39

PARTICLES BEHAVING AS WAVES

VP39.2.1. **IDENTIFY:** This problem involves electron diffraction and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $a\sin\theta = m\lambda$, $K = eV$.

EXECUTE: **(a)** We want the de Broglie wavelength. First use $K = eV$ and $K = p^2/2m$ to find p, and then use $\lambda = h/p$ to find λ.

$$p = \sqrt{2mK} = \sqrt{2meV}.\ \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}.$$

Using $V = 69.0$ V gives $\lambda = 0.148$ nm.

(b) We want the minimum angle at which a diffraction maximum occurs. Using $m = 1$, $a\sin\theta = m\lambda$ gives $\theta_{\text{min}} = \arcsin(\lambda/d) = \arcsin(0.148/0.172) = 59.1°$.

EVALUATE: There is no other angle at which a maximum occurs. We can use $K = p^2/2m$ because the electron is nonrelativistic at this energy which is much less than its rest energy of 0.511 MeV.

VP39.2.2. **IDENTIFY:** This problem involves electron diffraction and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $d\sin\theta = m\lambda$, $K = eV$.

EXECUTE: **(a)** We want the kinetic energy. $K = eV = e(36.5\text{ V}) = 36.5\text{ eV} = 5.85 \times 10^{-18}$ J.

(b) We want the de Broglie wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = 0.202\text{ nm}.$$

(c) We want the atomic spacing d. $d\sin\theta = m\lambda$ gives d = (0.203 nm)/(sin 48.0°) = 0.273 nm.

EVALUATE: We can use $K = p^2/2m$ because the electron's speed is much less than c.

VP39.2.3. **IDENTIFY:** This problem involves electron diffraction and the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $d\sin\theta = m\lambda$, $K = eV$.

EXECUTE: **(a)** We want the de Broglie wavelength. Solving $d\sin\theta = m\lambda$ with $m = 2$ gives

λ = (0.218 nm)(sin 75.0°)/2 = 0.105 nm.

(b) We want the accelerating voltage V. Solving $eV = K = p^2/2m$ and using $\lambda = h/p$ gives

$$V = \frac{(h/\lambda)^2}{2me} = \frac{h^2}{2me\lambda^2} = \frac{h^2}{2me(0.105\text{ nm})^2} = 136\text{ V}.$$

EVALUATE: The electron's kinetic energy is K = 136 eV, so it is not relativistic.

VP39.2.4. **IDENTIFY:** This problem involves the de Broglie wavelength of a proton.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $K = eV$.

EXECUTE: **(a)** We want the de Broglie wavelength. Using the given speed gives $\lambda = h/mv = 1.67$ pm.
(b) We want the accelerating voltage V. $eV = K = \frac{1}{2}mv^2$. Solving for V using the known quantities gives $V = 295$ V.
EVALUATE: If a proton and electron have comparable speeds, the proton has a *much smaller* de Broglie wavelength because it is much more massive than the electron.

VP39.6.1. **IDENTIFY:** The problem involves the energy due to electron transitions in a hypothetical atom.
SET UP and **EXECUTE:** We want the wavelengths of the emitted light in each case. The possible transitions are $5 \to 0$ and $5 \to 2 \to 0$, and $\Delta E = hc/\lambda$.
$\underline{5 \to 0}$: $\Delta E = 5.00$ eV – 0 = 5.00 eV. $\lambda = hc/\Delta E = hc/(5.00 \text{ eV}) = 248$ nm.
$\underline{5 \to 2}$: $\Delta E = 5.00$ eV – 2.00 = 3.00 eV. $\lambda = hc/\Delta E = hc/(3.00 \text{ eV}) = 414$ nm.
$\underline{2 \to 0}$: $\Delta E = 2.00$ eV – 0 = 2.00 eV. $\lambda = hc/\Delta E = hc/(2.00 \text{ eV}) = 620$ nm.
EVALUATE: The wavelengths emitted by a gas of this atom would be 248 nm, 414 nm, and 620 nm.

VP39.6.2. **IDENTIFY:** The problem involves the energy due to electron transitions in a hypothetical atom.
SET UP: $\Delta E = hc/\lambda$.
EXECUTE: **(a)** $\Delta E_{1\to\text{grd}} = hc/\lambda_1 = hc/(385 \text{ nm}) = 3.22 \text{ eV}$. Therefore $E_1 = 3.22$ eV relative to ground.
$\Delta E_{2\to 1} = hc/\lambda_2 = hc/(674 \text{ nm}) = 1.84 \text{ eV}$. Therefore $E_2 = 1.84$ eV relative to E_1. Relative to ground we have $E_2 = 1.84 \text{ eV} + 3.22 \text{ eV} = 5.06 \text{ eV}$.
(b) $\lambda = hc/\Delta E = hc/(5.06 \text{ eV}) = 245$ nm.
EVALUATE: Note that the energy difference between adjacent levels gets smaller for higher and higher levels.

VP39.6.3. **IDENTIFY:** The problem involves electron transitions in a Bohr hydrogen atom.
SET UP: $\Delta E = hc/\lambda$. For the Bohr hydrogen atom, $E_n = -(13.60 \text{ eV})/n^2$. We want the energy and wavelength of the emitted photon.
EXECUTE: **(a)** $5 \to 3$: $\Delta E = (-13.60 \text{ eV})(1/5^2 - 1/3^2) = 0.967$ eV. $\lambda = hc/\Delta E = hc/(0.967 \text{ eV})$ = 1.28 μm.
(b) $4 \to 2$: $\Delta E = (-13.60 \text{ eV})(1/4^2 - 1/2^2) = 2.55$ eV. $\lambda = hc/\Delta E = hc/(2.55 \text{ eV}) = 487$ nm.
(c) $3 \to 1$: $\Delta E = (-13.60 \text{ eV})(1/3^2 - 1/1^2) = 12.1$ eV. $\lambda = hc/\Delta E = hc/(12.1 \text{ eV}) = 103$ nm.
EVALUATE: Note that as the energy difference increases, the wavelength of the emitted photon decreases. This is reasonable because shorter wavelength photons have more energy than long wavelength photons.

VP39.6.4. **IDENTIFY:** The problem involves the energy due to electron transitions in a hydrogen atom.
SET UP: $\Delta E = hc/\lambda$. For the Bohr hydrogen atom, $E_n = -(13.60 \text{ eV})/n^2$, $K_n = (13.60 \text{ eV})/n^2$, and $U_n = -(27.20 \text{ eV})/n^2$.
EXECUTE: **(a)** We want the difference in kinetic energy. $\Delta E = K_6 - K_2 = (13.60 \text{ eV})(1/6^2 - 1/2^2)$ $= -3.02$ eV.
(b) We want the difference in potential energy. $\Delta U = U_6 - U_2 = (-27.20 \text{ eV})(1/6^2 - 1/2^2) = +3.02 \text{ eV}$.
(c) We want the wavelength of the photon. $\Delta E = hc/\lambda$ gives $\lambda = hc/\Delta E = hc/(6.04 \text{ eV} - 3.02 \text{ eV})$ = 411 nm.
EVALUATE: Check: $\Delta E = K_6 - K_2 = (-13.60 \text{ eV})(1/6^2 - 1/2^2) = +3.02 \text{ eV}$, as we used in part (c).

VP39.8.1. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.
SET UP: $I = \sigma T^4$, Wien law: $\lambda_\text{m} = \dfrac{2.90\times 10^{-3} \text{ m}\cdot\text{K}}{T}$.
EXECUTE: **(a)** We want the peak wavelength. Using $T = 3590$ K, the Wien law gives 808 nm. This wavelength is greater than that of visible light, so it is in the *infrared* region.
(b) We want the intensity I. Using $T = 3590$ K, $I = \sigma T^4$ gives 9.42×10^6 W/m^2.

EVALUATE: Betelgeuse is a red giant. It is red because it radiates most of its visible light in the red end of spectrum.

VP39.8.2. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.

SET UP: $I = \sigma T^4$, Wien law: $\lambda_m = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$.

EXECUTE: **(a)** We want the temperature. Solving $I = \sigma T^4$ for T and using $I = 78.0\ \text{MW/m}^2$ gives $T = 6090$ K.

(b) We want the peak wavelength. Using $T = 6090$ in the Wien law gives $\lambda_m = 476$ nm.

(c) This wavelength is in the visible (to humans) part of the electromagnetic spectrum.

EVALUATE: This blackbody would be bluish because the peak wavelength is toward the blue end of the spectrum.

VP39.8.3. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.

SET UP: $I = \sigma T^4$, Wien law: $\lambda_m = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$.

EXECUTE: **(a)** We want the temperature. Using the Wien law with the peak wavelength at 239 nm gives $T = 12{,}100$ K.

(b) We want the power per unit area. Using $I = \sigma T^4$ with $T = 12{,}100$ K gives $I = 1.23 \times 10^9\ \text{W/m}^2 = 1.23\ \text{GW/m}^2$.

EVALUATE: Rigel is a very hot star. Its peak wavelength is in the ultraviolet part of the electromagnetic spectrum.

VP39.8.4 **IDENTIFY:** This problem involves blackbody radiation, the Planck radiation law, and the Wien law.

SET UP: $I = \sigma T^4$, Planck law: $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}$, Wien law: $\lambda_m = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$.

EXECUTE: **(a)** We want the peak wavelength. Using the Wien law with $T = 3040$ K gives $\lambda_m = 954$ nm.

(b) We want I. The intensity within the range $\Delta\lambda$ is $I \approx I(\lambda)\Delta\lambda$ if $\Delta\lambda$ is small, as it is in this case. Using the Planck law gives

$$I = I(\lambda)\Delta\lambda = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT}-1\right)}\Delta\lambda.$$

Using $\lambda = 954$ nm, $\Delta\lambda = 12.0$ nm, and $T = 3040$ K gives $I = 40.1\ \text{kW/m}^2$.

EVALUATE: The total intensity the star radiates is $I = \sigma T^4 = 4.84\ \text{MW/m}^2$ at $T = 3040$ K. So the fraction in the 12 nm range is only $(40.1\ \text{kW/m}^2)/(4.84\ \text{MW/m}^2) = 0.83\%$.

39.1. **IDENTIFY** and **SET UP:** $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. For an electron, $m = 9.11\times10^{-31}$ kg. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\lambda = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(4.70\times10^{6}\ \text{m/s})} = 1.55\times10^{-10}\ \text{m} = 0.155\ \text{nm}.$

(b) λ is proportional to $\dfrac{1}{m}$, so $\lambda_p = \lambda_e\left(\dfrac{m_e}{m_p}\right) = (1.55\times10^{-10}\ \text{m})\left(\dfrac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 8.46\times10^{-14}\ \text{m}.$

EVALUATE: For the same speed the proton has a smaller de Broglie wavelength.

39.2. **IDENTIFY** and **SET UP:** For a photon, $E = \dfrac{hc}{\lambda}$. For an electron or alpha particle, $p = \dfrac{h}{\lambda}$ and $E = \dfrac{p^2}{2m}$, so $E = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{0.20\times10^{-9}\ \text{m}} = 6.2\ \text{keV}.$

(b) $E = \dfrac{h^2}{2m\lambda^2} = \left(\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}}\right)^2 \dfrac{1}{2(9.11\times10^{-31}\ \text{kg})} = 6.03\times10^{-18}\ \text{J} = 38\ \text{eV}.$

(c) $E_{\text{alpha}} = E_{\text{e}}\left(\dfrac{m_{\text{e}}}{m_{\text{alpha}}}\right) = (38\ \text{eV})\left(\dfrac{9.11\times10^{-31}\ \text{kg}}{6.64\times10^{-27}\ \text{kg}}\right) = 5.2\times10^{-3}\ \text{eV}.$

EVALUATE: For a given wavelength a photon has much more energy than an electron, which in turn has more energy than a alpha particle.

39.3. **IDENTIFY:** For a particle with mass, $\lambda = \dfrac{h}{p}$ and $K = \dfrac{p^2}{2m}.$

SET UP: $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}.$

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{h}{\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(2.80\times10^{-10}\ \text{m})} = 2.37\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \dfrac{p^2}{2m} = \dfrac{(2.37\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 3.08\times10^{-18}\ \text{J} = 19.3\ \text{eV}.$

EVALUATE: This wavelength is on the order of the size of an atom. This energy is on the order of the energy of an electron in an atom.

39.4. **IDENTIFY:** For a particle with mass, $\lambda = \dfrac{h}{p}$ and $E = \dfrac{p^2}{2m}.$

SET UP: $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}.$

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(6.64\times10^{-27}\ \text{kg})\,(4.20\times10^{6}\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}} = 7.02\times10^{-15}\ \text{m}.$

EVALUATE: This wavelength is on the order of the size of a nucleus.

39.5. **IDENTIFY** and **SET UP:** The de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}.$

EXECUTE: The de Broglie wavelength is the same for the proton and the electron, so $\dfrac{h}{m_{\text{e}}v_{\text{e}}} = \dfrac{h}{m_{\text{p}}v_{\text{p}}}.$

$v_{\text{p}} = v_{\text{e}}(m_{\text{e}}/m_{\text{p}}) = (8.00\times10^{6}\ \text{m/s})[(9.109\times10^{-31}\ \text{kg})/(1.6726\times10^{-27}\ \text{kg})] = 4360\ \text{m/s} = 4.36\ \text{km/s}.$

EVALUATE: The proton and electron have the same de Broglie wavelength and the same momentum, but very different speeds because $m_{\text{p}} >> m_{\text{e}}$.

39.6. **IDENTIFY:** This problem is about the de Broglie wavelength.

SET UP: $\lambda = h/p$, $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}$. An electron has mass $9.11\times10^{-31}\ \text{kg}$.

EXECUTE: **(a)** For a nonrelativistic particle, $K = \dfrac{p^2}{2m}$, so $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2Km}}.$

(b) $(6.63\times10^{-34}\ \text{J}\cdot\text{s})/\sqrt{2(800\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})(9.11\times10^{-31}\ \text{kg})} = 4.34\times10^{-11}\ \text{m}.$

EVALUATE: The de Broglie wavelength decreases when the kinetic energy of the particle increases.

39.7. **IDENTIFY:** This problem is about the de Broglie wavelength.

SET UP: $\lambda = h/p$, $K = p^2/2m$, $p = m\gamma v$, $K = mc^2(\gamma - 1)$, $v = c\sqrt{1-1/\gamma^2}$. We want the de Broglie wavelength in each case.

EXECUTE: **(a)** $\lambda = h/p = h/(50.0\ \text{kg})(2.0\ \text{m/s}) = 6.6\times10^{-36}\ \text{m}.$

(b) The kinetic energy of the electron (2.0 MeV) is considerably greater than the rest energy (0.511 MeV) of the electron, so we must use the relativistic equations. $K = mc^2(\gamma - 1)$ gives $2.0 \text{ MeV} = (0.511 \text{ MeV})(\gamma - 1)$, so $\gamma = 4.914$. Now find the wavelength. Combining $\lambda = h/p$, $p = m\gamma v$, and $v = c\sqrt{1 - 1/\gamma^2}$ gives $\lambda = \dfrac{h}{mc\sqrt{\gamma^2 - 1}}$. Using $\gamma = 4.914$ gives $v = 5.0 \times 10^{-13}$ m/s.

