14

PERIODIC MOTION

VP14.3.1. **IDENTIFY:** The glider undergoes SHM on the spring.

SET UP: For SHM, $T = 1/f$, $\omega = 2\pi f$, $f = \dfrac{1}{2\pi}\sqrt{k/m}$, and $F = kx$ for an ideal spring.

EXECUTE: **(a)** $\omega = 2\pi f = 2\pi(4.15\text{ Hz}) = 26.1$ rad/s. $T = 1/f = 1/(4.15\text{ Hz}) = 0.241$ s.

(b) Use $f = \dfrac{1}{2\pi}\sqrt{k/m}$ to solve for k: $k = 4\pi^2 f^2 m = 4\pi^2(4.15\text{ Hz})^2(0.400\text{ kg}) = 272$ N/m.

(c) $F = kx = (272\text{ N/m})(0.0200\text{ m}) = 5.44$ N.

EVALUATE: The force in part (c) is around a pound.

VP14.3.2. **IDENTIFY:** The puck is executing SHM on the spring.

SET UP: For SHM $T = 1/f$, $T = 2\pi\sqrt{m/k}$, and $a_{max} = A\omega^2$. Our target variables are the mass of the puck and the amplitude of the oscillations.

EXECUTE: **(a)** Solve $T = 2\pi\sqrt{m/k}$ for m: $m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(1.20\text{ s})^2(4.50\text{ N/m})}{4\pi^2} = 0.164$ kg.

(b) Solve $a_{max} = A\omega^2$ for A: $A = \dfrac{a_{max}}{\omega^2} = \dfrac{a_{max}}{(2\pi/T)^2} = \dfrac{1.20\text{ m/s}^2}{[2\pi/(1.20\text{ s})]^2} = 0.0438\text{ m} = 4.38$ cm.

EVALUATE: The maximum acceleration occurs at the instants that the puck has stopped moving, which are at the extremes of its motion when $x = \pm A$.

VP14.3.3. **IDENTIFY:** The piston is moving in SHM.

SET UP: For SHM, $x(t) = A\cos(\omega t + \phi)$ and $v_{max} = A\omega$. We know that the frequency is 50.0 Hz and at a certain instant $x = 0.0300$ m and $v = 12.5$ m/s; but we don't know if either x or v are positive or negative. The target variables are the amplitude of the motion and the maximum speed of the piston.

EXECUTE: **(a)** With $x = A\cos(\omega t + \phi)$, we have $v = dx/dt = -A\omega\sin(\omega t + \phi)$. Since we don't know the sign of x or v at the instant in question, so we can neglect the minus sign for v. Taking the ration of v/x gives $\dfrac{v}{x} = \dfrac{A\omega\sin(\omega t + \phi)}{A\cos(\omega t + \phi)} = \omega\tan(\omega t + \phi)$. We know this ratio at the instant in question, so we can use this result to find $(\omega t + \phi)$. Knowing this, we can use the known value of x to find the amplitude A.

Using the known values, first find $(\omega t + \phi)$. Solving $\dfrac{v}{x} = \omega\tan(\omega t + \phi)$ for $(\omega t + \phi)$ gives

$(\omega t + \phi) = \arctan\left(\dfrac{v}{x\omega}\right) = \arctan\left(\dfrac{v}{2\pi f x}\right)$. Using the known values we have $(\omega t + \phi)$ =

$\arctan\left(\dfrac{12.5\text{ m/s}}{2\pi(50.0\text{ Hz})(0.0300\text{ m})}\right)$ = 52.984°. Now use $x = A\cos(\omega t + \phi)$ to find A, giving 0.0300 m = A cos(52.984°), so $A = 0.0498$ m.

(b) $v_{max} = A\omega = 2\pi fA = 2\pi(50.0\text{ Hz})(0.0498\text{ m}) = 15.7\text{ m/s}$.

EVALUATE: From the information give, we don't know the exact position or velocity of the piston. It could be to the right or left of the origin moving either right or left. But none of this affects the amplitude or maximum speed.

VP14.3.4. **IDENTIFY:** The cat and platform oscillate together in SHM.

SET UP: We use $f = \frac{1}{2\pi}\sqrt{k/m}$ and $a_{max} = \omega^2 A$ with $\omega = 2\pi f$. The target variables are the frequency of vibration and the amplitude of the motion so that the acceleration that will not disturb the sleeping cat.

EXECUTE: **(a)** $f = \frac{1}{2\pi}\sqrt{k/m} = .\frac{1}{2\pi}\sqrt{\frac{185\text{ N/m}}{5.00\text{ kg}}} == 0.968\text{ Hz}.$.

(b) Use $a_{max} = \omega^2 A = (2\pi f)^2 A$ to solve for A: $A = \frac{a_{max}}{(2\pi f)^2} = \frac{1.52\text{ m/s}^2}{[2\pi(0.968\text{ Hz})]^2} = 0.0411\text{ m}$.

EVALUATE: This motion makes about one vibration per second with an amplitude of about 4 cm, so it is not particularly fast.

VP14.4.1. **IDENTIFY:** This problem deals with the energy of a glider attached to a spring and oscillating with SHM.

SET UP: We use $v_{max} = A\omega$, $\omega = \sqrt{k/m}$, $K = \frac{1}{2}mv^2$, and $U = \frac{1}{2}kx^2$. The target variables are the amplitude A of the motion, the total mechanical energy E of the system, and the potential energy and kinetic energy at a certain point.

EXECUTE: **(a)** Use $v_{max} = A\omega$ and $\omega = \sqrt{k/m}$ to solve for A. We get $A = \frac{v_{max}}{\omega}$, which gives

$A = v_{max}\sqrt{\frac{m}{k}} = (0.350\text{ m/s})\sqrt{\frac{0.150\text{ kg}}{8.00\text{ N/m}}} = 0.0479\text{ m}$.

(b) $E = K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.150\text{ kg})(0.350\text{ m/s})^2 = 9.19\times10^{-3}\text{ J}$.

(c) $U = \frac{1}{2}kx^2 = \frac{1}{2}(8.00\text{ N/m})(0.0300\text{ m})^2 = 3.60\times10^{-3}\text{ J}$.

$K = E - U = 9.19\times10^{-3}\text{ J} - 3.60\times10^{-3}\text{ J} = 5.59\times10^{-3}\text{ J}$.

EVALUATE: Another way to find the amplitude is to realize that $K_{max} = U_{max}$. Therefore $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$, which gives $A = \sqrt{\frac{mv_{max}^2}{k}} = \sqrt{\frac{(0.150\text{ kg})(0.350\text{ m/s})^2}{8.00\text{ N/m}}} = 0.479\text{ m}$, which agrees with your result.

VP14.4.2. **IDENTIFY:** The block oscillates in SHM on the spring.

SET UP: The total mechanical energy is $E = K + U$, where $K = \frac{1}{2}mv^2$. We also know that $\omega = \sqrt{k/m}$ and $v_{max} = A\omega$. Our target variables are the force constant of the spring and the speed of the block when the potential energy equals one-half the total mechanical energy.

EXECUTE: **(a)** When $x = 0$, $U = 0$ so $K = K_{max}$. Therefore $v = v_{max} = A\omega$. This tells us that $E = \frac{1}{2}mv_{max}^2 = \frac{1}{2}m(A\omega)^2$. Using $\omega = \sqrt{k/m}$, this becomes $E = \frac{1}{2}m\left(A\sqrt{\frac{k}{m}}\right)^2$, which gives

$k = \frac{2E}{A^2} = \frac{2(6.00\times10^{-2}\text{ J})}{(0.0440\text{ m})^2} = 62.0\text{ N/m}$.

(b) If $U = E/2$, then K must also equal $E/2$, so $\frac{1}{2}mv^2 = \frac{1}{2}E$. This gives $v = \sqrt{\frac{E}{m}} = \sqrt{\frac{6.00\times10^{-2}\text{ J}}{0.300\text{ kg}}} =$ 0.447 m/s.

EVALUATE: Note that we *cannot* say that $v = v_{max}/2$ when $K = K_{max}/2$ because K depends on the *square* of v.

VP14.4.3. **IDENTIFY:** The glider oscillates in SHM on the spring.

SET UP: The total mechanical energy is $E = K + U$, where $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. We know the total mechanical energy E of the system, the amplitude of the oscillations, and the maximum speed of the glider. We also know that $a_{max} = \omega^2 A$ and $\omega = \sqrt{k/m}$, $x(t) = A\cos(\omega t + \phi)$, and $a(t) = -\omega^2 A\cos(\omega t + \phi)$. Our target variables are the force constant of the spring, the mass m of the glider, maximum acceleration of the glider, and its acceleration when the potential energy is 3.00×10^{-3} J.

EXECUTE: **(a)** When $x = 0$, $U = 0$ so $K = K_{max}$ and $E = K_{max}$. So $\frac{1}{2}mv_{max}^2 = E$, which gives

$$m = \frac{2E}{v_{max}^2} = \frac{2(4.00\times10^{-3}\text{ J})}{(0.125\text{ m/s})^2} = 0.512\text{ kg}.$$

When $x = A$, $K = 0$ so $U = U_{max}$, so $E = U_{max} = \frac{1}{2}kA^2$ which gives $k = \frac{2E}{A^2} = \frac{2(4.00\times10^{-3}\text{ J})}{(0.0300\text{ m})^2} = 8.89$ N/m.

(b) $a_{max} = \omega^2 A = \frac{k}{m}A = \left(\frac{8.89\text{ N/m}}{0.512\text{ kg}}\right)(0.0300\text{ m}) = 0.521\text{ m/s}^2$.

(c) We can use if we can find the value of $\cos(\omega t + \phi)$. We know that $U = 3.00\times10^{-3}\text{ J} = \frac{1}{2}kx^2$ and that $x(t) = A\cos(\omega t + \phi)$. Therefore $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$, which gives $\cos(\omega t + \phi) = \sqrt{\frac{2U}{kA^2}}$. Now find the acceleration for this value of $\cos(\omega t + \phi)$. We can drop the minus sign since we only want the magnitude of the acceleration. $a(t) = \omega^2 A\cos(\omega t + \phi) = \omega^2 A\sqrt{\frac{2U}{kA^2}} = \frac{k}{m}\sqrt{\frac{2U}{k}}$. Using $k = 8.89$ N/m, $m = 0.512$ kg, and $U = 3.00\times10^{-3}$ J, we have $a = 0.451\text{ m/s}^2$.

EVALUATE: In part (c) the acceleration is less than the maximum of 0.521 m/s^2 from part (b), so our result is reasonable.

VP14.4.4. **IDENTIFY:** An object is in SHM, and the energy of the system is conserved.

SET UP: The total energy is $E = K + U$ and is constant, where $U = \frac{1}{2}kx^2$. We want to know the values of x when the kinetic energy is equal to 1/3 of the total mechanical energy and to 1/5 of the total mechanical energy.

EXECUTE: **(a)** If $K = 1/3\ E$, then $U = 2/3\ E$, and $E = \frac{1}{2}kA^2$. This gives $\frac{1}{2}kx^2 = \frac{2}{3}\left(\frac{1}{2}kA^2\right)$, so $x = \pm A\sqrt{\frac{2}{3}}$.

(b) Proceed as in part (a). If $K = 4/5\,E$, then $U = 1/5\,E$, which gives $\frac{1}{2}kx^2 = \frac{1}{5}\left(\frac{1}{2}kA^2\right)$, so $x = \pm\frac{A}{\sqrt{5}}$.

EVALUATE: We get square roots in our answers because U depends on the *square* of x and K depends on the *square* of v.

VP14.9.1. **IDENTIFY:** We are dealing with the small oscillations of a simple pendulum.

SET UP: $T = 1/f$ and $T = 2\pi\sqrt{L/g}$ for small oscillations. We want to find g on the alien planet, but first we need the period T.

EXECUTE: **(a)** $T = 1/f = 1/(0.609\text{ Hz}) = 1.64\text{ s}$.

(b) Solve $T = 2\pi\sqrt{L/g}$ for g, giving $g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.500\text{ m})}{(1.64\text{ s})^2} = 7.32\text{ m/s}^2$.

EVALUATE: This would be a very simple way to determine g since measurements of L and T are quite easy to make.

VP14.9.2. **IDENTIFY:** We are comparing the oscillation frequencies of a simple pendulum and an object attached to a spring.

SET UP: For the glider $\omega = \sqrt{k/m}$, and for a simple pendulum $\omega = \sqrt{g/L}$. The target variable is the length L of the pendulum so that it oscillates with the same frequency as the object attached to the spring.

EXECUTE: If the oscillation frequencies are the same, the angular frequencies must also be the same. So $\sqrt{k/m} = \sqrt{g/L}$, which gives $L = \frac{mg}{k} = \frac{(0.350\text{ kg})(9.80\text{ m/s}^2)}{8.75\text{ N/m}} = 0.392\text{ m}$.

EVALUATE: Our result is only accurate if the pendulum makes small oscillations.

VP14.9.3. **IDENTIFY:** The bicycle tire oscillates, but it is a *physical* pendulum, not a simple pendulum.

SET UP: The angular frequency is $\omega = \sqrt{\frac{Mgd}{I}}$ and $\omega = 2\pi f$.

EXECUTE: Using the given moment of inertia, we have $2\pi f = \sqrt{\frac{Mgd}{I}} = \sqrt{\frac{MgR}{2MR^2}} = \sqrt{\frac{g}{2R}}$. The oscillation frequency is $f = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$.

EVALUATE: If R is large, f is small, meaning that the wheel oscillates with a long period. This is analogous to a long simple pendulum, which oscillates slowly. If g were large, the frequency would be large since gravity could pull it back quickly from its extremes. Both cases suggest that our result is physically plausible.

VP14.9.4. **IDENTIFY:** The rod swings back and forth, but it is not a *simple* pendulum because its mass is spread out. Instead it is a *physical* pendulum.

SET UP: The period of swing is $T = 2\pi\sqrt{\frac{I}{mgd}}$. We want to find the moment of inertia of the rod.

EXECUTE: Use $T = 2\pi\sqrt{\frac{I}{mgd}}$ and solve for I, giving $I = mgd\left(\frac{T}{2\pi}\right)^2$. Therefore we get

$$I = (0.600\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m})\left(\frac{1.59\text{ s}}{2\pi}\right)^2 = 0.188\text{ kg}\cdot\text{m}^2.$$

EVALUATE: If this rod were uniform, its moment of inertia about the pivot would be $I = \frac{1}{3}ML^2 = \frac{1}{3}(0.600\text{ kg})(0.900\text{ m})^2 = 0.162\text{ kg}\cdot\text{m}^2$, which is *less than* we found. This is reasonable because the center of mass of this pendulum is *below* the midpoint of the rod, so it will have a larger moment of inertia about the pivot point than if it were uniform.

14.1. IDENTIFY: We want to relate the characteristics of various waves, such as the period, frequency, and angular frequency.

SET UP: The frequency f in Hz is the number of cycles per second. The angular frequency ω is $\omega = 2\pi f$ and has units of radians per second. The period T is the time for one cycle of the wave and has units of seconds. The period and frequency are related by $T = \dfrac{1}{f}$.

EXECUTE: **(a)** $T = \dfrac{1}{f} = \dfrac{1}{466\text{ Hz}} = 2.15\times10^{-3}\text{ s}$.

$\omega = 2\pi f = 2\pi(466\text{ Hz}) = 2.93\times10^{3}\text{ rad/s}$.

(b) $f = \dfrac{1}{T} = \dfrac{1}{50.0\times10^{-6}\text{ s}} = 2.00\times10^{4}\text{ Hz}$. $\omega = 2\pi f = 1.26\times10^{5}\text{ rad/s}$.

(c) $f = \dfrac{\omega}{2\pi}$ so f ranges from $\dfrac{2.7\times10^{15}\text{ rad/s}}{2\pi\text{ rad}} = 4.3\times10^{14}\text{ Hz}$ to $\dfrac{4.7\times10^{15}\text{ rad/s}}{2\pi\text{ rad}} = 7.5\times10^{14}\text{ Hz}$. $T = \dfrac{1}{f}$ so T ranges from $\dfrac{1}{7.5\times10^{14}\text{ Hz}} = 1.3\times10^{-15}\text{ s}$ to $\dfrac{1}{4.3\times10^{14}\text{ Hz}} = 2.3\times10^{-15}\text{ s}$.

(d) $T = \dfrac{1}{f} = \dfrac{1}{5.0\times10^{6}\text{ Hz}} = 2.0\times10^{-7}\text{ s}$ and $\omega = 2\pi f = 2\pi(5.0\times10^{6}\text{ Hz}) = 3.1\times10^{7}\text{ rad/s}$.

EVALUATE: Visible light has much higher frequency than either sounds we can hear or ultrasound. Ultrasound is sound with frequencies higher than what the ear can hear. Large f corresponds to small T.

14.2. IDENTIFY and SET UP: The amplitude is the maximum displacement from equilibrium. In one period the object goes from $x = +A$ to $x = -A$ and returns.

EXECUTE: **(a)** $A = 0.120\text{ m}$.

(b) $0.800\text{ s} = T/2$ so the period is 1.60 s.

(c) $f = \dfrac{1}{T} = 0.625\text{ Hz}$.

EVALUATE: Whenever the object is released from rest, its initial displacement equals the amplitude of its SHM.

14.3. IDENTIFY: The period is the time for one vibration and $\omega = \dfrac{2\pi}{T}$.

SET UP: The units of angular frequency are rad/s.

