

```
1 !date
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2
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Fri Apr 15 09:48:21 UTC 2022
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Please run the above line to refresh the date before your submission.

▼ CSCI-SHU 210 Data Structures

Recitation 10 Trees/Binary trees

You should work on the 5 tasks as written in the worksheet.

- For students who have recitation on Wednesday, you should submit your solutions by **Apr 15th** Friday 11:59pm.
- For students who have recitation on Thursday, you should submit your solutions by **Apr 16th** Saturday 11:59pm.
- For students who have recitation on Friday, you should submit your solutions by **Apr 17th** Sunday 11:59pm.

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Please submit the following items to the Gradescope:

- Your Colab notebook link (by clicking the Share button at the top-right corner of the Colab notebook, share to anyone)
- The printout of your run in Colab notebook in pdf format
- No late submission is permitted. All solutions must be from your own work. Total points of the assignment is 100.

▼ **LinkedQueue class (provided as a separate ADT. Use it only if you need to use it for your solution).**

```
1 class LinkedQueue:
2     """FIFO queue implementation using a singly linked
3
4     #----- nested _Node class ----
5     class _Node:
```

```

6         """Lightweight, nonpublic class for storing a s
7         __slots__ = '_element', '_next'           # strea
8
9         def __init__(self, element, next):
10             self._element = element
11             self._next = next
12
13     #----- queue methods -----
14     def __init__(self):
15         """Create an empty queue."""
16         self._head = None
17         self._tail = None
18         self._size = 0           # numbe
19
20     def __len__(self):
21         """Return the number of elements in the queue."""
22         return self._size
23
24     def is_empty(self):
25         """Return True if the queue is empty."""
26         return self._size == 0
27
28     def first(self):
29         """Return (but do not remove) the element at th
30
31         Raise Empty exception if the queue is empty.
32         """
33         if self.is_empty():
34             raise Exception('Queue is empty')
35         return self._head._element           # front
36
37     def dequeue(self):
38         """Remove and return the first element of the q
39
40         Raise Empty exception if the queue is empty.
41         """
42         if self.is_empty():
43             raise Exception('Queue is empty')
44         answer = self._head._element
45         self._head = self._head._next
46         self._size -= 1

```

```

47         if self.is_empty():                                # speci
48             self._tail = None                                # rem
49         return answer
50
51     def enqueue(self, e):
52         """Add an element to the back of queue."""
53         newest = self._Node(e, None)                            # node
54         if self.is_empty():
55             self._head = newest                                # spe
56         else:
57             self._tail._next = newest
58             self._tail = newest                                # updat
59             self._size += 1
60
61     def __str__(self):
62         result = []
63         curNode = self._head
64         while (curNode is not None):
65             result.append(str(curNode._element) + " -->")
66             curNode = curNode._next
67         result.append("None")
68         return "".join(result)

```

▼ The Binary Tree with OOP, as we defined in lecture

```

1 class Tree:
2     class TreeNode:
3         def __init__(self, element, parent = None, left
4             self._parent = parent
5             self._element = element
6             self._left = left
7             self._right = right
8
9         def element(self):
10             return self._element
11
12     #----- binary tree constructor
13     def __init__(self):
14         """Create an initially empty binary tree."""
15         self._root = None

```

```

16         self._size = 0
17
18     #----- public accessors -----
19     def __len__(self):
20         """Return the total number of elements in the t
21         return self._size
22
23     def is_root(self, node):
24         """Return True if a given node represents the r
25         return self._root == node
26
27     def is_leaf(self, node):
28         """Return True if a given node does not have an
29         return self.num_children(node) == 0
30
31     def is_empty(self):
32         """Return True if the tree is empty."""
33         return len(self) == 0
34
35     def __iter__(self):
36         """Generate an iteration of the tree's elements
37         for node in self.nodes():
38             yield node._element
39
40     def depth(self, node):
41         """Return the number of levels separating a giv
42         if self.is_root(node):
43             return 0
44         else:
45             return 1 + self.depth(self.parent(node))
46
47     def height(self, node = None):    # time is linear i
48         if node is None:
49             node = self._root
50         if self.is_leaf(node):
51             return 0
52         else:
53             if node._left:
54                 l = self.height(node._left)
55             else:
56                 l = 0

```

