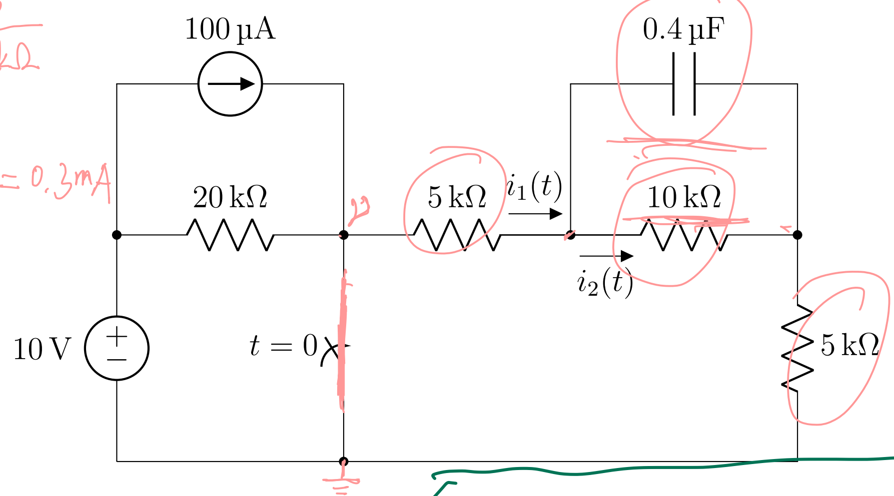
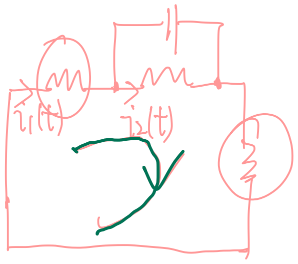
Exercises 07
-
First order circuits

Exercise 1 - RC circuit

①
$\frac{10V - v}{20k\Omega} + 100\mu A = \frac{v}{20k\Omega}$
$\Rightarrow v = 6V$
$i_1(0^-) = i_2(0^-) = \frac{v}{20k\Omega} = 0.3mA$



②

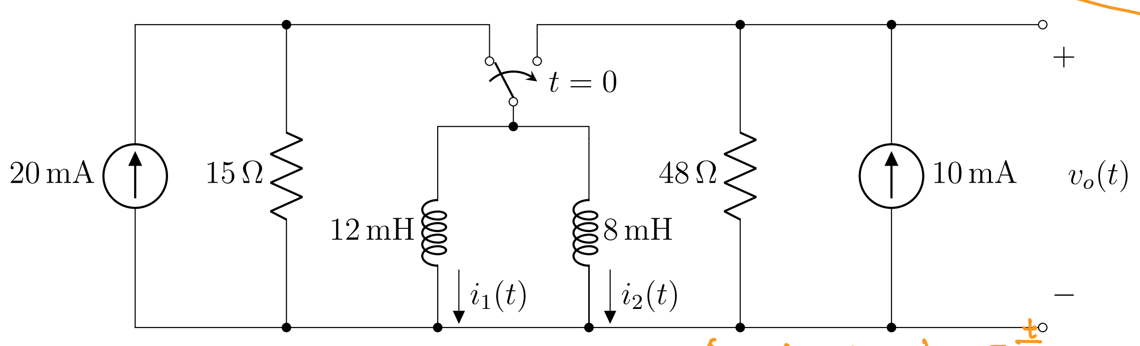


$i_2(0^+) = 0.3mA$
$i_1(0^+) = -0.3mA$

- Determine $i_1(0^-)$ and $i_2(0^-)$.
- Determine $i_1(0^+)$ and $i_2(0^+)$.
- Determine $i_1(\infty)$ and $i_2(\infty)$.
- Determine $i_1(t)$ and $i_2(t)$.

$i_1(t)\cdot 5k\Omega + i_2(t)\cdot 10k\Omega + i_1(t)\cdot 5k\Omega = 0$
$i_1(t)\cdot 10k\Omega + i_2(t)\cdot 10k\Omega = 0$
$i_1(t) = -i_2(t)$

Exercise 2 - RL circuit

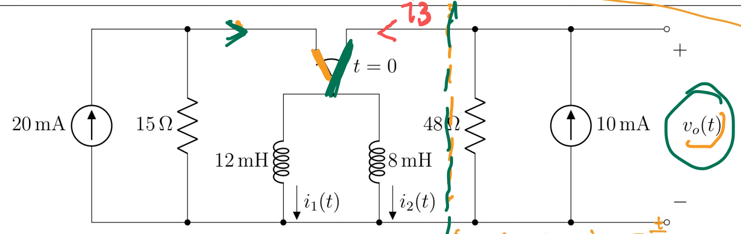


① $t<0$

- Determine $v_o(t)$, $i_1(t)$and $i_2(t)$.

