43

NUCLEAR PHYSICS

VP43.4.1. **IDENTIFY:** This problem is about the binding energy of an O-16 nucleus.

SET UP and **EXECUTE:** **(a)** We want the mass defect ΔM. Use the numerical values given in the text and Table 43.2. For O-16, $Z = 8$ and $A = 16$ so $N = 8$.

$\Delta M = ZM_{\text{H}} + Nm_{\text{n}} - {}^{A}_{Z}M.$

$\Delta M = 8(1.007825\text{ u}) - 8(1.008655\text{ u}) - 15.994915\text{ u} = 0.137005\text{ u}.$

(b) We want the binding energy. $E_{\text{B}} = \Delta Mc^2 = (0.137005\text{ u})(931.5\text{ MeV/u}) = 127.6\text{ MeV}.$

(c) We want the binding energy per nucleon. O-16 has 16 nucleons, so

$$E_{\text{B}}/\text{nucleon} = (127.6\text{ MeV})/16 = 7.976\text{ MeV/nucleon}.$$

EVALUATE: From the graph in Figure 43.2, we see that O-16 is more tightly bound that C-12 but less so than Ni-62.

VP43.4.2. **IDENTIFY:** This problem is about the energy of the Cu-63 nucleus.

SET UP and **EXECUTE:** We know that the mass defect is 0.5919378 u. **(a)** The binding energy E_{B} is the energy of the mass defect. $E_{\text{B}} = (0.5919378\text{ u})(931.5\text{ MeV/u}) = 551.4\text{ MeV}.$

(b) $E_{\text{B}}/\text{nucleon} = (551.4\text{ MeV})/(63\text{ nucleons}) = 8.752\text{ MeV/nucleon}.$

(c) We want the mass of the Cu-63 atom. Use Eq. (43.10) with $Z = 29$ and $A = 63$, so $N = 34$.

EVALUATE: Note that the C-63 mass is about 63 u, but *not quite*.

VP43.4.3. **IDENTIFY:** This problem is about nuclear binding energy.

SET UP: The binding energy per nucleon is E_{B}/A where E_{B} is the energy of the mass defect. Use the given mass defects to calculate E_{B}.

EXECUTE: **(a)** As-75. $E_{\text{B}}/A = (0.7005604\text{ u})(931.5\text{ MeV/u})/75 = 8.701\text{ MeV/nucleon}.$

(b) Sm-150. Use the same method as in (a) with $A = 150$, giving $E_{\text{B}}/A = 8.262\text{ MeV/nucleon}.$

(c) Ra-225. $A = 225$ gives $E_{\text{B}}/A = 7.668\text{ MeV/nucleon}.$

(d) As A increases from 75 to 225, E_{B}/A decreases from 8.701 MeV/nucleon to 7.668 MeV/nucleon. This behavior agrees with Figure 43.2.

EVALUATE: Note that the *total* binding energy increases as A increases, but the binding energy *per nucleon* decreases.

VP43.4.4. **IDENTIFY:** In this problem, we use the semiempirical mass formula to calculate the binding energy.

SET UP: Eq. (43.11) gives the formula and the constants $C_1, C_2, \ldots, C_5$ are given following it. We follow the procedure of Example 43.4.

EXECUTE: **(a)** Ru-100. $A = 100$, $Z = 44$. Using Eq. (43.11) gives the following terms.

$C_1A = (15.75\text{ MeV})(100) = 1575$

$-C_2A^{2/3} = -(17.80\text{ MeV})(100^{2/3}) = -383.5$

$-C_3Z(Z-1)/A^{1/3} = -(0.7100\text{ MeV})(44)(43)/(100^{1/3}) = -289.4\text{ MeV}$

$-C_4(A-2Z)^2/A = -(23.69\text{ MeV})(100-88)^2/(100) = -34.11\text{ MeV}$

$+C_5A^{-4/3} = (39\text{ MeV})(100^{-4/3}) = 0.084\text{ MeV}.$

Adding all these terms gives $E_B = 868.1$ MeV. $E_B/A = (868.1 \text{ MeV})/100 = 8.681$ MeV/nucleon.
(b) Hg-200. $A = 200$, $Z = 80$. Use the same procedure as in part (a), giving $E_B = 1584$ MeV,
$E_B/A = (1584 \text{ MeV})/(200) = 7.922$ MeV/nucleon.
EVALUATE: **(c)** From Figure 43.2: For $A = 100$, $E_B/A \approx 8.66$ MeV/nucleon, and we got 8.681 MeV/nucleon.
For $A = 200$, $E_B/A \approx 7.85$ MeV/nucleon, and we got 79.22 MeV/nucleon. Our results agree closely with those in the figure.

VP43.7.1. **IDENTIFY:** This problem is about energy in radioactive decay.
SET UP and **EXECUTE:** **(a)** We want the energy released. Refer to the decay shown in the problem. Using the given masses, the mass difference is 241.056827 u – (237.048172 u + 4.002603 u) = 0.0060520 u. The energy released is the energy of this mass, which is given by (0.006052 u)(931.5 MeV/u) = 5.637 MeV.
(b) We want the kinetic energy of the alpha particle. The rest energy of He is (4 u)(931.5 MeV/u) = 3726 MeV, which is much greater than the energy released. Therefore we do not need to use relativity. Momentum conservation gives $m_\alpha v_\alpha = m_{Np}v_{Np}$, so $v_\alpha/v_{Np} = m_{Np}/m_\alpha$. Using the masses gives $v_{Np} = (4.003/237.0)v_\alpha = 0.016890v_\alpha$. Energy conservation gives $K_\alpha + K_{Np} = 5.637$ MeV. Using $K = \frac{1}{2} mv^2$ and taking the ratio of kinetic energies gives $K_\alpha/K_{Np} = 59.22$. Adding the kinetic energies gives $K_\alpha + K_\alpha/59.22$
$= 5.637$ MeV. $K_\alpha = 5.543$ MeV.
(c) We want the speed of the alpha particle. Use the result from (b) and $K = \frac{1}{2} mv^2$ and solve for v, giving $v_\alpha = 1.63 \times 10^7$ m/s.
EVALUATE: $v/c = 0.16/3.0 = 0.053$, so v is about 5% the speed of light, which means that it was OK to neglect relativity.

VP43.7.2. **IDENTIFY:** This problem is about the nuclear decay of Be-8 to two alpha particles.
SET UP: First find the mass defect and use it to find the energy released. The alpha particles each get one-half of this energy.
EXECUTE: **(a)** We want the kinetic energy. The mass defect is $m_{Be} - 2m_{He}$, which gives [8.0053051 u – 2(4.0026033 u)](931.5 MeV/u) = 0.09175 MeV. Each alpha particle gets half of this, so $K_\alpha = 0.0459$ MeV.
(b) We want the speed of the alpha particles. The rest energy of the alpha is (4.0026 u)(931.5 MeV/u) = 3730 MeV, which is much greater than its kinetic energy. So we do not need to use relativity. Use $K = \frac{1}{2} mv^2$ with $K = 0.0459$ MeV, giving $v_\alpha = 1.49 \times 10^6$ m/s.
EVALUATE: $v/c = 0.0149/3.0 = 0.00497$, so $v \ll c$. Our neglect of relativity was justified.

VP43.7.3. **IDENTIFY:** This problem is about β^- decay.
SET UP: β^- decay is possible if the mass of the original neutral atom is greater than the mass of the final atom. The energy released is the energy of the difference in mass.
EXECUTE: **(a)** The mass of Be-12 is greater than the mass of B-12, so this decay is possible. The energy released is E = (12.026922 u – 12.014353 u)(931.5 MeV/u) = 11.71 MeV.
(b) The mass of Ar-33 is greater than the mass of Cl-33, so this decay is not possible.
(c) The mass of Br-82 is greater than the mass of Kr-82, so this decay is possible. As in part (a), E = (81.9168018 u – 81.9134812 u)(931.5 MeV/u) = 3.093 MeV.
EVALUATE: The greater the mass difference, the greater the energy that is released.

VP43.7.4. **IDENTIFY:** This problem is about β^+ decay.
SET UP: β^+ decay is possible if the mass of the original neutral atom is greater than the mass of the final atom by twice the electron mass. The mass of the electron is 0.000548580 u, so the mass of the original atom must be greater than that of the final atom by 2(0.000548580 u) = 0.001097160 u. The energy released is the energy of the difference in mass minus two electron masses.

EXECUTE: (a) The mass difference is 45.960198 – 45.952627 u = 0.0075710 u. This difference is greater than twice the electron mass, so the decay is possible. The energy released is

$$E = (0.0075710\text{ u} - 0.001097160\text{ u})(931.5\text{ MeV/u}) = 6.030\text{ MeV}.$$

(b) Δm = 133.908514 u – 133.904508 u = 0.0040060 u > $2m_e$, so the decay is possible. The energy is E = (0.0040060 u – 0.00109716 u)(931.5 MeV/u) = 2.710 MeV.

(c) Δm = 66.928202 u – 66.9271275 u = 0.0010745 u < $2m_e$, so β^+ decay is not possible. The original atom's mass is greater than that of the final atom, so electron capture is possible.

EVALUATE: If the mass of the original atom is greater than the mass of the final atom, then β^+ decay and electron capture are *both* possible, so both may occur. A number of radionuclides decay by more than one mode.

VP43.9.1. **IDENTIFY:** This problem deals with radioactive decay. The half-life is $T_{1/2}$ = 15.0 h and the initial activity is dN/dt = –1.60 μCi.

SET UP and **EXECUTE: (a)** $T_{\text{mean}} = T_{1/2}/(\ln 2)$ = (15.0 h)/(ln 2) = 21.6 h = 7.79×10^4 s.

(b) $\lambda = (\ln 2)/T_{1/2}$ = (ln 2)/(15.0 h) = $4.621 \times 10^{-2}\text{ h}^{-1} = 1.28 \times 10^{-5}\text{ s}^{-1}$.

(c) We want the initial number N_0 of Na-24 nuclei. We know the initial activity, so we make use of that.

$$N = N_0 e^{-\lambda t}$$

$$dN/dt = -\lambda N_0 e^{-\lambda t} = -\lambda N_0$$

$$N_0 = -\frac{1}{\lambda}\frac{dN}{dt} = -\frac{1}{1.28\times10^{-5}\text{ s}^{-1}}(-1.50\ \mu\text{Ci})(3.70\times10^{10}\text{ s}^{-1}) = 4.32\times10^{9}.$$

(d) We want dN/dt after 24.0 h. From our work in part (c), we can see that

$$dN/dt = dN/dt)_0\ e^{-\lambda t} = (1.50\ \mu\text{Ci})\ e^{-(0.04621\text{ h}^{-1})(24.0\text{ h})} = 0.495\ \mu\text{Ci}.$$

EVALUATE: In 15 h, dN/dt would be 0.75 μCi, and our result in part (d) is less than that, so it is reasonable.

VP43.9.2. **IDENTIFY:** This problem deals with radioactive decay. The half-life is $T_{1/2}$ = 35.0 h and the initial activity is dN/dt = –0.514 μCi.

SET UP and **EXECUTE: (a)** $\lambda = (\ln 2)/T_{1/2}$ = (ln 2)/(35.0 h) = $1.98 \times 10^{-2}\text{ h}^{-1} = 5.50 \times 10^{-6}\text{ s}^{-1}$.

(b) We want the present number N_0 of Nb-95 nuclei.

$$N = N_0 e^{-\lambda t}$$

$$dN/dt = -\lambda N_0 e^{-\lambda t} = -\lambda N_0$$

Call t = 0 the present instant when dN/dt = –0.514 μCi. Solve the above equation for N_0.

$$N_0 = -\frac{1}{\lambda}\frac{dN}{dt} = -\frac{1}{5.50\times10^{-6}\text{ s}^{-1}}(-0.514\ \mu\text{Ci})(3.70\times10^{10}\text{ s}^{-1}) = 3.46\times10^{9}.$$

(c) We want the initial number of Nb-95 nuclei. The 3.46×10^9 nuclei from part (b) is 23.8% of the original number of nuclei. So $0.238N_0 = 3.46 \times 10^9$, which gives $N_0 = 1.45 \times 10^{10}$.

(d) We want the time to now. Solve for t and use the results we have found.

$$N = N_0 e^{-\lambda t}$$

$$t = -\frac{\ln(N/N_0)}{\lambda} = \frac{\ln\left(\dfrac{3.46\times10^{9}}{1.45\times10^{10}}\right)}{0.0198\text{ h}^{-1}} = 72.5\text{ h}.$$

EVALUATE: The number of Mo-95 nuclei present now is $(0.762)(1.45 \times 10^{10}) = 1.11 \times 10^{10}$. The sum of nuclei now is $1.11 \times 10^{10} + 3.46 \times 10^9 = 1.45 \times 10^{10}$, which agrees with our result in (c).

VP43.9.3. **IDENTIFY:** We are dealing with radioactive decay. The initial activity is 0.415 μCi, and 45.0 s later it is 0.121 μCi.

SET UP and **EXECUTE:** **(a)** We want the decay constant and half-life. If we call R the activity, we have seen that it decreases exponentially. Use this fact and solve for the decay constant and then use it to find the half-life.

$$R = R_0 e^{-\lambda t}$$

$$\lambda = -\frac{\ln(R/R_0)}{t} = -\frac{\ln\left(\dfrac{0.121\ \mu\text{Ci}}{0.425\ \mu\text{Ci}}\right)}{45.0\ \text{s}} = 0.0279\ \text{s}^{-1}.$$

$$T_{1/2} = (\ln 2)/\lambda = (\ln 2)/(0.279\ \text{s}^{-1}) = 24.8\ \text{s}.$$

(b) We want the number of nuclei in an excited state. These are the nuclei that give off the radiation, so we want to find the number of undecayed nuclei initially and 45.0 s later.

Initially:

$$dN/dt = -\lambda N_0 e^{-\lambda t} = -\lambda N_0$$

$$(-0.425\ \mu\text{Ci})(3.70 \times 10^{10}\ \text{s}^{-1}) = -(0.0279\ \text{s}^{-1})N_0$$

$$N_0 = 5.63 \times 10^5.$$

After 45.0 s: Using the same approach gives $(-0.121\ \mu\text{Ci})(3.70 \times 10^{10}\ \text{s}^{-1}) = -(0.0279\ \text{s}^{-1})N_0$

$N_0 = 1.60 \times 10^5$.

EVALUATE: The number at 45.0 s is less than the initial number, so our result is reasonable.

VP43.9.4. **IDENTIFY:** This problem is about carbon-14 dating.

SET UP: The half-life of C-14 is 5730 y. If we call R the activity level per gram, it follows from previous work that $R = R_0 e^{-\lambda t}$.

