35

GEOMETRIC OPTICS

VP35.2.1. **IDENTIFY:** This problem is about double-slit interference of light.

SET UP: Constructive interference: $d\sin\theta_m = m\lambda$, for small angles: $y_m = R\dfrac{m\lambda}{d}$.

EXECUTE: **(a)** We want λ. For small angles we can use $y_m = R\dfrac{m\lambda}{d}$. Solve for λ_2.

$$\lambda_2 = \frac{y_2 d}{2R} = \frac{(11.4\text{ mm})(0.180\text{ mm})}{2(1500\text{ mm})} = 684\text{ nm}.$$

(b) We want the distance to the $m = -3$ fringe. This will be the same as the distance to the $m = +3$ fringe, so we can ignore the minus sign. $\dfrac{y_3}{y_2} = \dfrac{R(3\lambda/d)}{R(2\lambda/d)} = \dfrac{3}{2}$. $y_3 = (3/2)y_{-2} = (3/3)(11.4\text{ mm}) = 17.1\text{ mm}$.

EVALUATE: The small-angle approximation works only fairly close to the central maximum where bright fringes are evenly spaced.

VP35.2.2. **IDENTIFY:** This problem deals with the interference of radiowaves from two sources.

SET UP: Maxima occur when $d\sin\theta_m = m\lambda$, minima occur when $d\sin\theta_m = \left(m + \dfrac{1}{2}\right)\lambda$, $f\lambda = c$. We want the angles to the maxima and minima.

EXECUTE: **(a)** Maxima: First find λ: $\lambda = c/f = c/(1.05\text{ MHz}) = 285.7$ m. Now use $d\sin\theta_m = m\lambda$, $m = 0, \pm1, \pm2, \ldots$ Solve for $\sin\theta_m$, giving $\sin\theta_m = \dfrac{m\lambda}{d} = m\left(\dfrac{285.7\text{ m}}{810\text{ m}}\right) = 0.3527m$.

$m = 0$: $\sin\theta_0 = 0 \rightarrow \theta_0 = 0°$

$m = 1$: $\sin\theta_1 = (0.3527)(1) = 0.3527 \rightarrow \theta_1 = 20.7°$

$m = 2$: $\sin\theta_2 = (0.3527)(2) = 0.7021 \rightarrow \theta_2 = 44.9°$

$m = 3$: $\sin\theta_3 = (0.3527)(3) = 1.06 \rightarrow$ not possible

(b) Maxima: Use $d\sin\theta_m = \left(m + \dfrac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$

$m = 0$: $\sin\theta_0 = (0.3527)(0.500) = 0.1764 \rightarrow \theta_0 = 10.2°$

$m = 1$: $\sin\theta_1 = (0.3527)(1.5) = 0.5291 \rightarrow \theta_1 = 31.9°$

$m = 2$: $\sin\theta_2 = (0.3527)(2.5) = 0.8818 \rightarrow \theta_2 = 61.9°$

$m = 3$: $\sin\theta_3 = (0.3527)(3.5) = 1.23 \rightarrow$ not possible

EVALUATE: The minima are between the maxima, but not midway between them.

VP35.2.3. **IDENTIFY:** We have double slit interference.

SET UP: $y_m = R\dfrac{m\lambda}{d}$.

EXECUTE: **(a)** We want d. Solve $y_m = R\dfrac{m\lambda}{d}$ for d. $d = \dfrac{m\lambda R}{y_m} = \dfrac{3(685\text{ nm})(2100\text{ mm})}{9.15\text{ mm}} = 0.472\text{ mm}$.

(b) We want y. $\dfrac{y_g}{y_r} = \dfrac{R\dfrac{m\lambda_g}{d}}{R\dfrac{m\lambda_r}{d}} = \dfrac{\lambda_g}{\lambda_r} = \dfrac{515\text{ nm}}{685\text{ nm}} = 0.7518$, so y_g = (9.15 mm)(0.7518) = 6.88 mm.

EVALUATE: Since $y \propto \lambda$, as λ decreases, the bright fringes get closer together, which agrees with our result in part (b).

VP35.2.4. **IDENTIFY:** We have double slit interference.

SET UP: $d\sin\theta_m = m\lambda$ (bright fringes), $d\sin\theta_m = \left(m+\dfrac{1}{2}\right)\lambda$ (dark spots).

EXECUTE: **(a)** We want λ. $\lambda = \dfrac{d\sin\theta_m}{m} = \dfrac{(0.370\text{ mm})\sin(0.407°)}{4} = 657\text{ nm}$.

(b) We want θ_2 for a dark spot. $\sin\theta_2 = \dfrac{\left(2+\dfrac{1}{2}\right)\lambda}{d} = \dfrac{(2.5)(657\text{ nm})}{0.370\text{ mm}}$. θ_2 = 0.254°.

EVALUATE: The bright (and dark) spots are equally spaced for small angles but not for large ones.

VP35.3.1. **IDENTIFY:** This problem is about the intensity of a two-source interference pattern.

SET UP: $I = I_0\cos^2(\phi/2)$, $\phi/2 = \pi d\sin\theta/\lambda$, $f\lambda = c$.

EXECUTE: **(a)** We want I. First find λ. $\lambda = c/f$ = c/(59.3 MHz) = 5.059 m. $\phi/2 = \pi d\sin\theta/\lambda$ = π(13.0 m)(sin 5.00°)/(5.059 m) = 0.7036 rad = 40.313°. Now use $I = I_0\cos^2(\phi/2)$ to find the intensity. $I = I_0\cos^2(\phi/2)$ = (0.0330 W/m^2) cos^2(40.313°) = = 0.0192 W/m^2.

(b) We want θ_{min}. $I = I_0\cos^2(\phi/2) = \dfrac{I_0}{2}$, so $\cos(\phi/2) = 1/\sqrt{2}$. $\phi/2$ = 45.0° = π/4 rad. $\phi/2 = \pi d\sin\theta/\lambda$ = π/4, so $\sin\theta = \lambda/4d =$ (5.059 m)/[4(13.0 m)] = 0.09729. θ_{min} = 5.58°.

EVALUATE: At 5.00° I = 0.0192 W/m^2, which is a little more than $I_0/2$. So at a slightly larger angle I will equal $I_0/2$. This agrees with our result because we found θ = 5.58° for $I = I_0/2$.

VP35.3.2. **IDENTIFY:** This problem is about the intensity of a double-slit interference pattern.

SET UP: $I = I_0\cos^2(\phi/2)$, $\phi/2 = \pi d\sin\theta/\lambda$, for small angles $y_m = R\dfrac{m\lambda}{d}$

EXECUTE: **(a)** We want I. For small angles $\sin\theta \approx \tan\theta \approx \dfrac{y}{R}$, so $\dfrac{\phi}{2} = \dfrac{\pi d\sin\theta}{\lambda} \approx \dfrac{\pi dy}{\lambda R}$ $= \dfrac{\pi(0.230\text{ mm})(6.50\text{ mm})}{(655\text{ nm})(1.75\text{ m})} = 4.0974$ rad = 234.8°. $I = I_0\cos^2(\phi/2)$ = (0.520 W/m^2) cos^2(234.8°) = 0.173 W/m^2.

(b) We want y. $I = I_0\cos^2(\phi/2) = \dfrac{I_0}{4}$, $\cos(\phi/2) = 1/2$, so $\phi/2$ = 60° = π/3 rad. Therefore $\dfrac{\phi}{2} = \dfrac{\pi}{3} \approx \dfrac{\pi dy}{\lambda R}$, which gives $y = \dfrac{\lambda R}{3d} = \dfrac{(655\text{ nm})(1.75\text{ m})}{3(0.230\text{ mm})} = 1.66\text{ mm}$.

EVALUATE: We must be careful to convert angles in radians to degrees to use most calculators.

VP35.3.3. **IDENTIFY:** This problem involves the intensity of the interference pattern of two sources of radiowaves.

SET UP: $I = I_0 \cos^2(\phi/2)$, $\phi/2 = \pi d \sin\theta/\lambda$.

EXECUTE: **(a)** We want λ. Use $I = I_0 \cos^2(\phi/2)$ and $\phi/2 = \pi d \sin\theta/\lambda$ to find ϕ and then use that to find λ. At 6.00° we have 0.0303 W/m^2 = (0.0540 W/m^2) cos^2(ϕ/2). ϕ/2 = 41.49° = 0.7241 rad. Now get λ: $\phi/2 = \pi d \sin\theta/\lambda$ gives $\lambda = \dfrac{\pi d \sin\theta}{\phi/2} = \dfrac{\pi(8.00\text{ m})\sin(6.00°)}{0.7241\text{ rad}} = 3.63\text{ m}.$

(b) We want I at θ = 12.0°. $\phi/2 = \pi d \sin\theta/\lambda$ = π(8.00 m)(sin 12.0°)/(3.63 m) = 1.440 rad = 8253°.

$I = I_0 \cos^2(\phi/2)$ = (0.0540 W/m^2) cos^2(82.53°) = 9.14×10^{-4} W/m^2.

EVALUATE: It would not be good to use the small-angle approximation in this case with θ = 12°.

VP35.3.4. **IDENTIFY:** This problem is about the intensity of a two-slit interference pattern.

SET UP: $I = I_0 \cos^2(\phi/2)$, $\phi/2 = \pi d \sin\theta/\lambda$, $\sin\theta \approx y/R$ (small-angle approximation).

EXECUTE: **(a)** We want λ. First find ϕ/2, then use it to find p using $\sin\theta \approx y/R$. $I = I_0 \cos^2(\phi/2)$ gives $0.0900I_0 = I_0$ cos^2(ϕ /), so ϕ /2 = 72.54° = 1.266 rad. Now use $\phi/2 = \pi d \sin\theta/\lambda$ and solve for λ using $\sin\theta \approx y/R$. $\lambda = \dfrac{\pi d(y/R)}{\phi/2} = \dfrac{\pi(0.110\text{ mm})(10.5\text{ mm})/(5.00\text{ m})}{1.266\text{ rad}} = 573\text{ nm}.$

(b) We want y. Find $\phi/2$: $0.300I_0 = I_0 \cos^2\phi/2$ gives $\phi/2$ = 56.79° = 0.9918 rad. Now find y. Since y will be less than 10.5 mm, we can use the small-angle approximation. Solve $\phi/2 = \pi d \sin\theta/\lambda$ for y using $\sin\theta \approx y/R$. $y = \dfrac{\lambda(\phi/2)R}{\pi d}$. Take ratios giving $\dfrac{y_2}{y_1} = \dfrac{\dfrac{\lambda(\phi_2/2)R}{\pi d}}{\dfrac{\lambda(\phi_1/2)R}{\pi d}} = \dfrac{\phi_2/2}{\phi_1/2} = \dfrac{0.9912}{1.266} = 0.7829.$

y_2 = (0.7829)(10.5 mm) = 8.22 mm.

EVALUATE: We find $y_2 < y_1$, as we expect.

VP35.6.1. **IDENTIFY:** This problem is about thin-film interference.

SET UP: Refer to Fig. 35.15 in the textbook. Call h the thickness of the paper and t the thickness at point x. A half-cycle phase shift occurs at reflection at the lower plate. The dark fringes occur at points of destructive interference, for which $2t = m\lambda$ due to the half-cycle phase shift.

