42

MOLECULES AND CONDENSED MATTER

VP42.3.1. **IDENTIFY:** This problem involves the rotational energy of a molecule.

SET UP and **EXECUTE:** **(a)** We want the reduced mass. Use the given masses in the equation

$$m_r = \frac{m_1 m_2}{m_1 + m_2}.$$

The result is $m_r = 1.240 \times 10^{-26}$ kg.

(b) We want the moment of inertia. Use $r_0 = 0.1154$ nm and the result of part (a). The result is

$I = m_r r_0^2 = 1.651 \times 10^{-46}$ kg·m^2.

(c) We want the energies of the rotational states $l = 0, 1, 2$.

$$E_l = l(l+1)\frac{\hbar^2}{2I}$$

$$E_0 = 0$$

$$E_1 = 1(1+1)\frac{\hbar^2}{2I} = 4.203 \times 10^{-4} \text{ eV} = 0.4203 \text{ meV}.$$

$$E_2 = 2(2+1)\frac{\hbar^2}{2I} = 1.261 \times 10^{-3} \text{ eV} = 1.261 \text{ meV}.$$

EVALUATE: Note that molecular rotational energies are of the order of *milli*electron-volts, whereas energy states for electrons in atoms are around a few eV.

VP42.3.2. **IDENTIFY:** We are dealing with the rotational energy levels of a molecule.

SET UP: The following equations apply:

$$E_l = l(l+1)\frac{\hbar^2}{2I}, \quad m_r = \frac{m_1 m_2}{m_1 + m_2}, \quad I = m_r r_0^2.$$

EXECUTE: **(a)** We want the moment of inertia. Solve the E_l equation for I for the $l = 2$ state.

$$I = \frac{3\hbar^2}{E_2} = \frac{3\hbar^2}{7.90 \text{ meV}} = 2.64 \times 10^{-47} \text{ kg·m}^2.$$

(b) We want the reduced mass. Use the two given masses in the equation for m_r, giving

$$m_r = 1.627 \times 10^{-27} \text{ kg}.$$

(c) We want the distance r_0 between the two nuclei. Solve $I = m_r r_0^2$ for r_0 and use the results of parts (a) and (b). This gives

$$r_0 = \sqrt{I/m_r} = 0.127 \text{ nm}.$$

EVALUATE: Note that the chlorine has only a small effect on the reduced mass which is essentially the mass of the hydrogen atom.

VP42.3.3. **IDENTIFY:** This problem is about the rotational energy levels of the SiO molecule.

SET UP: The following equations apply:

$$E_l = l(l+1)\frac{\hbar^2}{2I}, \quad m_r = \frac{m_1 m_2}{m_1 + m_2}, \quad I = m_r r_0^2.$$

EXECUTE: **(a)** We want I. The energy of the emitted photon is equal to the energy difference between the $l = 0$ and $l = 1$ rotational levels. The photon energy is $E = hc/\lambda$. Therefore

$$E_1 - E_0 = 1(1+1)\frac{\hbar^2}{2I} - 0 = \frac{\hbar^2}{I}$$

Solving for I gives

$$I = \frac{\hbar\lambda}{2\pi c} = \frac{\hbar(6.882 \text{ mm})}{2\pi c} = 3.853 \times 10^{-46} \text{ kg} \cdot \text{m}^2.$$

(b) We want the reduced mass. Use the given masses.

$$m_r = \frac{m_1 m_2}{m_1 + m_2} = 1.690 \times 10^{-26} \text{ kg}.$$

(c) We want r_0. Solve $I = m_r r_0^2$ for r_0 and use the answers from parts (a) and (b), which gives r_0 = 0.1510 nm.

EVALUATE: The result in part (c) is quite reasonable for atomic distances in molecules.

VP42.3.4. **IDENTIFY:** This problem involves the vibrational and rotational energy levels in the CO molecule.

SET UP: When the molecule makes a transition to a lower energy state, the energy E_{ph} of the emitted photon is equal to the *magnitude* of the energy difference between the two states. Using Eq. (42.9) this photon energy is

$$E_{ph} = l_1(l_1+1)\frac{\hbar^2}{2I} + \left(n_1 + \frac{1}{2}\right)\hbar\omega - \left[l_2(l_2+1)\frac{\hbar^2}{2I} + \left(n_2 + \frac{1}{2}\right)\hbar\omega\right]$$

$$E_{ph} = \left[l_1(l_1+1) - l_2(l_2+1)\right]\frac{\hbar^2}{2I} + (n_1 - n_2)\hbar\omega$$

$n_1 - n_2 = +1$. Using this fact and some results from Example 42.3, we have

$$(n_1 - n_2)\hbar\omega = 0.2690 \text{ eV}, \quad \frac{\hbar^2}{2I} = 0.2395 \text{ meV}$$

The photon energy is therefore given by

$$E_{ph} = \left[l_1(l_1+1) - l_2(l_2+1)\right](0.2395 \text{ meV}) + 0.2690 \text{ eV}$$

EXECUTE: **(a)** Initial state is $l_1 = 3$. $\Delta l = -1$, so $l_2 = 2$.

$3 \rightarrow 2$ transition:

$$E_{ph} = \left[3(3+1) - 2(2+1)\right](0.2395 \text{ meV}) + 0.2690 \text{ eV} = 1.4370 \text{ meV} + 0.2690 \text{ eV} = 0.270437 \text{ eV}.$$

$$f = E_{ph}/h = (0.270437 \text{ eV})/h = 6.535 \times 10^{13} \text{ Hz}.$$
$$\lambda = c/f = 4.585 \ \mu\text{m}.$$

(b) Initial state is $l_1 = 2$. $\Delta l = -1$, so $l_2 = 1$.

$2 \rightarrow 1$ transition:

$$E_{ph} = \left[2(2+1) - 1(1+1)\right](0.2395 \text{ meV}) + 0.2690 \text{ eV} = 0.9580 \text{ meV} + 0.2690 \text{ eV} - 0.269958 \text{ eV}$$

$$f = E_{ph}/h = (0.269958 \text{ eV})/h = 6.538 \times 10^{13} \text{ Hz}.$$
$$\lambda = c/f = 4.593 \ \mu\text{m}.$$

EVALUATE: The wavelengths are very close together because the energy differences between states are very small since they are due only to *rotational* transitions which are low energy.

VP42.7.1. **IDENTIFY:** This problem involves the Fermi-Dirac distribution.
SET UP: Eq. (42.16) gives the probability $f(E)$ that a given energy state is occupied. The target variable is the energy for each probability. Solve for E in each case.
EXECUTE: **(a)** $f(E) = 0.33$. Solve for E.

$$0.33 = \frac{1}{e^{(E-E_F)/kT} + 1}$$

Separate the exponential and use logarithms, giving

$$(E - E_F)/kT = \ln 2.02$$

$$E = E_F + 0.71kT.$$

(b) $f(E) = 0.90$. Follow the same procedure using 0.90 instead of 0.33, giving $E = E_F - 2.2kT$.
EVALUATE: If $E = E_F$, $f(E) = 0.50$, for $f(E) > 0.50$, $E < E_F$, and for $f(E) < 0.50$, $E > E_F$. Our results are consistent with these conditions.

VP42.7.2. **IDENTIFY:** This problem involves the Fermi-Dirac distribution.
SET UP: Eq. (42.16) gives the probability $f(E)$ that a given energy state is occupied. The target variable is the probability for each specified energy.
EXECUTE: **(a)** $E = E_F - 0.0250$ eV. Using Eq. (42.16) gives

$$f(E) = \frac{1}{e^{(E_F - 0.0250 \text{ eV} - E_F)/kT} + 1}$$

The exponent of e is $(0.0250 \text{ eV})/kT = 0.9902$, so $f(E) = 1/(e^{-0.9902} + 1) = 0.729$.
(b) $E = E_F + 0.0400$ eV. Using Eq. (42.16) and following the same procedure gives $f(E) = 1/(e^{1.5843} + 1) = 0.170$.
(c) $E = E_F + 0.100$ eV. Using the same procedure gives $f(E) = 0.0187$.
EVALUATE: The probability decreases as E increases above the Fermi energy.

VP42.7.3. **IDENTIFY:** This problem involves the Fermi energy and the density of states.
SET UP and EXECUTE: **(a)** We want the Fermi energy at absolute zero. Using Eq. (42.19) with N/V given in the problem, we get

$$E_{F0} = \frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3} = 5.5 \text{ eV}.$$

(b) The speed is the target variable. The kinetic energy K is equal to the Fermi energy. Solve $K = E_F$ for v and use the result from part (a).

$$v = \sqrt{\frac{2E_F}{m}} = 1.4\times10^6 \text{ m/s}.$$

(c) The target variable is the density of states. Use Eq. (42.15) with $V = 1.0$ cm³ and $E = 5.5$ eV from part (a). The result is

$$g(E) = \frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2} = 9.9\times10^{40} \text{ states/J} = 1.6\times10^{22} \text{ states/eV}.$$

EVALUATE: The electron speeds are not great enough to require the use of special relativity.

VP42.7.4. **IDENTIFY:** This problem deals with the Fermi energy.
SET UP: From the previous problem we have $E_{F0} = 5.5$ eV. The target variable is the electron energy for the given probabilities.
EXECUTE: **(a)** $f(E) = 0.92$. Solve Eq. (42.16) for E using logarithms.

$$0.92 = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$$E - E_F = kT \ln(0.08696)$$

$$E = 5.4 \text{ eV}.$$

(b) $f(E) = 1.0 \times 10^{-4}$. Use the same procedure as in part (a) except $f(E) = 0.00010$, which gives $E = 5.7$ eV.

EVALUATE: Our results are consistent with problem VP42.7.1. When $E > E_F$, $f(E) < 0.50$, and when $E < E_F$, $f(E) > 0.50$.

VP42.9.1. **IDENTIFY:** This problem is about the Fermi energy and the band gap in a semiconductor.

SET UP and EXECUTE: For an electron at the bottom of the conduction band, $E = E_F + E_g/2$, so $E - E_F = E_g/2$. The target variable is the probability $f(E)$. Use Eq. (42.16) for $f(E)$.

(a) $E_g = 0.500$ eV. $(E - E_F)/kT = E_g/2kT = (0.500 \text{ eV})/2kT = 10.18$. Eq. (42.16) gives $f(E) = 1/(e^{10.18} + 1) = 3.79 \times 10^{-5}$.

(b) $E_g = 1.50$ eV. $(E - E_F)/kT = E_g/2kT = (1.50 \text{ eV})/2kT = 30.54$. Eq. (42.16) gives $f(E) = 1/(e^{30.54} + 1) = 5.46 \times 10^{-14}$.

EVALUATE: The probability decreases as the gap widens.

VP42.9.2. **IDENTIFY:** This problem is about the band gap in a semiconductor.

SET UP: $E = E_F + E_g/2$, so $E - E_F = E_g/2$. The target variable is the probability $f(E)$. Use Eq. (42.16) for $f(E)$. We want the ratio of the probabilities at different temperatures.

EXECUTE: **(a)** $E_g = 0.400$ eV. <u>At 315 K</u>: $E_g/2kT = (0.400 \text{ eV})/[2k(315 \text{ K})] = 7.3682$. $f(E)_{315} = 1/(e^{7.3682} + 1) = 6.3058 \times 10^{-4}$.

<u>At 295 K</u>: The exponent is 7.8677, so $f(E) = 1/(e^{7.8677} + 1) = 3.8290 \times 10^{-4}$.

Now take the ratio of the probabilities using the results we just found. This gives

$$\frac{f(E)_{315}}{f(E)_{295}} = \frac{6.3058 \times 10^{-4}}{3.8290 \times 10^{-4}} = 1.65.$$

(b) $E_g = 0.800$ eV. Use the same procedure as for part (a). <u>At 315 K</u>: $E_g/2kT = 14.7364$ which gives $f(E) = 3.98165 \times 10^{-7}$.

<u>At 295 K</u>: $E_g/2kT = 15.7354$ which gives $f(E) = 1.46623 \times 10^{-7}$.

$$\frac{f(E)_{315}}{f(E)_{295}} = \frac{3.98165 \times 10^{-7}}{1.46623 \times 10^{-7}} = 2.72.$$

EVALUATE: At $T = 315$ K the probability is greater than at 295 K, which is in agreement with Example 42.9.

VP42.9.3. **IDENTIFY:** We are dealing with the energy gap in a semiconductor.

