21

ELECTRIC CHARGE AND ELECTRIC FIELD

VP21.4.1. **IDENTIFY:** In this problem we use Coulomb's law to calculate the electric force between charges.

SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. We want the total force on q_3. Start with a sketch showing the charge arrangement, as in Fig. VP21.4.1a.

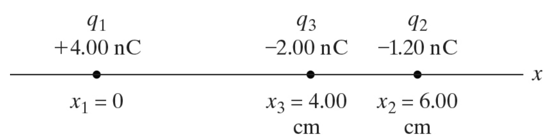


Figure VP21.4.1a

EXECUTE: We know that q_1 attracts q_3 because they have opposite signs, and q_2 repels q_3 because they have the same sign (both are negative). Sketch the forces on q_3 due to the other charges (see Fig. VP21.4.1b). Using $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$, we add the forces since they are both in the same direction. $F_{tot} =$ $F_{1\text{ on }3} + F_{2\text{ on }3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_3|}{r_{13}^2} + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2q_3|}{r_{23}^2} = \dfrac{|q_3|}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{r_{13}^2} + \dfrac{|q_2|}{r_{23}^2}\right)$. Using the given charges, $r_{13} =$ 4.00 cm = 0.0400 m, and r_{23} = 2.00 cm = 0.0200 m gives $F_{tot} = 9.89\times10^{-5}$ N = 98.9 μN. This force is in the $-x$ direction, so we can express it as $98.9\ \mu\text{N}\hat{\boldsymbol{i}}$.

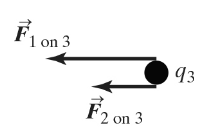


Figure VP21.4.1b

EVALUATE: It is always very helpful to make sketches like those shown here to get charge arrangements, distances, and force directions clear.

VP21.4.2. **IDENTIFY:** This problem requires the calculation of the electric force between charges, so we use Coulomb's law.

SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. Start with a sketch showing the charge arrangement, as in Fig. VP21.4.2a.

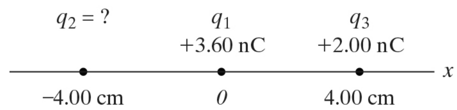


Figure VP21.4.2a

EXECUTE: **(a)** We want the force that q_1 exerts on q_3.

$F_{1\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2} = \frac{1}{4\pi\epsilon_0}\frac{(3.60\text{ nC})(2.00\text{ nC})}{(0.0400\text{ m})^2} = 40.5\ \mu\text{N}$. The direction is to the right (+x). Now sketch the forces on q_3 (Fig. 21.4.2b).

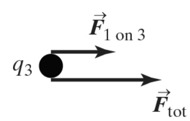


Figure VP21.4.2b

(b) We want the force that q_2 exerts on q_3. $F_{2\text{ on }3} = 54.0\ \mu\text{N} - 40.5\ \mu\text{N} = 13.5\ \mu\text{N}$ in the +x direction.

(c) We want q_2. $F_{2\text{ on }3} = \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r_{23}^2}$. Since q_2 repels q_3, q_2 must be positive. Solving for q_2 and using $r_{23} = 8.00\text{ cm} = 0.0800\text{ m}$, $q_3 = 2.00\text{ nC}$, and the result from part (b) gives $q_2 = +4.82\text{ nC}$.

EVALUATE: Always make sketches like those shown here to get charge arrangements, distances, and force directions clear.

VP21.4.3. **IDENTIFY:** This problem requires the calculation of the electric force between charges, so we use Coulomb's law.

SET UP: $F = \frac{1}{4\pi\epsilon_0}\frac{|qQ|}{r^2}$. Start with a sketch showing the charge arrangement, as in Fig. VP21.4.3a.

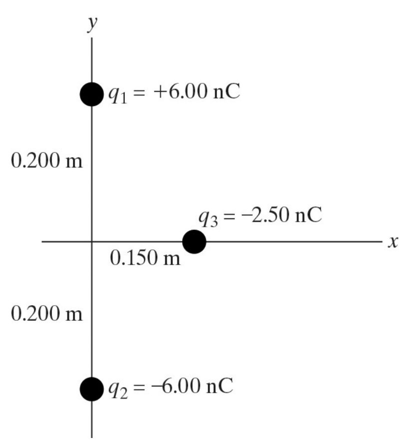


Figure VP21.4.3a

EXECUTE: Start with a careful sketch showing the forces on q_3 shown in Fig. VP21.4.3b. Since q_1 and q_2 have the same magnitude, the figure shows that the x components cancel and the y components add. So $F_{\text{tot}} = 2F_{1\text{ on }3}\sin\theta = \dfrac{2}{4\pi\epsilon_0}\dfrac{|q_1q_3|}{r_{13}^2}\sin\theta$. $\theta = \arctan\left(\dfrac{0.200\text{ m}}{0.150\text{ m}}\right) = 53.13°$.

$r_{13}^2 = (0.150\text{ m})^2 + (0.200\text{ m})^2 = 0.625\text{ m}^2$. Using these results, $q_1 = 6.00$ nC, and $q_3 = -2.50$ nC gives $F_{\text{tot}} = 3.45$ N in the $+y$ direction.

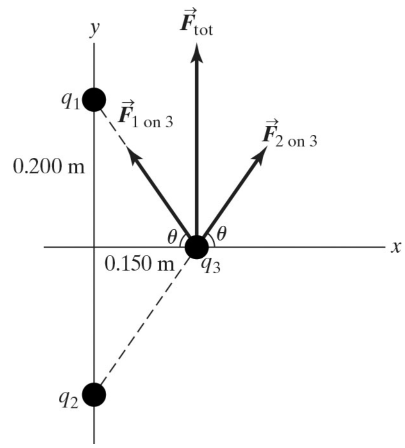


Figure VP21.4.3b

EVALUATE: Before doing any calculations, always make a vector drawing as in the figure. It can save a lot of unnecessary arithmetic and algebra.

VP21.4.4. **IDENTIFY:** We use Coulomb's law to calculate electric forces between point charges. Begin with a clear sketch of the charge arrangement, as in Fig. VP21.4.4a.

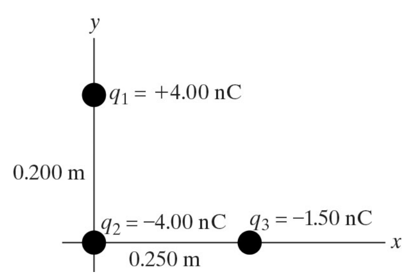


Figure VP21.4.4a

SET UP: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|qQ|}{r^2}$. We want the components of the total electric force on q_3. Fig. VP21.4.4b shows the two forces on q_3.

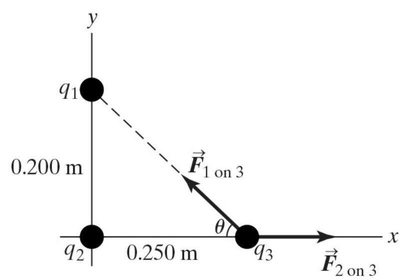


Figure VP21.4.4b

EXECUTE: $F_x = F_{2\text{ on }3} - F_{1\text{ on }3}\cos\theta$. Using Coulomb's law gives

$F_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2 q_3|}{r_{23}^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_3|}{r_{13}^2}\cos\theta$. $r_{13}^2 = (0.250\text{ m})^2 + (0.200\text{ m})^2 = 0.1025\text{ m}^2$.

$\theta = \arctan\left(\dfrac{0.200\text{ m}}{0.250\text{ m}}\right) = 38.66°$. Using $r_{23} = 0.250$ m and the given charges, gives $F_x = 0.451\ \mu$N.

$F_y = F_{1\text{ on }3}\sin\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_3|}{r_{13}^2}\sin\theta = 0.329\ \mu\text{N}$.

EVALUATE: Always start with careful sketches showing the charges and the force vectors.

VP21.10.1 **IDENTIFY:** We want to find the total electric field due to two point charges.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

EXECUTE: **(a)** First sketch the charge arrangement and the electric fields at the point (0, 0.100 m) (Fig. VP21.10.1a). Both fields point in the $-y$ direction, so $E_y = E_1 + E_2$ and $E_x = 0$. Using $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$

gives $E_y = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$. Using the given charges and $r_1 = r_2 = 0.100$ m gives $E_y = -8090$ N/C.

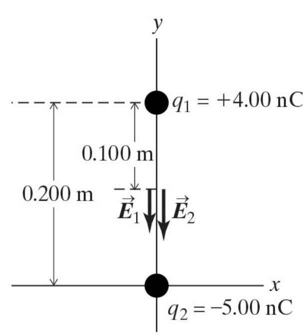


Figure VP21.10.1a

(b) Sketch the electric fields at the point (0, 0.400 m) as in Fig. VP21.10.1b. As in part (a), $E_x = 0$. Now the fields point in opposite directions, so $E_y = E_1 - E_2$. This gives $E_y = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$.

Using $r_1 = 0.200$ m and $r_2 = 0.400$ m, we get $E_y = 618$ N/C.

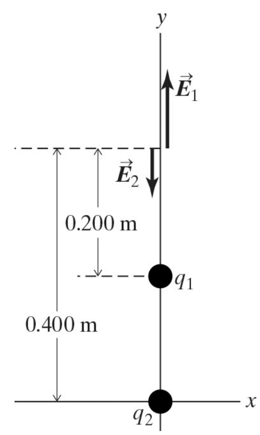


Figure VP21.10.1b

(c) Sketch the electric fields at the point (0.200 m, 0) as in Fig. VP21.10.1c. From this figure, we can see that $E_x = E_{1x} - |E_{2x}| = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}\cos\theta - \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$. $\theta = \arctan\left(\dfrac{0.200\text{ m}}{0.20\text{ m}}\right) = 45.0°$.

$r_1^2 = (0.200\text{ m})^2 + (0.200\text{ m})^2 = 0.0800\text{ m}^2$. Using these results and $r_2 = 0.200$ m, we get $E_x = -806$ N/C. From the figure we see that $E_y = -|E_{1y}| = -E_1\sin\theta = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}\sin\theta$. Using r_1 = 0.0800 m gives $E_y = -318$ N/C.

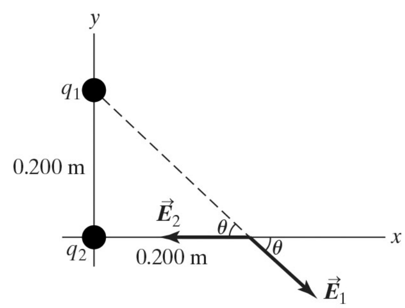


Figure VP21.10.1c

Evaluate: Careful sketches are especially important when working with two-dimensional charge configurations.

VP21.10.2. **Identify:** We are dealing with the electric field due to two point charges.

Set Up: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. First sketch the charge arrangement, as in Fig. 21.10.2a.

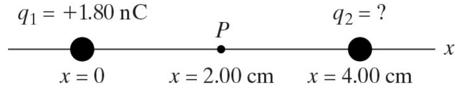


Figure VP21.10.2a

Execute: **(a)** We want the field due to q_1 at point P. Using q_1 = 1.80 nC and r_1 = 0.0200 m, $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2}$ gives $E_1 = 4.05\times10^4$ N/C in the $+x$ direction.

(c) Fig. VP21.10b shows the known electric fields at P. We want to find E_2. $E_{tot} = E_1 - E_2$, which gives $E_2 = E_{tot} - E_1 = 6.75\times10^4\text{ N/C} - 4.05\times10^4\text{ N/C} = 2.70\times10^4\text{ N/C}$ in the $+x$ direction.

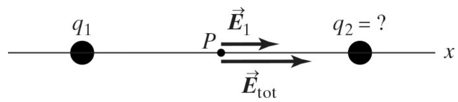


Figure VP21.10.2b

(c) The field E_2 must point in the $+x$ direction, which is toward q_2, so q_2 must be negative. $E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2|}{r_2^2}$. Solve for $|q_2|$, use $r_2 = 0.0200$ m and the result from part (b), so $q_2 = -1.20$ nC.

EVALUATE: Careful when finding electric fields. The components can be negative but the magnitude cannot be negative.

VP21.10.3. **IDENTIFY:** We view the hydrogen atom by modeling the orbital electron as a ring of charge centered on the proton.

SET UP: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. The total electric field is the field of the electron and the proton. We want the total field at $x = a$.

EXECUTE: **(a)** <u>Electron</u>: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{ea}{(a^2+a^2)^{3/2}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{2^{3/2}a^2}$. The electron is negative, so this field points *toward* the proton, which gives $E_x = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e}{2^{3/2}a^2}$.

<u>Proton</u>: $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{a^2}$ away from the proton.

$E_{tot} = E_p + E_e = E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{a^2} - \dfrac{1}{4\pi\epsilon_0}\dfrac{e}{2^{3/2}a^2} = \dfrac{e}{4\pi\epsilon_0 a^2}\left(1-\dfrac{1}{2\sqrt{2}}\right)$.

(b) $1-\dfrac{1}{2\sqrt{2}} > 0$, so the total field points *away from* the proton.

EVALUATE: The total field is less than the proton's field because the electron partially cancel it.

VP21.10.4. **IDENTIFY:** We want the electric field caused by a charged rod. We need calculus to do this calculation.

SET UP: Fig. VP21.10.4 shows the set up of the rod along the x-axis. The uniform linear charge density of the rod is Q/L.

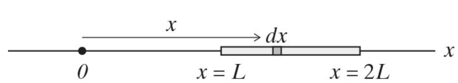


Figure VP21.10.4

EXECUTE: **(a)** $dq = (Q/L)dx$.

(b) $dE_x = -\dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{x^2} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/L)dx}{x^2}$. $dE_y = 0$.

(c) $E_x = \int dE_x = \displaystyle\int_L^{2L} -\dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/L)dx}{x^2} = -\dfrac{Q}{8\pi\epsilon_0 L^2}$.

EVALUATE: If we treat the rod as a point charge at its center, we get $E_x = -\frac{Q}{4\pi\epsilon_0 (3L/2)^2} = -\frac{Q}{9\pi\epsilon_0 L^2}$, which is *not* the same as our result. So we cannot simplify the rod as a point charge.

VP21.14.1. IDENTIFY: This problem involves an electric dipole in an external electric field.

SET UP: We want the torque on the dipole and its potential energy.

EXECUTE: **(a)** $\tau = pE\sin\phi = (6.13\times10^{-30}\,\text{C}\cdot\text{m})(3.00\times10^{5}\text{ N/C})\sin 50.0° = 1.41\times10^{-24}\text{ N}\cdot\text{m}.$

(b) $U = -pE\cos\phi = -1.18\times10^{-24}\text{ J},$ using the same values as in part (a).

EVALUATE: These results are very small, but a molecule consists of very small charges that are very close together.

VP21.14.2. IDENTIFY This problem involves an electric dipole in an external electric field.

SET UP: We want the charges that make up the dipole. $\tau = pE\sin\phi$ where $p = qd$.

EXECUTE: Solve for q: $q = \frac{\tau}{dE\sin\phi} = \frac{6.60\times10^{-26}\text{ N}\cdot\text{m}}{(1.10\times10^{-10}\text{ m})(8.50\times10^{4}\text{ N/C})(\sin 90°)} = 7.06\times10^{-21}\text{ C}.$

EVALUATE: Each end of the dipole has charge of this magnitude but opposite signs.

VP21.14.3. IDENTIFY: We are dealing with the potential energy of an electric dipole in an external electric field.

SET UP: $U = -pE\cos\phi.$

EXECUTE: We want p. $\Delta U = U_2 - U_1 = -pE\cos 0° = (-pE\cos 180°) = -2pE,$ so the work required is $W = +2pE$. Therefore $p = \frac{W}{2E} = \frac{4.60\times10^{-25}\text{ J}}{2(1.20\times10^{5}\text{ N/C})} = 1.92\times10^{-30}\text{ C}\cdot\text{m}.$

EVALUATE: If the charges are 0.50×10^{-11} m apart (about 1/10 the radius of an atom), the charges are each $q = p/d =$ $(1.92\times10^{-30}\text{ C}\cdot\text{m})/(0.50\times10^{-11}\text{ m}) = 3.8\times10^{-19}\text{ C}.$

VP21.14.4. IDENTIFY: We are looking at a KBr dipole having a dipole moment of $p = 3.50\times10^{-29}\text{ C}\cdot\text{m}.$

SET UP and EXECUTE: **(a)** We want the distance between the ions. $p = qd = ed$, so $d = p/e$. Using the given p gives $d = 2.19\times10^{-10}$ m.

(b) We want to find the point on the dipole axis where the electric field is $E = 8.00\times10^{4}$ N/C. There are two possibilities to consider: the point is between the ions or it is outside the dipole. Between the ions: The field is weakest midway between the ions. At that point

$$E_{\text{min}} = 2\left(\frac{1}{4\pi\epsilon_0}\frac{e}{(d/2)^2}\right) = 2.40\times10^{11}\text{ N/C}$$

This is less than 8.00×10^{4} N/C, so the point must be *outside* the dipole.

Outside the dipole: In this case the fields due to the two ions point in opposite directions. Call x the distance from the center of the dipole to the desired point and $2a$ the length of the dipole. The total field is $E = \frac{1}{4\pi\epsilon_0}\frac{e}{(x-a)^2} - \frac{1}{4\pi\epsilon_0}\frac{e}{(x+a)^2} = \frac{e}{4\pi\epsilon_0}\frac{4ax}{(x+a)^2(x-a)^2}$. The electric field at this point is 8.00×10^{4} N/C, which is much much less than E_{min} inside the dipole. Therefore this point must be very far from the center of the dipole, so $x >> a$. In this case, $x + a \approx x$ and $x - a \approx x$, so the equation for E simplifies to $E \approx \frac{e}{4\pi\epsilon_0}\frac{4ax}{x^4} = \frac{ea}{\pi\epsilon_0 x^3}$. Using $a = d/2$ and solving for x gives $x = \left(\frac{ed/2}{\pi\epsilon_0 E}\right)^{1/3}$. Using $d = 2.19\times10^{-10}$ m and $E = 8.00\times10^{4}$ N/C gives $x = 1.99\times10^{-8}$ m.

EVALUATE: Between the ions of the dipole the field is very strong since we are very close to the charges and the fields point in the same direction. Outside the dipole the fields are in opposite directions and partially cancel each other so the net field is much weaker. Our approximation in part (b) that $x \ll a$ is reasonable $x \approx 100d$.

21.1. **(a) IDENTIFY** and **SET UP:** Use the charge of one electron $(-1.602\times10^{-19}\text{ C})$ to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e}=\frac{-3.20\times10^{-9}\text{ C}}{-1.602\times10^{-19}\text{ C/electron}}=2.00\times10^{10}\text{ electrons.}$$

(b) IDENTIFY and **SET UP:** Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is $207\times10^{-3}\text{ kg/mol}$, so the number of moles in $8.00\times10^{-3}\text{ kg}$ is $n=\dfrac{m_{\text{tot}}}{M}=\dfrac{8.00\times10^{-3}\text{ kg}}{207\times10^{-3}\text{ kg/mol}}=0.03865\text{ mol.}$ N_A (Avogadro's number) is the number of atoms in 1 mole, so the number of lead atoms is $N=nN_\text{A}=(0.03865\text{ mol})(6.022\times10^{23}\text{ atoms/mol})=2.328\times10^{22}\text{ atoms.}$ The number of excess electrons per lead atom is $\dfrac{2.00\times10^{10}\text{ electrons}}{2.328\times10^{22}\text{ atoms}}=8.59\times10^{-13}.$

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

21.2. **IDENTIFY:** The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e=1.60\times10^{-19}\text{ C}$.

EXECUTE: The rate of charge flow is $20{,}000\text{ C/s}$ and $t=100\,\mu\text{s}=1.00\times10^{-4}\text{ s}$.

$Q=(20{,}000\text{ C/s})(1.00\times10^{-4}\text{ s})=2.00\text{ C}$. The number of electrons is $n_\text{e}=\dfrac{Q}{1.60\times10^{-19}\text{ C}}=1.25\times10^{19}.$

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. **IDENTIFY** and **SET UP:** A proton has charge $+e$ and an electron has charge $-e$, with $e=1.60\times10^{-19}\text{ C}$. The force between them has magnitude $F=k\dfrac{|q_1q_2|}{r^2}=k\dfrac{e^2}{r^2}$ and is attractive since the charges have opposite sign. A proton has mass $m_\text{p}=1.67\times10^{-27}\text{ kg}$ and an electron has mass $9.11\times10^{-31}\text{ kg}$. The acceleration is related to the net force $\vec{F}$ by $\vec{F}=m\vec{a}$.

EXECUTE: $F=k\dfrac{e^2}{r^2}=(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\text{ C})^2}{(2.0\times10^{-10}\text{ m})^2}=5.75\times10^{-9}\text{ N.}$

Proton: $a_\text{p}=\dfrac{F}{m_\text{p}}=\dfrac{5.75\times10^{-9}\text{ N}}{1.67\times10^{-27}\text{ kg}}=3.4\times10^{18}\text{ m/s}^2.$

Electron: $a_\text{e}=\dfrac{F}{m_\text{e}}=\dfrac{5.75\times10^{-9}\text{ N}}{9.11\times10^{-31}\text{ kg}}=6.3\times10^{21}\text{ m/s}^2$

The proton has an initial acceleration of $3.4\times10^{18}\text{ m/s}^2$ toward the electron and the electron has an initial acceleration of $6.3\times10^{21}\text{ m/s}^2$ toward the proton.

EVALUATE: The force the electron exerts on the proton is equal in magnitude to the force the proton exerts on the electron, but the accelerations of the two particles are very different because their masses are very different.

21.4. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the $+y$-direction.

EXECUTE: $F_{2\text{ on }1} = \dfrac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\ \text{C})(2.0\times10^{-6}\ \text{C})}{(0.60\ \text{m})^2} = 0.100\ \text{N}$. $(F_{2\text{ on }1})_x = 0$ and $(F_{2\text{ on }1})_y = +0.100\ \text{N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $(F_{Q\text{ on }1})_x = -0.23\ \text{N}$ and . $(F_{Q\text{ on }1})_y = 0.17\ \text{N}$.. $F_x = (F_{2\text{ on }1})_x + (F_{Q\text{ on }1})_x = -0.23\ \text{N}$.