EXECUTE: **(a)** We want h. $2t = m\lambda$, so $t = m\lambda/2$ (m = 0, 1, 2, …) As in Fig. 35.15 in the textbook, $t/x = h/l$, so $h = lt/x = l\,m\lambda/2$. The fringes are 1.30 mm apart. The first one is at x = 0, so the next one is at x = 1.30 mm. The m = 1 dark fringe is at t_1 = (1) λ /2 = (565 nm)/2 = 282.5 nm. So $\tan\theta = t_1/x_1 = h/l$, which gives $h = lt_1/x_1$ = (8.00 cm)(282.5 nm)/(1.30 mm) = 0.0174 mm.

(b) We want the fringe spacing. There is still a half-cycle phase shift at reflection because $n_{\text{glass}} > n_{\text{ethanol}}$, but $\lambda_n = \dfrac{\lambda_0}{n_{\text{ethanol}}}$. The m = 0 fringe is at x = 0. The m = 1 fringe is at x_1 (which we do not know).

$2t_1 = = \dfrac{\lambda_0}{n}$. So $t_1 = \lambda/2n$ = (565 nm)/[2(136)] = 207.7 nm. As before, $t_1/x_1 = h/l$, so $x_{-1} = t_1 l/h$. This gives x_1 = (207.7 nm)(8.00 cm)/(0.0174 mm) = 0.956 mm. This is the distance between the m = 0 fringe and the m = 1 fringe, so it is the fringe spacing.

EVALUATE: The replacement of ethanol for air made significant changes in the fringe pattern because it changed the wavelength of light in the region between the glass plates.

VP35.6.2. **IDENTIFY:** We have thin-film interference.

SET UP: We want to know which wavelengths of visible light have destructive and constructive interference. In both cases there is a half-cycle phase shift during reflection because n_{glass} is greater than both n_{air} and $n_{\text{carbon-tet}}$, so for constructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_m$ and for destructive interference $2t = m\lambda_m$.

EXECUTE: (a) Air film. Solve for λ, which gives $\lambda_m = \frac{2t}{m + 1/2}$.

Constructive: Use $2t = \left(m + \frac{1}{2}\right)\lambda_m$.

$m = 0$: $\lambda_0 = 4t = 4(146 \text{ nm}) = 584$ nm (not visible)

$m = 1$: $\lambda_1 = 2(146 \text{ nm})/1.5 = 194$ nm (not visible)

Therefore no visible light interferes constructively.

Destructive: Use $2t = m\lambda_m$.

$m = 1$: $\lambda_1 = 2t = 292$ nm (not visible)

No visible light interferes destructively.

(b) Carbon tetrachloride film. Follow the same procedure as in (a) except $\lambda_n = \frac{\lambda_0}{1.46}$. The results are: No wavelengths interfere constructively, but λ = 426 nm interferes destructively.

EVALUATE: By varying t we could get different wavelengths to interfere.

VP35.6.3. IDENTIFY: We have thin-film interference.

SET UP: We want to know which wavelengths of visible light have destructive and constructive interference. In both cases there is a half-cycle phase shift during reflection because n_{plastic} is greater than both n_{air} and n_{ethanol}, so for constructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_m$ and for destructive interference $2t = m\lambda_m$.

EXECUTE: (a) Air film. Solve for λ, which gives.

Constructive: Use $2t = \left(m + \frac{1}{2}\right)\lambda_m$.

$m = 0$: $\lambda_0 = 4t = 4(185 \text{ nm}) = 740$ nm (visible)

$m = 1$: $\lambda_1 = 2(185 \text{ nm})/1.5 = 247$ nm (not visible)

Therefore only 740 nm interferes constructively.

Destructive: Use $2t = m\lambda_m$.

$m = 1$: $\lambda_1 = 2t = 370$ nm (not visible)

No visible light interferes destructively.

(b) Carbon tetrachloride film. Follow the same procedure as in (a) except $\lambda_n = \frac{\lambda_0}{1.46}$. The results are: No wavelengths interfere constructively, but λ = 503 nm interferes destructively.

EVALUATE: We are only considering interference of rays reflected off the top and bottom of the gap between the sheets.

VP35.6.4. IDENTIFY: We have thin-film interference.

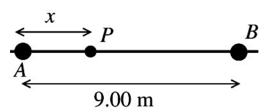
SET UP: There is a half-cycle phase reversal at the upper surface of the grease but not at the lower surface. For constructive interference $2t = \left(m + \frac{1}{2}\right)\lambda_m$, $\lambda_n = \frac{\lambda_0}{n}$.

EXECUTE: (a) We want the smallest possible thickness. $2t = \left(m + \frac{1}{2}\right)\lambda_m$. The minimum thickness is for $m = 0$, so $t_{\text{min}} = \lambda/4n_{\text{glass}} =$ (565 nm)/[4(1.60)] = 88.3 nm.

(b) We want the second smallest possible thickness. This is for $m = 1$. $t = \left(1 + \frac{1}{2}\right)\frac{\lambda_m}{2} = 265\ \text{nm}.$

EVALUATE: The second smallest thickness is one wavelength (in the grease) greater than the smallest thickness.

35.1. **IDENTIFY:** Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from A to P and from B to P. For constructive interference this path difference is an integer multiple of the wavelength.
SET UP: Consider Figure 35.1.



The distance of point P from each coherent source is $r_A = x$ and $r_B = 9.00\ \text{m} - x$.

Figure 35.1

EXECUTE: The path difference is $r_B - r_A = 9.00\ \text{m} - 2x$.

$r_B - r_A = m\lambda,\ m = 0,\ \pm 1, \pm 2,\ \ldots$

$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8\ \text{m/s}}{120 \times 10^6\ \text{Hz}} = 2.50\ \text{m}.$

Thus $9.00\ \text{m} - 2x = m(2.50\ \text{m})$ and $x = \frac{9.00\ \text{m} - m(2.50\ \text{m})}{2} = 4.50\ \text{m} - (1.25\ \text{m})m.$ x must lie in the range 0 to 9.00 m since P is said to be between the two antennas.

$m = 0$ gives $x = 4.50\ \text{m}.$
$m = +1$ gives $x = 4.50\ \text{m} - 1.25\ \text{m} = 3.25\ \text{m}.$
$m = +2$ gives $x = 4.50\ \text{m} - 2.50\ \text{m} = 2.00\ \text{m}.$
$m = +3$ gives $x = 4.50\ \text{m} - 3.75\ \text{m} = 0.75\ \text{m}.$
$m = -1$ gives $x = 4.50\ \text{m} + 1.25\ \text{m} = 5.75\ \text{m}.$
$m = -2$ gives $x = 4.50\ \text{m} + 2.50\ \text{m} = 7.00\ \text{m}.$
$m = -3$ gives $x = 4.50\ \text{m} + 3.75\ \text{m} = 8.25\ \text{m}.$

All other values of m give values of x out of the allowed range. Constructive interference will occur for $x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, and 8.25 m.

EVALUATE: Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

35.2. **IDENTIFY:** For destructive interference the path difference is $(m + \frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots.$ The longest wavelength is for $m = 0$. For constructive interference the path difference is $m\lambda,\ m = 0, \pm 1, \pm 2, \ldots.$The longest wavelength is for $m = 1$.
SET UP: The path difference is 120 m.

EXECUTE: **(a)** For destructive interference $\frac{\lambda}{2} = 120\ \text{m} \Rightarrow \lambda = 240\ \text{m}.$

(b) The longest wavelength for constructive interference is $\lambda = 120\ \text{m}.$

EVALUATE: The path difference doesn't depend on the distance of point Q from B.

35.3. **IDENTIFY:** If the path difference between the two waves is equal to a whole number of wavelengths, constructive interference occurs, but if it is an odd number of half-wavelengths, destructive interference occurs.
SET UP: We calculate the distance traveled by both waves and subtract them to find the path difference.

EXECUTE: Call P_1 the distance from the right speaker to the observer and P_2 the distance from the left speaker to the observer.

(a) $P_1 = 8.0\text{ m}$ and $P_2 = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10.0\text{ m}$. The path distance is $\Delta P = P_2 - P_1 = 10.0\text{ m} - 8.0\text{ m} = 2.0\text{ m}$.

(b) The path distance is one wavelength, so constructive interference occurs.

(c) $P_1 = 17.0\text{ m}$ and $P_2 = \sqrt{(6.0\text{ m})^2 + (17.0\text{ m})^2} = 18.0\text{ m}$. The path difference is $18.0\text{ m} - 17.0\text{ m} = 1.0\text{ m}$, which is one-half wavelength, so destructive interference occurs.

EVALUATE: Constructive interference also occurs if the path difference $2\lambda, 3\lambda, 4\lambda$, etc., and destructive interference occurs if it is $\lambda/2, 3\lambda/2, 5\lambda/2$, etc.

35.4. **IDENTIFY:** For constructive interference the path difference d is related to λ by $d = m\lambda,\ m = 0, 1, 2, \ldots$ For destructive interference $d = (m + \frac{1}{2})\lambda,\ m = 0, 1, 2, \ldots$

SET UP: $d = 2040\text{ nm}$.

EXECUTE: **(a)** The brightest wavelengths are when constructive interference occurs:

$$d = m\lambda_m \Rightarrow \lambda_m = \frac{d}{m} \Rightarrow \lambda_3 = \frac{2040\text{ nm}}{3} = 680\text{ nm},\ \lambda_4 = \frac{2040\text{ nm}}{4} = 510\text{ nm and } \lambda_5 = \frac{2040\text{ nm}}{5} = 408\text{ nm}.$$

(b) The path-length difference is the same, so the wavelengths are the same as part (a).

(c) $d = (m + \frac{1}{2})\lambda_m$ so $\lambda_m = \dfrac{d}{m + \frac{1}{2}} = \dfrac{2040\text{ nm}}{m + \frac{1}{2}}$. The visible wavelengths are $\lambda_3 = 583\text{ nm}$ and $\lambda_4 = 453\text{ nm}$.

EVALUATE: The wavelengths for destructive interference are between those for constructive interference.

35.5. **IDENTIFY:** This problem is about interference of electromagnetic waves.

SET UP: For constructive interference, the path difference $r_2 - r_1$ must be equal to $m\lambda$.

EXECUTE: **(a)** How many points have constructive interference? $r_2 - r_1 = m\lambda$. Call x the distance from A, so the distance from B is $40.0\text{ m} - x$. The path difference is $(40.0\text{ m} - x) - x = 40.0\text{ m} - 2x$, so we must have $40.0\ m - 2x = m\lambda$, where $m = 0, 1, 2, \ldots$. Therefore $x = 20.0\text{ m} - \dfrac{m\lambda}{2} = 20.0\text{ m} - (350\text{ m})m$.

For $m = 0$, $x = 20.0$ m, which is the midpoint between the speakers. The maximum m for a positive x is $m = 5$, for which $x = 2.50$ m. So there are 5 points between A and the midpoint. There are also 5 points on the other side of the midpoint. The total is therefore 5 + 5 + 1 = 11 points.

(b) The minimum x is for $m = 5$, so $x_{min} = 2.50$ m.

EVALUATE: There are also points at which destructive interference occurs when the path difference is an odd multiple of $\lambda/2$.

35.6. **IDENTIFY:** We have double-slit interference of light.

SET UP: We want the distance between the $m = 49$ and $m = 50$ maxima. $d\sin\theta_m = m\lambda$ (all angles), $y_m = R\dfrac{m\lambda}{d}$ (small angles), $y_m = R\tan\theta_m$.