EXECUTE: The period is $\dfrac{0.50\text{ s}}{440} = 1.14\times10^{-3}\text{ s}$ and the angular frequency is $\omega = \dfrac{2\pi}{T} = 5.53\times10^{3}\text{ rad/s}$.

EVALUATE: There are 880 vibrations in 1.0 s, so $f = 880\text{ Hz}$. This is equal to $1/T$.

14.4. IDENTIFY: The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.

SET UP: The maximum x is 10.0 cm. The time for one cycle is 16.0 s.

EXECUTE: **(a)** $T = 16.0\text{ s}$ so $f = \dfrac{1}{T} = 0.0625\text{ Hz}$.

(b) $A = 10.0\text{ cm}$.

(c) $T = 16.0\text{ s}$

(d) $\omega = 2\pi f = 0.393$ rad/s

EVALUATE: After one cycle the motion repeats.

14.5. **IDENTIFY:** This displacement is $\frac{1}{4}$ of a period.

SET UP: $T = 1/f = 0.250$ s.

EXECUTE: $t = 0.0625$ s

EVALUATE: The time is the same for $x = A$ to $x = 0$, for $x = 0$ to $x = -A$, for $x = -A$ to $x = 0$ and for $x = 0$ to $x = A$.

14.6. **IDENTIFY:** The swing moves back and forth and can be approximated as a simple pendulum.

SET UP: Estimate: The length is about 6 ft ≈ 2.0 m. The time between pushes is the period T of swing, where $T = 2\pi\sqrt{L/g}$. We want to find the period.

EXECUTE: $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{\dfrac{2.0\text{ m}}{9.80\text{ m/s}^2}} = 2.8$ s.

EVALUATE: A period of around 3 s seems reasonable for a small playground swing.

14.7. **IDENTIFY** and **SET UP:** The period is the time for one cycle. A is the maximum value of x.

EXECUTE: **(a)** From the figure with the problem, $T = 0.800$ s.

(b) $f = \dfrac{1}{T} = 1.25$ Hz.

(c) $\omega = 2\pi f = 7.85$ rad/s.

(d) From the figure with the problem, $A = 3.0$ cm.

(e) $T = 2\pi\sqrt{\dfrac{m}{k}}$, so $k = m\left(\dfrac{2\pi}{T}\right)^2 = (2.40\text{ kg})\left(\dfrac{2\pi}{0.800\text{ s}}\right)^2 = 148$ N/m.

EVALUATE: The amplitude shown on the graph does not change with time, so there must be little or no friction in this system.

14.8. **IDENTIFY:** Apply $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$.

SET UP: The period will be twice the interval between the times at which the glider is at the equilibrium position.

EXECUTE: $k = \omega^2 m = \left(\dfrac{2\pi}{T}\right)^2 m = \left(\dfrac{2\pi}{2(2.60\text{ s})}\right)^2 (0.200\text{ kg}) = 0.292\text{ N/m}$.

EVALUATE: $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$, so $1\text{ N/m} = 1\text{ kg/s}^2$.

14.9. **IDENTIFY** and **SET UP:** Use $T = 1/f$ to calculate T, $\omega = 2\pi f$ to calculate ω, and $\omega = \sqrt{k/m}$ for m.

EXECUTE: **(a)** $T = 1/f = 1/6.00\text{ Hz} = 0.167$ s.

(b) $\omega = 2\pi f = 2\pi(6.00\text{ Hz}) = 37.7$ rad/s.

(c) $\omega = \sqrt{k/m}$ implies $m = k/\omega^2 = (120\text{ N/m})/(37.7\text{ rad/s})^2 = 0.0844$ kg.

EVALUATE: We can verify that k/ω^2 has units of mass.

14.10. **IDENTIFY:** The mass and frequency are related by $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

SET UP: $f\sqrt{m} = \dfrac{\sqrt{k}}{2\pi} =$ constant, so $f_1\sqrt{m_1} = f_2\sqrt{m_2}$.

EXECUTE: **(a)** $m_1 = 0.750\text{ kg}$, $f_1 = 1.75\text{ Hz}$ and $m_2 = 0.750\text{ kg} + 0.220\text{ kg} = 0.970\text{ kg}.$

$$f_2 = f_1\sqrt{\frac{m_1}{m_2}} = (1.75\text{ Hz})\sqrt{\frac{0.750\text{ kg}}{0.970\text{ kg}}} = 1.54\text{ Hz}.$$

(b) $m_2 = 0.750\text{ kg} - 0.220\text{ kg} = 0.530\text{ kg}.$ $f_2 = (1.75\text{ Hz})\sqrt{\dfrac{0.750\text{ kg}}{0.530\text{ kg}}} = 2.08\text{ Hz}.$

EVALUATE: When the mass increases the frequency decreases, and when the mass decreases the frequency increases.

14.11. **IDENTIFY:** For SHM the motion is sinusoidal.
SET UP: $x(t) = A\cos(\omega t)$.

EXECUTE: $x(t) = A\cos(\omega t)$, where $A = 0.320\text{ m}$ and $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.900\text{ s}} = 6.981\text{ rad/s}.$

(a) $x = 0.320\text{ m}$ at $t_1 = 0$. Let t_2 be the instant when $x = 0.160\text{ m}$. Then we have

$0.160\text{ m} = (0.320\text{ m})\cos(\omega t_2)$. $\cos(\omega t_2) = 0.500.$ $\omega t_2 = 1.047\text{ rad}.$ $t_2 = \dfrac{1.047\text{ rad}}{6.981\text{ rad/s}} = 0.150\text{ s}.$ It takes $t_2 - t_1 = 0.150\text{ s}.$

(b) Let t_3 be when $x = 0$. Then we have $\cos(\omega t_3) = 0$ and $\omega t_3 = 1.571\text{ rad}.$ $t_3 = \dfrac{1.571\text{ rad}}{6.981\text{ rad/s}} = 0.225\text{ s}.$

It takes $t_3 - t_2 = 0.225\text{ s} - 0.150\text{ s} = 0.0750\text{ s}.$

EVALUATE: Note that it takes twice as long to go from $x = 0.320\text{ m}$ to $x = 0.160\text{ m}$ than to go from $x = 0.160\text{ m}$ to $x = 0$, even though the two distances are the same, because the speeds are different over the two distances.

14.12. **IDENTIFY:** For SHM the restoring force is directly proportional to the displacement and the system obeys Newton's second law.

SET UP: $F_x = ma_x$ and $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}.$

EXECUTE: $F_x = ma_x$ gives $a_x = -\dfrac{kx}{m}$, so $\dfrac{k}{m} = -\dfrac{a_x}{x} = -\dfrac{-5.30\text{ m/s}^2}{0.280\text{ m}} = 18.93\text{ s}^{-2}.$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{18.93\text{ s}^{-2}} = 0.692\text{ Hz}$$

EVALUATE: The period is around 1.5 s, so this is a rather slow vibration.

14.13. **IDENTIFY:** Use $A = \sqrt{x_0^2 + \dfrac{v_{0x}^2}{\omega^2}}$ to calculate A. The initial position and velocity of the block determine ϕ. $x(t)$ is given by $x = A\cos(\omega t + \phi)$.

SET UP: $\cos\theta$ is zero when $\theta = \pm\pi/2$ and $\sin(\pi/2) = 1$.

EXECUTE: **(a)** From $A = \sqrt{x_0^2 + \dfrac{v_{0x}^2}{\omega^2}}$, $A = \left|\dfrac{v_0}{\omega}\right| = \left|\dfrac{v_0}{\sqrt{k/m}}\right| = 0.98\text{ m}.$

(b) Since $x(0) = 0$, $x = A\cos(\omega t + \phi)$ requires $\phi = \pm\frac{\pi}{2}$. Since the block is initially moving to the left, $v_{0x} < 0$ and $v_{0x} = -\omega A\sin\phi$ requires that $\sin\phi > 0$, so $\phi = +\frac{\pi}{2}$.

(c) $\cos(\omega t + \pi/2) = -\sin\omega t$, so $x = (-0.98\text{ m})\sin[(12.2\text{ rad/s})t]$.

EVALUATE: The $x(t)$ result in part (c) does give $x = 0$ at $t = 0$ and $x < 0$ for t slightly greater than zero.

14.14. **IDENTIFY** and **SET UP:** We are given k, m, x_0, and v_0. Use $A = \sqrt{x_0^2 + v_{0x}^2/\omega^2} = \sqrt{x_0^2 + mv_{0x}^2/k}$, $\phi = \arctan(-v_{0x}/\omega x_0)$, and $x = A\cos(\omega t + \phi)$.

EXECUTE: **(a)** $A = \sqrt{x_0^2 + v_{0x}^2/\omega^2} = \sqrt{x_0^2 + mv_{0x}^2/k}$:

$A = \sqrt{(0.200\text{ m})^2 + (2.00\text{ kg})(-4.00\text{ m/s})^2/(300\text{ N/m})} = 0.383\text{ m}$

(b) $\phi = \arctan(-v_{0x}/\omega x_0)$:

$\omega = \sqrt{k/m} = \sqrt{(300\text{ N/m})/2.00\text{ kg}} = 12.25\text{ rad/s}$

$$\phi = \arctan\left(-\frac{(-4.00\text{ m/s})}{(12.25\text{ rad/s})(0.200\text{ m})}\right) = \arctan(+1.633) = 58.5^\circ\ (\text{or } 1.02\text{ rad})$$

(c) $x = A\cos(\omega t + \phi)$ gives $x = (0.383\text{ m})\cos([12.2\text{rad/s}]t + 1.02\text{ rad})$

EVALUATE: At $t = 0$ the block is displaced 0.200 m from equilibrium but is moving, so $A > 0.200$ m. Since $v_x = -\omega A\cos(\omega t + \phi)$, a phase angle ϕ in the range $0 < \phi < 90^\circ$ gives $v_{0x} < 0$.

14.15. **IDENTIFY:** The block oscillates in SHM.

SET UP: For the given initial conditions $x(t) = A\cos\omega t$ where $\omega = \sqrt{k/m} = 2\pi/T$ and $T = 2\pi\sqrt{m/k}$.

EXECUTE: **(a)** We want to find the time when $x(t) = A/2$ for the first time.
$x(t) = A\cos\omega t = A/2$, so $\omega t = 60^\circ = \pi/3$ rad, which gives $t = \pi/3\omega$. Since $\omega = 2\pi/T$, we have

$$t = \frac{\pi}{3\left(\frac{2\pi}{T}\right)} = T/6.$$

(b) We want to find the time when the block first $v_{max}/2$. The velocity is $v = dx/dt$, which gives $\frac{d(A\cos\omega t)}{dt} = -A\omega\sin\omega t = -v_{\text{max}}\sin\omega t$. We can drop the minus sign because we are interested only in the speed. Therefore $v = \frac{1}{2}v_{\text{max}} = v_{\text{max}}\sin\omega t$, so $\sin\omega t = \frac{1}{2}$, which means that $\omega t = 30^\circ = \pi/6$. Using $\omega = 2\pi/T$, we get $t = \frac{\pi/6}{\omega} = \frac{\pi}{6\left(\frac{2\pi}{T}\right)} = T/12.$

(c) The answer is no, the block does not reach $v_{max}/2$ when $x = A/2$.

EVALUATE: We can check our result using energy conservation. The total mechanical energy E is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$. We know that $v_{max} = A\omega = A\sqrt{k/m}$. Use energy to find x when $v = v_{max}/2$, which gives $\frac{1}{2}m\left(\frac{v_{\text{max}}}{2}\right)^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. Using $v_{max} = A\omega = A\sqrt{k/m}$, this becomes $\frac{1}{2}m\left(\frac{A\sqrt{k/m}}{2}\right)^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. Squaring and solving for x gives $x = \frac{\sqrt{3}}{2}A$, which is *not* equal to $A/2$. This agrees with our result in part (c).

14.16. **IDENTIFY:** The motion is SHM, and in each case the motion described is one-half of a complete cycle.

SET UP: For SHM, $x = A\cos(\omega t)$ and $\omega = \frac{2\pi}{T}$.

EXECUTE: **(a)** The time is half a period. The period is independent of the amplitude, so it still takes 2.70 s.

(b) $x = 0.090\text{ m}$ at time t_1. $T = 5.40\text{ s}$ and $\omega = \dfrac{2\pi}{T} = 1.164\text{ rad/s}$. $x_1 = A\cos(\omega t_1)$. $\cos(\omega t_1) = 0.500$. $\omega t_1 = 1.047\text{ rad}$ and $t_1 = 0.8997\text{ s}$. $x = -0.090\text{ m}$ at time t_2. $\cos(\omega t_2) = -0.500\text{ m}$. $\omega t_2 = 2.094\text{ rad}$ and $t_2 = 1.800\text{ s}$. The elapsed time is $t_2 - t_1 = 1.800\text{ s} - 0.8997\text{ s} = 0.900\text{ s}$.

EVALUATE: It takes less time to travel from ±0.090 m in (b) than it originally did because the block has larger speed at ± 0.090 m with the increased amplitude.

14.17. **IDENTIFY:** Apply $T = 2\pi\sqrt{\dfrac{m}{k}}$. Use the information about the empty chair to calculate k.

SET UP: When $m = 42.5\text{ kg}$, $T = 1.30\text{ s}$.

EXECUTE: Empty chair: $T = 2\pi\sqrt{\dfrac{m}{k}}$. gives $k = \dfrac{4\pi^2 m}{T^2} = \dfrac{4\pi^2(42.5\text{ kg})}{(1.30\text{ s})^2} = 993\text{ N/m}$.

With person in chair: $T = 2\pi\sqrt{\dfrac{m}{k}}$ gives $m = \dfrac{T^2 k}{4\pi^2} = \dfrac{(2.54\text{ s})^2(993\text{ N/m})}{4\pi^2} = 162\text{ kg}$ and

$m_{\text{person}} = 162\text{ kg} - 42.5\text{ kg} = 120\text{ kg}$.

EVALUATE: For the same spring, when the mass increases, the period increases.

14.18. **IDENTIFY and SET UP:** Use $T = 2\pi\sqrt{\dfrac{m}{k}}$ for T and $a_x = -\dfrac{k}{m}x$ to relate a_x and k.

EXECUTE: $T = 2\pi\sqrt{\dfrac{m}{k}}$, $m = 0.400\text{ kg}$

Use $a_x = -1.80\text{ m/s}^2$ to calculate k: $-kx = ma_x$ gives

$$k = -\frac{ma_x}{x} = -\frac{(0.400\text{ kg})(-1.80\text{ m/s}^2)}{0.300\text{ m}} = +2.40\text{ N/m, so } T = 2\pi\sqrt{\frac{m}{k}} = 2.57\text{ s.}$$

EVALUATE: a_x is negative when x is positive. ma_x/x has units of N/m and $\sqrt{m/k}$ has units of seconds.

14.19. **IDENTIFY:** $T = 2\pi\sqrt{\dfrac{m}{k}}$. $a_x = -\dfrac{k}{m}x$ so $a_{\text{max}} = \dfrac{k}{m}A$. $F = -kx$.

SET UP: a_x is proportional to x so a_x goes through one cycle when the displacement goes through one cycle. From the graph, one cycle of a_x extends from $t = 0.10\text{ s}$ to $t = 0.30\text{ s}$, so the period is $T = 0.20\text{ s}$. $k = 2.50\text{ N/cm} = 250\text{ N/m}$. From the graph the maximum acceleration is 12.0 m/s^2.

EXECUTE: **(a)** $T = 2\pi\sqrt{\dfrac{m}{k}}$ gives $m = k\left(\dfrac{T}{2\pi}\right)^2 = (250\text{ N/m})\left(\dfrac{0.20\text{ s}}{2\pi}\right)^2 = 0.253\text{ kg}$

(b) $A = \dfrac{ma_{\text{max}}}{k} = \dfrac{(0.253\text{ kg})(12.0\text{ m/s}^2)}{250\text{ N/m}} = 0.0121\text{ m} = 1.21\text{ cm}$

(c) $F_{\text{max}} = kA = (250\text{ N/m})(0.0121\text{ m}) = 3.03\text{ N}$.

EVALUATE: We can also calculate the maximum force from the maximum acceleration: $F_{\text{max}} = ma_{\text{max}} = (0.253\text{ kg})(12.0\text{ m/s}^2) = 3.04\text{ N}$, which agrees with our previous results.

14.20. **IDENTIFY:** The general expression for $v_x(t)$ is $v_x(t) = -\omega A\sin(\omega t + \phi)$. We can determine ω and A by comparing the equation in the problem to the general form.

SET UP: $\omega = 4.71\text{ rad/s}$. $\omega A = 3.60\text{ cm/s} = 0.0360\text{ m/s}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\text{ rad}}{4.71\text{ rad/s}} = 1.33\text{ s}$

(b) $A = \dfrac{0.0360 \text{ m/s}}{\omega} = \dfrac{0.0360 \text{ m/s}}{4.71 \text{ rad/s}} = 7.64\times10^{-3} \text{ m} = 7.64 \text{ mm}$

(c) $a_{\text{max}} = \omega^2 A = (4.71 \text{ rad/s})^2(7.64\times10^{-3} \text{ m}) = 0.169 \text{ m/s}^2$

(d) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (0.500 \text{ kg})(4.71 \text{ rad/s})^2 = 11.1 \text{ N/m}.$

EVALUATE: The overall negative sign in the expression for $v_x(t)$ and the factor of $-\pi/2$ both are related to the phase factor ϕ in the general expression.

