CSCI-SHU 210 Data Structures

Recitation 3 Recursion

You have a series of tasks in front of you. Complete them! Everyone should code on their own computer, but you are encouraged to talk to others, and seek help from each other and from the TA/LA.

Important:

1. Analyzing the output for recursive programs;
2. Determining the big O complexity for recursive programs;
3. Understand “Break large problem into smaller problems + induction”;
4. Understand what type of problem branching can solve.

Problem 1

Recursion output analysis

What is the output for the following recursive program? Don’t run it, first try to guess.

def f(n):

```
if n > 0:
        f(n-1)
        print(n, end = " ")
        f(n-1)
```

f(4)

Answer: 1 2 1 3 1 2 1 4 1 2 1 3 1 2 1

Problem 2

Determine big-O complexity for the following code snippets:

def func1(N):

```
if N < 1:
        return
else:
        for i in range(5*N):
                print("hi")
        func1(N - 5)
```

Answer: O(N^2)

def func2(N):

```
if n < 1:
        return
else:
        func2(N - 1)
        func2(N - 1)
        for i in range(N):
                print("*")
```

Answer: O(2^N) or O(2^N*N)

Problem 3

Palindrome (Recursive version)

Implement function palindrome( ): this function assesses whether an input String is indeed a palindrome.

Important:

1. Check the string letter by letter, no string.reverse( )
2. Use recursion to break large problem into smaller problem.

```
def palindrome_recursive(string, index):
        """
        # Complete the palindrome algorithm --- with recursion
        # Think about how to break a large problem into smaller sub problems.
        What is our base case in this problem?

        # Another way to ask: what is our smallest problem?
```

```
        How to get to this smallest problem?

        :param string: String -- the string to check whether it is a palindrome
        :param index: Int -- additional parameter for recursion tracking

        :return: True if @string is palindrome, False otherwise
        """
        if index >= len(string) // 2:
                return True
        if string[index] != string[len(string) - index - 1]:
                return False
        return palindrome_recursive(string, index + 1)



def main():
        s1 = "nodevillivedon"
        s2 = "livenoevil!liveonevil"
        s3 = "beliveivileb"
        r1 = palindrome_recursive(s1, 0)
        r2 = palindrome_recursive(s2, 0)
        r3 = palindrome_recursive(s3, 0)
        print("s1 is", r1)    # Should be True
        print("s2 is", r2)    # Should be True
        print("s3 is", r3)    # Should be False

main()
```

```
s1 is True
s2 is True
s3 is False
```

Problem 4

Tower of Hanoi

Your task is to code the famous Tower of Hanoi problem. Complete function hanoi( ), so the disks move correctly when you run the program.

Important:

1. We have graphical demo for this question.
2. Start, goal, mid parameters represent three poles.
3. To move a disk, call function game.move(num_disks, start, goal)
4. Use recursion to break large problem into smaller problem.

```
from tkinter import Tk, Canvas

def hanoi(num_disks, start, goal, mid):
        """
        Implement the tower of hanoi algorithm.
        If implemented correctly, a window should pop up, and disks will move as desir
```

```
        If implemented correctly, a "hint" should pop up, and disks will move as desir
        to move a disk, you can call
        game.move(num_disks, start, goal)

        :param num_disks: number of disks in this function call.
        :param start: Int -- start moving from this pole
        :param goal: Int -- move to this pole
        :param mid: Int -- Intermediate (useless) pole for this move.

        :return: nothing to return
        """
        # Method 1
        if num_disks <= 0:
            return

        hanoi(num_disks-1, start, mid, goal)
        game.move(num_disks, start, goal)
        hanoi(num_disks-1, mid, goal, start)


        # Method 2
        #if num_disks == 1:
                #game.move(num_disks, start, goal)

        #else:
                #hanoi(num_disks-1, start, mid, goal)
                #game.move(num_disks, start, goal)
                #hanoi(num_disks-1, mid, goal, start)



        # Method 3
        #if num_disks == 1:
                #game.move(num_disks, start, goal)
                #return

        #hanoi(num_disks-1, start, mid, goal)
        #game.move(num_disks, start, goal)
        #hanoi(num_disks-1, mid, goal, start)




# The graphical interface
class Tkhanoi:

        # Create our objects
        def __init__(self, n, bitmap = None):
                self.n = n
                self.tk = tk = Tk()
                self.canvas = c = Canvas(tk)
                c.pack()
                width, height = tk.getint(c['width']), tk.getint(c['height'])

                # Add background
                if bitmap:
```

```
                self.bitmap = c.create_bitmap(width//2, height//2,
                                              bitmap=bitmap,
                                              foreground='blu

        # Generate pegs
        pegwidth = 10
        pegheight = height//2
        pegdist = width//3
        x1, y1 = (pegdist-pegwidth)//2, height*1//3
        x2, y2 = x1+pegwidth, y1+pegheight
        self.pegs = []
        p = c.create_rectangle(x1, y1, x2, y2, fill='black')
        self.pegs.append(p)
        x1, x2 = x1+pegdist, x2+pegdist
        p = c.create_rectangle(x1, y1, x2, y2, fill='black')
        self.pegs.append(p)
        x1, x2 = x1+pegdist, x2+pegdist
        p = c.create_rectangle(x1, y1, x2, y2, fill='black')
        self.pegs.append(p)
        self.tk.update()

        # Generate pieces
        pieceheight = pegheight//16
        maxpiecewidth = pegdist*2//3
        minpiecewidth = 2*pegwidth
        self.pegstate = [[], [], []]
        self.pieces = {}
        x1, y1 = (pegdist-maxpiecewidth)//2, y2-pieceheight-2
        x2, y2 = x1+maxpiecewidth, y1+pieceheight
        dx = (maxpiecewidth-minpiecewidth) // (2*max(1, n-1))
        for i in range(n, 0, -1):
            p = c.create_rectangle(x1, y1, x2, y2, fill='red')
            self.pieces[i] = p
            self.pegstate[0].append(i)
            x1, x2 = x1 + dx, x2-dx
            y1, y2 = y1 - pieceheight-2, y2-pieceheight-2
            self.tk.update()
            self.tk.after(25)

    def run(self):
        hanoi(self.n, 0, 2, 1)

    # Reporting callback for the actual hanoi function
    def move(self, i, a, b):
        if self.pegstate[a][-1] != i: raise RuntimeError # Assertion
        del self.pegstate[a][-1]
        p = self.pieces[i]
        c = self.canvas

        # Lift the piece above peg a
        ax1, ay1, ax2, ay2 = c.bbox(self.pegs[a])
        while 1:
            x1, y1, x2, y2 = c.bbox(p)
            if y2 < ay1: break
            c.move(p, 0, -1)
```

```
                self.tk.update()

            # Move it towards peg b
            bx1, by1, bx2, by2 = c.bbox(self.pegs[b])
            newcenter = (bx1+bx2)//2
            while 1:
                x1, y1, x2, y2 = c.bbox(p)
                center = (x1+x2)//2
                if center == newcenter: break
                if center > newcenter: c.move(p, -1, 0)
                else: c.move(p, 1, 0)
                self.tk.update()

            # Move it down on top of the previous piece
            pieceheight = y2-y1
            newbottom = by2 - pieceheight*len(self.pegstate[b]) - 2
            while 1:
                x1, y1, x2, y2 = c.bbox(p)
                if y2 >= newbottom: break
                c.move(p, 0, 1)
                self.tk.update()

            # Update peg state
            self.pegstate[b].append(i)


bitmap = None
game = Tkhanoi(6, bitmap)
game.run()
```

```
---------------------------------------------------------------------------
TclError                                  Traceback (most recent call last)
<ipython-input-2-a82b9b7715a8> in <module>()
    142
    143 bitmap = None
--> 144 game = Tkhanoi(6, bitmap)
    145 game.run()

1 frames
/usr/lib/python3.7/tkinter/__init__.py in __init__(self, screenName, baseName, className,
useTk, sync, use)
   2021                 baseName = baseName + ext
   2022         interactive = 0
-> 2023         self.tk = _tkinter.create(screenName, baseName, className, interactive,
wantobjects, useTk, sync, use)
   2024         if useTk:
   2025             self._loadtk()

TclError: no display name and no $DISPLAY environment variable
```

