30

INDUCTANCE

VP30.4.1. **IDENTIFY:** This problem is about mutual inductance.

SET UP: $M = -\dfrac{N_2\Phi_2}{i_1}$, $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$.

EXECUTE: **(a)** We want the mutual inductance. $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$. Solve for *M*, giving $M = \dfrac{\mathcal{E}_2}{di_1/dt}$ = (0.130 mV)/(2.00 A/s) = 65.0 μH.

(b) We want the flux due to i_1. $M = -\dfrac{N_2\Phi_2}{i_1}$. Using the answer to (a) and the given quantities gives $\Phi_2 = 1.56\times10^{-3}$ Wb.

EVALUATE: Each coil induces an emf in the other one. That's why it's called *mutual* inductance.

VP30.4.2. **IDENTIFY:** This problem is about mutual inductance.

SET UP: $M = -\dfrac{N_2\Phi_2}{i_1}$, $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$. We want the magnitude of $\mathcal{E}_2$.

EXECUTE: **(a)** $\mathcal{E}_2 = M\dfrac{di_1}{dt}$ = (48.0 μH)(7.00 A/s) = 0.336 mV.

(b) Since *di/dt* = 0, $\mathcal{E}_2$ = 0.

(c) $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$ = (48.0 μH)(3.00 A/s) = 0.144 mV.

EVALUATE: The signs of $\mathcal{E}_2$ would be reversed in (a) and (c) because *di/dt* has opposite signs.

VP30.4.3. **IDENTIFY:** This problem involves the self-inductance of a toroidal solenoid.

SET UP: $L = N\dfrac{\Phi_B}{I}$, $\mathcal{E} = L\dfrac{di}{dt}$.

EXECUTE: **(a)** We want the flux when *i* = 12.0 A. Solve $L = N\dfrac{\Phi_B}{I}$ for Φ_B. This gives $\Phi_B = Li/N$ = (76.0 μH)(12.0 A)/465 = 1.96 μWb.

(b) We want the magnitude of the emf. $\mathcal{E} = L\dfrac{di}{dt}$ = (76.0 μH)(55.0 A/s) = 4.18 mV.

EVALUATE: It is not the amount of flux that causes an induced emf, but rather the *rate of change* of the flux.

VP30.4.4. **IDENTIFY:** This problem is about self-inductance.

SET UP: $\mathcal{E} = -L\dfrac{di}{dt}$.

EXECUTE: **(a)** We want L. $\mathcal{E} = L\frac{di}{dt}$. $L = \frac{\mathcal{E}}{di/dt} =$ (3.70 mV)/(145 A/s) = 25.5 μH.

(b) We want $\mathcal{E}$ at t = 2.00 s. i = (225 A/s^2)t^2, so di/dt = (450 A/s)t. Now evaluate $\mathcal{E} = -L\frac{di}{dt}$ at t = 2.00 s. $\mathcal{E} = -L\frac{di}{dt}$ = –(25.5 μH)[(450 A/s)(2.00 s)] = –23.0 mV. The minus sign means that its direction is opposite that of the current.

EVALUATE: The current is increasing, so the induced emf opposes this increase, which means that it opposes the current.

VP30.7.1. **IDENTIFY:** This is about an R-L circuit.

SET UP: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$.

EXECUTE: **(a)** Resistance is the target variable. When t = 80.0 μs, $i = 0.750 i_{max}$. $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$:

$0.750 i_{max} = i_{max}\left(1 - e^{-(R/L)(80.0\ \mu s)}\right)$. Use logarithms to solve for R giving

$R = -\frac{L\ln(0.250)}{t} = -\frac{(4.90\text{ mH})\ln(0.250)}{80.0\mu\text{s}}$ = 84.9 Ω.

(b) We want the current 80.0 μs after closing the switch. At 80.0 μs the current is 75.0% of its maximum value, so $i = (0.750)\frac{\mathcal{E}}{R} = (0.750)\frac{24.0\text{ V}}{84.9\ \Omega} = 0.212$ A.

EVALUATE: As a check we can use the equation for the current. Putting all these values into $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$, we get i = 0.212. which is in agreement with our result in (b).

VP30.7.2. **IDENTIFY:** This problem is about an R-L circuit.

SET UP: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$, $U_L = \frac{1}{2}Li^2$. We want the current, di/dt, and stored energy in the inductor at time t = 0.700 ms.

EXECUTE: **(a)** Current: Use $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$ with the given quantities and t = 0.700 ms. The result is i = 0.190 A.

(b) di/dt: Take the derivative of $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$. $di/dt = \frac{\mathcal{E}}{L}e^{-(R/L)t}$. Now evaluate it at t = 0.700 ms, giving di/dt = +229 A/s.

(c) Energy: Evaluate $U_L = \frac{1}{2}Li^2 = \frac{1}{2}$(37.5 μH)(0.190 A)^2 = 0.679 mJ.

EVALUATE: The energy does not remain in the inductor if the current decreases. Unlike a capacitor, an inductor cannot store energy if you remove it from a circuit.

VP30.7.3. **IDENTIFY:** This is an R-L circuit.

SET UP: $i = I_0 e^{-(R/L)t}$, $U_L = \frac{1}{2}Li^2$. We want the current, di/dt, and the energy in the inductor at t = 0.700 ms.

EXECUTE: **(a)** Current: Evaluate $i = I_0 e^{-(R/L)t}$ at t = 0.700 ms, giving i = 0.476 A.

(b) di/dt: Take the derivative of $i = I_0 e^{-(R/L)t}$ and evaluate it at t = 0.700 ms. This gives $di/dt = -\frac{\mathcal{E}}{L}e^{-(R/L)t}$ = –229 A/s.

(c) Energy: Evaluate $U_L = \frac{1}{2}Li^2 = \frac{1}{2}(37.5\ \mu\text{H})(0.476\ \text{A})^2 = 4.26$ mJ.

EVALUATE: *di/dt* is negative because the current is decreasing with time. U_L decreases as the current decreases.

VP30.7.4. IDENTIFY: We have an *R-L* circuit.

SET UP: $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$, $\mathcal{E} = -L\frac{di}{dt}$, $V_R = Ri$.

EXECUTE: **(a)** We want the time when $V_L = V_R$. $V_L = L\frac{di}{dt}$ and $V_R = Ri$. Equate these potentials and solve for *t*. Take *di/dt* to get V_L, giving $L\frac{di}{dt} = \mathcal{E}e^{-(R/L)t}$. Equating potentials gives $R\left(\frac{\mathcal{E}}{R}\right)\left(1 - e^{-(R/L)t}\right) = \mathcal{E}e^{-(R/L)t}$. Use logarithms to solve for *t*, giving $t = (L/R)\ln 2$.

(b) We want the current at the time in (a). $i = \frac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$. During the solution in (a), we found that $e^{-(R/L)t} = \frac{1}{2}$. Therefore $i = \frac{\mathcal{E}}{2R}$.

(c) We want *di/dt* at this time. When $V_{-L} = V_R$, each is $\mathcal{E}/2$, so $L\frac{di}{dt} = \frac{\mathcal{E}}{2}$. Thus $\frac{di}{dt} = \frac{\mathcal{E}}{2L}$.

EVALUATE: Check for (b): When $V_L = V_R$, $V_R = \mathcal{E}/2 = Ri$, so $i = \frac{\mathcal{E}}{2R}$ as we found.

VP30.10.1. IDENTIFY: This is an *L-C* circuit.

SET UP: $\omega = 1/\sqrt{LC}$, $U_L = \frac{1}{2}Li^2$, $U_C = \frac{Q^2}{2C}$.

EXECUTE: **(a)** We want *C*. $2\pi f = \omega = 1/\sqrt{LC}$. Solve for *C* and use the given values. $C = \frac{1}{L(2\pi f)^2}$ = 5.12 mF.

(b) We want the maximum current. By energy conservation $U_{L,\max} = U_{C,\max}$. $\frac{1}{2}Li_{\max}^2 = \frac{Q_{\max}^2}{2C}$. Solve for $i_{\max}$ and use the given quantities. $i_{\max} = \frac{Q_{\max}}{\sqrt{LC}}$ = 14.0 A.

EVALUATE: The maximum charge occurs when there is no current in the circuit because there is no energy in the inductor at that instant.

VP30.10.2. IDENTIFY: This is an *L-C* circuit.

SET UP: $2\pi f = \omega = 1/\sqrt{LC}$, $T = 1/f$, $U_L = \frac{1}{2}Li^2$, $U_C = \frac{Q^2}{2C}$.

EXECUTE: **(a)** We want the time when the capacitor first discharges. This time is $\frac{1}{4}$ of a period. $T = 1/f = 2\pi/\omega$. Using $\omega = 1/\sqrt{LC}$, we get $t = \frac{1}{4}T = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{2}\sqrt{LC}$. Using the values for *L* and *C* gives $t = 0.220$ ms.

(b) We want the current when the capacitor is uncharged. By energy conservation $U_{L,\max} = U_{C,\max}$. $\frac{1}{2}Li_{\max}^2 = \frac{Q_{\max}^2}{2C}$. Solve for $i_{\max}$ and use the given quantities. $i_{\max} = \frac{Q_{\max}}{\sqrt{LC}}$ = 2.85 A.

EVALUATE: The current in (b) is the maximum current.

VP30.10.3. **IDENTIFY:** This is an *L-C* circuit.

SET UP: $U_L = \frac{1}{2}Li^2$, $U_C = \frac{Q^2}{2C}$, $\omega = 1/\sqrt{LC}$. $U_L + U_C = 0.800$ J. When $U_L = U_C$, $Q = 5.30$ mC and $i = 8.00$ A.

EXECUTE: **(a)** We want *C*. Solve $U_C = \frac{Q^2}{2C}$ for *C*. $C = \frac{Q^2}{2U_C} = \frac{(5.30\text{ mC})^2}{2(0.400\text{ J})} = 35.1\,\mu\text{F}$.

(b) We want *L*. Solve $U_L = \frac{1}{2}Li^2$ for *L*. $L = \frac{2U_L}{i^2} = \frac{2(0.400\text{ mC})}{(8.00\text{ A})^2} = 12.5\text{ mH}$.

(c) We want ω. Use $\omega = 1/\sqrt{LC}$ = 1510 rad/s using the values of *L* and *C* from parts (a) and (b).

EVALUATE: If this circuit has absolutely no resistance, the sum $U_L + U_C$ will always be 0.800 J.

VP30.10.4. **IDENTIFY:** This is an *L-R-C* circuit, so we have damped oscillations.

SET UP: $\omega_0 = 1/\sqrt{LC}$, $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$.

EXECUTE: **(a)** We want *C*. Use $\omega_0 = 1/\sqrt{LC}$ and solve for *C*, giving

$$C = \frac{1}{L\omega_0^2} = \frac{1}{(42.0\text{ mH})(624\text{ rad/s})^2} = 61.1\,\mu\text{F}.$$

(b) We want *R*. For damped oscillations, $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$. Solve for *R*:

$R = 2L\sqrt{\omega_0^2 - \omega'^2}$. Using $\omega' = 208$ rad/s, $\omega_0 = 624$ rad/s, and $L = 42.0$ mH, we get $R = 49.4\ \Omega$.

EVALUATE: As we see, the resistance makes a very large difference in the angular frequency.

30.1. **IDENTIFY** and **SET UP:** Apply $|\mathcal{E}_2| = M\left|\frac{di_1}{dt}\right|$ and $|\mathcal{E}_1| = M\left|\frac{di_2}{dt}\right|$.

EXECUTE: **(a)** $|\mathcal{E}_2| = M\left|\frac{di_1}{dt}\right| = (3.25\times10^{-4}\text{ H})(830\text{ A/s}) = 0.270\text{ V}$; yes, it is constant.

(b) $|\mathcal{E}_1| = M\left|\frac{di_2}{dt}\right|$; *M* is a property of the pair of coils so is the same as in part (a). Thus $|\mathcal{E}_1| = 0.270$ V.

EVALUATE: The induced emf is the same in either case. A constant di/dt produces a constant emf.

30.2. **IDENTIFY:** $\mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right|$ and $\mathcal{E}_2 = M\left|\frac{\Delta i_1}{\Delta t}\right|$. $M = \left|\frac{N_2\Phi_{B2}}{i_1}\right|$, where Φ_{B2} is the flux through one turn of the second coil.

SET UP: *M* is the same whether we consider an emf induced in coil 1 or in coil 2.

EXECUTE: **(a)** $M = \frac{\mathcal{E}_2}{|\Delta i_1/\Delta t|} = \frac{1.65\times10^{-3}\text{ V}}{0.242\text{ A/s}} = 6.82\times10^{-3}\text{ H} = 6.82\text{ mH}$.

(b) $\Phi_{B2} = \frac{Mi_1}{N_2} = \frac{(6.82\times10^{-3}\text{ H})(1.20\text{ A})}{25} = 3.27\times10^{-4}\text{ Wb}$.

(c) $\mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right| = (6.82\times10^{-3}\text{ H})(0.360\text{ A/s}) = 2.46\times10^{-3}\text{ V} = 2.46\text{ mV}$.

EVALUATE: We can express *M* either in terms of the total flux through one coil produced by a current in the other coil, or in terms of the emf induced in one coil by a changing current in the other coil.

30.3. IDENTIFY and SET UP: Apply $M = \dfrac{N_2\Phi_{B2}}{i_1}$.

EXECUTE: (a) $M = \dfrac{N_2\Phi_{B2}}{i_1} = \dfrac{400(0.0320\text{ Wb})}{6.52\text{ A}} = 1.96\text{ H}.$

(b) $M = \dfrac{N_1\Phi_{B1}}{i_2}$ so $\Phi_{B1} = \dfrac{Mi_2}{N_1} = \dfrac{(1.96\text{ H})(2.54\text{ A})}{700} = 7.11\times10^{-3}\text{ Wb}.$

EVALUATE: M relates the current in one coil to the flux through the other coil. Eq. (30.5) shows that M is the same for a pair of coils, no matter which one has the current and which one has the flux.

30.4. IDENTIFY: Changing flux from one object induces an emf in another object.

(a) SET UP: The magnetic field due to a solenoid is $B = \mu_0 nI$.

EXECUTE: The above formula gives

$$B_1 = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(300)(0.120\text{ A})}{0.250\text{ m}} = 1.81\times10^{-4}\text{ T}.$$

The average flux through each turn of the inner solenoid is therefore

$$\Phi_B = B_1A = (1.81\times10^{-4}\text{ T})\pi(0.0100\text{ m})^2 = 5.68\times10^{-8}\text{ Wb}.$$

(b) SET UP: The flux is the same through each turn of both solenoids due to the geometry, so

$$M = \frac{N_2\Phi_{B,2}}{i_1} = \frac{N_2\Phi_{B,1}}{i_1}.$$

EXECUTE: $M = \dfrac{(25)(5.68\times10^{-8}\text{ Wb})}{0.120\text{ A}} = 1.18\times10^{-5}\text{ H}.$

(c) SET UP: The induced emf is $\mathcal{E}_2 = -M\dfrac{di_1}{dt}$.

EXECUTE: $\mathcal{E}_2 = -(1.18\times10^{-5}\text{ H})(1750\text{ A/s}) = -0.0207\text{ V}.$

EVALUATE: A mutual inductance around 10^{-5} H is not unreasonable.

30.5. IDENTIFY: We can relate the known self-inductance of the toroidal solenoid to its geometry to calculate the number of coils it has. Knowing the induced emf, we can find the rate of change of the current.

SET UP: Example 30.3 shows that the self-inductance of a toroidal solenoid is $L = \dfrac{\mu_0 N^2 A}{2\pi r}$. The voltage across the coil is related to the rate at which the current in it is changing by $\mathcal{E} = L\left|\dfrac{di}{dt}\right|$.

