13

GRAVITATION

VP13.5.1. **IDENTIFY:** This problem involves gravitational potential energy.

SET UP: The minimum amount of work is just the magnitude of the spacecraft's initial potential energy at the earth's surface. $U_{grav} = -Gm_1m_2/r$.

EXECUTE: $U_{grav} = -Gm_1m_2/r = -Gm_sm_E/R_E$. The magnitude of this energy is equal to the work, so

$$W = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(478\ \text{kg})\left(5.97\times10^{24}\ \text{kg}\right)}{6.37\times10^6\ \text{m}} = 2.99\times10^{10}\ \text{J}.$$

EVALUATE: Technically this is the energy given to the earth-satellite system, but it essentially all goes to the satellite because the earth is so massive that it does not change its kinetic energy.

VP13.5.2. **IDENTIFY:** This problem involves gravitational potential energy. Energy conservation applies.

SET UP: No air resistance, so $K_1 + U_1 = K_2 + U_2$, $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the debris, R the radius of the earth, and m_E its mass. Call point 1 the original location of the debris and point 2 the surface of the earth.

EXECUTE: $K_1 = 0$, so $-Gmm_E/(3R/2) = -Gmm_E/R + \frac{1}{2}mv^2$, which gives $v = \sqrt{\frac{2Gm_E}{3R}}$. Using $R = 6.37\times10^6$ m and $m_E = 5.97\times10^{24}$ kg gives $v = 6.46\times10^3$ m/s.

EVALUATE: The equation $\sqrt{\frac{2Gm_E}{3R}}$ does not contain the mass of the satellite, so *any* object dropped from the same height would reach the earth's surface with the same speed.

VP13.5.3. **IDENTIFY:** This problem involves gravitational potential energy. Energy conservation applies.

SET UP: No air resistance, so $K_1 + U_1 = K_2 + U_2$, $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the probe, R the radius of Mars, and M its mass. Call point 1 at the surface of Mars and point 2 at the maximum height of the probe. At that point, the probe's distance from the center of Mars is h and its speed is zero. Call v the speed at the surface.

EXECUTE: $K_1 + U_1 = K_2 + U_2$ gives $\frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{h}$. Using $M = 6.42\times10^{23}$ kg, $R = 3.39\times10^6$ m, and $v = 3.00\times10^3$ m/s, we find $h = 5.27\times10^6$ m from the center of Mars. The height H above the surface is $H = h - R = 5.27\times10^6\ \text{m} - 3.39\times10^6\ \text{m} = 1.88\times10^6$ m.

EVALUATE: The answer is independent of the mass of the probe since m cancels from the equations, so *any* probe would reach that height if it started at 3000 m/s.

VP13.5.4. **IDENTIFY:** This problem involves gravitational potential energy. Energy conservation applies.

SET UP: No air resistance, so $K_1 + U_1 = K_2 + U_2$, $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the spacecraft, R the radius of the earth and M its mass. Call point 1 at the surface of the earth and point 2 at the height of the spacecraft when it has reached the desired speed. Call v_1 its initial speed at the surface and v_2 the desired speed when it has reached the desired height r from the earth's center. We want to find v_1.

EXECUTE: Energy conservation gives $\frac{1}{2}mv_1^2 - \frac{GmM}{R} = \frac{1}{2}mv_2^2 - \frac{GmM}{r}$. Using $v_2 = 8.50\times10^3$ m/s, $r = 2.50\times10^8$ m, $M = 5.97\times10^{24}$ kg, and $R = 6.37\times10^6$ m, we get $v_1 = 1.39\times10^4$ m/s = 13.9 km/s.

EVALUATE: Any spacecraft, regardless of mass, would need the same launch speed since the mass m cancels from the equations.

VP13.6.1. **IDENTIFY:** This problem deals with satellite orbits. The force of gravity applies, as well as Newton's second law for circular motion.

SET UP: $F = Gm_1m_2/r^2$. $\sum F = m\frac{v^2}{r}$ for circular motion. Let m be the satellite mass, v its speed, and M the earth's mass. $M = 5.97\times10^{24}$ kg and $v = 4.00\times10^3$ m/s.

EXECUTE: **(a)** $\sum F = m\frac{v^2}{r}$ gives $\frac{GmM}{r^2} = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{GM}{v^2} = 2.49\times10^7$ m.

(b) The height H above the surface is $H = r - R = 2.49\times10^7\text{ m} - 6.37\times10^6\text{ m} = 1.85\times10^7$ m.

(c) $v = 2\pi r/T$, so $T = 2\pi r/v = 2\pi(2.49\times10^7\text{ m})/(4.00\times10^3\text{ m/s}) = 3.91\times10^4\text{ s} = 10.9$ h.

EVALUATE: The results are independent of the mass of the satellite.

VP13.6.2. **IDENTIFY:** This problem deals with satellite orbits. The force of gravity applies, as well as Newton's second law for circular motion.

SET UP: $F = Gm_1m_2/r^2$. $\sum F = m\frac{v^2}{r}$ for circular motion. Let m be the satellite mass, v its speed, T its orbital period, and M the mass of Mars.

EXECUTE: **(a)** We want the radius r of the orbit. $\sum F = m\frac{v^2}{r}$ gives $\frac{GmM}{r^2} = \frac{m\left(\frac{2\pi r}{T}\right)}{r}$. Solving for r gives $r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$. Using $M = 6.42\times10^{23}$ kg and $T = 24.66$ h $= 8.8776\times10^4$ s gives $r = 2.04\times10^7$ m.

(b) Using the $r = 2.04\times10^7$ m and $R = 3.39\times10^6$ m gives $v = 2\pi r/T = 1.45\times10^3$ m/s.

EVALUATE: The orbital radius does not depend on the mass of the satellite, but it *does* depend on the mass of the planet. On the earth the radius would be larger since $r \propto M^{1/3}$.

VP13.6.3. **IDENTIFY:** This problem deals with orbits around the sun and involves gravitational potential energy and kinetic energy. Newton's second law also applies.

SET UP: $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the spacecraft, M the mass of the sun, r_1 the radius of the outer orbit and r_2 that of the inner orbit. Apply $\sum F = m\frac{v^2}{r}$ to the spacecraft.

EXECUTE: **(a)** First find the kinetic energy in both orbits. $\sum F = m\frac{v^2}{r}$ to the spacecraft gives

$\frac{GmM}{r^2} = \frac{mv^2}{r} = \frac{2}{r}\left(\frac{1}{2}mv^2\right) = \frac{2K}{r}$, so $K = \frac{GmM}{2r}$. Therefore change in kinetic energy is

$\Delta K = K_2 - K_1 = \frac{GmM}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$. Using m = 1000 kg, $M = 1.99\times 10^{30}$ kg, $r_1 = 1.50\times 10^{11}$ m, and r_2 = 1.08×10^{11} m, we get $\Delta K = 1.72\times 10^{11}$ J.

(b) $U_2 - U_1 = -\frac{GmM}{r_2} - \left(-\frac{GmM}{r_1}\right) = GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = -3.44\times 10^{11}$ J.

(c) $E = K + U$, so $\Delta E = \Delta K + \Delta U = 1.72\times 10^{11}$ J + (-3.44×10^{11} J) = -1.72×10^{11} J.

EVALUATE: The spacecraft lost twice as much potential energy as it gained in kinetic energy, so it had a net loss equal to the kinetic energy gain.

VP13.6.4 **IDENTIFY:** This problem deals with orbits around the sun and involves gravitational potential energy and kinetic energy. Newton's second law also applies.

SET UP: $U_{grav} = -Gm_1m_2/r$, $K = \frac{1}{2}mv^2$. Let m be the mass of the spacecraft and M the mass of the earth. Apply $\sum F = m\frac{v^2}{r}$ to the spacecraft.

EXECUTE: **(a)** The work you would need to do is equal to the change in energy of the satellite. The potential energy does not change because r is the same, so the change in energy is the change in the kinetic energy of the satellite. Apply $\sum F = m\frac{v^2}{r}$ for a circular orbit.

$6.37\times 10^6\text{ m} = \frac{2}{r}\left(\frac{1}{2}mv^2\right) = \frac{2K}{r}$, which gives $K = \frac{GmM}{2r}$.

$\Delta K = K_2 - K_1 = \frac{GmM}{2r} - \frac{1}{2}mv_1^2$, where v_1 is the initial speed. Using m = 1500 kg, $M = 5.97\times 10^{24}$ kg, v_1 = 7500 m/s, and $r = 3.50\times 10^6$ m + 6.37×10^6 m = 9.87×10^6 m, we find that $\Delta E = \Delta K = -1.19\times 10^{10}$ J.

(b) The minimum energy you would have to give the satellite for it to escape earth would be its total energy, which is $E_1 = K_1 + U_1 = \frac{1}{2}mv_1^2 - \frac{GmM}{r}$. Using the numbers from part (a), this energy is $E_1 = -1.83\times 10^{10}$ J. The work you would have to would be $+1.83\times 10^{10}$ J.

EVALUATE: Orbital parameters such as period, speed, and radius do not depend on the mass of the satellite, but energies do depend on that mass.

VP13.9.1. **IDENTIFY:** We are dealing with a comet in noncircular orbit.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the sun (or central star for another solar system). $a = ea + r_p$, where e is the eccentricity of the orbit. Refer to Fig. 13.18 in the textbook.

EXECUTE: **(a)** We want a, so we use $2a = = r_p + r_a = 6.00\times 10^9$ m + 3.00×10^{12} m, so $a = 1.50\times 10^{12}$ m.

(b) $a = ea + r_p$, so $e = (a - r_p)/a = 1 - r_p/a = 1 - (6.00\times 10^9\text{ m})/(1.50\times 10^{12}\text{ m}) = 0.996$.

(c) Use $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ with a from (a) and $m_S = 1.99\times 10^{30}$ kg gives $T = 1.00\times 10^9$ s = 31.8 y.

EVALUATE: This orbital period is roughly the same as that of Saturn.

VP13.9.2. **IDENTIFY:** We are dealing with an asteroid in a noncircular orbit.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the sun (or central star for another solar system). $a = ea + r_p$, where e is the eccentricity of the orbit. Refer to Fig. 13.18 in the textbook. $T = 7.85$ y = 2.4775×10^8 s, and $e = 0.250$.

EXECUTE: **(a)** Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to find a. Solve for a: $a = \left[\dfrac{T^2 Gm_S}{4\pi^2}\right]^{1/3}$. Using T from above and $m_S = 1.99\times10^{30}$ kg gives $a = 5.91\times10^{11}$ m.

(b) $a = ea + r_p \quad\rightarrow\quad r_p = a(1-e) = (5.91\times10^{11}\text{ m})(1-0.250) = 4.43\times10^{11}$ m.

(c) $r_a = 2a - r_p = 2(5.91\times10^{11}\text{ m}) - 4.43\times10^{11}\text{ m} = 7.39\times10^{11}$ m.

EVALUATE: This orbit is considerably more eccentric than those of the major planets. Its aphelion distance is about 1.7 times greater than its perihelion distance.

VP13.9.3. **IDENTIFY:** We are dealing with a satellite in a noncircular orbit.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the sun (or central star for another solar system). $a = ea + r_p$, where e is the eccentricity of the orbit. Refer to Fig. 13.18 in the textbook. The minor planet makes 2 orbits in the time it takes Neptune to make 3 orbits, so the minor planet must have a longer period than Neptune: $T_{mp} = 3/2\ T_N$, and its eccentricity is $e = 0.330$.

EXECUTE: **(a)** We want to find the semi-major axis a. If we take the ratio of the period of Neptune and minor planet, the factors in common will cancel.

$$\frac{T_{mp}}{T_N} = \frac{\dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}}{\dfrac{2\pi r_N^{3/2}}{\sqrt{Gm_S}}} = \left(\frac{a}{r_N}\right)^{3/2} = \frac{3}{2} \quad\rightarrow\quad (a/r_N)^{3/2} = 3/2 \quad\rightarrow\quad a = r_N(3/2)^{2/3}$$

$a = (4.50\times10^{12}\text{ m})(3/2)^{2/3} = 5.90\times10^{12}$ m.

(b) $a = ea + r_p \quad\rightarrow\quad r_p = a(1-e) = (5.90\times10^{12}\text{ m})(1-0.330) = 3.95\times10^{12}$ m.

EVALUATE: At perihelion, this comet is *inside* Neptune's orbit. The eccentricity of its orbit is much greater than that of Neptune's orbit.

VP13.9.4. **IDENTIFY:** We are dealing with a planet in a noncircular orbit around another star.

SET UP: The semi-major axis a is $2a = r_p + r_a$, where r_p is the perihelion distance and r_a is the aphelion distance. $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, where m_S is the mass of the central star. $a = ea + r_p$, where e is the eccentricity of the orbit. In this case, $T = 4.39$ d $= 3.7930\times10^5$ s. Refer to Fig. 13.18 in the textbook.

EXECUTE: **(a)** We want the distance of the planet from its star at perigee (point of closest approach). $a = ea + r_p \quad\rightarrow\quad r_p = a(1-e) = (7.41\times10^9\text{ m})(1-0.173) = 6.13\times10^9$ m.

(b) We want the mass of HATS-43, so we use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$. Solving for m_S gives $m_S = \dfrac{4\pi^2 a^3}{GT^2}$. Using $a = 7.41\times10^9$ m and $T = 3.793\times10^5$ s, we get $m_S = 1.67\times10^{30}$ kg.

EVALUATE: This planet is considerably closer to its star than Mercury is to our sun. Comparing the mass of this star to that of our sun gives $m_{HATS}/m_{sun} = 1.67/1.99 = 0.839$, so the mass of this star is about 84% the mass of our sun.

13.1. **IDENTIFY** and **SET UP:** Use the law of gravitation, $F_g = \dfrac{Gm_1m_2}{r^2}$, to determine F_g.

EXECUTE: $F_{\text{S on M}} = G\dfrac{m_\text{S}m_\text{M}}{r_\text{SM}^2}$ (S = sun, M = moon); $F_{\text{E on M}} = G\dfrac{m_\text{E}m_\text{M}}{r_\text{EM}^2}$ (E = earth)

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(G\frac{m_\text{S}m_\text{M}}{r_\text{SM}^2}\right)\left(\frac{r_\text{EM}^2}{Gm_\text{E}m_\text{M}}\right) = \frac{m_\text{S}}{m_\text{E}}\left(\frac{r_\text{EM}}{r_\text{SM}}\right)^2$$

r_EM, the radius of the moon's orbit around the earth is given in Appendix F as 3.84×10^8 m. The moon is much closer to the earth than it is to the sun, so take the distance r_SM of the moon from the sun to be r_SE, the radius of the earth's orbit around the sun.

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(\frac{1.99\times10^{30}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{3.84\times10^8\ \text{m}}{1.50\times10^{11}\ \text{m}}\right)^2 = 2.18.$$

EVALUATE: The force exerted by the sun is larger than the force exerted by the earth. The moon's motion is a combination of orbiting the sun and orbiting the earth.

13.2. **IDENTIFY:** We want to calculate the gravitational force between two persons.

SET UP: Estimates: Mass of instructor is 75 kg, your mass is 70 kg, distance between the two of you is 3.0 m. Approximation: Treat both of you as point-masses. Use $F_\text{g} = \dfrac{Gm_1m_2}{r^2}$. We want to approximate the gravitational force that your physics instructor exerts on you.

EXECUTE: $F_\text{instr} = Gm_1m_2/r^2 = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(75\ \text{kg})(70\ \text{kg})/(3.0\ \text{m})^2 = 3.9\times10^{-8}$ N.

$F_\text{earth} = mg = (70\ \text{kg})(9.80\ \text{m/s}^2) = 690$ N.

EVALUATE: $F_\text{instr}/F_\text{earth} = (3.9\times10^{-8}\ \text{N})/(690\ \text{N}) = 5.7\times10^{-11}$ so F_instr is insignificant compared to the force the earth exerts on you.

13.3. **IDENTIFY:** The gravitational attraction of the astronauts on each other causes them to accelerate toward each other, so Newton's second law of motion applies to their motion.

SET UP: The net force on each astronaut is the gravity force exerted by the other astronaut. Call the astronauts *A* and *B*, where $m_A = 65$ kg and $m_B = 72$ kg. $F_\text{g} = Gm_1m_2/r^2$ and $\Sigma F = ma$.

EXECUTE: **(a)** The free-body diagram for astronaut *A* is given in Figure 13.3(a) and for astronaut *B* in Figure 13.3(b) (next page).

