

THE SECOND LAW OF THERMODYNAMICS

VP20.1.1. IDENTIFY: We are dealing the efficiency of a diesel engine.

SET UP: Efficiency = $e = W/Q_H$ and $Q_H = W + Q_C$.

EXECUTE: (a) We want the heat taken in, which is Q_H . Solve $e = W/Q_H$ for Q_H giving

$$Q_H = \frac{W}{e} = \frac{1.24 \times 10^4 \text{ J}}{0.180} = 6.89 \times 10^4 \text{ J}.$$

(b) $Q_C = W + Q_H = 6.89 \times 10^4 \text{ J} - 1.24 \times 10^4 \text{ J} = 5.65 \times 10^4 \text{ J}$.

EVALUATE: Only 18% of the heat taken in is converted to work. This is around the efficiency of automobile engines.

VP20.1.2. IDENTIFY: We are dealing with the efficiency of a heat engine.

SET UP and EXECUTE: $e = 1 - \left| \frac{Q_C}{Q_H} \right| = 1 - \frac{3.16 \times 10^4 \text{ J}}{3.82 \times 10^4 \text{ J}} = 0.173.$

EVALUATE: Only 17.3% of the heat input is converted to work; the rest is rejected from the engine as waste heat.

VP20.1.3. IDENTIFY: We are dealing with the efficiency of a gasoline heat engine.

SET UP: The efficiency is $e = 0.196$ and $Q_C = 4.96 \times 10^8 \text{ J}$ in 20 min.

EXECUTE: (a) We want to find Q_H . $e = 1 - \left| \frac{Q_C}{Q_H} \right| = 1 - \frac{4.96 \times 10^8 \text{ J}}{Q_H} = 0.196$. $Q_H = 6.17 \times 10^8 \text{ J}$.

(b) We want the work W . $W = Q_H - Q_C = 6.17 \times 10^8 \text{ J} - 4.96 \times 10^8 \text{ J} = 1.21 \times 10^8 \text{ J}$.

EVALUATE: The power output is $P = W/t = (1.21 \times 10^8 \text{ J}) / [(20)(60) \text{ s}] = 1.01 \times 10^5 \text{ W} = 101 \text{ kW}$.

VP20.1.4. IDENTIFY: A heat engine has a power output of $1.10 \times 10^5 \text{ W}$ and gets its heat input from the combustion of gasoline.

SET UP and EXECUTE: The heat of combustion of gasoline is $5.0 \times 10^7 \text{ J/kg}$.

(a) We want to see how much work this engine does in one hour. The power output is the rate at which the engine does work, so the work it does in one hour is $W = Pt = (1.10 \times 10^5 \text{ W})(3600 \text{ s}) = 3.96 \times 10^8 \text{ J}$.

(b) We want the heat input. $Q_H = (34 \text{ kg})(5.0 \times 10^7 \text{ J/kg}) = 1.7 \times 10^9 \text{ J}$.

(c) We want the efficiency. $e = \frac{W}{Q_H} = \frac{3.96 \times 10^8 \text{ J}}{1.7 \times 10^9 \text{ J}} = 0.23.$

EVALUATE: The heat wasted each hour is $Q_H - W = 1.7 \times 10^9 \text{ J} - 3.96 \times 10^8 \text{ J} = 1.3 \times 10^9 \text{ J}$.

VP20.4.1. IDENTIFY: This problem deals with a Carnot heat engine.

SET UP: For any heat engine $e = W/Q_H$ and $Q_H = W + Q_C$, and for a Carnot engine $e = 1 - \frac{T_C}{T_H}$.

EXECUTE: (a) We want the efficiency. $e = \frac{W}{Q_H} = \frac{1.68 \times 10^4 \text{ J}}{8.00 \times 10^4 \text{ J}} = 0.210$.

(b) We want Q_C . $Q_H = W + Q_C$, so $Q_C = 8.00 \times 10^4 \text{ J} - 1.68 \times 10^4 \text{ J} = 6.32 \times 10^4 \text{ J}$.

(c) We want T_H . $e = 1 - \frac{T_C}{T_H}$ gives $0.210 = 1 - \frac{298 \text{ K}}{T_H}$. $T_H = 377 \text{ K} = 104^\circ\text{C}$.

EVALUATE: The temperature must be in Kelvins to use these formulas.

VP20.4.2. IDENTIFY: We are analyzing a Carnot heat engine.

SET UP and EXECUTE: (a) We want the efficiency. $e = 1 - \frac{T_C}{T_H} = 1 - (200 \text{ K})/(500 \text{ K}) = 0.60$.

(b) Segment ab is the same as in Example 20.3, so $W_{ab} = 576 \text{ J}$.

$W_{bc} = nC_V(T_H - T_C) = (0.200 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(500 \text{ K} - 200 \text{ K}) = 1250 \text{ J}$.

$W_{cd} = nRT_C \ln(V_d/V_c) = (0.200 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(200 \text{ K}) \ln(1/2) = -231 \text{ J}$.

$W_{da} = nC_V(T_C - T_H) = (0.200 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(200 \text{ K} - 500 \text{ K}) = -1250 \text{ J}$.

EVALUATE: The total work is $576 \text{ J} + 1250 \text{ J} - 231 \text{ J} - 1250 \text{ J} = 345 \text{ J}$. This is much more work than in Example 20.3 due to the greater temperature difference between the hot and cold reservoirs. We get much more work from the same heat input of 576 J .

VP20.4.3. IDENTIFY: This problem is about a refrigerator operating on a Carnot cycle. $T_C = -10.0^\circ\text{C} = 263.15 \text{ K}$ and $T_H = 25.0^\circ\text{C} = 298.15 \text{ K}$.

SET UP and EXECUTE: (a) We want the coefficient of performance K .

$$K = \frac{T_C}{T_H - T_C} = \frac{263.15 \text{ K}}{298 \text{ K} - 263 \text{ K}} = 7.52.$$

(b) We want the work input. $K = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{|Q_C|}{W}$. $W = \frac{|Q_C|}{K} = \frac{4.00 \times 10^6 \text{ J}}{7.52} = 5.32 \times 10^5 \text{ J}$.

EVALUATE: $Q_H = W + |Q_C| = 5.32 \times 10^5 \text{ J} + 4.00 \times 10^6 \text{ J} = 4.53 \times 10^6 \text{ J}$. Each cycle this refrigerator discharges $4.53 \times 10^6 \text{ J}$ of heat into the room.

VP20.4.4. IDENTIFY: We are dealing with a Carnot heat engine. We know that $V_b = 2V_a$.

SET UP and EXECUTE: (a) We want the work done during the isothermal expansion ab .

$$W_{ab} = \int_a^b p dV = \int_a^b \frac{nRT}{V} dV = nRT_H \ln(V_b/V_a) = nRT_H \ln 2.$$

(b) We want the work done during the adiabatic expansion bc . $Q = 0$, so $W_{bc} = Q_{bc}$. So

$$W_{bc} = nC_V \Delta T_{bc} = n \left(\frac{3}{2} R \right) (T_H - T_C) = \frac{3}{2} nR(T_H - T_C).$$

(c) Given that $W_{ab} = W_{bc}$, we want e and the ratio of the cold to hot temperatures. Using the results from parts (a) and (b) gives $nRT_H \ln 2 = \frac{3}{2} nR(T_H - T_C)$, which gives $\frac{T_C}{T_H} = 1 - \frac{2}{3} \ln 2$. The efficiency is

$$e = 1 - \frac{T_C}{T_H} = 1 - \left(1 - \frac{2}{3} \ln 2 \right) = \frac{2}{3} \ln 2 = 0.462.$$

EVALUATE: This is quite an efficient engine. It is not necessarily true that $W_{ab} = W_{bc}$ for any Carnot engine.

VP20.10.1. IDENTIFY: We want to calculate the entropy changes due to melting, boiling, and temperature change.

SET UP and EXECUTE: For constant temperature $\Delta S = \frac{Q}{T}$, and for changing temperature $\Delta S = \int \frac{dQ}{T}$.

We want the entropy change in each case.

(a) The temperature stays constant, so $\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(1.00 \text{ kg})(1.042 \times 10^5 \text{ J/kg})}{159 \text{ K}} = 655 \text{ J/K}$.

(b) The temperature increases, so $\Delta S = \int \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc \ln(T_2 / T_1) = 1920 \text{ J/K}$ using $T_1 = 159 \text{ K}$, $T_2 = 159 \text{ K}$, $m = 1.00 \text{ kg}$, and $c = 2428 \text{ J/kg} \cdot \text{K}$.

(c) The temperature stays constant, so $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(1.00 \text{ kg})(8.54 \times 10^5 \text{ J/kg})}{351 \text{ K}} = 2430 \text{ J/K}$.

EVALUATE: The greatest increase is for boiling because $L_v \gg L_f$.

VP20.10.2. IDENTIFY: This problem involves the entropy change due to helium expansion by two different processes. We want to compare the entropy change in the two cases.

SET UP and EXECUTE: For constant temperature $\Delta S = \frac{Q}{T}$, and for changing temperature $\Delta S = \int \frac{dQ}{T}$.

(a) For an isothermal expansion we use $\Delta S = \frac{Q}{T}$. Since T is constant, $\Delta U = 0$, so $Q = W$. This gives

$$Q = W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \ln(V_2 / V_1). \quad \Delta S = \frac{Q}{T} = \frac{nRT \ln(V_2 / V_1)}{T} = nR \ln(V_2 / V_1).$$

Putting in the numbers gives $\Delta S = (5.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K}) \ln\left(\frac{0.360 \text{ m}^3}{0.120 \text{ m}^3}\right) = 45.7 \text{ J/K}$.

(b) Start with a pV -diagram showing the process (Fig. VP20.10.2). Break the process into its two segments: ab is isobaric and bc is isochoric. We know that $T_a = T_c = 20.0^\circ\text{C}$ and $V_c = V_b = 3V_a$.

Segment ab (constant pressure): The temperature is not constant, so we use $\Delta S = \int \frac{dQ}{T}$. The first law of thermodynamics (in its differential form) gives $dQ = dU + dW$. The pressure is constant, so $pV = nRT$ gives $p dV = nR dT$. Using this in the entropy calculation gives

$$dS_{ab} = \frac{dQ}{T} = \frac{dU + dW}{T} = \frac{nC_V dT}{T} + \frac{pdV}{T} = \frac{nC_V dT}{T} + \frac{nR dT}{T}.$$

$$\Delta S_{ab} = \int_{T_a}^{T_b} \left(\frac{nC_V}{T} + \frac{nR}{T} \right) dT = nC_V \ln(T_b / T_a) + nR \ln(T_b / T_a).$$

Segment bc (constant volume): Since the volume is constant, $dW = 0$, so $dQ = dU = nC_V dT$. Therefore

$$\Delta S_{bc} = \int \frac{dQ}{T} = \int \frac{dU}{T} = \int_{T_b}^{T_c} \frac{nC_V dT}{T} = nC_V \ln(T_c / T_b).$$

The total entropy change is $\Delta S_{ac} = \Delta S_{ab} + \Delta S_{bc} = nC_V \ln(T_b / T_a) + nR \ln(T_b / T_a) + nC_V \ln(T_c / T_b)$. But $T_a = T_c = 20.0^\circ\text{C}$, so $T_c / T_b = T_a / T_b$ which means that the first and third terms in the last sum are equal but have opposite signs, so they cancel. This leaves $\Delta S_{ac} = nR \ln(T_b / T_a)$. Using $pV = nRT$ for

constant pressure tells us that $\frac{T_b}{T_a} = \frac{V_b}{V_a} = 3$. Thus the total entropy change is

$$\Delta S_{ac} = nR \ln 3 = (5.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K}) \ln 3 = 45.7 \text{ J/K}.$$

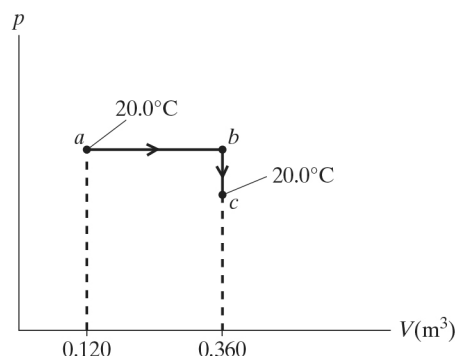


Figure VP20 10.2

EVALUATE: Notice that the entropy change is the same for both paths between the same initial and final states.

VP20.10.3. IDENTIFY: We are dealing the entropy change of a heat engine that is *not* using a Carnot cycle.

SET UP: For a constant temperature process $\Delta S = \frac{Q}{T}$. For one cycle, we want the entropy changes of the gas in the engine, the hot reservoir, and the cold reservoir. The two reservoirs are assumed to be so large that they do not change their temperatures as the engine runs. $T_H = 260^\circ\text{C} = 533.15\text{ K}$ and $T_C = 20^\circ\text{C} = 293.15\text{ K}$.

EXECUTE: (a) $\Delta S = 0$ for a reversible cycle.

(b) The engine takes a quantity of heat Q_H out of the hot reservoir, so

$$\Delta S_{\text{hot res}} = \frac{Q}{T} = \frac{-Q_H}{T_H} = \frac{-8.00 \times 10^4 \text{ J}}{533.15 \text{ K}} = -150 \text{ J/K}.$$

(b) The engine transfers a quantity of heat Q_C into the hot reservoir, so

$$\Delta S_{\text{cold res}} = \frac{Q}{T} = \frac{+Q_C}{T_H} = \frac{+6.40 \times 10^4 \text{ J}}{293.15 \text{ K}} = +218 \text{ J/K}.$$

(c) $\Delta S_{\text{tot}} = \Delta S_{\text{engine}} + \Delta S_{\text{hot res}} + \Delta S_{\text{cold res}} = 0 - 150 \text{ J/K} + 218 \text{ J/K} = +68 \text{ J/K}$. The entropy of the system *increases*.

EVALUATE: The entropy of the universe (that is, the total entropy) increases.

VP20.10.4. IDENTIFY: We are looking at the entropy change for an *irreversible* process.

SET UP: For ice melting at constant temperature $\Delta S = \frac{Q}{T}$, and for water changing temperature

$\Delta S = \int \frac{dQ}{T}$. We want the entropy change of the ice and of the water and the total entropy change for the entire system. $T_1 = 0.0^\circ\text{C} = 273 \text{ K}$ and $T_2 = 95.0^\circ\text{C} = 368 \text{ K}$.

EXECUTE: (a) $\Delta S_{\text{ice}} = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(1.00 \text{ kg})(3.34 \times 10^5 \text{ J/kg})}{273.15 \text{ K}} = +1220 \text{ J/K}$.

(b) $\Delta S_{\text{water}} = \int \frac{dQ}{T} = \int_{368 \text{ K}}^{273 \text{ K}} \frac{mcdT}{T} = (0.839 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{273 \text{ K}}{368 \text{ K}}\right) = -1050 \text{ J/K}$.

(c) $\Delta S_{\text{system}} = +1220 \text{ J/K} - 1050 \text{ J/K} = +170 \text{ J/K}$. The entropy increases.

EVALUATE: This is an irreversible process so the entropy should increase, as we've found.

20.1. IDENTIFY: For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $W = 2200 \text{ J}$. $|Q_C| = 4300 \text{ J}$.

EXECUTE: (a) $Q_H = W + |Q_C| = 6500 \text{ J}$.

(b) $e = \frac{2200 \text{ J}}{6500 \text{ J}} = 0.34 = 34\%$.

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W . But to calculate the efficiency we use the heat energy input, Q_H .

20.2. IDENTIFY: For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $|Q_H| = 9000 \text{ J}$. $|Q_C| = 6400 \text{ J}$.

