Circuits
-
Voltage and current laws

NYU 上海 SHANGHAI 纽约大学

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Context

上海纽约大学 NYU SHANGHAI

Analyzing a circuit

- we need to determine the **voltages** and **currents** in the circuit
- for resistors: once we know the voltage, we know the current (or conversely)
- for sources: more complicated. . .

Any kind of circuit can be analyzed

Additional rules

- Kirchhoff's current law (KCL) charge conservation
- Kirchhoff's voltage law (KVL) energy conservation.

Context

Wire

The wires connecting the different elements of a circuit will be considered to be **zero-resistance conductors** ideal.

It means that there is **no voltage drop** on a wire (whatever the current value)

Node

Definition: **a point** at which 2 or more elements are connected to

they share the same voltage.

Path/Branch/Loop

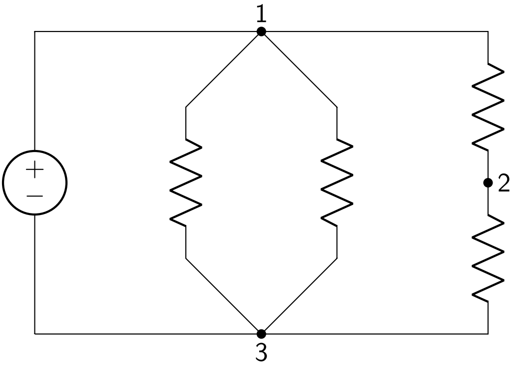
- Path: starting at a node, going through different connected elements and stopping at another node can be multiple elements
- Branch: single path, composed of one element only one path & one element
- Loop: path that stops at the starting node (closed path)

Context

The # elements should equal the # of branches

How many nodes and branches?

5 elements
↓
5 branches



6 Loops

3 nodes

from 1 to 3. 4 paths

Kirchhoff's current law

a restatement of charge conservation.

KCL

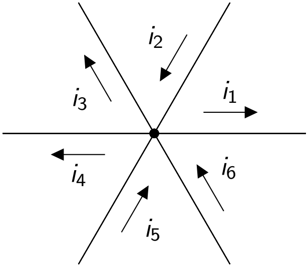
The algebraic sum of the currents **entering a node** (or a closed boundary) is **zero**

Interpretation

Conservation of charge:

- all the electrons have to go somewhere
- the current that goes in, has to come out