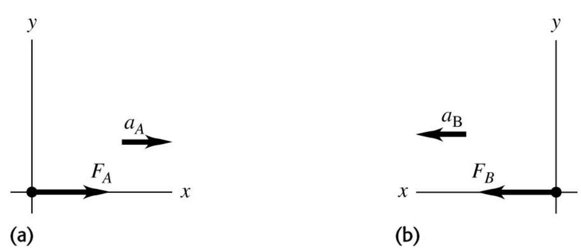


Figure 13.3

$\Sigma F_x = ma_x$ for *A* gives $F_A = m_Aa_A$ and $a_A = \dfrac{F_A}{m_A}$. And for *B*, $a_B = \dfrac{F_B}{m_B}$.

$$F_A = F_B = G\frac{m_Am_B}{r^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(65\ \text{kg})(72\ \text{kg})}{(20.0\ \text{m})^2} = 7.807\times10^{-10}\ \text{N}\ \text{so}$$

$$a_A = \frac{7.807\times10^{-10}\ \text{N}}{65\ \text{kg}} = 1.2\times10^{-11}\ \text{m/s}^2\ \text{and}\ a_B = \frac{7.807\times10^{-10}\ \text{N}}{72\ \text{kg}} = 1.1\times10^{-11}\ \text{m/s}^2.$$

(b) Using constant-acceleration kinematics, we have $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, which gives $x_A = \frac{1}{2}a_A t^2$ and $x_B = \frac{1}{2}a_B t^2$. $x_A + x_B = 20.0\text{ m}$, so $20.0\text{ m} = \frac{1}{2}(a_A + a_B)t^2$ and

$$t = \sqrt{\frac{2(20.0\text{ m})}{1.2\times10^{-11}\text{ m/s}^2 + 1.1\times10^{-11}\text{ m/s}^2}} = 1.32\times10^6\text{ s} = 15\text{ days.}$$

(c) Their accelerations would increase as they moved closer and the gravitational attraction between them increased.

EVALUATE: Even though the gravitational attraction of the astronauts is much weaker than ordinary forces on earth, if it were the only force acting on the astronauts, it would produce noticeable effects.

13.4. **IDENTIFY:** Apply $F_g = \dfrac{Gm_1m_2}{r^2}$, generalized to any pair of spherically symmetric objects.

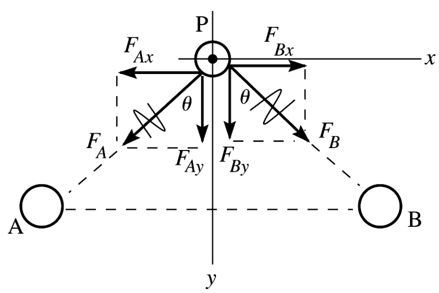
SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: The formula $F_g = \dfrac{Gm_1m_2}{r^2}$ applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

13.5. **IDENTIFY:** Use $F_g = \dfrac{Gm_1m_2}{r^2}$ to find the force exerted by each large sphere. Add these forces as vectors to get the net force and then use Newton's second law to calculate the acceleration.

SET UP: The forces are shown in Figure 13.5.



$\sin\theta = 0.80$
$\cos\theta = 0.60$
Take the origin of coordinate at point P.

Figure 13.5

EXECUTE: $F_A = G\dfrac{m_A m}{r^2} = G\dfrac{(0.26\text{ kg})(0.010\text{ kg})}{(0.100\text{ m})^2} = 1.735\times10^{-11}\text{ N}$

$F_B = G\dfrac{m_B m}{r^2} = 1.735\times10^{-11}\text{ N}$

$F_{Ax} = -F_A\sin\theta = -(1.735\times10^{-11}\text{ N})(0.80) = -1.39\times10^{-11}\text{ N}$

$F_{Ay} = +F_A\cos\theta = +(1.735\times10^{-11}\text{ N})(0.60) = +1.04\times10^{-11}\text{ N}$

$F_{Bx} = +F_B\sin\theta = +1.39\times10^{-11}\text{ N}$

$F_{By} = +F_B\cos\theta = +1.04\times10^{-11}\text{ N}$

$\Sigma F_x = ma_x$ gives $F_{Ax} + F_{Bx} = ma_x$

$0 = ma_x$ so $a_x = 0$

$\Sigma F_y = ma_y$ gives $F_{Ay} + F_{By} = ma_y$

$2(1.04\times10^{-11}\text{ N}) = (0.010\text{ kg})a_y$

$a_y = 2.1\times10^{-9}\text{ m/s}^2$, directed downward midway between A and B

EVALUATE: For ordinary size objects the gravitational force is very small, so the initial acceleration is very small. By symmetry there is no x-component of net force and the y-component is in the direction of the two large spheres, since they attract the small sphere.

13.6. IDENTIFY: The net force on A is the vector sum of the force due to B and the force due to C. In part (a), the two forces are in the same direction, but in (b) they are in opposite directions.

SET UP: Use coordinates where $+x$ is to the right. Each gravitational force is attractive, so is toward the mass exerting it. Treat the masses as uniform spheres, so the gravitational force is the same as for point masses with the same center-to-center distances. The free-body diagrams for (a) and (b) are given in Figures 13.6a and 13.6b. The gravitational force is $F_g = Gm_1m_2/r^2$.

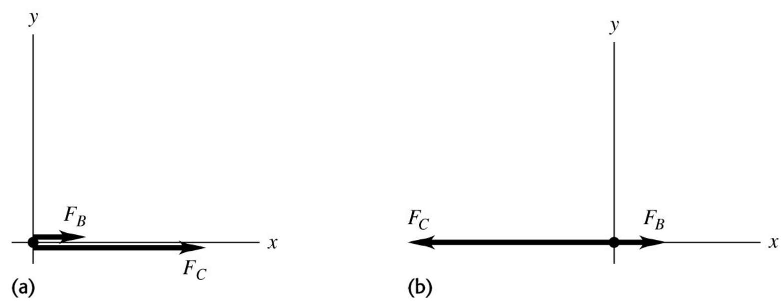


Figure 13.6

EXECUTE: (a) Calling F_B the force due to mass B and likewise for C, we have

$F_B = G\dfrac{m_Am_B}{r_{AB}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(2.00\ \text{kg})^2}{(0.50\ \text{m})^2} = 1.069\times10^{-9}\ \text{N}$ and

$F_C = G\dfrac{m_Am_C}{r_{AC}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(2.00\ \text{kg})^2}{(0.10\ \text{m})^2} = 2.669\times10^{-8}\ \text{N}.$ The net force is

$F_{\text{net},x} = F_{Bx} + F_{Cx} = 1.069\times10^{-9}\ \text{N} + 2.669\times10^{-8}\ \text{N} = 2.8\times10^{-8}\ \text{N}$ to the right.

(b) Following the same procedure as in (a), we have

$F_B = G\dfrac{m_Am_B}{r_{AB}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(2.00\ \text{kg})^2}{(0.40\ \text{m})^2} = 1.668\times10^{-9}\ \text{N}$

$F_C = G\dfrac{m_Am_C}{r_{AC}^2} = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(2.00\ \text{kg})^2}{(0.10\ \text{m})^2} = 2.669\times10^{-8}\ \text{N}$

$F_{\text{net},x} = F_{Bx} + F_{Cx} = 1.668\times10^{-9}\ \text{N} - 2.669\times10^{-8}\ \text{N} = -2.5\times10^{-8}\ \text{N}$

The net force on A is $2.5\times10^{-8}\ \text{N}$, to the left.

EVALUATE: As with any force, the gravitational force is a vector and must be treated like all other vectors. The formula $F_g = Gm_1m_2/r^2$ only gives the magnitude of this force.

13.7. IDENTIFY: The force exerted by the moon is the gravitational force, $F_g = \dfrac{Gm_Mm}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35\times10^{22}$ kg. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: (a) $F_{\text{moon}} = F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(7.35\times10^{22}\ \text{kg})(70\ \text{kg})}{(3.78\times10^8\ \text{m})^2} = 2.4\times10^{-3}\ \text{N}.$

(b) $F_{\text{earth}} = w = (70\ \text{kg})(9.80\ \text{m/s}^2) = 690\ \text{N}.$ $F_{\text{moon}}/F_{\text{earth}} = 3.5\times10^{-6}.$

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

13.8. **IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to find the force each point mass exerts on the particle, find the net force, and use Newton's second law to calculate the acceleration.

SET UP: Each force is attractive. The particle (mass m) is a distance $r_1 = 0.200$ m from $m_1 = 8.00$ kg and therefore a distance $r_2 = 0.300$ m from $m_2 = 12.0$ kg. Let $+x$ be toward the 12.0-kg mass.

EXECUTE: $F_1 = \dfrac{Gm_1m}{r_1^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(8.00\ \text{kg})m}{(0.200\ \text{m})^2} = (1.334\times10^{-8}\ \text{N/kg})m,$ in the $-x$-direction. $F_2 = \dfrac{Gm_2m}{r_2^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(12.0\ \text{kg})m}{(0.300\ \text{m})^2} = (8.893\times10^{-9}\ \text{N/kg})m,$ in the $+x$-direction. The net force is

$F_x = F_{1x} + F_{2x} = (-1.334\times10^{-8}\ \text{N/kg} + 8.893\times10^{-9}\ \text{N/kg})m = (-4.447\times10^{-9}\ \text{N/kg})m.$

$a_x = \dfrac{F_x}{m} = -4.45\times10^{-9}\ \text{m/s}^2.$ The acceleration is $4.45\times10^{-9}\ \text{m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

13.9. **IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.

SET UP: For the net force to be zero, the two forces on M must be in opposite directions. This is the case only when M is on the line connecting the two particles and between them. The free-body diagram for M is given in Figure 13.9. $m_1 = 3m$ and $m_2 = m$. If M is a distance x from m_1, it is a distance $1.00\ \text{m} - x$ from m_2.

EXECUTE: **(a)** $F_x = F_{1x} + F_{2x} = -G\dfrac{3mM}{x^2} + G\dfrac{mM}{(1.00\ \text{m} - x)^2} = 0.$ Cancelling and simplifying gives $3(1.00\ \text{m} - x)^2 = x^2$. Taking square roots gives $1.00\ \text{m} - x = \pm x/\sqrt{3}$. Since M is between the two particles, x must be less than 1.00 m and $x = \dfrac{1.00\ \text{m}}{1 + 1/\sqrt{3}} = 0.634$ m. M must be placed at a point that is 0.634 m from the particle of mass $3m$ and 0.366 m from the particle of mass m.

(b) (i) If M is displaced slightly to the right in Figure 13.9, the attractive force from m is larger than the force from $3m$ and the net force is to the right. If M is displaced slightly to the left in Figure 13.9, the attractive force from $3m$ is larger than the force from m and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.

(ii) If M is displaced a very small distance along the y-axis in Figure 13.9, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on M is zero is closer to the smaller mass.

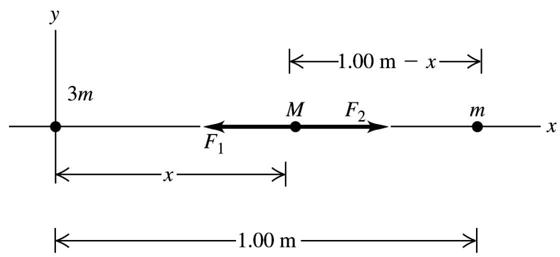


Figure 13.9

13.10. **IDENTIFY:** The force $\vec{F}_1$ exerted by m on M and the force $\vec{F}_2$ exerted by $2m$ on M are each given by $F_g = Gm_1m_2/r^2$ and the net force is the vector sum of these two forces.

SET UP: Each force is attractive. The forces on M in each region are sketched in Figure 13.10a. Let M be at coordinate x on the x-axis.

EXECUTE: **(a)** For the net force to be zero, $\vec{F}_1$ and $\vec{F}_2$ must be in opposite directions and this is the case only for $0 < x < L$. $\vec{F}_1 + \vec{F}_2 = 0$ then requires $F_1 = F_2$. $\dfrac{GmM}{x^2} = \dfrac{G(2m)M}{(L-x)^2}$. $2x^2 = (L-x)^2$ and $L - x = \pm\sqrt{2}x$. x must be less than L, so $x = \dfrac{L}{1+\sqrt{2}} = 0.414L$.

(b) For $x < 0$, $F_x > 0$. $F_x \to 0$ as $x \to -\infty$ and $F_x \to +\infty$ as $x \to 0$. For $x > L$, $F_x < 0$. as $x \to \infty$ and $F_x \to -\infty$ as $x \to L$. For $0 < x < 0.414L$, $F_x < 0$ and F_x increases from $-\infty$ to 0 as x goes from 0 to $0.414L$. For $0.414L < x < L$, $F_x > 0$ and F_x increases from 0 to $+\infty$ as x goes from $0.414L$ to L. The graph of F_x versus x is sketched in Figure 13.10b (next page).

EVALUATE: Any real object is not exactly a point so it is not possible to have both m and M exactly at $x = 0$ or $2m$ and M both exactly at $x = L$. But the magnitude of the gravitational force between two objects approaches infinity as the objects get very close together.

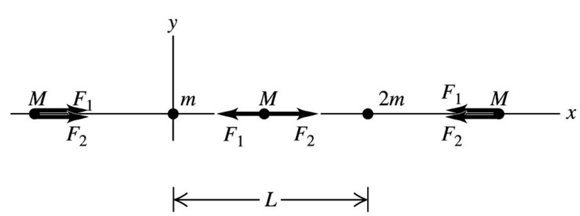


Figure 13.10a

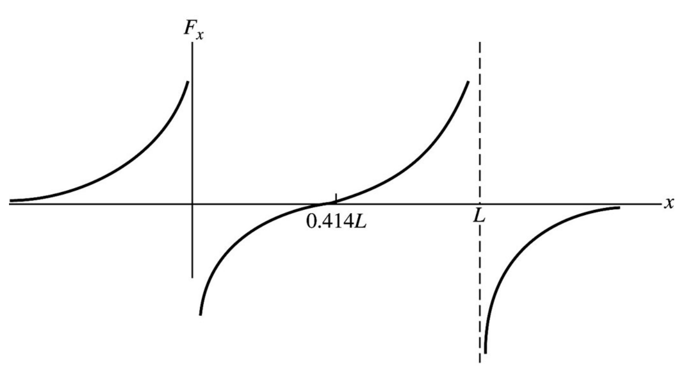


Figure 13.10b

13.11. IDENTIFY: $F_g = G\dfrac{mm_E}{r^2} = mg,$ so ssc where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E,$ where h is the distance of the object above the surface of the earth and $R_E = 6.37\times10^6$ m is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10},$ and so the distance above the surface of the earth is $(\sqrt{10}-1)R_E = 1.38\times10^7$ m.

EVALUATE: This height is about twice the radius of the earth.

13.12. IDENTIFY: Apply $g = G\dfrac{m_E}{r^2}$ to the earth and to Venus. $w = mg.$

SET UP: $g = \dfrac{Gm_E}{R_E^2} = 9.80\text{ m/s}^2.$ $m_V = 0.815m_E$ and $R_V = 0.949R_E.$ $w_E = mg_E = 75.0$ N.

EXECUTE: **(a)** $g_V = \dfrac{Gm_V}{R_V^2} = \dfrac{G(0.815m_E)}{(0.949R_E)^2} = 0.905\dfrac{Gm_E}{R_E^2} = 0.905g_E.$

(b) $w_V = mg_V = 0.905mg_E = (0.905)(75.0\text{ N}) = 67.9$ N.

EVALUATE: The mass of the rock is independent of its location but its weight equals the gravitational force on it and that depends on its location.

13.13. (a) IDENTIFY and **SET UP:** Apply $g = G\dfrac{m_E}{r^2}$ to the earth and to Titania. The acceleration due to gravity at the surface of Titania is given by $g_T = Gm_T/R_T^2,$ where m_T is its mass and R_T is its radius. For the earth, $g_E = Gm_E/R_E^2.$

EXECUTE: For Titania, $m_T = m_E/1700$ and $R_T = R_E/8,$ so

$$g_T = \frac{Gm_T}{R_T^2} = \frac{G(m_E/1700)}{(R_E/8)^2} = \left(\frac{64}{1700}\right)\frac{Gm_E}{R_E^2} = 0.0377g_E.$$

Since $g_E = 9.80\text{ m/s}^2,$ $g_T = (0.0377)(9.80\text{ m/s}^2) = 0.37\text{ m/s}^2.$

EVALUATE: g on Titania is much smaller than on earth. The smaller mass reduces g and is a greater effect than the smaller radius, which increases g.

(b) IDENTIFY and **SET UP:** Use density = mass/volume. Assume Titania is a sphere.

EXECUTE: From Section 13.2 we know that the average density of the earth is 5500 kg/m^3. For Titania

$$\rho_T = \frac{m_T}{\frac{4}{3}\pi R_T^3} = \frac{m_E/1700}{\frac{4}{3}\pi(R_E/8)^3} = \frac{512}{1700}\rho_E = \frac{512}{1700}(5500\text{ kg/m}^3) = 1700\text{ kg/m}^3.$$

EVALUATE: The average density of Titania is about a factor of 3 smaller than for earth. We can write $a_g = G\dfrac{m_E}{r^2}$ for Titania as $g_T = \frac{4}{3}\pi G R_T \rho_T.$ $g_T < g_E$ both because $\rho_T < \rho_E$ and $R_T < R_E.$

13.14. IDENTIFY: Apply $g = G\dfrac{m_E}{r^2}$ to Rhea.

SET UP: $\rho = m/V.$ The volume of a sphere is $V = \frac{4}{3}\pi R^3.$

EXECUTE: $M = \dfrac{gR^2}{G} = 2.32\times10^{21}$ kg and $\rho = \dfrac{M}{(4\pi/3)R^3} = 1.24\times10^3\text{ kg/m}^3.$

EVALUATE: The average density of Rhea is a bit less than one-fourth that of the earth.

