

THE FIRST LAW OF THERMODYNAMICS

VP19.5.1. IDENTIFY: We investigate several thermodynamic processes and apply the first law of thermodynamics.

SET UP: The first law of thermodynamics is $\Delta U = Q - W$.

EXECUTE: (a) $\Delta U = Q - W$: $2.50 \times 10^3 \text{ J} = Q - 3.25 \times 10^3 \text{ J}$. $Q = 5.75 \times 10^3 \text{ J}$.

(b) $\Delta U = Q - W$: $\Delta U = -7.00 \times 10^3 \text{ J} - 2.50 \times 10^4 \text{ J} = -3.20 \times 10^4 \text{ J}$.

(c) $\Delta U = Q - W$: $4.25 \times 10^3 \text{ J} = 2.40 \times 10^3 \text{ J} - W$. $W = -1.85 \times 10^3 \text{ J}$.

EVALUATE: Careful! Q is the heat put *into* the gas and W is the work done *by* the gas. If heat comes *out* of the gas, Q will be negative. If work is done *on* the gas, W will be negative.

VP19.5.2. IDENTIFY: We want to calculate the work that a gas does in various thermodynamic processes.

SET UP: $W = p\Delta V$ if pressure is constant. In general $W = \int p dV$. The work done by a gas is equal to the area under the curve on a pV -diagram.

EXECUTE: (a) The pressure is constant, so we use $W = p\Delta V = p(V_2 - V_1) = (6.20 \times 10^4 \text{ Pa})(4.50 \times 10^{-3} \text{ m}^3 - 2.00 \times 10^{-3} \text{ m}^3) = 155 \text{ J}$.

(b) $\Delta V = 0$ so $W = 0$.

(c) Pressure is constant so $W = p\Delta V = (1.25 \times 10^5 \text{ Pa})(-3.00 \times 10^{-3} \text{ m}^3) = -375 \text{ J}$.

(d) First sketch a pV -diagram of the process, as shown in Fig. VP19.5.2. The work done by the gas is the area under the curve. This area is made up of a triangle above a rectangle. The work is $W = A_{\text{triangle}} + A_{\text{rectangle}}$ which gives

$$W = \frac{1}{2}(2.50 \times 10^{-3} \text{ m}^3)(4.00 \times 10^5 \text{ Pa}) + (2.50 \times 10^{-3} \text{ m}^3)(1.50 \times 10^5 \text{ Pa}) = 875 \text{ J}.$$

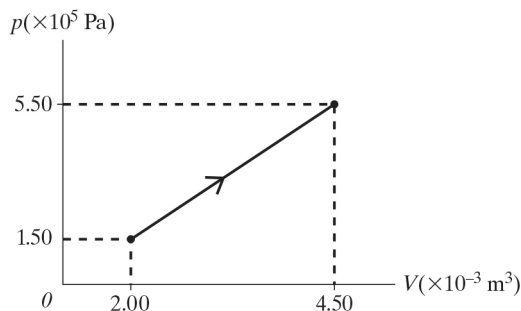


Figure VP19.5.2

EVALUATE: For more complicated curves, the area under the curve can be replaced by the definite integral $W = \int p dV$.

VP19.5.3. IDENTIFY: A gas undergoes various thermodynamic changes. We want to determine the work it does and the heat energy put into it. The ideal gas law and the first law of thermodynamics apply.

SET UP: We apply $\Delta U = Q - W$, $W = p\Delta V$ for constant pressure, and $pV = nRT$. We know that $p_1 = 2.00 \times 10^5 \text{ Pa}$ and $V_1 = 3.60 \times 10^{-3} \text{ m}^3$.

EXECUTE: (a) We want the work the gas does and its internal energy change. First find the work W done by the gas during the two processes 1 and 2. $W = W_1 + W_2$. This gives

$$W = p_1\Delta V_1 + p\Delta V_2 = p_1\Delta V_1 + 0 = (2.00 \times 10^5 \text{ Pa})(-1.20 \times 10^{-3} \text{ m}^3) = -2.40 \times 10^2 \text{ J}.$$

Now use $\Delta U = Q - W = 1560 \text{ J} - (-240 \text{ J}) = 1.80 \times 10^3 \text{ J}$.

(b) We want to find the work the gas does and how much heat flows into the gas. As before,

$$W = W_1 + W_2 = p_1\Delta V_1 + p_2\Delta V_2 = 0 + p_2\Delta V_2 = (6.00 \times 10^5 \text{ Pa})(-1.20 \times 10^{-3} \text{ m}^3) = -7.20 \times 10^2 \text{ J}.$$

Now apply $\Delta U = Q - W$ but we first need to find ΔU .

For the process in (a): $p_1V_1 = (2.00 \times 10^5 \text{ Pa})(3.60 \times 10^{-3} \text{ m}^3) = 720 \text{ J}$

$$p_2V_2 = (6.00 \times 10^5 \text{ Pa})(2.40 \times 10^{-3} \text{ m}^3) = 1440 \text{ J}$$

For the process in (b): $p_1V_1 = (2.00 \times 10^5 \text{ Pa})(3.60 \times 10^{-3} \text{ m}^3) = 720 \text{ J}$

$$p_2V_2 = (6.00 \times 10^5 \text{ Pa})(2.40 \times 10^{-3} \text{ m}^3) = 1440 \text{ J}$$

In both cases, the *change* in pV is $\Delta(pV) = 720 \text{ J}$. From $pV = nRT$ we see that $nR\Delta T = \Delta(pV)$.