EXECUTE: **(a)** We want the age of the sample. The activity per unit mass that you detect now is 113 decays per 20 min per 0.510 g of carbon. So R = [(113 decays)/(20 min)]/(0.510 g) = 0.18464 Bq/g. Use $R = R_0 e^{-\lambda t}$ with $\lambda = (\ln 2)/T_{1/2}$ and solve for t.

$$t = \frac{\ln(R/R_0)}{\lambda} = -\frac{T_{1/2}\ln(R/R_0)}{\ln 2} = -\frac{(5730\ \text{y})\ln\left(\dfrac{0.18464\ \text{Bq/g}}{0.255\ \text{Bq/g}}\right)}{45.0\ \text{s}} = 2670\ \text{y}.$$

(b) We want the current number of C-14 nuclei. Call $t = 0$ the present time. In your sample, the present activity is (113 decays)/(20 min) = 0.094167 Bq. At $t = 0$ (the present time), we have

$$N_0 = \frac{1}{\lambda}\frac{dN}{dt} = -\frac{T_{1/2}}{\ln 2}(-0.094167\ \text{Bq}) = \frac{(5730\ \text{y})(0.094167\ \text{Bq})}{\ln 2} = 2.46 \times 10^{10}.$$

EVALUATE: The age of your sample is less than one half-life, so its activity (0.18464 Bq/g) should be greater than half the original activity (0.255 Bq/g), which is what we have found.

43.1. **IDENTIFY** and **SET UP:** The pre-subscript is Z, the number of protons. The pre-superscript is the mass number A. $A = Z + N$, where N is the number of neutrons.

EXECUTE: **(a)** ${}^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.

(b) ${}^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.

(c) ${}^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.

EVALUATE: The number of protons determines the chemical element.

43.2. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the 1s level. Calculate the energy splitting between these states and relate this to the frequency of the photons.

SET UP: When the spin component is parallel to the field the interaction energy is $U = -\mu_z B$. When the spin component is antiparallel to the field the interaction energy is $U = +\mu_z B$. The transition energy for a transition between these two states is $\Delta E = 2\mu_z B$, where $\mu_z = 2.7928\mu_n$. The transition energy is related to the photon frequency by $\Delta E = hf$, so $2\mu_z B = hf$.

EXECUTE: $B = \dfrac{hf}{2\mu_z} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(22.7\times10^{6}\text{ Hz})}{2(2.7928)(5.051\times10^{-27}\text{ J/T})} = 0.533\text{ T}$

EVALUATE: This magnetic field is easily achievable. Photons of this frequency have wavelength $\lambda = c/f = 13.2$ m. These are radio waves.

43.3. **(a) IDENTIFY:** Find the energy equivalent of the mass defect.

SET UP: A $^{11}_{5}\text{B}$ atom has 5 protons, $11-5=6$ neutrons, and 5 electrons. The mass defect therefore is $\Delta M = 5m_p + 6m_n + 5m_e - M(^{11}_{5}\text{B})$.

EXECUTE: $\Delta M = 5(1.0072765\text{ u}) + 6(1.0086649\text{ u}) + 5(0.0005485799\text{ u}) - 11.009305\text{ u} = 0.08181\text{ u}$.

The energy equivalent is $E_B = (0.08181\text{ u})(931.5\text{ MeV/u}) = 76.21\text{ MeV}$.

(b) IDENTIFY and **SET UP:** Eq. (43.11): $E_B = C_1A - C_2A^{2/3} - C_3Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$.

The fifth term is zero since Z is odd but N is even. $A = 11$ and $Z = 5$.

EXECUTE:

$E_B = (15.75\text{ MeV})(11) - (17.80\text{ MeV})(11)^{2/3} - (0.7100\text{ MeV})5(4)/11^{1/3} - (23.69\text{ MeV})(11-10)^2/11$.

$$E_B = +173.25\text{ MeV} - 88.04\text{ MeV} - 6.38\text{ MeV} - 2.15\text{ MeV} = 76.68\text{ MeV}.$$

The percentage difference between the calculated and measured E_B is

$$\frac{76.68\text{ MeV} - 76.21\text{ MeV}}{76.21\text{ MeV}} = 0.6\%.$$

EVALUATE: Eq. (43.11) has a greater percentage accuracy for ^{62}Ni. The semi-empirical mass formula is more accurate for heavier nuclei.

43.4. **IDENTIFY:** The binding energy E_B is the difference between the energy of the constituent nuclides and final nuclide, converted to energy units.

SET UP: Use the masses in Table 43.2 and $m_n = 1.008665$ u for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

For $^{11}_{5}\text{B}$: $E_B(\text{B-11}) = 5m_H + 6m_n - m_{\text{B-11}}$.

For $^{11}_{6}\text{C}$: $E_B(\text{C-11}) = 6m_H + 5m_n - m_{\text{C-11}}$.

EXECUTE: **(a)** $^{11}_{5}\text{B}$: $E_B = [5(1.007825\text{ u}) + 6(1.008665\text{ u}) - 11.009305\text{ u}](931.5\text{ MeV/u}) = 76.21\text{ MeV}$.

$^{11}_{6}\text{C}$: $E_B = [6(1.007825\text{ u}) + 5(1.008665\text{ u}) - 11.011434\text{ u}](931.5\text{ MeV/u}) = 73.44\text{ MeV}$.

EVALUATE: **(b)** The $^{11}_{5}\text{B}$ has a larger binding energy than the $^{11}_{6}\text{C}$. This is probably due to the fact that $^{11}_{5}\text{B}$ has one fewer proton and one more neutron than $^{11}_{6}\text{C}$. The electrical repulsion between protons tends to decrease binding energy while the extra neutron tends to increase it without a corresponding electrical repulsion.

43.5. **IDENTIFY:** The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

SET UP: In terms of the masses of the particles involved, the binding energy is $E_B = (6m_H + 6m_n - m_{C-12})c^2$.

EXECUTE: **(a)** Using the values from Table 43.2, we get

$$E_B = [6(1.007825\text{ u}) + 6(1.008665\text{ u}) - 12.000000\text{ u}](931.5\text{ MeV/u}) = 92.16\text{ MeV}.$$

(b) The binding energy per nucleon is $(92.16\text{ MeV})/(12\text{ nucleons}) = 7.680\text{ MeV/nucleon}$.

(c) The energy of the C-12 nucleus is $(12.0000\ \text{u})(931.5\ \text{MeV/u}) = 11178\ \text{MeV}$. Therefore the percent of the mass that is binding energy is $\dfrac{92.16\ \text{MeV}}{11178\ \text{MeV}} = 0.8245\%$.

EVALUATE: The binding energy of $92.16\ \text{MeV}$ binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

43.6. **IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

SET UP: 1 u is equivalent to 931.5 MeV. $^{238}_{92}\text{U}$ has 92 protons, 146 neutrons and 238 nucleons.

EXECUTE: **(a)** $146m_\text{n} + 92m_\text{H} - m_\text{U} = 1.93\ \text{u}$.

(b) $1.80\times10^3\ \text{MeV}$.

(c) $7.57\ \text{MeV}$ per nucleon (using $931.5\ \text{MeV/u}$ and 238 nucleons).

EVALUATE: The binding energy per nucleon we calculated agrees with Figure 43.2 in the textbook.

43.7. **IDENTIFY:** This problem is about the binding energy of the atomic nucleus.

SET UP and **EXECUTE:** The target variable is the mass of a neutral Fe-56 atom. Since the atom is neutral, it contains all of its electrons. Use Eq. (43.10) and solve for the mass of Fe-56.

$$^{56}_{26}M = ZM_\text{H} + Nm_\text{n} - E_\text{B}/c^2$$

$$^{56}_{26}M = 26(1.007825\ \text{u}) + 30(1.008665\ \text{u}) - \left[(8.79\ \text{MeV})(56)/c^2\right]\left(\frac{1\ \text{u}}{931.5\ \text{MeV}/c^2}\right) = 55.935\ \text{u}.$$

EVALUATE: If there were no binding energy, the mass would be $26M_\text{H} + 30m_\text{n} = 56.463$ u. The binding energy makes a detectable difference in the mass of the atom.

43.8. **IDENTIFY:** The binding energy E_B is the difference between the energy of the constituent nuclides and final nuclide, converted to energy units.

SET UP: Use the masses in Table 43.2 and $m_\text{n} = 1.008665$ u for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

For an alpha particle: ^4_2He: $E_\text{B}(\alpha) = 2m_\text{H} + 2m_\text{n} - m_\text{He-4}$.

For $^{12}_6\text{C}$: $E_\text{B}(\text{C-12}) = 6m_\text{H} + 6m_\text{n} - m_\text{C-12}$.

EXECUTE: ^4_2He: $E_\text{B} = [2(1.007825\ \text{u}) + 2(1.008665\ \text{u}) - 4.002603\ \text{u}](931.5\ \text{MeV/u}) = 28.296\ \text{MeV}$, so 3 times the binding energy of the alpha particle is $3(28.296\ \text{MeV}) = 84.889\ \text{MeV}$.

$^{12}_6\text{C}$: $E_\text{B} = [6(1.007825\ \text{u}) + 6(1.008665\ \text{u}) - 12.000000\ \text{u}](931.5\ \text{MeV/u}) = 92.163\ \text{MeV}$.

EVALUATE: The binding energy of $^{12}_6\text{C}$ is greater than 3 times the binding energy of an alpha particle. This is reasonable since in addition to binding individual alpha particles, it takes additional energy to bind three of them in the nucleus to form carbon.

43.9. **IDENTIFY:** Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

SET UP: The binding energy of a deuteron is 2.224 MeV and the energy of the photon is $E = hc/\lambda$. Kinetic energy is $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** The energy of the photon is

$$E_\text{ph} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{3.50\times10^{-13}\ \text{m}} = 5.68\times10^{-13}\ \text{J}.$$

The binding of the deuteron is $E_\text{B} = 2.224\ \text{MeV} = 3.56\times10^{-13}\ \text{J}$. Therefore the kinetic energy is

$$K = (5.68 - 3.56)\times10^{-13}\ \text{J} = 2.12\times10^{-13}\ \text{J} = 1.32\ \text{MeV}.$$

(b) The particles share the energy equally, so each gets half. Solving the kinetic energy for v gives

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(1.06\times10^{-13}\ \text{J})}{1.6605\times10^{-27}\ \text{kg}}}=1.13\times10^{7}\ \text{m/s}.$$

EVALUATE: Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

43.10. **IDENTIFY:** The mass defect is the total mass of the constituents minus the mass of the atom.

SET UP: 1 u is equivalent to 931.5 MeV. ${}^{14}_{7}\text{N}$ has 7 protons and 7 neutrons. ${}^{4}_{2}\text{He}$ has 2 protons and 2 neutrons.

EXECUTE: **(a)** $7(m_\text{n}+m_\text{H})-m_\text{N}=0.112\ \text{u},$ which is 105 MeV, or 7.48 MeV per nucleon.

(b) Similarly, $2(m_\text{H}+m_\text{n})-m_\text{He}=0.03038\ \text{u}=28.3\ \text{MeV},$ or 7.07 MeV per nucleon.

EVALUATE: **(c)** The binding energy per nucleon is a little less for ${}^{4}_{2}\text{He}$ than for ${}^{14}_{7}\text{N}.$ This is in agreement with Figure 43.2 in the textbook.

43.11. **IDENTIFY:** We are looking at the decay of C-11.

SET UP and **EXECUTE:** **(a)** In positron decay, a proton changes to a neutron and an electron. So the number of protons decreases from 6 to 5 and the number of neutrons increases from 5 to 6. So the daughter nucleus contains 5 protons and 6 neutrons.

(b) For positron decay to occur, the mass of the parent must be at least two electron masses greater than the mass of the daughter. The daughter nucleus has 5 protons, so it is boron (B). The decay is

$${}^{11}_{6}\text{C}\to{}^{11}_{5}\text{B}+\beta^{+}+\nu_\text{e}.$$

Use Table 43.2 for the mass of boron and m_e = 0.000548580 u. The initial mass is 11.011433 u and the final mass is 11.009305 u + 2(0.000548580 u) = 11.01040216 u. The difference in mass is 11.01040216 u – 11.011433 u = –0.001030840 u. The energy released is the energy of the lost mass, which is (0.001030840 u)(931.5 MeV/u) = 0.960 MeV.

EVALUATE: Notice that we need to use *twice* the mass of the positron for positron decay.

43.12. **IDENTIFY:** Compare the total mass on each side of the reaction equation. Neglect the masses of the neutrino and antineutrino.

SET UP: 1 u is equivalent to 931.5 MeV.

EXECUTE: **(a)** The energy released is the energy equivalent of $m_\text{n}-m_\text{p}-m_\text{e}=8.40\times10^{-4}\ \text{u},$ or 783 keV.

(b) $m_\text{n}>m_\text{p},$ and the decay is not possible.

EVALUATE: β^- and β^+ particles have the same mass, equal to the mass of an electron.

43.13. **IDENTIFY:** In each case determine how the decay changes A and Z of the nucleus. The β^+ and β^- particles have charge but their nucleon number is $A=0.$

(a) SET UP: α-decay: Z decreases by 2, $A=N+Z$ decreases by 4 (an α particle is a ${}^{4}_{2}\text{He}$ nucleus).

EXECUTE: ${}^{239}_{94}\text{Pu}\to{}^{4}_{2}\text{He}+{}^{235}_{92}\text{U}.$

(b) SET UP: β^- decay: Z increases by 1, $A=N+Z$ remains the same (a β^- particle is an electron, ${}^{0}_{-1}\text{e}$).

EXECUTE: ${}^{24}_{11}\text{Na}\to{}^{0}_{-1}\text{e}+{}^{24}_{12}\text{Mg}.$

(c) SET UP: β^+ decay: Z decreases by 1, $A=N+Z$ remains the same (a β^+ particle is a positron, ${}^{0}_{+1}\text{e}$).

EXECUTE: ${}^{15}_{8}\text{O}\to{}^{0}_{+1}\text{e}+{}^{15}_{7}\text{N}.$

EVALUATE: In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.

43.14. IDENTIFY: The energy released is equal to the mass defect of the initial and final nuclei.
SET UP: The mass defect is equal to the difference between the initial and final masses of the constituent particles.
EXECUTE: **(a)** The mass defect is $238.050788\text{ u} - 234.043601\text{ u} - 4.002603\text{ u} = 0.004584\text{ u}$. The energy released is $(0.004584\text{ u})(931.5\text{ MeV/u}) = 4.270\text{ MeV}$.

(b) Take the ratio of the two kinetic energies, using the fact that $K = p^2/2m$:

$$\frac{K_{\text{Th}}}{K_{\alpha}} = \frac{\dfrac{p_{\text{Th}}^2}{2m_{\text{Th}}}}{\dfrac{p_{\alpha}^2}{2m_{\alpha}}} = \frac{m_{\alpha}}{m_{\text{Th}}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{\text{Th}} = \frac{4}{234+4}K_{\text{Total}} = \frac{4}{238}(4.270\text{ MeV}) = 0.07176\text{ MeV} = 1.148\times10^{-14}\text{ J}.$$

Solving for v in the kinetic energy gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.148\times10^{-14}\text{ J})}{(234.043601)(1.6605\times10^{-27}\text{ kg})}} = 2.431\times10^5\text{ m/s}.$$

EVALUATE: As we can see by the ratio of kinetic energies in part (b), the alpha particle will have a much higher kinetic energy than the thorium.