$F_y = (F_{2\text{ on }1})_y + (F_{Q\text{ on }1})_y = 0.100\ \text{N} + 0.17\ \text{N} = 0.27\ \text{N}$. The magnitude of the total force is

$F = \sqrt{(0.23\ \text{N})^2 + (0.27\ \text{N})^2} = 0.35\ \text{N}$. $\tan^{-1}\dfrac{0.23}{0.27} = 40°$, so $\vec{F}$ is 40° counterclockwise from the $+y$-axis, or 130° counterclockwise from the $+x$- axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.5. **IDENTIFY:** Each ion carries charge as it enters the axon.

SET UP: The total charge Q is the number N of ions times the charge of each one, which is e. So $Q = Ne$, where $e = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: The number N of ions is $N = (5.6\times10^{11}\ \text{ions/m})(1.5\times10^{-2}\ \text{m}) = 8.4\times10^9$ ions. The total charge Q carried by these ions is $Q = Ne = (8.4\times10^9)(1.60\times10^{-19}\ \text{C}) = 1.3\times10^{-9}\ \text{C} = 1.3\ \text{nC}$.

EVALUATE: The amount of charge is small, but these charges are close enough together to exert large forces on nearby charges.

21.6. **IDENTIFY:** Apply Coulomb's law and calculate the net charge q on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: $F = k\dfrac{|q_1 q_2|}{r^2}$ gives

$|q| = \sqrt{4\pi\epsilon_0 Fr^2} = \sqrt{4\pi\epsilon_0\,(3.33\times10^{-21}\ \text{N})(0.200\ \text{m})^2} = 1.217\times10^{-16}\ \text{C}$. Therefore, the total number of electrons required is $n = |q|/e = (1.217\times10^{-16}\ \text{C})/(1.60\times10^{-19}\ \text{C/electron}) = 760$ electrons.

EVALUATE: Each sphere has 760 excess electrons and each sphere has a net negative charge. The two like charges repel.

21.7. **IDENTIFY:** Apply $F = \dfrac{k|q_1q_2|}{r^2}$ and solve for r.

SET UP: $F = 650\ \text{N}$.

EXECUTE: $r = \sqrt{\dfrac{k|q_1q_2|}{F}} = \sqrt{\dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\ \text{C})^2}{650\ \text{N}}} = 3.7\times10^3\ \text{m} = 3.7\ \text{km}$

EVALUATE: Charged objects typically have net charges much less than 1 C.

21.8. **IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_\text{A} = 6.02\times10^{23}$ atoms/mol. $|q| = n'_\text{e}e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_\text{e} = n_\text{p} = (13)(N_\text{A})\left(\frac{0.0250\ \text{kg}}{0.026982\ \text{kg/mol}}\right) = 7.25\times10^{24}\ \text{electrons}.$$

(b) For a force of 1.00×10^4 N to act between the spheres, $F = 1.00\times10^4\ \text{N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$. This gives $|q| = \sqrt{4\pi\epsilon_0\,(1.00\times10^4\ \text{N})(0.800\ \text{m})^2} = 8.43\times10^{-4}\ \text{C}$. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27\times10^{15}$ electrons.

(c) $n'_e/n_e = 7.27\times10^{-10}$.

EVALUATE: When ordinary objects receive a net charge, the fractional change in the total number of electrons in the object is very small.

21.9. **IDENTIFY:** Apply Coulomb's law.

SET UP: Consider the force on one of the spheres.

EXECUTE: **(a)** $q_1 = q_2 = q$ and $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2} = \dfrac{q^2}{4\pi\epsilon_0 r^2}$, so

$$q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\ \text{m}\sqrt{\frac{0.220\ \text{N}}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 7.42\times10^{-7}\ \text{C (on each)}.$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \frac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \frac{1}{2}(7.42\times10^{-7}\ \text{C}) = 3.71\times10^{-7}\ \text{C}.$$

And then $q_2 = 4q_1 = 1.48\times10^{-6}$ C.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the spheres have equal charges or not.

21.10. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.10.

EXECUTE: $F_1 = k\dfrac{|q_1 q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-6}\ \text{N}.$

$$F_2 = k\frac{|q_2 q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-6}\ \text{N}.$$

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-6}$ N. The net force has magnitude 2.40×10^{-6} N and is in the $+x$-direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

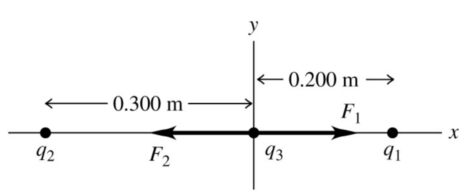


Figure 21.10

21.11. **IDENTIFY:** In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration: $a = F/m = (1/4\pi\epsilon_0)(e^2/r^2)/m.$

EXECUTE:

(a) $a = (9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2/[(0.00250\ \text{m})^2(1.67\times10^{-27}\ \text{kg})] = 2.21\times10^4\ \text{m/s}^2.$

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

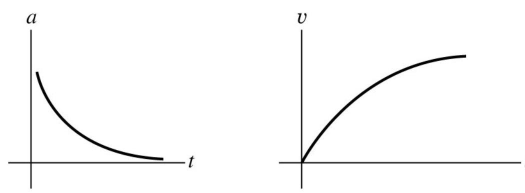


Figure 21.11

21.12. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: **(a)** $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1q_2|}{r^2}$ gives $0.600\ \text{N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(0.550\times10^{-6}\ \text{C})|q_2|}{(0.30\ \text{m})^2}$ and

$|q_2| = +1.09\times10^{-5}\ \text{C} =\ 10.9\ \mu\text{C}$. The force is attractive and $q_1 < 0,$ so $q_2 = +1.09\times10^{-5}\ \text{C}\ = +10.9\mu\text{C}.$

(b) $F = 0.600\ \text{N}$. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. **IDENTIFY:** Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\dfrac{|q_2q_3|}{r_2^2}$ and is in the $+x$-direction. $\vec{F}_1$ must be in the $-x$-direction, so q_1 must be positive. $F_1 = F_2$ gives $k\dfrac{|q_1||q_3|}{r_1^2} = k\dfrac{|q_2||q_3|}{r_2^2}.$

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\ \text{nC})\left(\frac{2.00\ \text{cm}}{4.00\ \text{cm}}\right)^2 = 0.750\ \text{nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_3.

21.14. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on Q.

SET UP: The force that q_1 exerts on Q is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The x-components cancel. We only need the y-components, and each charge contributes equally. $F_{1y} = F_{2y} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(2.0\times10^{-6}\ \text{C})(4.0\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2}\sin\alpha = -0.173\ \text{N}$ (since $\sin\alpha = 0.600$).

Therefore, the total force is $2F = 0.35\ \text{N},$ in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\ \mu\text{C}$ and q_2 is $+2.0\ \mu\text{C},$ then the net force is in the $+y$-direction.

21.15. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_1.

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the $+x$-direction.

EXECUTE: $F_2 = k\dfrac{|q_1q_2|}{r_{12}^2} = 6.749\times10^{-5}$ N, $F_3 = k\dfrac{|q_1q_3|}{r_{13}^2} = 1.124\times10^{-4}$ N. $F = F_2 + F_3 = 1.8\times10^{-4}$ N.

$F = 1.8\times10^{-4}$ N and is in the $+x$-direction.

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1\text{ on }3} = -\vec{F}_{3\text{ on }1}$, as required by Newton's third law.

21.16. **IDENTIFY** and **SET UP:** Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the x-coordinate of q_3.

EXECUTE: $\vec{F}_2$ is in the x-direction.

$F_2 = k\dfrac{|q_1q_2|}{r_{12}^2} = 3.37$ N, so $F_{2x} = +3.37$ N

$F_x = F_{2x} + F_{3x}$ and $F_x = -7.00$ N

$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37$ N

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$F_3 = k\dfrac{|q_1q_3|}{x^2}$, so $|x| = \sqrt{\dfrac{k|q_1q_3|}{F_3}} = 0.144$ m, so $x = -0.144$ m

EVALUATE: q_2 attracts q_1 in the $+x$-direction so q_3 must attract q_1 in the $-x$-direction, and q_3 is at negative x.

21.17. **IDENTIFY:** Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.17a.

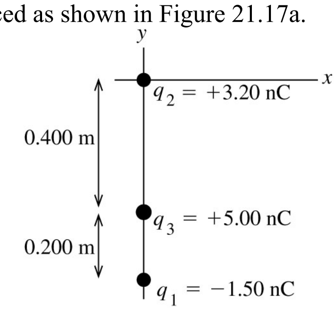
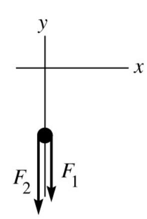


Figure 21.17a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.17b.



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2q_3|}{r_{23}^2}$$

Figure 21.17b

$$F_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times10^{-6}\ \text{N}$$

$$F_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times10^{-7}\ \text{N}$$

The resultant force is $\vec{\boldsymbol{R}} = \vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2$.

$R_x = 0.$

$R_y = -(F_1 + F_2) = -(1.685\times10^{-6}\ \text{N} + 8.988\times10^{-7}\ \text{N}) = -2.58\times10^{-6}\ \text{N}.$

The resultant force has magnitude $2.58\times10^{-6}\ \text{N}$ and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is repulsive.

21.18. **IDENTIFY:** We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-N combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$.

EXECUTE: **(a)** $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$.

O-H-N:

O^- - H^+: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.170\times10^{-9}\ \text{m})^2} = 7.96\times10^{-9}\ \text{N}$, attractive

O^- - N^-: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.280\times10^{-9}\ \text{m})^2} = 2.94\times10^{-9}\ \text{N}$, repulsive

N-H-N:

N^- - H^+: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.190\times10^{-9}\ \text{m})^2} = 6.38\times10^{-9}\ \text{N}$, attractive

N^- - N^-: $F = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.300\times10^{-9}\ \text{m})^2} = 2.56\times10^{-9}\ \text{N}$, repulsive

The total attractive force is $1.43\times10^{-8}\ \text{N}$ and the total repulsive force is $5.50\times10^{-9}\ \text{N}$. The net force is attractive and has magnitude $1.43\times10^{-8}\ \text{N} - 5.50\times10^{-9}\ \text{N} = 8.80\times10^{-9}\ \text{N}$.

(b) $F = k\dfrac{e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.0529\times10^{-9}\ \text{m})^2} = 8.22\times10^{-8}\ \text{N}.$

EVALUATE: The bonding force of the electron in the hydrogen atom is a factor of 10 larger than the bonding force of the adenine-thymine molecules.

21.19. **IDENTIFY:** We use Coulomb's law to find each electrical force and combine these forces to find the net force.

SET UP: In the O-H-O combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other O^-. In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^-. In the O-H-N combination the O^- is 0.180 nm from the H^+ and 0.290 nm from the other N^-. Like charges repel and unlike charges attract. The net force is the vector sum of the individual forces. The force due to each pair of charges is $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$.

EXECUTE: Using $F = k\dfrac{|q_1 q_2|}{r^2} = k\dfrac{e^2}{r^2}$, we find that the attractive forces are: O^-- H^+, 7.10×10^{-9} N; N^-- H^+, 6.37×10^{-9} N; O^-- H^+, 7.10×10^{-9} N. The total attractive force is 2.06×10^{-8} N. The repulsive forces are: O^- - O^-, 2.74×10^{-9} N; N^- - N^-, 2.56×10^{-9} N; O^- - N^-, 2.74×10^{-9} N. The total repulsive force is 8.04×10^{-9} N. The net force is attractive and has magnitude 1.26×10^{-8} N.

EVALUATE: The net force is attractive, as it should be if the molecule is to stay together.

21.20. **IDENTIFY:** Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

SET UP: A proton has mass $m_p = 1.67\times10^{-27}$ kg and charge $+e$. Let $+x$ be in the direction of motion of the proton.

EXECUTE: **(a)** $v_{0x} = 0$. $a = \dfrac{eE}{m_p}$. $x - x_0 = v_{0x}t + \dfrac{1}{2}a_x t^2$ gives $x - x_0 = \dfrac{1}{2}a_x t^2 = \dfrac{1}{2}\dfrac{eE}{m_p}t^2$. Solving for E gives $E = \dfrac{2(0.0160\text{ m})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(3.20\times10^{-6}\text{ s})^2} = 32.6$ N/C.

(b) $v_x = v_{0x} + a_x t = \dfrac{eE}{m_p}t = \dfrac{e}{m_p}\left(\dfrac{2(x-x_0)m_p}{et^2}\right)t = \dfrac{2(x-x_0)}{t} = \dfrac{2(0.0160\text{ m})}{3.20\times10^{-6}\text{ s}} = 1.00\times10^4$ m/s.

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

21.21. **IDENTIFY:** $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass 1.67×10^{-27} kg.

EXECUTE: **(a)** $F = (1.60\times10^{-19}\text{ C})(2.75\times10^3\text{ N/C}) = 4.40\times10^{-16}$ N.

(b) $a = \dfrac{F}{m} = \dfrac{4.40\times10^{-16}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 2.63\times10^{11}\text{ m/s}^2$.

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63\times10^{11}\text{ m/s}^2)(1.00\times10^{-6}\text{ s}) = 2.63\times10^5$ m/s.

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.22. **IDENTIFY:** For a point charge, $E = k\dfrac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: **(a)** The field is toward the negative charge so is downward.

$$E = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\text{ C}}{(0.250\text{ m})^2} = 719\text{ N/C}.$$

(b) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\text{ C})}{12.0\text{ N/C}}} = 1.94$ m.

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

21.23. **IDENTIFY:** The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) SET UP: First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50\times10^6 \text{ m/s})^2 + 2a(0.0320 \text{ m})$ and $a = 3.16\times10^{14} \text{ m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and

$E = ma/e = (1.67\times10^{-27} \text{ kg})(3.16\times10^{14} \text{ m/s}^2)/(1.60\times10^{-19} \text{ C}) = 3.30\times10^6 \text{ N/C}$, to the left.

(b) SET UP: Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50\times10^6 \text{ m/s})/(3.16\times10^{14} \text{ m/s}^2) = 1.42\times10^{-8} \text{ s} = 14.2 \text{ ns}$.

(c) SET UP: In part (a) we saw that the electric field is proportional to *m*, so we can use the ratio of the electric fields. $E_e/E_p = m_e/m_p$ and $E_e = (m_e/m_p)E_p$.

EXECUTE: $E_e = [(9.11\times10^{-31} \text{ kg})/(1.67\times10^{-27} \text{ kg})](3.30\times10^6 \text{ N/C}) = 1.80\times10^3 \text{ N/C}$, to the right.

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.24. IDENTIFY: Use constant acceleration equations to calculate the upward acceleration *a* and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let +*y* be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50 \text{ m})}{(3.00\times10^{-6} \text{ s})^2} = 1.00\times10^{12} \text{ m/s}^2.$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times10^{-31} \text{ kg})(1.00\times10^{12} \text{ m/s}^2)}{1.60\times10^{-19} \text{ C}} = 5.69 \text{ N/C}$$

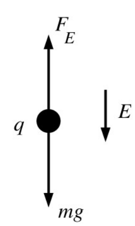
The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim10^{11}g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.25. IDENTIFY: The equation $\vec{F} = q\vec{E}$ relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.25. The weight is *mg*, downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.



$mg = |q|E$

Figure 21.25

EXECUTE: **(a)** $|q| = \frac{mg}{E} = \frac{(1.45\times10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{650 \text{ N/C}} = 2.19\times10^{-5} \text{ C}$ and $q = -21.9\ \mu\text{C}$.

(b) SET UP: The electrical force has magnitude $F_E = |q|E = eE$. The weight of a proton is $w = mg$. $F_E = w$ so $eE = mg$.

EXECUTE: $E = \dfrac{mg}{e} = \dfrac{(1.673\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}{1.602\times10^{-19}\ \text{C}} = 1.02\times10^{-7}\ \text{N/C}.$

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g$. In part (b) the $m/|q|$ ratio is much smaller $(\sim 10^{-8})$ than in part (a) $(\sim 10^{2})$ so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.26. **IDENTIFY:** The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu\text{C}$ charge is given in Figure 21.26. *FE* is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the +*x*-axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F_E = |q|E = (6.50\times10^{-6}\ \text{C})(1.85\times10^{8}\ \text{N/C}) = 1.20\times10^{3}\ \text{N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{26}\ \text{C})(8.75\times10^{-6}\ \text{C})}{(0.0250\ \text{m})^2} = 8.18\times10^{2}\ \text{N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382\ \text{N}.$

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^{3}\ \text{N}.$

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

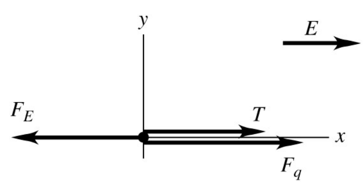
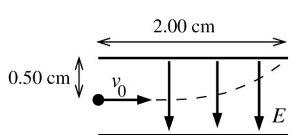


Figure 21.26

21.27. **IDENTIFY:** The equation $\vec{F} = q\vec{E}$ gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

SET UP: The motion is sketched in Figure 21.27a.



For an electron $q = -e$.

Figure 21.27a

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.27b.

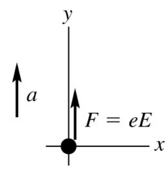


Figure 21.27b

EXECUTE: **(a)** $\sum F_y = ma_y$

$eE = ma$

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00\text{ cm}$ when $y - y_0 = +0.500\text{ cm}.$

<u>x-component</u>:

$v_{0x} = v_0 = 1.60\times10^6\text{ m/s},\ a_x = 0,\ x - x_0 = 0.0200\text{ m},\ t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}} = 1.25\times10^{-8}\text{ s}$$

In this same time t the electron travels 0.0050 m vertically.

<u>y-component</u>:

$t = 1.25\times10^{-8}\text{ s},\ v_{0y} = 0,\ y - y_0 = +0.0050\text{ m},\ a_y = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$$a_y = \frac{2(y - y_0)}{t^2} = \frac{2(0.0050\text{ m})}{(1.25\times10^{-8}\text{ s})^2} = 6.40\times10^{13}\text{ m/s}^2.$$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma.$

$$E = \frac{ma}{e} = \frac{(9.109\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 364\text{ N/C}$$

Note that the acceleration produced by the electric field is <u>much</u> larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the electron in this problem.

(b) $a = \dfrac{eE}{m_\text{p}} = \dfrac{(1.602\times10^{-19}\text{ C})(364\text{ N/C})}{1.673\times10^{-27}\text{ kg}} = 3.49\times10^{10}\text{ m/s}^2.$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times10^{-8}\text{ s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times10^{10}\text{ m/s}^2.$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.49\times10^{10}\text{ m/s}^2)(1.25\times10^{-8}\text{ s})^2 = -2.73\times10^{-6}\text{ m}.$

The displacement is $2.73\times10^{-6}\text{ m},$ downward.

EVALUATE: **(c)** The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

(d) In each case $a \gg g$ and it is reasonable to ignore the effects of gravity.

21.28. **IDENTIFY:** Apply constant acceleration equations to the motion of the electron.

SET UP: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

EXECUTE: Use the horizontal motion to find the time when the electron emerges from the field. $x - x_0 = 0.0200$ m, $a_x = 0$, $v_{0x} = 1.60\times10^6$ m/s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times10^{-8}$ s. Since $a_x = 0$, $v_x = 1.60\times10^6$ m/s. $y - y_0 = 0.0050$ m, $v_{0y} = 0$, $t = 1.25\times10^{-8}$ s. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5$ m/s. Then $v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6$ m/s.

EVALUATE: $v_y = v_{0y} + a_y t$ gives $a_y = 6.4\times10^{13}$ m/s^2. The electric field between the plates is $E = \dfrac{ma_y}{e} = \dfrac{(9.11\times10^{-31}\text{ kg})(6.4\times10^{13}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 364$ N/C. This is not a very large field.

21.29. IDENTIFY: Find the angle θ that $\hat{\boldsymbol{r}}$ makes with the $+x$-axis. Then $\hat{\boldsymbol{r}} = (\cos\theta)\hat{\boldsymbol{i}} + (\sin\theta)\hat{\boldsymbol{j}}$.

SET UP: $\tan\theta = y/x$.

EXECUTE: **(a)** $\tan^{-1}\left(\dfrac{-1.35}{0}\right) = -\dfrac{\pi}{2}$ rad. $\hat{\boldsymbol{r}} = -\hat{\boldsymbol{j}}$.

(b) $\tan^{-1}\left(\dfrac{12}{12}\right) = \dfrac{\pi}{4}$ rad. $\hat{\boldsymbol{r}} = \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{i}} + \dfrac{\sqrt{2}}{2}\hat{\boldsymbol{j}}$.

(c) $\tan^{-1}\left(\dfrac{2.6}{+1.10}\right) = 1.97\text{ rad} = 112.9°$. $\hat{\boldsymbol{r}} = -0.39\hat{\boldsymbol{i}} + 0.92\hat{\boldsymbol{j}}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{\boldsymbol{r}}$ is a unit vector, because $\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} = 1$.

21.30. IDENTIFY and SET UP: Use $\vec{E}$ in $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let $+x$ be east.

(a) EXECUTE: $F_x = |q|E = (1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = 2.403\times10^{-19}$ N.

$a_x = F_x/m = (2.403\times10^{-19}\text{ N})/(9.109\times10^{-31}\text{ kg}) = +2.638\times10^{11}\text{ m/s}^2$.

$v_{0x} = +4.50\times10^5$ m/s, $a_x = +2.638\times10^{11}$ m/s^2, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^5$ m/s.

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) EXECUTE: $F_x = -|q|E = -(1.602\times10^{-19}\text{ C})(1.50\text{ N/C}) = -2.403\times10^{-19}$ N.

$a_x = F_x/m = (-2.403\times10^{-19}\text{ N})/(1.673\times10^{-27}\text{ kg}) = -1.436\times10^{8}\text{ m/s}^2$.

$v_{0x} = +1.90\times10^4$ m/s, $a_x = -1.436\times10^8$ m/s^2, $x - x_0 = 0.375$ m, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^4$ m/s.

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.31. IDENTIFY: We want to find the force that a charged sphere exerts on a line of charge. By Newton's third law, this is also the force that the line exerts on the sphere, which is much easier to calculate. Fig. 21.31 shows the arrangement of the objects involved.

