17

TEMPERATURE AND HEAT

VP17.4.1. **IDENTIFY:** The rods expand (or contract) when their temperature is changed, so we are dealing with thermal expansion.

SET UP: $\Delta L = \alpha L_0 \Delta T$

EXECUTE: **(a)** We want to find α, the coefficient of linear expansion. Using $\Delta L = \alpha L_0 \Delta T$ gives

2.20×10^{-4} m $= \alpha(0.500$ m$)(37.0°C - 15.0°C)$, so $\alpha = 2.00 \times 10^{-5}$ (C°)$^{-1} = 2.00 \times 10^{-5}$ K^{-1}.

(b) We want the change in length. Using $\Delta L = \alpha L_0 \Delta T$ gives

$\Delta L = \left[2.00 \times 10^{-5} \text{ (C°)}^{-1} \right](0.300 \text{ m})(-20.0°C - 25.0°C) = -2.7 \times 10^{-4}$ m $= -0.27$ mm. Its length would

decrease by 0.27 mm.

EVALUATE: Increasing temperature causes thermal expansion, while decreasing temperature causes thermal contraction. In both cases, the relative change in length is normally very small.

VP17.4.2. **IDENTIFY:** The mug and ethanol both decrease in volume as their temperature is decreased, so we are dealing with thermal contraction.

SET UP: We are dealing with volume contraction, so we use $\Delta V = \beta V_0 \Delta T$. If the volume of the ethanol and the mug decreased by the same amount, the ethanol would continue to fill the mug. In that case we could not add any additional ethanol to the mug. But $\beta_{\text{ethanol}} > \beta_{\text{mug}}$, so the ethanol contracts more than the mug, leaving room to add additional ethanol. The amount of empty space after the contraction will be $V_{\text{empy}} = \Delta V_{\text{ethanol}} - \Delta V_{\text{mug}}$, which is what we want to find. Table 17.2 tells us that

$\beta_{\text{Cu}} = 5.1 \times 10^{-5} \text{(C°)}^{-1}$ and $\beta_{\text{ethanol}} = 75 \times 10^{-5} \text{(C°)}^{-1}$.

EXECUTE: $V_{\text{empy}} = \Delta V_{\text{ethanol}} - \Delta V_{\text{mug}} = \beta_e V_0 \Delta T - \beta_c V_0 \Delta T = (\beta_e - \beta_c) V_0 \Delta T$. Using the coefficients of volume expansion from Table 17.2, $V_0 = 250$ cm^3, and $\Delta T = -70.0°C$, we get $V_{\text{empty}} = -12$ cm^3. This is the net change in volume, and it is negative since the volumes have decreased. The available volume is 12 cm^3.

EVALUATE: If β_{ethanol} were less than β_{mug}, ethanol would spill out of the mug as the temperature fell because the mug would contract more than the ethanol.

VP17.4.3. **IDENTIFY:** The thermal stress is due to the contraction of the brass rod as its temperature is decreased.

SET UP: Thermal stress is $\dfrac{F}{A} = -Y\alpha\Delta T$. Table 17.1 gives $\alpha_{\text{brass}} = 2.0 \times 10^{-5} \text{(C°)}^{-1}$ and Y_{brass} is given in the problem. The target variable is the thermal stress.

EXECUTE: The force is $F = -AY\alpha\Delta T = -\pi r^2 Y\alpha\Delta T$. Using the given values for Y and α, $r = 5.00 \times 10^{-3}$ m, and $\Delta T = 13.0°C - 25.0°C = -12°C$, we get $F = 1700$ N.

EVALUATE: The bar tends to shrink when cooled, so the force prevent this. Therefore the forces are *tensile.*

VP17.4.4.   **IDENTIFY:**   The rods increase in length when their temperature is raised, so we are dealing with thermal expansion.

SET UP: The change in length of the combined rod is the sum of the changes of the two rods. So we know that $L = L_A + L_B$ and $\Delta L = \Delta L_A + \Delta L_B$, and we use $\Delta L = \alpha L_0 \Delta T$ for each rod. We want to solve for L_A in terms of the other quantities.

EXECUTE: $\Delta L = \Delta L_A + \Delta L_B = \alpha_A L_A \Delta T + \alpha_B (L - L_A) \Delta T$. Solve for L_A: $L_A = \dfrac{\alpha_B L - \dfrac{\Delta L}{\Delta T}}{\alpha_B - \alpha_A}$.

EVALUATE: The units may not look right. But realize that $\Delta L = \alpha L_0 \Delta T$ tells us that $\dfrac{\Delta L}{\Delta T} = \alpha L_0$ so the units in the numerator are those of αL, so the overall units are those of $\dfrac{\alpha L}{\alpha}$, which are units of length.

VP17.9.1.   **IDENTIFY:**   The heat lost by the aluminum is equal to the heat gained by the water, so the net heat transfer for the system is zero.

SET UP: The aluminum is hotter, so it loses heat and the water gains that heat. Use $Q = mc\Delta T$ for each substance. The target variable is the mass of the aluminum. From Table 17.3 we have $c_{Al} = 910$ J/kg $\cdot$ K and $c_w = 4190$ J/kg $\cdot$ K.

EXECUTE: Use $Q_{Al} + Q_w = 0$: $m_{Al} c_{Al} \Delta T_{Al} + m_w c_w \Delta T_w = 0$. Using the numbers gives $m_{Al} (910$ J/kg $\cdot$ K$)(-228.0$ K$) + (5.00$ kg$)(4190$ J/kg $\cdot$ K$)(2.0°$C$) = 0$, so $m_{Al} = 0.20$ kg.

EVALUATE: It is *not true* that $T_C = T_K$. But it *is true* that $\Delta T_C = \Delta T_K$ because the size of the Kelvin degree is the same as the size of the Celsius degree.

VP17.9.2.   **IDENTIFY:**   Heat is transferred to the copper cube to the ice cube, and this heat melts the ice. We are dealing with temperature changes and a change of phase for the ice from solid to liquid.

SET UP: The net heat transfer between the ice and copper is zero because the heat lost by the copper is gained by the ice cube. For temperature changes we use $Q = mc\Delta T$ and for the phase change we use $Q = mL_f$. The target variable is the initial temperature of the copper.

EXECUTE: Use $Q_{ice} + Q_{Cu} = 0$ and call T the initial temperature of the copper. This gives $m_{Cu} c_{Cu} \Delta T_{Cu} + m_i L_i = 0$. Using c_{Cu} from Table 17.3 and L_f from Table 17.4, we have $(0.460$ kg$)(390$ J/kg $\cdot$ K$)(0.00°$C $- T) + (7.50 \times 10^{-3}$ kg$)(334 \times 10^3$ J/kg$) = 0$, so $T = 14.0°$C.

EVALUATE: Just the melting a small ice cube can produce a large temperature change for the copper because c_{Cu} is small and L_f is large for water.

VP17.9.3.   **IDENTIFY:**   We mix ice and ethanol and wait until the mixture reaches an equilibrium temperature. The heat lost by the ethanol is equal to the heat gained by the ice, so the net heat transferred is zero.

SET UP: The target variable is the mass of the ice, m_i. For temperature changes we use $Q = mc\Delta T$ and for the phase change we use $Q = mL_f$. The heat transferred to the ice causes three changes: (1) increase in the ice temperature from $-5.00°$C to $0.00°$C, (2) melt the ice at $0.00°$C, and (3) increase the temperature of the melted ice from $0.00°$C to $10.0°$C.

EXECUTE: Use $Q_{ice} + Q_{ethanol} = 0$ and include the three changes for the ice listed above. This gives $m_i c_i \Delta T_i + m_i L_i + m_i c_w \Delta T_w + m_e c_e \Delta T_e = 0$. We know that $m_e = 1.60$ kg, and from Tables 17.3 and 17.4, we have $c_i = 2100$ J/kg $\cdot$ K, $c_w = 4190$ J/kg $\cdot$ K, $c_e = 2428$ J/kg $\cdot$ K, and $L_f = 334 \times 10^3$ J/kg. The temperature changes are $\Delta T_i = 5.00°$C, $\Delta T_w = 10.0°$C, and $\Delta T_e = -18.0°$C. Putting in the numbers and solving for m_i gives $m_i = 0.181$ kg.

EVALUATE: We must treat the ice in three stages. In addition to melting at $0°$C, the ice and liquid water have different specific heats so we cannot treat the ice change from $-5.00°$C to $10.0°$C in a single step.

VP17.9.4. **IDENTIFY:** The ice cools a silver ingot. Several possible outcomes are possible depending on the relative masses of the ice and silver and their initial temperatures: the ice could remain ice but at a higher temperature, the ice could partially melt and remain at 0.00°C, the ice could all melt and remain at 0.00°C, or the ice could all melt and increase its temperature above 0.00°C.

SET UP: For temperature changes use $Q = mc\Delta T$ and for the phase change use $Q = mL_f$. First see if there is enough heat in the silver to bring the ice up to its melting point temperature of 0.00°C and to melt it at 0.00°C. Use quantities from Tables 17.3 and 17.4 for water, ice, and silver. The target variable is the final equilibrium temperature T of the system.

EXECUTE: The heat to cool the silver down to 0.00°C is $Q = mc\Delta T$, which gives

$Q = (1.25 \text{ kg})(234 \text{ J/kg} \cdot \text{K})(315°\text{C}) = 9.214 \times 10^4 \text{ J}$. The heat needed to melt all the ice is $Q = mc\Delta T +$

$mL_f = (0.250 \text{ kg})(2100 \text{ J/kg} \cdot \text{K})(8.00°\text{C}) + (0.250 \text{ kg})(334 \times 10^3 \text{ J/kg}) = 8.77 \times 10^4 \text{ J}$. As we see, there is enough heat in the silver ingot to melt all the ice. Now we use the same procedure as in the previous problem except we have silver instead of ethanol. This gives $m_i c_i \Delta T_i + m_i L_i + m_i c_w \Delta T_w + m_s c_s \Delta T_s = 0$.

The temperature changes are $\Delta T_i = 8.00°\text{C}$, $\Delta T_w = T - 00.0°\text{C}$, and $\Delta T_s = T - 315°\text{C}$, $m_i = 0.250$ kg, and $m_s = 1.25$ kg. The result is $T = 3.31°\text{C}$, and all the ice melts.

EVALUATE: We must treat the ice in three stages. In addition to melting at 0°C, the ice and liquid water have different specific heats so we cannot treat the ice change from –8.00°C to 3.31°C in a single step.

VP17.15.1. **IDENTIFY:** The heat flows through the pane of glass, so we are dealing with heat conduction.

SET UP: The rate of heat flow is $H = kA\dfrac{T_H - T_C}{L}$.

EXECUTE: **(a)** We want the thermal conductivity k of the glass. Solve $H = kA\dfrac{T_H - T_C}{L}$ for k, which

gives $k = \dfrac{LH}{A(T_H - T_C)} = \dfrac{(0.00600 \text{ m})(1100 \text{ W})}{(0.500 \text{ m})^2(35.0 \text{ C}°)} = 0.754 \text{ W/m} \cdot \text{K}$.

(b) $H \propto 1/L$, so $\dfrac{H_9}{H_6} = \dfrac{1/9}{1/6} = \dfrac{2}{3}$, which gives $H_9 = \dfrac{2}{3}(1100 \text{ W}) = 733 \text{ W}$.

EVALUATE: Other ways to decrease the heat current would be to decrease the area of the pane or to use a double-pane window which traps a layer of air between two panes of glass. The air has a *much* lower thermal conductivity than glass.

VP17.15.2. **IDENTIFY:** Heat current flows through the two end-to-end rods (brass and lead), so we are dealing with heat conduction.

SET UP: Use $H = kA\dfrac{T_H - T_C}{L}$. Table 17.5 gives $k_{\text{brass}} = 109.0 \text{ W/m} \cdot \text{K}$ and $k_{\text{lead}} = 34.7 \text{ W/m} \cdot \text{K}$. The heat current is the same (6.00 W) in both rods. The target variables are the temperatures of the free ends of the two-rod system.

EXECUTE: **(a)** <u>Brass</u>: Using $H = kA\dfrac{T_H - T_C}{L}$ and putting in the numbers gives

$6.00 \text{ W} = (109.0 \text{ W/m} \cdot \text{K})(2.00 \times 10^{-4} \text{ m})\left(\dfrac{T_{\text{brass}} - 185°\text{C}}{0.250 \text{ m}}\right)$, so $T_{\text{brass}} = 254°\text{C}$.

(b) <u>Lead</u>: We follow the same procedure as for brass, giving

$6.00 \text{ W} = (34.7 \text{ W/m} \cdot \text{K})(2.00 \times 10^{-4} \text{ m})\left(\dfrac{185°\text{C} - T_{\text{lead}}}{0.250 \text{ m}}\right)$, so $T_{\text{lead}} = -31°\text{C}$.

EVALUATE: From $H = kA\dfrac{T_{\mathrm{H}} - T_{\mathrm{C}}}{L}$ we see that $\Delta T \propto \dfrac{1}{k}$. So, if all else is the same, an object with a small k should have a large temperature difference compared to one with a large k. Our results agree with this since $\Delta T_{\mathrm{lead}} > \Delta T_{\mathrm{brass}}$ and $k_{\mathrm{lead}} < k_{\mathrm{brass}}$.

VP17.15.3. IDENTIFY: We are dealing the radiation from a star.

SET UP: $H = Ae\sigma T^4$, where T must be in K, $e = 1$ for the star, and $A = 4\pi R^2$.

EXECUTE: $H = Ae\sigma T^4 = 4\pi R^2 \sigma T^4$, so solve for R, giving $R = \sqrt{\dfrac{H}{4\pi\sigma T^4}} = \dfrac{1}{T^2}\sqrt{\dfrac{H}{4\pi\sigma}}$. Putting in the

numbers gives $R = \dfrac{1}{(9940\ \mathrm{K})^2}\sqrt{\dfrac{9.7\times10^{27}\ \mathrm{W}}{4\pi\left(5.67\times10^{-8}\ \mathrm{W/m^2 \cdot K^4}\right)}} = 1.2\times10^9$ m.

$\dfrac{R_{\mathrm{Sirius}}}{R_{\mathrm{sun}}} = \dfrac{1.2\times10^9\ \mathrm{m}}{6.96\times10^8\ \mathrm{m}} = 1.7.$

EVALUATE: The surface area of Sirius is $(1.7)^2 = 2.9$ times as great as our sun, so Sirius is a luminous star.

VP17.15.4. IDENTIFY: The building radiates heat into the air, but the hot air also radiates heat back into the building. So we are looking for the *net* radiation, which will be *into* the building because its surface is less hot than the outside air.

SET UP: $H_{\mathrm{net}} = Ae\sigma\left(T^4 - T_{\mathrm{s}}^4\right)$, where temperatures must be in K and $e = 0.91$.

EXECUTE: $H_{\mathrm{net}} = (525\ \mathrm{m^2})(0.91)(5.67\times10^{-8}\ \mathrm{W/m^2 \cdot K^4})\left[(293\ \mathrm{K})^4 - (308\ \mathrm{K})^4\right] = -4.4\times10^4$ W. The

minus sign tells us that net heat flows *into* the building from the hot desert air at a rate of 4.4×10^4 W $=$ 44 kW.

EVALUATE: The heat flow into the building would be even greater during the day when the outside temperature could be 40°C or even higher.

17.1. IDENTIFY and SET UP: $T_{\mathrm{F}} = \frac{9}{5}T_{\mathrm{C}} + 32°$.

EXECUTE: (a) $T_{\mathrm{F}} = (9/5)(-62.8) + 32 = -81.0°$F.

(b) $T_{\mathrm{F}} = (9/5)(56.7) + 32 = 134.1°$F.

(c) $T_{\mathrm{F}} = (9/5)(31.1) + 32 = 88.0°$F.

EVALUATE: Fahrenheit degrees are smaller than Celsius degrees, so it takes more F° than C° to express the difference of a temperature from the ice point.

17.2. IDENTIFY and SET UP: To convert a temperature between °C and K use $T_{\mathrm{C}} = T_{\mathrm{K}} - 273.15$. To convert from °F to °C, subtract 32° and multiply by 5/9. To convert from °C to °F, multiply by 9/5 and add 32°. To convert a temperature difference, use that Celsius and Kelvin degrees are the same size and that 9 F° = 5 C°.

EXECUTE: (a) $T_{\mathrm{C}} = T_{\mathrm{K}} - 273.15 = 310 - 273.15 = 36.9°$C; $T_{\mathrm{F}} = \frac{9}{5}T_{\mathrm{C}} + 32° = \frac{9}{5}(36.9°) + 32° = 98.4°$F.

(b) $T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15 = 40 + 273.15 = 313$ K; $T_{\mathrm{F}} = \frac{9}{5}T_{\mathrm{C}} + 32° = \frac{9}{5}(40°) + 32° = 104°$F.

(c) 7 C° = 7 K; 7 C° = (7 C°)(9 F°/5 C°) = 13 F°.

(d) 4.0°C: $T_{\mathrm{F}} = \frac{9}{5}T_{\mathrm{C}} + 32° = \frac{9}{5}(4.0°) + 32° = 39.2°$F; $T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15 = 4.0 + 273.15 = 277$ K.

$-160°$C: $T_{\mathrm{F}} = \frac{9}{5}T_{\mathrm{C}} + 32° = \frac{9}{5}(-160°) + 32° = -256°$F; $T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15 = -160 + 273.15 = 113$ K.

(e) $T_{\mathrm{C}} = \frac{5}{9}(T_{\mathrm{F}} - 32°) = \frac{5}{9}(105° - 32°) = 41°$C; $T_{\mathrm{K}} = T_{\mathrm{C}} + 273.15 = 41 + 273.15 = 314$ K.

EVALUATE: Celsius-Fahrenheit conversions do not involve simple proportions due to the additive constant of 32°, but Celsius-Kelvin conversions require only simple addition/subtraction of 273.15.

17.3. **IDENTIFY:** Convert ΔT between different scales.

SET UP: ΔT is the same on the Celsius and Kelvin scales. $180 \text{ F}° = 100 \text{ C}°$, so $1 \text{ C}° = \frac{9}{5}\text{F}°$.

EXECUTE: **(a)** $\Delta T = 49.0 \text{ F}°$. $\Delta T = (49.0 \text{ F}°)\left(\dfrac{1 \text{ C}°}{\frac{9}{5} \text{ F}°}\right) = 27.2 \text{ C}°$.

(b) $\Delta T = -100 \text{ F}°$. $\Delta T = (-100.0 \text{ F}°)\left(\dfrac{1 \text{ C}°}{\frac{9}{5} \text{ F}°}\right) = -55.6 \text{ C}°$

EVALUATE: The magnitude of the temperature change is larger in F° than in C°.

17.4. **IDENTIFY:** This problem requires conversion of temperature scales. We want T_K as a function of T_F and T_F as a function of T_K.

SET UP and EXECUTE: We know that $T_K = T_C + 273.15$ and $T_C = \dfrac{5}{9}(T_F - 32°)$. Using these gives

$T_K = T_C + 273.15 = \dfrac{5}{9}(T_F - 32°) + 273.15$. This reduces to $T_K = \dfrac{5}{9}T_F + 255.37$.

Solve this result for T_F, giving $T_F = \dfrac{9}{5}T_K - 459.67$.

EVALUATE: The fractions 5/9 and 5/9 occur because the Kelvin degree is not the same size as the Fahrenheit degree; the Kelvin degree is larger.

17.5. **IDENTIFY:** Convert ΔT in kelvins to C° and to F°.

SET UP: $1 \text{ K} = 1 \text{ C}° = \frac{9}{5}\text{F}°$

EXECUTE: **(a)** $\Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(-10.0 \text{ C}°) = -18.0 \text{ F}°$

(b) $\Delta T_C = \Delta T_K = -10.0 \text{ C}°$

EVALUATE: Kelvin and Celsius degrees are the same size. Fahrenheit degrees are smaller, so it takes more of them to express a given ΔT value.

17.6. **IDENTIFY:** Set $T_C = T_F$ and $T_F = T_K$.

SET UP: $T_F = \frac{9}{5}T_C + 32°C$ and $T_K = T_C + 273.15 = \frac{5}{9}(T_F - 32°) + 273.15$

EXECUTE: **(a)** $T_F = T_C = T$ gives $T = \frac{9}{5}T + 32°$ and $T = -40°$; $-40°C = -40°F$.

(b) $T_F = T_K = T$ gives $T = \frac{5}{9}(T - 32°) + 273.15$ and $T = \frac{9}{4}(-(\frac{5}{9})(32°) + 273.15) = 575°$; $575°F = 575$ K.

EVALUATE: Since $T_K = T_C + 273.15$ there is no temperature at which Celsius and Kelvin thermometers agree.

17.7. **IDENTIFY:** When the volume is constant, $\dfrac{T_2}{T_1} = \dfrac{p_2}{p_1}$, for T in kelvins.

SET UP: $T_{\text{triple}} = 273.16$ K. Figure 17.7 in the textbook gives that the temperature at which CO_2 solidifies is $T_{CO_2} = 195$ K.

EXECUTE: $p_2 = p_1\left(\dfrac{T_2}{T_1}\right) = (1.35 \text{ atm})\left(\dfrac{195 \text{ K}}{273.16 \text{ K}}\right) = 0.964 \text{ atm}$

EVALUATE: The pressure decreases when T decreases.

17.8. **IDENTIFY:** Convert T_K to T_C and then convert T_C to T_F.

SET UP: $T_K = T_C + 273.15$ and $T_F = \frac{9}{5}T_C + 32°$.

