32

ELECTROMAGNETIC WAVES

VP32.2.1. **IDENTIFY:** This problem is about the properties of an electromagnetic wave.

SET UP: $E = cB$, the wave propagates in the direction of $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** We want E_{max}. $E_{max} = cB_{max} = c(4.30 \text{ mT}) = 1.29 \text{ MV/m}$.

(b) We want the wavelength. $\lambda = 2\pi/k = 2\pi/(2.50 \text{ Mrad/m}) = 2.51\ \mu\text{m}$.

(c) We want the frequency. $f = c/\lambda = c/(2.51\ \mu\text{m}) = 1.19\times10^{14}$ Hz.

(d) We want the direction. The direction is the same as $\vec{E}\times\vec{B}$, which is $+z$.

EVALUATE: This wave is not in the visible part of the spectrum.

VP32.2.2. **IDENTIFY:** This problem is about the properties of an electromagnetic wave.

SET UP: $E = cB$, the wave propagates in the direction of $\vec{E}\times\vec{B}$, $\omega/k = c$.

EXECUTE: **(a)** We want λ. $\lambda = 2\pi/k = 2\pi/(6.50 \text{ Mrad/m}) = 967$ nm.

(b) We want ω. $\omega/k = c$, so $\omega = ck = c(6.50 \text{ Mrad/m}) = 1.95\times10^{14}$ rad/s.

(c) Direction of travel? E is of the form $E = E_{max}\cos(kx - \omega t)$, so the wave is traveling in the $+y$-direction.

(d) We want B. $B = E/c = (2.46 \text{ MV/m})/c = 8.20$ mT. The wave travels in the direction of $\vec{E}\times\vec{B}$. We know that $\vec{E}$ is in the $+x$-direction and $\vec{E}\times\vec{B}$ is in the $+y$-direction. So by the right-hand rule for the cross product, $\vec{B}$ must be in the –z-direction. The full equation for $\vec{B}$ is therefore

$\vec{B} = -\hat{k}(8.20\times10^{-3}\text{ T})\cos[(6.50\times10^{6}\text{ rad/m})y - (1.95\times10^{15}\text{ rad/s})t]$.

EVALUATE: With a wavelength of 967 nm, this is not visible light.

VP32.2.3. **IDENTIFY:** This is an electromagnetic wave.

SET UP: $E = cB$, the wave propagates in the direction of $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** We want $\vec{B}$. $B = E/c = (4.20 \text{ MV/m})/c = 14.0$ mT. The direction of $\vec{E}$ is $+y$ and $\vec{E}\times\vec{B}$ is $+x$, so by the right-hand rule for the cross product, $\vec{B}$ must point in the $+z$-direction.

(b) We want $\vec{E}$ at $x = 1.85\ \mu$m. $E_y = E_{max}\cos kx$. $k = 2\pi/\lambda = 2\pi/(2.94\ \mu\text{m}) = 2.14$ Mrad/m. So $E_y = (4.20 \text{ MV/m})\cos[(2.137 \text{ Mrad/m})(1.85\ \mu\text{m})] = -2.89$ MV/m. The magnitude is 2.89 MV/m and the direction is in the $-y$-direction.

(c) We want $\vec{B}$ at $x = 1.85\ \mu$m. $B = E/c = (2.89 \text{ MV/m})/c = 9.63$ mT. The wave travels in the $+x$-direction and $\vec{E}$ is in the $-y$-direction, so $\vec{B}$ must be in the $-z$-direction by the right-hand rule for $\vec{E}\times\vec{B}$.

EVALUATE: Note that the wave has a large-amplitude electric field but a small-amplitude magnetic field. This is typical.

VP32.2.4. **IDENTIFY:** This problem deals with an electromagnetic wave in matter.

SET UP and **EXECUTE:** **(a)** We want f in vacuum. $f = c/\lambda = c/(1.16\ \mu\text{m}) = 2.59\times10^{14}$ Hz.

(b) We want f in the material. The frequency does not change, only the wavelength and speed change, so $f = 2.59\times10^{14}$ Hz.

(c) We want the speed in the material. Let v indicate vacuum. $\dfrac{v}{c} = \dfrac{f\lambda}{f\lambda_v} = \dfrac{\lambda}{\lambda_v}$.
$v = c\dfrac{\lambda}{\lambda_v} = c\dfrac{0.635\ \mu\text{m}}{1.16\ \mu\text{m}} = 1.64\times10^8$ m/s.

(d) We want K. $c/v = \sqrt{K} \rightarrow K = (c/v)^2$. Using v from (c) gives $K = 3.34$.

EVALUATE: The light slows down and the wavelength decreases as the light enters matter.

VP32.4.1. **IDENTIFY:** We are dealing with the energy in an electromagnetic wave.

SET UP: $E = cB$, $S = \dfrac{1}{\mu_0}EB$, $u = \epsilon_0 E^2$, $S = 11.0$ W/m^2.

EXECUTE: **(a)** We want the magnitudes of the fields. Combine $S = \dfrac{1}{\mu_0}EB$ and $E = cB$ to obtain $S = \dfrac{c}{\mu_0}B^2$. Solve for B and using the given numbers. $B = \sqrt{\mu_0 S/c} = 0.215\ \mu$T. $E = cB = 64.4$ V/m using the B we just found.

(b) We want the energy density. $u = \epsilon_0 E^2 = 36.7$ nJ/m^3 using the value of E that we just found.

EVALUATE: This energy density is typical of many electromagnetic waves.

VP32.4.2. **IDENTIFY:** This problem deals with the Poynting vector $\vec{S}$.

SET UP: $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$, $S = \dfrac{1}{\mu_0}EB$, $k = 2\pi/\lambda$. We want $\vec{S}$ at the following times.

EXECUTE: **(a)** At $x = 0$ and $t = 0$: $\vec{S} = \dfrac{E_{\text{max}}B_{\text{max}}}{\mu_0}\hat{i}$.

(b) At $x = \lambda/4$, $t = 0$: $E = E_{\text{max}}\cos(kx - 0) = E_{\text{max}}\cos\left[\left(\dfrac{2\pi}{\lambda}\right)\left(\dfrac{\lambda}{4}\right)\right] = E_{\text{max}}\cos(\pi/2) = 0$. So it follows that $S = 0$.

(c) At $x = \lambda/4$, $t = \pi/4\omega$:

$$E = E_{\text{max}}\cos(kx - \omega t) = E_{\text{max}}\cos\left[\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right) - \omega\left(\frac{\pi}{4\omega}\right)\right] = E_{\text{max}}\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = E_{\text{max}}\frac{1}{\sqrt{2}}.$$ And likewise $B = B_{\text{max}}\dfrac{1}{\sqrt{2}}$. Multiplying these two magnitudes together gives $\vec{S} = \dfrac{E_{\text{max}}B_{\text{max}}}{2\mu_0}\hat{i}$.

EVALUATE: It is clear that S also varies as the fields vary.

VP32.4.3. **IDENTIFY:** We are dealing with the power in electromagnetic waves.

SET UP: $E = cB$, $I = \dfrac{1}{2}\epsilon_0 cE_{\text{max}}^2$

EXECUTE: **(a)** We want B and I. Magnetic field: $B = E/c = (0.360\ \text{V/m})/c = 1.20$ nT.
Intensity: Use $I = \dfrac{1}{2}\epsilon_0 cE_{\text{max}}^2$ with $E_{\text{max}} = 0.360$ V/m, giving $I = 172\ \mu$W/m^2.

(b) We want the power radiated. $P = IA = I[(4\pi r^2)/2] = 156$ kW.

EVALUATE: The answer in (b) assumes that none of the energy is transformed into other types of energy (such as kinetic energy of air molecules) as the waves travel through the air.

VP32.4.4. **IDENTIFY:** This problem is about the energy in electromagnetic waves.

SET UP: $I = \dfrac{E_{\max}^2}{2\mu_0 c}$, $E = cB$.

EXECUTE: **(a)** We want the amplitudes of the fields. Solve $I = \dfrac{E_{\max}^2}{2\mu_0 c}$ for $E_{\max}$ and use $I = 1.36\ \text{kW/m}^2$. $E_{\max} = \sqrt{2\mu_0 c I} = 1.01$ kV/m. $B_{\max} = (1.01\ \text{kV/m})/c = 3.38\ \mu\text{T}$.

(b) We want the total power radiated by the sun. Use I from (a) and $r = 1.50\times10^{11}$ m. $P = IA = I(4\pi r^2) = 3.85\times10^{26}$ W.

EVALUATE: At half the earth-sun distance the intensity would be 4 times as great, so a planet there would be much hotter than the earth.

VP32.7.1. **IDENTIFY:** We are dealing with standing electromagnetic waves.

SET UP: $S = \dfrac{1}{\mu_0}EB$. We want the Poynting vector in the standing wave at the following times.

EXECUTE: **(a)** We want the maximum the Poynting vector can be. Using Eq. (32.34) and (32.35), we have $S = \dfrac{(2E_{\max}\sin kx \cos \omega t)(2B_{\max}\cos kx \sin \omega t)}{\mu_0}$. The largest that sin kx can be is 1, but then cos kx = 0, and likewise with $\sin \omega t$. The largest product is when sin kx = cos kx, which is when $kx = \pi/4$, and likewise for the $\sin \omega t$ factors. In this case, each factor is equal to $1/\sqrt{2}$. Since we have 4 such factors, the final result is $S_{\max} = \dfrac{1}{\mu_0}E_{\max}B_{\max}$. Using the given amplitudes gives $S_{\max} = 0.215\ \text{W/m}^2$.

(b) At $t = 0$: $\sin \omega t = 0$, so $S = 0$.

(c) At $x = 0.125$ mm, $t = 3.15 \times 10^{-13}$ s: As in Example 32.6, $S = \dfrac{E_{\max}B_{\max}\sin 2kx \sin 2\omega t)}{\mu_0}$. Using the given values for x, t, k, etc., we get $\vec{S} = 0.0907\ \text{W/m}^2\,\hat{\imath}$.

(d) Same approach except $t = 9.45 \times 10^{-13}$ s. $\vec{S} = -0.207\ \text{W/m}^2\,\hat{\imath}$.

EVALUATE: Note that $\vec{S}$ can have negative components.

VP32.7.2. **IDENTIFY:** This problem is about a standing electromagnetic wave.
SET UP: $E = cB$, $f\lambda = c$.

EXECUTE: **(a)** We want the wavelength. The nearest nodal plane is $\lambda/4$ from the conductor, so $\lambda/4$ = 3.60 mm. λ = 14.4 mm.
(b) We want the frequency. $f = c/\lambda = c/(14.4\ \text{mm}) = 2.08 \times 10^{10}$ Hz.