13.15. IDENTIFY: We are dealing with the acceleration due to gravity on another planet.

SET UP: We want to find g at the surface of a planet. At the surface, $g = GM/R^2$. Since we know the density ρ of the planet, we should put g in terms of ρ using $\rho = m/V$.

EXECUTE: **(a)** We want g at the surface of this planet. Using $m = \rho V = \rho(4/3\ \pi R^3)$, we have

$$g = \frac{Gm}{R^2} = \frac{G\rho\left(\frac{4}{3}\pi R^3\right)}{R^2} = \frac{4}{3}\pi G\rho R.$$ Now take the ratio of $g_{\text{planet}}/g_{\text{earth}}$ using the same density for both planets but $R = 1.25R_{\text{earth}}$. We get $\dfrac{g_p}{g_e} = \dfrac{\frac{4}{3}\pi G\rho R_p}{\frac{4}{3}\pi G\rho R_e} = \dfrac{R_p}{R_e} = \dfrac{1.25R_e}{R_e} = 1.25$, from which we get $g_p = 1.25$ $g_e = (1.25)(9.80\ \text{m/s}^2) = 12.3\ \text{m/s}^2$.

(b) We now want to change the density of this planet (a neat trick if you can do it!) so that g at its surface is the same as on earth. Use the result we found in part (a) for g: $g = \frac{4}{3}\pi G\rho R.$ Since $g_p = g_e$, we equate the two expressions for g, giving $\frac{4}{3}\pi G\rho_p R_p = \frac{4}{3}\pi G\rho_e R_e$. Solving for ρ_p gives

$$\rho_p = \frac{R_e}{R_p}\rho_e = \frac{R_e}{1.25R_e}\rho_e = 0.800\rho_e.$$ The planet should have 80.0% the density of earth.

EVALUATE: The planet is less dense than earth but it has more mass since it is larger, so g at its surface is the same as it is on earth.

13.16. **IDENTIFY:** The gravity of Io limits the height to which volcanic material will rise. The acceleration due to gravity at the surface of Io depends on its mass and radius.

SET UP: The radius of Io is $R = 1.821\times10^6$ m. Use coordinates where $+y$ is upward. At the maximum height, $v_{0y} = 0$, $a_y = -g_{\text{Io}}$, which is assumed to be constant. Therefore the constant-acceleration kinematics formulas apply. The acceleration due to gravity at Io's surface is given by $g_{\text{Io}} = Gm/R^2$.

EXECUTE: At the surface of Io, $g_{\text{Io}} = \dfrac{Gm}{R^2} = \dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.93\times10^{22}\ \text{kg})}{(1.821\times10^6\ \text{m})^2} = 1.797\ \text{m/s}^2$.

For constant acceleration (assumed), the equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ applies, so

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-1.797\ \text{m/s}^2)(5.00\times10^5\ \text{m})} = 1.3405\times10^3$ m/s. Now solve for $y - y_0$ when $v_{0y} = 1.3405\times10^3$ m/s and $a_y = -9.80\ \text{m/s}^2$. The equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{-(1.3405\times10^3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 9.17\times10^4\ \text{m} = 91.7\ \text{km}.$$

EVALUATE: Even though the mass of Io is around 100 times smaller than that of the earth, the acceleration due to gravity at its surface is only about 1/6 of that of the earth because Io's radius is much smaller than earth's radius.

13.17. **IDENTIFY:** The escape speed, as shown in Example 13.5, is $\sqrt{2GM/R}$.

SET UP: For Mars, $M = 6.42\times10^{23}$ kg and $R = 3.39\times10^6$ m. For Jupiter, $M = 1.90\times10^{27}$ kg and $R = 6.99\times10^7$ m.

EXECUTE: **(a)** $v = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})/(3.39\times10^6\ \text{m})} = 5.03\times10^3$ m/s.

(b) $v = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2(1.90\times10^{27}\ \text{kg})/(6.99\times10^7\ \text{m})} = 6.02\times10^4$ m/s.

(c) Both the kinetic energy and the gravitational potential energy are proportional to the mass of the spacecraft.

EVALUATE: Example 13.5 calculates the escape speed for earth to be 1.12×10^4 m/s. This is larger than our result for Mars and less than our result for Jupiter.

13.18. **IDENTIFY:** The kinetic energy is $K = \frac{1}{2}mv^2$ and the potential energy is $U = -\dfrac{GMm}{r}$.

SET UP: The mass of the earth is $M_E = 5.97 \times 10^{24}$ kg.

EXECUTE: **(a)** $K = \frac{1}{2}(629 \text{ kg})(3.33 \times 10^3 \text{ m/s})^2 = 3.49 \times 10^9$ J

(b) $U = -\dfrac{GM_E m}{r} = -\dfrac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(629 \text{ kg})}{2.87 \times 10^9 \text{ m}} = -8.73 \times 10^7$ J.

EVALUATE: The total energy $K + U$ is positive.

13.19. **IDENTIFY:** Mechanical energy is conserved. At the escape speed, the object has no kinetic energy when it is very far away from the planet.

SET UP: Call m the mass of the object, M the mass of the planet, and r its radius. $K_1 + U_1 = K_2 + U_2$, $K = ½ mv^2$, $U = –GmM/r$, $g = GM/r^2$.

EXECUTE: Energy conservation gives $½ mv^2 – GmM/r = 0 + 0$. $M = rv^2/2G$. Putting this into $g = GM/r^2$ gives $g = \dfrac{G\left(\dfrac{rv^2}{2G}\right)}{r^2} = \dfrac{v^2}{2r}$. Putting in the numbers gives

$g = (7.65 \times 10^3 \text{ m/s})^2/[2(3.24 \times 10^6 \text{ m})] = 9.03 \text{ m/s}^2$.

EVALUATE: This result is not very different from g on earth, so it is physically reasonable for a planet.

13.20. **IDENTIFY:** This problem involves gravitational potential energy.

SET UP: Estimate: Altitude is h = 30,000 ft (≈10,000 m) above the earth's surface. We want to find the percent change in the gravitational potential energy of the system (you and the earth) when you are at this altitude compared to when you are at the surface. Use $U = -\dfrac{Gm_E m}{r}$, where m is your mass.

EXECUTE: At the surface: $U_s = -\dfrac{Gm_E m}{R}$, and at altitude h: $U_h = -\dfrac{Gm_E m}{R_E + h}$. The fractional change in potential energy is $\dfrac{\Delta U}{U_s} = \dfrac{U_h - U_s}{U_s} = \dfrac{U_h}{U_s} - 1$. Using the expressions for U_h and U_s gives

$$\frac{\Delta U}{U_s} = \frac{-\dfrac{Gm_E m}{R_E + h}}{-\dfrac{Gm_E m}{R_E}} - 1 = \frac{R_E}{R_E + h} - 1 = \frac{1}{1 + h/R_E} - 1 = \left(1 + h/R_E\right)^{-1} - 1.$$ From Appendix B, we have

$(1+x)^n = 1^n + n1^{n-1}x + \dfrac{n(n-1)1^{n-2}}{2!}x^2 + \cdots$. If $x << 1$, we can neglect all the x^2 and higher terms, so for $n = –1$, we have $(1 + x)^{-1} \approx 1 – x$. In our case, $x = h/R_E << 1$, so $\dfrac{1}{1 + h/R_E} \approx 1 - \dfrac{h}{R_E}$, which yields

$\dfrac{\Delta U}{U_s} \approx \left(1 - \dfrac{h}{R_E}\right) - 1 \approx -\dfrac{h}{R_E}$. This gives $\dfrac{\Delta U}{U_s} \approx -\dfrac{10{,}000 \text{ m}}{6.37 \times 10^6 \text{ m}} \approx -1.6 \times 10^{-3}$. This result gives us only the *magnitude* of the change. Since U is negative, its magnitude decreases with altitude, but it is becoming *less negative*, so it is actually increasing. So the gravitational potential energy is 0.16% greater when you are in the plane.

EVALUATE: Fractional changes in gravitational potential energy only become apparent when $\Delta r / r$ is fairly large.

13.21. **IDENTIFY:** This problem involves the gravitational force and gravitational potential energy.

SET UP: $F_g = \dfrac{Gm_1 m_2}{r^2}$ and $U_g = -\dfrac{Gm_p m}{r}$. We know U_g and want to find the force F_g on you at the surface of the planet when $r = R$.

EXECUTE: Relate F_g and U_g: $F_g = \dfrac{Gmm_p}{R^2} = \left(\dfrac{Gmm_p}{R}\right)\dfrac{1}{R} = -\dfrac{U_g}{R} = -\dfrac{-1.20\times10^9 \text{ J}}{5.00\times10^6 \text{ m}} = 240 \text{ N}.$

EVALUATE: Knowing your mass, we could find g at the surface.

13.22. IDENTIFY: The satellite is in orbit, so we need to use Newton's second law, the gravitational force, and gravitational potential energy.

SET UP: Use $\sum F = m\dfrac{v^2}{R}$, $F_g = \dfrac{Gm_1m_2}{r^2}$, and $U_g = -\dfrac{Gm_pm}{r}$. We want to find r_B so that $K_B = 2K_A$.

EXECUTE: $\sum F = m\dfrac{v^2}{R}$ gives $\dfrac{Gmm_p}{r^2} = \dfrac{mv^2}{r} = \dfrac{2}{r}\left(\dfrac{1}{2}mv^2\right) = \dfrac{2K}{r}$, so $K = \dfrac{Gmm_p}{2r}$.

At A: $K_A = \dfrac{Gmm_p}{2r_A}$

At B: $K_B = \dfrac{Gmm_p}{2r_B} = 2K_A = \dfrac{2Gmm_p}{2r_A}$.

Solving the last equation for r_B gives $r_B = r_A/2$.

EVALUATE: It is reasonable that $r_B < r_A$ since when it is closer to the planet, the satellite must move faster to remain in orbit.

13.23. IDENTIFY: This problem involves the total energy of the earth-satellite system.

SET UP: Eq. 13.13: $E = -\dfrac{Gm_Em}{2r}$, where E is the total energy of the system. Also use $\sum F = m\dfrac{v^2}{R}$, $F_g = \dfrac{Gm_1m_2}{r^2}$, and $U_g = -\dfrac{Gm_pm}{r}$.

EXECUTE: $\sum F = m\dfrac{v^2}{R} = \dfrac{Gmm_E}{r^2}$ gives $v^2 = \dfrac{Gm_E}{r}$, so $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{Gm_E}{r}\right) = \dfrac{Gmm_E}{2r} =$ 2.00×10^6 J. Using Eq. 13.13 gives $E = -\dfrac{Gm_Em}{2r} = -K = -2.00\times10^6$ J. The total energy is $E = K + U$, so $U = E - K = -2.00\times10^6 \text{ J} - 2.00\times10^6 \text{ J} = -4.00\times10^6$ J.

EVALUATE: Note that $K = -\dfrac{1}{2}U$, which is a general result for orbits.

13.24. IDENTIFY: Newton's second law and his law of gravitation both apply to the satellite.

SET UP: $T = \dfrac{2\pi r}{v}$. r and v are also related by applying $\sum \vec{F} = m\vec{a}$ to the motion of the satellite. The satellite has $a_{rad} = v^2/R$, and the only force on the satellite is the gravitational force, $F_g = G\dfrac{m_Em}{r^2}$. $m_E = 5.97\times10^{24}$ kg.

EXECUTE: **(a)** The free-body diagram of the satellite is shown in Figure 13.24. $F_g = ma_{rad}$ gives $G\dfrac{m_Em}{r^2} = m\dfrac{v^2}{r}$, which simplifies to $G\dfrac{m_E}{r} = v^2$. Solving for r gives

$$r = \frac{Gm_E}{v^2} = \frac{(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24} \text{ kg})}{(6200 \text{ m/s})^2} = 1.04\times10^7 \text{ m}.$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.04\times10^7 \text{ m})}{6200 \text{ m/s}} = 1.05\times10^4 \text{ s} = 176 \text{ min} = 2.93 \text{ h}.$$

(b) $a_{rad} = \dfrac{v^2}{r} = \dfrac{(6200 \text{ m/s})^2}{1.04\times10^7 \text{ m}} = 3.70 \text{ m/s}^2.$

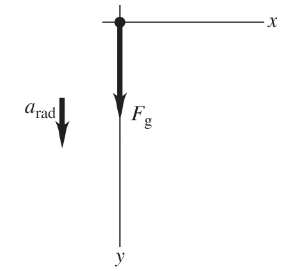
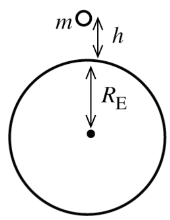


Figure 13.24

EVALUATE: The acceleration of the satellite is 38% of the acceleration due to gravity at the earth's surface.

13.25. **IDENTIFY:** Apply Newton's second law to the motion of the satellite and obtain an equation that relates the orbital speed v to the orbital radius r.

SET UP: The distances are shown in Figure 13.25a.

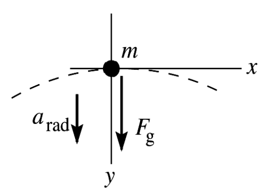


The radius of the orbit is $r = h + R_E$.

$r = 8.90\times10^5\ \text{m} + 6.37\times10^6\ \text{m} = 7.26\times10^6\ \text{m}$.

Figure 13.25a

The free-body diagram for the satellite is given in Figure 13.25b.



(a) EXECUTE: $\Sigma F_y = ma_y$

$F_g = ma_{rad}$

$$G\frac{mm_E}{r^2} = m\frac{v^2}{r}$$

Figure 13.25b

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{7.26\times10^6\ \text{m}}} = 7.408\times10^3\ \text{m/s}\ \text{ which rounds to 7410 m/s.}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(7.26\times10^6\ \text{m})}{7.408\times10^3\ \text{m/s}} = 6158\ \text{s} = 1.71\ \text{h}.$

EVALUATE: Note that $r = h + R_E$ is the radius of the orbit, measured from the center of the earth. For this satellite r is greater than for the satellite in Example 13.6, so its orbital speed is less.

13.26. **IDENTIFY:** The time to complete one orbit is the period T, given by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$. The speed v of the satellite is given by $v = \dfrac{2\pi r}{T}$.

SET UP: If h is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_E$. $R_E = 6.37\times10^6\ \text{m}$ and $m_E = 5.97\times10^{24}\ \text{kg}$.

EXECUTE: **(a)** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}} = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.37\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 5.93\times10^3\ \text{s} = 98.8\ \text{min}$

(b) $v = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.37\times10^6\ \text{m})}{5.93\times10^3\ \text{s}} = 7.50\times10^3\ \text{m/s} = 7.50\ \text{km/s}.$

EVALUATE: The satellite in Example 13.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in h corresponds to a small percentage change in r and the values of T and v for the two satellites do not differ very much.

13.27. **IDENTIFY:** We know orbital data (speed and orbital radius) for one satellite and want to use it to find the orbital speed of another satellite having a known orbital radius. Newton's second law and the law of universal gravitation apply to both satellites.

SET UP: For circular motion, $F_{\text{net}} = ma = mv^2/r$, which in this case is $G\dfrac{mm_p}{r^2} = m\dfrac{v^2}{r}$.

EXECUTE: Using $G\dfrac{mm_p}{r^2} = m\dfrac{v^2}{r}$, we get $Gm_p = rv^2 = \text{constant}$. $r_1v_1^2 = r_2v_2^2$.

$$v_2 = v_1\sqrt{\frac{r_1}{r_2}} = (4800\ \text{m/s})\sqrt{\frac{7.00\times10^7\ \text{m}}{3.00\times10^7\ \text{m}}} = 7330\ \text{m/s}.$$

EVALUATE: The more distant satellite moves slower than the closer satellite, which is reasonable since the planet's gravity decreases with distance. The masses of the satellites do not affect their orbits.

13.28. **IDENTIFY:** We can calculate the orbital period T from the number of revolutions per day. Then the period and the orbit radius are related by $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$.

SET UP: $m_E = 5.97\times10^{24}\ \text{kg}$ and $R_E = 6.37\times10^6\ \text{m}$. The height h of the orbit above the surface of the earth is related to the orbit radius r by $r = h + R_E$. $1\ \text{day} = 8.64\times10^4\ \text{s}$.