(c) This electron is not relativistic, so we use $K = p^2/2m$. Solving for v and using $\lambda = h/p$ gives $\lambda = \dfrac{h}{\sqrt{2mK}}$. Using $K = 20 \text{ eV} = 3.20 \times 10^{-18}$ J, we get $\lambda = 0.27$ nm.

EVALUATE: From part (a) we see that for ordinary everyday objects, the de Broglie wavelength is extremely small, much less than for things like electrons.

39.8. IDENTIFY and **SET UP:** Combining $E = \gamma mc^2$ and $E^2 = (mc^2)^2 + (pc)^2$ gives $p = mc\sqrt{\gamma^2 - 1}$.

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} = (h/mc)/\sqrt{\gamma^2 - 1} = 4.43 \times 10^{-12}$ m. (The incorrect nonrelativistic calculation gives 5.05×10^{-12} m.)

(b) $(h/mc)/\sqrt{\gamma^2 - 1} = 7.07 \times 10^{-13}$ m.

EVALUATE: The de Broglie wavelength decreases when the speed increases.

39.9. IDENTIFY and **SET UP:** Use $\lambda = h/p$.

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(5.00 \times 10^{-3} \text{ kg})(340 \text{ m/s})} = 3.90 \times 10^{-34}$ m.

EVALUATE: This wavelength is extremely short; the bullet will not exhibit wavelike properties.

39.10. IDENTIFY: Apply conservation of energy to relate the potential difference to the speed of the electrons.

SET UP: The mass of an electron is $m = 9.11 \times 10^{-31}$ kg. The energy of a photon is $E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** $\lambda = h/mv \to v = h/m\lambda$. Energy conservation gives $e\Delta V = \frac{1}{2}mv^2$.

$$\Delta V = \frac{mv^2}{2e} = \frac{m\left(\dfrac{h}{m\lambda}\right)^2}{2e} = \frac{h^2}{2em\lambda^2} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(1.60 \times 10^{-19} \text{ C})(9.11 \times 10^{-31} \text{ kg})\,(0.220 \times 10^{-9} \text{ m})^2} = 31.1 \text{ V}.$$

(b) $E_{\text{photon}} = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})\,(3.00 \times 10^8 \text{ m/s})}{0.220 \times 10^{-9} \text{ m}} = 9.035 \times 10^{-16}$ J. $e\Delta V = K = E_{\text{photon}}$ and

$$\Delta V = \frac{E_{\text{photon}}}{e} = \frac{9.035 \times 10^{-16} \text{ J}}{1.60 \times 10^{-19} \text{ C}} = 5650 \text{ V}.$$

EVALUATE: The electron in part (b) has wavelength $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$= \dfrac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(9.035 \times 10^{-16} \text{ J})}} = 0.0163$ nm, which is much shorter than the 0.220-nm wavelength of a photon of the same energy.

39.11. IDENTIFY: The acceleration gives momentum to the electrons. We can use this momentum to calculate their de Broglie wavelength.

SET UP: The kinetic energy K of the electron is related to the accelerating voltage V by $K = eV$. For an electron $E = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$ and $\lambda = \dfrac{h}{p}$. For a photon $E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** For an electron $p = \dfrac{h}{\lambda} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.00\times10^{-9}\ \text{m}} = 1.33\times10^{-25}\ \text{kg}\cdot\text{m/s}$ and

$E = \dfrac{p^2}{2m} = \dfrac{(1.33\times10^{-25}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 9.71\times10^{-21}\ \text{J}$. $V = \dfrac{K}{e} = \dfrac{9.71\times10^{-21}\ \text{J}}{1.60\times10^{-19}\ \text{C}} = 0.0607\ \text{V}$. The electrons would have kinetic energy 0.0607 eV.

(b) $E = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{5.00\times10^{-9}\ \text{m}} = 248\ \text{eV}$.

(c) $E = 9.71\times10^{-21}\ \text{J}$, so $\lambda = \dfrac{hc}{E} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{9.71\times10^{-21}\ \text{J}} = 20.5\ \mu\text{m}$.

EVALUATE: If they have the same wavelength, the photon has vastly more energy than the electron.

39.12. **IDENTIFY:** The electrons behave like waves and are diffracted by the slit.

SET UP: We use conservation of energy to find the speed of the electrons, and then use this speed to find their de Broglie wavelength, which is $\lambda = h/mv$. Finally we know that the first dark fringe for single-slit diffraction occurs when $a\sin\theta = \lambda$. The relativistic kinetic energy is $K = (\gamma-1)mc^2$.

EXECUTE: **(a)** The electrons gain kinetic energy K as they are accelerated through a potential difference V, so $eV = K = (\gamma-1)mc^2$. The potential difference is 0.100 kV, so eV = 0.100 keV. Therefore

$$eV = K = (\gamma-1)mc^2 = 0.100\ \text{keV}.$$

Solving for γ and using the fact that the rest energy of an electron is 0.511 MeV, we have

$$\gamma - 1 = (0.100\ \text{keV})/(0.511\ \text{MeV}) = (0.100\ \text{keV})/(511\ \text{keV}) = 1.96\times10^{-4}$$

so $\gamma << 1$ which means that we do not have to use special relativity.

(b) Use energy conservation to find the speed of the electron: $\frac{1}{2}mv^2 = eV$.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(100\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 5.93\times10^6\ \text{m/s}.$$

Now find the de Broglie wavelength:

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(5.93\times10^6\ \text{m/s})} = 1.23\times10^{-10}\ \text{m} = 0.123\ \text{nm}.$$

For the first single-slit dark fringe, we have $a\sin\theta = \lambda$, which gives

$$a = \frac{\lambda}{\sin\theta} = \frac{1.23\times10^{-10}\ \text{m}}{\sin(14.6°)} = 4.88\times10^{-10}\ \text{m} = 0.488\ \text{nm}.$$

EVALUATE: The slit width is around 4 times the de Broglie wavelength of the electron, and both are much smaller than the wavelength of visible light.

39.13. **IDENTIFY:** The intensity maxima are located by $d\sin\theta = m\lambda$. Use $\lambda = \dfrac{h}{p}$ for the wavelength of the neutrons. For a particle, $p = \sqrt{2mE}$.

SET UP: For a neutron, $m = 1.675\times10^{-27}\ \text{kg}$.

EXECUTE: For $m = 1$, $\lambda = d\sin\theta = \dfrac{h}{\sqrt{2mE}}$.

$$E = \frac{h^2}{2md^2\sin^2\theta} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.675\times10^{-27}\ \text{kg})(9.10\times10^{-11}\ \text{m})^2\sin^2(28.6°)} = 6.91\times10^{-20}\ \text{J} = 0.432\ \text{eV}.$$

EVALUATE: The neutrons have $\lambda = 0.0436\ \text{nm}$, comparable to the atomic spacing.

39.14. **IDENTIFY:** $\lambda = h/p$. Conservation of energy gives $eV = K = \dfrac{p^2}{2m}$, where V is the accelerating voltage.

SET UP: The electron mass is 9.11×10^{-31} kg and the proton mass is 1.67×10^{-27} kg.

EXECUTE: **(a)** $eV = K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$, so $V = \dfrac{(h/\lambda)^2}{2me} = 419$ V.

(b) The voltage is reduced by the ratio of the particle masses, $(419\text{ V})\dfrac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = 0.229$ V.

EVALUATE: $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. For the same λ, particles of greater mass have smaller E, so a smaller accelerating voltage is needed for protons.

39.15. **IDENTIFY:** We are comparing the wavelengths of a photon and an electron having the same energy.
SET UP: $\lambda = h/p$, $E = hc/\lambda$. We want the wavelengths.

EXECUTE: **(a)** Photon: $\lambda = \dfrac{hc}{E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})c}{6.00\text{ eV}} = 207$ nm.

(b) Electron: Combine $\lambda = h/p$ and $K = p^2/2m$ to get $\lambda = \dfrac{h}{\sqrt{2mK}}$. Using $K = 6.00\text{ eV} = 9.60\times10^{-19}$ J gives $\lambda = 0.501$ nm.

(c) The photon has a *much* longer wavelength than the electron.
EVALUATE: Notice that if a particle has the same kinetic energy as the energy of a photon, the particle has a much shorter wavelength.

39.16. **IDENTIFY:** We are comparing the energy of a photon and an electron having the same wavelength of 500 nm.

SET UP: $\lambda = h/p$, $E = hc/\lambda$, $K = p^2/2m$. We want the energy of each one.

EXECUTE: **(a)** Photon: Using the 500 nm wavelength gives $E = hc/\lambda = 2.48$ eV.

Electron: Combine $\lambda = h/p$ and $K = p^2/2m$ to get $K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$. Using 500 nm for the wavelength gives $K = 9.639\times10^{-25}\text{ J} = 6.02\times10^{-6}$ eV.

(b) The photon has much more energy than the electron.
EVALUATE: Note that of a photon and particle have the same wavelength, the photon has much more energy than the particle.

39.17. **(a) IDENTIFY:** If the particles are treated as point charges, $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}$.

SET UP: $q_1 = 2e$ (alpha particle); $q_2 = 82e$ (lead nucleus); r is given so we can solve for U.

EXECUTE: $U = (8.987\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(82)(1.602\times10^{-19}\text{ C})^2}{6.50\times10^{-14}\text{ m}} = 5.82\times10^{-13}$ J

$$U = 5.82\times10^{-13}\text{ J }(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 3.63\times10^6\text{ eV} = 3.63\text{ MeV}$$

(b) IDENTIFY: Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$.

SET UP: Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$.

EXECUTE: Conservation of energy thus says $K_1 = U_2 = 5.82\times10^{-13}\ \text{J} = 3.63\ \text{MeV}$.

(c) $K = \frac{1}{2}mv^2$, so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.82\times10^{-13}\ \text{J})}{6.64\times10^{-27}\ \text{kg}}} = 1.32\times10^7\ \text{m/s}$.

EVALUATE: $v/c = 0.044$, so it is ok to use the nonrelativistic expression to relate K and v. When the alpha particle stops, all its initial kinetic energy has been converted to electrostatic potential energy.

39.18. IDENTIFY: The kinetic energy of the alpha particle is all converted to electrical potential energy at closest approach. The force on the alpha particle is the electrical repulsion of the nucleus.

SET UP: The electrical potential energy of the system is $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r}$.

EXECUTE: **(a)** Equating the initial kinetic energy and the final potential energy and solving for the separation radius r gives

$$r = \frac{1}{4\pi\epsilon_0}\frac{(92e)(2e)}{K} = \frac{1}{4\pi\epsilon_0}\frac{(184)(1.60\times10^{-19}\ \text{C})^2}{(4.78\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 5.54\times10^{-14}\ \text{m}.$$

(b) The above result may be substituted into Coulomb's law. Alternatively, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulomb field is $F = \dfrac{|U|}{r}$.

$|U| = K$, so $F = \dfrac{K}{r} = \dfrac{(4.78\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/ev})}{(5.54\times10^{-14}\ \text{m})} = 13.8\ \text{N}$.

EVALUATE: The result in part (a) is comparable to the radius of a large nucleus, so it is reasonable. The force in part (b) is around 3 pounds, which is large enough to be easily felt by a person.

39.19. IDENTIFY and SET UP: Use the energy to calculate n for this state. Then use the Bohr equation, $L = n\hbar$, to calculate L.

EXECUTE: $E_n = -(13.6\ \text{eV})/n^2$, so this state has $n = \sqrt{13.6/1.51} = 3$. In the Bohr model, $L = n\hbar$, so for this state $L = 3\hbar = 3.16\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: We will find in Section 41.1 that the modern quantum mechanical description gives a different result.

39.20. IDENTIFY and SET UP: For a hydrogen atom $E_n = -\dfrac{13.6\ \text{eV}}{n^2}$. $\Delta E = \dfrac{hc}{\lambda}$, where ΔE is the magnitude of the energy change for the atom and λ is the wavelength of the photon that is absorbed or emitted.

EXECUTE: $\Delta E = E_3 - E_1 = -(13.6\ \text{eV})\left(\dfrac{1}{3^2} - \dfrac{1}{1^2}\right) = +12.09\ \text{eV}$.

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{12.09\ \text{eV}} = 102.6\ \text{nm}$, which rounds to 103 nm.

The frequency is $f = \dfrac{c}{\lambda} = \dfrac{2.998\times10^8\ \text{m/s}}{102.6\times10^{-9}\ \text{m}} = 2.92\times10^{15}\ \text{Hz}$.

EVALUATE: This photon is in the ultraviolet region of the electromagnetic spectrum.

39.21. IDENTIFY: The force between the electron and the nucleus in Be^{3+} is $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{Ze^2}{r^2}$, where $Z = 4$ is the nuclear charge. All the equations for the hydrogen atom apply to Be^{3+} if we replace e^2 by Ze^2.

(a) SET UP: Modify the energy equation for hydrogen, $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$ by replacing e^2 with Ze^2.

EXECUTE: $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$ (hydrogen) becomes

$$E_n = -\frac{1}{\epsilon_0^2}\frac{m(Ze^2)^2}{8n^2h^2} = Z^2\left(-\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}\right) = Z^2\left(-\frac{13.60\text{ eV}}{n^2}\right)(\text{for Be}^{3+}).$$

The ground-level energy of Be^{3+} is $E_1 = 16\left(-\dfrac{13.60\text{ eV}}{1^2}\right) = -218\text{ eV}.$

EVALUATE: The ground-level energy of Be^{3+} is $Z^2 = 16$ times the ground-level energy of H.

(b) SET UP: The ionization energy is the energy difference between the $n \to \infty$ level energy and the $n = 1$ level energy.

EXECUTE: The $n \to \infty$ level energy is zero, so the ionization energy of Be^{3+} is 218 eV.

EVALUATE: This is 16 times the ionization energy of hydrogen.

(c) SET UP: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$ just as for hydrogen but now R has a different value.

EXECUTE: $R_{\text{H}} = \dfrac{me^4}{8\epsilon_0^2 h^3 c} = 1.097\times10^7\text{ m}^{-1}$ for hydrogen becomes $R_{\text{Be}} = Z^2\dfrac{me^4}{8\epsilon_0^2 h^3 c}$

$= 16(1.097\times10^7\text{ m}^{-1}) = 1.755\times10^8\text{ m}^{-1}$ for Be^{3+}.

For $n = 2$ to $n = 1$, $\dfrac{1}{\lambda} = R_{\text{Be}}\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = 3R_{\text{Be}}/4.$

$$\lambda = 4/(3R_{\text{Be}}) = 4/(3(1.755\times10^8\text{ m}^{-1})) = 7.60\times10^{-9}\text{ m} = 7.60\text{ nm}.$$

EVALUATE: This wavelength is smaller by a factor of 16 compared to the wavelength for the corresponding transition in the hydrogen atom.