EXECUTE: **(a)** $T_C = 400 - 273.15 = 127°C$, $T_F = (9/5)(126.85) + 32 = 260°F$

(b) $T_C = 95 - 273.15 = -178°C$, $T_F = (9/5)(-178.15) + 32 = -289°F$

(c) $T_C = 1.55 \times 10^7 - 273.15 = 1.55 \times 10^7 °C$, $T_F = (9/5)(1.55 \times 10^7) + 32 = 2.79 \times 10^7 °F$

EVALUATE: All temperatures on the Kelvin scale are positive. T_C is negative if the temperature is below the freezing point of water.

17.9. **IDENTIFY** and **SET UP:** Fit the data to a straight line for $p(T)$ and use this equation to find T when $p = 0$.

EXECUTE: **(a)** If the pressure varies linearly with temperature, then $p_2 = p_1 + \gamma(T_2 - T_1)$.

$$\gamma = \frac{p_2 - p_1}{T_2 - T_1} = \frac{6.50\times10^4 \text{ Pa} - 4.80\times10^4 \text{ Pa}}{100°\text{C} - 0.01°\text{C}} = 170.0 \text{ Pa/C}°$$

Apply $p = p_1 + \gamma(T - T_1)$ with $T_1 = 0.01°\text{C}$ and $p = 0$ to solve for T.

$0 = p_1 + \gamma(T - T_1)$

$$T = T_1 - \frac{p_1}{\gamma} = 0.01°\text{C} - \frac{4.80\times10^4 \text{ Pa}}{170 \text{ Pa/C}°} = -282°\text{C}.$$

(b) Let $T_1 = 100°\text{C}$ and $T_2 = 0.01°\text{C}$; use $T_2/T_1 = p_2/p_1$ to calculate p_2, where T is in kelvins.

$$p_2 = p_1\left(\frac{T_2}{T_1}\right) = 6.50\times10^4 \text{ Pa}\left(\frac{0.01 + 273.15}{100 + 273.15}\right) = 4.76\times10^4 \text{ Pa; this differs from the } 4.80\times10^4 \text{ Pa that}$$

was measured so $T_2/T_1 = p_2/p_1$ is not precisely obeyed.

EVALUATE: The answer to part (a) is in reasonable agreement with the accepted value of $-273°\text{C}$.

17.10. **IDENTIFY:** Apply $T = T_{\text{triple}}\dfrac{p}{p_{\text{triple}}} = (273.16 \text{ K})\dfrac{p}{p_{\text{triple}}}$ and solve for p.

SET UP: $p_{\text{triple}} = 325$ mm of mercury

EXECUTE: $p = (325.0 \text{ mm of mercury})\left(\dfrac{373.15 \text{ K}}{273.16 \text{ K}}\right) = 444$ mm of mercury

EVALUATE: mm of mercury is a unit of pressure. Since $T = T_{\text{triple}}\dfrac{p}{p_{\text{triple}}} = (273.16 \text{ K})\dfrac{p}{p_{\text{triple}}}$ involves a

ratio of pressures, it is not necessary to convert the pressure to units of Pa.

17.11. **IDENTIFY:** $\Delta L = L_0 \alpha \Delta T$

SET UP: For steel, $\alpha = 1.2\times10^{-5} \text{ (C}°)^{-1}$

EXECUTE: $\Delta L = (1.2\times10^{-5}(\text{C}°)^{-1})(1410 \text{ m})(18.0°\text{C} - (-5.0°\text{C})) = +0.39 \text{ m}$

EVALUATE: The length increases when the temperature increases. The fractional increase is very small, since $\alpha \Delta T$ is small.

17.12. **IDENTIFY:** Apply $\Delta L = \alpha L_0 \Delta T$ and calculate ΔT. Then $T_2 = T_1 + \Delta T$, with $T_1 = 15.5°\text{C}$.

SET UP: Table 17.1 gives $\alpha = 1.2\times10^{-5} \text{ (C}°)^{-1}$ for steel.

EXECUTE: $\Delta T = \dfrac{\Delta L}{\alpha L_0} = \dfrac{0.471 \text{ ft}}{[1.2\times10^{-5} \text{ (C}°)^{-1}][1671 \text{ ft}]} = 23.5 \text{ C}°. \ T_2 = 15.5°\text{C} + 23.5 \text{ C}° = 39.0°\text{C}.$

EVALUATE: Since then the lengths enter in the ratio $\Delta L/L_0$, we can leave the lengths in ft.

17.13. **IDENTIFY:** Apply $L = L_0(1 + \alpha \Delta T)$ to the diameter D of the penny.

SET UP: $1 \text{ K} = 1 \text{ C}°$, so we can use temperatures in $°\text{C}$.

EXECUTE: Death Valley: $\alpha D_0 \Delta T = [2.6\times10^{-5} \text{ (C}°)^{-1}](1.90 \text{ cm})(28.0 \text{ C}°) = 1.4\times10^{-3}$ cm, so the

diameter is 1.9014 cm. Greenland: $\alpha D_0 \Delta T = -3.6\times10^{-3}$ cm, so the diameter is 1.8964 cm.

EVALUATE: When T increases the diameter increases and when T decreases the diameter decreases.

17.14.   **IDENTIFY:**   Apply $L = L_0(1 + \alpha \Delta T)$ to the diameter d of the rivet.

SET UP: For aluminum, $\alpha = 2.4 \times 10^{-5}$ $(\text{C}°)^{-1}$. Let d_0 be the diameter at –78.0°C and d be the diameter at 23.0°C.

EXECUTE: $d = d_0 + \Delta d = d_0(1 + \alpha \Delta T) = (0.4500 \text{ cm})(1 + (2.4 \times 10^{-5}(\text{C}°)^{-1}[23.0°\text{C} - (-78.0°\text{C})])$.

$d = 0.4511 \text{ cm} = 4.511 \text{ mm}$.

EVALUATE: We could have let d_0 be the diameter at 23.0°C and d be the diameter at −78.0°C. Then $\Delta T = -78.0°\text{C} - 23.0°\text{C}$.

17.15.   **IDENTIFY:**   Apply $\Delta V = V_0 \beta \Delta T$.

SET UP: For copper, $\beta = 5.1 \times 10^{-5}$ $(\text{C}°)^{-1}$. $\Delta V / V_0 = 0.150 \times 10^{-2}$.

EXECUTE: $\Delta T = \dfrac{\Delta V / V_0}{\beta} = \dfrac{0.150 \times 10^{-2}}{5.1 \times 10^{-5} \ (\text{C}°)^{-1}} = 29.4$ C°. $T_f = T_i + \Delta T = 49.4°\text{C}$.

EVALUATE: The volume increases when the temperature increases.

17.16.   **IDENTIFY:** $\Delta V = \beta V_0 \Delta T$. Use the diameter at −15°C to calculate the value of V_0 at that temperature.

SET UP: For a hemisphere of radius R, the volume is $V = \frac{2}{3} \pi R^3$. Table 17.2 gives $\beta = 7.2 \times 10^{-5}$ $(\text{C}°)^{-1}$ for aluminum.

EXECUTE: $V_0 = \frac{2}{3} \pi R^3 = \frac{2}{3} \pi (27.5 \text{ m})^3 = 4.356 \times 10^4 \text{ m}^3$.

$\Delta V = (7.2 \times 10^{-5} \ (\text{C}°)^{-1})(4.356 \times 10^4 \text{ m}^3)[35°\text{C} - (-15°\text{C})] = 160 \text{ m}^3$.

EVALUATE: We could also calculate $R = R_0(1 + \alpha \Delta T)$ and calculate the new V from R. The increase in volume is $V - V_0$, but we would have to be careful to avoid round-off errors when two large volumes of nearly the same size are subtracted.

17.17.   **IDENTIFY:**   Apply $\Delta V = V_0 \beta \Delta T$ to the volume of the flask and to the mercury. When heated, both the volume of the flask and the volume of the mercury increase.

SET UP: For mercury, $\beta_{\text{Hg}} = 18 \times 10^{-5}$ $(\text{C}°)^{-1}$.

8.95 cm³ of mercury overflows, so $\Delta V_{\text{Hg}} - \Delta V_{\text{glass}} = 8.95 \text{ cm}^3$.

EXECUTE: $\Delta V_{\text{Hg}} = V_0 \beta_{\text{Hg}} \Delta T = (1000.00 \text{ cm}^3)(18 \times 10^{-5} \ (\text{C}°)^{-1})(55.0 \text{ C}°) = 9.9 \text{ cm}^3$.

$\Delta V_{\text{glass}} = \Delta V_{\text{Hg}} - 8.95 \text{ cm}^3 = 0.95 \text{ cm}^3$. $\beta_{\text{glass}} = \dfrac{\Delta V_{\text{glass}}}{V_0 \Delta T} = \dfrac{0.95 \text{ cm}^3}{(1000.00 \text{ cm}^3)(55.0 \text{ C}°)} = 1.7 \times 10^{-5}$ $(\text{C}°)^{-1}$.

EVALUATE: The coefficient of volume expansion for the mercury is larger than for glass. When they are heated, both the volume of the mercury and the inside volume of the flask increase. But the increase for the mercury is greater and it no longer all fits inside the flask.

17.18.   **IDENTIFY:**   Apply $\Delta V = V_0 \beta \Delta T$ to the tank and to the ethanol.

SET UP: For ethanol, $\beta_e = 75 \times 10^{-5}$ $(\text{C}°)^{-1}$. For steel, $\beta_s = 3.6 \times 10^{-5}$ $(\text{C}°)^{-1}$.

EXECUTE: The volume change for the tank is

$\Delta V_s = V_0 \beta_s \Delta T = (1.90 \text{ m}^3)(3.6 \times 10^{-5}(\text{C}°)^{-1})(-14.0 \text{ C}°) = -9.576 \times 10^{-4} \text{ m}^3 = -0.9576 \text{ L}$.

The volume change for the ethanol is

$\Delta V_e = V_0 \beta_e \Delta T = (1.90 \text{ m}^3)(75 \times 10^{-5} \ (\text{C}°)^{-1})(-14.0 \text{ C}°) = -1.995 \times 10^{-2} \text{ m}^3 = -19.95 \text{ L}$.

The empty volume in the tank is $\Delta V_e - \Delta V_s = -19.95 \text{ L} - (-0.9576 \text{ L}) = -19.0 \text{ L}$. So 19.0 L of ethanol can be added to the tank.

EVALUATE: Both volumes decrease. But $\beta_e > \beta_s$, so the magnitude of the volume decrease for the ethanol is greater than it is for the tank.

17.19. **IDENTIFY** and **SET UP:** Use $\Delta A = 2\alpha A_0 \Delta T$ to calculate ΔA for the plate, and then $A = A_0 + \Delta A$.

EXECUTE: **(a)** $A_0 = \pi r_0^2 = \pi \left(\dfrac{1.350 \text{ cm}}{2}\right)^2 = 1.431 \text{ cm}^2$.

(b) Using $\Delta A = 2\alpha A_0 \Delta T$, we have

$\Delta A = 2(1.2 \times 10^{-5} \text{ C}^{\circ -1})(1.431 \text{ cm}^2)(175^\circ\text{C} - 25^\circ\text{C}) = 5.15 \times 10^{-3} \text{ cm}^2$ $A = A_0 + \Delta A = 1.436 \text{ cm}^2$.

EVALUATE: A hole in a flat metal plate expands when the metal is heated just as a piece of metal the same size as the hole would expand.

17.20. **IDENTIFY:** We are looking at the thermal expansion of the area of a metal plate.
SET UP: For length and width, we use $\Delta L = \alpha L_0 \Delta T$. We want to find a comparable formula for the change in the area of the plate. Fig. 17.20 shows the changes in the plate.
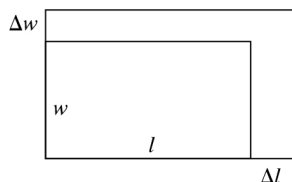
Figure 17.20

EXECUTE: The original area is $A_0 = \ell w$. The length expands by an amount $\Delta \ell = \alpha \ell \Delta T$ and width expands by $\Delta w = \alpha w \Delta T$. The expanded area is $A_2 = (\ell + \Delta \ell)(w + \Delta w) = \ell w + \ell \Delta w + w \Delta \ell + \Delta \ell \Delta w$. The change in area is $\Delta A = A_2 - A_0 = (\ell w + \ell \Delta w + w \Delta \ell + \Delta \ell \Delta w) - \ell w = \ell \Delta w + w \Delta \ell + \Delta \ell \Delta w$. Using $A_0 = \ell w$, $\Delta \ell = \alpha \ell \Delta T$, and $\Delta w = \alpha w \Delta T$, we get $\Delta A = \ell w \alpha \Delta T + w \ell \alpha \Delta T + \ell \alpha \Delta T w \alpha \Delta T = 2\alpha A_0 \Delta T + \ell \alpha \Delta T w \alpha \Delta T$.

The last term contains α^2, which is much smaller than α because α is typically very small. So we can drop the last term, leaving $\Delta A = 2\alpha A_0 \Delta T$.

EVALUATE: This result has the same form as linear and volume expansion. For volume expansion the coefficient of volume expansion is $\beta = 3\alpha$, and for area expansion the coefficient of area expansion is 2α.

17.21. **IDENTIFY:** Apply $\Delta L = L_0 \alpha \Delta T$ and stress $= F/A = -Y\alpha \Delta T$.

SET UP: For steel, $\alpha = 1.2 \times 10^{-5} \text{ (C}^\circ)^{-1}$ and $Y = 2.0 \times 10^{11}$ Pa.

EXECUTE: **(a)** $\Delta L = L_0 \alpha \Delta T = (12.0 \text{ m})(1.2 \times 10^{-5} \text{ (C}^\circ)^{-1})(42.0 \text{ C}^\circ) = 0.0060 \text{ m} = 6.0 \text{ mm}$.

(b) stress $= -Y\alpha \Delta T = -(2.0 \times 10^{11} \text{ Pa})(1.2 \times 10^{-5} \text{ (C}^\circ)^{-1})(42.0 \text{ C}^\circ) = -1.0 \times 10^8$ Pa. The minus sign means the stress is compressive.

EVALUATE: Commonly occurring temperature changes result in very small fractional changes in length but very large stresses if the length change is prevented from occurring.

17.22. **IDENTIFY:** Apply stress $= F/A = -Y\alpha \Delta T$ and solve for F.

SET UP: For brass, $Y = 0.9 \times 10^{11}$ Pa and $\alpha = 2.0 \times 10^{-5} \text{ (C}^\circ)^{-1}$.

EXECUTE: $F = -Y\alpha \Delta T A = -(0.9 \times 10^{11} \text{ Pa})(2.0 \times 10^{-5}(\text{C}^\circ)^{-1})(-110 \text{ C}^\circ)(2.01 \times 10^{-4} \text{ m}^2) = 4.0 \times 10^4$ N

EVALUATE: A large force is required. ΔT is negative and a positive tensile force is required.

17.23. **IDENTIFY:** We are dealing with the thermal expansion of two rods.

SET UP: Use $\Delta L = \alpha L_0 \Delta T$. From Table 17.1 we get the coefficients of linear expansion for aluminum and Invar. We know that $\Delta L_{\text{Al}} = 2\Delta L_{\text{Invar}}$ and $\Delta T_{\text{Al}} = \dfrac{1}{3}\Delta T_{\text{Invar}}$. We want $L_{\text{Al}}/L_{\text{Invar}}$.

EXECUTE: Apply $\Delta L = \alpha L_0 \Delta T$ to each bar and take the ratio of the length changes, giving

$\dfrac{\Delta L_{\text{Al}}}{\Delta L_{\text{Invar}}} = \dfrac{\alpha_{\text{Al}} L_{\text{Al}} \Delta T_{\text{Al}}}{\alpha_{\text{Invar}} L_{\text{Invar}} \Delta T_{\text{Invar}}}$. Solving for the length ratio gives $\dfrac{L_{\text{Al}}}{L_{\text{Invar}}} = \dfrac{\alpha_{\text{Invar}} \Delta L_{\text{Al}} \Delta T_{\text{Invar}}}{\alpha_{\text{Al}} \Delta L_{\text{Invar}} \Delta T_{\text{Al}}} = $

$\left(\dfrac{0.09}{2.4}\right)\left(\dfrac{\Delta L_{\text{Al}}}{\Delta L_{\text{Invar}}}\right)\left(\dfrac{3\Delta T_{\text{Al}}}{\Delta T_{\text{Al}}}\right) = 0.225,$ which we round to 0.2. This also gives $\dfrac{L_{\text{Invar}}}{L_{\text{Al}}} = \dfrac{1}{0.225} = 4.$

EVALUATE: The coefficient of linear expansion for Invar is much less than that of aluminum, so the Invar rod needs to be much longer than the aluminum rod to expand half as much with only 1/3 the temperature change.

17.24. **IDENTIFY:** The heat required is $Q = mc\Delta T$. $P = 200$ W $= 200$ J/s, which is energy divided by time.

SET UP: For water, $c = 4.19 \times 10^3$ J/kg·K.

EXECUTE: (a) $Q = mc\Delta T = (0.320 \text{ kg})(4.19 \times 10^3 \text{ J/kg·K})(60.0 \text{ C°}) = 8.04 \times 10^4$ J

(b) $t = \dfrac{8.04 \times 10^4 \text{ J}}{200.0 \text{ J/s}} = 402 \text{ s} = 6.7$ min

EVALUATE: 0.320 kg of water has volume 0.320 L. The time we calculated in part (b) is consistent with our everyday experience.

17.25. **IDENTIFY and SET UP:** Apply $Q = mc\,\Delta T$ to the kettle and water.

EXECUTE: kettle
$Q = mc\Delta T$, $c = 910$ J/kg·K (from Table 17.3)

$Q = (1.10 \text{ kg})(910 \text{ J/kg·K})(85.0°\text{C} - 20.0°\text{C}) = 6.5065 \times 10^4$ J

water
$Q = mc\Delta T$, $c = 4190$ J/kg·K (from Table 17.3)

$Q = (1.80 \text{ kg})(4190 \text{ J/kg·K})(85.0°\text{C} - 20.0°\text{C}) = 4.902 \times 10^5$ J

Total $Q = 6.5065 \times 10^4$ J $+ 4.902 \times 10^5$ J $= 5.55 \times 10^5$ J.

EVALUATE: Water has a much larger specific heat capacity than aluminum, so most of the heat goes into raising the temperature of the water.

17.26. **IDENTIFY and SET UP:** Use $Q = mc\,\Delta T$.

EXECUTE: (a) $Q = mc\Delta T$

$m = \frac{1}{2}(1.3 \times 10^{-3} \text{ kg}) = 0.65 \times 10^{-3}$ kg

$Q = (0.65 \times 10^{-3} \text{ kg})(1020 \text{ J/kg·K})\left(37°\text{C} - (-20°\text{C})\right) = 38$ J.

(b) 20 breaths/min (60 min/1 h) $= 1200$ breaths/h

So $Q = (1200)(38 \text{ J}) = 4.6 \times 10^4$ J.

EVALUATE: The heat loss rate is $Q/t = 13$ W.

17.27. **IDENTIFY:** Apply $Q = mc\Delta T$ to find the heat that would raise the temperature of the student's body 7 C°.

SET UP: 1 W $= 1$ J/s

EXECUTE: Find Q to raise the body temperature from $37°$C to $44°$C.

$Q = mc\Delta T = (70 \text{ kg})(3480 \text{ J/kg·K})(7 \text{ C°}) = 1.7 \times 10^6$ J.

$t = \dfrac{1.7 \times 10^6 \text{ J}}{1200 \text{ J/s}} = 1400 \text{ s} = 23 \text{ min.}$

EVALUATE: Heat removal mechanisms are essential to the well-being of a person.

17.28.   **IDENTIFY:** The heat input increases the temperature of 2.5 gal/min of water from 10°C to 49°C.

SET UP: 1.00 L of water has a mass of 1.00 kg, so
9.46 L/min $= (9.46 \text{ L/min})(1.00 \text{ kg/L})(1 \text{ min}/60 \text{ s}) = 0.158 \text{ kg/s}$. For water, $c = 4190 \text{ J/kg} \cdot \text{C}°$.

EXECUTE: $Q = mc\Delta T$ so $H = (Q/t) = (m/t)c\,\Delta T$. Putting in the numbers gives

$H = (0.158 \text{ kg/s})(4190 \text{ J/kg} \cdot \text{C}°)(49°\text{C} - 10°\text{C}) = 2.6 \times 10^4 \text{ W} = 26 \text{ kW}.$

EVALUATE: The power requirement is large, the equivalent of 260 100-watt light bulbs, but this large power is needed only for short periods of time. The rest of the time, the unit uses no energy, unlike a conventional water heater, which continues to replace lost heat even when hot water is not needed.

17.29.   **IDENTIFY:** Apply $Q = mc\Delta T$. $m = w/g$.

SET UP: The temperature change is $\Delta T = 18.0 \text{ K}$.

EXECUTE: $c = \dfrac{Q}{m\Delta T} = \dfrac{gQ}{w\Delta T} = \dfrac{(9.80 \text{ m/s}^2)(1.25 \times 10^4 \text{ J})}{(28.4 \text{ N})(18.0 \text{ K})} = 240 \text{ J/kg} \cdot \text{K}.$

EVALUATE: The value for c is similar to that for silver in Table 17.3, so it is a reasonable result.

17.30.   **IDENTIFY:** The work done by the brakes equals the initial kinetic energy of the train. Use the volume of the air to calculate its mass. Use $Q = mc\Delta T$ applied to the air to calculate ΔT for the air.

SET UP: $K = \frac{1}{2}mv^2$. $m = \rho V$.

EXECUTE: The initial kinetic energy of the train is $K = \frac{1}{2}(25{,}000 \text{ kg})(15.5 \text{ m/s})^2 = 3.00 \times 10^6 \text{ J}.$

Therefore, Q for the air is $3.00 \times 10^6 \text{ J}$.