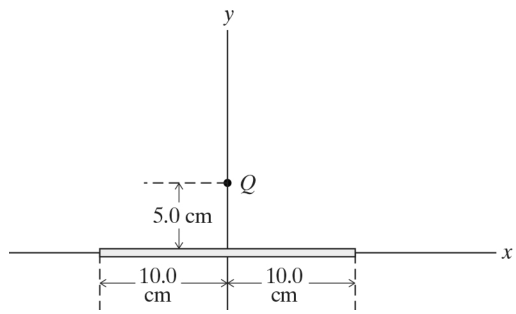


Figure 21.31

SET UP and EXECUTE: The small sphere is equivalent to a point charge. From the textbook we know that the electric field due to the line is $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}}$. The magnitude of the force that the line exerts on the sphere is $F_y = QE_y$. We know that $Q = -2.00\ \mu$C, a = 10.0 cm = 0.100 m, x = 5.00 cm = 0.0500 m, and λ = 4.80 nC/m. Using $F = \frac{Q}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}}$ for the given quantities, we get $F_y = 3.09\times10^{-3}$ N. The sphere is negative and the line is positive, so the sphere attracts the line, which means that the direction of the force is in the $+y$ direction.

EVALUATE: This is a small force, but for a very light line (such as a very thin wire) it could readily be observed.

21.32. **IDENTIFY:** The net electric field is the vector sum of the fields due to the individual charges.

SET UP: The electric field points toward negative charge and away from positive charge.

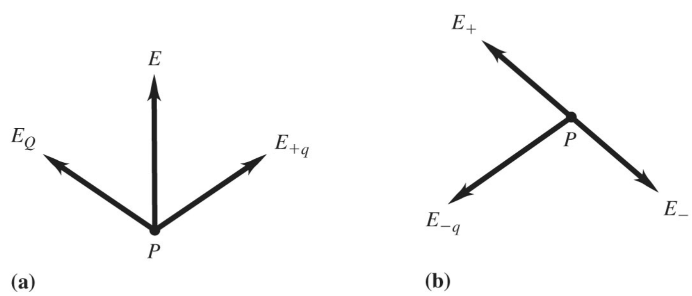


Figure 21.32

EXECUTE: **(a)** Figure 21.32(a) shows $\vec{E}_Q$ and $\vec{E}_{+q}$ at point P. $\vec{E}_Q$ must have the direction shown, to produce a resultant field in the specified direction. $\vec{E}_Q$ is toward Q, so Q is negative. In order for the horizontal components of the two fields to cancel, Q and q must have the same magnitude.

(b) No. If the lower charge were negative, its field would be in the direction shown in Figure 21.32(b). The two possible directions for the field of the upper charge, when it is positive ($\vec{E}_+$) or negative ($\vec{E}_-$), are shown. In neither case is the resultant field in the direction shown in the figure in the problem.

EVALUATE: When combining electric fields, it is always essential to pay attention to their directions.

21.33. **IDENTIFY and SET UP:** Two very long lines of charge are parallel to each other. At any point, the total electric field is the vector sum of the two fields. We want to find where the resultant field is zero. The resultant field can be zero only where the two fields have equal magnitudes but opposite directions.

Since the lines have opposite sign charge densities, in the region between the lines both fields point in the $+y$ direction, so they cannot cancel. Below the x-axis ($y < 0$) the fields are in opposite directions, but the field due to the lower line is always greater than the field due to the upper line, so they cannot cancel. For $y > 10.0$ cm, the fields are again in opposite directions. If we are close enough to the upper line, its field can equal in magnitude the field due to the lower line, so they can cancel. Call P the point on the y-axis at which the two field magnitudes are equal. Calling 1 the lower line and 2 the upper line, we have $|E_1| = |E_2|$ at point P. The field due to a very long line of charge is $E = \dfrac{\lambda}{2\pi\epsilon_0 x}$.

EXECUTE: Call y the y coordinate of P (and recalling that P is above the upper line), $|E_1| = |E_2|$ gives $\dfrac{\lambda_1}{2\pi\epsilon_0 y} = \dfrac{|\lambda_2|}{2\pi\epsilon_0 (y - 10.0\text{ cm})}$, which gives $\dfrac{8.00\,\mu\text{C/m}}{y} = \dfrac{4.00\,\mu\text{C/m}}{y - 10.0\text{ cm}}$, which gives $y = 20.0$ cm.

EVALUATE: If both lines had the same sign charge, the point of cancellation would be in the region between the lines.

21.34. **IDENTIFY:** Add the individual electric fields to obtain the net field.

SET UP: The electric field points away from positive charge and toward negative charge. The electric fields $\vec{E}_1$ and $\vec{E}_2$ add to form the net field $\vec{E}$.

EXECUTE: **(a)** The electric field is toward A at points B and C and the field is zero at A.

(b) The electric field is away from A at B and C. The field is zero at A.

(c) The field is horizontal and to the right at points A, B, and C.

EVALUATE: Compare your results to the field lines shown in Figure 21.28a and b in the textbook.

21.35. **IDENTIFY:** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).

SET UP: The placement of charges is sketched in Figure 21.35a.

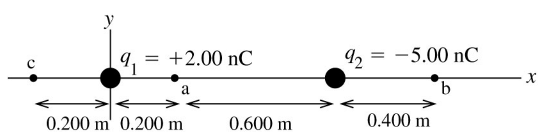


Figure 21.35a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.

(a) EXECUTE: (i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.35b.

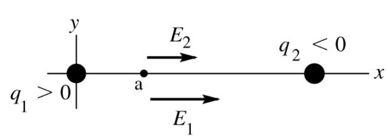


Figure 21.35b

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}.$$

$E_{1x} = 449.4\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = 124.8\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +449.4\ \text{N/C} + 124.8\ \text{N/C} = +574.2\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 574 N/C and is in the $+x$-direction.

(ii) At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.35c.

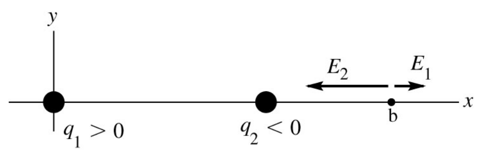


Figure 21.35c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 12.5\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$$

$E_{1x} = 12.5\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = -280.9\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +12.5\ \text{N/C} - 280.9\ \text{N/C} = -268.4\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 268 N/C and is in the $-x$-direction.

(iii) At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.35d.

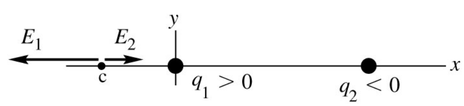


Figure 21.35d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$$

$E_{1x} = -449.4\ \text{N / C},\ E_{1y} = 0.$

$E_{2x} = +44.9\ \text{N / C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -449.4\ \text{N/C}\ + 44.9\ \text{N/C} = -404.5\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 404 N/C and is in the $-x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\text{ C})(574.2\text{ N/C}) = 9.20\times10^{-17}\text{ N}$, $-x$-direction.

(ii) $F = (1.602\times10^{-19}\text{ C})(268.4\text{ N/C}) = 4.30\times10^{-17}\text{ N}$, $+x$-direction.

(iii) $F = (1.602\times10^{-19}\text{ C})(404.5\text{ N/C}) = 6.48\times10^{-17}\text{ N}$, $+x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$- or $-x$-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were added because the fields were in the same direction and in (ii) and (iii) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.36. IDENTIFY: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ to calculate the force, using the electric field calculated in part (a).

(a) SET UP: The placement of charges is sketched in Figure 21.36a.

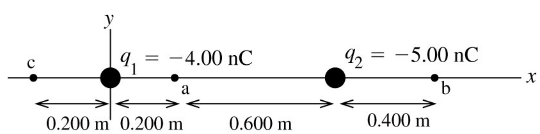


Figure 21.36a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.36b.

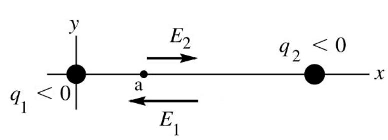


Figure 21.36b

EXECUTE: $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00\times10^{-9}\text{ C}}{(0.200\text{ m})^2} = 898.8\text{ N/C}.$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\text{ C}}{(0.600\text{ m})^2} = 124.8\text{ N/C}.$$

$E_{1x} = 898.8\text{ N/C}, E_{1y} = 0.$

$E_{2x} = 124.8\text{ N/C}, E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -898.8\text{ N/C} + 124.8\text{ N/C} = -774\text{ N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point a has magnitude 774 N/C and is in the –x-direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.36c.

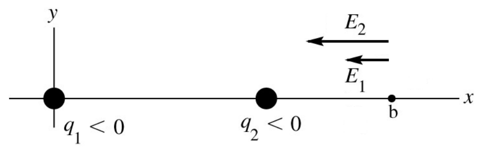


Figure 21.36c

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 24.97\ \text{N/C}.$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}.$

$E_{1x} = -24.97\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = -280.9\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = -24.97\ \text{N/C} - 280.9\ \text{N/C} = -305.9\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 306 N/C and is in the $-x$-direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.36d.

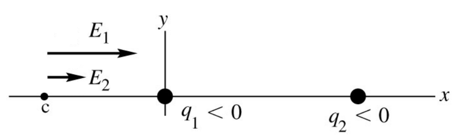


Figure 21.36d

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 898.8\ \text{N/C}.$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}.$

$E_{1x} = +898.8\ \text{N/C},\ E_{1y} = 0.$

$E_{2x} = +44.9\ \text{N/C},\ E_{2y} = 0.$

$E_x = E_{1x} + E_{2x} = +898.8\ \text{N/C} + 44.9\ \text{N/C} = +943.7\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} = 0.$

The resultant field at point b has magnitude 944 N/C and is in the $+x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(774\ \text{N/C}) = 1.24\times10^{-16}\ \text{N}$, +$x$-direction.

(ii) $F = (1.602\times10^{-19}\ \text{C})(305.9\ \text{N/C}) = 4.90\times10^{-17}\ \text{N}$, $+x$-direction.

(iii) $F = (1.602\times10^{-19}\ \text{C})(943.7\ \text{N/C}) = 1.51\times10^{-16}\ \text{N}$, $-x$-direction.

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the +*x*- or −*x*-direction depends on where the field point is relative to the charge that produces the field. In part (a), for (i) the field magnitudes were subtracted because the fields were in opposite directions and in (ii) and (iii) the field magnitudes were added because the two fields were in the same direction. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.37. IDENTIFY: $E = k\frac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges $q_1, q_2,$ and q_3, as shown in Figure 21.37(a). This figure also shows additional distances and angles. The electric fields at point *P* are shown in Figure 21.37(b). This figure also shows the *xy*-coordinates we will use and the *x*- and *y*-components of the fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\ \text{C}}{(0.100\ \text{m})^2} = 4.49\times10^6\ \text{N/C}.$

$E_2 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.00\times10^{-6}\ \text{C}}{(0.0600\ \text{m})^2} = 4.99\times10^6\ \text{N/C}.$

$E_y = E_{1y} + E_{2y} + E_{3y} = 0$ and $E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos 53.1° = 1.04\times10^7\ \text{N/C}.$

$E = 1.04\times10^7\ \text{N/C}$, toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The *x*-components of the fields of all three charges are in the same direction.

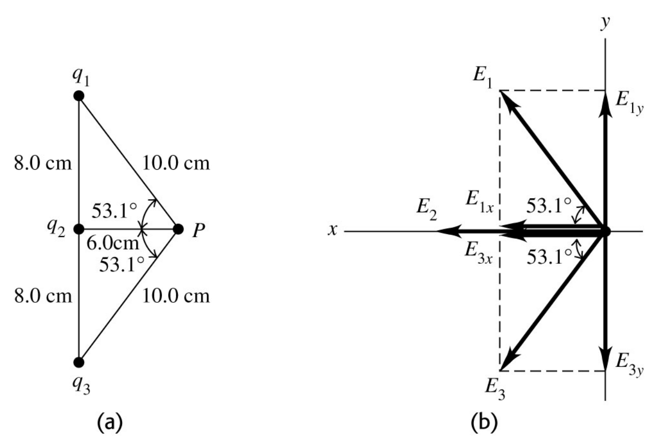


Figure 21.37

21.38. IDENTIFY: The net electric field is the vector sum of the individual fields.

SET UP: The distance from a corner to the center of the square is $r = \sqrt{(a/2)^2 + (a/2)^2} = a/\sqrt{2}$. The magnitude of the electric field due to each charge is the same and equal to $E_q = \frac{kq}{r^2} = 2\frac{kq}{a^2}$. All four *y*-components add and the *x*-components cancel.

EXECUTE: Each *y*-component is equal to $E_{qy} = -E_q\cos 45° = -\frac{E_q}{\sqrt{2}} = \frac{-2kq}{\sqrt{2}a^2} = -\frac{\sqrt{2}kq}{a^2}$. The resultant field is $\frac{4\sqrt{2}kq}{a^2}$, in the $-y$-direction.

EVALUATE: We must add the *y*-components of the fields, not their magnitudes.

21.39. **IDENTIFY:** For a point charge, $E = k\frac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge q in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1. Point B is 0.100 m from q_1 and 0.350 m from q_2.

EXECUTE: **(a)** The electric fields at point A due to the charges are shown in Figure 21.39(a).

$$E_1 = k\frac{|q_1|}{r_{A1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2.50\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{A2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 1.124\times10^4\ \text{N/C}.$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.

$E = E_2 - E_1 = 8.74\times10^3\ \text{N/C}$, to the right.

(b) The electric fields at point B are shown in Figure 21.39(b).

$$E_1 = k\frac{|q_1|}{r_{B1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 5.619\times10^3\ \text{N/C}.$$

$$E_2 = k\frac{|q_2|}{r_{B2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.350\ \text{m})^2} = 9.17\times10^2\ \text{N/C}.$$

Since the fields are in the same direction, we add their magnitudes to find the net field.

$E = E_1 + E_2 = 6.54\times10^3\ \text{N/C}$, to the right.

(c) At A, $E = 8.74\times10^3\ \text{N/C}$, to the right. The force on a proton placed at this point would be

$F = qE = (1.60\times10^{-19}\ \text{C})(8.74\times10^3\ \text{N/C}) = 1.40\times10^{-15}\ \text{N}$, to the right.

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

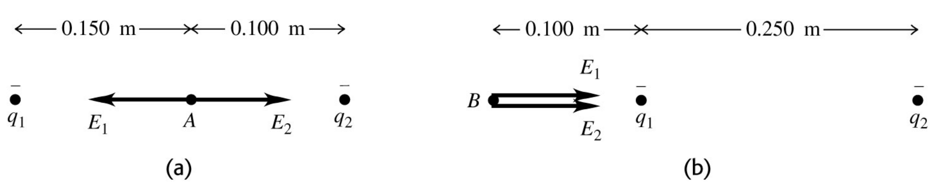


Figure 21.39

21.40. **IDENTIFY:** Apply $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{r} = \hat{j}$. For q_2, $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$, where θ is the angle between $\vec{E}_2$ and the $+x$-axis.

EXECUTE: **(a)** $\vec{E}_1 = \frac{q_1}{4\pi\epsilon_0 r_1^2}\hat{j} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\ \text{C})}{(0.0400\ \text{m})^2}\hat{j} = (-2.813\times10^4\ \text{N/C})\ \hat{j}$.

$\left|\vec{E}_2\right| = \frac{q_2}{4\pi\epsilon_0 r_2^2} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2 + (0.0400\ \text{m})^2} = 1.080\times10^4\ \text{N/C}$. The angle of $\vec{E}_2$, measured from the x-axis, is $180° - \tan^{-1}\left(\frac{4.00\ \text{cm}}{3.00\ \text{cm}}\right) = 126.9°$ Thus

$$\vec{E}_2 = (1.080\times10^4\ \text{N/C})(\hat{i}\cos126.9° + \hat{j}\sin126.9°) = (-6.485\times10^3\ \text{N/C})\hat{i} + (8.64\times10^3\ \text{N/C})\hat{j}.$$

(b) The resultant field is $\vec{E}_1+\vec{E}_2=(-6.485\times10^3\ \text{N/C})\hat{i}+(-2.813\times10^4\ \text{N/C}+8.64\times10^3\ \text{N/C})\hat{j}$.

$$\vec{E}_1+\vec{E}_2=(-6.485\times10^3\ \text{N/C})\hat{i}-(1.95\times10^4\ \text{N/C})\hat{j}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2, since q_2 is positive.

21.41. IDENTIFY: The forces the charges exert on each other are given by Coulomb's law. The net force on the proton is the vector sum of the forces due to the electrons.

SET UP: $q_e=-1.60\times10^{-19}$ C. $q_p=+1.60\times10^{-19}$ C. The net force is the vector sum of the forces exerted by each electron. Each force has magnitude $F=k\dfrac{|q_1q_2|}{r^2}=k\dfrac{e^2}{r^2}$ and is attractive so is directed toward the electron that exerts it.

EXECUTE: Each force has magnitude

$F_1=F_2=k\dfrac{|q_1q_2|}{r^2}=k\dfrac{e^2}{r^2}=\dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(1.50\times10^{-10}\ \text{m})^2}=1.023\times10^{-8}$ N. The vector force diagram is shown in Figure 21.41.

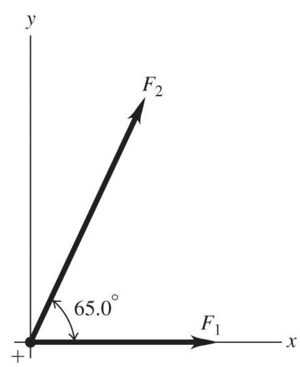


Figure 21. 41

Taking components, we get $F_{1x}=1.023\times10^{-8}$ N; $F_{1y}=0$. $F_{2x}=F_2\cos65.0°=4.32\times10^{-9}$ N; $F_{2y}=F_2\sin65.0°=9.27\times10^{-9}$ N. $F_x=F_{1x}+F_{2x}=1.46\times10^{-8}$ N; $F_y=F_{1y}+F_{2y}=9.27\times10^{-9}$ N. $F=\sqrt{F_x^2+F_y^2}=1.73\times10^{-8}$ N. $\tan\theta=\dfrac{F_y}{F_x}=\dfrac{9.27\times10^{-9}\ \text{N}}{1.46\times10^{-8}\ \text{N}}=0.6349$ which gives $\theta=32.4°$. The net force is 1.73×10^{-8} N and is directed toward a point midway between the two electrons.

EVALUATE: Note that the net force is less than the algebraic sum of the individual forces.

21.42. IDENTIFY: We can model a segment of the axon as a point charge.

SET UP: If the axon segment is modeled as a point charge, its electric field is $E=k\dfrac{q}{r^2}$. The electric field of a point charge is directed away from the charge if it is positive.

EXECUTE: (a) 5.6×10^{11} Na^+ ions enter per meter so in a $0.10\ \text{mm}=1.0\times10^{-4}$ m section, 5.6×10^7 Na^+ ions enter. This number of ions has charge $q=(5.6\times10^7)(1.60\times10^{-19}\ \text{C})=9.0\times10^{-12}$ C.

(b) $E=k\dfrac{|q|}{r^2}=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{9.0\times10^{-12}\ \text{C}}{(5.00\times10^{-2}\ \text{m})^2}=32$ N/C, directed away from the axon.

(c) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(9.0\times10^{-12}\ \text{C})}{1.0\times10^{-6}\ \text{N/C}}} = 280\ \text{m}.$

EVALUATE: The field in (b) is considerably smaller than ordinary laboratory electric fields.

21.43. **IDENTIFY:** The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.43a.

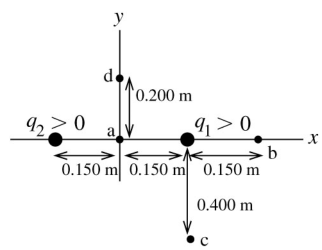
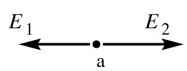


Figure 21.43a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.43b.

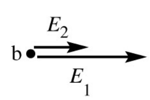


$E_1 = E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ with $r = 0.150$ m.

$E = E_2 - E_1 = 0;\ E_x = 0,\ E_y = 0.$

Figure 21.43b

(b) The two fields have the directions shown in Figure 21.43c.



$E = E_1 + E_2$, in the $+x$-direction.

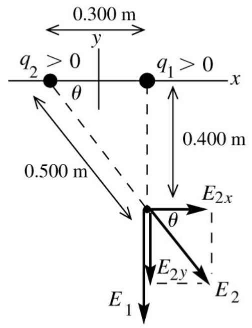
Figure 21.43c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2396.8\ \text{N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.450\ \text{m})^2} = 266.3\ \text{N/C}.$$

$E = E_1 + E_2 = 2396.8\ \text{N/C} + 266.3\ \text{N/C} = 2660\ \text{N/C};\ E_x = +2660\ \text{N/C},\ E_y = 0.$

(c) The two fields have the directions shown in Figure 21.43d.



$$\sin\theta = \frac{0.400\text{ m}}{0.500\text{ m}} = 0.800.$$

$$\cos\theta = \frac{0.300\text{ m}}{0.500\text{ m}} = 0.600.$$

Figure 21.43d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\text{ C}}{(0.400\text{ m})^2} = 337.1\text{ N/C}.$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\text{ C}}{(0.500\text{ m})^2} = 215.7\text{ N/C}.$$

$E_{1x} = 0,\ E_{1y} = -E_1 = -337.1\text{ N/C}.$

$E_{2x} = +E_2\cos\theta = +(215.7\text{ N/C})(0.600) = +129.4\text{ N/C}.$

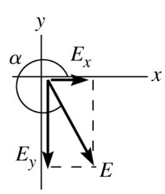
$E_{2y} = -E_2\sin\theta = -(215.7\text{ N/C})(0.800) = -172.6\text{ N/C}.$

$E_x = E_{1x} + E_{2x} = +129\text{ N/C}.$

$E_y = E_{1y} + E_{2y} = -337.1\text{ N/C} - 172.6\text{ N/C} = -510\text{ N/C}.$

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129\text{ N/C})^2 + (-510\text{ N/C})^2} = 526\text{ N/C}.$

$\vec{E}$ and its components are shown in Figure 21.43e.