EXECUTE: (a) Solving $L = \dfrac{\mu_0 N^2 A}{2\pi r}$ for N gives

$$N = \sqrt{\frac{2\pi rL}{\mu_0 A}} = \sqrt{\frac{2\pi(0.0600\text{ m})(2.50\times10^{-3}\text{ H})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.00\times10^{-4}\text{ m}^2)}} = 1940\text{ turns}.$$

(b) $\left|\dfrac{di}{dt}\right| = \dfrac{\mathcal{E}}{L} = \dfrac{2.00\text{ V}}{2.50\times10^{-3}\text{ H}} = 800\text{ A/s}.$

EVALUATE: The inductance is determined solely by how the coil is constructed. The induced emf depends on the rate at which the current through the coil is changing.

30.6. IDENTIFY: A changing current in an inductor induces an emf in it.

(a) SET UP: The self-inductance of a toroidal solenoid is $L = \dfrac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(500)^2(6.25\times 10^{-4}\ \text{m}^2)}{2\pi(0.0400\ \text{m})} = 7.81\times 10^{-4}\ \text{H}.$

(b) SET UP: The magnitude of the induced emf is $\mathcal{E} = L\left|\dfrac{di}{dt}\right|$.

EXECUTE: $\mathcal{E} = (7.81\times 10^{-4}\ \text{H})\left(\dfrac{5.00\ \text{A} - 2.00\ \text{A}}{3.00\times 10^{-3}\ \text{s}}\right) = 0.781\ \text{V}.$

(c) The current is decreasing, so the induced emf will be in the same direction as the current, which is from a to b, making b at a higher potential than a.

EVALUATE: This is a reasonable value for self-inductance, in the range of a mH.

30.7. IDENTIFY: $\mathcal{E} = L\left|\dfrac{\Delta i}{\Delta t}\right|$ and $L = \dfrac{N\Phi_B}{i}$.

SET UP: $\dfrac{\Delta i}{\Delta t} = 0.0640\ \text{A/s}.$

EXECUTE: **(a)** $L = \dfrac{\mathcal{E}}{|\Delta i/\Delta t|} = \dfrac{0.0160\ \text{V}}{0.0640\ \text{A/s}} = 0.250\ \text{H}.$

(b) The average flux through each turn is $\Phi_B = \dfrac{Li}{N} = \dfrac{(0.250\ \text{H})(0.720\ \text{A})}{400} = 4.50\times 10^{-4}\ \text{Wb}.$

EVALUATE: The self-induced emf depends on the rate of change of flux and therefore on the rate of change of the current, not on the value of the current.

30.8. IDENTIFY: Combine the two expressions for L: $L = N\Phi_B/i$ and $L = \mathcal{E}/|di/dt|$.

SET UP: Φ_B is the average flux through one turn of the solenoid.

EXECUTE: Solving for N we have $N = \mathcal{E}i/\Phi_B|di/dt| = \dfrac{(12.6\times 10^{-3}\ \text{V})(1.40\ \text{A})}{(0.00285\ \text{Wb})(0.0260\ \text{A/s})} = 238$ turns.

EVALUATE: The induced emf depends on the time rate of change of the total flux through the solenoid.

30.9. IDENTIFY and **SET UP:** Apply $|\mathcal{E}| = L|di/dt|$. Apply Lenz's law to determine the direction of the induced emf in the coil.

EXECUTE: **(a)** $|\mathcal{E}| = L|di/dt| = (0.260\ \text{H})(0.0180\ \text{A/s}) = 4.68\times 10^{-3}\ \text{V}.$

(b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the $+$ terminal at a.

EVALUATE: The induced emf is directed so as to oppose the decrease in the current.

30.10. IDENTIFY: Apply $\mathcal{E} = -L\dfrac{di}{dt}$.

SET UP: The induced emf points from low potential to high potential across the inductor.

EXECUTE: **(a)** The induced emf points from b to a, in the direction of the current. Therefore, the current is decreasing and the induced emf is directed to oppose this decrease.

(b) $|\mathcal{E}| = L|di/dt|$, so $|di/dt| = V_{ab}/L = (1.04\ \text{V})/(0.260\ \text{H}) = 4.00\ \text{A/s}.$ In 2.00 s the decrease in i is 8.00 A and the current at 2.00 s is 12.0 A − 8.0 A = 4.0 A.

EVALUATE: When the current is decreasing the end of the inductor where the current enters is at the lower potential. This agrees with our result and with Figure 30.6d in the textbook.

30.11. **IDENTIFY:** Use the definition of inductance and the geometry of a solenoid to derive its self-inductance.

SET UP: The magnetic field inside a solenoid is $B = \mu_0 \frac{N}{l} i$, and the definition of self-inductance is $L = \frac{N\Phi_B}{i}$.

EXECUTE: **(a)** $B = \mu_0 \frac{N}{l} i$, $L = \frac{N\Phi_B}{i}$, and $\Phi_B = \frac{\mu_0 NAi}{l}$. Combining these expressions gives $L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{l}$.

(b) $L = \frac{\mu_0 N^2 A}{l}$. $A = \pi r^2 = \pi(0.0750\times10^{-2}\ \text{m})^2 = 1.767\times10^{-6}\ \text{m}^2$.

$$L = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(50)^2(1.767\times10^{-6}\ \text{m}^2)}{5.00\times10^{-2}\ \text{m}} = 1.11\times10^{-7}\ \text{H} = 0.111\ \mu\text{H}.$$

EVALUATE: This is a physically reasonable value for self-inductance.

30.12. **IDENTIFY:** The changing current induces an emf in the solenoid.

SET UP: By definition of self-inductance, $L = \frac{N\Phi_B}{i}$. The magnitude of the induced emf is $|\mathcal{E}| = L\left|\frac{di}{dt}\right|$.

EXECUTE: $L = \frac{N\Phi_B}{i} = \frac{(800)(3.25\times10^{-3}\ \text{Wb})}{2.90\ \text{A}} = 0.8966\ \text{H}$.

$$\left|\frac{di}{dt}\right| = \frac{|\mathcal{E}|}{L} = \frac{6.20\times10^{-3}\ \text{V}}{0.8966\ \text{H}} = 6.92\times10^{-3}\ \text{A/s} = 6.92\ \text{mA/s}.$$

EVALUATE: An inductance of nearly a henry is rather large. For ordinary laboratory inductors, which are around a few millihenries, the current would have to be changing much faster to induce 6.2 mV.

30.13. **IDENTIFY:** This problem deals with a solenoid and Faraday's law.

SET UP: $B = \mu_0 nI$, $L = \mu_0 AN^2/l$, $\mathcal{E} = -L\frac{di}{dt}$. We want B. Use the given information to find n, and then use that result to find B. We only need to deal with magnitudes.

EXECUTE: $\mathcal{E}_L = L\frac{di}{dt} = \left(\mu_0 AN^2/l\right)\frac{di}{dt} = \left(\mu_0 ANn\right)\frac{di}{dt}$. Using the given numbers for $\mathcal{E}$, A, N, and di/dt gives $n = 0.0100/\mu_0$ turns/m. $B = \mu_0 nI = \mu_0\left(\frac{0.0100\ \text{turns/m}}{\mu_0}\right)(3.00\ \text{A}) = 0.0300\ \text{T}$.

EVALUATE: By planning a solution, one can sometimes avoid unnecessary arithmetic. In this case, we did not need to calculate with μ_0 because it cancels in the final step.

30.14. **IDENTIFY** and **SET UP:** The stored energy is $U = \frac{1}{2}LI^2$. The rate at which thermal energy is developed is $P = I^2R$.

EXECUTE: **(a)** $U = \frac{1}{2}LI^2 = \frac{1}{2}(12.0\ \text{H})(0.500\ \text{A})^2 = 1.50\ \text{J}$.

(b) $P = I^2R = (0.500\ \text{A})^2(180\ \Omega) = 45.0\ \text{W} = 45.0\ \text{J/s}$.

EVALUATE: **(c)** No. If I is constant then the stored energy U is constant. The energy being consumed by the resistance of the inductor comes from the emf source that maintains the current; it does not come from the energy stored in the inductor.

30.15. **IDENTIFY** and **SET UP:** Use $U_L = \frac{1}{2}LI^2$ to relate the energy stored to the inductance. Example 30.3 gives the inductance of a toroidal solenoid to be $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, so once we know L we can solve for N.

EXECUTE: $U = \frac{1}{2}LI^2$ so $L = \dfrac{2U}{I^2} = \dfrac{2(0.390\text{ J})}{(12.0\text{ A})^2} = 5.417\times10^{-3}\text{ H}$.

$$N = \sqrt{\frac{2\pi rL}{\mu_0 A}} = \sqrt{\frac{2\pi(0.150\text{ m})(5.417\times10^{-3}\text{ H})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^{-4}\text{ m}^2)}} = 2850.$$

EVALUATE: L and hence U increase according to the square of N.

30.16. **IDENTIFY:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a toroidal solenoid is $B = \dfrac{\mu_0 NI}{2\pi r}$.

EXECUTE: $B = \dfrac{\mu_0(300)(5.00\text{ A})}{2\pi(0.120\text{ m})} = 2.50\times10^{-3}\text{ T} = 2.50\text{ mT}$.

(b) SET UP: The self-inductance of a toroidal solenoid is $L = \dfrac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(300)^2(4.00\times10^{-4}\text{ m}^2)}{2\pi(0.120\text{ m})} = 6.00\times10^{-5}\text{ H}$.

(c) SET UP: The energy stored in an inductor is $U_L = \frac{1}{2}LI^2$.

EXECUTE: $U_L = \frac{1}{2}(6.00\times10^{-5}\text{ H})(5.00\text{ A})^2 = 7.50\times10^{-4}\text{ J}$.

(d) SET UP: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: $u = \dfrac{(2.50\times10^{-3}\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 2.49\text{ J/m}^3$.

EVALUATE: (e) $u = \dfrac{\text{energy}}{\text{volume}} = \dfrac{\text{energy}}{2\pi rA} = \dfrac{7.50\times10^{-4}\text{ J}}{2\pi(0.120\text{ m})(4.00\times10^{-4}\text{ m}^2)} = 2.49\text{ J/m}^3$.

An inductor stores its energy in the magnetic field inside of it.

30.17. **IDENTIFY:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a solenoid is $B = \mu_0 nI$.

EXECUTE: $B = \dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(400)(80.0\text{ A})}{0.250\text{ m}} = 0.161\text{ T}$.

(b) SET UP: The energy density in a magnetic field is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: $u = \dfrac{(0.161\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 1.03\times10^4\text{ J/m}^3$.

(c) SET UP: The total stored energy is $U = uV$.

EXECUTE: $U = uV = u(lA) = (1.03\times10^4\text{ J/m}^3)(0.250\text{ m})(0.500\times10^{-4}\text{ m}^2) = 0.129\text{ J}$.

(d) SET UP: The energy stored in an inductor is $U = \frac{1}{2}LI^2$.

EXECUTE: Solving for L and putting in the numbers gives

$$L = \frac{2U}{I^2} = \frac{2(0.129\ \text{J})}{(80.0\ \text{A})^2} = 4.02\times10^{-5}\ \text{H}.$$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.18. **IDENTIFY:** Energy $= Pt$. $U = \frac{1}{2}LI^2$.

SET UP: $P = 150\ \text{W} = 150\ \text{J/s}$.

EXECUTE: **(a)** Energy $= (150\ \text{W})(24\ \text{h})(3600\ \text{s/h}) = 1.296\times10^7\ \text{J}$, which rounds to $1.30\times10^7\ \text{J} = 13.0\ \text{MJ}$.

(b) $L = \frac{2U}{I^2} = \frac{2(1.296\times10^7\ \text{J})}{(80.0\ \text{A})^2} = 4.05\times10^3\ \text{H} = 4.05\ \text{kH}$.

EVALUATE: A large value of L and a large current would be required, just for one light bulb. Also, the resistance of the inductor would have to be very small, to avoid a large $P = I^2R$ rate of electrical energy loss.

30.19. **IDENTIFY:** The energy density depends on the strength of the magnetic field, and the energy depends on the volume in which the magnetic field exists.

SET UP: The energy density is $u = \dfrac{B^2}{2\mu_0}$.

EXECUTE: First find the energy density: $u = \dfrac{B^2}{2\mu_0} = \dfrac{(4.80\ \text{T})^2}{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 9.167\times10^6\ \text{J/m}^3$. The energy U in a volume V is $U = uV = (9.167\times10^6\ \text{J/m}^3)(10.0\times10^{-6}\ \text{m}^3) = 91.7\ \text{J}$.

EVALUATE: A field of 4.8 T is very strong, so this is a high energy density for a magnetic field.

30.20. **IDENTIFY:** This problem is about the energy density in electric and magnetic fields.

SET UP: $u_E = \dfrac{\epsilon_0 E^2}{2}$, $u_B = \dfrac{B^2}{2\mu_0}$.

EXECUTE: **(a)** We want E/B when $u_E = u_B$. Equate the energy densities. $\dfrac{\epsilon_0 E^2}{2} = \dfrac{B^2}{2\mu_0}$.

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.00\times10^8\ \text{V/m}\cdot\text{T}.$$

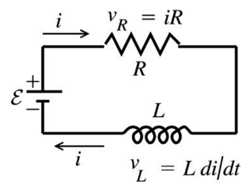
(b) We want B. Use the result from (a) giving $B = 1.67\ \mu\text{T}$.

EVALUATE: Note the units of E/B: $\dfrac{\text{V}}{\text{m}\cdot\text{T}} = \dfrac{\text{J/C}}{\text{m}\left(\dfrac{\text{N}}{\text{C}\cdot\text{m/s}}\right)} =$ m/s. Therefore,

$$\frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.00\times10^8\ \text{V/m}\cdot\text{T} = 3.00\times10^8\ \text{m/s},$$ which is the speed of light in vacuum.

30.21. **IDENTIFY:** Apply Kirchhoff's loop rule to the circuit. $i(t)$ is given by $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$.

SET UP: The circuit is sketched in Figure 30.21.



$\dfrac{di}{dt}$ is positive as the current increases from its initial value of zero.

Figure 30.21

EXECUTE: $\mathcal{E} - v_R - v_L = 0.$

$\mathcal{E} - iR - L\dfrac{di}{dt} = 0$ so $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t}).$

(a) Initially $(t = 0)$, $i = 0$ so $\mathcal{E} - L\dfrac{di}{dt} = 0.$

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} = \frac{6.00\text{ V}}{2.50\text{ H}} = 2.40\text{ A/s}.$$

(b) $\mathcal{E} - iR - L\dfrac{di}{dt} = 0.$ (Use this equation rather than $\dfrac{di}{dt} = \dfrac{\mathcal{E}}{L}e^{-(R/L)t}$ since i rather than t is given.)

Thus $\dfrac{di}{dt} = \dfrac{\mathcal{E} - iR}{L} = \dfrac{6.00\text{ V} - (0.500\text{ A})(8.00\ \Omega)}{2.50\text{ H}} = 0.800\text{ A/s}.$

(c) $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t}) = \left(\dfrac{6.00\text{ V}}{8.00\ \Omega}\right)(1 - e^{-(8.00\ \Omega/2.50\text{ H})(0.250\text{ s})}) = 0.750\text{ A}(1 - e^{-0.800}) = 0.413\text{ A}.$

(d) Final steady state means $t \to \infty$ and $\dfrac{di}{dt} \to 0$, so $\mathcal{E} - iR = 0.$

$$i = \frac{\mathcal{E}}{R} = \frac{6.00\text{ V}}{8.00\ \Omega} = 0.750\text{ A}.$$

EVALUATE: Our results agree with Figure 30.12 in the textbook. The current is initially zero and increases to its final value of $\mathcal{E}/R$. The slope of the current in the figure, which is di/dt, decreases with t.

30.22. **IDENTIFY:** With S_1 closed and S_2 open, the current builds up to a steady value. Then with S_1 open and S_2 closed, the current decreases exponentially.

SET UP: The decreasing current is $i = I_0 e^{-(R/L)t}$.