Kirchhoff's current law



KCL formula

$$-i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$$

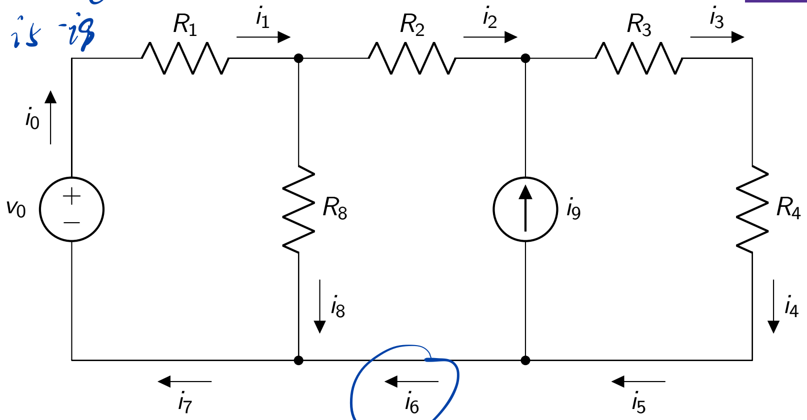
or

$$i_2 + i_5 + i_6 = i_1 + i_3 + i_4$$

Kirchhoff's current law

$\begin{cases} i_6 = i_7 - i_8 \\ i_6 = i_5 - i_9 \end{cases}$

i_6 is not necessary



How many nodes? How many branches?
Apply KCL to each node

5 nodes.
7 branches

$i_6 + i_9 = i_5$

$i_6 + i_8 = i_7$

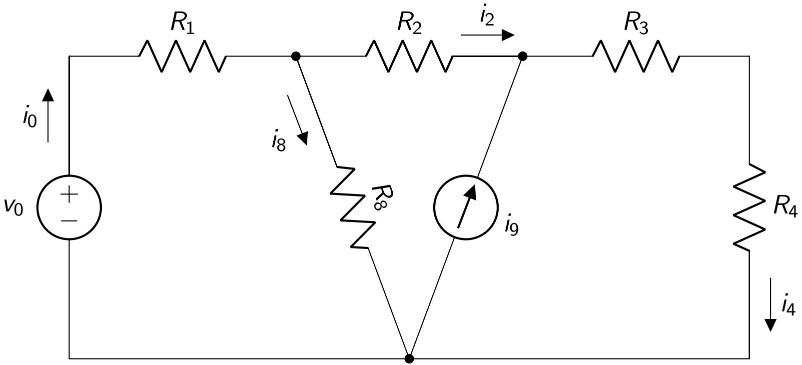
Kirchhoff's current law

- $i_0 = i_1$
- $i_1 = i_2 + i_8$
- $i_2 + i_9 = i_3$
- $i_3 = i_4$
- $i_4 = i_5$
- $i_5 = i_6 + i_9$
- $i_6 + i_8 = i_7$
- $i_7 = i_0$

Are there some redundant variables?
How about i_6?

Kirchhoff's current law

Better annotation



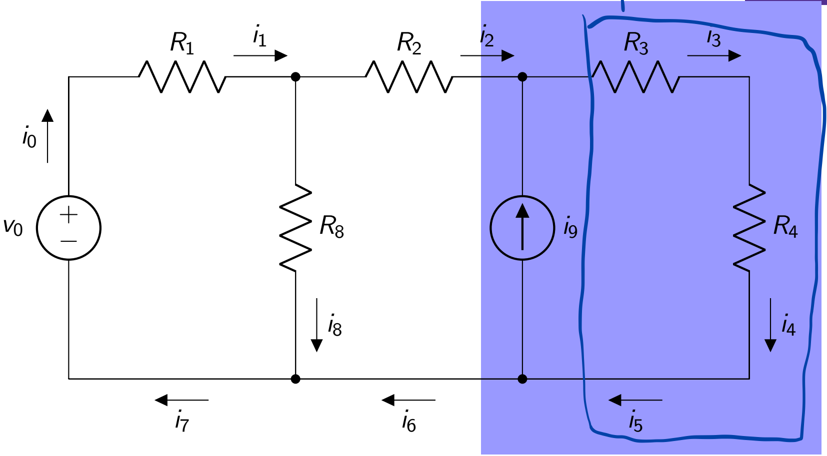
Kirchhoff's current law

- $i_0 = i_2 + i_8$
- $i_2 + i_9 = i_4$
- $i_4 + i_8 = i_0 + i_9$

Actually, only 2 equations are necessary. The third one can be obtained from the other two.

Kirchhoff's current law

$i_3 = i_5$



KCL on closed boundaries: $i_2 = i_6$

Kirchhoff's voltage law

$$\oint \vec{E} \cdot d\vec{l} = 0$$

KVL

The algebraic sum of **voltages** around a **loop** are **zero**

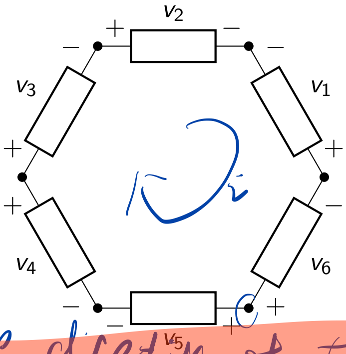
Interpretation

Conservation of energy:

- voltages are differences of electric potentials
- around a loop, the difference in electrical potential is zero