EXECUTE: (a) $W = 9000 \text{ J} - 6400 \text{ J} = 2600 \text{ J}$.

(b) $e = \frac{W}{Q_H} = \frac{2600 \text{ J}}{9000 \text{ J}} = 0.29 = 29\%$.

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W . But to calculate the efficiency we use the heat energy input, Q_H .

20.3. IDENTIFY and SET UP: The problem deals with a heat engine. $W = +3700 \text{ W}$ and $Q_H = +16,100 \text{ J}$.

Use $e = \frac{W}{Q_H} = 1 - \left| \frac{Q_C}{Q_H} \right|$ to calculate the efficiency e and $W = |Q_H| - |Q_C|$ to calculate $|Q_C|$.

Power = W/t .

EXECUTE: (a) $e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{3700 \text{ J}}{16,100 \text{ J}} = 0.23 = 23\%$.

(b) $W = Q = |Q_H| - |Q_C|$

Heat discarded is $|Q_C| = |Q_H| - W = 16,100 \text{ J} - 3700 \text{ J} = 12,400 \text{ J}$.

(c) Q_H is supplied by burning fuel; $Q_H = mL_c$ where L_c is the heat of combustion.

$m = \frac{Q_H}{L_c} = \frac{16,100 \text{ J}}{4.60 \times 10^4 \text{ J/g}} = 0.350 \text{ g}$.

(d) $W = 3700 \text{ J}$ per cycle

In $t = 1.00 \text{ s}$ the engine goes through 60.0 cycles.

$P = W/t = 60.0(3700 \text{ J})/1.00 \text{ s} = 222 \text{ kW}$

$P = (2.22 \times 10^5 \text{ W})(1 \text{ hp}/746 \text{ W}) = 298 \text{ hp}$

EVALUATE: $Q_C = -12,400 \text{ J}$. In one cycle $Q_{\text{tot}} = Q_C + Q_H = 3700 \text{ J}$. This equals W_{tot} for one cycle.

20.4. IDENTIFY: $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: For 1.00 s, $W = 180 \times 10^3 \text{ J}$.

EXECUTE: (a) $Q_H = \frac{W}{e} = \frac{180 \times 10^3 \text{ J}}{0.280} = 6.43 \times 10^5 \text{ J}$.

(b) $|Q_C| = |Q_H| - W = 6.43 \times 10^5 \text{ J} - 1.80 \times 10^5 \text{ J} = 4.63 \times 10^5 \text{ J}$.

EVALUATE: Of the $6.43 \times 10^5 \text{ J}$ of heat energy supplied to the engine each second, $1.80 \times 10^5 \text{ J}$ is converted to mechanical work and the remaining $4.63 \times 10^5 \text{ J}$ is discarded into the low temperature reservoir.

20.5. IDENTIFY: This cycle involves adiabatic (ab), isobaric (bc), and isochoric (ca) processes.

SET UP: ca is at constant volume, ab has $Q = 0$, and bc is at constant pressure. For a constant pressure

process $W = p\Delta V$ and $Q = nC_p \Delta T$. $pV = nRT$ gives $n\Delta T = \frac{p\Delta V}{R}$, so $Q = \left(\frac{C_p}{R} \right) p\Delta V$. If $\gamma = 1.40$

the gas is diatomic and $C_p = \frac{7}{2}R$. For a constant volume process $W = 0$ and $Q = nC_V \Delta T$. $pV = nRT$ gives $n\Delta T = \frac{V\Delta p}{R}$, so $Q = \left(\frac{C_V}{R}\right)V\Delta p$. For a diatomic ideal gas $C_V = \frac{5}{2}R$. $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$.

EXECUTE: (a) $V_b = 9.0 \times 10^{-3} \text{ m}^3$, $p_b = 1.5 \text{ atm}$ and $V_a = 2.0 \times 10^{-3} \text{ m}^3$. For an adiabatic process

$$p_a V_a^\gamma = p_b V_b^\gamma. \quad p_a = p_b \left(\frac{V_b}{V_a}\right)^\gamma = (1.5 \text{ atm}) \left(\frac{9.0 \times 10^{-3} \text{ m}^3}{2.0 \times 10^{-3} \text{ m}^3}\right)^{1.4} = 12.3 \text{ atm}.$$

(b) Heat enters the gas in process ca , since T increases.

$$Q = \left(\frac{C_V}{R}\right)V\Delta p = \left(\frac{5}{2}\right)(2.0 \times 10^{-3} \text{ m}^3)(12.3 \text{ atm} - 1.5 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm}) = 5470 \text{ J}. \quad Q_H = 5470 \text{ J}.$$

(c) Heat leaves the gas in process bc , since T decreases.

$$Q = \left(\frac{C_p}{R}\right)p\Delta V = \left(\frac{7}{2}\right)(1.5 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(-7.0 \times 10^{-3} \text{ m}^3) = -3723 \text{ J}. \quad Q_C = -3723 \text{ J}.$$

$$(d) \quad W = Q_H + Q_C = +5470 \text{ J} + (-3723 \text{ J}) = 1747 \text{ J}.$$

$$(e) \quad e = \frac{W}{Q_H} = \frac{1747 \text{ J}}{5470 \text{ J}} = 0.319 = 31.9\%.$$

EVALUATE: We did not use the number of moles of the gas.

20.6. IDENTIFY and SET UP: This problem deals with the work done by a heat engine. We want to find the work done during 100 cycles. $|Q_H| = W + |Q_C|$. $Q_C = mL_f$. We know that $Q_H = 8000 \text{ J}$.

EXECUTE: $Q_C = mL_f = (0.0180 \text{ kg})(334,000 \text{ J/kg}) = 6012 \text{ J}$. $W = |Q_H| - |Q_C| = 8000 \text{ J} - 6012 \text{ J} = 1990 \text{ J}$.

EVALUATE: The efficiency of this engine is $e = W/Q_H = (1990 \text{ J})/(8000 \text{ J}) = 0.249$, which is typical of many engines.

20.7. IDENTIFY and SET UP: We are dealing with an engine operating on the Otto cycle for which $\gamma = 7/5$ and $e = 0.63$. The compression ratio r , which we want to find, is given by $\frac{1}{r^{\gamma-1}} = 1 - e$.

EXECUTE: (a) We want r . Solving $\frac{1}{r^{\gamma-1}} = 1 - e$ for r gives $r^{\gamma-1} = \frac{1}{1-e}$, so $r^{7/5-1} = \frac{1}{1-0.63}$, which gives $r = 12$.

(b) We want the rejected and absorbed heat if the engine does 12.6 kJ of work per cycle. Solving $e = W/Q_H$ for Q_H gives $Q_H = W/e = (12.6 \text{ kJ})/(0.63) = 20 \text{ kJ}$.

Using $|Q_H| = W + |Q_C|$ gives $Q_C = Q_H - W = 20 \text{ kJ} - 12.6 \text{ kJ} = 7.4 \text{ kJ}$.

EVALUATE: This engine is indeed very efficient at 63%.

20.8. IDENTIFY: Apply $e = 1 - \frac{1}{r^{\gamma-1}}$. $e = 1 - \frac{|Q_C|}{|Q_H|}$.

SET UP: In part (b), $Q_H = 10,000 \text{ J}$. The heat discarded is $|Q_C|$.

EXECUTE: (a) $e = 1 - \frac{1}{9.50^{0.40}} = 0.594 = 59.4\%$.

(b) $|Q_C| = |Q_H|(1 - e) = (10,000 \text{ J})(1 - 0.594) = 4060 \text{ J}$.

EVALUATE: The work output of the engine is $W = |Q_H| - |Q_C| = 10,000 \text{ J} - 4060 \text{ J} = 5940 \text{ J}$.

20.9. IDENTIFY: For the Otto-cycle engine, $e = 1 - r^{1-\gamma}$.

SET UP: r is the compression ratio.

EXECUTE: (a) $e = 1 - (8.8)^{-0.40} = 0.581$, which rounds to 58%.

(b) $e = 1 - (9.6)^{20.40} = 0.595$ an increase of 1.4%.

EVALUATE: An increase in r gives an increase in e .

20.10. IDENTIFY: $|Q_H| = |Q_C| + |W|$. $K = \frac{|Q_C|}{|W|}$.

SET UP: For water, $c_w = 4190 \text{ J/kg} \cdot \text{K}$ and $L_f = 3.34 \times 10^5 \text{ J/kg}$. For ice, $c_{ice} = 2100 \text{ J/kg} \cdot \text{K}$.

EXECUTE: (a) $Q = mc_{ice}\Delta T_{ice} - mL_f + mc_w\Delta T_w$.

$$Q = (1.80 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K})(-5.0^\circ\text{C}) - 3.34 \times 10^5 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(-25.0^\circ\text{C})] = -8.09 \times 10^5 \text{ J}$$

$Q = -8.09 \times 10^5 \text{ J}$. Q is negative for the water since heat is removed from it.

(b) $|Q_C| = 8.09 \times 10^5 \text{ J}$. $W = \frac{|Q_C|}{K} = \frac{8.09 \times 10^5 \text{ J}}{2.40} = 3.37 \times 10^5 \text{ J}$.

(c) $|Q_H| = 8.09 \times 10^5 \text{ J} + 3.37 \times 10^5 \text{ J} = 1.15 \times 10^6 \text{ J}$.

EVALUATE: For this device, $Q_C > 0$ and $Q_H < 0$. More heat is rejected to the room than is removed from the water.

20.11. IDENTIFY: The heat $Q = mc\Delta T$ that comes out of the water to cool it to 5.0°C is Q_C for the refrigerator.

SET UP: For water 1.0 L has a mass of 1.0 kg and $c = 4.19 \times 10^3 \text{ J/kg} \cdot \text{C}^\circ$. $P = \frac{|W|}{t}$. The coefficient of performance is $K = \frac{|Q_C|}{|W|}$.

EXECUTE: $Q = mc\Delta T = (12.0 \text{ kg})(4.19 \times 10^3 \text{ J/kg} \cdot \text{C}^\circ)(5.0^\circ\text{C} - 31^\circ\text{C}) = -1.31 \times 10^6 \text{ J}$. $|Q_C| = 1.31 \times 10^6 \text{ J}$.

$$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{Pt} \text{ so } t = \frac{|Q_C|}{PK} = \frac{1.31 \times 10^6 \text{ J}}{(135 \text{ W})(2.25)} = 4313 \text{ s} = 71.88 \text{ min} = 1.20 \text{ h}.$$

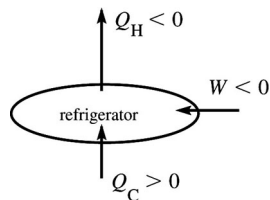
EVALUATE: 1.2 h seems like a reasonable time to cool down the dozen bottles.

20.12. IDENTIFY and SET UP: For the refrigerator $K = 2.10$ and $Q_C = +3.1 \times 10^4 \text{ J}$. Use $K = Q_C/|W|$ to calculate $|W|$ and then $W = Q_C + Q_H$ to calculate Q_H .

EXECUTE: (a) Coefficient of performance: $K = Q_C/|W|$.

$$|W| = Q_C/K = 3.10 \times 10^4 \text{ J} / 2.10 = 1.48 \times 10^4 \text{ J}.$$

(b) The operation of the device is illustrated in Figure 20.12.



$$W = Q_C + Q_H$$

$$Q_H = W - Q_C$$

$$Q_H = -1.48 \times 10^4 \text{ J} - 3.10 \times 10^4 \text{ J} = -4.58 \times 10^4 \text{ J (negative because heat goes out of the system)}$$

Figure 20.12

EVALUATE: $|Q_H| = |W| + |Q_C|$. The heat $|Q_H|$ delivered to the high temperature reservoir is greater than the heat taken in from the low temperature reservoir.

20.13. IDENTIFY: The hot reservoir of a Carnot engine is 72.0°C higher than the cold reservoir and this engine is 12.5% efficient. We want the temperature of the two reservoirs.

SET UP: For a Carnot engine $e = 1 - \frac{T_C}{T_H}$. $T_C = T_H - 72.0 \text{ K}$.

EXECUTE: $e = 1 - \frac{T_C}{T_H} = 1 - \frac{T_H - 72.0 \text{ K}}{T_H} = 0.125$ gives $T_H = 576 \text{ K}$, so $T_C = 576 \text{ K} - 72 \text{ K} = 504 \text{ K}$.

EVALUATE: Careful! The 72.0 C° is a temperature *difference*. Since Kelvin and Celsius degrees are the same size, so $\Delta T_K = \Delta T_C$. We should *not* convert the 72.0 C° to get 345 K .

20.14. IDENTIFY: $|W| = |Q_H| - |Q_C|$. $Q_C < 0$, $Q_H > 0$. $e = \frac{W}{Q_H}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 300 \text{ K}$, $T_H = 520 \text{ K}$. $|Q_H| = 6.45 \times 10^3 \text{ J}$.

EXECUTE: (a) $Q_C = -Q_H \left(\frac{T_C}{T_H} \right) = -(6.45 \times 10^3 \text{ J}) \left(\frac{300 \text{ K}}{520 \text{ K}} \right) = -3.72 \times 10^3 \text{ J}$.

(b) $|W| = |Q_H| - |Q_C| = 6.45 \times 10^3 \text{ J} - 3.72 \times 10^3 \text{ J} = 2.73 \times 10^3 \text{ J}$.

(c) $e = \frac{W}{Q_H} = \frac{2.73 \times 10^3 \text{ J}}{6.45 \times 10^3 \text{ J}} = 0.423 = 42.3\%$.

EVALUATE: We can verify that $e = 1 - T_C/T_H$ also gives $e = 42.3\%$.

20.15. IDENTIFY and SET UP: Use $W = Q_C + Q_H$ to calculate $|W|$. Since it is a Carnot device we can use

$\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}$ to relate the heat flows out of the reservoirs. The reservoir temperatures can be used in

$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ to calculate e .

EXECUTE: (a) The operation of the device is sketched in Figure 20.15.

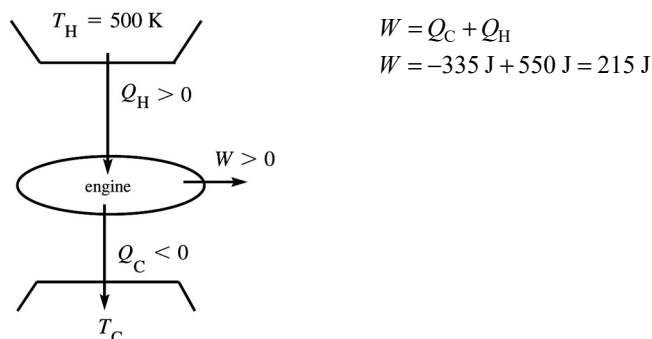


Figure 20.15

(b) For a Carnot cycle, $\frac{|Q_C|}{|Q_H|} = \frac{T_C}{T_H}$, which gives

$$T_C = T_H \frac{|Q_C|}{|Q_H|} = 620 \text{ K} \left(\frac{335 \text{ J}}{550 \text{ J}} \right) = 378 \text{ K}.$$

(c) $e_{\text{Carnot}} = 1 - T_C/T_H = 1 - 378 \text{ K}/620 \text{ K} = 0.390 = 39.0\%$.

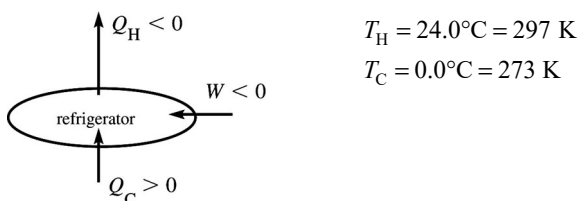
EVALUATE: We could use the fundamental definition of e , $e = \frac{W}{Q_H}$:

$$e = W/Q_H = (215 \text{ J})/(550 \text{ J}) = 39\%, \text{ which checks.}$$

20.16. IDENTIFY and SET UP: The device is a Carnot refrigerator.

We can use $W = Q_C + Q_H$ and $|Q_C|/|Q_H| = T_C/T_H$.