$m = \rho V = (1.20 \text{ kg/m}^3)(65.0 \text{ m})(20.0 \text{ m})(12.0 \text{ m}) = 1.87 \times 10^4 \text{ kg}.$ $Q = mc\Delta T$ gives

$\Delta T = \dfrac{Q}{mc} = \dfrac{3.00 \times 10^6 \text{ J}}{(1.87 \times 10^4 \text{ kg})(1020 \text{ J/kg} \cdot \text{K})} = 0.157 \text{ C}°.$

EVALUATE: The mass of air in the station is comparable to the mass of the train and the temperature rise is small.

17.31.   **IDENTIFY** and **SET UP:** Set the change in gravitational potential energy equal to the quantity of heat added to the water.

EXECUTE: The change in mechanical energy equals the decrease in gravitational potential energy, $\Delta U = -mgh$; $|\Delta U| = mgh$. $Q = |\Delta U| = mgh$ implies $mc\Delta T = mgh$

$\Delta T = gh/c = (9.80 \text{ m/s}^2)(225 \text{ m})/(4190 \text{ J/kg} \cdot \text{K}) = 0.526 \text{ K} = 0.526 \text{ C}°$

EVALUATE: Note that the answer is independent of the mass of the object. Note also the small change in temperature that corresponds to this large change in height!

17.32.   **IDENTIFY:** Set the energy delivered to the nail equal to $Q = mc\Delta T$ for the nail and solve for ΔT.

SET UP: For aluminum, $c = 0.91 \times 10^3 \text{ J/kg} \cdot \text{K}$. $K = \frac{1}{2}mv^2$.

EXECUTE: The kinetic energy of the hammer before it strikes the nail is

$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.80 \text{ kg})(7.80 \text{ m/s})^2 = 54.8 \text{ J}$. Each strike of the hammer transfers $0.60(54.8 \text{ J}) = 32.9 \text{ J}$, and with 10 strikes $Q = 329 \text{ J}$. $Q = mc\Delta T$ and

$\Delta T = \dfrac{Q}{mc} = \dfrac{329 \text{ J}}{(8.00 \times 10^{-3} \text{ kg})(0.91 \times 10^3 \text{ J/kg} \cdot \text{K})} = 45.2 \text{ C}°.$

EVALUATE: This agrees with our experience that hammered nails get noticeably warmer.

17.33. **IDENTIFY:** Some of the kinetic energy of the bullet is transformed through friction into heat, which raises the temperature of the water in the tank.

SET UP: Set the loss of kinetic energy of the bullet equal to the heat energy Q transferred to the water. $Q = mc\Delta T$. From Table 17.3, the specific heat of water is 4.19×10^3 J/kg·C°.

EXECUTE: The kinetic energy lost by the bullet is

$K_i - K_f = \frac{1}{2} m(v_i^2 - v_f^2) = \frac{1}{2}(15.0 \times 10^{-3} \text{ kg})[(865 \text{ m/s})^2 - (534 \text{ m/s})^2] = 3.47 \times 10^3$ J, so for the water

$Q = 3.47 \times 10^3$ J. $Q = mc\Delta T$ gives $\Delta T = \dfrac{Q}{mc} = \dfrac{3.47 \times 10^3 \text{ J}}{(13.5 \text{ kg})(4.19 \times 10^3 \text{ J/kg·C°})} = 0.0613$ C°.

EVALUATE: The heat energy required to change the temperature of ordinary-size objects is very large compared to the typical kinetic energies of moving objects.

17.34. **IDENTIFY:** The amount of heat lost by the boiling water is equal to the amount of heat gained by the water in the beaker.

SET UP: Calculate Q for each mass of water and set their algebraic sum equal to zero. Let the water you add have mass m. The 750 g of cold water has a temperature change of $+65$ C° and a heat flow Q_c. The mass m of water has a temperature change of -25 C° and a heat flow Q_h. $Q = mc\,\Delta T$.

EXECUTE: $Q_c = mc\,\Delta T = (0.750 \text{ kg})(4190 \text{ J/kg·C°})(65 \text{ C°}) = 2.043 \times 10^5$ J

$Q_h = mc\,\Delta T = m(4190 \text{ J/kg·C°})(-25 \text{ C°}) = -(1.048 \times 10^5 \text{ J/kg})m$

$Q_h + Q_c = 0$ so 2.043×10^5 J $+ (-1.048 \times 10^5 \text{ J/kg})m = 0$ and $m = 1.95$ kg $= 1950$ g.

EVALUATE: The amount of water we need to add (1950 g) is considerably greater than the water already in the beaker (750 g). This is reasonable because we want the final temperature to be closer to the 100°C temperature of the boiling water than to the original 10.0°C temperature of the original water.

17.35. **IDENTIFY and SET UP:** Heat comes out of the metal and into the water. The final temperature is in the range $0 < T < 100°C$, so there are no phase changes. $Q_{\text{system}} = 0$.

(a) EXECUTE: $Q_{\text{water}} + Q_{\text{metal}} = 0$

$m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} + m_{\text{metal}} c_{\text{metal}} \Delta T_{\text{metal}} = 0$

$(1.00 \text{ kg})(4190 \text{ J/kg·K})(2.0 \text{ C°}) + (0.500 \text{ kg})(c_{\text{metal}})(-78.0 \text{ C°}) = 0$

$c_{\text{metal}} = 215$ J/kg·K

(b) EVALUATE: Water has a larger specific heat capacity so stores more heat per degree of temperature change.

(c) If some heat went into the styrofoam then Q_{metal} should actually be larger than in part (a), so the true c_{metal} is larger than we calculated; the value we calculated would be smaller than the true value.

17.36. **IDENTIFY:** The heat that comes out of the person goes into the ice-water bath and causes some of the ice to melt.

SET UP: Normal body temperature is 98.6°F = 37.0°C, so for the person $\Delta T = -5$ C°. The ice-water bath stays at 0°C. A mass m of ice melts and $Q_{\text{ice}} = mL_f$. From Table 17.4, for water

$L_f = 334 \times 10^3$ J/kg.

EXECUTE: $Q_{\text{person}} = mc\Delta T = (70.0 \text{ kg})(3480 \text{ J/kg·C°})(-5.0 \text{ C°}) = -1.22 \times 10^6$ J. Therefore, the amount

of heat that goes into the ice is 1.22×10^6 J. $m_{\text{ice}} L_f = 1.22 \times 10^6$ J and $m_{\text{ice}} = \dfrac{1.22 \times 10^6 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 3.7$ kg.

EVALUATE: If less ice than this is used, all the ice melts and the temperature of the water in the bath rises above 0°C.

17.37.   **IDENTIFY:**   The amount of heat lost by the iron is equal to the amount of heat gained by the water. The water must first be heated to 100°C and then vaporized.

SET UP: The relevant equations are $Q = mc\Delta T$ and $Q = L_v m$. The specific heat of iron is $c_{iron} = 0.47 \times 10^3$ J/(kg·K), the specific heat of water is $c_{water} = 4.19 \times 10^3$ J/(kg·K), and the heat of vaporization of water is $L_v = 2256 \times 10^3$ J/kg.

EXECUTE: The iron cools: $Q_{iron} = m_i c_i \Delta T_i$.

The water warms and vaporizes: $Q_{water} = c_w m_w \Delta T_w + m_w L_{v_w} = m_w (c_w \Delta T_w + L_{v_w})$.

Assume that all of the heat lost by the iron is gained by the water so that $Q_{water} = -Q_{iron}$. Equating the respective expressions for each Q and solving for m_w we obtain

$$m_w = \frac{-m_i c_i \Delta T_i}{c_w \Delta T_w + L_{v_w}} = \frac{-(1.20 \text{ kg})(0.47 \times 10^3 \text{ J/kg·K})(120.0°C - 650.0°C)}{(4.19 \times 10^3 \text{ J/kg·K})(100.0°C - 15.0°C) + 2256 \times 10^3 \text{ J/kg}} = 0.114 \text{ kg}.$$

EVALUATE: Note that only a relatively small amount of water is required to cause a very large temperature change in the iron. This is due to the high heat of vaporization and specific heat of water, and the relatively low specific heat capacity of iron.

17.38.   **IDENTIFY:**   The initial temperature of the ice and water mixture is 0.0°C. Assume all the ice melts. We will know that assumption is incorrect if the final temperature we calculate is less than 0.0°C. The net Q for the system (can, water, ice and lead) is zero.

SET UP: For copper, $c_c = 390$ J/kg·K. For lead, $c_l = 130$ J/kg·K. For water, $c_w = 4.19 \times 10^3$ J/kg·K and $L_f = 3.34 \times 10^5$ J/kg.

EXECUTE: For the copper can, $Q_c = m_c c_c \Delta T_c = (0.100 \text{ kg})(390 \text{ J/kg·K})(T - 0.0°C) = (39.0 \text{ J/K})T$.

For the water, $Q_w = m_w c_w \Delta T_w = (0.160 \text{ kg})(4.19 \times 10^3 \text{ J/kg·K})(T - 0.0°C) = (670.4 \text{ J/K})T$.

For the ice, $Q_i = m_i L_f + m_i c_w \Delta T_w$

$Q_i = (0.018 \text{ kg})(3.34 \times 10^5 \text{ J/kg}) + (0.018 \text{ kg})(4.19 \times 10^3 \text{ J/kg·K})(T - 0.0°C) = 6012 \text{ J} + (75.4 \text{ J/K})T$

For the lead, $Q_l = m_l c_l \Delta T_l = (0.750 \text{ kg})(130 \text{ J/kg·K})(T - 255°C) = (97.5 \text{ J/K})T - 2.486 \times 10^4 \text{ J}$

$\Sigma Q = 0$ gives $(39.0 \text{ J/K})T + (670.4 \text{ J/K})T + 6012 \text{ J} + (75.4 \text{ J/K})T + (97.5 \text{ J/K})T - 2.486 \times 10^4 \text{ J} = 0$.

$$T = \frac{1.885 \times 10^4 \text{ J}}{882.3 \text{ J/K}} = 21.4°C.$$

EVALUATE: $T > 0.0°C$, which confirms that all the ice melts.

17.39.   **IDENTIFY:**   The heat lost by the cooling copper is absorbed by the water and the pot, which increases their temperatures.

SET UP: For copper, $c_c = 390$ J/kg·K. For iron, $c_i = 470$ J/kg·K. For water, $c_w = 4.19 \times 10^3$ J/kg·K.

EXECUTE: For the copper pot,
$Q_c = m_c c_c \, \Delta T_c = (0.500 \text{ kg})(390 \text{ J/kg·K})(T - 20.0°C) = (195 \text{ J/K})T - 3900 \text{ J}$. For the block of iron,
$Q_i = m_i c_i \, \Delta T_i = (0.250 \text{ kg})(470 \text{ J/kg·K})(T - 85.0°C) = (117.5 \text{ J/K})T - 9988 \text{ J}$. For the water,
$Q_w = m_w c_w \, \Delta T_w = (0.170 \text{ kg})(4190 \text{ J/kg·K})(T - 20.0°C) = (712.3 \text{ J/K})T - 1.425 \times 10^4 \text{ J}$. $\Sigma Q = 0$ gives
$(195 \text{ J/K})T - 3900 \text{ J} + (117.5 \text{ J/K})T - 9988 \text{ J} + (712.3 \text{ J/K})T - 1.425 \times 10^4 \text{ J}$.

$$T = \frac{2.814 \times 10^4 \text{ J}}{1025 \text{ J/K}} = 27.5°C.$$

EVALUATE: The basic principle behind this problem is conservation of energy: no energy is lost; it is only transferred.

17.40.   IDENTIFY: By energy conservation, the heat lost by the water is gained by the ice. This heat must first increase the temperature of the ice from −40.0°C to the melting point of 0.00°C, then melt the ice, and finally increase its temperature to 28.0°C. The target variable is the mass of the water m.

SET UP: $Q_{ice} = m_{ice}c_{ice}\Delta T_{ice} + m_{ice}L_f + m_{ice}c_w\Delta T_{melted\ ice}$ and $Q_{water} = mc_w\Delta T_w$.

EXECUTE: Using $Q_{ice} = m_{ice}c_{ice}\Delta T_{ice} + m_{ice}L_f + m_{ice}c_w\Delta T_{melted\ ice}$,   with the values given in the table in the text, we have $Q_{ice} = (0.200 \text{ kg})[2100 \text{ J/(kg} \cdot \text{C°)}](40.0\text{C°}) + (0.200 \text{ kg})(3.34\times10^5 \text{ J/kg})$

$+ (0.200 \text{ kg})[4190 \text{ J/(kg} \cdot \text{C°)}](28.0\text{C°}) = 1.071\times10^5 \text{ J}.$

$Q_{water} = mc_w\Delta T_w = m[4190 \text{ J/(kg} \cdot \text{C°)}](28.0°\text{C} - 80.0°\text{C}) = -(217{,}880 \text{ J/kg})m.$ $Q_{ice} + Q_{water} = 0$ gives

$1.071\times10^5 \text{ J} = (217{,}880 \text{ J/kg})m.$ $m = 0.491 \text{ kg}.$

EVALUATE: There is about twice as much water as ice because the water must provide the heat not only to melt the ice but also to increase its temperature.

17.41.   IDENTIFY: By energy conservation, the heat lost by the copper is gained by the ice. This heat must first increase the temperature of the ice from −20.0°C to the melting point of 0.00°C, then melt some of the ice. At the final thermal equilibrium state, there is ice and water, so the temperature must be 0.00°C. The target variable is the initial temperature of the copper.

SET UP: For temperature changes, $Q = mc\Delta T$ and for a phase change from solid to liquid $Q = mL_F$.

EXECUTE: For the ice,

$Q_{ice} = (2.00 \text{ kg})[2100 \text{ J/(kg} \cdot \text{C°)}](20.0\text{C°}) + (0.80 \text{ kg})(3.34\times10^5 \text{ J/kg}) = 3.512\times10^5 \text{ J}.$ For the copper, using the specific heat from the table in the text gives

$Q_{copper} = (6.00 \text{ kg})[390 \text{ J/(kg} \cdot \text{C°)}](0°\text{C} - T) = -(2.34\times10^3 \text{ J/C°})T.$ Setting the sum of the two heats equal to zero gives $3.512\times10^5 \text{ J} = (2.34\times10^3 \text{ J/C°})T,$ which gives $T = 150°\text{C}.$

EVALUATE: Since the copper has a smaller specific heat than that of ice, it must have been quite hot initially to provide the amount of heat needed.

17.42.   IDENTIFY: For a temperature change $Q = mc\Delta T$ and for the liquid to solid phase change $Q = -mL_f$.

SET UP: For water, $c = 4.19\times10^3 \text{ J/kg} \cdot \text{K}$ and $L_f = 3.34\times10^5 \text{ J/kg}.$

EXECUTE:

$Q = mc\Delta T - mL_f = (0.290 \text{ kg})[(4.19\times10^3 \text{ J/kg} \cdot \text{K})(-18.0 \text{ C°}) - 3.34\times10^5 \text{ J/kg}] = -1.187\times10^5 \text{ J},$ which rounds to $-1.19 \times 10^5 \text{ J}.$

The minus sign says $1.19\times10^5 \text{ J}$ must be removed from the water.

$(1.187\times10^5 \text{ J})\left(\dfrac{1 \text{ cal}}{4.186 \text{ J}}\right) = 2.84\times10^4 \text{ cal} = 28.4 \text{ kcal}.$

$(1.187\times10^5 \text{ J})\left(\dfrac{1 \text{ Btu}}{1055 \text{ J}}\right) = 113 \text{ Btu}.$

EVALUATE: $Q < 0$ when heat comes out of an object. The equation $Q = mc\Delta T$ puts in the correct sign automatically, from the sign of $\Delta T = T_f - T_i$.   But in $Q = \pm mL$ we must select the correct sign.

17.43.   IDENTIFY and SET UP: Use $Q = mc\ \Delta T$ for the temperature changes and $Q = mL$ for the phase changes.

EXECUTE: Heat must be added to do the following:

ice at −10.0°C → ice at 0°C

$Q_{ice} = mc_{ice}\Delta T = (18.0\times10^{-3} \text{ kg})(2100 \text{ J/kg} \cdot \text{K})(0°\text{C} - (-10.0°\text{C})) = 378 \text{ J}$

phase transition ice (0°C) → liquid water (0°C)(melting)

$Q_{melt} = +mL_f = (18.0\times10^{-3} \text{ kg})(334\times10^3 \text{ J/kg}) = 6.012\times10^3 \text{ J}$

water at 0°C (from melted ice) → water at 100°C

$Q_{water} = mc_{water}\Delta T = (18.0\times10^{-3} \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(100°\text{C} - 0°\text{C}) = 7.542\times10^{3} \text{ J}$

phase transition water $(100°\text{C}) \rightarrow$ steam $(100°\text{C})$(boiling)

$Q_{boil} = +mL_v = (18.0\times10^{-3} \text{ kg})(2256\times10^{3} \text{ J/kg}) = 4.0608\times10^{4} \text{ J}$

The total Q is $Q = 378 \text{ J} + 6.012\times10^{3} \text{ J} + 7.542\times10^{3} \text{ J} + 4.068\times10^{4} \text{ J} = 5.45\times10^{4} \text{ J}$

$(5.45\times10^{4} \text{ J})(1 \text{ cal}/4.186 \text{ J}) = 1.30\times10^{4} \text{ cal}$

$(5.45\times10^{4} \text{ J})(1 \text{ Btu}/1055 \text{ J}) = 51.7 \text{ Btu.}$

EVALUATE: Q is positive and heat must be added to the material. Note that more heat is needed for the liquid to gas phase change than for the temperature changes.

17.44. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change and $Q = +mL_f$ for the solid to liquid phase transition. The ice starts to melt when its temperature reaches $0.0°\text{C}$. The system stays at $0.00°\text{C}$ until all the ice has melted.

SET UP: For ice, $c = 2.10\times10^{3} \text{ J/kg}\cdot\text{K}$. For water, $L_f = 3.34\times10^{5} \text{ J/kg}$.

EXECUTE: **(a)** Q to raise the temperature of ice to $0.00°\text{C}$:

$Q = mc\Delta T = (0.550 \text{ kg})(2.10\times10^{3} \text{ J/kg}\cdot\text{K})(15.0 \text{ C°}) = 1.73\times10^{4} \text{ J.}$ $t = \dfrac{1.73\times10^{4} \text{ J}}{800.0 \text{ J/min}} = 21.7 \text{ min.}$

(b) To melt all the ice requires $Q = mL_f = (0.550 \text{ kg})(3.34\times10^{5} \text{ J/kg}) = 1.84\times10^{5} \text{ J.}$

$t = \dfrac{1.84\times10^{5} \text{ J}}{800.0 \text{ J/min}} = 230 \text{ min.}$ The total time after the start of the heating is 252 min.

(c) A graph of T versus t is sketched in Figure 17.44.

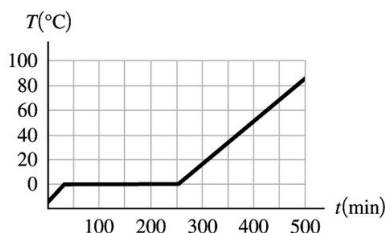**EVALUATE:** It takes much longer for the ice to melt than it takes the ice to reach the melting point.

Figure 17.44

17.45. **IDENTIFY and SET UP:** The heat that must be added to a lead bullet of mass m to melt it is $Q = mc\Delta T + mL_f$ ($mc\Delta T$ is the heat required to raise the temperature from $25°\text{C}$ to the melting point of $327.3°\text{C}$; mL_f is the heat required to make the solid $\rightarrow$ liquid phase change.) The kinetic energy of the bullet if its speed is v is $K = \frac{1}{2}mv^2$.

EXECUTE: $K = Q$ says $\frac{1}{2}mv^2 = mc\Delta T + mL_f$

$v = \sqrt{2(c\Delta T + L_f)}$

$v = \sqrt{2[(130 \text{ J/kg}\cdot\text{K})(327.3°\text{C} - 25°\text{C}) + 24.5\times10^{3} \text{ J/kg}]} = 357 \text{ m/s}$

EVALUATE: This is a typical speed for a rifle bullet. A bullet fired into a block of wood does partially melt, but in practice not all of the initial kinetic energy is converted to heat that remains in the bullet.

17.46. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the vapor $\rightarrow$ liquid phase transition, $Q = -mL_v$.

SET UP: For water, $L_v = 2.256\times10^{6} \text{ J/kg}$ and $c = 4.19\times10^{3} \text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $Q = +m(-L_v + c\Delta T)$

$Q = +(25.0 \times 10^{-3}$ kg$)(-2.256 \times 10^6$ J/kg $+ [4.19 \times 10^3$ J/kg $\cdot$ K$][-66.0$ C$^\circ]) = -6.33 \times 10^4$ J

(b) $Q = mc\Delta T = (25.0 \times 10^{-3}$ kg$)(4.19 \times 10^3$ J/kg $\cdot$ K$)(-66.0$ C$^\circ) = -6.91 \times 10^3$ J.

(c) The total heat released by the water that starts as steam is nearly a factor of ten larger than the heat released by water that starts at 100°C. Steam burns are much more severe than hot-water burns.

EVALUATE: For a given amount of material, the heat for a phase change is typically much more than the heat for a temperature change.

17.47. **IDENTIFY:** Use $Q = Mc\Delta T$ to find Q for a temperature rise from 34.0°C to 40.0°C. Set this equal to $Q = mL_v$ and solve for m, where m is the mass of water the camel would have to drink.

SET UP: $c = 3480$ J/kg $\cdot$ K and $L_v = 2.42 \times 10^6$ J/kg. For water, 1.00 kg has a volume 1.00 L. $M = 400$ kg is the mass of the camel.