EXECUTE: **(a)** $i = I_0 e^{-(R/L)t} = \dfrac{\mathcal{E}}{R}e^{-(R/L)t}$. $e^{-(R/L)t} = \dfrac{iR}{\mathcal{E}} = \dfrac{(0.280\text{ A})(15.0\ \Omega)}{6.30\text{ V}} = 0.6667.$

$\dfrac{Rt}{L} = -\ln(0.6667).$ $L = -\dfrac{Rt}{\ln(0.6667)} = -\dfrac{(15.0\ \Omega)(2.00\times10^{-3}\text{ s})}{\ln(0.6667)} = 0.0740\text{ H} = 74.0\text{ mH}.$

(b) $\dfrac{i}{I_0} = e^{-(R/L)t}$. $e^{-(R/L)t} = 0.0100.$ $\dfrac{Rt}{L} = -\ln(0.0100).$

$$t = -\frac{\ln(0.0100)L}{R} = -\frac{\ln(0.0100)(0.0740\text{ H})}{15.0\ \Omega} = 0.0227\text{ s} = 22.7\text{ ms}.$$

EVALUATE: Typical LR circuits change rapidly compared to human time scales, so 22.7 ms is not unusual.

30.23. **IDENTIFY:** $i = \mathcal{E}/R(1 - e^{-t/\tau})$, with $\tau = L/R$. The energy stored in the inductor is $U = \frac{1}{2}Li^2$.

SET UP: The maximum current occurs after a long time and is equal to $\mathcal{E}/R$.

EXECUTE: **(a)** $i_{\max} = \mathcal{E}/R$ so $i = i_{\max}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$ and $e^{-t/\tau} = \frac{1}{2}$. $-t/\tau = \ln(\frac{1}{2})$.

$$t = \frac{L\ln 2}{R} = \frac{(\ln 2)(1.25\times10^{-3}\text{ H})}{50.0\ \Omega} = 17.3\ \mu\text{s}.$$

(b) $U = \frac{1}{2}U_{\max}$ when $i = i_{\max}/\sqrt{2}$. $1 - e^{-t/\tau} = 1/\sqrt{2}$, so $e^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$.
$t = -L\ln(0.2929)/R = 30.7\ \mu\text{s}$.

EVALUATE: $\tau = L/R = 2.50\times10^{-5}\ \text{s} = 25.0\ \mu s$. The time in part (a) is 0.692τ and the time in part (b) is 1.23τ.

30.24. **IDENTIFY:** We have an *R-L* circuit.

SET UP: $\mathcal{E}_L = -L\dfrac{di}{dt}$, $i = I_{\max}\left(1 - e^{-(R/L)t}\right)$. We want the time *T* when *i* = 5.00 A. First find *R* and *L*.

EXECUTE: After a long time, $i = I_{\max}$ = 15.0 A, so $R = \mathcal{E}/I_{-\max}$ = (240 V)/(15.0 A) = 16.0 Ω. At time *T*, $V_R = Ri$ = (16.0 Ω)(5.00 A) = 80.0 V, so $\mathcal{E}_L$ = 240 V – 80.0 V = 160 V, so $L\dfrac{di}{dt}$ = 160 V. This gives *L*(20.0 A/s) = 160 V, so *L* = 8.00 H. Now find the current. Use $i = I_{\max}\left(1 - e^{-(R/L)t}\right)$, and solve for *T* using the known quantities. Using logarithms gives *T* = –(*L/R*) ln(2/3) = 0.203 s.

EVALUATE: $\tau = L/R$ = (8.00 H)/(16.0 Ω) = 0.500 s, so *T* is *not* the time constant for this circuit.

30.25. **IDENTIFY:** We have an *R-L* circuit.

SET UP: $U_L = \dfrac{1}{2}Li^2$. We want the voltage across the inductor when it contains 0.400 J of energy.

EXECUTE: Solve $U_L = \dfrac{1}{2}Li^2$ for *i*, giving $i = \sqrt{2U_L/L}$. $V_L = \mathcal{E} - V_R = \mathcal{E} - RI = \mathcal{E} - R\sqrt{2U_L/L}$. Using the given quantities gives V_L = 15.4 V.

EVALUATE: As time increases, *i* increases so U_L increases to a maximum value when $i = \mathcal{E}/R$ = 1.67 A.

30.26. **IDENTIFY:** This is an *R-L* circuit.

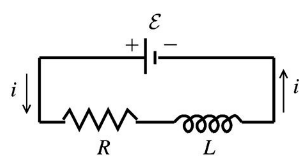
SET UP: We want the time *T* for the current to decrease from 12.0 A to 6.00 A. $i = I_0e^{-(R/L)t}$, $\mathcal{E}_L = -L\dfrac{di}{dt}$.

EXECUTE: After we close S_2 the current is $i = I_0e^{-(R/L)t}$. At time *T* the current is half its initial value, so $\dfrac{1}{2}I_0 = I_0e^{-(R/L)T}$. Use logarithms to solve for *T*, giving *T* = (*L/R*) ln 2. We see that we need to find *L/R*. At *t* = 0, $V_L = V_R$, so $-L\dfrac{di}{dt} = -Ri_0$. $L/R = i_0/(di/dt)$ = (12.0 A)/(36.0 A/s) = 0.333 s. Therefore *T* = (0.333 s) ln 2 = 0.231 s.

EVALUATE: The time constant *L/R* is the same with S_1 closed and S_2 open as it is with S_1 open and S_2 closed. But in the first case, the current increases with time but in the second case it decreases with time.

30.27. **IDENTIFY:** Apply the concepts of current decay in an *R-L* circuit. Apply the loop rule to the circuit. $i(t)$ is given by $i = I_0e^{-(R/L)t}$. The voltage across the resistor depends on *i* and the voltage across the inductor depends on di/dt.

SET UP: The circuit with S_1 closed and S_2 open is sketched in Figure 30.27a.



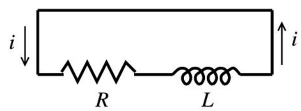
$$\mathcal{E} - iR - L\frac{di}{dt} = 0.$$

Figure 30.27a

Constant current established means $\frac{di}{dt} = 0.$

$$i = \frac{\mathcal{E}}{R} = \frac{60.0\ \text{V}}{240\ \Omega} = 0.250\ \text{A}.$$

EXECUTE: **(a)** The circuit with S_2 closed and S_1 open is shown in Figure 30.27b.



$i = I_0 e^{-(R/L)t}.$

At $t = 0, i = I_0 = 0.250\ \text{A}.$

Figure 30.27b

The inductor prevents an instantaneous change in the current; the current in the inductor just after S_2 is closed and S_1 is opened equals the current in the inductor just before this is done.

(b) $i = I_0 e^{-(R/L)t} = (0.250\ \text{A})e^{-(240\ \Omega/0.160\ \text{H})(4.00\times 10^{-4}\ \text{s})} = (0.250\ \text{A})e^{-0.600} = 0.137\ \text{A}.$

(c) See Figure 30.27c.

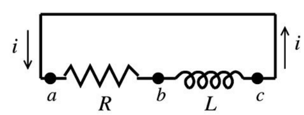


Figure 30.27c

If we trace around the loop in the direction of the current the potential falls as we travel through the resistor so it must rise as we pass through the inductor: $v_{ab} > 0$ and $v_{bc} < 0$. So point *c* is at a higher potential than point *b*.

$v_{ab} + v_{bc} = 0$ and $v_{bc} = -v_{ab}.$

Or, $v_{cb} = v_{ab} = iR = (0.137\ \text{A})(240\ \Omega) = 32.9\ \text{V}.$

(d) $i = I_0 e^{-(R/L)t}.$

$i = \frac{1}{2}I_0$ says $\frac{1}{2}I_0 = I_0 e^{-(R/L)t}$ and $\frac{1}{2} = e^{-(R/L)t}.$

Taking natural logs of both sides of this equation gives $\ln(\frac{1}{2}) = -Rt/L.$

$$t = \left(\frac{0.160\ \text{H}}{240\ \Omega}\right)\ln 2 = 4.62\times 10^{-4}\ \text{s}.$$

EVALUATE: The current decays, as shown in Figure 30.13 in the textbook. The time constant is $\tau = L/R = 6.67\times 10^{-4}$ s. The values of *t* in the problem are less than one time constant. At any instant the potential drop across the resistor (in the direction of the current) equals the potential rise across the inductor.

30.28. **IDENTIFY:** Apply $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t}).$

SET UP: $v_{ab} = iR.$ $v_{bc} = L\frac{di}{dt}.$ The current is increasing, so di/dt is positive.

EXECUTE: **(a)** At $t = 0,$ $i = 0.$ $v_{ab} = 0$ and $v_{bc} = 60$ V.

(b) As $t \to \infty,$ $i \to \mathcal{E}/R$ and $di/dt \to 0.$ $v_{ab} \to 60$ V and $v_{bc} \to 0.$

(c) When $i = 0.150$ A, $v_{ab} = iR = 36.0$ V and $v_{bc} = 60.0\ \text{V} - 36.0\ \text{V} = 24.0$ V.

EVALUATE: At all times, $\mathcal{E} = v_{ab} + v_{bc},$ as required by the loop rule.

30.29. **IDENTIFY:** With S_1 closed and S_2 open, the current builds up to a steady value.

SET UP: Applying Kirchhoff's loop rule gives $\mathcal{E} - iR - L\dfrac{di}{dt} = 0.$

EXECUTE: $v_R = \mathcal{E} - L\dfrac{di}{dt} = 18.0\text{ V} - (0.380\text{ H})(7.20\text{ A/s}) = 15.3\text{ V}.$

EVALUATE: The rest of the 18.0 V of the emf is across the inductor.

30.30. **IDENTIFY** and **SET UP:** The inductor opposes changes in current through it. $P = iV$, $P_R = i^2R$, $V = Ri$, $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** The inductor prevents an instantaneous build up of current, so the initial current is zero. The power supplied by the battery is $P = i\mathcal{E} = 0$ since $i_0 = 0$.

(b) The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$ and $i = \mathcal{E}/R$, so

$U_L = (1/2)(2.50\text{ H})[(6.00\text{ V})/(8.00\ \Omega)]^2 = 0.703\text{ J}$.

$P_R = i^2R = [(6.00\text{ V})/(8.00\ \Omega)]^2(8.00\ \Omega) = 4.50\text{ W}$.

The power supplied by the battery is

$P_{\mathcal{E}} = i\mathcal{E} = (\mathcal{E}/R)\mathcal{E} = (6.00\text{ V})^2/(8.00\ \Omega) = 4.50\text{ W}$.

EVALUATE: At steady-state the current is not changing so the potential difference across the inductor is zero. Therefore the power supplied by the battery is all consumed in the resistor, as we found.

30.31. **IDENTIFY:** This is an *R-L* circuit.

SET UP: $P = iV$. We want the power in the inductor.

EXECUTE: **(a)** At $t = 0$: $P_L = iV_L = 0$ because $i = 0$.

(b) As $t \to \infty$: The current is not changing, so $V_L = 0$. Therefore $P_L = iV_L = 0$.

(c) When $i = \mathcal{E}/2R$: $V_R = Ri = R\left(\dfrac{\mathcal{E}}{2R}\right) = \dfrac{\mathcal{E}}{2}$, so $V_L = \dfrac{\mathcal{E}}{2}$. $P_L = iV_L = \left(\dfrac{\mathcal{E}}{2R}\right)\left(\dfrac{\mathcal{E}}{2}\right) = \dfrac{\mathcal{E}^2}{4R}$.

EVALUATE: As $t \to \infty$, P_L becomes a maximum but its rate of change approaches zero.

30.32. **IDENTIFY:** An L-C circuit oscillates, with the energy going back and forth between the inductor and capacitor.

(a) SET UP: The frequency is $f = \dfrac{\omega}{2\pi}$ and $\omega = \dfrac{1}{\sqrt{LC}}$, giving $f = \dfrac{1}{2\pi\sqrt{LC}}$.

EXECUTE: $f = \dfrac{1}{2\pi\sqrt{(0.280\times10^{-3}\text{ H})(15.0\times10^{-6}\text{ F})}} = 2.456\times10^3\text{ Hz}$, which rounds to 2.46 kHz.

(b) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: $U = \frac{1}{2}(15.0\times10^{-6}\text{ F})(150.0\text{ V})^2 = 0.169\text{ J}$.

(c) SET UP: The current in the circuit is $i = -\omega Q\sin\omega t$, and the energy stored in the inductor is $U = \frac{1}{2}Li^2$.

EXECUTE: First find ω and Q. $\omega = 2\pi f = 2\pi(2456\text{ Hz}) = 1.543\times10^4\text{ rad/s}$.

$Q = CV = (15.0\times10^{-6}\text{ F})(150.0\text{ V}) = 2.25\times10^{-3}\text{ C}$. Now calculate the current:

$i = -(1.543\times10^4\text{ rad/s})(2.25\times10^{-3}\text{ C})\sin[(1.543\times10^4\text{ rad/s})(1.30\times10^{-3}\text{ s})]$. Notice that the argument of the sine is in *radians*, so convert it to degrees if necessary. The result is $i = -32.48\text{ A}$.

Now find the energy in the inductor: $U = \frac{1}{2}Li^2 = \frac{1}{2}(0.280\times10^{-3}\text{ H})(-32.48\text{ A})^2 = 0.148\text{ J}$.

EVALUATE: At the end of 1.30 ms, more of the energy is now in the inductor than in the capacitor.

30.33. IDENTIFY: Apply $\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$.

SET UP: $q = Q$ when $i = 0$. $i = i_{\text{max}}$ when $q = 0$. $1/\sqrt{LC} = 1917\ \text{s}^{-1}$.

EXECUTE: **(a)** $\frac{1}{2}Li_{\text{max}}^2 = \frac{Q^2}{2C}$.

$$Q = i_{\text{max}}\sqrt{LC} = (0.850\times10^{-3}\ \text{A})\sqrt{(0.0850\ \text{H})(3.20\times10^{-6}\ \text{F})} = 4.43\times10^{-7}\ \text{C}.$$

(b) $q = \sqrt{Q^2 - LCi^2} = \sqrt{(4.43\times10^{-7}\ \text{C})^2 - \left(\frac{5.00\times10^{-4}\,\text{A}}{1917\ \text{s}^{-1}}\right)^2} = 3.58\times10^{-7}\ \text{C}.$

EVALUATE: The value of q calculated in part (b) is less than the maximum value Q calculated in part (a).

30.34. IDENTIFY: The energy moves back and forth between the inductor and capacitor.

(a) SET UP: The period is $T = \frac{1}{f} = \frac{1}{\omega/2\pi} = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$.

EXECUTE: Solving for L gives

$$L = \frac{T^2}{4\pi^2 C} = \frac{(8.60\times10^{-5}\ \text{s})^2}{4\pi^2(7.50\times10^{-9}\ \text{F})} = 2.50\times10^{-2}\ \text{H} = 25.0\ \text{mH}.$$

(b) SET UP: The charge on a capacitor is $Q = CV$.

EXECUTE: $Q = CV = (7.50\times10^{-9}\ \text{F})(12.0\ \text{V}) = 9.00\times10^{-8}\ \text{C}.$

(c) SET UP: The stored energy is $U = Q^2/2C$.

EXECUTE: $U = \frac{(9.00\times10^{-8}\ \text{C})^2}{2(7.50\times10^{-9}\ \text{F})} = 5.40\times10^{-7}\ \text{J}.$

(d) SET UP: The maximum current occurs when the capacitor is discharged, so the inductor has all the initial energy. $U_L + U_C = U_{\text{Total}}$. $\frac{1}{2}LI^2 + 0 = U_{\text{Total}}$.

EXECUTE: Solve for the current:

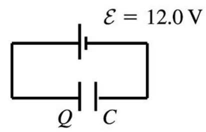
$$I = \sqrt{\frac{2U_{\text{Total}}}{L}} = \sqrt{\frac{2(5.40\times10^{-7}\ \text{J})}{2.50\times10^{-2}\ \text{H}}} = 6.58\times10^{-3}\ \text{A} = 6.58\ \text{mA}.$$

EVALUATE: The energy oscillates back and forth forever. However, if there is any resistance in the circuit, no matter how small, all this energy will eventually be dissipated as thermal energy in the resistor.