SET UP: The target variable is the width of the energy gap E_g at different temperatures. Combining $E - E_F = E_g/2$ with Eq. (42.16) gives

$$f(E) = \frac{1}{e^{E_g/2kT} + 1}$$

Solve this equation for E_g using logarithms.

$$E_g = 2kT \ln\left(1 + \frac{1}{f(E)}\right) = 2kT \ln(527.32).$$

EXECUTE: **(a)** $T = 305$ K. Using this temperature in the equation we just derived gives $E_g = 0.329$ eV.

(b) $T = 325$ K. Using the same procedure gives $E_g = 0.351$ eV.

EVALUATE: As the temperature increases, the gap width also increases.

VP42.9.4. **IDENTIFY:** We are dealing with the energy gap in a semiconductor.

SET UP: The target variable is the probability at different temperatures. Figure VP42.9.4 helps to visualize how the various quantities are related. In this case, $E = E_F + E_g/4$. Therefore $E - E_F = E_g/4 = (0.300 \text{ eV})/4 = 0.0750$ eV and $(E - E_F)/kT = (0.0750 \text{ eV})/kT = (870.37 \text{ K})/T$.
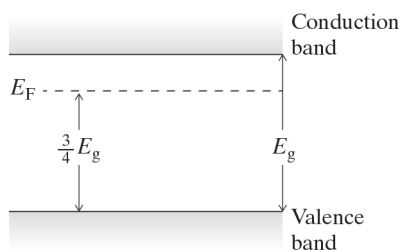
Figure VP42.9.4

EXECUTE: Use Eq. (42.16).

(a) $T = 275$ K. $(E - E_F)/kT = (870.37$ K$)/T = (870.37$ K$)/(275$ K$) = 3.1650$. Therefore

$$f(E) = \frac{1}{e^{3.1650} + 1} = 0.0405.$$

(b) $T = 325$ K. $(E - E_F)/kT = (870.37$ K$)/T = (870.37$ K$)/(325$ K$) = 2.67806$. Therefore

$$f(E) = \frac{1}{e^{2.67806} + 1} = 0.0643.$$

EVALUATE: As the temperature increases, so does the probability of finding electrons in the conduction band.

42.1. **IDENTIFY and SET UP:** $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r}$. The binding energy of the molecule is equal to U plus the

ionization energy of K minus the electron affinity of Br.

EXECUTE: **(a)** $U = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r} = -5.0$ eV.

(b) -5.0 eV $+ (4.3$ eV $- 3.5$ eV$) = -4.2$ eV.

EVALUATE: We expect the magnitude of the binding energy to be somewhat less than this estimate. At this separation the two ions don't behave exactly like point charges and U is smaller in magnitude than our estimate. The experimental value for the binding energy is -4.0 eV, which is smaller in magnitude than our estimate.

42.2. **IDENTIFY:** The energy decrease of the molecule or atom is equal to the energy of the emitted photon. From this energy, we can calculate the wavelength of the photon.

SET UP: $\Delta E = \dfrac{hc}{\lambda}$. Use Figure 32.4 in the textbook to find out the region of the electromagnetic

spectrum in which each wavelength lies.

(a) EXECUTE: $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{0.198\ \text{eV}} = 6.26\ \mu\text{m}.$

EVALUATE: This radiation is in the infrared.

(b) EXECUTE: $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{7.80\ \text{eV}} = 159\ \text{nm}.$

EVALUATE: This radiation is in the ultraviolet.

(c) EXECUTE: $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{4.80\times10^{-3}\ \text{eV}} = 0.258\ \text{mm}.$

EVALUATE: This radiation is in the microwave region.

42.3.   **IDENTIFY:**   The energy of the photon is equal to the energy difference between the $l = 1$ and $l = 2$ states. This energy determines its wavelength.

SET UP: The reduced mass of the molecule is $m_r = \dfrac{m_H m_H}{m_H + m_H} = \frac{1}{2} m_H$, its moment of inertia is

$I = m_r r_0^2$, the photon energy is $\Delta E = \dfrac{hc}{\lambda}$, and the energy of the state l is $E_l = l(l+1)\dfrac{\hbar^2}{2I}$.

EXECUTE: $I = m_r r_0^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(0.074 \times 10^{-9} \text{ m})^2 = 4.57 \times 10^{-48} \text{ kg} \cdot \text{m}^2$. Using

$E_l = l(l+1)\dfrac{\hbar^2}{2I}$, the energy levels are

$E_2 = 6\dfrac{\hbar^2}{2I} = 6\dfrac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{2(4.57 \times 10^{-48} \text{ kg} \cdot \text{m}^2)} = 6(1.218 \times 10^{-21} \text{ J}) = 7.307 \times 10^{-21} \text{ J}$ and

$E_1 = 2\dfrac{\hbar^2}{2I} = 2(1.218 \times 10^{-21} \text{ J}) = 2.436 \times 10^{-21} \text{ J}. \ \Delta E = E_2 - E_1 = 4.87 \times 10^{-21} \text{ J}.$ Using $\Delta E = \dfrac{hc}{\lambda}$ gives

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{4.871 \times 10^{-21} \text{ J}} = 4.08 \times 10^{-5} \text{ m} = 40.8 \ \mu\text{m}.$

EVALUATE: This wavelength is much longer than that of visible light.

42.4.   **IDENTIFY:**   The energy absorbed by the photon is equal to the energy difference between the two rotational states.

SET UP: The rotational energy of a molecule is $E_l = l(l+1)\dfrac{\hbar^2}{2I}$ and the energy of the photon is the

energy difference ΔE between the states, so $\Delta E = hc/\lambda$. The transition is from the $l = 3$ state to the $l = 4$ state.

EXECUTE: Using the rotational energy formula, the energy difference between the two states is

$$\Delta E = E_4 - E_3 = 4(4+1)\dfrac{\hbar^2}{2I} - 3(3+1)\dfrac{\hbar^2}{2I} = 4\dfrac{\hbar^2}{I}.$$

This is the energy absorbed by the photon, so $\Delta E = hc/\lambda$ gives $hc/\lambda = 4\hbar^2/I$. Solving for λ gives $\lambda = \pi^2 Ic/h = \pi^2 (4.6 \times 10^{-48} \text{ kg} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})/(6.626 \times 10^{-34} \text{ J} \cdot \text{s}) = 2.05 \times 10^{-5} \text{ m} = 20.6 \ \mu\text{m}$, which rounds to $21 \ \mu\text{m}$.

EVALUATE: Rotational energy changes are much less than transitions between n-shells, so the wavelengths involved are much longer for rotational transitions.

42.5.   **IDENTIFY:**   The energy given to the photon comes from a transition between rotational states.

SET UP: The rotational energy of a molecule is $E = l(l+1)\dfrac{\hbar^2}{2I}$ and the energy of the photon is

$E = hc/\lambda$.

EXECUTE: Use the energy formula, the energy difference between the $l = 3$ and $l = 1$ rotational levels

of the molecule is $\Delta E = \dfrac{\hbar^2}{2I}[3(3+1) - 1(1+1)] = \dfrac{5\hbar^2}{I}$. Since $\Delta E = hc/\lambda$, we get $hc/\lambda = 5\hbar^2/I$. Solving for

I gives

$$I = \dfrac{5\hbar\lambda}{2\pi c} = \dfrac{5(1.055 \times 10^{-34} \text{ J} \cdot \text{s})(1.780 \text{ nm})}{2\pi(3.00 \times 10^8 \text{ m/s})} = 4.981 \times 10^{-52} \text{ kg} \cdot \text{m}^2.$$

Using $I = m_r r_0^2$, we can solve for r_0:

$$r_0 = \sqrt{\dfrac{I(m_N + m_H)}{m_N m_H}} = \sqrt{\dfrac{(4.981 \times 10^{-52} \text{ kg} \cdot \text{m}^2)(2.33 \times 10^{-26} \text{ kg} + 1.67 \times 10^{-27} \text{ kg})}{(2.33 \times 10^{-26} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}} = 5.65 \times 10^{-13} \text{ m}.$$

EVALUATE: This separation is much smaller than the diameter of a typical atom and is not very realistic. But we are treating a *hypothetical* NH molecule.

42.6. **IDENTIFY:** $I = m_1 r_1^2 + m_2 r_2^2$. Since the two atoms are identical, the center of mass is midway between them.
SET UP: Each atom has a mass m and is at a distance $L/2$ from the center of mass.
EXECUTE: The moment of inertia is $2(m)(L/2)^2 = mL^2/2 = 2.21 \times 10^{-44}$ kg $\cdot$ m^2.
EVALUATE: $r_0 = L$ and $m_r = m/2$, so $I = m_r r_0^2$ gives the same result.

42.7. **IDENTIFY and SET UP:** Set $K = E_1$ from Example 42.2. Use $K = \frac{1}{2} I \omega^2$ to solve for ω and $v = r\omega$ to solve for v.
EXECUTE: **(a)** From Example 42.2, $E_1 = 0.479$ meV $= 7.674 \times 10^{-23}$ J and $I = 1.449 \times 10^{-46}$ kg $\cdot$ m^2.
$K = \frac{1}{2} I \omega^2$ and $K = E$ gives $\omega = \sqrt{2E_1/I} = 1.03 \times 10^{12}$ rad/s.
(b) $v_1 = r_1 \omega_1 = (0.0644 \times 10^{-9} \text{ m})(1.03 \times 10^{12} \text{ rad/s}) = 66.3$ m/s (carbon).
$v_2 = r_2 \omega_2 = (0.0484 \times 10^{-9} \text{ m})(1.03 \times 10^{12} \text{ rad/s}) = 49.8$ m/s (oxygen).
(c) $T = 2\pi/\omega = 6.10 \times 10^{-12}$ s.
EVALUATE: Even for fast rotation rates, $v \ll c$.

42.8. **IDENTIFY:** Find ΔE for the transition and compute λ from $\Delta E = hc/\lambda$.
SET UP: From Example 42.2, $E_l = l(l+1)\dfrac{\hbar^2}{2I}$, with $\dfrac{\hbar^2}{2I} = 0.2395 \times 10^{-3}$ eV. $\Delta E = 0.2690$ eV is the spacing between vibrational levels. Thus $E_n = (n + \frac{1}{2})\hbar\omega$, with $\hbar\omega = 0.2690$ eV. The total vibrational and rotational energy is $E = E_n + E_l = (n + \frac{1}{2})\hbar\omega + l(l+1)\dfrac{\hbar^2}{2I}$.
EXECUTE: **(a)** $n = 0 \to n = 1$ and $l = 2 \to l = 3$.
For $n = 0$, $l = 2$, $E_i = \frac{1}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

For $n = 1$, $l = 3$, $E_f = \frac{3}{2}\hbar\omega + 12\left(\dfrac{\hbar^2}{2I}\right)$.

$\Delta E = E_f - E_i = \hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right) = 0.2690$ eV $+ 6(0.2395 \times 10^{-3}$ eV$) = 0.2704$ eV.

$\dfrac{hc}{\lambda} = \Delta E$ so $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2704 \text{ eV}} = 4.586 \times 10^{-6}$ m $= 4.586\,\mu$m.

(b) $n = 0 \to n = 1$ and $l = 3 \to l = 2$.
For $n = 0$, $l = 3$, $E_i = \frac{1}{2}\hbar\omega + 12\left(\dfrac{\hbar^2}{2I}\right)$.

For $n = 1$, $l = 2$, $E_f = \frac{3}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

$\Delta E = E_f - E_i = \hbar\omega - 6\left(\dfrac{\hbar^2}{2I}\right) = 0.2690$ eV $- 6(0.2395 \times 10^{-3}$ eV$) = 0.2676$ eV.

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2676 \text{ eV}} = 4.634 \times 10^{-6}$ m $= 4.634\,\mu$m.

(c) $n = 0 \rightarrow n = 1$ and $l = 4 \rightarrow l = 3$.

For $n = 0, l = 4$, $E_i = \frac{1}{2}\hbar\omega + 20\left(\frac{\hbar^2}{2I}\right)$.

For $n = 1, l = 3$, $E_f = \frac{3}{2}\hbar\omega + 12\left(\frac{\hbar^2}{2I}\right)$.

$\Delta E = E_f - E_i = \hbar\omega - 8\left(\frac{\hbar^2}{2I}\right) = 0.2690 \text{ eV} - 8(0.2395 \times 10^{-3} \text{ eV}) = 0.2671 \text{ eV}.$

$\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{0.2671 \text{ eV}} = 4.643 \times 10^{-6} \text{ m} = 4.643 \,\mu\text{m}.$

EVALUATE: All three transitions are for $n = 0 \rightarrow n = 1$. The spacing between vibrational levels is larger than the spacing between rotational levels, so the difference in λ for the various rotational transitions is small. When the transition is to a larger l, $\Delta E > \hbar\omega$ and when the transition is to a smaller l, $\Delta E < \hbar\omega$.