```

57         if node._right:
58             r = self.height(node._right)
59         else:
60             r = 0
61         return 1 + max(l, r)
62
63     def nodes(self):
64         """Generate an iteration of the tree's nodes."""
65         return self.preorder()
66
67     def preorder(self):
68         """Generate a preorder iteration of nodes in th
69         if not self.is_empty():
70             for node in self._subtree_preorder(self._ro
71                 yield node
72
73     def _subtree_preorder(self, node):
74         """Generate a preorder iteration of nodes in su
75         yield node
76         for c in self.children(node):
77             for other in self._subtree_preorder(c):
78                 yield other
79
80     def postorder(self):
81         """Generate a postorder iteration of nodes in t
82         if not self.is_empty():
83             for node in self._subtree_postorder(self._r
84                 yield node
85
86     def _subtree_postorder(self, node):
87         """Generate a postorder iteration of nodes in s
88         for c in self.children(node):
89             for other in self._subtree_postorder(c):
90                 yield other
91         yield node
92     def inorder(self):
93         """Generate an inorder iteration of nodes in th
94         if not self.is_empty():
95             for node in self._subtree_inorder(self._root)
96                 yield node
97

```

```

98     def _subtree_inorder(self, node):
99         """Generate an inorder iteration of nodes in s
100         if node._left is not None:          # if left c
101             for other in self._subtree_inorder(node._left
102                 yield other
103         yield node                          # visi
104         if node._right is not None:         # if right
105             for other in self._subtree_inorder(node._right
106                 yield other
107
108
109     def breadthfirst(self):
110         """Generate a breadth-first iteration of the nodes
111         if not self.is_empty():
112             fringe = LinkedQueue()          # known
113             fringe.enqueue(self._root)      # starting
114             while not fringe.is_empty():
115                 node = fringe.dequeue()     # r
116                 yield node                 # r
117                 for c in self.children(node):
118                     fringe.enqueue(c)      # add
119
120
121     def root(self):
122         """Return the root of the tree (or None if tree is empty)
123         return self._root
124
125     def parent(self, node):
126         """Return node's parent (or None if node is the root)
127         return node._parent
128
129     def left(self, node):
130         """Return node's left child (or None if no left child)
131         return node._left
132
133     def right(self, node):
134         """Return node's right child (or None if no right child)
135         return node._right
136
137     def children(self, node):
138         """Generate an iteration of nodes representing the children of node

```

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139         if node._left is not None:
140             yield node._left
141         if node._right is not None:
142             yield node._right
143
144     def num_children(self, node):
145         """Return the number of children of a given node
146         count = 0
147         if node._left is not None:      # left child exists
148             count += 1
149         if node._right is not None:     # right child exists
150             count += 1
151         return count
152
153     def sibling(self, node):
154         """Return a node representing given node's sibling
155         parent = node._parent
156         if parent is None:              # p must be root
157             return None
158         else:
159             if node == parent._left:
160                 return parent._right    # possibly
161             else:
162                 return parent._left     # possibly
163
164     #----- nonpublic mutators -----
165     def add_root(self, e):
166         """Place element e at the root of an empty tree
167
168         Raise ValueError if tree nonempty.
169         """
170         if self._root is not None:
171             raise ValueError('Root exists')
172         self._size = 1
173         self._root = self.TreeNode(e)
174         return self._root
175
176     def add_left(self, node, e):
177         """Create a new left child for a given node, store it
178
179         Return the new node.

```

```

180         Raise ValueError if node already has a left chi
181         """
182         if node._left is not None:
183             raise ValueError('Left child exists')
184         self._size += 1
185         node._left = self.TreeNode(e, node)
186         return node._left
187
188     def add_right(self, node, e):
189         """Create a new right child for a given node, s
190
191         Return the new node.
192         Raise ValueError if node already has a right ch
193         """
194         if node._right is not None:
195             raise ValueError('Right child exists')
196         self._size += 1
197         node._right = self.TreeNode(e, node)
198         return node._right
199
200     def _replace(self, node, e):
201         """Replace the element at given node with e, an
202         old = node._element
203         node._element = e
204         return old
205
206     def _delete(self, node):
207         """Delete the given node, and replace it with i
208
209         Return the element that had been stored at the
210         Raise ValueError if node has two children.
211         """
212         if self.num_children(node) == 2:
213             raise ValueError('Node has two children')
214         child = node._left if node._left else node._rig
215         if child is not None:
216             child._parent = node._parent          # child's
217         if node is self._root:
218             self._root = child                    # child beco
219         else:
220             parent = node._parent