$i(t) = i(\infty) + \left(i(0^-) - i(\infty)\right)\cdot e^{-\frac{t}{\tau}}$ $\tau = \frac{L}{R}$

Exercise 2 - RL circuit



- Determine $v_o(t)$, $i_1(t)$ and $i_2(t)$.

$t<0$

$i(t) = i(\infty) + \left(i(0^-) - i(\infty)\right)\cdot e^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{R}$

① $t<0$

$i_1(0^-) = i_1(0^+)$
$i_2(0^-) = i_2(0^+)$

$V_0(0^-) = 10mA \cdot 48\Omega = 0.48V$

$$\begin{cases} i_1(0^-) = i_1(-\infty) + \frac{1}{12mH}\cdot\int_{-\infty}^{0^-} v(t')\,dt' & i_1(-\infty)=0 \\ i_2(0^-) = i_2(-\infty) + \frac{1}{8mH}\int_{-\infty}^{0^-} v(t')\,dt' & i_2(-\infty)=0 \end{cases}$$

$$\Rightarrow \begin{cases} 12mH\cdot i_1(0^-) = 8mH\cdot i_2(0^-) \\ i_1(0^-) + i_2(0^-) = 20\,mA \end{cases} \Rightarrow \begin{cases} i_1(0^-) = 8\,mA \\ i_2(0^-) = 12\,mA \end{cases}$$

② $t>0$

$\left(\cancel{V_0(\infty) = 48\Omega\cdot 10mA = 0.48V}\right) \qquad V_0(\infty) = 0V$

$i_1(\infty) + i_2(\infty) = 10mA$

$$\begin{cases} i_1(\infty) = i_1(0^+) + \frac{1}{12mH}\int_{0^+}^{\infty} v(t')\,dt' \\ i_2(\infty) = i_2(0^+) + \frac{1}{8mH}\int_{0^+}^{\infty} v(t')\,dt' \end{cases} \Rightarrow 12mH\cdot\left(i_1(\infty) - i_1(0^+)\right) = 8mH\cdot\left(i_2(\infty) - i_2(0^+)\right)$$

$i_1(\infty) = 4mA \qquad i_2(\infty) = 6mA$

③ $L_{eq} = \frac{12mH\cdot 8mH}{12mH+8mH} = 4.8mH \qquad \tau = \frac{L}{R} = \frac{4.8mH}{48\Omega} = 0.1ms$

$$\begin{cases} i_1(t) = 4mA + (8mA - 4mA)\cdot e^{-t/\tau} = 4mA\cdot(1+e^{-t/\tau}) \\ i_2(t) = 6mA + (12mA - 6mA)\cdot e^{-t/\tau} = 6mA\cdot(1+e^{-t/\tau}) \\ V_0(t) = L_1\cdot\frac{d\,i_1(t)}{dt} = 12mH\cdot\left(4mA\cdot\frac{-1}{\tau}\cdot e^{-t/\tau}\right) = -0.48V\cdot e^{-t/\tau} \quad 0\le t<\infty \end{cases}$$

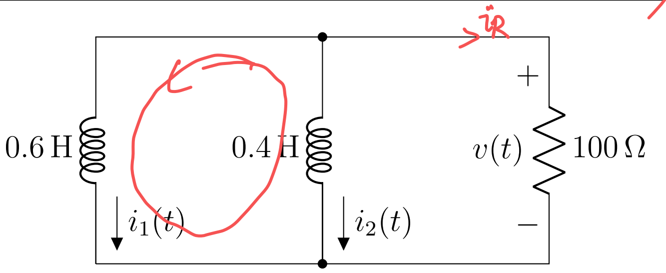
$i_L(t) = i_L(\infty) + (i_L(0^+) - i_L(\infty))\cdot e^{-t/\tau}$ } RL

$\Rightarrow v_L(t)$

$v_C(0^-) = v_C(0^+) \Rightarrow i_C(0^+)$, $i_C(\infty)$, $v_C(\infty)$ } $\Rightarrow v_C(t) = v_C(\infty) + (v_C(0^+) - v_C(\infty))\cdot e^{-t/\tau}$

$\Rightarrow i_C(t)$

Exercise 3 - Natural response RL circuit 1



At $t = 0^-$, the inductor currents are:

- $i_1(0^-) = 1\,\text{mA}$
- $i_2(0^-) = -1\,\text{mA}$

Determine $v(t)$ for $t \geq 0$.