(d) SET UP: Modify the Bohr equation for hydrogen, $r_n = \epsilon_0\dfrac{n^2h^2}{\pi me^2}$, by replacing e^2 with Ze^2.

EXECUTE: $r_n = \epsilon_0\dfrac{n^2h^2}{\pi m(Ze^2)}$ (Be^{3+}).

EVALUATE: For a given n the orbit radius for Be^{3+} is smaller by a factor of $Z = 4$ compared to the corresponding radius for hydrogen.

39.22. IDENTIFY and SET UP: In the Bohr model for hydrogen, the energy levels are $E_n = -\dfrac{13.60\text{ eV}}{n^2}$ and the orbital radii are $r_n = n^2a_0$.

EXECUTE: **(a)** $E_2 - E_1 = -(13.6\text{ eV})(1/2^2 - 1/1^2) = 10.20\text{ eV}.$

$E_{10} - E_9 = -(13.6\text{ eV})(1/10^2 - 1/9^2) = 0.03190\text{ eV}.$

(b) $E_{n+1} - E_n = -(13.6\text{ eV})\left[\dfrac{1}{(n+1)^2} - \dfrac{1}{n^2}\right] = -(13.6\text{ eV})\left[\dfrac{n^2-(n+1)^2}{n^2(n+1)^2}\right] = (13.6\text{ eV})\left[\dfrac{2n+1}{n^2(n+1)^2}\right].$

As n gets very large, the factor in brackets approaches $2n/n^4 = 2/n^3$, so the entire quantity approaches $(13.6\text{ eV})(2/n^3) = (27.2\text{ eV})/n^3$.

(c) $r_{n+1} - r_n = a_0\left[(n+1)^2 - n^2\right] = a_0(n^2 + 2n + 1 - n^2) = (2n+1)a_0$. As n gets larger, $2n + 1$ gets larger, so the radial distance between adjacent orbits increases.

EVALUATE: As n gets large, the energy difference between adjacent shells gets small, but the radial distance between adjacent shells gets large. In other words, the orbits get progressively farther apart, but their energy gets closer together.

39.23. **IDENTIFY:** Apply the equations for v_n and r_n: $v_n = \dfrac{e^2}{2\epsilon_0 nh}$, $r_n = \epsilon_0 \dfrac{n^2h^2}{\pi me^2}$.

SET UP: The orbital period for state n is the circumference of the orbit divided by the orbital speed.

EXECUTE: **(a)** $v_n = \dfrac{1}{\epsilon_0}\dfrac{e^2}{2nh}: n=1 \Rightarrow v_1 = \dfrac{(1.602\times10^{-19}\text{ C})^2}{\epsilon_0\, 2\,(6.626\times10^{-34}\text{ J}\cdot\text{s})} = 2.19\times10^6\text{ m/s}.$

$n=2 \Rightarrow v_2 = \dfrac{v_1}{2} = 1.09\times10^6\text{ m/s}.$ $n=3 \Rightarrow v_3 = \dfrac{v_1}{3} = 7.27\times10^5\text{ m/s}.$

(b) Orbital period $= \dfrac{2\pi r_n}{v_n} = \dfrac{2\epsilon_0\, n^2h^2/me^2}{1/\epsilon_0 \cdot e^2/2nh} = \dfrac{4\epsilon_0^2\, n^3h^3}{me^4}.$

$n=1 \Rightarrow T_1 = \dfrac{4\epsilon_0^2\,(6.626\times10^{-34}\text{ J}\cdot\text{s})^3}{(9.11\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ C})^4} = 1.53\times10^{-16}\text{ s}$

$n=2: T_2 = T_1(2)^3 = 1.22\times10^{-15}\text{ s}.$ $n=3: T_3 = T_1(3)^3 = 4.13\times10^{-15}\text{ s}.$

(c) number of orbits $= \dfrac{1.0\times10^{-8}\text{ s}}{1.22\times10^{-15}\text{ s}} = 8.2\times10^6.$

EVALUATE: The orbital speed is proportional to $1/n$, the orbital radius is proportional to n^2, and the orbital period is proportional to n^3.

39.24. **IDENTIFY:** This problem deals with the Bohr model of the atom.

SET UP: $K = p^2/2m$, $K_n = (13.60\text{ eV})/n^2$, $L_n = nh/2\pi$.

EXECUTE: **(a)** We want the kinetic energy. Using $K = p^2/2m$ with the given momentum, we get $K = 1.52$ eV.

(b) We want the angular momentum. First find n using the result from part (a). Solve the equation $K_n = (13.60\text{ eV})/n^2$ for n, giving $n = \sqrt{\dfrac{13.60\text{ eV}}{1.52\text{ eV}}} = 3.$ Now find L. $L = 3(h/2\pi) = 3.16\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}.$

(c) $n = 3$, as shown above.

EVALUATE: The electron is in the second excited state.

39.25. **IDENTIFY** and **SET UP:** The ionization threshold is at $E = 0$. The energy of an absorbed photon equals the energy gained by the atom and the energy of an emitted photon equals the energy lost by the atom.

EXECUTE: **(a)** $\Delta E = 0 - (-20\text{ eV}) = 20\text{ eV}.$

(b) When the atom in the $n=1$ level absorbs an 18-eV photon, the final level of the atom is $n = 4$. The possible transitions from $n=4$ and corresponding photon energies are $n=4 \to n=3$, 3 eV; $n=4 \to n=2$, 8 eV; $n=4 \to n=1$, 18 eV. Once the atom has gone to the $n=3$ level, the following transitions can occur: $n=3 \to n=2$, 5 eV; $n=3 \to n=1$, 15 eV. Once the atom has gone to the $n=2$ level, the following transition can occur: $n=2 \to n=1$, 10 eV. The possible energies of emitted photons are: 3 eV, 5 eV, 8 eV, 10 eV, 15 eV, and 18 eV.

(c) There is no energy level 8 eV higher in energy than the ground state, so the photon cannot be absorbed.

(d) The photon energies for $n=3 \to n=2$ and for $n=3 \to n=1$ are 5 eV and 15 eV. The photon energy for $n=4 \to n=3$ is 3 eV. The work function must have a value between 3 eV and 5 eV.

EVALUATE: The atom has discrete energy levels, so the energies of emitted or absorbed photons have only certain discrete energies.

39.26. **IDENTIFY:** We are investigating a positronium atom using the Bohr model.

SET UP and **EXECUTE:** **(a)** We want the reduced mass m_r.

$m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{m^2}{m + m} = m/2.$

(b) We want r. In the ground state, $n = 1$.

$r_n = \epsilon_0 \dfrac{n^2 h^2}{\pi m_r e^2} = 2a_0 = 0.106 \text{ nm}.$

(c) We want E_1. Using $m = m_r$ and $n = 1$ gives

$E_n = -\dfrac{me^2}{\epsilon_0^2\, 8n^2h^2} = \dfrac{1}{2}(-13.60 \text{ eV}) = -6.80 \text{ eV}.$

(d) We want the wavelength. The energy E is the energy difference between the levels. Find E and then solve for the wavelength.

$$E = \frac{hc}{\lambda} = (-6.80 \text{ eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 0.9444 \text{ eV}.$$

This gives $\lambda = 1310 \text{ nm}$.

EVALUATE: The magnitude of the ground state energy for positronium is less than that of hydrogen. Therefore the photons emitted during transitions in positronium have less energy (and hence longer wavelength) than in comparable transition in hydrogen.

39.27. **IDENTIFY** and **SET UP:** The wavelength of the photon is related to the transition energy $E_i - E_f$ of the atom by $E_i - E_f = \dfrac{hc}{\lambda}$ where $hc = 1.240 \times 10^{-6} \text{ eV} \cdot \text{m}$.

EXECUTE: **(a)** The minimum energy to ionize an atom is when the upper state in the transition has $E = 0$, so $E_1 = -17.50 \text{ eV}$. For $n = 5 \to n = 1$, $\lambda = 73.86 \text{ nm}$ and $E_5 - E_1 = \dfrac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{73.86 \times 10^{-9} \text{ m}}$ $= 16.79 \text{ eV}$. $E_5 = -17.50 \text{ eV} + 16.79 \text{ eV} = -0.71 \text{ eV}$. For $n = 4 \to n = 1$, $\lambda = 75.63 \text{ nm}$ and $E_4 = -1.10 \text{ eV}$. For $n = 3 \to n = 1$, $\lambda = 79.76 \text{ nm}$ and $E_3 = -1.95 \text{ eV}$. For $n = 2 \to n = 1$, $\lambda = 94.54 \text{ nm}$ and $E_2 = -4.38 \text{ eV}$.

(b) $E_i - E_f = E_4 - E_2 = -1.10 \text{ eV} - (-4.38 \text{ eV}) = 3.28 \text{ eV}$ and $\lambda = \dfrac{hc}{E_i - E_f} = \dfrac{1.240 \times 10^{-6} \text{ eV} \cdot \text{m}}{3.28 \text{ eV}}$ $= 378 \text{ nm}$.

EVALUATE: The $n = 4 \to n = 2$ transition energy is smaller than the $n = 4 \to n = 1$ transition energy so the wavelength is longer. In fact, this wavelength is longer than for any transition that ends in the $n = 1$ state.

39.28. **IDENTIFY** and **SET UP:** For the Lyman series the final state is $n = 1$ and the wavelengths are given by $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$, $n = 2, 3, \ldots$. For the Paschen series the final state is $n = 3$ and the wavelengths are given by $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n^2}\right)$, $n = 4, 5, \ldots$. $R = 1.097 \times 10^7 \text{ m}^{-1}$. The longest wavelength is for the smallest n and the shortest wavelength is for $n \to \infty$.

EXECUTE: Lyman: Longest: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = \dfrac{3R}{4}$. $\lambda = \dfrac{4}{3(1.097 \times 10^7 \text{ m}^{-1})} = 121.5 \text{ nm}$.

Shortest: $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{\infty^2}\right) = R$. $\lambda = \dfrac{1}{1.097 \times 10^7 \text{ m}^{-1}} = 91.16 \text{ nm}$.

Paschen: Longest: $\dfrac{1}{\lambda}=R\left(\dfrac{1}{3^2}-\dfrac{1}{4^2}\right)=\dfrac{7R}{144}$. $\lambda=\dfrac{144}{7(1.097\times10^7\ \text{m}^{-1})}=1875\ \text{nm}.$

Shortest: $\dfrac{1}{\lambda}=R\left(\dfrac{1}{3^2}-\dfrac{1}{\infty^2}\right)=\dfrac{R}{9}$. $\lambda=\dfrac{9}{1.097\times10^7\ \text{m}^{-1}}=820\ \text{nm}.$

EVALUATE: The Lyman series is in the ultraviolet. The Paschen series is in the infrared.

39.29. **IDENTIFY:** Apply conservation of energy to the system of atom and photon.

SET UP: The energy of a photon is $E_\gamma=\dfrac{hc}{\lambda}.$

EXECUTE: **(a)** $E_\gamma=\dfrac{hc}{\lambda}=\dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{8.60\times10^{-7}\ \text{m}}=2.31\times10^{-19}\ \text{J}=1.44\ \text{eV}.$ So the internal energy of the atom increases by $1.44\ \text{eV}$ to $E=-6.52\ \text{eV}+1.44\ \text{eV}=-5.08\ \text{eV}.$

(b) $E_\gamma=\dfrac{hc}{\lambda}=\dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{4.20\times10^{-7}\ \text{m}}=4.74\times10^{-19}\ \text{J}=2.96\ \text{eV}.$ So the final internal energy of the atom decreases to $E=-2.68\ \text{eV}-2.96\ \text{eV}=-5.64\ \text{eV}.$

EVALUATE: When an atom absorbs a photon the energy of the atom increases. When an atom emits a photon the energy of the atom decreases.

39.30. **IDENTIFY** and **SET UP:** Balmer's formula is $\dfrac{1}{\lambda}=R\left(\dfrac{1}{2^2}-\dfrac{1}{n^2}\right)$. For the H_γ spectral line $n=5$. Once we have λ, calculate f from $f=c/\lambda$ and E using $E=hf$.

EXECUTE: **(a)** $\dfrac{1}{\lambda}=R\left(\dfrac{1}{2^2}-\dfrac{1}{5^2}\right)=R\left(\dfrac{25-4}{100}\right)=R\left(\dfrac{21}{100}\right).$

Thus $\lambda=\dfrac{100}{21R}=\dfrac{100}{21(1.097\times10^7)}\ \text{m}=4.341\times10^{-7}\ \text{m}=434.1\ \text{nm}.$

(b) $f=\dfrac{c}{\lambda}=\dfrac{2.998\times10^8\ \text{m/s}}{4.341\times10^{-7}\ \text{m}}=6.906\times10^{14}\ \text{Hz}.$

(c) $E=hf=(6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.906\times10^{14}\ \text{Hz})=4.576\times10^{-19}\ \text{J}=2.856\ \text{eV}.$

EVALUATE: Section 39.3 shows that the longest wavelength in the Balmer series (H_α) is 656 nm and the shortest is 365 nm. Our result for H_γ falls within this range. The photon energies for hydrogen atom transitions are in the eV range, and our result is of this order.

39.31. **IDENTIFY:** We know the power of the laser beam, so we know the energy per second that it delivers. The wavelength of the light tells us the energy of each photon, so we can use that to calculate the number of photons delivered per second.

SET UP: The energy of each photon is $E=hf=\dfrac{hc}{\lambda}=\dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: $\lambda=10.6\times10^{-6}$ m, so $E=1.88\times10^{-20}$ J. $P=\dfrac{E_{\text{tot}}}{t}=\dfrac{NE}{t}$ so

$$\frac{N}{t}=\frac{P}{E}=\frac{0.100\times10^3\ \text{W}}{1.88\times10^{-20}\ \text{J}}=5.32\times10^{21}\ \text{photons/s}.$$

EVALUATE: At over 10^{21} photons per second, we can see why we do not detect individual photons.

39.32. **IDENTIFY:** We can calculate the energy of a photon from its wavelength. Knowing the intensity of the beam and the energy of a single photon, we can determine how many photons strike the blemish with each pulse.

SET UP: The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon. The photon beam is spread over an area $A = \pi r^2$ with $r = 2.5$ mm.

EXECUTE: **(a)** $\lambda = 585$ nm and $E = \dfrac{hc}{\lambda} = 3.40\times10^{-19}\ \text{J} = 2.12\ \text{eV}$.

(b) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$, so $N = \dfrac{Pt}{E} = \dfrac{(20.0\ \text{W})(0.45\times10^{-3}\ \text{s})}{3.40\times10^{-19}\ \text{J}} = 2.65\times10^{16}$ photons. These photons are spread over an area πr^2, so the number of photons per mm^2 is

$$\frac{2.65\times10^{16}\ \text{photons}}{\pi(2.5\ \text{mm})^2} = 1.35\times10^{15}\ \text{photons/mm}^2.$$

EVALUATE: With so many photons per mm^2, it is impossible to detect individual photons.