EXECUTE: The mass of water that the camel saves is
$$m = \frac{Mc\Delta T}{L_v} = \frac{(400 \text{ kg})(3480 \text{ J/kg} \cdot \text{K})(6.0 \text{ K})}{(2.42 \times 10^6 \text{ J/kg})} = 3.45 \text{ kg} \text{ which is a volume of 3.45 L.}$$

EVALUATE: This is nearly a gallon of water, so it is an appreciable savings.

17.48. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the liquid $\rightarrow$ vapor phase change, $Q = +mL_v$.

SET UP: The density of water is 1000 kg/m³.

EXECUTE: **(a)** The heat that goes into mass m of water to evaporate it is $Q = +mL_v$. The heat flow for the man is $Q = m_{man}c\Delta T$, where $\Delta T = -1.00$ C°. $\Sigma Q = 0$ so $mL_v + m_{man}c\Delta T = 0$ and
$$m = -\frac{m_{man}c\Delta T}{L_v} = -\frac{(70.0 \text{ kg})(3480 \text{ J/kg} \cdot \text{K})(-1.00 \text{ C}^\circ)}{2.42 \times 10^6 \text{ J/kg}} = 0.101 \text{ kg} = 101 \text{ g.}$$

(b) $V = \dfrac{m}{\rho} = \dfrac{0.101 \text{ kg}}{1000 \text{ kg/m}^3} = 1.01 \times 10^{-4}$ m³ $= 101$ cm³. This is about 35% of the volume of a soft-drink can.

EVALUATE: Fluid loss by evaporation from the skin can be significant.

17.49. **IDENTIFY:** The asteroid's kinetic energy is $K = \frac{1}{2}mv^2$. To boil the water, its temperature must be raised to 100.0°C and the heat needed for the phase change must be added to the water.

SET UP: For water, $c = 4190$ J/kg $\cdot$ K and $L_v = 2256 \times 10^3$ J/kg.

EXECUTE: $K = \frac{1}{2}(2.60 \times 10^{15}$ kg$)(32.0 \times 10^3$ m/s$)^2 = 1.33 \times 10^{24}$ J. $Q = mc\Delta T + mL_v$.
$$m = \frac{Q}{c\Delta T + L_v} = \frac{1.33 \times 10^{22} \text{ J}}{(4190 \text{ J/kg} \cdot \text{K})(90.0 \text{ K}) + 2256 \times 10^3 \text{ J/kg}} = 5.05 \times 10^{15} \text{ kg.}$$

EVALUATE: The mass of water boiled is 2.5 times the mass of water in Lake Superior.

17.50. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change. The net Q for the system (sample, can and water) is zero.

SET UP: For water, $c_w = 4.19 \times 10^3$ J/kg $\cdot$ K. For copper, $c_c = 390$ J/kg $\cdot$ K.

EXECUTE: For the water, $Q_w = m_w c_w \Delta T_w = (0.200$ kg$)(4.19 \times 10^3$ J/kg $\cdot$ K$)(7.1$ C$^\circ) = 5.95 \times 10^3$ J.

For the copper can, $Q_c = m_c c_c \Delta T_c = (0.150$ kg$)(390$ J/kg $\cdot$ K$)(7.1$ C$^\circ) = 415$ J.

For the sample, $Q_s = m_s c_s \Delta T_s = (0.085$ kg$)c_s(-73.9$ C$^\circ)$.

$\Sigma Q = 0$ gives $(0.085$ kg$)(-73.9$ C$^\circ)c_s + 415$ J $+ 5.95 \times 10^3$ J $= 0$. $c_s = 1.01 \times 10^3$ J/kg $\cdot$ K.

EVALUATE: Heat comes out of the sample and goes into the water and the can. The value of c_s we calculated is consistent with the values in Table 17.3.

17.51. **IDENTIFY** and **SET UP:** Heat flows out of the water and into the ice. The net heat flow for the system is zero. The ice warms to 0°C, melts, and then the water from the melted ice warms from 0°C to the final temperature.

EXECUTE: $Q_{\text{system}} = 0$; calculate Q for each component of the system: (Beaker has small mass says that $Q = mc\Delta T$ for beaker can be neglected.)

0.250 kg of water: cools from 75.0°C to 40.0°C

$Q_{\text{water}} = mc\Delta T = (0.250 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(40.0°\text{C} - 75.0°\text{C}) = -3.666 \times 10^4 \text{ J}.$

ice: warms to 0°C; melts; water from melted ice warms to 40.0°C

$Q_{\text{ice}} = mc_{\text{ice}}\Delta T + mL_{\text{f}} + mc_{\text{water}}\Delta T.$

$Q_{\text{ice}} = m[(2100 \text{ J/kg} \cdot \text{K})(0°\text{C} - (-20.0°\text{C})) + 334 \times 10^3 \text{ J/kg} + (4190 \text{ J/kg} \cdot \text{K})(40.0°\text{C} - 0°\text{C})].$

$Q_{\text{ice}} = (5.436 \times 10^5 \text{ J/kg})m.$ $Q_{\text{system}} = 0$ says $Q_{\text{water}} + Q_{\text{ice}} = 0.$ $-3.666 \times 10^4 \text{ J} + (5.436 \times 10^5 \text{ J/kg})m = 0.$

$m = \dfrac{3.666 \times 10^4 \text{ J}}{5.436 \times 10^5 \text{ J/kg}} = 0.0674 \text{ kg}.$

EVALUATE: Since the final temperature is 40.0°C we know that all the ice melts and the final system is all liquid water. The mass of ice added is much less than the mass of the 75°C water; the ice requires a large heat input for the phase change.

17.52. **IDENTIFY** and **SET UP:** Large block of ice implies that ice is left, so $T_2 = 0°\text{C}$ (final temperature). Heat comes out of the ingot and into the ice. The net heat flow is zero. The ingot has a temperature change and the ice has a phase change.

EXECUTE: $Q_{\text{system}} = 0$; calculate Q for each component of the system:

ingot

$Q_{\text{ingot}} = mc\Delta T = (4.00 \text{ kg})(234 \text{ J/kg} \cdot \text{K})(0°\text{C} - 750°\text{C}) = -7.02 \times 10^5 \text{ J}$

ice

$Q_{\text{ice}} = +mL_{\text{f}}$, where m is the mass of the ice that changes phase (melts)

$Q_{\text{system}} = 0$ says $Q_{\text{ingot}} + Q_{\text{ice}} = 0$

$-7.02 \times 10^5 \text{ J} + m(334 \times 10^3 \text{ J/kg}) = 0$

$m = \dfrac{7.02 \times 10^5 \text{ J}}{334 \times 10^3 \text{ J/kg}} = 2.10 \text{ kg}$

EVALUATE: The liquid produced by the phase change remains at 0°C since it is in contact with ice.

17.53. **IDENTIFY:** We mix liquids at different temperatures, so this is a problem in calorimetry.

SET UP: The unknown liquid cools down from 30.0°C to 14.0°C. The ice all melts and the resulting water increases from 0.0°C to 14.0°C. The heat gained by the ice all comes from the unknown liquid, so the net heat change for the mixture is zero. The ice goes through two changes: melting at 0.0°C followed by a temperature increase to 14.0°C. We use $Q = mc\Delta T$ and want to find the specific heat c of the unknown liquid.

EXECUTE: $Q_{\text{ice}} = Q_{\text{melt}} + Q_{\text{increase temp}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\Delta T_{\text{water}}$ and $Q_{\text{unknown}} = mc\Delta T_{\text{unknown}}$. Using $Q_{\text{ice}} + Q_{\text{unknown}} = 0$ gives $m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\Delta T_{\text{water}} + mc\Delta T_{\text{unknown}} = 0.$ Using the given masses and temperatures as well as $L_{\text{f}} = 334 \times 10^3 \text{ J/kg}$ and $c_{\text{water}} = 4190 \text{ J/kg} \cdot \text{K}$, we get $c = 2370 \text{ J/kg} \cdot \text{K}$.

EVALUATE: From Table 17.3 we see that this value is close to the specific heats of ethylene and glycol, so it is a reasonable result.

17.54. **IDENTIFY:** At steady state, the rate of heat flow is the same throughout both rods, as well as out of the boiling water and into the ice-water mixture. The heat that flows into the ice-water mixture goes only into melting ice since the temperature remains at 0.00°C.

SET UP: For steady state heat flow, $\dfrac{Q}{t} = \dfrac{kA\Delta T}{L}$. The heat to melt ice is $Q = mL_f$.

EXECUTE: **(a)** $\dfrac{Q}{t} = \dfrac{kA\Delta T}{L}$ is the same for both of the rods. Using the physical properties of brass and copper from the tables in the text, we have

$$\frac{[109.0 \text{ W/(m·K)}](100.0°C - T)}{0.300 \text{ m}} = \frac{[385.0 \text{ W/(m·K)}](T - 0.0°C)}{0.800 \text{ m}}.$$

Solving for T gives $T = 43.0°C$.

(b) The heat entering the ice-water mixture is

$$Q = \frac{kAt\Delta T}{L} = \frac{[109.0 \text{ W/(m·K)}](0.00500 \text{ m}^2)(300.0 \text{ s})(100.0°C - 43.0°C)}{0.300 \text{ m}}. \quad Q = 3.1065 \times 10^4 \text{ J. Then}$$

$Q = mL_f$ so $m = \dfrac{3.1065 \times 10^4 \text{ J}}{3.34 \times 10^5 \text{ J/kg}} = 0.0930 \text{ kg} = 93.0 \text{ g}.$

EVALUATE: The temperature of the interface between the two rods is between the two extremes (0°C and 100°C), but not midway between them.

17.55. **IDENTIFY:** A heat current flows through the two bars, so we are dealing with thermal conduction.

SET UP: Both bars have the same length and cross-sectional area, and we want to find the thermal conductivity of the unknown metal. The graph plots T versus T_H, so we need to relate these quantities to interpret the graph. For this we use $H = kA\dfrac{T_H - T_C}{L}$. At steady state, H is the same in both bars.

EXECUTE: For the copper bar $H = k_{Cu}A\dfrac{T_H - T}{L}$, and for the unknown bar $H = kA\dfrac{T - 0.0°C}{L}$.

Equating the heat currents and simplifying gives $k_{Cu}(T_H - T) = kT$. Solving for T gives $T = \dfrac{k_{Cu}}{k + k_{Cu}}T_H$,

so a graph of T versus T_H should be a straight line having slope $\dfrac{k_{Cu}}{k + k_{Cu}}$. Solving for k gives

$k = k_{Cu}\left(\dfrac{1}{\text{slope}} - 1\right)$. From Table 17.5, we have $k_{Cu} = 385$ W/mol·K, so

$k = (385 \text{ W/m·K})\left(\dfrac{1}{0.710} - 1\right) = 157 \text{ W/m·K}.$

EVALUATE: From Table 17.5, $k = 157$ W/m·K is between that of brass and aluminum, so our result is reasonable.

17.56. **IDENTIFY:** For a melting phase transition, $Q = mL_f$. The rate of heat conduction is $\dfrac{Q}{t} = \dfrac{kA(T_H - T_C)}{L}$.

SET UP: For water, $L_f = 3.34 \times 10^5$ J/kg.

EXECUTE: The heat conducted by the rod in 10.0 min is

$Q = mL_f = (8.50 \times 10^{-3} \text{ kg})(3.34 \times 10^5 \text{ J/kg}) = 2.84 \times 10^3 \text{ J}. \quad \dfrac{Q}{t} = \dfrac{2.84 \times 10^3 \text{ J}}{600 \text{ s}} = 4.73 \text{ W}.$

$k = \dfrac{(Q/t)L}{A(T_H - T_C)} = \dfrac{(4.73 \text{ W})(0.600 \text{ m})}{(1.25 \times 10^{-4} \text{ m}^2)(100 \text{ C}°)} = 227 \text{ W/m·K}.$

EVALUATE: The heat conducted by the rod is the heat that enters the ice and produces the phase change.

17.57.   **IDENTIFY** and **SET UP:**   Call the temperature at the interface between the wood and the styrofoam T. The heat current in each material is given by $H = kA(T_H − T_C)/L$.
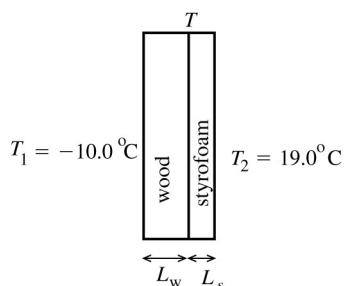
See Figure 17.57.

Heat current through the wood: $H_w = k_w A(T − T_1)L_w$

Heat current through the styrofoam: $H_s = k_s A(T_2 − T)/L_s$

Figure 17.57

In steady-state heat does not accumulate in either material. The same heat has to pass through both materials in succession, so $H_w = H_s$.

EXECUTE: **(a)** This implies $k_w A(T − T_1)/L_w = k_s A(T_2 − T)/L_s$

$k_w L_s (T − T_1) = k_s L_w (T_2 − T)$

$T = \dfrac{k_w L_s T_1 + k_s L_w T_2}{k_w L_s + k_s L_w} = \dfrac{−0.0176 \text{ W} \cdot °\text{C/K} + 0.01539 \text{ W} \cdot °\text{C/K}}{0.00257 \text{ W/K}} = −0.86°\text{C}.$

EVALUATE: The temperature at the junction is much closer in value to T_1 than to T_2. The styrofoam has a very small k, so a larger temperature gradient is required for than for wood to establish the same heat current.

(b) IDENTIFY and **SET UP:**   Heat flow per square meter is $\dfrac{H}{A} = k\left(\dfrac{T_H − T_C}{L}\right)$. We can calculate this either for the wood or for the styrofoam; the results must be the same.

EXECUTE: Wood: $\dfrac{H_w}{A} = k_w \dfrac{T − T_1}{L_w} = (0.080 \text{ W/m} \cdot \text{K})\dfrac{−0.86°\text{C} − (−10.0°\text{C})}{0.030 \text{ m}} = 24 \text{ W/m}^2.$

Styrofoam: $\dfrac{H_s}{A} = k_s \dfrac{T_2 − T}{L_s} = (0.027 \text{ W/m} \cdot \text{K})\dfrac{19.0°\text{C} − (−0.86°\text{C})}{0.022 \text{ m}} = 24 \text{ W/m}^2.$

EVALUATE: H must be the same for both materials and our numerical results show this. Both materials are good insulators and the heat flow is very small.

17.58.   **IDENTIFY:**   This problem is about heat flow, so we use $\dfrac{Q}{t} = \dfrac{kA(T_H − T_C)}{L}$.

SET UP: $T_H − T_C = 175°\text{C} − 35°\text{C}$. $1 \text{ K} = 1 \text{ C°}$, so there is no need to convert the temperatures to kelvins.

EXECUTE: **(a)** $\dfrac{Q}{t} = \dfrac{(0.040 \text{ W/m} \cdot \text{K})(1.40 \text{ m}^2)(175°\text{C} − 35°\text{C})}{4.0 \times 10^{-2} \text{ m}} = 196 \text{ W}.$

(b) The power input must be 196 W, to replace the heat conducted through the walls.

EVALUATE: The heat current is small because k is small for fiberglass.

17.59.   **IDENTIFY:**   We compare the thermal conductivity of several materials.

SET UP and EXECUTE: **(a)** Touch metal, glass, and wood. The result is that the metal is the coldest and the wood the warmest.

(b) From Table 17.5: $k_{Cu} = 385 \text{ W/m} \cdot \text{K}$, k_{wood} is between 0.12 W/m·K and 0.04 W/m·K, and $k_{glass} = 0.8$ W/m·K. Our ranking agrees with these thermal conductivities.

EVALUATE: The large variation in the thermal conductivity of wood is due to the density of the wood. The more air that is trapped within its fibers, the poorer it is at conducting heat because $k_{air} = 0.024$ W/m·K, making a very poor conductor of heat (but therefore an excellent insulator).

17.60. **IDENTIFY:** $\dfrac{Q}{t} = \dfrac{kA\Delta T}{L}$. Q/t is the same for both sections of the rod.

SET UP: For copper, $k_c = 385$ W/m·K. For steel, $k_s = 50.2$ W/m·K.

EXECUTE: **(a)** For the copper section, $\dfrac{Q}{t} = \dfrac{(385 \text{ W/m·K})(4.00\times10^{-4} \text{ m}^2)(100°\text{C}-65.0°\text{C})}{1.00 \text{ m}} = 5.39$ J/s.

(b) For the steel section, $L = \dfrac{kA\Delta T}{(Q/t)} = \dfrac{(50.2 \text{ W/m·K})(4.00\times10^{-4} \text{ m}^2)(65.0°\text{C}-0°\text{C})}{5.39 \text{ J/s}} = 0.242$ m.

EVALUATE: The thermal conductivity for steel is much less than that for copper, so for the same ΔT and A, a smaller L for steel would be needed for the same heat current as in copper.

17.61. **IDENTIFY and SET UP:** The heat conducted through the bottom of the pot goes into the water at 100°C to convert it to steam at 100°C. We can calculate the amount of heat flow from the mass of material that changes phase. Then use $H = kA(T_H - T_C)/L$ to calculate T_H, the temperature of the lower surface of the pan.

EXECUTE: $Q = mL_v = (0.390 \text{ kg})(2256\times10^3 \text{ J/kg}) = 8.798\times10^5$ J

$H = Q/t = 8.798\times10^5$ J/180 s $= 4.888\times10^3$ J/s

Then $H = kA(T_H - T_C)/L$ says that $T_H - T_C = \dfrac{HL}{kA} = \dfrac{(4.888\times10^3 \text{ J/s})(8.50\times10^{-3} \text{ m})}{(50.2 \text{ W/m·K})(0.150 \text{ m}^2)} = 5.52$ C°

$T_H = T_C + 5.52 \text{ C}° = 100°\text{C} + 5.52 \text{ C}° = 105.5°\text{C}$.

EVALUATE: The larger $T_H - T_C$ is the larger H is and the faster the water boils.

17.62. **IDENTIFY:** Apply $H = kA(T_H - T_C)/L$ and solve for A.

SET UP: The area of each circular end of a cylinder is related to the diameter D by $A = \pi R^2 = \pi(D/2)^2$. For steel, $k = 50.2$ W/m·K. The boiling water has $T = 100°\text{C}$, so $\Delta T = 300$ K.

EXECUTE: $\dfrac{Q}{t} = kA\dfrac{\Delta T}{L}$ and $190 \text{ J/s} = (50.2 \text{ W/m·K})A\left(\dfrac{300 \text{ K}}{0.500 \text{ m}}\right)$. This gives $A = 6.308\times10^{-3}$ m²,

and $D = \sqrt{4A/\pi} = \sqrt{4(6.308\times10^{-3} \text{ m}^2)/\pi} = 8.96\times10^{-2}$ m $= 8.96$ cm.

EVALUATE: H increases when A increases.

17.63. **IDENTIFY:** Assume the temperatures of the surfaces of the window are the outside and inside temperatures. Use the concept of thermal resistance. For part (b) use the fact that when insulating materials are in layers, the R values are additive.

SET UP: From Table 17.5, $k = 0.8$ W/m·K for glass. $R = L/k$.

EXECUTE: **(a)** For the glass, $R_{glass} = \dfrac{5.20\times10^{-3} \text{ m}}{0.8 \text{ W/m·K}} = 6.50\times10^{-3}$ m²·K/W.

$H = \dfrac{A(T_H - T_C)}{R} = \dfrac{(1.40 \text{ m})(2.50 \text{ m})(39.5 \text{ K})}{6.50\times10^{-3} \text{ m}^2 \cdot \text{K/W}} = 2.1\times10^4$ W

(b) For the paper, $R_{paper} = \dfrac{0.750\times10^{-3} \text{ m}}{0.05 \text{ W/m·K}} = 0.015$ m²·K/W. The total R is

$R = R_{glass} + R_{paper} = 0.0215$ m²·K/W. $H = \dfrac{A(T_H - T_C)}{R} = \dfrac{(1.40 \text{ m})(2.50 \text{ m})(39.5 \text{ K})}{0.0215 \text{ m}^2 \cdot \text{K/W}} = 6.4\times10^3$ W.

EVALUATE: The layer of paper decreases the rate of heat loss by a factor of about 3.

17.64. **IDENTIFY:** The rate of energy radiated per unit area is $\dfrac{H}{A} = e\sigma T^4$.

SET UP: A perfect blackbody has $e = 1$.

EXECUTE: **(a)** $\dfrac{H}{A} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(273 \text{ K})^4 = 315 \text{ W/m}^2$

(b) $\dfrac{H}{A} = (1)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(2730 \text{ K})^4 = 3.15 \times 10^6 \text{ W/m}^2$

EVALUATE: When the Kelvin temperature increases by a factor of 10 the rate of energy radiation increases by a factor of 10^4.

17.65. **IDENTIFY:** Use $H = Ae\sigma T^4$ to calculate A.

SET UP: $H = Ae\sigma T^4$ so $A = H / e\sigma T^4$

150-W and all electrical energy consumed is radiated says $H = 150 \text{ W}$.

EXECUTE: $A = \dfrac{150 \text{ W}}{(0.35)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(2450 \text{ K})^4} = 2.1 \times 10^{-4} \text{ m}^2 (1 \times 10^4 \text{ cm}^2 / 1 \text{ m}^2) = 2.1 \text{ cm}^2$

EVALUATE: Light-bulb filaments are often in the shape of a tightly wound coil to increase the surface area; larger A means a larger radiated power H.

17.66. **IDENTIFY:** The net heat current is $H = Ae\sigma(T^4 - T_s^4)$. A power input equal to H is required to maintain constant temperature of the sphere.

SET UP: The surface area of a sphere is $4\pi r^2$.

EXECUTE: $H = 4\pi(0.0150 \text{ m})^2(0.35)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(3000 \text{ K})^4 - (290 \text{ K})^4] = 4.54 \times 10^3 \text{ W}$.