30.35. IDENTIFY and SET UP: The angular frequency is given by $\omega = \frac{1}{\sqrt{LC}}$. $q(t)$ and $i(t)$ are given by $q = Q\cos(\omega t + \phi)$ and $i = -\omega Q\sin(\omega t + \phi)$. The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2 = q^2/2C$. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.50\ \text{H})(6.00\times10^{-5}\ \text{F})}} = 105.4\ \text{rad/s}$, which rounds to $105\ \text{rad/s}$. The period is given by $T = \frac{2\pi}{\omega} = \frac{2\pi}{105.4\ \text{rad/s}} = 0.0596\ \text{s}.$

(b) The circuit containing the battery and capacitor is sketched in Figure 30.35.



$$\mathcal{E} - \frac{Q}{C} = 0.$$

$$Q = \mathcal{E}C = (12.0\text{ V})(6.00\times10^{-5}\text{ F}) = 7.20\times10^{-4}\text{ C}.$$

Figure 30.35

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.00\times10^{-5}\text{ F})(12.0\text{ V})^2 = 4.32\times10^{-3}\text{ J}.$

(d) $q = Q\cos(\omega t + \phi)$ (Eq. 30.21).

$q = Q$ at $t = 0$ so $\phi = 0$.

$q = Q\cos\omega t = (7.20\times10^{-4}\text{ C})\cos([105.4\text{ rad/s}][0.0230\text{ s}]) = -5.42\times10^{-4}\text{ C}.$

The minus sign means that the capacitor has discharged fully and then partially charged again by the current maintained by the inductor; the plate that initially had positive charge now has negative charge and the plate that initially had negative charge now has positive charge.

(e) The current is $i = -\omega Q\sin(\omega t + \phi)$.

$i = -(105\text{ rad/s})(7.20\times10^{-4}\text{ C})\sin[(105.4\text{ rad/s})(0.0230\text{ s})] = -0.050\text{ A}.$

The negative sign means the current is counterclockwise in Figure 30.15 in the textbook.

or

$\frac{1}{2}Li^2 + \frac{q^2}{2C} = \frac{Q^2}{2C}$ gives $i = \pm\sqrt{\frac{1}{LC}}\sqrt{Q^2 - q^2}$ (Eq. 30.26).

$i = \pm(105\text{ rad/s})\sqrt{(7.20\times10^{-4}\text{ C})^2 - (-5.42\times10^{-4}\text{ C})^2} = \pm 0.050\text{ A},$ which checks.

(f) $U_C = \frac{q^2}{2C} = \frac{(-5.42\times10^{-4}\text{ C})^2}{2(6.00\times10^{-5}\text{ F})} = 2.45\times10^{-3}\text{ J}.$

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(1.50\text{ H})(0.050\text{ A})^2 = 1.87\times10^{-3}\text{ J}.$

EVALUATE: Note that $U_C + U_L = 2.45\times10^{-3}\text{ J} + 1.87\times10^{-3}\text{ J} = 4.32\times10^{-3}\text{ J}.$

This agrees with the total energy initially stored in the capacitor,

$$U = \frac{Q^2}{2C} = \frac{(7.20\times10^{-4}\text{ C})^2}{2(6.00\times10^{-5}\text{ F})} = 4.32\times10^{-3}\text{ J}.$$

Energy is conserved. At some times there is energy stored in both the capacitor and the inductor. When $i = 0$ all the energy is stored in the capacitor and when $q = 0$ all the energy is stored in the inductor. But at all times the total energy stored is the same.

30.36. IDENTIFY: $\omega = \frac{1}{\sqrt{LC}} = 2\pi f$.

SET UP: ω is the angular frequency in rad/s and *f* is the corresponding frequency in Hz.

EXECUTE: **(a)** $L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(1.6\times10^6\text{ Hz})^2(4.18\times10^{-12}\text{ F})} = 2.37\times10^{-3}\text{ H}.$

(b) The maximum capacitance corresponds to the minimum frequency.

$$C_{\max} = \frac{1}{4\pi^2 f_{\min}^2 L} = \frac{1}{4\pi^2(5.40\times10^5\text{ Hz})^2(2.37\times10^{-3}\text{ H})} = 3.67\times10^{-11}\text{ F} = 36.7\text{ pF}.$$

EVALUATE: To vary *f* by a factor of three (approximately the range in this problem), *C* must be varied by a factor of nine.

30.37. IDENTIFY: Apply energy conservation and $\omega = \frac{1}{\sqrt{LC}}$ and $i = -\omega Q\sin(\omega t + \phi)$.

SET UP: If *I* is the maximum current, $\frac{1}{2}LI^2 = \frac{Q^2}{2C}$. For the inductor, $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\frac{1}{2}LI^2 = \dfrac{Q^2}{2C}$ gives $Q = I\sqrt{LC} = (0.750\text{ A})\sqrt{(0.0800\text{ H})(1.25\times10^{-9}\text{ F})} = 7.50\times10^{-6}\text{ C}.$

(b) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.0800\text{ H})(1.25\times10^{-9}\text{ F})}} = 1.00\times10^5\text{ rad/s}.\quad f = \dfrac{\omega}{2\pi} = 1.59\times10^4\text{ Hz}.$

(c) $q = Q$ at $t = 0$ means $\phi = 0$. $i = -\omega Q\sin(\omega t)$, so

$i = -(1.00\times10^5\text{ rad/s})(7.50\times10^{-6}\text{ C})\sin[(1.00\times10^5\text{ rad/s})(2.50\times10^{-3}\text{ s})] = 0.7279\text{ A}.$

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(0.0800\text{ H})(0.7279\text{ A})^2 = 0.0212\text{ J}.$

EVALUATE: The total energy of the system is $\frac{1}{2}LI^2 = 0.0225\text{ J}$. At $t = 2.50\text{ ms}$, the current is close to its maximum value and most of the system's energy is stored in the inductor.

30.38. **IDENTIFY:** The presence of resistance in an *L-R-C* circuit affects the frequency of oscillation and causes the amplitude of the oscillations to decrease over time.

(a) SET UP: The frequency of damped oscillations is $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}.$

EXECUTE: $\omega' = \sqrt{\dfrac{1}{(22\times10^{-3}\text{ H})(15.0\times10^{-9}\text{ F})} - \dfrac{(75.0\ \Omega)^2}{4(22\times10^{-3}\text{ H})^2}} = 5.5\times10^4\text{ rad/s}.$

The frequency *f* is $f = \dfrac{\omega}{2\pi} = \dfrac{5.50\times10^4\text{ rad/s}}{2\pi} = 8.76\times10^3\text{ Hz} = 8.76\text{ kHz}.$

(b) SET UP: The amplitude decreases as $A(t) = A_0\, e^{-(R/2L)t}$.

EXECUTE: Solving for *t* and putting in the numbers gives:

$$t = \frac{-2L\ln(A/A_0)}{R} = \frac{-2(22.0\times10^{-3}\text{ H})\ln(0.100)}{75.0\ \Omega} = 1.35\times10^{-3}\text{ s} = 1.35\text{ ms}.$$

(c) SET UP: At critical damping, $R = \sqrt{4L/C}$.

EXECUTE: $R = \sqrt{\dfrac{4(22.0\times10^{-3}\text{ H})}{15.0\times10^{-9}\text{ F}}} = 2420\ \Omega.$

EVALUATE: The frequency with damping is almost the same as the resonance frequency of this circuit $(1/\sqrt{LC})$, which is plausible because the $75\text{-}\Omega$ resistance is considerably less than the $2420\ \Omega$ required for critical damping.

30.39. **IDENTIFY:** Evaluate $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}.$

SET UP: The angular frequency of the circuit is ω'.

EXECUTE: **(a)** When $R = 0$, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\text{ H})(2.50\times10^{-5}\text{ F})}} = 298\text{ rad/s}.$

(b) We want $\dfrac{\omega'}{\omega_0} = 0.95$, so $\dfrac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \dfrac{R^2C}{4L} = (0.95)^2$. This gives

$R = \sqrt{\dfrac{4L}{C}(1-(0.95)^2)} = \sqrt{\dfrac{4(0.450\text{ H})(0.0975)}{(2.50\times10^{-5}\text{ F})}} = 83.8\ \Omega.$

EVALUATE: When *R* increases, the angular frequency decreases and approaches zero as $R \to 2\sqrt{L/C}$.

30.40. **IDENTIFY** and **SET UP:** Eq. (30.28) is $q = Ae^{-(R/2L)t}\cos\left(\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}t + \phi\right)$. We first find *A* and ϕ using the given information.

EXECUTE: **(a)** The charge is a maximum at $t = 0$, so $A = q_0 = 2.80\times10^{-4}$ C and $\phi = 0$.

(b) At the end of the first oscillation, $\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t = 2\pi$. Solving for t gives

$$t = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} = \frac{2\pi}{\sqrt{\frac{1}{(0.400\text{ H})(7.00\ \mu\text{F})} - \frac{(320\ \Omega)^2}{4(0.400\text{ H})^2}}} = 0.0142\text{ s} = 14.2\text{ ms}.$$

(c) In Eq. (30.28), at the end of the first oscillation, the cosine factor is equal to 1, so the charge is $q = q_0 e^{-(R/2L)t} = (2.80\times10^{-4}\text{ C})\, e^{-(320\ \Omega)(0.0142\text{ s})/[2(0.400\text{ H})]} = 9.75\times10^{-7}\text{ C}$.

EVALUATE: The charge on the capacitor is only 0.35% of its initial value.

30.41. **IDENTIFY:** This problem is about a solenoid inductor.

SET UP and **EXECUTE:** **(a)** Estimate: Diameter = 7.0 mm.

(b) Estimate: With no overlap of the wires, N(0.812 mm) = 4.0 cm. N = (40 mm)/(0.812 mm) = 49, which we round to 50 turns.

(c) If B is constant, $L = \mu_0 AN^2/l = \mu_0[\pi(0.406\text{ mm})^2](50^2)/(4.0\text{ cm}) = 3.0\ \mu\text{H}$.

(d) $U_L = \frac{1}{2}Li^2 = (1/2)(3.0\ \mu\text{H})(1.0\text{ A})^2 = 1.5\ \mu\text{J}$.

EVALUATE: Some lab inductors have an inductance of a few microhenries.

30.42. **IDENTIFY:** This is an R-L circuit and $i(t)$ is given by $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$.

SET UP: When $t \to \infty$, $i \to i_\text{f} = V/R$.

EXECUTE: **(a)** $R = \frac{V}{i_\text{f}} = \frac{16.0\text{ V}}{6.45\times10^{-3}\text{ A}} = 2481\ \Omega$, which rounds to 2480 Ω.

(b) $i = i_\text{f}(1 - e^{-(R/L)t})$ so $\frac{Rt}{L} = -\ln(1 - i/i_\text{f})$ and $L = \frac{-Rt}{\ln(1 - i/i_\text{f})} = \frac{-(2481\ \Omega)(9.40\times10^{-4}\text{ s})}{\ln(1 - (4.86/6.45))} = 1.67\text{ H}$.

EVALUATE: The current after a long time depends only on R and is independent of L. The value of R/L determines how rapidly the final value of i is reached.

30.43. **IDENTIFY:** This problem involves electromagnetic induction and self-inductance.

SET UP and **EXECUTE:** $L = \frac{\Phi_B}{i}$, $B = \frac{\mu_0 I}{2r}$, $\mathcal{E} = -\frac{d\Phi_B}{dt}$.. **(a)** We want the self-inductance.

$$\Phi_B = BA = \left(\frac{\mu_0 I}{2r}\right)(\pi r^2) = \frac{\mu_0 \pi r i}{2}.\quad L = \frac{\Phi_B}{i} = \frac{\mu_0 \pi r i/2}{i} = \frac{\mu_0 \pi r}{2} = \frac{\mu_0 \pi(3.0\text{ cm})}{2} = 59\text{ nH}.$$

(b) We want the maximum emf. $|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d}{dt}\left(\frac{\mu_0 \pi r i}{2}\right) = \frac{\mu_0 \pi r}{2}\frac{di}{dt}$. $i(t) = I_0 \sin(2\pi f t)$ gives

$\mathcal{E} = \mu_0 \pi^2 r I_0 f \cos(2\pi f t)$. $\mathcal{E}_\text{max} = \mu_0 \pi^2 r I_0 f = \ = \mu_0 \pi^2 (0300\text{ m})(1.20\text{ A})(60.0\text{ Hz}) = 26.8\ \mu\text{V}$.

EVALUATE: This only an estimate but it suggests a rather small induced voltage.

30.44. **IDENTIFY:** Apply $\mathcal{E} = -L\frac{di}{dt}$ and $Li = N\Phi_B$.

SET UP: Φ_B is the flux through one turn.

EXECUTE: **(a)** $\mathcal{E} = -L\dfrac{di}{dt} = -(7.50\times10^{-3}\ \text{H})\dfrac{d}{dt}\{(0.680\ \text{A})\cos[\pi t/(0.0250\ \text{s})]\}.$

$\mathcal{E} = (7.50\times10^{-3}\ \text{H})(0.680\ \text{A})\dfrac{\pi}{0.0250\ \text{s}}\sin[\pi t/(0.0250\ \text{s})].$ Therefore,

$\mathcal{E}_{\text{max}} = (7.50\times10^{-3}\ \text{H})(0.680\ \text{A})\dfrac{\pi}{0.0250\ \text{s}} = 0.641\ \text{V}.$

(b) $\Phi_{B\,\text{max}} = \dfrac{Li_{\text{max}}}{N} = \dfrac{(7.50\times10^{-3}\ \text{H})(0.680\ \text{A})}{400} = 1.28\times10^{-5}\ \text{Wb} = 12.8\ \mu\text{Wb}.$

(c) $\mathcal{E}(t) = -L\dfrac{di}{dt} = (7.50\times10^{-3}\ \text{H})(0.680\ \text{A})(\pi/0.0250\ \text{s})\sin[\pi t/(0.0250\ \text{s})].$

$\mathcal{E}(t) = (0.641\ \text{V})\sin[(125.6\ \text{s}^{-1})t].$ Therefore, at $t = 0.0180$ s,

$\mathcal{E}(0.0180\ \text{s}) = (0.641\ \text{V})\sin[(125.6\ \text{s}^{-1})(0.0180\ \text{s})] = 0.494\ \text{V}.$ The magnitude of the induced emf is 0.494 V.

EVALUATE: The maximum emf is when $i = 0$ and at this instant $\Phi_B = 0$.

30.45. **IDENTIFY:** Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1\ \text{m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\ \text{J/m}^3$. The total energy stored in volume V of the sunspot is $U_B = u_B V$. The mass of the material in volume V of the sunspot is $m = \rho V$. $K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4$ m/s.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.46. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: The energy stored is $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.$

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r}l\,dr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 il}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 il}{2\pi}\ln(b/a).$

(d) $L = \dfrac{N\Phi_B}{i} = l\dfrac{\mu_0}{2\pi}\ln(b/a).$

(e) $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}l\dfrac{\mu_0}{2\pi}\ln(b/a)i^2 = \dfrac{\mu_0 li^2}{4\pi}\ln(b/a).$

EVALUATE: The magnetic field between the conductors is due only to the current in the inner conductor.

30.47. **IDENTIFY:** $U = \frac{1}{2}LI^2$. The self-inductance of a solenoid is found in Exercise 30.15 to be $L = \dfrac{\mu_0 AN^2}{l}$.

SET UP: The length l of the solenoid is the number of turns divided by the turns per unit length.

EXECUTE: **(a)** $L = \dfrac{2U}{I^2} = \dfrac{2(10.0\ \text{J})}{(2.00\ \text{A})^2} = 5.00\ \text{H}.$

(b) $L = \dfrac{\mu_0 AN^2}{l}$. If α is the number of turns per unit length, then $N = \alpha l$ and $L = \mu_0 A\alpha^2 l$. For this coil $\alpha = 10$ coils/mm $= 10\times10^3$ coils/m. Solving for l gives

$$l = \frac{L}{\mu_0 A\alpha^2} = \frac{5.00 \text{ H}}{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})\pi(0.0200 \text{ m})^2(10\times10^3 \text{ coils/m})^2} = 31.7 \text{ m}.$$ This is not a practical length for laboratory use.