EXECUTE: The satellite moves 15.65 revolutions in $8.64\times10^4\ \text{s}$, so the time for 1.00 revolution is

$T = \dfrac{8.64\times10^4\ \text{s}}{15.65} = 5.52\times10^3\ \text{s}$. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives

$r = \left(\dfrac{Gm_E T^2}{4\pi^2}\right)^{1/3} = \left(\dfrac{[6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2][5.97\times10^{24}\ \text{kg}][5.52\times10^3\ \text{s}]^2}{4\pi^2}\right)^{1/3}$. $r = 6.75\times10^6\ \text{m}$ and

$h = r - R_E = 3.8\times10^5\ \text{m} = 380\ \text{km}.$

EVALUATE: The period of this satellite is slightly larger than the period for the satellite in Example 13.6 and the altitude of this satellite is therefore somewhat greater.

13.29. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to the motion of the baseball. $v = \dfrac{2\pi r}{T}$.

SET UP: $r_D = 6\times10^3\ \text{m}$.

EXECUTE: **(a)** $F_g = ma_{\text{rad}}$ gives $G\dfrac{m_D m}{r_D^2} = m\dfrac{v^2}{r_D}$.

$$v = \sqrt{\frac{Gm_D}{r_D}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.5\times10^{15}\ \text{kg})}{6\times10^3\ \text{m}}} = 4.07\ \text{m/s}\ \text{ which rounds to 4.1 m/s.}$$

$4.1\ \text{m/s} = 9.1\ \text{mph}$, which is easy to achieve.

(b) $T=\dfrac{2\pi r}{v}=\dfrac{2\pi(6\times10^3\ \text{m})}{4.07\ \text{m/s}}=9263\ \text{s}=154.4\ \text{min}=2.6\ \text{h}.$ The game would last a very long time indeed!

EVALUATE: The speed v is relative to the center of Deimos. The baseball would already have some speed before we throw it because of the rotational motion of Deimos.

13.30. IDENTIFY: $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$, where m_{star} is the mass of the star. $v=\dfrac{2\pi r}{T}$.

SET UP: $3.09\ \text{days}=2.67\times10^5\ \text{s}.$ The orbit radius of Mercury is $5.79\times10^{10}\ \text{m}.$ The mass of our sun is $1.99\times10^{30}\ \text{kg}.$

EXECUTE: **(a)** $T=2.67\times10^5\ \text{s}.$ $r=(5.79\times10^{10}\ \text{m})/9=6.43\times10^9\ \text{m}.$ $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$ gives

$$m_{\text{star}}=\frac{4\pi^2r^3}{T^2G}=\frac{4\pi^2(6.43\times10^9\ \text{m})^3}{(2.67\times10^5\ \text{s})^2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}=2.21\times10^{30}\ \text{kg}.\quad \frac{m_{\text{star}}}{m_{\text{sun}}}=1.11,\ \text{so}$$

$m_{\text{star}}=1.11m_{\text{sun}}.$

(b) $v=\dfrac{2\pi r}{T}=\dfrac{2\pi(6.43\times10^9\ \text{m})}{2.67\times10^5\ \text{s}}=1.51\times10^5\ \text{m/s}=151\ \text{km/s}$

EVALUATE: The orbital period of Mercury is 88.0 d. The period for this planet is much less primarily because the orbit radius is much less and also because the mass of the star is greater than the mass of our sun.

13.31. IDENTIFY: The orbital speed is given by $v=\sqrt{Gm/r}$, where m is the mass of the star. The orbital period is given by $T=\dfrac{2\pi r}{v}$.

SET UP: The sun has mass $m_{\text{S}}=1.99\times10^{30}\ \text{kg}.$ The orbit radius of the earth is $1.50\times10^{11}\ \text{m}.$

EXECUTE: **(a)** $v=\sqrt{Gm/r}.$

$$v=\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.85\times1.99\times10^{30}\ \text{kg})/((1.50\times10^{11}\ \text{m})(0.11))}=8.27\times10^4\ \text{m/s}.$$

(b) $2\pi r/v=1.25\times10^6\ \text{s}=14.5\ \text{days}$ (about two weeks).

EVALUATE: The orbital period is less than the 88-day orbital period of Mercury; this planet is orbiting very close to its star, compared to the orbital radius of Mercury.

13.32. IDENTIFY: The period of each satellite is given by $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{p}}}}$. Set up a ratio involving T and r.

SET UP: $T=\dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{p}}}}$ gives $\dfrac{T}{r^{3/2}}=\dfrac{2\pi}{\sqrt{Gm_{\text{p}}}}=\text{constant}$, so $\dfrac{T_1}{r_1^{3/2}}=\dfrac{T_2}{r_2^{3/2}}$.

EXECUTE: $T_2=T_1\left(\dfrac{r_2}{r_1}\right)^{3/2}=(6.39\ \text{days})\left(\dfrac{48{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2}=24.5\ \text{days}.$ For the other satellite,

$$T_2=(6.39\ \text{days})\left(\frac{64{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2}=37.7\ \text{days}.$$

EVALUATE: T increases when r increases.

13.33. IDENTIFY: Kepler's third law applies.

SET UP: $T=\dfrac{2\pi a^{3/2}}{\sqrt{Gm_{\text{S}}}}$, $d_{\min}=a(1-e)$, $d_{\max}=a(1+e)$.

EXECUTE: (a) Kepler's third law gives

$$T=\frac{2\pi a^{3/2}}{\sqrt{Gm_S}}=\frac{2\pi(5.91\times10^{12}\text{ m})^{3/2}}{\sqrt{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}}=7.84\times10^9\text{ s }[(1\text{ y})/(3.156\times10^7\text{ s})]=248\text{ y}.$$

(b) $d_{min}=a(1-e)=(5.91\times10^{12}\text{ m})(1-0.249)=4.44\times10^{12}\text{ m}$; $d_{max}=a(1+e)=7.38\times10^{12}\text{ m}$.

EVALUATE: $d_{max}=1.66d_{min}$, which is *much* greater than for the earth's orbit since the earth moves in a much more circular orbit than Pluto.

13.34. IDENTIFY: Knowing the orbital radius and orbital period of a satellite, we can calculate the mass of the object about which it is revolving.

SET UP: The radius of the orbit is $r=10.5\times10^9\text{ m}$ and its period is $T=6.3\text{ days}=5.443\times10^5\text{ s}$. The mass of the sun is $m_S=1.99\times10^{30}\text{ kg}$. The orbital period is given by $T=\frac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$.

EXECUTE: Solving $T=\frac{2\pi r^{3/2}}{\sqrt{Gm_{HD}}}$ for the mass of the star gives

$$m_{HD}=\frac{4\pi^2r^3}{T^2G}=\frac{4\pi^2(10.5\times10^9\text{ m})^3}{(5.443\times10^5\text{ s})^2(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)}=2.3\times10^{30}\text{ kg},\text{ which is } m_{HD}=1.2m_S.$$

EVALUATE: The mass of the star is only 20% greater than that of our sun, yet the orbital period of the planet is much shorter than that of the earth, so the planet must be much closer to the star than the earth is.

13.35. IDENTIFY: We are dealing with the gravitational force due to spherical shells.

SET UP: Outside a uniform spherical shell, $F_g=\frac{Gm_1m_2}{r^2}$, and inside of it $F_g=0$. Our target variable is the net force F at various distances from the center, where $F=F_A+F_B$.

EXECUTE: (a) At $r=2.00$ m, the point mass is inside both shells, so $F=0$.

(b) At $r=5.00$ m, the mass is between the shells, so $F_B=0$, so $F=F_A$. This gives

$$F=F_A=\frac{Gmm_A}{r^2}=\frac{G(0.0200\text{ kg})(20.0\text{ kg})}{(5.00\text{ m})^2}=1.07\times10^{-12}\text{ N}.$$

(c) At $r=8.00$ m, the mass is outside of both shells. We treat each one as a point mass at its center.

$$F=\frac{G(m_A+m_B)}{r^2}=\frac{G(0.0200\text{ kg})(60.0\text{ kg})}{(8.00\text{ m})^2}=1.25\times10^{-12}\text{ N}.$$

EVALUATE: If a shell is not uniform, we cannot treat it as a point mass at its center. And the force on a mass inside of it is not necessarily zero.

13.36. IDENTIFY: Section 13.6 states that for a point mass outside a spherical shell the gravitational force is the same as if all the mass of the shell were concentrated at its center. It also states that for a point inside a spherical shell the force is zero.

SET UP: For $r=5.01\text{ m}$ the point mass is outside the shell and for $r=4.99\text{ m}$ and $r=2.72\text{ m}$ the point mass is inside the shell.

EXECUTE: (a) (i) $F_g=\frac{Gm_1m_2}{r^2}=(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(1000.0\text{ kg})(2.00\text{ kg})}{(5.01\text{ m})^2}=5.31\times10^{-9}\text{ N}.$ (ii) $F_g=0.$ (iii) $F_g=0.$

(b) For $r<5.00\text{ m}$ the force is zero and for $r>5.00\text{ m}$ the force is proportional to $1/r^2$. The graph of F_g versus r is sketched in Figure 13.36.

EVALUATE: Inside the shell the gravitational potential energy is constant and the force on a point mass inside the shell is zero.

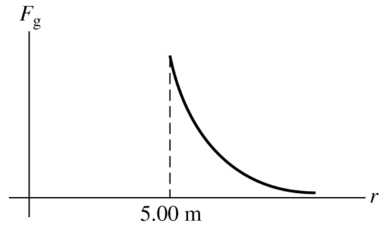


Figure 13.36

13.37. IDENTIFY: Section 13.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance r from the center of a uniform sphere, where r is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, where M is the mass of the sphere and R is its radius.

The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = M\left(\dfrac{r}{R}\right)^3$. $r = 5.01\text{ m}$ is outside the sphere and $r = 2.50\text{ m}$ is inside the sphere. $F_g = \dfrac{Gm_1m_2}{r^2}$.

EXECUTE: **(a)** (i) $F_g = \dfrac{GMm}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\text{ kg})(2.00\text{ kg})}{(5.01\text{ m})^2} = 5.31\times10^{-9}\text{ N}$.

(ii) $F_g = \dfrac{GM'm}{r^2}$. $M' = M\left(\dfrac{r}{R}\right)^3 = (1000.0\text{ kg})\left(\dfrac{2.50\text{ m}}{5.00\text{ m}}\right)^3 = 125\text{ kg}$.

$F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(125\text{ kg})(2.00\text{ kg})}{(2.50\text{ m})^2} = 2.67\times10^{-9}\text{ N}$.

(b) $F_g = \dfrac{GM(r/R)^3 m}{r^2} = \left(\dfrac{GMm}{R^3}\right)r$ for $r < R$ and $F_g = \dfrac{GMm}{r^2}$ for $r > R$. The graph of F_g versus r is sketched in Figure 13.37.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

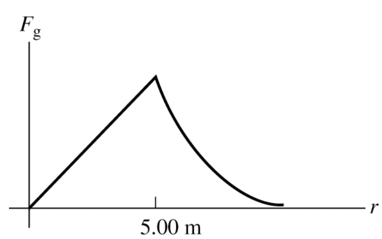


Figure 13.37

13.38. IDENTIFY: The gravitational potential energy of a pair of point masses is $U = -G\dfrac{m_1m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find U.

SET UP: Divide the rod into differential masses dm at position l, measured from the right end of the rod. $dm = dl(M/L)$.

EXECUTE: **(a)** $U = -\dfrac{Gm\,dm}{l+x} = -\dfrac{GmM}{L}\dfrac{dl}{l+x}.$

Integrating, $U = -\dfrac{GmM}{L}\int_0^L \dfrac{dl}{l+x} = -\dfrac{GmM}{L}\ln\left(1+\dfrac{L}{x}\right)$. For $x \gg L$, the natural logarithm is $\sim(L/x)$, and $U \to -GmM/x$.

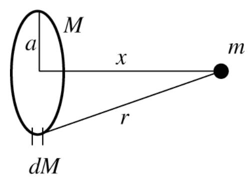
(b) The x-component of the gravitational force on the sphere is

$F_x = -\dfrac{\partial U}{\partial x} = \dfrac{GmM}{L}\dfrac{(-L/x^2)}{1+(L/x)} = -\dfrac{GmM}{x^2+Lx}$, with the minus sign indicating an attractive force. As $x \gg L$, the denominator in the above expression approaches x^2, and $F_x \to -GmM/x^2$, as expected.

EVALUATE: When x is much larger than L the rod can be treated as a point mass, and our results for U and F_x do reduce to the correct expression when $x \gg L$.

13.39. **IDENTIFY:** Find the potential due to a small segment of the ring and integrate over the entire ring to find the total U.

(a) SET UP:



Divide the ring up into small segments dM, as indicated in Figure 13.39.

Figure 13.39

EXECUTE: The gravitational potential energy of dM and m is $dU = -GmdM/r$.

The total gravitational potential energy of the ring and particle is $U = \int dU = -Gm\int dM/r$.

But $r = \sqrt{x^2+a^2}$ is the same for all segments of the ring, so

$U = -\dfrac{Gm}{r}\int dM = -\dfrac{GmM}{r} = -\dfrac{GmM}{\sqrt{x^2+a^2}}.$

(b) EVALUATE: When $x \gg a$, $\sqrt{x^2+a^2} \to \sqrt{x^2} = x$ and $U = -GmM/x$. This is the gravitational potential energy of two point masses separated by a distance x. This is the expected result.

(c) IDENTIFY and **SET UP:** Use $F_x = -dU/dx$ with $U(x)$ from part (a) to calculate F_x.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(-\dfrac{GmM}{\sqrt{x^2+a^2}}\right)$

$F_x = +GmM\dfrac{d}{dx}(x^2+a^2)^{-1/2} = GmM\left(-\dfrac{1}{2}(2x)(x^2+a^2)^{-3/2}\right)$

$F_x = -GmMx/(x^2+a^2)^{3/2}$; the minus sign means the force is attractive.

EVALUATE: **(d)** For $x \gg a$, $(x^2+a^2)^{3/2} \to (x^2)^{3/2} = x^3$

Then $F_x = -GmMx/x^3 = -GmM/x^2$. This is the force between two point masses separated by a distance x and is the expected result.

(e) For $x = 0$, $U = -GMm/a$. Each small segment of the ring is the same distance from the center and the potential is the same as that due to a point charge of mass M located at a distance a.

For $x = 0$, $F_x = 0$. When the particle is at the center of the ring, symmetrically placed segments of the ring exert equal and opposite forces and the total force exerted by the ring is zero.

13.40. **IDENTIFY:** We are investigating the gravitational field $\vec{g}$ for a spherical shell.

SET UP: The field is defined as $\vec{g} = \dfrac{\vec{F}_g}{m}$. We want to find $\vec{g}$ inside and outside a spherical shell.

EXECUTE: **(a)** Inside ($r < R$) the shell $\vec{F}_g = 0$, so $\vec{g} = 0$.

(b) Outside ($r > R$) the shell behaves like a point mass at its center, so $g = \dfrac{GmM / r^2}{m} = \dfrac{GM}{r^2}$.

EVALUATE: The gravitational field obeys an inverse-square law.

13.41. **IDENTIFY** and **SET UP:** At the north pole, $F_g = w_0 = mg_0$, where g_0 is given by $g = G\dfrac{m_E}{r^2}$ applied to Neptune. At the equator, the apparent weight is given by $w = w_0 - mv^2/R$. The orbital speed v is obtained from the rotational period using $v = 2\pi R/T$.

EXECUTE: **(a)** $g_0 = Gm/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.02\times10^{26}\ \text{kg})/(2.46\times10^7\ \text{m})^2 = 11.25\ \text{m/s}^2$. This agrees with the value of g given in the problem.

$F = w_0 = mg_0 = (3.00\ \text{kg})(11.25\ \text{m/s}^2) = 33.74\ \text{N}$, which rounds to 33.7 N. This is the true weight of the object.

(b) We have $w = w_0 - mv^2/R$

$T = \dfrac{2\pi r}{v}$ gives $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(2.46\times10^7\ \text{m})}{(16\ \text{h})(3600\ \text{s/1 h})} = 2.683\times10^3\ \text{m/s}$

$v^2/R = (2.683\times10^3\ \text{m/s})^2/(2.46\times10^7\ \text{m}) = 0.2927\ \text{m/s}^2$

Then $w = 33.74\ \text{N} - (3.00\ \text{kg})(0.2927\ \text{m/s}^2) = 32.9\ \text{N}$.

EVALUATE: The apparent weight is less than the true weight. This effect is larger on Neptune than on earth.

13.42. **IDENTIFY:** At the North Pole, Sneezy has no circular motion and therefore no acceleration. But at the equator he has acceleration toward the center of the earth due to the earth's rotation.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg, radius $R_E = 6.37\times10^6$ m and rotational period $T = 24\ \text{hr} = 8.64\times10^4$ s. Use coordinates for which the $+y$ direction is toward the center of the earth. The free-body diagram for Sneezy at the equator is given in Figure 13.42. The radial acceleration due to Sneezy's circular motion at the equator is $a_{rad} = \dfrac{4\pi^2 R}{T^2}$, and Newton's second law applies to Sneezy.