14.21. IDENTIFY: Compare the specific $x(t)$ given in the problem to the general form $x = A\cos(\omega t + \phi)$.

SET UP: $A = 7.40$ cm, $\omega = 4.16$ rad/s, and $\phi = -2.42$ rad.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.16 \text{ rad/s}} = 1.51 \text{ s}.$

(b) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (1.50 \text{ kg})(4.16 \text{ rad/s})^2 = 26.0 \text{ N/m}$

(c) $v_{\text{max}} = \omega A = (4.16 \text{ rad/s})(7.40 \text{ cm}) = 30.8 \text{ cm/s}$

(d) $F_x = -kx$ so $F_{\text{max}} = kA = (26.0 \text{ N/m})(0.0740 \text{ m}) = 1.92 \text{ N}.$

(e) $x(t)$ evaluated at $t = 1.00$ s gives $x = -0.0125$ m. $v_x = -\omega A\sin(\omega t + \phi) = 30.4$ cm/s.

$a_x = -kx/m = -\omega^2 x = +0.216 \text{ m/s}^2.$

(f) $F_x = -kx = -(26.0 \text{ N/m})(-0.0125 \text{ m}) = +0.325 \text{ N}$

EVALUATE: The maximum speed occurs when $x = 0$ and the maximum force is when $x = \pm A$.

14.22. IDENTIFY: The frequency of vibration of a spring depends on the mass attached to the spring. Differences in frequency are due to differences in mass, so by measuring the frequencies we can determine the mass of the virus, which is the target variable.

SET UP: The frequency of vibration is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}.$

EXECUTE: **(a)** The frequency without the virus is $f_{\text{s}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_{\text{s}}}}$, and the frequency with the virus is

$f_{\text{s+v}} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_{\text{s}} + m_{\text{v}}}}$. $\dfrac{f_{\text{s+v}}}{f_{\text{s}}} = \left(\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_{\text{s}} + m_{\text{v}}}}\right)\left(2\pi\sqrt{\dfrac{m_{\text{s}}}{k}}\right) = \sqrt{\dfrac{m_{\text{s}}}{m_{\text{s}} + m_{\text{v}}}} = \dfrac{1}{\sqrt{1 + m_{\text{v}}/m_{\text{s}}}}.$

(b) $\left(\dfrac{f_{\text{s+v}}}{f_{\text{s}}}\right)^2 = \dfrac{1}{1 + m_{\text{v}}/m_{\text{s}}}$. Solving for m_{v} gives

$m_{\text{v}} = m_{\text{s}}\left(\left[\dfrac{f_{\text{s}}}{f_{\text{s+v}}}\right]^2 - 1\right) = (2.10\times10^{-16} \text{ g})\left(\left[\dfrac{2.00\times10^{15} \text{ Hz}}{2.87\times10^{14} \text{ Hz}}\right]^2 - 1\right) = 9.99\times10^{-15} \text{ g},$ or

$m_{\text{v}} = 9.99$ femtograms.

EVALUATE: When the mass increases, the frequency of oscillation increases.

14.23. IDENTIFY: The *jerk* is defined as *da/dt*. We want to investigate the jerk for an object in SHM.

SET UP: $j_x = da_x/dt$, $v_x = -A\omega\sin\omega t$, and $a_x = dv_x/dt$.

EXECUTE: **(a)** First find a_x: $a_x = dv_x/dt = d(-A\omega\sin\omega t)/dt = -\omega^2 A\cos\omega t$. Now find j_x:

$j_x = da_x/dt = d(-\omega^2 A\cos\omega t)/dt = \omega^3 A\sin\omega t.$

(b) The jerk has its largest positive value when $\sin\omega t$ is 1, so $\omega t = \pi/2$. Since $v_x = -A\omega\sin\omega t$, integration tells us that $x(t) = A\cos\omega t$. So when j_x is a positive maximum, $x = A\cos(\pi/2) = 0$.

(c) The jerk is most negative when $\sin\omega t$ is –1, so $\omega t = 3\pi/2$. So when j_x is most negative, $x = A\cos(3\pi/2) = 0$.

(d) The jerk is zero when $\omega t = 0, \pi, 2\pi, \ldots$. At these values, cos ωt is either +1 or –1, so $x = \pm A$.
(e) We know that $v_x = -Kj_x$ where $K = +0.040\ \text{s}^2$ and that $v_x = -A\omega \sin \omega t$. Using our result for j_x from part (a), we get $v_x = -K\omega^3 A \sin \omega t$. Equating these two expressions for v_x gives $K\omega^3 A = \omega A$, so $\omega = 1/\sqrt{K} = 1/\sqrt{0.040\ \text{s}^2} = 5.0$ rad/s. The period is $T = 2\pi/\omega$, which gives $T = 2\pi/(5.0\ \text{rad/s}) = 1.3$ s.

EVALUATE: The jerk also has simple harmonic behavior.

14.24. IDENTIFY: The mechanical energy of the system is conserved. The maximum acceleration occurs at the maximum displacement and the motion is SHM.

SET UP: Energy conservation gives $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$, $T = 2\pi\sqrt{\frac{m}{k}}$, and $a_{\max} = \frac{kA}{m}$.

EXECUTE: **(a)** From the graph, we read off $T = 16.0$ s and $A = 10.0$ cm $= 0.100$ m. $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$ gives $v_{\max} = A\sqrt{\frac{k}{m}}$. $T = 2\pi\sqrt{\frac{m}{k}}$, so $\sqrt{\frac{k}{m}} = \frac{2\pi}{T}$. Therefore

$$v_{\max} = A\left(\frac{2\pi}{T}\right) = (0.100\ \text{m})\left(\frac{2\pi}{16.0\ \text{s}}\right) = 0.0393\ \text{m/s}.$$

(b) $a_{\max} = \frac{kA}{m} = \left(\frac{2\pi}{T}\right)^2 A = \left(\frac{2\pi}{16.0\ \text{s}}\right)^2 (0.100\ \text{m}) = 0.0154\ \text{m/s}^2$

EVALUATE: The acceleration is much less than g.

14.25. IDENTIFY: The mechanical energy of the system is conserved. The maximum acceleration occurs at the maximum displacement and the motion is SHM.

SET UP: Energy conservation gives $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$ and $a_{\max} = \frac{kA}{m}$.

EXECUTE: $A = 0.165$ m. $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$ gives $\frac{k}{m} = \left(\frac{v_{\max}}{A}\right)^2 = \left(\frac{3.90\ \text{m/s}}{0.165\ \text{m}}\right)^2 = 558.7\ \text{s}^{-2}$.

$$a_{\max} = \frac{kA}{m} = (558.7\ \text{s}^{-2})(0.165\ \text{m}) = 92.2\ \text{m/s}^2.$$

EVALUATE: The acceleration is much greater than g.

14.26. IDENTIFY: The mechanical energy of the system is conserved, Newton's second law applies and the motion is SHM.

SET UP: Energy conservation gives $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, $F_x = ma_x$, $F_x = -kx$, and the period is $T = 2\pi\sqrt{\frac{m}{k}}$.

EXECUTE: Solving $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ for v_x gives $v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$. $T = 2\pi\sqrt{\frac{m}{k}}$, so $\sqrt{\frac{k}{m}} = \frac{2\pi}{T} = \frac{2\pi}{3.20\ \text{s}} = 1.963\ \text{s}^{-1}$. $v_x = \pm(1.963\ \text{s}^{-1})\sqrt{(0.250\ \text{m})^2 - (0.160\ \text{m})^2} = \pm 0.377\ \text{m/s}$.

$$a_x = -\frac{kx}{m} = -(1.963\ \text{s}^{-1})^2\ (0.160\ \text{m}) = -0.617\ \text{m/s}^2.$$

EVALUATE: The block is on the positive side of the equilibrium position $(x = 0)$. If $v_x = +0.377$ m/s, the block is moving in the positive direction and slowing down since the acceleration is in the negative direction. If $v_x = -0.377$ m/s, the block is moving in the negative direction and speeding up.

14.27. **IDENTIFY** and **SET UP:** Use $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$. $x = \pm A$ when $v_x = 0$ and $v_x = \pm v_{\text{max}}$ when $x = 0$.

EXECUTE: **(a)** $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$E = \frac{1}{2}(0.150 \text{ kg})(0.400 \text{ m/s})^2 + \frac{1}{2}(300 \text{ N/m})(0.012 \text{ m})^2 = 0.0336 \text{ J}.$

(b) $E = \frac{1}{2}kA^2$ so $A = \sqrt{2E/k} = \sqrt{2(0.0336 \text{ J})/(300 \text{ N/m})} = 0.0150 \text{ m}$

(c) $E = \frac{1}{2}mv_{\text{max}}^2$ so $v_{\text{max}} = \sqrt{2E/m} = \sqrt{2(0.0336 \text{ J})/(0.150 \text{ kg})} = 0.669 \text{ m/s}.$

EVALUATE: The total energy E is constant but is transferred between kinetic and potential energy during the motion.

14.28. **IDENTIFY** and **SET UP:** Use $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ to relate K and U. U depends on x and K depends on v_x.

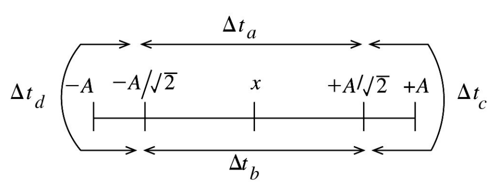
EXECUTE: **(a)** $U + K = E$, so $U = K$ says that $2U = E$

$2\left(\frac{1}{2}kx^2\right) = \frac{1}{2}kA^2$ and $x = \pm A/\sqrt{2}$; magnitude is $A/\sqrt{2}$

But $U = K$ also implies that $2K = E$

$2\left(\frac{1}{2}mv_x^2\right) = \frac{1}{2}kA^2$ and $v_x = \pm\sqrt{k/m}A/\sqrt{2} = \pm\omega A/\sqrt{2}$; magnitude is $\omega A/\sqrt{2}$.

(b) In one cycle x goes from A to 0 to $-A$ to 0 to $+A$. Thus $x = +A\sqrt{2}$ twice and $x = -A/\sqrt{2}$ twice in each cycle. Therefore, $U = K$ four times each cycle. The time between $U = K$ occurrences is the time Δt_a for $x_1 = +A/\sqrt{2}$ to $x_2 = -A\sqrt{2}$, time Δt_b for $x_1 = -A/\sqrt{2}$ to $x_2 = +A/\sqrt{2}$, time Δt_c for $x_1 = +A/\sqrt{2}$ to $x_2 = +A\sqrt{2}$, or the time Δt_d for $x_1 = -A/\sqrt{2}$ to $x_2 = -A/\sqrt{2}$, as shown in Figure 14.28.



$\Delta t_a = \Delta t_b$

$\Delta t_c = \Delta t_d$

Figure 14.28

Calculation of Δt_a:

Specify x in $x = A\cos\omega t$ (choose $\phi = 0$ so $x = A$ at $t = 0$) and solve for t.

$x_1 = +A/\sqrt{2}$ implies $A/\sqrt{2} = A\cos(\omega t_1)$

$\cos\omega t_1 = 1/\sqrt{2}$ so $\omega t_1 = \arccos(1/\sqrt{2}) = \pi/4 \text{ rad}$

$t_1 = \pi/4\omega$

$x_2 = -A/\sqrt{2}$ implies $-A/\sqrt{2} = A\cos(\omega t_2)$

$\cos\omega t_2 = -1/\sqrt{2}$ so $\omega t_1 = 3\pi/4 \text{ rad}$

$t_2 = 3\pi/4\omega$

$\Delta t_a = t_2 - t_1 = 3\pi/4\omega - \pi/4\omega = \pi/2\omega$ (Note that this is $T/4$, one-fourth period.)

Calculation of Δt_d:

$x_1 = -A/\sqrt{2}$ implies $t_1 = 3\pi/4\omega$

$x_2 = -A/\sqrt{2}$, t_2 is the *next* time after t_1 that gives $\cos\omega t_2 = -1/\sqrt{2}$

Thus $\omega t_2 = \omega t_1 + \pi/2 = 5\pi/4$ and $t_2 = 5\pi/4\omega$

$\Delta t_d = t_2 - t_1 = 5\pi/4\omega - 3\pi/4\omega = \pi/2\omega,$ so is the same as Δt_a.
Therfore the occurrences of $K = U$ are equally spaced in time, with a time interval between them of $\pi/2\omega$.
EVALUATE: This is one-fourth T, as it must be if there are 4 equally spaced occurrences each period.
(c) EXECUTE: $x = A/2$ and $U + K = E$

$K = E - U = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}k(A/2)^2 = \frac{1}{2}kA^2 - \frac{1}{8}kA^2 = 3kA^2/8$

Then $\dfrac{K}{E} = \dfrac{3kA^2/8}{\frac{1}{2}kA^2} = \dfrac{3}{4}$ and $\dfrac{U}{E} = \dfrac{\frac{1}{8}kA^2}{\frac{1}{2}kA^2} = \dfrac{1}{4}$

EVALUATE: At $x = 0$ all the energy is kinetic and at $x = \pm A$ all the energy is potiential. But $K = U$ does not occur at $x = \pm A/2$, since U is not linear in x.

14.29. **IDENTIFY:** Velocity and position are related by $E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$. Acceleration and position are related by $-kx = ma_x$.
SET UP: The maximum speed is at $x = 0$ and the maximum magnitude of acceleration is at $x = \pm A$.
EXECUTE: **(a)** For $x = 0$, $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$ and $v_{\text{max}} = A\sqrt{\dfrac{k}{m}} = (0.040\text{ m})\sqrt{\dfrac{450\text{ N/m}}{0.500\text{ kg}}} = 1.20\text{ m/s}$

(b) $v_x = \pm\sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2} = \pm\sqrt{\dfrac{450\text{ N/m}}{0.500\text{ kg}}}\sqrt{(0.040\text{ m})^2 - (0.015\text{ m})^2} = \pm 1.11\text{ m/s}.$

The speed is $v = 1.11$ m/s.

(c) For $x = \pm A$, $a_{\text{max}} = \dfrac{k}{m}A = \left(\dfrac{450\text{ N/m}}{0.500\text{ kg}}\right)(0.040\text{ m}) = 36\text{ m/s}^2$

(d) $a_x = -\dfrac{kx}{m} = -\dfrac{(450\text{ N/m})(-0.015\text{ m})}{0.500\text{ kg}} = +13.5\text{ m/s}^2$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(450\text{ N/m})(0.040\text{ m})^2 = 0.360\text{ J}$

EVALUATE: The speed and acceleration at $x = -0.015$ m are less than their maximum values.

14.30. **IDENTIFY:** The block moves in SHM attached to a spring.
SET UP: We are looking at the block's average speed and maximum speed. $v_{\text{av}} = d/t$, $v_{\text{max}} = A\omega$, $\omega = \sqrt{\dfrac{k}{m}}$ and $T = 2\pi\sqrt{\dfrac{m}{k}}$.

EXECUTE: **(a)** During one cycle the block moves through a total distance of $d = 4A$ in time T, so

$v_{\text{av}} = \dfrac{d}{t} = \dfrac{4A}{T} = \dfrac{4A}{2\pi\sqrt{\dfrac{m}{k}}} = \dfrac{2A}{\pi}\sqrt{\dfrac{k}{m}}.$

(b) $v_{\text{max}} = A\omega = A\sqrt{\dfrac{k}{m}}$. Using this fact and our result from part (a), we have

$v_{\text{av}} = \dfrac{2A}{\pi}\sqrt{\dfrac{k}{m}} = \left(A\sqrt{\dfrac{k}{m}}\right)\left(\dfrac{2}{\pi}\right) = \dfrac{2}{\pi}v_{\text{max}}.$

(c) $\dfrac{v_{\text{av}}}{\frac{1}{2}v_{\text{max}}} = \dfrac{\frac{2}{\pi}v_{\text{max}}}{\frac{1}{2}v_{\text{max}}} = \dfrac{4}{\pi} > 1$, so $v_{\text{av}} > \dfrac{v_{\text{max}}}{2}$. This suggests that the block spends more time traveling at speeds greater than $v_{\text{max}}/2$.

EVALUATE: The answer in (c) is reasonable because $v_{\text{av}} = (2/\pi)v_{\text{max}} < v_{\text{max}}$.

14.31. **IDENTIFY:** A block moves in SHM on a spring.

SET UP: $x(t) = A\cos(\omega t + \phi)$, $v(t) = dx/dt$. We want to know how far the block is from the origin when its speed is one-half of its maximum speed.

EXECUTE: First find $v(t)$: $v(t) = dx/dt = \dfrac{d\left(A\cos(\omega t+\phi)\right)}{dt} = -A\omega\sin(\omega t+\phi)$. Now find the time when $v = v_{max}/2$. We can drop the minus sign since we are interested only in the speed, not its direction. We also realize that $A\omega = v_{max}$. This gives $\dfrac{v_{max}}{2} = v_{max}\sin(\omega t+\phi)$, so $\sin(\omega t+\phi) = \dfrac{1}{2}$, which gives $(\omega t+\phi) = 30° = \dfrac{\pi}{6}$. At this time $x = A\cos(\omega t+\phi) = A\cos\dfrac{\pi}{6} = A\dfrac{\sqrt{3}}{2} \approx 0.866A$. The distance is $x \approx 0.866A$, which is greater than $A/2$.

EVALUATE: The block changes its speed most rapidly when it is near $x = A$ because a_x is greatest then.

14.32. **IDENTIFY:** Newton's second law applies to the system, and mechanical energy is conserved.

SET UP: $\Sigma F_x = ma_x$, $K_1 + U_1 = K_2 + U_2$, $U = \frac{1}{2}kx^2$, $F_{spring} = -kx$.