(c) We want the amplitude of E in the first nodal plane of B. The nodal planes of B are antinodal planes of E, so at this point $E = E_{\max} = cB_{\max} = c(0.120\ \mu\text{T}) = 36.0$ V/m.
EVALUATE: The electric and magnetic fields are out of phase by $\pi/2$ in this polarized light.

VP32.7.3. **IDENTIFY:** We are looking at standing electromagnetic waves in a cavity.
SET UP: The walls are at $x = 0$ and $x = 4.50$ cm, $\lambda_n = 2L/n$, $f\lambda = c$.

EXECUTE: **(a)** We want the frequencies and wavelengths. Use $\lambda_n = 2L/n$.

$\lambda_1 = 2L/1 = 2L = 9.00$ cm. $f_1 = c/(9.00\ \text{cm}) = 3.33$ GHz.

$\lambda_2 = L = 4.50$ cm. $f_2 = c/(4.50\ \text{cm}) = 6.67$ GHz.

$\lambda_3 = 2L/3 = 3.00$ cm. $f_3 = c/(3.00\ \text{cm}) = 10.0$ GHz.

(b) We want the nodal planes for E. The nodal planes are $\lambda/2$ apart.

For λ_1 : 0, 4.50 cm.

For λ_2 : 0, 2.25 cm, 4.50 cm.

For λ_3 : 0, 1.50 cm, 3.00 cm, 4.50 cm.

EVALUATE: The antinodal planes for E are midway between the nodal planes.

VP32.7.4. **IDENTIFY:** We are looking at standing electromagnetic waves in a cavity.
SET UP: The walls are 48.8 cm apart, λ = 12.2 cm. $\lambda_n = 2L/n$, $f\lambda = c$
EXECUTE: **(a)** What is the frequency? $f = c/\lambda = c/(12.2 \text{ cm}) = 2.46$ GHz.
(b) How many antinodal planes between the walls? Nodes occur when $\lambda_n = 2L/n$. $n_{\text{max}} = 2L/\lambda$ = 2(48.8 cm)/(12.2 cm) = 8. The antinodal planes are between the nodal planes. There are 9 nodes including the node at x = 0, so there are 8 antinodal planes.
EVALUATE: If nodal (and antinodal) planes are too far apart, there could be cold (and hot) spots in the oven.

32.1. **IDENTIFY:** Since the speed is constant, distance $x = ct$.
SET UP: The speed of light is $c = 3.00\times10^8$ m/s. $1 \text{ y} = 3.156\times10^7$ s.
EXECUTE: **(a)** $t = \dfrac{x}{c} = \dfrac{3.84\times10^8 \text{ m}}{3.00\times10^8 \text{ m/s}} = 1.28 \text{ s.}$
(b) $x = ct = (3.00\times10^8 \text{ m/s})(8.61 \text{ y})(3.156\times10^7 \text{ s/y}) = 8.15\times10^{16} \text{ m} = 8.15\times10^{13} \text{ km.}$
EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.

32.2. **IDENTIFY:** Find the direction of propagation of an electromagnetic wave if we know the directions of the electric and magnetic fields.
SET UP: The direction of propagation of an electromagnetic wave is in the direction of $\vec{E}\times\vec{B}$, which is related to the directions of $\vec{E}$ and $\vec{B}$ according to the right-hand rule for the cross product. The directions of $\vec{E}$ and $\vec{B}$ in each case are shown in Figure 32.2.

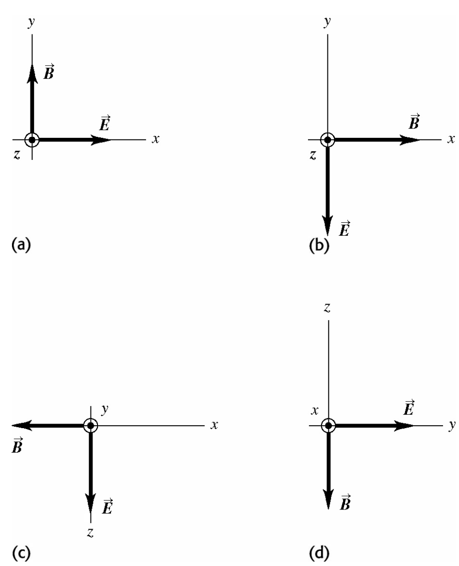


Figure 32.2

EXECUTE: **(a)** The wave is propagating in the $+z$-direction.
(b) $+z$-direction.
(c) $-y$-direction.
(d) $-x$-direction.
EVALUATE: In each case, the direction of propagation is perpendicular to the plane of $\vec{E}$ and $\vec{B}$.

32.3. **IDENTIFY:** $E_{max} = cB_{max}$. $\vec{E}\times\vec{B}$ is in the direction of propagation.

SET UP: $c = 3.00\times10^8$ m/s. $E_{max} = 4.00$ V/m.

EXECUTE: $B_{max} = E_{max}/c = 1.33\times10^{-8}$ T. For $\vec{E}$ in the $+x$-direction, $\vec{E}\times\vec{B}$ is in the $+z$-direction when $\vec{B}$ is in the $+y$-direction.

EVALUATE: $\vec{E}$, $\vec{B}$, and the direction of propagation are all mutually perpendicular.

32.4. **IDENTIFY** and **SET UP:** The direction of propagation is given by $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** $\hat{S} = \hat{i}\times(-\hat{j}) = -\hat{k}$.

(b) $\hat{S} = \hat{j}\times\hat{i} = -\hat{k}$.

(c) $\hat{S} = (-\hat{k})\times(-\hat{i}) = \hat{j}$.

(d) $\hat{S} = \hat{i}\times(-\hat{k}) = \hat{j}$.

EVALUATE: In each case the directions of $\vec{E}$, $\vec{B}$, and the direction of propagation are all mutually perpendicular.

32.5. **IDENTIFY:** Knowing the wavelength and speed of x rays, find their frequency, period, and wave number. All electromagnetic waves travel through vacuum at the speed of light.

SET UP: $c = 3.00\times10^8$ m/s. $c = f\lambda$. $T = \dfrac{1}{f}$. $k = \dfrac{2\pi}{\lambda}$.

EXECUTE: $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{0.10\times10^{-9} \text{ m}} = 3.0\times10^{18}$ Hz,

$$T = \frac{1}{f} = \frac{1}{3.0\times10^{18} \text{ Hz}} = 3.3\times10^{-19} \text{ s}, \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.10\times10^{-9} \text{ m}} = 6.3\times10^{10} \text{ m}^{-1}.$$

EVALUATE: The frequency of the x rays is much higher than the frequency of visible light, so their period is much shorter.

32.6. **IDENTIFY:** $c = f\lambda$ and $k = \dfrac{2\pi}{\lambda}$.

SET UP: $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda}$. UVA: 7.50×10^{14} Hz to 9.38×10^{14} Hz. UVB: 9.38×10^{14} Hz to 1.07×10^{15} Hz.

(b) $k = \dfrac{2\pi}{\lambda}$. UVA: 1.57×10^7 rad/m to 1.96×10^7 rad/m. UVB: 1.96×10^7 rad/m to 2.24×10^7 rad/m.

EVALUATE: Larger λ corresponds to smaller f and k.

32.7. **IDENTIFY:** Electromagnetic waves propagate through air at essentially the speed of light. Therefore, if we know their wavelength, we can calculate their frequency or vice versa.

SET UP: The wave speed is $c = 3.00\times10^8$ m/s. $c = f\lambda$.

EXECUTE: **(a)** (i) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^3 \text{ m}} = 6.0\times10^4$ Hz.

(ii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-6} \text{ m}} = 6.0\times10^{13}$ Hz.

(iii) $f = \dfrac{3.00\times10^8 \text{ m/s}}{5.0\times10^{-9} \text{ m}} = 6.0\times10^{16}$ Hz.

(b) (i) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{6.50\times10^{21} \text{ Hz}} = 4.62\times10^{-14} \text{ m} = 4.62\times10^{-5}$ nm.

(ii) $\lambda = \dfrac{3.00\times10^8 \text{ m/s}}{590\times10^3 \text{ Hz}} = 508 \text{ m} = 5.08\times10^{11} \text{ nm}.$

EVALUATE: Electromagnetic waves cover a huge range in frequency and wavelength.

32.8. IDENTIFY: $c = f\lambda$. $E_{\max} = cB_{\max}$. Apply Eqs. (32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{435\times10^{-9} \text{ m}} = 6.90\times10^{14} \text{ Hz}.$

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{2.70\times10^{-3} \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 9.00\times10^{-12} \text{ T}.$

(c) $k = \dfrac{2\pi}{\lambda} = 1.44\times10^7$ rad/m. $\omega = 2\pi f = 4.34\times10^{15}$ rad/s. If $\vec{E}(z,t) = \hat{\imath}E_{\max}\cos(kz+\omega t)$, then $\vec{B}(z,t) = -\hat{\jmath}B_{\max}\cos(kz+\omega t)$, so that $\vec{E}\times\vec{B}$ will be in the $-\hat{k}$-direction.

$\vec{E}(z,t) = \hat{\imath}(2.70\times10^{-3} \text{ V/m})\cos[(1.44\times10^7 \text{ rad/m})z + (4.34\times10^{15} \text{ rad/s})t]$ and

$\vec{B}(z,t) = -\hat{\jmath}(9.00\times10^{-12} \text{ T})\cos[(1.44\times10^7 \text{ rad/m})z + (4.34\times10^{15} \text{ rad/s})t]$.

EVALUATE: The directions of $\vec{E}$ and $\vec{B}$ and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz+\omega t$ since the wave is traveling in the $-z$-direction. Waves for visible light have very high frequencies.

32.9. IDENTIFY and **SET UP:** Compare the $\vec{E}(y,t)$ given in the problem to the general form given by Eq. (32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

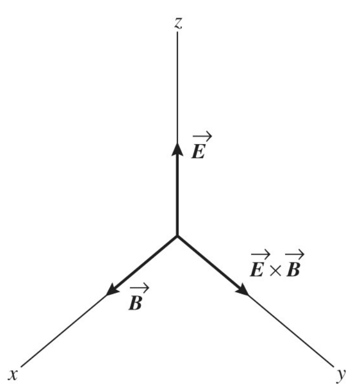
EXECUTE: **(a)** The equation for the electric field contains the factor $\cos(ky-\omega t)$ so the wave is traveling in the $+y$-direction.

(b) $\vec{E}(y,t) = (3.10\times10^5 \text{ V/m})\hat{k}\cos[ky - (12.65\times10^{12} \text{ rad/s})t]$.

Comparing to Eq. (32.17) gives $\omega = 12.65\times10^{12}$ rad/s

$\omega = 2\pi f = \dfrac{2\pi c}{\lambda}$ so $\lambda = \dfrac{2\pi c}{\omega} = \dfrac{2\pi(2.998\times10^8 \text{ m/s})}{(12.65\times10^{12} \text{ rad/s})} = 1.49\times10^{-4} \text{ m}.$

(c)



$\vec{E}\times\vec{B}$ must be in the $+y$-direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $+z$-direction then $\vec{B}$ must be in the $+x$-direction, as shown in Figure 32.9.