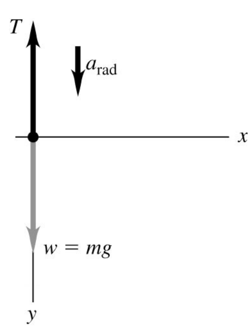


Figure 13.42

EXECUTE: At the north pole Sneezy has $a = 0$ and $T = w = 395.0$ N (the gravitational force exerted by the earth). Sneezy has mass $m = w/g = 40.31$ kg. At the equator Sneezy is traveling in a circular path and has radial acceleration $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(6.37\times10^6\text{ m})}{(8.64\times10^4\text{ s})^2} = 0.03369\text{ m/s}^2$. Newton's second law $\Sigma F_y = ma_y$ gives $w - T = ma_{\text{rad}}$. Solving for T gives

$T = w - ma_{\text{rad}} = m(g - a_{\text{rad}}) = (40.31\text{ kg})(9.80\text{ m/s}^2 - 0.03369\text{ m/s}^2) = 394\text{ N}$.

EVALUATE: At the equator Sneezy has an inward acceleration and the outward tension is less than the true weight, since there is a net inward force.

13.43. IDENTIFY: The orbital speed for an object a distance r from an object of mass M is $v = \sqrt{\dfrac{GM}{r}}$. The mass M of a black hole and its Schwarzschild radius R_{S} are related by $R_{\text{S}} = \dfrac{2GM}{c^2}$.

SET UP: $c = 3.00\times10^8$ m/s. 1 ly $= 9.461\times10^{15}$ m.

EXECUTE:

(a) $M = \dfrac{rv^2}{G} = \dfrac{(7.5\text{ ly})(9.461\times10^{15}\text{ m/ly})(200\times10^3\text{ m/s})^2}{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 4.3\times10^{37}\text{ kg} = 2.1\times10^7\text{ M}_{\text{S}}$.

(b) No, the object has a mass very much greater than 50 solar masses.

(c) $R_{\text{S}} = \dfrac{2GM}{c^2} = \dfrac{2v^2 r}{c^2} = 6.32\times10^{10}\text{ m}$, which does fit.

EVALUATE: The Schwarzschild radius of a black hole is approximately the same as the radius of Mercury's orbit around the sun.

13.44. IDENTIFY: The clumps orbit the black hole. Their speed, orbit radius and orbital period are related by $v = \dfrac{2\pi r}{T}$. Their orbit radius and period are related to the mass M of the black hole by $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$. The radius of the black hole's event horizon is related to the mass of the black hole by $R_{\text{S}} = \dfrac{2GM}{c^2}$.

SET UP: $v = 3.00\times10^7$ m/s. $T = 27\text{ h} = 9.72\times10^4$ s. $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $r = \dfrac{vT}{2\pi} = \dfrac{(3.00\times10^7\text{ m/s})(9.72\times10^4\text{ s})}{2\pi} = 4.64\times10^{11}\text{ m}$.

(b) $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ gives $M = \dfrac{4\pi^2 r^3}{GT^2} = \dfrac{4\pi^2(4.64\times10^{11}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(9.72\times10^4\text{ s})^2} = 6.26\times10^{36}\text{ kg}$.

$= 3.15\times10^6 M_S$, where M_S is the mass of our sun

(c) $R_{\text{S}} = \dfrac{2GM}{c^2} = \dfrac{2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(6.26\times10^{36}\text{ kg})}{(3.00\times10^8\text{ m/s})^2} = 9.28\times10^9\text{ m}$

EVALUATE: The black hole has a mass that is about 3×10^6 solar masses.

13.45. IDENTIFY: Use $F_{\text{g}} = Gm_1m_2/r^2$ to find each gravitational force. Each force is attractive. In part (b) apply conservation of energy.

SET UP: For a pair of masses m_1 and m_2 with separation r, $U = -G\dfrac{m_1m_2}{r}$.

EXECUTE: **(a)** From symmetry, the net gravitational force will be in the direction 45° from the x-axis (bisecting the x- and y-axes), with magnitude

$$F = (6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0150\text{ kg})\left[\frac{(2.0\text{ kg})}{(2(0.50\text{ m})^2)} + 2\frac{(1.0\text{ kg})}{(0.50\text{ m})^2}\sin 45°\right] = 9.67\times10^{-12}\text{ N}$$

(b) The initial displacement is so large that the initial potential energy may be taken to be zero. From the work-energy theorem, $\frac{1}{2}mv^2 = Gm\left[\frac{(2.0\text{ kg})}{\sqrt{2}\,(0.50\text{ m})} + 2\frac{(1.0\text{ kg})}{(0.50\text{ m})}\right]$. Cancelling the factor of m, solving for v, and using the numerical values gives $v = 3.02\times10^{-5}$ m/s.

EVALUATE: The result in part (b) is independent of the mass of the particle. It would take the particle a long time to reach point P.

13.46. **IDENTIFY:** Use $g = G\frac{m_E}{r^2}$ to calculate g for Europa. The acceleration of a particle moving in a circular path is $a_{rad} = r\omega^2$.

SET UP: In $a_{rad} = r\omega^2$, ω must be in rad/s. For Europa, $R = 1.560\times10^6$ m.

EXECUTE: $g = \frac{Gm}{R^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(4.80\times10^{22}\text{ kg})}{(1.560\times10^6\text{ m})^2} = 1.316\text{ m/s}^2$. $g = a_{rad}$ gives

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{1.316\text{ m/s}^2}{4.25\text{ m}}} = (0.5565\text{ rad/s})\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 5.31\text{ rpm}.$$

EVALUATE: The radius of Europa is about one-fourth that of the earth and its mass is about one-hundredth that of earth, so g on Europa is much less than g on earth. The lander would have some spatial extent so different points on it would be different distances from the rotation axis and a_{rad} would have different values. For the ω we calculated, $a_{rad} = g$ at a point that is precisely 4.25 m from the rotation axis.

13.47. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the motion of the two spheres.

SET UP: Denote the 50.0-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2.

EXECUTE: **(a)** Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1v_1 = m_2v_2$.

(b) (i) From the work-energy theorem in the form $K_i + U_i = K_f + U_f$, with the initial kinetic energy $K_i = 0$ and $U = -G\frac{m_1m_2}{r}$, $Gm_1m_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right] = \frac{1}{2}(m_1v_1^2 + m_2v_2^2)$. Using the conservation of momentum relation $m_1v_1 = m_2v_2$ to eliminate v_2 in favor of v_1 and simplifying yields $v_1^2 = \frac{2Gm_2^2}{m_1+m_2}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$, with a similar expression for v_2. Substitution of numerical values gives $v_1 = 1.49\times10^{-5}$ m/s, $v_2 = 7.46\times10^{-6}$ m/s. (ii) The magnitude of the relative velocity is the sum of the speeds, 2.24×10^{-5} m/s.

(c) The distance the centers of the spheres travel (x_1 and x_2) is proportional to their acceleration, and $\frac{x_1}{x_2} = \frac{a_1}{a_2} = \frac{m_2}{m_1}$, or $x_1 = 2x_2$. When the spheres finally make contact, their centers will be a distance of $2r$ apart, or $x_1 + x_2 + 2r = 40$ m, or $2x_2 + x_2 + 2r = 40$ m. Thus, $x_2 = 40/3\text{ m} - 2r/3$, and $x_1 = 80/3\text{ m} - 4r/3$. The point of contact of the surfaces is $80/3\text{ m} - r/3 = 26.6$ m from the initial position of the center of the 50.0-kg sphere.

EVALUATE: The result $x_1/x_2 = 2$ can also be obtained from the conservation of momentum result that $\frac{v_1}{v_2} = \frac{m_2}{m_1}$, at every point in the motion.

EVALUATE: The work done by the attractive gravity forces is negative. The work you do is positive.

13.48. **IDENTIFY:** The gravitational pulls of Titan and Saturn on the *Huygens* probe should be in opposite directions and of equal magnitudes to cancel.

SET UP: The mass of Saturn is $m_S = 5.68\times10^{26}$ kg. When the probe is a distance d from the center of Titan it is a distance $1.22\times10^9\ \text{m} - d$ from the center of Saturn. The magnitude of the gravitational force is given by $F_{\text{grav}} = GmM/r^2$.

EXECUTE: Equal gravity forces means the two gravitational pulls on the probe must balance, so $G\dfrac{mm_T}{d^2} = G\dfrac{mm_S}{(1.22\times10^9\ \text{m} - d)^2}$. Simplifying, this becomes $d = \sqrt{\dfrac{m_T}{m_S}}(1.22\times10^9\ \text{m} - d)$. Using the masses from the text and solving for d we get

$d = \sqrt{\dfrac{1.35\times10^{23}\ \text{kg}}{5.68\times10^{26}\ \text{kg}}}(1.22\times10^9\ \text{m} - d) = (0.0154)(1.22\times10^9\ \text{m} - d)$, so $d = 1.85\times10^7\ \text{m} = 1.85\times10^4\ \text{km}$.

EVALUATE: For the forces to balance, the probe must be much closer to Titan than to Saturn since Titan's mass is much smaller than that of Saturn.

13.49. **IDENTIFY** and **SET UP:** **(a)** To stay above the same point on the surface of the earth the orbital period of the satellite must equal the orbital period of the earth:

$T = 1\ \text{d}(24\ \text{h}/1\ \text{d})(3600\ \text{s}/1\ \text{h}) = 8.64\times10^4\ \text{s}$. The equation $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives the relation between the orbit radius and the period.

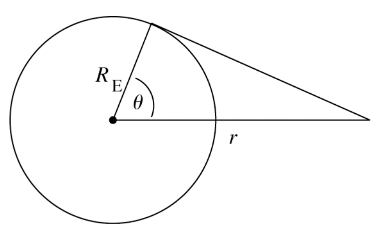
EXECUTE: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives $T^2 = \dfrac{4\pi^2 r^3}{Gm_E}$. Solving for r gives

$$r = \left(\frac{T^2 Gm_E}{4\pi^2}\right)^{1/3} = \left(\frac{(8.64\times10^4\ \text{s})^2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{4\pi^2}\right)^{1/3} = 4.23\times10^7\ \text{m}.$$

This is the radius of the orbit; it is related to the height h above the earth's surface and the radius R_E of the earth by $r = h + R_E$. Thus $h = r - R_E = 4.23\times10^7\ \text{m} - 6.37\times10^6\ \text{m} = 3.59\times10^7\ \text{m}$.

EVALUATE: The orbital speed of the geosynchronous satellite is $2\pi r/T = 3080$ m/s. The altitude is much larger and the speed is much less than for the satellite in Example 13.6.

(b) Consider Figure 13.49.



$$\cos\theta = \frac{R_E}{r} = \frac{6.37\times10^6\ \text{m}}{4.23\times10^7\ \text{m}}$$

$\theta = 81.3°$

Figure 13.49

A line from the satellite is tangent to a point on the earth that is at an angle of $81.3°$ above the equator. The sketch shows that points at higher latitudes are blocked by the earth from viewing the satellite.

13.50. **IDENTIFY:** We are dealing with satellites orbiting two different planets.

SET UP: The period is $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}$. We want to compare the period around two different planets. The planets have the same average density ρ, so first express T in terms of ρ.

EXECUTE: $m = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = \frac{4}{3}\pi\rho R^3$. Now find *T*. The satellites are just above the surface, so *r* = *R*. In this case $T = \frac{2\pi R^{3/2}}{\sqrt{Gm_\text{p}}} = \frac{2\pi R^{3/2}}{\sqrt{G\left(\frac{4}{3}\pi\rho R^3\right)}} = \frac{2\pi}{\sqrt{4G\pi\rho/3}}$. This result depends only on ρ, which is the same for both planets. Therefore *T* is the same in both cases, so $T_A = T_B$.

EVALUATE: The planets have the same density, but not the same mass. The smaller planet has less mass than the larger planet, but since it is smaller the satellite orbits closer to it so both satellites have the same period.

13.51. IDENTIFY: From Example 13.5, the escape speed is $v = \sqrt{\frac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$.

EXECUTE: In terms of the density ρ, the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$v = \sqrt{(8\pi/3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2500\ \text{kg/m}^3)(150\times10^3\ \text{m})^2} = 177\ \text{m/s}.$

EVALUATE: This is much less than the escape speed for the earth, 11,200 m/s.

13.52. IDENTIFY: Apply $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_\text{E}}}$ to relate the orbital period *T* and M_P, the planet's mass, and then use $w = \frac{Gm_\text{E}m}{r^2}$ applied to the planet to calculate the astronaut's weight.

SET UP: The radius of the orbit of the lander is $5.75\times10^5\ \text{m} + 4.80\times10^6\ \text{m}$.

EXECUTE: From $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_\text{E}}}$, we get $T^2 = \frac{4\pi^2 r^3}{GM_\text{P}}$ and

$$M_\text{P} = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(5.75\times10^5\ \text{m} + 4.80\times10^6\ \text{m})^3}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.8\times10^3\ \text{s})^2} = 2.731\times10^{24}\ \text{kg},$$

or about half the earth's mass. Now we can find the astronaut's weight on the surface from $w = \frac{Gm_\text{E}m}{r^2}$. (The landing on the north pole removes any need to account for centripetal acceleration.)

$$w = \frac{GM_\text{p}m_\text{a}}{r_\text{p}^2} = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.731\times10^{24}\ \text{kg})(85.6\ \text{kg})}{(4.80\times10^6\ \text{m})^2} = 677\ \text{N}.$$

EVALUATE: At the surface of the earth the weight of the astronaut would be 839 N.

13.53. IDENTIFY: Apply the law of gravitation to the astronaut at the north pole to calculate the mass of the planet. Then apply $\Sigma\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}$ to the astronaut, with $a_\text{rad} = \frac{4\pi^2 R}{T^2}$, toward the center of the planet, to calculate the period *T*. Apply $T = \frac{2\pi r^{3/2}}{\sqrt{Gm_\text{E}}}$ to the satellite in order to calculate its orbital period.

SET UP: Get radius of X: $\frac{1}{4}(2\pi R) = 18{,}850\ \text{km}$ and $R = 1.20\times10^7\ \text{m}$. Astronaut mass: $m = \frac{w}{g} = \frac{943\ \text{N}}{9.80\ \text{m/s}^2} = 96.2\ \text{kg}.$

EXECUTE: $\dfrac{GmM_X}{R^2} = w,$ where $w = 915.0$ N.

$$M_X = \frac{mg_x R^2}{Gm} = \frac{(915\text{ N})(1.20\times 10^7\text{ m})^2}{(6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(96.2\text{ kg})} = 2.05\times 10^{25}\text{ kg}$$

Apply Newton's second law to the astronaut on a scale at the equator of X. $F_{\text{grav}} - F_{\text{scale}} = ma_{\text{rad}}$, so

$F_{\text{grav}} - F_{\text{scale}} = \dfrac{4\pi^2 mR}{T^2}$. $915.0\text{ N} - 850.0\text{ N} = \dfrac{4\pi^2(96.2\text{ kg})(1.20\times 10^7\text{ m})}{T^2}$ and

$T = 2.65\times 10^4\text{ s}\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 7.36\text{ h}.$

(b) For the satellite,

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_X}} = \sqrt{\frac{4\pi^2(1.20\times 10^7\text{ m} + 2.0\times 10^6\text{ m})^3}{(6.67\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.05\times 10^{25}\text{ kg})}} = 8.90\times 10^3\text{ s} = 2.47\text{ hours.}$$

EVALUATE: The acceleration of gravity at the surface of the planet is $g_X = \dfrac{915.0\text{ N}}{96.2\text{ kg}} = 9.51\text{ m/s}^2$, similar to the value on earth. The radius of the planet is about twice that of earth. The planet rotates more rapidly than earth and the length of a day is about one-third what it is on earth.

13.54. IDENTIFY: We are dealing with a planet that is spherically symmetric but not radially uniform. We will need to integrate to find the mass as a function of distance *r* from the center. To determine the equation for the density we use the slope-intercept form of the equation of a straight line, which is $y = mx + b$.

SET UP: The graph in Fig. 13.9 slopes downward. Its slope is $m = -\dfrac{(13{,}000 - 3000)\text{ kg/m}^3}{R_E} = -\dfrac{10{,}000\text{ kg/m}^3}{R_E}$ and its *y*-intercept is $b = 13{,}000\text{ kg/m}^3$. Using the slope-intercept form of a straight-line equation ($y = mx + b$), the density is $\rho(r) = -\dfrac{10{,}000\text{ kg/m}^3}{R_E}r + 13{,}000\text{ kg/m}^3$, which we simplify as $\rho(r) = mr + b$ for convenience while integrating.