(a) The operation of the device is sketched in Figure 20.16.

**Figure 20.16**

The amount of heat taken out of the water to make the liquid \rightarrow solid phase change is

$Q = -mL_f = -(85.0 \text{ kg})(334 \times 10^3 \text{ J/kg}) = -2.84 \times 10^7 \text{ J}$. This amount of heat must go into the working substance of the refrigerator, so $Q_C = +2.84 \times 10^7 \text{ J}$. For Carnot cycle $|Q_C|/|Q_H| = T_C/T_H$.

EXECUTE: $|Q_H| = |Q_C|(T_H/T_C) = 2.84 \times 10^7 \text{ J}(297 \text{ K}/273 \text{ K}) = 3.09 \times 10^7 \text{ J}$.

(b) $W = Q_C + Q_H = +2.84 \times 10^7 \text{ J} - 3.09 \times 10^7 \text{ J} = -2.5 \times 10^6 \text{ J}$

EVALUATE: W is negative because this much energy must be supplied to the refrigerator rather than obtained from it. Note that in $W = Q_C + Q_H$ we must use Kelvin temperatures.

20.17. IDENTIFY: $e = \frac{W}{Q_H}$ for any engine. For the Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 20.0^\circ\text{C} + 273.15 \text{ K} = 293.15 \text{ K}$

EXECUTE: (a) $Q_H = \frac{W}{e} = \frac{2.5 \times 10^4 \text{ J}}{0.66} = 3.79 \times 10^4 \text{ J}$, which rounds to $3.8 \times 10^4 \text{ J}$.

(b) $W = Q_H + Q_C$ so $Q_C = W - Q_H = 2.5 \times 10^4 \text{ J} - 3.79 \times 10^4 \text{ J} = -1.29 \times 10^4 \text{ J}$.

$T_H = -T_C \frac{Q_H}{Q_C} = -(293.15 \text{ K}) \left(\frac{3.79 \times 10^4 \text{ J}}{-1.29 \times 10^4 \text{ J}} \right) = 861 \text{ K} = 590^\circ\text{C}$.

EVALUATE: For a heat engine, $W > 0$, $Q_H > 0$ and $Q_C < 0$.

20.18. IDENTIFY: The theoretical maximum performance coefficient is a $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$. $K = \frac{|Q_C|}{|W|}$. $|Q_C|$ is

the heat removed from the water to convert it to ice. For the water, $|Q| = mc_w \Delta T + mL_f$.

SET UP: $T_C = -5.0^\circ\text{C} = 268 \text{ K}$. $T_H = 20.0^\circ\text{C} = 293 \text{ K}$. $c_w = 4190 \text{ J/kg} \cdot \text{K}$ and $L_f = 334 \times 10^3 \text{ J/kg}$.

EXECUTE: (a) In one year the freezer operates $(5 \text{ h/day})(365 \text{ days}) = 1825 \text{ h}$.

$P = \frac{730 \text{ kWh}}{1825 \text{ h}} = 0.400 \text{ kW} = 400 \text{ W}$.

(b) $K_{\text{Carnot}} = \frac{268 \text{ K}}{293 \text{ K} - 268 \text{ K}} = 10.7$.

(c) $|W| = Pt = (400 \text{ W})(3600 \text{ s}) = 1.44 \times 10^6 \text{ J}$. $|Q_C| = K|W| = 1.54 \times 10^7 \text{ J}$. $|Q| = mc_w \Delta T + mL_f$ gives

$m = \frac{|Q_C|}{c_w \Delta T + L_f} = \frac{1.54 \times 10^7 \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(20.0 \text{ K}) + 334 \times 10^3 \text{ J/kg}} = 36.9 \text{ kg}$.

EVALUATE: For any actual device, $K < K_{\text{Carnot}}$, $|Q_C|$ is less than we calculated and the freezer makes less ice in one hour than the mass we calculated in part (c).

20.19. IDENTIFY: $|Q_H| = |W| + |Q_C|$. $Q_H < 0$, $Q_C > 0$. $K = \frac{|Q_C|}{|W|}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 270 \text{ K}$, $T_H = 320 \text{ K}$. $|Q_C| = 415 \text{ J}$.

EXECUTE: (a) $Q_H = -\left(\frac{T_H}{T_C}\right)Q_C = -\left(\frac{320\text{ K}}{270\text{ K}}\right)(415\text{ J}) = -492\text{ J}.$

(b) For one cycle, $|W| = |Q_H| - |Q_C| = 492\text{ J} - 415\text{ J} = 77\text{ J}.$ $P = \frac{(165)(77\text{ J})}{60\text{ s}} = 212\text{ W}.$

(c) $K = \frac{|Q_C|}{|W|} = \frac{415\text{ J}}{77\text{ J}} = 5.4.$

EVALUATE: The amount of heat energy $|Q_H|$ delivered to the high-temperature reservoir is greater than the amount of heat energy $|Q_C|$ removed from the low-temperature reservoir.

20.20. IDENTIFY: $W = Q_C + Q_H.$ For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}.$ For the ice to liquid water phase transition,

$$Q = mL_f.$$

SET UP: For water, $L_f = 334 \times 10^3\text{ J/kg}.$

EXECUTE: $Q_C = -mL_f = -(0.0400\text{ kg})(334 \times 10^3\text{ J/kg}) = -1.336 \times 10^4\text{ J}.$ $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ gives

$$Q_H = -(T_H / T_C)Q_C = -(-1.336 \times 10^4\text{ J})[(373.15\text{ K}) / (273.15\text{ K})] = +1.825 \times 10^4\text{ J}.$$

$$W = Q_C + Q_H = 4.89 \times 10^3\text{ J}.$$

EVALUATE: For a heat engine, Q_C is negative and Q_H is positive. The heat that comes out of the engine ($Q < 0$) goes into the ice ($Q > 0$).

20.21. IDENTIFY and SET UP: We are looking at two refrigerators, A and B , operating on a Carnot cycle. The coefficient of performance of a Carnot refrigerator is $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = \frac{T_C}{\Delta T}.$ We know that

$K_A = K_B + 0.16K_B = 1.16K_B,$ $\Delta T_B = \Delta T_A + 0.30\Delta T_A = 1.30\Delta T_A,$ and $T_C(B) = 180\text{ K}.$ We want to find the cold reservoir temperature for A , $T_C(A).$

EXECUTE: $K_A = \frac{T_C(A)}{\Delta T_A}$ and $K_B = \frac{T_C(B)}{\Delta T_B},$ so $\frac{K_A}{K_B} = \frac{\frac{T_C(A)}{\Delta T_A}}{\frac{T_C(B)}{\Delta T_B}}.$ Using the given information, we have

$$\frac{1.16K_B}{K_B} = \frac{T_C(A)}{T_C(B)} \frac{\Delta T_B}{\Delta T_A} = \frac{T_C(A)}{T_C(B)} \frac{1.30\Delta T_A}{\Delta T_A}.$$
 This gives $1.16 = \frac{1.30T_C(A)}{180\text{ K}},$ so $T_C(A) = 161\text{ K}.$

EVALUATE: All temperatures must be in Kelvins.

20.22. IDENTIFY: Apply $Q_{\text{system}} = 0$ to calculate the final temperature. $Q = mc\Delta T.$ $\Delta S = mc \ln(T_2/T_1)$ when an object undergoes a temperature change.

SET UP: For water $c = 4190\text{ J/kg} \cdot \text{K}.$ Boiling water has $T = 100.0^\circ\text{C} = 373\text{ K}.$

EXECUTE: (a) The heat transfer between 100°C water and 30°C water occurs over a finite temperature difference and the process is irreversible.

(b) $(195\text{ kg})c(T_2 - 30.0^\circ\text{C}) + (5.00\text{ kg})c(T_2 - 100^\circ\text{C}) = 0.$ $T_2 = 31.75^\circ\text{C} = 304.90\text{ K}.$

(c) $\Delta S = (195\text{ kg})(4190\text{ J/kg} \cdot \text{K}) \ln\left(\frac{304.90\text{ K}}{303.15\text{ K}}\right) + (5.00\text{ kg})(4190\text{ J/kg} \cdot \text{K}) \ln\left(\frac{304.90\text{ K}}{373.15\text{ K}}\right) = 471\text{ J/K}.$

EVALUATE: $\Delta S_{\text{system}} > 0,$ as it should for an irreversible process.

20.23. IDENTIFY: $\Delta S = \frac{Q}{T}$ for each object, where T must be in kelvins. The temperature of each object remains constant.

SET UP: For water, $L_f = 3.34 \times 10^5 \text{ J/kg}$.

EXECUTE: (a) The heat flow into the ice is $Q = mL_f = (0.350 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 1.17 \times 10^5 \text{ J}$. The heat flow occurs at $T = 273 \text{ K}$, so $\Delta S = \frac{Q}{T} = \frac{1.17 \times 10^5 \text{ J}}{273 \text{ K}} = 429 \text{ J/K}$. Q is positive and ΔS is positive.

(b) $Q = -1.17 \times 10^5 \text{ J}$ flows out of the heat source, at $T = 298 \text{ K}$. $\Delta S = \frac{Q}{T} = \frac{-1.17 \times 10^5 \text{ J}}{298 \text{ K}} = -393 \text{ J/K}$. Q is negative and ΔS is negative.

(c) $\Delta S_{\text{tot}} = 429 \text{ J/K} + (-393 \text{ J/K}) = +36 \text{ J/K}$.

EVALUATE: For the total isolated system, $\Delta S > 0$ and the process is irreversible.

20.24. IDENTIFY: $Q = mc\Delta T$ for the water. $\Delta S = mc \ln(T_2/T_1)$ when an object undergoes a temperature change. $\Delta S = Q/T$ for an isothermal process.

SET UP: For water, $c = 4190 \text{ J/kg} \cdot \text{K}$. $85.0^\circ\text{C} = 358.2 \text{ K}$. $20.0^\circ\text{C} = 293.2 \text{ K}$.

EXECUTE: (a) $\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{293.2 \text{ K}}{358.2 \text{ K}}\right) = -210 \text{ J/K}$. Heat comes out of the water and its entropy decreases.

(b) $Q = mc\Delta T = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(-65.0 \text{ K}) = -6.81 \times 10^4 \text{ J}$. The amount of heat that goes into the air is $+6.81 \times 10^4 \text{ J}$. For the air, $\Delta S = \frac{Q}{T} = \frac{+6.81 \times 10^4 \text{ J}}{293.1 \text{ K}} = +232 \text{ J/K}$.

$\Delta S_{\text{system}} = -210 \text{ J/K} + 232 \text{ J/K} = +22 \text{ J/K}$.

EVALUATE: $\Delta S_{\text{system}} > 0$ and the process is irreversible.

20.25. IDENTIFY: The process is at constant temperature, so $\Delta S = \frac{Q}{T}$. $\Delta U = Q - W$.

SET UP: For an isothermal process of an ideal gas, $\Delta U = 0$ and $Q = W$. For a compression, $\Delta V < 0$ and $W < 0$.

EXECUTE: $Q = W = -1850 \text{ J}$. $\Delta S = \frac{-1850 \text{ J}}{293 \text{ K}} = -6.31 \text{ J/K}$.

EVALUATE: The entropy change of the gas is negative. Heat must be removed from the gas during the compression to keep its temperature constant and therefore the gas is not an isolated system.

20.26. IDENTIFY and SET UP: The initial and final states are at the same temperature, at the normal boiling point of 4.216 K . Calculate the entropy change for the irreversible process by considering a reversible isothermal process that connects the same two states, since ΔS is path independent and depends only on the initial and final states. For the reversible isothermal process we can use $\Delta S = Q/T$.

The heat flow for the helium is $Q = -mL_v$, negative since in condensation heat flows out of the helium.

The heat of vaporization L_v is given in Table 17.4 and is $L_v = 20.9 \times 10^3 \text{ J/kg}$.

EXECUTE: $Q = -mL_v = -(0.130 \text{ kg})(20.9 \times 10^3 \text{ J/kg}) = -2717 \text{ J}$
 $\Delta S = Q/T = -2717 \text{ J}/4.216 \text{ K} = -644 \text{ J/K}$.

EVALUATE: The system we considered is the 0.130 kg of helium; ΔS is the entropy change of the helium. This is not an isolated system since heat must flow out of it into some other material. Our result that $\Delta S < 0$ doesn't violate the second law since it is not an isolated system. The material that receives the heat that flows out of the helium would have a positive entropy change and the total entropy change would be positive.

20.27. IDENTIFY: Each phase transition occurs at constant temperature and $\Delta S = \frac{Q}{T}$. $Q = mL_v$.

SET UP: For vaporization of water, $L_v = 2256 \times 10^3 \text{ J/kg}$.

EXECUTE: (a) $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(1.00 \text{ kg})(2256 \times 10^3 \text{ J/kg})}{(373.15 \text{ K})} = 6.05 \times 10^3 \text{ J/K}$. Note that this is the

change of entropy of the water as it changes to steam.

(b) The magnitude of the entropy change is roughly five times the value found in Example 20.5.

EVALUATE: Water is less ordered (more random) than ice, but water is far less random than steam; a consideration of the density changes indicates why this should be so.

20.28. IDENTIFY and SET UP: (a) The velocity distribution of the Maxwell-Boltzmann distribution,

$f(v) = 4\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$, depends only on T , so in an isothermal process it does not change.

(b) EXECUTE: Calculate the change in the number of available microscopic states and apply Eq. (20.23). Following the reasoning of Example 20.11, the number of possible positions available to each molecule is altered by a factor of 3 (becomes larger). Hence the number of microscopic states the gas occupies at volume $3V$ is $w_2 = (3)^N w_1$, where N is the number of molecules and w_1 is the number of possible microscopic states at the start of the process, where the volume is V . Then, using

$\Delta S = k \ln(w_2/w_1)$, we have $\Delta S = k \ln(w_2/w_1) = k \ln(3)^N = Nk \ln(3) = nN_A k \ln(3) = nR \ln(3)$.

$\Delta S = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K}) \ln(3) = +18.3 \text{ J/K}$.

(c) IDENTIFY and SET UP: For an isothermal reversible process $\Delta S = Q/T$.

EXECUTE: Calculate W and then use the first law to calculate Q .

$\Delta T = 0$ implies that $\Delta U = 0$, since system is an ideal gas.

Then by $\Delta U = Q - W$, $Q = W$.

For an isothermal process, $W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} (nRT/V) dV = nRT \ln(V_2/V_1)$.

Thus $Q = nRT \ln(V_2/V_1)$ and $\Delta S = Q/T = nR \ln(V_2/V_1)$.

$\Delta S = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K}) \ln(3V_1/V_1) = +18.3 \text{ J/K}$.

EVALUATE: This is the same result as obtained in part (b).

20.29. IDENTIFY: For a free expansion, $\Delta S = nR \ln(V_2/V_1)$.

SET UP: $V_1 = 2.40 \text{ L} = 2.40 \times 10^{-3} \text{ m}^3$.

EXECUTE: $\Delta S = (0.100 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K}) \ln\left(\frac{425 \text{ m}^3}{2.40 \times 10^{-3} \text{ m}^3}\right) = 10.0 \text{ J/K}$.

EVALUATE: $\Delta S_{\text{system}} > 0$ and the free expansion is irreversible.