43.15. IDENTIFY: This problem is about β^+ decay.
SET UP and **EXECUTE:** For β^+ decay, the mass of the decay products must be at least two electron-masses greater than the mass of the original atom. The decay in this case is

$${}^{12}_{7}\text{N} \rightarrow {}^{12}_{6}\text{C} + \beta^+ + \nu_e.$$

The target variable is the mass M of the N-12. Using the energy released given in the problem, we have $M = m_{\text{C}} + 2m_{\text{e}} + 16.316\text{ MeV}/c^2$. Using Table 43.2 gives
$M = 12.000000\text{ u} + 2(0.000548580\text{ u}) + (16.316\text{ MeV})[\text{u}/(931.5\text{ MeV})] = 12.018611\text{ u}$.
EVALUATE: Check: M is greater than the mass of C-12, as it should be.

43.16. IDENTIFY: In each reaction the nucleon number and the total charge are conserved.
SET UP: An α particle has charge $+2e$ and nucleon number 4. An electron has charge $-e$ and nucleon number zero. A positron has charge $+e$ and nucleon number zero.

EXECUTE: **(a)** A proton changes to a neutron, so the emitted particle is a positron (β^+).

(b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

(c) A neutron changes to a proton, so the emitted particle is an electron (β^-).

EVALUATE: We have considered the conservation laws. We have not determined if the decays are energetically allowed.

43.17. IDENTIFY: Determine the energy released during tritium decay.
SET UP: In beta decay an electron, e^-, is emitted by the nucleus. The beta decay reaction is ${}^{3}_{1}\text{H} \rightarrow \text{e}^- + {}^{3}_{2}\text{He}$. If neutral atom masses are used, ${}^{3}_{1}\text{H}$ includes one electron and ${}^{3}_{2}\text{He}$ includes two electrons. One electron mass cancels and the other electron mass in ${}^{3}_{2}\text{He}$ represents the emitted electron. Or, we can subtract the electron masses and use the nuclear masses. The atomic mass of ${}^{3}_{2}\text{He}$ is 3.016029 u.

EXECUTE: **(a)** The mass of the ${}^{3}_{1}\text{H}$ nucleus is $3.016049\text{ u} - 0.000549\text{ u} = 3.015500\text{ u}$. The mass of the ${}^{3}_{2}\text{He}$ nucleus is $3.016029\text{ u} - 2(0.000549\text{ u}) = 3.014931\text{ u}$. The nuclear mass of ${}^{3}_{2}\text{He}$ plus the mass of the emitted electron is $3.014931\text{ u} + 0.000549\text{ u} = 3.015480\text{ u}$. This is slightly less than the nuclear mass for ${}^{3}_{1}\text{H}$, so the decay is energetically allowed.

(b) The mass decrease in the decay is $3.015500\text{ u} - 3.015480\text{ u} = 2.0\times10^{-5}\text{ u}$. Note that this can also be calculated as $m({}^{3}_{1}\text{H}) - m({}^{4}_{2}\text{He})$, where atomic masses are used. The energy released is $(2.0\times10^{-5}\text{ u})(931.5\text{ MeV/u}) = 0.019\text{ MeV}$. The total kinetic energy of the decay products is 0.019 MeV,
or 19 keV.

EVALUATE: The energy is not shared equally by the decay products because they have unequal masses.

43.18. **IDENTIFY:** This problem involves radioactive decay and half-life.

SET UP and **EXECUTE:** The target variable is the number of decays in $0.500T_{1/2}$. We are working with the half-life of the isotope, so it is convenient to express the decay in terms of base 2 as follows:

$$N = N_0e^{-\lambda t} = N_0 2^{-t/T_{1/2}} = N_0 2^{-(0.500T_{1/2})/T_{1/2}} = N_0 2^{-0.500}.$$

The number of decays is $\Delta N = N_0 - N = N_0 - N_0e^{-0.500} = 0.293N_0$.

EVALUATE: Note that ΔN is *not* $N_0/4$.

43.19. **IDENTIFY** and **SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$ The mass of a single nucleus is $124m_\text{p} = 2.07\times10^{-25}\text{ kg}$.

$$|dN/dt| = 0.350\text{ Ci} = 1.30\times10^{10}\text{ Bq},\quad |dN/dt| = \lambda N.$$

EXECUTE: $N = \dfrac{6.13\times10^{-3}\text{ kg}}{2.07\times10^{-25}\text{ kg}} = 2.96\times10^{22}$; $\lambda = \dfrac{|dN/dt|}{N} = \dfrac{1.30\times10^{10}\text{ Bq}}{2.96\times10^{22}} = 4.39\times10^{-13}\text{ s}^{-1}$.

$$T_{1/2} = \frac{\ln 2}{\lambda} = 1.58\times10^{12}\text{ s} = 5.01\times10^{4}\text{ y}.$$

EVALUATE: Since $T_{1/2}$ is very large, the activity changes very slowly.

43.20. **IDENTIFY:** The equation $N = N_0e^{-\lambda t}$ can be written as $N = N_0 2^{-t/T_{1/2}}$.

SET UP: The amount of elapsed time since the source was created is roughly 2.5 years.

EXECUTE: The current activity is $N = (5000\text{ Ci})2^{-(2.5\text{ yr})/(5.271\text{ yr})} = 3600\text{ Ci}$. The source is barely usable.

EVALUATE: Alternatively, we could calculate $\lambda = \dfrac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$ and use $N = N_0e^{-\lambda t}$ directly to obtain the same answer.

43.21. **IDENTIFY:** From the known half-life, we can find the decay constant, the rate of decay, and the activity.

SET UP: $\lambda = \dfrac{\ln 2}{T_{1/2}}$. $T_{1/2} = 4.47\times10^{9}\text{ yr} = 1.41\times10^{17}\text{ s}$. The activity is $\left|\dfrac{dN}{dt}\right| = \lambda N$. The mass of one ${}^{238}\text{U}$ is approximately $238m_\text{p}$. $1\text{ Ci} = 3.70\times10^{10}\text{ decays/s}$.

EXECUTE: **(a)** $\lambda = \dfrac{\ln 2}{1.41\times10^{17}\text{ s}} = 4.92\times10^{-18}\text{ s}^{-1}$.

(b) $N = \dfrac{|dN/dt|}{\lambda} = \dfrac{3.70\times10^{10}\text{ Bq}}{4.92\times10^{-18}\text{ s}^{-1}} = 7.52\times10^{27}$ nuclei. The mass m of uranium is the number of nuclei times the mass of each one. $m = (7.52\times10^{27})(238)(1.67\times10^{-27}\text{ kg}) = 2.99\times10^{3}\text{ kg}$.

(c) $N = \frac{10.0\times10^{-3}\text{ kg}}{238m_\text{p}} = \frac{10.0\times10^{-3}\text{ kg}}{238(1.67\times10^{-27}\text{ kg})} = 2.52\times10^{22}$ nuclei.

$\left|\frac{dN}{dt}\right| = \lambda N = (4.92\times10^{-18}\text{ s}^{-1})(2.52\times10^{22}) = 1.24\times10^{5}$ decays/s.

EVALUATE: Because ^{238}U has a very long half-life, it requires a large amount (about 3000 kg) to have an activity of a 1.0 Ci.

43.22. IDENTIFY: From the half-life and mass of an isotope, we can find its initial activity rate. Then using the half-life, we can find its activity rate at a later time.

SET UP: The activity $|dN/dt| = \lambda N$. $\lambda = \frac{\ln 2}{T_{1/2}}$. The mass of one ^{103}Pd nucleus is $103m_\text{p}$. In a time of one half-life the number of radioactive nuclei and the activity decrease by a factor of 2.

EXECUTE: **(a)** $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(17\text{ days})(24\text{ h/day})(3600\text{ s/h})} = 4.7\times10^{-7}\text{ s}^{-1}$.

$N = \frac{0.250\times10^{-3}\text{ kg}}{103m_\text{p}} = 1.45\times10^{21}$. $|dN/dt| = (4.7\times10^{-7}\text{ s}^{-1})(1.45\times10^{21}) = 6.8\times10^{14}$ Bq.

(b) 68 days is $4T_{1/2}$ so the activity is $(6.8\times10^{14}\text{ Bq})/2^4 = 4.2\times10^{13}$ Bq.

EVALUATE: At the end of 4 half-lives, the activity rate is less than a tenth of its initial rate.

43.23. IDENTIFY and SET UP: As discussed in Section 43.4, the activity $A = |dN/dt|$ obeys the same decay equation as $N(t)$: $A = A_0 e^{-\lambda t}$. For ^{14}C, $T_{1/2} = 5730$ y and $\lambda = \ln 2/T_{1/2}$ so $A = A_0 e^{-(\ln 2)t/T_{1/2}}$; calculate A at each t; $A_0 = 184$ decays/min.

EXECUTE: **(a)** For t = 1000 y, we have A = (184 decays/min)$e^{-(\ln 2)(1000\text{ y})/(5730\text{ y})}$ = 163 decays/min.
(b) For t = 50,000 y, the same equation gives A = 0.435 decays/min.
EVALUATE: The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor of $(\frac{1}{2})^{8.73}$.

43.24. IDENTIFY and SET UP: The decay rate decreases by a factor of 2 in a time of one half-life.

EXECUTE: **(a)** 24 days is 3 half-lives, so the activity at the end of that time is $(325\text{ Bq})/(2^3) = 40.6$ Bq, which rounds to 41 Bq.
(b) The activity is proportional to the number of radioactive nuclei, so the percent is

$$\frac{17.0\text{ Bq}}{40.6\text{ Bq}} = 0.42 = 42\%.$$

(c) $^{131}_{53}\text{I} \to {}^{0}_{-1}\text{e} + {}^{131}_{54}\text{Xe}$. The nucleus $^{131}_{54}\text{Xe}$ is produced.

EVALUATE: Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

43.25. IDENTIFY: This problem is about radioactive decay.
SET UP: We are working with the half-life of the isotope, so it is convenient to express the decay in terms of base 2 when needed. $N = N_0 e^{-\lambda t} = N_0 2^{-t/T_{1/2}}$

EXECUTE: **(a)** We want to know the number of nuclei initially present. We know the initial rate of decay, which is dN/dt, so

$$dN/dt = d(N_0 e^{-\lambda t})/dt = -\lambda N_0 e^{-\lambda t}.$$

At $t = 0$, $dN/dt = -8.0 \times 10^{16}$ Bq. Solving for N_0 gives

$N_0 = \frac{1}{\lambda}\frac{dN}{dt} = -\frac{T_{1/2}}{0.693}\frac{dN}{dt} = -\frac{(64.0)(3600\text{ s})}{0.693}\left(-8.0\times10^{16}\text{ Bq}\right) = 2.7\times10^{22}$ nuclei.

(b) We want the number of nuclei remaining at the end of 12.0 days. Use base 2 for simplicity.

$$N = N_0 2^{-t/T_{1/2}} = (2.66\times10^{22})2^{-(12.0)(24.\text{h})(64.0\text{h})} = 1.2\times10^{21}\text{nuclei.}$$

EVALUATE: Note that 12.0 days is 4.5 half-lives, so $N/N_0 = 2^{-4.5}$.

43.26. IDENTIFY: Apply $|dN/dt| = \lambda N$ to calculate N, the number of radioactive nuclei originally present in the spill. Since the activity A is proportional to the number of radioactive nuclei, $N = N_0 e^{-\lambda t}$ leads to $A = A_0 e^{-\lambda t}$, where A is the activity.

SET UP: The mass of one ^{131}Ba nucleus is about 131 u.

EXECUTE: **(a)** $\left|\dfrac{dN}{dt}\right| = 400\ \mu\text{Ci} = (400\times10^{-6})(3.70\times10^{10}\ \text{s}^{-1}) = 1.48\times10^{7}$ decays/s.

$$T_{1/2} = \frac{\ln 2}{\lambda} \rightarrow \lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12\ \text{d})(86{,}400\ \text{s/d})} = 6.69\times10^{-7}\ \text{s}^{-1}.$$

$\left|\dfrac{dN}{dt}\right| = \lambda N \Rightarrow N = \dfrac{|dN/dt|}{\lambda} = \dfrac{1.48\times10^{7}\ \text{decays/s}}{6.69\times10^{-7}\text{s}^{-1}} = 2.21\times10^{13}$ nuclei. The mass of these ^{131}Ba nuclei is

$$m = (2.21\times10^{13}\ \text{nuclei})\times(131\times1.66\times10^{-27}\ \text{kg/nucleus}) = 4.8\times10^{-12}\ \text{kg} = 4.8\times10^{-9}\ \text{g} = 4.8\ \text{ng}.$$

(b) $A = A_0 e^{-\lambda t}$. $1\ \mu\text{Ci} = (400\ \mu\text{Ci})\, e^{-\lambda t}$. $\ln(1/400) = -\lambda t$.

$$t = -\frac{\ln(1/400)}{\lambda} = -\frac{\ln(1/400)}{6.69\times10^{-7}\text{s}^{-1}} = 8.96\times10^{6}\ \text{s}\left(\frac{1\ \text{d}}{86{,}400\ \text{s}}\right) = 104\ \text{days}.$$

EVALUATE: The time is about 8.7 half-lives and the activity after that time is $(400\ \mu\text{Ci})\left(\tfrac{1}{2}\right)^{8.7} \approx 1\mu\text{Ci}$.

43.27. IDENTIFY: Apply $A = A_0 e^{-\lambda t}$ and $\lambda = \ln 2/T_{1/2}$.

SET UP: $\ln e^{x} = x$.

EXECUTE: $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$. $-\dfrac{(\ln 2)t}{T_{1/2}} = \ln(A/A_0)$.

$$T_{1/2} = -\frac{(\ln 2)t}{\ln(A/A_0)} = -\frac{(\ln 2)(4.00\ \text{days})}{\ln(3091/8318)} = 2.80\ \text{days}.$$

EVALUATE: The activity has decreased by more than half and the elapsed time is more than one half-life.

43.28. IDENTIFY: Apply $A = A_0 e^{-\lambda t}$.

SET UP: From Example 43.9, $\lambda = 3.83\times10^{-12}\ \text{s}^{-1} = 1.21\times10^{-4}\ \text{y}^{-1}$ for radiocarbon.