$i_1(0^-) + i_2(0^-) = i_3(t) = 0$ No current in the resistor.

There is no reason to have non-zero i_3.

The current will continue flowing between the inductors

$v(t) = 0 \quad t \geq 0$

Exercise 4 - Natural response RL circuit 2

With the same circuit, the initial conditions are now:

- $i_1(0^-) = 2\,\text{mA}$
- $i_2(0^-) = -1\,\text{mA}$

Determine $v(t)$ for $t \geq 0$.

$i_R(t) = -i_1(t) - i_2(t)$ $\quad i_1(0^-) = i_1(0^+) = 2mA$ $\quad i_2(0^+) = -1mA$

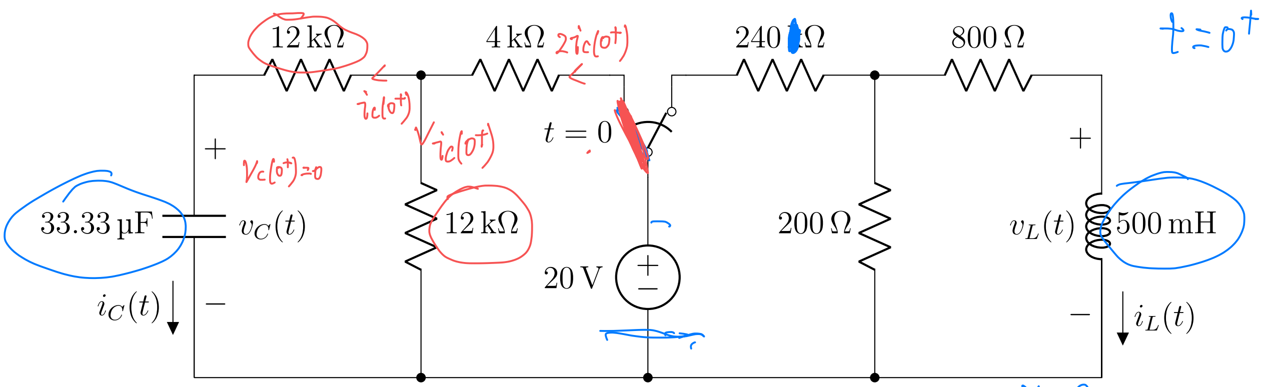
$i_R(0^+) = -1mA \Rightarrow v(0^+) = -0.1V$

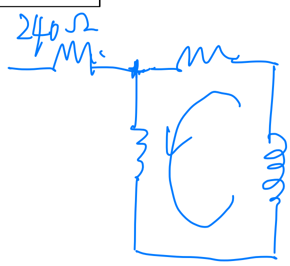
$i_1(\infty) = i_1(0^+) + \frac{1}{0.6H}\int_{0^+}^{\infty} v(t')\,dt'$

$i_2(\infty) = i_2(0^+) + \frac{1}{0.4H}\int_{0^+}^{\infty} v(t')\,dt'$

$\Rightarrow 0.6H\cdot(i_1(\infty) - i_1(0^+)) = 0.4H\cdot(i_2(\infty) - i_2(0^+))$

Exercise 5 - Complex circuit 1





Determine $v_C(t)$, $i_C(t)$, $v_L(t)$ and $i_L(t)$ for $t \geq 0$.