Figure 32.9

$k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{c} = \dfrac{12.65\times10^{12} \text{ rad/s}}{2.998\times10^8 \text{ m/s}} = 4.22\times10^4 \text{ rad/m}.$

$E_{\max} = 3.10\times10^5$ V/m.

Then $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{3.10\times10^5\ \text{V/m}}{2.998\times10^8\ \text{m/s}} = 1.03\times10^{-3}\ \text{T}.$

Using Eq. (32.17) and the fact that $\vec{B}$ is in the $+\hat{\imath}$-direction when $\vec{E}$ is in the $+\hat{k}$-direction,

$\vec{B} = +(1.03\times10^{-3}\ \text{T})\hat{\imath}\cos[(4.22\times10^4\ \text{rad/m})y - (12.65\times10^{12}\ \text{rad/s})t].$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.10. **IDENTIFY:** For an electromagnetic wave propagating in the negative x-direction, $E = E_{\max}\cos(kx+\omega t)$. $\omega = 2\pi f$ and $k = \dfrac{2\pi}{\lambda}$. $T = \dfrac{1}{f}$. $E_{\max} = cB_{\max}$.

SET UP: $E_{\max} = 375$ V/m, $k = 1.99\times10^7$ rad/m, and $\omega = 5.97\times10^{15}$ rad/s.

EXECUTE: **(a)** $c = \omega/k = (5.97\times10^{15}\ \text{rad/s})/(1.99\times10^7\ \text{rad/m}) = 3.00\times10^8\ \text{m/s}$. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

(b) $E_{\max} = 375$ V/m, the amplitude of the given cosine function for E. $B_{\max} = \dfrac{E_{\max}}{c} = 1.25\ \mu\text{T}$.

(c) $f = \dfrac{\omega}{2\pi} = 9.50\times10^{14}$ Hz. $\lambda = \dfrac{2\pi}{k} = 3.16\times10^{-7}\ \text{m} = 316$ nm. $T = \dfrac{1}{f} = 1.05\times10^{-15}$ s. This wavelength is too short to be visible.

EVALUATE: $c = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\left(\dfrac{2\pi}{k}\right) = \dfrac{\omega}{k}$ is an alternative expression for the wave speed.

32.11. **IDENTIFY** and **SET UP:** $c = f\lambda$ allows calculation of λ. $k = 2\pi/\lambda$ and $\omega = 2\pi f$. $E_{\max} = cB_{\max}$ relates the electric and magnetic field amplitudes.

EXECUTE: **(a)** $c = f\lambda$ so $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8\ \text{m/s}}{830\times10^3\ \text{Hz}} = 361\ \text{m}.$

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi\ \text{rad}}{361\ \text{m}} = 0.0174\ \text{rad/m}.$

(c) $\omega = 2\pi f = (2\pi)(830\times10^3\ \text{Hz}) = 5.22\times10^6\ \text{rad/s}.$

(d) Eq. (32.18): $E_{\max} = cB_{\max} = (2.998\times10^8\ \text{m/s})(4.82\times10^{-11}\ \text{T}) = 0.0144\ \text{V/m}.$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio broadcast band. The electric and magnetic fields in the wave are very weak.

32.12. **IDENTIFY:** Apply $v = \dfrac{c}{\sqrt{KK_{\text{m}}}}$. $E_{\max} = cB_{\max}$. $v = f\lambda$.

SET UP: $K = 3.64$. $K_{\text{m}} = 5.18$.

EXECUTE: **(a)** $v = \dfrac{c}{\sqrt{KK_{\text{m}}}} = \dfrac{(3.00\times10^8\ \text{m/s})}{\sqrt{(3.64)(5.18)}} = 6.91\times10^7\ \text{m/s}.$

(b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times10^7\ \text{m/s}}{65.0\ \text{Hz}} = 1.06\times10^6\ \text{m}.$

(c) $B_{\max} = \dfrac{E_{\max}}{v} = \dfrac{7.20\times10^{-3}\ \text{V/m}}{6.91\times10^7\ \text{m/s}} = 1.04\times10^{-10}\ \text{T}.$

EVALUATE: The wave travels slower in this material than in air.

32.13. **IDENTIFY** and **SET UP:** $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ to calculate n and K.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{2.17\times10^8\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 3.81\times10^{-7}\ \text{m}.$

(b) $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 5.26\times10^{-7}\ \text{m}.$

(c) $n = \dfrac{c}{v} = \dfrac{2.998\times10^{8}\ \text{m/s}}{2.17\times10^{8}\ \text{m/s}} = 1.38.$

(d) $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ so $K = n^2 = (1.38)^2 = 1.90.$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so n is always greater than unity.

32.14. **IDENTIFY:** We want to find the amount of energy given to each receptor cell and the amplitude of the magnetic field at the cell.

SET UP: Intensity is average power per unit area and power is energy per unit time. $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$, $I = P/A$, and $E_{\text{max}} = cB_{\text{max}}$.

EXECUTE: **(a)** For the beam, the energy is $U = Pt = (2.0\times10^{12}\ \text{W})(4.0\times10^{-9}\ \text{s}) = 8.0\times10^{3}\ \text{J} = 8.0\ \text{kJ}$. This energy is spread uniformly over 100 cells, so the energy given to each cell is 80 J.

(b) The cross-sectional area of each cell is $A = \pi r^2$, with $r = 2.5\times10^{-6}$ m.

$$I = \frac{P}{A} = \frac{2.0\times10^{12}\ \text{W}}{(100)\pi(2.5\times10^{-6}\ \text{m})^2} = 1.0\times10^{21}\ \text{W/m}^2.$$

(c) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times10^{21}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8}\ \text{m/s})}} = 8.7\times10^{11}\ \text{V/m}.$

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 2.9\times10^{3}\ \text{T}.$$

EVALUATE: Both the electric field and magnetic field are very strong compared to ordinary fields.

32.15. **IDENTIFY:** $I = P/A$. $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$. $E_{\text{max}} = cB_{\text{max}}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\epsilon_0 = 8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{(0.05)(75\ \text{W})}{4\pi(3.0\times10^{-2}\ \text{m})^2} = 330\ \text{W/m}^2.$

(b) $E_{\text{max}} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(330\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8}\ \text{m/s})}} = 500\ \text{V/m}.$

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 1.7\times10^{-6}\ \text{T} = 1.7\ \mu\text{T}.$$

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.16. **IDENTIFY:** The intensity of the electromagnetic wave is given by $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2 = \epsilon_0 cE_{\text{rms}}^2$. The total energy passing through a window of area A during a time t is IAt.

SET UP: $\epsilon_0 = 8.85\times10^{-12}$ F/m.

EXECUTE: Use the fact that energy $= \epsilon_0 cE_{\text{rms}}^2 At$.

Energy $= (8.85\times10^{-12}\ \text{F/m})(3.00\times10^{8}\ \text{m/s})(0.0400\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 6.37\times10^{-5}\ \text{J} = 63.7\ \mu\text{J}.$

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

32.17. **IDENTIFY:** $I = P_{\text{av}}/A$.

SET UP: At a distance r from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.

EXECUTE: $P_{av} = I(4\pi r^2) = (5.0\times10^3\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\ \text{m})^2 = 2.5\times10^{25}\ \text{W}.$

EVALUATE: The intensity decreases with distance from the star as $1/r^2$.

32.18. **IDENTIFY** and **SET UP:** $I = \frac{1}{2}\epsilon_0 cE_{max}{}^2$. $E_{max} = cB_{max}$. At the earth the power radiated by the sun is spread over an area of $4\pi r^2$, where $r = 1.50\times10^{11}$ m is the distance from the earth to the sun. $P = IA$.

EXECUTE: **(a)** $E_{max} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.4\times10^3\ \text{W/m}^2)}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 1.03\times10^3\ \text{N/C}.$

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{1.03\times10^3\ \text{N/C}}{3.00\times10^8\ \text{m/s}} = 3.43\times10^{-6}\ \text{T}.$

(b) $P = I(4\pi r^2) = (1.4\times10^3\ \text{W/m}^2)(4\pi)(1.50\times10^{11}\ \text{m})^2 = 4.0\times10^{26}\ \text{W}.$

EVALUATE: The intensity of the magnetic field of the light waves from the sun is about 1/10 the earth's magnetic field.

32.19. **IDENTIFY:** This problem is about the power carried by an electromagnetic wave.

SET UP: $I = \dfrac{1}{2}\epsilon_0\, cE_{max}^2$, $I = P_{av}/A$. We want the average power output of the source.

EXECUTE: The data is plotted as E_{max} versus $1/r$, so we need to relate those quantities to interpret the graph. We know that $I = P_{av}/A$ and $I = \dfrac{1}{2}\epsilon_0\, cE_{max}^2$. Equate the intensities and solve for E_{max}.

$\dfrac{1}{2}\epsilon_0\, cE_{max}^2 = \dfrac{P_{av}}{A} = \dfrac{P_{av}}{4\pi r^2}$. $E_{max} = \sqrt{\dfrac{2P_{av}}{4\pi\epsilon_0\, c}}\dfrac{1}{r}$. A graph of E_{max} versus $1/r$ should be a straight line having slope equal to $\sqrt{\dfrac{2P_{av}}{4\pi\epsilon_0\, c}}\dfrac{1}{r}$. Solve for P_{av}. $P_{av} = \dfrac{4\pi\epsilon_0\, c}{2}(\text{slope})^2$. Using the given slope, we get $P_{av} = 93.8$ W.

EVALUATE: This power is similar to an ordinary 100-W light bulb.

32.20. **IDENTIFY** and **SET UP**: $c = f\lambda$, $E_{max} = cB_{max}$ and $I = E_{max}B_{max}/2\mu_0$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^8\ \text{Hz}.$

(b) $B_{max} = \dfrac{E_{max}}{c} = \dfrac{0.0540\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.80\times10^{-10}\ \text{T}.$

(c) $I = S_{av} = \dfrac{E_{max}B_{max}}{2\mu_0} = \dfrac{(0.0540\ \text{V/m})(1.80\times10^{-10}\ \text{T})}{2\mu_0} = 3.87\times10^{-6}\ \text{W/m}^2.$

EVALUATE: Alternatively, $I = \frac{1}{2}\epsilon_0\, cE_{max}^2$.

32.21. **IDENTIFY:** $P_{av} = IA$ and $I = E_{max}^2/2\mu_0 c$

SET UP: The surface area of a sphere is $A = 4\pi r^2$.