EXECUTE: The activity of the sample is $\dfrac{2690\ \text{decays/min}}{(60\ \text{s/min})(0.500\ \text{kg})} = 89.7\ \text{Bq/kg}$, while the activity of atmospheric carbon is 255 Bq/kg (see Example 43.9). The age of the sample is then

$$t = -\frac{\ln(89.7/225)}{\lambda} = -\frac{\ln(89.7/225)}{1.21\times10^{-4}/\text{y}} = 8640\ \text{y}.$$

EVALUATE: For ^{14}C, $T_{1/2} = 5730$ y. The age is more than one half-life and the activity per kg of carbon is less than half the value when the tree died.

43.29. IDENTIFY and SET UP: Apply $|dN/dt| = \lambda N$ with $\lambda = \ln 2/T_{1/2}$. In one half-life, one half of the nuclei decay.

EXECUTE: **(a)** $\left|\dfrac{dN}{dt}\right| = 7.56\times10^{11}\ \text{Bq} = 7.56\times10^{11}$ decays/s.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(30.8\text{ min})(60\text{ s/min})} = 3.75\times10^{-4}\text{ s}^{-1}.$$

$$N_0 = \frac{1}{\lambda}\left|\frac{dN}{dt}\right| = \frac{7.56\times10^{11}\text{ decay/s}}{3.75\times10^{-4}\text{ s}^{-1}} = 2.02\times10^{15}\text{ nuclei.}$$

(b) The number of nuclei left after one half-life is $\frac{N_0}{2} = 1.01\times10^{15}$ nuclei, and the activity is half: $\left|\frac{dN}{dt}\right| = 3.78\times10^{11}$ decays/s.

(c) After three half-lives (92.4 minutes) there is an eighth of the original amount, so $N = 2.53\times10^{14}$ nuclei, and an eighth of the activity: $\left|\frac{dN}{dt}\right| = 9.45\times10^{10}$ decays/s.

EVALUATE: Since the activity is proportional to the number of radioactive nuclei that are present, the activity is halved in one half-life.

43.30. **IDENTIFY** and **SET UP:** $1\text{ Gy} = 1\text{ J/kg}$ and is the SI unit of absorbed dose. $1\text{ rad} = 0.010\text{ Gy}$. Sv is the SI unit for equivalent dose. Equivalent dose $=$ RBE $\times$ absorbed dose. Rem is the equivalent dose when the absorbed dose is in rad. For x rays, $\text{RBE} = 1.0$. For protons, $\text{RBE} = 10$.

EXECUTE: **(a)** 5.0 Gy, 500 rad. $\text{RBE} = 1.0$, so equivalent dose $=$ absorbed dose. 5.0 Sv and 500 rem.

(b) $(70.0\text{ kg})(5.0\text{ J/kg}) = 350\text{ J}$.

(c) The absorbed dose and total absorbed energy are the same but the equivalent dose is 10 times larger. So the answers are: 5.0 Gy, 500 rad, 50 Sv, 5000 rem, 350 J.

EVALUATE: The same energy deposited by protons as x rays is ten times greater in its biological effect.

43.31. **IDENTIFY** and **SET UP:** The unit for absorbed dose is $1\text{ rad} = 0.01\text{ J/kg} = 0.01\text{ Gy}$. Equivalent dose in rem is RBE times absorbed dose in rad.

EXECUTE: $1\text{ rad} = 10^{-2}\text{ Gy}$, so $1\text{ Gy} = 100\text{ rad}$ and the dose was 500 rad.

$$\text{rem} = (\text{rad})(\text{RBE}) = (500\text{ rad})(4.0) = 2000\text{ rem.}\quad 1\text{ Gy} = 1\text{ J/kg, so } 5.0\text{ J/kg.}$$

EVALUATE: Gy, rad, and J/kg are all units of absorbed dose. Rem is a unit of equivalent dose, which depends on the RBE of the radiation.

43.32. **IDENTIFY** and **SET UP:** The unit for absorbed dose is $1\text{ rad} = 0.01\text{ J/kg} = 0.01\text{ Gy}$. Equivalent dose in rem is RBE times absorbed dose in rad.

EXECUTE: **(a)** $\text{rem} = \text{rad}\times\text{RBE}$. $300 = x(10)$ and $x = 30$ rad.

(b) 1 rad deposits 0.010 J/kg, so 30 rad deposit 0.30 J/kg. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is $(0.025\text{ kg})(0.30\text{ J/kg}) = 7.5\times10^{-3}\text{ J} = 7.5\text{ mJ}$.

(c) $\text{RBE} = 1$ for β-rays, so rem $=$ rad. Therefore 30 rad $=$ 30 rem.

EVALUATE: The same absorbed dose produces a larger equivalent dose when the radiation is neutrons than when it is electrons.

43.33. **IDENTIFY** and **SET UP:** For x rays $\text{RBE} = 1$ and the equivalent dose equals the absorbed dose.

EXECUTE: **(a)** 175 krad $=$ 175 krem $=$ 1.75 kGy $=$ 1.75 kSv. $(1.75\times10^3\text{ J/kg})(0.220\text{ kg}) = 385\text{ J}$.

(b) 175 krad $=$ 1.75 kGy; $(1.50)(175\text{ krad}) = 262.5\text{ krem} = 2.625\text{ kSv}$. The energy deposited would be 385 J, the same as in (a).

EVALUATE: The energy required to raise the temperature of 0.220 kg of water 1 C° is 922 J, and 385 J is less than this. The energy deposited corresponds to a very small amount of heating.

43.34. **IDENTIFY** and **SET UP:** For x rays $\text{RBE} = 1$ so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

EXECUTE: One whole-body scan delivers $(75\text{ kg})(12\times10^{-3}\text{ J/kg}) = 0.90\text{ J}$. One chest x ray delivers $(5.0\text{ kg})(0.20\times10^{-3}\text{ J/kg}) = 1.0\times10^{-3}\text{ J}$. It takes $\dfrac{0.90\text{ J}}{1.0\times10^{-3}\text{ J}} = 900$ chest x rays to deliver the same total energy.

EVALUATE: For the CT scan the equivalent dose is much larger, and it is applied to the whole body.

43.35. **IDENTIFY:** This problem looks at the biological effects of radiation.

SET UP: $N = N_0 e^{-\lambda t}$, $\lambda = 0.693/T_{½} = 0.693/(29\text{ y}) = 0.0239\text{ y}^{-1}$.

EXECUTE: **(a)** We want the absorbed dose during one year from 1.0 μg of Sr-90. First find the number of decays ΔN that occur during one year. Each decay releases 1.1 MeV of energy. The mass of a Sr-90 atom is approximately $38m_p + 52m_n + 38m_e = 1.5067\times10^{-25}$ kg. If N_0 is the number of Sr-90 atoms in the 1.0-μg sample, then $(1.5067\times10^{-25}\text{ kg})N_0 = 1.0\ \mu\text{g}$, which gives $N_0 = 6.637\times10^{15}$ atoms. At the end of one year, the number of atoms N that are left will be $N = N_0e^{-\lambda(1\text{ y})}$. The number of decays during the year is $N_0 - N = N_0 - N_0e^{-\lambda(1\text{ y})} = (6.637\times10^{15})[1 - e^{-(0.0239/\text{y})(1\text{ y})}] = 1.567\times10^{14}$ decays. The total energy E absorbed by these decays is $E = (1.567\times10^{14})(1.1\text{ MeV}) = 27.59$ J. This energy is delivered to 50 kg of body tissue, so the absorbed dose is (27.59 J)/(50 kg) = 0.55 J/kg = 0.55 Gy. Since 1 rad = 0.01 Gy, this dose is also 55 rad.

(b) We want the equivalent dose. Equivalent dose (in Sv) = RBE × absorbed dose (in Gy). From Table 43.3, the RBE for gamma rays is 1, and the RBE for electrons is given in the problem as 1.0. So the maximum equivalent dose is (1.0)(0.55 Gy) = 0.55 Sv. Since 1 rem = 0.01 Sv, we can also say that the equivalent dose is 55 rem.

EVALUATE: The longer the exposure lasts, the greater the dose because more decays take place.

43.36. **IDENTIFY:** 1 rem = 0.01 Sv. Equivalent dose in rem equals RBE times the absorbed dose in rad. 1 rad = 0.01 J/kg. To change the temperature of water, $Q = mc\Delta T$.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $5.4\text{ Sv}(100\text{ rem/Sv}) = 540\text{ rem}$.

(b) The RBE of 1 gives an absorbed dose of 540 rad.

(c) The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4\text{ Gy})(65\text{ kg}) = 351\text{ J}$. The energy required to raise the temperature of 65 kg by 0.010° C is $(65\text{ kg})(4190\text{ J/kg}\cdot\text{K})(0.01\text{ C}^\circ) = 3\text{ kJ}$.

EVALUATE: The amount of energy received corresponds to a very small heating of his body.

43.37. **IDENTIFY:** Each photon delivers energy. The energy of a single photon depends on its wavelength.

SET UP: equivalent dose (rem) = RBE×absorbed dose (rad). 1 rad = 0.010 J/kg. For x rays, RBE = 1. Each photon has energy $E = \dfrac{hc}{\lambda}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.0200\times10^{-9}\text{ m}} = 9.94\times10^{-15}\text{ J}$. The absorbed energy is

$$(5.00\times10^{10}\text{ photons})(9.94\times10^{-15}\text{ J/photon}) = 4.97\times10^{-4}\text{ J} = 0.497\text{ mJ}.$$

(b) The absorbed dose is $\dfrac{4.97\times10^{-4}\text{ J}}{0.600\text{ kg}} = 8.28\times10^{-4}\text{ J/kg} = 0.0828\text{ rad}$. Since RBE = 1, the equivalent dose is 0.0828 rem.

EVALUATE: The amount of energy absorbed is rather small (only $\frac{1}{2}$ mJ), but it is absorbed by only 600 g of tissue.

43.38. **IDENTIFY:** The reaction energy Q is $Q = (M_A + M_B - M_C - M_D)c^2$.

SET UP: $Q = (m_{\text{H-1}} + m_{\text{H-3}} - m_{\text{H-2}} - m_{\text{H-2}})c^2$. Use the masses in Table 43.2 and $m_n = 1.008665$ u for the neutron. The masses in Table 43.2 are for neutral atoms and therefore include the mass of the electrons. 1 u is equivalent to 931.5 MeV.

EXECUTE: $Q = [1.007825\text{ u} + 3.016049\text{ u} - 2(2.014102\text{ u})](931.5\text{ MeV/u}) = -4.033\text{ MeV}.$

EVALUATE: Since Q is negative, the initial energy is less than the final energy, so energy had to be put into the system to cause the reaction to occur. Therefore, this is an endoergic reaction.

43.39. **(a) IDENTIFY and SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = +2+9-4 = 7$ and $Z = +1+4-2 = 3.$ Thus X is ${}^{7}_{3}\text{Li}$ and the reaction is ${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be} = {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}.$

(b) IDENTIFY and SET UP: Calculate the mass decrease and find its energy equivalent.

EXECUTE: If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2. 1 u is equivalent to 931.5 MeV.

$${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be has mass } 2.014102\text{ u} + 9.012182\text{ u} = 11.26284\text{ u}.$$

$${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He has mass } 7.016005\text{ u} + 4.002603\text{ u} = 11.018608\text{ u}.$$

The mass decrease is $11.026284\text{ u} - 11.018608\text{ u} = 0.007676\text{ u}.$

This corresponds to an energy release of $(0.007676\text{ u})(931.5\text{ MeV/1 u}) = 7.150\text{ MeV}.$

(c) IDENTIFY and SET UP: Estimate the threshold energy by calculating the Coulomb potential energy when the ${}^{2}_{1}\text{H}$ and ${}^{9}_{4}\text{Be}$ nuclei just touch. Obtain the nuclear radii from $R = R_0A^{1/3}.$

EXECUTE: The radius R_{Be} of the ${}^{9}_{4}\text{Be}$ nucleus is $R_{\text{Be}} = (1.2\times10^{-15}\text{ m})(9)^{1/3} = 2.5\times10^{-15}\text{ m}.$

The radius R_{H} of the ${}^{2}_{1}\text{H}$ nucleus is $R_{\text{H}} = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.5\times10^{-15}\text{ m}.$

The nuclei touch when their center-to-center separation is

$R = R_{\text{Be}} + R_{\text{H}} = 4.0\times10^{-15}\text{ m}.$

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e(4e)}{r}.$$

$$U = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.602\times10^{-19}\text{ C})^2}{(4.0\times10^{-15}\text{ m})(1.602\times10^{-19}\text{ J/eV})} = 1.4\text{ MeV}.$$

This is an estimate of the threshold energy for this reaction.

EVALUATE: The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

43.40. **IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the reaction.

SET UP: 1 u is equivalent to $931.5\text{ MeV}.$

EXECUTE: $m_{{}^{3}_{2}\text{He}} + m_{{}^{2}_{1}\text{H}} - m_{{}^{4}_{2}\text{He}} - m_{{}^{1}_{1}\text{H}} = 1.97\times10^{-2}\text{ u},$ so the energy released is $18.4\text{ MeV}.$

EVALUATE: Using neutral atom masses includes three electron masses on each side of the reaction equation and the same result is obtained as if nuclear masses had been used.

43.41. **IDENTIFY and SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to $931.5\text{ MeV}.$ The mass of one ${}^{235}\text{U}$ nucleus is $235m_{\text{p}}.$

EXECUTE: **(a)** ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \to {}^{144}_{56}\text{Ba} + {}^{89}_{36}\text{Kr} + 3{}^{1}_{0}\text{n}.$ We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is $\Delta M = m({}^{235}_{92}\text{U}) + m({}^{1}_{0}\text{n}) - [m({}^{144}_{56}\text{Ba}) + m({}^{89}_{36}\text{Kr}) + 3m({}^{1}_{0}\text{n})],$

$\Delta M = 235.043930\text{ u} + 1.0086649\text{ u} - 143.922953\text{ u} - 88.917631\text{ u} - 3(1.0086649\text{ u})$, $\Delta M = 0.1860\text{ u}$. The energy released is $(0.1860\text{ u})(931.5\text{ MeV/u}) = 173.3\text{ MeV}$.

(b) The number of ^{235}U nuclei in 1.00 g is $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_{\text{p}}} = 2.55\times10^{21}$. The energy released per gram is $(173.3\text{ MeV/nucleus})(2.55\times10^{21}\text{ nuclei/g}) = 4.42\times10^{23}\text{ MeV/g}$.

EVALUATE: The energy released is $7.1\times10^{10}\text{ J/kg}$. This is much larger than typical heats of combustion, which are about $5\times10^{4}\text{ J/kg}$.