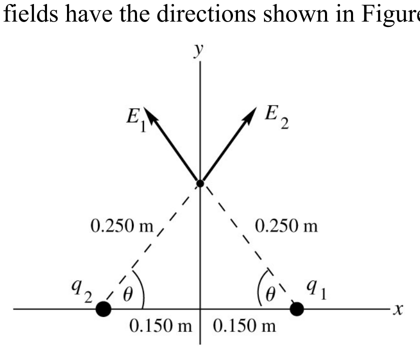
$$\tan\alpha = \frac{E_y}{E_x}.$$

$$\tan\alpha = \frac{-510\text{ N/C}}{+129\text{ N/C}} = -3.953.$$

$\alpha = 284°$, counterclockwise from $+x$-axis.

Figure 21.43e

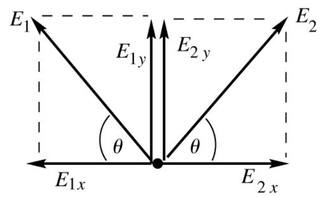
(d) The two fields have the directions shown in Figure 21.43f.



$$\sin\theta = \frac{0.200\text{ m}}{0.250\text{ m}} = 0.800.$$

Figure 21.43f

The components of the two fields are shown in Figure 21.43g.



$E_1 = E_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{6.00\times10^{-9}\ \text{C}}{(0.250\ \text{m})^2}$.

$E_1 = E_2 = 862.8\ \text{N/C}$.

Figure 21.43g

$E_{1x} = -E_1\cos\theta,\ E_{2x} = +E_2\cos\theta.$

$E_x = E_{1x} + E_{2x} = 0.$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = +E_2\sin\theta.$

$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1\sin\theta = 2(862.8\ \text{N/C})(0.800) = 1380\ \text{N/C}.$

$E = 1380$ N/C, in the $+y$-direction.

EVALUATE: Point *a* is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point *b* is to the right of both charges and both electric fields are in the $+x$-direction and the resultant field is in this direction. At point *c* both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the fourth quadrant. At point *d* both fields have an upward component but by symmetry they have equal and opposite x-components so the net field is in the $+y$-direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.44. **IDENTIFY:** Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.44.

EXECUTE: $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{(6.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}(-\hat{i}) = -150\hat{i}\ \text{N/C}.$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\ \text{C})\left(\frac{1}{(1.00\ \text{m})^2}(0.600)\hat{i} + \frac{1}{(1.00\ \text{m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\text{N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\ \text{N/C})\hat{i} + (28.8\ \text{N/C})\hat{j}$. $E = \sqrt{(128.4\ \text{N/C})^2 + (28.8\ \text{N/C})^2} = 131.6\ \text{N/C}$ at $\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6^\circ$ above the $-x$-axis and therefore 167.4° counterclockwise from the $+x$-axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

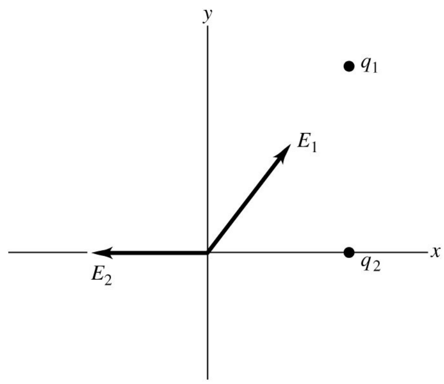


Figure 21.44

21.45. **IDENTIFY and SET UP:** We are dealing with the net electric field produced by three very large sheets of charge. The magnitude of the field due to each sheet is $E = \sigma / 2\epsilon_0$ and is independent of the distance from the sheet. $\vec{E}_A$ points away from sheet A, $\vec{E}_B$ points toward sheet B, and $\vec{E}_C$ points away from sheet C. Draw the sheet arrangement, locate point P midway between B and C, and show the fields at P (see Fig. 21.45). All the fields are along the x-axis.

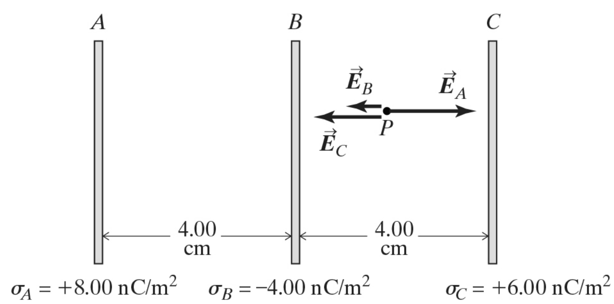


Figure 21.45

EXECUTE: Using the directions shown in the figure, we have $E_{Px} = \dfrac{|\sigma_A|}{2\epsilon_0} - \dfrac{|\sigma_B|}{2\epsilon_0} - \dfrac{|\sigma_C|}{2\epsilon_0} =$

$\dfrac{8.00\ \text{nC/m}^2 - 4.00\ \text{nC/m}^2 - 6.00\ \text{nC/m}^2}{2\epsilon_0} = -113$ N/C. The magnitude is 113 N/C and the direction is in the $-x$ direction (to the left).

EVALUATE: Since the electric fields are independent of distance from the sheet, the resultant field would would be the same *anywhere* between sheets B and C.

21.46. **IDENTIFY and SET UP:** We are dealing with the electric fields of two point charges. First sketch the charge arrangement as in Fig. 21.46. We want to find q_2 so that the resultant electric field at point P is zero. The magnitude of the field due to a point charge is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$. The two fields at P cancel, so

$E_1 = E_2$.

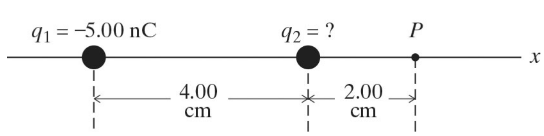


Figure 21.46

EXECUTE: $\frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2}$ gives $\frac{5.00\text{ nC}}{(6.00\text{ cm})^2} = \frac{q_2}{(2.00\text{ cm})^2}$, so $q_2 = +0.556$ nC. At point P E_1 points to the left, so E_2 must point to the right, which tells us that q_2 is positive.

EVALUATE: There was no need to convert to standard SI units because most of the quantities cancel.

21.47. IDENTIFY: We are dealing with the resultant electric field of two point charges. Sketch the charge arrangement as in Fig. 21.47.

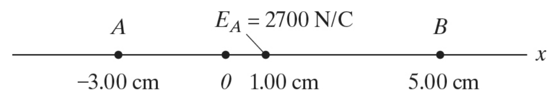


Figure 21.47

SET UP: We know that $|q_A| = 2|q_B|$, $E_A = 2700$ N/C (magnitude only) at $x = 1.00$ cm, and $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$. In each case, we want to find the resultant electric field at the origin.

EXECUTE: **(a)** Both A and B are positive. First find q_A: $E_A = \frac{1}{4\pi\epsilon_0}\frac{q_A}{r^2} = 2700\text{ N/C}$, and $r = 4.00$ cm = 0.0400 m, which gives $q_A = 0.48053$ nC. Therefore $q_B = 2q_A = 0.96106$ nC. At the origin, the two fields point in opposite directions, so $E = E_A - E_B = \frac{1}{4\pi\epsilon_0}\left[\frac{0.48053\text{ nC}}{(0.0300\text{ m})^2} - \frac{0.96106\text{ nC}}{(0.0500\text{ m})^2}\right] = 1340\text{ N/C}$, in the $+x$ direction.

(b) Both A and B are negative. The calculation is the same as in part (a) except the field is in the opposite direction: $E = 1340$ N/C in the $-x$ direction.

(c) A is positive and B is negative. Both fields point in the $+x$ direction, so the magnitudes add, giving $E = E_A + E_B = \frac{1}{4\pi\epsilon_0}\left[\frac{0.48053\text{ nC}}{(0.0300\text{ m})^2} + \frac{0.96106\text{ nC}}{(0.0500\text{ m})^2}\right] = 8260$ N/C in the $+x$ direction.

(d) A is negative and B is positive. Both fields point in the $-x$ direction but E has the same magnitude as in part (c), so $E = 8260$ N/C in the $-x$ direction.

EVALUATE: It is always best to sketch the charge arrangement to visualize the direction of the electric fields.

21.48. IDENTIFY: For a long straight wire, $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\frac{1}{2\pi\epsilon_0} = 1.80\times10^{10}\text{ N}\cdot\text{m}^2/\text{C}^2$.

EXECUTE: Solve $E = \frac{\lambda}{2\pi\epsilon_0 r}$ for r: $r = \frac{3.20\times10^{-10}\text{ C/m}}{2\pi\epsilon_0(2.50\text{ N/C})} = 2.30\text{ m}$.

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

21.49. IDENTIFY: For a ring of charge, the magnitude of the electric field is given by $E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$. Use $\vec{F} = q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q = 0.125\times10^{-9}$ C, $a = 0.025$ m and $x = 0.400$ m.

EXECUTE: **(a)** $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}\hat{i} = (7.0\text{ N/C})\hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(-2.50\times10^{-6}\ \text{C})(7.0\ \text{N/C})\hat{\boldsymbol{i}} = (1.75\times10^{-5}\ \text{N})\hat{\boldsymbol{i}}.$

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

21.50. **(a) IDENTIFY:** The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}}.$$

EXECUTE: $E = \dfrac{(18.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(175\times10^{-9}\ \text{C/m})}{(0.0600\ \text{m})\sqrt{\left(\dfrac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}} = 3.03\times10^4\ \text{N/C},$ directed upward.

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius a of the circle using $2\pi a = l.$

EXECUTE: Solving for a gives $a = l/2\pi = (8.50\ \text{cm})/2\pi = 1.353\ \text{cm}.$

The charge on this circle is $Q = \lambda l = (175\ \text{nC/m})(0.0850\ \text{m}) = 14.88\ \text{nC}.$

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(14.88\times10^{-9}\ \text{C/m})(0.0600\ \text{m})}{\left[(0.0600\ \text{m})^2+(0.01353\ \text{m})^2\right]^{3/2}}$$

$$E = 3.45\times10^4\ \text{N/C},\ \text{upward}.$$

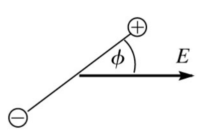
EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.51. **(a) IDENTIFY** and **SET UP:** Use $p = qd$ to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.

EXECUTE: $p = qd = (4.5\times10^{-9}\ \text{C})(3.1\times10^{-3}\ \text{m}) = 1.4\times10^{-11}\ \text{C}\cdot\text{m}.$ The direction of $\vec{p}$ is from q_1 toward q_2.

(b) IDENTIFY and **SET UP:** Use $\tau = pE\sin\phi$ to relate the magnitudes of the torque and field.

EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.51, so



$$E = \frac{\tau}{p\sin\phi}.$$

$$E = \frac{7.2\times10^{-9}\ \text{N}\cdot\text{m}}{(1.4\times10^{-11}\ \text{C}\cdot\text{m})\sin 36.9^\circ} = 860\ \text{N/C}.$$

Figure 21. 51

EVALUATE: The equation $\tau = pE\sin\phi$ gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8}\ \text{N}\cdot\text{m}.$ The torque for the orientation of the dipole in the problem is less than this maximum.

21.52. **(a) IDENTIFY:** The potential energy is given by $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi.$

SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi,$ where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0°) = -pE$.
perpendicular: $\phi = 90°$ and $U(90°) = 0$.

$\Delta U = U(90°) - U(0°) = pE = (5.0\times10^{-30}\ \text{C}\cdot\text{m})(1.6\times10^{6}\ \text{N/C}) = 8.0\times10^{-24}\ \text{J}$.

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24}\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})} = 0.39\ \text{K}$.

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.53. IDENTIFY: The torque on a dipole in an electric field is given by $\vec{\tau} = \vec{p}\times\vec{E}$.

SET UP: $\tau = pE\sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: **(a)** The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.53(a).
(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.
(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.53(b).

EVALUATE: The field of the dipole is directed from the + charge toward the − charge.

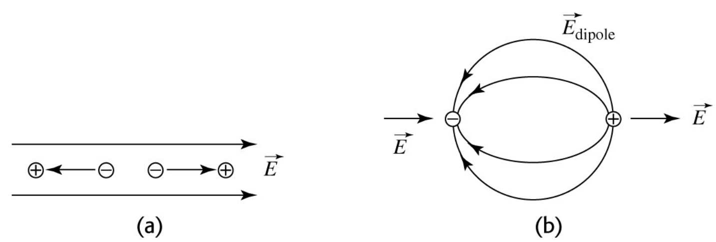


Figure 21. 53

21.54. IDENTIFY: Calculate the electric field due to the dipole and then apply $\vec{F} = q\vec{E}$.

SET UP: The field of a dipole is $E_{\text{dipole}}(x) = \dfrac{p}{2\pi\epsilon_0 x^3}$.

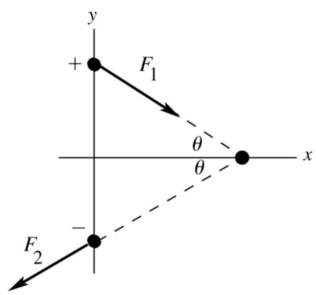
EXECUTE: $E_{\text{dipole}} = \dfrac{6.17\times10^{-30}\ \text{C}\cdot\text{m}}{2\pi\epsilon_0 (3.0\times10^{-9}\ \text{m})^3} = 4.11\times10^{6}\ \text{N/C}$. The electric force is

$F = qE = (1.60\times10^{-19}\ \text{C})(4.11\times10^{6}\ \text{N/C}) = 6.58\times10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).

EVALUATE: $\vec{E}_{\text{dipole}}$ is in the direction of $\vec{p}$, so is in the $+x$-direction. The charge q of the ion is negative, so $\vec{F}$ is directed opposite to $\vec{E}$ and is therefore in the $-x$-direction.

21.55. (a) IDENTIFY: Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.

SET UP: The two forces on each charge in the dipole are shown in Figure 21.55a.



$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$.

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$.

Figure 21. 55a

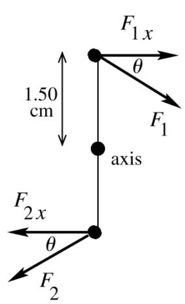
EXECUTE: $F_1 = k\dfrac{|qq'|}{r^2} = k\dfrac{(5.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times10^3\ \text{N}$.

$F_{1y} = -F_1\sin\theta = -842.6\ \text{N}$.

$F_{2y} = -842.6\ \text{N}$ so $F_y = F_{1y} + F_{2y} = -1680\ \text{N}$ (in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).

EVALUATE: The x-components cancel and the y-components add.

(b) SET UP: Refer to Figure 21.55b.



The y-components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

Figure 21. 55b

EXECUTE: $F_{1x} = F_1\cos\theta = 743.1\ \text{N}$.

$\tau = 2(F_{1x})(0.0150\ \text{m}) = 22.3\ \text{N}\cdot\text{m}$, clockwise.

EVALUATE: The electric field produced by the $-10.00\ \mu\text{C}$ charge is not uniform so $\tau = pE\sin\phi$ does not apply.

21.56. IDENTIFY: An electric dipole is in an external electric field. We want to know about the torque on this dipole and its electric potential energy due to this field.

SET UP: The torque is $\vec{\tau} = \vec{p}\times\vec{E}$ and the potential energy is $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$.

EXECUTE: **(a)** We want the orientation of the dipole that will produce the maximum torque into the paper. Fig. 21.56a shows the orientation so that the torque will be into the page. The magnitude of the torque is $\tau = pE\sin\phi$, which is a maximum when $\phi = 90°$. Fig. 21.56b shows this orientation, for which $\vec{p}$ is downward. The potential energy at this angle is $U = -pE\cos\phi = -pE\cos 90° = 0$.

Figures 21.56a and 21.56b

(b) We want the orientation so that the torque is zero and the potential energy is a maximum. The magnitude of the torque is $\tau = pE\sin\phi$, which is zero for $\phi = 0°$ or 180°.

For $\phi = 0°$: $U = -pE \cos 0° = -pE$.

For $\phi = 180°$: $U = -pE \cos 180° = +pE$.

As we can see, the orientation for maximum potential energy is $\phi = 180°$. At this angle, $\vec{p}$ points opposite to the electric field. To find the type of equilibrium, imagine displacing the dipole a small angle from the $\phi = 180°$ equilibrium position. Fig. 21.56c shows the dipole and the electric forces on the dipole. As the figure shows, the torques tend to rotate the dipole *away from* the equilibrium position, so this is an *unstable* equilibrium.

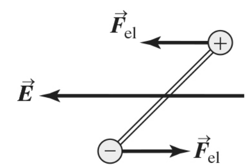
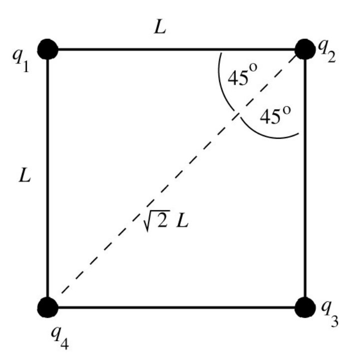


Figure 21.56c

EVALUATE: A system is in a stable equilibrium state when its potential energy is a minimum (like a ball in a valley) and an unstable state when the potential energy is a maximum (like a ball at the top of a peak). In our case, $U = +pE$ (a maximum) when $\phi = 180°$, so this is an unstable equilibrium. The state $U = -pE$ (a minimum) when $\phi = 0°$ is a stable state. To show this, use the procedure we followed with Fig. 21.56c except displace the dipole a small angle from the $\phi = 0°$ state to see which way the torques tend to turn the dipole.

21.57. **IDENTIFY:** Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

SET UP: The charges are placed as shown in Figure 21.57a.



$q_1 = q_2 = q_3 = q_4 = Q$

Figure 21.57a

Consider forces on q_4. The free-body diagram is given in Figure 21.57b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$-direction.

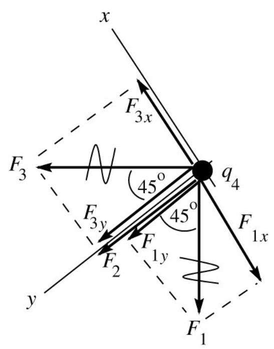


Figure 21.57b

EXECUTE: **(a)** $F_3 = F_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{L^2}.$

$F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{2L^2}.$

$F_{1x} = -F_1 \sin 45° = -F_1/\sqrt{2}.$

$F_{1y} = +F_1 \cos 45° = +F_1/\sqrt{2}.$

$F_{3x} = +F_3 \sin 45° = +F_3/\sqrt{2}.$

$F_{3y} = +F_3 \cos 45° = +F_3/\sqrt{2}.$

$F_{2x} = 0, F_{2y} = F_2.$

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0.$

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\frac{1}{4\pi\epsilon_0}\frac{Q^2}{L^2} + \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2L^2} = \frac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2}).$$

$R = \dfrac{Q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2}).$ Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.58. **IDENTIFY:** Apply $F = \dfrac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance d to the left of q_1, so it is a distance $d + 0.600$ m from q_2.

EXECUTE: $F_1 = F_2$ gives $\dfrac{kq|q_1|}{d^2} = \dfrac{kq|q_2|}{(d+0.600\text{ m})^2}.$

$d = \pm\sqrt{\dfrac{|q_1|}{|q_2|}}(d+0.600\text{ m}) = \pm(0.8452)(d+0.600\text{ m}).$ d must be positive, so

$d = \dfrac{(0.8452)(0.600\text{ m})}{1-0.8452} = 3.27\text{ m}.$ The net force would be zero when $+q$ is at $x = -3.27$ m.

EVALUATE: When $+q$ is at $x = -3.27$ m, $\vec{F}_1$ is in the $-x$-direction and $\vec{F}_2$ is in the $+x$-direction.

21.59. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3.

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.59.

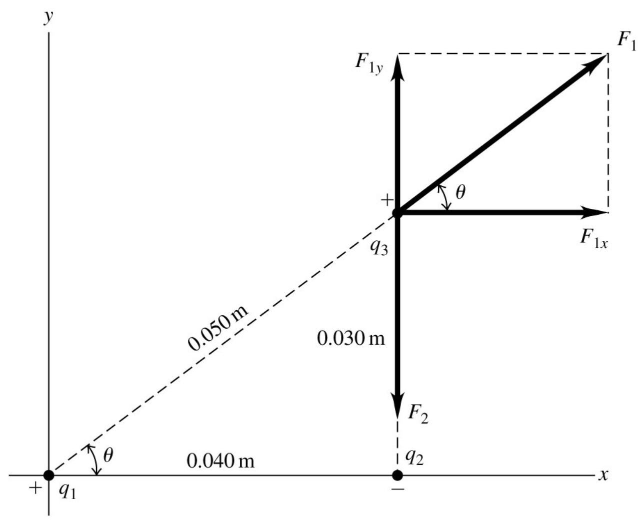


Figure 21.59

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0500\ \text{m})^2} = 1.079\times10^{-4}\ \text{N}.$

$\theta = 36.9°$. $F_{1x} = +F_1\cos\theta = 8.63\times10^{-5}\ \text{N}$. $F_{1y} = +F_1\sin\theta = 6.48\times10^{-5}\ \text{N}$.

$$F_2 = k\frac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2} = 1.20\times10^{-4}\ \text{N}.$$

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20\times10^{-4}\ \text{N}$. $F_x = F_{1x} + F_{2x} = 8.63\times10^{-5}\ \text{N}$.

$F_y = F_{1y} + F_{2y} = 6.48\times10^{-5}\ \text{N} + (-1.20\times10^{-4}\ \text{N}) = -5.52\times10^{-5}\ \text{N}$.

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02\times10^{-4}\ \text{N}$. $\tan\phi = \left|\dfrac{F_y}{F_x}\right| = 0.640$. $\phi = 32.6°$, below the +*x*-axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.60. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.60. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_\text{e} = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\dfrac{mg\sin\theta}{\cos\theta} = F_\text{e} = \dfrac{kq^2}{d^2}$. But

$\tan\theta \approx \sin\theta = \dfrac{d}{2L}$, so $d^3 = \dfrac{2kq^2L}{mg}$ and $d = \left(\dfrac{q^2L}{2\pi\epsilon_0\, mg}\right)^{1/3}$.

EVALUATE: *d* increases when *q* increases.

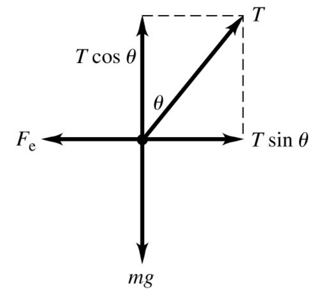


Figure 21.60

21.61. **IDENTIFY:** Use Coulomb's law for the force that one sphere exerts on the other and apply the first condition of equilibrium to one of the spheres.