EXECUTE: $P_{av} = S_{av}A = \left(\dfrac{E_{max}^2}{2c\mu_0}\right)(4\pi r^2)$. $E_{max} = \sqrt{\dfrac{P_{av}c\mu_0}{2\pi r^2}} = \sqrt{\dfrac{(60.0\ \text{W})(3.00\times10^8\ \text{m/s})\mu_0}{2\pi(5.00\ \text{m})^2}} = 12.0\ \text{V/m}.$

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{12.0\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 4.00\times10^{-8}\ \text{T}.$

EVALUATE: E_{max} and B_{max} are both inversely proportional to the distance from the source.

32.22. **IDENTIFY:** The intensity and the energy density of an electromagnetic wave depend on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I = P_{av}/A$, and the average radiation pressure is $P_{av} = 2I/c$, where $I = \frac{1}{2}\epsilon_0 cE_{max}^2$. The energy density is $u = \epsilon_0 E^2$.

EXECUTE: **(a)** $I = P_{av}/A = \dfrac{777{,}000 \text{ W}}{2\pi(5000 \text{ m})^2} = 0.004947 \text{ W/m}^2$.

$$p_{rad} = 2I/c = \frac{2(0.004947 \text{ W/m}^2)}{3.00\times 10^8 \text{ m/s}} = 3.30\times 10^{-11} \text{ Pa}.$$

(b) $I = \frac{1}{2}\epsilon_0 cE_{max}^2$ gives

$$E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.004947 \text{ W/m}^2)}{(8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times 10^8 \text{ m/s})}} = 1.93 \text{ N/C}.$$

$$B_{max} = E_{max}/c = (1.93 \text{ N/C})/(3.00\times 10^8 \text{ m/s}) = 6.43\times 10^{-9} \text{ T}.$$

(c) $u = \epsilon_0 E^2$, so $u_{av} = \epsilon_0 (E_{rms})^2$ and $E_{rms} = \dfrac{E_{max}}{\sqrt{2}}$, so

$$u_{av} = \frac{\epsilon_0 E_{max}^2}{2} = \frac{(8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(1.93 \text{ N/C})^2}{2} = 1.65\times 10^{-11} \text{ J/m}^3.$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.23. **IDENTIFY:** We know the greatest intensity that the eye can safely receive.

SET UP: $I = \dfrac{P}{A}$. $I = \frac{1}{2}\epsilon_0 cE_{max}^2$. $E_{max} = cB_{max}$.

EXECUTE: **(a)** $P = IA = (1.0\times 10^2 \text{ W/m}^2)\pi(0.75\times 10^{-3} \text{ m})^2 = 1.8\times 10^{-4} \text{ W} = 0.18 \text{ mW}$.

(b) $E = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(1.0\times 10^2 \text{ W/m}^2)}{(8.85\times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times 10^8 \text{ m/s})}} = 274 \text{ V/m}$. $B_{max} = \dfrac{E_{max}}{c} = 9.13\times 10^{-7} \text{ T}$.

(c) $P = 0.18 \text{ mW} = 0.18 \text{ mJ/s}$.

(d) $I = (1.0\times 10^2 \text{ W/m}^2)\left(\dfrac{1 \text{ m}}{10^2 \text{ cm}}\right)^2 = 0.010 \text{ W/cm}^2$.

EVALUATE: Both the electric and magnetic fields are quite weak compared to normal laboratory fields.

32.24. **IDENTIFY:** Apply $p_{rad} = \dfrac{I}{c}$ and $p_{rad} = \dfrac{2I}{c}$. The average momentum density is given by $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2}$ with S replaced by $S_{av} = I$.

SET UP: 1 atm $= 1.013\times 10^5$ Pa.

EXECUTE: **(a)** Absorbed light: $p_{rad} = \dfrac{I}{c} = \dfrac{2500 \text{ W/m}^2}{3.0\times 10^8 \text{ m/s}} = 8.33\times 10^{-6} \text{ Pa}$. Then

$$p_{rad} = \frac{8.33\times 10^{-6} \text{ Pa}}{1.013\times 10^5 \text{ Pa/atm}} = 8.23\times 10^{-11} \text{ atm}.$$

(b) Reflecting light: $p_{rad} = \dfrac{2I}{c} = \dfrac{2(2500 \text{ W/m}^2)}{3.0\times 10^8 \text{ m/s}} = 1.67\times 10^{-5} \text{ Pa}$. Then

$$p_{rad} = \frac{1.67\times 10^{-5} \text{ Pa}}{1.013\times 10^5 \text{ Pa/atm}} = 1.65\times 10^{-10} \text{ atm}.$$

(c) The momentum density is $\dfrac{dp}{dV}=\dfrac{S_{av}}{c^2}=\dfrac{2500\ \text{W/m}^2}{(3.0\times10^8\ \text{m/s})^2}=2.78\times10^{-14}\ \text{kg/m}^2\cdot\text{s}.$

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.25. **IDENTIFY:** We know the wavelength and power of the laser beam, as well as the area over which it acts.

SET UP: $P = IA$. $A=\pi r^2$. $E_{max}=cB_{max}$. The intensity $I=S_{av}$ is related to the maximum electric field by $I=\frac{1}{2}\epsilon_0 cE_{max}^2$. The average energy density u_{av} is related to the intensity I by $I=u_{av}c$.

EXECUTE: **(a)** $I=\dfrac{P}{A}=\dfrac{0.500\times10^{-3}\ \text{W}}{\pi(0.500\times10^{-3}\ \text{m})^2}=637\ \text{W/m}^2.$

(b) $E_{max}=\sqrt{\dfrac{2I}{\epsilon_0 c}}=\sqrt{\dfrac{2(637\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}}=693\ \text{V/m}.$ $B_{max}=\dfrac{E_{max}}{c}=2.31\ \mu\text{T}.$

(c) $u_{av}=\dfrac{I}{c}=\dfrac{637\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}}=2.12\times10^{-6}\ \text{J/m}^3.$

EVALUATE: The fields are very weak, so a cubic meter of space contains only about $2\ \mu\text{J}$ of energy.

32.26. **IDENTIFY:** This problem deals with radiation pressure.

SET UP: $p_{rad}=\dfrac{2I}{c}$, $I = P_{av}/A$. We want the average laser output power. Let subscripts L refer to the laser and D refer to the disk. Also let d be the thickness of the disk, r be the radius of the disk and R the radius of the laser beam. Fig. 32.26 illustrates the arrangement.

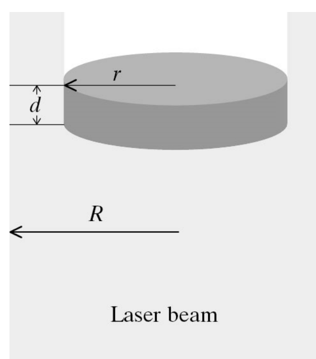


Figure 32.26

EXECUTE: **(a)** The force that the laser exerts on the disk must equal the weight of the disk. The laser force is due to radiation pressure p_{rad}, so $p_{rad}A_D = p_{rad}(\pi r^2) = mg = \rho V_D g=\rho(\pi r^2 d)g.$ $p_{rad}=\rho g d.$ Using $p_{rad}=\dfrac{2I}{c}$ gives $\dfrac{2I}{c}=\rho g d.$ Using $I = P_{av}/A_L$ with $A_L = \pi R^2$, we get $\dfrac{2}{c}\left(\dfrac{P_{av}}{\pi R^2}\right)=\rho g d.$ Solving for P_{av} and using the given values, we get $P_{av}=\dfrac{\pi R^2\rho g c d}{2}=33.3\ \text{W}.$

(b) The power does not depend on the radius of the disk, so the answer is the same as in part (a).

EVALUATE: The result in (b) may be surprising. But doubling r increases the laser force on the disk by a factor of $2^2 = 4$ and it also increase the weight of the disk by the same factor. This result would *not* be true if $r \geq 1.00$ mm, however, because the disk radius would be greater than the laser beam radius. In that case, doubling r would increase the weight of the disk but would not increase the laser force.

32.27. **IDENTIFY:** The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $(2\frac{1}{2})\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: **(a)** Since $(2\frac{1}{2})\lambda = L$, we have $L = (5/2)(12.2\text{ cm}) = 30.5$ cm.

(b) Solving for the frequency gives $f = c/\lambda = (3.00\times10^8\text{ m/s})/(0.122\text{ m}) = 2.46\times10^9$ Hz.

(c) $L = 35.5$ cm in this case. $(2\frac{1}{2})\lambda = L$, so $\lambda = 2L/5 = 2(35.5\text{ cm})/5 = 14.2$ cm.

$f = c/\lambda = (3.00\times10^8\text{ m/s})/(0.142\text{ m}) = 2.11\times10^9$ Hz.

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.28. **IDENTIFY:** The nodal planes of $\vec{E}$ and $\vec{B}$ are located by Eqs. (32.26) and (32.27).

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{75.0\times10^6\text{ Hz}} = 4.00$ m.

EXECUTE: **(a)** $\Delta x = \dfrac{\lambda}{2} = 2.00$ m.

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is $\dfrac{\lambda}{4} = \dfrac{\Delta x}{2} = \dfrac{2.00\text{ m}}{2} = 1.00$ m.

EVALUATE: The nodal planes of $\vec{B}$ are separated by a distance $\lambda/2$ and are midway between the nodal planes of $\vec{E}$.

32.29. **IDENTIFY:** We are looking at standing electromagnetic waves in a cavity.

SET UP: Nodal planes are a half wavelength apart. $\lambda_n = 2L/n$. We want the distance L between the walls.

EXECUTE: Use $\lambda_n = 2L/n$. For the two frequencies, the values of n differ by 1. $\frac{1}{2}\lambda_n = \frac{1}{2}\left(\frac{2L}{n}\right) = 1.50$ cm. $\frac{1}{2}\lambda_{n+1} = \frac{1}{2}\left(\frac{2L}{n+1}\right) = 1.25$ cm. Solving for L gives $L = 7.50$ cm.

EVALUATE: The antinodal planes have the same spacing as the nodal planes.

32.30. **IDENTIFY:** Evaluate the partial derivatives of the expressions for $E_y(x,t)$ and $B_z(x,t)$.

SET UP: $\dfrac{\partial}{\partial x}\cos(kx-\omega t) = -k\sin(kx-\omega t)$, $\dfrac{\partial}{\partial t}\cos(kx-\omega t) = \omega\sin(kx-\omega t)\cdot$

$\dfrac{\partial}{\partial x}\sin(kx-\omega t) = k\cos(kx-\omega t)$, $\dfrac{\partial}{\partial t}\sin(kx-\omega t) = -\omega\cos(kx-\omega t)$.