42.9. **IDENTIFY** and **SET UP:** The energy of a rotational level with quantum number l is $E_l = l(l+1)\hbar^2/2I$.

$I = m_r r^2$, with the reduced mass m_r given by $m_r = \frac{m_1 m_2}{m_1 + m_2}$. Calculate I and ΔE and then use $\Delta E = hc/\lambda$ to find λ.

EXECUTE: (a) $m_r = \frac{m_1 m_2}{m_1 + m_2} = \frac{m_{Li} m_H}{m_{Li} + m_H} = \frac{(1.17 \times 10^{-26} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{1.17 \times 10^{-26} \text{ kg} + 1.67 \times 10^{-27} \text{ kg}} = 1.461 \times 10^{-27} \text{ kg}.$

$I = m_r r^2 = (1.461 \times 10^{-27} \text{ kg})(0.159 \times 10^{-9} \text{ m})^2 = 3.694 \times 10^{-47} \text{ kg} \cdot \text{m}^2.$

$l = 3 : E = 3(4)\left(\frac{\hbar^2}{2I}\right) = 6\left(\frac{\hbar^2}{I}\right).$

$l = 4 : E = 4(5)\left(\frac{\hbar^2}{2I}\right) = 10\left(\frac{\hbar^2}{I}\right).$

$\Delta E = E_4 - E_3 = 4\left(\frac{\hbar^2}{I}\right) = 4\left(\frac{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{3.694 \times 10^{-47} \text{ kg} \cdot \text{m}^2}\right) = 1.20 \times 10^{-21} \text{ J} = 7.49 \times 10^{-3} \text{ eV}.$

(b) $\Delta E = hc/\lambda$, so $\lambda = \frac{hc}{\Delta E} = \frac{(4.136 \times 10^{-15} \text{ eV})(2.998 \times 10^8 \text{ m/ s})}{7.49 \times 10^{-3} \text{ eV}} = 166 \,\mu\text{m}.$

EVALUATE: LiH has a smaller reduced mass than CO and λ is somewhat smaller here than the λ calculated for CO in Example 42.2.

42.10. **IDENTIFY:** For a $n \rightarrow n - 1$ vibrational transition, $\Delta E = \hbar\sqrt{\frac{k'}{m_r}}$. ΔE is related to λ of the photon by

$\Delta E = \frac{hc}{\lambda}.$

SET UP: $m_r = \frac{m_{Na} m_{Cl}}{m_{Na} + m_{Cl}}.$

EXECUTE: $\Delta E = \frac{hc}{\lambda} = \hbar\sqrt{k'/m_r}$, and solving for k', $k' = \left(\frac{2\pi c}{\lambda}\right)^2 m_r = 205 \text{ N/m}.$

EVALUATE: The value of k' we calculated for NaCl is comparable to that of a fairly stiff lab spring.

42.11. **IDENTIFY:** The vibrational energy of the molecule is related to its force constant and reduced mass, while the rotational energy depends on its moment of inertia, which in turn depends on the reduced mass.

SET UP: The vibrational energy is $E_n = (n + \frac{1}{2})\hbar\omega = (n + \frac{1}{2})\hbar\sqrt{\frac{k'}{m_r}}$ and the rotational energy is

$$E_l = l(l+1)\frac{\hbar^2}{2I}.$$

EXECUTE: For a vibrational transition, we have $\Delta E_v = \hbar\sqrt{\frac{k'}{m_r}}$, so we first need to find m_r. The energy

for a rotational transition is $\Delta E_R = \frac{\hbar^2}{2I}[2(2+1) - 1(1+1)] = \frac{2\hbar^2}{I}$. Solving for I and using the fact that

$I = m_r r_0^2$, we have $m_r r_0^2 = \frac{2\hbar^2}{\Delta E_R}$, which gives

$$m_r = \frac{2\hbar^2}{r_0^2 \Delta E_R} = \frac{2(1.055 \times 10^{-34} \text{ J} \cdot \text{s})(6.583 \times 10^{-16} \text{ eV} \cdot \text{s})}{(0.8860 \times 10^{-9} \text{ m})^2 (8.841 \times 10^{-4} \text{ eV})} = 2.0014 \times 10^{-28} \text{ kg}.$$

Now look at the vibrational transition to find the force constant.

$$\Delta E_v = \hbar\sqrt{\frac{k'}{m_r}} \Rightarrow k' = \frac{m_r(\Delta E_v)^2}{\hbar^2} = \frac{(2.0014 \times 10^{-28} \text{ kg})(0.2560 \text{ eV})^2}{(6.583 \times 10^{-16} \text{ eV} \cdot \text{s})^2} = 30.27 \text{ N/m}.$$

EVALUATE: This would be a rather weak spring in the laboratory.

42.12. **IDENTIFY and SET UP:** For an average spacing a, the density is $\rho = m/a^3$, where m is the average of the ionic masses.

EXECUTE: **(a)** $a^3 = \frac{m}{\rho} = \frac{(6.49 \times 10^{-26} \text{ kg} + 1.33 \times 10^{-25} \text{ kg})/2}{(2.75 \times 10^3 \text{ kg/m}^3)} = 3.60 \times 10^{-29} \text{ m}^3$, and

$a = 3.30 \times 10^{-10} \text{ m} = 0.330 \text{ nm}.$

EVALUATE: **(b)** Exercise 42.15 says that the average spacing for NaCl is 0.282 nm. The larger (higher atomic number) atoms have the larger spacing.

42.13. **IDENTIFY and SET UP:** Find the volume occupied by each atom. The density is the average mass of Na and Cl divided by this volume.

EXECUTE: Each atom occupies a cube with side length 0.282 nm. Therefore, the volume occupied by each atom is $V = (0.282 \times 10^{-9} \text{ m})^3 = 2.24 \times 10^{-29} \text{ m}^3$. In NaCl there are equal numbers of Na and Cl atoms, so the average mass of the atoms in the crystal is

$$m = \frac{1}{2}(m_{Na} + m_{Cl}) = \frac{1}{2}(3.82 \times 10^{-26} \text{ kg} + 5.89 \times 10^{-26} \text{ kg}) = 4.855 \times 10^{-26} \text{ kg}.$$

The density then is $\rho = \frac{m}{V} = \frac{4.855 \times 10^{-26} \text{ kg}}{2.24 \times 10^{-29} \text{ m}^3} = 2.17 \times 10^3 \text{ kg/m}^3.$

EVALUATE: The density of water is $1.00 \times 10^3 \text{ kg/m}^3$, so our result is reasonable.

42.14. **IDENTIFY and SET UP:** $\Delta E = \frac{hc}{\lambda}$, where ΔE is the band gap.

EXECUTE: **(a)** $\lambda = \frac{hc}{\Delta E} = 2.27 \times 10^{-7} \text{ m} = 227 \text{ nm}$, in the ultraviolet.

EVALUATE: **(b)** Visible light lacks enough energy to excite the electrons into the conduction band, so visible light passes through the diamond unabsorbed.
(c) Impurities can lower the gap energy making it easier for the material to absorb shorter wavelength visible light. This allows longer wavelength visible light to pass through, giving the diamond color.

42.15. **IDENTIFY:** The energy gap is the energy of the maximum-wavelength photon.
SET UP: The energy difference is equal to the energy of the photon, so $\Delta E = hc/\lambda$.
EXECUTE: **(a)** Using the photon wavelength to find the energy difference gives
$$\Delta E = hc/\lambda = (4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})/(1.11 \times 10^{-6} \text{ m}) = 1.12 \text{ eV}.$$
(b) A wavelength of $1.11 \, \mu\text{m} = 1110$ nm is in the infrared, shorter than that of visible light.

EVALUATE: Since visible photons have more than enough energy to excite electrons from the valence to the conduction band, visible light will be absorbed, which makes silicon opaque.

42.16. **IDENTIFY and SET UP:** The energy ΔE deposited when a photon with wavelength λ is absorbed is
$$\Delta E = \frac{hc}{\lambda}.$$
EXECUTE: $\Delta E = \dfrac{hc}{\lambda} = \dfrac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{9.31 \times 10^{-13} \text{ m}} = 2.14 \times 10^{-13} \text{ J} = 1.34 \times 10^6 \text{ eV}$. So the number

of electrons that can be excited to the conduction band is $n = \dfrac{1.34 \times 10^6 \text{ eV}}{1.12 \text{ eV}} = 1.20 \times 10^6$ electrons.

EVALUATE: A photon of wavelength
$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.13 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{1.12 \text{ eV}} = 1.11 \times 10^{-6} \text{ m} = 1110$ nm can excite one electron. This
photon is in the infrared.

42.17. **IDENTIFY:** The density of states is given by $g(E) = \dfrac{(2m)^{3/2} V}{2\pi^2 \hbar^3} E^{1/2}$.

SET UP: $m = 9.11 \times 10^{-31}$ kg, the mass of an electron.
EXECUTE:
$$g(E) = \frac{(2m)^{3/2} V}{2\pi^2 \hbar^3} E^{1/2} = \frac{[2(9.11 \times 10^{-31} \text{ kg})]^{3/2} (1.0 \times 10^{-6} \text{ m}^3)(5.0 \text{ eV})^{1/2} (1.60 \times 10^{-19} \text{ J/eV})^{1/2}}{2\pi^2 (1.054 \times 10^{-34} \text{ J} \cdot \text{s})^3}.$$
$g(E) = (9.5 \times 10^{40} \text{ states/J})(1.60 \times 10^{-19} \text{ J/eV}) = 1.5 \times 10^{22}$ states/eV.
EVALUATE: For a metal the density of states expressed as states/eV is very large.

42.18. **IDENTIFY:** At absolute zero, the average free-electron energy is $E_{av} = \dfrac{3}{5} E_F$. The speed v is related to
E_{av} by $\frac{1}{2} m v^2 = E_{av}$.
SET UP: $k = 1.38 \times 10^{-23}$ J/K.
EXECUTE: **(a)** $E_{av} = \dfrac{3}{5} E_F = 1.94$ eV.

(b) $v = \sqrt{2 E_{av}/m} = \sqrt{\dfrac{2(1.94 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{9.11 \times 10^{-31} \text{ kg}}} = 8.25 \times 10^5$ m/s.

(c) $\dfrac{E_F}{k} = \dfrac{(3.23 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{(1.38 \times 10^{-23} \text{ J/K})} = 3.74 \times 10^4$ K.

EVALUATE: The Fermi energy of sodium is less than that of copper. Therefore, the values of E_{av} and v
we have calculated for sodium are less than those calculated for copper in Example 42.7.

42.19. **IDENTIFY:** This problem deals with the density of states $g(E)$.
SET UP: The original energy is 1.60 eV and we want to know the new energy. From Eq. (42.15) we see that $g(E)$ is proportional to the square root of E.
EXECUTE: **(a)** $g(E)$ is doubled. Since $g(E)$ is proportional to the square root of the energy, E must increase by a factor of 4 to double $g(E)$, so $E = 4(1.60 \text{ eV}) = 6.40$ eV.

(b) $g(E)$ is halved. We need to decrease $g(E)$ by a factor of $\frac{1}{2}$, so E must decrease by a factor of $\frac{1}{4}$, so

$E = (1.60 \text{ eV})/4 = 0.400 \text{ eV}$.

EVALUATE: A larger fractional change in E is required than the fractional change in $g(E)$.

42.20. **IDENTIFY:** This problem involves the Fermi-Dirac distribution.

SET UP: Eq. (42.16) gives this distribution. We know that $kT = 0.25E_F$ and we want the probability $f(E)$.

EXECUTE: (a) $E = 0.50E_F$. Using Eq. (42.16) gives

$$f(0.50E_F) = \frac{1}{e^{(0.50E_F - E_F)} + 1} = \frac{1}{e^{-2} + 1} = 0.88.$$

(b) $E = 1.50E_F$. Use the same procedure as in (a) with $E = 1.50E_F$. $f(1.50E_F) = 1/(e^2 + 1) = 0.12$.

EVALUATE: Our results agree with the curve for $kT = \frac{1}{4} E_F$ in Figure 42.23.

42.21. **IDENTIFY and SET UP:** The electron contribution to the molar heat capacity at constant volume of a

metal is $C_V = \left(\dfrac{\pi^2 kT}{2E_F} \right) R$.

EXECUTE: (a) $C_V = \dfrac{\pi^2 (1.381 \times 10^{-23} \text{ J/K})(300 \text{ K})}{2(5.48 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})} R = 0.0233R$.