EVALUATE: Since $3000 \text{ K} > 290 \text{ K}$ and H is proportional to T^4, the rate of emission of heat energy is much greater than the rate of absorption of heat energy from the surroundings.

17.67. **IDENTIFY:** Apply $H = Ae\sigma T^4$ and calculate A.

SET UP: For a sphere of radius R, $A = 4\pi R^2$. $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$. The radius of the earth is $R_E = 6.37 \times 10^6 \text{ m}$, the radius of the sun is $R_{sun} = 6.96 \times 10^8 \text{ m}$, and the distance between the earth and the sun is $r = 1.50 \times 10^{11} \text{ m}$.

EXECUTE: The radius is found from $R = \sqrt{\dfrac{A}{4\pi}} = \sqrt{\dfrac{H / (\sigma T^4)}{4\pi}} = \sqrt{\dfrac{H}{4\pi\sigma}} \dfrac{1}{T^2}$.

(a) $R_a = \sqrt{\dfrac{(2.7 \times 10^{32} \text{ W})}{4\pi(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)}} \dfrac{1}{(11{,}000 \text{ K})^2} = 1.61 \times 10^{11} \text{ m}$

(b) $R_b = \sqrt{\dfrac{(2.10 \times 10^{23} \text{ W})}{4\pi(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)}} \dfrac{1}{(10{,}000 \text{ K})^2} = 5.43 \times 10^6 \text{ m}$

EVALUATE: **(c)** The radius of Procyon B is comparable to that of the earth, and the radius of Rigel is comparable to the earth-sun distance.

17.68. **IDENTIFY:** This problem deals with the thermal expansion for water in the range of 0°C to 10°C.

SET UP: Use Fig. 17.12 in the textbook. From analytic geometry, a parabola with vertex at the point (h,k) and opening upward is $(x - h)^2 = 4p(y - k)$ with $p > 0$. In this case, $h = 4.0°\text{C}$ and $k = 1.00003$ cm^3. We want to find the equation for $V(T)$ with T in degrees Celsius.

EXECUTE: **(a)** Using the equation for a parabola gives $(T - 4.0°\text{C})^2 = 4p(V - 1.00003 \text{ cm}^3)$. We need to find $4p$. From the graph, $V = 1.00015$ cm^3 when $T = 0°\text{C}$, so $(-4.0°\text{C})^2 = 4p(1.00015 - 1.00003)\text{cm}^3$, which gives $4p = 1.33 \times 10^5$ (C°)^2/cm^3. Therefore

$(T-4.0°C)^2 = \left[1.33 \times 10^5 (C°)^2 / cm^3\right](V - 1.00003 \, cm^3).$ Now solve for V which gives

$V = \left[7.5 \times 10^{-6} cm^3 / (C°)^2\right](T - 4.0°C)^2 + 1.00003 \, cm^3.$ From this result, we see that $A = 1.00003 \, cm^3$

and $B = 7.5 \times 10^{-6} cm^3 / (C°)^2$.

(b) Use $dV = \beta(T_C)V dt$, where $V = A + B(T - 4.0°C)^2$ and $V \approx 1 \, cm^3$ (because the graph in the Fig. 17.12 is for 1 gram of water), which gives $dV = d\left[A + B(T - 4.0°C)^2\right] = 2B(T - 4°C)dT.$ Therefore

$\beta(T) = 2B(T - 4°C).$

At 1.0°C: $\beta = 2\left[7.5 \times 10^{-6} cm^3 / (C°)^2\right](1.0°C - 4.0°C) = -4.5 \times 10^{-5} / C°.$

At 4.0°C: $\beta = 0$.

At 7.0°C: $\beta = 4.5 \times 10^{-5} / C°$.

At 10.0°C: $\beta = 9.0 \times 10^{-5} / C°.$

EVALUATE: With $V = 1.0 \, cm^3$, $\beta = dV/dT$ is the slope of the graph in Fig. 17.12. From this graph, we can see that the slope is zero when $T = 4.0°C$. Also when $T = 1.0°C$ and $7.0°C$ (3C° within 4.0°C), the slope has the same magnitude but different sign. This also agrees with our result.

17.69. **IDENTIFY:** Use $\Delta L = L_0 \alpha \Delta T$ to find the change in diameter of the sphere and the change in length of the cable. Set the sum of these two increases in length equal to 2.00 mm.

SET UP: $\alpha_{brass} = 2.0 \times 10^{-5} \, K^{-1}$ and $\alpha_{steel} = 1.2 \times 10^{-5} \, K^{-1}.$

EXECUTE: $\Delta L = (\alpha_{brass}L_{0,brass} + \alpha_{steel}L_{0,steel})\Delta T.$

$\Delta T = \dfrac{2.00 \times 10^{-3} \, m}{(2.0 \times 10^{-5} \, K^{-1})(0.350 \, m) + (1.2 \times 10^{-5} \, K^{-1})(10.5 \, m)} = 15.0 \, C°. \quad T_2 = T_1 + \Delta T = 35.0°C.$

EVALUATE: The change in diameter of the brass sphere is 0.10 mm. This is small, but should not be neglected.

17.70. **IDENTIFY:** The tension in the wire is related to the speed of waves on the wire, and it also related to the thermal stress in the wire.

SET UP: The thermal stress is $\dfrac{F}{A} = -Y\alpha \Delta T$, $v = f\lambda$, and $v = \sqrt{\dfrac{F}{\mu}}$. In the fundamental mode $\lambda = 2L.$

We know that $L = 0.400 \, m$ and at 20.0°C, $f_1 = 440$ Hz.

EXECUTE: (a) We want the tension F in the wire when $f_1 = 440$ Hz at 20.0°C. We know that $\lambda = 2L = 0.800 \, m$ and $v = \sqrt{\dfrac{F}{\mu}}$, where $\mu = m/L = (0.00250 \, kg)/(0.400 \, m) = 6.25 \times 10^{-3} \, kg/m.$ Solving $v = \sqrt{\dfrac{F}{\mu}}$

for F and using $v = f\lambda$ gives $F = \mu(f\lambda)^2$. For the numbers here we have

$F = (6.25 \times 10^{-3} \, kg/m)\left[(440 \, Hz)(0.800 \, m)\right]^2 = 774 \, N.$

(b) Now $f_1 = 460$ Hz and we want to find the temperature of the wire. The thermal stress is

$\dfrac{F}{A} = -Y\alpha \Delta T$, so we need F and A to find ΔT. The wavelength is the same as in part (a) because the

wire has not changed length, but the frequency has changed. As before, we use $v = f\lambda$ and $v = \sqrt{\dfrac{F}{\mu}}$ to

get F, giving $F = \mu(f\lambda)^2$. The numbers are the same as before except that $f = 460$ Hz instead of 440 Hz. The result is $F = 846.4 \, N.$ (Note that the tension has *increased*.) Now find the cross-sectional area A of the wire. We know its mass, length, and density, so we use $m = \rho V = \rho A L$ which gives $A = \dfrac{m}{\rho L}$

$$= \frac{0.00250 \text{ kg}}{(7800 \text{ kg/m}^3)(0.400 \text{ m})} = 8.013 \times 10^{-7} \text{ m}^2.$$ We are now ready to use $\frac{F}{A} = -Y\alpha\Delta T$ to find ΔT. The

F in that equation is not the tension. Rather it is the *difference* in tension when the temperature is changed by ΔT. It is the force needed to maintain the length of the wire. The wire would tend to

contract so the tension prevents this. Therefore $F = 846.4 \text{ N} - 774 \text{ N} = 72 \text{ N}$. Solving $\frac{F}{A} = -Y\alpha\Delta T$ for

ΔT gives $\Delta T = -\frac{F}{AY\alpha}$. For the numbers here we get

$$\Delta T = -\frac{72 \text{ N}}{(8.013 \times 10^{-7} \text{m}^2)(20 \times 10^{10} \text{ Pa})(1.2 \times 10^{-5} \text{ K})} = -37 \text{ C}°.$$ The new temperature is $T = T_0 + \Delta T = $

$20.0°\text{C} + (-37 \text{ C}°) = -17°\text{C}.$

EVALUATE: Heating the wire would have caused it to expand which would have decreased the tension and hence lowered the fundamental frequency.

17.71. **IDENTIFY:** We are dealing the thermal expansion of a volume of liquid when heat is added to it.
SET UP: Use $Q = mc\Delta T$, $\Delta V = \beta L_0 \Delta T$, and $m = \rho V$.
EXECUTE: Using the above equations gives $Q = mc\Delta T = \rho V c\Delta T$. From $\Delta V = \beta L_0 \Delta T$ we

have $\Delta T = \frac{\Delta V}{\beta \rho}$, so $Q = \rho V c \frac{\Delta V}{\beta V}$. Solving for c gives $c = \frac{Q\beta}{\rho \Delta V}$.

EVALUATE: Our result indicates that if c is large, a large amount of heat Q will be needed to change its volume. This is a reasonable result.

17.72. **IDENTIFY:** In this problem, we are dealing with the effect of thermal expansion on the period of a simple pendulum.
SET UP: At 20.0°C the length L of the copper wire is 3.00 m. We want to find the percent change in the

period if the temperature is increased to 220°C. We know that $T = 2\pi\sqrt{\frac{L}{g}}$ and $\Delta L = \alpha L_0 \Delta T$, and we

want $\frac{\Delta T}{T_1}$.

EXECUTE: At 20.0°C: $T_1 = 2\pi\sqrt{\frac{L}{g}}$. At 22.0°C: $T_2 = 2\pi\sqrt{\frac{L_2}{g}} = 2\pi\sqrt{\frac{L + \Delta L}{g}}$. Using $\Delta L = \alpha L_0 \Delta T$ gives

$T_2 = 2\pi\sqrt{\frac{L + \Delta L}{g}} = 2\pi\sqrt{\frac{L + \alpha L \Delta T}{g}} = 2\pi\sqrt{\frac{L}{g}}\sqrt{1 + \alpha\Delta T}$. We want $\frac{\Delta T}{T_1}$ which is $\frac{\Delta T}{T_1} = $

$\frac{T_2 - T_1}{T_1} = \frac{T_2}{T_1} - 1 = \frac{2\pi\sqrt{\frac{L}{g}}\sqrt{1 + \alpha\Delta T}}{2\pi\sqrt{\frac{L}{g}}} - 1 = \sqrt{1 + \alpha\Delta T} - 1.$ For copper we have

$\alpha\Delta T = (1.7 \times 10^{-5} \text{ K}^{-1})(200 \text{ K}) = 3.4 \times 10^{-3}$, which is much less than 1. Therefore we can use the

approximation $(1 + x)^n \approx 1 + nx$, which in this case is $\sqrt{1 + x} = (1 + x)^{1/2} \approx 1 + x/2$, where $x = \alpha\Delta T$.

This gives $\frac{\Delta T}{T_1} = \sqrt{1 + \alpha\Delta T} - 1 \approx \frac{\alpha\Delta T}{2} = \frac{3.4 \times 10^{-3}}{2} = 0.0017 = 0.17\%.$

EVALUATE: Even for a temperature change as large as 200°C, the percent change is only 0.17%. This illustrates that thermal expansion typically produces very small fractional length changes. Note that the fact that the pendulum bob was fused quartz played no role in the solution; it was extraneous information.

17.73. **IDENTIFY** and **SET UP:** Use the temperature difference in M° and in C° between the melting and boiling points of mercury to relate M° to C°. Also adjust for the different zero points on the two scales to get an equation for T_M in terms of T_C.

(a) EXECUTE: normal melting point of mercury: $-39°C = 0.0°M$

normal boiling point of mercury: $357°C = 100.0°M$

$100.0 \text{ M}° = 396 \text{ C}°$ so $1 \text{ M}° = 3.96 \text{ C}°$

Zero on the M scale is -39 on the C scale, so to obtain T_C multiply T_M by 3.96 and then subtract 39°:

$T_C = 3.96 T_M - 39°$

Solving for T_M gives $T_M = \frac{1}{3.96}(T_C + 39°)$

The normal boiling point of water is 100°C; $T_M = \frac{1}{3.96}(100° + 39°) = 35.1°M$.

(b) $10.0 \text{ M}° = 39.6 \text{ C}°$

EVALUATE: A M° is larger than a C° since it takes fewer of them to express the difference between the boiling and melting points for mercury.

17.74. **IDENTIFY:** $v = \sqrt{F/\mu} = \sqrt{FL/m}$. For the fundamental, $\lambda = 2L$ and $f = \frac{v}{\lambda} = \frac{1}{2}\sqrt{\frac{F}{mL}}$. F, v and λ change when T changes because L changes. $\Delta L = L\alpha\Delta T$, where L is the original length.

SET UP: For copper, $\alpha = 1.7 \times 10^{-5} \text{ (C}°)^{-1}$.

EXECUTE: **(a)** We can use differentials to find the frequency change because all length changes are small percents. $\Delta f \approx \frac{\partial f}{\partial L}\Delta L$ (only L changes due to heating).

$\Delta f = \frac{1}{2}\frac{1}{2}(F/mL)^{-1/2}(F/m)(-1/L^2)\Delta L = -\frac{1}{2}\left(\frac{1}{2}\sqrt{\frac{F}{mL}}\right)\frac{\Delta L}{L} = -\frac{1}{2}f\frac{\Delta L}{L}$.

$\Delta f = -\frac{1}{2}(\alpha\Delta T)f = -\frac{1}{2}(1.7 \times 10^{-5} \text{ (C}°)^{-1})(40 \text{ C}°)(440 \text{ Hz}) = -0.15 \text{ Hz}$. The frequency decreases since the length increases.

(b) $\Delta v = \frac{\partial v}{\partial L}\Delta L$.

$\frac{\Delta v}{v} = \frac{\frac{1}{2}(FL/m)^{-1/2}(F/m)\Delta L}{\sqrt{FL/m}} = \frac{\Delta L}{2L} = \frac{\alpha\Delta T}{2} = \frac{1}{2}(1.7 \times 10^{-5} \text{ (C}°)^{-1})(40 \text{ C}°) = 3.4 \times 10^{-4} = 0.034\%$.

(c) $\lambda = 2L$ so $\Delta\lambda = 2\Delta L \rightarrow \frac{\Delta\lambda}{\lambda} = \frac{2\Delta L}{2L} = \frac{\Delta L}{L} = \alpha\Delta T$.

$\frac{\Delta\lambda}{\lambda} = (1.7 \times 10^{-5} \text{ (C}°)^{-1})(40 \text{ C}°) = 6.8 \times 10^{-4} = 0.068\%$. λ increases.

EVALUATE: The wave speed and wavelength increase when the length increases and the frequency decreases. The percentage change in the frequency is -0.034%. The fractional change in all these quantities is very small.

17.75. **IDENTIFY** and **SET UP:** Use $\Delta V = V_0\beta\Delta T$ for the volume expansion of the oil and of the cup. Both the volume of the cup and the volume of the olive oil increase when the temperature increases, but β is larger for the oil so it expands more. When the oil starts to overflow, $\Delta V_{oil} = \Delta V_{glass} + (3.00 \times 10^{-3} \text{ m})A$, where A is the cross-sectional area of the cup.

EXECUTE: $\Delta V_{oil} = V_{0,oil}\beta_{oil}\Delta T = (9.7 \text{ cm})A\beta_{oil}\Delta T$. $\Delta V_{glass} = V_{0,glass}\beta_{glass}\Delta T = (10.0 \text{ cm})A\beta_{glass}\Delta T$.

$(9.7 \text{ cm})A\beta_{oil}\Delta T = (10.0 \text{ cm})A\beta_{glass}\Delta T + (0.300 \text{ cm})A$. The A divides out. Solving for ΔT gives

$\Delta T = 47.4 \text{ C}°$. $T_2 = T_1 + \Delta T = 69.4°C$.

EVALUATE: If the expansion of the cup is neglected, the olive oil will have expanded to fill the cup when $(0.300 \text{ cm})A = (9.7 \text{ cm})A\beta_{oil}\Delta T$, so $\Delta T = 45.5 \text{ C}°$ and $T_2 = 77.5°\text{C}$. Our result is somewhat higher than this. The cup also expands but not as much since $\beta_{glass} << \beta_{oil}$.

17.76.   **IDENTIFY:**   As the tape changes temperature, the distances between the markings will increase, thus making the readings inaccurate.

SET UP: For steel, $\alpha = 1.2 \times 10^{-5} (\text{C}°)^{-1}$. The two points that match the length of the object are 25.970 m apart at $20.0°\text{C}$. Find the distance between them at $5.00°\text{C}$. For linear expansion, $L = L_0(1 + \alpha \Delta T)$.

EXECUTE: $L = L_0(1 + \alpha \Delta T) = (25.970 \text{ m})[1 + (1.2 \times 10^{-5} (\text{C}°)^{-1})(5.00°\text{C} - 20.0°\text{C})] = 25.965 \text{ m}$. The true distance between the points is 25.965 m.

EVALUATE: The error in measurement is 25.970 m – 25.965 m = 0.005 m = 5 mm. This is not likely to be a very serious error in a measurement of nearly 30 m. If greater precision is needed, some sort of laser measuring device would probably be used.

17.77.   **IDENTIFY** and **SET UP:**   Call the metals A and B. Use the data given to calculate α for each metal.
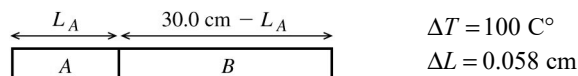
EXECUTE: $\Delta L = L_0 \alpha \Delta T$ so $\alpha = \Delta L/(L_0 \Delta T)$

metal A: $\alpha_A = \dfrac{\Delta L}{L_0 \Delta T} = \dfrac{0.0650 \text{ cm}}{(30.0 \text{ cm})(100 \text{ C}°)} = 2.167 \times 10^{-5} (\text{C}°)^{-1}$

metal B: $\alpha_B = \dfrac{\Delta L}{L_0 \Delta T} = \dfrac{0.0350 \text{ cm}}{(30.0 \text{ cm})(100 \text{ C}°)} = 1.167 \times 10^{-5} (\text{C}°)^{-1}$

EVALUATE: L_0 and ΔT are the same, so the rod that expands the most has the larger α.

IDENTIFY and **SET UP:**   Now consider the composite rod (Figure 17.77). Apply $\Delta L = L_0 \alpha \Delta T$. The target variables are L_A and L_B, the lengths of the metals A and B in the composite rod.

Figure 17.77

EXECUTE: $\Delta L = \Delta L_A + \Delta L_B = (\alpha_A L_A + \alpha_B L_B)\Delta T$

$\Delta L/\Delta T = \alpha_A L_A + \alpha_B (0.300 \text{ m} - L_A)$

$L_A = \dfrac{\Delta L/\Delta T - (0.300 \text{ m})\alpha_B}{\alpha_A - \alpha_B} = \dfrac{(0.058 \times 10^{-2} \text{ m})/(100 \text{ C}°) - (0.300 \text{ m})(1.167 \times 10^{-5} (\text{C}°)^{-1})}{1.00 \times 10^{-5} (\text{C}°)^{-1}} = 23.0 \text{ cm}$

$L_B = 30.0 \text{ cm} - L_A = 30.0 \text{ cm} - 23.0 \text{ cm} = 7.0 \text{ cm}$

EVALUATE: The expansion of the composite rod is similar to that of rod A, so the composite rod is mostly metal A.

17.78.   **IDENTIFY:**   The copper sphere is radiating energy and is therefore cooling down. So we are dealing with heat capacity and thermal radiation.

SET UP: The rate at which heat is radiated is $H = Ae\sigma T^4$ and $Q = mc\Delta T$. For this sphere $e = 1.00$ and $c = 390 \text{ J/kg} \cdot \text{K}$, and $m = \rho V$. The radiated energy in time t is Ht.

EXECUTE: **(a)** We want the time t that it takes the sphere to cool by 1.00 K due to the radiation, assuming that H remains constant. The heat for the copper sphere to cool down by 1 K is $Q = mc\Delta T = \rho Vc\Delta T$, so $Ht = Q$. This gives $A\sigma T^4 t = \rho Vc\Delta T$. Using $A = 4\pi R^2$ and $V = \dfrac{4}{3}\pi R^3$, the previous equation becomes $\sigma T^4 t = \dfrac{\rho Rc\Delta T}{3}$. Solving for t using $T = 300 \text{ K}$, $c = 390 \text{ J/kg} \cdot \text{K}$, $\Delta T = 1 \text{ K}$, $\rho = 8900 \text{ kg/m}^3$, and $R = 0.0500 \text{ m}$, we get $t = 126 \text{ s} = 2.10 \text{ min}$.

(b) $\dfrac{\Delta H}{H} = \dfrac{H_2 - H_1}{H_1} = \dfrac{H_2}{H_1} - 1 = \dfrac{A\sigma T_2^4}{A\sigma T_1^4} - 1 = \left(\dfrac{299\text{ K}}{300\text{ K}}\right)^4 - 1 = -0.0133.$

EVALUATE: The fractional change in H is so small that neglecting it was a reasonable option. The fractional change is negative because the sphere is decreasing in temperature so the rate of radiation is decreasing.

17.79. **IDENTIFY:**   The change in length due to heating is $\Delta L_T = L_0 \alpha \Delta T$ and this need not equal ΔL. The

change in length due to the tension is $\Delta L_F = \dfrac{FL_0}{AY}$. Set $\Delta L = \Delta L_F + \Delta L_T$.

SET UP: $\alpha_{\text{brass}} = 2.0 \times 10^{-5}\ (\text{C}°)^{-1}$. $\alpha_{\text{steel}} = 1.5 \times 10^{-5}\ (\text{C}°)^{-1}$. $Y_{\text{steel}} = 20 \times 10^{10}$ Pa.

EXECUTE: **(a)** The change in length is due to the tension and heating. $\dfrac{\Delta L}{L_0} = \dfrac{F}{AY} + \alpha \Delta T$. Solving for

F/A, $\dfrac{F}{A} = Y\left(\dfrac{\Delta L}{L_0} - \alpha \Delta T\right)$.