EXECUTE: (a) $M(r) = \int \rho(r')dV = \int \rho(r')4\pi r'^2 dr' = \int_0^r (mr' + b)4\pi r'^2 dr' = \pi m r^4 + \dfrac{4\pi b r^3}{3}$. Substitute for *m*, *b* and R_E. $M(r) = -(4.932\times 10^{-3}\text{ kg/m}^4)r^4 + (5.4454\times 10^4\text{ kg/m}^3)r^3$ for $r \le R_E$.

(b) For $r \le R_E$ we use the result from (a) for the mass enclosed within a sphere of radius *r*. To find the force the mass in this shell exerts on a mass *m* a distance *r* from the center, we use $F_g = \dfrac{GmM(r)}{r^2} =$

$\dfrac{Gm\left[-(4.932\times 10^{-3}\text{ kg/m}^4)r^4 + (5.4454\times 10^4\text{ kg/m}^3)r^3\right]}{r^2}$, which gives

$F_g = Gm\left[-(4.932\times 10^{-3}\text{ kg/m}^4)r^2 + (5.4454\times 10^4\text{ kg/m}^3)r\right]$. Putting in for *G* gives

$F_g = m\left[-(3.29\times 10^{-13}\text{ N/kg}\cdot\text{m}^2)r^2 + (3.63\times 10^{-6}\text{ N/kg}\cdot\text{m})r\right]$

For $r \ge R_E$, evaluate the final equation in part (a) for $M(r)$ when $r = R_E = 6.37\times 10^6$ m. This gives 5.95×10^{24} kg. Outside the earth, we can treat it as a point mass at its center, so we use $F_g = \dfrac{GmM}{r^2} =$

$\dfrac{Gm(5.95\times 10^{24}\text{ kg})}{r^2} = \dfrac{m(3.79\times 10^{14}\text{ N}\cdot\text{m}^2/\text{kg})}{r^2}$.

EVALUATE: The mass we calculated in part (a) for the earth (5.95×10^{24} kg) is extremely close to the actual value of 5.97×10^{24} kg , so our model is very good. In addition, in the equation for F_g from part (b), $F_g = m\left[-(3.29\times10^{-13}\ \text{N/kg}\cdot\text{m}^2)r^2 + (3.63\times10^{-6}\ \text{N/kg}\cdot\text{m})r\right]$, the quantity in square brackets should give the acceleration due to gravity at the surface of the earth if we use $r = 6.37\times10^6$ m . Doing so gives a value of 9.79 m/s², which is extremely close to 9.80 m/s².

13.55. IDENTIFY: The free-fall time of the rock will give us the acceleration due to gravity at the surface of the planet. Applying Newton's second law and the law of universal gravitation will give us the mass of the planet since we know its radius.

SET UP: For constant acceleration, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. At the surface of the planet, Newton's second law gives $m_{\text{rock}}g = \dfrac{Gm_{\text{rock}}m_{\text{p}}}{R_{\text{p}}^2}$.

EXECUTE: First find $a_y = g$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(1.90\ \text{m})}{(0.480\ \text{s})^2} = 16.49\ \text{m/s}^2 = g$.

$g = 16.49\ \text{m/s}^2$. $m_{\text{p}} = \dfrac{gR_{\text{p}}^2}{G} = \dfrac{(16.49\ \text{m/s})(8.60\times10^7\ \text{m})^2}{6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 1.83\times10^{27}$ kg.

EVALUATE: The planet's mass is over 100 times that of the earth, which is reasonable since it is larger (in size) than the earth yet has a greater acceleration due to gravity at its surface.

13.56. IDENTIFY: Use the measurements of the motion of the rock to calculate g_{M}, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{\text{rad}}$ and $T = \dfrac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward. $g_{\text{M}} = G\dfrac{m_{\text{M}}}{R_{\text{M}}^2}$. The radius of Mongo is $R_{\text{M}} = \dfrac{c}{2\pi} = \dfrac{2.00\times10^8\ \text{m}}{2\pi} = 3.18\times10^7$ m. The ship moves in an orbit of radius $r = 3.18\times10^7\ \text{m} + 3.00\times10^7\ \text{m} = 6.18\times10^7$ m.

EXECUTE: **(a)** $v_{0y} = +12.0$ m/s, $v_y = -12.0$ m/s, $a_y = -g_{\text{M}}$ and $t = 4.80$ s. $v_y = v_{0y} + a_y t$ gives

$$-g_{\text{M}} = \frac{v_y - v_{0y}}{t} = \frac{-12.0\ \text{m/s} - 12.0\ \text{m/s}}{4.80\ \text{s}} \text{ and } g_{\text{M}} = 5.00\ \text{m/s}^2.$$

$$m_{\text{M}} = \frac{g_{\text{M}}R_{\text{M}}^2}{G} = \frac{(5.00\ \text{m/s}^2)(3.18\times10^7\ \text{m})^2}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 7.577\times10^{25}\ \text{kg which rounds to } 7.58\times10^{25}\ \text{kg}.$$

(b) $F_g = ma_{\text{rad}}$ gives $G\dfrac{m_{\text{M}}m}{r^2} = m\dfrac{v^2}{r}$ and $v^2 = \dfrac{Gm_{\text{M}}}{r}$.

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_{\text{M}}}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\text{M}}}} = \frac{2\pi(6.18\times10^7\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.577\times10^{25}\ \text{kg})}}$$

$T = 4.293\times10^4\ \text{s} = 11.9$ h.

EVALUATE: $R_{\text{M}} = 5.0R_{\text{E}}$ and $m_{\text{M}} = 12.7m_{\text{E}}$, so $g_{\text{M}} = \dfrac{12.7}{(5.0)^2}g_{\text{E}} = 0.508g_{\text{E}}$, which agrees with the value calculated in part (a).

13.57. IDENTIFY: Use the orbital speed and altitude to find the mass of the planet. Use this mass and the planet's radius to find g at the surface. Use projectile motion to find the horizontal range x, where $x = \dfrac{v_0^2\sin(2\alpha)}{g}$.

SET UP: For an object in a circular orbit, $v = \sqrt{GM/r}$. $g = GM/r^2$. Call r the orbital radius and R the radius of the planet.

EXECUTE: $v = \sqrt{GM/r}$ gives $M = rv^2/G$. Using this to find g gives $g = GM/R^2 = G(rv^2/G)/R^2 = v^2r/R^2 = (4900\text{ m/s})^2(4.48\times10^6\text{ m} + 6.30\times10^5\text{ m})/(4.48\times10^6\text{ m})^2 = 6.113$ m/s². Now use this acceleration to find the horizontal range.

$$x = \frac{v_0^2\sin(2\alpha)}{g} = (12.6\text{ m/s})^2\sin[2(30.8°)]/(6.113\text{ m/s}^2) = 22.8\text{ m}.$$

EVALUATE: On this planet, $g = 0.624g_E$, so the range is about 1.6 times what it would be on earth.

13.58. IDENTIFY: The 0.100 kg sphere has gravitational potential energy due to the other two spheres. Its mechanical energy is conserved.

SET UP: From energy conservation, $K_1 + U_1 = K_2 + U_2$, where $K = \frac{1}{2}mv^2$, and $U = -GmM/r$.

EXECUTE: Using $K_1 + U_1 = K_2 + U_2$, we have $K_1 = 0$, $m_A = 5.00$ kg, $m_B = 10.0$ kg and $m = 0.100$ kg.

$$U_1 = -\frac{Gmm_A}{r_{A1}} - \frac{Gmm_B}{r_{B1}} = -(6.674\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.100\text{ kg})\left(\frac{5.00\text{ kg}}{0.400\text{ m}} + \frac{10.0\text{ kg}}{0.600\text{ m}}\right)$$

$U_1 = -1.9466\times10^{-10}$ J.

$$U_2 = -\frac{Gmm_A}{r_{A2}} - \frac{Gmm_B}{r_{B2}} = -(6.674\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.100\text{ kg})\left(\frac{5.00\text{ kg}}{0.800\text{ m}} + \frac{10.0\text{ kg}}{0.200\text{ m}}\right)$$

$U_2 = -3.7541\times10^{-10}$ J. $K_2 = U_1 - U_2 = -1.9466\times10^{-10}\text{ J} - (-3.7541\times10^{-10}\text{ J}) = 1.8075\times10^{-10}$ J.

$$\frac{1}{2}mv^2 = K_2 \text{ and } v = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(1.8075\times10^{-10}\text{ J})}{0.100\text{ kg}}} = 6.01\times10^{-5}\text{ m/s}.$$

EVALUATE: The kinetic energy gained by the sphere is equal to the loss in its potential energy.

13.59. IDENTIFY: This problem involves the gravitational flux, as defined in Ex. 13.40.

SET UP: From Ex. 13.40, the gravitational field is $g = GM/r^2$ outside of a spherically symmetric object of mass M. The flux is $\Phi_g = gA$. We want the flux through a spherical surface outside the object.

EXECUTE: $\Phi_g = gA = \left(\frac{GM}{r^2}\right)(4\pi r^2) = 4\pi GM$.

EVALUATE: The answer does *not* depend on the radius of the spherical surface since the r^2 factors cancel out.

13.60. IDENTIFY: In this problem we must calculate the gravitational force on a point mass M due to a uniform rod, so we need to use the gravitational force formula and integrate.

SET UP: Follow the hint in the problem. The magnitude of the force dF on M due to a tiny mass element dm is $dF_g = \frac{GMdm}{r^2}$. We integrate to find the total force. Fig. 13.60 shows the arrangement of masses and the force. Call the y-axis along the rod with the x-axis perpendicular to it at its midpoint. M is on the x-axis. We want to find the force on M.

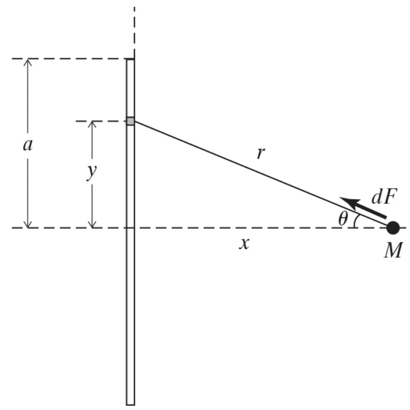


Figure 13.60

EXECUTE: **(a)** The rod is uniform. Therefore for every mass element *dm* above the *x*-axis, there is an equal element *dm* below the *x*-axis. The mass elements above the *x*-axis give *M* an upward pull and those below the *x*-axis give it a downward pull, and these pulls cancel. The mass elements also give a pull to the left (toward the rod), and these pulls add. Each of these pulls has an *x*-component $dF_x = dF\ \cos\theta$. Since the *y*-components cancel, the net force on *M* is due only to the *x*-components, so $F = \int dF_x$. We use $dF = \dfrac{GMdm}{r^2}$, where $\cos\theta = x/r$ and $r = \sqrt{x^2+y^2}$. The mass element *dm* is the mass of a tiny segment of rod of length *dy*, so $dm = \rho dy$. Putting this information into the integral gives $F = \int_{-a}^{a} \dfrac{GM}{x^2+y^2}\dfrac{x}{\sqrt{x^2+y^2}}\rho dy \ = \ GM\rho x\int_{-a}^{a}\dfrac{dy}{\left(x^2+y^2\right)^{3/2}}$. Using the integral tables in Appendix B gives

$$F = \ GM\rho x\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\Bigg|_{-a}^{+a} = \frac{2GM\rho}{x}\frac{a}{\sqrt{x^2+a^2}}$$

. In the problem statement we are given that *r* is the perpendicular distance from the center of the rod to *M*, which is *x* in our result. So rewriting in terms of *r* gives $F = \dfrac{2GM\rho a}{r\sqrt{r^2+a^2}}$. The direction is toward the rod.

(b) If $a >> r$, the factor $\dfrac{a}{\sqrt{r^2+a^2}} \to 1$, so $F \to \dfrac{2GM\rho}{r}$.

(c) For $a >> r$, we use the result in part (b), so the field is $g = \dfrac{F}{M} = \dfrac{\frac{2GM\rho}{r}}{M} = \dfrac{2G\rho}{r}$.

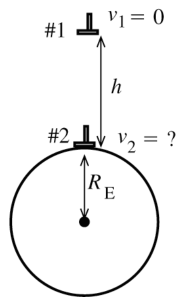
(d) For $a >> r$, we use the result in part (b), so the field is $g = \dfrac{2G\rho}{r}$ and the flux is

$\Phi_g = gA = \left(\dfrac{2G\rho}{r}\right)(2\pi rL) = 4\pi G\rho L$. The mass inside the cylinder is $m_{\text{inside}} = \rho L$, so the flux is

$\Phi_g = 4\pi G m_{\text{inside}}$.

EVALUATE: In part (b) we see that the force obeys an inverse *r* law instead of an inverse square law.

13.61. **IDENTIFY** and **SET UP:** Apply conservation of energy. We must use $U = -\dfrac{GMm}{r}$ for the gravitational potential energy since *h* is not small compared to R_E.



As indicated in Figure 13.61, take point 1 to be where the hammer is released and point 2 to be just above the surface of the earth, so $r_1 = R_E + h$ and $r_2 = R_E$.

Figure 13.61

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

Only gravity does work, so $W_{other} = 0$.

$K_1 = 0, \quad K_2 = \frac{1}{2}mv_2^2$

$$U_1 = -G\frac{mm_E}{r_1} = -\frac{Gmm_E}{h + R_E}, \quad U_2 = -G\frac{mm_E}{r_2} = -\frac{Gmm_E}{R_E}$$

Thus, $-G\dfrac{mm_E}{h + R_E} = \dfrac{1}{2}mv_2^2 - G\dfrac{mm_E}{R_E}$

$$v_2^2 = 2Gm_E\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right) = \frac{2Gm_E}{R_E(R_E + h)}(R_E + h - R_E) = \frac{2Gm_E h}{R_E(R_E + h)}$$

$$v_2 = \sqrt{\frac{2Gm_E h}{R_E(R_E + h)}}$$

EVALUATE: If $h \to \infty$, $v_2 \to \sqrt{2Gm_E/R_E}$, which equals the escape speed. In this limit this event is the reverse of an object being projected upward from the surface with the escape speed. If $h \ll R_E$, then $v_2 = \sqrt{2Gm_E h/R_E^2} = \sqrt{2gh}$, the same result if *mgh* is used for *U*.

13.62. **IDENTIFY:** In orbit the total mechanical energy of the satellite is $E = -\dfrac{Gm_E m}{2R_E}$. $U = -G\dfrac{m_E m}{r}$. $W = E_2 - E_1$.

SET UP: $U \to 0$ as $r \to \infty$.

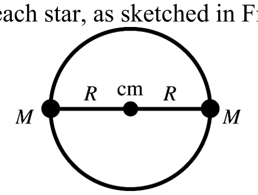
EXECUTE: **(a)** The energy the satellite has as it sits on the surface of the Earth is $E_1 = \dfrac{-GmM_E}{R_E}$. The energy it has when it is in orbit at a radius $R \approx R_E$ is $E_2 = \dfrac{-GmM_E}{2R_E}$. The work needed to put it in orbit is the difference between these: $W = E_2 - E_1 = \dfrac{GmM_E}{2R_E}$.

(b) The total energy of the satellite far away from the earth is zero, so the additional work needed is $0 - \left(\dfrac{-GmM_E}{2R_E}\right) = \dfrac{GmM_E}{2R_E}$.

EVALUATE: **(c)** The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

13.63. **IDENTIFY:** Use $F_g = Gm_1m_2/r^2$ to calculate F_g. Apply Newton's second law to circular motion of each star to find the orbital speed and period. Apply the conservation of energy to calculate the energy input (work) required to separate the two stars to infinity.

(a) SET UP: The cm is midway between the two stars since they have equal masses. Let R be the orbit radius for each star, as sketched in Figure 13.63.



The two stars are separated by a distance $2R$, so

$$F_g = GM^2/(2R)^2 = GM^2/4R^2$$

Figure 13.63

(b) EXECUTE: $F_g = ma_{rad}$

$GM^2/4R^2 = M(v^2/R)$ so $v = \sqrt{GM/4R}$

And $T = 2\pi R/v = 2\pi R\sqrt{4R/GM} = 4\pi\sqrt{R^3/GM}$

(c) SET UP: Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the system of the two stars. Separate to infinity implies $K_2 = 0$ and $U_2 = 0$.

EXECUTE: $K_1 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 2\left(\frac{1}{2}M\right)(GM/4R) = GM^2/4R$

$U_1 = -GM^2/2R$

Thus the energy required is $W_{other} = -(K_1 + U_1) = -(GM^2/4R - GM^2/2R) = GM^2/4R.$

EVALUATE: The closer the stars are and the greater their mass, the larger their orbital speed, the shorter their orbital period and the greater the energy required to separate them.

13.64. IDENTIFY: In the center of mass coordinate system, $r_{cm} = 0.$ Apply $\vec{F} = m\vec{a}$ to each star, where F is the gravitational force of one star on the other and $a = a_{rad} = \dfrac{4\pi^2 R}{T^2}.$

SET UP: $v = \dfrac{2\pi R}{T}$ allows R to be calculated from v and T.