(b) The electron contribution found in part (a) is $0.0233R = 0.194 \text{ J/mol} \cdot \text{K}$. This is

$0.194/25.3 = 7.67 \times 10^{-3} = 0.767\%$ of the total C_V.

EVALUATE: (c) Only a small fraction of C_V is due to the electrons. Most of C_V is due to the

vibrational motion of the ions.

42.22. **IDENTIFY and SET UP:** The probability $f(E)$ that a state with energy E is occupied is given by the

Fermi-Dirac distribution $f(E) = \dfrac{1}{e^{(E - E_F)/kT} + 1}$, where E_F is the Fermi energy. In this case, $E = 2E_F$

$= 2(kT_F)$.

EXECUTE: $f(2E) = \dfrac{1}{e^{(2E_F - E_F)/kT_F} + 1} = \dfrac{1}{e^{E_F/kT_F} + 1} = \dfrac{1}{e^{kT_F/kT_F} + 1} = \dfrac{1}{e + 1} = 0.269.$

EVALUATE: A probability of $0.269 \approx 27\%$ is about 1 in 4. As E increases, the probability gets smaller.

42.23. **IDENTIFY:** The probability is given by the Fermi-Dirac distribution.

SET UP: The Fermi-Dirac distribution is $f(E) = \dfrac{1}{e^{(E - E_F)/kT} + 1}$.

EXECUTE: We calculate the value of $f(E)$, where $E = 8.520 \text{ eV}$, $E_F = 8.500 \text{ eV}$,

$k = 1.38 \times 10^{-23} \text{ J/K} = 8.625 \times 10^{-5} \text{ eV/K}$, and $T = 20°C = 293 \text{ K}$. The result is $f(E) = 0.312 = 31.2\%$.

EVALUATE: Since the energy is close to the Fermi energy, the probability is quite high that the state is occupied by an electron.

42.24. **IDENTIFY and SET UP:** Follow the procedure of Example 42.9. Evaluate $f(E) = \dfrac{1}{e^{(E - E_F)/kT} + 1}$ for

$E - E_F = E_g/2$, where E_g is the band gap.

EXECUTE: (a) The probabilities are 1.78×10^{-7}, 2.37×10^{-6}, and 1.51×10^{-5}.

(b) It can be shown that the Fermi-Dirac distribution, $f(E) = \dfrac{1}{e^{(E - E_F)/kT} + 1}$, has the property that

$f(E_F - E) = 1 - f(E)$, and so the probability that a state at the top of the valence band is occupied is the

same as the probability that a state of the bottom of the conduction band is filled (this result depends on

having the Fermi energy in the middle of the gap). Therefore, the probabilities at each T are the same as in part (a).

EVALUATE: The probabilities increase with temperature.

42.25. **IDENTIFY:** Use the Fermi-Dirac distribution $f(E) = \dfrac{1}{e^{(E-E_F)/kT} + 1}$. Solve for $E - E_F$.

SET UP: $e^{(E-E_F)/kT} = \dfrac{1}{f(E)} - 1$.

The problem states that $f(E) = 4.4 \times 10^{-4}$ for E at the bottom of the conduction band.

EXECUTE: $e^{(E-E_F)/kT} = \dfrac{1}{4.4 \times 10^{-4}} - 1 = 2.272 \times 10^3$.

$E - E_F = kT \ln(2.272 \times 10^3) = (1.3807 \times 10^{-23} \text{ J/T})(300 \text{ K}) \ln(2.272 \times 10^3) = 3.201 \times 10^{-20}$ J $= 0.20$ eV.

$E_F = E - 0.20$ eV; the Fermi level is 0.20 eV below the bottom of the conduction band.

EVALUATE: The energy gap between the Fermi level and bottom of the conduction band is large compared to kT at $T = 300$ K and as a result $f(E)$ is small.

42.26. **IDENTIFY:** The wavelength of the photon to be detected depends on its energy.

SET UP: $\Delta E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136 \times 10^{-15} \text{ eV} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{0.67 \text{ eV}} = 1.9 \ \mu\text{m}$.

(b) $\lambda = (1.9 \ \mu\text{m})\left(\dfrac{0.67 \text{ eV}}{1.12 \text{ eV}}\right) = 1.1 \ \mu\text{m}$.

EVALUATE: Both of these photons are in the infrared.

42.27. **IDENTIFY:** Knowing the saturation current of a *p-n* junction at a given temperature, we want to find the current at that temperature for various voltages.

SET UP: $I = I_S(e^{eV/kT} - 1)$.

EXECUTE: **(a)** (i) For $V = 1.00$ mV, $\dfrac{eV}{kT} = \dfrac{(1.602 \times 10^{-19} \text{ C})(1.00 \times 10^{-3} \text{ V})}{(1.381 \times 10^{-23} \text{ J/K})(290 \text{ K})} = 0.0400$.

$I = (0.500 \text{ mA})(e^{0.0400} - 1) = 0.0204$ mA.

(ii) For $V = -1.00$ mV, $\dfrac{eV}{kT} = -0.0400$. $I = (0.500 \text{ mA})(e^{-0.0400} - 1) = -0.0196$ mA.

(iii) For $V = 100$ mV, $\dfrac{eV}{kT} = 4.00$. $I = (0.500 \text{ mA})(e^{4.00} - 1) = 26.8$ mA.

(iv) For $V = -100$ mV, $\dfrac{eV}{kT} = -4.00$. $I = (0.500 \text{ mA})(e^{-4.00} - 1) = -0.491$ mA.

EVALUATE: **(b)** For small V, between ± 1.00 mV, $R = V/I$ is approximately constant and the diode obeys Ohm's law to a good approximation. For larger V the deviation from Ohm's law is substantial.

42.28. **IDENTIFY:** The current depends on the voltage across the diode and its temperature, so the resistance also depends on these quantities.

SET UP: The current is $I = I_S(e^{eV/kT} - 1)$ and the resistance is $R = V/I$.

EXECUTE: **(a)** The resistance is $R = \dfrac{V}{I} = \dfrac{V}{I_S(e^{eV/kT} - 1)}$. The exponent is

$\dfrac{eV}{kT} = \dfrac{e(0.0850 \text{ V})}{(8.625 \times 10^{-5} \text{ eV/K})(293 \text{ K})} = 3.3635$, giving $R = \dfrac{85.0 \text{ mV}}{(0.950 \text{ mA})(e^{3.3635} - 1)} = 3.21 \ \Omega$.

(b) In this case, the exponent is $\dfrac{eV}{kT} = \dfrac{e(-0.050\ \text{V})}{(8.625\times10^{-5}\ \text{eV/K})(293\ \text{K})} = -1.979$

which gives $R = \dfrac{-50.0\ \text{mV}}{(0.950\ \text{mA})(e^{-1.979}-1)} = 61.1\ \Omega.$

EVALUATE: Reversing the voltage can make a considerable change in the resistance of a diode.

42.29. **IDENTIFY** and **SET UP:** The voltage-current relation is given by $I = I_s(e^{eV/kT}-1)$. Use the current for $V = +15.0\ \text{mV}$ to solve for the constant I_s.

EXECUTE: **(a)** Find $I_s: V = +15.0\times10^{-3}\ \text{V}$ gives $I = 9.25\times10^{-3}\ \text{A}.$

$\dfrac{eV}{kT} = \dfrac{(1.602\times10^{-19}\ \text{C})(15.0\times10^{-3}\ \text{V})}{(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})} = 0.5800.$

$I_s = \dfrac{I}{e^{eV/kT}-1} = \dfrac{9.25\times10^{-3}\ \text{A}}{e^{0.5800}-1} = 1.177\times10^{-2} = 11.77\ \text{mA}.$

Then can calculate I for $V = 10.0\ \text{mV}$: $\dfrac{eV}{kT} = \dfrac{(1.602\times10^{-19}\ \text{C})(10.0\times10^{-3}\ \text{V})}{(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})} = 0.3867.$

$I = I_s(e^{eV/kT}-1) = (11.77\ \text{mA})(e^{0.3867}-1) = 5.56\ \text{mA}.$

(b) $\dfrac{eV}{kT}$ has the same magnitude as in part (a) but now V is negative so $\dfrac{eV}{kT}$ is negative.

$\underline{V = -15.0\ \text{mV}} : \dfrac{eV}{kT} = -0.5800$ and $I = I_s(e^{eV/kT}-1) = (11.77\ \text{mA})(e^{-0.5800}-1) = -5.18\ \text{mA}.$

$\underline{V = -10.0\ \text{mV}} : \dfrac{eV}{kT} = -0.3867$ and $I = I_s(e^{eV/kT}-1) = (11.77\ \text{mA})(e^{-0.3867}-1) = -3.77\ \text{mA}.$

EVALUATE: There is a directional asymmetry in the current, with a forward-bias voltage producing more current than a reverse-bias voltage of the same magnitude, but the voltage is small enough for the asymmetry not be pronounced.

42.30. **IDENTIFY:** Apply the equation $I = I_S(e^{eV/kT}-1).$

SET UP: $I_S = 6.40\ \text{mA}.$ $\ln e^x = x.$

EXECUTE: **(a)** Solving $I = I_S(e^{eV/kT}-1)$ for the voltage as a function of current gives

$V = \dfrac{kT}{e}\ln\left(\dfrac{I}{I_S}+1\right) = \dfrac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{1.602\times10^{-19}\ \text{C}}\ln\left(\dfrac{40.0\ \text{mA}}{6.40\ \text{mA}}+1\right) = 0.0512\ \text{V} = 51.2\ \text{mV}.$

(b) Using the result from part (a), the quantity $e^{eV/kT} = 7.242,$ so for a reverse-bias voltage of the same magnitude, $I = I_S(e^{-eV/kT}-1) = (6.40\ \text{mA})\left(\dfrac{1}{7.242}-1\right) = -5.52\ \text{mA}.$

EVALUATE: The reverse bias current for a given magnitude of voltage is much less than the forward bias current.

42.31. **IDENTIFY:** In this problem we are dealing with a junction diode.

SET UP: We want the resistance of the diode. The current through the junction is given by Eq. (42.22) and $T = 17°\text{C} = 290\ \text{K}.$ Using this equation to find the resistance gives

$$R = \dfrac{V}{I} = \dfrac{V}{I_S\left(e^{eV/kT}-1\right)}.$$

EXECUTE: Forward bias: $V = +0.20\ \text{V}.$

$$R = \dfrac{0.20\ \text{V}}{(10\ \mu\text{A})\left(e^{e(0.20\ \text{V})/k(290\ \text{K})}-1\right)} = 6.7\ \Omega.$$

Reverse bias: $V = -0.20$ V. Use the same procedure with $V = -0.20$ V, giving $R = 2.0 \times 10^4\ \Omega$.

EVALUATE: In reverse bias the resistance is around 3000 times greater than in forward bias.

42.32. **IDENTIFY** and **SET UP:** $E_l = l(l+1)\dfrac{\hbar^2}{2I}$. ΔE for the molecule is related to λ for the photon by

$$\Delta E = \frac{hc}{\lambda}.$$

EXECUTE: $E_2 = 3\dfrac{\hbar^2}{I}$ and $E_1 = \dfrac{\hbar^2}{I}$, so $\Delta E = \dfrac{2\hbar^2}{I}$. Using $\lambda = 54.3\ \mu$m, we get

$$I = \frac{2\hbar^2}{\Delta E} = \frac{h\lambda}{2\pi^2 c} = 6.08 \times 10^{-48}\ \text{kg} \cdot \text{m}^2.$$

EVALUATE: The I we calculated is approximately a factor of 24 times smaller than I calculated for the CO molecule in Example 42.2.

42.33. **IDENTIFY** and **SET UP:** From Chapter 21, the electric dipole moment is $p = qd$, where the dipole consists of charges $\pm q$ separated by distance d.

EXECUTE: **(a)** Point charges $+e$ and $-e$ separated by distance d, so
$$p = ed = (1.602 \times 10^{-19}\ \text{C})(0.24 \times 10^{-9}\ \text{m}) = 3.8 \times 10^{-29}\ \text{C} \cdot \text{m}.$$

(b) $p = qd$, so $q = \dfrac{p}{d} = \dfrac{3.0 \times 10^{-29}\ \text{C} \cdot \text{m}}{0.24 \times 10^{-9}\ \text{m}} = 1.3 \times 10^{-19}\ \text{C}.$

(c) $\dfrac{q}{e} = \dfrac{1.3 \times 10^{-19}\ \text{C}}{1.602 \times 10^{-19}\ \text{C}} = 0.81.$

(d) $q = \dfrac{p}{d} = \dfrac{1.5 \times 10^{-30}\ \text{C} \cdot \text{m}}{0.16 \times 10^{-9}\ \text{m}} = 9.37 \times 10^{-21}\ \text{C}.$

$$\frac{q}{e} = \frac{9.37 \times 10^{-21}\ \text{C}}{1.602 \times 10^{-19}\ \text{C}} = 0.058.$$

EVALUATE: The fractional ionic character for the bond in HI is much less than the fractional ionic character for the bond in NaCl. The bond in HI is mostly covalent and not very ionic.