20.30. IDENTIFY: For a Carnot engine, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$. $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. $|W| = |Q_H| - |Q_C|$. $Q_H > 0$, $Q_C < 0$.

$pV = nRT$.

SET UP: The work done by the engine each cycle is $mg\Delta y$, with $m = 15.0 \text{ kg}$ and $\Delta y = 2.00 \text{ m}$.

$T_H = 773 \text{ K}$. $Q_H = 500 \text{ J}$.

EXECUTE: (a) The pV diagram is sketched in Figure 20.30.

(b) $W = mg\Delta y = (15.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = 294 \text{ J}$. $|Q_C| = |Q_H| - |W| = 500 \text{ J} - 294 \text{ J} = 206 \text{ J}$, and $Q_C = -206 \text{ J}$.

$T_C = -T_H \left(\frac{Q_C}{Q_H} \right) = -(773 \text{ K}) \left(\frac{-206 \text{ J}}{500 \text{ J}} \right) = 318 \text{ K} = 45^\circ\text{C}$.

(c) $e = 1 - \frac{T_C}{T_H} = 1 - \frac{318 \text{ K}}{773 \text{ K}} = 0.589 = 58.9\%$.

(d) $|Q_C| = 206 \text{ J}$.

(e) The maximum pressure is for state *a*. This is also where the volume is a minimum, so

$$V_a = 5.00 \text{ L} = 5.00 \times 10^{-3} \text{ m}^3. \quad T_a = T_H = 773 \text{ K}.$$

$$p_a = \frac{nRT_a}{V_a} = \frac{(2.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(773 \text{ K})}{5.00 \times 10^{-3} \text{ m}^3} = 2.57 \times 10^6 \text{ Pa}.$$

EVALUATE: We can verify that $e = \frac{W}{Q_H}$ gives the same value for *e* as calculated in part (c).

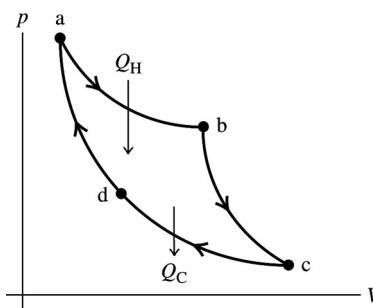


Figure 20.30

20.31. IDENTIFY: The total work that must be done is $W_{\text{tot}} = mg\Delta y$. $|W| = |Q_H| - |Q_C|$. $Q_H > 0$, $W > 0$ and

$$Q_C < 0. \text{ For a Carnot cycle, } \frac{Q_C}{Q_H} = -\frac{T_C}{T_H}.$$

SET UP: $T_C = 373 \text{ K}$, $T_H = 773 \text{ K}$. $|Q_H| = 250 \text{ J}$.

EXECUTE: (a) $Q_C = -Q_H \left(\frac{T_C}{T_H} \right) = -(250 \text{ J}) \left(\frac{373 \text{ K}}{773 \text{ K}} \right) = -121 \text{ J}$.

(b) $|W| = 250 \text{ J} - 121 \text{ J} = 129 \text{ J}$. This is the work done in one cycle.

$$W_{\text{tot}} = (500 \text{ kg})(9.80 \text{ m/s}^2)(100 \text{ m}) = 4.90 \times 10^5 \text{ J}. \text{ The number of cycles required is}$$

$$\frac{W_{\text{tot}}}{|W|} = \frac{4.90 \times 10^5 \text{ J}}{129 \text{ J/cycle}} = 3.80 \times 10^3 \text{ cycles}.$$

EVALUATE: In $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$, the temperatures must be in kelvins.

20.32. IDENTIFY: The same amount of heat that enters the person's body also leaves the body, but these transfers of heat occur at different temperatures, so the person's entropy changes.

SET UP: We are asked to find the entropy change of the person. The person is not an isolated system. In 1.0 s, $0.80(80 \text{ J}) = 64 \text{ J}$ of heat enters the person's body at $37^\circ\text{C} = 310 \text{ K}$. This amount of heat leaves

the person at a temperature of $30^\circ\text{C} = 303 \text{ K}$. $\Delta S = \frac{Q}{T}$.

EXECUTE: For the person, $\Delta S = \frac{+64 \text{ J}}{310 \text{ K}} + \frac{-64 \text{ J}}{303 \text{ K}} = -4.8 \times 10^{-3} \text{ J/K}$.

EVALUATE: The entropy of the person can decrease without violating the second law of thermodynamics because the person isn't an isolated system.

- 20.33. IDENTIFY:** We know the efficiency of this Carnot engine, the heat it absorbs at the hot reservoir and the temperature of the hot reservoir.

SET UP: For a heat engine $e = \frac{W}{|Q_H|}$ and $Q_H + Q_C = W$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$. $Q_C < 0$, $W >$

0, and $Q_H > 0$. $T_H = 135^\circ\text{C} = 408\text{ K}$. In each cycle, $|Q_H|$ leaves the hot reservoir and $|Q_C|$ enters the cold reservoir. The work done on the water equals its increase in gravitational potential energy, mgh .

EXECUTE: (a) $e = \frac{W}{Q_H}$ so $W = eQ_H = (0.220)(410\text{ J}) = 90.2\text{ J}$.

(b) $Q_C = W - Q_H = 90.2\text{ J} - 410\text{ J} = -319.8\text{ J}$, which rounds to -320 J .

(c) $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ so $T_C = -T_H\left(\frac{Q_C}{Q_H}\right) = -(408\text{ K})\left(\frac{-319.8\text{ J}}{410\text{ J}}\right) = 318\text{ K} = 45^\circ\text{C}$.

(d) $\Delta S = \frac{-|Q_H|}{T_H} + \frac{|Q_C|}{T_C} = \frac{-410\text{ J}}{408\text{ K}} + \frac{319.8\text{ J}}{318\text{ K}} = 0$. The Carnot cycle is reversible and $\Delta S = 0$.

(e) $W = mgh$ so $m = \frac{W}{gh} = \frac{90.2\text{ J}}{(9.80\text{ m/s}^2)(35.0\text{ m})} = 0.263\text{ kg} = 263\text{ g}$.

EVALUATE: The Carnot cycle is reversible so $\Delta S = 0$ for the world. However some parts of the world gain entropy while other parts lose it, making the sum equal to zero.

- 20.34. IDENTIFY:** Use the ideal gas law to calculate p and V for each state. Use the first law and specific

expressions for Q , W , and ΔU for each process. Use $e = \frac{W}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$ to calculate e . Q_H is the net heat

flow into the gas.

SET UP: $\gamma = 1.40$

$C_V = R/(\gamma - 1) = 20.79\text{ J/mol} \cdot \text{K}$; $C_p = C_V + R = 29.10\text{ J/mol} \cdot \text{K}$. The cycle is sketched in Figure 20.34.

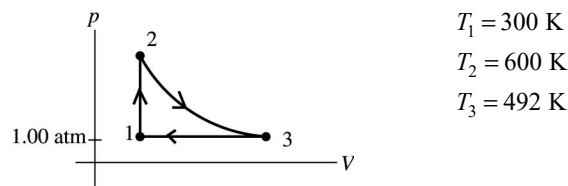


Figure 20.34

EXECUTE: (a) Point 1:

$p_1 = 1.00\text{ atm} = 1.013 \times 10^5\text{ Pa}$ (given); $pV = nRT$;

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.350\text{ mol})(8.3145\text{ J/mol} \cdot \text{K})(300\text{ K})}{1.013 \times 10^5\text{ Pa}} = 8.62 \times 10^{-3}\text{ m}^3$$

Point 2:

Process $1 \rightarrow 2$ is at constant volume so $V_2 = V_1 = 8.62 \times 10^{-3}\text{ m}^3$

$pV = nRT$ and n , R , V constant implies $p_1/T_1 = p_2/T_2$

$$p_2 = p_1(T_2/T_1) = (1.00\text{ atm})(600\text{ K}/300\text{ K}) = 2.00\text{ atm} = 2.03 \times 10^5\text{ Pa}$$

Point 3:

Consider the process $3 \rightarrow 1$, since it is simpler than $2 \rightarrow 3$.

Process $3 \rightarrow 1$ is at constant pressure so $p_3 = p_1 = 1.00\text{ atm} = 1.013 \times 10^5\text{ Pa}$

$pV = nRT$ and n, R, p constant implies $V_1/T_1 = V_3/T_3$

$$V_3 = V_1(T_3/T_1) = (8.62 \times 10^{-3} \text{ m}^3)(492 \text{ K}/300 \text{ K}) = 14.1 \times 10^{-3} \text{ m}^3$$

(b) Process 1 \rightarrow 2:

Constant volume ($\Delta V = 0$)

$$Q = nC_V \Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 300 \text{ K}) = 2180 \text{ J}$$

$$\Delta V = 0 \text{ and } W = 0. \text{ Then } \Delta U = Q - W = 2180 \text{ J}$$

Process 2 \rightarrow 3:

Adiabatic means $Q = 0$.

$$\Delta U = nC_V \Delta T \text{ (any process), so}$$

$$\Delta U = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 600 \text{ K}) = -780 \text{ J}$$

Then $\Delta U = Q - W$ gives $W = Q - \Delta U = 1780 \text{ J}$. (It is correct for W to be positive since ΔV is positive.)

Process 3 \rightarrow 1:

For constant pressure

$$W = p\Delta V = (1.013 \times 10^5 \text{ Pa})(8.62 \times 10^{-3} \text{ m}^3 - 14.1 \times 10^{-3} \text{ m}^3) = -560 \text{ J}$$

or $W = nR\Delta T = (0.350 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -560 \text{ J}$, which checks. (It is correct for W to be negative, since ΔV is negative for this process.)

$$Q = nC_p \Delta T = (0.350 \text{ mol})(29.10 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1960 \text{ J}$$

$$\Delta U = Q - W = -1960 \text{ J} - (-560 \text{ J}) = -1400 \text{ J}$$

or $\Delta U = nC_V \Delta T = (0.350 \text{ mol})(20.79 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 492 \text{ K}) = -1400 \text{ J}$, which checks.

$$\text{(c) } W_{\text{net}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1} = 0 + 780 \text{ J} - 560 \text{ J} = +220 \text{ J}$$

$$\text{(d) } Q_{\text{net}} = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} + Q_{3 \rightarrow 1} = 2180 \text{ J} + 0 - 1960 \text{ J} = +220 \text{ J}$$

$$\text{(e) } e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{220 \text{ J}}{2180 \text{ J}} = 0.101 = 10.1\%.$$

$$e(\text{Carnot}) = 1 - T_C/T_H = 1 - 300 \text{ K}/600 \text{ K} = 0.500.$$

EVALUATE: For a cycle $\Delta U = 0$, so by $\Delta U = Q - W$ it must be that $Q_{\text{net}} = W_{\text{net}}$ for a cycle. We can also check that $\Delta U_{\text{net}} = 0$: $\Delta U_{\text{net}} = \Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} + \Delta U_{3 \rightarrow 1} = 2180 \text{ J} - 1050 \text{ J} - 1130 \text{ J} = 0$
 $e < e(\text{Carnot})$, as it must.

20.35. IDENTIFY: The same amount of heat that enters the person's body also leaves the body, but these transfers of heat occur at different temperatures, so the person's entropy changes.

SET UP: 1 food calorie = 1000 cal = 4186 J. The heat enters the person's body at $37^\circ\text{C} = 310 \text{ K}$ and leaves at a temperature of $30^\circ\text{C} = 303 \text{ K}$. $\Delta S = \frac{Q}{T}$.

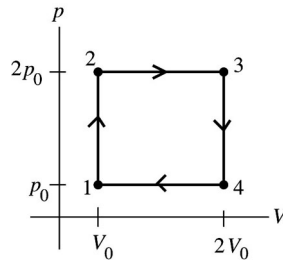
$$\text{EXECUTE: } |Q| = (0.80)(2.50 \text{ g})(9.3 \text{ food calorie/g}) \left(\frac{4186 \text{ J}}{1 \text{ food calorie}} \right) = 7.79 \times 10^4 \text{ J}.$$

$$\Delta S = \frac{+7.79 \times 10^4 \text{ J}}{310 \text{ K}} + \frac{-7.79 \times 10^4 \text{ J}}{303 \text{ K}} = -5.8 \text{ J/K. Your body's entropy decreases.}$$

EVALUATE: The entropy of your body can decrease without violating the second law of thermodynamics because you are not an isolated system.

20.36. IDENTIFY: Use $e = \frac{W}{Q_H}$ to calculate e .

SET UP: The cycle is sketched in Figure 20.36.



$$C_V = 5R/2$$

$$\text{for an ideal gas } C_p = C_V + R = 7R/2$$

Figure 20.36

SET UP: Calculate Q and W for each process.

Process 1 \rightarrow 2:

$$\Delta V = 0 \text{ implies } W = 0$$

$$\Delta V = 0 \text{ implies } Q = nC_V\Delta T = nC_V(T_2 - T_1)$$

$$\text{But } pV = nRT \text{ and } V \text{ constant says } p_1V = nRT_1 \text{ and } p_2V = nRT_2.$$

$$\text{Thus } (p_2 - p_1)V = nR(T_2 - T_1); \quad V\Delta p = nR\Delta T \text{ (true when } V \text{ is constant).}$$

$$\text{Then } Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)V\Delta p = (C_V/R)V_0(2p_0 - p_0) = (C_V/R)p_0V_0. \quad (Q > 0; \text{ heat is absorbed by the gas.})$$

Process 2 \rightarrow 3:

$$\Delta p = 0 \text{ so } W = p\Delta V = p(V_3 - V_2) = 2p_0(2V_0 - V_0) = 2p_0V_0 \quad (W \text{ is positive since } V \text{ increases.})$$

$$\Delta p = 0 \text{ implies } Q = nC_p\Delta T = nC_p(T_2 - T_1).$$

$$\text{But } pV = nRT \text{ and } p \text{ constant says } pV_1 = nRT_1 \text{ and } pV_2 = nRT_2.$$

$$\text{Thus } p(V_2 - V_1) = nR(T_2 - T_1); \quad p\Delta V = nR\Delta T \text{ (true when } p \text{ is constant).}$$

$$\text{Then } Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(2V_0 - V_0) = (C_p/R)p_0V_0. \quad (Q > 0; \text{ heat is absorbed by the gas.})$$

Process 3 \rightarrow 4:

$$\Delta V = 0 \text{ implies } W = 0$$

$$\Delta V = 0 \text{ so}$$

$$Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)(2V_0)(p_0 - 2p_0) = -2(C_V/R)p_0V_0$$

$$(Q < 0 \text{ so heat is rejected by the gas.})$$

Process 4 \rightarrow 1:

$$\Delta p = 0 \text{ so } W = p\Delta V = p(V_1 - V_4) = p_0(V_0 - 2V_0) = -p_0V_0 \quad (W \text{ is negative since } V \text{ decreases})$$

$$\Delta p = 0 \text{ so } Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(V_0 - 2V_0) = -(C_p/R)p_0V_0 \quad (Q < 0 \text{ so}$$

$$\text{heat is rejected by the gas.})$$

Total work performed by the gas during the cycle:

$$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} + W_{4 \rightarrow 1} = 0 + 2p_0V_0 + 0 - p_0V_0 = p_0V_0$$

(Note that W_{tot} equals the area enclosed by the cycle in the pV -diagram.)