EVALUATE: The number of turns is $N = (31.7 \text{ m})(10\times10^3 \text{ coils/m}) = 3.17\times10^5$ turns. The length of wire in the solenoid is the circumference C of one turn times the number of turns.
$C = \pi d = \pi(4.00\times10^{-2} \text{ m}) = 0.126$ m. The length of wire is
$(0.126 \text{ m})(3.17\times10^5) = 4.0\times10^4 \text{ m} = 40$ km. This length of wire will have a large resistance and I^2R electrical energy loses will be very large.

30.48. IDENTIFY and SET UP: Eq. (30.14) is $i = \dfrac{\mathcal{E}}{R}(1-e^{-Rt/L})$, $P_R = i^2R$, $\mathcal{E}_L = -L\dfrac{di}{dt}$.

EXECUTE: (a) Using Eq. (30.14) in the power consumed in the resistor gives

$$P_R = i^2R = \left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]^2 R = \frac{\mathcal{E}^2}{R}(1-2e^{-Rt/L}+e^{-2Rt/L}).$$

After a long time, that is $t\to\infty$, the exponential terms all go to zero and the power approaches its maximum value of $\dfrac{\mathcal{E}^2}{R}$.

(b) The power in the inductor is

$$P_L = i\mathcal{E}_L = i\left(L\frac{di}{dt}\right) = \left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]L\frac{d}{dt}\left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right] = \left[\frac{\mathcal{E}}{R}(1-e^{-Rt/L})\right]L\left(\frac{\mathcal{E}}{L}\right)e^{-Rt/L}.$$

$$P_L = \frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})e^{-Rt/L}.$$

(c) $P_L(0) = 0$ since $1-e^0 = 0$. $P_L(t\to\infty) = 0$ since $e^{-Rt/L}\to 0$ as $t\to\infty$.

(d) P_L is a maximum when $dP_L/dt = 0$. Taking the time derivative of P_L from (b), we have

$$\frac{\mathcal{E}^2}{R}\left[\left(\frac{R}{L}e^{-Rt/L}\right)(e^{-Rt/L}) - \frac{R}{L}(1-e^{-Rt/L})(e^{-Rt/L})\right] = \frac{\mathcal{E}^2}{L}e^{-Rt/L}(e^{-Rt/L}-1+e^{-Rt/L}) = 0.$$

$2e^{-Rt/L} = 1.$

$t = -(L/R)\ln(1/2) = (L/R)\ln 2.$

At this instant, $e^{-Rt/L} = \frac{1}{2}$. Using the result from (b), we have

$$P_L = \frac{\mathcal{E}^2}{R}(1-e^{-Rt/L})e^{-Rt/L} = \frac{\mathcal{E}^2}{R}\left(1-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}\frac{\mathcal{E}^2}{R}.$$

(e) $P_\varepsilon = i\mathcal{E} = \dfrac{\mathcal{E}^2}{R}(1-e^{-Rt/L})$. The maximum power is $\dfrac{\mathcal{E}^2}{R}$ as $t\to\infty$.

EVALUATE: As time gets very large, current approaches a steady-state value, so the potential across an inductor approaches zero since the current through it is not changing.

30.49. IDENTIFY and SET UP: Use $U_C = \frac{1}{2}CV_C^2$ (energy stored in a capacitor) to solve for C. Then use $\omega = \dfrac{1}{\sqrt{LC}}$ and $\omega = 2\pi f$ to solve for the L that gives the desired current oscillation frequency.

EXECUTE: $V_C = 12.0$ V; $U_C = \frac{1}{2}CV_C^2$ so $C = 2U_C/V_C^2 = 2(0.0160 \text{ J})/(12.0 \text{ V})^2 = 222\ \mu\text{F}$.

$f = \frac{1}{2\pi\sqrt{LC}}$ so $L = \frac{1}{(2\pi f)^2 C}$.

$f = 3500$ Hz gives $L = 9.31\ \mu$H.

EVALUATE: f is in Hz and ω is in rad/s; we must be careful not to confuse the two.

30.50. IDENTIFY and SET UP: Apply $\mathcal{E}_L = -L\frac{di}{dt}$ and $V_R = Ri$. An inductor opposes a change in current through it. Kirchhoff's rules apply.

EXECUTE: **(a)** At the instant the switch is closed, the inductor will not allow any current through it, so all the current goes through R_1. So $i_1 = i_2 = \mathcal{E}/R_1 = (96.0\text{ V})/(12.0\ \Omega) = 8.00$ A. $i_3 = 0$.

(b) After a long time, steady-state is reached, so $di_3/dt = 0$ and $\mathcal{E}_L = -L\frac{di_3}{dt} = 0$. In this case, the potential across R_1 and across R_2 is 96.0 V. Therefore

$i_2 = (96.0\text{ V})/(12.0\ \Omega) = 8.00$ A.

$i_3 = (96.0\text{ V})/(16.0\ \Omega) = 6.00$ A.

$i_1 = i_2 + i_3 = 8.00\text{ A} + 6.00\text{ A} = 14.00$ A.

(c) Apply Kirchhoff's loop rule, giving

$$\mathcal{E} - i_3R_2 - L\frac{di_3}{dt} = 0.$$

Separating variables and integrating gives

$$\int_0^t -\frac{R_2}{L}dt' = \int_0^{i_3} \frac{1}{i_3' - \mathcal{E}/R_2}di_3'.$$

Carrying out the integration and solving for t gives

$$-\frac{R_2}{L}t = \ln\left(\frac{i_3 - \mathcal{E}/R_2}{-\mathcal{E}/R_2}\right).$$

$$t = \frac{L}{R_2}\ln\left(\frac{\mathcal{E}/R_2}{\mathcal{E}/R_2 - i_3}\right) = \frac{0.300\text{ H}}{16.0\ \Omega}\ln\left(\frac{96.0\text{ V}}{96.0\text{ V} - (3.00\text{ A})(16.0\ \Omega)}\right) = 0.0130\text{ s} = 13.0\text{ ms}.$$

(d) $i_2 = \mathcal{E}/R_1 = (96.0\text{ V})/(12.0\ \Omega) = 8.00$ A. $i_1 = i_2 + i_3 = 8.00\text{ A} + 3.00\text{ A} = 11.0$ A.

EVALUATE: At steady-state, the potential drop across an inductor is zero if it has no resistance. Initially the inductor acts like an open circuit because it will not allow current to flow through it.

30.51. IDENTIFY: This problem involves induction and magnetic torque.

SET UP and EXECUTE: **(a)** We want the force the bar exerts on the spool. The torques due to the force of the bar and the magnetic force balance so $Fa = \mu B\sin\theta = i(\pi a^2)NB\sin\theta$. This gives $F = \pi aiNB\sin\theta = \pi(0.0500\text{ m})(1.00\text{ A})(500)(2.00\text{ T})(\sin 45°) = 111$ N.

(b) We want the time when the force is zero. At this time, the magnetic torque equals the counter torque, so $i\pi a^2NB\sin\theta = \tau$. The current is $i = i_0e^{-(R/L)t}$, so $i_0e^{-(R/L)t}\ \pi a^2NB\sin\theta = \tau$. Isolate the exponential: $e^{-(R/L)t} = \frac{\tau}{i_0\pi a^2NB\sin\theta}$. Using the given numerical values gives $e^{-(R/L)t} = 0.09005$. Solving using logarithms gives $t = -(L/R)\ln(0.09005) = 37.1$ ms.

(c) We want the angular acceleration after a long time. Using $\Sigma\tau = I\alpha$, we have $\tau_{\text{mag}} = I\alpha$.

$\tau_{\text{mag}} = \mu BN\sin\theta = i\pi a^2BN\sin\theta$. $I = \frac{1}{2}Ma^2$. So $\tau_{\text{mag}} = I\alpha$ gives $i\pi a^2BN\sin\theta = \frac{1}{2}Ma^2\alpha$.

$\alpha = \frac{2i\pi BN\sin\theta}{M}$. After a long time, $i = 1.00$ A, as in part (a). Using the given quantities we have $\alpha = 4000\text{ rad/s}^2$.

EVALUATE: Be careful not to confuse μ (the magnetic moment) with μ_0 (the magnetic constant).

30.52. **IDENTIFY** and **SET UP:** Apply Kirchhoff's rules. $V_R = Ri$ and $\mathcal{E}_L = -L\dfrac{di}{dt}$.

EXECUTE: **(a)** Immediately after the switch is closed, the inductor will not allow any current in it, so all the current flows through R_2. At that instant, the equivalent circuit consists of R_1 and R_2 in series with each other and connected across the terminals of the battery. Ohm's law gives
$i_1 = i_2 = \mathcal{E}/(R_1 + R_2) = (48.0\text{ V})/(14\ \Omega) = 3.43\text{ A}$.

The current through the inductor is zero, so $i_3 = 0$.
(b) After a long time, steady-state has been achieved, so the potential across the inductor is zero. Therefore it acts like a short circuit, so no current flows through R_2. The equivalent circuit consists of R_3 connected across the terminals of the battery. By Ohm's law
$i_1 = i_3 = \mathcal{E}/R_1 = (48.0\text{ V})/(8.00\ \Omega) = 6.00\text{ A}$. $i_2 = 0$.
(c) Use Kirchhoff's loop rule. A loop around the left-hand section of the circuit gives
$\mathcal{E} - i_1R_1 - i_2R = 0$ (Eq. 1).
A loop around the right-hand section of the circuit gives
$-i_2R_2 + L\,di_3/dt = 0$ (Eq. 2).
Kirchhoff's junction rule gives
$i_1 = i_2 + i_3$ (Eq. 3).

Combining Eq. 1 and Eq. 3 and rearranging gives $i_2 = \dfrac{\mathcal{E} - i_3R_1}{R_1 + R_2}$. Putting this result into Eq. 2 gives

$-\left(\dfrac{\mathcal{E} - i_3R_1}{R_1 + R_2}\right)R_2 + L\dfrac{di_3}{dt} = 0$. Separating variables and integrating gives

$$\int_0^{i_3} \frac{di_3'}{i_3' - \mathcal{E}/R_1} = -\int_0^t \frac{R_1R_2}{L(R_1 + R_2)}\,dt'.$$

$$\ln\left(\frac{i_3 - \mathcal{E}/R_1}{-\mathcal{E}/R_1}\right) = -\frac{R_1R_2}{L(R_1 + R_2)}t.$$

$$i_3 = \frac{\mathcal{E}}{R_1}\left(1 - e^{-R_1R_2t/L(R_1+R_2)}\right).$$

(d) Solve for t when $i_3 = (\mathcal{E}/R_1)/2$.

$$t = \frac{L(R_1 + R_2)}{R_1R_2}\ln\left(\frac{\mathcal{E}/R_1}{\mathcal{E}/R_1 - \mathcal{E}/2R_1}\right) = \frac{L(R_1 + R_2)}{R_1R_2}\ln 2 = \frac{(0.200\text{ H})(14.00\ \Omega)}{(8.00\ \Omega)(6.00\ \Omega)}\ln 2 = 0.0404\text{ s}.$$

(e) $i_2 = \dfrac{\mathcal{E} - i_3R_1}{R_1 + R_2} = \dfrac{48.0\text{ V} - (3.00\text{ A})(8.00\ \Omega)}{14.00\ \Omega} = 1.71\text{ A}$.

$i_1 = i_2 + i_3 = 1.71\text{ A} + 3.00\text{ A} = 4.71\text{ A}$.

EVALUATE: The inductor initially acted like an open circuit, but at steady-state it acted like a short circuit. If it had resistance, it would not have behaved like a short circuit at steady-state.

30.53. **IDENTIFY:** In this problem we treat damping in an L-R-C circuit.

SET UP and **EXECUTE:** **(a)** We want the charge that flows onto the capacitor. Apply $\mathcal{E}_{\text{av}} = N\dfrac{\Delta\Phi_B}{\Delta t}$.

$\Delta B = B - 0 = B$, so $\dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{BA}{\Delta t}$. $\mathcal{E}_{\text{av}} = N\dfrac{BA}{\Delta t} = IR$. $I = \dfrac{NBA}{R\Delta t}$. $Q = I\Delta t = \left(\dfrac{NBA}{R\Delta t}\right)\Delta t = \dfrac{NBA}{R}$. $A = \pi r^2$. R $= (0.0333\ \Omega/\text{m})(2\pi r)N = 0.6277\ \Omega$. Using the area, this R, and the other given quantities, we get $Q =$ 15.0 mC.

(b) We want the time for the capacitor to fully discharge for the first time. Apply Kirchhoff's loop rule, giving $L\dfrac{di}{dt} + Ri + \dfrac{q}{C} = 0$, which we can express as $\dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{LC} = 0$. The solution to this differential equation is $q(t) = Ae^{-(R/2L)t}\cos\left(\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\,t+\phi\right)$. At $t = 0$, $q = Q_0$ and $i = 0$, so $A = Q_0$ and $\phi = \pi/2$. When the capacitor discharges for the first time, $q = 0$, which occurs when $\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}\,t$ $= \pi/2$. Therefore $t = \dfrac{\pi}{2\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}}$. We know that $R = 0.6277\ \Omega$ and $C = 10.0\ \mu\text{F}$, so we need L. Using $L = \mu_0 AN^2/l$ with the given quantities we get $L = 355.3\ \mu\text{H}$. Using these values gives $t = 93.8\ \mu\text{S}$.

(c) We want the frequency f'. Using $f' = \frac{\omega'}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}}$ gives $f' = 2.67$ kHz.

(d) We want the energy in the capacitor when $t = 0$. $U_0 = \frac{Q_0^2}{2C} = 11.3$ J using $Q_0 = 15.0$ mC from (a).

(e) We want the time for the maximum capacitor energy to be 10.0% of its initial energy. First find the charge for which the energy is 10.0% of the initial energy. $\frac{U}{U_0} = \frac{Q^2/2C}{Q_0^2/2C} = \left(\frac{Q}{Q_0}\right)^2 = 0.100$, so $Q = Q_0\sqrt{0.100}$. Now use the equation for $q(t)$. When q is a maximum, $t = 0$ so the cosine factor is equal to one. Using $Q = Q_0\sqrt{0.100}$ gives $Q_0 e^{-(R/2L)t} = Q_0\sqrt{0.100}$. Solve for t giving $t = -\frac{2L}{R}\ln\left(\sqrt{0.100}\right)$ $= 1.30$ ms.

EVALUATE: The charge and current continue to oscillate but with decreasing amplitude.

30.54. IDENTIFY: The initial energy stored in the capacitor is shared between the inductor and the capacitor.

SET UP: The potential across the capacitor and inductor is always the same, so $\frac{q}{C} = L\left|\frac{di}{dt}\right|$. The capacitor energy is $U_C = \frac{q^2}{2C} = \frac{1}{2}CV^2$, and the inductor energy is $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** The initial energy in the capacitor is $U_0 = \frac{1}{2}Cv_0^2 = \frac{1}{2}(6.40\text{ nF})(24.0\text{ V})^2 = 1.84\ \mu\text{J}$. This energy is shared between the inductor and the capacitor. The energy in the capacitor at this time is

$$U_C = \frac{q^2}{2C} = \frac{(0.0800\ \mu\text{C})^2}{2(6.40\text{ nF})} = 0.500\ \mu\text{J}.$$

The energy remaining in the inductor is
$U_L = U_0 - U_C = 1.84\ \mu\text{J} - 0.500\ \mu\text{J} = 1.34\ \mu\text{J}$.

The energy in the inductor is $U_L = \frac{1}{2}Li^2$, so

$$i = \sqrt{\frac{2U_L}{L}} = \sqrt{\frac{2(1.34\times 10^{-6}\text{ J})}{0.0660\text{ H}}} = 6.37\times 10^{-3}\text{ A} = 6.37\text{ mA}.$$

(b) When the capacitor charge is 0.0800 μC, we found that the energy stored in the capacitor is 0.500 μJ.

$$U_C = \frac{1}{2}Cv_C^2 \;\rightarrow\; v_C = \sqrt{\frac{2U_0}{C}} = \sqrt{\frac{2(0.500\ \mu\text{J})}{6.40\text{ nF}}} = 12.5\text{ V}.$$

The potential across the inductor and capacitor is the same, so $v_L = 12.5$ V.