EXECUTE: **(a)** The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1.$

(b) The forces on each star are equal in magnitude, so the product of the mass and the radial accelerations are equal: $\dfrac{4\pi^2 M_1R_1}{T_1^2} = \dfrac{4\pi^2 M_2R_2}{T_2^2}.$ From the result of part (a), the numerators of these expressions are equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric form desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \dfrac{GM_1M_2}{(R_1 + R_2)^2}$, so that equivalent expressions for the period are $M_2T^2 = \dfrac{4\pi^2 R_1(R_1 + R_2)^2}{G}$ and $M_1T^2 = \dfrac{4\pi^2 R_2(R_1 + R_2)^2}{G}.$

Adding the expressions gives $(M_1 + M_2)T^2 = \dfrac{4\pi^2(R_1 + R_2)^3}{G}$ or $T = \dfrac{2\pi(R_1 + R_2)^{3/2}}{\sqrt{G(M_1 + M_2)}}.$

(c) First we must find the radii of each orbit given the speed and period data. In a circular orbit,

$v=\frac{2\pi R}{T}$, or $R=\frac{vT}{2\pi}$. Thus $R_\alpha=\frac{(36\times10^3\ \text{m/s})(137\ \text{d})(86{,}400\ \text{s/d})}{2\pi}=6.78\times10^{10}$ m and

$R_\beta=\frac{(12\times10^3\ \text{m/s})(137\ \text{d})(86{,}400\ \text{s/d})}{2\pi}=2.26\times10^{10}$ m. Now find the sum of the masses.

$(M_\alpha+M_\beta)=\frac{4\pi^2(R_\alpha+R_\beta)^3}{T^2G}$. Inserting the values of T and the radii gives

$(M_\alpha+M_\beta)=\frac{4\pi^2(6.78\times10^{10}\ \text{m}+2.26\times10^{10}\ \text{m})^3}{[(137\ \text{d})(86{,}400\ \text{s/d})]^2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}=3.12\times10^{30}$ kg. Since

$M_\beta=M_\alpha R_\alpha/R_\beta=3M_\alpha$, $4M_\alpha=3.12\times10^{30}$ kg, or $M_\alpha=7.80\times10^{29}$ kg, and $M_\beta=2.34\times10^{30}$ kg.

(d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b): $R_\beta=(M_\alpha/M_\beta)R_\alpha=(0.67/3.8)R_\alpha=(0.176)R_\alpha$, $R_\alpha+R_\beta=\sqrt[3]{\frac{(M_\alpha+M_\beta)T^2G}{4\pi^2}}$. For Monocerotis, inserting the values for M and T gives $R_\alpha=1.9\times10^9$ m, $v_\alpha=4.4\times10^2$ km/s and for the black hole $R_\beta=34\times10^8$ m, $v_\beta=77$ km/s.

EVALUATE: Since T is the same, v is smaller when R is smaller.

13.65. IDENTIFY and SET UP: Use conservation of energy, $K_1+U_1+W_{\text{other}}=K_2+U_2$. The gravity force exerted by the sun is the only force that does work on the comet, so $W_{\text{other}}=0$.

EXECUTE: $K_1=\frac{1}{2}mv_1^2$, $v_1=2.0\times10^4$ m/s

$U_1=-Gm_\text{S}m/r_1$, where $r_1=2.5\times10^{11}$ m

$K_2=\frac{1}{2}mv_2^2$

$U_2=-Gm_\text{S}m/r_2$, $r_2=5.0\times10^{10}$ m

$\frac{1}{2}mv_1^2-Gm_\text{S}m/r_1=\frac{1}{2}mv_2^2-Gm_\text{S}m/r_2$

$$v_2^2=v_1^2+2Gm_\text{S}\left(\frac{1}{r_2}-\frac{1}{r_1}\right)=v_1^2+2Gm_\text{S}\left(\frac{r_1-r_2}{r_1r_2}\right)$$

$v_2=6.8\times10^4$ m/s

EVALUATE: The comet has greater speed when it is closer to the sun.

13.66. IDENTIFY: $g=\frac{GM}{R^2}$, where M and R are the mass and radius of the planet.

SET UP: Let m_U and R_U be the mass and radius of Uranus and let g_U be the acceleration due to gravity at its poles. The orbit radius of Miranda is $r=h+R_\text{U}$, where $h=1.04\times10^8$ m is the altitude of Miranda above the surface of Uranus.

EXECUTE: **(a)** From the value of g at the poles,

$m_\text{U}=\frac{g_\text{U}R_\text{U}^2}{G}=\frac{(9.0\ \text{m/s}^2)(2.5360\times10^7\ \text{m})^2}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}=8.674\times10^{25}$ kg which rounds to 8.7×10^{25} kg.

(b) $Gm_\text{U}/r^2=g_\text{U}(R_\text{U}/r)^2=0.35\ \text{m/s}^2$.

(c) $Gm_\text{M}/R_\text{M}^2=0.079\ \text{m/s}^2$.

EVALUATE: **(d)** No. Both the object and Miranda are in orbit together around Uranus, due to the gravitational force of Uranus. The object has additional force toward Miranda.

13.67. **(a) IDENTIFY** and **SET UP:** Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_a + r_p = R_E + h_a + R_E + h_p$, where h_a and h_p are the heights at apogee and perigee, respectively.

$a = R_E + (h_a + h_p)/2$

$a = 6.37\times10^6\ \text{m} + (400\times10^3\ \text{m} + 4000\times10^3\ \text{m})/2 = 8.57\times10^6\ \text{m}$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = \frac{2\pi(8.57\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 7.90\times10^3\ \text{s}$$

(b) Conservation of angular momentum gives $r_a v_a = r_p v_p$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{6.37\times10^6\ \text{m} + 4.00\times10^6\ \text{m}}{6.37\times10^6\ \text{m} + 4.00\times10^5\ \text{m}} = 1.53.$$

(c) Conservation of energy applied to apogee and perigee gives $K_a + U_a = K_p + U_p$

$\frac{1}{2}mv_a^2 - Gm_E m/r_a = \frac{1}{2}mv_p^2 - Gm_E m/r_p$

$v_p^2 - v_a^2 = 2Gm_E(1/r_p - 1/r_a) = 2Gm_E(r_a - r_p)/r_a r_p$

But $v_p = 1.532 v_a$, so $1.347 v_a^2 = 2Gm_E(r_a - r_p)/r_a r_p$

$v_a = 5.51\times10^3\ \text{m/s}$, $v_p = 8.43\times10^3\ \text{m/s}$

(d) Need v so that $E = 0$, where $E = K + U$.

at perigee: $\frac{1}{2}mv_p^2 - Gm_E m/r_p = 0$

$$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})/(6.77\times10^6\ \text{m})} = 1.085\times10^4\ \text{m/s}$$

This means an increase of $1.085\times10^4\ \text{m/s} - 8.43\times10^3\ \text{m/s} = 2.42\times10^3\ \text{m/s}$.

at apogee:

$$v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})/(1.037\times10^7\ \text{m})} = 8.763\times10^3\ \text{m/s}$$

This means an increase of $8.763\times10^3\ \text{m/s} - 5.51\times10^3\ \text{m/s} = 3.25\times10^3\ \text{m/s}$.

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

13.68. **IDENTIFY:** The engines do work on the rocket and change its kinetic energy and gravitational potential energy.

SET UP: Call M the mass of the earth and m the mass of the rocket. $U_g = -GMm/r$, $K = \frac{1}{2}mMG/r$ for a circular orbit, $K_1 + U_1 + W_{other} = K_2 + U_2$.

EXECUTE: **(a)** For a circular orbit, using $K = \frac{1}{2}mMG/r$ gives the difference in kinetic energy: $K_2 - K_1 = \frac{1}{2}mMG(1/r_2 - 1/r_1)$. Using the given numbers $m = 5000$ kg, $M = 5.97\times10^{24}$ kg, $r_2 = 8.80\times10^6$ m, and $r_1 = 7.20\times10^6$ m, we get $K_2 - K_1 = -2.52\times10^{10}$ J. The minus sign tells us that the kinetic energy decreases.

(b) $U_2 - U_1 = -GmM/r_2 - (-GmM/r_1) = GmM(1/r_1 - 1/r_2) = -2(K_2 - K_1) = +5.03\times10^{10}$ J. The plus sign means that the energy increases.

(c) $K_1 + U_1 + W_{other} = K_2 + U_2$ gives

$W_{other} = (K_2 - K_1) + (U_2 - U_1) = -2.51\times10^{10}\ \text{J} + 5.03\times10^{10}\ \text{J} = +2.51\times10^{10}\ \text{J}$.

EVALUATE: In part (b), the potential energy increases because it becomes less negative. The work is positive because the total energy increases.

13.69. **IDENTIFY** and **SET UP:** Apply conservation of energy, $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, and solve for W_{other}. Only $r = h + R_{\text{E}}$ is given, so use $v = \sqrt{GM/r}$ to relate *r* and *v*.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$U_1 = -Gm_{\text{M}}m/r_1$, where m_{M} is the mass of Mars and $r_1 = R_{\text{M}} + h$, where R_{M} is the radius of Mars and $h = 2000\times10^3$ m.

$$U_1 = -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(5000\ \text{kg})}{3.39\times10^6\ \text{m} + 2000\times10^3\ \text{m}} = -3.97230\times10^{10}\ \text{J}$$

$U_2 = -Gm_{\text{M}}m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(5000\ \text{kg})}{3.39\times10^6\ \text{m} + 4000\times10^3\ \text{m}} = -2.89725\times10^{10}\ \text{J}$$

For a circular orbit $v = \sqrt{Gm_{\text{M}}/r}$, with the mass of Mars rather than the mass of the earth.

Using this gives $K = \frac{1}{2}mv^2 = \frac{1}{2}m(Gm_{\text{M}}/r) = \frac{1}{2}Gm_{\text{M}}m/r$, so $K = -\frac{1}{2}U$.

$K_1 = -\frac{1}{2}U_1 = +1.98615\times10^{10}$ J and $K_2 = -\frac{1}{2}U_2 = +1.44863\times10^{10}$ J

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$W_{\text{other}} = (K_2 - K_1) + (U_2 - U)$

$W_{\text{other}} = (1.44863\times10^{10}\ \text{J} - 1.98615\times10^{10}\ \text{J}) + (+3.97230\times10^{10}\ \text{J} - 2.89725\times10^{10}\ \text{J})$

$W_{\text{other}} = 5.38\times10^9$ J.

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.

13.70. **IDENTIFY:** The engines do work on the rocket and change its kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mMG/r = -\frac{1}{2}U_{\text{g}}$ for a circular orbit; $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ and $U_{\text{g}} = -2K$. Combining these two equations gives $K_1 - 2K_1 + W_{\text{other}} = K_2 - 2K_2$, so $K_2 = K_1 - W_{\text{other}}$. This gives $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - W_{\text{other}}$. Solving for v_2 gives

$v_2 = \sqrt{v_1^2 - \dfrac{2W_{\text{other}}}{m}} = \sqrt{(9640\ \text{m/s})^2 - \dfrac{2(-7.50\times10^9\ \text{J})}{848\ \text{kg}}}$ = 10,500 m/s, which is greater than v_1, so the speed increases.

EVALUATE: The work is negative, yet the speed increases. The potential energy decreases (becomes more negative), so the total energy decreases. Due to the friction, the satellite will go to a lower orbit (closer to the earth), so it must have a greater speed to remain in orbit.

13.71. **IDENTIFY:** Integrate $dm = \rho dV$ to find the mass of the planet. Outside the planet, the planet behaves like a point mass, so at the surface $g = GM/R^2$.

SET UP: A thin spherical shell with thickness dr has volume $dV = 4\pi r^2 dr$. The earth has radius $R_{\text{E}} = 6.37\times10^6$ m.

EXECUTE: Get $M: M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr$. The density is $\rho = \rho_0 - br$, where $\rho_0 = 15.0\times10^3\ \text{kg/m}^3$ at the center and at the surface, $\rho_{\text{S}} = 2.0\times10^3\ \text{kg/m}^3$, so $b = \dfrac{\rho_0 - \rho_s}{R}$.

$$M = \int_0^R (\rho_0 - br)\, 4\pi r^2 dr = \frac{4\pi}{3}\rho_0 R^3 - \pi b R^4 = \frac{4}{3}\pi R^3 \rho_0 - \pi R^4 \left(\frac{\rho_0 - \rho_s}{R}\right) = \pi R^3 \left(\frac{1}{3}\rho_0 + \rho_s\right) \text{ and}$$

$M = 5.71 \times 10^{24}$ kg. Then $g = \dfrac{GM}{R^2} = \dfrac{G\pi R^3(\frac{1}{3}\rho_0 + \rho_s)}{R^2} = \pi R G\left(\dfrac{1}{3}\rho_0 + \rho_s\right).$

$$g = \pi(6.37 \times 10^6 \text{ m})(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)\left(\frac{15.0 \times 10^3 \text{ kg/m}^3}{3} + 2.0 \times 10^3 \text{ kg/m}^3\right).$$

$g = 9.34 \text{ m/s}^2.$

EVALUATE: The average density of the planet is

$\rho_{av} = \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3(5.71 \times 10^{24} \text{ kg})}{4\pi(6.37 \times 10^6 \text{ m})^3} = 5.27 \times 10^3 \text{ kg/m}^3.$ Note that this is not $(\rho_0 + \rho_s)/2.$

13.72. IDENTIFY and SET UP: Use $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to calculate a.

$T = 30{,}000 \text{ y}(3.156 \times 10^7 \text{ s/1 y}) = 9.468 \times 10^{11} \text{ s}$

EXECUTE: $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, which gives $T^2 = \dfrac{4\pi^2 a^3}{Gm_S}$, so $a = \left(\dfrac{Gm_S T^2}{4\pi^2}\right)^{1/3} = 1.4 \times 10^{14}$ m.

EVALUATE: The average orbit radius of Pluto is 5.9×10^{12} m (Appendix F); the semi-major axis for this comet is larger by a factor of 24. Converting to meters gives 4.3 light years = $4.3(9.461 \times 10^{15} \text{ m})$ = 4.1×10^{16} m. The distance of Alpha Centauri is larger by a factor of 300. The orbit of the comet extends well past Pluto but is well within the distance to Alpha Centauri.

13.73. IDENTIFY: The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the sphere. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Figure E13.35 in the textbook, toward the ring.

SET UP: Divide the ring into infinitesimal elements with mass dM.

EXECUTE: Each mass element dM of the ring exerts a force of magnitude $\dfrac{(Gm)dM}{a^2 + x^2}$ on the sphere,and the x-component of this force is $\dfrac{GmdM}{a^2 + x^2}\dfrac{x}{\sqrt{a^2 + x^2}} = \dfrac{GmdMx}{(a^2 + x^2)^{3/2}}.$

Therefore, the force on the sphere is $GmMx/(a^2 + x^2)^{3/2}$, in the $-x$-direction. The sphere attracts the ring with a force of the same magnitude.

EVALUATE: As $x \gg a$ the denominator approaches x^3 and $F \to \dfrac{GMm}{x^2}$, as expected.

13.74. IDENTIFY and SET UP: Use $F_g = Gm_1 m_2/r^2$ to calculate the force between the point mass and a small segment of the semicircle.

EXECUTE: The radius of the semicircle is $R = L/\pi.$

Divide the semicircle up into small segments of length $R\, d\theta$, as shown in Figure 13.74.

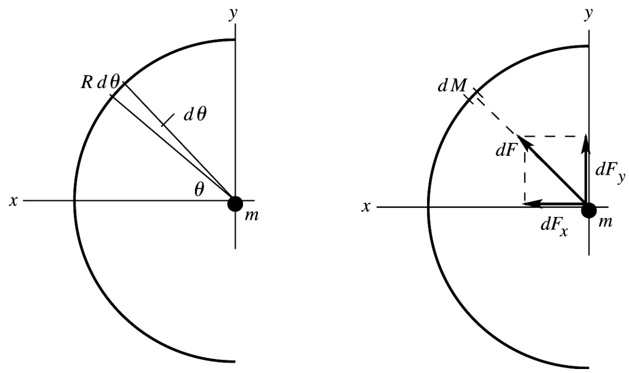


Figure 13.74

$dM = (M/L)R\,d\theta = (M/\pi)\,d\theta$

$d\vec{F}$ is the gravity force on m exerted by dM.

$\int dF_y = 0$; the y-components from the upper half of the semicircle cancel the y-components from the lower half.

The x-components are all in the $+x$-direction and all add.

$$dF = G\frac{mdM}{R^2}$$

$$dF_x = G\frac{mdM}{R^2}\cos\theta = \frac{Gm\pi M}{L^2}\cos\theta\,d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{Gm\pi M}{L^2}(2)a$$

$$F = \frac{2\pi GmM}{L^2}$$

EVALUATE: If the semicircle were replaced by a point mass M at $x = R$, the gravity force would be $GmM/R^2 = \pi^2 GmM/L^2$. This is $\pi/2$ times larger than the force exerted by the semicirclar wire. For the semicircle it is the x-components that add, and the sum is less than if the force magnitudes were added.