EXECUTE: Assume $\vec{E} = E_{\max}\hat{\boldsymbol{j}}\cos(kx-\omega t)$ and $\vec{B} = B_{\max}\hat{\boldsymbol{k}}\cos(kx-\omega t+\phi)$, with $-\pi < \phi < \pi$. Eq. (32.12) is $\dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t}$. This gives $kE_{\max}\sin(kx-\omega t) = +\omega B_{\max}\sin(kx-\omega t+\phi)$, so $\phi = 0$, and $kE_{\max} = \omega B_{\max}$, so $E_{\max} = \dfrac{\omega}{k}B_{\max} = \dfrac{2\pi f}{2\pi/\lambda}B_{\max} = f\lambda B_{\max} = cB_{\max}$. Similarly for Eq. (32.14), $-\dfrac{\partial B_z}{\partial x} = \epsilon_0\mu_0\dfrac{\partial E_y}{\partial t}$ gives $kB_{\max}\sin(kx-\omega t+\phi) = \epsilon_0\mu_0\omega E_{\max}\sin(kx-\omega t)$, so $\phi = 0$ and $kB_{\max} = \epsilon_0\mu_0\omega E_{\max}$, so $B_{\max} = \dfrac{\epsilon_0\mu_0\omega}{k}E_{\max} = \dfrac{2\pi f}{c^2 2\pi/\lambda}E_{\max} = \dfrac{f\lambda}{c^2}E_{\max} = \dfrac{1}{c}E_{\max}$.

EVALUATE: The $\vec{E}$ and $\vec{B}$ fields must oscillate in phase.

32.31. IDENTIFY: We know the wavelength and power of a laser beam as well as the area over which it acts and the duration of a pulse.

SET UP: The energy is $U = Pt$. For absorption the radiation pressure is $\frac{I}{c}$, where $I = \frac{P}{A}$. The wavelength in the eye is $\lambda = \frac{\lambda_0}{n}$. $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$ and $E_{\max} = cB_{\max}$.

EXECUTE: **(a)** $U = Pt = (250\times10^{-3}\text{ W})(1.50\times10^{-3}\text{ s}) = 3.75\times10^{-4}\text{ J} = 0.375\text{ mJ}$.

(b) $I = \frac{P}{A} = \frac{250\times10^{-3}\text{ W}}{\pi(255\times10^{-6}\text{ m})^2} = 1.22\times10^6\text{ W/m}^2$. The average pressure is

$\frac{I}{c} = \frac{1.22\times10^6\text{ W/m}^2}{3.00\times10^8\text{ m/s}} = 4.08\times10^{-3}\text{ Pa}$.

(c) $\lambda = \frac{\lambda_0}{n} = \frac{810\text{ nm}}{1.34} = 604\text{ nm}$. $f = \frac{v}{\lambda} = \frac{c}{\lambda_0} = \frac{3.00\times10^8\text{ m/s}}{810\times10^{-9}\text{ m}} = 3.70\times10^{14}\text{ Hz}$; f is the same in the air and in the vitreous humor.

(d) $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(1.22\times10^6\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})}} = 3.03\times10^4\text{ V/m}$.

$B_{\max} = \frac{E_{\max}}{c} = 1.01\times10^{-4}\text{ T}$.

EVALUATE: The intensity of the beam is high, as it must be to weld tissue, but the pressure it exerts on the retina is only around 10^{-8} that of atmospheric pressure. The magnetic field in the beam is about twice that of the earth's magnetic field.

32.32. IDENTIFY: This problem involves the energy of the waves in a microwave oven.

SET UP and **EXECUTE:** **(a)** Estimate: 1 minute.

(b) We want the heat. Use $Q = mc\Delta T$. T_1 (room temperature) = 20°C, T_2 = 100°C, m = mass of 237 mL of water = 237 g = 0.237 kg, c = 4190 J/kg·K. Q = 79 kJ.

(c) We want the power. $P = Q/t$ = (79 kJ)/(60 s) = 1.3 kW.

(d) We want the average intensity. P = 2.6 kW. $I_{av} = P_{av}/A$. Estimate: Cup diameter = 7.5 cm, so r = 3.75 cm. I_{av} = (2.6 kW)/[π(0.0375 m)²] = 600 kW/m².

(e) We want $E_{\max}$. Use $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$, solve for $E_{\max}$, and use I = 600 kW/m². This gives $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = 21\text{ kV/m}$.

EVALUATE: Since all the estimates are reasonable values, our result for $E_{\max}$ should be fairly reasonable.

32.33. IDENTIFY: The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{av}/A = \frac{1}{2}\epsilon_0 cE_{\max}^2$, and the force is $F = p_{rad}A$ where $p_{rad} = I/c$.

EXECUTE: **(a)** $I = P_{av}/A = (25{,}000\text{ W})/[4\pi(5.75\times10^5\text{ m})^2] = 6.02\times10^{-9}\text{ W/m}^2$.

(b) $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$, so $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(6.02\times10^{-9}\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})}} = 2.13\times10^{-3}\text{ N/C}$.

$B_{\max} = E_{\max}/c = (2.13\times10^{-3}\text{ N/C})/(3.00\times10^8\text{ m/s}) = 7.10\times10^{-12}\text{ T}$.

(c) $F = p_{rad}A = (I/c)A = (6.02\times10^{-9}\text{ W/m}^2)(0.150\text{ m})(0.400\text{ m})/(3.00\times10^8\text{ m/s}) = 1.20\times10^{-18}\text{ N}$.

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.34. **IDENTIFY:** We are dealing with electromagnetic waves from a moving magnet. In one cycle, the magnet starts right at the coil, then moves 10 cm away, and then moves back to where it started.

SET UP and **EXECUTE:** **(a)** One cycle lasts ½ second. In ½ cycle, $\Delta B = B$ and $\Delta T = \frac{1}{2}\left(\frac{1}{2}\text{ s}\right) = \frac{1}{4}$ s. Using these results and the given numbers gives $\frac{|\Delta\Phi_B|}{\Delta t} = \frac{A\Delta B}{\Delta t} = \frac{AB}{\Delta t} = \frac{\pi r^2 B}{\Delta t} = 50\ \mu\text{Wb/s}$.

(b) We want the average magnitude of E within the loop. Using $\oint E dl = \frac{d\Phi_B}{dt}$ gives $E2\pi r = \frac{|\Delta\Phi_B|}{\Delta t}$.

Solving for E and using the result from part (a) with r = 2.0 cm gives $E = \frac{1}{2\pi r}\frac{|\Delta\Phi_B|}{\Delta t} = 400\ \mu\text{V/m}$.

(c) We want the intensity. Eq. (32.28): $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$. Using $B = E/c$ this becomes $S = \frac{E^2}{\mu_0 c}$. Using E = 400 μV/m, this gives S = 0.42 nW/m^2.

(d) We want the total power. $P = SA = S(2\pi rl)$ = (0.42 nW/m^2)(2π)(0.0200 m)(0.10 m) = 5.3 pW.

EVALUATE: The radiated energy is extremely small because the fields are weak and the back-and-forth motion is very slow.

32.35. **IDENTIFY:** $I = P_{\text{av}}/A$. For an absorbing surface, the radiation pressure is $p_{\text{rad}} = \frac{I}{c}$.

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{\text{av}}/(4\pi r^2)$.

EXECUTE: **(a)** At the sun's surface: $I = \frac{P_{\text{av}}}{4\pi R^2} = \frac{3.9\times10^{26}\text{ W}}{4\pi(6.96\times10^8\text{ m})^2} = 6.4\times10^7\text{ W/m}^2$ and $p_{\text{rad}} = \frac{I}{c} = \frac{6.4\times10^7\text{ W/m}^2}{3.00\times10^8\text{ m/s}} = 0.21\text{ Pa}$.

Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6\times10^8\text{ W/m}^2$ and $p_{\text{rad}} = 0.85\text{ Pa}$. At the top of the earth's atmosphere, the measured sunlight intensity is 1400 W/m^2 and $p_{\text{rad}} = 5\times10^{-6}$ Pa, which is about 100,000 times less than the values above.

EVALUATE: **(b)** The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.36. **(a) IDENTIFY** and **SET UP:** Calculate I and then use $I = \frac{E_{\max}^2}{2\mu_0 c}$ to calculate $E_{\max}$ and $E_{\max} = cB_{\max}$ to calculate $B_{\max}$.

EXECUTE: The intensity is power per unit area: $I = \frac{P}{A} = \frac{5.80\times10^{-3}\text{ W}}{\pi(1.25\times10^{-3}\text{ m})^2} = 1182\text{ W/m}^2$.

$I = \frac{E_{\max}^2}{2\mu_0 c}$, so $E_{\max} = \sqrt{2\mu_0 cI}$. $E_{\max} = \sqrt{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.998\times10^8\text{ m/s})(1182\text{ W/m}^2)} = 943.5\text{ V/m}$, which rounds to 943 V/m.

$B_{\max} = \frac{E_{\max}}{c} = \frac{943.5\text{ V/m}}{2.998\times10^8\text{ m/s}} = 3.148\times10^{-6}\text{ T}$, which rounds to 3.15 μT.

EVALUATE: The magnetic field amplitude is quite small compared to laboratory fields.

(b) IDENTIFY and **SET UP:** $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = \frac{B^2}{2\mu_0}$ give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\max}\cos(kx - \omega t)$ and the average value of $\cos^2(kx - \omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\max}^2 = \frac{1}{4}(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(943.5\ \text{V/m})^2 = 1.97\times10^{-6}\ \text{J/m}^3 = 1.97\ \mu\text{J/m}^3$. The energy density in the magnetic field is $u_B = \dfrac{B^2}{2\mu_0}$. The average value is

$$u_{B,\text{av}} = \frac{B_{\max}^2}{4\mu_0} = \frac{(3.148\times10^{-6}\ \text{T})^2}{4(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 1.97\times10^{-6}\ \text{J/m}^3 = 1.97\ \mu\text{J/m}^3.$$

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and **SET UP:** The total energy in this length of beam is the average energy density $(u_{\text{av}} = u_{E,\text{av}} + u_{B,\text{av}} = 3.94\times10^{-6}\ \text{J/m}^3)$ times the volume of this part of the beam.

EXECUTE: $U = u_{\text{av}}LA = (3.94\times10^{-6}\ \text{J/m}^3)(1.00\ \text{m})\pi(1.25\times10^{-3}\ \text{m})^2 = 1.93\times10^{-11}\ \text{J}$.

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00$ m: $U = P\left(\dfrac{L}{c}\right) = (5.80\times10^{-3}\ \text{W})\left(\dfrac{1.00\ \text{m}}{2.998\times10^8\ \text{m/s}}\right) = 1.93\times10^{-11}\ \text{J}$, which checks.

32.37. IDENTIFY: The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = p_{\text{rad}}A = (I/c)A$, while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$. Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{\text{net}} = I\alpha$, to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{\text{abs}} = p_{\text{rad}}A = (I/c)A = \dfrac{\frac{1}{2}\epsilon_0 cE_{\max}^2 A}{c} = \frac{1}{2}\epsilon_0 AE_{\max}^2$.