① $i_L(0^-) = \frac{20V}{240\Omega + 160\Omega}\cdot\frac{200\Omega}{200\Omega + 800\Omega} = 10mA = i_L(0^+)$

$v_L(0^+) = -10mA\cdot(200\Omega + 800\Omega) = -10V$

$i_L(\infty) = 0A \quad v_L(\infty) = 0V \quad R_{eq} = 1000\Omega \quad \tau = \frac{L}{R} = \frac{0.5H}{1000\Omega} = 0.5ms$

$i_L(t) = 10mA\cdot e^{-t/\tau} \quad v_L(t) = -10V\cdot e^{-t/\tau}$

② $v_C(0^-) = 0V = v_C(0^+) \quad v_C(\infty) = 20V\cdot\frac{12k\Omega}{12k\Omega + 4k\Omega} = 15V \quad i_C(\infty) = 0$

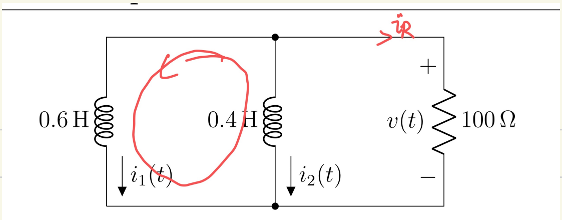
$20V = 4k\Omega\cdot 2i_C(0^+) + 12k\Omega\cdot i_C(0^+) \Rightarrow i_C(0^+) = 1mA$

$R_{eq} = 12k\Omega + (12k\Omega \,\|\, 4k\Omega) = 15k\Omega \quad \tau = R\cdot C = 15k\Omega\cdot 33.33\mu F = 0.5s$

$v_C(t) = 15V + (-15V)\cdot e^{-t/\tau} = 15V\cdot(1 - e^{-t/\tau}) \quad i_C(t) = C\cdot\frac{dv_C(t)}{dt}$

$= 33.33\mu F\cdot 15V\cdot\frac{1}{\tau}\cdot e^{-t/\tau}$

$=$



$i_R(\infty) = 0$

$i_1(\infty) \neq 0$

$i_2(\infty) \neq 0$

Exercise 4 - Natural response RL circuit 2

With the same circuit, the initial conditions are now:

- $i_1(0^-) = 2\,\mathrm{mA}$
- $i_2(0^-) = -1\,\mathrm{mA}$

Determine $v(t)$ for $t \geq 0$.

$i_R(t) = -i_1(t) - i_2(t) \quad i_1(0^-) = i_1(0^+) = 2mA \quad i_2(0^+) = -1mA$

$i_R(0^+) = -1mA \Rightarrow v(0^+) = -0.1V$

$$\begin{cases} i_1(\infty) = i_1(0^+) + \frac{1}{0.6H}\int_{0^+}^{\infty} v(t')\,dt' \\ i_2(\infty) = i_2(0^+) + \frac{1}{0.4H}\int_{0^+}^{\infty} v(t')\,dt' \end{cases} \Rightarrow 0.6H\cdot\left(i_1(\infty) - i_1(0^+)\right) = 0.4H\cdot\left(i_2(\infty) - i_2(0^+)\right) \quad ①$$

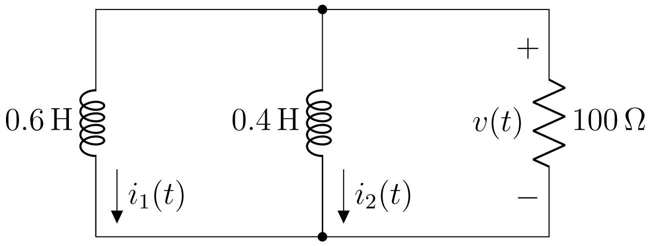
$$i_R(\infty) = -i_1(\infty) - i_2(\infty) = 0 \quad ② \quad \Rightarrow i_1(\infty) = 1.6mA \quad i_2(\infty) = -1.6mA$$

$$L_{eq} = \frac{0.6H\cdot 0.4H}{0.6H + 0.4H} = 0.24H \qquad \tau = \frac{L_{eq}}{R} = \frac{0.24H}{100\,\Omega} = 2.4ms$$

$$i_1(t) = 1.6mA + 0.4mA\cdot e^{-t/\tau} \qquad i_2(t) = -1.6mA + (-1mA + 1.6mA)\cdot e^{-t/\tau}$$

$$v(t) = \left(-i_1(t) - i_2(t)\right)\cdot 100\,\Omega = -100mV\cdot e^{-t/\tau}$$

Exercise 3 - Natural response RL circuit 1



At $t = 0^-$, the inductor currents are:

- $i_1(0^-) = 1\,\mathrm{mA}$
- $i_2(0^-) = -1\,\mathrm{mA}$

Determine $v(t)$ for $t \geq 0$.