42.34. **IDENTIFY** and **SET UP:** For an $l \to l-1$ transition, the frequency of the emitted photon is

$$f = \frac{\Delta E}{h} = \frac{\Delta E}{2\pi\hbar} = \frac{l\hbar}{2\pi I}, \text{ so } \Delta E = \frac{l\hbar^2}{I}.$$

$$I = m_r r_0^2.\ m_r = \frac{(3.82 \times 10^{-26}\ \text{kg})(3.15 \times 10^{-26}\ \text{kg})}{3.82 \times 10^{-26}\ \text{kg} + 3.15 \times 10^{-26}\ \text{kg}} = 1.726 \times 10^{-26}\ \text{kg}.$$

EXECUTE: $I = \dfrac{\hbar^2 l}{\Delta E} = \dfrac{hl\lambda}{4\pi^2 c} = 6.43 \times 10^{-46}\ \text{kg} \cdot \text{m}^2$ and from $I = m_r r_0^2$ the separation is

$$r_0 = \sqrt{\frac{I}{m_r}} = 0.193\ \text{nm}.$$

EVALUATE: Section 42.1 says $r_0 = 0.24$ nm for NaCl. Our result for NaF is smaller than this. This makes sense, since F is a smaller atom than Cl.

42.35. **IDENTIFY:** $E_{ex} = \dfrac{L^2}{2I} = \dfrac{\hbar^2 l(l+1)}{2I}$. $E_g = 0$ $(l=0)$, and there is an additional multiplicative factor of $2l+1$ because for each l state there are really $(2l+1)$ m_l states with the same energy.

SET UP: From Example 42.2, $I = 1.449 \times 10^{-46}\ \text{kg} \cdot \text{m}^2$ for CO.

EXECUTE: (a) $\frac{n_l}{n_0} = (2l+1)e^{-\hbar^2 l(l+1)/(2IkT)}$.

(b) (i) $E_{l=1} = \frac{\hbar^2(1)(1+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)} = 7.67\times10^{-23}$ J. $\frac{E_{l=1}}{kT} = \frac{7.67\times10^{-23}\text{ J}}{(1.38\times10^{-23}\text{ J/K})(300\text{ K})} = 0.0185$.

$$(2l+1) = 3, \text{ so } \frac{n_{l=1}}{n_0} = (3)e^{-0.0185} = 2.95.$$

(ii) $\frac{E_{l=2}}{kT} = \frac{\hbar^2(2)(2+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)(1.38\times10^{-23}\text{ J/K})(300\text{ K})} = 0.0556.$ $(2l+1) = 5$, so

$$\frac{n_{l=1}}{n_0} = (5)(e^{-0.0556}) = 4.73.$$

(iii) $\frac{E_{l=10}}{kT} = \frac{\hbar^2(10)(10+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)(1.38\times10^{-23}\text{ J/K})(300\text{ K})} = 1.02.$

$$(2l+1) = 21, \text{ so } \frac{n_{l=10}}{n_0} = (21)(e^{-1.02}) = 7.57.$$

(iv) $\frac{E_{l=20}}{kT} = \frac{\hbar^2(20)(20+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)(1.38\times10^{-23}\text{ J/K})(300\text{ K})} = 3.90.$ $(2l+1) = 41$, so

$$\frac{n_{l=20}}{n_0} = (41)e^{-3.90} = 0.833.$$

(v) $\frac{E_{l=50}}{kT} = \frac{\hbar^2(50)(50+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)(1.38\times10^{-23}\text{ J/K})(300\text{ K})} = 23.7.$ $(2l+1) = 101$, so

$$\frac{n_{l=50}}{n_0} = (101)e^{-23.7} = 5.38\times10^{-9}.$$

EVALUATE: (c) There is a competing effect between the $(2l+1)$ term and the decaying exponential. The $2l+1$ term dominates for small l, while the exponential term dominates for large l.

42.36. IDENTIFY: The ratio n_l/n_0 will be largest when its derivative with respect to l is zero.

SET UP: From Problem 42.35, we know that $n_l/n_0 = (2l+1)e^{-\hbar^2 l(l+1)/2IkT}$. Set $\frac{\partial(n_l/n_0)}{\partial l} = 0$ and solve for l.

EXECUTE: (a) $\frac{\partial}{\partial l}\left[(2l+1)e^{-\hbar^2 l(l+1)/2IkT}\right] = 2e^{-\hbar^2 l(l+1)/2IkT} - \frac{(2l+1)^2\hbar^2}{2IkT}e^{-\hbar^2 l(l+1)/2IkT} = 0.$

The exponentials cannot be zero, so

$$2 - \frac{(2l+1)^2\hbar^2}{2IkT} = 0 \quad \rightarrow \quad l_{max} = \frac{\sqrt{IkT}}{\hbar} - \frac{1}{2}.$$

(b) Using $I = 1.449\times10^{-46}$ kg·m² for CO from Example 42.2 gives

$$l_{max} = \frac{\sqrt{(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)(1.38\times10^{-23}\text{ J/K})(300\text{ K})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} - \frac{1}{2} = 6.84 \approx 7.$$

EVALUATE: Since $l_{max} \propto \sqrt{I}$, it would have a different value for molecules other than CO.

42.37. IDENTIFY: This problem is about the power in a diode.
SET UP: We use Eq. (42.22) and $P = IV$.
EXECUTE: (a) We want the power.
$$P = IV = I_S\left(e^{eV/kT} - 1\right)V = (1.2\times10^{-11}\text{ A})\left(e^{e(0.6\text{ V})/k(300\text{ K})} - 1\right)(0.6V) = 85\text{ mW}.$$

(b) We want V when $P = 500$ mA $= 0.5$ A. We can solve this by trial-and-error. First put the equation in the convenient form

$$P = (1.2 \times 10^{-11} \text{ A})\left(e^{(38.65 \text{ V}^{-1})V} - 1\right)V$$

We know that V must be greater than 0.6 V. Trying $V = 0.8$ V gives $P = 260$ W. Trying $V = 0.7$ V gives $P = 4.7$ W. Trying $V = 0.65$ V gives $P = 635$ mW. Trying $V = 0.64$ V gives $P = 425$ mW. Trying $V = 0.645$ V gives $P = 520$ mW, which is close enough to 500 mA. So the maximum voltage is $V = 0.645$ V. To one significant figure, the answer is $V = 0.6$ V.

(c) We want the current when $V = 0.645$ V. $I = P/V = (520 \text{ mW})/(0.645 \text{ V}) = 0.81$ A.

EVALUATE: Note in part (b) how sensitive the power is to small changes in V due to the exponential.

42.38. **IDENTIFY:** The rotational energy levels are given by $E_l = l(l+1)\dfrac{\hbar^2}{2I}$. The transition energy ΔE for the molecule and λ for the photon are related by $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: From Example 42.2, $I_{CO} = 1.449 \times 10^{-46}$ kg $\cdot$ m^2.

EXECUTE: **(a)** $E_{l=1} = \dfrac{\hbar^2 l(l+1)}{2I} = \dfrac{(1.054 \times 10^{-34} \text{ J} \cdot \text{s})^2 (1)(1+1)}{2(1.449 \times 10^{-46} \text{ kg} \cdot \text{m}^2)} = 7.67 \times 10^{-23}$ J. $E_{l=0} = 0$.

$\Delta E = 7.67 \times 10^{-23}$ J $= 4.79 \times 10^{-4}$ eV. $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{(7.67 \times 10^{-23} \text{ J})}$

$= 2.59 \times 10^{-3}$ m $= 2.59$ mm.

EVALUATE: **(b)** Let's compare the value of kT when $T = 20K$ to that of ΔE for the $l = 1 \rightarrow l = 0$ rotational transition: $kT = (1.38 \times 10^{-23} \text{ J/K})(20 \text{ K}) = 2.76 \times 10^{-22}$ J.

$\Delta E = 7.67 \times 10^{-23}$ J (from part (a)). So $\dfrac{kT}{\Delta E} = 3.60$. Therefore, although T is quite small, there is still

plenty of energy to excite CO molecules into the first rotational level. This allows astronomers to detect the 2.59 mm wavelength radiation from such molecular clouds.

42.39. **IDENTIFY:** The vibrational energy levels are given by $E_n = (n + \frac{1}{2})\hbar\sqrt{\dfrac{k'}{m_r}}$. The zero-point energy is

$$E_0 = \frac{1}{2}\hbar\sqrt{\dfrac{2k'}{m_H}}.$$

SET UP: For H_2, $m_r = \dfrac{m_H}{2}$.

EXECUTE: $E_0 = \frac{1}{2}(1.054 \times 10^{-34} \text{ J} \cdot \text{s})\sqrt{\dfrac{2(576 \text{ N/m})}{1.67 \times 10^{-27} \text{ kg}}} = 4.38 \times 10^{-20}$ J $= 0.274$ eV.

EVALUATE: This is much less than the magnitude of the H_2 bond energy.

42.40. **IDENTIFY:** $\Delta E = hf = \hbar\sqrt{\dfrac{k'}{m_r}}$.

SET UP: $m_r = \dfrac{m_O m_H}{m_O + m_H} = 1.57 \times 10^{-27}$ kg.

EXECUTE: The vibration frequency is $f = \dfrac{\Delta E}{h} = 1.12 \times 10^{14}$ Hz. The force constant is

$$k' = (2\pi f)^2 m_r = 777 \text{ N/m}.$$

EVALUATE: This would be a fairly stiff spring in an ordinary physics lab.

42.41. **IDENTIFY** and **SET UP:** Use $I = m_r r_0^2$ to calculate I. The energy levels are given by

$E_{nl} = l(l+1)\left(\dfrac{\hbar^2}{2I}\right) + (n+\tfrac{1}{2})\hbar\sqrt{\dfrac{k'}{m_r}}$. The transition energy ΔE is related to the photon wavelength by

$\Delta E = hc/\lambda$.

EXECUTE: **(a)** $m_r = \dfrac{m_H m_I}{m_H + m_I} = \dfrac{(1.67\times10^{-27}\ \text{kg})(2.11\times10^{-25}\ \text{kg})}{1.67\times10^{-27}\ \text{kg} + 2.11\times10^{-25}\ \text{kg}} = 1.657\times10^{-27}\ \text{kg}$.

$I = m_r r_0^2 = (1.657\times10^{-27}\ \text{kg})(0.160\times10^{-9}\ \text{m})^2 = 4.24\times10^{-47}\ \text{kg}\cdot\text{m}^2$.

(b) The energy levels are $E_{nl} = l(l+1)\left(\dfrac{\hbar^2}{2I}\right) + (n+\tfrac{1}{2})\hbar\sqrt{\dfrac{k'}{m_r}}$.

$$\sqrt{\dfrac{k'}{m}} = \omega = 2\pi f, \ \ \text{so}\ \ E_{nl} = l(l+1)\left(\dfrac{\hbar^2}{2I}\right) + (n+\tfrac{1}{2})hf.$$

(i) Transition $n = 1 \to n = 0, l = 1 \to l = 0$:

$$\Delta E = (2-0)\left(\dfrac{\hbar^2}{2I}\right) + (1+\tfrac{1}{2}-\tfrac{1}{2})hf = \dfrac{\hbar^2}{I} + hf.$$

$$\Delta E = \dfrac{hc}{\lambda}, \ \ \text{so}\ \ \lambda = \dfrac{hc}{\Delta E} = \dfrac{hc}{(\hbar^2/I) + hf} = \dfrac{c}{(\hbar/2\pi I) + f}.$$

$$\dfrac{\hbar}{2\pi I} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(4.24\times10^{-47}\ \text{kg}\cdot\text{m}^2)} = 3.960\times10^{11}\ \text{Hz}.$$

$$\lambda = \dfrac{c}{(\hbar/2\pi I) + f} = \dfrac{2.998\times10^8\ \text{m/s}}{3.960\times10^{11}\ \text{Hz} + 6.93\times10^{13}\ \text{Hz}} = 4.30\ \mu\text{m}.$$

(ii) Transition $n = 1 \to n = 0, l = 2 \to l = 1$:

$$\Delta E = (6-2)\left(\dfrac{\hbar^2}{2I}\right) + hf = \dfrac{2\hbar^2}{I} + hf.$$

$$\lambda = \dfrac{c}{2(\hbar/2\pi I) + f} = \dfrac{2.998\times10^8\ \text{m/s}}{2(3.960\times10^{11}\ \text{Hz}) + 6.93\times10^{13}\ \text{Hz}} = 4.28\ \mu\text{m}.$$

(iii) Transition $n = 2 \to n = 1, l = 2 \to l = 3$:

$$\Delta E = (6-12)\left(\dfrac{\hbar^2}{2I}\right) + hf = -\dfrac{3\hbar^2}{I} + hf.$$

$$\lambda = \dfrac{c}{-3(\hbar/2\pi I) + f} = \dfrac{2.998\times10^8\ \text{m/s}}{-3(3.960\times10^{11}\ \text{Hz}) + 6.93\times10^{13}\ \text{Hz}} = 4.40\ \mu\text{m}.$$

EVALUATE: The vibrational energy change for the $n = 1 \to n = 0$ transition is the same as for the $n = 2 \to n = 1$ transition. The rotational energies are much smaller than the vibrational energies, so the wavelengths for all three transitions don't differ much.