43.42. **IDENTIFY:** The charge and the nucleon number are conserved. The energy of the photon must be at least as large as the energy equivalent of the mass increase in the reaction.
SET UP: 1 u is equivalent to 931.5 MeV.
EXECUTE: **(a)** ${}^{28}_{14}\text{Si} + \gamma \Rightarrow {}^{24}_{12}\text{Mg} + {}^{A}_{Z}\text{X}$. $A + 24 = 28$ so $A = 4$. $Z + 12 = 14$ so $Z = 2$. X is an α particle.
(b) $-\Delta m = m({}^{24}_{12}\text{Mg}) + m({}^{4}_{2}\text{He}) - m({}^{28}_{14}\text{Si}) = 23.985042\text{ u} + 4.002603\text{ u} - 27.976927\text{ u} = 0.010718\text{ u}$.
$E_\gamma = (-\Delta m)c^2 = (0.010718\text{ u})(931.5\text{ MeV/u}) = 9.984\text{ MeV}$.
EVALUATE: The wavelength of the photon is
$\lambda = \dfrac{hc}{E} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{9.984\times10^{6}\text{ eV}} = 1.24\times10^{-13}\text{ m} = 1.24\times10^{-4}\text{ nm}$. This is a gamma ray photon.

43.43. **IDENTIFY:** Charge and the number of nucleons are conserved in the reaction. The energy absorbed or released is determined by the mass change in the reaction.
SET UP: 1 u is equivalent to 931.5 MeV.
EXECUTE: **(a)** $Z = 3 + 2 - 0 = 5$ and $A = 4 + 7 - 1 = 10$.
(b) The nuclide is a boron nucleus, and z and so 2.79 MeV of energy is absorbed.
EVALUATE: The absorbed energy must come from the initial kinetic energy of the reactants.

43.44. **IDENTIFY:** We are looking at the energy released during radioactive decay.
SET UP: The decay is ${}^{10}_{4}\text{Be} \to {}^{10}_{5}\text{B} + \beta + \bar{\nu}_{\text{e}}$.
EXECUTE: **(a)** The target variable is the released energy. Beta-minus decay can occur only if the mass of the original atom is greater than the mass of the final atom, which in this case is true. Use Table 43.2. $\Delta m = m_{\text{i}} - m_{\text{f}} = 10.013535\text{ u} - 10.012937 = 0.000598\text{ u}$. The energy E released is

$$E = (0.000598\text{ u})[(931.5\text{ MeV})/\text{u}] = 0.557\text{ MeV}.$$

(b) We want the speed of the beta-minus particle. The released energy is a bit larger than the rest energy of the electron (0.511 MeV), so we need to use relativity. First find γ and then use it to find v.

$$E = K = mc^2(\gamma - 1)$$

$$\gamma = \frac{K}{mc^2} + 1 = \frac{0.577\text{ MeV}}{0.511\text{ MeV}} + 1 = 2.09$$

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(2.09)^2} = 0.87c.$$

EVALUATE: Special relativity definitely cannot be neglected at such a high speed.

43.45. **IDENTIFY** and **SET UP:** $m = \rho V$. $1\text{ gal} = 3.788\text{ L} = 3.788\times10^{-3}\text{ m}^3$. The mass of a ^{235}U nucleus is $235m_{\text{p}}$. $1\text{ MeV} = 1.60\times10^{-13}\text{ J}$.

EXECUTE: **(a)** For 1 gallon, $m = \rho V = (737\text{ kg/m}^3)(3.788\times10^{-3}\text{ m}^3) = 2.79\text{ kg} = 2.79\times10^{3}\text{ g}$.

$$\frac{1.3\times10^{8}\text{ J/gal}}{2.79\times10^{3}\text{ g/gal}} = 4.7\times10^{4}\text{ J/g}.$$

(b) 1 g contains $\dfrac{1.00\times10^{-3}\text{ kg}}{235m_\text{p}}=2.55\times10^{21}$ nuclei.

$$(200\text{ MeV/nucleus})(1.60\times10^{-13}\text{ J/MeV})(2.55\times10^{21}\text{ nuclei})=8.2\times10^{10}\text{ J/g}.$$

(c) A mass of $6m_\text{p}$ produces 26.7 MeV.

$$\frac{(26.7\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{6m_\text{p}}=4.26\times10^{14}\text{ J/kg}=4.26\times10^{11}\text{ J/g}.$$

(d) The total energy available would be $(1.99\times10^{30}\text{ kg})(4.7\times10^{7}\text{ J/kg})=9.4\times10^{37}\text{ J}$.

$$\text{Power}=\frac{\text{energy}}{t}\text{ so }t=\frac{\text{energy}}{\text{power}}=\frac{9.4\times10^{37}\text{ J}}{3.86\times10^{26}\text{ W}}=2.4\times10^{11}\text{ s}=7600\text{ y}.$$

EVALUATE: If the mass of the sun were all proton fuel, it would contain enough fuel to last

$$(7600\text{ y})\left(\frac{4.3\times10^{11}\text{ J/g}}{4.7\times10^{4}\text{ J/g}}\right)=7.0\times10^{10}\text{ y}.$$

43.46. **IDENTIFY:** The minimum energy to remove a proton from the nucleus is equal to the energy difference between the two states of the nucleus (before and after proton removal).

(a) SET UP: ${}^{12}_{6}\text{C}={}^{1}_{1}\text{H}+{}^{11}_{5}\text{B}$. $\Delta m=m({}^{1}_{1}\text{H})+m({}^{11}_{5}\text{B})-m({}^{12}_{6}\text{C})$. The electron masses cancel when neutral atom masses are used.

EXECUTE: $\Delta m=1.007825\text{ u}+11.009305\text{ u}-12.000000\text{ u}=0.01713\text{ u}$. The energy equivalent of this mass increase is $(0.01713\text{ u})(931.5\text{ MeV/u})=16.0\text{ MeV}$.

(b) SET UP and **EXECUTE:** We follow the same procedure as in part (a).

$$\Delta M=6M_\text{H}+6M_\text{n}-{}^{12}_{6}M=6(1.007825\text{ u})+6(1.008665\text{ u})-12.000000\text{ u}=0.09894\text{ u}.$$

$$E_\text{B}=(0.09894\text{ u})(931.5\text{ MeV/u})=92.16\text{ MeV}.\ \frac{E_\text{B}}{A}=7.68\text{ MeV/u}.$$

EVALUATE: The proton removal energy is about twice the binding energy per nucleon.

43.47. **IDENTIFY:** The minimum energy to remove a proton or a neutron from the nucleus is equal to the energy difference between the two states of the nucleus, before and after removal.

(a) SET UP: ${}^{17}_{8}\text{O}={}^{1}_{0}\text{n}+{}^{16}_{8}\text{O}$. $\Delta m=m({}^{1}_{0}\text{n})+m({}^{16}_{8}\text{O})-m({}^{17}_{8}\text{O})$. The electron masses cancel when neutral atom masses are used.

EXECUTE: $\Delta m=1.008665\text{ u}+15.994915\text{ u}-16.999132\text{ u}=0.004448\text{ u}$. The energy equivalent of this mass increase is $(0.004448\text{ u})(931.5\text{ MeV/u})=4.14\text{ MeV}$.

(b) SET UP and **EXECUTE:** Following the same procedure as in part (a) gives

$$\Delta M=8M_\text{H}+9M_\text{n}-{}^{17}_{8}M=8(1.007825\text{ u})+9(1.008665\text{ u})-16.999132\text{ u}=0.1415\text{ u}.$$

$$E_\text{B}=(0.1415\text{ u})(931.5\text{ MeV/u})=131.8\text{ MeV}.\ \frac{E_\text{B}}{A}=7.75\text{ MeV/nucleon}.$$

EVALUATE: The neutron removal energy is about half the binding energy per nucleon.

43.48. **IDENTIFY:** This problem is about the heat released by radioactive decay.

SET UP and **EXECUTE:** **(a)** We want the number of U-238 atoms in 1 kg of mantle material. The mass of U-238 is about $238m_\text{p}=3.975\times10^{-25}$ kg. If N is the number of U-238 atoms in 31 μg of material, then $(3.975\times10^{-25}\text{ kg})N=31\times10^{-9}$ kg. So $N=7.800\times10^{16}$ atoms of U-238 in 31 μg of U-238 which is also the number of U-238 atoms in 1 kg of mantle material.

(b) We want the decay constant. $\lambda=0.693/T_{1/2}$. $T_{1/2}=4.47\times10^{9}\text{ y}=1.4\times10^{17}$ s. Using this gives $\lambda=4.9\times10^{-18}\text{ s}^{-1}$.

(c) Multiplying our results as directed gives $\lambda N=0.38$ decays/s·kg.

(d) Converting units gives

$$\left(0.38\frac{\text{decays/s}}{\text{kg}}\right)(52\text{ MeV/decay})\left(1.60\times10^{-13}\text{ J/MeV}\right)=3.2\times10^{-12}\text{ W/kg.}$$

(e) We want the power. Using the result from part (d) and the information in the problem gives

$$P=\left(3.2\times10^{-12}\text{ W/kg}\right)\left[\frac{2}{3}\left(6\times10^{24}\text{ kg}\right)\right]=1.3\times10^{13}\text{ W}\approx10\text{ TW.}$$

(f) Use the power from part (e). $0.39E_{\text{tot}}$ = 13 TW, so E_{tot} = 30 TW.

(g) We want the total power. $P_{\text{tot}} - P_{\text{out}} = P_{\text{net}}$, so $P_{\text{tot}} = P_{\text{net}} + P_{\text{out}} = P_{\text{rad}} + P_{\text{rad}} = 2P_{\text{rad}}$ = 60 TW.

EVALUATE: The power from radioactivity comes largely from the mantle and the power from left over heat comes largely from the hot cores.

43.49. IDENTIFY: Use the decay scheme and half-life of ^{90}Sr to find out the product of its decay and the amount left after a given time.

SET UP: The particle emitted in β^- decay is an electron, ${}^{0}_{-1}\text{e}$. In a time of one half-life, the number of radioactive nuclei decreases by a factor of 2. $6.25\% = \frac{1}{16} = 2^{-4}$.

EXECUTE: **(a)** ${}^{90}_{38}\text{Sr}\to{}^{0}_{-1}\text{e}+{}^{90}_{39}\text{Y}$. The daughter nucleus is ${}^{90}_{39}\text{Y}$.

(b) 56 y is $2T_{1/2}$ so $N = N_0/2^2 = N_0/4$; 25% is left.

(c) $\frac{N}{N_0}=2^{-n}$; $\frac{N}{N_0}=6.25\%=\frac{1}{16}=2^{-4}$, so $t = 4T_{1/2} = 112$ y.

EVALUATE: After half a century, $\frac{1}{4}$ of the ^{90}Sr would still be left!

43.50. IDENTIFY: Calculate the mass defect for the decay. Example 43.5 uses conservation of linear momentum to determine how the released energy is divided between the decay partners.

SET UP: 1 u is equivalent to 931.5 MeV.

EXECUTE: The α-particle will have $\frac{226}{230}$ of the released energy (see Example 43.5).

$$\frac{226}{230}(m_{\text{Th}}-m_{\text{Ra}}-m_{\text{He-4}})=5.032\times10^{-3}\text{ u or 4.69 MeV.}$$

EVALUATE: Most of the released energy goes to the α particle, since its mass is much less than that of the daughter nucleus.

43.51. (a) IDENTIFY and SET UP: The heavier nucleus will decay into the lighter one.

EXECUTE: ${}^{25}_{13}\text{Al}$ will decay into ${}^{25}_{12}\text{Mg}$.

(b) IDENTIFY and SET UP: Determine the emitted particle by balancing A and Z in the decay reaction.

EXECUTE: This gives ${}^{25}_{13}\text{Al}\to{}^{25}_{12}\text{Mg}+{}^{0}_{+1}\text{e}$. The emitted particle must have charge $+e$ and its nucleon number must be zero. Therefore, it is a β^+ particle, a positron.

(c) IDENTIFY and SET UP: Calculate the energy defect ΔM for the reaction and find the energy equivalent of ΔM. Use the nuclear masses for ${}^{25}_{13}\text{Al}$ and ${}^{25}_{12}\text{Mg}$, to avoid confusion in including the correct number of electrons if neutral atom masses are used.

EXECUTE: The nuclear mass for ${}^{25}_{13}\text{Al}$ is

$M_{\text{nuc}}({}^{25}_{13}\text{Al}) = 24.990428\text{ u} - 13(0.000548580\text{ u}) = 24.983296\text{ u}.$

The nuclear mass for ${}^{25}_{12}\text{Mg}$ is $M_{\text{nuc}}({}^{25}_{12}\text{Mg}) = 24.985837\text{ u} - 12(0.000548580\text{ u}) = 24.979254\text{ u}.$

The mass defect for the reaction is

$\Delta M = M_{\text{nuc}}({}^{25}_{13}\text{Al}) - M_{\text{nuc}}({}^{25}_{12}\text{Mg}) - M({}^{0}_{+1}\text{e}) = 24.983296\text{ u} - 24.979254\text{ u} - 0.00054858\text{ u} = 0.003493\text{ u}.$

$$Q=(\Delta M)c^2=0.003493\text{ u}(931.5\text{ MeV/1 u})=3.254\text{ MeV.}$$

EVALUATE: The mass decreases in the decay and energy is released. Note: ${}^{25}_{13}\text{Al}$ can also decay into ${}^{25}_{12}\text{Mg}$ by the electron capture: ${}^{25}_{13}\text{Al} + {}^{0}_{-1}\text{e} \rightarrow {}^{25}_{12}\text{Mg}$. The ${}^{0}_{-1}$ electron in the reaction is an orbital electron in the neutral ${}^{25}_{13}\text{Al}$ atom. The mass defect can be calculated using the nuclear masses:

$\Delta M = M_{\text{nuc}}({}^{25}_{13}\text{Al}) + M({}^{0}_{-1}\text{e}) - M_{\text{nuc}}({}^{25}_{12}\text{Mg}) = 24.983296\ \text{u} + 0.00054858\ \text{u} - 24.979254\ \text{u} = 0.004591\ \text{u}.$

$Q = (\Delta M)c^2 = (0.004591\ \text{u})(931.5\ \text{MeV/1 u}) = 4.277\ \text{MeV}.$ The mass decreases in the decay and energy is released.

43.52. **IDENTIFY:** This problem is about nuclear fusion reactions in the sun.

SET UP and **EXECUTE:** Follow the directions for each part. **(a)** We want the number N of fusion reactions per second. One alpha particle is produced for each cycle of the proton-proton chain, and each cycle produces 27.73 MeV. Using the given power, we have

$$(26.73\ \text{MeV/cycle})\left(1.60\times10^{-13}\ \text{J/MeV}\right)N = 3.8\times10^{26}\ \text{W}$$

$N = 8.885 \times 10^{37}$ reactions/s, which rounds to $N = 8.9 \times 10^{37}$ reactions/s.

(b) Each cycle produces 2 neutrinos, so $N_\text{v} = 1.8 \times 10^{38}$ neutrinos/s.