39.33. **IDENTIFY** and **SET UP:** The number of photons emitted each second is the total energy emitted divided by the energy of one photon. The energy of one photon is given by $E = hc/\lambda$. $E = Pt$ gives the energy emitted by the laser in time t.

EXECUTE: In 1.00 s the energy emitted by the laser is $(7.50\times10^{-3}\ \text{W})(1.00\ \text{s}) = 7.50\times10^{-3}\ \text{J}$.

The energy of each photon is $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{10.6\times10^{-6}\ \text{m}} = 1.874\times10^{-20}\ \text{J}$.

Therefore $\dfrac{7.50\times10^{-3}\ \text{J/s}}{1.874\times10^{-20}\ \text{J/photon}} = 4.00\times10^{17}$ photons/s.

EVALUATE: The number of photons emitted per second is extremely large.

39.34. **IDENTIFY** and **SET UP:** Visible light has wavelengths from about 380 nm to about 750 nm. The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: **(a)** 193 nm is shorter than the shortest wavelength of visible light so is in the ultraviolet.

(b) $E = \dfrac{hc}{\lambda} = 1.03\times10^{-18}\ \text{J} = 6.44\ \text{eV}$.

(c) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$, so $N = \dfrac{Pt}{E} = \dfrac{(1.50\times10^{-3}\ \text{W})(12.0\times10^{-9}\ \text{s})}{1.03\times10^{-18}\ \text{J}} = 1.75\times10^{7}$ photons.

EVALUATE: A very small amount of energy is delivered to the lens in each pulse, but this still corresponds to a large number of photons.

39.35. **IDENTIFY:** Apply the equation $\dfrac{n_{\text{ex}}}{n_{\text{g}}} = e^{-(E_{\text{ex}}-E_{\text{g}})/kT}$ from the section on the laser.

SET UP: The energy of each of these excited states above the ground state is hc/λ, where λ is the wavelength of the photon emitted in the transition from the excited state to the ground state.

EXECUTE: $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(E_{2P3/2} - E_{2P1/2})/KT}$. From the diagram

$$\Delta E_{3/2-\text{g}} = \frac{hc}{\lambda_1} = \frac{(6.626\times10^{-34}\ \text{J})(2.998\times10^{8}\ \text{m/s})}{5.890\times10^{-7}\ \text{m}} = 3.373\times10^{-19}\ \text{J}.$$

$$\Delta E_{1/2-\text{g}} = \frac{hc}{\lambda_2} = \frac{(6.626\times10^{-34}\ \text{J})(2.998\times10^{8}\ \text{m/s})}{5.896\times10^{-7}\ \text{m}} = 3.369\times10^{-19}\ \text{J}.$$

So $\Delta E_{3/2-1/2} = 3.373\times10^{-19}\ \text{J} - 3.369\times10^{-19}\ \text{J} = 4.00\times10^{-22}\ \text{J}.$

$\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(4.00\times10^{-22}\ \text{J})/(1.38\times10^{-23}\text{J/K}\cdot500\ \text{K})} = 0.944.$ So more atoms are in the $2P_{1/2}$ state.

EVALUATE: At this temperature $kT = 6.9\times10^{-21}$ J. This is greater than the energy separation between the states, so an atom has almost equal probability for being in either state, with only a small preference for the lower energy state.

39.36. **IDENTIFY:** This problem involves blackbody radiation and the Wien law.

SET UP and **EXECUTE:** $I = \sigma T^4$, Wien law: $\lambda_m = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{T}$. We want the change in the peak wavelength if the intensity *I* increases by a factor of 16. From $I = \sigma T^4$ we see that if *I* changes by a factor of 16, so does T^4. Thus *T* increases by a factor of 2 because $2^4 = 16$. From the Wien law, if *T* increases by a factor of 2, the peak wavelength decreases by a factor of $\frac{1}{2}$, so it is *halved*.

EVALUATE: All the temperatures must be in kelvin units.

39.37. **IDENTIFY:** Energy radiates at the rate $H = Ae\sigma T^4$.

SET UP: The surface area of a cylinder of radius *r* and length *l* is $A = 2\pi rl$.

EXECUTE: **(a)** $T = \left(\dfrac{H}{Ae\sigma}\right)^{1/4} = \left(\dfrac{100\ \text{W}}{2\pi(0.20\times10^{-3}\ \text{m})(0.30\ \text{m})(0.26)(5.671\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right)^{1/4}.$

$T = 2.06\times10^3\ \text{K}.$

(b) $\lambda_\text{m}T = 2.90\times10^{-3}\ \text{m}\cdot\text{K};\ \lambda_\text{m} = 1410\ \text{nm}.$

EVALUATE: **(c)** λ_m is in the infrared. The incandescent bulb is not a very efficient source of visible light because much of the emitted radiation is in the infrared.

39.38. **IDENTIFY:** Apply Wien's displacement law and $c = f\lambda$.

SET UP: *T* in kelvins gives λ in meters.

EXECUTE: **(a)** $\lambda_\text{m} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{3.00\ \text{K}} = 0.966\ \text{mm}$, and $f = \dfrac{c}{\lambda_\text{m}} = 3.10\times10^{11}\ \text{Hz}.$

(b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to 9.66 μm and raises the frequency by the same factor, to 3.10×10^{13} Hz.

(c) Similarly, $\lambda_\text{m} = 966\ \text{nm}$ and $f = 3.10\times10^{14}$ Hz.

EVALUATE: λ_m decreases when *T* increases, as explained in the textbook.

39.39. **IDENTIFY** and **SET UP:** The wavelength λ_m where the Planck distribution peaks is given by Wien's displacement law, $\lambda_\text{m}T = 2.90\times10^{-3}\ \text{m}\cdot\text{K}.$

EXECUTE: $\lambda_\text{m} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{2.728\ \text{K}} = 1.06\times10^{-3}\ \text{m} = 1.06\ \text{mm}.$

EVALUATE: This wavelength is in the microwave portion of the electromagnetic spectrum. This radiation is often referred to as the "microwave background" (Chapter 44). Note that in Wien's law, *T* must be in kelvins.

39.40. **IDENTIFY:** Apply Wien's displacement law.

SET UP: $\lambda_\text{m}T = 2.90\times10^{-3}\ \text{m}\cdot\text{K}.$

EXECUTE: For 10.0-μm infrared: $T = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{10.0\times10^{-6}\ \text{m}} = 290\ \text{K}.$

<u>For 600-nm visible</u>: $T = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{600\times10^{-9}\ \text{m}} = 4830\ \text{K}.$

<u>For 100-nm ultraviolet</u>: $T = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_\text{m}} = \dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{100\times10^{-9}\ \text{m}} = 29{,}000\ \text{K}.$

EVALUATE: Most materials would melt (or burn or vaporize) before reaching 29,000 K!

39.41. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law and Wien's displacement law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$. Wien's displacement law tells us that the peak-intensity wavelength is $\lambda_\text{m} = (\text{constant})/T$.

EXECUTE: **(a)** The hot and cool stars radiate the same total power, so the Stefan-Boltzmann law gives $\sigma A_\text{h} T_\text{h}^4 = \sigma A_\text{c} T_\text{c}^4 \Rightarrow 4\pi R_\text{h}^2 T_\text{h}^4 = 4\pi R_\text{c}^2 T_\text{c}^4 = 4\pi(3R_\text{h})^2 T_\text{c}^4 \Rightarrow T_\text{h}^4 = 9T^4 \Rightarrow T_\text{h} = T\sqrt{3} = 1.7T$, rounded to two significant digits.

(b) Using Wien's law, we take the ratio of the wavelengths, giving $\dfrac{\lambda_\text{m}(\text{hot})}{\lambda_\text{m}(\text{cool})} = \dfrac{T_\text{c}}{T_\text{h}} = \dfrac{T}{T\sqrt{3}} = \dfrac{1}{\sqrt{3}} = 0.58$, rounded to two significant digits.

EVALUATE: Although the hot star has only 1/9 the surface area of the cool star, its absolute temperature has to be only 1.7 times as great to radiate the same amount of energy.

39.42. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** $I = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(24{,}000\ \text{K})^4 = 1.9\times10^{10}\ \text{W/m}^2.$

(b) Wien's law gives $\lambda_\text{m} = (0.00290\ \text{m}\cdot\text{K})/(24{,}000\ \text{K}) = 1.2\times10^{-7}\,\text{m} = 20\ \text{nm}.$

This is not visible since the wavelength is less than 400 nm.

(c) $P = AI \Rightarrow 4\pi R^2 = P/I = (1.00\times10^{25}\ \text{W})/(1.9\times10^{10}\ \text{W/m}^2)$, which gives

$$R_\text{Sirius} = 6.51\times10^6\ \text{m} = 6510\ \text{km}.$$

$R_\text{Sirius}/R_\text{sun} = (6.51\times10^6\ \text{m})/(6.96\times10^9\ \text{m}) = 0.0093$, which gives

$$R_\text{Sirius} = 0.0093\ R_\text{sun} \approx 1\%\ R_\text{sun}.$$

(d) Using the Stefan-Boltzmann law, we have

$$\frac{P_\text{sun}}{P_\text{Sirius}} = \frac{\sigma A_\text{sun} T_\text{sun}^4}{\sigma A_\text{Sirius} T_\text{Sirius}^4} = \frac{4\pi R_\text{sun}^2 T_\text{sun}^4}{4\pi R_\text{Sirius}^2 T_\text{Sirius}^4}$$

$$= \left(\frac{R_\text{sun}}{R_\text{Sirius}}\right)^2 \left(\frac{T_\text{sun}}{T_\text{Sirius}}\right)^4 \cdot \frac{P_\text{sun}}{P_\text{Sirius}} = \left(\frac{R_\text{sun}}{0.00935 R_\text{sun}}\right)^2 \left(\frac{5800\ \text{K}}{24{,}000\ \text{K}}\right)^4 = 39.$$

EVALUATE: Even though the absolute surface temperature of Sirius B is about 4 times that of our sun, it radiates only 1/39 times as much energy per second as our sun because it is so small.

39.43. **IDENTIFY:** Apply the Heisenberg uncertainty principle.

SET UP: $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $(\Delta x)(m\Delta v_x) \ge \hbar/2$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $\hbar/2$ (for the minimum uncertainty) gives $v_x = \dfrac{\hbar}{2m(0.010)\Delta x}$

$= \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(9.109\times10^{-31}\ \text{kg})(0.010)(0.30\times10^{-4}\ \text{m})} = 19.3\ \text{m/s}$, which rounds to 19 m/s.

(b) Taking the ratio of the equation for the proton to the equation for the electron gives

$$v_{\rm p} = \frac{m_{\rm e}}{m_{\rm p}} v_{\rm e} = \frac{9.109\times10^{-31}\ \text{kg}}{1.673\times10^{-27}\ \text{kg}} \cdot (19.3\ \text{m/s}) = 10.5\ \text{mm/s, which rounds to } 11\ \text{mm/s.}$$

EVALUATE: For a given Δp_x, Δv_x is smaller for a proton than for an electron, since the proton has larger mass.

39.44. **IDENTIFY:** Since we know only that the mosquito is somewhere in the room, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is an uncertainty in its momentum.
SET UP: The uncertainty principle is $\Delta x \Delta p_x \ge \hbar/2$.

EXECUTE: **(a)** You know the mosquito is somewhere in the room, so the maximum uncertainty in its horizontal position is $\Delta x = 5.0$ m.

(b) The uncertainty principle gives $\Delta x \Delta p_x \ge \hbar/2$, and $\Delta p_x = m\Delta v_x$ since we know the mosquito's mass. This gives $\Delta x\, m\Delta v_x \ge \hbar/2$, which we can solve for Δv_x to get the minimum uncertainty in v_x.

$$\Delta v_x = \frac{\hbar}{2m\Delta x} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.5\times10^{-6}\ \text{kg})(5.0\ \text{m})} = 7.0\times10^{-30}\ \text{m/s, which is hardly a serious impediment!}$$

EVALUATE: For something as "large" as a mosquito, the uncertainty principle places a negligible limitation on our ability to measure its speed.

39.45. **IDENTIFY** and **SET UP:** The Heisenberg Uncertainty Principle says $\Delta x \Delta p_x \ge \hbar/2$. The minimum allowed $\Delta x \Delta p_x$ is $\hbar/2$. $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $m\Delta x \Delta v_x = \hbar/2$. $\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.67\times10^{-27}\ \text{kg})(2.0\times10^{-12}\ \text{m})} = 1.6\times10^4$ m/s.

(b) $\Delta x = \dfrac{\hbar}{2m\Delta v_x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(9.11\times10^{-31}\ \text{kg})(0.250\ \text{m/s})} = 2.3\times10^{-4}$ m.

EVALUATE: The smaller Δx is, the larger Δv_x must be.

39.46. **IDENTIFY:** Since we know that the marble is somewhere on the table, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is therefore an uncertainty in its momentum.
SET UP: The uncertainty principle is $\Delta x \Delta p_x \ge \hbar/2$.

EXECUTE: **(a)** Since the marble is somewhere on the table, the maximum uncertainty in its horizontal position is $\Delta x = 1.75$ m.

(b) Following the same procedure as in part (b) of Problem 39.44, the minimum uncertainty in the horizontal velocity of the marble is $\Delta v_x = \dfrac{\hbar}{2m\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(0.0100\ \text{kg})(1.75\ \text{m})} = 3.01\times10^{-33}$ m/s.

(c) The uncertainty principle tells us that we cannot know that the marble's horizontal velocity is *exactly* zero, so the smallest we could measure it to be is 3.01×10^{-33} m/s, from part (b). The longest time it could remain on the table is the time to travel the full width of the table (1.75 m), so $t = x/v_x$ $= (1.75\ \text{m})/(3.01\times10^{-33}\ \text{m/s}) = 5.81\times10^{32}\ \text{s} = 1.84\times10^{25}$ years. Since the universe is about 14×10^9 years old, this time is about $\dfrac{1.8\times10^{25}\ \text{yr}}{14\times10^9\ \text{yr}} \approx 1.3\times10^{15}$ times the age of the universe! Don't hold your breath!

EVALUATE: For household objects, the uncertainty principle places a negligible limitation on our ability to measure their speed.

39.47. **IDENTIFY:** The Heisenberg uncertainty principle tells us that $\Delta x \Delta p_x \geq \hbar/2$.

SET UP: We can treat the standard deviation as a direct measure of uncertainty.

EXECUTE: Here $\Delta x \Delta p_x = (1.2\times10^{-10}\ \text{m})(3.0\times10^{-25}\ \text{kg}\cdot\text{m/s}) = 3.6\times10^{-35}\ \text{J}\cdot\text{s}$, but $\hbar/2 = 5.28\times10^{-35}\ \text{J}\cdot\text{s}$. Therefore $\Delta x \Delta p_x < \hbar/2$, so the claim is *not valid*.

EVALUATE: The uncertainty product $\Delta x \Delta p_x$ must increase by a factor of about 1.5 to become consistent with the Heisenberg uncertainty principle.

39.48. **IDENTIFY:** Apply conservation of momentum to the system of atom and emitted photon.