Problem 5

All Possible Combinations problem

Implement a recursive approach to show all the teams that can be created from a group (out of n things choose k at a time). Implement the recursive showTeams( ), given a group of players, and the size of the team, display all the possible combinations of players.

Important:

1. Combination is different from permutation. This is a combination problem.
2. There are $\frac{n!}{k!(n-k)!}$ combinations (Choose k out of n) [1, 2], [2, 1] are the same combinations.
3. There are $\frac{n!}{(n-k)!}$ permutations (Choose k out of n) [1, 2], [2, 1] are different permutations.
4. Understand what is a help function.

Example Input:

players = [“Dey”, “Ruowen”, “Josh”, “Kinder”, “Mario”, “Rock”, “LOL”] # 7 players

show_team_driver(players, 2) # Choose 2 from 7

Should output:

['Rock', 'LOL'] ['Mario', 'LOL'] ['Mario', 'Rock'] ['Kinder', 'LOL'] ['Kinder', 'Rock'] ['Kinder', 'Mario'] ['Josh', 'LOL'] ['Josh', 'Rock'] ['Josh', 'Mario'] ['Josh', 'Kinder'] ['Ruowen', 'LOL'] ['Ruowen', 'Rock'] ['Ruowen', 'Mario'] ['Ruowen', 'Kinder'] ['Ruowen', 'Josh'] ['Dey', 'LOL'] ['Dey', 'Rock'] ['Dey', 'Mario'] ['Dey', 'Kinder'] ['Dey', 'Josh'] ['Dey', 'Ruowen']

Another example Input:

players = [“Dey”, “Ruowen”, “Josh”, “Kinder”, “Mario”, “Rock”, “LOL”] # 7 players

show_team_driver(players, 4) # Choose 4 from 7

Should output:

['Kinder', 'Mario', 'Rock', 'LOL'] ['Josh', 'Mario', 'Rock', 'LOL'] ['Josh', 'Kinder', 'Rock', 'LOL'] ['Josh', 'Kinder', 'Mario', 'LOL'] ['Josh', 'Kinder', 'Mario', 'Rock'] ['Ruowen', 'Mario', 'Rock', 'LOL'] ['Ruowen', 'Kinder', 'Rock', 'LOL'] ['Ruowen', 'Kinder', 'Mario', 'LOL'] ['Ruowen', 'Kinder', 'Mario', 'Rock'] ['Ruowen', 'Josh', 'Rock', 'LOL'] ['Ruowen', 'Josh', 'Mario', 'LOL'] ['Ruowen', 'Josh', 'Mario', 'Rock'] ['Ruowen', 'Josh', 'Kinder', 'LOL'] ['Ruowen', 'Josh', 'Kinder', 'Rock'] ['Ruowen', 'Josh', 'Kinder', 'Mario'] ['Professor Day', 'Mario', 'Rock', 'LOL'] ['Professor Day', 'Kinder', 'Rock', 'LOL'] ['Professor Day', 'Kinder', 'Mario', 'LOL'] ['Professor Day', 'Kinder', 'Mario', 'Rock'] ['Professor Day', 'Josh', 'Rock', 'LOL'] ['Professor Day', 'Josh', 'Mario', 'LOL'] ['Professor Day', 'Josh', 'Mario', 'Rock'] ['Professor Day', 'Josh', 'Kinder', 'LOL'] ['Professor Day', 'Josh', 'Kinder', 'Rock'] ['Professor Day', 'Josh', 'Kinder', 'Mario'] ['Professor Day',