Total heat absorbed by the gas during the cycle (Q_H):

Heat is absorbed in processes 1 \rightarrow 2 and 2 \rightarrow 3.

$$Q_H = Q_{1 \rightarrow 2} + Q_{2 \rightarrow 3} = \frac{C_V}{R}p_0V_0 + 2\frac{C_p}{R}p_0V_0 = \left(\frac{C_V + 2C_p}{R}\right)p_0V_0$$

$$\text{But } C_p = C_V + R \text{ so } Q_H = \frac{C_V + 2(C_V + R)}{R}p_0V_0 = \left(\frac{3C_V + 2R}{R}\right)p_0V_0.$$

Total heat rejected by the gas during the cycle (Q_C):

Heat is rejected in processes $3 \rightarrow 4$ and $4 \rightarrow 1$.

$$Q_C = Q_{3 \rightarrow 4} + Q_{4 \rightarrow 1} = -2 \frac{C_V}{R} p_0 V_0 - \frac{C_p}{R} p_0 V_0 = - \left(\frac{2C_V + C_p}{R} \right) p_0 V_0$$

$$\text{But } C_p = C_V + R \text{ so } Q_C = - \frac{2C_V + (C_V + R)}{R} p_0 V_0 = - \left(\frac{3C_V + R}{R} \right) p_0 V_0.$$

$$\text{Efficiency: } e = \frac{W}{Q_H} = \frac{p_0 V_0}{[(3C_V + 2R)/R](p_0 V_0)} = \frac{R}{3C_V + 2R} = \frac{R}{3(5R/2) + 2R} = \frac{2}{19}. \quad e = 0.105 = 10.5\%.$$

EVALUATE: As a check on the calculations note that

$$Q_C + Q_H = - \left(\frac{3C_V + R}{R} \right) p_0 V_0 + \left(\frac{3C_V + 2R}{R} \right) p_0 V_0 = p_0 V_0 = W, \text{ as it should.}$$

- 20.37. IDENTIFY:** $pV = nRT$, so pV is constant when T is constant. Use the appropriate expression to calculate Q and W for each process in the cycle. $e = \frac{W}{Q_H}$.

SET UP: For an ideal diatomic gas, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$.

EXECUTE: (a) $p_a V_a = 2.0 \times 10^3 \text{ J}$. $p_b V_b = 2.0 \times 10^3 \text{ J}$. $pV = nRT$ so $p_a V_a = p_b V_b$ says $T_a = T_b$.

(b) For an isothermal process, $Q = W = nRT \ln(V_2/V_1)$. ab is a compression, with $V_b < V_a$, so $Q < 0$ and

heat is rejected. bc is at constant pressure, so $Q = nC_p \Delta T = \frac{C_p}{R} p \Delta V$. ΔV is positive, so $Q > 0$ and heat

is absorbed. ca is at constant volume, so $Q = nC_V \Delta T = \frac{C_V}{R} V \Delta p$. Δp is negative, so $Q < 0$ and heat is rejected.

$$\text{(c) } T_a = \frac{p_a V_a}{nR} = \frac{2.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 241 \text{ K}. \quad T_b = \frac{p_b V_b}{nR} = T_a = 241 \text{ K}.$$

$$T_c = \frac{p_c V_c}{nR} = \frac{4.0 \times 10^3 \text{ J}}{(1.00)(8.314 \text{ J/mol} \cdot \text{K})} = 481 \text{ K}.$$

$$\text{(d) } Q_{ab} = nRT \ln \left(\frac{V_b}{V_a} \right) = (1.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) \ln \left(\frac{0.0050 \text{ m}^3}{0.010 \text{ m}^3} \right) = -1.39 \times 10^3 \text{ J}.$$

$$Q_{bc} = nC_p \Delta T = (1.00) \left(\frac{7}{2} \right) (8.314 \text{ J/mol} \cdot \text{K})(241 \text{ K}) = 7.01 \times 10^3 \text{ J}.$$

$$Q_{ca} = nC_V \Delta T = (1.00) \left(\frac{5}{2} \right) (8.314 \text{ J/mol} \cdot \text{K})(-241 \text{ K}) = -5.01 \times 10^3 \text{ J}.$$

$$Q_{\text{net}} = Q_{ab} + Q_{bc} + Q_{ca} = 610 \text{ J}. \quad W_{\text{net}} = Q_{\text{net}} = 610 \text{ J}.$$

$$\text{(e) } e = \frac{W}{Q_H} = \frac{610 \text{ J}}{7.01 \times 10^3 \text{ J}} = 0.087 = 8.7\%.$$

EVALUATE: We can calculate W for each process in the cycle. $W_{ab} = Q_{ab} = -1.39 \times 10^3 \text{ J}$.

$W_{bc} = p \Delta V = (4.0 \times 10^5 \text{ Pa})(0.0050 \text{ m}^3) = 2.00 \times 10^3 \text{ J}$. $W_{ca} = 0$. $W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 610 \text{ J}$, which does equal Q_{net} .

- 20.38. IDENTIFY and SET UP:** This problem involves the entropy change of ice water and the surrounding air in a room. Estimate: The cup is $\frac{1}{4}$ ice and $\frac{3}{4}$ water all at 0°C when we go to bed. Convert pints to liters and cubic meters: 1 gal = 3.788 L and 8 pints = 1 gal, so 1 pint = $(3.788 \text{ L})/8 = 0.474 \text{ L} = 4.74 \times 10^{-4} \text{ m}^3$. From Table 12.1, $\rho_w = 1000 \text{ kg/m}^3$ and $\rho_{\text{ice}} = 920 \text{ kg/m}^3$, so the masses are $m_{\text{ice}} = (920 \text{ kg/m}^3)(\frac{1}{4})(4.74 \times 10^{-4} \text{ m}^3) = 0.109 \text{ kg}$, $m_w = (1000 \text{ kg/m}^3)(\frac{3}{4})(4.74 \times 10^{-4} \text{ m}^3) = 0.355 \text{ kg}$. In the

morning, the cup is full of liquid water at 22.2°C. During the night, the water-ice mixture went from 0.0°C to 22.2°C. $\Delta S = Q/T$ (for constant temperature) and $\Delta S = \int \frac{dQ}{T}$ (if the temperature varies).

EXECUTE: (a) We want the entropy change of the ice-water mixture. It starts at 0.0°C and ends up at 22.2°C. $\Delta S_{\text{tot}} = \Delta S_{\text{ice}} + \Delta S_{\text{water}}$. The entropy change of the original ice is the sum of the entropy change as it melts and the entropy change of the resulting water as it increases from 0.0°C to 22.2°C.

$$\Delta S_{\text{melt ice}} = \frac{Q}{T} = \frac{mL_f}{T} = \frac{(0.109 \text{ kg})(334 \times 10^3 \text{ J/kg})}{273 \text{ K}} = +133.2 \text{ J/K}.$$

$$\Delta S_{\text{melted ice}} = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{m_i c_w dT}{T} = m_i c_w \ln(T_2/T_1). \text{ Using } m_i = 0.109 \text{ kg}, c_w = 4190 \text{ J/kg} \cdot \text{K}, T_1 = 0.0^\circ\text{C} =$$

273.15 K, $T_2 = 22.2^\circ\text{C} = 295.35 \text{ K}$, we get $\Delta S_{\text{melted ice}} = 35.7 \text{ J/K}$. The total entropy change of the ice is $\Delta S_{\text{ice}} = 133.2 \text{ J/K} + 35.7 \text{ J/K} = 169 \text{ J/K}$. The entropy change of the original water in the glass is

$$\Delta S_{\text{water}} = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{m_w c_w dT}{T} = m_w c_w \ln T_2/T_1. \text{ Using } T_1 = 0.0^\circ\text{C} = 273.15 \text{ K}, T_2 = 22.2^\circ\text{C} = 295.35 \text{ K},$$

$m_w = 0.355 \text{ kg}$, we get $\Delta S_{\text{water}} = 116.2 \text{ J/K}$. The total entropy change is $\Delta S_{\text{tot}} = \Delta S_{\text{ice}} + \Delta S_{\text{water}} = 169 \text{ J/K} + 116 \text{ J/K} = +285 \text{ J/K}$.

(b) The air remains at a constant 22.2°C = 295.35 K, so $\Delta S_{\text{air}} = Q/T_{\text{air}}$. Q is the *total* heat that flows out of the air into the ice-water mixture in the plastic cup. This heat melts the ice and then increases the temperature of *all* the water from 0.0°C to 22.2°C. Therefore $Q = Q_{\text{melt ice}} + Q_{\text{heat all water to } 22.2^\circ\text{C}} =$

$m_i L_f + m_{\text{all water}} c_w \Delta T_w$. This gives

$Q = (0.109 \text{ kg} + 0.355 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(22.2^\circ\text{C}) = 7.96 \times 10^4 \text{ J}$. This heat comes *out* of the air, so

$$\Delta S_{\text{air}} = \frac{Q}{T_{\text{air}}} = \frac{-7.96 \times 10^4 \text{ J}}{285.35 \text{ K}} = -279 \text{ J/K}. \text{ The total entropy change is } -279 \text{ J/K} + 285 \text{ J/K} = +6 \text{ J/K}, \text{ a}$$

positive change.

EVALUATE: The heating of the ice/water mixture is irreversible, so $\Delta S > 0$.

20.39. IDENTIFY: $T_b = T_c$ and is equal to the maximum temperature. Use the ideal gas law to calculate T_a .

Apply the appropriate expression to calculate Q for each process. $e = \frac{W}{Q_H}$. $\Delta U = 0$ for a complete cycle

and for an isothermal process of an ideal gas.

SET UP: For helium, $C_V = 3R/2$ and $C_p = 5R/2$. The maximum efficiency is for a Carnot cycle, and

$$e_{\text{Carnot}} = 1 - T_C/T_H.$$

EXECUTE: (a) $Q_{\text{in}} = Q_{ab} + Q_{bc}$. $Q_{\text{out}} = Q_{ca}$. $T_{\text{max}} = T_b = T_c = 327^\circ\text{C} = 600 \text{ K}$.

$$\frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} \rightarrow T_a = \frac{p_a}{p_b} T_b = \frac{1}{3} (600 \text{ K}) = 200 \text{ K}.$$

$$p_b V_b = nRT_b \rightarrow V_b = \frac{nRT_b}{p_b} = \frac{(2 \text{ moles})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{ K})}{3.0 \times 10^5 \text{ Pa}} = 0.0332 \text{ m}^3.$$

$$\frac{p_b V_b}{T_b} = \frac{p_c V_c}{T_c} \rightarrow V_c = V_b \frac{p_b}{p_c} = (0.0332 \text{ m}^3) \left(\frac{3}{1} \right) = 0.0997 \text{ m}^3 = V_a.$$

$$Q_{ab} = nC_V \Delta T_{ab} = (2 \text{ mol}) \left(\frac{3}{2} \right) (8.31 \text{ J/mol} \cdot \text{K})(400 \text{ K}) = 9.97 \times 10^3 \text{ J}.$$

$$Q_{bc} = W_{bc} = \int_b^c p dV = \int_b^c \frac{nRT_b}{V} dV = nRT_b \ln \frac{V_c}{V_b} = nRT_b \ln 3.$$

$$Q_{bc} = (2.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{ K}) \ln 3 = 1.10 \times 10^4 \text{ J}. \quad Q_{in} = Q_{ab} + Q_{bc} = 2.10 \times 10^4 \text{ J}.$$

$$Q_{out} = Q_{ca} = nC_p \Delta T_{ca} = (2.00 \text{ mol}) \left(\frac{5}{2} \right) (8.31 \text{ J/mol} \cdot \text{K})(400 \text{ K}) = 1.66 \times 10^4 \text{ J}.$$

$$(b) \quad Q = \Delta U + W = 0 + W \rightarrow W = Q_{in} - Q_{out} = 2.10 \times 10^4 \text{ J} - 1.66 \times 10^4 \text{ J} = 4.4 \times 10^3 \text{ J}.$$

$$e = W/Q_{in} = \frac{4.4 \times 10^3 \text{ J}}{2.10 \times 10^4 \text{ J}} = 0.21 = 21\%.$$

$$(c) \quad e_{\max} = e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{200 \text{ K}}{600 \text{ K}} = 0.67 = 67\%.$$

EVALUATE: The thermal efficiency of this cycle is about one-third of the efficiency of a Carnot cycle that operates between the same two temperatures.

20.40. IDENTIFY: Since there is temperature difference between the inside and outside of your body, you can use it as a heat engine.

SET UP: For a heat engine $e = \frac{W}{Q_H}$. For a Carnot engine $e = 1 - \frac{T_C}{T_H}$. Gravitational potential energy is

$$U_{\text{grav}} = mgh. \quad 1 \text{ food calorie} = 1000 \text{ cal} = 4186 \text{ J}.$$

EXECUTE: (a) $e = 1 - \frac{T_C}{T_H} = 1 - \frac{303 \text{ K}}{310 \text{ K}} = 0.0226 = 2.26\%$. This engine has a very low thermal efficiency.

(b) $U_{\text{grav}} = mgh = (2.50 \text{ kg})(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 29.4 \text{ J}$. This equals the work output of the engine.

$$e = \frac{W}{Q_H} \quad \text{so} \quad Q_H = \frac{W}{e} = \frac{29.4 \text{ J}}{0.0226} = 1.30 \times 10^3 \text{ J}.$$

(c) Since 80% of food energy goes into heat, you must eat food with a food energy of

$$\frac{1.30 \times 10^3 \text{ J}}{0.80} = 1.63 \times 10^3 \text{ J}. \quad \text{Each candy bar gives } (350 \text{ food calorie})(4186 \text{ J/food calorie}) = 1.47 \times 10^6 \text{ J}.$$

$$\text{The number of candy bars required is } \frac{1.63 \times 10^3 \text{ J}}{1.47 \times 10^6 \text{ J/candy bar}} = 1.11 \times 10^{-3} \text{ candy bars}.$$

EVALUATE: A large amount of mechanical work must be done to use up the energy from one candy bar.

20.41. IDENTIFY: $e_{\max} = e_{\text{Carnot}} = 1 - T_C / T_H$. $e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t}$. $W = Q_H + Q_C$ so $\frac{W}{t} = \frac{Q_C}{t} + \frac{Q_H}{t}$. For a temperature change $Q = mc\Delta T$.

SET UP: $T_H = 300.15 \text{ K}$, $T_C = 279.15 \text{ K}$. For water, $\rho = 1000 \text{ kg/m}^3$, so a mass of 1 kg has a volume of 1 L. For water, $c = 4190 \text{ J/kg} \cdot \text{K}$.

$$\text{EXECUTE: (a)} \quad e = 1 - \frac{279.15 \text{ K}}{300.15 \text{ K}} = 7.0\%.$$

$$(b) \quad \frac{Q_H}{t} = \frac{P_{\text{out}}}{e} = \frac{210 \text{ kW}}{0.070} = 3.0 \text{ MW}. \quad \frac{|Q_C|}{t} = \frac{Q_H}{t} - \frac{W}{t} = 3.0 \text{ MW} - 210 \text{ kW} = 2.8 \text{ MW}.$$

$$(c) \quad \frac{m}{t} = \frac{|Q_C|/t}{c\Delta T} = \frac{(2.8 \times 10^6 \text{ W})(3600 \text{ s/h})}{(4190 \text{ J/kg} \cdot \text{K})(4 \text{ K})} = 6 \times 10^5 \text{ kg/h} = 6 \times 10^5 \text{ L/h}.$$

EVALUATE: The efficiency is small since T_C and T_H don't differ greatly.

20.42. IDENTIFY: Use $\Delta U = Q - W$ and the appropriate expressions for Q , W and ΔU for each type of process. $pV = nRT$ relates ΔT to p and V values. $e = \frac{W}{Q_H}$, where Q_H is the heat that enters the gas during the cycle.

SET UP: For a monatomic ideal gas, $C_p = \frac{5}{2}R$ and $C_v = \frac{3}{2}R$.