(c) We want the number of neutrinos that hit the earth per second. Assume that the neutrinos travel outward uniformly in all directions. Call N_E the number that hit the earth, r the earth-sun distance, and r_E the radius of the earth. This gives

$$\frac{N_\text{E}}{N_\nu} = \frac{\pi r_\text{E}^2}{4\pi r^2}$$

$$N_\text{E} = N_\nu\left(\frac{r_\text{E}}{2r}\right)^2 = \left(1.8\times10^{38}\ \text{neutrinos/s}\right)\left[\frac{6370\ \text{kg}}{2(150\times10^6\ \text{km})}\right] = 8.1\times10^{28}\ \text{neutrinos/s}.$$

(d) The area presented to the sun is the earth's cross-sectional area, so

$$A = \pi r_\text{E}^2 = \pi(6370\ \text{km})^2 = 1.3\times10^{18}\ \text{cm}^2.$$

(e) Dividing as specified gives

$$\frac{8.1\times10^{28}\ \text{neutrinos/s}}{1.3\times10^{18}\ \text{cm}^2} = 6\times10^{10}\ \text{neutrinos/s}\cdot\text{cm}^2.$$

EVALUATE: The number in part (e) is 60 billion neutrinos/s through each square centimeter! Most of them travel straight through without hitting anything.

43.53. **IDENTIFY** and **SET UP:** The amount of kinetic energy released is the energy equivalent of the mass change in the decay. $m_\text{e} = 0.0005486\ \text{u}$ and the atomic mass of ${}^{14}_{7}\text{N}$ is 14.003074 u. The energy equivalent of 1 u is 931.5 MeV. ${}^{14}\text{C}$ has a half-life of $T_{1/2} = 5730\ \text{y} = 1.81\times10^{11}\ \text{s}$. The RBE for an electron is 1.0.

EXECUTE: **(a)** ${}^{14}_{6}\text{C} \rightarrow \text{e}^- + {}^{14}_{7}\text{N} + \overline{\nu}_\text{e}.$

(b) The mass decrease is $\Delta M = m({}^{14}_{6}\text{C}) - [m_\text{e} + m({}^{14}_{7}\text{N})]$. Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of ${}^{14}_{6}\text{C}$ is $14.003242\ \text{u} - 6m_\text{e} = 13.999950\ \text{u}$. The nuclear mass of ${}^{14}_{7}\text{N}$ is $14.003074\ \text{u} - 7m_\text{e} = 13.999234\ \text{u}$.

$\Delta M = 13.999950\ \text{u} - 13.999234\ \text{u} - 0.000549\ \text{u} = 1.67\times10^{-4}\ \text{u}.$ The energy equivalent of ΔM is 0.156 MeV.

(c) The mass of carbon is $(0.18)(75\ \text{kg}) = 13.5\ \text{kg}$. From Example 43.9, the activity due to 1 g of carbon in a living organism is 0.255 Bq. The number of decay/s due to 13.5 kg of carbon is

$$(13.5\times10^3\ \text{g})(0.255\ \text{Bq/g}) = 3.4\times10^3\ \text{decays/s}.$$

(d) Each decay releases 0.156 MeV so 3.4×10^3 decays/s releases $530\ \text{MeV/s} = 8.5\times10^{-11}\ \text{J/s}$.

(e) The total energy absorbed in 1 year is $(8.5\times10^{-11}\ \text{J/s})(3.156\times10^{7}\ \text{s}) = 2.7\times10^{-3}\ \text{J}$. The absorbed dose is $\dfrac{2.7\times10^{-3}\ \text{J}}{75\ \text{kg}} = 3.6\times10^{-5}\ \text{J/kg} = 36\ \mu\text{Gy} = 3.6\ \text{mrad}$. With $\text{RBE} = 1.0$, the equivalent dose is $36\ \mu\text{Sv} = 3.6\ \text{mrem}$.

EVALUATE: Section 43.5 says that background radiation exposure is about 1.0 mSv per year. The radiation dose calculated in this problem is much less than this.

43.54. **IDENTIFY** and **SET UP:** $m_\pi = 264m_e = 2.40\times10^{-28}\ \text{kg}$. The total energy of the two photons equals the rest mass energy $m_\pi c^2$ of the pion.

EXECUTE: **(a)** $E_{\text{ph}} = \frac{1}{2}m_\pi c^2 = \frac{1}{2}(2.40\times10^{-28}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = 1.08\times10^{-11}\ \text{J} = 67.5\ \text{MeV}$.

$$E_{\text{ph}} = \frac{hc}{\lambda},\ \text{so}\ \lambda = \frac{hc}{E_{\text{ph}}} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{67.5\times10^{6}\ \text{eV}} = 1.84\times10^{-14}\ \text{m} = 18.4\ \text{fm}.$$

These are gamma ray photons, so they have $\text{RBE} = 1.0$.

(b) Each pion delivers $2(1.08\times10^{-11}\ \text{J}) = 2.16\times10^{-11}\ \text{J}$.

The absorbed dose is $200\ \text{rad} = 2.00\ \text{Gy} = 2.00\ \text{J/kg}$.

The energy deposited is $(25\times10^{-3}\ \text{kg})(2.00\ \text{J/kg}) = 0.050\ \text{J}$.

The number of π^0 mesons needed is $\dfrac{0.050\ \text{J}}{2.16\times10^{-11}\ \text{J/meson}} = 2.3\times10^{9}$ mesons.

EVALUATE: Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same λ and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and $E = pc$.

43.55. **IDENTIFY** and **SET UP:** The mass defect is E_B/c^2.

EXECUTE: $m_{^{11}_{6}\text{C}} - m_{^{11}_{5}\text{B}} - 2m_e = 1.03\times10^{-3}\ \text{u}$. Decay is energetically possible.

EVALUATE: The energy released in the decay is $(1.03\times10^{-3}\ \text{u})(931.5\ \text{MeV/u}) = 0.959\ \text{MeV}$.

43.56. **IDENTIFY:** Assume the activity is constant during the year and use the given value of the activity to find the number of decays that occur in one year. Absorbed dose is the energy absorbed per mass of tissue. Equivalent dose is RBE times absorbed dose.

SET UP: For α particles, $\text{RBE} = 20$ (from Table 43.3).

EXECUTE: $(0.52\times10^{-6}\ \text{Ci})(3.7\times10^{10}\ \text{Bq/Ci})(3.156\times10^{7}\ \text{s}) = 6.07\times10^{11}\ \alpha$ particles. The absorbed dose is $\dfrac{(6.07\times10^{11})(4.0\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}{(0.50\ \text{kg})} = 0.78\ \text{Gy} = 78\ \text{rad}$. The equivalent dose is $(20)(78\ \text{rad}) = 1600\ \text{rem}$.

EVALUATE: The equivalent dose is 16 Sv. This is large enough for significant damage to the person.

43.57. **IDENTIFY** and **SET UP:** One-half of the sample decays in a time of $T_{1/2}$.

EXECUTE: **(a)** $\dfrac{10\times10^{9}\ \text{y}}{200{,}000\ \text{y}} = 5.0\times10^{4}$.

(b) $\left(\frac{1}{2}\right)^{5.0\times10^4}$. This exponent is too large for most hand-held calculators. But $\left(\frac{1}{2}\right) = 10^{-0.301}$, so $\left(\frac{1}{2}\right)^{5.0\times10^4} = (10^{-0.301})^{5.0\times10^4} = 10^{-15{,}000}$.

EVALUATE: For $N = 1$ after 16 billion years, $N_0 = 10^{15{,}000}$. The mass of this many ^{99}Tc nuclei would be $(99)(1.66\times10^{-27}\ \text{kg})(10^{15{,}000}) = 10^{14{,}750}\ \text{kg}$, which is immense, far greater than the mass of any star.

43.58. **IDENTIFY:** One rad of absorbed dose is 0.01 J/kg. The equivalent dose in rem is the absorbed dose in rad times the RBE. For part (c) apply $|dN/dt| = \lambda N$ with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

SET UP: For α particles, RBE = 20 (Table 43.3).

EXECUTE: **(a)** $(7.75\times10^{12})(4.77\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})/(70.0\ \text{kg}) = 0.0846\ \text{Gy} = 8.46\ \text{rad}$.

(b) $(20)(8.46\ \text{rad}) = 169\ \text{rem}$.

(c) $\left|\dfrac{dN}{dt}\right| = \dfrac{m}{Am_{\text{p}}}\dfrac{\ln(2)}{T_{1/2}} = 1.17\times10^{9}\ \text{Bq} = 31.6\ \text{mCi}$.

(d) $t = \dfrac{7.75\times10^{12}}{1.17\times10^{9}\ \text{Bq}} = 6.62\times10^{3}\ \text{s}$, which is about 1.8 hours.

EVALUATE: The time in part (d) is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

43.59. **IDENTIFY:** Use $N = N_0 e^{-\lambda t}$ to relate the initial number of radioactive nuclei, N_0, to the number, N, left after time t.

SET UP: We have to be careful; after ^{87}Rb has undergone radioactive decay it is no longer a rubidium atom. Let N_{85} be the number of ^{85}Rb atoms; this number doesn't change. Let N_0 be the number of ^{87}Rb atoms on earth when the solar system was formed. Let N be the present number of ^{87}Rb atoms.

EXECUTE: The present measurements say that $0.2783 = N/(N + N_{85})$.

$(N + N_{85})(0.2783) = N$, so $N = 0.3856N_{85}$. The percentage we are asked to calculate is $N_0/(N_0 + N_{85})$.

N and N_0 are related by $N = N_0 e^{-\lambda t}$ so $N_0 = e^{+\lambda t}N$.

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{Ne^{\lambda t}}{Ne^{\lambda t} + N_{85}} = \dfrac{(0.3855e^{\lambda t})N_{85}}{(0.3856e^{\lambda t})N_{85} + N_{85}} = \dfrac{0.3856e^{\lambda t}}{0.3856e^{\lambda t} + 1}$.

$$t = 4.6\times10^{9}\ \text{y};\ \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{4.75\times10^{10}\ \text{y}} = 1.459\times10^{-11}\ \text{y}^{-1}.$$

$$e^{\lambda t} = e^{(1.459\times10^{-11}\ \text{y}^{-1})(4.6\times10^{9}\ \text{y})} = e^{0.06711} = 1.0694.$$

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{(0.3856)(1.0694)}{(0.3856)(1.0694) + 1} = 29.2\%$.

EVALUATE: The half-life for ^{87}Rb is a factor of 10 larger than the age of the solar system, so only a small fraction of the ^{87}Rb nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

43.60. **IDENTIFY:** Apply $N = N_0 e^{-\lambda t}$, with $\lambda = \dfrac{\ln 2}{T_{1/2}}$.

SET UP: Let 1 refer to $^{15}_{8}\text{O}$ and 2 refer to $^{19}_{8}\text{O}$. $\dfrac{N_1}{N_2} = \dfrac{e^{-\lambda_1 t}}{e^{-\lambda_2 t}}$, since N_0 is the same for the two isotopes.

$$e^{-\lambda t} = e^{-(\ln 2/T_{1/2})t} = (e^{-\ln 2})^{t/T_{1/2}} = (\tfrac{1}{2})^{t/T_{1/2}}.\ \ \frac{N_1}{N_2} = \left(\tfrac{1}{2}\right)^{(t/(T_{1/2})_1)/(t/(T_{1/2})_2)} = 2^{t\left(\frac{1}{(T_{1/2})_2} - \frac{1}{(T_{1/2})_1}\right)}.$$

EXECUTE: **(a)** After 3.0 min = 180 s, the ratio of the number of nuclei is

$$N_1/N_2 = \frac{2^{-180/122.2}}{2^{-180/26.9}} = 2^{(180)\left(\frac{1}{26.9} - \frac{1}{122.2}\right)} = 2^{(180)(0.02899)} = 37.2.$$

(b) After 12.0 min = 720 s, the ratio is $N_1/N_2 = 2^{(720)(002899)} = 1.92\times10^{6}$.

EVALUATE: The $^{19}_{8}\text{O}$ nuclei decay at a greater rate, so the ratio $N(^{15}_{8}\text{O})/N(^{19}_{8}\text{O})$ increases with time.

43.61. **IDENTIFY** and **SET UP:** Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

EXECUTE: **(a)** First find the number of decays each second:

$2.6\times10^{-4}\ \text{Ci}\left(\dfrac{3.70\times10^{10}\ \text{decays/s}}{1\ \text{Ci}}\right)=9.6\times10^{6}\ \text{decays/s}.$ The average energy per decay is $1.25\ \text{MeV},$ and one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is $\frac{1}{2}(9.6\times10^{6}\ \text{decays/s})(1.25\times10^{6}\ \text{eV/decay})(1.602\times10^{-19}\ \text{J/eV})=9.6\times10^{-7}\ \text{J/s}.$

(b) The absorbed dose is the energy absorbed divided by the mass of the tissue:

$$\frac{9.6\times10^{-7}\ \text{J/s}}{0.200\ \text{kg}}=(4.8\times10^{-6}\ \text{J/kg}\cdot\text{s})(1\ \text{rad}/(0.01\ \text{J/kg}))=4.8\times10^{-4}\ \text{rad/s}.$$

(c) equivalent dose (REM) = RBE × absorbed dose (rad). In one second the equivalent dose is

$$(0.70)(4.8\times10^{-4}\ \text{rad})=3.4\times10^{-4}\ \text{rem}.$$

(d) $(200\ \text{rem})/(3.4\times10^{-4}\ \text{rem/s})=(5.9\times10^{5}\ \text{s})(1\ \text{h}/3600\ \text{s})=164\ \text{h}=6.9\ \text{days}.$

EVALUATE: The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200-rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

43.62. **IDENTIFY:** This problem deals with nuclear fusion and the p-p I chain in the sun.

SET UP: The first reaction is $2\,{}^{1}_{1}\text{H}\rightarrow{}^{2}_{1}\text{H}+\beta^{+}+\nu_{\text{e}}.$ Follow the directions for each part.

EXECUTE: **(a)** We want the total energy E_1 of the deuteron and neutrino. First find the energy Q released in the fusion reaction, treating the neutrino as essentially massless. From Table 43.2, the deuteron mass is m_d = 2.014102 u. $Q = (2m_p - m_d)c^2$ = [2(1.007276 u) – 2.014102 u)(931.5 MeV/u) = 0.419 Mev. This is the energy shared between the deuteron, the positron, and the neutrino. But photons from electron-positron annihilation each had energy equal to the rest energy of the electron, so the positron must have been at rest after the first reaction. Therefore all the energy E_1 = 0.419 MeV is shared by the deuteron and neutrino.