SET UP: The placement of the spheres is sketched in Figure 21.61a.

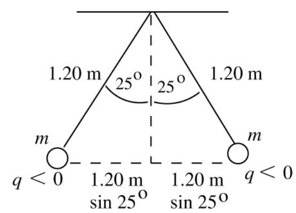


Figure 21.61a

EXECUTE: **(a)** The free-body diagrams for each sphere are given in Figure 21.61b.

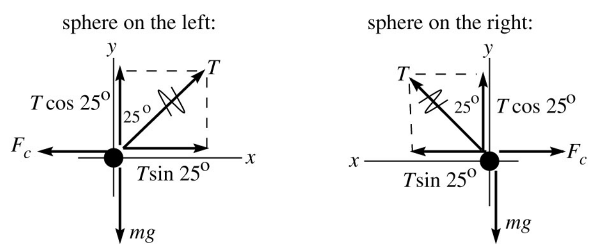


Figure 21.61b

F_c is the repulsive Coulomb force exerted by one sphere on the other.

(b) From either force diagram in part (a): $\sum F_y = ma_y$.

$T\cos 25.0° - mg = 0$ and $T = \dfrac{mg}{\cos 25.0°}$.

$\sum F_x = ma_x$.

$T\sin 25.0° - F_c = 0$ and $F_c = T\sin 25.0°$.

Use the first equation to eliminate T in the second: $F_c = (mg/\cos 25.0°)(\sin 25.0°) = mg\tan 25.0°$.

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}.$$

Combine this with $F_c = mg\tan 25.0°$ and get $mg\tan 25.0° = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$.

$$q = (2.40\text{ m})\sin 25.0° \sqrt{\frac{mg\tan 25.0°}{(1/4\pi \epsilon_0)}}.$$

$$q = (2.40\text{ m})\sin 25.0° \sqrt{\frac{(15.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 25.0°}{8.988\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.80\times10^{-6}\text{ C}.$$

(c) The separation between the two spheres is given by $2L\sin\theta$. $q = 2.80\mu\text{C}$ as found in part (b). $F_c = (1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2$ and $F_c = mg\tan\theta$. Thus $(1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2 = mg\tan\theta$.

$$(\sin\theta)^2\tan\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4L^2mg} =$$

$$(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(2.80\times10^{-6}\text{ C})^2}{4(0.600\text{ m})^2(15.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)} = 0.3328.$$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan\theta \approx \sin\theta$. With this approximation the equation becomes $\sin^3\theta = 0.3328$ and $\sin\theta = 0.6930$, so $\theta = 43.9°$. Now refine this guess:

θ	$\sin^2\theta\tan\theta$	
45.0°	0.5000	
40.0°	0.3467	
39.6°	0.3361	
39.5°	0.3335	
39.4°	0.3309	so $\theta = 39.5°$.

EVALUATE: The expression in part (c) says $\theta \to 0$ as $L \to \infty$ and $\theta \to 90°$ as $L \to 0$. When L is decreased from the value in part (a), θ increases.

21.62. IDENTIFY and SET UP: The horizontal electric field exerts a force on the moving sphere. The field is in the same direction as the velocity of the sphere, so it will increase the sphere's speed as the sphere falls. We want to know the magnitude E of the electric field. It points toward the east, so it does not affect the vertical velocity of the sphere. It does, however, give the sphere horizontal acceleration. We know the initial and final speeds of the sphere.

EXECUTE: First find the time to fall 60.0 cm from rest. Then use that time to find the vertical velocity v_y just as the sphere reaches the ground. Using $y = \frac{1}{2}gt^2$ gives $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(0.600\text{ m})}{9.80\text{ m/s}^2}} = 0.3499\text{ s}$. $v_y = gt = (9.80\text{ m/s}^2)(0.3499\text{ s}) = 3.429\text{ m/s}$. Now find v_x just as the sphere reaches the ground. Using $\Sigma F_x = ma_x$ and $F_x = qE$ gives $a_x = qE/m$. The horizontal velocity is $v_x = v_{0x} + a_xt = v_0 + (qE/m)t$. At ground level $v = 5.00$ m/s, so $\sqrt{v_x^2 + v_y^2} = 5.00\text{ m/s}$. Squaring gives $v_x^2 + v_y^2 = 25.0\text{ m}^2/\text{s}^2$. Using our results for v_x and v_y, this becomes $(v_0 + qEt/m)^2 + v_y^2 = 25.0\text{ m}^2/\text{s}^2$. Solving for E gives $E = \frac{m}{qt}\left(\sqrt{25.0\text{ m}^2/\text{s}^2 - v_y^2} - v_0\right)$. Using $m = 0.500\text{ g} = 5.00\times10^{-4}\text{ kg}$, $q = 5.00\ \mu\text{C}$, $t = 0.3499$ s, $v_y = 3.429$ m/s, and $v_0 = 2.00$ m/s, we get $E = 468\text{ N/C}$.

EVALUATE: We can check using the work-energy theorem. $W_{\text{tot}} = W_g + W_E$. $W_g = mgy = (0.500\text{ g})(9.80\text{ m/s}^2)(0.600\text{ m}) = 2.94\times10^{-3}\text{ J}$. $W_E = qEx$, where $x = v_0t + \frac{1}{2}a_xt^2$. Using $a_x = qE/m$ and putting in $q = 5.00\ \mu$C, $E = 468$ N/C, and $m = 0.500$ g, we get $x = 0.986$ m. Therefore $W_E = (0.500\ \mu\text{C})(468\text{ N/C})(0.986\text{ m}) = 2.308\times10^{-3}\text{ J}$. Adding gives $W_{\text{tot}} = 5.25\times10^{-3}\text{ J}$. The kinetic energy change is $K_2 - K_1 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(v^2 - v_0^2\right)$, which gives

$\Delta K = \frac{1}{2}(0.500\text{ g})\left[(5.00\text{ m/s})^2 - (2.00\text{ m/s})^2\right] = 5.25\times10^{-3}$ J. Our result is consistent with the work-energy theorem.

21.63. **IDENTIFY:** The electric field exerts a horizontal force away from the wall on the ball. When the ball hangs at rest, the forces on it (gravity, the tension in the string, and the electric force due to the field) add to zero.

SET UP: The ball is in equilibrium, so for it $\sum F_x = 0$ and $\sum F_y = 0$. The force diagram for the ball is given in Figure 21.63. F_E is the force exerted by the electric field. $\vec{F} = q\vec{E}$. Since the electric field is horizontal, $\vec{F}_E$ is horizontal. Use the coordinates shown in the figure. The tension in the string has been replaced by its x- and y-components.

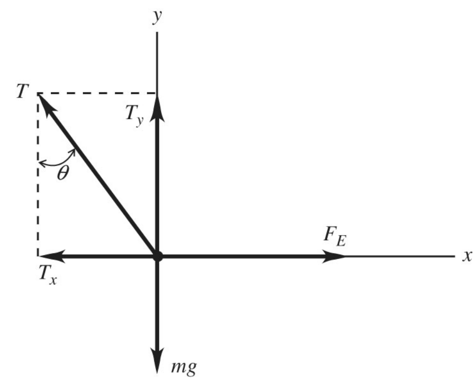


Figure 21.63

EXECUTE: $\sum F_y = 0$ gives $T_y - mg = 0$. $T\cos\theta - mg = 0$ and $T = \frac{mg}{\cos\theta}$. $\sum F_x = 0$ gives $F_E - T_x = 0$. $F_E - T\sin\theta = 0$. Combing the equations and solving for F_E gives

$F_E = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta = (12.3\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(\tan 17.4°) = 3.78\times10^{-2}$ N. $F_E = |q|E$ so

$E = \frac{F_E}{|q|} = \frac{3.78\times10^{-2}\text{ N}}{1.11\times10^{-6}\text{ C}} = 3.41\times10^4$ N/C. Since q is negative and $\vec{F}_E$ is to the right, $\vec{E}$ is to the left in the figure.

EVALUATE: The larger the electric field E the greater the angle the string makes with the wall.

21.64. **IDENTIFY:** A charged sphere is released in a vertical electric field and accelerates upward.

SET UP: We want the time it takes the sphere to travel upward a distance d. The forces on it are mg downward and qE upward. $\sum F_y = ma_y$ applies to the sphere.

EXECUTE: The vertical distance it travels in time t is $d = \frac{1}{2}at^2$. Using $\sum F_y = ma_y$ we get $qE - mg = ma$, so $a = \frac{qE - mg}{m}$. Therefore $d = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{qE - mg}{m}\right)t^2$. Solving for t gives

$t = \sqrt{\frac{2md}{qE - mg}}$.

EVALUATE: We must have $qE > mg$ or the sphere would not travel upward.

21.65. **IDENTIFY and SET UP:** We model the interaction as being due to a negative charge on the balloon and an equal positive charge in the ceiling due to the positive charge left behind when the negative charge on the comb repels electrons in the ceiling. We are assuming that the negative charge in the ceiling is far enough away to be ignored. Furthermore, we model both of these charges as point charges separated by 500 μm and apply Coulomb's law to find the magnitude of the electric force. $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1 q_2|}{r^2}$.

EXECUTE: **(a)** $F_{\text{el}} = mg$. $\dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2} = mg$. $q = \sqrt{\dfrac{mgr^2}{1/4\pi\epsilon_0}}$. Using m = 0.004 kg and r = 500 μm gives q = 1 nC.

(b) $Q = 10q = 10.0$ nC $= Ne$, so $N = Q/e$ (10 nC)/ $(1.6\times10^{-19}$ C$) \approx 6\times10^{10}$ electrons.

EVALUATE: This is a very rough approximation because the charges in the ceiling and balloon are not concentrated to a point as we assumed in our model, and the negative charges in the ceiling are not really far enough away to be ignored.

21.66. **IDENTIFY:** We want to estimate the amount of charge in a penny made of zinc with a mass of 2.5 g.

SET UP: The gram atomic mass of zinc is 65.38 g/mol, and a zinc atom contains 30 electrons.

EXECUTE: **(a)** $(2.5\text{ g})\left(\dfrac{1\text{ mol}}{65.38\text{ g}}\right)\left(\dfrac{6.022\times10^{23}\text{ atoms}}{\text{mol}}\right)\left(\dfrac{30\text{ electrons}}{\text{atom}}\right) = 6.9\times10^{23}$ electrons.

(b) $q = (6.9\times10^{23}\text{ electrons})\left(\dfrac{1.60\times10^{-19}\text{ C}}{\text{electron}}\right) = 1.1\times10^{5}\text{ C}$.

(c) Coulomb's law: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(1.1\times10^{5}\text{ C})^2}{(0.020\text{ m})^2} = 2.7\times10^{23}\text{ N}$.

(d) Estimate: 2 million leaves.

(e) There would be 6.9×10^{23} leaves and each tree would contain 2×10^{6} leaves, so the number of trees in the forest would be $(6.9\times10^{23}\text{ leaves})\left(\dfrac{1\text{ tree}}{2\times10^{6}\text{ leaves}}\right) = 3.5\times10^{17}$ trees. Think of each tree as being at the center of a 10 m by 10 m square, so the area for each tree would be 100 m^2. The area of this forest would be $(3.5\times10^{17}\text{ trees})\left(\dfrac{100\text{ m}^2}{1\text{ tree}}\right) = 3.5\times10^{19}\text{ m}^2$.

(f) $\dfrac{A_{\text{trees}}}{A_{\text{earth}}} = \dfrac{3.5\times10^{19}\text{ m}^2}{4\pi(6.37\times10^{6}\text{ m}^2)} \approx 70{,}000$. The trees would occupy an area about 70,000 times the surface area of the earth.

EVALUATE: The penny contains about 110,000 C of negative charge, but it also contains 110,000 C of positive charge. That is why it doesn't blow itself apart.

21.67. **IDENTIFY:** For a point charge, $E = k\dfrac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500$ nC and $q_2 = +8.00$ nC. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.67. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\dfrac{0.500\text{ nC}}{x^2} = k\dfrac{8.00\text{ nC}}{(1.20\text{ m} - x)^2}$.

$16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24$ m is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

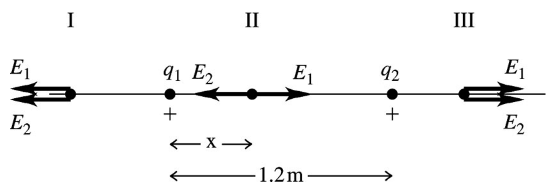


Figure 21.67

21.68. IDENTIFY: The net electric field at the origin is the vector sum of the fields due to the two charges.

SET UP: $E = k\dfrac{|q|}{r^2}$. $\vec{E}$ is toward a negative charge and away from a positive charge. At the origin, $\vec{E}_1$ due to the -3.00 nC charge is in the $+x$-direction, toward the charge.

EXECUTE: (a)

$E_1 = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times10^{-9}\ \text{C})}{(1.20\ \text{m})^2} = 18.73\ \text{N/C}$, so $E_{1x} = +18.73\ \text{N/C}$. $E_x = E_{1x} + E_{2x}$.

$E_x = +45.0\ \text{N/C}$, so $E_{2x} = E_x - E_{1x} = +45.0\ \text{N/C} - 18.73\ \text{N/C} = 26.27\ \text{N/C}$. $\vec{E}$ is away from Q so Q is positive. Using $E_2 = k\dfrac{|Q|}{r^2}$ gives $|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(26.27\ \text{N/C})(0.600\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.05\times10^{-9}\ \text{C} = 1.05\ \text{nC}$. Since Q is positive, $Q = +1.05$ nC.

(b) $E_x = -45.0\ \text{N/C}$, so $E_{2x} = E_x - E_{1x} = -45.0\ \text{N/C} - 18.73\ \text{N/C} = -63.73\ \text{N/C}$. $\vec{E}$ is toward Q so Q is negative. $|Q| = \dfrac{E_2 r^2}{k} = \dfrac{(63.73\ \text{N/C})(0.600\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.55\times10^{-9}\ \text{C} = 2.55\ \text{nC}$. Since Q is negative, we have $Q = -2.55$ nC.

EVALUATE: The equation $E = k\dfrac{|q|}{r^2}$ gives only the *magnitude* of the electric field. When combining fields, you still must figure out whether to add or subtract the magnitudes depending on the direction in which the fields point.

21.69. IDENTIFY: For equilibrium, the forces must balance. The electrical force is given by Coulomb's law.

SET UP: Set up axes so that the charge $+Q$ is located at $x = 0$, the charge $+4Q$ is located at $x = d$, and the unknown charge that is required to produce equilibrium, q, is located at a position $x = a$. Apply $F = k\dfrac{|q_1 q_2|}{r^2}$ to each pair of charges to obtain eqilibrium.

EXECUTE: For a charge q to be in equilbrium, it must be placed between the two given positive charges $(0 < a < d)$ and the magnitude of the force between q and $+Q$ must be equal to the magnitude of the force between q and $+4Q$: $k\dfrac{|q|Q}{a^2} = k\dfrac{4|q|Q}{(d-a)^2}$. Solving for a we obtain $(d-a) = \pm 4a$, which has $a = \dfrac{d}{3}$ as its only root in the required interval $(0 < a < d)$. Furthermore, to conteract the repulsive force between $+Q$ and $+4Q$ the charge q must be negative $(q = -|q|)$. The condition that $+Q$ is in equilibrium gives us $k\dfrac{-qQ}{(d/3)^2} = k\dfrac{4Q^2}{d^2}$. Solving for q we obtain $q = -\dfrac{4}{9}Q$.

EVALUATE: We have shown that both q and $+Q$ are in equilibrium provided that $a=\frac{d}{3}$ and $q=-\frac{4}{9}Q$. To make sure that the problem is well posed, we should check that these conditions also place the charge $+4Q$ is in equilbrium. We can do this by showing that $k\frac{-4qQ}{(d-a)^2}$ is equal to $k\frac{4Q^2}{d^2}$ when the given values for both a and q are substituted.

21.70. **IDENTIFY** and **SET UP:** Like charges repel and unlike charges attract, and Coulomb's law applies. The positions of the three charges are sketched in Figure 21.70(a) and each force acting on q_3 is shown. The distance between q_1 and q_3 is 5.00 cm.

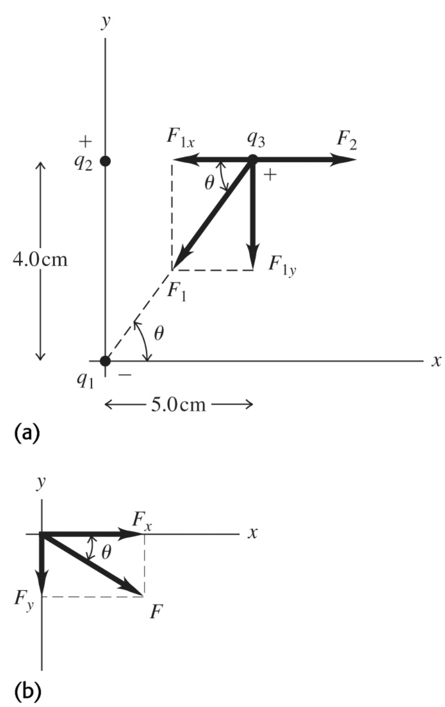


Figure 21.70

EXECUTE: **(a)** $F_1=k\frac{|q_1q_3|}{r_1^2}=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(5.00\times10^{-2}\ \text{m})^2}=5.394\times10^{-5}\ \text{N}.$

$F_{1x}=-F_1\cos\theta=-(5.394\times10^{-5}\ \text{N})(0.600)=-3.236\times10^{-5}\ \text{N}.$

$F_{1y}=-F_1\sin\theta=-(5.394\times10^{-5}\ \text{N})(0.800)=-4.315\times10^{-5}\ \text{N}.$

$F_2=k\frac{|q_2q_3|}{r_2^2}=(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2.00\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(3.00\times10^{-2}\ \text{m})^2}=9.989\times10^{-5}\ \text{N}.$

$F_{2x}=9.989\times10^{-5}\ \text{N};\ F_{2y}=0.$

$F_x=F_{1x}+F_{2x}=9.989\times10^{-5}\ \text{N}+(-3.236\times10^{-5}\ \text{N})=6.75\times10^{-5}\ \text{N};$

$F_y=F_{1y}+F_{2y}=-4.32\times10^{-5}\ \text{N}.$

(b) $\vec{F}$ and its components are shown in Figure 21.70(b).

$F = \sqrt{F_x^2 + F_y^2} = 8.01\times10^{-5}$ N. $\tan\theta = \left|\dfrac{F_y}{F_x}\right| = 0.640$ and $\theta = 32.6°$. $\vec{F}$ is 327° counterclockwise from the $+x$-axis.

EVALUATE: The equation $F = k\dfrac{|q_1q_2|}{r^2}$ gives only the magnitude of the force. We must find the direction by deciding if the force between the charges is attractive or repulsive.

21.71. **IDENTIFY:** Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.71a.

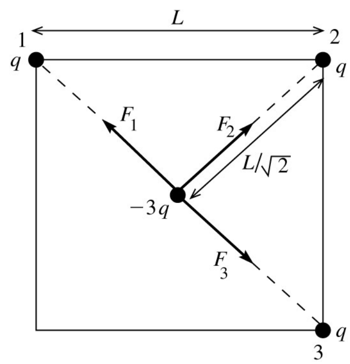


Figure 21. 71a

EXECUTE: $\vec{F}_1 + \vec{F}_3 = 0$, so the net force is $\vec{F} = \vec{F}_2$.

$F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(L/\sqrt{2})^2} = \dfrac{6q^2}{4\pi\epsilon_0 L^2}$, away from the vacant corner.

(b) SET UP: The forces are sketched in Figure 21.71b.

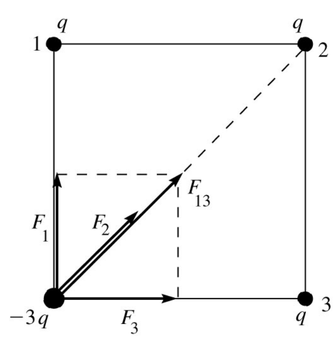


Figure 21. 71b

EXECUTE: $F_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{(\sqrt{2}L)^2} = \dfrac{3q^2}{4\pi\epsilon_0(2L^2)}$.

$F_1 = F_3 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q(3q)}{L^2} = \dfrac{3q^2}{4\pi\epsilon_0 L^2}$.

The vector sum of F_1 and F_3 is $F_{13} = \sqrt{F_1^2 + F_3^2}$.

$F_{13} = \sqrt{2}F_1 = \dfrac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}$; $\vec{F}_{13}$ and $\vec{F}_2$ are in the same direction.

$F = F_{13} + F_2 = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2} + \dfrac{1}{2}\right)$, and is directed toward the center of the square.

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

21.72. **IDENTIFY:** For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Qq_1|}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma =$ $(0.00500\text{ kg})(324\text{ m/s}^2) = 1.62\text{ N}$. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q. $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\text{ N}}{2\left(\dfrac{2.25\text{ cm}}{3.00\text{ cm}}\right)} = 1.08\text{ N}$. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs. $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|q_1}{r^2}$ and

$$q_1 = \frac{r^2F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}|Q|} = \frac{(0.0300\text{ m})^2(1.08\text{ N})}{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\text{ C})} = 6.17\times10^{-8}\text{ C}.$$

$q_2 = -q_1 = -6.17\times10^{-8}\text{ C}$.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.73. **IDENTIFY:** The small bags of protons behave like point-masses and point-charges since they are extremely far apart.

SET UP: For point-particles, we use Newton's formula for universal gravitation $(F = Gm_1m_2/r^2)$ and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: **(a)** $(0.0010\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 6.0\times10^{23}$ protons.

(b) Using Coulomb's law, where the separation is twice the radius of the earth, we have

$F_{\text{electrical}} = (9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.0\times10^{23}\times1.60\times10^{-19}\text{ C})^2/(2\times6.37\times10^6\text{ m})^2 = 5.1\times10^5\text{ N}$.

$F_{\text{grav}} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(0.0010\text{ kg})^2/(2\times6.37\times10^6\text{ m})^2 = 4.1\times10^{-31}\text{ N}$.

EVALUATE: **(c)** The electrical force (≈200,000 lb!) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.74. **IDENTIFY:** The positive sphere will be deflected in the direction of the electric field but the negative sphere will be deflected in the direction opposite to the electric field. Since the spheres hang at rest, they are in equilibrium so the forces on them must balance. The external forces on each sphere are gravity, the tension in the string, the force due to the uniform electric field and the electric force due to the other sphere.