(a) ab : The temperature changes by the same factor as the volume, and so

$$Q = nC_p\Delta T = \frac{C_p}{R}p_a(V_a - V_b) = (2.5)(3.00 \times 10^5 \text{ Pa})(0.300 \text{ m}^3) = 2.25 \times 10^5 \text{ J}.$$

The work $p\Delta V$ is the same except for the factor of $\frac{5}{2}$, so $W = 0.90 \times 10^5 \text{ J}$.

$$\Delta U = Q - W = 1.35 \times 10^5 \text{ J}.$$

bc : The temperature now changes in proportion to the pressure change, and

$$Q = \frac{3}{2}(p_c - p_b)V_b = (1.5)(-2.00 \times 10^5 \text{ Pa})(0.800 \text{ m}^3) = -2.40 \times 10^5 \text{ J}, \text{ and the work is zero}$$

$$(\Delta V = 0). \Delta U = Q - W = -2.40 \times 10^5 \text{ J}.$$

ca : The easiest way to do this is to find the work done first; W will be the negative of area in the p - V plane bounded by the line representing the process ca and the verticals from points a and c . The area of this trapezoid is $\frac{1}{2}(3.00 \times 10^5 \text{ Pa} + 1.00 \times 10^5 \text{ Pa})(0.800 \text{ m}^3 - 0.500 \text{ m}^3) = 6.00 \times 10^4 \text{ J}$ and so the work is $-0.60 \times 10^5 \text{ J}$. ΔU must be $1.05 \times 10^5 \text{ J}$ (since $\Delta U = 0$ for the cycle, anticipating part (b)), and so Q must be $\Delta U + W = 0.45 \times 10^5 \text{ J}$.

(b) See above; $Q = W = 0.30 \times 10^5 \text{ J}$, $\Delta U = 0$.

(c) The heat added, during process ab and ca , is $2.25 \times 10^5 \text{ J} + 0.45 \times 10^5 \text{ J} = 2.70 \times 10^5 \text{ J}$ and the

$$\text{efficiency is } e = \frac{W}{Q_H} = \frac{0.30 \times 10^5}{2.70 \times 10^5} = 0.111 = 11.1\%.$$

EVALUATE: For any cycle, $\Delta U = 0$ and $Q = W$.

20.43. IDENTIFY: Use $pV = nRT$. Apply the expressions for Q and W that apply to each type of process.

$$e = \frac{W}{Q_H}.$$

SET UP: For O_2 , $C_v = 20.85 \text{ J/mol} \cdot \text{K}$ and $C_p = 29.17 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) $p_1 = 2.00 \text{ atm}$, $V_1 = 4.00 \text{ L}$, $T_1 = 300 \text{ K}$.

$$p_2 = 2.00 \text{ atm}. \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}. \quad V_2 = \left(\frac{T_2}{T_1}\right)V_1 = \left(\frac{450 \text{ K}}{300 \text{ K}}\right)(4.00 \text{ L}) = 6.00 \text{ L}.$$

$$V_3 = 6.00 \text{ L}. \quad \frac{p_2}{T_2} = \frac{p_3}{T_3}. \quad p_3 = \left(\frac{T_3}{T_2}\right)p_2 = \left(\frac{250 \text{ K}}{450 \text{ K}}\right)(2.00 \text{ atm}) = 1.11 \text{ atm}.$$

$$V_4 = 4.00 \text{ L}. \quad p_3V_3 = p_4V_4. \quad p_4 = p_3\left(\frac{V_3}{V_4}\right) = (1.11 \text{ atm})\left(\frac{6.00 \text{ L}}{4.00 \text{ L}}\right) = 1.67 \text{ atm}.$$

These processes are shown in Figure 20.43.

$$(b) \quad n = \frac{p_1V_1}{RT_1} = \frac{(2.00 \text{ atm})(4.00 \text{ L})}{(0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K})(300 \text{ K})} = 0.325 \text{ mol}$$

$$\text{Process } 1 \rightarrow 2: \quad W = p\Delta V = nR\Delta T = (0.325 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(150 \text{ K}) = 405 \text{ J}.$$

$$Q = nC_p\Delta T = (0.325 \text{ mol})(29.17 \text{ J/mol} \cdot \text{K})(150 \text{ K}) = 1422 \text{ J}.$$

$$\text{Process } 2 \rightarrow 3: \quad W = 0. \quad Q = nC_v\Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(-200 \text{ K}) = -1355 \text{ J}.$$

Process 3 \rightarrow 4: $\Delta U = 0$ and

$$Q = W = nRT_3 \ln\left(\frac{V_4}{V_3}\right) = (0.325 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(250 \text{ K}) \ln\left(\frac{4.00 \text{ L}}{6.00 \text{ L}}\right) = -274 \text{ J}.$$

Process 4 \rightarrow 1: $W = 0$. $Q = nC_V \Delta T = (0.325 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(50 \text{ K}) = 339 \text{ J}$.

(c) $W = 405 \text{ J} - 274 \text{ J} = 131 \text{ J}$.

$$(d) e = \frac{W}{Q_H} = \frac{131 \text{ J}}{1422 \text{ J} + 339 \text{ J}} = 0.0744 = 7.44\%.$$

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{250 \text{ K}}{450 \text{ K}} = 0.444 = 44.4\%; \quad e_{\text{Carnot}} \text{ is much larger.}$$

EVALUATE: $Q_{\text{tot}} = +1422 \text{ J} + (-1355 \text{ J}) + (-274 \text{ J}) + 339 \text{ J} = 132 \text{ J}$. This is equal to W_{tot} , apart from a slight difference due to rounding. For a cycle, $W_{\text{tot}} = Q_{\text{tot}}$, since $\Delta U = 0$.

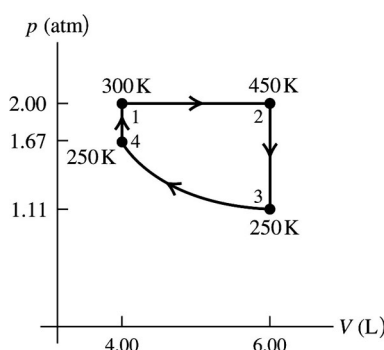


Figure 20.43

20.44. IDENTIFY: $e = \frac{W}{Q_H}$. 1 day = 8.64×10^4 s. For the river water, $Q = mc\Delta T$, where the heat that goes into

the water is the heat Q_C rejected by the engine. The density of water is 1000 kg/m^3 . When an object undergoes a temperature change, $\Delta S = mc \ln(T_2/T_1)$.

SET UP: $18.0^\circ\text{C} = 291.1 \text{ K}$. $18.5^\circ\text{C} = 291.6 \text{ K}$.

EXECUTE: (a) $Q_H = \frac{W}{e}$ so $P_H = \frac{P_W}{e} = \frac{1000 \text{ MW}}{0.40} = 2.50 \times 10^3 \text{ MW}$.

(b) The heat input in one day is $(2.50 \times 10^9 \text{ W})(8.64 \times 10^4 \text{ s}) = 2.16 \times 10^{14} \text{ J}$. The mass of coal used per

day is $\frac{2.16 \times 10^{14} \text{ J}}{2.65 \times 10^7 \text{ J/kg}} = 8.15 \times 10^6 \text{ kg}$.

(c) $|Q_H| = |W| + |Q_C|$. $|Q_C| = |Q_H| - |W|$. $P_C = P_H - P_W = 2.50 \times 10^3 \text{ MW} - 1000 \text{ MW} = 1.50 \times 10^3 \text{ MW}$.

(d) The heat input to the river is $1.50 \times 10^9 \text{ J/s}$. $Q = mc\Delta T$ and $\Delta T = 0.5^\circ\text{C}$ gives

$$m = \frac{Q}{c\Delta T} = \frac{1.50 \times 10^9 \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(0.5 \text{ K})} = 7.16 \times 10^5 \text{ kg}. \quad V = \frac{m}{\rho} = 716 \text{ m}^3. \quad \text{The river flow rate must be } 716 \text{ m}^3/\text{s}.$$

(e) In one second, $7.16 \times 10^5 \text{ kg}$ of water goes from 291.1 K to 291.6 K .

$$\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) = (7.16 \times 10^5 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln\left(\frac{291.6 \text{ K}}{291.1 \text{ K}}\right) = 5.1 \times 10^6 \text{ J/K}.$$

EVALUATE: The entropy of the river increases because heat flows into it. The mass of coal used per second is huge.

- 20.45. IDENTIFY:** The efficiency of the composite engine is $e_{12} = \frac{W_1 + W_2}{Q_{H1}}$, where Q_{H1} is the heat input to the first engine and W_1 and W_2 are the work outputs of the two engines. For any heat engine, $W = Q_C + Q_H$, and for a Carnot engine, $\frac{Q_{\text{low}}}{Q_{\text{high}}} = -\frac{T_{\text{low}}}{T_{\text{high}}}$, where Q_{low} and Q_{high} are the heat flows at the two reservoirs that have temperatures T_{low} and T_{high} .

SET UP: $Q_{\text{high},2} = -Q_{\text{low},1}$, $T_{\text{low},1} = T'$, $T_{\text{high},1} = T_H$, $T_{\text{low},2} = T_C$ and $T_{\text{high},2} = T'$.

EXECUTE: $e_{12} = \frac{W_1 + W_2}{Q_{H1}} = \frac{Q_{\text{high},1} + Q_{\text{low},1} + Q_{\text{high},2} + Q_{\text{low},2}}{Q_{\text{high},1}}$. Since $Q_{\text{high},2} = -Q_{\text{low},1}$, this reduces to

$$e_{12} = 1 + \frac{Q_{\text{low},2}}{Q_{\text{high},1}}. \quad Q_{\text{low},2} = -Q_{\text{high},2} = -Q_{\text{high},1} \frac{T_{\text{low},2}}{T_{\text{high},2}} = -Q_{\text{low},1} \frac{T_C}{T'} = -Q_{\text{high},1} \left(\frac{T_{\text{low},1}}{T_{\text{high},1}} \right) \frac{T_C}{T'} = -Q_{\text{high},1} \left(\frac{T'}{T_H} \right) \frac{T_C}{T'}. \quad \text{This gives}$$

$$e_{12} = 1 - \frac{T_C}{T_H}. \quad \text{The efficiency of the composite system is the same as that of the original engine.}$$

EVALUATE: The overall efficiency is independent of the value of the intermediate temperature T' .

- 20.46. IDENTIFY:** We are dealing with the efficiency of an automobile engine.

SET UP: (a) Estimate: 45 miles/gallon (a Prius).

EXECUTE: (b) The combustion releases 120 MJ/gal. At 40 mph the engine is turning at 3000 rpm which is 3000 cycles/min. We want to know how many joules/cycle are released. At 40 mph, the time to travel 45 mi is $t = x/v = (45 \text{ mi})/(40 \text{ mi/h}) = 1.125 \text{ h} = 67.5 \text{ min}$. The energy released during this time is 1 gal = 120 MJ. In 67.5 min the number of cycles the engine turns through is $(3000 \text{ cycles/min})(67.5 \text{ min}) = 2.025 \times 10^5 \text{ cycles}$. The energy per cycle is $\frac{120 \text{ MJ}}{2.025 \times 10^5 \text{ cycles}} = 593 \text{ J/cycle}$.

(c) We want the power the car engine supplies at 40 mph. The power P is the energy E divided by the time t , so $P = E/t$. At 20% efficiency, $E = (0.20)(593 \text{ J/cycle}) = 118.5 \text{ J/cycle}$ and $3000 \text{ cycles/min} = 50 \text{ cycles/s}$, so the time for one cycle is $t = 1/50 \text{ s}$. So $P = E/t = (118.5 \text{ J})/(1/50 \text{ s}) = 5900 \text{ W}$.

(d) Convert to horsepower: $(5900 \text{ W})(1 \text{ hp}/746 \text{ W}) = 7.9 \text{ hp}$.

EVALUATE: Based on the “power climb” in Example 6.10, the car power is about $\frac{5900 \text{ W}}{241 \text{ W}} = 24$ times as great as the power climber.

- 20.47. (a) IDENTIFY and SET UP:** Calculate e from $e = 1 - 1/(r^{\gamma-1})$, Q_C from $e = (Q_H + Q_C)/Q_H$, and then W from $W = Q_C + Q_H$.

EXECUTE: $e = 1 - 1/(r^{\gamma-1}) = 1 - 1/(10.6^{0.4}) = 0.6111$

$e = (Q_H + Q_C)/Q_H$ and we are given $Q_H = 200 \text{ J}$; calculate Q_C .

$Q_C = (e - 1)Q_H = (0.6111 - 1)(200 \text{ J}) = -78 \text{ J}$. (negative, since corresponds to heat leaving)

Then $W = Q_C + Q_H = -78 \text{ J} + 200 \text{ J} = 122 \text{ J}$. (positive, in agreement with Figure 20.6 in the text)

EVALUATE: Q_H , $W > 0$, and $Q_C < 0$ for an engine cycle.

(b) IDENTIFY and SET UP: The stroke times the bore equals the change in volume. The initial volume is the final volume V times the compression ratio r . Combining these two expressions gives an equation for V . For each cylinder of area $A = \pi(d/2)^2$ the piston moves 0.0864 m and the volume changes from rV to V , as shown in Figure 20.47a.

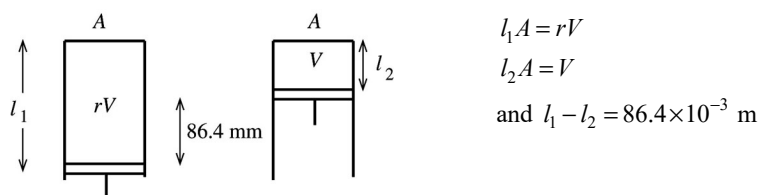


Figure 20.47a

EXECUTE: $l_1 A - l_2 A = rV - V$ and $(l_1 - l_2)A = (r - 1)V$

$$V = \frac{(l_1 - l_2)A}{r - 1} = \frac{(86.4 \times 10^{-3} \text{ m})\pi(41.25 \times 10^{-3} \text{ m})^2}{10.6 - 1} = 4.811 \times 10^{-5} \text{ m}^3$$

At point a the volume is $rV = 10.6(4.811 \times 10^{-5} \text{ m}^3) = 5.10 \times 10^{-4} \text{ m}^3$.

(c) IDENTIFY and SET UP: The processes in the Otto cycle are either constant volume or adiabatic. Use the Q_H that is given to calculate ΔT for process bc . Use $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ and $pV = nRT$ to relate p , V and T for the adiabatic processes ab and cd .

EXECUTE: Point a : $T_a = 300 \text{ K}$, $p_a = 8.50 \times 10^4 \text{ Pa}$ and $V_a = 5.10 \times 10^{-4} \text{ m}^3$.