'Ruowen', 'Rock', 'LOL'] ['Professor Day', 'Ruowen', 'Mario', 'LOL'] ['Professor Day', 'Ruowen', 'Mario', 'Rock'] ['Professor Day', 'Ruowen', 'Kinder', 'LOL'] ['Professor Day', 'Ruowen', 'Kinder', 'Rock'] ['Professor Day', 'Ruowen', 'Kinder', 'Mario'] ['Professor Day', 'Ruowen', 'Josh', 'LOL'] ['Professor Day', 'Ruowen', 'Josh', 'Rock'] ['Professor Day', 'Ruowen', 'Josh', 'Mario'] ['Professor Day', 'Ruowen', 'Josh', 'Kinder']

```
import copy
def show_team(names, team_size):
    help_show_team(names, team_size, [], 0)

def help_show_team(names, team_size, result_list, position):
    """
    # Base case 1: we get enough person in the result_list.
    # Base case 2: we have checked all the players.

    # Create two branches
    # Branch 1 add current person to result_list
    # Branch 2 does not add current person to result_list(copy)
    # Move on to the next person

    :param names: List[String] -- list of players
    :param team_size: Int -- choose how many players
    :param result_list: List[String] -- Additional list parameter for recursion
    :param position: Int -- Additional index parameter for recursion

    :return: Nothing to return
    :print: All the combinations players
    """
    if len(result_list) == team_size:
        print(result_list)
        return
    if position >= len(names):
        return


    result_list_copy = copy.deepcopy(result_list)
    result_list_copy.append(names[position])
    help_show_team(names, team_size, result_list, position + 1)
    help_show_team(names, team_size, result_list_copy, position + 1)


players = ["Dey", "Ruowen", "Josh", "Kinder", "Mario", "Rock", "LOL"]
show_team(players, 2)
```

```
['Rock', 'LOL']
['Mario', 'LOL']
['Mario', 'Rock']
['Kinder', 'LOL']
['Kinder', 'Rock']
['Kinder', 'Mario']
['Josh', 'LOL']
['Josh', 'Rock']
```

```
['Josh', 'Mario']
['Josh', 'Kinder']
['Ruowen', 'LOL']
['Ruowen', 'Rock']
['Ruowen', 'Mario']
['Ruowen', 'Kinder']
['Ruowen', 'Josh']
['Dey', 'LOL']
['Dey', 'Rock']
['Dey', 'Mario']
['Dey', 'Kinder']
['Dey', 'Josh']
['Dey', 'Ruowen']
```

Problem 6

Binary Search

Complete function binary_search( ): this function uses a binary search to determine whether an ordered list contains a specified value. We will implement two versions of binary search:

1. Recursive
2. Iterative

```
import random
def binary_search_rec(x, sorted_list):
    # this function uses binary search to determine whether an ordered array
    # contains a specified value.
    # return True if value x is in the list
    # return False if value x is not in the list
    # If you need, you can use a helper function.
    # TO DO
    return helper_binary_search_rec(x,sorted_list,0,len(sorted_list)-1)

def helper_binary_search_rec(x, sorted_list, low, high):
    # this function uses binary search to determine whether an ordered array
    # contains a specified value.
    # return True if value x is in the list
    # return False if value x is not in the list
    # TO DO
    if (low <= high):
        mid = (low + high) // 2
        if (sorted_list[mid] == x):
            return True
        elif (sorted_list[mid] > x):
            return helper_binary_search_rec(x, sorted_list, low, mid - 1)
        else:
            return helper_binary_search_rec(x, sorted_list, mid + 1, high)
    return False

def binary_search_iter(x, sorted_list):
    # TO DO
    # return True if value x is in the list
```

```
        # return False if value x is not in the list
        low = 0
        high = len(sorted_list)-1
        # TO DO
        # return True if value x is in the list
        # return False if value x is not in the list
        while (low <= high):
                mid = (low + high) // 2
                if (sorted_list[mid] == x):
                        return True
                elif (sorted_list[mid] > x):
                        high = mid - 1
                else:
                        low = mid + 1
        return False

def main():
        sorted_list = []
        for i in range(100):
                sorted_list.append(random.randint(0, 100))
        sorted_list.sort()

        print("Testing recursive binary search ...")
        for i in range(5):
                value = random.randint(0, 100)
                answer = binary_search_rec(value, sorted_list)
                if (answer == True):
                        print("List contains value", value)
                else:
                        print("List does not contain value", value)

        print("Testing iterative binary search ...")
        for i in range(5):
                value = random.randint(0, 100)
                answer = binary_search_iter(value, sorted_list)
                if (answer == True):
                        print("List contains value", value)
                else:
                        print("List does not contain value", value)

main()
```

```
Testing recursive binary search ...
List contains value 99
List does not contain value 100
List contains value 60
List does not contain value 31
List contains value 22
Testing iterative binary search ...
List does not contain value 4
List does not contain value 9
List does not contain value 43
List does not contain value 9
List contains value 18
```

Problem 7 (Optional)

Use Turtle module draw a Tree (**Use recursion**)

Turtle module is a python built in module. Turtle module draws lines by moving the cursor.

For example

import turtle

t = turtle.Turtle() # Initialize the turtle

t.left(30) # The turtle turns left 30 degrees

t.right(30) # The turtle turns right 30 degrees

t.forward(20) # The turtle moves forward 20 pixels, leave a line on the path.

t.backward(30) # The turtle moves backward 30 pixels, leave a line on the path.

… and more! In this recitation, that's all we need."""

With the mind set of recursion, let's break down this problem.

1. Move forward
2. Make a turn, aim to the direction for the first branch
3. Recursion for a smaller problem
4. Make a turn, aim to the direction for the second branch
5. Recursion for a smaller problem
6. Make a turn, aim to the direction for coming back
7. Come back