For a totally reflecting surface, the force will be twice as great, which is $\epsilon_0 cE_{\max}^2$. The net torque is therefore $\tau_{\text{net}} = F_{\text{refl}}(L/2) - F_{\text{abs}}(L/2) = \epsilon_0 AE_{\max}^2 L/4$.

Newton's second law for rotation gives $\tau_{\text{net}} = I\alpha$. $\epsilon_0 AE_{\max}^2 L/4 = 2m(L/2)^2\alpha$.

Solving for α gives

$$\alpha = \epsilon_0 AE_{\max}^2/(2mL) = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0150\ \text{m})^2(1.25\ \text{N/C})^2}{(2)(0.00400\ \text{kg})(1.00\ \text{m})} = 3.89\times10^{-13}\ \text{rad/s}^2.$$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.38. IDENTIFY: The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = \text{S}_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\max}^2$.

EXECUTE: Since $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$, $E_{\max} \propto \sqrt{I}$. A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and $(0.200\ \text{m})/(10.0\ \text{m}) = 1/50$, we have $I_{0.20} = 50^2 I_{10}$. Since $E_{\max} \propto \sqrt{I}$, we have $E_{0.20} = 50E_{10} = (50)(3.50\ \text{N/C}) = 175\ \text{N/C}$.

EVALUATE: While the intensity increases by a factor of $50^2 = 2500$, the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.39. **IDENTIFY** and **SET UP:** In the wire the electric field is related to the current density by $\vec{E} = \rho\vec{J}$. Use Ampere's law to calculate $\vec{B}$. The Poynting vector is given by $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ and $\vec{P} = \mathbf{r}\vec{S}\cdot d\vec{A}$ relates the energy flow through a surface to $\vec{S}$.

EXECUTE: **(a)** The direction of $\vec{E}$ is parallel to the axis of the cylinder, in the direction of the current. $E = \rho J = \rho I/\pi a^2$. ($E$ is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.39a; take the current to be coming out of the page.

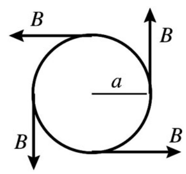


Figure 32.39a

Apply Ampere's law to a circle of radius a.

$\mathbf{r}\vec{B}\cdot d\vec{l} = B(2\pi ra)$

$I_{\text{encl}} = I.$

$\mathbf{r}\vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi a) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi a}.$

The direction of $\vec{B}$ is counterclockwise around the circle.

(c) The directions of $\vec{E}$ and $\vec{B}$ are shown in Figure 32.39b.

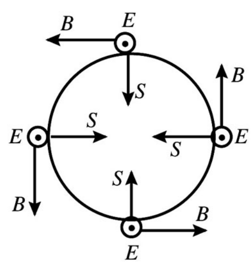


Figure 32.39b

The direction of $\vec{S} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}.$

is radially inward.

$$S = \frac{1}{\mu_0}EB = \frac{1}{\mu_0}\left(\frac{\rho I}{\pi a^2}\right)\left(\frac{\mu_0 I}{2\pi a}\right).$$

$$S = \frac{\rho I^2}{2\pi^2 a^3}.$$

EVALUATE: **(d)** Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi al) = \dfrac{\rho I^2}{2\pi^2 a^3}(2\pi al) = \dfrac{\rho l I^2}{\pi a^2}$. But $R = \dfrac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since $\vec{S}$ is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.40. **IDENTIFY:** The changing magnetic field of the electromagnetic wave produces a changing flux through the wire loop, which induces an emf in the loop. The wavelength of the wave is much greater than the diameter of the loop, so we can treat the magnetic field as being uniform over the area of the loop.

SET UP: $\Phi_B = B\pi r^2 = \pi r^2 B_{\text{max}}\cos(kx - \omega t)$, taking x for the direction of propagation of the wave. Faraday's law says $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The intensity of the wave is $I = \dfrac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \dfrac{c}{2\mu_0}B_{\text{max}}^2$, and $f = \dfrac{c}{\lambda}$.

EXECUTE: $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \omega B_{\max}\sin(kx-\omega t)\pi r^2$. $|\mathcal{E}|_{\max} = 2\pi f B_{\max}\pi r^2$.

$f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{6.90\ \text{m}} = 4.348\times10^7\ \text{Hz}$. Solving $I = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{c}{2\mu_0}B_{\max}^2$ for $B_{\max}$ gives

$B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.0275\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}}} = 1.518\times10^{-8}\ \text{T}$.

$|\mathcal{E}|_{\max} = 2\pi(4.348\times10^7\ \text{Hz})(1.518\times10^{-8}\ \text{T})\pi(0.075\ \text{m})^2 = 7.33\times10^{-2}\ \text{V} = 73.3\ \text{mV}$.

EVALUATE: This voltage is quite small compared to everyday voltages, so it normally would not be noticed. But in very delicate laboratory work, it could be large enough to take into consideration.

32.41. **IDENTIFY:** The nodal planes are one-half wavelength apart.
SET UP: The nodal planes of B are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots,$ which are $\lambda/2$ apart.

EXECUTE: **(a)** The wavelength is $\lambda = c/f = (2.998\times10^8\ \text{m/s})/(110.0\times10^6\ \text{Hz}) = 2.725\ \text{m}$. So the nodal planes are at $(2.725\ \text{m})/2 = 1.363\ \text{m}$ apart.

(b) For the nodal planes of E, we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.725\ \text{m})/2 = 10.90\ \text{m}$.

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.42. **IDENTIFY:** This problem involves an L-R-C ac circuit, electromagnetic waves, and Faraday's law.

SET UP: $\mathcal{E} = -N\frac{d\Phi_B}{dt}$, $B = E/c$, $k = \omega/c$. B_x must have the same mathematical form as E_y.

EXECUTE: **(a)** We want the flux. $dA = adz$. $\Phi_B = \int_{-a/2}^{a/2} B_x a dz = aB_{\max}\int_{-a/2}^{a/2}\cos(kx-\omega t)dz$. This gives

$\Phi_B = \frac{aB_{\max}}{k}\left[\sin(ka/2-\omega t)+\sin(ka/2+\omega t)\right] = \frac{aE_{\max}}{\omega}\left[2\sin(ka/2)\cos\omega t\right]$.

(b) We want the magnitude of the emf. Using the result from part (a) we get $\mathcal{E} = N\frac{d\Phi_B}{dt}$ = $\frac{d}{dt}\left(\frac{aE_{\max}}{\omega}\left[2\sin(ka/2)\cos\omega t\right]\right) = 2NaE_{\max}\sin(ka/2)\sin\omega t$.

(c) We want C. At resonance, $2\pi f_0 = \omega_0 = 1/\sqrt{LC}$. Solve for C and put in the given numbers using f_0 = 4.00 MHz and L = 78.0 μH. The result is $C = \frac{1}{L(2\pi f_0)^2}$ = 20.3 pF.

(d) We want the rms current. The circuit is at resonance, so $i = V/Z = V/R$. (Note that we are using i instead of I for the current amplitude so as not to confuse it with the intensity I.) Using the result from part (b) gives $V = \mathcal{E}_{\max} = 2NaE_{\max}$. Now find $E_{\max}$ using the intensity $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$. This gives

$E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}}$, so $V = 2NaE_{\max} = 2Na\sqrt{\frac{2I}{\epsilon_0 c}}$. Thus $i = \frac{V}{R} = \frac{2Na}{R}\sqrt{\frac{2I}{\epsilon_0 c}}$. Finally $i_{\text{rms}} = i/\sqrt{2}$, so we get

$i_{\text{rms}} = \frac{2Na}{R\sqrt{2}}\sqrt{\frac{2I}{\epsilon_0 c}} = \frac{2Na}{R}\sqrt{\frac{I}{\epsilon_0 c}}$. Using the numbers in the problem gives i_{rms} = 19.4 A.

EVALUATE: At resonance, Z is a minimum so the current amplitude is a maximum.

32.43. **IDENTIFY:** The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/r = GmM/r^2$, where M is the mass of the earth and m is the mass of the satellite. The intensity I of the wave is $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\max}{}^2$, and by definition, $I = P_{\text{av}}/A$.

EXECUTE: **(a)** The period of the orbit is 12 hr. Applying Newton's second law to the satellite gives $mv^2/r = GmM/r^2$, which gives $\dfrac{m(2\pi r/T)^2}{r} = \dfrac{GmM}{r^2}$. Solving for r, we get

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(12\times3600\ \text{s})^2}{4\pi^2}\right]^{1/3} = 2.66\times10^7\ \text{m}.$$

The height above the surface is $h = 2.66\times10^7\ \text{m} - 6.37\times10^6\ \text{m} = 2.02\times10^7\ \text{m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I = P_{\text{av}}/A = P_{\text{av}}/(2\pi h^2) = (25.0\ \text{W})/[2\pi(2.02\times10^7\ \text{m})^2] = 9.75\times10^{-15}\ \text{W/m}^2$$

(b) $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\max}^2$, so

$$E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(9.75\times10^{-15}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}} = 2.71\times10^{-6}\ \text{N/C}.$$

$$B_{\max} = E_{\max}/c = (2.71\times10^{-6}\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 9.03\times10^{-15}\ \text{T}.$$

$$t = d/c = (2.02\times10^7\ \text{m})/(3.00\times10^8\ \text{m/s}) = 0.0673\ \text{s}.$$

(c) $p_{\text{rad}} = I/c = (9.75\times10^{-15}\ \text{W/m}^2)/(3.00\times10^8\ \text{m/s}) = 3.25\times10^{-23}\ \text{Pa}.$

(d) $\lambda = c/f = (3.00\times10^8\ \text{m/s})/(1575.42\times10^6\ \text{Hz}) = 0.190\ \text{m}.$

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.44. **IDENTIFY:** For a totally reflective surface the radiation pressure is $\dfrac{2I}{c}$. Find the force due to this pressure and express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_g = G\dfrac{mM_{\text{sun}}}{r^2}$.

SET UP: The mass of the sun is $M_{\text{sun}} = 1.99\times10^{30}\ \text{kg}$. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{\text{rad}} = \left(\dfrac{2I}{c}\right)A$, where A is the area of the sail. $I = \dfrac{P}{4\pi r^2}$, where r is the distance of the sail from the sun. $F_{\text{rad}} = \left(\dfrac{2A}{c}\right)\left(\dfrac{P}{4\pi r^2}\right) = \dfrac{PA}{2\pi r^2 c}\cdot F_{\text{rad}} = F_g$ so $\dfrac{PA}{2\pi r^2 c} = G\dfrac{mM_{\text{sun}}}{r^2}$.

$$A = \frac{2\pi cGmM_{\text{sun}}}{P} = \frac{2\pi(3.00\times10^8\ \text{m/s})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10{,}000\ \text{kg})(1.99\times10^{30}\ \text{kg})}{3.9\times10^{26}\ \text{W}}.$$

$A = 6.42\times10^6\ \text{m}^2 = 6.42\ \text{km}^2.$

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{\text{rad}} = F_g$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.45. **IDENTIFY and SET UP:** The gravitational force is given by $F_g = G\dfrac{mM}{r^2}$. Express the mass of the particle in terms of its density and volume. The radiation pressure is given by $p_{rad} = \dfrac{I}{c}$; relate the power output L of the sun to the intensity at a distance r. The radiation force is the pressure times the cross-sectional area of the particle.