EXECUTE: **(a)** $\Sigma F_x = ma_x$ gives $ma_x = -kx$, which gives

$(0.300 \text{ kg})(-12.0 \text{ m/s}^2) = -k(0.240 \text{ m})$.

Solving for k gives $k = 15.0$ N/m.

(b) Applying $K_1 + U_1 = K_2 + U_2$ gives $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$.

Putting in the numbers gives

$(0.300 \text{ kg})(4.00 \text{ m/s})^2 + (15.0 \text{ N/m})(0.240 \text{ m})^2 = (15.0 \text{ N/m})A^2$,

so $A = 0.61449$ m, which rounds to 0.614 m.

(c) The kinetic energy is maximum when the potential energy is zero, which is when $x = 0$. Therefore $\frac{1}{2}kA^2 = \frac{1}{2}mv^2$, which gives

$(15.0 \text{ N/m})(0.61449 \text{ m})^2 = (0.300 \text{ kg})v^2$

$v = 4.345$ m/s which rounds to 4.35 m/s.

(d) The maximum force occurs when $x = A$, so Newton's second law gives $F_{max} = ma_{max} = kA$.

$(15.0 \text{ N/m})(0.61449 \text{ m}) = (0.300 \text{ kg})a_{max}$, which gives $a_{max} = 30.7 \text{ m/s}^2$.

EVALUATE: It is frequently necessary to use a combination of energy conservation and Newton's laws.

14.33. **IDENTIFY:** Conservation of energy says $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ and Newton's second law says $-kx = ma_x$.

SET UP: Let $+x$ be to the right. Let the mass of the object be m.

EXECUTE: $k = -\dfrac{ma_x}{x} = -m\left(\dfrac{-8.40 \text{ m/s}^2}{0.600 \text{ m}}\right) = (14.0 \text{ s}^{-2})m.$

$A = \sqrt{x^2 + (m/k)v^2} = \sqrt{(0.600 \text{ m})^2 + \left(\dfrac{m}{(14.0 \text{ s}^{-2})m}\right)(2.20 \text{ m/s})^2} = 0.840 \text{ m}$. The object will therefore travel $0.840 \text{ m} - 0.600 \text{ m} = 0.240 \text{ ma}$ to the right before stopping at its maximum amplitude.

EVALUATE: The acceleration is not constant and we cannot use the constant acceleration kinematic equations.

14.34. **IDENTIFY:** The mechanical energy (the sum of the kinetic energy and potential energy) is conserved.

SET UP: $K + U = E$, with $E = \frac{1}{2}kA^2$ and $U = \frac{1}{2}kx^2$

EXECUTE: $U = K$ says $2U = E$. This gives $2(\frac{1}{2}kx^2) = \frac{1}{2}kA^2$, so $x = A/\sqrt{2}$.

EVALUATE: When $x = A/2$ the kinetic energy is three times the elastic potential energy.

14.35. **IDENTIFY** and **SET UP:** Velocity, position, and total energy are related by $E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x{}^2 + \frac{1}{2}kx^2$. Acceleration and position are related by $-kx = ma_x$. The maximum magnitude of acceleration is at $x = \pm A$.

EXECUTE: **(a)** $E = \frac{1}{2}mv_x{}^2 + \frac{1}{2}kx^2 = \frac{1}{2}(2.00\text{ kg})(-4.00\text{ m/s})^2 + \frac{1}{2}(315\text{ N/m})(+0.200\text{ m})^2$.

$E = 16.0\text{ J} + 6.3\text{ J} = 22.3\text{ J}$. $E = \frac{1}{2}kA^2$ and

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(22.3\text{ J})}{315\text{ N/m}}} = 0.376\text{ m}.$$

(b) $a_{\max} = \dfrac{k}{m}A = \left(\dfrac{315\text{ N/m}}{2.00\text{ kg}}\right)(0.376\text{ m}) = 59.2\text{ m/s}^2$

(c) $F_{\max} = ma_{\max} = (2.00\text{ kg})(59.2\text{ m/s}^2) = 118\text{ N}$. Or, $F_x = -kx$ gives $F_{\max} = kA = (315\text{ N/m})(0.376\text{ m}) = 118\text{ N}$, which checks.

EVALUATE: The maximum force and maximum acceleration occur when the displacement is maximum and the velocity is zero.

14.36. **IDENTIFY:** Use the amount the spring is stretched by the weight of the fish to calculate the force constant *k* of the spring. $T = 2\pi\sqrt{m/k}$. $v_{\max} = \omega A = 2\pi fA$.

SET UP: When the fish hangs at rest the upward spring force $|F_x| = kx$ equals the weight *mg* of the fish. $f = 1/T$. The amplitude of the SHM is 0.0500 m.

EXECUTE: **(a)** $mg = kx$ so $k = \dfrac{mg}{x} = \dfrac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{0.180\text{ m}} = 3.54\times 10^3\text{ N/m}$.

(b) $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{65.0\text{ kg}}{3.54\times 10^3\text{ N/m}}} = 0.8514\text{ s}$ which rounds to 0.851 s.

(c) $v_{\max} = 2\pi fA = \dfrac{2\pi A}{T} = \dfrac{2\pi(0.0500\text{ m})}{0.8514\text{ s}} = 0.369\text{ m/s}$.

EVALUATE: Note that *T* depends only on *m* and *k* and is independent of the distance the fish is pulled down. But $v_{\max}$ does depend on this distance.

14.37. **IDENTIFY:** Initially part of the energy is kinetic energy and part is potential energy in the stretched spring. When $x = \pm A$ all the energy is potential energy and when the glider has its maximum speed all the energy is kinetic energy. The total energy of the system remains constant during the motion.

SET UP: Initially $v_x = \pm 0.815\text{ m/s}$ and $x = \pm 0.0300\text{ m}$.

EXECUTE: **(a)** Initially the energy of the system is

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.175\text{ kg})(0.815\text{ m/s})^2 + \frac{1}{2}(155\text{ N/m})(0.0300\text{ m})^2 = 0.128\text{ J}$. $\frac{1}{2}kA^2 = E$ and

$A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(0.128\text{ J})}{155\text{ N/m}}} = 0.0406\text{ m} = 4.06\text{ cm}$.

(b) $\frac{1}{2}mv_{\max}^2 = E$ and $v_{\max} = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(0.128\text{ J})}{0.175\text{ kg}}} = 1.21\text{ m/s}$.

(c) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{155\text{ N/m}}{0.175\text{ kg}}} = 29.8\text{ rad/s}$.

EVALUATE: The amplitude and the maximum speed depend on the total energy of the system but the angular frequency is independent of the amount of energy in the system and just depends on the force constant of the spring and the mass of the object.

14.38. **IDENTIFY:** The torsion constant κ is defined by $\tau_z = -\kappa\theta$. $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ and $T = 1/f$. $\theta(t) = \Theta\cos(\omega t + \phi)$.

SET UP: For the disk, $I = \frac{1}{2}MR^2$. $\tau_z = -FR$. At $t = 0$, $\theta = \Theta = 3.34° = 0.0583$ rad, so $\phi = 0$.

EXECUTE: **(a)** $\kappa = -\frac{\tau_z}{\theta} = -\frac{-FR}{0.0583\text{ rad}} = +\frac{(4.23\text{ N})(0.120\text{ m})}{0.0583\text{ rad}} = 8.71\text{ N}\cdot\text{m/rad}$

(b) $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} = \frac{1}{2\pi}\sqrt{\frac{2\kappa}{MR^2}} = \frac{1}{2\pi}\sqrt{\frac{2(8.71\text{ N}\cdot\text{m/rad})}{(6.50\text{ kg})(0.120\text{ m})^2}} = 2.17\text{ Hz}$. $T = 1/f = 0.461$ s.

(c) $\omega = 2\pi f = 13.6$ rad/s. $\theta(t) = (3.34°)\cos([13.6\text{ rad/s}]t)$.

EVALUATE: The frequency and period are independent of the initial angular displacement, so long as this displacement is small.

14.39. **IDENTIFY:** $K = \frac{1}{2}mv^2$, $U_{\text{grav}} = mgy$ and $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: At the lowest point of the motion, the spring is stretched an amount $2A$.

EXECUTE: **(a)** At the top of the motion, the spring is unstretched and so has no potential energy, the cat is not moving and so has no kinetic energy, and the gravitational potential energy relative to the bottom is $2mgA = 2(4.00\text{ kg})(9.80\text{ m/s}^2)(0.050\text{ m}) = 3.92$ J. This is the total energy, and is the same total for each part.

(b) $U_{\text{grav}} = 0, K = 0$, so $U_{\text{spring}} = 3.92$ J.

(c) At equilibrium the spring is stretched half as much as it was for part (a), and so $U_{\text{spring}} = \frac{1}{4}(3.92\text{ J}) = 0.98$ J, $U_{\text{grav}} = \frac{1}{2}(3.92\text{ J}) = 1.96$ J, and so $K = 0.98$ J.

EVALUATE: During the motion, work done by the forces transfers energy among the forms kinetic energy, gravitational potential energy and elastic potential energy.

14.40. **IDENTIFY:** $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ and $T = 1/f$ says $T = 2\pi\sqrt{\frac{I}{\kappa}}$.

SET UP: $I = \frac{1}{2}mR^2$.

EXECUTE: Solving $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ for κ in terms of the period,

$$\kappa = \left(\frac{2\pi}{T}\right)^2 I = \left(\frac{2\pi}{1.00\text{ s}}\right)^2\left[\frac{1}{2}(2.00\times10^{-3}\text{ kg})(2.20\times10^{-2}\text{ m})^2\right] = 1.91\times10^{-5}\text{ N}\cdot\text{m/rad}.$$

EVALUATE: The longer the period, the smaller the torsion constant.

14.41. **IDENTIFY and SET UP:** The number of ticks per second tells us the period and therefore the frequency. We can use a formula from Table 9.2 to calculate *I*. Then $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ allows us to calculate the torsion constant κ.

EXECUTE: Ticks four times each second implies 0.25 s per tick. Each tick is half a period, so $T = 0.50$ s and $f = 1/T = 1/0.50\text{ s} = 2.00$ Hz.

(a) Thin rim implies $I = MR^2$ (from Table 9.2).
$I = (0.900\times10^{-3}\text{ kg})(0.55\times10^{-2}\text{ m})^2 = 2.7\times10^{-8}\text{ kg}\cdot\text{m}^2$

(b) $T = 2\pi\sqrt{I/\kappa}$ so $\kappa = I(2\pi/T)^2 = (2.7\times10^{-8}\text{ kg}\cdot\text{m}^2)(2\pi/0.50\text{ s})^2 = 4.3\times10^{-6}\text{ N}\cdot\text{m/rad}$

EVALUATE: Both *I* and κ are small numbers.

14.42. **IDENTIFY:** $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$.

SET UP: $f = 165/(265\text{ s})$, the number of oscillations per second.

EXECUTE: $I = \dfrac{\kappa}{(2\pi f)^2} = \dfrac{0.450\text{ N}\cdot\text{m/rad}}{[2\pi(165)/(265\text{ s})]^2} = 0.0294\text{ kg}\cdot\text{m}^2.$

EVALUATE: For a larger I, f is smaller.

14.43. **IDENTIFY:** $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\dfrac{T}{4} = \dfrac{\pi}{2}\sqrt{\dfrac{L}{g}} = 0.25\text{ s}.$

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on L and g.

14.44. **IDENTIFY:** We are looking at a real pendulum for which the simplified formulas are not completely accurate.

SET UP: Equation (14.35), using only the first correction term, is $T = 2\pi\sqrt{\dfrac{L}{g}}\left(1+\dfrac{1}{4}\sin^2\dfrac{\Theta}{2}\right)$. We want to find Θ so that the simplified formula $T = 2\pi\sqrt{\dfrac{L}{g}}$ is in error by 2.0%, which means that $\dfrac{\Delta T}{T} = 0.020$.

EXECUTE: $\dfrac{\Delta T}{T} = \dfrac{T_{\text{large }\Theta} - T_{\text{small }\Theta}}{T_{\text{large }\Theta}} = 1 - \dfrac{T_{\text{small }\Theta}}{T_{\text{large }\Theta}} = 1 - \dfrac{2\pi\sqrt{\dfrac{L}{g}}}{2\pi\sqrt{\dfrac{L}{g}}\left(1+\dfrac{1}{4}\sin^2\dfrac{\Theta}{2}\right)}$, which simplifies to

$0.020 = \dfrac{\dfrac{1}{4}\sin^2\dfrac{\Theta}{2}}{1+\dfrac{1}{4}\sin^2\dfrac{\Theta}{2}}$. Solving for $\sin^2\dfrac{\Theta}{2}$ gives $\sin^2\dfrac{\Theta}{2} = 8.163\times10^{-2}$, so $\Theta = 33°$.

EVALUATE: For a 1% error, the maximum angle could be $\Theta = 23°$. These results show that it is adequate to use the simplified formula in a large number of cases.

14.45. **IDENTIFY:** Since the cord is much longer than the height of the object, the system can be modeled as a simple pendulum. We will assume the amplitude of swing is small, so that $T = 2\pi\sqrt{\dfrac{L}{g}}$.

SET UP: The number of swings per second is the frequency $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$.

EXECUTE: $f = \dfrac{1}{2\pi}\sqrt{\dfrac{9.80\text{ m/s}^2}{1.50\text{ m}}} = 0.407$ swings per second.

EVALUATE: The period and frequency are both independent of the mass of the object.

14.46. **IDENTIFY:** Use $T = 2\pi\sqrt{L/g}$ to relate the period to g.

SET UP: Let the period on earth be $T_\text{E} = 2\pi\sqrt{L/g_\text{E}}$, where $g_\text{E} = 9.80\text{ m/s}^2$, the value on earth.

Let the period on Mars be $T_\text{M} = 2\pi\sqrt{L/g_\text{M}}$, where $g_\text{M} = 3.71\text{ m/s}^2$, the value on Mars.

We can eliminate L, which we don't know, by taking a ratio:

EXECUTE: $\dfrac{T_\text{M}}{T_\text{E}} = 2\pi\sqrt{\dfrac{L}{g_\text{M}}}\dfrac{1}{2\pi}\sqrt{\dfrac{g_\text{E}}{L}} = \sqrt{\dfrac{g_\text{E}}{g_\text{M}}}.$

$$T_M = T_E\sqrt{\frac{g_E}{g_M}} = (1.60\text{ s})\sqrt{\frac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}} = 2.60\text{ s.}$$

EVALUATE: Gravity is weaker on Mars so the period of the pendulum is longer there.

14.47. **IDENTIFY:** Apply $T = 2\pi\sqrt{L/g}$

SET UP: The period of the pendulum is $T = (136\text{ s})/100 = 1.36\text{ s}$.

EXECUTE: $g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2(0.500\text{ m})}{(1.36\text{ s})^2} = 10.7\text{ m/s}^2.$

EVALUATE: The same pendulum on earth, where g is smaller, would have a larger period.

14.48. **IDENTIFY** and **SET UP:** The period is for the time for one cycle. The angular amplitude is the maximum value of θ.

EXECUTE: **(a)** From the graph with the problem,

$T = 1.60$ s. $f = \dfrac{1}{T} = 0.625$ Hz. $\omega = 2\pi f = 3.93$ rad/s. From the graph we also determine that the amplitude is 6 degrees.

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $L = g\left(\dfrac{T}{2\pi}\right)^2 = (9.80\text{ m/s}^2)\left(\dfrac{1.60\text{ s}}{2\pi}\right)^2 = 0.635\text{ m}.$

(c) No. The graph is unchanged if the mass of the bob is changed while the length of the pendulum and amplitude of swing are kept constant. The period is independent of the mass of the bob.

EVALUATE: The amplitude of the graph in the problem does not decrease over the time shown, so there must be little or no friction in this pendulum.

14.49. **IDENTIFY:** $a_{tan} = L\alpha$, $a_{rad} = L\omega^2$ and $a = \sqrt{a_{tan}^2 + a_{rad}^2}$. Use energy conservation in parts (b) and (c).

SET UP: Just after the sphere is released, $\omega = 0$ and $a_{rad} = 0$. When the rod is vertical, $a_{tan} = 0$.

EXECUTE: **(a)** The forces and acceleration are shown in Figure 14.49(a). $a_{rad} = 0$ so $a = a_{tan} = g\sin\theta$.

(b) The forces and acceleration are shown in Figure 14.49(b). In this case, the sphere has radial and tangential acceleration, so we need to use $a = \sqrt{a_{tan}^2 + a_{rad}^2}$. Use energy conservation, calling point 1 the instant that the sphere is released from from rest at angle θ and point 2 the instant the rod makes an angle ϕ with the vertical. This gives $U_1 = U_2 + K_2$, so $mgL(1-\cos\theta) = mgL(1-\cos\phi) + \frac{1}{2}mv^2$. Solving for v^2 gives $v^2 = 2gL(\cos\phi - \cos\theta)$. Therefore $a_{rad} = \dfrac{v^2}{L} = 2g(\cos\phi - \cos\theta)$. To find a_{tan}, apply $\Sigma\vec{\tau} = I\alpha$: $mgL\sin\phi = mL^2\alpha = mL(L\alpha) = mL\tan\alpha$, which gives $a_{tan} = g\sin\phi$. The magnitude of the acceleration is $a = \sqrt{a_{tan}^2 + a_{rad}^2} = \sqrt{(g\sin\phi)^2 + [2g(\cos\phi - \cos\theta)]^2}$, which simplifies to $a = g\sqrt{\sin^2\phi + 4(\cos\phi - \cos\theta)^2}$. In this case $\phi = \theta/2$, so the acceleration is

$a = g\sqrt{\sin^2(\theta/2) + 4[\cos(\theta/2) - \cos\theta]^2}$.