(b) The neutrino is relativistic but the deuteron is not. If the neutrino is massless, then $E_v = pc$. Thus momentum conservation gives $p_d = p_v$, so $m_d v_d = E_v/c$. Energy conservation gives $E_d + E_v = E_1$, so

$\frac{1}{2}m_{\text{d}}v_{\text{d}}^2+E_\nu=E_1.$

(c) We want E_v/E_1. Solve the momentum and energy equations in part (b) for E_v. This leads to the following quadratic equation:

$$E_\nu^2+2m_{\text{d}}c^2E_\nu-2m_{\text{d}}c^2E_1=0.$$

Using the positive root with the quadratic formula and doing some algebra gives

$$E_\nu=m_{\text{d}}c^2\left(-1+\sqrt{1+\frac{2E_1}{m_dc^2}}\right).$$

Using m_d = 2.014102 u and E_1 = 0.419 MeV, we E_v/E_1 = 0.41895/0.419 = 0.99989 = 99.989%.

(d) As we just saw, the neutrino essentially carries off all the energy. The first reaction occurs twice for each p-p I chain, so they carry off $2E_1$ = 2(0.419 MeV) = 0.838 MeV.

(e) In Section 43.8, it was calculated that the p-p chain releases 26.73 MeV of energy. So the percent that the neutrinos carry away is (0.838 MeV)/(26.73 MeV) = 3.1%.

EVALUATE: The 3.1% of the energy is large enough to be measurable. If neutrinos were not taken into account, the energy calculations for the sun would be off by about 3%.

43.63. **IDENTIFY:** This problem investigates the p-p II fusion chain in the sun.

SET UP: Follow the directions in each part and use Table 43.2 for nuclide masses. The third step of the chain is

$$^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^7_4\text{Be} + \gamma.$$

EXECUTE: **(a)** We want the energy E_γ of the photon. Realize that $E_\gamma = Q$. From Table 43.2 we have:
Mass of He-3 = 3.016029 u
Mass of He-4 = 4.002603 u
Mass of Be-7 = 7.016930 u
The nuclide masses include the electrons, but since we'll be subtracting their effect subtracts out.
Therefore $E_\gamma = Q$ = (3.016029 u + 4.002603 u – 7.016930 u)(931.5 MeV/u) = 1.59 MeV.
(b) We want the energy E_ν of the neutrino in the following electron-capture of Be-7.

$$^7_4\text{Be} + \text{e}^- \rightarrow {}^7_3\text{Li} + \nu_\text{e}.$$

Using the given mass of Li-7 and the Be-7 mass from Table 43.2, we have

E_ν = (7.016930 u – 7.016003 u)(931.5 MeV/u) = 0.864 MeV.

(c) We want the energy Q that is released. Using the given reaction, we have

Q = [1.007825 u + 7.016003 u – 2(4.002603 u)](931.5 MeV/u) = 17.35 MeV.

(d) We want the speed v_α of the alpha particles. The alpha particles have equal speeds, so for each one K_α = (17.35 MeV)/2 = 8.675 MeV. Use the relativistic kinetic energy $K = mc^2(\gamma - 1)$. Solving for γ gives

$$\gamma = 1 + K_\alpha / m_\alpha c^2 = 1 + \frac{8.675 \text{ MeV}}{(4.002603 \text{ u})(931.5 \text{ MeV/u})} = 1.0023267.$$

Using this result gives

$$v = c\sqrt{1 - 1/\gamma^2} = 0.0681c.$$

(e) We want the total energy. The overall p-p II chain can be summarized as

$$4\,^1_1\text{H} + {}^4_2\text{He} \rightarrow 2\,^4_2\text{He} + 2\,\nu_\text{e} + \gamma.$$

Using masses from Table 43.2 gives Q = [4(1.007825 u) – 4.002603 u](931.5 MeV/u) = 26.7 MeV.
(f) The p-p I releases 26.73 MeV, so E_II/E_I = (26.7 MeV)/(26.73 MeV) = 99.9%. They are about the same.
(g) We want E_ν/E_tot. Using the results from (b) and (f) gives E_ν/E_I = (0.864 MeV)/(26.7 MeV) = 3.2%.

EVALUATE: The amount of energy per reaction is the same for p-p I and p-p II, but p-p II occurs less frequently so it contributes less to the sun's total energy production.

43.64. **IDENTIFY:** We are looking at the p-p III chain fusion in our sun.

SET UP: Follow the directions in each part and use Table 43.2 for nuclide masses.

EXECUTE: **(a)** We want the energy Q released. Looking at the reaction given in the problem, we see that Q = [4(1.007825 u) – 4.002603 u](931.5 MeV/u) = 26.73 MeV.
(b) We want the neutrino energy. The mass of B-8 is 8.024607 u and the mass of Be-8 + β^+ is 8.0053051 u. So E_ν = (8.024607 u – 8.0053051 u)(931.5 MeV/u) = 18.0 MeV.
(c) We want E_ν/E_tot. Using the results from (a) and (b) gives E_ν/E_tot = (18.0 MeV)/(26.73 MeV) = 67.3%.

EVALUATE: Comparing this result to those of problems 43.62 and 43.63, we see that in the p-p III chain the neutrino carries away a might higher percent of the energy than in the other two chains.

43.65. **IDENTIFY** and **SET UP:** The number of radioactive nuclei left after time t is given by $N = N_0 e^{-\lambda t}$. The problem says $N/N_0 = 0.29$; solve for t.

EXECUTE: $0.29 = e^{-\lambda t}$ so $\ln(0.29) = -\lambda t$ and $t = -\ln(0.29)/\lambda$. Example 43.9 gives

$$\lambda = 1.209 \times 10^{-4} \text{ y}^{-1} \text{ for } ^{14}\text{C. Thus } t = \frac{-\ln(0.29)}{1.209 \times 10^{-4} \text{ y}} = 1.0 \times 10^4 \text{ y}.$$

EVALUATE: The half-life of ^{14}C is 5730 y, so our calculated t is about 1.75 half-lives, so the fraction remaining is around $\left(\frac{1}{2}\right)^{1.75} = 0.30.$

43.66. IDENTIFY: In terms of the number ΔN of cesium atoms that decay in one week and the mass $m = 1.0\text{ kg},$ the equivalent dose is $3.5\text{ Sv} = \dfrac{\Delta N}{m}((\text{RBE})_\gamma \text{E}_\gamma + (\text{RBE})_\text{e}\text{E}_\text{e}).$

SET UP: $1\text{ day} = 8.64\times10^4\text{ s}.$ $1\text{ year} = 3.156\times10^7\text{ s}.$

EXECUTE: $3.5\text{ Sv} = \dfrac{\Delta N}{m}((1)(0.66\text{ MeV}) + (1.5)(0.51\text{ MeV})) = \dfrac{\Delta N}{m}(2.283\times10^{-13}\text{ J}),$ so

$$\Delta N = \frac{(1.0\text{ kg})(3.5\text{ Sv})}{(2.283\times10^{-13}\text{ J})} = 1.535\times10^{13}.\ \ \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{(30.07\text{ y})(3.156\times10^7\text{ sec/y})} = 7.30\times10^{-10}\text{ sec}^{-1}.$$

$$\Delta N = |dN/dt|t = \lambda N t,\ \text{so}\ N = \frac{\Delta N}{\lambda t} = \frac{1.535\times10^{13}}{(7.30\times10^{-10}\text{ s}^{-1})(7\text{ days})(8.64\times10^4\text{ s/day})} = 3.48\times10^{16}.$$

EVALUATE: We have assumed that $|dN/dt|$ is constant during a time of one week. That is a very good approximation, since the half-life is much greater than one week.

43.67. (a) IDENTIFY and SET UP: Use $R = R_0 A^{1/3}$ to calculate the radius R of a ${}^2_1\text{H}$ nucleus. Calculate the Coulomb potential energy $U = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$ of the two nuclei when they just touch.

EXECUTE: The radius of ${}^2_1\text{H}$ is $R = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.51\times10^{-15}\text{ m}.$ The barrier energy is the Coulomb potential energy of two ${}^2_1\text{H}$ nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\text{ C})^2}{2(1.51\times10^{-15}\text{ m})} = 7.64\times10^{-14}\text{ J} = 0.48\text{ MeV}.$$

(b) IDENTIFY and SET UP: Find the energy equivalent of the mass decrease.

EXECUTE: ${}^2_1\text{H} + {}^2_1\text{H} \to {}^3_2\text{He} + {}^1_0\text{n}$

If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

$${}^2_1\text{H} + {}^2_1\text{H}\ \text{has mass}\ 2(2.014102\text{ u}) = 4.028204\text{ u}$$

$${}^3_2\text{He} + {}^1_0\text{n}\ \text{has mass}\ 3.016029\text{ u} + 1.008665\text{ u} = 4.024694\text{ u}$$

The mass decrease is $4.028204\text{ u} - 4.024694\text{ u} = 3.510\times10^{-3}\text{ u}.$ This corresponds to a liberated energy of $(3.510\times10^{-3}\text{ u})(931.5\text{ MeV/u}) = 3.270\text{ MeV},$ or $(3.270\times10^6\text{ eV})(1.602\times10^{-19}\text{ J/eV}) = 5.239\times10^{-13}\text{ J}.$

(c) IDENTIFY and SET UP: We know the energy released when two ${}^2_1\text{H}$ nuclei fuse. Find the number of reactions obtained with one mole of ${}^2_1\text{H}.$

EXECUTE: Each reaction takes two ${}^2_1\text{H}$ nuclei. Each mole of D_2 has 6.022×10^{23} molecules, so 6.022×10^{23} pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is $(6.022\times10^{23})(5.239\times10^{-13}\text{ J}) = 3.155\times10^{11}\text{ J/mol}.$

EVALUATE: The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

43.68. **IDENTIFY:** The energy of the photon is equal to the sum of the kinetic energies of the proton and the neutron plus the binding energy of the deuteron.

SET UP: $\frac{hc}{\lambda} = K_p + K_n + E_B$. Since the proton and neutron have equal kinetic energy, this equation becomes $hc/\lambda = 2K_p + E_B = 2(\frac{1}{2} m_p v^2) + E_B$. Solving for v^2 gives $v^2 = \frac{hc}{m_p} \cdot \frac{1}{\lambda} - \frac{E_B}{m_p}$.

EXECUTE: **(a)** Figure 43.68 shows the graph of v^2 versus $1/\lambda$ for the data given in the problem. As shown above, the equation for v^2 versus $1/\lambda$ is $v^2 = \frac{hc}{m_p} \cdot \frac{1}{\lambda} - \frac{E_B}{m_p}$. The graph of v^2 versus $1/\lambda$ should be a straight line with slope equal to hc/m_p and y-intercept equal to $-E_B/m_p$. The slope of the best-fit straight line for our graph is 119.9 m^3/s^2 and the y-intercept is -2.156×10^{14} m^2/s^2.

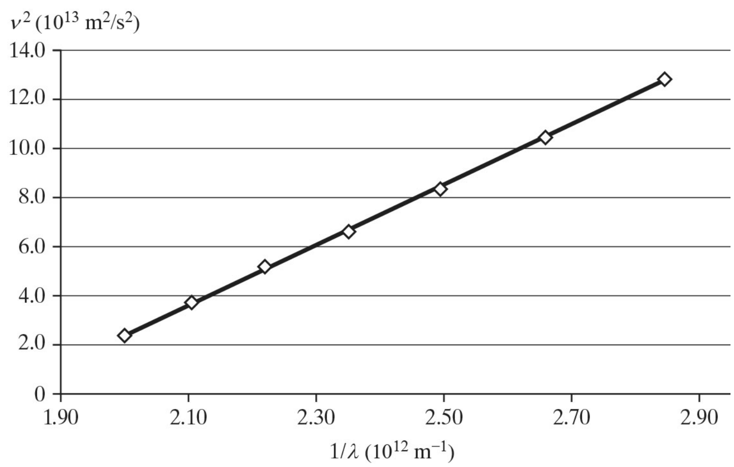

Figure 43.68

(b) Using the values for the slope and y-intercept for our graph, we have

$$hc/m_p = \text{slope} \rightarrow m_p = hc/(\text{slope}).$$

$$m_p = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})/(119.9\ \text{m}^3/\text{s}^2) = 1.66 \times 10^{-27}\ \text{kg}.$$

The y-intercept gives us the binding energy: $-E_B/m_p = y$-intercept, so $E_B = -m_p(y\text{-intercept})$.

$$E_B = -(1.66 \times 10^{-27}\ \text{kg})(-2.156 \times 10^{14}\ \text{m}^2/\text{s}^2) = 3.58 \times 10^{-13}\ \text{J} = 2.23 \times 10^6\ \text{eV} = 2.23\ \text{MeV}.$$

EVALUATE: The binding energy of a deuteron is $E_B = (m_{H\text{-}1} + m_n - m_D)c^2$, so $E_B = (1.007825\ \text{u} + 1.008665\ \text{u} - 2.014102\ \text{u})(931.5\ \text{MeV/u}) = 2.22$ MeV, so we are *very* close. Our results for m_p are *very* close to the accepted value for the proton mass of 1.67×10^{-27} kg.

43.69. **IDENTIFY:** Apply $\left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$, with $\lambda = \frac{\ln 2}{T_{1/2}}$.

SET UP: $\ln|dN/dt| = \ln \lambda N_0 - \lambda t$.

EXECUTE: **(a)** A least-squares fit to log of the activity *vs.* time gives a slope of magnitude

$$\lambda = 0.5995\ \text{h}^{-1},\ \text{for a half-life of } \frac{\ln 2}{\lambda} = 1.16\ \text{h}.$$

(b) The initial activity is $N_0\lambda$, and this gives $N_0 = \frac{(2.00 \times 10^4\ \text{Bq})}{(0.5995\ \text{hr}^{-1})(1\ \text{hr}/3600\ \text{s})} = 1.20 \times 10^8$.

(c) $N = N_0 e^{-\lambda t} = 1.81 \times 10^6$.

EVALUATE: The activity decreases by about $\frac{1}{2}$ in the first hour, so the half-life is about 1 hour.

43.70. **IDENTIFY:** We cannot tell much from the raw data, but we know for radioactive decay of a single nuclide the activity rate A decreases as $A = A_0 e^{-\lambda t} = |dN/dt|$.

SET UP: $\left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$ for a single nuclide. Since $A = A_0 e^{-\lambda t}$, it follows that $\ln A = \ln A_0 - \lambda t$.

Therefore if a single nuclide is present, a graph of ln A versus t should be a straight line with slope equal to $-\lambda$. We start with a graph of ln A versus t for the data given in the problem. Figure 43.70a shows this graph.