Point b : $V_b = V_a/r = 4.81 \times 10^{-5} \text{ m}^3$. Process $a \rightarrow b$ is adiabatic, so $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$.

$$T_a (rV)^{\gamma-1} = T_b V^{\gamma-1}$$

$$T_b = T_a r^{\gamma-1} = 300 \text{ K}(10.6)^{0.4} = 771 \text{ K}$$

$$pV = nRT \text{ so } pV/T = nR = \text{constant, so } p_a V_a/T_a = p_b V_b/T_b$$

$$p_b = p_a (V_a/V_b)(T_b/T_a) = (8.50 \times 10^4 \text{ Pa})(rV/V)(771 \text{ K}/300 \text{ K}) = 2.32 \times 10^6 \text{ Pa}$$

Point c : Process $b \rightarrow c$ is at constant volume, so $V_c = V_b = 4.81 \times 10^{-5} \text{ m}^3$

$Q_H = nC_V \Delta T = nC_V (T_c - T_b)$. The problem specifies $Q_H = 200 \text{ J}$; use to calculate T_c . First use the p , V , T values at point a to calculate the number of moles n .

$$n = \frac{pV}{RT} = \frac{(8.50 \times 10^4 \text{ Pa})(5.10 \times 10^{-4} \text{ m}^3)}{(8.3145 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 0.01738 \text{ mol}$$

$$\text{Then } T_c - T_b = \frac{Q_H}{nC_V} = \frac{200 \text{ J}}{(0.01738 \text{ mol})(20.5 \text{ J/mol} \cdot \text{K})} = 561.3 \text{ K, and}$$

$$T_c = T_b + 561.3 \text{ K} = 771 \text{ K} + 561 \text{ K} = 1332 \text{ K}$$

$$p/T = nR/V = \text{constant so } p_b/T_b = p_c/T_c$$

$$p_c = p_b (T_c/T_b) = (2.32 \times 10^6 \text{ Pa})(1332 \text{ K}/771 \text{ K}) = 4.01 \times 10^6 \text{ Pa}$$

Point d : $V_d = V_a = 5.10 \times 10^{-4} \text{ m}^3$

Process $c \rightarrow d$ is adiabatic, so $T_d V_d^{\gamma-1} = T_c V_c^{\gamma-1}$

$$T_d (rV)^{\gamma-1} = T_c V^{\gamma-1}$$

$$T_d = T_c / r^{\gamma-1} = 1332 \text{ K}/10.6^{0.4} = 518 \text{ K}$$

$$p_c V_c/T_c = p_d V_d/T_d$$

$$p_d = p_c (V_c/V_d)(T_d/T_c) = (4.01 \times 10^6 \text{ Pa})(V/rV)(518 \text{ K}/1332 \text{ K}) = 1.47 \times 10^5 \text{ Pa}$$

EVALUATE: Can look at process $d \rightarrow a$ as a check.

$$Q_C = nC_V (T_a - T_d) = (0.01738 \text{ mol})(20.5 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 518 \text{ K}) = -78 \text{ J, which agrees with part (a).}$$

The cycle is sketched in Figure 20.47b.

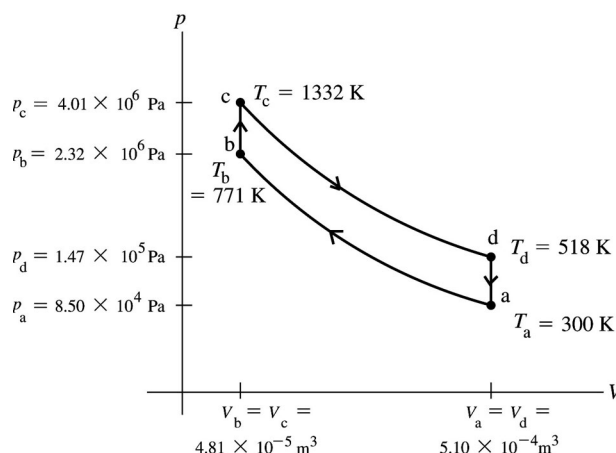


Figure 20.47b

(d) IDENTIFY and SET UP: The Carnot efficiency is given by $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. T_H is the highest

temperature reached in the cycle and T_C is the lowest.

EXECUTE: From part (a) the efficiency of this Otto cycle is $e = 0.611 = 61.1\%$.

The efficiency of a Carnot cycle operating between 1332 K and 300 K is

$e_{\text{Carnot}} = 1 - T_C/T_H = 1 - 300 \text{ K}/1332 \text{ K} = 0.775 = 77.5\%$, which is larger.

EVALUATE: The second law of thermodynamics requires that $e \leq e_{\text{Carnot}}$, and our result obeys this law.

20.48. IDENTIFY: We want to estimate the entropy of the air in a typical room.

SET UP: (a) Estimate: My office volume is $V = (12 \text{ ft})(15 \text{ ft})(8.0 \text{ ft}) = 1440 \text{ ft}^3 = 41 \text{ m}^3$.

EXECUTE: (b) $n/(41 \text{ m}^3) = (1 \text{ mol})/(22.4 \text{ L})$, which gives $n = 1800 \text{ mol}$.

(c) $N = nN_A = (1800 \text{ mol})(6.022 \times 10^{23} \text{ molecules/mol}) = 1.1 \times 10^{27} \text{ molecules}$.

(d) $S = k \ln w$. In this case, $N^N = w$, so $S = k \ln N^N = kN \ln N$. For the numbers here, we get $S = (1.38 \times 10^{-23} \text{ J/K})(1.1 \times 10^{27}) \ln(1.1 \times 10^{27}) = 9.3 \times 10^5 \text{ J/K}$.

EVALUATE: As N increases, there are more possible states, so S also increases.

20.49. IDENTIFY and SET UP: A refrigerator is like a heat engine run in reverse. In the pV -diagram shown with the figure, heat enters the gas during parts ab and bc of the cycle, and leaves during ca . Treating H_2 as a diatomic gas, we know that $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Segment bc is isochoric, so $Q_{bc} = nC_V \Delta T$.

Segment ca is isobaric, so $Q_{ca} = nC_p \Delta T$. Segment ab is isothermal, so $Q_{ab} = nRT \ln(V_b/V_a)$. The

coefficient of performance of a refrigerator is $K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$, and $pV = nRT$ applies.

Calculate the values for Q_C and Q_H and use the definition of K . Use $1000 \text{ L} = 1 \text{ m}^3$ and work in units of $\text{L} \cdot \text{atm}$.

EXECUTE: Use $pV = nRT$ to find p_b . Since ab is isothermal, $p_a V_a = p_b V_b$, which gives

$$p_b = (0.700 \text{ atm})(0.0300 \text{ m}^3)/(0.100 \text{ m}^3) = 0.210 \text{ atm}.$$

$Q_C = Q_{ab} + Q_{bc}$, so we need to calculate these quantities.

$$Q_{ab} = nRT \ln(V_b/V_a) = p_a V_a \ln(V_b/V_a) = (0.700 \text{ atm})(30.0 \text{ L}) \ln[(100 \text{ L})/(30 \text{ L})] = 25.2834 \text{ L} \cdot \text{atm}.$$

$$Q_{bc} = nC_V \Delta T_{bc} = n \left(\frac{5}{2}R\right) \Delta T_{bc} = \frac{5}{2} V_b \Delta p_{bc} = (5/2)(100 \text{ L})(0.700 \text{ atm} - 0.210 \text{ atm}) = 122.5 \text{ L} \cdot \text{atm}.$$

$$\text{Therefore } Q_C = Q_{ab} + Q_{bc} = 25.2834 \text{ L} \cdot \text{atm} + 122.500 \text{ L} \cdot \text{atm} = 147.7834 \text{ L} \cdot \text{atm}.$$

$$Q_H = Q_{ca} = nC_p \Delta T_{ca} = n\left(\frac{7}{2}R\right) \Delta T_{ca} = \frac{7}{2} p_c \Delta T_{ca} = (7/2)(0.700 \text{ atm})(30.0 \text{ L} - 100.0 \text{ L}) = -171.500 \text{ L} \cdot \text{atm}.$$

$$\text{Now get } K: K = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{147.7834 \text{ L} \cdot \text{atm}}{(171.500 \text{ L} \cdot \text{atm} - 147.7834 \text{ L} \cdot \text{atm})} = 6.23.$$

EVALUATE: K is greater than 1, which it must be. Efficiencies are less than 1.

- 20.50. IDENTIFY and SET UP:** A person radiates heat from the surface of her body which is at a constant temperature of $T = 30.0^\circ\text{C}$ into air at $T_s = 20.0^\circ\text{C}$. In 1.0 s, heat $Ae\sigma T^4$ flows from the person into the room and heat $Ae\sigma T_s^4$ flows out of the room into the person. The heat flows into and out of the room occur at a temperature of T_s . At constant temperature, $\Delta S = Q/T$.

EXECUTE: For the room,

$$\Delta S = \frac{Ae\sigma T^4}{T_s} - \frac{Ae\sigma T_s^4}{T_s} = \frac{Ae\sigma(T^4 - T_s^4)}{T_s}$$

$$\Delta S = \frac{(1.85 \text{ m}^2)(1.00)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.0)[(303 \text{ K})^4 - (293 \text{ K})^4]}{293 \text{ K}} = 0.379 \text{ J/K}$$

EVALUATE: The entropy change is positive since the air in the room becomes more disordered. In addition, this process is irreversible; heat from the cool room will not spontaneously flow into the warmer person and increase her temperature even further. Heat flows only from hot to cold.

- 20.51. IDENTIFY:** Use $\Delta S = mc \ln(T_2/T_1)$ for an isothermal process. For the value of T for which ΔS is a maximum, $d(\Delta S)/dT = 0$.

SET UP: The heat flow for a temperature change is $Q = mc\Delta T$.

EXECUTE: (a) As in Example 20.10, the entropy change of the first object is $m_1 c_1 \ln(T/T_1)$ and that of the second is $m_2 c_2 \ln(T'/T_2)$, and so the net entropy change is as given. Neglecting heat transfer to the surroundings, $Q_1 + Q_2 = 0$, $m_1 c_1 (T - T_1) + m_2 c_2 (T' - T_2) = 0$, which is the given expression.

(b) Solving the energy-conservation relation for T' and substituting into the expression for ΔS gives

$$\Delta S = m_1 c_1 \ln\left(\frac{T}{T_1}\right) + m_2 c_2 \ln\left(1 - \frac{m_1 c_1}{m_2 c_2} \left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right).$$

Differentiating with respect to T and setting the derivative equal to 0 gives $0 = \frac{m_1 c_1}{T} + \frac{(m_2 c_2)(m_1 c_1/m_2 c_2)(-1/T_2)}{\left(1 - (m_1 c_1/m_2 c_2)\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)}$. This may be solved for

$$T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}.$$

Using this value for T in the conservation of energy expression in part (a) and solving for T' gives $T' = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$. Therefore, $T = T'$ when ΔS is a maximum.

EVALUATE: (c) The final state of the system will be that for which no further entropy change is possible. If $T < T'$, it is possible for the temperatures to approach each other while increasing the total entropy, but when $T = T'$, no further spontaneous heat exchange is possible.

- 20.52. IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change.

SET UP: For water, $c = 4190 \text{ J/kg} \cdot \text{K}$. $20^\circ\text{C} = 293.15 \text{ K}$, $78^\circ\text{C} = 351.15 \text{ K}$ and $120^\circ\text{C} = 393.15 \text{ K}$.

EXECUTE:

$$\text{(a) } \Delta S = mc \ln(T_2/T_1) = (250 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln(351.15 \text{ K}/293.15 \text{ K}) = 189 \text{ J/K}.$$

$$\text{(b) } \Delta S = \frac{-mc\Delta T}{T_{\text{element}}} = \frac{-(250 \times 10^{-3} \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(351.15 \text{ K} - 293.15 \text{ K})}{393.15 \text{ K}} = -155 \text{ J/K}.$$

(c) The sum of the result of parts (a) and (b) is $\Delta S_{\text{system}} = 34.6 \text{ J/K}$. (Carry extra figures when subtraction is involved.)

EVALUATE: (d) Heating a liquid is not reversible. Whatever the energy source for the heating element, heat is being delivered at a higher temperature than that of the water, and the entropy loss of the source will be less in magnitude than the entropy gain of the water. The net entropy change is positive.

- 20.53. IDENTIFY and SET UP:** The most efficient heat engine operating between any two given temperatures is the Carnot engine, and its efficiency is $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$.

EXECUTE: (a) For prototype A, $e_{\text{max}} = e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - (320 \text{ K})/(450 \text{ K}) = 0.289 = 28.9\%$. By

similar calculations, we get the following:

A: $e_{\text{max}} = 0.289 = 28.9\%$

B: $e_{\text{max}} = 0.383 = 38.3\%$

C: $e_{\text{max}} = 0.538 = 53.8\%$

D: $e_{\text{max}} = 0.244 = 24.4\%$

(b) Engine C claims a maximum efficiency of 56%, which is greater than the maximum possible for its temperature range, so it is impossible.

(c) We get the following ratios:

A: $e_{\text{claimed}}/e_{\text{max}} = 0.21/0.289 = 0.73$

B: $e_{\text{claimed}}/e_{\text{max}} = 0.35/0.383 = 0.90$

D: $e_{\text{claimed}}/e_{\text{max}} = 0.20/0.244 = 0.82$

In decreasing order, we have B, D, A.

EVALUATE: Engine B is not only the most efficient, it is also closest to its maximum possible efficiency for its temperature range. Engines A and D have nearly the same efficiency, but D comes somewhat closer to its theoretical maximum than does A.

- 20.54. IDENTIFY and SET UP:** In terms of power, $K = \frac{|Q_C|/t}{|W|/t}$. $1 \text{ Btu} = 1055 \text{ J}$ and $1 \text{ W} \cdot \text{h} = 3600 \text{ J}$.

EXECUTE: (a) $1 \text{ W} \cdot \text{h} = (3600 \text{ J})(1 \text{ Btu}/1055 \text{ J}) = 3.412 \text{ Btu}$, so $\text{EER} = (3.412 \text{ Btu}/\text{W} \cdot \text{h})K$.

(b) $T_H = 95^\circ\text{F} = 35^\circ\text{C} = 308.1 \text{ K}$; $T_C = 80^\circ\text{F} = 26.7^\circ\text{C} = 299.8 \text{ K}$.

For a Carnot air conditioner, $K = T_C/(T_H - T_C) = (299.8 \text{ K})/(308.1 \text{ K} - 299.8 \text{ K}) = 36$. The corresponding EER is

$\text{EER} = (3.412 \text{ Btu}/\text{W} \cdot \text{h})(36) = 120 \text{ Btu}/\text{W} \cdot \text{h}$.

(c) $K = \text{EER}/3.412 = 10.9/3.412 = 3.195$

$|W| = |Q_C|/K = (1.9 \times 10^{10} \text{ J})/(3.195) = 5.95 \times 10^9 \text{ J} [(1 \text{ kWh})/(3.6 \times 10^6 \text{ J})] = 1650 \text{ kWh}$. The cost is $\$0.153(1650 \text{ kWh}) = \253 .

(d) Using the same approach as in (c) gives the following values:

$K = 4.279$, $W = 1233 \text{ kWh}$, cost = \$189, savings = $\$253 - \$189 = \$64$ per year.

EVALUATE: Whether it is worth it to replace your air conditioner depends on what it costs. In 10 years you would save \$640 (probably more since utility rates tend to rise over time). But would the unit last 10 years?

- 20.55. IDENTIFY and SET UP:** The cycle consists of two isochoric processes (ab and cd) and two isobaric processes (bc and da). Use $Q = nC_V\Delta T$ and $Q = nC_p\Delta T$ for these processes. For an ideal monatomic gas (argon), $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. Use $R = 8.3145 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) Using the equations listed above, the heat transfers are as follows:

$Q_{ab} = (3/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(300.0 \text{ K} - 250.0 \text{ K}) = 2.494 \text{ kJ}$

$Q_{bc} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(380.0 \text{ K} - 300.0 \text{ K}) = 6.652 \text{ kJ}$

$$Q_{cd} = (3/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(316.7 \text{ K} - 380.0 \text{ K}) = -3.158 \text{ kJ}$$

$$Q_{da} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(250.0 \text{ K} - 316.7 \text{ K}) = -5.546 \text{ kJ}$$

The efficiency of this cycle is $e = \frac{W}{|Q_{in}|} = \frac{|Q_{in}| - |Q_{out}|}{|Q_{in}|} = 1 - \frac{|Q_{out}|}{|Q_{in}|}$. This gives

$$e = 1 - \frac{3.158 \text{ kJ} + 5.546 \text{ kJ}}{2.494 \text{ kJ} + 6.652 \text{ kJ}} = 0.0483 = 4.83\%.$$

(b) If we double the number of moles, all the values of Q will double, but the factor of 2 cancels out, so the efficiency remains the same.