EXECUTE: **(a)** The gravitational force is $F_g = G\dfrac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho \tfrac{4}{3}\pi R^3$. Thus $F_g = \dfrac{4\rho G\pi M R^3}{3r^2}$.

(b) For a totally absorbing surface $p_{rad} = \dfrac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation a distance r from the sun is $I = \dfrac{L}{4\pi r^2}$. Thus $p_{rad} = \dfrac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the direction of propagation of the radiation, so $F_{rad} = p_{rad}A_\perp$, where $A_\perp = \pi R^2$ is the component of area of the particle perpendicular to the radiation direction. Thus

$$F_{rad} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}.$$

(c) $F_g = F_{rad}$.

$$\frac{4\rho G\pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}.$$

$$\left(\frac{4\rho G\pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G\pi M}.$$

$$R = \frac{3(3.9\times10^{26}\ \text{W})}{16(2.998\times10^{8}\ \text{m/s})(3000\ \text{kg/m}^3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\pi(1.99\times10^{30}\ \text{kg})}.$$

$R = 1.9\times10^{-7}\ \text{m} = 0.19\ \mu\text{m}$.

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R.

(d) $\dfrac{F_{rad}}{F_g} = \left(\dfrac{LR^2}{4cr^2}\right)\left(\dfrac{3r^2}{4\rho G\pi M R^3}\right) = \dfrac{3L}{16c\rho G\pi M R}$. F_{rad} is proportional to R^2 and F_g is proportional to R^3, so this ratio is proportional to $1/R$. If $R < 0.20\ \mu\text{m}$ then $F_{rad} > F_g$ and the radiation force will drive the particles out of the solar system.

32.46. **IDENTIFY and SET UP:** The intensity of an electromagnetic wave can be expressed in many ways, including $I = \dfrac{P}{A} = \tfrac{1}{2}\epsilon_0 cE_{max}^2 = \dfrac{cp_{rad}}{2}$, with the last way valid at a totally reflecting surface. In addition, the average energy density u in a wave is $u = \tfrac{1}{2}\epsilon_0 E_{max}^2$. Also, $B = E/c$ and $p = \dfrac{F_\perp}{A}$.

EXECUTE: For each laser, we calculate the beam intensity using formula that is appropriate for the information we know about the beam.

<u>Laser A</u>: $I = P/A = (2.6\ \text{W})/[\pi(1.3\times10^{-3}\ \text{m})^2] = 4.9\times10^5\ \text{W/m}^2$.

<u>Laser B</u>: $I = \tfrac{1}{2}\epsilon_0 cE_{max}^2 = (1/2)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})(480\ \text{V/m})^2 = 310\ \text{W/m}^2$.

<u>Laser C</u>: Combining $I = \tfrac{1}{2}\epsilon_0 cE_{max}^2$ and $B_{max} = E_{max}/c$, we get

$I = \tfrac{1}{2}\epsilon_0 c^3 B_{max}^2 = (1/2)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})^3(8.7\times10^{-6}\ \text{T})^2 = 9000\ \text{W/m}^2$.

<u>Laser D</u>: The surface is totally reflecting, so

$I = \dfrac{cp_{rad}}{2} = \dfrac{cF_\perp}{2A} = (3.00\times10^8\ \text{m/s})(6.0\times10^{-8}\ \text{N})/[2\pi(0.90\times10^{-3}\ \text{m})^2] = 3.5\times10^7\ \text{W/m}^2$.

Laser E: Combining $I = \frac{1}{2}\epsilon_0\, cE_{\text{max}}^2$ and $u = \frac{1}{2}\epsilon_0\, E_{\text{max}}^2$ gives $I = \frac{1}{2}\epsilon_0\, c\left(\frac{2u}{\epsilon_0}\right) = cu,$ so

$I = (3.0\times10^8\text{ m/s})(3.0\times10^{-7}\text{ J/m}^3) = 90\text{ W/m}^2$.
In order of increasing intensity, we have E, B, C, A, D.
EVALUATE: The laser intensities vary a great deal. But even the least intense one is around 10 times as intense as a 100-W lightbulb viewed at 1 m, if the 100 W all went into light (which it certainly does *not*).

32.47. IDENTIFY and SET UP: The intensity of the light beam is $I = \frac{1}{2}\epsilon_0\, cE_{\text{max}}^2$.

EXECUTE: **(a)** A graph of I versus E_{max}^2 should be a straight line having slope equal to $\frac{1}{2}\epsilon_0\, c$.

(b) Using the slope of the graph given with the problem, we have $\frac{1}{2}\epsilon_0\, c = 1.33\times10^{-3}\text{ J/(V}^2\cdot\text{s)}$.

Solving for c gives $c = 2[1.33\times10^{-3}\text{ J/(V}^2\cdot\text{s)}]/(8.854\times10^{-12}\text{ C}^2\text{/N}\cdot\text{m}^2) = 3.00\times10^8\text{ m/s}$.

EVALUATE: This result is nearly identical to the speed of light in vacuum.

32.48. IDENTIFY and SET UP: The spacing between antinodes is $\lambda/2$, and $f\lambda = c$.

EXECUTE: The antinode spacing is $d = \lambda/2 = \frac{c}{2}\cdot\frac{1}{f}$. Therefore a graph of d versus $1/f$ should be a straight line having a slope equal to $c/2$. Figure 32.48 shows the graph of d versus $1/f$.

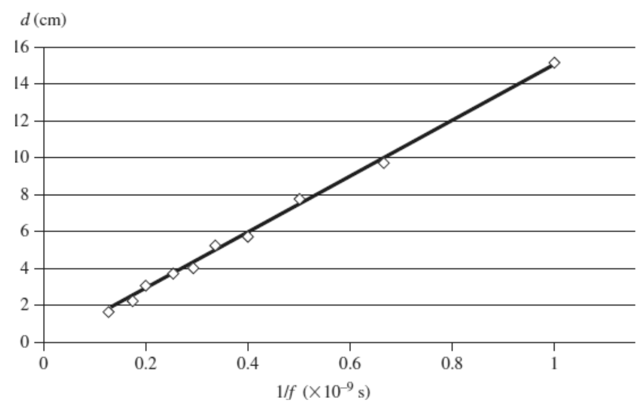


Figure 32.48

The slope of the best-fit line is 15.204×10^9 cm/s = 15.204×10^7 m/s, so $c/2 = 15.204\times10^7$ m/s, which gives $c = 3.0\times10^8$ m/s.
EVALUATE: This result is *very* close to the well-established value for the speed of light in vacuum.

32.49. IDENTIFY: This problem involves radiation pressure.
SET UP: Part of the incident beam is absorbed and part is reflected. The intensity of the incident beam is I_0, that of the transmitted beam is $eI_{–0}$, the that of the reflected beam is $(1 – e)I_0$ as shown in Fig. 32.49.

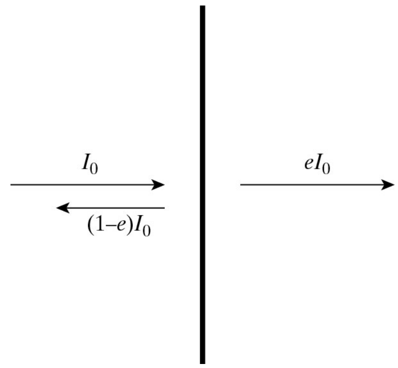


Figure 32.49

EXECUTE: (a) The radiation pressure due to the transmitted part of the beam is $p_{tr} = I/c$ and the pressure due to the reflected part is $p_{ab} = 2I/c$. In terms of the incident intensity, these pressures are $p_{tr} = 2eI_0/c$ and $p_{ab} = 2(1-e)I_0/c$. The total radiation pressure is the sum of these, which is $p_{rad} = \frac{I_0}{c}(2-e)$

EVALUATE: Check: For a perfect absorber all the radiation is absorbed, so $e = 1$. In this case, $p_{rad} = \frac{I_0}{c}(2-e) = = \frac{I_0}{c}(2-1) = \frac{I_0}{c}$, as we have seen in the textbook. For a perfect reflector none of the incident beam is absorbed, so $e = 0$ and $p_{rad} = \frac{I_0}{c}(2-0) = \frac{2I_0}{c}$, as we have seen. Both extremes agree with the results in the textbook.

EXECUTE: (b) We want the force due to radiation. $F_{rad} = p_{rad}A$ and $p_{rad} = \frac{I_0}{c}(2-e)$.
$F_{rad} = \frac{I_0}{c}(2-e)\pi r^2$. Using $I_0 = 1.4\ \text{kW/m}^2$ and $r = 4.0\ \mu\text{m}$, we get $F_{rad} = 3.3\times10^{-16}$ N.
(c) We want F_{rad}/F_{grav}. $\frac{F_{rad}}{F_{grav}} = \frac{F_{rad}}{GmM_{sun}/r^2}$. Using $m = 1.0\times10^{-13}$ kg, $M_{sun} = 1.99\times10^{30}$ kg, $r = 1.5\times10^{11}$ m, and the result from part (b), we get $F_{rad}/F_{grav} = 0.55$.

EVALUATE: The force of the sun's radiation is about half as great as its gravitational force. Light particles of the same size would feel the same pressure but less gravity, so they could be blown away by the sun. Also closer to the sun the radiation is more intense, so the radiation pressure could blow more particles away from the sun.

32.50. IDENTIFY: This problem is about electromagnetic waves.

SET UP and EXECUTE: (a) Evaluate $\oint \vec{E}\cdot d\hat{l}$. $E_z = 0$ and the path is in the xz-plane, so only E_x contributes to the integral. Therefore $\oint \vec{E}\cdot d\hat{l} = \int_{-h/2}^{h/2} E_x(z=0)dx + \int_{h/2}^{-h/2} E_x(z=\lambda/2)dx$. Using the given waves gives $\oint \vec{E}\cdot d\hat{l} = \int_{-h/2}^{h/2} E\cos(0-\omega t)dx + \int_{h/2}^{-h/2} E\cos(k\lambda/2-\omega t)dx$. Integrating gives $\oint \vec{E}\cdot d\hat{l}$ $= hE\left[\cos\omega t - \left(\cos(k\lambda/2)\cos\omega t\right) + \sin(k\lambda/2)\sin\omega t\right]$. $\frac{k\lambda}{2} = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right) = \pi$. $\sin\pi = 0$ and $\cos\pi = -1$, so our result reduces to $\oint \vec{E}\cdot d\hat{l} = 2hE\cos\omega t$.