(c) The forces and acceleration are shown in Figure 14.49(c). Calling point 2 the lowest part of the swing, $U_1 = K_2$ gives $mgL(1-\cos\theta) = \frac{1}{2}mv^2$ and $v = \sqrt{2gL(1-\cos\theta)}$. Using the formula derived in part (b) with $\phi = 0°$, the acceleration is $a = g\sqrt{\sin^2 0° + 4(\cos 0° - \cos\theta)^2} = 2g(1-\cos\theta)$.

EVALUATE: As the rod moves toward the vertical, v increases, a_{rad} increases and a_{tan} decreases. The result in (c) agrees with the fact that $a_{\text{tan}} = 0$ when the rod is vertical because in that case $a = a_{\text{rad}} =$
$\dfrac{v^2}{L} = \dfrac{\left(\sqrt{2gL(1-\cos\theta)}\right)^2}{L} = 2g(1-\cos\theta)$, which is what we found in part (c).

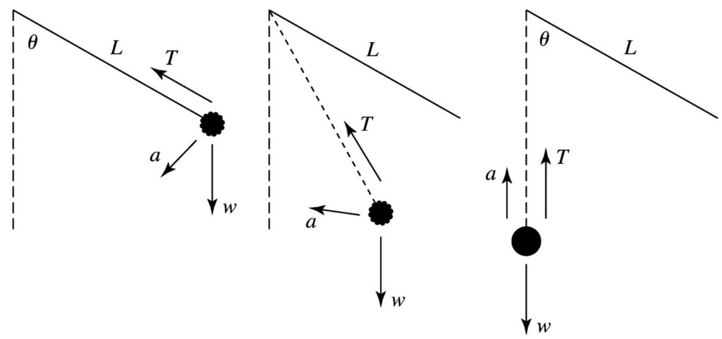


Figure 14.49

14.50. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: From the parallel axis theorem, the moment of inertia of the hoop about the nail is $I = MR^2 + MR^2 = 2MR^2$. $d = R$.

EXECUTE: Solving for R, $R = gT^2/8\pi^2 = 0.496$ m.

EVALUATE: A simple pendulum of length $L = R$ has period $T = 2\pi\sqrt{R/g}$. The hoop has a period that is larger by a factor of $\sqrt{2}$.

14.51. **IDENTIFY:** Pendulum A can be treated as a simple pendulum. Pendulum B is a physical pendulum.

SET UP: For pendulum B the distance d from the axis to the center of gravity is $3L/4$. $I = \frac{1}{3}(m/2)L^2$ for a bar of mass $m/2$ and the axis at one end. For a small ball of mass $m/2$ at a distance L from the axis, $I_{\text{ball}} = (m/2)L^2$.

EXECUTE: Pendulum A: $T_A = 2\pi\sqrt{\dfrac{L}{g}}$.

Pendulum B: $I = I_{\text{bar}} + I_{\text{ball}} = \frac{1}{3}(m/2)L^2 + (m/2)L^2 = \frac{2}{3}mL^2$.

$T_B = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{\frac{2}{3}mL^2}{mg(3L/4)}} = 2\pi\sqrt{\dfrac{L}{g}}\sqrt{\dfrac{2}{3}\cdot\dfrac{4}{3}} = \sqrt{\dfrac{8}{9}}\left(2\pi\sqrt{\dfrac{L}{g}}\right) = 0.943T_A$. The period is longer for pendulum A.

EVALUATE: Example 14.9 shows that for the bar alone, $T = \sqrt{\dfrac{2}{3}}T_A = 0.816T_A$. Adding the ball of equal mass to the end of the rod increases the period compared to that for the rod alone.

14.52. **IDENTIFY:** Apply $T = 2\pi\sqrt{\dfrac{I}{mgd}}$ to calculate I and conservation of energy to calculate the maximum angular speed, Ω_{max}.

SET UP: $d = 0.250$ m. In part (b), $y_{\text{i}} = d(1-\cos\Theta)$, with $\Theta = 0.400$ rad and $y_{\text{f}} = 0$.

EXECUTE: **(a)** Solving $T = 2\pi\sqrt{\frac{I}{mgd}}$ for *I*, we get

$$I = \left(\frac{T}{2\pi}\right)^2 mgd = \left(\frac{0.940\text{ s}}{2\pi}\right)^2 (1.80\text{ kg})(9.80\text{ m/s}^2)(0.250\text{ m}) = 0.0987\text{ kg}\cdot\text{m}^2.$$

(b) The small-angle approximation will not give three-figure accuracy for $\Theta = 0.400$ rad. From energy considerations, $mgd(1-\cos\Theta) = \frac{1}{2}I\Omega_{\text{max}}^2$. Expressing Ω_{max} in terms of the period of small-angle oscillations, this becomes

$$\Omega_{\text{max}} = \sqrt{2\left(\frac{2\pi}{T}\right)^2 (1-\cos\Theta)} = \sqrt{2\left(\frac{2\pi}{0.940\text{ s}}\right)^2 [1-\cos(0.400\text{ rad})]} = 2.66\text{ rad/s}.$$

EVALUATE: The time for the motion in part (b) is $t = T/4$, so $\Omega_{\text{av}} = \Delta\theta/\Delta t = (0.400\text{ rad})/(0.235\text{ s}) =$ 1.70 rad/s. Ω increases during the motion and the final Ω is larger than the average Ω.

14.53. **IDENTIFY:** Pendulum *A* can be treated as a simple pendulum. Pendulum *B* is a physical pendulum. Use the parallel-axis theorem to find the moment of inertia of the ball in *B* for an axis at the top of the string.
SET UP: For pendulum *B* the center of gravity is at the center of the ball, so $d = L$. For a solid sphere with an axis through its center, $I_{\text{cm}} = \frac{2}{5}MR^2$. $R = L/2$ and $I_{\text{cm}} = \frac{1}{10}ML^2$.

EXECUTE: Pendulum *A*: $T_A = 2\pi\sqrt{\frac{L}{g}}$.

Pendulum *B*: The parallel-axis theorem says $I = I_{\text{cm}} + ML^2 = \frac{11}{10}ML^2$.

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{11ML^2}{10MgL}} = \sqrt{\frac{11}{10}}\left(2\pi\sqrt{\frac{L}{g}}\right) = \sqrt{\frac{11}{10}}T_A = 1.05T_A.$$ It takes pendulum *B* longer to completea swing.

EVALUATE: The center of the ball is the same distance from the top of the string for both pendulums, but the mass is distributed differently and *I* is larger for pendulum *B*, even though the masses are the same.

14.54. **IDENTIFY:** The ornament is a physical pendulum, so $T = 2\pi\sqrt{I/mgd}$. *T* is the target variable.

SET UP: $I = 5MR^2/3$, the moment of inertia about an axis at the edge of the sphere. *d* is the distance from the axis to the center of gravity, which is at the center of the sphere, so $d = R$.

EXECUTE: $T = 2\pi\sqrt{5/3}\sqrt{R/g} = 2\pi\sqrt{5/3}\sqrt{0.050\text{ m}/(9.80\text{ m/s}^2)} = 0.58\text{ s}.$

EVALUATE: A simple pendulum of length $R = 0.050$ m has period 0.45 s; the period of the physical pendulum is longer.

14.55. **IDENTIFY:** The object is moving with damped SHM.
SET UP: We want to know the angular frequency ω' if the damping constant is one-half the critical value. $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$, where *b* is the damping constant and $b_{\text{crit}} = 2\sqrt{km}$. If there is no damping $(b = 0)$, then $\omega = \sqrt{k/m}$.

EXECUTE: If $b = \frac{1}{2}b_{\text{crit}} = \frac{1}{2}\left(2\sqrt{km}\right)$, the angular frequency becomes

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{k}{m} - \frac{\left[\frac{1}{2}\left(2\sqrt{km}\right)\right]^2}{4m^2}} = \sqrt{\frac{k}{m} - \frac{k}{4m}} = \sqrt{\frac{3}{4}\frac{k}{m}} = \omega\frac{\sqrt{3}}{2}.$$

EVALUATE: Notice that if $b = \frac{1}{2}b_{\text{crit}}$, it does *not* follow that $\omega' = \frac{\omega}{2}$.

14.56. IDENTIFY: From a small damping force, $A_2 = A_1 e^{-(b/2m)t}$.

SET UP: $\ln(e^{-x}) = -x$

EXECUTE: $b = \frac{2m}{t}\ln\left(\frac{A_1}{A_2}\right) = \frac{2(0.050\text{ kg})}{(5.00\text{ s})}\ln\left(\frac{0.300\text{ m}}{0.100\text{ m}}\right) = 0.0220\text{ kg/s}.$

EVALUATE: As a check, note that the oscillation frequency is the same as the undamped frequency to $4.8\times10^{-3}\%$, so our assumption of a small damping force is valid.

14.57. IDENTIFY and SET UP: Use $\omega' = \sqrt{(k/m)-(b^2/4m^2)}$ to calculate ω', and then $f' = \omega'/2\pi$.

(a) EXECUTE: $\omega' = \sqrt{(k/m)-(b^2/4m^2)} = \sqrt{\frac{2.50\text{ N/m}}{0.300\text{ kg}} - \frac{(0.900\text{ kg/s})^2}{4(0.300\text{ kg})^2}} = 2.47\text{ rad/s}$

$f' = \omega'/2\pi = (2.47\text{ rad/s})/2\pi = 0.393\text{ Hz}$

(b) IDENTIFY and SET UP: The condition for critical damping is $b = 2\sqrt{km}$.

EXECUTE: $b = 2\sqrt{(2.50\text{ N/m})(0.300\text{ kg})} = 1.73\text{ kg/s}$

EVALUATE: The value of b in part (a) is less than the critical damping value found in part (b). With no damping, the frequency is $f = 0.459\text{ Hz}$; the damping reduces the oscillation frequency.

14.58. IDENTIFY: The graph with the problem shows that the amplitude of vibration is decreasing, so the system must be losing mechanical energy.

SET UP: The mechanical energy is $E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$.

EXECUTE: (a) When $|x|$ is a maximum and the tangent to the curve is horizontal the speed of the mass is zero. This occurs at $t = 0$, $t = 1.0\text{ s}$, $t = 2.0\text{ s}$, $t = 3.0\text{ s}$ and $t = 4.0\text{ s}$.

(b) At $t = 0$, $v_x = 0$ and $x = 7.0\text{ cm}$ so $E_0 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(0.070\text{ m})^2 = 0.55\text{ J}$.

(c) At $t = 1.0\text{ s}$, $v_x = 0$ and $x = -6.0\text{ cm}$ so $E_1 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(-0.060\text{ m})^2 = 0.405\text{ J}$. At $t = 4.0\text{ s}$, $v_x = 0$ and $x = 3.0\text{ cm}$ so $E_4 = \frac{1}{2}kx^2 = \frac{1}{2}(225\text{ N/m})(0.030\text{ m})^2 = 0.101\text{ J}$. The mechanical energy "lost" is $E_1 - E_4 = 0.30\text{ J}$. The mechanical energy lost was converted to other forms of energy by nonconservative forces, such as friction, air resistance, and other dissipative forces.

EVALUATE: After a while the mass will come to rest and then all its initial mechanical energy will have been "lost" because it will have been converted to other forms of energy by nonconservative forces.

14.59. IDENTIFY: Apply $A = \frac{F_{\text{max}}}{\sqrt{\left(k - m\omega_{\text{d}}^2\right)^2 + b^2\omega_{\text{d}}^2}}$.

SET UP: $\omega_{\text{d}} = \sqrt{k/m}$ corresponds to resonance, and in this case $A = \frac{F_{\text{max}}}{\sqrt{\left(k - m\omega_{\text{d}}^2\right)^2 + b^2\omega_{\text{d}}^2}}$ reduces to $A = F_{\text{max}}/b\omega_{\text{d}}$.

EXECUTE: (a) $A_1/3$

(b) $2A_1$

EVALUATE: Note that the resonance frequency is independent of the value of b. (See Figure 14.28 in the textbook).

14.60. **IDENTIFY:** We are dealing with forced oscillations and resonance.

SET UP: Equation (14.46) gives $A = \dfrac{F_{\text{max}}}{\sqrt{\left(k - m\omega_{\text{d}}^2\right)^2 + b^2\omega_{\text{d}}^2}}$, the amplitude A of a forced oscillator where $\omega = \sqrt{k/m}$ and F_{max} is the maximum value of the driving force. We want to investigate how varying the damping constant b affects the amplitude.

EXECUTE: **(a)** In this case, $b = 0.20\sqrt{km}$ and $\omega_{\text{d}} = \omega = \sqrt{k/m}$, so A is

$$A = \frac{F_{\text{max}}}{\sqrt{\left(k - m(k/m)\right)^2 + \left(0.20\sqrt{km}\right)^2\left(\sqrt{k/m}\right)^2}} = \frac{F_{\text{max}}}{0.20k} = \frac{5F_{\text{max}}}{k}.$$

(b) In this case, $b = 0.40\sqrt{km}$ and $\omega_{\text{d}} = \omega = \sqrt{k/m}$, so A is

$$A = \frac{F_{\text{max}}}{\sqrt{\left(0.40\sqrt{km}\right)^2 (k/m)}} = \frac{F_{\text{max}}}{0.40k} = \frac{2.5F_{\text{max}}}{k}.$$

(c) For $b = 0.20\sqrt{km}$ and $\omega_{\text{d}} = \dfrac{\omega}{2} = \dfrac{1}{2}\sqrt{k/m}$, A is $A = \dfrac{F_{\text{max}}}{\sqrt{\left(k - m(\omega^2/4)\right)^2 + \left(0.20\sqrt{km}\right)^2\left(\frac{1}{2}\sqrt{k/m}\right)^2}}$

$$= \frac{F_{\text{max}}}{\sqrt{\left(k - m(k/4m)\right)^2 + (0.040km)(k/4m)}} = \frac{F_{\text{max}}}{\sqrt{(3k/4)^2 + (0.010)k^2}} = \frac{F_{\text{max}}}{0.20k} = \frac{1.3F_{\text{max}}}{k}.$$

$$\frac{A_\omega}{A_{\omega/2}} = \frac{5F_{\text{max}}/k}{1.32F_{\text{max}}k} = 3.8.$$

For $b = 0.40\sqrt{km}$ and $\omega_{\text{d}} = \dfrac{\omega}{2} = \dfrac{1}{2}\sqrt{k/m}$, $A = \dfrac{F_{\text{max}}}{\sqrt{(3k/4)^2 + \left(0.40\sqrt{km}\right)^2\left(\frac{1}{2}\sqrt{k/m}\right)^2}} =$

$$\frac{F_{\text{max}}}{\sqrt{(3k/4)^2 + (0.040)k^2}} = \frac{0.13F_{\text{max}}}{k}. \quad \frac{A_\omega}{A_{\omega/2}} = \frac{2.5F_{\text{max}}/k}{0.13F_{\text{max}}k} = 1.9.$$

EVALUATE: The amplitude increases by a larger factor for $b = 0.20\sqrt{km}$ than for $b = 0.40\sqrt{km}$, which agrees with Fig. 14.28.

14.61. **IDENTIFY:** Two objects are in SHM on different springs, and we want to compare their maximum speed and maximum acceleration.

SET UP: We know that $v_{\text{max}} = A\omega = A\sqrt{k/m}$ and $a_{\text{max}} = \omega^2 A = (k/m)A.$

EXECUTE: **(a)** $\dfrac{v_{\text{max},A}}{v_{\text{max},B}} = \dfrac{\sqrt{\dfrac{k_A}{m_A}}A_A}{\sqrt{\dfrac{k_B}{m_B}}A_B} = \sqrt{\left(\dfrac{k_A}{k_B}\right)\left(\dfrac{m_B}{m_A}\right)}\left(\dfrac{A_A}{A_B}\right) = \sqrt{\left(\dfrac{9k_B}{k_B}\right)\left(\dfrac{4m_A}{m_A}\right)}\left(\dfrac{2A_B}{A_B}\right) = 12.$

(b) $\dfrac{a_{\text{max},A}}{a_{\text{max},B}} = \dfrac{\dfrac{k_A}{m_A}A_A}{\dfrac{k_B}{m_B}A_B} = \left(\dfrac{k_A}{k_B}\right)\left(\dfrac{m_B}{m_A}\right)\left(\dfrac{A_A}{A_B}\right) = \left(\dfrac{9k_B}{k_B}\right)\left(\dfrac{4m_A}{m_A}\right)\left(\dfrac{2A_B}{A_B}\right) = 72.$

EVALUATE: The ratio of the accelerations is considerably greater than that of the speeds because the acceleration depends on k/m while the speed depends on $\sqrt{k/m}$.

14.62. **IDENTIFY:** Apply $x(t) = A\cos(\omega t + \phi)$

SET UP: $x = A$ at $t = 0$, so $\phi = 0$. $A = 6.00$ cm. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.300\text{ s}} = 20.9$ rad/s, so $x(t) = (6.00\text{ cm})\cos[(20.9\text{ rad/s})t]$.