Kirchhoff's voltage law

Better follow the PSC



PSC followed:
$-v_1 - v_4 - v_6 + v_5 + v_3 + v_2 = 0$

we can only assume one direction of the current

KVL formula

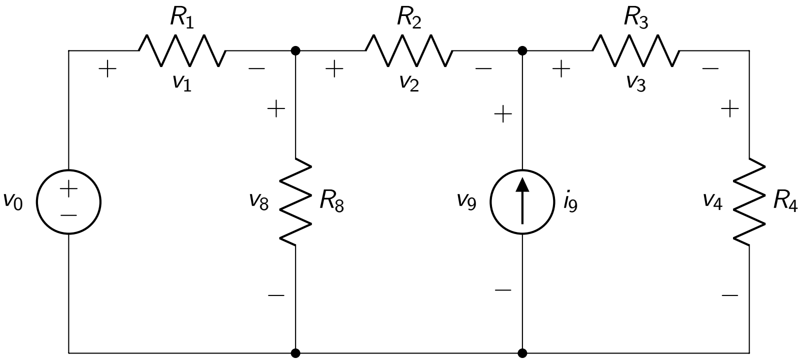
but the polarity can be defined by us

$$-v_1 + v_2 + v_3 - v_4 + v_5 - v_6 = 0$$

or

$$v_1 + v_4 + v_6 = v_2 + v_3 + v_5$$

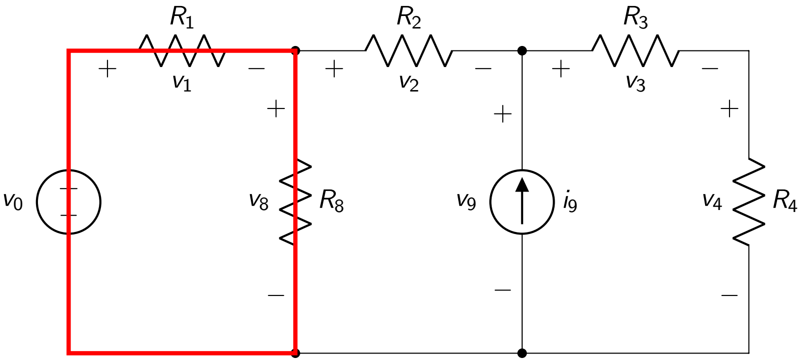
Kirchhoff's voltage law



6 Loops

How many loops?
Apply KVL to each loop

Kirchhoff's voltage law



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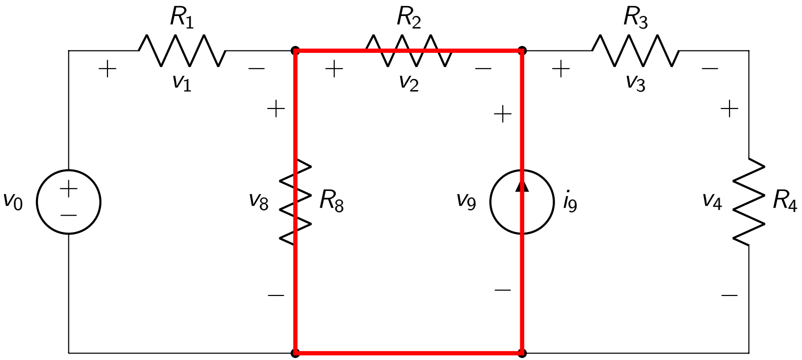
Apply KVL to each loop

Kirchhoff's voltage law



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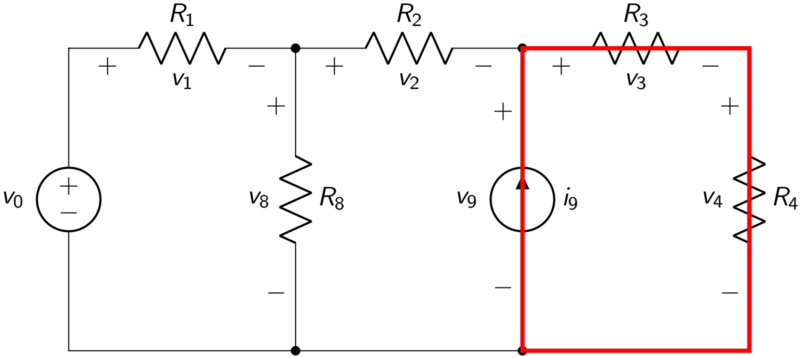
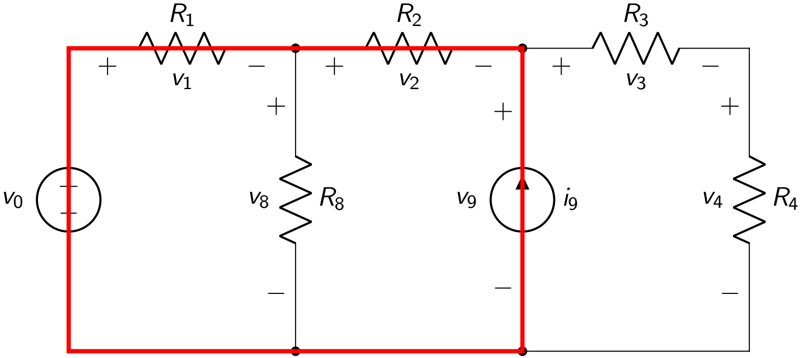
Kirchhoff's voltage law