EXECUTE: First find the slit spacing d. Near the center we can use $y_m = R\dfrac{m\lambda}{d}$. We know the distance between adjacent fringes, so $\Delta m = 1$. $\Delta y_m = R\dfrac{\Delta m\lambda}{d} = \dfrac{R\lambda}{d}$. Solving for d gives $d = \dfrac{R\lambda}{\Delta y} = (200\text{ cm})$ $(500\text{ nm})/(3.53\text{ cm}) = 2.833\times10^4\text{ nm}$. For the distant fringes (large n) we cannot use the small-angle approximation, so we use $d\sin\theta_m = m\lambda$. $\sin\theta_{49} = 49\lambda/d$ and $\sin\theta_{50} = 50\lambda/d$. Using the known values for d and λ gives $\theta_{49} = 59.87°$ and $\theta_{50} = 61.94°$. Using $y_m = R\tan\theta_m$ with R = 200 cm gives $\Delta y = y_{50} - y_{49} = R\left(\tan\theta_{50} - \tan\theta_{49}\right) = 30.6$ cm.

EVALUATE: Clearly the small-angle approximation cannot be used for fringes far from the center.

35.7. **IDENTIFY:** The value of y_{20} is much smaller than R and the approximate expression $y_m = R\dfrac{m\lambda}{d}$ is accurate.

SET UP: $y_{20} = 10.6\times10^{-3}$ m.

EXECUTE: $d = \dfrac{20R\lambda}{y_{20}} = \dfrac{(20)(1.20\text{ m})(502\times10^{-9}\text{ m})}{10.6\times10^{-3}\text{ m}} = 1.14\times10^{-3}\text{ m} = 1.14\text{ mm}.$

EVALUATE: $\tan\theta_{20} = \dfrac{y_{20}}{R}$ so $\theta_{20} = 0.51°$ and the approximation $\sin\theta_{20} \approx \tan\theta_{20}$ is very accurate.

35.8. **IDENTIFY:** Since the dark fringes are equally spaced, $R \gg y_m$, the angles are small and the dark bands are located by $y_{m+\frac{1}{2}} = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: The separation between adjacent dark bands is $\Delta y = \dfrac{R\lambda}{d}$.

EXECUTE: $\Delta y = \dfrac{R\lambda}{d} \Rightarrow d = \dfrac{R\lambda}{\Delta y} = \dfrac{(1.80\text{ m})(4.50\times10^{-7}\text{ m})}{3.90\times10^{-3}\text{ m}} = 2.08\times10^{-4}\text{ m} = 0.208\text{ mm}.$

EVALUATE: When the separation between the slits decreases, the separation between dark fringes increases.

35.9. **IDENTIFY** and **SET UP:** The dark lines correspond to destructive interference and hence are located by $d\sin\theta = (m+\frac{1}{2})\lambda$ so $\sin\theta = \dfrac{(m+\frac{1}{2})\lambda}{d}$, $m = 0, \pm1, \pm2, \ldots.$

Solve for θ that locates the second and third dark lines. Use to find the distance of each of the dark lines from the center of the screen.

EXECUTE: 1st dark line is for $m = 0$.

2nd dark line is for $m = 1$ and $\sin\theta_1 = \dfrac{3\lambda}{2d} = \dfrac{3(500\times10^{-9}\text{ m})}{2(0.450\times10^{-3}\text{m})} = 1.667\times10^{-3}$ and $\theta_1 = 1.667\times10^{-3}$ rad.

3rd dark line is for $m = 2$ and $\sin\theta_2 = \dfrac{5\lambda}{2d} = \dfrac{5(500\times10^{-9}\text{ m})}{2(0.450\times10^{-3}\text{m})} = 2.778\times10^{-3}$ and $\theta_2 = 2.778\times10^{-3}$ rad.

(Note that θ_1 and θ_2 are small so that the approximation $\theta \approx \sin\theta \approx \tan\theta$ is valid.) The distance of each dark line from the center of the central bright band is given by $y_m = R\tan\theta$, where $R = 0.850$ m is the distance to the screen.

$\tan\theta \approx \theta$ so $y_m = R\theta_m$.

$y_1 = R\theta_1 = (0.750\text{ m})(1.667\times10^{-3}\text{ rad}) = 1.25\times10^{-3}\text{ m}.$

$y_2 = R\theta_2 = (0.750\text{ m})(2.778\times10^{-3}\text{ rad}) = 2.08\times10^{-3}\text{ m}.$

$\Delta y = y_2 - y_1 = 2.08\times10^{-3}\text{ m} - 1.25\times10^{-3}\text{ m} = 0.83\text{ mm}.$

EVALUATE: Since θ_1 and θ_2 are very small we could have used $y_m = R\dfrac{m\lambda}{d}$, generalized to destructive interference: $y_m = R(m+\frac{1}{2})\lambda/d$.

35.10. **IDENTIFY:** We have two-slit interference.

SET UP: Bright fringes occur when $d\sin\theta_m = m\lambda$, $f\lambda = c$. We want the distance d between the slits. We can vary the frequency, and we know that $f_m = 5.60\times10^{12}$ Hz and $f_{m+1} = 7.47\times10^{12}$ Hz.

EXECUTE: For the first frequency $d\sin\theta_m = m\lambda = m\dfrac{c}{f_m}$. For the next higher frequency $d\sin\theta_{m+1} = (m+1)\dfrac{c}{f_{m+1}}$. Both maxima occur at the same place, so $d\sin\theta$ is the same. Thus $m\dfrac{c}{f_m} = (m+1)\dfrac{c}{f_{m+1}}$. Using the known frequencies gives $m = 3$. Now we can find d. $d\sin\theta_3 = 3\dfrac{c}{f_m}$. Using $\theta = 60.0°$ and f_m gives $d = 0.186$ mm.

EVALUATE: We cannot use the small-angle approximation for an angle as large as 60°.

35.11. **IDENTIFY:** Bright fringes are located at angles θ given by $d\sin\theta = m\lambda$.

SET UP: The largest value $\sin\theta$ can have is 1.00.

EXECUTE: **(a)** $m = \dfrac{d\sin\theta}{\lambda}$. For $\sin\theta = 1$, $m = \dfrac{d}{\lambda} = \dfrac{0.0116\times10^{-3}\text{ m}}{5.85\times10^{-7}\text{ m}} = 19.8$. Therefore, the largest m for fringes on the screen is $m = 19$. There are $2(19)+1 = 39$ bright fringes, the central one and 19 above and 19 below it.

(b) The most distant fringe has $m = \pm19$. $\sin\theta = m\dfrac{\lambda}{d} = \pm19\left(\dfrac{5.85\times10^{-7}\text{ m}}{0.0116\times10^{-3}\text{ m}}\right) = \pm0.958$ and $\theta = \pm73.3°$.

EVALUATE: For small θ the spacing Δy between adjacent fringes is constant but this is no longer the case for larger angles.

35.12. **IDENTIFY:** The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference $y_m = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: $d = 0.200\times10^{-3}$ m. $R = 4.00$ m.

EXECUTE: The distance between any two successive minima is $y_{m+1} - y_m = R\dfrac{\lambda}{d} = (4.00\text{ m})\dfrac{(400\times10^{-9}\text{m})}{(0.200\times10^{-3}\text{ m})} = 8.00$ mm. Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

EVALUATE: For small angles, when $y_m << R$, the interference minima are equally spaced.

35.13. **IDENTIFY** and **SET UP:** The dark lines are located by $d\sin\theta = (m+\frac{1}{2})\lambda$. The distance of each line from the center of the screen is given by $y = R\tan\theta$.

EXECUTE: First dark line is for $m = 0$ and $d\sin\theta_1 = \lambda/2$.

$\sin\theta_1 = \dfrac{\lambda}{2d} = \dfrac{550\times10^{-9}\text{ m}}{2(1.80\times10^{-6}\text{ m})} = 0.1528$ and $\theta_1 = 8.789°$. Second dark line is for $m = 1$ and $d\sin\theta_2 = 3\lambda/2$.

$\sin\theta_2 = \dfrac{3\lambda}{2d} = 3\left(\dfrac{550\times10^{-9}\text{ m}}{2(1.80\times10^{-6}\text{ m})}\right) = 0.4583$ and $\theta_2 = 27.28°$.

$y_1 = R\tan\theta_1 = (0.350\text{ m})\tan 8.789° = 0.0541$ m.

$y_2 = R\tan\theta_2 = (0.350\text{ m})\tan 27.28° = 0.1805$ m.

The distance between the lines is $\Delta y = y_2 - y_1 = 0.1805\text{ m} - 0.0541\text{ m} = 0.126\text{ m} = 12.6$ cm.

EVALUATE: $\sin\theta_1 = 0.1528$ and $\tan\theta_1 = 0.1546$. $\sin\theta_2 = 0.4583$ and $\tan\theta_2 = 0.5157$. As the angle increases, $\sin\theta \approx \tan\theta$ becomes a poorer approximation.

35.14. IDENTIFY: For small angles: $y_m = R\dfrac{m\lambda}{d}$.

SET UP: First-order means $m = 1$.

EXECUTE: The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d}\Delta\lambda = \frac{(4.00\text{ m})(1)}{(0.300\times 10^{-3}\text{ m})}(660-470)\times(10^{-9}\text{ m}) = 2.53\times 10^{-3}\text{ m} = 2.53\text{ mm}.$$

EVALUATE: The separation between these fringes for different wavelengths increases when the slit separation decreases.

35.15. IDENTIFY and SET UP: Use the information given about the bright fringe and $y_m = R\dfrac{m\lambda}{d}$ to find the distance d between the two slits. Then use $d\sin\theta = (m+\frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$ and $y = R\tan\theta$ to calculate λ for which there is a first-order dark fringe at this same place on the screen.

EXECUTE: $y_1 = \dfrac{R\lambda_1}{d}$, so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\text{ m})(600\times 10^{-9}\text{ m})}{4.84\times 10^{-3}\text{ m}} = 3.72\times 10^{-4}$ m. (R is much greater than d, so $y_m = R\dfrac{m\lambda}{d}$ is valid.) The dark fringes are located by $d\sin\theta = (m+\frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$. The first-order dark fringe is located by $\sin\theta = \lambda_2/2d$, where λ_2 is the wavelength we are seeking.

$$y = R\tan\theta \approx R\sin\theta = \frac{\lambda_2 R}{2d}.$$

We want λ_2 such that $y = y_1$. This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200$ nm.

EVALUATE: For $\lambda = 600$ nm the path difference from the two slits to this point on the screen is 600 nm. For this same path difference (point on the screen) the path difference is $\lambda/2$ when $\lambda = 1200$ nm.

35.16. IDENTIFY: Bright fringes are located at $y_m = R\dfrac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m+\frac{1}{2})\lambda$ and $y = R\tan\theta$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^{8}\text{ m/s}}{6.32\times 10^{14}\text{ Hz}} = 4.75\times 10^{-7}$ m. For the third bright fringe (not counting the central bright spot), $m = 3$. For the third dark fringe, $m = 2$.

EXECUTE: **(a)** $d = \dfrac{m\lambda R}{y_m} = \dfrac{3(4.75\times 10^{-7}\text{ m})(0.850\text{ m})}{0.0311\text{ m}} = 3.89\times 10^{-5}\text{ m} = 0.0389$ mm.