EXECUTE: $t = 0$ at $x = 6.00$ cm. $x = -1.50$ cm when $-1.50\text{ cm} = (6.00\text{ cm})\cos[(20.9\text{ rad/s})t]$.

$$t = \left(\frac{1}{20.9\text{ rad/s}}\right)\arccos\left(\frac{1.50\text{ cm}}{6.00\text{ cm}}\right) = 0.0872\text{ s. It takes }0.0872\text{ s.}$$

EVALUATE: It takes $t = T/4 = 0.075$ s to go from $x = 6.00$ cm to $x = 0$ and 0.150 s to go from $x = +6.00$ cm to $x = -6.00$ cm. Our result is between these values, as it should be.

14.63. **IDENTIFY** and **SET UP:** Calculate x using $x = A\cos(\omega t + \phi)$. Use T to find ω and x_0 to calculate ϕ.

EXECUTE: At $t = 0$, $x = 0$ and the object is traveling in the $-x$-direction, so $\phi = \pi/2$ rad.

Thus $x = A\cos(\omega t + \pi/2)$.

$T = 2\pi/\omega$ so $\omega = 2\pi/T = 2\pi/1.20\text{ s} = 5.236$ rad/s

$x = (0.600\text{ m})\cos[(5.236\text{ rad/s})(0.480\text{ s}) + \pi/2] = -0.353$ m.

The distance of the object from the equilibrium position is 0.353 m.

EVALUATE: It takes the object time $t = T/2 = 0.600$ s to return to $x = 0$, so at $t = 0.480$ s it is still at negative x.

14.64. **IDENTIFY:** $T = 2\pi\sqrt{\dfrac{m}{k}}$. The period changes when the mass changes.

SET UP: M is the mass of the empty car and the mass of the loaded car is $M + 250$ kg.

EXECUTE: The period of the empty car is $T_E = 2\pi\sqrt{\dfrac{M}{k}}$. The period of the loaded car is

$$T_L = 2\pi\sqrt{\frac{M + 250\text{ kg}}{k}}.\quad k = \frac{(250\text{ kg})(9.80\text{ m/s}^2)}{4.00\times 10^{-2}\text{ m}} = 6.125\times 10^4\text{ N/m}$$

$$M = \left(\frac{T_L}{2\pi}\right)^2 k - 250\text{ kg} = \left(\frac{1.92\text{ s}}{2\pi}\right)^2 (6.125\times 10^4\text{ N/m}) - 250\text{ kg} = 5.469\times 10^3\text{ kg.}$$

$$T_E = 2\pi\sqrt{\frac{5.469\times 10^3\text{ kg}}{6.125\times 10^4\text{ N/m}}} = 1.88\text{ s.}$$

EVALUATE: When the mass decreases, the period decreases.

14.65. **IDENTIFY:** An object is executing SHM on a spring.

SET UP: We want to change the spring so that the amplitude A_2 is half the original amplitude A_1 and the mechanical energy E_2 is 4 times its original value E_1. Our target variables are the new force constant k_2 and the new maximum speed $v_{2,max}$ of the object. We know that $U_{max} = \frac{1}{2}kA^2$, $E = K + U = K_{max} = E_{max}$, and $v_{max} = A\omega = A\sqrt{k/m}$.

EXECUTE: **(a)** We want to relate k_2 to k_1. The original and final mechanical energies are $E_1 = \frac{1}{2}k_1A_1^2$ and $E_2 = \frac{1}{2}k_2A_2^2 = 4E_1 = 4\left(\frac{1}{2}k_1A_1^2\right)$, so $k_2 = 4\left(\dfrac{A_1}{A_2}\right)^2 k_1 = 4\left(\dfrac{2A_2}{A_2}\right)^2 k_1 = 16k_1$.

(b) Using $v_{max} = A\omega = A\sqrt{k/m}$ and taking the ratio of the maximum speeds gives

$$\frac{v_{2,max}}{v_{1,max}} = \frac{\sqrt{\frac{k_2}{m}}A_2}{\sqrt{\frac{k_1}{m}}A_1} = \sqrt{\frac{k_2}{k_1}}\left(\frac{A_2}{A_1}\right) = \sqrt{\frac{16k_1}{k_1}}\left(\frac{A_2}{2A_2}\right) = 2,\text{ so } v_{2,max} = 2v_{1,max}.$$

EVALUATE: $E = \frac{1}{2}mv_{\max}^2$, so if we increase E by a factor of 4, $v_{\max}^2$ must increase by a factor of 4, so $v_{\max}$ must increase by a factor of 2, which is what we found in part (b).

14.66. IDENTIFY: In SHM, $a_{\max} = \frac{k}{m_{\text{tot}}}A$. Apply $\sum \vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\sum F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{\max} = \left(\frac{k}{M+m}\right)A$. Set $a_{\max} = a$ and solve for A: $\mu_s g = \left(\frac{k}{M+m}\right)A$ and $A = \frac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.

14.67. IDENTIFY: A block is moving with SHM on a spring. Using measurements of its maximum speed and mass, we will use graphical interpretation.

SET UP: $v_{\max} = A\omega = A\sqrt{k/m}$. Our target variable is the force constant of the spring. The data is plotted as $v_{\max}^2$ versus $1/m$, so we need to find a relation between these quantities to interpret the graph.

EXECUTE: Solving $v_{\max} = A\omega = A\sqrt{k/m}$ gives $v_{\max}^2 = (A^2k)(1/m)$, so a graph of $v_{\max}^2$ versus $1/m$ should be a straight line having slope equal to A^2k. Therefore A^2k = slope = 8.62 $\text{N}\cdot\text{m}$, which gives k = (8.62 $\text{N}\cdot\text{m}$)/(0.120 m)^2 = 599 N/m.

EVALUATE: Converting 599 N/m to lb/in gives k = 3.42 lb/in. which is reasonable for a rather stiff spring.

14.68. IDENTIFY: Two blocks oscillate on a spring in SHM. One of them is accelerated only by static friction, so we'll need to use Newton's second law in addition to the principles of SHM.

SET UP: The maximum friction force on the upper block occurs when its acceleration is a maximum, and that occurs at $a_{\max} = \omega^2 A$ for the SHM. For the block not to slip, $f_s = \mu_s n$. The target variable is the coefficient of static friction between the two blocks if the maximum distance that the spring can move from its equilibrium position without causing slipping is d = 8.8 cm. We know that $a_{\max} = \omega^2 A$ and $T = 2\pi\sqrt{m/k}$ and will need to use $\sum F_x = ma_x$.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4.50 \text{ kg}}{150 \text{ N/m}}} = 1.09$ s.

(b) At d = 8.8 cm, f_s is at its maximum value, so $f_s = \mu_s n = \mu_s mg$. Only friction is accelerating the upper block, so $\sum F_x = ma_x$ gives $\mu_s mg = ma_x$. Now look at the system of two blocks (of total mass $m + M$) that are oscillating together. For the system, $\omega = \sqrt{k/m_{\text{total}}} = \sqrt{k/(m+M)}$, so $a_{\max} = \omega^2 A = \left(\sqrt{k/(m+M)}\right)^2 d = \frac{kd}{m+M}$. Going back to $\sum F_x = ma_x$ gives $\mu_s mg = ma_{\max} = m\left(\frac{kd}{m+M}\right)$. Which gives $\mu_s = \frac{kd}{g(m+M)} = \frac{(150 \text{ N/m})(0.088 \text{ m})}{(9.80 \text{ m/s}^2)(4.50 \text{ kg})} = 0.30$.

EVALUATE: According to Table 5.1, this value is about the same as for rubber on wet concrete, so it is reasonable.

14.69. IDENTIFY: The largest downward acceleration the ball can have is g whereas the downward acceleration of the tray depends on the spring force. When the downward acceleration of the tray is greater than g, then the ball leaves the tray. $y(t) = A\cos(\omega t + \phi)$.

SET UP: The downward force exerted by the spring is $F = kd$, where d is the distance of the object above the equilibrium point. The downward acceleration of the tray has magnitude $\frac{F}{m} = \frac{kd}{m}$, where m is the total mass of the ball and tray. $x = A$ at $t = 0$, so the phase angle ϕ is zero and $+x$ is downward.

EXECUTE: **(a)** $\frac{kd}{m} = g$ gives $d = \frac{mg}{k} = \frac{(1.775 \text{ kg})(9.80 \text{ m/s}^2)}{185 \text{ N/m}} = 9.40 \text{ cm}$. This point is 9.40 cm above the equilibrium point so is $9.40 \text{ cm} + 15.0 \text{ cm} = 24.4 \text{ cm}$ above point A.

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{185 \text{ N/m}}{1.775 \text{ kg}}} = 10.2 \text{ rad/s}$. The point in (a) is above the equilibrium point so $x = -9.40$ cm.

$x = A\cos(\omega t)$ gives $\omega t = \arccos\left(\frac{x}{A}\right) = \arccos\left(\frac{-9.40 \text{ cm}}{15.0 \text{ cm}}\right) = 2.25 \text{ rad}$. $t = \frac{2.25 \text{ rad}}{10.2 \text{ rad/s}} = 0.221 \text{ s}$.

(c) $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$ gives $v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\frac{185 \text{ N/m}}{1.775 \text{ kg}}([0.150 \text{ m}]^2 - [-0.0940 \text{ m}]^2)} = 1.19 \text{ m/s}$.

EVALUATE: The period is $T = 2\pi\sqrt{\frac{m}{k}} = 0.615 \text{ s}$. To go from the lowest point to the highest point takes time $T/2 = 0.308$ s. The time in (b) is less than this, as it should be.

14.70. **IDENTIFY:** Apply conservation of linear momentum to the collision and conservation of energy to the motion after the collision. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and $T = \frac{1}{f}$.

SET UP: The object returns to the equilibrium position in time $T/2$.

EXECUTE: **(a)** Momentum conservation during the collision: $mv_0 = (2m)V$.

$$V = \frac{1}{2}v_0 = \frac{1}{2}(2.00 \text{ m/s}) = 1.00 \text{ m/s}.$$

Energy conservation after the collision: $\frac{1}{2}MV^2 = \frac{1}{2}kx^2$.

$$x = \sqrt{\frac{MV^2}{k}} = \sqrt{\frac{(20.0 \text{ kg})(1.00 \text{ m/s})^2}{170.0 \text{ N/m}}} = 0.343 \text{ m (amplitude)}$$

$\omega = 2\pi f = \sqrt{k/M}$. $f = \frac{1}{2\pi}\sqrt{k/M} = \frac{1}{2\pi}\sqrt{\frac{170.0 \text{ N/m}}{20.0 \text{ kg}}} = 0.464 \text{ Hz}$. $T = \frac{1}{f} = \frac{1}{0.464 \text{ Hz}} = 2.16 \text{ s}$.

(b) It takes $1/2$ period to first return: $\frac{1}{2}(2.16 \text{ s}) = 1.08 \text{ s}$.

EVALUATE: The total mechanical energy of the system determines the amplitude. The frequency and period depend only on the force constant of the spring and the mass that is attached to the spring.

14.71. **IDENTIFY** and **SET UP:** The bounce frequency is given by $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and the pendulum frequency by $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$. Use the relation between these two frequencies that is specified in the problem to calculate the equilibrium length L of the spring, when the apple hangs at rest on the end of the spring.

EXECUTE: Vertical SHM: $f_b = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

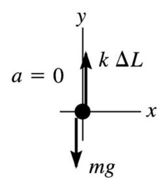
Pendulum motion (small amplitude): $f_p = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

The problem specifies that $f_\text{p} = \frac{1}{2} f_\text{b}$, so $\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2}\frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Thus $g/L = k/4m$, which gives $L = 4gm/k = 4w/k = 4(1.00\text{ N})/1.50\text{ N/m} = 2.67\text{ m}$.

EVALUATE: This is the *stretched* length of the spring, its length when the apple is hanging from it. (Note: Small angle of swing means *v* is small as the apple passes through the lowest point, so a_rad is small and the component of *mg* perpendicular to the spring is small. Thus the amount the spring is stretched changes very little as the apple swings back and forth.)

IDENTIFY: Use Newton's second law to calculate the distance the spring is stretched from its unstretched length when the apple hangs from it.

SET UP: The free-body diagram for the apple hanging at rest on the end of the spring is given in Figure 14.71.



EXECUTE: $\sum F_y = ma_y$

$k\Delta L - mg = 0$

$\Delta L = mg/k = w/k = 1.00\text{ N}/1.50\text{ N/m} = 0.667\text{ m}.$

Thus the unstretched length of the spring is

$$2.67\text{ m} - 0.67\text{ m} = 2.00\text{ m}.$$

Figure 14.71

EVALUATE: The spring shortens to its unstretched length when the apple is removed.

14.72. IDENTIFY: The vertical forces on the floating object must sum to zero. The buoyant force *B* applied to the object by the liquid is given by Archimedes's principle. The motion is SHM if the net force on the object is of the form $F_y = -ky$ and then $T = 2\pi\sqrt{m/k}$.

SET UP: Take $+y$ to be downward.

EXECUTE: **(a)** $V_\text{submerged} = LA,$ where *L* is the vertical distance from the surface of the liquid to the bottom of the object. Archimedes's principle states $\rho g LA = Mg,$ so $L = \frac{M}{\rho A}.$

(b) The buoyant force is $\rho gA(L + y) = Mg + F,$ where *y* is the additional distance the object moves downward. Using the result of part (a) and solving for *y* gives $y = \frac{F}{\rho g A}.$

(c) The net force is $F_\text{net} = Mg - \rho gA(L + y) = -\rho gAy.\ k = \rho gA,$ and the period of oscillation is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{\rho g A}}.$$

EVALUATE: The force *F* determines the amplitude of the motion but the period does not depend on how much force was applied.

14.73. IDENTIFY: The object oscillates as a physical pendulum, so $f = \frac{1}{2\pi}\sqrt{\frac{m_\text{object} gd}{I}}$. Use the parallel-axis theorem, $I = I_\text{cm} + Md^2,$ to find the moment of inertia of each stick about an axis at the hook.

SET UP: The center of mass of the square object is at its geometrical center, so its distance from the hook is $L\cos 45° = L/\sqrt{2}$. The center of mass of each stick is at its geometrical center. For each stick, $I_\text{cm} = \frac{1}{12} mL^2.$

EXECUTE: The parallel-axis theorem gives I for each stick for an axis at the center of the square to be $\frac{1}{12}mL^2 + m(L/2)^2 = \frac{1}{3}mL^2$ and the total I for this axis is $\frac{4}{3}mL^2$. For the entire object and an axis at the hook, applying the parallel-axis theorem again to the object of mass $4m$ gives
$I = \frac{4}{3}mL^2 + 4m(L/\sqrt{2})^2 = \frac{10}{3}mL^2$.

$$f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}}gd}{I}} = \frac{1}{2\pi}\sqrt{\frac{4m_{\text{object}}gL/\sqrt{2}}{\frac{10}{3}m_{\text{object}}L^2}} = \sqrt{\frac{6}{5\sqrt{2}}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) = 0.921\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right).$$

EVALUATE: Just as for a simple pendulum, the frequency is independent of the mass. A simple pendulum of length L has frequency $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ and this object has a frequency that is slightly less than this.

14.74. **IDENTIFY:** Conservation of energy says $K + U = E$.

SET UP: $U = \frac{1}{2}kx^2$ and $E = U_{\text{max}} = \frac{1}{2}kA^2$.

EXECUTE: **(a)** The graph is given in Figure 14.74. The following answers are found algebraically, to be used as a check on the graphical method.

(b) $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.200 \text{ J})}{(10.0 \text{ N/m})}} = 0.200 \text{ m}.$

(c) $\frac{E}{4} = 0.050 \text{ J}.$

(d) $U = \frac{1}{2}E.\ \ x = \frac{A}{\sqrt{2}} = 0.141 \text{ m}.$

(e) From Eq. (14.18), using $v_0 = \sqrt{\frac{2K_0}{m}}$ and $x_0 = -\sqrt{\frac{2U_0}{k}}$, $-\frac{v_0}{\omega x_0} = \frac{\sqrt{(2K_0/m)}}{\sqrt{(k/m)}\sqrt{(2U_0/k)}} = \sqrt{\frac{K_0}{U_0}} = \sqrt{0.429}$

and $\phi = \arctan(\sqrt{0.429}) = 3.72$ rad.

EVALUATE: The dependence of U on x is not linear and $U = \frac{1}{2}U_{\text{max}}$ does not occur at $x = \frac{1}{2}x_{\text{max}}$.

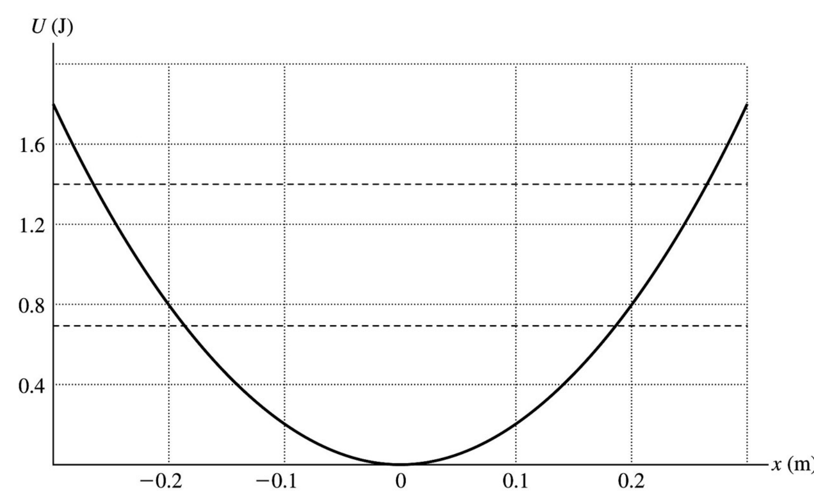


Figure 14.74

14.75. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$ so the period changes because the mass changes.