```



```

221         if node is parent._left:
222             parent._left = child
223         else:
224             parent._right = child
225     self._size -= 1
226     return node._element
227
228 def _attach(self, node, t1, t2):
229     """Attach trees t1 and t2, respectively, as the
230
231     As a side effect, set t1 and t2 to empty.
232     Raise TypeError if trees t1 and t2 do not match
233     Raise ValueError if node already has a child. (
234     """
235     if not self.is_leaf(node):
236         raise ValueError('Node must be leaf')
237     if not type(self) is type(t1) is type(t2):      #
238         raise TypeError('Tree types must match')
239     self._size += len(t1) + len(t2)
240     if not t1.is_empty():          # attached t1 as
241         t1._root._parent = node
242         node._left = t1._root
243         t1._root = None            # set t1 instan
244         t1._size = 0
245     if not t2.is_empty():          # attached t2 as
246         t2._root._parent = node
247         node._right = t2._root
248         t2._root = None            # set t2 instan
249         t2._size = 0

```

```

1 def pretty_print(tree):
2     # ----- Need to enter height to w
3     levels = tree.height() + 1
4     print("Levels:", levels)
5     print_internal([tree._root], 1, levels)
6
7 def print_internal(this_level_nodes, current_level, max
8     if (len(this_level_nodes) == 0 or all_elements_are_
9         return # Base case of recursion: out of nodes,
10
11     floor = max_level - current_level;

```

```

12     endgeLines = 2 ** max(floor - 1, 0);
13     firstSpaces = 2 ** floor - 1;
14     betweenSpaces = 2 ** (floor + 1) - 1;
15     print_spaces(firstSpaces)
16     next_level_nodes = []
17     for node in this_level_nodes:
18         if (node is not None):
19             print(node._element, end = "")
20             next_level_nodes.append(node._left)
21             next_level_nodes.append(node._right)
22         else:
23             next_level_nodes.append(None)
24             next_level_nodes.append(None)
25             print_spaces(1)
26
27     print_spaces(betweenSpaces)
28     print()
29     for i in range(1, endgeLines + 1):
30         for j in range(0, len(this_level_nodes)):
31             print_spaces(firstSpaces - i)
32             if (this_level_nodes[j] == None):
33                 print_spaces(endgeLines + endgeLine
34                             continue
35             if (this_level_nodes[j]._left != None):
36                 print("/", end = "")
37             else:
38                 print_spaces(1)
39             print_spaces(i + i - 1)
40             if (this_level_nodes[j]._right != None):
41                 print("\\", end = "")
42             else:
43                 print_spaces(1)
44             print_spaces(endgeLines + endgeLines - i)
45     print()
46
47     print_internal(next_level_nodes, current_level + 1,
48
49 def all_elements_are_None(list_of_nodes):
50     for each in list_of_nodes:
51         if each is not None:
52             return False

```

```

53     return True
54
55 def print_spaces(number):
56     for i in range(number):
57         print(" ", end = "")

```

Task 1: create and perform some operations on a Binary Search Tree

```

1 class BinarySearchTree(Tree):
2
3     #----- nonpublic utilitie
4     def _subtree_search(self, node, v):
5         """Return the node having value v, or last node
6         if v == node._element:
7             return node
8         elif v < node._element:
9             if node._left is not None:
10                 return self._subtree_search(node._left,
11             else:
12                 if node._right is not None:
13                     return self._subtree_search(node._right
14         return node
15
16     def _subtree_first_node(self, node):
17         """Return the node that contains the first item
18         walk = node
19         while walk._left is not None: #
20             walk = walk._left
21         return walk
22
23     def _subtree_last_node(self, node):
24         """Return the node that contains the last item
25         walk = node
26         while walk._right is not None: #
27             walk = walk._right
28         return walk
29
30     #----- public methods providing Bin