42.42. **IDENTIFY:** The frequency is proportional to the reciprocal of the square root of the reduced mass. The transition energy ΔE and the wavelength of the light emitted are related by $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: $f_0 = 1.24\times10^{14}\ \text{Hz}$.

EXECUTE: **(a)** In terms of the atomic masses, the frequency of the isotope with the deuterium atom is

$f = f_0\left(\dfrac{m_F m_H/(m_H + m_F)}{m_F m_D/(m_D + m_F)}\right)^{1/2} = f_0\left(\dfrac{1 + (m_F/m_D)}{1 + (m_F/m_H)}\right)^{1/2}$. Using f_0 and the given masses,

$f = 8.99\times10^{13}\ \text{Hz}$.

(b) For the molecule, $\Delta E = hf$. $hf = \dfrac{hc}{\lambda}$, so $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{8.99 \times 10^{13} \text{ Hz}} = 3.34 \times 10^{-6}$ m $= 3340$ nm. This

wavelength is in the infrared.

EVALUATE: The vibrational frequency of the molecule equals the frequency of the light that is emitted.

42.43. **IDENTIFY:** E_{F0} is given by $E_{F0} = \dfrac{3^{2/3} \pi^{4/3} \hbar^2 n^{2/3}}{2m}$. Since potassium is a metal and E does not change

much with T for metals, we approximate E_F by E_{F0}, so $E_F = \dfrac{3^{2/3} \pi^{4/3} \hbar^2 n^{2/3}}{2m}$.

SET UP: The number of atoms per m^3 is ρ/m. If each atom contributes one free electron, the electron

concentration is $n = \dfrac{\rho}{m} = \dfrac{851 \text{ kg/m}^3}{6.49 \times 10^{-26} \text{ kg}} = 1.31 \times 10^{28}$ electrons/m^3.

EXECUTE: $E_F = \dfrac{3^{2/3} \pi^{4/3} (1.054 \times 10^{-34} \text{ J} \cdot \text{s})^2 (1.31 \times 10^{28} / \text{m}^3)^{2/3}}{2(9.11 \times 10^{-31} \text{ kg})} = 3.24 \times 10^{-19}$ J $= 2.03$ eV.

EVALUATE: The E_F we calculated for potassium is about a factor of three smaller than the E_F for

copper that was calculated in Example 42.7.

42.44. **IDENTIFY** and **SET UP:** At r where U_{tot} is a minimum, $\dfrac{d}{dr} U_{\text{tot}} = 0$.

EXECUTE: **(a)** $\dfrac{d}{dr} U_{\text{tot}} = \dfrac{\alpha e^2}{4\pi\epsilon_0} \dfrac{1}{r^2} - 8A \dfrac{1}{r^9}$. Setting this equal to zero when $r = r_0$ gives $r_0^7 = \dfrac{8A4\pi\epsilon_0}{\alpha e^2}$

and so $U_{\text{tot}} = \dfrac{\alpha e^2}{4\pi\epsilon_0} \left(-\dfrac{1}{r} + \dfrac{r_0^7}{8r^8} \right)$. At $r = r_0$, $U_{\text{tot}} = -\dfrac{7\alpha e^2}{32\pi\epsilon_0 r_0} = -1.26 \times 10^{-18}$ J $= -7.85$ eV.

(b) To remove a Na^+Cl^- ion pair from the crystal requires 7.85 eV. When neutral Na and Cl atoms are

formed from the Na^+ and Cl^- atoms there is a net release of energy -5.14 eV $+ 3.61$ eV $= -1.53$ eV,

so the net energy required to remove a neutral Na Cl pair from the crystal is

$$7.85 \text{ eV} - 1.53 \text{ eV} = 6.32 \text{ eV}.$$

EVALUATE: Our calculation is in good agreement with the experimental value.

42.45. **IDENTIFY** and **SET UP:** Use the description of the bcc lattice in Figure 42.11c in the textbook to

calculate the number of atoms per unit cell and then the number of atoms per unit volume.

EXECUTE: **(a)** Each unit cell has one atom at its center and 8 atoms at its corners that are each shared

by 8 other unit cells. So there are $1 + 8/8 = 2$ atoms per unit cell.

$$\dfrac{n}{V} = \dfrac{2}{(0.35 \times 10^{-9} \text{ m})^3} = 4.66 \times 10^{-8} \text{ atoms/m}^3.$$

(b) $E_{F0} = \dfrac{3^{2/3} \pi^{4/3} \hbar^2}{2m} \left(\dfrac{N}{V} \right)^{2/3}$.

In this equation N/V is the number of free electrons per m^3. But the problem says to assume one free

electron per atom, so this is the same as n/V calculated in part (a).

$m = 9.109 \times 10^{-31}$ kg (the electron mass), so $E_{F0} = 7.563 \times 10^{-19}$ J $= 4.7$ eV.

EVALUATE: Our result for metallic lithium is similar to that calculated for copper in Example 42.7.

42.46. **(a) IDENTIFY:** The rotational energy levels are given by $E_l = l(l+1) \dfrac{\hbar^2}{2I}$. The photon wavelength λ is

related to the transition energy of the atom by $\Delta E = \dfrac{hc}{\lambda}$.

SET UP: For emission, $\Delta l = -1$. For such a transition, from state l to state $l-1$, we have

$\Delta E_l = [l(l+1) - (l-1)l]\dfrac{\hbar^2}{2I} = \dfrac{l\hbar^2}{I}$. The difference in transition energies for adjacent lines in the spectrum

is $\Delta E = \Delta E_l - \Delta E_{l-1} = \dfrac{\hbar^2}{I}$.

EXECUTE: The transition energies corresponding to the observed wavelengths are 3.29×10^{-21} J,

2.87×10^{-21} J, 2.47×10^{-21} J, 2.06×10^{-21} J, and 1.65×10^{-21} J. The average spacing of these energies is

0.410×10^{-21} J. Then, $\dfrac{\hbar^2}{I} = 0.410 \times 10^{-21}$ J, from which $I = 2.71 \times 10^{-47}$ kg $\cdot$ m^2.

EVALUATE: With $\dfrac{\hbar^2}{I} = 0.410 \times 10^{-21}$ J and $\Delta E_l = \dfrac{l\hbar^2}{I}$, we find that these wavelengths correspond to

transitions from levels 8, 7, 6, 5, and 4 to the respective next lower levels.

(b) IDENTIFY: Each transition is from the level l to the level $l-1$. The rotational energies are given by

$E_l = l(l+1)\dfrac{\hbar^2}{2I}$. The transition energy is related to the photon wavelength by $\Delta E = hc/\lambda$.

SET UP: $E_l = l(l+1)\hbar^2/2I$, so $\Delta E = E_l - E_{l-1} = [l(l+1) - l(l-1)]\left(\dfrac{\hbar^2}{2I}\right) = l\left(\dfrac{\hbar^2}{I}\right)$.

EXECUTE: $l\left(\dfrac{\hbar^2}{I}\right) = \dfrac{\hbar c}{\lambda}$.

$l = \dfrac{2\pi c I}{\hbar \lambda} = \dfrac{2\pi (2.998 \times 10^8 \text{ m/s})(2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2)}{(1.055 \times 10^{-34} \text{ J} \cdot \text{s})\lambda} = \dfrac{4.843 \times 10^{-4} \text{ m}}{\lambda}$.

For $\lambda = 60.4 \ \mu$m, $l = \dfrac{4.843 \times 10^{-4} \text{ m}}{60.4 \times 10^{-6} \text{ m}} = 8$.

For $\lambda = 69.0 \ \mu$m, $l = \dfrac{4.843 \times 10^{-4} \text{ m}}{69.0 \times 10^{-6} \text{ m}} = 7$.

For $\lambda = 80.4 \ \mu$m, $l = \dfrac{4.843 \times 10^{-4} \text{ m}}{80.4 \times 10^{-6} \text{ m}} = 6$.

For $\lambda = 96.4 \ \mu$m, $l = \dfrac{4.843 \times 10^{-4} \text{ m}}{96.4 \times 10^{-6} \text{ m}} = 5$.

For $\lambda = 120.4 \ \mu$m, $l = \dfrac{4.843 \times 10^{-4} \text{ m}}{120.4 \times 10^{-6} \text{ m}} = 4$.

EVALUATE: In each case l is an integer, as it must be.

(c) IDENTIFY: The rotational energies of a molecule depend on its moment of inertia, which in turn depends on the separation between the atoms in the molecule.

SET UP: Part (a) gives $I = 2.71 \times 10^{-47}$ kg $\cdot$ m^2. $I = m_r r^2$. Calculate m_r and solve for r.

EXECUTE: $m_r = \dfrac{m_H m_{Cl}}{m_H + m_{Cl}} = \dfrac{(1.67 \times 10^{-27} \text{ kg})(5.81 \times 10^{-26} \text{ kg})}{1.67 \times 10^{-27} \text{ kg} + 5.81 \times 10^{-26} \text{ kg}} = 1.623 \times 10^{-27}$ kg.

$r = \sqrt{\dfrac{I}{m_r}} = \sqrt{\dfrac{2.71 \times 10^{-47} \text{ kg} \cdot \text{m}^2}{1.623 \times 10^{-27} \text{ kg}}} = 1.29 \times 10^{-10}$ m $= 0.129$ nm.

EVALUATE: This is a typical atomic separation for a diatomic molecule; see Example 42.2 for the corresponding distance for CO.

(d) IDENTIFY and SET UP: The longest λ implies the smallest ΔE, and this is for the transition from $l = 1$ to $l = 0$.

EXECUTE: $\Delta E = l\left(\dfrac{\hbar^2}{I}\right) = (1)\dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{2.71\times10^{-47}\text{ kg}\cdot\text{m}^2} = 4.099\times10^{-22}\text{ J.}$

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{4.099\times10^{-22}\text{ J}} = 4.85\times10^{-4}\text{ m} = 485\,\mu\text{m.}$

EVALUATE: This is longer than any wavelengths in part (b).

42.47.   **IDENTIFY** and **SET UP:** The occupation probability $f(E)$ is $f(E) = \dfrac{1}{e^{(E-E_{\text{F}})/kT}+1}$.