(b) We want the flux. $dA = h\,dz$. $BdA = h\,dz$. B_y is positive and A_y is negative, so the flux is negative. As in (a), we make use of $k\lambda/2 = \pi$.

$$\Phi_B = -\int B_y dA = -\int_0^{\lambda/2} B\cos(kz-\omega t)h\,dz = -\frac{Bh}{k}\left[\sin\left(k\lambda/2-\omega t\right) - \sin(-\omega t)\right] = -\frac{2Bh}{k}\sin\omega t.$$

(c) We want to find dB in terms of E and c. Using the result of (b), $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{2Bh}{k}\cos\omega t$. In part (a) we found that $\mathcal{E} = \oint \vec{E}\cdot d\hat{l} = 2hE\cos\omega t$. Equating these two expressions gives $E = cB$.

(d) Follow the directions in the problem: reverse the sign of the sine function and add the two terms. For $\vec{E}$ this gives

$\vec{E} = \vec{E}_R + \vec{E}_L = E\left[\cos(kz-\omega t)\hat{\imath} + \sin(kz-\omega t)\hat{\jmath}\right] + E\left[\cos(kz-\omega t)\hat{\imath} - \sin(kz-\omega t)\hat{\jmath}\right]$, which reduces to $\vec{E} = 2E\cos(kz-\omega t)\hat{\imath}$. The same procedure for $\vec{B}$ leads to $\vec{B} = 2B\cos(kz-\omega t)\hat{\jmath}$.

(e) We want the E and B. $S = \dfrac{E_{\max}^2}{\mu_0 c} = I$. Solve for $E_{\max}$ and use the given intensity.

$E_{\max} = \sqrt{\mu_0 cI} = \sqrt{\mu_0 c(100\ \text{W/m}^2)} = 194\ \text{V/m}$. $B_{\max} = E_{\max}/c = (194\ \text{V/m})/c = 0.647\ \mu\text{T}$.

EVALUATE: Note that B is a weak magnetic field but E is a considerably stronger electric field.

32.51. IDENTIFY: The orbiting particle has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11\times10^{-31}\ \text{kg}$ and a proton has mass $m_p = 1.67\times10^{-27}\ \text{kg}$.

EXECUTE: **(a)** $\left[\dfrac{q^2a^2}{6\pi\epsilon_0 c^3}\right] = \dfrac{\text{C}^2(\text{m/s}^2)^2}{(\text{C}^2/\text{N}\cdot\text{m}^2)(\text{m/s})^3} = \dfrac{\text{N}\cdot\text{m}}{\text{s}} = \dfrac{\text{J}}{\text{s}} = \text{W} = \left[\dfrac{dE}{dt}\right]$.

(b) For a proton moving in a circle, the acceleration is

$a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(6.00\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}{(1.67\times10^{-27}\ \text{kg})(0.75\ \text{m})} = 1.53\times10^{15}\ \text{m/s}^2$. The rate at which it emits energy because of its acceleration is

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\epsilon_0 c^3} = \frac{(1.6\times10^{-19}\ \text{C})^2(1.53\times10^{15}\ \text{m/s}^2)^2}{6\pi\epsilon_0 (3.0\times10^8\ \text{m/s})^3} = 1.33\times10^{-23}\ \text{J/s} = 8.32\times10^{-5}\ \text{eV/s}.$$

Therefore, the fraction of its energy that it radiates every second is

$$\frac{(dE/dt)(1\ \text{s})}{E} = \frac{8.32\times10^{-5}\ \text{eV}}{6.00\times10^6\ \text{eV}} = 1.39\times10^{-11}.$$

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses: $E_e = E_p\dfrac{m_e}{m_p} = (6.00\times10^6\ \text{eV})\dfrac{(9.11\times10^{-31}\ \text{kg})}{(1.67\times10^{-27}\ \text{kg})} = 3273\ \text{eV}$.

Therefore, the fraction of its energy that it radiates every second is

$$\frac{(dE/dt)(1\ \text{s})}{E} = \frac{8.32\times10^{-5}\ \text{eV}}{3273\ \text{eV}} = 2.54\times10^{-8}.$$

EVALUATE: The proton has speed $v = \sqrt{\dfrac{2E}{m_p}} = \sqrt{\dfrac{2(6.0\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{1.67\times10^{-27}\ \text{kg}}} = 3.39\times10^7\ \text{m/s}$.

The electron has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.52 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.52. **IDENTIFY:** The electron has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $1\text{ eV} = 1.60\times10^{-19}$ C. An electron has $|q| = e = 1.60\times10^{-19}$ C.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is $a = \dfrac{v^2}{R} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \dfrac{2(13.6\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(9.11\times10^{-31}\text{ kg})(5.29\times10^{-11}\text{ m})} = 9.03\times10^{22}\text{ m/s}^2$. Then using the formula for the rate of energy emission given in Problem 32.51: $\dfrac{dE}{dt} = \dfrac{q^2a^2}{6\pi\epsilon_0 c^3} = \dfrac{(1.60\times10^{-19}\text{ C})^2(9.03\times10^{22}\text{ m/s}^2)^2}{6\pi\epsilon_0(3.00\times10^8\text{ m/s})^3} = 4.64\times10^{-8}\text{ J/s} = 2.89\times10^{11}\text{ eV/s}$. This large value of $\dfrac{dE}{dt}$ would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.51 must not apply to electrons in atoms.

32.53. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E_y(x,t) = E_{\max}e^{-k_C x}\cos(k_C x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max}(-k_C)e^{-k_C x}\cos(k_C x - \omega t) + E_{\max}(-k_C)e^{-k_C x}\sin(k_C x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}(+k_C^2)e^{-k_C x}\cos(k_C x - \omega t) + E_{\max}(+k_C^2)e^{-k_C x}\sin(k_C x - \omega t).$$

$$+E_{\max}(+k_C^2)e^{-k_C x}\sin(k_C x - \omega t) + E_{\max}(-k_C^2)e^{-k_C x}\cos(k_C x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = +2E_{\max}k_C^2 e^{-k_C x}\sin(k_C x - \omega t). \quad \frac{\partial E_y}{\partial t} = +E_{\max}e^{-k_C x}\,\omega\sin(k_C x - \omega t).$$

Setting $\dfrac{\partial^2 E_y}{\partial x^2} = \dfrac{\mu\partial E_y}{\rho\partial t}$ gives $2E_{\max}k_C^2 e^{-k_C x}\sin(k_C x - \omega t) = \mu/\rho E_{\max}e^{-k_C x}\,\omega\sin(k_C x - \omega t)$. This will only be true if $\dfrac{2k_C^2}{\omega} = \dfrac{\mu}{\rho}$, or $k_C = \sqrt{\dfrac{\omega\mu}{2\rho}}$.

(b) The energy in the wave is dissipated by the i^2R heating of the conductor.

(c) $E_y = \dfrac{E_{y0}}{e} \Rightarrow k_C x = 1$, $x = \dfrac{1}{k_C} = \sqrt{\dfrac{2\rho}{\omega\mu}} = \sqrt{\dfrac{2(1.72\times10^{-8}\ \Omega\cdot\text{m})}{2\pi(1.0\times10^6\text{ Hz})\mu_0}} = 6.60\times10^{-5}$ m.

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

32.54. **IDENTIFY** and **SET UP:** Since 60 Hz is in the range 25 Hz to 3 kHz, we use the formula $E_{\max} = \dfrac{350}{f}$ V/m, where f is in kHz. The intensity is $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$.

EXECUTE: The maximum electric field is $E_{\max} = \dfrac{350}{f}$ V/m $= \dfrac{350}{0.060}$ V/m = 5800 V/m. Now find the intensity for the maximum field.

$I = \frac{1}{2}\epsilon_0 cE_{\max}^2 = (1/2)(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\text{ m/s})(5800\text{ V/m})^2 = 4.5\times10^4\text{ W/m}^2$

$= 45\text{ kW/m}^2$, which is choice (c).

EVALUATE: At higher frequencies the intensity would be less because the maximum electric field, which is inversely proportional to the frequency, would be smaller.

32.55. **IDENTIFY and SET UP:** The maximum electric field is proportional to $1/f$, and the intensity is proportional to E_{max}^2.

EXECUTE: Since E_{max} is proportional to $1/f$, doubling f decreases the maximum field by $\frac{1}{2}$. Because the intensity is proportional to E_{max}^2, decreasing E_{max} by a factor of $\frac{1}{2}$ will decrease the intensity by a factor of $(\frac{1}{2})^2 = \frac{1}{4}$, which is choice (d).

EVALUATE: Higher frequencies could be more harmful, so we tolerate lower fields at higher frequency.

32.56. **IDENTIFY and SET UP:** In the frequency range 25 Hz to 3 kHz, for a given frequency the maximum electric field is $E_{\text{max}} = 350/f$ and the maximum electric field is $B_{\text{max}} = 5/f$. $B = cE$.

EXECUTE: For the electric field, the maximum intensity at a frequency f is $I_{\text{max}} = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2$. Since $E_{\text{max}} = 350/f$, the intensity is $I_{\text{max}}^E = \frac{1}{2}\epsilon_0 c\left(\frac{350}{f}\right)^2$.

The intensity in terms of the magnetic field is $I = \frac{1}{2}\epsilon_0 cE_{\text{max}}^2 = \frac{1}{2}\epsilon_0 c(B_{\text{max}}c)^2 = \frac{1}{2}\epsilon_0 c^3 B_{\text{max}}^2$, where we have used $E_{\text{max}} = cB_{\text{max}}$. The maximum magnetic field is $B_{\text{max}} = 5/f$, so the maximum intensity for this magnetic field is $I_{\text{max}}^B = \frac{1}{2}\epsilon_0 c^3\left(\frac{5}{f}\right)^2$. Taking the ratio of the two intensities gives

$$\frac{I_{\text{max}}^E}{I_{\text{max}}^B} = \frac{\frac{1}{2}\epsilon_0 c\left(\frac{350}{f}\right)^2}{\frac{1}{2}\epsilon_0 c^3\left(\frac{5}{f}\right)^2} = \frac{1}{c^2}\left(\frac{350}{5}\right)^2 = 5.4\times10^{-14}.$$

The allowed intensity using the electric field limitation is *much* less than the allowed intensity using the magnetic field limitation, which is choice (b).

EVALUATE: The magnetic force on a charge due to an electromagnetic wave is normally much less than the electric force, so the intensity allowed for the electric field is much less than for the magnetic field.