SET UP: $\dfrac{dm}{dt} = -2.00\times10^{-3}$ kg/s. The rate of change of the period is $\dfrac{dT}{dt}$.

EXECUTE: **(a)** When the bucket is half full, $m = 7.00$ kg. $T = 2\pi\sqrt{\dfrac{7.00\text{ kg}}{450\text{ N/m}}} = 0.784$ s.

(b) $\dfrac{dT}{dt} = \dfrac{2\pi}{\sqrt{k}}\dfrac{d}{dt}(m^{1/2}) = \dfrac{2\pi}{\sqrt{k}}\tfrac{1}{2}m^{-1/2}\dfrac{dm}{dt} = \dfrac{\pi}{\sqrt{mk}}\dfrac{dm}{dt}$.

$\dfrac{dT}{dt} = \dfrac{\pi}{\sqrt{(7.00\text{ kg})(450\text{ N/m})}}(-2.00\times10^{-3}\text{ kg/s}) = -1.12\times10^{-4}$ s per s. $\dfrac{dT}{dt}$ is negative, so the period is getting shorter.

(c) The shortest period is when all the water has leaked out and $m = 2.00$ kg. In that case, $T = 2\pi\sqrt{m/k} = 0.419$ s.

EVALUATE: The rate at which the period changes is not constant but instead increases in time, even though the rate at which the water flows out is constant.

14.76. **IDENTIFY:** We are looking at molecular vibrations of the HI molecule.

SET UP: The classical frequency is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$, where $m \approx m_{\text{proton}} = 1.6726\times10^{-27}$ kg.

EXECUTE: **(a)** We want the force constant of the HI molecule, assuming that only the H atom moves significantly. Solve $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ for k, giving $k = 4\pi^2 mf^2$ from which we have

$k = 4\pi^2(1.6726\times10^{-27}\text{ kg})(7\times10^{13}\text{ Hz})^2$ = 300k N/m.

(b) $E_{\text{vibr}} = \dfrac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2E_{\text{vibr}}}{m}} = \sqrt{\dfrac{2(5\times10^{-20}\text{ J})}{1.6726\times10^{-27}\text{ kg}}} = 7700$ m/s – 8 km/s.

(c) $E = \dfrac{1}{2}kA^2$ so $A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(5\times10^{-20}\text{ J})}{300\text{ N/m}}} \approx 2\times10^{-11}$ m. This result is a little more than 1/10 the equilibrium distance.

EVALUATE: These results are not bad for a rough calculation.

14.77. **IDENTIFY** and **SET UP:** Measure x from the equilibrium position of the object, where the gravity and spring forces balance. Let $+x$ be downward.

(a) Use conservation of energy $\tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2$ to relate v_x and x. Use $T = 2\pi\sqrt{\dfrac{m}{k}}$ to relate T to k/m.

EXECUTE: $\tfrac{1}{2}mv_x^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2$

For $x = 0, \tfrac{1}{2}mv_x^2 = \tfrac{1}{2}kA^2$ and $v = A\sqrt{k/m}$, just as for horizontal SHM. We can use the period to calculate $\sqrt{k/m}: T = 2\pi\sqrt{m/k}$ implies $\sqrt{k/m} = 2\pi/T$. Thus $v = 2\pi A/T = 2\pi(0.100\text{ m})/4.20\text{ s} = 0.150$ m/s.

(b) **IDENTIFY** and **SET UP:** Use $a_x = -\dfrac{k}{m}x$ to relate a_x and x.

EXECUTE: $ma_x = -kx$ so $a_x = -(k/m)x$

$+x$-direction is downward, so here $x = -0.050$ m

$a_x = -(2\pi/T)^2(-0.050\text{ m}) = +(2\pi/4.20\text{ s})^2(0.050\text{ m}) = 0.112\text{ m/s}^2$ (positive, so direction is downward)

(c) **IDENTIFY** and **SET UP:** Use $x = A\cos(\omega t + \phi)$ to relate x and t. The time asked for is twice the time it takes to go from $x = 0$ to $x = +0.050$ m.

EXECUTE: $x(t) = A\cos(\omega t + \phi)$

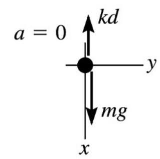
Let $\phi = -\pi/2,$ so $x = 0$ at $t = 0.$ Then $x = A\cos(\omega t - \pi/2) = A\sin\omega t = A\sin(2\pi t/T).$ Find the time t that gives $x = +0.050$ m: $0.050 \text{ m} = (0.100 \text{ m}) \sin(2\pi t/T)$

$2\pi t/T = \arcsin(0.50) = \pi/6$ and $t = T/12 = 4.20 \text{ s}/12 = 0.350$ s

The time asked for in the problem is twice this, 0.700 s.

(d) IDENTIFY: The problem is asking for the distance *d* that the spring stretches when the object hangs at rest from it. Apply Newton's second law to the object.

SET UP: The free-body diagram for the object is given in Figure 14.77.



EXECUTE: $\sum F_x = ma_x$

$mg - kd = 0$

$d = (m/k)g$

Figure 14.77

But $\sqrt{k/m} = 2\pi/T$ (part (a)) and $m/k = (T/2\pi)^2$

$$d = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{4.20 \text{ s}}{2\pi}\right)^2 (9.80 \text{ m/s}^2) = 4.38 \text{ m}.$$

EVALUATE: When the displacement is upward (part (b)), the acceleration is downward. The mass of the partridge is never entered into the calculation. We used just the ratio *k/m,* that is determined from *T.*

14.78. IDENTIFY: The rod is a physical pendulum. We will need to do graphical analysis.

SET UP: The period of a physical pendulum is $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, where *d* is the distance from the rotation axis to the center of gravity of the rod. We want to find I_{cm} about the center of mass of the rod, which is at its center. Therefore we need to derive a relation between I_{cm} and *d* so we can interpret the graph.

EXECUTE: In the formula $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, *I* is the moment of inertia of the rod about the rotation axis, which is a distance *d* from the center of the rod. But we want I_{cm} about the center of gravity of the rod. Using the parallel-axis theorem from Chapter 9, we have $I = I_{cm} + md^2.$ Therefore the period is

$T = 2\pi\sqrt{\dfrac{I_{cm} + md^2}{mgd}}$. Solving for T^2 gives $T^2 = 4\pi^2\left(\dfrac{I_{cm} + md^2}{mgd}\right) = \dfrac{4\pi^2 I_{cm}}{mgd} + \dfrac{4\pi^2 d}{g}$. Rearranging gives

$T^2 - \dfrac{4\pi^2 d}{g} = \left(\dfrac{4\pi^2 I_{cm}}{mg}\right)\dfrac{1}{d}.$ Now we can see that a graph of $T^2 - \dfrac{4\pi^2 d}{g}$ versus $1/d$ should be a straight line having slope equal to $\dfrac{4\pi^2 I_{cm}}{mg}$. Therefore $\dfrac{4\pi^2 I_{cm}}{mg} = \text{slope}$. Solving for I_{cm} gives us

$I_{cm} = mg(\text{slope})/4\pi^2 = (0.400 \text{ kg})(9.80 \text{ m/s}^2)(0.320 \text{ m}\cdot\text{s}^2)/(4\pi^2) = 0.0318 \text{ kg}\cdot\text{m}^2$.

EVALUATE: If this rod were uniform, its moment of inertia about its center of gravity would be

$I = \dfrac{1}{12}ML^2 = \dfrac{1}{12}(0.400 \text{ kg})(0.800 \text{ m})^2 = 0.0213 \text{ kg}\cdot\text{m}^2$. This is less than we found above, so the nonuniform rod must have more mass toward its ends than a uniform rod. Its density must increase with distance from the center.

14.79. **IDENTIFY:** Apply conservation of linear momentum to the collision between the steak and the pan. Then apply conservation of energy to the motion after the collision to find the amplitude of the subsequent SHM. Use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period.

(a) SET UP: First find the speed of the steak just before it strikes the pan. Use a coordinate system with $+y$ downward.

$v_{0y} = 0$ (released from the rest); $y - y_0 = 0.40$ m; $a_y = +9.80$ m/s^2; $v_y = ?$

$v_y^2 = v_{0_y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = +\sqrt{2a_y(y - y_0)} = +\sqrt{2(9.80 \text{ m/s}^2)(0.40 \text{ m})} = +2.80$ m/s

SET UP: Apply conservation of momentum to the collision between the steak and the pan. After the collision the steak and the pan are moving together with common velocity v_2. Let *A* be the steak and *B* be the pan. The system before and after the collision is shown in Figure 14.79.

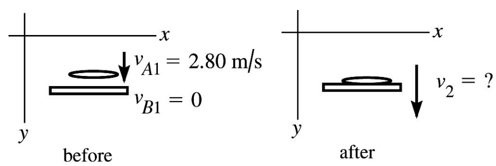


Figure 14.79

EXECUTE: P_y conserved: $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$

$m_A v_{A1} = (m_A + m_B)v_2$

$$v_2 = \left(\frac{m_A}{m_A + m_B}\right)v_{A1} = \left(\frac{2.2 \text{ kg}}{2.2 \text{ kg} + 0.20 \text{ kg}}\right)(2.80 \text{ m/s}) = 2.57 \text{ m/s}$$

(b) SET UP: Conservation of energy applied to the SHM gives: $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ where v_0 and x_0 are the initial speed and displacement of the object and where the displacement is measured from the equilibrium position of the object.

EXECUTE: The weight of the steak will stretch the spring an additional distance *d* given by $kd = mg$ so $d = \frac{mg}{k} = \frac{(2.2 \text{ kg})(9.80 \text{ m/s}^2)}{400 \text{ N/m}} = 0.0539$ m. So just after the steak hits the pan, before the pan has had time to move, the steak plus pan is 0.0539 m above the equilibrium position of the combined object. Thus $x_0 = 0.0539$ m. From part (a) $v_0 = 2.57$ m/s, the speed of the combined object just after the collision. Then $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ gives

$$A = \sqrt{\frac{mv_0^2 + kx_0^2}{k}} = \sqrt{\frac{2.4 \text{ kg}(2.57 \text{ m/s})^2 + (400 \text{ N/m})(0.0539 \text{ m})^2}{400 \text{ N/m}}} = 0.21 \text{ m}$$

(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\frac{2.4 \text{ kg}}{400 \text{ N/m}}} = 0.49$ s

EVALUATE: The amplitude is less than the initial height of the steak above the pan because mechanical energy is lost in the inelastic collision.

14.80. **IDENTIFY:** $F_x = -kx$ allows us to calculate *k*. $T = 2\pi\sqrt{m/k}$. $x(t) = A\cos(\omega t + \phi)$. $F_{\text{net}} = -kx$.

SET UP: Let $\phi = \pi/2$ so $x(t) = A\sin(\omega t)$. At $t = 0$, $x = 0$ and the object is moving downward. When the object is below the equilibrium position, F_{spring} is upward.

EXECUTE: **(a)** Solving $T = 2\pi\sqrt{m/k}$ for m, and using $k = \dfrac{F}{\Delta l}$ gives

$$m = \left(\frac{T}{2\pi}\right)^2 \frac{F}{\Delta l} = \left(\frac{1.00\text{ s}}{2\pi}\right)^2 \frac{40.0\text{ N}}{0.250\text{ m}} = 4.05\text{ kg}.$$

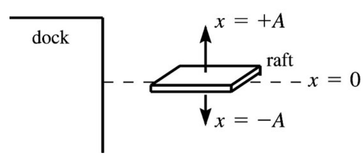
(b) $t = (0.35)T,$ and so $x = -A\sin[2\pi(0.35)] = -0.0405$ m. Since $t > T/4,$ the mass has already passed the lowest point of its motion, and is on the way up.

(c) Taking upward forces to be positive, $F_{\text{spring}} - mg = -kx,$ where x is the displacement from equilibrium, so $F_{\text{spring}} = -(160\text{ N/m})(-0.030\text{ m}) + (4.05\text{ kg})(9.80\text{ m/s}^2) = 44.5\text{ N}.$

EVALUATE: When the object is below the equilibrium position the net force is upward and the upward spring force is larger in magnitude than the downward weight of the object.

14.81. **IDENTIFY**: Use $x = A\cos(\omega t + \phi)$ to relate x and t. $T = 3.5$ s.

SET UP: The motion of the raft is sketched in Figure 14.81.



Let the raft be at $x = +A$ when $t = 0$. Then $\phi = 0$ and $x(t) = A\cos\omega t.$

Figure 14.81

EXECUTE: Calculate the time it takes the raft to move from $x = +A = +0.200$ m to $x = A - 0.100\text{ m} = 0.100$ m.

Write the equation for $x(t)$ in terms of T rather than ω: $\omega = 2\pi/T$ gives that $x(t) = A\cos(2\pi t/T)$

$x = A$ at $t = 0$

$x = 0.100$ m implies $0.100\text{ m} = (0.200\text{ m})\cos(2\pi t/T)$

$\cos(2\pi t/T) = 0.500$ so $2\pi t/T = \arccos(0.500) = 1.047$ rad

$t = (T/2\pi)(1.047\text{ rad}) = (3.5\text{ s}/2\pi)(1.047\text{ rad}) = 0.583$ s

This is the time for the raft to move down from $x = 0.200$ m to $x = 0.100$ m. But people can also get off while the raft is moving up from $x = 0.100$ m to $x = 0.200$ m, so during each period of the motion the time the people have to get off is $2t = 2(0.583\text{ s}) = 1.17$ s.

EVALUATE: The time to go from $x = 0$ to $x = A$ and return is $T/2 = 1.75$ s. The time to go from $x = A/2$ to A and return is less than this.

14.82. **IDENTIFY**: $T = 2\pi/\omega$. $F_r(r) = -kr$ to determine k.

SET UP: Example 13.10 derives $F_r(r) = -\dfrac{GM_{\text{E}}m}{R_{\text{E}}^3}r.$

EXECUTE: $F_r(r) = -\dfrac{GM_{\text{E}}m}{R_{\text{E}}^3}r$ is in the form of $F = -kx$, with x replaced by r, so the motion is simple harmonic. $k = \dfrac{GM_{\text{E}}m}{R_{\text{E}}^3}$. $\omega^2 = \dfrac{k}{m} = \dfrac{GM_{\text{E}}}{R_{\text{E}}^3} = \dfrac{g}{R_{\text{E}}}$. The period is then

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_{\text{E}}}{g}} = 2\pi\sqrt{\frac{6.37\times 10^6\text{ m}}{9.80\text{ m/s}^2}} = 5070\text{ s, or } 84.5\text{ min.}$$

EVALUATE: The period is independent of the mass of the object but does depend on R_{E}, which is also the amplitude of the motion.

14.83. **IDENTIFY**: During the collision, linear momentum is conserved. After the collision, mechanical energy is conserved and the motion is SHM.

SET UP: The linear momentum is $p_x = mv_x$, the kinetic energy is $\frac{1}{2}mv^2$, and the potential energy is $\frac{1}{2}kx^2$. The period is $T = 2\pi\sqrt{\frac{m}{k}}$, which is the target variable.

EXECUTE: Apply conservation of linear momentum to the collision:

$(8.00\times10^{-3}\text{ kg})(280\text{ m/s}) = (1.00\text{ kg})v$. $v = 2.24$ m/s. This is $v_{\max}$ for the SHM. $A = 0.150$ m (given).So $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$. $k = \left(\frac{v_{\max}}{A}\right)^2 m = \left(\frac{2.24\text{ m/s}}{0.150\text{ m}}\right)^2 (1.00\text{ kg}) = 223.0$ N/m.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.00\text{ kg}}{223.0\text{ N/m}}} = 0.421\text{ s.}$$

EVALUATE: This block would weigh about 2 pounds, which is rather heavy, but the spring constant is large enough to keep the period within an easily observable range.

14.84. **IDENTIFY:** Newton's second law, in both its linear and rotational form, applies to this system. The motion is SHM.

SET UP: $\sum F = ma_{\text{cm}}$ and $\sum\tau = I\alpha$, where $I = \frac{2}{5}MR^2$ for a solid sphere, and $R\alpha = a_{\text{cm}}$ with no slipping.

EXECUTE: For each sphere, $f_s R = \left(\frac{2}{5}MR^2\right)\alpha$. $R\alpha = a_{\text{cm}}$. $f_s = \frac{2}{5}Ma_{\text{cm}}$. For the system of two spheres, $2f_s - kx = -2Ma_{\text{cm}}$. $\frac{4}{5}Ma_{\text{cm}} - kx = -2Ma_{\text{cm}}$. $kx = \frac{14}{5}Ma_{\text{cm}}$ and $a_{\text{cm}} = \frac{5}{14}\left(\frac{k}{M}\right)x$. $a_x = -\frac{5}{14}\left(\frac{k}{M}\right)x$.

$a_x = -\omega^2 x$ so $\omega = \sqrt{\frac{5k}{14M}}$. $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{14M}{5k}} = 2\pi\sqrt{\frac{14(0.800\text{ kg})}{5(160\text{ N/m})}} = 0.743$ s.

EVALUATE: If the surface were smooth, there would be no rolling, but the presence of friction provides the torque to cause the spheres to rotate.

14.85. **IDENTIFY**: Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2 g h_0 = \frac{1}{2}m_2 v^2$ and

$$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s.}$$

Momentum conservation during collision: $m_2 v = (m_2 + m_3)V$ and

$$V = \frac{m_2 v}{m_2 + m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s.}$$

Energy conservation during upward swing: $Mgh_f = \frac{1}{2}MV^2$ and

$$h_f = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm.}$$

Figure 14.85 shows how the maximum angular displacement is calculated from h_f. $\cos\theta = \dfrac{48.4\text{ cm}}{50.0\text{ cm}}$ and $\theta = 14.5°$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}} = \dfrac{1}{2\pi}\sqrt{\dfrac{9.80\text{ m/s}^2}{0.500\text{ m}}} = 0.705\text{ Hz}$.