SET UP: Assume the atom is initially at rest. For a photon $E = \dfrac{hc}{\lambda}$ and $p = \dfrac{h}{\lambda}$.

EXECUTE: **(a)** Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \dfrac{h}{\lambda} \Rightarrow v = \dfrac{h}{m\lambda}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{h}{m\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2}$.

(c) $\dfrac{K}{E} = \dfrac{h^2}{2m\lambda^2}\cdot\dfrac{\lambda}{hc} = \dfrac{h}{2mc\lambda}$. Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

(d) $E = 10.2\ \text{eV} \Rightarrow \lambda = \dfrac{hc}{E} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(10.2\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 1.22\times10^{-7}\ \text{m} = 122\ \text{nm}.$

$$K = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.67\times10^{-27}\ \text{kg})(1.22\times10^{-7}\ \text{m})^2} = 8.84\times10^{-27}\ \text{J} = 5.53\times10^{-8}\ \text{eV}.$$

$\dfrac{K}{E} = \dfrac{5.53\times10^{-8}\ \text{eV}}{10.2\ \text{eV}} = 5.42\times10^{-9}$. This is quite small so recoil can be neglected.

EVALUATE: For emission of photons with ultraviolet or longer wavelengths the recoil kinetic energy of the atom is much less than the energy of the emitted photon.

39.49. **(a) IDENTIFY** and **SET UP:** Apply the equation for the reduced mass, $m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{207 m_e m_p}{207 m_e + m_p}$, where m_e denotes the electron mass.

EXECUTE: $m_r = \dfrac{207(9.109\times10^{-31}\ \text{kg})(1.673\times10^{-27}\ \text{kg})}{207(9.109\times10^{-31}\ \text{kg}) + 1.673\times10^{-27}\ \text{kg}} = 1.69\times10^{-28}\ \text{kg}.$

(b) IDENTIFY: In the energy equation $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$, replace $m = m_e$ by m_r: $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{m_r e^4}{8n^2h^2}$.

SET UP: Write as $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8n^2h^2}\right)$, since we know that $\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8h^2} = 13.60\ \text{eV}$. Here m_H denotes the reduced mass for the hydrogen atom; $m_H = (0.99946)(9.109\times10^{-31}\ \text{kg}) = 9.104\times10^{-31}\ \text{kg}$.

EXECUTE: $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{13.60\ \text{eV}}{n^2}\right)$.

$$E_1 = \frac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(-13.60\ \text{eV}) = 186(-13.60\ \text{eV}) = -2.53\ \text{keV}.$$

(c) SET UP: From part (b), $E_n = \left(\frac{m_r}{m_H}\right)\left(-\frac{R_H ch}{n^2}\right)$, where $R_H = 1.097\times10^7\ \text{m}^{-1}$ is the Rydberg constant for the hydrogen atom. Use this result in $\frac{hc}{\lambda} = E_i - E_f$ to find an expression for $1/\lambda$. The initial level for the transition is the $n_i = 2$ level and the final level is the $n_f = 1$ level.

EXECUTE: $\frac{hc}{\lambda} = \frac{m_r}{m_H}\left[-\frac{R_H ch}{n_i^2} - \left(-\frac{R_H ch}{n_f^2}\right)\right]$.

$\frac{1}{\lambda} = \frac{m_r}{m_H} R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$.

$\frac{1}{\lambda} = \frac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(1.097\times10^7\ \text{m}^{-1})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.527\times10^9\ \text{m}^{-1}$.

$\lambda = 0.655\ \text{nm}$.

EVALUATE: From Example 39.6, the wavelength of the radiation emitted in this transition in hydrogen is 122 nm. The wavelength for muonium is $\frac{m_H}{m_r} = 5.39\times10^{-3}$ times this. The reduced mass for hydrogen is very close to the electron mass because the electron mass is much less then the proton mass: $m_p/m_e = 1836$. The muon mass is $207m_e = 1.886\times10^{-28}\ \text{kg}$. The proton is only about 10 times more massive than the muon, so the reduced mass is somewhat smaller than the muon mass. The muon–proton atom has much more strongly bound energy levels and much shorter wavelengths in its spectrum than for hydrogen.

39.50. IDENTIFY: This problem involves the energy levels in the Bohr atom.

SET UP: $E_n = -(13.60\ \text{eV})/n^2$, $E = hc/\lambda$. We want the wavelength.

EXECUTE: The energy of the photon is the sum of the energy to ionize the atom plus the kinetic energy of the electron. So $hc/\lambda = (13.60\ \text{eV})/n^2 + K$. Using $n = 3$ and $K = 8.00$ eV and solving for the wavelength gives $\lambda = 130\ \text{nm}$.

EVALUATE: This light is not visible to humans.

39.51. IDENTIFY: This problem involves the Pickering emission series and the Bohr atom.

SET UP: $\frac{1}{\lambda} = (1.097\times10^7\ \text{m}^{-1})\left[\frac{1}{4} - \frac{1}{(n/2)^2}\right]$, $n = 5, 6, \ldots$

EXECUTE: (a) We want the longest and shortest wavelengths of the Pickering series. For the longest wavelength (the least energy), the transition is between adjacent levels, so $n = 5$.

$$\frac{1}{\lambda} = (1.097\times10^7\ \text{m}^{-1})\left[\frac{1}{4} - \frac{1}{(5/2)^2}\right].\ \lambda = 1013\ \text{nm}.$$

For shortest wavelength, n approaches infinity.

$$\frac{1}{\lambda} = (1.097\times10^7\ \text{m}^{-1})\left[\frac{1}{4} - \frac{1}{(\infty/2)^2}\right].\ \lambda = 364.6\ \text{nm}.$$

(b) Shortest wavelength: The transition is between n_L and infinity. The Bohr energy is $\Delta E = E_\infty - E_{n_L}$.

$\frac{hc}{\lambda_{\min}} = \frac{Z^2 E_1}{n^2}$. $E_1 = 13.60\ \text{eV}$ and $\lambda_{\min} = 364.6\ \text{nm}$ which gives $Z^2/n^2 = 0.250$. Try values of n to find a Z that satisfies this equation.

$n = 1$: This cannot work because Z must be a whole number.

$n = 2$: This gives $Z = 1$, which is hydrogen which we know is *not* the atom.

n = 3: Z is not a whole number.
n = 4: This gives Z = 2 and ends on level 4, so n_L = 4, Z = 2 (helium).
EVALUATE: Find n for the $\lambda = 1013$ nm transition using the Bohr model for a Z = 2 atom.

$$\Delta E = \frac{hc}{\lambda} = Z^2(13.60 \text{ eV})\left(\frac{1}{4^2} - \frac{1}{n^2}\right).$$

Using $\lambda = 1013$ nm and Z = 2 gives n = 5, so the transition is from the n = 5 state to the n = 4 state.

39.52. **IDENTIFY** and **SET UP:** The de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$, so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n. Then $\lambda = h\left(\frac{2\pi r_n}{nh}\right) = \frac{2\pi r_n}{n}$.

EXECUTE: **(a)** For $n = 1, \lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529\times10^{-10}$ m, so

$\lambda = 2\pi(0.529\times10^{-10} \text{ m}) = 3.32\times10^{-10}$ m.

$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.

(b) For $n = 4, \lambda = 2\pi r_4/4$.

$r_n = n^2 a_0$ so $r_4 = 16a_0$.

$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32\times10^{-10} \text{ m}) = 1.33\times10^{-9}$ m.

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\frac{1}{n} = \frac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n, the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.53. **(a) IDENTIFY** and **SET UP:** The photon energy is given to the electron in the atom. Some of this energy overcomes the binding energy of the atom and what is left appears as kinetic energy of the free electron. Apply $hf = E_f - E_i$, the energy given to the electron in the atom when a photon is absorbed.

EXECUTE: The energy of one photon is $\frac{hc}{\lambda} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{85.5\times10^{-9} \text{ m}}$.

$$\frac{hc}{\lambda} = 2.323\times10^{-18} \text{ J}(1 \text{ eV}/1.602\times10^{-19} \text{ J}) = 14.50 \text{ eV}.$$

The final energy of the electron is $E_f = E_i + hf$. In the ground state of the hydrogen atom the energy of the electron is $E_i = -13.60$ eV. Thus $E_f = -13.60 \text{ eV} + 14.50 \text{ eV} = 0.90$ eV.

EVALUATE: **(b)** At thermal equilibrium a few atoms will be in the $n = 2$ excited levels, which have an energy of $-13.6 \text{ eV}/4 = -3.40 \text{ eV}, 10.2$ eV greater than the energy of the ground state. If an electron with $E = -3.40$ eV gains 14.5 eV from the absorbed photon, it will end up with $14.5 \text{ eV} - 3.4 \text{ eV} = 11.1$ eV of kinetic energy.

39.54. **IDENTIFY:** We are dealing with transitions between states having very close energies.
SET UP: E_n is the energy of state n relative to ground, so the photon energies due to the transitions are $E_1 = hc/\lambda_1$ and $E_2 = hc/\lambda_2$.

EXECUTE: (a)

$$\Delta E = |E_1 - E_2| = hc\left|\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right| = hc\left|\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right| \approx \frac{hc\Delta\lambda}{\lambda^2}$$

(b) Using the given numbers gives

$$\Delta E \approx \frac{hc(589.6\text{ nm} - 589.0\text{ nm})}{(589.6\text{ nm})^2} = 0.002\text{ eV}.$$

EVALUATE: Using 589.0 nm in the denominator in part (b) would give the same answer.

39.55. **IDENTIFY:** Assuming that Betelgeuse radiates like a perfect blackbody, Wien's displacement and the Stefan-Boltzmann law apply to its radiation.

SET UP: Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times 10^{-3}\text{ m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First use Wien's law to find the peak wavelength:

$$\lambda_{\text{m}} = (2.90\times 10^{-3}\text{ m}\cdot\text{K})/(3000\text{ K}) = 9.667\times 10^{-7}\text{ m}.$$

Call N the number of photons/second radiated. $N\times$(energy per photon) $= IA = \sigma A T^4$.

$$N(hc/\lambda_{\text{m}}) = \sigma A T^4.\quad N = \frac{\lambda_{\text{m}}\sigma A T^4}{hc}.$$

$$N = \frac{(9.667\times 10^{-7}\text{ m})(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(4\pi)(600\times 6.96\times 10^{8}\text{ m})^2(3000\text{ K})^4}{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(3.00\times 10^{8}\text{ m/s})}.$$

$$N = 5\times 10^{49}\text{ photons/s}.$$

(b) $\dfrac{I_{\text{B}}A_{\text{B}}}{I_{\text{S}}A_{\text{S}}} = \dfrac{\sigma A_{\text{B}}T_{\text{B}}^4}{\sigma A_{\text{S}}T_{\text{S}}^4} = \dfrac{4\pi R_{\text{B}}^2 T_{\text{B}}^4}{4\pi R_{\text{S}}^2 T_{\text{S}}^4} = \left(\dfrac{600 R_{\text{S}}}{R_{\text{S}}}\right)^2\left(\dfrac{3000\text{ K}}{5800\text{ K}}\right)^4 = 3\times 10^4.$

EVALUATE: Betelgeuse radiates 30,000 times as much energy per second as does our sun!

39.56. **IDENTIFY:** The diffraction grating allows us to determine the peak-intensity wavelength of the light. Then Wien's displacement law allows us to calculate the temperature of the blackbody, and the Stefan-Boltzmann law allows us to calculate the rate at which it radiates energy.

SET UP: The bright spots for a diffraction grating occur when $d\sin\theta = m\lambda$. Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times 10^{-3}\text{ m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$. The area of a sphere is $A = 4\pi r^2$.

EXECUTE: **(a)** First find the wavelength of the light:

$$\lambda = d\sin\theta = [1/(385{,}000\text{ lines/m})]\sin(14.4°) = 6.459\times 10^{-7}\text{ m}.$$

Now use Wien's law to find the temperature: $T = (2.90\times 10^{-3}\text{ m}\cdot\text{K})/(6.459\times 10^{-7}\text{ m}) = 4490\text{ K}.$

(b) The energy radiated by the blackbody is equal to the power times the time, giving $U = Pt = IAt = \sigma A T^4 t$, which gives

$t = U/(\sigma A T^4) = (12.0\times 10^6\text{ J})/[(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(4\pi)(0.0750\text{ m})^2(4490\text{ K})^4] = 7.37\text{ s}.$

EVALUATE: By ordinary standards, this blackbody is very hot, so it does not take long to radiate 12.0 MJ of energy.

39.57. **IDENTIFY:** We are applying quantum principles to our moon.

SET UP and **EXECUTE:** **(a)** We want the de Broglie wavelength of the moon. First find the speed. $v = 2\pi r/t$. Using the orbital radius r from Appendix F and t = 27.3 d = (27.3)(86,400 s), we have v = 1.0229 km/s. Now use $\lambda = h/p = h/mv$ with the mass m given in the problem and the speed we just found, giving $\lambda = 8.81\times 10^{-60}$ m.

(b) We want the acceleration of the moon. Apply Newton's second law and universal gravitation to the moon. We also know that the angular momentum is $L = M_{moon}vR = mh/2\pi$, so $v = mh/2\pi M_{moon}R$. Using these relationships gives

$$\frac{GM_{earth}M_{moon}}{R^2} = \frac{M_{moon}v^2}{R}$$

$$\frac{GM_{earth}}{R} = v^2 = \left(\frac{mh}{2\pi M_{moon}R}\right)^2$$

$$R = m^2\left(\frac{h^2}{4\pi^2 M_{moon}^2 GM_{earth}}\right) = m^2\, a_{moon}$$

$$a_{moon} = \frac{h^2}{4\pi^2 M_{moon}^2 GM_{earth}}.$$

(c) We want a_{moon}. Using the numbers given in the problem for the masses, our result from part (b) gives $a_{moon} = 5.17 \times 10^{-129}\ m$ m/s^2.

(d) We want m. Using $R_m = m^2 a_{moon}$ and the result from part (c) gives $m = 2.73 \times 10^{68}$.

(e) We want E_0. From $E = -E_0/m$ we get $E_0 = -m^2E$. Using $E = K + U$ gives

$$E = \frac{1}{2}I\omega^2 - \frac{GM_{moon}M_{earth}}{R} = \frac{1}{2}M_{moon}R^2\left(\frac{2\pi}{T}\right)^2 - \frac{GM_{moon}M_{earth}}{R}.$$

Using $E_0 = -m^2E$ with the above result and $m = 2.73 \times 10^{68}$, we get $E_0 = -2.81 \times 10^{165}$ J.

Evaluate: According to our result, the moon is not even close to its ground state.

39.58. **Identify:** Combine $I = \sigma T^4$, $P = IA$, and $\Delta E = Pt$.

Set Up: In the Stefan-Boltzmann law the temperature must be in kelvins. 400°C = 673 K.