(b) The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that ΔL is not zero, but is the amount $\alpha_{\text{brass}} L_0 \Delta T$ that the brass expands, and so

$\dfrac{F}{A} = Y_{\text{steel}}(\alpha_{\text{brass}} - \alpha_{\text{steel}})\Delta T = (20 \times 10^{10}\text{ Pa})(2.0 \times 10^{-5}\ (\text{C}°)^{-1} - 1.2 \times 10^{-5}\ (\text{C}°)^{-1})(120\text{ C}°) = 1.92 \times 10^8$ Pa.

EVALUATE: The length of the brass bar increases more than the length of the steel wires. The wires remain taut and are under tension when the temperature of the system is raised above 20°C.

17.80. **IDENTIFY** and **SET UP:** $v = \sqrt{F/\mu}$. The coefficient of linear expansion α is defined by $\Delta L = L_0 \alpha \Delta T$.

This can be combined with $Y = \dfrac{F/A}{\Delta L/L_0}$ to give $\Delta F = -Y\alpha A \Delta T$ for the change in tension when the

temperature changes by ΔT. Combine the two equations and solve for α.

EXECUTE: $v_1 = \sqrt{F/\mu}$, $v_1^2 = F/\mu$ and $F = \mu v_1^2$

The length and hence μ stay the same but the tension decreases by $\Delta F = -Y\alpha A \Delta T$.

$v_2 = \sqrt{(F + \Delta F)/\mu} = \sqrt{(F - Y\alpha A \Delta T)/\mu}$

$v_2^2 = F/\mu - Y\alpha A \Delta T/\mu = v_1^2 - Y\alpha A \Delta T/\mu$

And $\mu = m/L$ so $A/\mu = AL/m = V/m = 1/\rho$. ($A$ is the cross-sectional area of the wire, V is the volume

of a length L.) Thus $v_1^2 - v_2^2 = \alpha(Y\Delta T/\rho)$ and $\alpha = \dfrac{v_1^2 - v_2^2}{(Y/\rho)\Delta T}$.

EVALUATE: When T increases the tension decreases and v decreases.

17.81. **(a) IDENTIFY** and **SET UP:**   The diameter of the ring undergoes linear expansion (increases with T) just like a solid steel disk of the same diameter as the hole in the ring. Heat the ring to make its diameter equal to 2.5020 in.

EXECUTE: $\Delta L = \alpha L_0 \Delta T$ so $\Delta T = \dfrac{\Delta L}{L_0 \alpha} = \dfrac{0.0020\text{ in.}}{(2.5000\text{ in.})(1.2 \times 10^{-5}(\text{C}°)^{-1})} = 66.7\text{ C}°$

$T = T_0 + \Delta T = 20.0°\text{C} + 66.7\text{ C}° = 87°\text{C}$

(b) IDENTIFY and **SET UP:**   Apply the linear expansion equation to the diameter of the brass shaft and to the diameter of the hole in the steel ring.

EXECUTE: $L = L_0(1 + \alpha \Delta T)$

Want L_s (steel) $= L_b$ (brass) for the same ΔT for both materials: $L_{0s}(1+\alpha_s\Delta T) = L_{0b}(1+\alpha_b\Delta T)$ so

$L_{0s} + L_{0s}\alpha_s\Delta T = L_{0b} + L_{0b}\alpha_b\Delta T$

$\Delta T = \dfrac{L_{0b} - L_{0s}}{L_{0s}\alpha_s - L_{0b}\alpha_b} = \dfrac{2.5020 \text{ in.} - 2.5000 \text{ in.}}{(2.5000 \text{ in.})(1.2\times10^{-5}(\text{C}°)^{-1}) - (2.5020 \text{ in.})(2.0\times10^{-5}(\text{C}°)^{-1})}$

$\Delta T = \dfrac{0.0020}{3.00\times10^{-5} - 5.00\times10^{-5}} \text{ C}° = -100 \text{ C}°$

$T = T_0 + \Delta T = 20.0°\text{C} - 100 \text{ C}° = -80°\text{C}$

EVALUATE: Both diameters decrease when the temperature is lowered but the diameter of the brass shaft decreases more since $\alpha_b > \alpha_s$; $|\Delta L_b| - |\Delta L_s| = 0.0020$ in.

17.82. **IDENTIFY:** Calculate the total food energy value for one doughnut. $K = \frac{1}{2}mv^2$.

SET UP: 1 cal = 4.186 J

EXECUTE: **(a)** The energy is $(2.0 \text{ g})(4.0 \text{ kcal/g}) + (17.0 \text{ g})(4.0 \text{ kcal/g}) + (7.0 \text{ g})(9.0 \text{ kcal/g}) = 139$ kcal.
The time required is $(139 \text{ kcal})/(510 \text{ kcal/h}) = 0.273 \text{ h} = 16.4$ min.

(b) $v = \sqrt{2K/m} = \sqrt{2(139\times10^3 \text{ cal})(4.186 \text{ J/cal})/(60 \text{ kg})} = 139 \text{ m/s} = 501$ km/h.

EVALUATE: When we set $K = Q$, we must express Q in J, so we can solve for v in m/s.

17.83. **IDENTIFY:** We mix an unknown liquid at 90.0°C with water at 0.0°C and measure the final equilibrium temperature T. The heat lost by the unknown liquid is equal to the heat gained by the water, so $Q_{\text{net}} = 0$.

SET UP: $Q_{\text{net}} = Q_{\text{water}} + Q_{\text{unknown}}$. We need to relate the mass of the water m_w to T in order to interpret the graph of m_w versus T^{-1}. Let x refer to the unknown liquid. We want to find c_x. For temperature changes, we use $Q = mc\Delta T$.

EXECUTE: Using $Q_{\text{net}} = Q_{\text{water}} + Q_{\text{unknown}} = 0$ gives $m_w c_w \Delta T_w + m_x c_x \Delta T_x = 0$. Solve for m_w as a function

of T^{-1}. $m_w c_w (T - 0.0°\text{C}) = m_x c_x (90.0°\text{C} - T)$, so $m_w = \left(\dfrac{m_x c_x}{c_w}\right)\left(\dfrac{90.0°\text{C}}{T} - 1\right)$. The graph of m_w versus

T^{-1} should be a straight line having slope equal to $\dfrac{m_x c_x 90.0°\text{C}}{c_w}$. Solving for c_x gives

$c_x = \dfrac{c_w(\text{slope})}{m_x(90.0°\text{C})} = \dfrac{(4190 \text{ J/kg}\cdot\text{K})(2.15 \text{ kg}\cdot\text{C}°)}{(0.050 \text{ kg})(90.0°\text{C})} = 2000 \text{ J/kg}\cdot\text{K}$.

EVALUATE: From Table 17.3 we see that c_x is about half that of water, but close to that of ice, ethanol, and ethylene glycol, so we get a reasonable result.

17.84. **IDENTIFY:** $Q_{\text{system}} = 0$. Assume that the normal melting point of iron is above 745°C so the iron initially is solid.

SET UP: For water, $c = 4190 \text{ J/kg}\cdot\text{K}$ and $L_v = 2256\times10^3$ J/kg. For solid iron, $c = 470 \text{ J/kg}\cdot\text{K}$.

EXECUTE: The heat released when the iron slug cools to 100°C is $Q = mc\Delta T =$

$(0.1000 \text{ kg})(470 \text{ J/kg}\cdot\text{K})(645 \text{ K}) = 3.03\times10^4$ J. The heat absorbed when the temperature of the water is

raised to 100°C is $Q = mc\Delta T = (0.0850 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(80.0 \text{ K}) = 2.85\times10^4$ J. This is less than the

heat released from the iron and $3.03\times10^4 \text{ J} - 2.85\times10^4 \text{ J} = 1.81\times10^3$ J of heat is available for converting

some of the liquid water at 100°C to vapor. The mass m of water that boils is

$m = \dfrac{1.81\times10^3 \text{ J}}{2256\times10^3 \text{ J/kg}} = 8.01\times10^{-4} \text{ kg} = 0.801$ g.

(a) The final temperature is 100°C.

(b) There is $85.0 \text{ g} - 0.801 \text{ g} = 84.2$ g of liquid water remaining, so the final mass of the iron and remaining water is 184.2 g.

EVALUATE: If we ignore the phase change of the water and write $m_{\text{iron}}c_{\text{iron}}(T - 745°C) + m_{\text{water}}c_{\text{water}}(T - 20.0°C) = 0,$ when we solve for T we will get a value slightly larger than 100°C. That result is unphysical and tells us that some of the water changes phase.

17.85. **IDENTIFY** and **SET UP:** To calculate Q, use $Q = mc\,\Delta T$ in the form $dQ = nC\,dT$ and integrate, using $C(T)$ given in the problem. C_{av} is obtained from $C = \dfrac{1}{n}\dfrac{dQ}{dT}$ using the finite temperature range instead of an infinitesimal dT.

EXECUTE: **(a)** $dQ = nCdT$

$$Q = n\int_{T_1}^{T_2} C\,dT = n\int_{T_1}^{T_2} k(T^3/\theta^3)dT = (nk/\theta^3)\int_{T_1}^{T_2} T^3\,dt = (nk/\theta^3)\left(\tfrac{1}{4}T^4\Big|_{T_1}^{T_2}\right)$$

$$Q = \frac{nk}{4\theta^3}\left(T_2^4 - T_1^4\right) = \frac{(1.50\text{ mol})(1940\text{ J/mol}\cdot\text{K})}{4(281\text{ K})^3}\left[(40.0\text{ K})^4 - (10.0\text{ K})^4\right] = 83.6\text{ J}.$$

(b) $C_{\text{av}} = \dfrac{1}{n}\dfrac{\Delta Q}{\Delta T} = \dfrac{1}{1.50\text{ mol}}\left(\dfrac{83.6\text{ J}}{40.0\text{ K} - 10.0\text{ K}}\right) = 1.86\text{ J/mol}\cdot\text{K}$

(c) $C = k(T/\theta)^3 = (1940\text{ J/mol}\cdot\text{K})(40.0\text{ K}/281\text{ K})^3 = 5.60\text{ J/mol}\cdot\text{K}$

EVALUATE: C is increasing with T, so C at the upper end of the temperature integral is larger than its average value over the interval.

17.86. **IDENTIFY:** We are looking at the thermal radiation from the logs in a campfire.

SET UP and EXECUTE: The net radiation rate is $H_{\text{net}} = Ae\sigma\left(T^4 - T_s^4\right).$

(a) Estimate: Log diameter is 25 cm = 0.25 m and length is 0.50 m.

(b) The total area is that of the two faces plus the side: $A = 2A_{\text{face}} + A_{\text{side}} = 2\pi R^2 + 2\pi RL$. For our log, we have $A = 2\pi(0.125\text{ m})^2 + 2\pi(0.125\text{ m})(0.50\text{ m}) = 0.49\text{ m}^2$.

(c) We want the net radiated power for $T = 700°C = 973$ K, $T_s = 20.0°C = 293$ K, and $e = 1$. Therefore $H_{\text{net}} = Ae\sigma\left(T^4 - T_s^4\right) = (0.49\text{ m}^2)(5.67\times10\text{ W/m}^2\cdot\text{K}^4)\left[(973\text{ K})^4 - (293\text{ K})^4\right]$ so we get $H_{\text{net}} = 2.5\times10^4$ W.

EVALUATE: On a chilly night when the temperature is 0°C (273 K) the rate of radiation would be even greater. As any camper knows, campfires can get very hot and radiate a lot of energy, as we have discovered.

17.87. **IDENTIFY:** Apply $Q = mc\Delta T$ to the air in the room.

SET UP: The mass of air in the room is $m = \rho V = (1.20\text{ kg/m}^3)(3200\text{ m}^3) = 3840$ kg. 1 W = 1 J/s.

EXECUTE: **(a)** $Q = (3000\text{ s})(140\text{ students})(100\text{ J/s}\cdot\text{student}) = 4.20\times10^7$ J.

(b) $Q = mc\Delta T.$ $\Delta T = \dfrac{Q}{mc} = \dfrac{4.20\times10^7\text{ J}}{(3840\text{ kg})(1020\text{ J/kg}\cdot\text{K})} = 10.7$ C°.

(c) $\Delta T = (10.7\text{ C}°)\left(\dfrac{280\text{ W}}{100\text{ W}}\right) = 30.0$ C°.

EVALUATE: In the absence of a cooling mechanism for the air, the air temperature would rise significantly.

17.88. **IDENTIFY:** $dQ = nCdT$ so for the temperature change $T_1 \to T_2$, $Q = n\int_{T_1}^{T_2} C(T)dT.$

SET UP: $\int dT = T$ and $\int TdT = \tfrac{1}{2}T^2$. Express T_1 and T_2 in kelvins: $T_1 = 300$ K, $T_2 = 500$ K.

EXECUTE: Denoting C by $C = a + bT$, a and b independent of temperature, integration gives

$$Q = n\left[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)\right].$$

$Q = (3.00 \text{ mol})[(29.5 \text{ J / mol} \cdot \text{K})(500 \text{ K} - 300 \text{ K}) + (4.10 \times 10^{-3} \text{ J / mol} \cdot \text{K}^2)((500 \text{ K})^2 - (300 \text{ K})^2)].$

$Q = 1.97 \times 10^4 \text{ J}.$

EVALUATE: If C is assumed to have the constant value $29.5 \text{ J/mol} \cdot \text{K}$, then $Q = 1.77 \times 10^4 \text{ J}$ for this temperature change. At $T_1 = 300 \text{ K}$, $C = 32.0 \text{ J/mol} \cdot \text{K}$ and at $T_2 = 500 \text{ K}$, $C = 33.6 \text{ J/mol} \cdot \text{K}$. The average value of C is $32.8 \text{ J/mol} \cdot \text{K}$. If C is assumed to be constant and to have this average value, then $Q = 1.97 \times 10^4 \text{ J}$, which is equal to the correct value.

17.89. **IDENTIFY:** The energy generated in the body is used to evaporate water, which prevents the body from overheating.
SET UP: Energy is (power)(time); calculate the heat energy Q produced in one hour. The mass m of water that vaporizes is related to Q by $Q = mL_v$. 1.0 kg of water has a volume of 1.0 L.
EXECUTE: **(a)** $Q = (0.80)(500 \text{ W})(3600 \text{ s}) = 1.44 \times 10^6 \text{ J}$. The mass of water that evaporates each hour is $m = \dfrac{Q}{L_v} = \dfrac{1.44 \times 10^6 \text{ J}}{2.42 \times 10^6 \text{ J/kg}} = 0.60 \text{ kg}$.
(b) $(0.60 \text{ kg/h})(1.0 \text{ L/kg}) = 0.60 \text{ L/h}$. The number of bottles of water is

$\dfrac{0.60 \text{ L/h}}{0.750 \text{ L/bottle}} = 0.80 \text{ bottles/h}.$

EVALUATE: It is not unreasonable to drink 8/10 of a bottle of water per hour during vigorous exercise.

17.90. **IDENTIFY:** If it cannot be gotten rid of in some way, the metabolic energy transformed to heat will increase the temperature of the body.
SET UP: From Problem 17.89, $Q = 1.44 \times 10^6 \text{ J}$ and $m = 70 \text{ kg}$. $Q = mc\Delta T$. Convert the temperature change in C° to F° using that 9 F° = 5 C°.
EXECUTE: **(a)** $Q = mc\Delta T$ so $\Delta T = \dfrac{Q}{mc} = \dfrac{1.44 \times 10^6 \text{ J}}{(70 \text{ kg})(3480 \text{ J/kg} \cdot \text{C}°)} = 5.9 \text{ C}°.$
(b) $\Delta T = (5.9°\text{C})\left(\dfrac{9 \text{ F}°}{5 \text{ C}°}\right) = 10.6°\text{F}$. $T = 98.6°\text{F} + 10.6 \text{ F}° = 109°\text{F}.$

EVALUATE: A temperature this high can cause heat stroke and be lethal.

17.91. **IDENTIFY and SET UP:** The heat produced from the reaction is $Q_{\text{reaction}} = mL_{\text{reaction}}$, where L_{reaction} is the heat of reaction of the chemicals.
$Q_{\text{reaction}} = W + \Delta U_{\text{spray}}$
EXECUTE: For a mass m of spray, $W = \frac{1}{2}mv^2 = \frac{1}{2}m(19 \text{ m/s})^2 = (180.5 \text{ J/kg})m$ and
$\Delta U_{\text{spray}} = Q_{\text{spray}} = mc\Delta T = m(4190 \text{ J/kg} \cdot \text{K})(100°\text{C} - 20°\text{C}) = (335,200 \text{ J/kg})m.$
Then $Q_{\text{reaction}} = (180 \text{ J/kg} + 335,200 \text{ J/kg})m = (335,380 \text{ J/kg})m$ and $Q_{\text{reaction}} = mL_{\text{reaction}}$ implies
$mL_{\text{reaction}} = (335,380 \text{ J/kg})m.$

The mass m divides out and $L_{\text{reaction}} = 3.4 \times 10^5 \text{ J/kg}.$

EVALUATE: The amount of energy converted to work is negligible for the two significant figures to which the answer should be expressed. Almost all of the energy produced in the reaction goes into heating the compound.

17.92. **IDENTIFY:** The bike peddler generates energy, 70% of which goes into melting the ice. We are dealing with a change of phase of the ice and the power generated by turning the bicycle wheel.
SET UP: The bike peddler generates $25.0 \text{ N} \cdot \text{m}$ of torque while peddling to turn the bike wheel at 30.0 rev/min = 3.142 rad/s. The power due to this torque is $P = \tau\omega$, the energy E it produces in time t is $E = Pt$, the heat Q to melt ice is $Q = mL_f$, and the heat to raise the water temperature is $Q = mc\Delta T$.

EXECUTE: **(a)** The target variable is the time to melt 3.00 kg of ice at 0.0°C. All the ice is at 0.0°C so all the energy from peddling is used only to melt the ice. This heat is $Q = mL_f =$
$(3.00 \text{ kg})(334\times10^3 \text{ J/kg}) = 1.00\times10^6 \text{ J}$. The peddling power is $P = \tau\omega = (25.0 \text{ N} \cdot \text{m})(3.142 \text{ rad/s}) =$
78.55 W. Only 70% of this power is used to melt ice, so the available power is (0.70)(78.55 W) = 55.0
W. The heat from this power in time t is $Q = Pt$, and this must be $1.00\times10^6 \text{ J}$ to melt the ice. Therefore
$1.00\times10^6 \text{ J} = (55.0 \text{ W})t$, so $t = 1.8\times10^4 \text{ s} = 5.1 \text{ h}$.

(b) The additional heat is required to raise the water temperature from 0.0°C to 10.5°C. We use
$Q = mc\Delta T$ where m is the mass of 6.0 L of water plus 3.00 kg of melted ice. One liter is 6000 cm³, and
each cm³ has a mass of 1.00 g, so 6.0 L of water has a mass of 6000 g = 6.0 kg. The total water mass is
9.0 kg. The heat is $Q = mc\Delta T = (9.00 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(10.5 \text{ C°}) = 3.96\times10^5 \text{ J}$. The cyclist is
providing 55.0 W of power, so $(55.0 \text{ W})t = 3.96\times10^5 \text{ J}$, which gives $t = 7200 \text{ s} = 2.0 \text{ h}$.

EVALUATE: It takes about 2½ times as long to melt the ice than to heat all the water by 10.5°C. This
difference in time occurs because L_f is large for water.

17.93. **IDENTIFY:** The heat lost by the water is equal to the amount of heat gained by the ice. First calculate
the amount of heat the water could give up if it is cooled to 0.0°C. Then see how much heat it would
take to melt all of the ice. If the heat to melt the ice is less than the heat the water would give up, the ice
all melts and then the resulting water is heated to some final temperature.
SET UP: $Q = mc \Delta T$ and $Q = mL_f$.
EXECUTE: **(a)** Heat from water if cooled to 0.0°C: $Q = mc \Delta T$
$Q = mc \Delta T = (1.50 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(28.0 \text{ K}) = 1.760 \times 10^5 \text{ J}$
Heat to melt all of the ice: $Q = mc \Delta T + mL_f = m(c\Delta T + L_f)$
$Q = (0.600 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K})(22.0 \text{ K}) + 3.34 \times 10^5 \text{ J/kg}] = 2.276 \times 10^5 \text{ J}$
Since the heat required to melt all the ice is greater than the heat available by cooling the water to 0.0°C,
not all the ice will melt.

(b) Since not all the ice melts, the final temperature of the water (and ice) will be 0.0°C. So the heat
from the water will melt only part of the ice. Call m the mass of the melted ice. Therefore
$Q_{\text{from water}} = 1.760 \times 10^5 \text{ J} = (0.600 \text{ kg})(2100 \text{ J/kg} \cdot \text{K})(22.0 \text{ K}) + m(3.34 \times 10^5 \text{ J/kg})$, which gives $m =$
0.444 kg, which is the amount of ice that melts. The mass of ice remaining is 0.600 kg – 0.444 kg =
0.156 kg. The final temperature will be 0.0°C since some ice remains in the water.
EVALUATE: An alternative approach would be to assume that all the ice melts and find the final
temperature of the water in the container. This actually comes out to be negative, which is not possible
if all the ice melts. Therefore not all the ice could have melted. Once you know this, proceed as in part
(b).