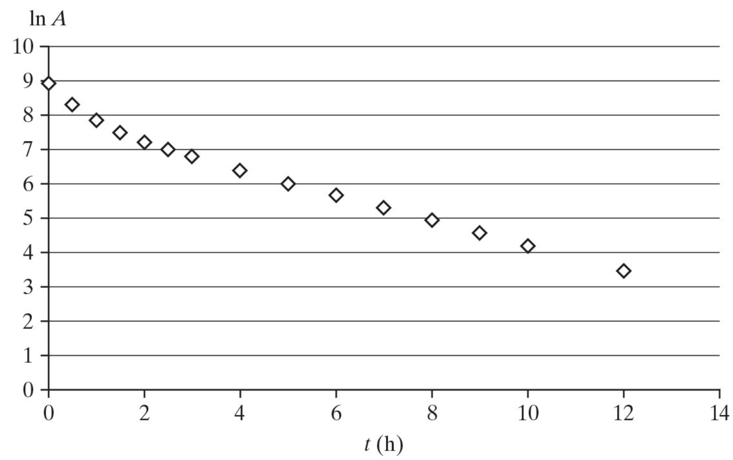

Figure 43.70a

EXECUTE: **(a)** The graph in Figure 43.70a is not a straight line, which suggests that the sample contains more than one nuclide. However after about 4.0 h, the graph does become a straight line. This suggests that a short-lived nuclide has decreased in activity, leaving only a single long-lived nuclide. To investigate the long-lived nuclide, we assume that nearly all the decays from 5.0 h on are due to this nuclide. So we make a graph of log A versus t for the decays from 5.0 h to 12.0 h. The result is shown in Figure 43.70b. This graph is a straight line, suggesting that our hypothesis is correct.

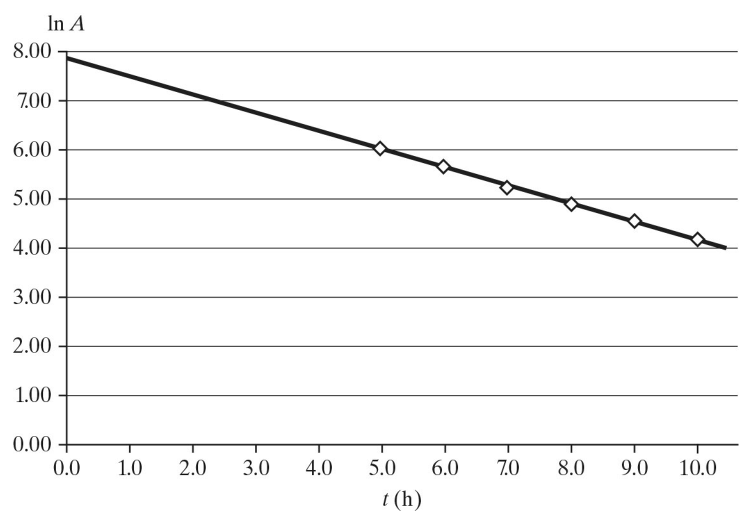

Figure 43.70b

To investigate the short-lived nuclide (or nuclides), we use our graph in Figure 43.70b to determine the activity of the long-lived nuclide during the first 4.0 h. We then subtract these numbers from the total decay rate to determine the decay rate due to the short-lived nuclide. The table shows these results.

Time (h)	Total rate (dec/s)	*A* (dec/s) for long-lived nuclide	ln *A* for long-lived nuclide	*A* (dec/s) for short-lived nuclide	ln *A* for short-lived nuclide
0	7500	2440	7.8	5060	8.53
0.5	4120	2320	7.75	1800	7.50
1.0	2570	1720	7.45	850	6.75
1.5	1790	1410	7.25	380	5.94
2.0	1350	1210	7.1	140	4.94
2.5	1070	992	6.9	78	4.36
3.0	872	6.7	812	60	4.09
4.0	596	572	6.35	24	3.18

Now we use the data in our table to graph ln *A* versus *t* for the short-lived nuclide. This graph is shown in Figure 43.70c. The last two points on the graph are unreliable because the decay rate is very small. For the rest of the points, the graph is a straight line. Therefore, since our two graphs of ln *A* versus *t* yield straight lines, it appears that our sample contains a minimum of two different nuclides, one with a short half-life and one with a long half-life.

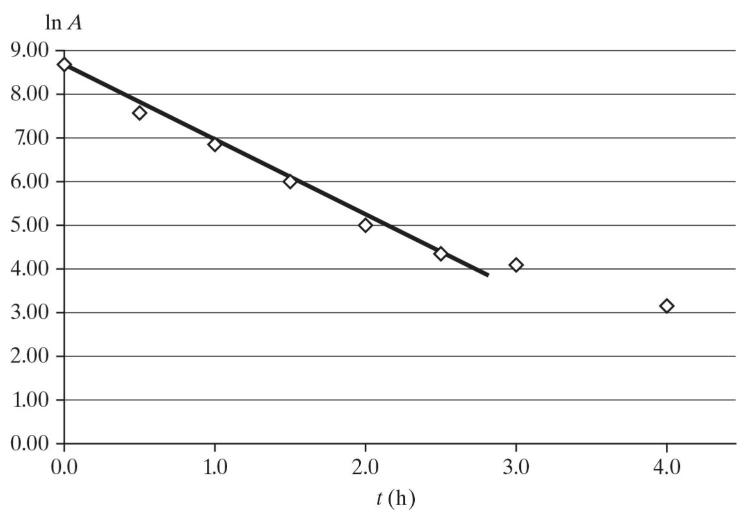

Figure 43.70c

(b) Long-lived nuclide: The slope of the graph in Figure 43.70b is $-0.3636\ \text{h}^{-1}$, so $\lambda_{\text{long}} = -\text{slope} = 0.3636\ \text{h}^{-1}$. Using $\lambda = (\ln 2)/T_{1/2}$, the half-life is $T_{1/2} = (\ln 2)/\lambda_{\text{long}} = (\ln 2)/(0.3636\ \text{h}^{-1}) = 1.9$ h.

Short-lived nuclide: The slope of the graph in Figure 43.70c is $-1.78\ \text{h}^{-1}$, so $\lambda_{\text{short}} = 1.78\ \text{h}^{-1}$. The half-life is $T_{1/2} = (\ln 2)/(1.78\ \text{h}^{-1}) = 0.39$ h.

(c) $\left|\dfrac{dN}{dt}\right| = \lambda N$, so $N = \dfrac{|dN/dt|}{\lambda}$.

Long-lived nuclide: $N_0 = (2440\ \text{Bq})/[(0.3636\ \text{h}^{-1})(1\ \text{h}/3600\ \text{s})] = 2.4 \times 10^7$ nuclei.

Short-lived nuclide: $N_0 = (5060\ \text{Bq})/[(1.78\ \text{h}^{-1})(1\ \text{h}/3600\ \text{s})] = 1.0 \times 10^7$ nuclei.

(d) Use $N = N_0 e^{-\lambda t}$ for each nuclide.

Long-lived nuclide: $N = N_0 e^{-\lambda_{\text{long}} t} = (2.4 \times 10^7)\, e^{-(0.3636\ \text{h}^{-1})(5.0\ \text{h})} = 3.9 \times 10^6$ nuclei.

Short-lived nuclide: $N = N_0 e^{-\lambda_{\text{short}} t} = (1.0 \times 10^7)\, e^{-(1.78\ \text{h}^{-1})(5.0\ \text{h})} = 1.4 \times 10^3$ nuclei.

EVALUATE: After 5.0 h, the number of shorter-lived nuclei is much less than the number of longer-lived nuclei. The ratio of the number of short-lived to the number of long-lived nuclei is $\dfrac{N_{\text{short}}}{N_{\text{long}}} = \dfrac{1.0 \times 10^7}{2.4 \times 10^7} \dfrac{e^{-\lambda_{\text{short}} t}}{e^{-\lambda_{\text{long}} t}} = 0.42 e^{-(\lambda_{\text{short}} - \lambda_{\text{long}})t}$. Since $\lambda_{\text{short}} > \lambda_{\text{long}}$, this ratio keeps decreasing with time.

43.71. IDENTIFY: Apply $A = A_0 e^{-\lambda t}$, where A is the activity and $\lambda = (\ln 2)/T_{1/2}$. This equation can be written as $A = A_0 2^{-(t/T_{1/2})}$. The activity of the engine oil is proportional to the mass worn from the piston rings.

SET UP: $1\ \text{Ci} = 3.7 \times 10^{10}$ Bq.

EXECUTE: The activity of the original iron, after 1000 hours of operation, would be $(9.4 \times 10^{-6}\ \text{Ci})\,(3.7 \times 10^{10}\ \text{Bq/Ci}) 2^{-(1000\ \text{h})/[(45\ \text{d})(24\ \text{h/d})]} = 1.8306 \times 10^5$ Bq. The activity of the oil is 84 Bq, or 4.5886×10^{-4} of the total iron activity, and this must be the fraction of the mass worn, or mass of 4.59×10^{-2} g. The rate at which the piston rings lost their mass is then 4.59×10^{-5} g/h.

EVALUATE: This method is very sensitive and can measure very small amounts of wear.

43.72. IDENTIFY and **SET UP:** Follow the procedure outlined in the problem. Solve the differential equation $dN_2/dt = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2$, where N_{10} is the initial number of ${}^{234}_{92}\text{U}$ nuclei and N_2 is the number of ${}^{230}_{88}\text{Th}$ nuclei. Assume a solution of the form $N_2(t) = N_{10}(h_1 e^{-\lambda_1 t} + h_2 e^{-\lambda_2 t})$ and follow the suggestions in the problem.

EXECUTE: **(a)** $N_2(0) = 0 = N_{10}(h_1 + h_2) \rightarrow h_2 = -h_1$.

(b) Take the derivative of $N_2(t)$: $dN_2/dt = N_{10}(-\lambda_1 h_1 e^{-\lambda_1 t} - \lambda_2 h_2 e^{-\lambda_2 t})$. Now substitute dN/dt and $N(t)$ into the original differential equation.

$$N_{10}(-\lambda_1 h_1 e^{-\lambda_1 t} - \lambda_2 h_2 e^{-\lambda_2 t}) = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 (h_1 e^{-\lambda_1 t} + h_2 e^{-\lambda_2 t}).$$

Collect coefficients of $e^{-\lambda_1 t}$ and $e^{-\lambda_2 t}$.

$$N_{10}(\lambda_1 h_1 + \lambda_1 - \lambda_2 h_2) e^{-\lambda_1 t} + N_{10}(\lambda_2 h_2 - \lambda_2 h_2) e^{-\lambda_2 t} = 0.$$

Setting the coefficients equal to zero gives $h_1 = \dfrac{\lambda_1}{\lambda_2 - \lambda_1}$. Putting this result into the equation for $N_2(t)$ and using the fact that $h_2 = -h_1$ gives $N_2(t) = \dfrac{\lambda_1}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$.

(c) The initial number of ${}^{234}_{92}\text{U}$ atoms is

$$N_{10} = (30.0\ \text{g})[(6.022 \times 10^{23}\ \text{atoms})/(234\ \text{g})] = 7.7205 \times 10^{22}\ \text{atoms}.$$

Using $\lambda = (\ln 2)/T_{1/2}$, we get $\lambda_1 = (\ln 2)/(2.46 \times 10^5\ \text{y})$ and $\lambda_2 = (\ln 2)/(7.54 \times 10^4\ \text{y})$, which gives $\dfrac{\lambda_1}{\lambda_2 - \lambda_1} = 0.44197$. At time $t = 2.46 \times 10^5$ y, the number N_2 of ${}^{230}_{88}\text{Th}$ atoms is

$$N_2 = (7.7205 \times 10^{22}\ \text{atoms})(0.44197)[e^{-(\ln 2)} - e^{(\ln 2)(2.46)/(0.754)}] = 1.3506 \times 10^{22}\ \text{atoms}.$$

The mass M of all these ${}^{230}_{88}\text{Th}$ atoms is

$$M = (1.3506 \times 10^{22}\ \text{atoms})[(1\ \text{mol})/(6.022 \times 10^{23}\ \text{atoms})][(230\ \text{g})/\text{mol}] = 5.16\ \text{g}.$$

EVALUATE: The ${}^{234}_{92}\text{U}$ decays with a half-life of 2.46×10^5 y, so at this time half the original uranium, or 15.0 g, is still left but only 5.16 g of ${}^{230}_{88}\text{Th}$ is present because it has continued to decay after being formed.

43.73. **IDENTIFY and SET UP:** The reaction is ${}^{130}_{52}\text{Te} + \text{X} \rightarrow {}^{131}_{52}\text{Te}.$

EXECUTE: X must have no charge since Z remains 52, and it must increase the atomic weight from 130 to 131, so it must be a neutron, which is choice (a).

EVALUATE: The other reactions in the choices all start out with the wrong isotope, so they cannot be correct.

43.74. **IDENTIFY and SET UP:** The reaction is ${}^{131}_{52}\text{Te} \rightarrow {}^{131}_{53}\text{I} + \text{X}.$

EXECUTE: The mass number 131 does not change, but the atomic number goes from 52 to 53, so the nucleus gained a charge of +1 (or lost a charge of –1). Therefore X must must be a β^- particle, which is choice (b).

EVALUATE: Alpha decay would change the mass number A by 4 units, β^+ decay would decrease the atomic number Z by 1 unit, and gamma decay would not affect Z.

43.75. **IDENTIFY and SET UP:** A thyroid treatment administers 3.7 GBq of ${}^{131}_{53}\text{I},$ which has a half-life of 8.04 days. $\lambda = (\ln 2)/T_{1/2}$ and $|dN/dt| = \lambda N = \dfrac{\ln 2}{T_{1/2}}N.$

EXECUTE: Solve for N and put in the numbers.

$N = \dfrac{T_{1/2}\,|dN/dt|}{\ln 2}$ = (8.04 d)(24 ×3600 s/d)(3.7 ×10^9 decays/s)/(ln 2) = 3.7 ×10^{15} atoms, which is choice (d).

EVALUATE: This amount is (3.7 ×10^{15} atoms)/(6.02 ×10^{23} atoms/mol) = 6.1 ×10^{-9} mol ≈ 6 nanomoles.

43.76. **IDENTIFY and SET UP:** The reaction is ${}^{123}_{52}\text{Te} + \text{p} \rightarrow {}^{123}_{53}\text{I} + \text{n}.$ Calculate the reaction energy Q to find out if the reaction is exoergic or endoergic.

EXECUTE: $Q = (M_{\text{Te}} + M_{\text{p}} - M_{\text{I}} - M_{\text{n}})c^2$. Using the values from the problem gives
Q = (122.904270 u + 1.007825 u – 122.905589 u – 1.008665 u)c^2 = –0.002159 uc^2, so the reaction is endoergic. We must put in energy to cause the reaction, which means that the proton must have a minimum kinetic energy, so choice (d) is correct.

EVALUATE: $|Q|$ = (0.002159 u)(931.5 MeV/u) = 2.011 MeV, so the proton must have at least 2.011 MeV of kinetic energy to cause the reaction.

43.77. **IDENTIFY and SET UP:** ${}^{131}_{53}\text{I}$ undergoes β^- decay, but ${}^{123}_{53}\text{I}$ undergoes gamma decay.

EXECUTE: From Table 43.3 we see that the RBE for gamma rays is 1, but the RBE for electrons is 1.0-1.5, so the electrons could cause more tissue damage than the gamma rays. This makes choice (b) the best one.

EVALUATE: The higher the RBE, the more likely it is that tissue damage could occur.