```

```

31     def first(self):
32         """Return the first node (smallest node) in the
33         return self._subtree_first_node(self.root()) if
34
35     def last(self):
36         """Return the last node (largest node) in the t
37         return self._subtree_last_node(self.root()) if
38
39     def before(self, node):
40         """Return the node that is just before the give
41
42         Return None if the given node is the first node
43         """
44         if node._left is not None:
45             return self._subtree_last_node(node._left)
46         else:
47             # walk upward
48             walk = node
49             above = walk._parent
50             while above is not None and walk == above._
51                 walk = above
52                 above = walk._parent
53             return above
54
55     def after(self, node):
56         """Return the node that is just after the given
57
58         Return None if the given node is the last node.
59         """
60         if node._right is not None:
61             return self._subtree_first_node(node._right)
62         else:
63             walk = node
64             above = walk._parent
65             while above is not None and walk == above._
66                 walk = above
67                 above = walk._parent
68             return above
69
70     def delete(self, node):
71         """Remove the given node."""

```

```

72         if node._left and node._right:                # node
73             replacement = self._subtree_last_node(node)
74             self._replace(node, replacement._element)
75             node = replacement
76         # now node has at most one child
77         self._delete(node)
78         #self._rebalance_delete(parent)
79
80     #----- public methods for accessing
81     def get_node(self, v):
82         """Return the node associated with value (raise
83         if self.is_empty():
84             raise Exception('Tree is empty')
85         else:
86             node = self._subtree_search(self._root, v)
87             if v != node._element:
88                 raise ValueError('Not found: ' + repr(v)
89             return node
90
91     def insert(self, v):
92         """Insert value v into the Binary Search Tree"""
93         if self.is_empty():
94             leaf = self.add_root(v)          # from BinaryTr
95         else:
96             node = self._subtree_search(self._root, v)
97             if node._element < v:
98                 leaf = self.add_right(node, v)          #
99             else:
100                 leaf = self.add_left(node, v)           #
101             self._rebalance_insert(leaf)                #
102
103     def delete_value(self, v):
104         """Remove the node within the Tree that contain
105         if not self.is_empty():
106             node = self._subtree_search(self._root, v)
107             if v == node._element:
108                 self.delete(node)
109             return
110         raise ValueError('Not found: ' + repr(v))
111
112     def _rebalance_insert(self, p):          # Do nothing in

```

```

113         pass
114
115     def _rebalance_delete(self, p):        # Do nothing in
116         pass
117
118     def __iter__(self):
119         """Generate an iteration of all values in order
120         node = self.first()
121         while node is not None:
122             yield node._element
123             node = self.after(node)
124
125     def __reversed__(self):
126         """Generate an iteration of all values in rever
127         node = self.last()
128         while node is not None:
129             yield node._element
130             node = self.before(node)
131
132     ## Task 2 ##
133     def minimum(self):
134         return self.first()._element
135
136     ## Task 3 ##
137     def second_minimum(self):
138         return self.after(self.first())._element
139
140     ## Task 4 ##
141     def is_valid(self):
142         if self.is_empty():
143             return True
144         walk=self.first()
145         # future=self.after(walk
146         while walk:
147             temp=self.after(walk)
148             if temp and temp._element<=walk._element:
149                 return False
150             walk=temp
151         return True
152
153

```

```

154     ## Task 5 ##
155     def iter_range(self, start, end):
156         begin=self._subtree_search(self._root, start)
157         if begin._element<start:
158             begin=self.after(begin)
159         last=self._subtree_search(self._root, end)
160         if last._element > end:
161             last=self.before(last)
162         walk=begin
163         while walk!=last:
164             yield walk._element
165             walk=self.after(walk)
166         yield(last._element)
167
168
169
170

1 print("-----Task 1 Build BST-----")
2 # Constuct a BST
3 # 1. Insert 0, 1, 2, 3, 4 into the tree.
4 # 2. Get the Node of 2 by calling get_node(self, value)
5 # 3. Use before(self, node) function to get node of 1.
6 # 4. Use after(self, node) function to get node of 3.
7 # 5. Delete 0, 1, 2, 3, 4 from the tree.
8
9 ## Task 1 ##
10 t=BinarySearchTree()
11 t.insert(0)
12 t.insert(1)
13 t.insert(2)
14 t.insert(3)
15 t.insert(4)
16 # t.pretty_print()
17 pretty_print(t)
18 temp=t.get_node(2)
19 print(temp._element)
20 # pretty_print(t)
21
22 before_temp=t.before(temp)
23 # print(before_temp)
24 # pretty_print(t)