(b) $\sin\theta = (2+\frac{1}{2})\dfrac{\lambda}{d} = (2.5)\left(\dfrac{4.75\times 10^{-7}\text{ m}}{3.89\times 10^{-5}\text{ m}}\right) = 0.0305$ and $\theta = 1.75°$.

$y = R\tan\theta = (85.0\text{ cm})\tan 1.75° = 2.60$ cm.

EVALUATE: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

35.17. IDENTIFY: Use $I = I_0\cos^2(\phi/2)$ with $\phi = (2\pi/\lambda)(r_2 - r_1)$.

SET UP: ϕ is the phase difference and $(r_2 - r_1)$ is the path difference.

EXECUTE: **(a)** $I = I_0(\cos 30.0°)^2 = 0.750 I_0$.

(b) $60.0° = (\pi/3)$ rad. $(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80$ nm.

EVALUATE: $\phi = 360°/6$ and $(r_2 - r_1) = \lambda/6.$

35.18. **(a) IDENTIFY and SET UP:** The minima are located at angles θ given by $d\sin\theta = (m+\frac{1}{2})\lambda.$ The first minimum corresponds to $m = 0.$ Solve for $\theta.$ Then the distance on the screen is $y = R\tan\theta.$

EXECUTE: $\sin\theta = \dfrac{\lambda}{2d} = \dfrac{660\times10^{-9}\ \text{m}}{2(0.260\times10^{-3}\ \text{m})} = 1.27\times10^{-3}$ and $\theta = 1.27\times10^{-3}$ rad.

$y = (0.900\ \text{m})\tan(1.27\times10^{-3}\ \text{rad}) = 1.14\times10^{-3}\ \text{m} = 1.14$ mm.

(b) IDENTIFY and SET UP: The equation $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$ gives the intensity I as a function of the position y on the screen. Set $I = I_0/2$ and solve for y.

EXECUTE: $I = \frac{1}{2}I_0$ says $\cos^2\left(\dfrac{\pi dy}{\lambda R}\right) = \frac{1}{2}.$

$\cos\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{\sqrt{2}}$ so $\dfrac{\pi dy}{\lambda R} = \dfrac{\pi}{4}$ rad.

$y = \dfrac{\lambda R}{4d} = \dfrac{(660\times10^{-9}\ \text{m})(0.900\ \text{m})}{4(0.260\times10^{-3}\ \text{m})} = 5.71\times10^{-4}\ \text{m} = 0.571$ mm.

EVALUATE: $I = I_0/2$ at a point on the screen midway between where $I = I_0$ and $I = 0.$

35.19. **IDENTIFY and SET UP:** The phase difference ϕ is given by $\phi = (2\pi d/\lambda)\sin\theta.$

EXECUTE: $\phi = [2\pi(0.340\times10^{-3}\ \text{m})/(500\times10^{-9}\ \text{m})\sin 23.0° = 1670$ rad.

EVALUATE: The *m*th bright fringe occurs when $\phi = 2\pi m,$ so there are a large number of bright fringes within $23.0°$ from the centerline. Note that the equation $\phi = (2\pi d/\lambda)\sin\theta$ gives ϕ in radians.

35.20. **IDENTIFY:** Light from the two slits interferes on the screen. The bright and dark fringes are very close together compared to the distance between the screen and the slits, so we can use the small-angle approximation. $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2).$ The intensity is $I = I_0\cos^2\left(\dfrac{\phi}{2}\right).$

SET UP: The intensity is $I = I_0\cos^2\left(\dfrac{\phi}{2}\right).$ $\dfrac{\phi}{2} = \dfrac{\pi d\sin\theta}{\lambda},$ but for small angles $\dfrac{\phi}{2} \approx \dfrac{\pi dy}{R\lambda}.$

EXECUTE: At the first minim, $y = 3.00$ mm and $\phi/2 = \pi/2$. At $y = 2.00$ mm, which is 2/3 of 3.00 mm, $\phi/2 = (2/3)(\pi/2) = \pi/3 = 60°$. Therefore the intensity at $x = 2.00$ mm is

$I = (0.0600\ \text{W/m}^2)\cos^2(60°) = 0.0150\ \text{W/m}^2.$

(b) Using the same reasoning as in (a), 1.50 mm is $\frac{1}{2}$ of 3.00 m, so $\phi/2 = (1/2)(\pi/2) = \pi/4 = 45°$. So

$I = (0.0600\ \text{W/m}^2)\cos^2(45°) = 0.0300\ \text{W/m}^2.$

EVALUATE: As a check, we could first find λ and then use it to find the intensities. At the first minimum, $\phi/2 = \pi/2 = \pi dy/R\lambda,$ which gives $\lambda = 2dy/R = 5.40\times10^{-4}$ mm. Now use this to calculate the intensities using $I = I_0\cos^2\left(\dfrac{\phi}{2}\right)$ and $\dfrac{\phi}{2} \approx \dfrac{\pi dy}{R\lambda}.$

35.21. IDENTIFY: The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{1.20\times10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2) = \dfrac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}$.

(b) $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right) = I_0 \cos^2\left(\dfrac{4.52 \text{ rad}}{2}\right) = 0.404 I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.21. IDENTIFY: The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{1.20\times10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2) = \dfrac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}$.

(b) $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right) = I_0 \cos^2\left(\dfrac{4.52 \text{ rad}}{2}\right) = 0.404 I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.22. IDENTIFY: We are dealing with thin-film interference with an air film of variable thickness.

SET UP: There is no phase-reversal at the upper air surface, but there is one at the lower air surface because $n_{glass} > n_{air}$. There is constructive interference when $t_1 = 650$ nm, and the next value of t for which constructive interference occurs is $t_2 = 910$ nm. $2t = \left(m + \dfrac{1}{2}\right)\lambda$, $\lambda_n = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** We want wavelength of the light in air. Using $2t = \left(m + \dfrac{1}{2}\right)\lambda$ at both thicknesses gives $2t_1 = \left(m + \dfrac{1}{2}\right)\lambda$ and $2t_2 = \left(m + \dfrac{3}{2}\right)\lambda$. Taking t_2/t_1 gives $m = 2$. From this we get $\lambda = 520$ nm.

(b) We want the minimum thickness t_{min}. For the smallest thickness, $2t = \lambda/2$, so $t = \dfrac{\lambda}{4} = 130$ nm.

EVALUATE: Be careful of the half-cycle phase shift when the light reflects off the glass at the bottom of the air film.

35.23. IDENTIFY: Consider interference between rays reflected at the upper and lower surfaces of the film. Consider phase difference due to the path difference of $2t$ and any phase differences due to phase changes upon reflection.

SET UP: Consider Figure 35.23.

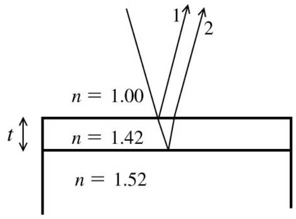


Figure 35.23

Both rays (1) and (2) undergo a $180°$ phase change on reflection, so there is no net phase difference introduced and the condition for destructive interference is $2t = (m + \frac{1}{2})\lambda$.

EXECUTE: $t = \dfrac{(m+\frac{1}{2})\lambda}{2}$; thinnest film says $m = 0$ so $t = \dfrac{\lambda}{4}$.

X and $t = \dfrac{\lambda_0}{4(1.42)} = \dfrac{650\times10^{-9}\text{ m}}{4(1.42)} = 1.14\times10^{-7}\text{ m} = 114\text{ nm}.$

EVALUATE: We compared the path difference to the wavelength in the film, since that is where the path difference occurs.

35.24. **IDENTIFY:** Require destructive interference for light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air $(n = 1.00)$ reflects from the film $(n = 2.62)$ and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film $(n = 2.62)$ reflects from glass $(n = 1.62)$ and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{505\text{ nm}}{2.62}$.

EXECUTE: **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{505\text{ nm}}{2[2.62]}\right) = (96.4\text{ nm})m.$ The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2, 3$ and 4: 192 nm, 289 nm, and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda_0/2n$. Compare this to Problem 35.23, where the minimum thickness for destructive interference is $t = \lambda_0/4n$.

35.25. **IDENTIFY:** The light reflected from the top of the TiO_2 film interferes with the light reflected from the top of the glass surface. These waves are out of phase due to the path difference in the film and the phase differences caused by reflection.

SET UP: There is a π phase change at the TiO_2 surface but none at the glass surface, so for destructive interference the path difference must be $m\lambda$ in the film.

EXECUTE: **(a)** Calling T the thickness of the film gives $2T = m\lambda_0/n$, which yields $T = m\lambda_0/(2n)$. Substituting the numbers gives

$T = m\ (520.0\text{ nm})/[2(2.62)] = 99.237\text{ nm}.$

T must be greater than 1036 nm, so $m = 11$, which gives $T = 1091.6\text{ nm}$, since we want to know the minimum thickness to add.

$\Delta T = 1091.6\text{ nm} - 1036\text{ nm} = 55.6\text{ nm}.$

(b) (i) Path difference $= 2T = 2(1092\text{ nm}) = 2184\text{ nm} = 2180\text{ nm}.$

(ii) The wavelength in the film is $\lambda = \lambda_0/n = (520.0\text{ nm})/2.62 = 198.5\text{ nm}.$

Path difference $= (2180\text{ nm})/[(198.5\text{ nm})/\text{wavelength}] = 11.0$ wavelengths.

EVALUATE: Because the path difference in the film is 11.0 wavelengths, the light reflected off the top of the film will be 180° out of phase with the light that traveled through the film and was reflected off the glass due to the phase change at reflection off the top of the film.

35.26. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections. For destructive interference the total phase difference is an integer number of half cycles.

SET UP: The reflection at the top surface of the film produces a half-cycle phase shift. There is no phase shift at the reflection at the bottom surface.

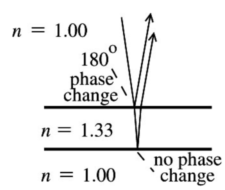
EXECUTE: **(a)** Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{550\text{ nm}}{4(1.70)} = 80.9\text{ nm}.$

(b) The next smallest thickness for constructive interference is with another half wavelength thickness added. $t = \dfrac{3\lambda}{4} = \dfrac{3\lambda_0}{4n} = \dfrac{3(550\text{ nm})}{4(1.70)} = 243\text{ nm}.$

EVALUATE: Note that we must compare the path difference to the wavelength in the film.

35.27. **IDENTIFY:** Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.

SET UP: Consider Figure 35.27.



There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

Figure 35.27

EXECUTE: **(a)** The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to the reflections. Hence the condition for constructive interference is $2t = (m+\frac{1}{2})(\lambda_0/n),\ m = 0, 1, 2, \ldots$ Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(290\text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{771.4\text{ nm}}{m+\frac{1}{2}}$$

for $m = 0, \lambda = 1543$ nm; for $m = 1, \lambda = 514$ nm; for $m = 2, \lambda = 308$ nm;… Only 514 nm is in the visible region; the color for this wavelength is green.

(b) $\lambda_0 = \dfrac{2tn}{m+\frac{1}{2}} = \dfrac{2(340\text{ nm})(1.33)}{m+\frac{1}{2}} = \dfrac{904.4\text{ nm}}{m+\frac{1}{2}}$

for $m = 0, \lambda = 1809$ nm; for $m = 1, \lambda = 603$ nm; for $m = 2, \lambda = 362$ nm;… Only 603 nm is in the visible region; the color for this wavelength is orange.

EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

35.28. **IDENTIFY** and **SET UP:** Since the film reflects 575 nm strongly, we must have constructive interference at that wavelength. The light reflected from the air-benzene interface experiences a 180° phase inversion (since $n_{\text{air}} < n_{\text{benzene}}$), but the light reflected from the benzene-water interface does not experience a phase inversion (since $n_{\text{benzene}} > n_{\text{water}}$). Thus, the condition for constructive interference is $2t = m\dfrac{\lambda}{2n}$, where $m = 1,\ 3,\ 5,\ldots$ and $\dfrac{\lambda}{n}$ is the wavelength of the light in the benzene (which is where the path-difference occurs).

EXECUTE: The minimum required thickness occurs when $m = 1$, so $t = \dfrac{\lambda}{4n} = \dfrac{575\ \text{nm}}{4(1.50)} = 95.8\ \text{nm}.$

EVALUATE: Since the path difference occurs within the benzene, and not within the water, the exact value of the index of refraction of water is not needed (provided we know that $n_{\text{benzene}} > n_{\text{water}}$).

35.29. **IDENTIFY:** Require destructive interference between light reflected from the two points on the disc.

SET UP: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is $2t = (m + \frac{1}{2})\lambda$, where t is the depth of the pit. $\lambda = \dfrac{\lambda_0}{n}$. The minimum pit depth is for $m = 0$.

EXECUTE: $2t = \dfrac{\lambda}{2}$. $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{790\ \text{nm}}{4(1.8)} = 110\ \text{nm} = 0.11\ \mu\text{m}.$

EVALUATE: The path difference occurs in the plastic substrate and we must compare the wavelength in the substrate to the path difference.

35.30. **IDENTIFY:** Apply $y = m(\lambda/2)$.

SET UP: $m = 818$. Since the fringes move in opposite directions, the two people move the mirror in opposite directions.

EXECUTE: **(a)** For Jan, the total shift was $y_1 = \dfrac{m\lambda_1}{2} = \dfrac{818(6.06\times10^{-7}\ \text{m})}{2} = 2.48\times10^{-4}\,\text{m}$. For Linda, the total shift was $y_2 = \dfrac{m\lambda_2}{2} = \dfrac{818(5.02\times10^{-7}\ \text{m})}{2} = 2.05\times10^{-4}\ \text{m}.$

(b) The net displacement of the mirror is the difference of the above values:

$\Delta y = y_1 - y_2 = 0.248\ \text{mm} - 0.205\ \text{mm} = 0.043\ \text{mm}.$

EVALUATE: The person using the larger wavelength moves the mirror the greater distance.

35.31. **IDENTIFY** and **SET UP:** Apply $y = m(\lambda/2)$ and calculate y for $m = 1800$.

EXECUTE: $y = m(\lambda/2) = 1800(633\times10^{-9}\ \text{m})/2 = 5.70\times10^{-4}\ \text{m} = 0.570\ \text{mm}.$

EVALUATE: A small displacement of the mirror corresponds to many wavelengths and a large number of fringes cross the line.

35.32. **IDENTIFY:** This problem involves the interference of waves.

SET UP and **EXECUTE:** **(a)** We want E. $I = \dfrac{1}{2}\epsilon_0\, cE^2_{\text{max}} = P/A.$

$$E = \sqrt{\frac{2I}{\epsilon_0\, c}} = \sqrt{\frac{2(100\ \text{kW/cm}^2)}{\epsilon_0\, c}} = 870\ \text{kV/m}.$$

(b) Estimate the phase difference δ. $\dfrac{\delta}{2\pi} = \dfrac{\Delta l}{\lambda}$. The phase difference accumulates with each round trip. There are 280 round trips, each with an up-and-back segment. So the total for Δl is $\Delta l = 2(280)(10^{-18}\text{m})$. Therefore $\delta = 2\pi\dfrac{\Delta l}{\lambda} = \dfrac{2\pi(560\times10^{-18}\ \text{m})}{1064\ \text{nm}} = 3.3\times10^{-9}\ \text{rad}.$

(c) We want $E_{\min}$. Eq. (35.7) gives $E_p = 2E\left|\cos\frac{\phi}{2}\right|$. $\phi = \pi + \delta$, so

$$E_p = 2E\left|\cos\frac{\pi+\delta}{2}\right| = 2E\sin\frac{\delta}{2} \approx 2E\frac{\delta}{2} = E\delta = (870\text{ kV/m})(3.3\times10^{-9}\text{rad}) = 2.9\times10^{-3}\text{V/m}.$$

EVALUATE: The minimum field strength is about 3 mN/C, which is very small.

35.33. **IDENTIFY:** The two scratches are parallel slits, so the light that passes through them produces an interference pattern. However, the light is traveling through a medium (plastic) that is different from air.

SET UP: The central bright fringe is bordered by a dark fringe on each side of it. At these dark fringes, $d\ \sin\theta = \frac{1}{2}\lambda/n$, where n is the refractive index of the plastic.

EXECUTE: First use geometry to find the angles at which the two dark fringes occur. At the first dark fringe $\tan\theta = [(5.82\text{ mm})/2]/(3250\text{ mm})$, giving $\theta = \pm 0.0513°$.

For destructive interference, we have $d\ \sin\theta = \frac{1}{2}\lambda/n$ and

$n = \lambda/(2d\sin\theta) = (632.8\text{ nm})/[2(0.000225\text{ m})(\sin\ 0.0513°)] = 1.57.$

EVALUATE: The wavelength of the light in the plastic is reduced compared to what it would be in air.

35.34. **IDENTIFY:** Consider the interference between light reflected from the top and bottom surfaces of the air film between the lens and the glass plate. Introducing a liquid between the lens and the plate just changes the wavelength from λ_0 to $\frac{\lambda_0}{n}$, where n is the refractive index of the liquid.

SET UP: For maximum intensity, with a net half-cycle phase shift due to reflections, $2t = (m+\frac{1}{2})\lambda$, where λ is the wavelength in the film. $t = R - \sqrt{R^2 - r^2}$.

EXECUTE: $\frac{(2m+1)\lambda}{4} = R - \sqrt{R^2-r^2} \Rightarrow \sqrt{R^2-r^2} = R - \frac{(2m+1)\lambda}{4}$

$$\Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2} \Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2}$$

$$\Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}},\ \text{for } R \gg \lambda.$$

$\lambda = \lambda_0/n$, where λ_0 is the wavelength in air. Therefore, if r_0 is the radius of the third bright ring when air is between the lens and plate, the radius with water between the lens and plate is

$$r = \frac{r_0}{\sqrt{n}} = \frac{0.640\text{ mm}}{\sqrt{1.33}} = 0.555\text{ mm}.$$

EVALUATE: The refractive index of the water is less than that of the glass plate, so the phase changes on reflection are the same as when air is in the space.

35.35. **IDENTIFY** and **SET UP:** Consider the interference of the rays reflected from each side of the film. At the front of the film light in air reflects off the film $(n = 1.432)$ and there is a 180° phase shift. At the back of the film light in the film $(n = 1.432)$ reflects off the glass $(n = 1.62)$ and there is a 180° phase shift. Therefore, the reflections introduce no net phase shift. The path difference is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \frac{\lambda_{\text{air}}}{n}$.

EXECUTE: **(a)** Since there is no net phase difference produced by the reflections, the condition for destructive interference is $2t = (m+\frac{1}{2})\lambda.\ t = (m+\frac{1}{2})\frac{\lambda}{2}$ and the minimum thickness is

$$t = \frac{\lambda}{4} = \frac{\lambda_{\text{air}}}{4n} = \frac{550\text{ nm}}{4(1.432)} = 96.0\text{ nm}.$$

(b) For destructive interference, $2t = (m+\frac{1}{2})\dfrac{\lambda_{\text{air}}}{n}$ and $\lambda_{\text{air}} = \dfrac{2tn}{m+\frac{1}{2}} = \dfrac{275\text{ nm}}{m+\frac{1}{2}}$. $m = 0$: $\lambda_{\text{air}} = 550$ nm.

$m = 1$: $\lambda_{\text{air}} = 183$ nm. All other λ_{air} values are shorter. For constructive interference, $2t = m\dfrac{\lambda_{\text{air}}}{n}$ and

$\lambda_{\text{air}} = \dfrac{2tn}{m} = \dfrac{275\text{ nm}}{m}$. For $m = 1$, $\lambda_{\text{air}} = 275$ nm and all other λ_{air} values are shorter.

EVALUATE: The only visible wavelength in air for which there is destructive interference is 550 nm. There are no visible wavelengths in air for which there is constructive interference.

35.36. **IDENTIFY** and **SET UP:** Consider reflection from either side of the film. **(a)** At the front of the film, light in air $(n = 1.00)$ reflects off the film $(n = 1.45)$ and there is a 180° phase shift. At the back of the film, light in the film $(n = 1.45)$ reflects off the cornea $(n = 1.38)$ and there is no phase shift. The reflections produce a net 180° phase difference so the condition for constructive interference is $2t = (m+\frac{1}{2})\lambda$, where $\lambda = \dfrac{\lambda_{\text{air}}}{n}$. $t = (m+\frac{1}{2})\dfrac{\lambda_{\text{air}}}{2n}$.

EXECUTE: The minimum thickness is for $m = 0$, and is given by $t = \dfrac{\lambda_{\text{air}}}{4n} = \dfrac{600\text{ nm}}{4(1.45)} = 103$ nm

(103.4 nm with less rounding).

(b) $\lambda_{\text{air}} = \dfrac{2nt}{m+\frac{1}{2}} = \dfrac{2(1.45)(103.4\text{ nm})}{m+\frac{1}{2}} = \dfrac{300\text{ nm}}{m+\frac{1}{2}}$. For $m = 0$, $\lambda_{\text{air}} = 600$ nm. For $m = 1$, $\lambda_{\text{air}} = 200$ nm

and all other values are smaller. No other visible wavelengths are reinforced. The condition for destructive interference is $2t = m\dfrac{\lambda_{\text{air}}}{n}$. $\lambda = \dfrac{2tn}{m} = \dfrac{300\text{ nm}}{m}$. For $m = 1$, $\lambda_{\text{air}} = 300$ nm and all other values are shorter. There are no visible wavelengths for which there is destructive interference.

(c) Now both rays have a 180° phase change on reflection and the reflections don't introduce any net phase shift. The expression for constructive interference in parts (a) and (b) now gives destructive interference and the expression in (a) and (b) for destructive interference now gives constructive interference. The only visible wavelength for which there will be destructive interference is 600 nm and there are no visible wavelengths for which there will be constructive interference.

EVALUATE: Changing the net phase shift due to the reflections can convert the interference for a particular thickness from constructive to destructive, and vice versa.

35.37. **IDENTIFY:** The insertion of the metal foil produces a wedge of air, which is an air film of varying thickness. This film causes a path difference between light reflected off the top and bottom of this film.

SET UP: The two sheets of glass are sketched in Figure 35.37. The thickness of the air wedge at a distance x from the line of contact is $t = x\tan\theta$. Consider rays 1 and 2 that are reflected from the top and bottom surfaces, respectively, of the air film. Ray 1 has no phase change when it reflects and ray 2 has a 180° phase change when it reflects, so the reflections introduce a net 180° phase difference. The path difference is $2t$ and the wavelength in the film is $\lambda = \lambda_{\text{air}}$.