Exercise 4 - Natural response RL circuit 2

With the same circuit, the initial conditions are now:

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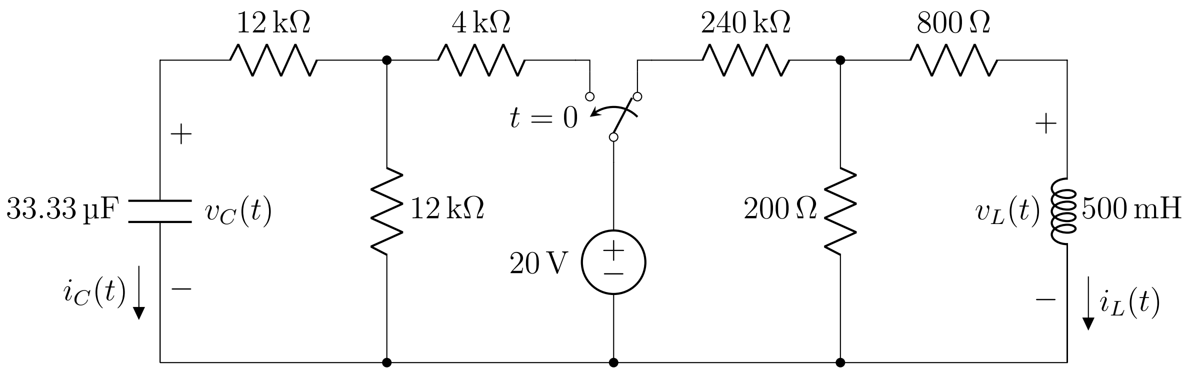
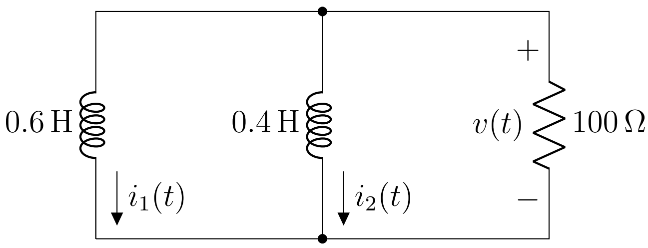
Determine $v(t)$ for $t \geq 0$.

Exercise 5 - Complex circuit 1



Determine $v_C(t)$, $i_C(t)$, $v_L(t)$ and $i_L(t)$ for $t \geq 0$.

Exercise 3 - Natural response RL circuit 1



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Exercise 4 - Natural response RL circuit 2

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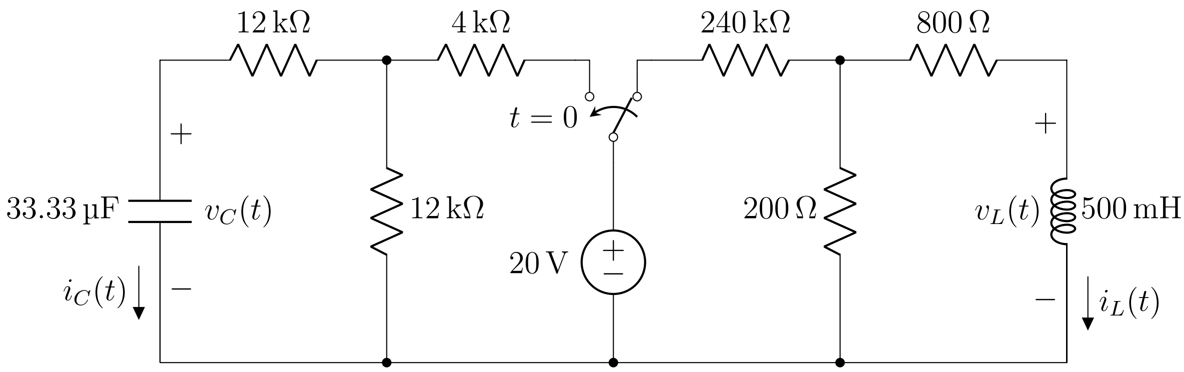
Determine $v(t)$ for $t \geq 0$.

Exercise 5 - Complex circuit 1



Determine $v_C(t)$, $i_C(t)$, $v_L(t)$ and $i_L(t)$ for $t \geq 0$.