Execute: $t = \dfrac{\Delta E}{A\sigma T^4} = \dfrac{100\text{ J}}{(4.00\times10^{-6}\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(673\text{ K})^4} = 2.15\times10^3\text{ s}$

$= 35.8\text{ min} = 0.597\text{ h}.$

Evaluate: The power is $P = 46.5$ mW. Since the area of the hole is small, the rate at which the cavity radiates energy through the hole is very small.

39.59. **Identify** and **Set Up:** Follow the procedures specified in the problem.

Execute: **(a)** $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $\lambda = \dfrac{c}{f} \Rightarrow I(f) = \dfrac{2\pi hc^2}{(c/f)^5(e^{hf/kT}-1)} = \dfrac{2\pi hf^5}{c^3(e^{hf/kT}-1)}.$

(b) $\displaystyle\int_0^\infty I(\lambda)\,d\lambda = \int_\infty^0 I(f)\,df\left(\frac{-c}{f^2}\right) = \int_0^\infty \frac{2\pi hf^3\,df}{c^2(e^{hf/kT}-1)} = \frac{2\pi(kT)^4}{c^2h^3}\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{2\pi(kT)^4}{c^2h^3}\frac{1}{240}(2\pi)^4$

$= \dfrac{(2\pi)^5(kT)^4}{240h^3c^2} = \dfrac{2\pi^5k^4T^4}{15c^2h^3}.$

(c) The expression $\dfrac{2\pi^5k^4}{15h^3c^2} = \sigma$ as shown in Eq. (39.28). Plugging in the values for the constants we get $\sigma = 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4$.

Evaluate: The Planck radiation law, $I(\lambda) = \dfrac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$, predicts the Stefan-Boltzmann law, $I = \sigma T^4$.

39.60. **IDENTIFY:** $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$. From Chapter 36, if $\lambda \ll a$ then the width w of the central maximum is $w = 2\frac{R\lambda}{a}$, where $R = 2.5\text{ m}$ and a is the width of the slit.

SET UP: $v_x = \sqrt{\frac{2E}{m}}$, since the beam is traveling in the x-direction and $\Delta v_y \ll v_x$.

EXECUTE: **(a)** $\lambda = \frac{h}{\sqrt{2mE}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\text{ kg})(40\text{ eV})(1.60\times10^{-19}\text{ J/eV})}} = 1.94\times10^{-10}\text{ m}.$

(b) $\frac{R}{v} = \frac{R}{\sqrt{2E/m}} = \frac{(2.5\text{ m})(9.11\times10^{-31}\text{ kg})^{1/2}}{\sqrt{2(40\text{ eV})(1.6\times10^{-19}\text{ J/eV})}} = 6.67\times10^{-7}\text{ s}.$

(c) The width w is $w = 2R\frac{\lambda}{a}$' and $w = \Delta v_y t = \Delta p_y t/m$, where t is the time found in part (b) and a is the slit width. Combining the expressions for w, $\Delta p_y = \frac{2m\lambda R}{at} = 2.65\times10^{-28}\text{ kg}\cdot\text{m/s}.$

(d) $\Delta y = \frac{\hbar}{2\Delta p_y} = 0.20\ \mu\text{m}$, which is the same order of magnitude of the width of the slit.

EVALUATE: For these electrons $\lambda = 1.94\times10^{-10}\text{ m}$. This is much smaller than a and the approximate expression $w = \frac{2R\lambda}{a}$ is very accurate. Also, $v_x = \sqrt{\frac{2E}{m}} = 3.75\times10^6\text{ m/s}$. $\Delta v_y = \frac{\Delta p_y}{m} = 2.9\times10^2\text{ m/s}$, so it is the case that $v_x \gg \Delta v_y$.

39.61. **IDENTIFY:** This problem involves the de Broglie wavelength.
SET UP and **EXECUTE:** Use $\lambda = h/mv$. **(a)** Estimate: 0.5 mm/s.
(b) Using $m = 85$ kg gives $\lambda = h/mv = h/(85\text{ kg})(0.5\text{ mm/s}) = 1.6\times10^{-32}$ m.
(c) $\lambda = h/mv = h/(0.5\text{ mg})(0.5\text{ mm/s}) = 2.7\times10^{-24}$ m.
(d) $\lambda = h/mv = (1\text{ J·s})/(0.5\text{ mg})(1\text{ m/s}) = 2\times10^6$ m = 2000 km.
(e) $\lambda = h/mv = (1\text{ J·s})/(85\text{ kg})(2.5\text{ m/s}) = 4.7$ mm.
(f) We want the speed. Estimate: Doorway is 1.0 m wide. For single-slit diffraction, $\lambda = a\sin\theta$ $= (1.0\text{ m})\sin 5° = 0.087$ m. $\lambda = h/mv = (1\text{ J·s})/[(0.145\text{ kg})(0.087\text{ m})] = 80$ m/s.
EVALUATE: From our results we can see that the wave nature of particles is not apparent for everyday-size things but only for particles of the scale of electrons, protons, atoms, and molecules.

39.62. **IDENTIFY:** The de Broglie wavelength of the electrons must be such that the first diffraction minimum occurs at $\theta = 20.0°$.

SET UP: The single-slit diffraction minima occur at angles θ given by $a\sin\theta = m\lambda$. $p = \frac{h}{\lambda}$.

EXECUTE: **(a)** $\lambda = a\sin\theta = (300\times10^{-9}\text{ m})(\sin 20°) = 1.0261\times10^{-7}\text{ m}$. $\lambda = h/mv \to v = h/m\lambda$.

$$v = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(1.0261\times10^{-7}\text{ m})} = 7.09\times10^3\text{ m/s} = 7.09\text{ km/s}.$$

(b) No electrons strike the screen at the location of the second diffraction minimum. $a\sin\theta_2 = 2\lambda$.

$$\sin\theta_2 = \pm2\frac{\lambda}{a} = \pm2\left(\frac{1.0261\times10^{-7}\text{ m}}{3.00\times10^{-7}\text{ m}}\right) = \pm0.684.\quad \theta_2 = \pm43.2°.$$

EVALUATE: The intensity distribution in the diffraction pattern depends on the wavelength λ and is the same for light of wavelength λ as for electrons with de Broglie wavelength λ.

39.63. **IDENTIFY:** The electrons behave like waves and produce a double-slit interference pattern after passing through the slits.
SET UP: The first angle at which destructive interference occurs is given by $d\sin\theta = \lambda/2$. The de Broglie wavelength of each of the electrons is $\lambda = h/mv$.
EXECUTE: **(a)** First find the wavelength of the electrons. For the first dark fringe, we have $d\sin\theta = \lambda/2$, which gives (1.25 nm)(sin 18.0°) = $\lambda/2$, and λ = 0.7725 nm. Now solve the de Broglie wavelength equation for the speed of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.7725\times10^{-9}\ \text{m})} = 9.42\times10^{5}\ \text{m/s}$$

which is about 0.3% the speed of light, so they are *nonrelativistic*.
(b) Energy conservation gives $eV = \frac{1}{2}mv^2$ and $V = mv^2/2e$

$= (9.11\times10^{-31}\ \text{kg})(9.42\times10^{5}\ \text{m/s})^2/[2(1.60\times10^{-19}\ \text{C})] = 2.52\ \text{V}.$

EVALUATE: The de Broglie wavelength of the electrons is comparable to the separation of the slits.

30.64. **IDENTIFY:** The de Broglie wavelength of the electrons must equal the wavelength of the light.
SET UP: The maxima in the two-slit interference pattern are located by $d\sin\theta = m\lambda$. For an electron, $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$.

EXECUTE: $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(20.0\times10^{-6}\ \text{m})\sin(0.0300\ \text{rad})}{2} = 300\ \text{nm}.$ The velocity of an electron with this wavelength is given by $\lambda = h/p$. $v = \dfrac{p}{m} = \dfrac{h}{m\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(300\times10^{-9}\ \text{m})} = 2.43\times10^{3}\ \text{m/s}$ = 2.43 km/s.
Since this velocity is much smaller than c we can calculate the energy of the electron classically, so $K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.43\times10^{3}\ \text{m/s})^2 = 2.68\times10^{-24}\ \text{J} = 16.7\times10^{-6}\ \text{eV} = 16.7\ \mu\text{eV}.$

EVALUATE: The energy of the photons of this wavelength is $E = \dfrac{hc}{\lambda} = 4.14\ \text{eV}.$ The photons and electrons have the same wavelength but a photon has around 250,000 times as much energy as an electron.

39.65. **IDENTIFY:** Both the electrons and photons behave like waves and exhibit single-slit diffraction after passing through their respective slits.
SET UP: The energy of the photon is $E = hc/\lambda$ and the de Broglie wavelength of the electron is $\lambda = h/mv = h/p$. Destructive interference for a single slit first occurs when $a\sin\theta = \lambda$.
EXECUTE: **(a)** For the photon: $\lambda = hc/E$ and $a\sin\theta = \lambda$. Since the a and θ are the same for the photons and electrons, they must both have the same wavelength. Equating these two expressions for λ gives $a\sin\theta = hc/E$. For the electron, $\lambda = h/p = \dfrac{h}{\sqrt{2mK}}$ and $a\ \Delta \to 0$, Equating these two expressions for λ gives $a\sin\theta = \dfrac{h}{\sqrt{2mK}}$. Equating the two expressions for $a\sin\theta$ gives $hc/E = \dfrac{h}{\sqrt{2mK}}$, which gives $E = c\sqrt{2mK} = (4.05\times10^{-7}\ \text{J}^{1/2})\sqrt{K}$.
(b) $\dfrac{E}{K} = \dfrac{c\sqrt{2mK}}{K} = \sqrt{\dfrac{2mc^2}{K}}$. Since $v \ll c$, $mc^2 > K$, so the square root is >1. Therefore $E/K > 1$, meaning that the photon has more energy than the electron.
EVALUATE: When a photon and a particle have the same wavelength, the photon has more energy than the particle.

39.66. **IDENTIFY and SET UP:** The de Broglie wavelength of the blood cell is $\lambda = \dfrac{h}{mv}$.

EXECUTE: $\lambda = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.00\times10^{-14}\ \text{kg})(4.00\times10^{-3}\ \text{m/s})} = 1.66\times10^{-17}$ m.

EVALUATE: We need not be concerned about wave behavior.

39.67. **IDENTIFY and SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** $\lambda = \dfrac{h}{p} = \dfrac{h\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{mv} \Rightarrow \lambda^2 m^2 v^2 = h^2\left(1-\dfrac{v^2}{c^2}\right) = h^2 - \dfrac{h^2 v^2}{c^2} \Rightarrow \lambda^2 m^2 v^2 + h^2\dfrac{v^2}{c^2} = h^2$

$\Rightarrow v^2 = \dfrac{h^2}{\left(\lambda^2 m^2 + \dfrac{h^2}{c^2}\right)} = \dfrac{c^2}{\left(\dfrac{\lambda^2 m^2 c^2}{h^2} + 1\right)} \Rightarrow v = \dfrac{c}{\left(1+\left(\dfrac{mc\lambda}{h}\right)^2\right)^{1/2}}.$

(b) $v = \dfrac{c}{\left(1+\left(\dfrac{\lambda}{(h/mc)}\right)^2\right)^{1/2}} \approx c\left(1-\dfrac{1}{2}\left(\dfrac{mc\lambda}{h}\right)^2\right) = (1-\Delta)c.\ \ \Delta = \dfrac{m^2c^2\lambda^2}{2h^2}.$

(c) $\lambda = 1.00\times10^{-15}\,\text{m} \ll \dfrac{h}{mc}.\ \ \Delta = \dfrac{(9.11\times10^{-31}\ \text{kg})^2(3.00\times10^{8}\ \text{m/s})^2(1.00\times10^{-15}\ \text{m})^2}{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2} = 8.50\times10^{-8}$

$\Rightarrow v = (1-\Delta)c = (1-8.50\times10^{-8})c.$

EVALUATE: As $\Delta \to 0$, $v \to c$ and $\lambda \to 0$.

39.68. **IDENTIFY and SET UP:** The minimum uncertainty product is $\Delta x \Delta p_x = \hbar/2$. $\Delta x = r_1$, where r_1 is the radius of the $n=1$ Bohr orbit. In the $n=1$ Bohr orbit, $mv_1 r_1 = \dfrac{h}{2\pi}$ and $p_1 = mv_1 = \dfrac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{\hbar}{2r_1} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(0.529\times10^{-10}\ \text{m})} = 1.0\times10^{-24}\ \text{kg}\cdot\text{m/s}$. This is the same as the magnitude of the momentum of the electron in the $n=1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

39.69. **IDENTIFY and SET UP:** Combining the two equations in the hint gives $pc = \sqrt{K(K+2mc^2)}$ and $\lambda = \dfrac{hc}{\sqrt{K(K+2mc^2)}}$.

EXECUTE: **(a)** With $K = 3mc^2$ this becomes $\lambda = \dfrac{hc}{\sqrt{3mc^2(3mc^2+2mc^2)}} = \dfrac{h}{\sqrt{15}mc}$.

(b) (i) $K = 3mc^2 = 3(9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})^2 = 2.456\times10^{-13}\ \text{J} = 1.53\ \text{MeV}$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{15}(9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})} = 6.26\times10^{-13}\ \text{m}.$$

(ii) K is proportional to m, so for a proton $K = (m_\text{p}/m_\text{e})(1.53\ \text{MeV}) = 1836(1.53\ \text{MeV}) = 2810\ \text{MeV}$.

λ is proportional to $1/m$, so for a proton

$\lambda = (m_\text{e}/m_\text{p})(6.26\times10^{-13}\ \text{m}) = (1/1836)(6.26\times10^{-13}\ \text{m}) = 3.41\times10^{-16}$ m.

EVALUATE: The proton has a larger rest mass energy so its kinetic energy is larger when $K = 3mc^2$. The proton also has larger momentum so has a smaller λ.

39.70. **IDENTIFY:** Apply the Heisenberg uncertainty principle. Consider only one component of position and momentum.

SET UP: $\Delta x \Delta p_x \geq \hbar/2$. Take $\Delta x \approx 5.0\times10^{-15}$ m. $K = E - mc^2$. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2(5.0\times10^{-15}\ \text{m})} = 1.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 = 3.3\times10^{-14}\ \text{J} = 0.21\ \text{MeV}.$

EVALUATE: **(c)** The result of part (b), about 2×10^5 eV, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.71. **(a) IDENTIFY** and **SET UP:** $\Delta x \Delta p_x \geq \hbar/2$. Estimate Δx as $\Delta x \approx 5.0\times10^{-15}$ m.

EXECUTE: Then the minimum allowed Δp_x is $\Delta p_x \approx \dfrac{\hbar}{2\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(5.0\times10^{-15}\ \text{m})} = 1.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) IDENTIFY and **SET UP:** Assume $p \approx 1.1\times10^{-20}$ kg · m/s. Use $E^2 = (mc^2)^2 + (pc)^2$ to calculate E, and then $K = E - mc^2$.