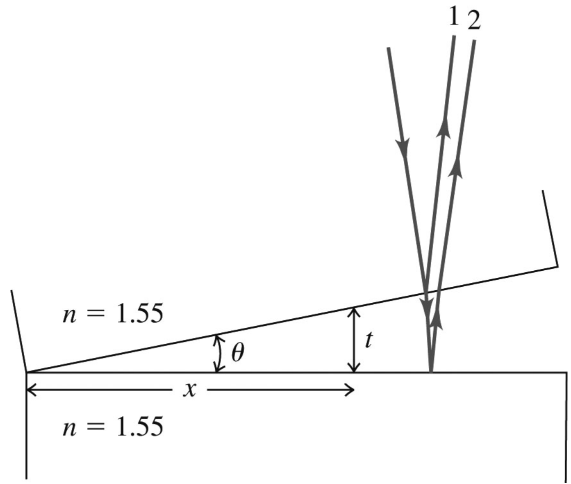


Figure 35.37

EXECUTE: **(a)** Since there is a 180° phase difference from the reflections, the condition for constructive interference is $2t = (m + \frac{1}{2})\lambda$. The positions of first enhancement correspond to $m = 0$ and $2t = \dfrac{\lambda}{2}$. $x \tan\theta = \dfrac{\lambda}{4}$. θ is a constant, so $\dfrac{x_1}{\lambda_1} = \dfrac{x_2}{\lambda_2}$. $x_1 = 1.15$ mm, $\lambda_1 = 400.0$ nm. $x_2 = x_1\left(\dfrac{\lambda_2}{\lambda_1}\right)$. For $\lambda_2 = 550$ nm (green), $x_2 = (1.15\text{ mm})\left(\dfrac{550\text{ nm}}{400\text{ nm}}\right) = 1.58$ mm. For $\lambda_2 = 600$ nm (orange), $x_2 = (1.15\text{ mm})\left(\dfrac{600\text{ nm}}{400\text{ nm}}\right) = 1.72$ mm.

(b) The positions of next enhancement correspond to $m = 1$ and $2t = \dfrac{3\lambda}{2}$. $x \tan\theta = \dfrac{3\lambda}{4}$. The values of x are 3 times what they are in part (a). Violet: 3.45 mm; green: 4.74 mm; orange: 5.16 mm.

(c) $\tan\theta = \dfrac{\lambda}{4x} = \dfrac{400.0\times10^{-9}\text{ m}}{4(1.15\times10^{-3}\text{ m})} = 8.70\times10^{-5}$. $\tan\theta = \dfrac{t_{\text{foil}}}{11.0\text{ cm}}$, so $t_{\text{foil}} = 9.57\times10^{-4}\text{ cm} = 9.57\ \mu\text{m}$.

EVALUATE: The thickness of the foil must be very small to cause these observable interference effects. If it is too thick, the film is no longer a "thin film."

35.38. **IDENTIFY:** We are dealing with two-source interference.

SET UP: For constructive interference $d\sin\theta_m = m\lambda$. $y_m = R\tan\theta_m$. For small angles $y_m = R\dfrac{m\lambda}{d}$.

EXECUTE: **(a)** We want the distance d between slits. The small-angle approximation is justified in this case. Adjacent fringes differ by $\Delta m = 1$, so $\Delta y_m = R\dfrac{\Delta m\lambda}{d} = \dfrac{R\lambda}{d}$. Solving for d gives $d = \dfrac{R\lambda}{\Delta y} =$ (1 m)(650 nm)/(1 cm) = 65 μm.

(b) No. It seems much too small to do at home.

(c) We want the distance between the points of enhanced sound (constructive interference). For sound of frequency 1.0 kHz, $\lambda = v/f$ = (344 m/s)(1.0 kHz) = 0.344 m, d = 40 cm = 0.40 m. We cannot use the small-angle approximation because d is only slightly larger than λ. Using $d\sin\theta_m = m\lambda$ gives $(0.40\text{ m})\sin\theta_1 = 0.344$ m, so $\theta_1 = 59°$. $y_1 = R\tan\theta_1 = (2\text{ m})\tan 59° = 3.3$ m. There is no θ_2 so the distance between the desired points in 3.3 m.

(d) We want d and f so that $\Delta y = 1.75$ m. Thus $y_1 = 1.75$ m, so $\tan\theta_1 = \dfrac{y_1}{R} = \dfrac{1.75\text{ m}}{2\text{ m}}$ which gives $\theta_1 = 41°$. Using $d\sin\theta_m = m\lambda$ gives $d\sin 41° = \lambda$. A frequency of 1.0 kHz is easily audible, so we use that which makes $\lambda = 0.344$ m. Thus $d \sin 41° = 0.344$ m, so $d = 0.52$ m = 52 cm.

EVALUATE: **(e)** The 52-cm spacing in part (d) would be easily achieved, and 1.0 kHz is clearly within human hearing.

35.39. **IDENTIFY:** The liquid alters the wavelength of the light and that affects the locations of the interference minima.

SET UP: The interference minima are located by $d\sin\theta = (m+\tfrac{1}{2})\lambda$. For a liquid with refractive index n, $\lambda_{\text{liq}} = \dfrac{\lambda_{\text{air}}}{n}$.

EXECUTE: $\dfrac{\sin\theta}{\lambda} = \dfrac{(m+\tfrac{1}{2})}{d} =$ constant, so $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and

$$n = \frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liq}}} = \frac{\sin 35.20°}{\sin 19.46°} = 1.730.$$

EVALUATE: In the liquid the wavelength is shorter and $\sin\theta = (m+\tfrac{1}{2})\dfrac{\lambda}{d}$ gives a smaller θ than in air, for the same m.

35.40. **IDENTIFY:** As the brass is heated, thermal expansion will cause the two slits to move farther apart.

SET UP: For destructive interference, $d\ \sin\theta = \lambda/2$. The change in separation due to thermal expansion is $dw = \alpha w_0\, dT$, where w is the distance between the slits.

EXECUTE: The first dark fringe is at $d \sin\ \theta = \lambda/2 \Rightarrow \sin\theta = \lambda/2d$.

Call $d \equiv w$ for these calculations to avoid confusion with the differential. $\sin\theta = \lambda/2w$.

Taking differentials gives $d(\sin\theta) = d(\lambda/2w)$ and $\cos\theta\, d\theta = -\lambda/2\ dw/w^2$. For thermal expansion, $dw = \alpha w_0 dT$, which gives $\cos\theta\, d\theta = -\dfrac{\lambda}{2}\dfrac{\alpha w_0 dT}{w_0^2} = -\dfrac{\lambda\alpha dT}{2w_0}$. Solving for $d\theta$ gives $d\theta = -\dfrac{\lambda\alpha dT}{2w_0\cos\theta_0}$.

Get λ: $w_0\sin\theta_0 = \lambda/2 \to \lambda = 2w_0\sin\theta_0$. Substituting this quantity into the equation for $d\theta$ gives

$$d\theta = -\frac{2w_0\sin\theta_0\,\alpha dT}{2w_0\cos\theta_0} = -\tan\theta_0\,\alpha dT.$$

$$d\theta = -\tan(26.6°)(2.0\times10^{-5}\ \text{K}^{-1})(115\ \text{K}) = -0.001152\ \text{rad} = -0.066°.$$

The minus sign tells us that the dark fringes move closer together.

EVALUATE: We can also see that the dark fringes move closer together because $\sin\theta$ is proportional to $1/d$, so as d increases due to expansion, θ decreases.

35.41. **IDENTIFY:** For destructive interference, $d = r_2 - r_1 = (m+\tfrac{1}{2})\lambda$.

SET UP: $r_2 - r_1 = \sqrt{(200\text{ m})^2 + x^2} - x$.

EXECUTE: $(200\text{ m})^2 + x^2 = x^2 + \left[(m+\tfrac{1}{2})\lambda\right]^2 + 2x(m+\tfrac{1}{2})\lambda$.

$x = \dfrac{20{,}000\text{ m}^2}{(m+\tfrac{1}{2})\lambda} - \tfrac{1}{2}(m+\tfrac{1}{2})\lambda$. The wavelength is calculated by $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{5.80\times10^6\text{ Hz}} = 51.7$ m.

$m=0: x = 761$ m; $m=1: x = 219$ m; $m=2: x = 90.1$ m; $m=3; x = 20.0$ m.

EVALUATE: For $m = 3$, $d = 3.5\lambda = 181$ m. The maximum possible path difference is the separation of 200 m between the sources.

35.42. **IDENTIFY:** For destructive interference the net phase difference must be $180°$, which is one-half a period, or $\lambda/2$. Part of this phase difference is due to the fact that the speakers are 1/4 of a period out of phase, and the rest is due to the path difference between the sound from the two speakers.

SET UP: The phase of *A* is 90° or, $\lambda/4$, ahead of *B*. At points above the centerline, points are closer to *A* than to *B* and the signal from *A* gains phase relative to *B* because of the path difference. Destructive interference will occur when $d\sin\theta = (m+\frac{1}{4})\lambda,\ m = 0, 1, 2, \ldots.$ At points at an angle θ below the centerline, the signal from *B* gains phase relative to *A* because of the phase difference. Destructive interference will occur when $d\sin\theta = (m+\frac{3}{4})\lambda,\ m = 0, 1, 2, \ldots.\ \lambda = \dfrac{v}{f}.$

EXECUTE: $\lambda = \dfrac{340\text{ m/s}}{444\text{ Hz}} = 0.766\text{ m}.$

<u>Points above the centerline</u>: $\sin\theta = (m+\frac{1}{4})\dfrac{\lambda}{d} = (m+\frac{1}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{1}{4}).\quad m=0:\ \theta = 3.14°;$

$m=1:\ \theta = 15.9°;\ m=2:\ \theta = 29.5°;\ m=3:\ \theta = 45.4°;\ m=4:\ \theta = 68.6°.$

<u>Points below the centerline</u>: $\sin\theta = (m+\frac{3}{4})\dfrac{\lambda}{d} = (m+\frac{3}{4})\left(\dfrac{0.766\text{ m}}{3.50\text{ m}}\right) = 0.219(m+\frac{3}{4}).\quad m=0:\ \theta = 9.45°;$

$m=1:\ \theta = 22.5°;\ m=2:\ \theta = 37.0°;\ m=3:\ \theta = 55.2°.$

EVALUATE: It is *not* always true that the path difference for destructive interference must be $(m+\frac{1}{2})\lambda$, but it *is* always true that the phase difference must be 180° (or odd multiples of 180°).

35.43. **IDENTIFY and SET UP:** Consider interference between rays reflected from the upper and lower surfaces of the film to relate the thickness of the film to the wavelengths for which there is destructive interference. The thermal expansion of the film changes the thickness of the film when the temperature changes.

EXECUTE: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.

Smallest nonzero thickness is given by $t = \lambda/2n$.

At $20.0°\text{C},\ t_0 = (582.4\text{ nm})/[(2)(1.750)] = 166.4\text{ nm}.$

At $170°\text{C},\ t = (588.5\text{ nm})/[(2)(1.750)] = 168.1\text{ nm}.$

$t = t_0(1+\alpha\Delta T)$ so

$\alpha = (t-t_0)/(t_0\Delta T) = (1.7\text{ nm})/[(166.4\text{ nm})(150\text{C}°)] = 6.8\times10^{-5}(\text{C}°)^{-1}.$

EVALUATE: When the film is heated its thickness increases, and it takes a larger wavelength in the film to equal $2t$. The value we calculated for α is the same order of magnitude as those given in Table 17.1.