17.94. **IDENTIFY:** The amount of heat lost by the soft drink and mug is equal to the heat gained by the ice.
The ice must first be heated to 0.0°C, then melted, and finally the resulting water heated to the final
temperature of the system.
SET UP: Assume that all the ice melts. If we calculate $T_f < 0$, we will know this assumption is
incorrect. For aluminum, $c_a = 910 \text{ J/kg} \cdot \text{K}$. For water, $L_f = 3.35\times10^5 \text{ J/kg}$ and
$c_w = 4.19\times10^3 \text{ J/kg} \cdot \text{K}$. For ice, $c_i = 2100 \text{ J/kg} \cdot \text{K}$. The density of water is $1.00\times10^3 \text{ kg/m}^3$, so 1.00
L of water has mass 1.00 kg.
EXECUTE: For the soft drink:
$Q_w = m_w c_w \Delta T_w = (2.00 \text{ kg})(4.19\times10^3 \text{ J/kg} \cdot \text{K})(T - 20.0°C) = (8380 \text{ J/K})T - 1.676\times10^5 \text{ J}$.
For the mug: $Q_a = m_a c_a \Delta T_a = (0.257 \text{ kg})(910 \text{ J/kg} \cdot \text{K})(T - 20.0°C) = (234 \text{ J/K})T - 4.68\times10^3 \text{ J}$.
For the ice: $Q_i = m_i c_i \Delta T_i + m_i L_f + m_i c_w \Delta T_w$.

$Q_i = (0.120 \text{ kg})[(2100 \text{ J/kg} \cdot \text{K})(15.0 \text{ C}°) + 3.34 \times 10^5 \text{ J/kg} + (4.19 \times 10^3 \text{ J/kg} \cdot \text{K})(T - 0 \text{ C}°)].$

$Q_i = 4.386 \times 10^4 \text{ J} + (503 \text{ J/K})T.$

$\Sigma Q = 0$ gives

$(8380 \text{ J/K})T - 1.676 \times 10^5 \text{ J} + (234 \text{ J/K})T - 4.68 \times 10^3 \text{ J} + 4.386 \times 10^4 \text{ J} + (503 \text{ J/K})T = 0.$

$T = \dfrac{1.284 \times 10^5 \text{ J}}{9117 \text{ J/K}} = 14.1°\text{C}.$

EVALUATE: $T > 0°\text{C}$, so our assumption that all the ice melts is correct. Note that the ice and the water from the melted ice have different specific heat capacities.

17.95. **IDENTIFY** and **SET UP:** Assume that all the ice melts and that all the steam condenses. If we calculate a final temperature T that is outside the range 0°C to 100°C then we know that this assumption is incorrect. Calculate Q for each piece of the system and then set the total $Q_{\text{system}} = 0$.

EXECUTE: **(a)** <u>Copper can</u> (changes temperature from 0.0° to T; no phase change):
$Q_{\text{can}} = mc\Delta T = (0.446 \text{ kg})(390 \text{ J/kg} \cdot \text{K})(T - 0.0°\text{C}) = (173.9 \text{ J/K})T$

<u>Ice</u> (melting phase change and then the water produced warms to T):
$Q_{\text{ice}} = +mL_f + mc\Delta T = (0.0950 \text{ kg})(334 \times 10^3 \text{ J/kg}) + (0.0950 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 0.0°\text{C})$

$Q_{\text{ice}} = 3.173 \times 10^4 \text{ J} + (398.0 \text{ J/K})T.$

<u>Steam</u> (condenses to liquid and then water produced cools to T):
$Q_{\text{steam}} = -mL_v + mc\Delta T = -(0.0350 \text{ kg})(2256 \times 10^3 \text{ J/kg}) + (0.0350 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(T - 100.0°\text{C})$

$Q_{\text{steam}} = -7.896 \times 10^4 \text{ J} + (146.6 \text{ J/K})T - 1.466 \times 10^4 \text{ J} = -9.362 \times 10^4 \text{ J} + (146.6 \text{ J/K})T$

$Q_{\text{system}} = 0$ implies $Q_{\text{can}} + Q_{\text{ice}} + Q_{\text{steam}} = 0.$

$(173.9 \text{ J/K})T + 3.173 \times 10^4 \text{ J} + (398.0 \text{ J/K})T - 9.362 \times 10^4 \text{ J} + (146.6 \text{ J/K})T = 0$

$(718.5 \text{ J/K})T = 6.189 \times 10^4 \text{ J}$

$T = \dfrac{6.189 \times 10^4 \text{ J}}{718.5 \text{ J/K}} = 86.1°\text{C}.$

(b) No ice, no steam, and $0.0950 \text{ kg} + 0.0350 \text{ kg} = 0.130 \text{ kg}$ of liquid water.

EVALUATE: This temperature is between 0°C and 100°C so our assumptions about the phase changes being complete were correct.

17.96. **IDENTIFY:** The final amount of ice is less than the initial mass of water, so water remains and the final temperature is 0°C. The ice added warms to 0°C and heat comes out of water to convert that water to ice. Conservation of energy says $Q_i + Q_w = 0$, where Q_i and Q_w are the heat flows for the ice that is added and for the water that freezes.

SET UP: Let m_i be the mass of ice that is added and m_w is the mass of water that freezes. The mass of ice increases by 0.434 kg, so $m_i + m_w = 0.434 \text{ kg}$. For water, $L_f = 334 \times 10^3 \text{ J/kg}$ and for ice $c_i = 2100 \text{ J/kg} \cdot \text{K}$. Heat comes out of the water when it freezes, so $Q_w = -mL_f$.

EXECUTE: $Q_i + Q_w = 0$ gives $m_i c_i (15.0 \text{ C}°) + (-m_w L_f) = 0$, $m_w = 0.434 \text{ kg} - m_i$, so
$m_i c_i (15.0 \text{ C}°) + (-0.434 \text{ kg} + m_i) L_f = 0.$

$m_i = \dfrac{(0.434 \text{ kg})L_f}{c_i(15.0 \text{ C}°) + L_f} = \dfrac{(0.434 \text{ kg})(334 \times 10^3 \text{ J/kg})}{(2100 \text{ J/kg} \cdot \text{K})(15.0 \text{ K}) + 334 \times 10^3 \text{ J/kg}} = 0.397 \text{ kg}.$ 0.397 kg of ice was added.

EVALUATE: The mass of water that froze when the ice at $-15.0\text{C}°$ was added was
$0.884 \text{ kg} - 0.450 \text{ kg} - 0.397 \text{ kg} = 0.037 \text{ kg}.$

17.97.   **IDENTIFY** and **SET UP:**   Heat comes out of the steam when it changes phase and heat goes into the water and causes its temperature to rise. $Q_{\text{system}} = 0$. First determine what phases are present after the system has come to a uniform final temperature.

EXECUTE: (a)   Heat that must be removed from steam if all of it condenses is

$Q = -mL_{\text{v}} = -(0.0400 \text{ kg})(2256 \times 10^3 \text{ J/kg}) = -9.02 \times 10^4 \text{ J}$

Heat absorbed by the water if it heats all the way to the boiling point of 100°C:

$Q = mc\Delta T = (0.200 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(50.0 \text{ C}°) = 4.19 \times 10^4 \text{ J}$

(b) Mass of steam that condenses is $m = Q/L_{\text{v}} = 4.19 \times 10^4 \text{ J}/2256 \times 10^3 \text{ J/kg} = 0.0186 \text{ kg}$.

Thus there is $0.0400 \text{ kg} - 0.0186 \text{ kg} = 0.0214 \text{ kg}$ of steam left. The amount of liquid water is $0.0186 \text{ kg} + 0.200 \text{ kg} = 0.219 \text{ kg}$.

EVALUATE: The water can't absorb enough heat for all the steam to condense. Steam is left and the final temperature then must be 100°C.

17.98.   **IDENTIFY:**   Heat is conducted out of the body. At steady state, the rate of heat flow is the same in both layers (fat and fur).

SET UP: Since the model is only a crude approximation to a bear, we will make the simplifying assumption that the surface area of each layer is constant and given by the surface area of a sphere of radius 1.5 m. Let the temperature of the fat-air boundary be T. A section of the two layers is sketched in Figure 17.98. A Kelvin degree is the same size as a Celsius degree, so $\text{W/m} \cdot \text{K}$ and $\text{W/m} \cdot \text{C}°$ are equivalent units. At steady state the heat current through each layer is equal to 50 W. The area of each layer is $A = 4\pi r^2$, with $r = 0.75 \text{ m}$. Use $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$.
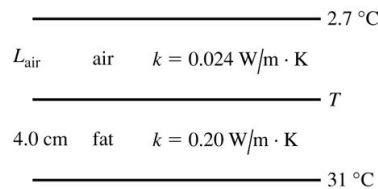

Figure 17.98

EXECUTE: (a) Apply $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$ to the fat layer and solve for $T_{\text{C}} = T$. For the fat layer

$T_{\text{H}} = 31°\text{C}$. $T = T_{\text{H}} - \dfrac{HL}{kA} = 31°\text{C} - \dfrac{(50 \text{ W})(4.0 \times 10^{-2} \text{ m})}{(0.20 \text{ W/m} \cdot \text{K})(4\pi)(0.75 \text{ m})^2} = 31°\text{C} - 1.4°\text{C} = 29.6°\text{C}$.

(b) Apply $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$ to the air layer and solve for $L = L_{\text{air}}$. For the air layer $T_{\text{H}} = T = 29.6°\text{C}$ and

$T_{\text{C}} = 2.7°\text{C}$. $L = \dfrac{kA(T_{\text{H}} - T_{\text{C}})}{H} = \dfrac{(0.024 \text{ W/m} \cdot \text{K})(4\pi)(0.75 \text{ m})^2(29.6°\text{C} - 2.7°\text{C})}{50 \text{ W}} = 9.1 \text{ cm}$.

EVALUATE: The thermal conductivity of air is much lass than the thermal conductivity of fat, so the temperature gradient for the air must be much larger to achieve the same heat current. So, most of the temperature difference is across the air layer.

17.99.   **IDENTIFY:**   Apply $H = kA\dfrac{T_{\text{H}} - T_{\text{C}}}{L}$.

SET UP: For the glass use $L = 12.45 \text{ cm}$, to account for the thermal resistance of the air films on either side of the glass.

EXECUTE: **(a)** $H = (0.120 \text{ W/m} \cdot \text{K}) (2.00 \times 0.95 \text{ m}^2) \left(\dfrac{28.0 \text{ C}°}{5.0 \times 10^{-2} \text{ m} + 1.8 \times 10^{-2} \text{ m}} \right) = 93.9 \text{ W}.$

(b) The heat flow through the wood part of the door is reduced by a factor of $1 - \dfrac{(0.50)^2}{(2.00 \times 0.95)} = 0.868,$

so it becomes 81.5 W. The heat flow through the glass is

$H_{\text{glass}} = (0.80 \text{ W/m} \cdot \text{K})(0.50 \text{ m})^2 \left(\dfrac{28.0 \text{ C}°}{12.45 \times 10^{-2} \text{ m}} \right) = 45.0 \text{ W},$ and so the ratio is $\dfrac{81.5 + 45.0}{93.9} = 1.35.$

EVALUATE: The single-pane window produces a significant increase in heat loss through the door. (See Problem 17.101).

17.100. **IDENTIFY:** This problem deals with thermal stress.

SET UP: The metal bar tends to expand as its temperature rises from 0°C to T, but it cannot do this because it is confined by the concrete slab which does not expand appreciably. The force F on the bar due to this thermal stress is the normal force for the static friction force that supports the bar and the container it is lifting. Our target variable is the mass m of the heaviest container this system can support, and the thermal stress is $F / A = -Y\alpha\Delta T$.

EXECUTE: $f_S = w_{\text{bar}} + w_{\text{container}}$. The force magnitude is $F = AY\alpha\Delta T = AY\alpha T$. For the heaviest container, static friction is a maximum, so $f_S = \mu_S n$, where $n = F$. The area A is the area of contact between the slab and the cylindrical bar, so $A = 2\pi r d$. Using M for the mass of the bar and m for the mass of the container, we have $\mu_S(2\pi r d)Y\alpha T = Mg + mg$. Solving for m gives

$m = M + 2\pi\mu_S r d Y\alpha T / g.$

EVALUATE: The slab would need to be very strong to support a reasonably heavy container (including its contents).

17.101. **IDENTIFY and SET UP:** Use H written in terms of the thermal resistance R: $H = A\Delta T / R$, where $R = L/k$ and $R = R_1 + R_2 + \dots$ (additive).

EXECUTE: <u>single pane:</u> $R_s = R_{\text{glass}} + R_{\text{film}}$, where $R_{\text{film}} = 0.15 \text{ m}^2 \cdot \text{K/W}$ is the combined thermal resistance of the air films on the room and outdoor surfaces of the window.

$R_{\text{glass}} = L/k = (4.2 \times 10^{-3} \text{ m})/(0.80 \text{ W/m} \cdot \text{K}) = 0.00525 \text{ m}^2 \cdot \text{K/W}$

Thus $R_s = 0.00525 \text{ m}^2 \cdot \text{K/W} + 0.15 \text{ m}^2 \cdot \text{K/W} = 0.1553 \text{ m}^2 \cdot \text{K/W}.$

<u>double pane:</u> $R_d = 2R_{\text{glass}} + R_{\text{air}} + R_{\text{film}}$, where R_{air} is the thermal resistance of the air space between the panes. $R_{\text{air}} = L/k = (7.0 \times 10^{-3} \text{ m})/(0.024 \text{ W/m} \cdot \text{K}) = 0.2917 \text{ m}^2 \cdot \text{K/W}$

Thus $R_d = 2(0.00525 \text{ m}^2 \cdot \text{K/W}) + 0.2917 \text{ m}^2 \cdot \text{K/W} + 0.15 \text{ m}^2 \cdot \text{K/W} = 0.4522 \text{ m}^2 \cdot \text{K/W}$

$H_s = A\Delta T / R_s, H_d = A\Delta T / R_d$, so $H_s / H_d = R_d / R_s$ (since A and ΔT are same for both)

$H_s / H_d = (0.4522 \text{ m}^2 \cdot \text{K/W})/(0.1553 \text{ m}^2 \cdot \text{K/W}) = 2.9$

EVALUATE: The heat loss is about a factor of 3 less for the double-pane window. The increase in R for a double pane is due mostly to the thermal resistance of the air space between the panes.

17.102. **IDENTIFY:** Apply $H = \dfrac{kA\Delta T}{L}$ to each rod. Conservation of energy requires that the heat current through the copper equals the sum of the heat currents through the brass and the steel.

SET UP: Denote the quantities for copper, brass and steel by 1, 2, and 3, respectively, and denote the temperature at the junction by T_0.

EXECUTE: **(a)** $H_1 = H_2 + H_3$. Using $H = kA(T_H - T_C)/L$ and dividing by the common area gives

$\frac{k_1}{L_1}(100°C - T_0) = \frac{k_2}{L_2}T_0 + \frac{k_3}{L_3}T_0$. Solving for T_0 gives $T_0 = \frac{(k_1/L_1)}{(k_1/L_1) + (k_2/L_2) + (k_3/L_3)}(100°C)$.

Substitution of numerical values gives $T_0 = 78.4°C$.

(b) Using $H = \frac{kA}{L}\Delta T$ for each rod, with $\Delta T_1 = 21.6\ C°$, $\Delta T_2 = \Delta T_3 = 78.4\ C°$ gives

$H_1 = 12.8$ W, $H_2 = 9.50$ W and $H_3 = 3.30$ W.

EVALUATE: In part (b), H_1 is seen to be the sum of H_2 and H_3.

17.103. **IDENTIFY:** The jogger radiates heat but the air radiates heat back into the jogger.

SET UP: The emissivity of a human body is taken to be 1.0. In the equation for the radiation heat current, $H_{net} = Ae\sigma(T^4 - T_s^4)$, the temperatures must be in kelvins.

EXECUTE: (a) $P_{jog} = (0.80)(1300\ W) = 1.04 \times 10^3$ J/s.

(b) $H_{net} = Ae\sigma(T^4 - T_s^4)$, which gives

$H_{net} = (1.85\ m^2)(1.00)(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)([306\ K]^4 - [313\ K]^4) = -87.1$ W. The person gains 87.1 J of heat each second by radiation.

(c) The total excess heat per second is 1040 J/s $+ 87$ J/s $= 1130$ J/s.

(d) In 1 min $= 60$ s, the runner must dispose of $(60\ s)(1130\ J/s) = 6.78 \times 10^4$ J. If this much heat goes to evaporate water, the mass m of water that evaporates in one minute is given by $Q = mL_v$, so

$m = \frac{Q}{L_v} = \frac{6.78 \times 10^4\ J}{2.42 \times 10^6\ J/kg} = 0.028$ kg $= 28$ g.

(e) In a half-hour, or 30 minutes, the runner loses $(30\ min)(0.028\ kg/min) = 0.84$ kg. The runner must drink 0.84 L, which is $\frac{0.84\ L}{0.750\ L/bottle} = 1.1$ bottles.

EVALUATE: The person *gains* heat by radiation since the air temperature is greater than his skin temperature.

17.104. **IDENTIFY:** The nonmechanical part of the basal metabolic rate (i.e., the heat) leaves the body by radiation from the surface.

SET UP: In the radiation equation, $H_{net} = Ae\sigma(T^4 - T_s^4)$, the temperatures must be in kelvins; $e = 1.0$, $T = 30°C = 303$ K, and $T_s = 18°C = 291$ K. Call the basal metabolic rate BMR.

EXECUTE: (a) $H_{net} = Ae\sigma(T^4 - T_s^4)$.

$H_{net} = (2.0\ m^2)(1.0)(5.67 \times 10^{-8}\ W/m^2 \cdot K^4)([303\ K]^4 - [291\ K]^4) = 140$ W.

(b) $(0.80)BMR = 140$ W, so BMR $= 180$ W.

EVALUATE: If the emissivity of the skin were less than 1.0, the body would radiate less so the BMR would have to be lower than we found in (b).

17.105. **(a) IDENTIFY and EXECUTE:** Heat must be conducted from the water to cool it to 0°C and to cause the phase transition. The entire volume of water is not at the phase transition temperature, just the upper surface that is in contact with the ice sheet.

(b) IDENTIFY: The heat that must leave the water in order for it to freeze must be conducted through the layer of ice that has already been formed.

SET UP: Consider a section of ice that has area A. At time t let the thickness be h. Consider a short time interval t to $t + dt$. Let the thickness that freezes in this time be dh. The mass of the section that freezes in the time interval dt is $dm = \rho\ dV = \rho A\ dh$. The heat that must be conducted away from this mass of water to freeze it is $dQ = dmL_f = (\rho AL_f)dh$. $H = dQ/dt = kA(\Delta T/h)$, so the heat dQ conducted in time

dt throughout the thickness *h* that is already there is $dQ = kA\left(\dfrac{T_H - T_C}{h}\right)dt$. Solve for *dh* in terms of *dt*

and integrate to get an expression relating *h* and *t*.

EXECUTE: Equate these expressions for *dQ*.

$$\rho A L_f dh = kA\left(\frac{T_H - T_C}{h}\right)dt$$

$$h\,dh = \left(\frac{k(T_H - T_C)}{\rho L_f}\right)dt$$

Integrate from $t = 0$ to time *t*. At $t = 0$ the thickness *h* is zero.

$$\int_0^h h\,dh = [k(T_H - T_C)/\rho L_f]\int_0^t dt$$

$$\tfrac{1}{2}h^2 = \frac{k(T_H - T_C)}{\rho L_f}t \text{ and } h = \sqrt{\frac{2k(T_H - T_C)}{\rho L_f}}\sqrt{t}$$

The thickness after time *t* is proportional to $\sqrt{t}$.

(c) The expression in part (b) gives

$$t = \frac{h^2 \rho L_f}{2k(T_H - T_C)} = \frac{(0.25 \text{ m})^2(920 \text{ kg/m}^3)(334 \times 10^3 \text{ J/kg})}{2(1.6 \text{ W/m} \cdot \text{K})(0°C - (-10°C))} = 6.0 \times 10^5 \text{ s}$$

$t = 170$ h.

(d) Find *t* for $h = 40$ m. *t* is proportional to h^2, so $t = (40 \text{ m}/0.25 \text{ m})^2(6.00 \times 10^5 \text{ s}) = 1.5 \times 10^{10}$ s. This is about 500 years. With our current climate this will not happen.

EVALUATE: As the ice sheet gets thicker, the rate of heat conduction through it decreases. Part (d) shows that it takes a very long time for a moderately deep lake to totally freeze.

17.106. **IDENTIFY:** $I_1 r_1^2 = I_2 r_2^2$. Apply $H = Ae\sigma T^4$ to the sun.

SET UP: $I_1 = 1.50 \times 10^3$ W/m² when $r = 1.50 \times 10^{11}$ m.

**EXECUTE:** **(a)** The energy flux at the surface of the sun is

$$I_2 = (1.50 \times 10^3 \text{ W/m}^2)\left(\frac{1.50 \times 10^{11} \text{ m}}{6.96 \times 10^8 \text{ m}}\right)^2 = 6.97 \times 10^7 \text{ W/m}^2.$$

(b) Solving $H = Ae\sigma T^4$ with $e = 1$, $T = \left[\dfrac{H}{A}\dfrac{1}{\sigma}\right]^{\frac{1}{4}} = \left[\dfrac{6.97 \times 10^7 \text{ W/m}^2}{5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4}\right]^{\frac{1}{4}} = 5920$ K.

EVALUATE: The total power output of the sun is $P = 4\pi r_2^2 I_2 = 4 \times 10^{26}$ W.

17.107. **IDENTIFY and SET UP:** Use $H_{net} = e\sigma A(T^4 - T_s^4)$ to find the net heat current into the can due to radiation. Use $Q = Ht$ to find the heat that goes into the liquid helium, set this equal to *mL* and solve for the mass *m* of helium that changes phase.

EXECUTE: Calculate the net rate of radiation of heat from the can. $H_{net} = Ae\sigma(T^4 - T_s^4)$.
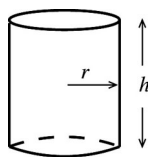
The surface area of the cylindrical can is $A = 2\pi rh + 2\pi r^2$.
(See Figure 17.107.)