13.75. **IDENTIFY:** Compare F_E to Hooke's law.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $R_E = 6.37\times10^6$ m.

EXECUTE: **(a)** For $F_x = -kx$, $U = \frac{1}{2}kx^2$. The force here is in the same form, so by analogy $U(r) = \frac{Gm_E m}{2R_E^3}r^2$. This is also given by the integral of F_g from 0 to r with respect to distance.

(b) From part (a), the initial gravitational potential energy is $\frac{Gm_E m}{2R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives $v^2 = \frac{Gm_E}{R_E}$, so $v = 7.91\times10^3$ m/s.

EVALUATE: When $r = 0$, $U(r) = 0$, as specified in the problem.

13.76. **IDENTIFY:** Kepler's third law applies to the planets.

SET UP: $T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$.

EXECUTE: Squaring $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ gives $T^2 = \left(\dfrac{4\pi^2}{Gm_S}\right)a^3$. If we graph T^2 versus a^3, the equation is of the slope-y-intercept form of a straight line, $y = mx + b$. In this case, the slope is $(4\pi^2/Gm_S)$, and the y-intercept is zero. Take logs of both sides of the equation in Kepler's third law, giving $\log T = \log\left[\left(\dfrac{2\pi}{\sqrt{Gm_S}}\right)a^{3/2}\right]$, which can be written as $\log T = \dfrac{3}{2}\log a + \log\left(\dfrac{2\pi}{\sqrt{Gm_s}}\right)$. Therefore graphing $\log T$ versus $\log a$ should yield a straight line. Figure 13.76 shows this graph.

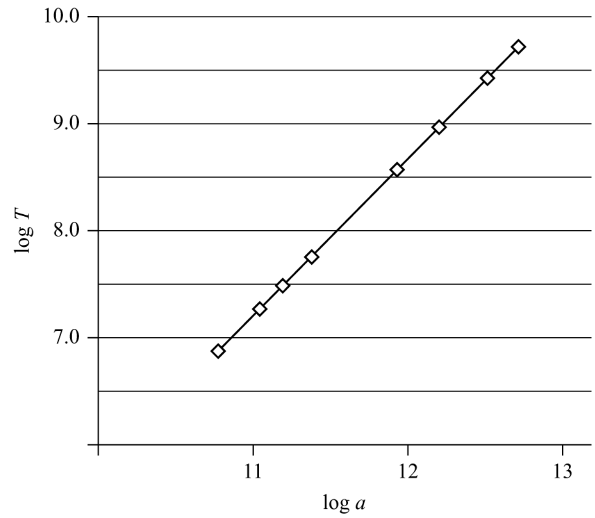


Figure 13.76

(b) In the slope-y-intercept form $y = mx + b$, the slope is 3/2. The best-fit equation of the graph in the figure is $y = 1.4986x - 9.2476$. Since 1.4986 rounds to 1.50, which is equal to 3/2, our graph has the expected slope.

(c) The y-intercept is $\log\left(\dfrac{2\pi}{\sqrt{Gm_S}}\right) = -9.2476$. Solving for m_S, we get $\dfrac{2\pi}{\sqrt{Gm_S}} = 10^{-9.2476}$. Squaring and solving for m_S gives $m_S = 4\pi^2[10^{2(9.2476)}]/G = 1.85\times10^{30}$ kg. From Appendix F, $m_S = 1.99\times10^{30}$ kg. Our result agrees to within about 7% with the value in Appendix F.

(d) Solving Kepler's third law for a gives $a = \left(\dfrac{T\sqrt{Gm_s}}{2\pi}\right)^{2/3}$. Expressing T in seconds, we get $T = 1325.4$ d (24 h/d)(3600 s/h) $= 1.145\times10^8$ s. Putting in $G = 6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ and $m_S = 1.99\times10^{30}$ kg, we get $a = 3.53\times10^{11}$ m $= 353\times10^6$ km. From the table with the problem, we see that Vesta's orbit lies between that of Mars and Jupiter.

EVALUATE: The asteroid belt lies between Mars and Jupiter, which is why Vesta is usually considered an asteroid.

13.77. IDENTIFY and SET UP: At the surface of a planet, $g = \dfrac{GM}{R^2}$, and average density is $\rho = m/V$, where $V = 4/3\ \pi R^3$ for a sphere.

EXECUTE: We have expressions for g and M: $g = \dfrac{GM}{R^2}$ and $M = \rho V = \rho\left(\dfrac{4}{3}\pi R^3\right)$. Combining them we get $g = \dfrac{G\rho\left(\dfrac{4}{3}\pi R^3\right)}{R^2} = \dfrac{4\pi G\rho R}{3}$. Using $R = D/2$ gives $g = \dfrac{2\pi G\rho D}{3}$.

(a) A graph of g versus D is shown in Figure 13.77. As this graph shows, the densities vary considerably and show no apparent pattern.

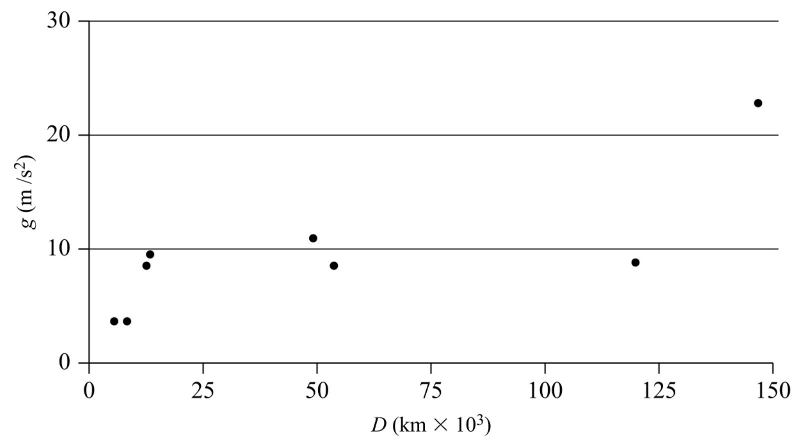


Figure 13.77

(b) Using the equation we just derived, $g = \dfrac{2\pi G\rho D}{3}$, we solve for ρ and use the values from the table given in the problem. For example, for Mercury we have

$\rho = \dfrac{3g}{2\pi DG} = \dfrac{3(3.7\ \text{m/s}^2)}{2\pi(4.879\times10^6\ \text{m})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 5400\ \text{kg/m}^3$. Continuing the calculations and putting the results in order of decreasing density, we get the following results.

Earth: 5500 kg/m^3
Mercury: 5400 kg/m^3
Venus: 5300 kg/m^3
Mars: 3900 kg/m^3
Neptune: 1600 kg/m^3
Uranus: 1200 kg/m^3
Jupiter: 1200 kg/m^3
Saturn: 534 kg/m^3

(c) For several reasons, it is reasonable that the other planets would be denser toward their centers. Gravity is stronger at close distances, so it would compress matter near the center. In addition, during the formation of planets, heavy elements would tend to sink toward the center and displace light elements, much as a rock sinks in water. This variation in density would have no effect on our analysis however, since the planets are still spherically symmetric.

(d) $g = \dfrac{2\pi G\rho D}{3} = \dfrac{2\pi(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.20536\times10^8\ \text{m})(5500\ \text{kg/m}^3)}{3} = 93\ \text{m/s}^2$.

EVALUATE: Saturn is less dense than water, so it would float if we could throw it into our ocean (which of course is impossible since it is much larger than the earth). This low density is the reason that g at its "surface" is less than g at the earth's surface, even though the mass of Saturn is much greater than that of the earth. Also note in our results in (b) that the inner four planets are much denser than the outer four (the gas giants), with the earth being the densest of all.

13.78. **IDENTIFY** and **SET UP:** $L = MvR$, $v = 2\pi R/T$.

EXECUTE: (a) Combining the two equations gives $L = mvR = m(2\pi R/T)R = 2\pi mR^2/T$.

(b) We use our formula with the quantities in Appendix F. For example, for Mercury we have $L = 2\pi(3.30\times10^{23}\text{ kg})(5.79\times10^{10}\text{ m})^2/[(88.0\text{ d})(24\text{ h/d})(3600\text{ s/h})] = 9.14\times10^{38}\ \text{kg}\cdot\text{m}^2/\text{s}$. Similar calculations give the following results.

Mercury: 9.14×10^{38} kg · m^2/s
Venus: 1.84×10^{40} kg · m^2/s
Earth: 2.67×10^{40} kg · m^2/s
Mars: 3.53×10^{39} kg · m^2/s
Jupiter: 1.93×10^{43} kg · m^2/s
Saturn: 7.86×10^{42} kg · m^2/s
Uranus: 1.73×10^{42} kg · m^2/s
Neptune: 2.50×10^{42} kg · m^2/s

The total angular momentum is the sum of all of these since the planets all move in the same direction around the sun. $L_{\text{tot}} = 3.13\times10^{43}\ \text{kg}\cdot\text{m}^2/\text{s}$.

(c) Treating the sun as a uniform solid sphere, we have

$$L = I\omega = \left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right) = \frac{4\pi MR^2}{5T} = \frac{4\pi(1.99\times10^{30}\text{ kg})(6.96\times10^8\text{ m})^2}{5(24.6\text{ d})(24\text{ h/d})(3600\text{ s/h})} = 1.14\times10^{42}\ \text{kg}\cdot\text{m}^2/\text{s}.$$

(d) $L_S/L_P = (1.14\times10^{42}\ \text{kg}\cdot\text{m}^2/\text{s})/(3.14\times10^{43}\ \text{kg}\cdot\text{m}^2/\text{s}) = 0.0363$. The angular momentum of the sun is only 3.63% of the angular momentum of the planets. From Appendix F, the total mass of the planets is $m_P = 2.669\times10^{27}$ kg, so $m_S/m_P = (1.99\times10^{30}\text{ kg})/(2.669\times10^{27}\text{ kg}) = 746$, so the sun is 746 times as massive as the planets combined.

EVALUATE: The sun contains nearly all the mass in the solar system, yet it has only 3.6% of the angular momentum. There appears to be no progressive pattern in the angular momentum of the planets as we go from the inner plants to the outer ones.

13.79. **IDENTIFY:** Apply $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$ to the transfer orbit.

SET UP: The orbit radius for earth is $r_E = 1.50\times10^{11}$ m and for Mars it is $r_M = 2.28\times10^{11}$ m. From Figure 13.18 in the textbook, $a = \frac{1}{2}(r_E + r_M)$.

EXECUTE: **(a)** To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Figure 13.18 in the textbook, the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from $E = -Gm_Sm/2r$, the energy of the spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion.

(b) The time will be half the period as given in $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_S}}$, with the semimajor axis equal to

$a = \frac{1}{2}(r_E + r_M) = 1.89\times10^{11}$ ma so

$$t = \frac{T}{2} = \frac{\pi\ (1.89\times10^{11}\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}} = 2.24\times10^7\text{ s} = 259\text{ days},$$ which is more than $8\frac{1}{2}$ months.

(c) During this time, Mars will pass through an angle of $(360°)\dfrac{(2.24\times 10^7\text{ s})}{(687\text{ d})(86{,}400\text{ s/d})}=135.9°,$ and the spacecraft passes through an angle of $180°,$ so the angle between the earth-sun line and the Mars-sun line must be $44.1°$.

EVALUATE: The period T for the transfer orbit is 526 days, the average of the orbital periods for earth and Mars.

13.80. IDENTIFY: Apply $\Sigma\vec{F}=m\vec{a}$ to each ear.

SET UP: Denote the orbit radius as r and the distance from this radius to either ear as δ. Each ear, of mass m, can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by F.

EXECUTE: The force equation for either ear is $\dfrac{GMm}{(r+\delta)^2}-F=m\omega^2(r+\delta),$ where δ can be of either sign. Replace the product $m\omega^2$ with the value for $\delta=0,$ $m\omega^2=GMm/r^3,$ and solve for F:

$$F=(GMm)\left[\frac{r+\delta}{r^3}-\frac{1}{(r+\delta)^2}\right]=\frac{GMm}{r^3}\left[r+\delta-r(1+(\delta/r)^{-2}\right].$$

Using the binomial theorem to expand the term in square brackets in powers of δ/r,

$$F\approx\frac{GMm}{r^3}[r+\delta-r(1-2(\delta/r))]=\frac{GMm}{r^3}(3\delta)=2.1\text{ kN}.$$

This tension is much larger than that which could be sustained by human tissue, and the astronaut is in trouble.

(b) The center of gravity is not the center of mass. The gravity force on the two ears is not the same.

EVALUATE: The tension between her ears is proportional to their separation.

13.81. IDENTIFY: As suggested in the problem, divide the disk into rings of radius r and thickness dr.

SET UP: Each ring has an area $dA=2\pi r\,dr$ and mass $dM=\dfrac{M}{\pi a^2}dA=\dfrac{2M}{a^2}r\,dr.$

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then $(Gm\,dM)(x/(r^2+x^2)^{3/2}).$ The contribution dF to the force is $dF=\dfrac{2GMmx}{a^2}\dfrac{rdr}{(x^2+r^2)^{3/2}}.$

The total force F is then the integral over the range of r;

$$F=\int dF=\frac{2GMmx}{a^2}\int_0^a\frac{r}{(x^2+r^2)^{3/2}}dr.$$

The integral (either by looking in a table or making the substitution $u=r^2+a^2$) is

$$\int_0^a\frac{r}{(x^2+r^2)^{3/2}}dr=\left[\frac{1}{x}-\frac{1}{\sqrt{a^2+x^2}}\right]=\frac{1}{x}\left[1-\frac{x}{\sqrt{a^2+x^2}}\right].$$

Substitution yields the result $F=\dfrac{2GMm}{a^2}\left[1-\dfrac{x}{\sqrt{a^2+x^2}}\right].$ The force on m is directed toward the center of the ring. The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}}=(1+(a/x)^2)^{-1/2}\approx 1-\frac{1}{2}\left(\frac{a}{x}\right)^2$$

if $x\gg a,$ where the binomial expansion has been used. Substitution of this into the above form gives

$F\approx\dfrac{GMm}{x^2},$ as it should.

EVALUATE: As $x\to 0,$ the force approaches a constant.

13.82. **IDENTIFY** and **SET UP:** Use $\rho = m/V$ to calculate the density of the planet and then use the table given in the problem to estimate its composition.

EXECUTE: Using $\rho = m/V$ gives

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{7.9m_{\text{E}}}{\frac{4}{3}\pi(2.3R_{\text{E}})^3} = \frac{7.9}{(2.3)^3}\frac{m_{\text{E}}}{\frac{4}{3}\pi(R_{\text{E}})^3} = 0.65\rho_{\text{E}}.$$

From the table, this density is in the range 0.4–0.9 times the density of the earth, so the planet probably has an iron core with a rock mantle and some lighter elements, which is choice (c).

EVALUATE: A method such as this gives only an estimation of the composition of a planet.

13.83. **IDENTIFY** and **SET UP:** Use $g = GM/R^2$.

EXECUTE: $g = GM/R^2 = G(7.9m_{\text{E}})/(2.3R_{\text{E}})^2 = [(7.9)/(2.3)^2](Gm_{\text{E}}/R_{\text{E}}^2) = 1.5g_{\text{E}}$, which is choice (c).

EVALUATE: Even though this planet has 7.9 times the mass of the earth, g at its surface is only $1.5g_{\text{E}}$ because the planet is 2.3 times the radius of the earth, which makes the surface farther away from its center than is the case with the earth.

13.84. **IDENTIFY** and **SET UP:** Apply Newton's second law and the law of universal gravitation to the planet, calling m the mass of the planet, M the mass of the star, r the orbital radius, and T the time for one orbit. $\Sigma F = ma$, $F_{\text{g}} = Gm_1m_2/r^2$, $a_{\text{rad}} = mv^2/r$, $v = 2\pi r/T$.

EXECUTE: $\dfrac{GmM}{r^2} = \dfrac{mv^2}{r} = \dfrac{m(2\pi r/T)^2}{r} = \dfrac{4\pi^2 mr}{T^2}$. Now solve for r, which gives

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{G(0.70M_{\text{sun}})\left(\frac{9.5}{365}T_{\text{earth}}\right)^2}{4\pi^2} = (0.70)\left(\frac{9.5}{365}\right)^2\left(\frac{GM_{\text{sun}}T_{\text{earth}}^2}{4\pi^2}\right) = 0.000474r_{\text{earth}}^3$$

$r = (0.000474)^{1/3}r_{\text{earth}} = 0.078r_{\text{earth}}$, which is choice (b).

EVALUATE: The planet takes only 9.5 days for one orbit, yet the star has 70% the mass of our sun, so the planet must be very close to the star compared to the earth. And this is, in fact, what we have found, since r for this planet iss 7.8% the distance of the earth from the sun.