35.44. **IDENTIFY:** The maximum intensity occurs at all the points of constructive interference. At these points, the path difference between waves from the two transmitters is an integral number of wavelengths.

SET UP: For constructive interference, $\sin\theta = m\lambda/d$.

EXECUTE: **(a)** First find the wavelength of the UHF waves:

$\lambda = c/f = (3.00\times10^8\text{ m/s})/(1575.42\text{ MHz}) = 0.1904\text{ m}.$

For maximum intensity $(\pi d\ \sin\theta)/\lambda = m\pi$, so

$\sin\theta = m\lambda/d = m[(0.1904\text{ m})/(5.18\text{ m})] = 0.03676m.$

The maximum possible *m* would be for $\theta = 90°$, or $\sin\theta = 1$, so

$m_{\max} = d/\lambda = (5.18\text{ m})/(0.1904\text{ m}) = 27.2$,

which must be ±27 since *m* is an integer. The total number of maxima is 27 on either side of the central fringe, plus the central fringe, for a total of $27+27+1\ = 55$ bright fringes.

(b) Using sin $\theta = m\lambda/d$, where $m = 0, \pm1, \pm2, \text{and} \pm3$, we have

$\sin\theta = m\lambda/d = m[(0.1904\text{ m})/(5.18\text{ m})] = 0.03676m.$

$m = 0$: $\sin\theta = 0$, which gives $\theta = 0°$.

$m = \pm 1$: $\sin\theta = \pm(0.03676)(1)$, which gives $\theta = \pm 2.11°$.

$m = \pm 2$: $\sin\theta = \pm(0.03676)(2)$, which gives $\theta = \pm 4.22°$.

$m = \pm 3$: $\sin\theta = \pm(0.03676)(3)$, which gives $\theta = \pm 6.33°$.

(c) $I = I_0 \cos^2\left(\dfrac{\pi d \sin\theta}{\lambda}\right) = (2.00\text{W/m}^2)\cos^2\left[\dfrac{\pi(5.18\text{ m})\sin(4.65°)}{0.1904\text{ m}}\right] = 1.28\text{ W/m}^2.$

EVALUATE: Notice that $\sin\theta$ increases in integer steps, but θ only increases in integer steps for small θ.

35.45. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections.

SET UP: There is just one half-cycle phase change upon reflection, so for constructive interference $2t = (m_1 + \frac{1}{2})\lambda_1 = (m_2 + \frac{1}{2})\lambda_2$, where these wavelengths are in the glass. The two different wavelengths differ by just one m-value, $m_2 = m_1 - 1$.

EXECUTE: $(m_1 + \frac{1}{2})\lambda_1 = (m_1 - \frac{1}{2})\lambda_2 \Rightarrow m_1(\lambda_2 - \lambda_1) = \dfrac{\lambda_1 + \lambda_2}{2} \Rightarrow m_1 = \dfrac{\lambda_1 + \lambda_2}{2(\lambda_2 - \lambda_1)}.$

$m_1 = \dfrac{477.0\text{ nm} + 540.6\text{ nm}}{2(540.6\text{ nm} - 477.0\text{ nm})} = 8.$ $2t = \left(8 + \dfrac{1}{2}\right)\dfrac{\lambda_{01}}{n} \Rightarrow t = \dfrac{17(477.0\text{ nm})}{4(1.52)} = 1334\text{ nm}.$

EVALUATE: Now that we have t we can calculate all the other wavelengths for which there is constructive interference.

35.46. **IDENTIFY:** Light reflected from the top of the coating interferes with light reflected from the bottom of the coating, so we have thin-film interference.

SET UP: For maximum transmission in (a) we want minimum reflection. For minimum transmission in (b) we want maximum reflection. A half-cycle phase shift occurs at the air-coating surface but not at the coating-plastic surface. Thus for minimum reflection we must have $2t = m\dfrac{\lambda_0}{n}$, and for maximum reflection we must have $2t = (m + \frac{1}{2})\dfrac{\lambda_0}{n}$, where t is the thickness of the coating and n is the index of refraction of the coating. We want the thinnest coating possible, so we use m = 1 in (a) and m = 0 in (b).

EXECUTE: **(a)** $2t = m\dfrac{\lambda_0}{n}$ gives $t = m\dfrac{\lambda_0}{2n}$ = (1)(510 nm)/[2(1.65)] = 155 nm.

(b) $2t = (m + \frac{1}{2})\dfrac{\lambda_0}{n}$ gives $t = (m + \frac{1}{2})\dfrac{\lambda_0}{2n}$ = (1/2)(510 nm)/[2(1.65)] = 77.3 nm.

EVALUATE: The thickness in (b) is $\frac{1}{2}$ the thickness in (a) because the path differences differ by a factor of one-half of a wavelength.

35.47. **IDENTIFY** and **SET UP:** At the $m = 3$ bright fringe for the red light there must be destructive interference at this same θ for the other wavelength.

EXECUTE: For constructive interference: $d\sin\theta = m\lambda_1 \Rightarrow d\sin\theta = 3(700\text{ nm}) = 2100\text{ nm}$. For destructive interference: $d\sin\theta = (m + \frac{1}{2})\lambda_2 \Rightarrow \lambda_2 = \dfrac{d\sin\theta}{m + \frac{1}{2}} = \dfrac{2100\text{ nm}}{m + \frac{1}{2}}$. So the possible wavelengths are $\lambda_2 = 600$ nm, for $m = 3$, and $\lambda_2 = 467$ nm, for $m = 4$.

EVALUATE: Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.48. **IDENTIFY:** Require constructive interference for the reflection from the top and bottom surfaces of each cytoplasm layer and each guanine layer.

SET UP: At the water (or cytoplasm) to guanine interface, there is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams, just due to the reflections.

EXECUTE: For the guanine layers:

$$2t_g = (m+\tfrac{1}{2})\frac{\lambda}{n_g} \Rightarrow \lambda = \frac{2t_g n_g}{(m+\frac{1}{2})} = \frac{2(74\text{ nm})(1.80)}{(m+\frac{1}{2})} = \frac{266\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm } (m=0).$$

For the cytoplasm layers:

$$2t_c = (m+\tfrac{1}{2})\frac{\lambda}{n_c} \Rightarrow \lambda = \frac{2t_c n_c}{(m+\frac{1}{2})} = \frac{2(100\text{ nm})(1.333)}{(m+\frac{1}{2})} = \frac{267\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm } (m=0).$$

(b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

(c) At different angles, the path length in the layers changes (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

EVALUATE: The thickness of the guanine and cytoplasm layers are inversely proportional to their refractive indices $\left(\frac{100}{74} = \frac{1.80}{1.333}\right)$, so both kinds of layers produce constructive interference for the same wavelength in air.

35.49. **IDENTIFY:** Dark fringes occur because the path difference is one-half of a wavelength.

SET UP: At the first dark fringe, $d\sin\theta = \lambda/2$. The intensity at any angle θ is given by $I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$.

EXECUTE: **(a)** At the first dark fringe, we have $d\sin\theta = \lambda/2$. $d/\lambda = 1/(2\sin 19.0°) = 1.54$.

(b) $I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) = \frac{I_0}{10} \Rightarrow \cos\left(\frac{\pi d\sin\theta}{\lambda}\right) = \frac{1}{\sqrt{10}}$. $\frac{\pi d\sin\theta}{\lambda} = \arccos\left(\frac{1}{\sqrt{10}}\right) = 71.57° = 1.249$ rad.

Using the result from part (a), that $d/\lambda = 1.54$, we have $\pi(1.54)\sin\theta = 1.249$. $\sin\theta = 0.2589$, so $\theta = \pm 15.0°$.

EVALUATE: Since the first dark fringes occur at $\pm 19.0°$, it is reasonable that at 15° the intensity is reduced to only $1/10$ of its maximum central value.

35.50. **IDENTIFY:** Light from the two slits interferes on the screen. We can use the small-angle approximation because we are only looking at closely spaced bright fringes near the center of the pattern.

SET UP: For small angles, the bright fringes are at positions on the screen given by $y = R\frac{m\lambda}{d}$. R, λ, and d are all fixed, and the bright fringes are adjacent ones.

EXECUTE: **(a)** The fringe spacidng is $\Delta y = \frac{R\lambda\Delta m}{d}$ and $\Delta m = 1$ because the bright fringes are adjacent. This equation can be written as $\Delta y = R\lambda\cdot\frac{1}{d}$. From this result, we see that a graph of Δy versus $1/d$ should be a straight line having a slope equal to $R\lambda$.

(b) We use points (9.20 mm^{-1}, 5.0 mm) and (2.00 mm^{-1}, 1.0 mm) to calculate the slope, giving slope = $\frac{(5.0-1.0)\text{ mm}}{(9.20-2.00)\text{ mm}^{-1}} = 0.5556\text{ mm}^2$. Since λR = slope, we have

λ = (slope)/R = (0.5556 mm^2)/(900 mm) = 6.2×10^{-4} mm = 6.2×10^{-7} m = 620 nm.

(Answers may vary a bit depending on accuracy in reading the graph.)

EVALUATE: This wavelength is well within the range of visible light. According to Figure 32.4 in the textbook, this light should be yellow-orange.

35.51. **IDENTIFY:** The wave from A travels a longer distance than the wave from B to reach point P, so the two waves will be out of phase when they reach P. For constructive interference, the path difference should be a whole-number multiple of the wavelength.
SET UP: To reach point P, the wave from A travels 240.0 m and the wave from B travels a distance x. The path difference for these two waves is 240.0 m $-x$. For any wave, $\lambda f = v$. Intensity maxima occur at x = 210.0 m, 216.0 m, and 222.0 m, and there are others.
EXECUTE: **(a)** The distance between adjacent intensity maxima is λ. The three given values of x are 6.0 m apart, so the wavelength must be 6.0 m. The frequency is $f = c/\lambda = c/(6.0\text{ m}) = 5.0\times10^7\text{ Hz} = 50\text{ MHz}$.
(b) Destructive interference occurs when 240.0 m $- x = (m+\frac{1}{2})\lambda$, which gives

x = 240.0 m $-$ $(m+\frac{1}{2})$ (6.0 m).

The largest x occurs when $m = 0$, so x = 240.0 m – 3.0 m = 237.0 m.
EVALUATE: According to Figure 32.4 in the textbook, a wave having a wavelength of 6.0 m is in the radiowave region of the electromagnetic spectrum, which is consistent with the fact that you are using short-wave radio antennas.

35.52. **IDENTIFY:** Assume that the glass is horizontal. The light that travels through the glass and reflects off of its lower surface interferes with incident the light that reflects off the upper surface of the glass. The glass behaves like a thin film. A half-cycle phase change occurs at the upper surface but not at the lower surface because $n_{air} < n_{glass}$.
SET UP: For constructive interference with this glass, $2t = (m+\frac{1}{2})\,(\lambda_0/n)$.

EXECUTE: **(a)** For the 386-nm light: $2t = (m+\frac{1}{2})$ [(386 nm)/n].

For the 496-nm light: $2t = (m+1+\frac{1}{2})$ [(496 nm)/n] = $(m+\frac{3}{2})$ [(386 nm)/n].

Taking the ratio of these two equations gives

$$\frac{m+\frac{1}{2}}{m+\frac{3}{2}} = \frac{386\text{ nm}}{497\text{ nm}} = 1.285.$$

Solving for m gives $m = 3$. Now find t using the equation for the shorter-wavelength light.
$2t = (3+\frac{1}{2})$ [(386 nm)/(1.40)] $\rightarrow$ t = 620 nm.