EXECUTE: $E = \sqrt{(mc^2)^2 + (pc)^2}$. $mc^2 = (9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 8.187\times10^{-14}\ \text{J}.$

$pc = (1.1\times10^{-20}\ \text{kg}\cdot\text{m/s})(2.998\times10^8\ \text{m/s}) = 3.165\times10^{-12}\ \text{J}.$

$E = \sqrt{(8.187\times10^{-14}\ \text{J})^2 + (3.165\times10^{-12}\ \text{J})^2} = 3.166\times10^{-12}\ \text{J}.$

$K = E - mc^2 = 3.166\times10^{-12}\ \text{J} - 8.187\times10^{-14}\ \text{J} = 3.084\times10^{-12}\ \text{J}\times(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 19\ \text{MeV}.$

(c) IDENTIFY and **SET UP:** The Coulomb potential energy for a pair of point charges is given by $U = -kq_1q_2/r$. The proton has charge $+e$ and the electron has charge $-e$.

EXECUTE: $U = -\dfrac{ke^2}{r} = -\dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{5.0\times10^{-15}\ \text{m}} = -4.6\times10^{-14}\ \text{J} = -0.29\ \text{MeV}.$

EVALUATE: The kinetic energy of the electron required by the uncertainty principle would be much larger than the magnitude of the negative Coulomb potential energy. The total energy of the electron would be large and positive and the electron could not be bound within the nucleus.

39.72. **IDENTIFY** and **SET UP:** $\Delta E \Delta t \geq \hbar/2$. Take the minimum uncertainty product, so $\Delta E = \dfrac{\hbar}{2\Delta t}$, with $\Delta t = 8.4\times10^{-17}$ s. $m = 264m_e$. $\Delta m = \dfrac{\Delta E}{c^2}$.

EXECUTE: $\Delta E = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(8.4\times10^{-17}\ \text{s})} = 6.28\times10^{-19}\ \text{J}.$ $\Delta m = \dfrac{6.28\times10^{-19}\ \text{J}}{(3.00\times10^8\ \text{m/s})^2} = 7.0\times10^{-36}\ \text{kg}.$

$$\frac{\Delta m}{m} = \frac{7.0\times10^{-36}\ \text{kg}}{(264)(9.11\times10^{-31}\ \text{kg})} = 2.9\times10^{-8}.$$

EVALUATE: The fractional uncertainty in the mass is very small.

39.73. **IDENTIFY** and **SET UP:** Use $\lambda = h/p$ to relate your wavelength and speed.

EXECUTE: **(a)** $\lambda = \dfrac{h}{mv}$, so $v = \dfrac{h}{m\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(60.0\ \text{kg})(1.0\ \text{m})} = 1.1\times10^{-35}\ \text{m/s}.$

(b) $t = \dfrac{\text{distance}}{\text{speed}} = \dfrac{0.80\ \text{m}}{1.1\times10^{-35}\ \text{m/s}} = 7.3\times10^{34}\ \text{s}(1\ \text{y}/3.156\times10^7\ \text{s}) = 2.3\times10^{27}\ \text{y}.$

Since you walk through doorways much more quickly than this, you will not experience diffraction effects.

EVALUATE: A 1-kg object moving at 1 m/s has a de Broglie wavelength $\lambda = 6.6\times10^{-34}\text{ m},$ which is exceedingly small. An object like you has a very, very small λ at ordinary speeds and does not exhibit wavelike properties.

39.74. IDENTIFY: The transition energy E for the atom and the wavelength λ of the emitted photon are related by $E = \dfrac{hc}{\lambda}.$ Apply the Heisenberg uncertainty principle in the form $\Delta E\Delta t \ge \dfrac{\hbar}{2}.$

SET UP: Assume the minimum possible value for the uncertainty product, so that $\Delta E\Delta t = \dfrac{\hbar}{2}.$

EXECUTE: **(a)** $E = 2.58\text{ eV} = 4.13\times10^{-19}\text{ J}$, with a wavelength of $\lambda = \dfrac{hc}{E} = 4.82\times10^{-7}\text{m} = 482\text{ nm}.$

(b) $\Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})}{2(1.64\times10^{-7}\text{ s})} = 3.22\times10^{-28}\text{ J} = 2.01\times10^{-9}\text{ eV}.$

(c) $\lambda E = hc$, so $(\Delta\lambda)E + \lambda\Delta E = 0$, and $|\Delta E/E| = |\Delta\lambda/\lambda|,$ so

$$\Delta\lambda = \lambda|\Delta E/E| = (4.82\times10^{-7}\text{ m})\left(\frac{3.22\times10^{-28}\text{ J}}{4.13\times10^{-19}\text{ J}}\right) = 3.75\times10^{-16}\text{ m} = 3.75\times10^{-7}\text{ nm}.$$

EVALUATE: The finite lifetime of the excited state gives rise to a small spread in the wavelength of the emitted light.

39.75. IDENTIFY: Assume both the x rays and electrons are at normal incidence and scatter from the surface plane of the crystal, so the maxima are located by $d\sin\theta = m\lambda,$ where d is the separation between adjacent atoms in the surface plane.

SET UP: Let primed variables refer to the electrons. $\lambda' = \dfrac{h}{p'} = \dfrac{h}{\sqrt{2mE'}}.$

EXECUTE: $\sin\theta' = \dfrac{\lambda'}{\lambda}\sin\theta$, and $\lambda' = (h/p') = (h/\sqrt{2mE'})$, and so $\theta' = \arcsin\left(\dfrac{h}{\lambda\sqrt{2mE'}}\sin\theta\right).$

$$\theta' = \arcsin\left(\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(\sin 35.8°)}{(3.00\times10^{-11}\text{ m})\sqrt{2(9.11\times10^{-31}\text{ kg})(4.50\times10^{+3}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}\right) = 20.9°.$$

EVALUATE: The x rays and electrons have different wavelengths and the $m = 1$ maxima occur at different angles.

39.76. IDENTIFY: The photon is emitted as the atom returns to the lower energy state. The duration of the excited state limits the energy of that state due to the uncertainty principle.

SET UP: The wavelength λ of the photon is related to the transition energy E of the atom by $E = \dfrac{hc}{\lambda}.$

$\Delta E\ \Delta t \ge \hbar/2.$ The minimum uncertainty in energy is $\Delta E \ge \dfrac{\hbar}{2\Delta t}.$

EXECUTE: **(a)** The photon energy equals the transition energy of the atom, 3.50 eV.

$$\lambda = \frac{hc}{E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.50\text{ eV}} = 355\text{ nm}.$$

(b) $\Delta E = \dfrac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(2.0\times10^{-6}\text{ s})} = 2.6\times10^{-29}\text{ J} = 1.6\times10^{-10}\text{ eV}.$

EVALUATE: The uncertainty in the energy could be larger than that found in (b), but never smaller.

39.77. IDENTIFY: The wave (light or electron matter wave) having less energy will cause less damage to the virus.

SET UP: For a photon $E_{\text{ph}} = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{\lambda}$. For an electron $E_{\text{e}} = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{5.00\times10^{-9}\ \text{m}} = 248\ \text{eV}.$

(b) $E_{\text{e}} = \dfrac{h^2}{2m\lambda^2} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(5.00\times10^{-9}\ \text{m})^2} = 9.65\times10^{-21}\ \text{J} = 0.0603\ \text{eV}.$

EVALUATE: The electron has much less energy than a photon of the same wavelength and therefore would cause much less damage to the virus.

39.78. **IDENTIFY** and **SET UP:** Assume $px \approx h$ and use this to express E as a function of x. E is a minimum for that x that satisfies $\dfrac{dE}{dx} = 0.$

EXECUTE: **(a)** Using the given approximation, $E = \dfrac{1}{2}\left[(h/x)^2/m + kx^2\right]$, so $(dE/dx) = kx - (h^2/mx^3)$, and the minimum energy occurs when $kx = (h^2/mx^3)$, or $x^2 = \dfrac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

EVALUATE: **(b)** $U = \frac{1}{2}kx^2 = \dfrac{h}{2}\sqrt{\dfrac{k}{m}}$. $K = \dfrac{p^2}{2m} = \dfrac{h^2}{2mx^2} = \dfrac{h}{2}\sqrt{\dfrac{k}{m}}$. At this x the kinetic and potential energies are the same.

39.79. **(a) IDENTIFY** and **SET UP:** $U = A|x|$. $F_x = -dU/dx$ relates force and potential. The slope of the function $A|x|$ is not continuous at $x = 0$, so we must consider the regions $x > 0$ and $x < 0$ separately.

EXECUTE: For $x > 0, |x| = x$, so $U = Ax$ and $F = -\dfrac{d(Ax)}{dx} = -A$. For $x < 0$, $|x| = -x$, so $U = -Ax$ and $F = -\dfrac{d(-Ax)}{dx} = +A$. We can write this result as $F = -A|x|/x$, valid for all x except for $x = 0$.

(b) IDENTIFY and **SET UP:** Use the uncertainty principle, expressed as $\Delta p\Delta x \approx h$, and as in Problem 39.78 estimate Δp by p and Δx by x. Use this to write the energy E of the particle as a function of x. Find the value of x that gives the minimum E and then find the minimum E.

EXECUTE: $E = K + U = \dfrac{p^2}{2m} + A|x|.$

$$px \approx h, \text{so } p \approx h/x.$$

Then $E \approx \dfrac{h^2}{2mx^2} + A|x|.$

For $x > 0, E = \dfrac{h^2}{2mx^2} + Ax.$

To find the value of x that gives minimum E set $\dfrac{dE}{dx} = 0.$

$$0 = \frac{-2h^2}{2mx^3} + A.$$

$$x^3 = \frac{h^2}{mA} \text{ and } x = \left(\frac{h^2}{mA}\right)^{\frac{1}{3}}.$$

With this x the minimum E is

$$E=\frac{h^2}{2m}\left(\frac{mA}{h^2}\right)^{2/3}+A\left(\frac{h^2}{mA}\right)^{1/3}=\frac{1}{2}h^{2/3}m^{-1/3}A^{2/3}+h^{2/3}m^{-1/3}A^{2/3}.$$

$$E=\frac{3}{2}\left(\frac{h^2A^2}{m}\right)^{1/3}.$$

EVALUATE: The potential well is shaped like a V. The larger A is, the steeper the slope of U and the smaller the region to which the particle is confined and the greater is its energy. Note that for the x that minimizes E, $2K=U$.

39.80. **(a) IDENTIFY and SET UP:** Let the y-direction be from the thrower to the catcher, and let the x-direction be horizontal and perpendicular to the y-direction. A cube with volume $V=125\text{ cm}^3=0.125\times10^{-3}\text{ m}^3$ has side length $l=V^{1/3}=(0.125\times10^{-3}\text{ m}^3)^{1/3}=0.050\text{ m}$. Thus estimate Δx as $\Delta x\approx 0.050\text{ m}$. Use the uncertainty principle to estimate Δp_x.

EXECUTE: $\Delta x\Delta p_x\ge\hbar/2$ then gives $\Delta p_x\approx\dfrac{\hbar}{2\Delta x}=\dfrac{0.01055\text{ J}\cdot\text{s}}{2(0.050\text{ m})}=0.11\text{ kg}\cdot\text{m/s}$. (The value of $\hbar$ in this other universe has been used.)

(b) IDENTIFY and SET UP: $\Delta x=(\Delta v_x)t$ is the uncertainty in the x-coordinate of the ball when it reaches the catcher, where t is the time it takes the ball to reach the second student.
Obtain Δv_x from Δp_x.

EXECUTE: The uncertainty in the ball's horizontal velocity is $\Delta v_x=\dfrac{\Delta p_x}{m}=\dfrac{0.11\text{ kg}\cdot\text{m/s}}{0.25\text{ kg}}=0.42\text{ m/s}$.

The time it takes the ball to travel to the second student is $t=\dfrac{12\text{ m}}{6.0\text{ m/s}}=2.0\text{ s}$. The uncertainty in the x-coordinate of the ball when it reaches the second student that is introduced by Δv_x is $\Delta x=(\Delta v_x)t=(0.42\text{ m/s})(2.0\text{ s})=0.84\text{ m}$. The ball could miss the second student by about 0.84 m.

EVALUATE: A game of catch would be very different in this universe. We don't notice the effects of the uncertainty principle in everyday life because h is so small.

39.81. **IDENTIFY and SET UP:** For hydrogen-like atoms (1 electron and Z protons), the energy levels are $E_n=(-13.60\text{ eV})Z^2/n^2$, with $n=1$ for the ground state. The energy of a photon is $E=hc/\lambda$.

EXECUTE: **(a)** The least energy absorbed is between the ground state ($n=1$) and the $n=2$ state, which gives the longest wavelength. So $\Delta E_{1\to2}=hc/\lambda$. Using the energy levels for this atom, we have

$(-13.6\text{ eV})Z^2\left(\dfrac{1}{2^2}-\dfrac{1}{1^2}\right)=\dfrac{hc}{\lambda}\;\to\;(10.20\text{ eV})Z^2=hc/\lambda$. Solving Z gives

$$Z=\sqrt{\frac{hc}{(10.20\text{ eV})\lambda}}=\sqrt{\frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{(10.20\text{ eV})(13.56\times10^{-9}\text{ m})}}=3.0.$$

(b) The next shortest wavelength is between the $n=3$ and $n=1$ states.

$$\Delta E_{1\to3}=(-13.6\text{ eV})(3)^2\left(\frac{1}{3^2}-\frac{1}{1^2}\right)=\frac{hc}{\lambda}.$$

Solving for λ gives
$\lambda=(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})/(108.8\text{ eV})=11.40\text{ nm}$.

(c) By energy conservation, $E_{\text{photon}}=E_{\text{ionization}}+K_{\text{el}}$. The ionization energy is the minimum energy to completely remove an electron from the atom, which is from the $n=1$ state to the $n=\infty$ state. Therefore $E_{\text{ionization}}=(13.60\text{ eV})Z^2=(13.60\text{ eV})(9)$. Therefore the kinetic energy of the electron is

$$K_{\text{el}}=E_{\text{photon}}-E_{\text{ionization}}=hc/\lambda-E_{\text{ionization}}.$$

$$K_{\text{el}}=(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})/(6.78\times10^{-9}\text{ m})-(13.60\text{ eV})(9)=60.5\text{ eV}.$$

EVALUATE: The energy levels for a $Z = 3$ atom are 9 times as great as for the comparable energy levels in hydrogen, so the wavelengths of the absorbed light are much shorter than they would be for comparable transitions in hydrogen.

39.82. IDENTIFY and SET UP: The kinetic energy of the electron is $K = eV_{ac}$. The first-order maximum in the Davisson-Germer experiment occurs when $d\sin\theta = \lambda$. The de Broglie wavelength of an electron is $\lambda = h/p$, and its kinetic energy is $K = p^2/2m$. Therefore its momentum is $p = \sqrt{2mK}$, which means its de Broglie wavelength can be expressed as $\lambda = h/\sqrt{2mK}$.

EXECUTE: **(a)** Figure 39.82 shows the graph of $\sin\theta$ versus $1/\sqrt{V_{ac}}$ for the data in the problem. The slope of the best-fit graph is 3.522 $V^{1/2}$.