Figure 17.107

$A = 2\pi r(h+r) = 2\pi(0.045 \text{ m})(0.250 \text{ m} + 0.045 \text{ m}) = 0.08341 \text{ m}^2$.

$H_{\text{net}} = (0.08341 \text{ m}^2)(0.200)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)((4.22 \text{ K})^4 - (77.3 \text{ K})^4)$

$H_{\text{net}} = -0.0338 \text{ W}$ (the minus sign says that the net heat current is into the can). The heat that is put into the can by radiation in one hour is $Q = -(H_{\text{net}})t = (0.0338 \text{ W})(3600 \text{ s}) = 121.7 \text{ J}$. This heat boils a mass

m of helium according to the equation $Q = mL_f$, so $m = \dfrac{Q}{L_f} = \dfrac{121.7 \text{ J}}{2.09 \times 10^4 \text{ J/kg}} = 5.82 \times 10^{-3} \text{ kg} = 5.82 \text{ g}$.

EVALUATE: In the expression for the net heat current into the can the temperature of the surroundings is raised to the fourth power. The rate at which the helium boils away increases by about a factor of $(293/77)^4 = 210$ if the walls surrounding the can are at room temperature rather than at the temperature of the liquid nitrogen.

17.108. **IDENTIFY:** We have blackbody radiation. The sphere at 41.0°C (314 K) radiates into the box, but the box at 30.0°C (303 K) radiates back into the sphere. All temperatures must be in kelvins.

SET UP: The rate at which heat is radiated by a blackbody in surroundings is $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$. The target variable is the emissivity e for part (a). We know that $H_{\text{net}} = 0.660$ J/s for the sphere, and $A = 4\pi r^2$ for a sphere.

EXECUTE: **(a)** Using $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$, we put in the numbers and solve for the emissivity e. Using $H_{\text{net}} = 0.660$ J/s, $A = 4\pi r^2 = 4\pi(0.0320 \text{ m})^2 = 0.012868 \text{ m}^2$, $T = 314$ K, $T_S = 303$ K, and $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$, we get $e = 0.700$. Note that this number has no units.

(b) Using $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$, we get

$H = (0.700)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.012868 \text{ m}^2)[(355 \text{ K})^4 - (303 \text{ K})^4] = 3.81 \text{ J/s} = 3.81 \text{ W}$.

EVALUATE: The ratio of powers is (3.81 W)/(0.660 W) = 5.77. This is much less than 2^4, which is 16. The temperature must be in kelvins, so a temperature of 82°C (355 K) is not double 41°C (314 K). The temperature ratio is only 355/314 = 1.13.

17.109. **IDENTIFY:** The latent heat of fusion L_f is defined by $Q = mL_f$ for the solid $\rightarrow$ liquid phase transition. For a temperature change, $Q = mc\Delta T$.

SET UP: At $t = 1$ min the sample is at its melting point and at $t = 2.5$ min all the sample has melted.

EXECUTE: **(a)** It takes 1.5 min for all the sample to melt once its melting point is reached and the heat input during this time interval is $(1.5 \text{ min})(10.0 \times 10^3 \text{ J/min}) = 1.50 \times 10^4 \text{ J}$. $Q = mL_f$.

$L_f = \dfrac{Q}{m} = \dfrac{1.50 \times 10^4 \text{ J}}{0.500 \text{ kg}} = 3.00 \times 10^4 \text{ J/kg}$.

(b) The liquid's temperature rises 30 C° in 1.5 min. $Q = mc\Delta T$.

$c_{\text{liquid}} = \dfrac{Q}{m\Delta T} = \dfrac{1.50 \times 10^4 \text{ J}}{(0.500 \text{ kg})(30 \text{ C°})} = 1.00 \times 10^3 \text{ J/kg} \cdot \text{K}$.

The solid's temperature rises 15 C° in 1.0 min. $c_{\text{solid}} = \dfrac{Q}{m\Delta T} = \dfrac{1.00 \times 10^4 \text{ J}}{(0.500 \text{ kg})(15 \text{ C°})} = 1.33 \times 10^3 \text{ J/kg} \cdot \text{K}$.

EVALUATE: The specific heat capacities for the liquid and solid states are different. The values of c and L_f that we calculated are within the range of values in Tables 17.3 and 17.4.

17.110. **IDENTIFY:** The heat lost by the water is equal to the heat gained by the liquid and the cup. The specific heat capacities do not change over the temperature ranges involved. No phase changes are involved.

SET UP: $Q = mc\,\Delta T$, and $Q_L + Q_m + Q_w = 0$, where L is for the liquid, m is for the metal, and w is for water.

EXECUTE: For the first experiment, 0.500 kg of the liquid are used. $Q_L + Q_m + Q_w = 0$ gives

$0 = (0.500 \text{ kg})c_L(58.1°C - 20.0°C) + (0.200 \text{ kg})c_m(58.1°C - 20.0°C)$

$+ (0.500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(58.1°\text{C} - 80.0°\text{C})$

For the second experiment, 1.00 kg of the liquid is used. $Q_L + Q_m + Q_w = 0$ gives
$0 = (1.00 \text{ kg})c_L(49.3°\text{C} - 20.0°\text{C}) + (0.200 \text{ kg})c_m(49.3°\text{C} - 20.0°\text{C})$
$+ (0.500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(49.3°\text{C} - 80.0°\text{C})$

The two equations from the two experiments simplify to
$19.050c_L + 7.620c_m = 45,880.5$
$29.3c_L + 5.86c_m = 64,316.5$
Solving these two equations simultaneously gives $c_m = 1067 \text{ J/kg} \cdot \text{K}$, which rounds to $1070 \text{ J/kg} \cdot \text{K}$, and c_L
$= 1981.6 \text{ J/kg} \cdot \text{K}$, which rounds to $1980 \text{ J/kg} \cdot \text{K}$.

EVALUATE: The liquid has about half the specific heat capacity of water. The metal has a specific heat capacity of $1070 \text{ J/kg} \cdot \text{K}$, which is a bit more than that of aluminum. So both answers are physically reasonable.

17.111. **IDENTIFY:** At steady state, the heat current in both bars is the same when they are connected end-to-end. The heat to melt the ice is the heat conducted through the bars.

SET UP: $Q = mL$ and $H = kA\dfrac{T_H - T_C}{L}$.

EXECUTE: With bar A alone: 0.109 kg of ice melts in 45.0 min = (45.0)(60) s. Therefore the heat current is $H = mL_f/t = (0.109 \text{ kg})(334 \times 10^5 \text{ J/kg})/[(45.0)(60) \text{ s}] = 13.48 \text{ J/s} = 13.48 \text{ W}$. Applying this result to the heat flow in bar A gives $H = kA\dfrac{T_H - T_C}{L}$. Solving for k_A gives $k_A = HL/A(T_H - T_C)$. Numerically we get
$k_A = (13.48 \text{ W})(0.400 \text{ m})/[(2.50 \times 10^{-4} \text{ m}^2)(100 \text{ C}°)] = 215.7 \text{ W/m} \cdot \text{K}$, which rounds to $216 \text{ W/m} \cdot \text{K}$.
With the two bars end-to-end: The heat current is the same in both bars, so $H_A = H_B$. Using
$H = kA\dfrac{T_H - T_C}{L}$ for each bar, we get $\dfrac{k_A A(100°\text{C} - 62.4°\text{C})}{L} = \dfrac{k_B A(62.4°\text{C} - 0°\text{C})}{L}$. Using our result for
k_A and canceling A and L, we get $k_B = 130 \text{ W/m} \cdot \text{K}$.
EVALUATE: $k_A = 216 \text{ W/m} \cdot \text{K}$, which is slightly larger than that of aluminum, and $k_B = 130 \text{ W/m} \cdot \text{K}$, which is between that of aluminum and brass. Therefore these results are physically reasonable.

17.112. **IDENTIFY:** The heat of combustion of propane is used to melt ice, so a change of state is involved.
SET UP: The heat to melt ice is $Q = mL_f$. This heat comes from the combustion of propane, which releases 25.6 MJ/L. Only 30% of this heat is used to melt the ice. As the ice melts it is drawn off at 500 mL/min, so that is the rate at which the ice is melting. The propane tank holds 18 L of propane. One liter of water has a mass of 1.00 kg. We want to find out how long the propane tank can keep burning fuel before it runs out.
EXECUTE: The water melts at 500 L/min, which is 0.500 kg/min or 0.008333 kg/s. Therefore the rate at which the propane is providing heat to the ice is $Q/t = mL_f$. This gives
$Q/t = (0.008333 \text{ kg/s})(334 \times 10^3 \text{ J/kg}) = 2.783 \times 10^3 \text{ J/s}$. This heat comes from the combustion of propane. The total heat available from an 18-L tank is $(18 \text{ L})(25.6 \times 10^6 \text{ J/L})$, but only 30% of this heat is available to melt ice. So the available heat is $(0.30)(18 \text{ L})(25.6 \times 10^6 \text{ J/L}) = 1.3824 \times 10^8 \text{ J}$. To use up all the propane in the tank, $(Q/t)t = E_{\text{available}}$. Putting in the values we just found gives $(2.783 \times 10^3 \text{ J/s})t$
$= 1.3824 \times 10^8 \text{ J}$, so $t = 5.0 \times 10^4 \text{s} = 830 \text{ min} = 14 \text{ h}$.
EVALUATE: The tank would melt even more ice if more than 30% of its heat went to melting ice.

17.113. **IDENTIFY:** Heat flows through the frustrum of a cone shown in the figure in the textbook. We want to derive an expression for the heat current H through this cone.
SET UP and EXECUTE: Start with $H = kA\dfrac{T_H - T_C}{L}$ and follow the suggestions in the problem.
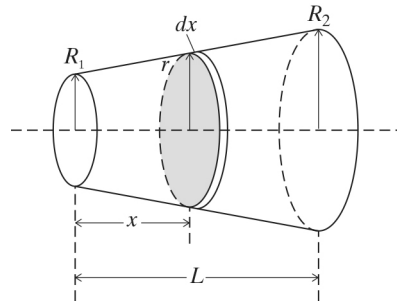
Figure 17.113

Figure 17.113 shown here shows the cone oriented along the x-axis. We break it up into tiny slabs of radius r and thickness dx, as shown. We write the heat current formula for differential differences in temperature and thickness. We also realize that at steady state H is the same at all the slabs, so it is constant. The equation for H becomes $H = kA\dfrac{dT}{dx}$ for the heat current through a slab of thickness dx.

The cross-sectional area is $A = \pi r^2$, but r changes with x. So we need to relate r to x. To do so, realize that the side of the cone is a straight line and its equation will give r as a function of x. Using the slope-intercept form of a straight-line equation, $r = Bx + C$. When $x = 0$, $r = R_1$ and when $x = L$, $r = R_2$. The first of these conditions tells us that $C = R_1$, and the second says that $B = \dfrac{R_2 - R_1}{L}$. We can now write the heat flow equation as $H = k\pi(Bx+C)^2 \dfrac{dT}{dx}$. Separating variables gives $\dfrac{Hdx}{(Bx+C)^2} = k\pi dT$.

Integrating gives $H\displaystyle\int_0^L \dfrac{dx}{(Bx+C)^2} = \displaystyle\int_{T_{\mathrm{H}}}^{T_{\mathrm{C}}} k\pi dT$. The temperature integral just gives $k\pi\Delta T$ but we need to complete the left-hand-side integral. To do so, let $Bx + C = u$, so $du = Bdx$. The integral becomes $\displaystyle\int \dfrac{(1/B)du}{u^2} = -\dfrac{1}{B}u^{-1} = -\dfrac{1}{Bu}$. Returning to the original variables, we have

$H\displaystyle\int_0^L \dfrac{dx}{(Bx+C)^2} = H\left(-\dfrac{1}{B(Bx+C)}\right)\Bigg|_0^L = -\dfrac{H}{B}\left[\dfrac{1}{BL+C} - \dfrac{1}{C}\right]$. Carrying out the algebra and putting in the values for B and C gives $HL\left(\dfrac{1}{R_1 R_2}\right) = k\pi\Delta T$. Solving for H gives $H = \dfrac{k\pi R_1 R_2}{L}(T_{\mathrm{H}} - T_{\mathrm{C}})$.

EVALUATE: If the cone were reduced to a solid cylinder of radius R_1, H would be $H_1 = k\pi R_1^2 \dfrac{\Delta T}{L}$, and if it were a solid cylinder of radius R_2, H would be $H_2 = k\pi R_2^2 \dfrac{\Delta T}{L}$. The value we found

$H = \dfrac{k\pi R_1 R_2}{L}(T_{\mathrm{H}} - T_{\mathrm{C}})$ is between these two extremes, so it is plausible.

17.114. **IDENTIFY:** The rate in (iv) is given by $H_{\mathrm{net}} = e\sigma A(T^4 - T_s^4)$, with $T = 309$ K and $T_s = 320$ K. The heat absorbed in the evaporation of water is $Q = mL$.

SET UP: $m = \rho V$, so $\dfrac{m}{V} = \rho$.

EXECUTE: **(a)** The rates are: (i) 280 W,
(ii) $(54 \text{ J/h} \cdot \text{C}° \cdot \text{m}^2)(1.5 \text{ m}^2)(11 \text{ C}°)/(3600 \text{ s/h}) = 0.248$ W,
(iii) $(1400 \text{ W/m}^2)(1.5 \text{ m}^2) = 2.10 \times 10^3$ W,

(iv) $(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.5 \text{ m}^2)((320 \text{ K})^4 - (309 \text{ K})^4) = 116 \text{ W}.$

The total is 2.50 kW, with the largest portion due to radiation from the sun.

(b) $\dfrac{P}{\rho L_v} = \dfrac{2.50 \times 10^3 \text{ W}}{(1000 \text{ kg/m}^3)(2.42 \times 10^6 \text{ J/kg} \cdot \text{K})} = 1.03 \times 10^{-6} \text{ m}^3/\text{s}.$ This is equal to 3.72 L/h.

(c) Redoing the above calculations with $e = 0$ and the decreased area gives a power of 945 W and a corresponding evaporation rate of 1.4 L/h. Wearing reflective clothing helps a good deal. Large areas of loose-weave clothing also facilitate evaporation.

EVALUATE: The radiant energy from the sun absorbed by the area covered by clothing is assumed to be zero, since $e \approx 0$ for the clothing and the clothing reflects almost all the radiant energy incident on it. For the same reason, the exposed skin area is the area used in applying $H_{\text{net}} = e\sigma A(T^4 - T_s^4)$.

17.115.   IDENTIFY: Apply the equation $H = kA(T_H - T_C)/L.$ For a cylindrical surface, the area A in this equation is a function of the distance r from the central axis, and the material must be considered as a series of shells with thickness dr and a temperature difference dT between the inside and outside of the shell. The heat current will be a constant, and must be found by integrating a differential equation.

SET UP: The surface area of the curved side of a cylinder is $2\pi rL.$ When $x \ll 1$, $\ln(1+x) \approx x.$

EXECUTE: (a) For a cylindrical shell, $H = kA(T_H - T_C)/L$ becomes $H = k(2\pi rL)\dfrac{dT}{dr}$ or $\dfrac{Hdr}{2\pi r} = kLdT.$

Between the limits $r = a$ and $r = b$, this integrates to $\dfrac{H}{2\pi}\ln(b/a) = kL(T_2 - T_1),$ or $H = \dfrac{2\pi kL(T_2 - T_1)}{\ln(b/a)}.$

(b) Using $H = k(2\pi rL)\dfrac{dT}{dr}$ from part (a), we integrate: $\displaystyle\int_a^r \dfrac{Hdr}{k2\pi rL} = -\int_{T_2}^T dT,$ which gives

$\dfrac{H}{2\pi kL}\ln(r/a) = -(T - T_2) = T_2 - T.$ Solving for T and using H from part (a) gives

$T = T_2 - \dfrac{H}{2\pi kL}\ln(r/a) = \dfrac{(T_2 - T_1)2\pi kL\ln(r/a)}{2\pi kL\ln(b/a)} = T_2 - \dfrac{(T_2 - T_1)\ln(r/a)}{\ln(b/a)},$ which can also be written as

$T(r) = T_2 + (T_1 - T_2)\dfrac{\ln(r/a)}{\ln(b/a)}.$

(c) For a thin-walled cylinder, $a \approx b$, so $\dfrac{b-a}{a} \ll 1.$ We can write $\dfrac{b}{a} = 1 + \dfrac{b-a}{a},$ and the log term

becomes $\ln\left(\dfrac{b}{a}\right) = \ln\left(1 + \dfrac{b-a}{a}\right) \approx \dfrac{b-a}{a}$ using $\ln(1+x) \approx x$ for $x \ll 1.$ Therefore the rate of heat flow

becomes $H = \dfrac{(T_2 - T_1)2\pi kL}{\dfrac{b-a}{a}} = k(2\pi aL)\dfrac{T_2 - T_1}{b-a}.$ This is equivalent to Eq. (17.21) in which $A = 2\pi aL,$

which is the surface area of the thin cylindrical shell of radius a and length $L.$

(d) For steady-state heat flow, the rate of flow through the cork is the same as through the Styrofoam. Using our result from part (a), we have $H_C = H_S$, which gives

$(140°\text{C} - T)2\pi k_C L/(\ln 4/2) = (T - 15°\text{C})2\pi k_S L/(\ln 6/4)$

Cancelling $2\pi L$ and using the given values for k_C and k_S, we get $T = 73°\text{C}.$

(e) Use the result of part (a) for $H.$

$H = \dfrac{(140°\text{C} - 73°\text{C})2\pi(0.0400 \text{ W/m} \cdot \text{K})(2.00 \text{ m})}{\ln(4/2)} = 49 \text{ W}.$

EVALUATE: As a check, calculate H in the Styrofoam. $\Delta T = 73°\text{C} - 15°\text{C} = 58 \text{ C}°.$

$H_S = \dfrac{2\pi(2.00 \text{ m})(0.027 \text{ W/m} \cdot \text{K})}{\ln(6.00/4.00)}(58 \text{ C}°) = 49 \text{ W}.$ This is the same as we just found, which it should be

for steady-state flow.

17.116. **IDENTIFY:** The cryoprotectant must cool from 22°C to –20°C, then it must freeze, and finally it must cool from –20°C to 77 K (–196°C).

SET UP: We use $Q = mc\,\Delta T$ and $Q = mL_{\mathrm{f}}$. $Q_{\mathrm{tot}} = Q_1 + Q_2 + Q_3$.

EXECUTE: $Q_1 = mc\,\Delta T = (35 \times 10^{-3}\ \mathrm{kg})(4500\ \mathrm{J/kg \cdot K})(42\ \mathrm{C°}) = 6.6\ \mathrm{kJ}$

$Q_2 = mL_{\mathrm{f}} = (35 \times 10^{-3}\ \mathrm{kg})(2.80 \times 10^5\ \mathrm{J/kg}) = 9.8\ \mathrm{kJ}$
$Q_3 = mc\,\Delta T = (35 \times 10^{-3}\ \mathrm{kg})(2000\ \mathrm{J/kg \cdot K})[-196°C - (-20°C)] = 12.3\ \mathrm{kJ}$

$Q_{\mathrm{tot}} = 6.6\ \mathrm{kJ} + 9.8\ \mathrm{kJ} + 12.3\ \mathrm{kJ} = 28.7\ \mathrm{kJ} \approx 29\ \mathrm{kJ} = 2.9 \times 10^4\ \mathrm{J}$, which is choice (b).

EVALUATE: This is a large amount of heat for such a small sample, but it must change its temperature from 22°C to –196°C and also freeze.

17.117. **IDENTIFY** and **SET UP:** The graph shows that the specific heat of the solid decreases with temperature, so its average value is less than the 2.0×10^3 J/kg·K shown in the table.

EXECUTE: Since the average specific heat is less than the value in the table, less heat will need to come out of it to bring it into equilibrium with the cold plate. Therefore a shorter time will be needed for it to come to equilibrium with the cold plate, which is choice (a).

EVALUATE: The average value of the specific heat is about 1500 J/kg·K between –20°C and –200°C, so the difference in time could be important enough to be concerned about. As we saw in the previous problem, the heat to cool the cryoprotectant from –20°C down to –196°C was the largest contribution to the total heat.

17.118. **IDENTIFY:** The rate of heat conduction should be the same in both cases, but the area in the area in the second case is 4 times the area in the first case since the linear dimensions are both doubled.

SET UP: $H = kA\dfrac{T_{\mathrm{H}} - T_{\mathrm{C}}}{L}$, $H_1 = H_2$, $A_2 = 4A_1$, k and $T_{\mathrm{H}} - T_{\mathrm{C}}$ are the same in both cases.

EXECUTE: Calculate H_1 and H_2 and equate them: $kA_1\dfrac{\Delta T}{L_1} = kA_2\dfrac{\Delta T}{L_2} = k(4A_1)\dfrac{\Delta T}{L_2}$. Cancelling common factors gives $L_2 = 4L_1$, so the layer should be 4 times as thick as it was for the smaller plates, which is choice (d).

EVALUATE: If we doubled only one dimension, then L_2 would have to be only twice as thick as for the smaller plates.

17.119. **IDENTIFY** and **SET UP:** Heat from the cryoprotectant *and* the environment enters the cold plate. You measure the amount of heat that enters the cold plate, and assume that it all came from the cryoprotectant.

EXECUTE: You think that more heat entered the plate from the cryoprotectant than actually did so. This will make you think that the specific heat of the cryoprotectant is greater than it actually is, which is choice (a).

EVALUATE: Heat from the environment could also be entering the cryoprotectant, but since the cold plate is on average colder than the cryoprotectant, more heat will enter the cold plate than will enter the cryoprotectant, so the two effects will not cancel each other out. You will still measure a specific heat that is greater than the actual value.