(c) Using the same procedure as in (a), the revised numbers are

$$Q_{ab} = 2.494 \text{ kJ (unchanged)}$$

$$Q_{bc} = 38.247 \text{ kJ}$$

$$Q_{cd} = -6.316 \text{ kJ}$$

$$Q_{da} = -31.878 \text{ kJ}$$

As in part (a), the efficiency of this cycle is $e = 1 - \frac{|Q_{out}|}{|Q_{in}|}$, which gives

$$e = 1 - \frac{31.878 \text{ kJ} + 6.316 \text{ kJ}}{38.247 \text{ kJ} + 2.494 \text{ kJ}} = 0.0625 = 6.25\%.$$

(d) In symbolic form, we have $Q_{ab} = +2.494 \text{ kJ (unchanged)}$

$$Q_{bc} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(T_c - 300.0 \text{ K}), \text{ which is positive.}$$

$$Q_{cd} = (3/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(T_d - T_c), \text{ which is negative.}$$

$$Q_{da} = (5/2)(4.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(250.0 \text{ K} - T_d), \text{ which is positive.}$$

Using these values, the efficiency becomes $e = 1 - \frac{3(T_c - T_d) + 5(T_d - 250.0 \text{ K})}{150 + 5(T_c - 300.0 \text{ K})}$. Using the fact that $T_c =$

$1.20T_d$ and simplifying, we get $e = \frac{0.40T_d - 100 \text{ K}}{6.00T_d - 1350 \text{ K}}$. As $T_d \rightarrow \infty$, $e \rightarrow 0.40/6.00 = 0.0667 = 6.67\%$.

EVALUATE: In (c), the Carnot efficiency for the temperature extremes given would be

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - (250 \text{ K})/(760 \text{ K}) = 0.67 = 67\%, \text{ which is 10 times the maximum efficiency of your}$$

engine. Maybe you need a new design!

20.56. IDENTIFY: We are dealing with an ideal Stirling heat engine.

SET UP: First make a pV -diagram, as shown in Fig. 20.56.

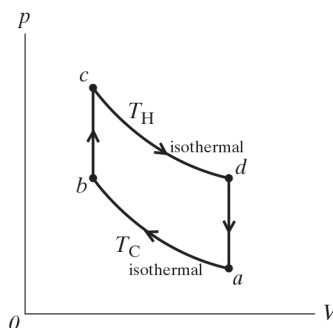


Figure 20.56

EXECUTE: (a) Both ab and cd are isothermal, and both bc and da are isochoric. Using $pV = nRT$ and taking ratios gives

bc: $\frac{p_c V_c}{p_b V_b} = \frac{T_c}{T_b} = \frac{T_H}{T_C}$. Since $V_b = V_c$, we have $\frac{p_c}{p_b} = \frac{T_H}{T_C}$.

da: Since $V_d = V_a$, we get $\frac{p_d}{p_a} = \frac{T_H}{T_C}$.

Equating the two expressions for T_H/T_C gives $\frac{p_d}{p_a} = \frac{p_c}{p_b}$. Rearranging gives $\frac{p_b}{p_a} = \frac{p_c}{p_d} = \text{CR}$.

(b) We want the work done in segment cd . $W_{cd} = \int_c^d p dV = \int_c^d \frac{nRT_H}{V} dV = nRT_H \ln(V_d/V_c)$. Using $pV =$

nRT we have $\frac{V_d}{V_c} = \frac{nRT_H/p_d}{nRT_H/p_c} = \frac{p_c}{p_d} = \text{CR}$. Using this result we get $W_{cd} = nRT_H \ln(\text{CR})$.

(c) We want W_{ab} . $W_{ab} = \int_a^b p dV = \int_a^b \frac{nRT_C}{V} dV = nRT_C \ln(V_b/V_a)$. As in part (b), we take ratios giving

$\frac{V_b}{V_a} = \frac{nRT_b/p_b}{nRT_a/p_a} = \frac{p_a}{p_b} = \frac{1}{\text{CR}}$. So $W_{ab} = nRT_C \ln(1/\text{CR}) = -nRT_C \ln(\text{CR})$.

(d) We want the power output of the engine, which is the rate at which it does work, so $P = W/t$. With a frequency of operation of 100 Hz, each cycle takes 1/100 s. Work is done only in segments ab and cd since the other segments are isochoric. Using our results from parts (b) and (c) gives $W_{\text{tot}} = W_{cd} + W_{ab} = nRT_H \ln(\text{CR}) - nRT_C \ln(\text{CR}) = nR \ln(\text{CR})(T_H - T_C)$, so the power output is $P = W/t = nR \ln(\text{CR})(T_H - T_C)/t$. Using $n = 1$ mol, $\text{CR} = 10$, $T_C = 20^\circ\text{C} = 293$ K, $T_H = 100^\circ\text{C} = 373$ K, and $t = 1/100$ s, we get $P = 1.53 \times 10^5$ W = 153 kW.

EVALUATE: If this were a Carnot engine operating between the same temperature extremes, its efficiency would be $e = 1 - (293 \text{ K})/(373 \text{ K}) = 0.214$. For the Stirling engine, heat enters the gas during segments bc and cd , so $Q_{\text{in}} = Q_{bc} + Q_{cd}$. Using the numbers for part (c), we have $Q_{bc} = nC_V(T_H - T_C) = n(3/2 R)(T_H - T_C) = 997.7$ J. During segment cd , T is constant so the internal energy U is constant. By the first law of thermodynamics, this means that $Q_{\text{in}} = Q_{cd} = W_{cd} = nRT_H \ln(\text{CR}) = 7141$ J. Thus during one cycle $Q_{\text{in}} = 997.7 \text{ J} + 7141 \text{ J} = 8138$ J. The work during one cycle is $W = Pt = (153 \text{ kW})(1/100 \text{ s}) = 1530$ J. So the efficiency is $e = W/Q_{\text{in}} = \frac{1530 \text{ J}}{8138 \text{ J}} = 0.188$. As expected, this engine is not as efficient as a

Carnot engine operating between the same temperature extremes.

20.57. IDENTIFY and SET UP: Once you cut through all the extraterrestrial description, this problem boils down to an ordinary Carnot heat engine. First list all the known information and then summarize it on a pV -diagram as shown in Fig. 20.57. We know the following information:

$T_d = 123 \text{ K} = \text{ambient temperature} = T_a$

$p_b = 20.3 \text{ kPa}$

$p_d = \text{ambient pressure because the gauge pressure is zero at } d$

Trigger volume is V_a .

Combustion starts at point b .

$r_a = 1/3 r_d \rightarrow r_d = 3r_a \rightarrow V_d = 27V_a$

$V_b = 1/2 V_a \rightarrow V_a = 2V_b$

$\gamma = 1.30$

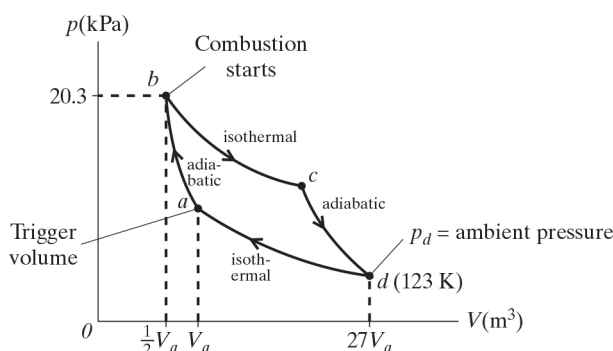


Figure 20.57

EXECUTE: (a) We want the combustion temperature, which is T_b . Since da is isothermal, $T_a = T_d = 123$ K. Since ab is adiabatic, we know that $T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}$. Using $V_b = \frac{1}{2} V_a$, we have

$$T_b = T_a (V_a / V_b)^{\gamma-1} = (123 \text{ K}) \left(\frac{2V_a}{V_b} \right)^{1.30-1} = 151 \text{ K}.$$

(b) We want the ambient pressure, which is p_d . Since ab is adiabatic, we use $p_a V_a^\gamma = p_b V_b^\gamma$ to get p_a . Then use $p_a V_a = p_d V_d$ to find p_d since da is isothermal.

$$\text{Find } p_a: p_a = p_b (V_b / V_a)^\gamma = (20.3 \text{ kPa}) \left(\frac{V_b}{2V_b} \right)^{1.30} = 8.244 \text{ kPa}.$$

Find p_d : $p_d = p_a \left(\frac{V_a}{V_d} \right) = (8.244 \text{ kPa}) \left(\frac{V_a}{27V_a} \right) = 0.305 \text{ kPa} = 305 \text{ Pa}$. At point d the gauge pressure is zero, so 305 Pa is the ambient pressure.

(c) We want the heat input Q_{in} each minute. The work that is done is $W = 60 \text{ kJ/h} = 1.00 \text{ kJ/min}$. For a Carnot engine $e = \frac{W}{Q_{\text{in}}} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_d}{T_b}$. Using the result from part (a) and the given information, we

$$\text{have } \frac{1.00 \text{ kJ}}{Q_{\text{in}}} = 1 - \frac{123 \text{ K}}{151 \text{ K}}, \text{ so } Q_{\text{in}} = 5.39 \text{ kJ}.$$

(d) We want the heat rejected Q_{out} each minute. $Q_{\text{in}} = W + Q_{\text{out}}$, so $5.39 \text{ kJ} = 1.00 \text{ kJ} + Q_{\text{out}}$, which gives $Q_{\text{out}} = 4.39 \text{ kJ}$.

EVALUATE: The efficiency of this engine is $e = W/Q_{\text{in}} = (1.00 \text{ kJ})/(5.39 \text{ kJ}) = 0.185$. Sporons are about as efficient at doing work as automobile engines on Earth.

20.58. IDENTIFY: Calculate Q_C and Q_H in terms of p and V at each point. Use the ideal gas law and the pressure-volume relation for adiabatic processes for an ideal gas. $e = 1 - \frac{|Q_C|}{|Q_H|}$.

SET UP: For an ideal gas, $C_p = C_v + R$, and taking air to be diatomic, $C_p = \frac{7}{2} R$, $C_v = \frac{5}{2} R$, and $\gamma = \frac{7}{5}$.

EXECUTE: (a) Referring to Figure 20.7 in the textbook, $Q_H = n \frac{7}{2} R (T_c - T_b) = \frac{7}{2} (p_c V_c - p_b V_b)$. Similarly, $Q_C = n \frac{5}{2} R (p_a V_a - p_d V_d)$. What needs to be done is to find the relations between the product of the

pressure and the volume at the four points. For an ideal gas, $\frac{p_c V_c}{T_c} = \frac{p_b V_b}{T_b}$ so $p_c V_c = p_b V_b \left(\frac{T_c}{T_b} \right)$. For a compression ratio r , and given that for the Diesel cycle the process ab is adiabatic,

$$p_b V_b = p_a V_a \left(\frac{V_a}{V_b} \right)^{\gamma-1} = p_a V_a r^{\gamma-1}. \text{ Similarly, } p_d V_d = p_c V_c \left(\frac{V_c}{V_d} \right)^{\gamma-1}. \text{ Note that the last result uses the fact}$$

that process da is isochoric, and $V_d = V_a$; also, $p_c = p_b$ (process bc is isobaric), and so $V_c = V_b \left(\frac{T_c}{T_a} \right)$.

Then,

$$\frac{V_c}{V_a} = \frac{T_c}{T_b} \cdot \frac{V_b}{V_a} = \frac{T_b}{T_a} \cdot \frac{T_a}{T_b} \cdot \frac{V_a}{V_b} = \frac{T_c}{T_a} \cdot \left(\frac{T_a V_a^{\gamma-1}}{T_b V_b^{\gamma-1}} \right) \left(\frac{V_a}{V_b} \right)^{-\gamma} = \frac{T_c}{T_a} r^{\gamma}$$

Combining the above results, $p_d V_d = p_a V_a \left(\frac{T_c}{T_a} \right)^{\gamma} r^{\gamma-\gamma^2}$. Substitution of the above results into

$$e = 1 - \frac{|Q_C|}{|Q_H|} \text{ gives } e = 1 - \frac{5 \left[\left(\frac{T_c}{T_a} \right)^{\gamma} r^{\gamma-\gamma^2} - 1 \right]}{\left(\frac{T_c}{T_a} \right)^{\gamma} - r^{\gamma-1}}.$$

(b) $e = 1 - \frac{1}{1.4} \left[\frac{(5.002)r^{-0.56} - 1}{(3.167) - r^{0.40}} \right]$, where $\frac{T_c}{T_a} = 3.167$ and $\gamma = 1.40$ have been used. Substitution of

$r = 21.0$ yields $e = 0.708 = 70.8\%$.

EVALUATE: The efficiency for an Otto cycle with $r = 21.0$ and $\gamma = 1.40$ is

$e = 1 - r^{1-\gamma} = 1 - (21.0)^{-0.40} = 70.4\%$. This efficiency is very close to the value for the Diesel cycle.

20.59. IDENTIFY and SET UP: The Carnot efficiency is $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. Solve for T_C to get the temperature for

the desired efficiency. Then use the graph to find the depth at which the water is at that temperature.

EXECUTE: Solving $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$ for T_C gives $T_C = T_H(1 - e) = (300 \text{ K})(1 - 0.065) = 280.5 \text{ K} =$

7.5°C . From the graph, we see that this temperature occurs at a depth of about 400 m, which is choice (b).

EVALUATE: This depth is over 1200 ft, so deep water is essential for such a power plant.

20.60. IDENTIFY and SET UP: This power plant produces energy (i.e., does work) at a rate of 10 MW and is 6.5% efficient. This means that the 10 MW is 6.5% of the heat input, Q_H , per second.

EXECUTE: $W/t = 0.065 Q_H/t$, so $Q_H/t = (10 \text{ MW})/(0.065)$. The entropy change per second is $[Q/T_H]/t = [(10 \text{ MW})/(0.065)]/(300 \text{ K}) = 5.1 \times 10^5 \text{ J/K}$, which is choice (b).

EVALUATE: The entropy increases because the ammonia gets more disordered as it vaporizes, so our answer is plausible.

20.61. IDENTIFY and SET UP: Both the warm and cold reservoirs are so large that they do not change temperature as heat is added or lost, so $\Delta S = Q/T$. For a Carnot engine, $|Q_C/Q_H| = |T_C/T_H|$.

EXECUTE: $\Delta S_{\text{warm}} = Q_H/T_H$ and $\Delta S_{\text{cold}} = Q_C/T_C$. But for a Carnot engine, $|Q_H/T_H| = |Q_C/T_C|$. Therefore $\Delta S_{\text{warm}} = -\Delta S_{\text{cold}}$, which is choice (d).

EVALUATE: Our result means that the net entropy change for a cycle is zero. An ideal Carnot engine is reversible, so the entropy change in a cycle is zero, which agrees with our result.

20.62. IDENTIFY and SET UP: For a cycle, $Q_H = W + Q_C$ and $e = W/Q_H$.

EXECUTE: For this engine, $W/t = 10 \text{ MW}$ and $Q_C/t = 165 \text{ MW}$, so $Q_H/t = W/t + Q_C/t = 175 \text{ W}$. The efficiency is $e = (W/t)/(Q_C/t) = (10 \text{ MW})/(175 \text{ MW}) = 0.057 = 5.7\%$, choice (a).

EVALUATE: An actual heat engine is always less efficient than the theoretical limit due to friction as well as internal effects in the gas which prevent it from behaving exactly like an ideal gas.