Using $v_L = \mathcal{E}_L = \left|L\frac{di}{dt}\right|$, we have $\frac{di}{dt} = \frac{v_L}{L} = \frac{12.5\text{ V}}{0.0660\text{ H}} = 189$ A/s.

EVALUATE: When the capacitor contains 0.0800 μC of charge, the energy in the inductor is 1.34 μJ. Therefore the current is $U_L = \frac{1}{2}Li^2$, so $i = \sqrt{\frac{2U_L}{L}} = \sqrt{\frac{2(1.34\ \mu\text{J})}{0.0660\ \text{H}}} = 6.4$ mA. The current is only 6.4 mA but is changing at a rate of 189 A/s. However, it only changes at that rate for a tiny fraction of a second.

30.55. **IDENTIFY:** Apply energy conservation to the circuit.

SET UP: For a capacitor $V = q/C$ and $U = q^2/2C$. For an inductor $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $V_{\text{max}} = \frac{Q}{C} = \frac{6.00\times10^{-6}\ \text{C}}{2.50\times10^{-4}\ \text{F}} = 0.0240\ \text{V}$.

(b) $\frac{1}{2}Li_{\text{max}}^2 = \frac{Q^2}{2C}$, so $i_{\text{max}} = \frac{Q}{\sqrt{LC}} = \frac{6.00\times10^{-6}\ \text{C}}{\sqrt{(0.0600\ \text{H})(2.50\times10^{-4}\ \text{F})}} = 1.55\times10^{-3}\ \text{A}$.

(c) $U_{\text{max}} = \frac{1}{2}Li_{\text{max}}^2 = \frac{1}{2}(0.0600\ \text{H})(1.55\times10^{-3}\ \text{A})^2 = 7.21\times10^{-8}\ \text{J}$.

(d) If $i = \frac{1}{2}i_{\text{max}}$ then $U_L = \frac{1}{4}U_{\text{max}} = 1.80\times10^{-8}\ \text{J}$ and $U_C = \frac{3}{4}U_{\text{max}} = \frac{(\sqrt{(3/4)}Q)^2}{2C} = \frac{q^2}{2C}$. This gives $q = \sqrt{\frac{3}{4}}Q = 5.20\times10^{-6}\ \text{C}$.

EVALUATE: $U_{\text{max}} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$ for all times.

30.56. **IDENTIFY:** The total energy is shared between the inductor and the capacitor.

SET UP: The potential across the capacitor and inductor is always the same, so $\frac{q}{C} = L\left|\frac{di}{dt}\right|$. The capacitor energy is $U_C = \frac{q^2}{2C}$ and the inductor energy is $U_L = \frac{1}{2}Li^2$.

EXECUTE: The total energy is $\frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q_{\text{max}}^2}{2C} = \frac{1}{2}CV_{\text{max}}^2$.

$$q = LC\left|\frac{di}{dt}\right| = (0.330\ \text{H})(5.90\times10^{-4}\ \text{F})(73.0\ \text{A/s}) = 1.421\times10^{-2}\ \text{C}.$$

$$\frac{1}{2}CV_{\text{max}}^2 = \frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{(1.421\times10^{-2}\ \text{C})^2}{2(5.90\times10^{-4}\ \text{F})} + \frac{1}{2}(0.330\ \text{H})(2.50\ \text{A})^2 = 1.202\ \text{J}.$$

$$V_{\text{max}} = \sqrt{\frac{2(1.202\ \text{J})}{5.90\times10^{-4}\ \text{F})}} = 63.8\ \text{V}.$$

EVALUATE: By energy conservation, the maximum energy stored in the inductor will be 1.202 J, and this will occur at the instants when the capacitor is uncharged.

30.57. **IDENTIFY:** The current through an inductor doesn't change abruptly. After a long time the current isn't changing and the voltage across each inductor is zero.

SET UP: For part (c) combine the inductors.

EXECUTE: **(a)** Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

(b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to the circuit shown in Figure 30.57a. $I = (20.0\ \text{V})/(75.0\ \Omega) = 0.267$ A. The voltage between points a and b is zero, so the voltmeter reads zero.

(c) Combine the inductor network into its equivalent, as shown in Figure 30.57b. $R = 75.0\ \Omega$ is the equivalent resistance. The current is $i = (\mathcal{E}/R)(1 - e^{-t/\tau})$ with $\tau = L/R = (10.8\text{ mH})/(75.0\ \Omega) = 0.144\text{ ms}$. $\mathcal{E} = 20.0\text{ V}$, $R = 75.0\ \Omega$, $t = 0.115\text{ ms}$ so $i = 0.147\text{ A}$. $V_R = iR = (0.147\text{ A})(75.0\ \Omega) = 11.0\text{ V}$. $20.0\text{ V} - V_R - V_L = 0$ and $V_L = 20.0\text{ V} - V_R = 9.0\text{ V}$. The ammeter reads 0.147 A and the voltmeter reads 9.0 V.

EVALUATE: The current through the battery increases from zero to a final value of 0.267 A. The voltage across the inductor network drops from 20.0 V to zero.

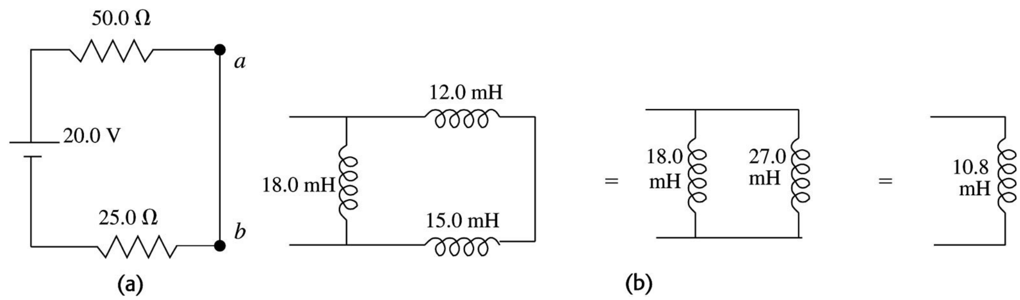


Figure 30.57

30.58. **IDENTIFY:** At $t = 0$, $i = 0$ through each inductor. At $t \to \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: **(a)** Initially the inductor blocks current through it, so the simplified equivalent circuit is shown in Figure 30.58a. $i = \dfrac{\mathcal{E}}{R} = \dfrac{50\text{ V}}{150\ \Omega} = 0.333\text{ A}$. $V_1 = (100\ \Omega)(0.333\text{ A}) = 33.3\text{ V}$.

$V_4 = (50\ \Omega)(0.333\text{ A}) = 16.7\text{ V}$. $V_3 = 0$ since no current flows through it. $V_2 = V_4 = 16.7\text{ V}$, since the inductor is in parallel with the $50\text{-}\Omega$ resistor. $A_1 = A_3 = 0.333\text{ A}$, $A_2 = 0$.

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. The simplified circuit is sketched in Figure 30.58b. $i = \mathcal{E}/R = \dfrac{50\text{ V}}{130\ \Omega} = 0.385\text{ A}$.

$V_1 = (100\ \Omega)(0.385\text{ A}) = 38.5\text{ V}$; $V_2 = 0$; $V_3 = V_4 = 50\text{ V} - 38.5\text{ V} = 11.5\text{ V}$.

$i_1 = 0.385\text{ A}$; $i_2 = \dfrac{11.5\text{ V}}{75\ \Omega} = 0.153\text{ A}$; $i_3 = \dfrac{11.5\text{ V}}{50\ \Omega} = 0.230\text{ A}$.

EVALUATE: Just after the switch is closed the current through the battery is 0.333 A. After a long time the current through the battery is 0.385 A. After a long time there is an additional current path, the equivalent resistance of the circuit is decreased and the current has increased.

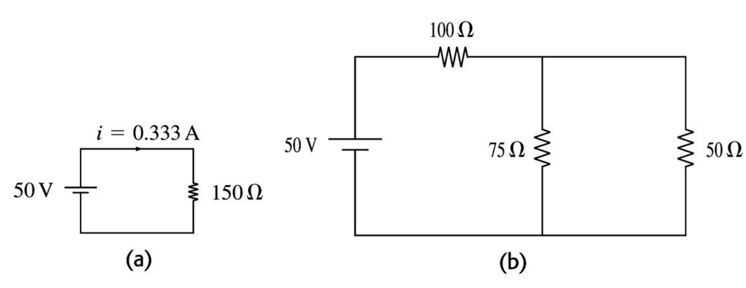


Figure 30.58

30.59. **IDENTIFY** and **SET UP:** The current in an R-L circuit is given by $i = i_0 e^{-Rt/L}$, where R is the total resistance. In our measurements, the current is one-half the initial current, so $i = i_0/2$.

EXECUTE: **(a)** Taking natural logarithms of the current equation, with $R = R_L + R_{ext}$ and $i = i_0/2$, we get

$\ln(i/i_0) = -Rt/L$.

$\ln(1/2) = -(R_L + R_{ext})t_{half}/L$.

$\ln 2 = t_{half}(R_L + R_{ext})/L$.

where t_{half} is the time for the current to decrease to half its initial value. Solving for $1/t_{half}$ gives $\dfrac{1}{t_{half}} = \dfrac{R_{ext}}{L\ln 2} + \dfrac{R_L}{L\ln 2}$. Therefore a graph of $1/t_{half}$ versus R_{ext} should be a straight line having a slope equal to $1/(L \ln 2)$ and a y-intercept equal to $R_L/(L \ln 2)$. Figure 30.59 shows this graph.

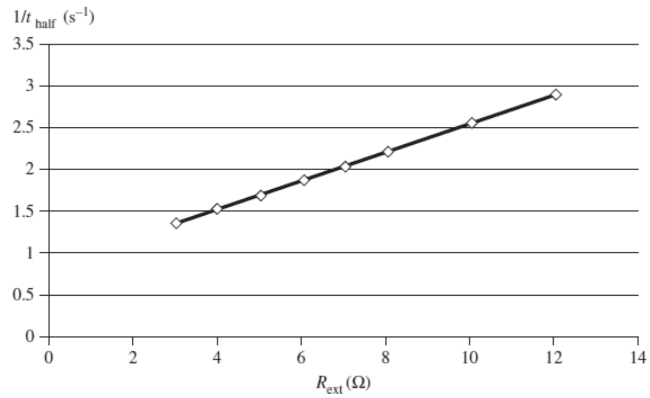


Figure 30.59

(b) The best-fit equation for the line in the graph is $\dfrac{1}{t_{half}} = 0.1692\ (\Omega\cdot\text{s})^{-1}R_{ext} + 0.8524\ \text{s}^{-1}$. Using the slope and solving for L gives $L = \dfrac{1}{(\text{slope})\ln 2} = \dfrac{1}{[0.1692\ (\Omega\cdot\text{s})^{-1}]\ln 2} = 8.53\ \text{H}$, which rounds to 8.5 H.

Now use the y-intercept and solve for R_L.

$\dfrac{R_L}{L\ln 2} = y\text{-intercept}$, so $R_L = (y\text{-intercept})(L\ln 2) = (0.8524\ \text{s}^{-1})(8.53\ \text{H})\ln 2 = 5.04\ \Omega$, which rounds to 5.0 Ω.

(c) $U_L = \frac{1}{2}Li^2 = (1/2)(8.53\ \text{H})(20.0\ \text{A})^2 = 1.7\times10^3\ \text{J} = 1.7\ \text{kJ}$.

$P_R = i^2R = (20.0\ \text{A})^2(5.04\ \Omega) = 2.0\times10^3\ \text{W} = 2.0\ \text{kW}$.

EVALUATE: Whether the 5.0-Ω resistance of this inductor would be significant would depend on the external resistance in the circuit. For the data of this problem, the solenoid resistance would definitely be significant for the external resistances used.

30.60. **IDENTIFY:** Closing S_2 and simultaneously opening S_1 produces an L-C circuit with initial current through the inductor of 3.50 A. When the current is a maximum the charge q on the capacitor is zero and when the charge q is a maximum the current is zero. Conservation of energy says that the maximum energy $\frac{1}{2}Li_{max}^2$ stored in the inductor equals the maximum energy $\frac{1}{2}\dfrac{q_{max}^2}{C}$ stored in the capacitor.

SET UP: $i_{max} = 3.50\ \text{A}$, the current in the inductor just after the switch is closed.

EXECUTE: **(a)** $\frac{1}{2}Li_{max}^2 = \frac{1}{2}\dfrac{q_{max}^2}{C}$.

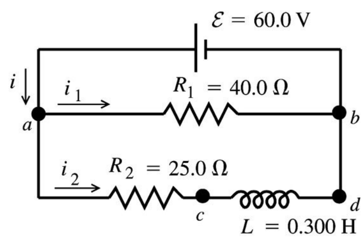
$q_{max} = (\sqrt{LC})i_{max} = \sqrt{(2.0\times10^{-3}\text{ H})(5.0\times10^{-6}\text{ F})}(3.50\text{ A}) = 3.50\times10^{-4}\text{ C} = 0.350\text{ mC}.$

(b) When q is maximum, $i = 0$.

EVALUATE: In the final circuit the current will oscillate.

30.61. **IDENTIFY:** Apply the loop rule to each parallel branch. The voltage across a resistor is given by iR and the voltage across an inductor is given by $L|di/dt|$. The rate of change of current through the inductor is limited.

SET UP: With S closed the circuit is sketched in Figure 30.61a.



The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is closed the current in the inductor has not had time to increase from zero, so $i_2 = 0$.

Figure 30.61a

EXECUTE : **(a)** $\mathcal{E} - v_{ab} = 0$, so $v_{ab} = 60.0$ V.

(b) The voltage drops across R, as we travel through the resistor in the direction of the current, so point a is at higher potential.

(c) $i_2 = 0$ so $v_{R_2} = i_2R_2 = 0$.

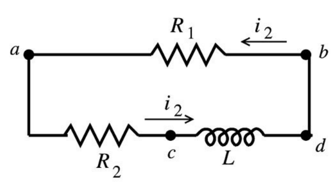
$\mathcal{E} - v_{R_2} - v_L = 0$ so $v_L = \mathcal{E} = 60.0$ V.

(d) The voltage rises when we go from b to a through the emf, so it must drop when we go from a to b through the inductor. Point c must be at higher potential than point d.

(e) After the switch has been closed a long time, $\frac{di_2}{dt} \to 0$ so $v_L = 0$. Then $\mathcal{E} - v_{R_2} = 0$ and $i_2R_2 = \mathcal{E}$

so $i_2 = \frac{\mathcal{E}}{R_2} = \frac{60.0\text{ V}}{25.0\ \Omega} = 2.40$ A.

SET UP: The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is opened again the current through the inductor hasn't had time to change and is still $i_2 = 2.40$ A. The circuit is sketched in Figure 30.61b.



EXECUTE: The current through R_1 is $i_2 = 2.40$ A in the direction b to a.

Thus $v_{ab} = -i_2R_1 = -(2.40\text{ A})(40.0\ \Omega)$.

$v_{ab} = -96.0$ V.

Figure 30.61b

(f) Point where current enters resistor is at higher potential; point b is at higher potential.

(g) $v_L - v_{R_1} - v_{R_2} = 0$.

$v_L = v_{R_1} + v_{R_2}$.

$v_{R_1} = -v_{ab} = 96.0$ V; $v_{R_2} = i_2R_2 = (2.40\text{ A})(25.0\ \Omega) = 60.0$ V.

Then $v_L = v_{R_1} + v_{R_2} = 96.0\text{ V} + 60.0\text{ V} = 156$ V.

As you travel counterclockwise around the circuit in the direction of the current, the voltage drops across each resistor, so it must rise across the inductor and point *d* is at higher potential than point *c*. The current is decreasing, so the induced emf in the inductor is directed in the direction of the current. Thus, $v_{cd} = -156$ V.

(h) Point *d* is at higher potential.