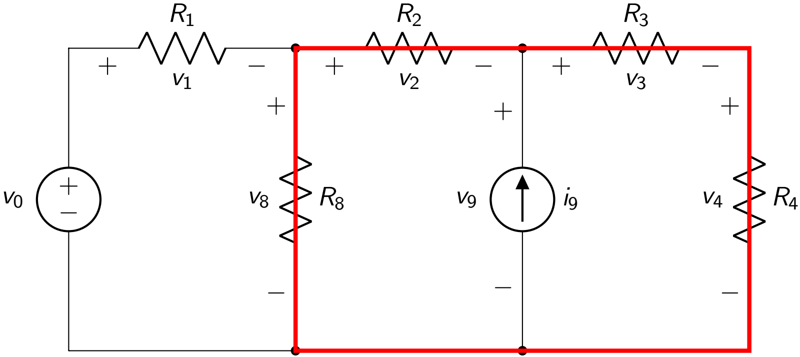
How many loops?
Apply KVL to each loop

Kirchhoff's voltage law



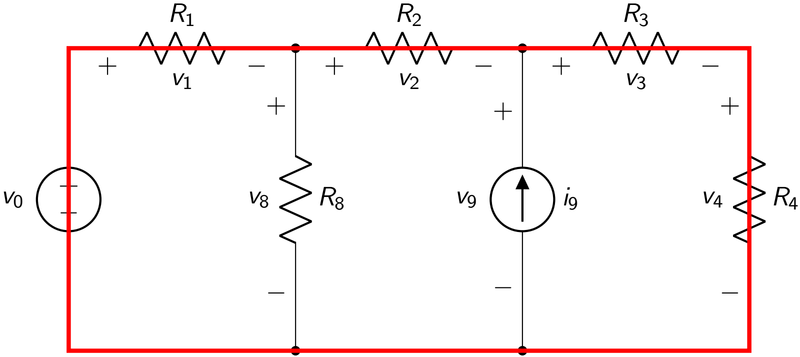
How many loops?
Apply KVL to each loop

Kirchhoff's voltage law



How many loops?
Apply KVL to each loop

Kirchhoff's voltage law



How many loops?
Apply KVL to each loop

Kirchhoff's voltage law

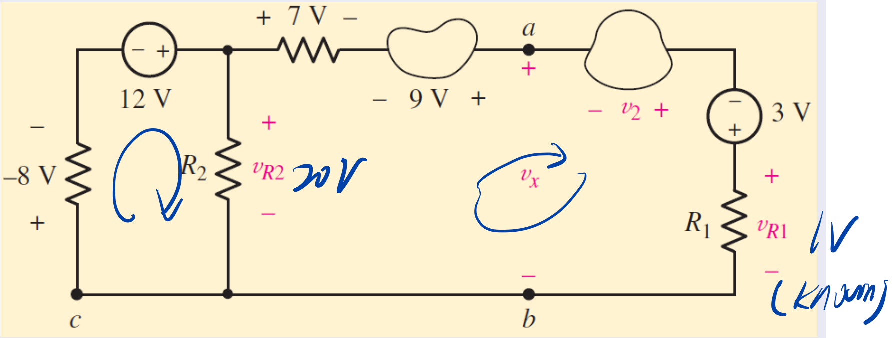
- $v_0 = v_1 + v_8$
- $v_8 = v_2 + v_9$
- $v_9 = v_3 + v_4$
- $v_0 = v_1 + v_2 + v_9$
- $v_8 = v_2 + v_3 + v_4$
- $v_0 = v_1 + v_2 + v_3 + v_4$

Actually, only 3 equations are necessary. The three others can be deduced from the other ones.

Examples I

$-24V$

When $v_{R1} = 1V$, determine v_{R2} and v_2.

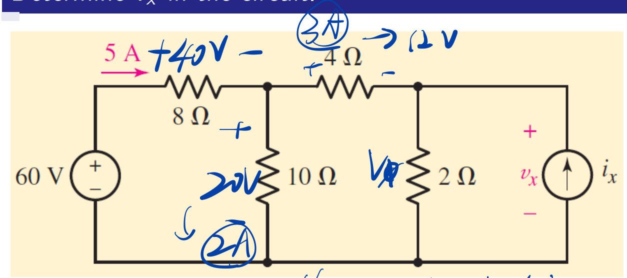


$-1 + V_{R_2} - 7V + 9V + V_2 + 3V = 0$

Examples II

Combine ohm's Law. KVL& KCL.

Determine v_x in the circuit.



we're actually calculating voltage across the 2Ω. resistor.

∴ finally $V_x = 8V$

How to analyze circuits by KCL and KVL?

1. Identify the goal of the problem.
   Label i or v on the circuit diagram.
2. Collect the known information.
   The nodes, branches, ...
3. Devise a plan
   KCL and KVL and Ohm's law
4. Construct an appropriate set of equations.
   Summing the currents flowing into the node.
5. Determine if additional information is required.
6. Attempt a solution
7. Verify the solution. Is it reasonable or expected?
   It is always worth the effort to recheck our work.