NAME:

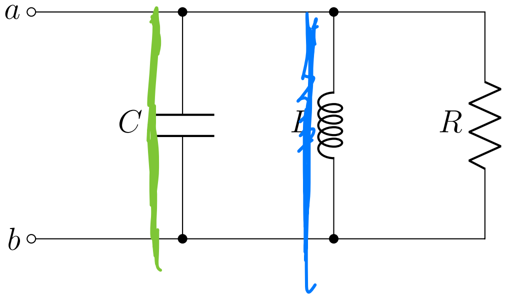
Homework 3
-
Sinusoidal analysis

Deadline: Wednesday, 11 May 2022, 11:55 PM

You can send your solutions in electronic version to NYU Brightspace/Assignment. **No extended deadline!**

Exercise 1 - Equivalent impedance/admittance

growth of functions



$\propto 0$ $\quad \infty$

$j\omega L$, $\frac{1}{j\omega C}$, R

$\propto \infty$ $\quad \propto 0$

$\omega \to 0$

- Determine the equivalent impedance of this network, as a function of the angular frequency ω.
- Which element is it equivalent to when $\omega \to 0$?
- Which element is it equivalent to when $\omega \to \infty$?
- Which element is it equivalent to when $\omega = \dfrac{1}{\sqrt{LC}}$?

$\omega \to \infty$

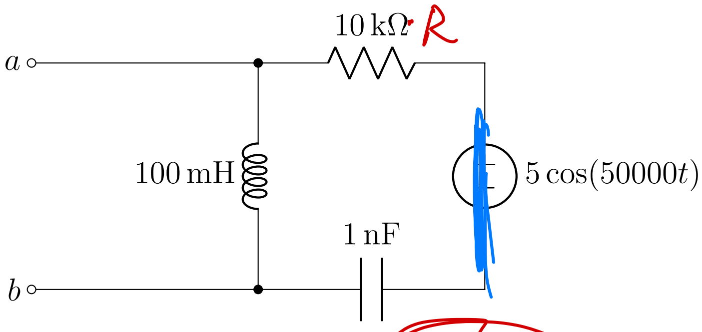
$$Z_{ab} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{R \cdot j\omega L}{\underbrace{j\omega L}_{\propto 0} + R - \underbrace{\omega^2 LC}_{\propto 0}}$$

$\omega \to 0 \qquad Z_{ab} \propto \dfrac{R \cdot j\omega L}{R} = j\omega L \qquad$ An inductor

$\omega \to \infty \qquad \dfrac{1}{R} \propto 0 \quad \dfrac{1}{j\omega L} \propto 0$ compared to $j\omega C \qquad Z_{ab} = \dfrac{1}{j\omega C} \qquad$ A capacitor

$\omega = \dfrac{1}{\sqrt{LC}} \qquad Z_{ab} = R \qquad$ A resistor

Exercise 2 - Thévenin equivalence

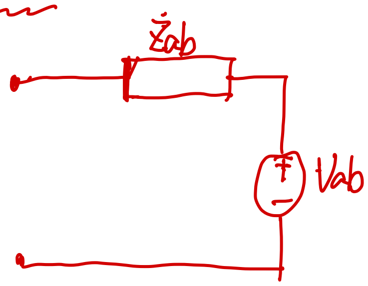


- Determine the equivalent Thévenin circuit in the phasor domain.

① $\omega = 50000$ rad/s $\quad V_s = 5\angle 0^\circ$ V $\quad Z_L = j\omega L = j5000\,\Omega \quad Z_C = \frac{1}{j\omega C} = -j2000\,\Omega$

$$V_{ab} = V_s \cdot \frac{Z_L}{R + Z_L + Z_C} = \frac{250}{109}j + \frac{75}{109}\ \text{V} =$$