SET UP: The electric force on one sphere due to the other is $F_C = k\dfrac{|q^2|}{r^2}$ in the horizontal direction, the force on it due to the uniform electric field is $F_E = qE$ in the horizontal direction, the gravitational force is mg vertically downward and the force due to the string is T directed along the string. For equilibrium $\sum F_x = 0$ and $\sum F_y = 0$.

EXECUTE: **(a)** The positive sphere is deflected in the same direction as the electric field, so the one that is deflected to the left is positive.

(b) The separation between the two spheres is $2(0.530 \text{ m})\sin 29.0^\circ = 0.5139 \text{ m}$.

$F_C = k\dfrac{|q^2|}{r^2} = \dfrac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(72.0\times10^{-9} \text{ C})^2}{(0.5139 \text{ m})^2} = 1.765\times10^{-4} \text{ N}$. $F_E = qE$. $\sum F_y = 0$ gives

$T\cos 29.0^\circ - mg = 0$ so $T = \dfrac{mg}{\cos 29.0^\circ}$. $\sum F_x = 0$ gives $T\sin 29.0^\circ + F_C - F_E = 0$.

$mg\tan 29.0^\circ + F_C = qE$. Combining the equations and solving for E gives

$$E = \frac{mg\tan 29.0^\circ + F_C}{q} = \frac{(6.80\times10^{-6} \text{ kg})(9.80 \text{ m/s}^2)\tan 29.0^\circ + 1.765\times10^{-4} \text{ N}}{72.0\times10^{-9} \text{ C}} = 2.96\times10^3 \text{ N/C}.$$

EVALUATE: Since the charges have opposite signs, they attract each other, which tends to reduce the angle between the strings. Therefore if their charges were negligibly small, the angle between the strings would be greater than 58.0°.

21.75. IDENTIFY: The only external force acting on the electron is the electrical attraction of the proton, and its acceleration is toward the center of its circular path (that is, toward the proton). Newton's second law applies to the electron and Coulomb's law gives the electrical force on it due to the proton.

SET UP: Newton's second law gives $F_C = m\dfrac{v^2}{r}$. Using the electrical force for F_C gives $k\dfrac{e^2}{r^2} = m\dfrac{v^2}{r}$.

EXECUTE: Solving for v gives $v = \sqrt{\dfrac{ke^2}{mr}} = \sqrt{\dfrac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})^2}{(9.109\times10^{-31} \text{ kg})(5.29\times10^{-11} \text{ m})}} = 2.19\times10^6 \text{ m/s}$.

EVALUATE: This speed is less than 1% the speed of light, so it is reasonably safe to use Newtonian physics.

21.76. IDENTIFY: A uniformly charged horizontal disk exerts an upward electric force on a small charged sphere. Gravity exerts a downward force on the sphere.

SET UP: The magnitude of the electric force on the sphere is $F = QE$, where E is the electric field a distance z above the center of the disk and is given by $E = \dfrac{\sigma}{2\epsilon_0}\left(1 - \dfrac{1}{\sqrt{(R/z)^2 + 1}}\right)$.

EXECUTE: **(a)** The net upward force is $F = F_{\text{el}} + F_{\text{g}} = \dfrac{Q\sigma}{2\epsilon_0}\left(1 - \dfrac{1}{\sqrt{(R/z)^2 + 1}}\right) - Mg$.

(b) We want the height h above the disk at which the sphere hovers. This will occur when the electric force is equal to the weight of the sphere, so $\dfrac{Q\sigma}{2\epsilon_0}\left(1 - \dfrac{1}{\sqrt{(R/z)^2 + 1}}\right) - Mg = 0$. Now solve for z.

Rearranging gives $1 - \dfrac{1}{\sqrt{(R/z)^2 + 1}} = \dfrac{2Mg\epsilon_0}{Q\sigma} \equiv v$, where we have used the v defined in the problem.

We now need to solve the equation $1 - v = \dfrac{1}{\sqrt{(R/h)^2 + 1}}$ for h, where h is the height at which the sphere hovers. Doing some algebra yields $h = \dfrac{R}{\sqrt{1/(1-v)^2 - 1}} = \dfrac{R(1-v)}{\sqrt{v(2-v)}}$.

(c) We want to find h if $M = 100$ g $= 0.100$ kg, $Q = 1.00\ \mu$C, $R = 5.00$ cm $= 0.0500$ m, and $\sigma = 10.0\ \text{nC/cm}^2 = 1.00\times10^{-4}\ \text{C/m}^2$. Using $v = \dfrac{2Mg\epsilon_0}{Q\sigma}$ and $h = \dfrac{R(1-v)}{\sqrt{v(2-v)}}$ with these numbers, we get $v = 0.1735$, which gives $h = 0.0734$ m $= 7.34$ cm.

EVALUATE: A 100-gram object hovering about 7 cm from the disk could easily be observed in a student laboratory.

21.77. IDENTIFY: $\vec{E} = \dfrac{\vec{F}_0}{q_0}$ gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the x- and y-components of the motion, just as for projectile motion.

SET UP: The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE.$ Use coordinates where positive y is downward. Then applying $\sum\vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m.$ In these coordinates the initial velocity has components $v_x = +v_0\cos\alpha$ and $v_y = +v_0\sin\alpha,$ as shown in Figure 21.77a.

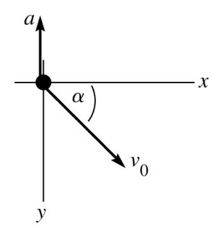


Figure 21.77a

EXECUTE: **(a)** Finding $h_{\max}$: At $y = h_{\max}$ the y-component of the velocity is zero.

$v_y = 0, v_{0y} = v_0\sin\alpha, a_y = -eE/m, y - y_0 = h_{\max} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0).$

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y}.$

$h_{\max} = \dfrac{-v_0^2\sin^2\alpha}{2(-eE/m)} = \dfrac{mv_0^2\sin^2\alpha}{2eE}.$

(b) Use the vertical motion to find the time t: $y - y_0 = 0, v_{0y} = v_0\sin\alpha, a_y = -eE/m, t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2.$

With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(v_0\sin\alpha)}{-eE/m} = \dfrac{2mv_0\sin\alpha}{eE}.$

Then use the x-component motion to find d: $a_x = 0, v_{0x} = v_0\cos\alpha, t = 2mv_0\sin\alpha/eE, x - x_0 = d = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $d = v_0\cos\alpha\left(\dfrac{2mv_0\sin\alpha}{eE}\right) = \dfrac{mv_0^2 2\sin\alpha\cos\alpha}{eE} = \dfrac{mv_0^2\sin 2\alpha}{eE}.$

(c) The trajectory of the proton is sketched in Figure 21.77b.

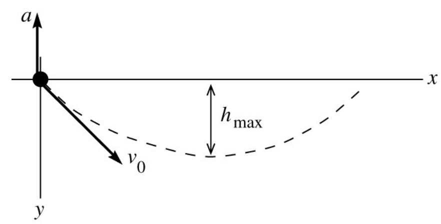


Figure 21.77b

(d) Use the expression in part (a): $h_{max} = \dfrac{[(4.00\times10^5 \text{ m/s})(\sin 30.0°)]^2(1.673\times10^{-27} \text{ kg})}{2(1.602\times10^{-19} \text{ C})(500 \text{ N/C})} = 0.418 \text{ m}.$

Use the expression in part (b): $d = \dfrac{(1.673\times10^{-27} \text{ kg})(4.00\times10^5 \text{ m/s})^2 \sin 60.0°}{(1.602\times10^{-19} \text{ C})(500 \text{ N/C})} = 2.89 \text{ m}.$

EVALUATE: In part (a), $a_y = -eE/m = -4.8\times10^{10} \text{ m/s}^2$. This is much larger in magnitude than g, the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. h_{max} and d increase when α or v_0 increase and decrease when E increases.

21.78. **IDENTIFY:** The electric field is vertically downward and the charged object is deflected downward, so it must be positively charged. While the object is between the plates, it is accelerated downward by the electric field. Once it is past the plates, it moves downward with a constant vertical velocity which is the same downward velocity it acquired while between the plates. Its horizontal velocity remains constant at v_0 throughout its motion. The forces on the object are all constant, so its acceleration is constant; therefore we can use the standard kinematics equations. Newton's second law applies to the object.

SET UP: Call the x-axis positive to the right and the y-axis positive downward. The equations $\vec{E} = \dfrac{\vec{F}_0}{q_0}$, $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, $v_y = v_{0y} + a_y t$, $x = v_x t$, and $\Sigma F_y = ma_y$ all apply. $v_x = v_0$ = constant.

EXECUTE: *Time through the plates:* $t = x/v_x = x/v_0$ = (0.260 m)/(5000 m/s) = 5.20×10^{-5} s.

Vertical deflection between the plates: $\Delta y_1 = y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2 = \frac{1}{2}(qE/m)t^2$

$\Delta y_1 = \frac{1}{2}(800 \text{ N/C})(5.20\times10^{-5} \text{ s})^2(q/m) = (1.0816\times10^{-6} \text{ kg}\cdot\text{m/C})(q/m)$.

v_y as the object just emerges from the plates:
$v_y = v_{0y} + a_y t = (qE/m)t = (q/m)(800 \text{ N/C})(5.20\times10^{-5} \text{ s}) = (0.04160 \text{ kg}\cdot\text{m/C}\cdot\text{s})(q/m)$. (This is the initial vertical velocity for the next step.)
Time to travel 56.0 cm: $t = x/v_x$ = (0.560 m)/(5000 m/s) = 1.120×10^{-4} s.
Vertical deflection after leaving the plates:
$\Delta y_2 = v_{0y}t = (0.04160 \text{ kg}\cdot\text{m/C}\cdot\text{s})(q/m)(1.120\times10^{-4} \text{ s}) = (4.6592\times10^{-6} \text{ kg}\cdot\text{m/C})(q/m)$.
Total vertical deflection:
$d = \Delta y_1 + \Delta y_2$.
1.25 cm = 0.0125 m = $(1.0816\times10^{-6} \text{ kg}\cdot\text{m/C})(q/m) + (4.6592\times10^{-6} \text{ kg}\cdot\text{m/C})(q/m)$.
q/m = 2180 C/kg.
EVALUATE: The charge on 1.0 kg is so huge that it could not be dealt with in a laboratory. But this is a tiny object, more likely with a mass in the range of 1.0 μg, so its charge would be (2180 C/kg)(10^{-9} kg) = 2.18×10^{-6} C ≈ 2 μC. That amount of charge could be used in an experiment.

21.79. **IDENTIFY:** Divide the charge distribution into infinitesimal segments of length dx'. Calculate E_x and E_y due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx' is $dQ = (Q/a)dx'$. The distance between a segment at x' and a point at x on the x-axis is $x - x'$ since $x > a$.

EXECUTE: **(a)** $dE_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{dQ}{(x-x')^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(Q/a)dx'}{(x-x')^2}$. Integrating with respect to x' over the length of the charge distribution gives

$$E_x = \frac{1}{4\pi\epsilon_0}\int_0^a \frac{(Q/a)dx'}{(x-x')^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{a}{x(x-a)} = \frac{1}{4\pi\epsilon_0}\frac{Q}{x(x-a)}.\ Ey = 0.$$

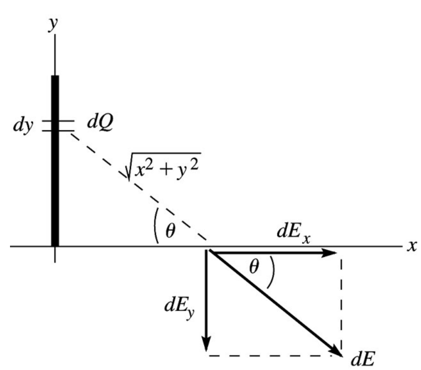
(b) At the location of the charge, $x = r + a$, so $E_x = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{(r+a)(r+a-a)} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r(r+a)}$.

Using $\vec{F} = q\vec{E}$, we have $\vec{F} = q\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r(r+a)}\hat{\boldsymbol{i}}$.

EVALUATE: **(c)** For $r \gg a$, $r + a \to r$, so the magnitude of the force becomes $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{r^2}$. The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.80. **IDENTIFY:** Use $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$ to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.10. Use $\vec{E} = \dfrac{\vec{F}}{q_0}$ to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.80.



$$\sin\theta = \frac{y}{\sqrt{x^2+y^2}}.$$

$$\cos\theta = \frac{x}{\sqrt{x^2+y^2}}.$$

Figure 21.80

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ = Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{dQ}{x^2+y^2}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{dy}{x^2+y^2}\right)$.

$$dE_x = dE\cos\theta = \frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{(x^2+y^2)^{3/2}}\right).$$

$$dE_y = -dE\sin\theta = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{(x^2+y^2)^{3/2}}\right).$$

$$E_x = \int dE_x = -\frac{Qx}{4\pi\epsilon_0 a}\int_0^a \frac{dy}{(x^2+y^2)^{3/2}} = \frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a = \frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}.$$

$$E_y = \int dEy = -\frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{ydy}{(x^2+y^2)^{3/2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a = -\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right).$$

(b) $\vec{F} = q_0\vec{E}.$

$$F_x = -qE_x = \frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\ F_y = -qE_y = \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right).$$

(c) For $x \gg a$, $\dfrac{1}{\sqrt{x^2+a^2}} = \dfrac{1}{x}\left(1+\dfrac{a^2}{x^2}\right)^{-1/2} = \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right) = \dfrac{1}{x}-\dfrac{a^2}{2x^3}.$

$$F_x \approx -\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y \approx \frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right) = \frac{qQa}{8\pi\epsilon_0 x^3}.$$

EVALUATE: For $x \gg a$, $F_y \ll F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x \gg a$ the charge distribution Q acts like a point charge.

21.81. IDENTIFY: Apply $E = \dfrac{\sigma}{2\epsilon_0}[1-(R^2/x^2+1)^{-1/2}].$

SET UP: $\sigma = Q/A = Q/\pi R^2$. $(1+y^2)^{-1/2} \approx 1-y^2/2$, when $y^2 \ll 1$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{2\epsilon_0}[1-(R^2/x^2+1)^{-1/2}]$ gives

$$E = \frac{7.00\ \text{pC}/\pi(0.025\ \text{m})^2}{2\epsilon_0}\left[1-\left(\frac{(0.025\ \text{m})^2}{(0.200\ \text{m})^2}+1\right)^{-1/2}\right] = 1.56\ \text{N/C, in the } +x\text{-direction.}$$

(b) For $x \gg R$, $E = \dfrac{\sigma}{2\epsilon_0}[1-(1-R^2/2x^2+\cdots)] \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x^2} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x^2} = \dfrac{Q}{4\pi\epsilon_0 x^2}.$

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x = 0.200$ m, the percent difference is $\dfrac{(1.58-1.56)}{1.56} = 0.01 = 1\%$. For $x = 0.100$ m,

$E_{\text{disk}} = 6.00$ N/C and $E_{\text{point}} = 6.30$ N/C, so the percent difference is $\dfrac{(6.30-6.00)}{6.30} = 0.047 \approx 5\%.$

EVALUATE: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x = 10.0$ cm, $R/x = 25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.82. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere, with x horizontal and y vertical.

SET UP: The free-body diagram for the sphere is given in Figure 21.82. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E = \dfrac{\sigma}{2\epsilon_0}.$

EXECUTE: $\sum F_y = 0$ gives $T\cos\alpha = mg$ and $T = \dfrac{mg}{\cos\alpha}$. $\sum F_x = 0$ gives $T\sin\alpha = \dfrac{q\sigma}{2\epsilon_0}$ and

$T = \dfrac{q\sigma}{2\epsilon_0 \sin\alpha}$. Combining these two equations we have $\dfrac{mg}{\cos\alpha} = \dfrac{q\sigma}{2\epsilon_0 \sin\alpha}$ and $\tan\alpha = \dfrac{q\sigma}{2\epsilon_0\, mg}$.

Therefore, $\alpha = \arctan\left(\dfrac{q\sigma}{2\epsilon_0\, mg}\right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

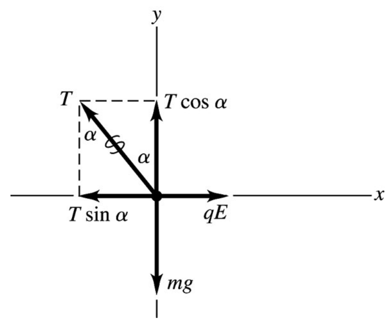
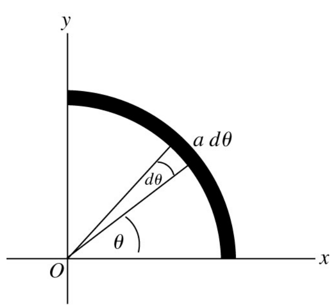


Figure 21.82

21.83. IDENTIFY: Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the x- and y-components of the total field.

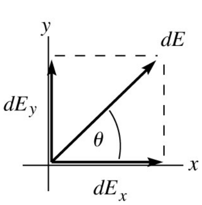
SET UP: Consider the small segment shown in Figure 21.83a.



EXECUTE: A small segment that subtends angle $d\theta$ has length $a\,d\theta$ and contains charge $dQ = \left(\dfrac{a d\theta}{\frac{1}{2}\pi a}\right)Q = \dfrac{2Q}{\pi}d\theta.$ ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

Figure 21.83a

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.83b, along with its components.



$$dE = \frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}.$$

$$dE = \frac{Q}{2\pi^2\epsilon_0\, a^2}d\theta.$$

Figure 21.83b

$dE_x = dE\cos\theta = (Q/2\pi^2 \epsilon_0 a^2)\cos\theta d\theta.$

$E_x = \int dE_x = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}(\sin\theta\big|_0^{\pi/2}) = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}.$

$dE_y = dE\sin\theta = (Q/2\pi^2 \epsilon_0 a^2)\sin\theta d\theta.$

$E_y = \int dE_y = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}(-\cos\theta\big|_0^{\pi/2}) = \dfrac{Q}{2\pi^2 \epsilon_0 a^2}.$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.84. IDENTIFY: We must add the electric field components of the positive half and the negative half.

SET UP: From Problem 21.83, the electric field due to the quarter-circle section of positive charge has components $E_x = +\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$, $E_y = -\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$. The field due to the quarter-circle section of negative charge has components $E_x = +\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$, $E_y = +\dfrac{Q}{2\pi^2 \epsilon_0 a^2}$.

EXECUTE: The components of the resultant field are the sum of the *x*- and *y*-components of the fields due to each half of the semicircle. The *y*-components cancel, but the *x*-components add, giving $E_x = +\dfrac{Q}{\pi^2 \epsilon_0 a^2}$, in the $+x$-direction.

EVALUATE: Even though the net charge on the semicircle is zero, the field it produces is *not* zero because of the way the charge is arranged.

21.85. IDENTIFY: Each wire produces an electric field at *P* due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{\text{net}} = 2E_1\cos 45° = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at *P* is *eE*.

EXECUTE: **(a)** The net field is $E_{\text{net}} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x\sqrt{x^2+a^2}}\cos 45°.$

$$E_{\text{net}} = \frac{2(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\ \text{C})\cos 45°}{(0.600\ \text{m})\sqrt{(0.600\ \text{m})^2+(0.600\ \text{m})^2}} = 6.25\times10^4\ \text{N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right at point *P*.

(b) $F = eE = (1.60\times10^{-19}\ \text{C})(6.25\times10^4\ \text{N/C}) = 1.00\times10^{-14}\ \text{N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.86. IDENTIFY: Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these, $E_{\text{net}} = \dfrac{\sigma_B}{2\epsilon_0} \pm \dfrac{\sigma_A}{2\epsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\epsilon_0}$, where we use the $+$ or $-$ sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The two fields oppose and the field of *B* is stronger than that of *A*, so

$$E_{\text{net}} = \frac{\sigma_B}{2\epsilon_0} - \frac{\sigma_A}{2\epsilon_0} = \frac{\sigma_B - \sigma_A}{2\epsilon_0} = \frac{11.6\ \mu\text{C/m}^2 - 8.80\ \mu\text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.58\times10^5\ \text{N/C},\ \text{to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}} = (11.6\ \mu\text{C/m}^2 + 8.80\ \mu\text{C/m}^2)/2\epsilon_0 = 1.15\times10^6\ \text{N/C},\ \text{to the right.}$$

(c) The fields add but now point to the left, so $E_{\text{net}} = 1.15\times10^6$ N/C, to the left.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.87. **IDENTIFY:** Apply the formula for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - 1/\sqrt{(R/x)^2 + 1}\right].$$

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma.$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2+1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2+1}\right]\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$

$\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2+1} - 1/\sqrt{(R_2/x)^2+1}\right)\dfrac{|x|}{x}\hat{\boldsymbol{i}}.$ The electric field is in the $+x$-direction at points above

the disk and in the $-x$-direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{\boldsymbol{i}}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2+1} = \dfrac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives

$\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\dfrac{|x|^2}{x}\hat{\boldsymbol{i}} = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{\boldsymbol{i}}.$ Sufficiently close means that $(x/R_1)^2 \ll 1$.

(d) $F_x = -qE_x = -\dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x.$ The force is in the form of Hooke's law: $F_x = -kx$, with

$$k = \frac{q\sigma}{2\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).\quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$$

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.88. **IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0$ cm be the horizontal distance the drop travels and $d = 0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and call $a_y = a$.

EXECUTE: **(a)** Find the time of flight: $t = D/v = (0.020\ \text{m})/(50\ \text{m/s}) = 4.00\times10^{-4}$ s. $d = \dfrac{1}{2}at^2$.

$a = \dfrac{2d}{t^2} = \dfrac{2(3.00\times10^{-4}\ \text{m})}{(4.00\times10^{-4}\ \text{s})^2} = 3750\ \text{m/s}^2$. Then $a = F/m = qE/m$ gives

$q = ma/E = \dfrac{(1.4\times10^{-11}\ \text{kg})(3750\ \text{m/s}^2)}{8.00\times10^4\ \text{N/C}} = 6.56\times10^{-13}$ C, which rounds to 6.6×10^{-13} s.