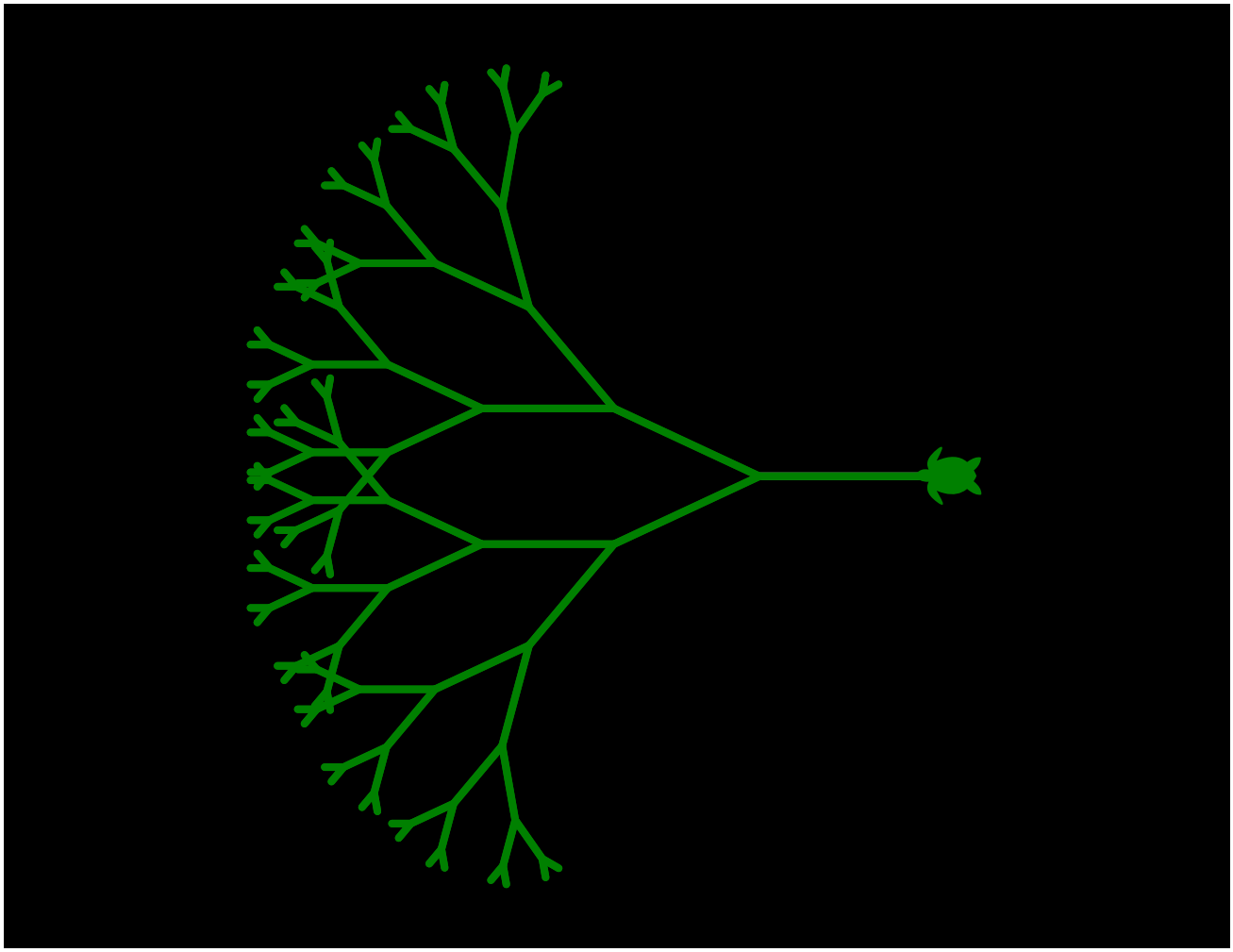
Base case: If the branch is too small, stop.
Otherwise, we should create two new branches (two recursions, two smaller problems)

```
!pip3  install  ColabTurtle
```

```
Collecting ColabTurtle
  Downloading ColabTurtle-2.1.0.tar.gz (6.8 kB)
Building wheels for collected packages: ColabTurtle
  Building wheel for ColabTurtle (setup.py) ... done
  Created wheel for ColabTurtle: filename=ColabTurtle-2.1.0-py3-none-any.whl size=7657 sha256
  Stored in directory: /root/.cache/pip/wheels/0d/ab/65/cc4478508751448dfb4ecb20a6533082855c2
Successfully built ColabTurtle
Installing collected packages: ColabTurtle
Successfully installed ColabTurtle-2.1.0
```

```
from  ColabTurtle  import  Turtle
```

```
from colabturtle import turtle

def draw_tree(branchLen,t):
        """
        Figure out the tree pattern, then display the recursion tree.
        You may have to play/tune with angles/lengths to draw a pretty tree.

        :param branchLen: Int -- Length of this branch. Should reduce every recursion.
        :param t: turtle.Turtle -- Instance of turtle module. We can call turtle functi

        :return: Nothing to return
        """
        pass
        if branchLen <=5:
            return
        t.forward(branchLen)
        t.right(25)
        draw_tree(branchLen-15,t)
        t.left(50)
        draw_tree(branchLen-15,t)
        t.right(25)
        t.backward(branchLen)




def main():
        Turtle.initializeTurtle(initial_speed=10)
        Turtle.left(90)
        Turtle.backward(100)
        Turtle.color("green")
        draw_tree(100,Turtle)        # Drawing tree with branchLen = 100

main()
```



✓ 15 秒 完成时间：17:15