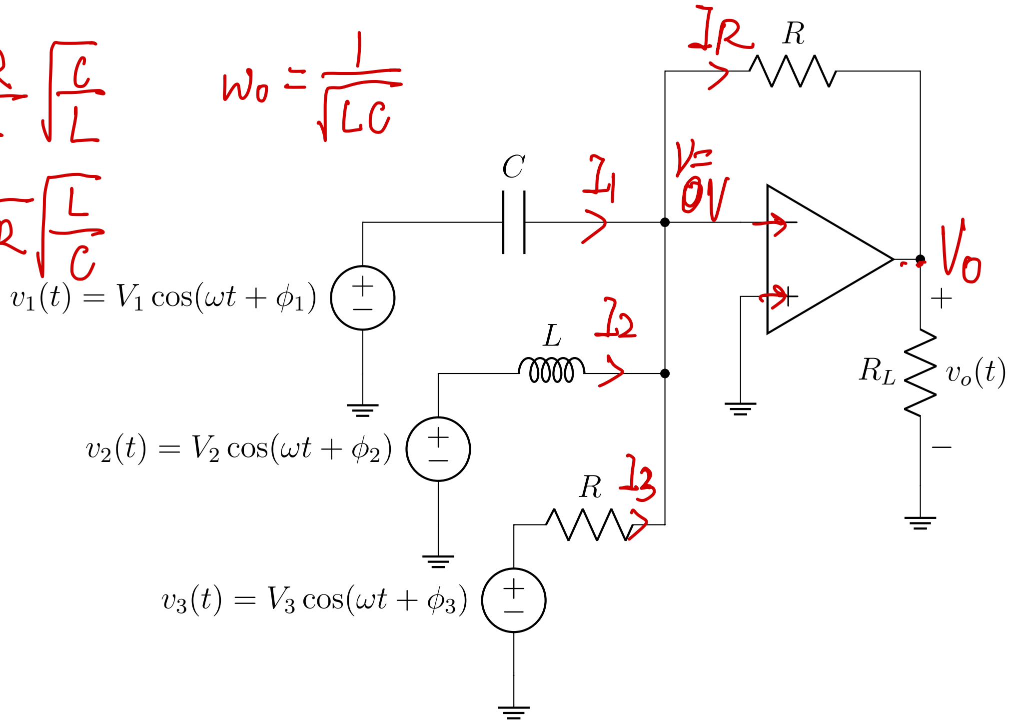
$$Z_{ab} = Z_L \parallel (R + Z_C) = 10^3 \cdot \left(\frac{25}{109} + j\frac{470}{109}\right)$$



Exercise 3 - Op Amp

series $\frac{R}{2}\sqrt{\frac{C}{L}}$

parallel $\frac{1}{2R}\sqrt{\frac{L}{C}}$



The operational amplifier is supposed to be ideal.

Questions:

1. Determine the phasor $\mathbf{V}_o$ of $v_o(t)$ as a function of R, L, C, ω, $\mathbf{V}_1$ ($V_1\underline{/\phi_1}$), $\mathbf{V}_2$ ($V_2\underline{/\phi_2}$) and $\mathbf{V}_3$ ($V_3\underline{/\phi_3}$)
2. Supposing that $v_1(t) = v_2(t)$, which angular frequency leads to $v_o(t) = -v_3(t)$?
3. Supposing that $v_2(t) = 0$ and $v_1(t) = v_3(t)$, which frequency leads to

$v_o(t) = \frac{\sqrt{2}}{2}V_1\cos(\omega t + \phi_1 + 225°)$ $V_o = \frac{\sqrt{2}}{2}V_1\angle\phi_1 + 225°$

1) $I_R = I_1 + I_2 + I_3 = \frac{0 - V_o}{R} = \frac{V_1}{\frac{1}{j\omega C}} + \frac{V_2}{j\omega L} + \frac{V_3}{R}$

$\Rightarrow V_o = -R\cdot\left(j\omega C V_1 + \frac{1}{j\omega L}V_2 + \frac{1}{R}V_3\right)$

2) $v_1(t) = v_2(t) \Rightarrow V_1 = V_2$ $V_o = -R\cdot\left(j\omega C V_1 + \frac{1}{j\omega L}V_1 + \frac{1}{R}V_3\right) = -V_3$

$j\omega C \cancel{V_1} = -\frac{1}{j\omega L}\cancel{V_1} \Rightarrow -j\omega L\cdot j\omega C = 1 \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

3) $V_2 = 0$ & $V_1 = V_3$ $V_o = -R\left(j\omega C V_1 + \frac{1}{R}V_1\right) = V_1(-1 - jR\omega C)$

$= \frac{\sqrt{2}}{2}V_1\angle\phi_1 + 225°$ $-\frac{1}{2}(1+j)V_1$ $-V_1(1+jR\omega C)$

Assume $\phi_1 = 0$ $V_o = \frac{\sqrt{2}}{2}V_1\angle 225° = V_1(-1 - jR\omega C) \Rightarrow \frac{\sqrt{2}}{2}\cdot\left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)V_1 \overset{?}{=} -V_1(1+jR\omega C)$

$\Rightarrow R\omega C = 1 \Rightarrow \omega = \frac{1}{RC}$