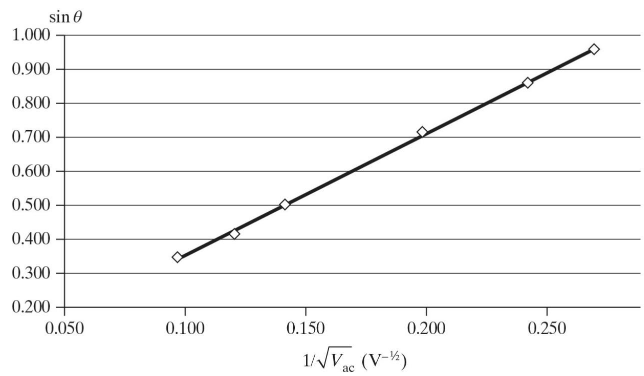

Figure 39.82

(b) At the first maximum, we have $d\sin\theta = \lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}} = \dfrac{h}{\sqrt{2meV_{ac}}}$, which we can write as $\sin\theta = \dfrac{h}{d\sqrt{2me}} \cdot \dfrac{1}{\sqrt{V_{ac}}}$. From this result, we see that a graph of $\sin\theta$ versus $1/\sqrt{V_{ac}}$ should be a straight line having slope equal to $\dfrac{h}{d\sqrt{2me}}$. Solving for d gives $d = \dfrac{h}{(\text{slope})\sqrt{2me}}$, which gives

$$d = \frac{4.136\times10^{-15}\ \text{J}\cdot\text{s}}{(3.522\ \text{V}^{1/2})\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})}} = 3.48\times10^{-10}\ \text{m} = 0.348\ \text{nm}.$$

EVALUATE: Atom spacing in crystals are typically around a few tenths of a nanometer, so these results are plausible.

39.83. IDENTIFY and SET UP: The power radiated by an ideal blackbody is $P = \sigma AT^4$. Wien's displacement law, $\lambda_m T = 2.90\times10^{-3}\ \text{m}\cdot\text{K}$, applies to the stars. The surface area of a star is $A = 4\pi R^2$, and $R_{sun} = 6.96\times10^8$ m.

EXECUTE: **(a)** Calculate the radiated power for each star using $P = \sigma AT^4$. For Polaris we have $P = \sigma AT^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)[(46)(6.96\times10^8\ \text{m})]^2(6015\ \text{K})^4 = 9.56\times10^{29}$ W.

Repeating this calculation for the other stars gives us the following results.

Polaris: $P = 9.56\times10^{29}$ W

Vega: $P = 2.19\times10^{28}$ W

Antares: $P = 3.60\times10^{31}$ W

α Centauri B: $P = 1.98\times10^{26}$ W

Antares has the greatest radiated power.

(b) Apply Wien's displacement law, $\lambda_{\mathrm{m}}T = 2.90\times10^{-3}\ \mathrm{m\cdot K}$, and solve for λ_{m}. For example, for Polaris we have $\lambda_{\mathrm{m}} = \dfrac{2.90\times10^{-3}\ \mathrm{m\cdot K}}{T} = \dfrac{2.90\times10^{-3}\ \mathrm{m\cdot K}}{6015\ \mathrm{K}} = 4.82\times10^{-7}\ \mathrm{m} = 482\ \mathrm{nm}.$ Repeating this calculation for the other stars gives the following results.

Polaris: $\lambda_{\mathrm{m}} = 482$ nm
Vega: $\lambda_{\mathrm{m}} = 302$ nm
Antares: $\lambda_{\mathrm{m}} = 853$ nm
α Centauri B: $\lambda_{\mathrm{m}} = 551$ nm

The visible range is 380 nm to 750 nm, so Polaris and α Centauri B radiate chiefly in the visible range.
(c) By comparing the results in part (a), we see that only α Centauri B radiates less than our sun.
EVALUATE: The power radiated by a star depends on its surface area *and* its surface temperature. Vega, a very hot star, radiates less than the much cooler Antares because Antares has over 300 times the radius of Vega and therefore over 300^2 times the surface area of Vega. A hot star is not necessarily brighter than a cool star.

39.84. **IDENTIFY:** This problem involves blackbody radiation and the Planck radiation law.
SET UP: We have $r = 1.23$ m and use Equation 39.24. The device captures all photons within 1% of the central peak E_0. For such a small range, we can treat the emittance as constant over

$$E_0 \pm \Delta E, \Delta E = 0.0100E_0.\ \ E = hc/\lambda,\ \ I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)},\ I = \int I(\lambda)\, d\,\lambda.$$

EXECUTE: **(a)** Restructure $\int I(\lambda) d\,\lambda$ as $\int I(E)\, dE.$ Using $E = hc/\lambda$ gives $d\lambda = -\lambda^2\, dE/hc.$

$$I = \int I(E)\, dE = \int \frac{2\pi hc^2\left(-\lambda^2\ dE/hc\right)}{\lambda^5\left(e^{E/kT} - 1\right)} = -\int \frac{2\pi E^3}{h^3c^2\left(e^{e/kT} - 1\right)} dE.$$

Therefore it follows that

$$I(E) = \frac{2\pi E^3}{h^3\, c^2\left(e^{E/kT} - 1\right)}.$$

(b) We want to find the E_0 that maximizes $I(E)$ and then use it to find $I_{\mathrm{max}} = I(E_0)$. Set $dI(E)/dt = 0$ for a maximum, using the result from part (a). This gives $\dfrac{\left(e^{E/kT} - 1\right)\left(3\,E^2\right) - E^3\left(1/kT\right)e^{E/kT}}{\left(e^{E/kT} - 1\right)^2} = 0,$ which simplifies to $3 - 3\mathrm{e}^{-E/kT} - E/kT = 0.$

Letting $x = E/kT$, the final equation we must solve is $3 - x = 3e^{-x}$. Using trial-and-error (which gives an answer rather quickly) or appropriate software, the result is $x = 2.821$, so $E/kT = 2.821$, which gives $E_0 = 2.821kT$. Now use this result to find $I_{\mathrm{max}} = I(E_0)$, giving

$$I_{\mathrm{max}} = \frac{2\pi(2.821kT)^3}{h^3c^3(2.821 - 1)} = \frac{2.843\pi k^3T^3}{h^3c^2}.$$

(c) We want the power and current at $T = 1000$ K. At $T = 1000$ K, $E_0 = 2.821kT = 3.896\times10^{-20}$ J. The photocells capture all photons within 1% of E_0, so the full range of energy is $E_0 \pm 1\%E_0$, which is 2% E_0. So $\Delta E = (0.020)(3.896\times10^{-20}\ \mathrm{J}) = 7.792\times10^{-22}$ J. $P = IA = I(E)(\Delta E)A$. Using given numbers and the values we have found gives

$$p = \left(\frac{2.843\pi k^3T^3}{h^3c^3}\right)(\Delta E)\left(4\pi r^2\right) = 13.3\ \mathrm{kW}.$$

Solving $P = I^2R$ for I and using this result gives $I = 3.64$ A.

(d) We want the power and current if $T = 5000$ K. Follow the same procedure as in part (c), giving $E_0 = 2.821kT = (2.821)k(5000\text{ K}) = 1.946 \times 10^{-19}$ J, $\Delta E = 0.020E_0 = 3.893 \times 10^{-21}$ J, $P = 8.29$ MW, and $I = 91.1$ A.

EVALUATE: In (c) and (d) we see that increasing the temperature by a factor of 5 increased the power by a factor of $5^4 = 625$ because the power is proportional to T^4.

39.85. **IDENTIFY:** We are looking at the energy and momentum in particle collision at nonrelativistic speeds. An alpha particle collides with a gold nucleus at rest.

SET UP: Energy and momentum are conserved during the collision. Use $K = p^2/2m$ to express p in terms of K.

EXECUTE: **(a)** We want V (the gold) speed. Energy conservation: $K = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$. Momentum conservation: $\sqrt{2mK} = MV - mv$ Solving for V gives $V = \sqrt{8mK}/(m+M)$.

(b) We want $\Delta K(\alpha)/K$. The kinetic energy lost by the alpha particle is equal to the kinetic energy gained by the gold nucleus. Using this fact gives

$$\frac{\Delta K(\alpha)}{K(\alpha)} = \frac{\frac{1}{2}MV^2}{K} = \frac{\frac{1}{2}\left(\frac{\sqrt{8mK}}{m+M}\right)^2}{K} = \frac{4mM}{(m+M)^2}.$$

(c) From part (b) we know that the fractional energy lost depends only on the masses, so the result is *independent* of the initial kinetic energy. Therefore the answer is *yes*.

(d) We want V and the fractional energy lost to the gold if $K = 5.00$ MeV. Using the given numbers for m, M, and K in $V = \sqrt{8mK}/(m+M)$ from part (a) and converting 5.00 MeV to joules gives $V = 1.49 \times 10^6$ m/s $= 0.00497c$. From part (b), the fraction lost is $4mM/(m + M)^2 = 0.182 = 18.2\%$.

(e) We want v, so that $V = 0.10c$ by using nonrelativistic physics. First find K using $V = \sqrt{8mK}/(m+M)$, giving $K = [V(m + M)]^2/8m$. Classically, $K = \frac{1}{2}mv^2$. Equating this to the previous result and solving for v gives $v = V(1 + M/m)/2$. Using $V = 0.10c$ and the given masses gives $v = 1.04c$.

EVALUATE: **(f)** This speed is not possible, so we would need to use the relativistic equations.

39.86. **IDENTIFY:** Follow the steps specified in the hint.

SET UP: The value of Δx_i that minimizes Δx_f satisfies $\dfrac{d(\Delta x_f)}{d(\Delta x_i)} = 0.$

EXECUTE: Time of flight of the marble, from a free-fall kinematic equation is just $t = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{2(25.0\text{ m})}{9.80\text{ m/s}^2}} = 2.26\text{ s}.$ $\Delta x_f = \Delta x_i + (\Delta v_x)t = \Delta x_i + \left(\dfrac{\Delta p_x}{m}\right)t = \dfrac{\hbar t}{2\Delta x_i m} + \Delta x_i.$ To minimize Δx_f with respect to Δx_i, $\dfrac{d(\Delta x_f)}{d(\Delta x_i)} = 0 = \dfrac{-\hbar t}{2m(\Delta x_i)^2} + 1 \Rightarrow \Delta x_i(\text{min}) = \sqrt{\left(\dfrac{\hbar t}{2m}\right)}$

$$\Rightarrow \Delta x_f(\text{min}) = \sqrt{\frac{\hbar t}{2m}} + \sqrt{\frac{\hbar t}{2m}} = \sqrt{\frac{2\hbar t}{m}} = \sqrt{\frac{2(1.055\times 10^{-34}\text{ J}\cdot\text{s})(2.26\text{ s})}{0.0200\text{ kg}}} = 1.54\times 10^{-16}\text{ m}$$

$$= 1.54\times 10^{-7}\text{ nm}.$$

EVALUATE: The uncertainty introduced by the uncertainty principle is completely negligible in this situation.

39.87. **IDENTIFY** and **SET UP:** The period was found in Exercise 39.23b: $T = \dfrac{4\epsilon_0^2 n^3 h^3}{me^4}$. The equation

$E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{me^4}{8n^2h^2}$ gives the energy of state n of a hydrogen atom.

EXECUTE: **(a)** The frequency is $f = \dfrac{1}{T} = \dfrac{me^4}{4\epsilon_0^2 n^3 h^3}$.

(b) The equation $hf = E_\text{i} - E_\text{f}$ tells us that $f = \dfrac{1}{h}(E_2 - E_1)$. So $f = \dfrac{me^4}{8\epsilon_0^2 h^3}\left(\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}\right)$. If $n_2 = n$ and $n_1 = n+1$, then $\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2} = \dfrac{1}{n^2} - \dfrac{1}{(n+1)^2} = \dfrac{1}{n^2}\left(1 - \dfrac{1}{(1+1/n)^2}\right) \approx \dfrac{1}{n^2}\left(1 - \left(1 - \dfrac{2}{n} + \ldots\right)\right) = \dfrac{2}{n^3}$. Therefore, for large n, $f \approx \dfrac{me^4}{4\epsilon_0^2 n^3 h^3}$.

EVALUATE: We have shown that for large n we obtain the classical result that the frequency of revolution of the electron is equal to the frequency of the radiation it emits.

39.88. **IDENTIFY** and **SET UP:** The de Broglie wavelength of the helium is $\lambda = h/p$. Its kinetic energy is $K = p^2/2m$, so $p = \sqrt{2mK}$. Therefore its de Broglie wavelength can be expressed as $\lambda = h/\sqrt{2mK}$. The kinetic energy of the ions acquired during acceleration is $K = eV = p^2/2m$.

EXECUTE: Express the wavelength in terms of V, giving $\lambda = h/\sqrt{2mK} = h/\sqrt{2meV}$. From this we see that a large mass m results in a small (short) wavelength, which is choice (b).

EVALUATE: Because helium is 7300 times heavier than an electron and because $\lambda \propto 1/\sqrt{m}$, the wavelength for helium would be $1/\sqrt{7300} = 0.012$ times the wavelength of an electron.

39.89. **IDENTIFY:** Calculate the accelerating potential V need to produce a helium ion with a wavelength of 0.1 pm to see if that potential lies within the range of 10-50 kV.

SET UP: The de Broglie wavelength of the helium ion is $\lambda = h/p$, so $p = h/\lambda$. By energy conservation, $K = eV = p^2/2m$.

EXECUTE: Combining the above equations gives

$$eV = K = p^2/2m = \frac{(h/\lambda)^2}{2m}, \text{ so } V = \frac{(h/\lambda)^2}{2me}.$$

$$V = \frac{\left[(6.626\times10^{-34}\ \text{J}\cdot\text{s})/(0.1\times10^{-12}\ \text{m})\right]^2}{2(7300)(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})} = 2.1\times10^4\ \text{V} = 21\ \text{kV}.$$

This voltage is within the 10-50 kV range, so choice (a) is correct.

EVALUATE: A large voltage is required because the desired wavelength is small.

39.90. **IDENTIFY** and **SET UP:** Electric and magnetic fields act on electrical charges.

EXECUTE: Focusing particles requires electric and magnetic forces, so they must have charge, which makes choice (c) correct.

EVALUATE: All particles have wave properties, so choice (a) is not correct. Helium is an inert gas, so it normally does form molecules, so that rules out choice (b). The mass difference between a helium atom and a helium ion is negligible because the electron is 7300 times lighter than a helium ion, which eliminates choice (d).

39.91. **IDENTIFY** and **SET UP:** The ion loses 0.2 MeV/μm, and its energy can be determined only to within 6 keV. Call x the minimum difference in thickness that can be discerned, and realize that 0.2 MeV = 200 keV.

EXECUTE: (0.2 MeV/μm)x = 6 keV. Solving for x gives x = (6 keV)/(200 keV/μm) = 0.03 μm, which makes choice (a) the correct one.

EVALUATE: Greater precision in determining the energy of the ion would allow one to discern smaller features.