(b) Use the equations and calculations above: if $v \to v/2$, then $t \to 2t$, so $a \to a/4$, which means that $q \to q/4$, so $q = (6.56 \times 10^{-13}\ \text{s})/4 = 1.64 \times 10^{-13}$ s, which rounds to 1.6×10^{-13} s.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.89 **IDENTIFY:** The net force on the third sphere is the vector sum of the forces due to the other two charges. Coulomb's law gives the forces.

SET UP: $F = k\dfrac{|q_1 q_2|}{r^2}$.

EXECUTE: **(a)** Between the two fixed charges, the electric forces on the third sphere q_3 are in opposite directions and have magnitude 4.50 N in the $+x$-direction. Applying Coulomb's law gives
$4.50\ \text{N} = k[q_1(4.00\ \mu\text{C})/(0.200\ \text{m})^2 - q_2(4.00\ \mu\text{C})/(0.200\ \text{m})^2]$.
Simplifying gives $q_1 - q_2 = 5.00\ \mu$C.
With q_3 at $x = +0.600$ m, the electric forces on q_3 are all in the $+x$-direction and add to 3.50 N. As before, Coulomb's law gives
$3.50\ \text{N} = k[q_1(4.00\ \mu\text{C})/(0.600\ \text{m})^2 + q_2(4.00\ \mu\text{C})/(0.200\ \text{m})^2]$.
Simplifying gives $q_1 + 9q_2 = 35.0\ \mu$C.
Solving the two equations simultaneously gives $q_1 = 8.00\ \mu$C and $q_2 = 3.00\ \mu$C.
(b) Both forces on q_3 are in the $-x$-direction, so their magnitudes add. Factoring out common factors and using the values for q_1 and q_2 we just found, Coulomb's law gives
$F_{\text{net}} = kq_3\,[q_1/(0.200\ \text{m})^2 + q_2/(0.600\ \text{m})^2]$.
$F_{\text{net}} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,[(8.00\ \mu\text{C})/(0.200\ \text{m})^2 + (3.00\ \mu\text{C})/(0.600\ \text{m})^2] = 7.49$ N, and it is in the $-x$-direction.
(c) The forces are in opposite direction and add to zero, so
$0 = kq_1q_3/x^2 - kq_2q_3/(0.400\ \text{m} - x)^2$.
$(0.400\ \text{m} - x)^2 = (q_2/q_1)x^2$.
Taking square roots of both sides gives
$0.400\ \text{m} - x = \pm x\sqrt{q_2/q_1} = \pm 0.6124x$.

Solving for x, we get two values: $x = 0.248$ m and $x = 1.03$ m. The charge q_3 must be between the other two charges for the forces on it to balance. Only the first value is between the two charges, so it is the correct one: $x = 0.248$ m.

EVALUATE: Check the answers in part (a) by substituting these values back into the original equations. $8.00\ \mu\text{C} - 3.00\ \mu\text{C} = 5.00\ \mu\text{C}$ and $8.00\ \mu\text{C} + 9(3.00\ \mu\text{C}) = 35.0\ \mu\text{C}$, so the answers check in both equations. In part (c), the second root, $x = 1.03$ m, has some meaning. The condition we imposed to solve the problem was that the magnitudes of the two forces were equal. This happens at $x = 0.248$ mn, but it also happens at $x = 1.03$ m. However at the second root the forces are both in the $+x$-direction and therefore cannot cancel.

21.90. **IDENTIFY** and **SET UP:** The electric field E_x produced by a uniform ring of charge, for points on an axis perpendicular to the plane of the ring at its center, is $E_x = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$, where a is the radius of the ring, x is the distance from its center along the axis, and Q is the total charge on the ring.

EXECUTE: **(a)** Far from the ring, at large values of x, the ring can be considered as a point-charge, so its electric field would be $E = kQ/x^2$. Therefore $Ex^2 = kQ$, which is a constant. From the graph (a) in the problem, we read off that at large distances $Ex^2 = 45\ \text{N}\cdot\text{m}^2/\text{C}$, which is equal to kQ, so we have
$Q = (45\ \text{N}\cdot\text{m}^2/\text{C})/k = 5.0 \times 10^{-9}\ \text{C} = 5.0$ nC.

(b) The electric field along the axis a distance x from the ring is $E_x = \dfrac{kQx}{(x^2 + a^2)^{3/2}}$. Very close to the ring, $x^2 << a^2$, so the formula becomes $Ex = kQx/a^3$. Therefore $E/x = kQ/a^3$, which is a constant. From graph (b) in the problem, E/x approaches $700\ \text{N/C}\cdot\text{m}$ as x approaches zero. So $kQ/a^3 = 700\ \text{N/C}\cdot\text{m}$, which gives

$a = [kQ/(700\ \text{N/C}\cdot\text{m})]^{1/3} = [(45\ \text{N}\cdot\text{m}^2/\text{C})/(700\ \text{N/C}\cdot\text{m})]^{1/3} = 0.40\ \text{m} = 40\ \text{cm}$.

EVALUATE: It is physically reasonable that a ring 40 cm in radius could carry 5.0 nC of charge.

21.91. IDENTIFY: An infinite positively charged sheet has a round hole cut in it. A particle with mass m and charge $-q$ is dropped from the center of the hole and falls. The net force on it is the electric force upward and gravity downward.

SET UP and EXECUTE: (a) We can think of the sheet with the hole as a combination of an infinite positive sheet and a negative disk of radius R, both having the same magnitude charge density σ, so the net field E_z is $E_z = E_{\text{sheet}} + E_{\text{disk}}$. Calling $+z$ upward, the net field a distance Δ below the center of the hole is $E_z = -\dfrac{\sigma}{2\epsilon_0} + \dfrac{\sigma}{2\epsilon_0}\left[1 - \dfrac{1}{\sqrt{1+(R/\Delta)^2}}\right] = -\dfrac{\sigma}{2\epsilon_0}\left[\dfrac{1}{\sqrt{1+(R/\Delta)^2}}\right]$, and it points downward.

(b) As the particle falls the net electric field *increases* because the field of the negative hole decreases while that of the positive sheet increases. The maximum field occurs very far from the hole, and that field is just due to the sheet, which is $E_z = -\dfrac{\sigma}{2\epsilon_0}$. The maximum mass for which the particle will not fall indefinitely is given by $F_{\text{el}} = m_{\max}g$, so $qE_z = m_{\max}g$, and $\dfrac{q\sigma}{2\epsilon_0} = m_{\max}g$. Thus $m_{\max} = \dfrac{q\sigma}{2g\epsilon_0}$.

(c) We want the work done by the electric field as the particle drops from $z = 0$ to $z = -\Delta$ if $m < m_{\max}$. The force on the particle is upward but its displacement is downward, so the work is negative. Therefore $W_{\text{E}} = -\int qE_z dz = -\int_0^{-\Delta} \dfrac{q\sigma}{2\epsilon_0\sqrt{1+(R/z)^2}}dz = -\dfrac{q\sigma}{2\epsilon_0}\int_0^{-\Delta}\dfrac{z\,dz}{\sqrt{z^2+R^2}}$. The integration can be done using tables or by letting $u = z^2 + R^2$ and $du = 2z\,dz$. Either way, the result is

$W_{\text{E}} = -\dfrac{q\sigma}{2\epsilon_0}\sqrt{z^2+R^2}\Big|_0^{-\Delta} = -\dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2} - R\right)$.

(d) Gravity is constant, so $W_{\text{g}} = +mg\Delta$.

(e) We want the depth to which the particle will fall. The work-energy theorem $W_{\text{tot}} = K_2 - K_1$ gives $W_{\text{g}} + W_{\text{E}} = K_2 - K_1$. $K_1 = 0$, so we have $mg\Delta - \dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2} - R\right) = \dfrac{1}{2}mv^2$. At the maximum depth the particle remains at rest, so solve this result for Δ when $v = 0$ and call the result $\Delta_{\max}$. Doing so we have $mg\Delta - \dfrac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2} - R\right) = 0$. So $\Delta_{\max} = \dfrac{mgq\sigma R}{\epsilon_0\left[(q\sigma/2\epsilon_0)^2 - (mg)^2\right]}$. Finally we express this result in terms of $\alpha = m/m_{\max}$. Using $m_{\max}g = \dfrac{q\sigma}{2\epsilon_0}$ gives $\alpha = \dfrac{m}{\dfrac{q\sigma}{2g\epsilon_0}} = \dfrac{2g\epsilon_0 m}{q\sigma}$. Thus $mg = \dfrac{q\sigma\alpha}{2\epsilon_0}$.

Putting this into our result for $\Delta_{\max}$ gives $\Delta_{\max} = \dfrac{mgq\sigma R}{\epsilon_0\left[(mg/\alpha)^2 - (mg)^2\right]}$. Substituting $mg = \dfrac{q\sigma\alpha}{2\epsilon_0}$ and simplifying gives $\Delta_{\max} = \dfrac{2R\alpha}{1-\alpha^2}$.

(f) We want to find v as a function of Δ if $\Delta < \Delta_{\max}$. Solve $mg\Delta - \frac{q\sigma}{2\epsilon_0}\left(\sqrt{\Delta^2+R^2}-R\right) = \frac{1}{2}mv^2$ for v.

Using $m_{\max}g = \frac{q\sigma}{2\epsilon_0}$ gives $mg\Delta - m_{\max}g\left(\sqrt{\Delta^2+R^2}-R\right) = \frac{1}{2}mv^2$. Dividing by m and using

$\alpha = m/m_{\max}$ gives $v^2 = 2g\Delta - \frac{2g}{\alpha}\left(\sqrt{\Delta^2+R^2}-R\right)$, so $v = \sqrt{2g\left(\Delta - \frac{\sqrt{\Delta^2+R^2}-R}{\alpha}\right)}$.

(g) We want $m_{\max}$ and $\Delta_{\max}$. Using the given quantities, with $\sigma = 1.00\ \text{nC/cm}^2 = 1.00\times10^{-5}\ \text{C/m}^2$, the results from parts (b) and (e) give $m_{\max}$ = 57.7 g and $\Delta_{\max}$ = 10.7 cm.

EVALUATE: It may seem strange that the electric field gets *stronger* as we get farther from the sheet. But the closer we get to the sheet, the greater the effect of the hole. As we get farther from the sheet the effect of the hole diminishes.

21.92. **IDENTIFY:** The charges at the ends of the stationary rod exert electric forces on the charges at the ends of the second rod that is free to rotate. These forces produce torques, which cause the second rod to spin about its center.

SET UP: The magnitude of the electric force is $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$.

EXECUTE: **(a)** We want the force $\vec{F}_1$ on the upper right charge of the movable rod due to the charge on the left side of the fixed rod. Fig. 21.92a shows this arrangement and the force $\vec{F}_1$. Use the law of cosines to find the distance r_1. $r_1^2 = (L/2)^2 + (L/2)^2 - 2(L/2)^2\cos(180° - \theta)$. This gives

$r_1^2 = (L^2/2)(1+\cos\theta)$, so $F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{(L^2/2)(1+\cos\theta)} = \frac{2}{4\pi\epsilon_0}\frac{Q^2}{L^2(1+\cos\theta)}$. Using Fig. 21.92c, we see that the components of $\vec{F}_1$ are $F_{1x} = F_1\cos\phi = F_1\cos(\theta/2)$ and

$F_{1y} = F_1\sin\phi = F_1\sin(\theta/2)$. Therefore $\vec{F}_1 = \frac{Q^2}{2\pi\epsilon_0 L^2(1+\cos\theta)}\left[\cos(\theta/2)\hat{\boldsymbol{i}} + \sin(\theta/2)\hat{\boldsymbol{j}}\right]$.

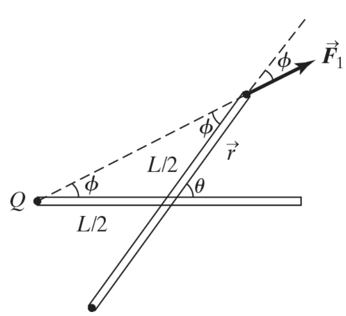


Figure 21.92a

(b) We want the force $\vec{F}_2$ on the upper right charge of the movable rod due to the charge on the right side of the fixed rod. Fig. 21.92b shows this arrangement and for force $\vec{F}_2$. The law of cosines gives r_2:

$r_2^2 = (L/2)^2 + (L/2)^2 - 2(L/2)^2\cos\theta = (L^2/2)(1-\cos\theta)$. Coulomb's law gives $F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2}$

$= \frac{1}{4\pi\epsilon_0}\frac{Q^2}{(L^2/2)(1-\cos\theta)} = \frac{2}{4\pi\epsilon_0}\frac{Q^2}{L^2(1-\cos\theta)}$. Using Fig. 21.92d, we see that $2\phi + \theta = 180°$, so

$\phi = 180° - \theta/2$. Therefore the components of $\vec{F}_2$ are

$F_{2x} = -F_2 \cos\phi = -F_2 \cos(90° - \theta/2) = -F_2 \sin(\theta/2)$ and
$F_{2y} = F_2 \sin\phi = -F_2 \sin(90° - \theta/2) = F_2 \cos(\theta/2)$. In vector form, the force is

$$\vec{F}_2 = \frac{Q^2}{2\pi\epsilon_0 L^2(1-\cos\theta)}\left[-\sin(\theta/2)\hat{\boldsymbol{i}} + \cos(\theta/2)\hat{\boldsymbol{j}}\right].$$

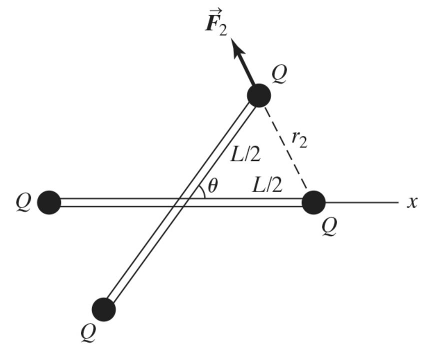


Figure 21.92b

(c) We want the torque due to $\vec{F}_1$. Use $\tau = rF\sin\phi$ for the quantities shown in Fig. 21.92c. $r = L/2$ and extends from the center of the rods to the upper end of the movable rod. From the triangle in the figure, we see that $2\phi + (180° - \theta) = 180°$, so $\phi = \theta/2$. Using $\tau = rF\sin\phi$, we have

$$\tau_1 = \frac{L}{2}\left(\frac{2}{4\pi\epsilon_0}\frac{Q^2}{L^2(1+\cos\theta)}\right)\sin(\theta/2) = \frac{1}{4\pi\epsilon_0}\frac{Q^2\sin(\theta/2)}{L(1+\cos\theta)}, \text{ clockwise.}$$

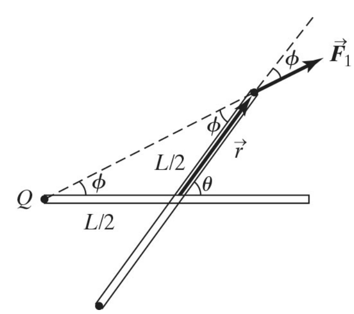


Figure 21.92c

(d) We want the torque due to $\vec{F}_2$. See Fig. 21.92d for the geometry. As in part (c), use $\tau = rF\sin\phi$, where $r = L/2$. From the figure, we see that $2\phi + \theta = 180°$, so $\phi = (180° - \theta)/2 = 90° - \theta/2$. So

$\sin\phi = \cos(\theta/2)$. The torque is $\tau_2 = \dfrac{L}{2}\left(\dfrac{2}{4\pi\epsilon_0}\dfrac{Q^2}{L^2(1-\cos\theta)}\right)\cos(\theta/2) = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2\cos(\theta/2)}{L(1-\cos\theta)}$,

counterclockwise.

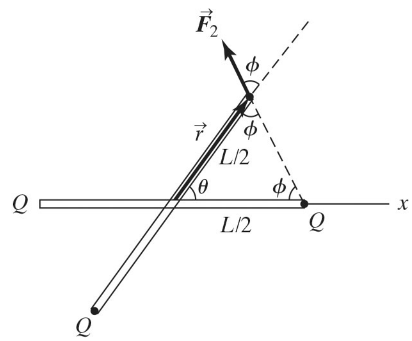


Figure 21.92d

(e) Torque τ_3 is on the lower charge of the movable rod due to the left charge on the fixed rod. This torque is the same as τ_2 and is also counterclockwise, so $\tau_3 = \tau_2$. Torque τ_4 is on the lower charge of the movable rod due to the right charge on the fixed rod. This torque is the same as τ_1 and is also clockwise, so $\tau_4 = \tau_1$. Calling counterclockwise positive, the total torque is

$\tau_{\text{tot}} = -\tau_1 + \tau_2 + \tau_3 - \tau_4 = 2\tau_2 - 2\tau_1 = 2(\tau_2 - \tau_1)$.

$$\tau_{\text{tot}} = 2\left[\frac{1}{4\pi\epsilon_0}\frac{Q^2\cos(\theta/2)}{L(1-\cos\theta)} - \frac{1}{4\pi\epsilon_0}\frac{Q^2\sin(\theta/2)}{L(1+\cos\theta)}\right] = \frac{Q^2}{2\pi\epsilon_0 L}\left[\frac{\cos(\theta/2)}{1-\cos\theta} - \frac{\sin(\theta/2)}{1+\cos\theta}\right].$$

(f) We want the net torque for small angular displacements from the equilibrium position of $\theta = \pi/2$ rad. Letting $\theta = \frac{\pi}{2} - \epsilon$, where $\epsilon \ll 1$ rad, we use $\tau_{\text{net}} = \frac{Q^2}{2\pi\epsilon_0 L}\left[\frac{\cos(\theta/2)}{1-\cos\theta} - \frac{\sin(\theta/2)}{1+\cos\theta}\right]$ with

$\theta = \frac{\pi}{2} - \epsilon$ and keep only terms with powers lower than ϵ^2. Using the power series in Appendix B we have the following expansions.

$$\cos(\theta/2) \to \cos\left(\frac{\pi/2-\epsilon}{2}\right) = \cos\left(\frac{\pi}{4} - \frac{\epsilon}{2}\right) = \frac{1}{\sqrt{2}}\cos\frac{\epsilon}{2} + \frac{1}{\sqrt{2}}\sin\frac{\epsilon}{2} \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\epsilon^2}{8}\right) + \frac{\epsilon}{2\sqrt{2}}.$$

$$\sin(\theta/2) \to \sin\left(\frac{\pi/2-\epsilon}{2}\right) = \sin\left(\frac{\pi}{4} - \frac{\epsilon}{2}\right) = \frac{1}{\sqrt{2}}\cos\frac{\epsilon}{2} - \frac{1}{\sqrt{2}}\sin\frac{\epsilon}{2} \approx \frac{1}{\sqrt{2}}\left(1 - \frac{\epsilon^2}{8}\right) - \frac{\epsilon}{2\sqrt{2}}.$$

Both denominators are approximately equal to 1 for small $\mathcal{E}$. Using these results for the torque gives $\tau \approx \frac{Q^2}{2\pi\epsilon_0 L}\left\{\frac{1}{\sqrt{2}}\left(1 - \frac{\epsilon^2}{8}\right) + \frac{\epsilon}{2\sqrt{2}} - \left[\frac{1}{\sqrt{2}}\left(1 - \frac{\epsilon^2}{8}\right) - \frac{\epsilon}{2\sqrt{2}}\right]\right\}$, which reduces to $\tau = \left(\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 L}\right)\epsilon$.

(g) The torque tends to return the rod to the equilibrium position, so Newton's second law for rotation gives $I\frac{d^2\epsilon}{dt^2} = -\tau$. Using our result from part (f) gives $\frac{d^2\epsilon}{dt^2} = -\left(\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 LI}\right)\epsilon$. From this we see

that $\omega = \sqrt{\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 LI}}$, so $f = \frac{1}{2\pi}\sqrt{\frac{Q^2}{2\sqrt{2}\pi\epsilon_0 LI}}$. In this result, I is the moment of inertia of the two

small balls at the ends of the bar, which is $I = 2M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{2}$. Using this value for I gives

$$f = \frac{Q}{2\pi\sqrt{\pi\sqrt{2}\epsilon_0 ML^3}}.$$

EVALUATE: According to our result in (f), a large Q gives a high frequency while a large M or L gives a low frequency. These results are physically reasonable.

21.93. **IDENTIFY:** Two parallel rods of length L carry equal positive charge Q and are a distance a apart. We want to find the electric force that each one exerts on the other.

SET UP: We break the rods up into infinitesimal segments and integrate to find the total force that one of them exerts on the other. The force that dq_1 exerts on dq_2 is $dF = \frac{1}{4\pi\epsilon_0}\frac{dq_1 dq_2}{r_{12}^2}$. Fig. 21.93 shows the charge elements.

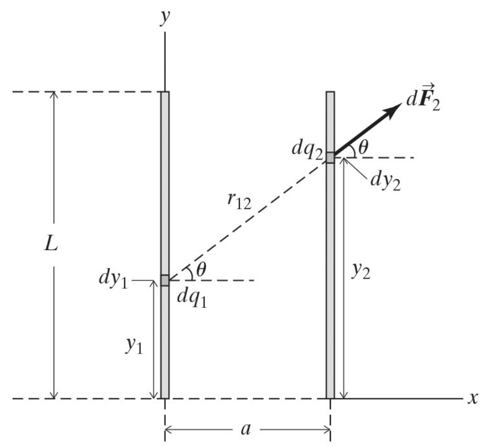


Figure 21.93

EXECUTE: (a) Because both rods have uniform charge distribution, every charge element above the midpoint of rod 1 will have an identical element below the midpoint. So the y components of the force on rod 2 cancel but the x components add.