EXECUTE: **(a)** Figure 42.47 shows the graph of E versus $\ln\{[1/f(E)]-1\}$ for the data given in the problem. The slope of the best-fit straight line is 0.445 eV, and the y-intercept is 1.80 eV.
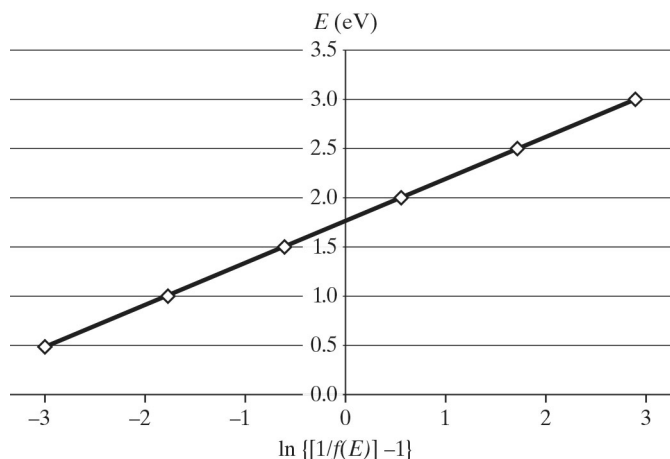

Figure 42.47

(b) Solve the $f(E)$ equation for E, giving $e^{(E-E_{\text{F}})/kT} = 1/f(E)-1$. Now take natural logarithms of both side of the equation, giving $(E-E_{\text{F}})/kT = \ln\{[1/f(E)]-1\}$, which gives $E = kT\ln\{[1/f(E)]-1\}+E_{\text{F}}$. From this we see that a graph of E versus $\ln\{[1/f(E)]-1\}$ should be a straight line having a slope equal to kT and a y-intercept equal to E_{F}. From our graph, we get $E_{\text{F}} = y$-intercept $= 1.80$ eV.
The slope is equal to kT, so $T = $ (slope)$/k = (0.445$ eV$)/(1.38\times10^{-23}$ J/K$)$
$$= (7.129\times10^{-20}\text{ J})/(1.38\times10^{-23}\text{ J/K})$$
$$= 5170\text{ K.}$$

EVALUATE: In Example 42.7, the Fermi energy for copper was found to be 7.03 eV, so our result of 1.80 eV seems plausible.

42.48.   **IDENTIFY** and **SET UP:** We assume that the equation $I = I_{\text{S}}(e^{eV/kT}-1)$ applies for this p-n junction. Use the given information for I and V to calculate the saturation current I_{S} and the temperature T for the junction.

EXECUTE: Put the numbers given in the problem into the equation for the current.

0.407 mA $= I_{\text{S}}(e^{(1.60\times10^{-19}\text{ C})(0.00500\text{ V})/(1.38\times10^{-23}\text{ J/K})T}-1) = I_{\text{S}}(e^{(57.97\text{ K})/T})-1.$

-0.338 mA $= I_{\text{S}}(e^{(1.60\times10^{-19}\text{ C})(-0.00500\text{ V})/(1.38\times10^{-23}\text{ J/K})T}-1) = I_{\text{S}}(e^{(-57.97\text{ K})/T})-1.$

Dividing the two equations gives $\dfrac{0.407}{0.338} = \dfrac{(e^{(57.97\text{ K})/T})-1}{(e^{(-57.97\text{ K})/T})-1}.$

Cross-multiplying and simplifying gives $e^{(57.97\text{ K})/T} - 2.204 + 1.204e^{-(57.97\text{ K})/T} = 0.$
Letting $x = e^{(57.97\text{ K})/T}$, this equation becomes $x - 2.204 + 1.204x^{-1} = 0.$

Multiplying by x gives the quadratic equation $x^2 - 2.204x + 1.204 = 0$.
The two solutions are $x = 1.00$ or $x = 1.204$. The $x = 1.00$ solution would require $T = \infty$, which is not physically possible, so $x = 1.204$ is the solution to use. This gives $1.204 = e^{(57.97 \text{ K})/T}$. Taking natural logarithms of both sides gives $\ln(1.204) = (57.97 \text{ K})/T$, so $T = (57.97 \text{ K})/\ln(1.204) = 312$ K.

The saturation current is $I_S = \dfrac{I}{e^{eV/kT} - 1} = \dfrac{0.407 \text{ mA}}{e^{(1.60\times10^{-19} \text{ C})(0.00500 \text{ V})/[(1.38\times10^{-23} \text{ J/K})(312 \text{ K})]} - 1} = 1.99$ mA.

EVALUATE: 312 K = 39°C = 102°F, which is quite a bit warmer than room temperature ($\approx$20°C). A current of about 2 mA is not unusual in laboratory apparatus.

42.49. **IDENTIFY:** This problem involves a semiconductor device used to make a half-wave rectifier.
SET UP: Eq. (42.22) gives the current through the junction of the semiconductor. The saturation current is 5×10^{-13} A and $T = 300$ K.
EXECUTE: **(a)** We want the minimum V_{out}, so $I = 1\ \mu$A. Solve Eq. (42.22) for V. (Careful! In this equation, e is the charge of an electron, not the base for natural logarithms.)
$$V_{\text{out}} = \frac{kT}{e}\ln\left(1 + \frac{I}{I_S}\right) = \frac{k(300 \text{ K})}{e}\ln\left(1 + \frac{1\ \mu\text{A}}{5\times10^{-13} \text{ A}}\right) = 0.375 \text{ V}.$$
(b) We want the maximum V_{out}. Combine Eq. (42.22) and $I = V/R_D$ and solve for V. Note that $e/kT = e/[k(300 \text{ K})] = 38.65$ V^{-1}.
$$V = R_D I = R_D I_S\left(e^{eV/kT} - 1\right) = (100\ \Omega)(5\times10^{-13} \text{ A})\left(e^{(38.65 \text{ V}^{-1})V} - 1\right).$$
Solve this equation by trial-and-error or using mathematical software. The result is $V_{\text{max}} = 0.6$ V.
(c) We want the current. $I = V/R_D = (0.6 \text{ V})/(100\ \Omega) = 0.006$ A.
(d) If $V_{\text{in}} > V_{\text{max}}$: $V_{\text{out}} = V_{\text{max}} = 0.6$ V.
If $V_{\text{in}} < V_{\text{max}}$: $V_{\text{out}} = V_{\text{in}}$.
EVALUATE: The exponential behavior makes I very sensitive to changes in V.

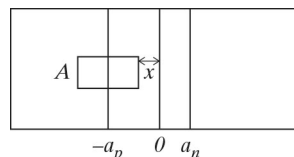**49.50.** **IDENTIFY:** We are dealing with a p-n junction diode.

Figure 42.50

SET UP and EXECUTE: **(a)** We want the electric field on the p side of the junction. Apply Gauss's law using the Gaussian surface shown in Figure 42.50 and follow the hint in the problem.
$\int KE \cos\theta\, dA = Q/\epsilon_0$, where $Q - eN_A(x - a_p)$.
$$KEA\cos\pi = \frac{-eN_A(a_p - x)A}{\epsilon_0}$$
$$E_x = -\frac{eN_A(x - a_p)}{K\epsilon_0}.$$
(b) We want the electric field on the n side of the junction. Use the same procedure as in part (a) except that the Gaussian surface lies to the right of $x = 0$ and extends into the n region. The result is
$$KEA\cos 0° = \frac{-eN_D(a_n - x)A}{\epsilon_0}$$
$$E_x = -\frac{eN_D(a_n - x)}{K\epsilon_0}.$$

(c) By continuity, at $x = 0$ the field must be the same for both expressions.

$$-\frac{eN_A(x-a_p)}{K\epsilon_0} = -\frac{eN_D(a_n-x)}{K\epsilon_0} \rightarrow N_A a_p = N_D a_n.$$

(d) We want $V(x)$. Use Eq. (23.18) as advised in the problem.

$$V(x) - V_0 = -\int E_x\, dx = -\int_{-a_p}^{-x} -\frac{eN_A}{K\epsilon_0}(x'-a_p)dx' = \frac{eN_A}{K\epsilon_0}\left(\frac{x^2}{2} + a_p x + \frac{3}{2}a_p^2\right).$$

Now use the requirement that $V(x) = 0$ when $x = -a_p$ to find V_0.

$$-V_0 = \frac{eN_A}{K\epsilon_0}\left(\frac{a_p^2}{2} - a_p^2 + \frac{3}{2}a_p^2\right) = \frac{eN_A}{K\epsilon_0}a_p^2.$$

Putting this result into our equation for $V(x)$ and combining terms gives

$$V(x) = \frac{eN_A}{2K\epsilon_0}(x+a_p)^2.$$

(e) We want $V(x)$ on the other side of the junction. Using the same approach as in part (d) gives

$$V(x) - V_0 - \int E_x\, dx = -\int_0^x -\frac{eN_D}{K\epsilon_0}(a_n-x')dx' = \frac{eN_D}{K\epsilon_0}\left(a_n x - \frac{x^2}{2}\right).$$

From part (d) we know $V(0)$ at $x = 0$. From our result above we know that $V(0) = V_0$. So

$$V_0 = \frac{eN_A}{2K\epsilon_0}a_p^2.$$

Using this in our result for $V(x)$ gives

$$V(x) = \frac{eN_A a_p^2}{2K\epsilon_0} - \frac{eN_D}{2K\epsilon_0}\left(x^2 - 2a_n x\right).$$

(f) We want the potential barrier $V_b = V(a_n) - V(-a_p)$. $V_b = V(a_n)$ because $V(-a_p) = 0$. Using the result from part (e) gives

$$V_b = \frac{eN_A a_p^2}{2K\epsilon_0} - \frac{eN_D}{2K\epsilon_0}\left(a_n^2 - 2a_n^2\right) = \frac{e}{2K\epsilon_0}\left(N_A a_p^2 + N_D a_n^2\right).$$

(g) We want a_p. Using the results from part (c) and the values for the variables given in the problem, we have

$$a_p = a_n\frac{N_D}{N_A} = 275 \text{ nm}.$$

(h) We want the peak electric field at $x = 0$. We can use either equation for E because of continuity and evaluate it at $x = 0$ with $K = 11.7$, giving

$$E = \frac{eN_D a_n}{K\epsilon_0} = 4.25 \text{ MV/m}.$$

(i) We want V_b. Use the results from part (f) and the given values and result from part (g). This gives

$$V_b = \frac{e}{2K\epsilon_0}\left(N_A a_p^2 + N_D a_n^2\right) = 0.701 \text{ V}.$$

EVALUATE: The electric field in part (h) is quite small compared to many lab fields.

42.51.   **IDENTIFY:**   We are looking at the vibrations of the CO_2 molecule.
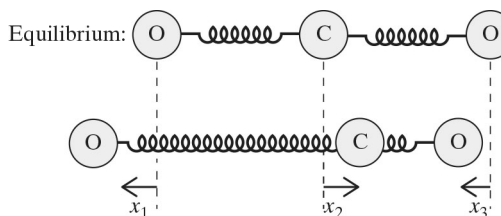

Figure 42.51

SET UP: According to the textbook description with Figure 42.10(c) for the asymmetric stretching mode, when carbon moves to the right, oxygen atoms move to the left, and when carbon moves to the left, they move to the right. Figure 42.51 shows the atoms in both their equilibrium configuration and displaced as described. First use Hooke's law to find the net force on each atom and then apply Newton's second law. Follow the instructions for each part of the problem.

EXECUTE: **(a)** <u>Force on the left oxygen atom</u>: This force is to the right. $F_{O1} = k'(x_2 + |x_1|)$. But x_1 is negative, so $|x_1| = -x_1$. Therefore, $F_{O1} = k'(x_2 - x_1) = -k'(x_1 - x_2)$.

<u>Force on the carbon atom</u>: The force due to the left-hand oxygen atom is to the left and the force due to the right-hand oxygen atom is also to the left. The net force is the sum of these two.

$F_C = -k'(x_2 + |x_1|) - k'(x_2 + |x_3|) = -k'(x_2 - x_1) - k'(x_2 - x_3) = k'(x_1 - 2x_2 + x_3)$.

<u>Force on the right oxygen atom</u>: This force is to the right.

$$F_{O2} = k'(x_2 + |x_3|) = k'(x_2 - x_3).$$

The corresponding equations for Newton's second law are

$$M_O \ddot{x}_1 = -k'(x_1 - x_2)$$
$$M_C \ddot{x}_2 = -k'(x_1 - 2x + x_3)$$
$$M_O \ddot{x}_3 = -k'(x_2 - x_3).$$

(b) Making the substitutions suggested in the problem, the Newton's second law equation become

$$M_O \ddot{X}_O = -k'(X_O + X_C)$$
$$-M_C \ddot{X}_C = k'(X_O + 2X_C + X_O) = 2k'(X_O + X_C)$$
$$M_O \ddot{X}_O = k'(-X_C - X_O) = -k'(X_C + X_O)$$

Combining these three equations, we find that the two independent equations are

$$M_O \ddot{X}_O = -k'(X_O + X_C)$$
$$M_C \ddot{X}_C = -2k'(X_O + X_C).$$

(c) Eliminating $X_C + X_O$ gives

$$M_C \ddot{X}_C = 2M_O \ddot{X}_O$$

Now solve this differential equation.

$$M_C \frac{d\dot{X}_C}{dt} = 2M_O \frac{d\dot{X}_O}{dt}$$

$$\dot{X}_C = \frac{2M_O}{M_C} \dot{X}_O$$

$$X_C = \frac{2M_O}{M_C} X_O.$$

(d) Following the directions gives

$$M_O \ddot{X}_O = -k'(X_O + X_C) = -k'\left(\frac{2M_O}{M_C} X_O + X_O\right)$$

$$\frac{M_O M_C}{M_C + 2M_O} \ddot{X}_O = -k'X_O$$

$$M_{\text{eff}} = \frac{M_O M_C}{M_C + 2M_O}.$$

(e) We want the angular frequency. From the differential equation in part (d) we recognize that
$\ddot{X}_O = -(k'/M_{\text{eff}})X_O$

$$\omega = \sqrt{k'/M_{\text{eff}}} = \sqrt{\frac{k'}{\frac{M_O M_C}{M_C + 2M_O}}} = \sqrt{\frac{k'(M_C + 2M)}{M_O M_C}}.$$

(f) We want the frequency f.

$$f = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{k'(M_C + 2M)}{M_O M_C}}.$$

Using $k' = 1860$ N/m, $M_O = 16$ u, and $M_C = 12$ u, we get $f = 8.06 \times 10^{13}$ Hz.
EVALUATE: This frequency is much lower than that of visible light.