EVALUATE: The voltage across R_1 is constant once the switch is closed. In the branch containing R_2, just after S is closed the voltage drop is all across *L* and after a long time it is all across R_2. Just after S is opened the same current flows in the single loop as had been flowing through the inductor and the sum of the voltage across the resistors equals the voltage across the inductor. This voltage dies away, as the energy stored in the inductor is dissipated in the resistors.

30.62. **IDENTIFY:** Apply the loop rule to the two loops. The current through the inductor doesn't change abruptly.

SET UP: For the inductor $|\mathcal{E}| = L\left|\dfrac{di}{dt}\right|$ and $\mathcal{E}$ is directed to oppose the change in current.

EXECUTE: **(a)** Switch is closed, then at some later time

$$\frac{di}{dt} = 50.0\text{ A/s} \Rightarrow v_{cd} = L\frac{di}{dt} = (0.300\text{ H})(50.0\text{ A/s}) = 15.0\text{ V}.$$

<u>The top circuit loop:</u> $60.0\text{ V} = i_1R_1 \Rightarrow i_1 = \dfrac{60.0\text{ V}}{40.0\ \Omega} = 1.50\text{ A}.$

<u>The bottom loop:</u> $60.0\text{ V} - i_2R_2 - 15.0\text{ V} = 0 \Rightarrow i_2 = \dfrac{45.0\text{ V}}{25.0\ \Omega} = 1.80\text{ A}.$

(b) After a long time: $i_2 = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A},$ and immediately when the switch is opened, the inductor maintains this current, so $i_1 = i_2 = 2.40$ A.

EVALUATE: The current through R_1 changes abruptly when the switch is closed.

30.63. **IDENTIFY** and **SET UP:** The circuit is sketched in Figure 30.63a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time *i* has reached its final value and $di/dt = 0$. The voltage across a resistor depends on *i* and the voltage across an inductor depends on di/dt.

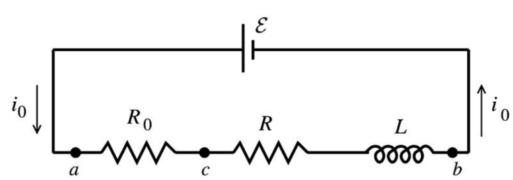


Figure 30.63a

EXECUTE: **(a)** At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$ so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0$ V. ($i_0R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\dfrac{di_0}{dt} \to 0$ so the potential $-L\dfrac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives $\mathcal{E} - i_0(R_0 + R) = 0$.

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0\text{ V}}{50.0\ \Omega + 150\ \Omega} = 0.180\text{ A}.$$

$v_{ac} = i_0 R_0 = (0.180 \text{ A})(50.0 \ \Omega) = 9.0 \text{ V}.$

Thus $v_{cb} = i_0 R + L\dfrac{di_0}{dt} = (0.180 \text{ A})(150 \ \Omega) + 0 = 27.0 \text{ V}.$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0.$

$\mathcal{E} - iR_0 - iR - L\dfrac{di}{dt} = 0.$

$L\dfrac{di}{dt} = \mathcal{E} - i(R_0 + R)$ and $\left(\dfrac{L}{R+R_0}\right)\dfrac{di}{dt} = -i + \dfrac{\mathcal{E}}{R+R_0}.$

$\dfrac{di}{-i + \mathcal{E}/(R+R_0)} = \left(\dfrac{R+R_0}{L}\right)dt.$

Integrate from $t = 0$, when $i = 0$, to t, when $i = i_0$:

$\displaystyle\int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R+R_0)} = \frac{R+R_0}{L}\int_0^t dt = -\ln\left[-i + \frac{\mathcal{E}}{R+R_0}\right]_0^{i_0} = \left(\frac{R+R_0}{L}\right)t,$ so

$\ln\left(-i_0 + \dfrac{\mathcal{E}}{R+R_0}\right) - \ln\left(\dfrac{\mathcal{E}}{R+R_0}\right) = -\left(\dfrac{R+R_0}{L}\right)t.$

$\ln\left(\dfrac{-i_0 + \mathcal{E}/(R+R_0)}{\mathcal{E}/(R+R_0)}\right) = -\left(\dfrac{R+R_0}{L}\right)t.$

Taking exponentials of both sides gives $\dfrac{-i_0 + \mathcal{E}/(R+R_0)}{\mathcal{E}/(R+R_0)} = e^{-(R+R_0)t/L}$ and $i_0 = \dfrac{\mathcal{E}}{R+R_0}(1 - e^{-(R+R_0)t/L}).$

Substituting in the numerical values gives $i_0 = \dfrac{36.0 \text{ V}}{50 \ \Omega + 150 \ \Omega}(1 - e^{-(200 \ \Omega/4.00 \text{ H})t}) = (0.180 \text{ A})(1 - e^{-t/0.020 \text{ s}}).$

At $t \to 0, i_0 = (0.180 \text{ A})(1 - 1) = 0$ (agrees with part (a)). At $t \to \infty, i_0 = (0.180 \text{ A})(1 - 0) = 0.180 \text{ A}$ (agrees with part (b)).

$v_{ac} = i_0 R_0 = \dfrac{\mathcal{E}R_0}{R+R_0}(1 - e^{-(R+R_0)t/L}) = 9.0 \text{ V}(1 - e^{-t/0.020 \text{ s}}).$

$v_{cb} = \mathcal{E} - v_{ac} = 36.0 \text{ V} - 9.0 \text{ V}(1 - e^{-t/0.020 \text{ s}}) = 9.0 \text{ V}(3.00 + e^{-t/0.020 \text{ s}}).$

At $t \to 0, v_{ac} = 0, v_{cb} = 36.0$ V (agrees with part (a)). At $t \to \infty, v_{ac} = 9.0$ V, $v_{cb} = 27.0$ V (agrees with part (b)). The graphs are given in Figure 30.63b.

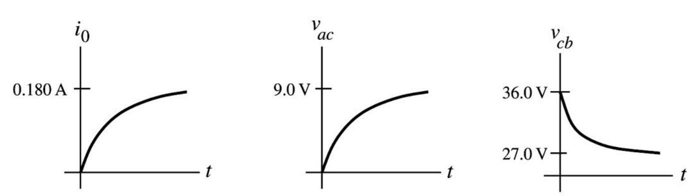


Figure 30.63b

EVALUATE: The expression for $i(t)$ we derived becomes $i = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$ if the two resistors R_0 and R in series are replaced by a single equivalent resistance $R_0 + R$.

30.64. IDENTIFY: Apply the loop rule. The current through the inductor doesn't change abruptly.

SET UP: With S_2 closed, v_{cb} must be zero.

EXECUTE: **(a)** Immediately after S_2 is closed, the inductor maintains the current $i = 0.180$ A through R. The loop rule around the outside of the circuit yields

$\mathcal{E} + \mathcal{E}_L - iR - i_0 R_0 = 36.0\text{ V} + (0.18\text{ A})(150\ \Omega) - (0.18\text{ A})(150\ \Omega) - i_0(50\ \Omega) = 0.$ $i_0 = \dfrac{36\text{ V}}{50\ \Omega} = 0.720\text{ A}.$

$v_{ac} = (0.72\text{ A})(50\ \Omega) = 36.0\text{ V}$ and $v_{cb} = 0.$

(b) After a long time, $v_{ac} = 36.0\text{ V},$ and $v_{cb} = 0.$ Thus $i_0 = \dfrac{\mathcal{E}}{R_0} = \dfrac{36.0\text{ V}}{50\ \Omega} = 0.720\text{ A},$ $i_R = 0$ and

$i_{s2} = 0.720\text{ A}.$

(c) $i_0 = 0.720\text{ A},$ $i_R(t) = \dfrac{\mathcal{E}}{R_{\text{total}}} e^{-(R/L)t}$ and $i_R(t) = (0.180\text{ A})e^{-(37.5\text{ s}^{-1})t}.$

$i_{s2}(t) = (0.720\text{ A}) - (0.180\text{ A})e^{-(37.5\text{ s}^{-1})t} = (0.180\text{ A})(4 - e^{-(37.5\text{ s}^{-1})t}).$ The graphs of the currents are given in Figure 30.64.

EVALUATE: R_0 is in a loop that contains just $\mathcal{E}$ and $R_0,$ so the current through R_0 is constant. After a long time the current through the inductor isn't changing and the voltage across the inductor is zero. Since v_{cb} is zero, the voltage across *R* must be zero and i_R becomes zero.

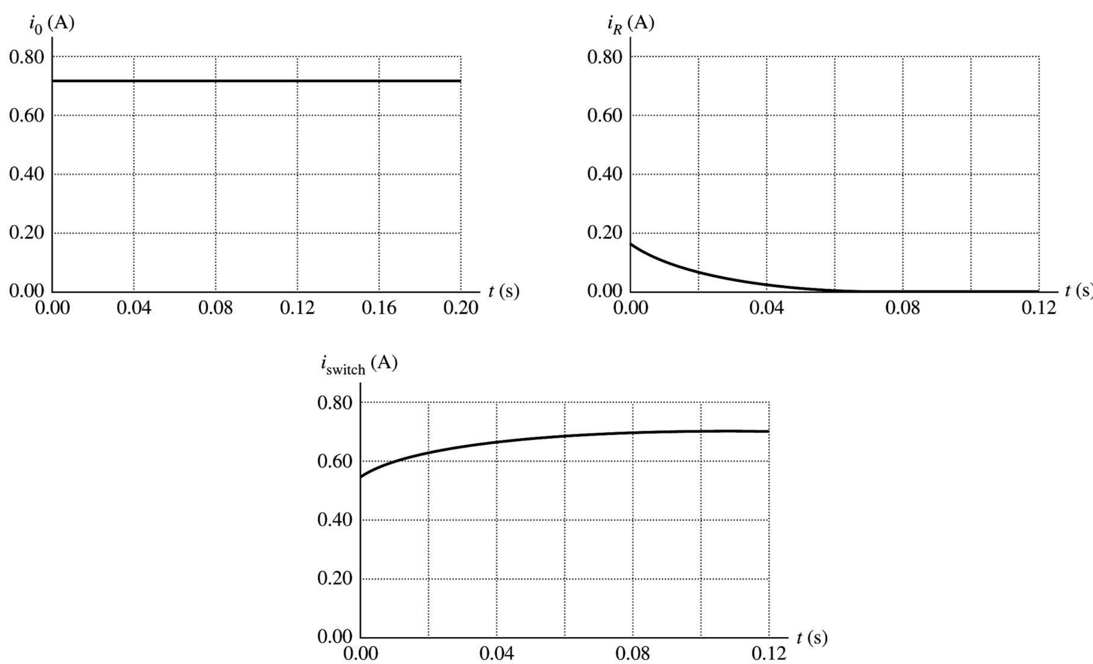


Figure 30.64

30.65. IDENTIFY: At $t = 0,$ $i = 0$ through each inductor. At $t \to \infty,$ the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: (a) Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the $40.0\text{-}\Omega$ and $15.0\text{-}\Omega$ resistors and is equal to $(25.0\text{ V})/(40.0\ \Omega + 15.0\ \Omega) = 0.455\text{ A}.$ $A_1 = A_4 = 0.455\text{ A};$ $A_2 = A_3 = 0.$

(b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit. The circuit is equivalent to the circuit sketched in Figure 30.65. $I = (25.0\text{ V})/(42.73\ \Omega) = 0.585\text{ A}.$ A_1 reads 0.585 A. The voltage across each parallel branch is $25.0\text{ V} - (0.585\text{ A})(40.0\ \Omega) = 1.60\text{ V}.$ A_2 reads $(1.60\text{ V})/(5.0\ \Omega) = 0.320\text{ A}.$ A_3 reads $(1.60\text{ V})/(10.0\ \Omega) = 0.160\text{ A}.$ A_4 reads $(1.60\text{ V})/(15.0\ \Omega) = 0.107\text{ A}.$

EVALUATE: Just after the switch is closed the current through the battery is 0.455 A. After a long time the current through the battery is 0.585 A. After a long time there are additional current paths, the equivalent resistance of the circuit is decreased and the current has increased.

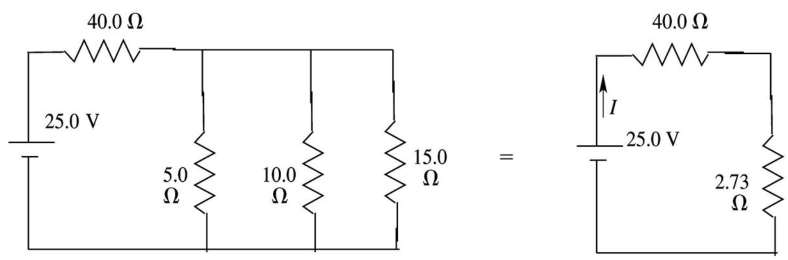


Figure 30.65

30.66. IDENTIFY: At steady state with the switch in position 1, no current flows to the capacitors and the inductors can be replaced by wires. Apply conservation of energy to the circuit with the switch in position 2.

SET UP: Replace the series combinations of inductors and capacitors by their equivalents.

EXECUTE: **(a)** At steady state $i=\dfrac{\mathcal{E}}{R}=\dfrac{75.0\text{ V}}{125\ \Omega}=0.600\text{ A}.$

(b) The equivalent circuit capacitance of the two capacitors is given by $\dfrac{1}{C_\text{s}}=\dfrac{1}{25\ \mu\text{F}}+\dfrac{1}{35\ \mu\text{F}}$ and

$C_\text{s}=14.6\ \mu\text{F}.$ $L_\text{s}=15.0\text{ mH}+5.0\text{ mH}=20.0\text{ mH}.$ The equivalent circuit is sketched in Figure 30.66a.

Energy conservation: $\dfrac{q^2}{2C}=\dfrac{1}{2}Li_0^2.$ $q=i_0\sqrt{LC}=(0.600\text{ A})\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})}=3.24\times10^{-4}\text{ C}.$

As shown in Figure 30.66b, the capacitors have their maximum charge at $t=T/4$.

$t=\frac{1}{4}T=\frac{1}{4}(2\pi\sqrt{LC})=\dfrac{\pi}{2}\sqrt{LC}=\dfrac{\pi}{2}\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})}=8.49\times10^{-4}\text{ s}.$

EVALUATE: With the switch closed the battery stores energy in the inductors. This then is the energy in the *L-C* circuit when the switch is in position 2.

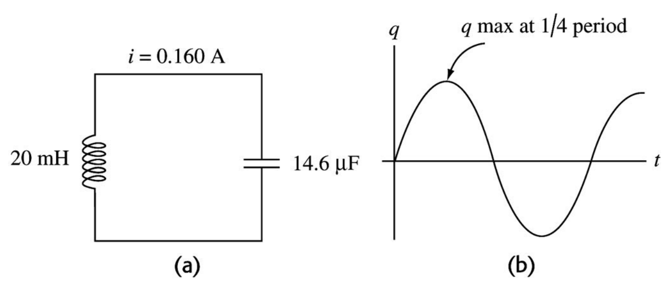


Figure 30.66

30.67. IDENTIFY and SET UP: Kirchhoff's loop rule applies, the emf across an inductor is $\mathcal{E}_L=-L\dfrac{di}{dt}$, the potential across a resistor is $V=Ri$, and the time constant for an *L-R* circuit is $\tau=L/R$.

EXECUTE: **(a)** First find the current as a function of time. The inductor has a resistance R_L which is in series with the 10.0-Ω resistor R. Apply Kirchhoff's loop rule to the circuit: $\mathcal{E}-iR-iR_L-L\dfrac{di}{dt}=0.$

Now separate variables and integrate.

$$\int_0^t -\frac{R+R_L}{L}dt' = \int_0^i \frac{di'}{i' - \varepsilon/(R+R_L)}.$$

$$-\frac{R+R_L}{L}t = \ln\left(\frac{i-\mathcal{E}/(R+R_L)}{-\mathcal{E}/(R+R_L)}\right).$$

$$i = \frac{\mathcal{E}}{R+R_L}(1-e^{-(R+R_L)t/L}).$$

The potential across the inductor is the sum of the potential due to the resistance and the potential due to the inductance, so $v_L = iR_L + L\,di/dt$. Using the equation we just found for the current i and taking $L\,di/dt$, we get $v_L = iR_L + L\frac{di}{dt} = \left[\frac{\mathcal{E}}{R+R_L}(1-e^{-(R+R_L)t/L})\right]R_L + \mathcal{E}e^{-(R+R_L)t/L}.$

Collecting terms and taking out common factors, the result is $v_L = \frac{\mathcal{E}}{R+R_L}(R_L + Re^{-(R+R_L)t/L}).$

(b) Initially there is no current in the circuit due to the inductor, so the potential across the resistance R is zero. Therefore the potential across the inductor is equal to the emf of the battery.

