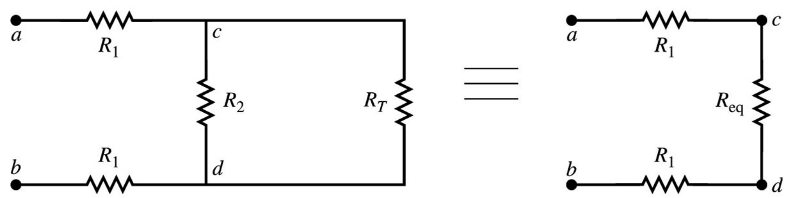


Figure 26.85

26.86. IDENTIFY and SET UP: $R = \dfrac{\rho L}{A}$.

EXECUTE: Solve for ρ: $\rho = \dfrac{AR}{L} = \dfrac{\pi r^2 R}{L} = \dfrac{\pi(0.3\ \text{nm})^2(1\times10^{11}\ \Omega)}{12\ \text{nm}} = 2.4\ \Omega\cdot\text{m}\ \Omega \approx 2\ \Omega\cdot\text{m}$, which is choice (c).

EVALUATE: According to the information in Table 25.1, this resistivity is much greater than that of conductors but much less than that of insulators. It is closer to that of semiconductors.

26.87. IDENTIFY and SET UP: The channels are all in parallel. For *n* identical resistors *R* in parallel, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \ldots = \dfrac{1}{R} + \dfrac{1}{R} + \ldots = \dfrac{n}{R}$, so $R_{eq} = R/n.\ I = jA.$

EXECUTE: $I = jA = V/\text{R}_{eq} = V/(R/n) = nV/R.$

$jR/V = n/A = (5\ \text{mA/cm}^2)(10^{11}\ \Omega)/(50\ \text{mV}) = 10^{10}/\text{cm}^2 = 100/\mu\text{m}^2$, which is choice (d).

EVALUATE: A density of 100 per μm^2 seems plausible, since these are microscopic structures.

26.88. IDENTIFY and SET UP: $\tau = RC$. The resistance is $1\times10^{11}\ \Omega$. *C* is the capacitance per area divided by the number density of channels, which is $100/\mu\text{m}^2$ from Problem 26.87.

EXECUTE: $C = (1\ \mu\text{F/cm}^2)/(100/\mu\text{m}^2) = 10^{-16}$ F. The time constant is

$\tau = RC = (1\times10^{11}\ \Omega)(10^{-16}\ \text{F}) = 1\times10^{-5}\ \text{s} = 10\ \mu s$, which is choice (b).

EVALUATE: This time constant is comparable to that of typical laboratory *RC* circuits.