```

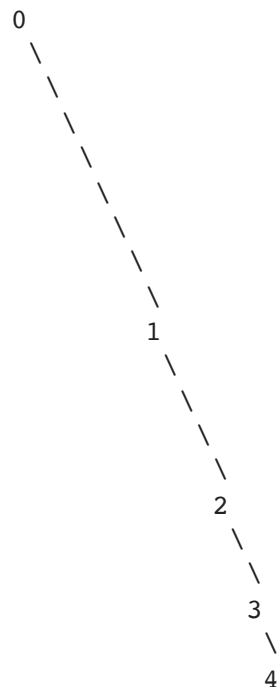
```

24 # pretty_print(t)
25 print(before_temp._element)
26
27
28 after_temp=t.after(temp)
29 # print(after_temp)
30 # pretty_print(t)
31 print(after_temp._element)
32
33
34 for i in t.inorder():
35     t.delete(i)
36 # pretty_print(t)
37
38

```

☞ -----Task 1 Build BST-----

Levels: 5



2
1
3

▼ Task 2: Minimum(self)

Implement function `minimum(self)` above. When called, the minimum element within the tree is returned.

1 ##### Below part is for task 2 to task 5 #####


```

2 # Construct a BST t2
3 #           3
4 #         /  \
5 #       /    \
6 #     /      \
7 #    /        \
8 #   1          7
9 #  /  \      /  \
10 # /    \  /    \
11 # 0      2 5      8
12 #       /  \  \
13 #      4  6   9
14 t2 = BinarySearchTree()
15 t2.insert(3)
16 t2.insert(1)
17 t2.insert(7)
18 t2.insert(0)
19 t2.insert(2)
20 t2.insert(5)
21 t2.insert(8)
22 t2.insert(4)
23 t2.insert(6)
24 t2.insert(9)
25 print("-----Task 2 minimum-----")
26 print("Minimum of tree is: ", t2.minimum(), ", Expected
-----Task 2 minimum-----
Minimum of tree is: 0 , Expected: 0

```

▼ Task 3: second_minimum(self)

Implement function `second_minimum(self)`. When called, the second smallest element within the tree is returned.

```

1 print("-----Task 3 second_minimum-----")
2 print("Second smallest of tree is: ", t2.second_minimum
-----Task 3 second_minimum-----
Second smallest of tree is: 1 , Expected: 1

```

▼ Task 4: is_valid(self)

Implement function `is_valid(self)`. When called, returns True if the self tree is a valid binary search tree. Returns false otherwise.

```

1 print("-----Task 4 is_valid-----")
2 print("Is the tree a valid BST?: ", t2.is_valid(), ", s
3
4 ##### Mess up t2 #####
5 node_three = BinarySearchTree.TreeNode(3, t2.last(), No
6 t2.last()._left = node_three
7 #
8 #
9 #
10 #
11 #
12 #
13 #
14 #
15 #
16 #
17 #
18 #
19 #
20 #
21 #
22 #
23 #
24 #
25 #
26 #
27 print("Is the tree a valid BST?: ", t2.is_valid(), ", s

-----Task 4 is_valid-----
Is the tree a valid BST?:  True , should be True
Is the tree a valid BST?:  False , should be False

```

```

      3
     /\
    /\  \
   /\   \
  1  7   \
 /\  /\   \
0 2 5 8   \
   /\  \   \
  4 6  9   \
           3

```

<=== Problem!

▼ Task 5: `iter_range(self, start, stop)`

Implement function `iter_range(self, start, stop)`. When called, Yield a generator that contains elements in order, such that `start <= elements <= stop`.

```

1 print("-----Task 5 iter_range-----")

```

```
2 t2 = BinarySearchTree()
3 t2.insert(3)
4 t2.insert(1)
5 t2.insert(7)
6 t2.insert(0)
7 t2.insert(2)
8 t2.insert(5)
9 t2.insert(8)
10 t2.insert(4)
11 t2.insert(6)
12 t2.insert(9)
13 print([x for x in t2.iter_range(3, 7)], ", Expected: [3

-----Task 5 iter_range-----
[3, 4, 5, 6, 7] , Expected: [3, 4, 5, 6, 7]
```