$v_L(0) = \frac{\mathcal{E}}{R+R_L}(R_L+R) = \mathcal{E} = 50.0$ V.

(c) As $t \to \infty,$ we know that $v_L = 20.0$ V. So $v_R = \mathcal{E} - v_L = 50.0$ V – 20.0 V = 30.0 V. The current in R is therefore $i = (30.0\text{ V})/(10.0\ \Omega) = 3.00$ A, which is also the current in the circuit.

(d) As $t \to \infty,$ the potential across the inductor is due only to its resistance R_L, the potential across it is 20.0 V, and the current through it is 3.00 A. Therefore $R_L = (20.0\text{ V})/(3.00\text{ A}) = 6.67\ \Omega$.

(e) The time constant for this circuit is $\tau = L/(R+R_L)$. Using the equation derived in (a) for v_L, at the end of one time constant v_L is $v_L = \frac{\mathcal{E}}{R+R_L}(R_L + Re^{-1}) = \frac{50.0\text{ V}}{16.67\ \Omega}\left[6.67\,\Omega + (10.0\,\Omega)e^{-1}\right] = 31.0\text{ V}.$

From the graph shown with the problem in the textbook, we read that $t = 2.4$ ms when $v_L = 31.0$ V. So the time constant is 2.4 ms. Solving $\tau = L/(R+R_L)$ for L gives

$L = \tau(R+R_L) = (2.4\text{ ms})(10.0\ \Omega + 6.67\ \Omega) = 40$ mH.

EVALUATE: In this case, the resistance of the inductor is close to the external resistance in the circuit, so it is significant and cannot be ignored.

30.68. **IDENTIFY** and **SET UP:** The current grows in the circuit after the switch is closed. In an R-L circuit the full emf initially is across the inductance and after a long time is totally across the resistance because the inductor opposes changes in the current through it. A solenoid in a circuit is represented as a resistance in series with an inductance. Apply the loop rule to the circuit; the voltage across a resistance is given by Ohm's law, and emf across an inductor is $\mathcal{E}_L = -L\frac{di}{dt}.$

EXECUTE: **(a)** In the R-L circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. As $t \to \infty$ the current in the circuit approaches its final, steady-state value. The final voltage across the solenoid is $iR_L,$ where I is the final current in the circuit. The potential across the external resistor R is 25.0 V after the switch has been closed for a very long time, which is when steady-state has been achieved. Using $V_R = iR$ and $i = \frac{\mathcal{E}}{R+R_L}$ gives $V_R = \frac{\mathcal{E}}{R+R_L}R.$ Solving for R_L gives

$R_L = R\left(\frac{\mathcal{E}}{V_R} - 1\right) = (50.0\ \Omega)\left(\frac{25.0\text{ V}}{25.0\text{ V}} - 1\right) = 0.$ The solenoid has no appreciable resistance.

(b) Kirchhoff's loop rule gives $\mathcal{E} - iR - L\frac{di}{dt} = 0.$ Separating variables and integrating gives

$$\int_0^i \frac{di'}{i' - \mathcal{E}/R} = -\int_0^t \frac{R}{L} dt' \quad \rightarrow \quad \ln\left(\frac{i - \mathcal{E}/R}{-\mathcal{E}/R}\right) = -\frac{R}{L}t.$$

$$i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L}) \quad \rightarrow \quad v_R = \mathcal{E}(1 - e^{-Rt/L}).$$

(c) At the end of the first time constant, we have $\mathcal{E}(1 - e^{-Rt/L}) = \mathcal{E}(1 - e^{-1})$ = (24.0 V)(1 – 1/*e*) = 15.8 V.

From the graph with the problem in the text, we determine that when v_R = 15.8 V, *t* = 8.0 ms, so the time constant is τ = 8.0 ms. Using $\tau = L/R$, we have $L = \tau R$ = (8.0 ms)(50.0 Ω) = 0.40 H.

(d) At steady-state the current is (25.0 V)/(50.0 Ω) = 0.500 A. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$ = (1/2)(0.40 H)(0.500 A)^2 = 0.050 J = 50 mJ.

EVALUATE: We found that the resistance of the inductor is zero. However that really just means that it is much less than the external resistance of 50.0 Ω, so it does not affect the measurements. In reality every inductor has *some* resistance since it is made out of real metal.

30.69. **IDENTIFY:** This problem deals with a solenoid that is within another solenoid.

SET UP: $B = \mu_0 nI$, $\Phi_B = BA\cos\phi = BA$ since the fields are uniform within an ideal solenoid. Refer to Fig. P30.69 in the textbook for the quantities involved.

EXECUTE: (a) We want the flux through the inner coil. The magnetic fields due to both solenoids point in the same direction, so $\Phi_{\text{in}} = B_1A_2 - B_2A_2 = \mu_0 In_1\pi a^2 - \mu_0 In_2\pi a^2$. Using $n = N/\lambda$ this reduces to $\Phi_{\text{in}} = \frac{\mu_0 I(N_1 - N_2)\pi a^2}{\lambda}$.

(b) We want the flux through the outer coil. $\Phi_{\text{out}} = B_1A_1 - B_2A_2 = \mu_0 In_1\pi b^2 - \mu_0 In_2\pi a^2$, which reduces to $\Phi_{\text{out}} = \frac{\mu_0 I\pi}{\lambda}(N_1b^2 - N_2a^2)$.

(c) We want *L*. $\Phi_{\text{in}} = B_1A_2 - B_2A_2 = \mu_0 In_1\pi a^2 - \mu_0 In_2\pi a^2$ and $\Phi_{\text{out}} = B_1A_1 - B_2A_2 = \mu_0 In_1\pi b^2 - \mu_0 In_2\pi a^2$, which reduce to $\Phi_{\text{in}} = \frac{\mu_0 I(N_1 - N_2)\pi a^2}{\lambda}$ and $\Phi_{\text{out}} = \frac{\mu_0 I\pi}{\lambda}(N_1b^2 - N_2a^2)$. The self-inductance is $L = \frac{\Phi_B}{I} = \frac{N_1\Phi_{\text{out}} + N_2\Phi_{\text{in}}}{I}$. Using the fluxes and simplifying gives $L = \frac{\mu_0\pi}{\lambda}(N_1^2b^2 - N_2^2a^2)$.

(d) We want the self-inductance if the inner current is reversed. In this case, B_2 reverses direction. $L = \frac{\Phi_B}{I} = \frac{N_1\Phi_{\text{out}} + N_2\Phi_{\text{in}}}{I}$. Substituting the fluxes with B_2 reversed and simplifying gives

$$L = \frac{\mu_0\pi}{\lambda}(N_1^2b^2 + 2N_1N_2a^2 + N_2^2a^2).$$

(e) We want *L* for the original configuration. Using $L = \frac{\mu_0\pi}{\lambda}(N_1^2b^2 - N_2^2a^2)$ with the same numbers gives *L* = 10.3 mH.

(f) We want *L* for the reversed configuration. Using $L = \frac{\mu_0\pi}{\lambda}(N_1^2b^2 + 2N_1N_2a^2 + N_2^2a^2)$ with λ = 20.0 cm, *a* = 1.00 cm, *b* = 2.00 cm, N_1 = 1200 turns, and N_2 = 750 turns, we get *L* = 16.0 mH.

EVALUATE: In case (d) if *a* = 0 (no inner coil), $L = \frac{\mu_0\pi}{\lambda}(N_1^2b^2) = \frac{\mu_0\pi b^2N_1^2}{\lambda}$, which is just the formula for the self-inductance of a single solenoid.

30.70. **IDENTIFY:** Apply $L = \dfrac{N\Phi_B}{i}$ to calculate L.

SET UP: In the air the magnetic field is $B_{\text{Air}} = \dfrac{\mu_0 N i}{W}$. In the liquid, $B_{\text{L}} = \dfrac{\mu N i}{W}$.

EXECUTE: **(a)** $\Phi_B = BA = B_{\text{L}} A_{\text{L}} + B_{\text{Air}} A_{\text{Air}} = \dfrac{\mu_0 N i}{W}[(D-d)W] + \dfrac{K\mu_0 N i}{W}(dW) = \mu_0 N i\left[(D-d)+Kd\right]$.

$$L = \frac{N\Phi_B}{i} = \mu_0 N^2[(D-d)+Kd] = L_0 - L_0\frac{d}{D} + L_{\text{f}}\frac{d}{D} = L_0 + \left(\frac{L_{\text{f}} - L_0}{D}\right)d.$$

$$d = \left(\frac{L - L_0}{L_{\text{f}} - L_0}\right)D, \text{ where } L_0 = \mu_0 N^2 D, \text{ and } L_{\text{f}} = K\mu_0 N^2 D.$$

(b) and **(c)** Using $K = \chi_{\text{m}} + 1$ we can find the inductance for any height $L = L_0\left(1 + \chi_{\text{m}}\dfrac{d}{D}\right)$.

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d = D/4$	0.63024 H	0.63000 H
$d = D/2$	0.63048 H	0.62999 H
$d = 3D/4$	0.63072 H	0.62999 H
$d = D$	0.63096 H	0.62998 H

The values $\chi_{\text{m}}(O_2) = 1.52\times10^{-3}$ and $\chi_{\text{m}}(\text{Hg}) = -2.9\times10^{-5}$ have been used.

EVALUATE: **(d)** The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

30.71. **IDENTIFY:** Apply Kirchhoff's loop rule to the to and bottom branches of the circuit.

SET UP: Just after the switch is closed the current through the inductor is zero and the charge on the capacitor is zero.

EXECUTE: **(a)** $\mathcal{E} - i_1 R_1 - L\dfrac{di_1}{dt} = 0 \Rightarrow i_1 = \dfrac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t})$.

$$\mathcal{E} - i_2 R_2 - \frac{q_2}{C} = 0 \Rightarrow -\frac{di_2}{dt}R_2 - \frac{i_2}{C} = 0 \Rightarrow i_2 = \frac{\mathcal{E}}{R_2}e^{-(1/R_2C)t}.$$

$$q_2 = \int_0^t i_2 dt' = -\frac{\mathcal{E}}{R_2}R_2 C e^{-(1/R_2C)t'}\Big|_0^t = \mathcal{E}C(1 - e^{-(1/R_2C)t}).$$

(b) $i_1(0)\dfrac{\mathcal{E}}{R_1}(1 - e^0) = 0, i_2 = \dfrac{\mathcal{E}}{R_2}e^0 = \dfrac{48.0\text{ V}}{5000\ \Omega} = 9.60\times10^{-3}$ A.

(c) As $t \to \infty$: $i_1(\infty) = \dfrac{\mathcal{E}}{R_1}(1 - e^{-\infty}) = \dfrac{\mathcal{E}}{R_1} = \dfrac{48.0\text{ V}}{25.0\ \Omega} = 1.92$ A, $i_2 = \dfrac{\mathcal{E}}{R_2}e^{-\infty} = 0$. A good definition of a "long time" is many time constants later.

(d) $i_1 = i_2 \Rightarrow \dfrac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = \dfrac{\mathcal{E}}{R_2}e^{-(1/R_2C)t} \Rightarrow (1 - e^{-(R_1/L)t}) = \dfrac{R_1}{R_2}e^{-(1/R_2C)t}$. Expanding the exponentials like $e^x = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \ldots$, we find: $\dfrac{R_1}{L}t - \dfrac{1}{2}\left(\dfrac{R_1}{L}\right)^2 t^2 + \ldots = \dfrac{R_1}{R_2}\left(1 - \dfrac{t}{RC} + \dfrac{t^2}{2R^2C^2} - \ldots\right)$ and

$t\left(\dfrac{R_1}{L} + \dfrac{R_1}{R_2{}^2 C}\right) + O(t^2) + \ldots = \dfrac{R_1}{R_2}$, if we have assumed that $t \ll 1$. Therefore:

$$t \approx \frac{1}{R_2}\left(\frac{1}{(1/L) + (1/R_2^2 C)}\right) = \left(\frac{LR_2C}{L + R_2^2 C}\right) = \left(\frac{(8.0\text{ H})(5000\ \Omega)(2.0\times10^{-5}\text{ F})}{8.0\text{ H} + (5000\ \Omega)^2(2.0\times10^{-5}\text{ F})}\right) = 1.6\times10^{-3}\text{s}.$$

(e) At $t = 1.57\times10^{-3}$ s: $i_1 = \frac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t}) = \frac{48\text{ V}}{25\ \Omega}(1-e^{-(25/8)t}) = 9.4\times10^{-3}$ A.

(f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$$i_{1/2} = 0.960\text{ A} = i_1 = \frac{\mathcal{E}}{R_1}(1-e^{-(R_1/L)t}) = (1.92\text{ A})(1-e^{-(R_1/L)t}).$$

$$e^{-(R_1/L)t} = 0.500 \Rightarrow t = -\frac{L}{R_1}\ln(0.5) = -\frac{8.0\text{ H}}{25\ \Omega}\ln(0.5) = 0.22\text{ s}.$$

EVALUATE: i_1 is initially zero and rises to a final value of 1.92 A. i_2 is initially 9.60 mA and falls to zero, q_2 is initially zero and rises to $q_2 = \mathcal{E}C = 960\ \mu$C.

30.72. **IDENTIFY** and **SET UP:** Apply $L = \frac{N\Phi_B}{i}$ to calculate *L*, then solve for the number of turns *N*. Treat the solenoid as being ideal.

EXECUTE: $L = \frac{N\Phi}{i} = \frac{N\mu_0 inA}{i} = \frac{\mu_0 N^2 A}{l}$. $N = \sqrt{\frac{Ll}{\mu_0 A}} = \sqrt{\frac{(4.4\text{ H})(2\text{ m})}{\mu_0\pi(0.5\text{ m})^2}} = 3000,$ which is choice (b).

EVALUATE: This solenoid is far from ideal since its diameter is half its length, but we can get a rough estimate of the number of coils.

30.73. **IDENTIFY** and **SET UP:** The current in the circuit is $i = i_0e^{-Rt/L}$. Solve for the time t_{half} for the current to reach one-half its original value.

EXECUTE: $i_0/2 = i_0e^{-Rt/L}$, so $t_{\text{half}} = (L/R)\ln 2 = [(4.4\text{ H})/(0.005\ \Omega)]\ln 2 = 610\text{ s} \approx 10$ min, choice (b).

EVALUATE: This result is true if no more of the magnet loses its superconductivity. If more of it does so, the time will be less than this because *R* will be greater.

30.74. **IDENTIFY** and **SET UP:** In Problem 30.73, we saw that $t_{\text{half}} = (L/R)\ln 2$.

EXECUTE: The resistance is increasing, and t_{half} is inversely proportional to *R*, so the time will be shorter, which is choice (a).

EVALUATE: How much shorter the time will be will depend on how fast the magnet is losing its superconductivity.

30.75. **IDENTIFY:** The magnetic energy stored in the magnet is converted into thermal energy which evaporates the liquid helium.

SET UP: The magnetic energy is $U_L = \frac{1}{2}Li^2$. The heat *Q* to evaporate a mass *m* of liquid is $Q = mL_v$.

EXECUTE: $\frac{1}{2}Li^2 = mL_v$. Solving for *m* gives

$m = Li^2/2L_v = (4.4\text{ H})(750\text{ A})^2/[2(20900\text{ J/kg})] = 59\text{ kg} \approx 60$ kg, which is choice (c).

EVALUATE: This is a lot of liquid helium! It is important to avoid quenches!