42.52. **IDENTIFY:** This problem deals with the energy levels of molecules.
SET UP: Follow the instructions with each part. To get five significant figures, we should use numbers of even greater precision.
EXECUTE: **(a)** We want the photon energies. Use $E = hc/\lambda$ to calculate the energies

$E = hc/\lambda = hc/(4.2680\ \mu\text{m}) = 0.29049$ eV.
$E = hc/(4.2753\ \mu\text{m}) = 0.290006$ eV.
$E = hc/(4.2826\ \mu\text{m}) = 0.289512$ eV.
$E = hc/(4.2972\ \mu\text{m}) = 0.288528$ eV.
$E = hc/(4.3046\ \mu\text{m}) = 0.288032$ eV.

(b) We want to know which one of the energies in part (a) is due to a transition from an $l = 0$ state. Eq. (42.9) gives the energy due to rotation and vibration. The selection rules are $\Delta l = \pm 1$ and $\Delta n = 1$.

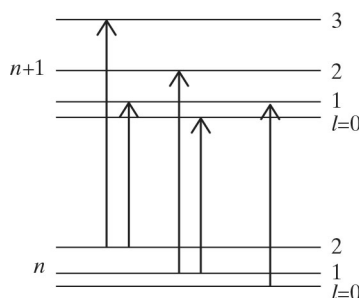$$E_{nl} = l(l+1)\frac{\hbar^2}{2I} + \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m_r}}$$

Figure 42.52

All the transitions are from the lowest energy states, which are $l = 0$, 1, 2. Figure 42.52 shows the five possible transitions from adjacent n states. The energy of a photon is equal to the energy difference between the two levels. Since Δn is the same for all these transitions, the energy difference between *rotational* states is what causes the different-energy photons. Therefore we need only look at the rotational energy differences between states. These are given by

$$\Delta E = E_{l_2} - E_{l_1} = \left[l_2(l_2+1) - l_1(l_1+1)\right]\frac{\hbar^2}{2I}$$

For the five transitions shown in Figure 42.52, the energy differences are

$$\Delta E_{0\to1} = \left[1(1+1) - 0\right]\frac{\hbar^2}{2I} = \frac{\hbar^2}{I}$$

$$\Delta E_{1\to0} = \left[0 - 2\right]\frac{\hbar^2}{2I} = -\frac{\hbar^2}{I}$$

$$\Delta E_{1\to2} = \left[2(2+1) - 1(1+1)\right]\frac{\hbar^2}{2I} = \frac{2\hbar^2}{I}$$

$$\Delta E_{2\to1} = \left[1(1+1) - 2(2+1)\right]\frac{\hbar^2}{2I} = \frac{-2\hbar^2}{I}$$

$$\Delta E_{2\to3} = \left[3(3+1) - 2(2+1)\right]\frac{\hbar^2}{2I} = \frac{3\hbar^2}{I}$$

The total energy difference is due to Δl and Δn. The photon energy E is therefore $E = \Delta E_n + \Delta E_l$. For the l transition in this situation, we have the following:

$$E_{0\to1} = \Delta E_n + \frac{\hbar^2}{I}$$

$$E_{1\to0} = \Delta E_n - \frac{\hbar^2}{I}$$

$$E_{1\to2} = \Delta E_n + \frac{2\hbar^2}{I}$$

$$E_{2\to1} = \Delta E_n - \frac{2\hbar^2}{I}$$

$$E_{2\to3} = \Delta E_n + \frac{3\hbar^2}{I}$$

Now rank the photon energies just calculated here and the photon energies calculated in part (a), from the lowest to the highest. By comparing the two rankings, we can tell which photon energy is due to a transition from an $l = 0$ state.

$$E_{2\to1} = \Delta E_n - \frac{2\hbar^2}{I} \qquad 0.288032 \text{ eV}$$

$$E_{1\to0} = \Delta E_n - \frac{\hbar^2}{I} \qquad 0.288528 \text{ eV}$$

$$E_{0\to1} = \Delta E_n + \frac{\hbar^2}{I} \qquad 0.289512 \text{ eV}$$

$$E_{1\to2} = \Delta E_n + \frac{2\hbar^2}{I} \qquad 0.290006 \text{ eV}$$

$$E_{2\to3} = \Delta E_n + \frac{3\hbar^2}{I} \qquad 0.290503 \text{ eV}$$

From this list, we see that energy from the $l = 0$ state is $E_{0\to1} = 0.289512$ eV.

(c) Using the results of part (b), we see that the second-highest energy states correspond, so $E_{1\to0}$ = 0.288528 eV.

(d) Use the results from parts (b) and (c). ΔE_n is the same for both transitions.

$$l = 0 \to l = 1: \quad 0.289512 \text{ eV} = \frac{\hbar^2}{I} + \Delta E_n$$

$$l = 1 \to l = 0: \quad 0.288528 \text{ eV} = \frac{\hbar^2}{I} + \Delta E_n$$

Add the equations and use Eq. (42.9) to find ΔE_n. Then solve for the desired quantity.

$$0.289512 \text{ eV} + 0.288528 \text{ eV} = 2\Delta E_n = 2\hbar\sqrt{k'/m_r}$$

$$\hbar\sqrt{k'/m_r} = 0.289020 \text{ eV}.$$

(e) We want I. Subtract the two equations at the beginning of part (d) and solve for I, giving

$$I = 1.41 \times 10^{-46} \text{ kg} \cdot \text{m}^2.$$

(f) What is the mystery gas? Use the information in Figure 42.52 in the textbook to calculate the moment of inertia of each molecule. Then compare these with what we found in part (e). Start with N_2.

$$I = m_r r_0^2 = \frac{m_1 m_2}{m_1 + m_2} r_0^2 = \frac{(14 \text{ u})(14 \text{ u})}{28 \text{ u}}(110 \text{ pm})^2 = 1.41 \times 10^{-46} \text{ kg} \cdot \text{m}^2.$$

This result matches the moment of inertia we found in part (e), so the gas is N_2.

(g) In part (f) we saw that $m_r = (14 \text{ u})(14 \text{ u})/(28 \text{ u}) = 7 \text{ u}$.

(h) Use the result from part (d) and solve for the spring constant, giving 2241 N/m.

EVALUATE: The spring constant we found would be a very large one for the types of springs found in most physics labs.

42.53.   IDENTIFY and **SET UP:** $p = -\dfrac{dE_{\text{tot}}}{dV}$. Relate E_{tot} to E_{F0} and evaluate the derivative.

EXECUTE: (a) $\dfrac{dE_{\text{tot}}}{dV} = \dfrac{3}{5}\left(\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\right)N^{5/3}\left(-\dfrac{2}{3}V^{-5/3}\right)$, so $p = \left(\dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\right)\left(\dfrac{N}{V}\right)^{5/3}$, as was to be shown.

(b) $N/V = 8.45 \times 10^{28} \text{ m}^{-3}$.

$$p = \left(\frac{3^{2/3}\pi^{4/3}(1.055 \times 10^{-34} \text{ J} \cdot \text{s})^2}{5(9.109 \times 10^{-31} \text{ kg})}\right)(8.45 \times 10^{28} \text{ m}^{-3})^{5/3} = 3.81 \times 10^{10} \text{ Pa} = 3.76 \times 10^5 \text{ atm}.$$

EVALUATE: (c) Normal atmospheric pressure is about 10^5 Pa, so these pressures are extremely large. The electrons are held in the metal by the attractive force exerted on them by the copper ions.

42.54.   IDENTIFY and **SET UP:** From Problem 42.53, $p = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\dfrac{N}{V}\right)^{5/3}$. Use this expression to calculate dp/dV.

EXECUTE: (a) $B = -V\dfrac{dp}{dV} = -V\left[\dfrac{5}{3} \cdot \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m} \cdot \left(\dfrac{N}{V}\right)^{2/3}\left(\dfrac{-N}{V^2}\right)\right] = \dfrac{5}{3}p$.

(b) $\dfrac{N}{V} = 8.45 \times 10^{28} \text{ m}^{-3}$. $B = \dfrac{5}{3} \cdot \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{5m}(8.45 \times 10^{28} \text{ m}^{-3})^{5/3} = 6.33 \times 10^{10} \text{ Pa}$.

EVALUATE: (c) The fraction of B due to the free electrons is $\dfrac{6.33 \times 10^{10} \text{ Pa}}{1.4 \times 10^{11} \text{ Pa}} = 0.45$. The copper ions themselves make up the remaining fraction.

42.55.   IDENTIFY and **SET UP:** Follow the steps specified in the problem.

EXECUTE: (a) $E_{F0} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\dfrac{N}{V}\right)^{2/3}$. Let $E_{F0} = \dfrac{1}{100}mc^2$.

$$\left(\frac{N}{V}\right) = \left[\frac{2m^2c^2}{(100)3^{2/3}\pi^{4/3}\hbar^2}\right]^{3/2} = \frac{2^{3/2}m^3c^3}{100^{3/2}3\pi^2\hbar^3} = \frac{2^{3/2}m^3c^3}{3000\pi^2\hbar^3} = 1.67 \times 10^{33} \text{ m}^{-3}.$$

(b) $\dfrac{8.45 \times 10^{28} \text{ m}^{-3}}{1.67 \times 10^{33} \text{ m}^{-3}} = 5.06 \times 10^{-5}$. Since the real concentration of electrons in copper is less than one part in 10^{-4} of the concentration where relativistic effects are important, it is safe to ignore relativistic effects for most applications.

(c) The number of electrons is $N_e = \dfrac{6(2\times10^{30}\text{ kg})}{1.99\times10^{-26}\text{ kg}} = 6.03\times10^{56}$. The concentration is

$$\frac{N_e}{V} = \frac{6.03\times10^{56}}{\frac{4}{3}\pi(6.00\times10^6\text{ m})^3} = 6.66\times10^{35}\text{ m}^{-3}.$$

EVALUATE: **(d)** Comparing this to the result from part (a) $\dfrac{6.66\times10^{35}\text{ m}^{-3}}{1.67\times10^{33}\text{ m}^{-3}} \cong 400,$ so relativistic effects

will be very important.

42.56.   **IDENTIFY** and **SET UP:**   The sensitivity is defined as $\Delta V/\Delta T$.
EXECUTE: From the graph shown with the introduction, we see that for a current of 0.1 A (100 mA),
the voltage at 25°C is about 0.75 V, and at 150°C it is about 0.55 V. Therefore the sensitivity is
$\dfrac{\Delta V}{\Delta T} = \dfrac{0.55\text{ V} - 0.75\text{ V}}{150°C - 25°C} = -1.6\text{ mV/C}° \approx -2.0\text{ mV/C}°.$ Therefore choice **(d)** is correct.
EVALUATE: If the temperature changes by 10 C°, V changes by about 20 mV, which can be significant.

42.57.   **IDENTIFY** and **SET UP:**   As the temperature increases, the electrons gain energy.
EXECUTE: As T increases, more electrons gain enough energy to jump into the conduction band, so for
a given voltage, more current will flow at a higher temperature than at a lower temperature. Therefore
choice **(b)** is correct.
EVALUATE: For ordinary resistors, increasing their temperature increases their resistance, so less
current would flow.

42.58.   **IDENTIFY** and **SET UP:**   On the graph shown with the introduction, draw a vertical line between the two
curves. This line represents a constant voltage.
EXECUTE: For V greater than about 0.2 V, the current at 150°C is greater than at 25°C. So increasing
the temperature at a constant voltage increases the current, which makes choice **(b)** correct.
EVALUATE: This result is consistent with what we found in Passage Problem 42.57.