(b) In light of the answer to part (a), we need only calculate the x components of for force on rod 2. We first find the force that dq_1 exerts on dq_2. Then we integrate over rod 1 to find the *total* force on dq_2 due rod 1. Using the geometry of the figure, we have $dF_{2x} = dF_2\cos\theta = \frac{1}{4\pi\epsilon_0}\frac{dq_1 dq_2}{r_{12}^2}\cos\theta$. Using

$dq_1 = \lambda dy_1$, $r_{12}^2 = (y_2 - y_1)^2 + a^2$, and $\cos\theta = \frac{a}{r_{12}} = \frac{a}{\sqrt{(y_2-y_1)^2 + a^2}}$, we have

$dF_{2x} = \left(\frac{1}{4\pi\epsilon_0}\int_{y_1=0}^{L}\frac{\lambda dy_1\cos\theta}{(y_2-y_1)^2+a^2}\right)dq_2 = \left(\frac{\lambda}{4\pi\epsilon_0}\int_{y_1=0}^{L}\frac{a\,dy_1}{\left[(y_2-y_1)^2+a^2\right]^{3/2}}\right)dq_2$. The integral can be done by letting $u = y_2 - y_1$, $du = -dy_1$. It is then in the form $\int\frac{-du}{(u^2+a^2)^{3/2}} = -\frac{1}{a^2}\frac{u}{\sqrt{u^2+a^2}}$. Doing this gives us $dF_{2x} = \left(\frac{\lambda a}{4\pi\epsilon_0}\right)\left(-\frac{1}{a^2}\right)\frac{y_2-y_1}{\sqrt{(y_2-y_1)^2+a^2}}\Bigg|_{y_1=0}^{L} dq_2$. Evaluating at the limits and using $dq_2 = \lambda dy_2$ gives $dF_{2x} = \left(-\frac{\lambda}{4\pi\epsilon_0 a}\right)\left[\frac{y_2-L}{\sqrt{(y_2-L)^2+a^2}} - \frac{y_2}{\sqrt{y_2^2+a^2}}\right]\lambda dy_2$. This result gives us the total x component of the force that rod 1 exerts on dq_2 in rod 2.

(c) We now integrate the result from part (b) over dy_2 to get the total x component of the force F_x on the full length of rod 2. $F_{2x} = \left(-\frac{\lambda^2}{4\pi\epsilon_0 a}\right)\int_0^L\left[\frac{y_2-L}{\sqrt{(y_2-L)^2+a^2}} - \frac{y_2}{\sqrt{y_2^2+a^2}}\right]dy_2$. We do the integral in two parts. To do the first one, let $u = y_2 - L$, $du = dy_2$, so it becomes $\int \frac{u\,du}{\sqrt{u^2+a^2}} = \sqrt{u^2+a^2}$. Evaluating at the limits, this part of the integral is $a - \sqrt{L^2+a^2}$. The second part of the integral is similar to the first part, and it is equal to $\sqrt{L^2+a^2} - a$. Combining these two results gives $F_{2x} = -\frac{\lambda^2}{4\pi\epsilon_0 a}\left[a - \sqrt{L^2+a^2} - \left(\sqrt{L^2+a^2} - a\right)\right] = \frac{\lambda^2}{2\pi\epsilon_0 a}\left(\sqrt{L^2+a^2} - a\right)$. Using $\lambda = Q/L$ this becomes $F_{2x} = \frac{Q^2}{2\pi\epsilon_0 L^2 a}\left(\sqrt{L^2+a^2} - a\right) = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)$. The force vector is $\vec{F} = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)\hat{\boldsymbol{i}}$.

(d) For $a >> L$: Use the result from part (c). $F_{2x} = \frac{Q^2}{2\pi\epsilon_0 L^2}\left(\sqrt{(L/a)^2+1} - 1\right)$. Since $a >> L$, $L/a << 1$, so we can use the approximation $\sqrt{1+x} \approx 1 + x/2$ where $x = (L/a)^2$. Doing so gives $F_{2x} \approx \frac{Q^2}{2\pi\epsilon_0 L^2}\left(1 + \frac{1}{2}(L/a)^2 - 1\right) = \frac{Q^2}{4\pi\epsilon_0 a^2}$, so $\vec{F} \approx \frac{Q^2}{4\pi\epsilon_0 a^2}\hat{\boldsymbol{i}}$. This result is reasonable because the two finite lines behave as point charges if they are far enough away from each other.

(e) $W = \lim\limits_{X\to\infty} \int_X^a F_x dx$. The work we do is the negative of the work done by the electric field. Using F_{2x} from part (c) with x replacing a in the formula, we have $W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\int_X^a \frac{\sqrt{L^2+x^2} - x}{x}dx.$

$W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\int_X^a \left(\frac{\sqrt{L^2+x^2}}{x} - 1\right)dx.$ Using tables for the first part of the integral gives

$$W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\left[\sqrt{x^2+L^2} - L\ln\left(\frac{L+\sqrt{x^2+L^2}}{x}\right) - x\right]\Bigg|_X^a \text{, which gives}$$

$$W = -\frac{Q^2}{2\pi\epsilon_0 L^2}\left\{\left[\sqrt{a^2+L^2} - L\ln\left(\frac{L+\sqrt{a^2+L^2}}{a}\right) - a\right] - \left[\sqrt{X^2+L^2} - L\ln\left(\frac{L+\sqrt{X^2+L^2}}{X}\right) - X\right]\right\}.$$

Taking the limit as $X \to \infty$ gives $W = \frac{Q^2}{2\pi\epsilon_0 L^2}\left[L\ln\left(\frac{L+\sqrt{a^2+L^2}}{a}\right) + a - \sqrt{a^2+L^2}\right].$

(f) We want the *relative* speed of the rods when they have moved very far apart, in which case their potential energy is zero. Since the rods have equal masses, they will have equal speeds by conservation of momentum. Therefore $U(x=a) = K(x\to\infty) = K_1 + K_2 = 2K_1 = 2\left(\frac{1}{2}mv^2\right) = mv^2$. We can use the result from part (e). $\frac{Q^2}{2\pi\epsilon_0 L^2}\left[L\ln\left(\frac{L+\sqrt{a^2+L^2}}{a}\right) + a - \sqrt{a^2+L^2}\right] = mv^2$. Solving for v using Q = 10.0 μC, L = 50.0 cm = 0.500 m, m = 500 g = 0.500 kg, a = 10.0 cm = 0.100 m, we get v = 10.735 m/s. This is the speed relative to the earth. The *relative speed* is the speed of one rod relative to the other. Since they are moving away from each other, their *relative* speed is 2(10.735 m/s) = 21.5 m/s.

EVALUATE: Evaluate our result in part (c) if $L \to 0$. For $x << 1$ we can use the approximation $\sqrt{1+x} \approx 1 + x/2$ where $x = (L/a)^2$. Doing so gives $F_{2x} \approx \dfrac{Q^2}{2\pi \epsilon_0 L^2}\left(1+\dfrac{1}{2}(L/a)^2 - 1\right) = \dfrac{Q^2}{4\pi \epsilon_0 a^2}$, so $\vec{F} \approx \dfrac{Q^2}{4\pi \epsilon_0 a^2}\hat{\boldsymbol{i}}$. This result is reasonable because the two rods behave like point charges if they are extremely short compared to the distance between them, so our result is reasonable. We can also check our result if the rods are extremely long ("infinite"). Using results from the text, we know that in that case $E_1 = \dfrac{\lambda}{2\pi \epsilon_0 r}$, so the force on rod 2 is $F_2 = E_1 Q_2 = E_1 \lambda L = \left(\dfrac{\lambda}{2\pi \epsilon_0 a}\right)\lambda L = \dfrac{\lambda^2 L}{2\pi \epsilon_0 a}$. Therefore the force per unit length on rod 2 is $\dfrac{F}{L} = \dfrac{\lambda^2}{2\pi \epsilon_0 a}$. Now look at our result $F = \dfrac{Q^2}{2\pi \epsilon_0 L^2}\left(\sqrt{(L/a)^2 + 1} - 1\right)$. $Q = \lambda L$ and for very large L the quantity in parentheses becomes L/a. Therefore the force becomes $F = \dfrac{(\lambda L)^2}{2\pi \epsilon_0 L^2}(L/a) = \dfrac{\lambda^2 L}{2\pi \epsilon_0 a}$, so the force per unit length is $\dfrac{F}{L} = \dfrac{\lambda^2}{2\pi \epsilon_0 a}$, which is what we just found. Therefore our result checks for very long rods.

21.94. IDENTIFY and SET UP: The electric field exerts a force (and hence a torque) on the charged ball at the end of the rod. This torque is always counterclockwise, so it causes the rod to increase its angular speed as the charged ball passes through the field.

EXECUTE: **(a)** From Fig. 21.94a, $r_x = a\cos\theta$ and $r_y = a\sin\theta$, so $\vec{r} = a\cos\theta\hat{\boldsymbol{i}} + a\sin\theta\hat{\boldsymbol{j}}$.

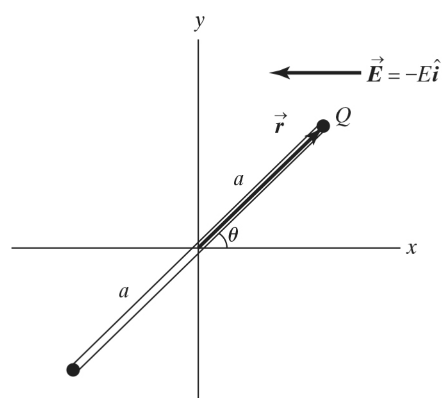


Figure 21.94a

(b) We want the torque on the rod when $0 \le \theta \le \pi$ (see Fig. 21.94b). The magnitude of the torque is $\tau = rF\sin\phi$. From the figure we see that $\phi = \pi - \theta$, so $\sin\phi = \sin(\pi - \theta) = \sin\theta$. $F = QE$ and $r = a$, so $\tau = aQE\sin\theta$. Using $\vec{\tau} = \vec{r} \times \vec{F}$ and the right-hand rule for the vector product, we see that the torque points along the $+z$-axis, so $\vec{\tau} = aQE\sin\theta\hat{\boldsymbol{k}}$.

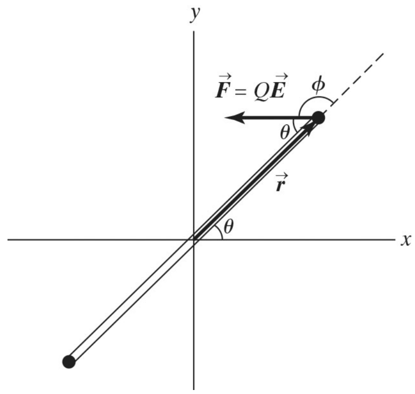


Figure 21.94b

(c) The torque is zero because the electric field is zero.
(d) $I_{\text{balls}} = 2Ma^2$, $I_{\text{rod}} = 0$ since it is very light.
(e) We want to use $\tau = -dU/d\theta$ to find the potential energy $U(\theta)$. Integrating and letting $U_0 = 0$ when $\theta = 0$ gives $U - U_0 = -\int \tau d\theta$.

$\underline{0 \le \theta \le \pi}$: $U - U_0 = U = -\int_0^\theta aQE\sin\theta d\theta = aQE\cos\theta\big|_0^\theta = aQE(\cos\theta - 1)$.

$\underline{\pi \le \theta \le 2\pi}$: $U = \text{constant} = U(\pi) = -2aQE$.

$\underline{2\pi \le \theta \le 3\pi}$: $U - U_0 = -\int_{2\pi}^{\theta} aQE\sin\theta d\theta = aQE\cos\theta\big|_{2\pi}^{\theta} = aQE(\cos\theta - 1)$, which gives $U = aQE(\cos\theta - 1) + (-2aQE) = aQE(\cos\theta - 3)$.

$\underline{3\pi \le \theta \le 4\pi}$: $U = \text{constant} = U(3\pi) = -4aQE$.

(f) Calling $\in$ the total energy, we have $\in = \frac{1}{2}I\omega^2 + U(\theta)$. We also know that $\tau = I\alpha = I\frac{d^2\theta}{dt^2} = I\frac{d\omega}{dt}$.

Combining this result with $\tau = -dU/d\theta$ gives $\frac{d\omega}{dt} = -\frac{1}{I}\frac{dU}{d\theta}$. Now look at the energy:

$\frac{d\in}{dt} = I\omega\frac{d\omega}{dt} + \frac{dU}{dt}$. The chain rule gives $\frac{dU}{dt} = \frac{dU}{d\theta}\frac{d\theta}{dt} = \omega\frac{dU}{d\theta}$. Using this and our result for $d\omega/dt$ gives $\frac{d\in}{dt} = I\omega\left(-\frac{1}{I}\frac{dU}{d\theta}\right) + \omega\frac{dU}{d\theta} = -\omega\frac{dU}{d\theta} + \omega\frac{dU}{d\theta} = 0$. Thus the energy is constant and therefore conserved.

(g) We want the angular speed ω at the n^{th} time that Q crosses the $-y$-axis. Doing calculations similar to those in part (e) gives us the following results:
At the first crossing: $\pi \le \theta \le 2\pi$: $U_1 = -2aQE$
At the second crossing: $3\pi \le \theta \le 4\pi$: $U_2 = -4aQE$
At the third crossing: $5\pi \le \theta \le 6\pi$: $U_3 = -6aQE$
At the fourth crossing: $7\pi \le \theta \le 8\pi$: $U_4 = -8aQE$

We can see that the pattern is $U_n = -2naQE$ ($n = 1, 2, 3, \ldots$). The total energy is $\in = \frac{1}{2}I\omega^2 + U(\theta)$.

Solving for ω gives $\omega = \sqrt{\frac{2(\in - U)}{I}}$. At the n^{th} crossing, we found that $U_n = -2naQE$, so

$\omega_n = \sqrt{\dfrac{2(\in - U_n)}{I}} = \sqrt{\dfrac{2(\in + 2naQE)}{2Ma^2}} = \sqrt{\dfrac{\in + 2naQE}{Ma^2}}$. But $\in = K + U$ and $K_0 = 0$ and $U_0 = 0$, so $\in = 0$.

Therefore $\omega_n = \sqrt{\dfrac{2nQE}{Ma}}$ $(n = 1, 2, 3, \cdots)$.

EVALUATE: The total energy is zero, but that does not mean that the rod stops moving. Remember that the potential energy is negative which cancels out the positive kinetic energy.

21.95. IDENTIFY: Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3, and add these force vectors to get the net force.

SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible force diagrams are sketched in Figure 21.95a.
Only the last picture can result in a net force in the $-x$-direction.

(b) $q_1 = -2.00\ \mu\text{C}, q_3 = +4.00\ \mu\text{C}$, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.95b.

$F_y = 0 = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1||q_3|}{(0.0400\text{ m})^2}\sin\theta_1 + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2||q_3|}{(0.0300\text{ m})^2}\sin\theta_2$. This gives

$q_2 = \dfrac{9}{16}|q_1|\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{9}{16}|q_1|\dfrac{3/5}{4/5} = \dfrac{27}{64}|q_1| = 0.843\ \mu\text{C}$.

(d) $F_x = F_{1x} + F_{2x}$ and $F_y = 0$, so $F = |q_3|\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{(0.0400\text{ m})^2}\dfrac{4}{5} + \dfrac{|q_2|}{(0.0300\text{ m})^2}\dfrac{3}{5}\right) = 56.2\text{ N}$.

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2.

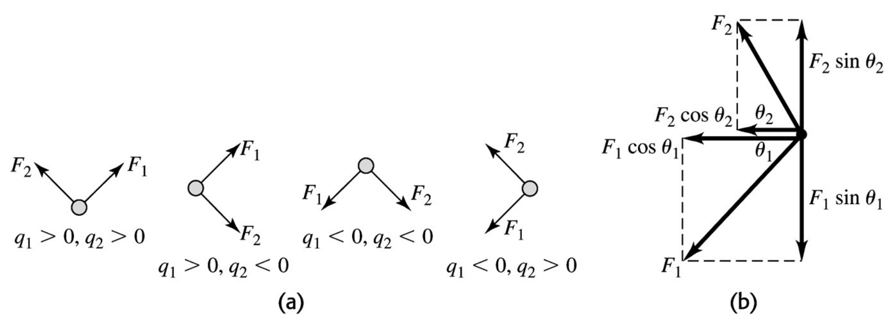


Figure 21.95

21.96. IDENTIFY: Calculate the electric field at P due to each charge and add these field vectors to get the net field.

SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible diagrams are sketched in Figure 21.96(a).
The first diagram is the only one in which the electric field must point in the negative y-direction.

(b) $q_1 = -3.00\ \mu\text{C}$, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 21.96(b). $\cos\theta_1 = \dfrac{5}{13}$,

$\sin\theta_1 = \dfrac{12}{13}$, $\cos\theta_2 = \dfrac{12}{13}$ and $\sin\theta_2 = \dfrac{5}{13}$. $E_x = 0 = -\dfrac{k|q_1|}{(0.050\text{ m})^2}\dfrac{5}{13} + \dfrac{k|q_2|}{(0.120\text{ m})^2}\dfrac{12}{13}$. This gives

$$\frac{k|q_2|}{(0.120\text{ m})^2}=\frac{k|q_1|}{(0.050\text{ m})^2}\frac{5}{12}.$$ Solving for $|q_2|$ gives $|q_2| = 7.2\ \mu\text{C}$, so $q_2 = -7.2\ \mu\text{C}$. Then

$$E_y = -\frac{k|q_1|}{(0.050\text{ m})^2}\frac{12}{13}-\frac{kq_2}{(0.120\text{ m})^2}\frac{5}{13} = -1.17\times10^7\text{ N/C}.\quad E = 1.17\times10^7\text{ N/C}.$$

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E.

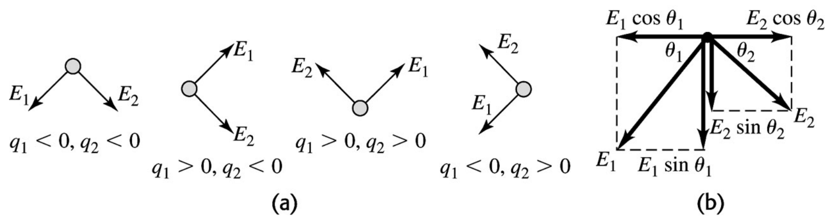


Figure 21.96

21.97. IDENTIFY: To find the electric field due to the second rod, divide that rod into infinitesimal segments of length dx, calculate the field dE due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F} = dq\,\vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.97. $dQ = (Q/L)dx'$.

EXECUTE: (a) $$dE = \frac{k\,dQ}{(x+a/2+L-x')^2}=\frac{kQ}{L}\frac{dx'}{(x+a/2+L-x')^2}.$$

$$E_x = \int_0^L dE_x = \frac{kQ}{L}\int_0^L\frac{dx'}{(x+a/2+L-x')^2} = \frac{kQ}{L}\left[\frac{1}{x+a/2+L-x'}\right]_0^L = \frac{kQ}{L}\left(\frac{1}{x+a/2}-\frac{1}{x+a/2+L}\right).$$

$$E_x = \frac{2kQ}{L}\left(\frac{1}{2x+a}-\frac{1}{2L+2x+a}\right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment dq of the first rod. This force is in the $+x$-direction. $dF = dq\,E$.

$$F = \int E\,dq = \int_{a/2}^{L+a/2}\frac{EQ}{L}dx = \frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\left(\frac{1}{2x+a}-\frac{1}{2L+2x+a}\right)dx.$$

$$F = \frac{2kQ^2}{L^2}\frac{1}{2}\left(\left[\ln(a+2x)\right]_{a/2}^{L+a/2}-\left[\ln(2L+2x+a)\right]_{a/2}^{L+a/2}\right) = \frac{kQ^2}{L^2}\ln\left[\left(\frac{a+2L+a}{2a}\right)\left(\frac{2L+2a}{4L+2a}\right)\right].$$

$$F = \frac{kQ^2}{L^2}\ln\left(\frac{(a+L)^2}{a(a+2L)}\right).$$

(c) For $a \gg L$, $$F = \frac{kQ^2}{L^2}\ln\left(\frac{a^2(1+L/a)^2}{a^2(1+2L/a)}\right) = \frac{kQ^2}{L^2}(2\ln(1+L/a)-\ln(1+2L/a)).$$

For small z, $\ln(1+z)\approx z-\frac{z^2}{2}$. Therefore, for $a \gg L$,

$$F \approx \frac{kQ^2}{L^2}\left[2\left(\frac{L}{a}-\frac{L^2}{2a^2}+\cdots\right)-\left(\frac{2L}{a}-\frac{2L^2}{a^2}+\cdots\right)\right]\approx\frac{kQ^2}{a^2}.$$

EVALUATE: The distance between adjacent ends of the rods is a. When $a \gg L$ the distance between the rods is much greater than their lengths and they interact as point charges.

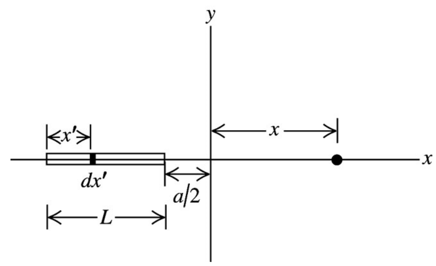


Figure 21.97

21.98. **IDENTIFY** and **SET UP:** The charge of n electrons is ne.
EXECUTE: The charge on the bee is $Q = ne$, so the number of missing electrons is $n = Q/e = (30\text{ pC})/e = (30\times10^{-12}\text{ C})/(1.60\times10^{-19}\text{ C}) = 1.88\times10^{8} \approx 1.9\times10^{8}$ electrons, which makes choice (a) correct.
EVALUATE: This charge is due to around 190 million electrons.

21.99. **IDENTIFY** and **SET UP:** One charge exerts a force on another charge without being in contact.
EXECUTE: Even though the bee does not touch the stem, the positive charges on the bee attract negative charges (electrons normally) in the stem. This pulls electrons toward the bee, leaving positive charge at the opposite end of the stem, which polarizes it. Thus choice (c) is correct.
EVALUATE: Choice (b) cannot be correct because the bee is positive and would therefore not attract the positive charges in the stem.

21.100. **IDENTIFY** and **SET UP:** Electric field lines begin on positive charges and end on negative charges.
EXECUTE: The flower and bee are both positive, so no field lines can end on either of them. This makes the figure in choice (c) the correct one.
EVALUATE: The net electric field is the vector sum of the field due to the bee and the field due to the flower. Somewhere between the bee and flower the fields cancel, depending on the relative amounts of charge on the bee and flower.

21.101. **IDENTIFY** and **SET UP:** Assume that the charge remains at the end of the stem and that the bees approach to 15 cm from this end of the stem. The electric field is $E = k\dfrac{|q|}{r^2}$.
EXECUTE: Using the numbers given, we have
$E = k\dfrac{|q|}{r^2} = (8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(40\times10^{-12}\text{ C})/(0.15\text{ m})^2 = 16\text{ N/C}$, which is choice (b).
EVALUATE: Even if the charge spread out a bit over the stem, the result would be in the neighborhood of the value we calculated.