(b) Solving $2t = (m+\frac{1}{2})\ (\lambda_0/n)$ for λ_0 gives $\lambda_0 = 2nt/(m+\frac{1}{2})$. The largest wavelength will be for $m = 0$, so $\lambda_0 = 2(1.40)(620\text{ nm})/(\frac{1}{2}) = 3470$ nm.

EVALUATE: Visible light is between approximately 400 nm and 700 nm, so the light in (b) is definitely not visible. According to Figure 32.4 in the textbook, it would be in the infrared region of the electromagnetic spectrum.

35.53. **IDENTIFY:** This problem involves the interference of sound waves.
SET UP and **EXECUTE:** Pressure nodes are displacement antinodes, so we have constructive interference at the points in question.
(a) We want the minimum radius r. For the minimum r, a wave traveling straight up and back will interfere with sound just leaving the source. So the path difference is $2r$ and we must have $2r = \lambda$, so $r = \lambda/2$.
(b) We want the number of pressure nodes. At 20°C v = 344 m/s, so $\lambda = v/f = 0.1376$ m. The path difference is $2\sqrt{r^2+(d/2)^2}-d$, so for constructive interference $2\sqrt{r^2+(d/2)^2}-d = m\lambda$.

$m = 0$: $2\sqrt{r^2+(d/2)^2} = d \rightarrow r = 0$, which is not possible.

$m = 1$: $2\sqrt{r^2+(d/2)^2}-d = \lambda$. Square and solve for d. $d = \dfrac{4r^2-\lambda^2}{2\lambda} = 84.1$ cm.

$m = 2$: $2\sqrt{r^2 + (d/2)^2} - d = 2\lambda$. $d = \dfrac{4(r^2 - \lambda^2)}{4\lambda} = 31.7$ cm.

$m = 3$: $2\sqrt{r^2 + (d/2)^2} - d = 3\lambda$. $d = \dfrac{4r^2 - 9\lambda^2}{6\lambda} = 9.64$ cm.

For $m \geq 4$ we get negative d. So there are 3 nodes on each side of the source, for a total of 6.

(c) $d = \pm 9.64$ cm, ± 31.7 cm, ± 84.1 cm.

(d) With helium, $v = 999$ m/s, so $\lambda = 0.3996$ m $= 40.0$ cm. Use the results from part (c).

$m = 1$: $d = \dfrac{4r^2 - \lambda^2}{2\lambda} = 11.3$ cm.

$m = 2$: $d = \dfrac{4(r^2 - \lambda^2)}{4\lambda}$ is negative, so not possible.

There are two nodes, at $d = \pm 11.3$ cm from the center.

EVALUATE: From the calculations for $m = 1$, if $\lambda > 4r$ there are no nodes.

35.54. **IDENTIFY:** This problem is about the interference due to three slits.

SET UP: $E_{\text{p}} = E\cos\omega t + E\cos(\omega t + \phi) + E\cos(\omega t - \phi)$.

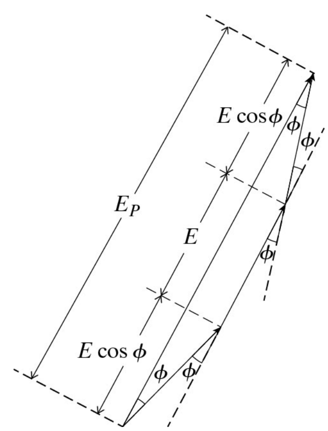


Figure 35.54

EXECUTE: **(a)** We want the amplitude E_{p}. Using the phasor diagram in Fig. 35.54, we see that $E_{\text{p}} = \cos\phi + E + E\cos\phi = E(1 + 2\cos\phi)$.

(b) We want I at P. $I = \frac{1}{2}\epsilon_0 c E_{\text{p}}^2 = \frac{1}{2}\left[E(1 + 2\cos\phi)\right]^2 \epsilon_0 c = \frac{1}{2}\epsilon_0 c E^2 (1 + 2\cos\phi)^2$. The maximum intensity occurs when $\phi = 0$, which gives $I_0 = 9\left(\frac{1}{2}\epsilon_0 c E^2\right)$. Thus $\frac{1}{2}\epsilon_0 c E^2 = \frac{I_0}{9}$. Using this result in our equation for I gives $I = \frac{1}{9} I_0 (1 + 2\cos\phi)^2$.

(c) Since $I \propto (1 + 2\cos\phi)^2$, the lesser maxima occur when $\cos\phi = -1$, for $\phi = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$.

(d) We want I. $I = \frac{1}{9} I_0 (1 - 2)^2 = \frac{I_0}{9}$.

(e) We want the dark fringes. Use the result from (b). $I = \frac{1}{9} I_0 (1 + 2\cos\phi)^2$. For the minima, $I = 0$, so $\cos\phi = -1/2$, so $\phi = \pm 120°$.

(f) From (c), lesser maxima occur when $\phi = \pm\pi, \pm 3\pi, \ldots$ The nearest to the center is when $\phi = \pm\pi$. $\phi = \pi = \dfrac{2\pi d \sin\theta}{\lambda}$. Using d = 0.200 mm and λ = 650 nm, this gives $\sin\theta = 0.001625$. For small angles, $\sin\theta \approx \tan\theta \approx \theta$. The distance on the screen is $y = R\tan\theta \approx R\theta$ = (1.00 m)(0.001625) = 1.63 mm.

(g) The first absolute maximum away from the center occurs for $\phi = 2\pi$. Using $\phi = 2\pi = \dfrac{2\pi d \sin\theta}{\lambda}$ gives $\sin\theta = 0.00325$. As above $y = R\tan\theta \approx R\theta$, which gives y = 3.25 mm.

EVALUATE: The angles are small near the center, so the first lesser maximum is midway between the central maximum and the first absolute maximum.

35.55. **IDENTIFY:** There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index.

SET UP: $\lambda = \dfrac{\lambda_0}{n}$ and $\Delta n = n_0(2.50\times10^{-5}\ (\text{C}°)^{-1})\Delta T$. $\Delta L = L_0(5.00\times10^{-6}\ (\text{C}°)^{-1})\Delta T$.

EXECUTE: The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by: $\Delta N_1 = \dfrac{2n_{\text{glass}}\Delta L}{\lambda_0} - \dfrac{2n_{\text{air}}\Delta L}{\lambda_0} = \dfrac{2(n_{\text{glass}}-1)L_0\alpha\Delta T}{\lambda_0}$ and

$$\Delta N_1 = \frac{2(1.48-1)(0.030\text{ m})(5.00\times10^{-6}/\text{C}°)(5.00\text{ C}°)}{5.89\times10^{-7}\text{ m}} = 1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2 = \frac{2\Delta n_{\text{glass}}L_0}{\lambda_0} = \frac{2(2.50\times10^{-5}/\text{C}°)(5.00\text{ C}°/\text{min})(1.00\text{ min})(0.0300\text{ m})}{5.89\times10^{-7}\text{ m}} = 12.73.$$

So, the total change in the number of wavelengths as the rod expands is
$\Delta N = 12.73 + 1.22 = 14.0$ fringes/minute.

EVALUATE: Both effects increase the number of wavelengths along the length of the rod. Both ΔL and Δn_{glass} are very small and the two effects can be considered separately.

35.56. **IDENTIFY:** Apply Snell's law to the refraction at the two surfaces of the prism. S_1 and S_2 serve as coherent sources so the fringe spacing is $\Delta y = \dfrac{R\lambda}{d}$, where d is the distance between S_1 and S_2.

SET UP: For small angles, $\sin\theta \approx \theta$, with θ expressed in radians.

EXECUTE: **(a)** Since we can approximate the angles of incidence on the prism as being small, Snell's law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\theta/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A$. At the plane of the source S_0, we can calculate the height of one image above the source:

$$\frac{d}{2} = \tan(\theta-\theta')a \approx (\theta-\theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1).$$

(b) To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\text{ m}+0.200\text{ m})\,(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})\,(3.50\times10^{-3}\text{ rad})\,(1.50-1.00)} = 1.57\times10^{-3}\text{ m}.$$

EVALUATE: The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.

35.57. **IDENTIFY** and **SET UP:** Interference occurs when two or more waves combine.
EXECUTE: All of the students now hear the tone, so no interference is occurring. Therefore only one speaker must be on, so the professor must have disconnected one of them. This makes choice (d) correct.
EVALUATE: Turning off one of the speakers would decrease the loudness of the sound, as was observed.

35.58. **IDENTIFY** and **SET UP:** Constructive interference occurs when a wave crest meets another crest, and descructive interference occurs when a crest meets a trough.
EXECUTE: The students who originally heard a loud tone were at a point of constructive interference, so a crest from one speaker met a crest from the other speaker, and those who originally heard nothing were at points where a crest met a trough. Now the students who originally heard a loud tone hear nothing, so at their point a crest meets a trough. The students who originally heard nothing are not at a point where a crest meets a crest. Since the speakers (and students) have not been moved, their phase relationship has been changed, which is choice (d).
EVALUATE: Since a point of constructive interference was turned into a point of destructive interference by the phase change, the phase change must have been π or 180°, equivalent to one-half a wavelength.

35.59. **IDENTIFY:** Moving one of the speakers increases the distance that its sound must travel to reach the listener. This changes the phase difference in the sound from the two speakers as it reaches the listeners.
SET UP: The movement of 0.34 m turned points of constructive interference into points of destructive interference, so that distance must be one-half of a wavelength. We use $v = f\lambda$ to find the frequency.
EXECUTE: $\lambda/2 = 0.34$ m, so $\lambda = 0.68$ m. $v = f\lambda$ gives $f = v/\lambda$ = (340 m/s)/(0.68 m) = 500 Hz, which is choice (c).
EVALUATE: If the professor moves the speaker an additional 0.34 m, the students will hear what they originally heard since that distance is a full wavelength.

35.60. **IDENTIFY** and **SET UP:** Since $v = f\lambda$, reducing the frequency half increases the wavelength by a factor of 2. For constructive interference, the path difference is $m\lambda$, where $m = 0, 1, 2, 3, \ldots$, and for destructive interference the path difference is $(m+\frac{1}{2})\lambda$, where $m = 0, 1, 2, 3, \ldots$.
EXECUTE: The new wavelength λ is twice as long as the original wavelength λ_0. Students who heard a loud tone before were at locations for which the path difference was $\lambda,\ 2\lambda,\ 3\lambda,\ 4\lambda,\ 5\lambda,\ \ldots$. But since the new wavelength is $\lambda = 2\lambda_0$, students who were at a path difference of λ_0 are now at a point where the path difference is $\lambda/2$, and those where it was $3\lambda_0$ are now where it is $3\lambda/2$, etc. So all those students for whom the path difference was $\lambda_0,\ 3\lambda_0,\ 5\lambda_0,\ 7\lambda_0,\ \ldots$ will hear nothing. For the students who are at points where the path difference was $2\lambda_0,\ 4\lambda_0,\ 6\lambda_0,\ \ldots$, the path difference is now $\lambda,\ 2\lambda,\ 3\lambda,\ \ldots$, so they will still hear a loud tone. Therefore choice (c) is correct.
EVALUATE: Students at various points in between those discussed here hear sound, but not of maximum loudness.