EVALUATE: $14.5° = 0.253$ rad. $\sin(0.253\text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and the equation $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$ is accurate.

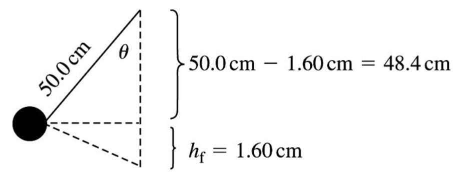


Figure 14.85

14.86. **IDENTIFY** and **SET UP:** $T = 2\pi\sqrt{\dfrac{I}{mgd}}$ gives the period for the bell and $T = 2\pi\sqrt{L/g}$ gives the period for the clapper.

EXECUTE: The bell swings as a physical pendulum so its period of oscillation is given by $T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{18.0\text{ kg}\cdot\text{m}^2/(34.0\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ m})} = 1.885\text{ s}$.

The clapper is a simple pendulum so its period is given by $T = 2\pi\sqrt{L/g}$.

Thus $L = g(T/2\pi)^2 = (9.80\text{ m/s}^2)(1.885\text{ s}/2\pi)^2 = 0.88\text{ m}$.

EVALUATE: If the cm of the bell were at the geometrical center of the bell, the bell would extend 1.20 m from the pivot, so the clapper is well inside the bell.

14.87. **IDENTIFY:** The motion is simple harmonic if the equation of motion for the angular oscillations is of the form $\dfrac{d^2\theta}{dt^2} = -\dfrac{\kappa}{I}\theta$, and in this case the period is $T = 2\pi\sqrt{I/\kappa}$.

SET UP: For a slender rod pivoted about its center, $I = \frac{1}{12}ML^2$.

EXECUTE: The torque on the rod about the pivot is $\tau = -\left(k\dfrac{L}{2}\theta\right)\dfrac{L}{2}$. $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$ gives

$\dfrac{d^2\theta}{dt^2} = -k\dfrac{L^2/4}{I}\theta = -\dfrac{3k}{M}\theta$. $\dfrac{d^2\theta}{dt^2}$ is proportional to θ and the motion is angular SHM. $\dfrac{\kappa}{I} = \dfrac{3k}{M}$, $T = 2\pi\sqrt{\dfrac{M}{3k}}$.

EVALUATE: The expression we used for the torque, $\tau = -\left(k\dfrac{L}{2}\theta\right)\dfrac{L}{2}$, is valid only when θ is small enough for $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.

14.88. **IDENTIFY:** The object oscillates as a physical pendulum, with $f = \dfrac{1}{2\pi}\sqrt{\dfrac{Mgd}{I}}$, where $M = 2m$ is the total mass of the object.

SET UP: The moment of inertia about the pivot is $2(1/3)ML^2 = (2/3)\,ML^2$, and the center of gravity when balanced is a distance $d = L/(2\sqrt{2})$ below the pivot.

EXECUTE: The frequency is $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6g}{4\sqrt{2}L}} = \frac{1}{4\pi}\sqrt{\frac{6g}{\sqrt{2}L}}$.

EVALUATE: If $f_{\text{sp}} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ is the frequency for a simple pendulum of length L,

$f = \frac{1}{2}\sqrt{\frac{6}{\sqrt{2}}}f_{\text{sp}} = 1.03 f_{\text{sp}}$.

14.89. **IDENTIFY:** The velocity is a sinusoidal function. From the graph we can read off the period and use it to calculate the other quantities.

SET UP: The period is the time for 1 cycle; after time T the motion repeats. The graph shows that $T = 1.60$ s and $v_{\text{max}} = 20.0$ cm/s. Mechanical energy is conserved, so $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, and Newton's second law applies to the mass.

EXECUTE: **(a)** $T = 1.60$ s (from the graph with the problem).

(b) $f = \frac{1}{T} = 0.625$ Hz.

(c) $\omega = 2\pi f = 3.93$ rad/s.

(d) $v_x = v_{\text{max}}$ when $x = 0$ so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2$. $A = v_{\text{max}}\sqrt{\frac{m}{k}}$. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ so $A = v_{\text{max}}/(2\pi f)$. From the graph in the problem, $v_{\text{max}} = 0.20$ m/s, so $A = \frac{0.20\text{ m/s}}{2\pi(0.625\text{ Hz})} = 0.051\text{ m} = 5.1\text{ cm}$. The mass is at $x = \pm A$ when $v_x = 0$, and this occurs at $t = 0.4$ s, 1.2 s, and 1.8 s.

(e) Newton's second law gives $-kx = ma_x$, so

$a_{\text{max}} = \frac{kA}{m} = (2\pi f)^2 A = (4\pi^2)(0.625\text{ Hz})^2(0.051\text{ m}) = 0.79\text{ m/s}^2 = 79\text{ cm/s}^2$. The acceleration is maximum when $x = \pm A$ and this occurs at the times given in (d).

(f) $T = 2\pi\sqrt{\frac{m}{k}}$ so $m = k\left(\frac{T}{2\pi}\right)^2 = (75\text{ N/m})\left(\frac{1.60\text{ s}}{2\pi}\right)^2 = 4.9\text{ kg}$.

EVALUATE: The speed is maximum at $x = 0$, when $a_x = 0$. The magnitude of the acceleration is maximum at $x = \pm A$, where $v_x = 0$.

14.90. **IDENTIFY** and **SET UP**: As stated in the problem, $T = 2\pi\sqrt{\frac{m + m_{\text{eff}}}{k}}$.

EXECUTE: **(a)** The graph of T^2 versus m is shown in Figure 14.90. Note that the times given in the table with the problem are for 10 oscillations, so we must divide each of them by 10 to get the period.

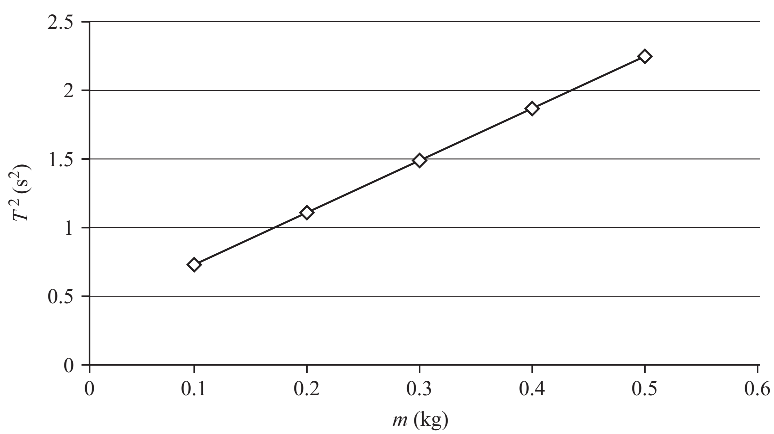


Figure 14.90

(b) Squaring the equation $T = 2\pi\sqrt{\dfrac{m+m_{\text{eff}}}{k}}$ and solving for T^2 in terms of m, we get

$T^2 = \left(\dfrac{4\pi^2}{k}\right)m + \dfrac{4\pi^2 m_{\text{eff}}}{k}.$

In the graph of T^2 versus m, the slope is $4\pi^2/k$ and the vertical intercept is $4\pi^2 m_{\text{eff}}/k$. The best-fit equation to our plotted points is $T^2 = (3.878\ \text{s}^2/\text{kg})m + 0.3492\ \text{s}^2$. Therefore we have
slope = $4\pi^2/k = 3.876\ \text{s}^2/\text{kg}$
$k = 4\pi^2/(3.876\ \text{s}^2/\text{kg}) = 10.19\ \text{kg/s}^2$ which rounds to $10.2\ \text{kg/s}^2 = 10.2$ N/m.
(c) Using the vertical intercept, we have
$4\pi^2 m_{\text{eff}}/k = 0.3492\ \text{s}^2$
$m_{\text{eff}} = (0.3492\ \text{s}^2)(10.19\ \text{kg/s}^2)/(4\pi^2) = 0.0901$ kg.
(d) The mass of the spring is 0.250 kg, so
$m_{\text{eff}}/m_{\text{spring}} = (0.0901\ \text{kg})/(0.250\ \text{kg}) = 0.360$.
Thus m_{eff} is 36% the mass of the spring.
(e) $T = 2\pi\sqrt{\dfrac{m+m_{\text{eff}}}{k}} = 2\pi\sqrt{\dfrac{0.450\ \text{kg}+0.0901\ \text{kg}}{10.19\ \text{kg/s}^2}} = 1.45$ s.
$f = 1/T = 1/(1.45\ \text{s}) = 0.691$ Hz.
$\omega = 2\pi f = 2\pi(0.691\ \text{H}) = 4.34$ rad/s.
EVALUATE: If the mass of a spring is comparable to the mass of the object oscillating from it, the mass of the spring can have a significant effect on the period. The result we found, that the effective mass is 0.36 times the mass of the spring, is in very good agreement with the result of challenge problem 14.93(c), which shows that the effective mass is 1/3 the mass of the spring.

14.91. **IDENTIFY** and **SET UP**: For small-amplitude oscillations, the period of a simple pendulum is $T = 2\pi\sqrt{L/g}$.

EXECUTE: **(a)** The graph of T^2 versus L is shown in Figure 14.91a. Using $T = 2\pi\sqrt{L/g}$, we solve for T^2 in terms of L, which gives $T^2 = \left(\dfrac{4\pi^2}{g}\right)L$. The graph of T^2 versus L should be a straight line having slope $4\pi^2/g$. The best-fit line for our data has the equation $T^2 = (3.9795\ \text{s}^2/\text{m})L + 0.6674\ \text{s}^2$. The quantity $4\pi^2/g = 4\pi^2/(9.80\ \text{m/s}^2) = 4.03\ \text{s}^2/\text{m}$. Our line has slope 3.98 s²/m, which is in very close agreement with the expected slope.

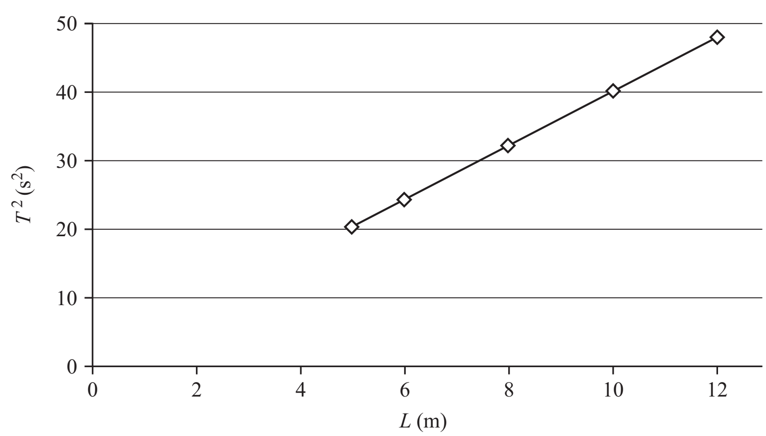


Figure 14.91a

(b) As L decreases, the angle the string makes with the vertical increases because the metal sphere is always released when it is touching the vertical wall. The formula $T = 2\pi\sqrt{L/g}$ is valid only for small angles. Figure 14.91b shows the graph of T/T_0 versus L.

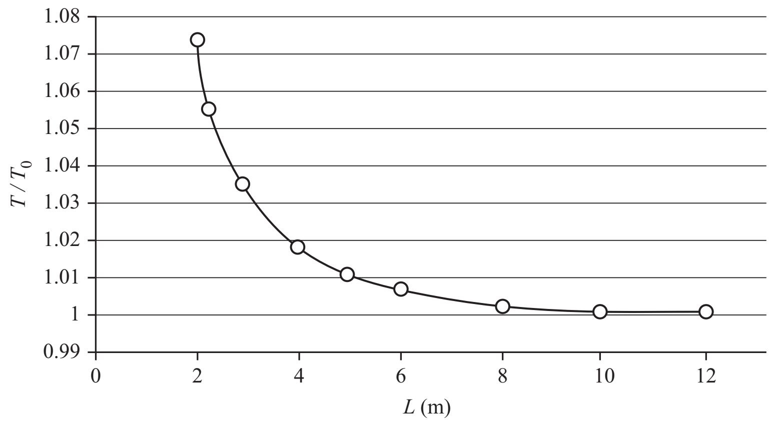


Figure 14.91b

(c) Since $T > T_0$, if T_0 is in error by 5%, $T/T_0 = 1.05$. From the graph in Figure 14.91b, that occurs for $L \approx 2.5$ m. In that case, $\sin\theta = (2.0\text{ m})/(2.5\text{ m}) = 0.80$, which gives $\theta = 53°$.

EVALUATE: Even for an angular amplitude of 53°, the error in using the formula $T = 2\pi\sqrt{L/g}$ is only 5%, so this formula is very useful in most situations. But for very large angular amplitudes it is not reliable.

14.92. IDENTIFY: In each situation, imagine the mass moves a distance Δx, the springs move distances Δx_1 and Δx_2, with forces $F_1 = -k_1\Delta x_1$, $F_2 = -k_2\Delta x_2$.

SET UP: Let Δx_1 and Δx_2 be positive if the springs are stretched, negative if compressed.

EXECUTE: **(a)** $\Delta x = \Delta x_1 = \Delta x_2$, $F = F_1 + F_2 = -(k_1 + k_2)\Delta x$, so $k_{\text{eff}} = k_1 + k_2$.

(b) Despite the orientation of the springs, and the fact that one will be compressed when the other is extended, $\Delta x = \Delta x_1 - \Delta x_2$ and both spring forces are in the same direction. The above result is still valid; $k_{\text{eff}} = k_1 + k_2$.

(c) For massless springs, the force on the block must be equal to the tension in any point of the spring combination, and $F = F_1 = F_2$. $\Delta x_1 = -\dfrac{F}{k_1}, \Delta x_2 == -\dfrac{F}{k_2}$, $\Delta x = -\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right)F = -\dfrac{k_1 + k_2}{k_1 k_2}F$ and

$$k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}.$$

(d) The result of part (c) shows that when a spring is cut in half, the effective spring constant doubles, and so the frequency increases by a factor of $\sqrt{2}$. Therefore $f_1/f_2 = 1/\sqrt{2}$.

EVALUATE: In cases (a) and (b) the effective force constant is greater than either k_1 or k_2 and in case (c) it is less.

14.93. **IDENTIFY**: Follow the procedure specified in the hint.

SET UP: Denote the position of a piece of the spring by l; $l=0$ is the fixed point and $l=L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l)=v\frac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time).

EXECUTE: **(a)** $dm=\frac{M}{L}dl$, and so $dK=\frac{1}{2}dm\,u^2=\frac{1}{2}\frac{Mv^2}{L^3}l^2\,dl$ and $K=\int dK=\frac{Mv^2}{2L^3}\int_0^L l^2\,dl=\frac{Mv^2}{6}$.

(b) $mv\frac{dv}{dt}+kx\frac{dx}{dt}=0$, or $ma+kx=0$, which is Eq. (14.4).

(c) m is replaced by $\frac{M}{3}$, so $\omega=\sqrt{\frac{3k}{M}}$ and $M'=\frac{M}{3}$.

EVALUATE: The effective mass of the spring is only one-third of its actual mass.

14.94. **IDENTIFY** and **SET UP**: The frequency is $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

EXECUTE: Use $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ to solve for the mass:

$m = k/(2\pi f)^2 = (1000\text{ N/m})/[2\pi(100\times10^3\text{ Hz})]^2 = 2.5\times10^{-9}\text{ kg} = 2.5\times10^{-6}\text{ g} = 2.5\ \mu\text{g}$, which is choice (c).

EVALUATE: This is a much smaller mass than we've dealt with in the previous problems, but we are looking at vibrations at the molecular level.

14.95. **IDENTIFY** and **SET UP**: The energy is constant, so it is equal to the potential energy when the speed is zero, so $E=\frac{1}{2}kA^2$.

EXECUTE: $E=\frac{1}{2}(1000\text{ N/m})(0.050\times10^{-9}\text{ m})^2 = 1.25\times10^{-18}\text{ J}$, which is closest to choice (a).

EVALUATE: This is a much smaller energy than we've dealt with in the previous problems, but we are looking at vibrations at the molecular level.

14.96. **IDENTIFY** and **SET UP**: The frequency is $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$, and we want $\frac{\Delta f}{f}$.

EXECUTE: $\frac{\Delta f}{f}=\frac{f-f_0}{f_0}$. Using $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and calling k the new force constant, we have

$$\frac{\Delta f}{f}=\frac{(1/2\pi)\sqrt{k/m}-(1/2\pi)\sqrt{k_0/m}}{(1/2\pi)\sqrt{k_0/m}}=\frac{\sqrt{k}-\sqrt{k_0}}{\sqrt{k_0}}=\sqrt{\frac{k}{k_0}}-1=\sqrt{\frac{1005\text{ N/m}}{1000\text{ N/m}}}-1$$

$\frac{\Delta f}{f}=2.5\times10^{-3}=0.25\%$, which is choice (b).

EVALUATE: Since $k_{\text{surf}}=5$ N/m is only 5/1000 = 0.5% of the original force constant, the effect on the frequency is even less, at 0.25%. This is reasonable because the frequency is proportional to the square root of the force constant.