Therefore for these two processes, ΔT is the same because $\Delta(pV)$ is the same, and if ΔT is the same,

ΔU must also be the same. So $\Delta U = 1.80 \times 10^3 \text{ J}$ as we found in part (a). Now we use $\Delta U = Q - W$:

$$1.80 \times 10^3 \text{ J} = Q - (-7.20 \times 10^2 \text{ J}), \text{ which gives } Q = 1.08 \times 10^3 \text{ J}.$$

EVALUATE: Note that ΔU depends only on ΔT , so if ΔT is the same ΔU must be the same, no matter what the process. ΔU is independent of path.

VP19.5.4. IDENTIFY: Heat flows into the solid CO_2 causing the gas to do work and change its internal energy. The first law of thermodynamics applies.

SET UP: $\Delta U = Q - W$, for sublimation $Q = mL_{\text{sub}}$, where $L_{\text{sub}} = 5.71 \times 10^5 \text{ J/kg}$. At constant pressure $W = p\Delta V$.

EXECUTE: (a) $Q = mL_{\text{sub}} = (6.0 \text{ kg})(5.71 \times 10^5 \text{ J/kg}) = 3.4 \times 10^6 \text{ J}$.

(b) We want the work. The pressure is constant so the work done is

$$W = p\Delta V = (1.01 \times 10^5 \text{ Pa})(3.4 \text{ m}^3 - 0.0040 \text{ m}^3) = 3.4 \times 10^5 \text{ J}.$$

(c) We want the change in internal energy. $\Delta U = Q - W = 3.4 \times 10^6 \text{ J} - 3.4 \times 10^5 \text{ J} = 3.1 \times 10^6 \text{ J}$.

EVALUATE: The internal energy change is due to an increase in the electrical potential energy of CO_2 molecules.

VP19.6.1. IDENTIFY: This problem involves molar heat capacity and internal energy for the monatomic gas neon.

SET UP: $Q = nC_V\Delta T$, $C_V = \frac{3}{2}R$ for a monatomic gas, $\Delta U = Q - W$. The target variable is the change in internal energy ΔU .

EXECUTE: (a) At constant volume, $W = 0$, so $\Delta U = Q = nC_V\Delta T = n\frac{3}{2}R\Delta T$. Using $n = 4.00 \text{ mol}$ and $\Delta T = 20.0 \text{ K}$ gives $\Delta U = 998 \text{ J}$.

(b) Since ΔU is independent of path, it is the same for constant volume and constant pressure processes (or any other process) if ΔT is the same. Therefore $\Delta U = Q_V = nC_V\Delta T = n\frac{3}{2}R\Delta T$. Using $n = 4.00$ mol and $\Delta T = 15.0$ K gives $\Delta U = 748$ J.

(c) Use the same reasoning as in part (b) since ΔU is independent of path. We have $\Delta U = n\frac{3}{2}R\Delta T$, and using $n = 4.00$ mol and $\Delta T = 12.0$ K gives $\Delta U = 599$ J.

EVALUATE: For any ideal gas processes, if ΔT is the same, ΔU is the same.

VP19.6.2. IDENTIFY: We are investigating the behavior of ideal N_2 gas. The ideal gas law and the first law of thermodynamics both apply.

SET UP: Use $pV = nRT$, $\Delta U = Q - W$, $Q = nC_V\Delta T$, and $C_V = \frac{3}{2}R$.

EXECUTE: (a) We want the initial volume of the gas. Solving $pV = nRT$ for V gives $V = nRT/p$. Using $n = 2.10$ mol, $T = 300$ K, and $p = 1.00 \times 10^5$ Pa gives $V = 5.24 \times 10^{-2}$ m³.

(b) (i) We want T_2 and ΔU when the pressure doubles at constant volume. For constant volume, $pV = nRT$ tells us that if p doubles, T also doubles, so $T_2 = 600$ K = 327°C .

For constant volume, $W = 0$, so $\Delta U = Q = nC_V\Delta T = n\frac{5}{2}R\Delta T$. Using $n = 2.10$ mol and $\Delta T = 300$ K gives $\Delta U = 1.31 \times 10^4$ J.

(ii) We want T_2 and ΔU when the volume doubles at constant pressure. For constant pressure, $pV = nRT$ tells us that if V doubles, T also doubles, so $T_2 = 600$ K = 327°C . By the same reasoning as in part (ii), $\Delta U = 1.31 \times 10^4$ J.

(iii) We want T_2 and ΔU when both pressure and volume double. From $pV = nRT$ we see that if both p and V double, the product pV increases by a factor of 4. Therefore T also increases by a factor of 4, so $T_2 = 4T_1 = 4(300 \text{ K}) = 1200$ K = 927°C .

Now $\Delta T = 1200 \text{ K} - 300 \text{ K} = 900$ K, which is 3 times the change in parts (i) and (ii), so ΔU is 3 times as large as before. Therefore $\Delta U = 3(1.31 \times 10^4 \text{ J}) = 3.93 \times 10^4$ J.

EVALUATE: Note that if we quadruple the Kelvin temperature we do *not* quadruple the Celsius temperature.

VP19.6.3. IDENTIFY: Ideal monatomic argon gas goes through changes in its temperature and pressure. We want to find the resulting change in its internal energy. The ideal gas law and first law of thermodynamics both apply.

SET UP: We use $pV = nRT$, $\Delta U = Q - W$, $\Delta U = nC_V\Delta T$ for any process, and $C_V = \frac{3}{2}R$ for a

monatomic gas. $pV = nRT$ tells us that $\Delta(pV) = nR\Delta T$, so $\Delta T = \frac{\Delta(pV)}{nR}$. This gives $\Delta U = nC_V\Delta T =$

$n\left(\frac{3}{2}R\right)\frac{\Delta(pV)}{nR} = \frac{3}{2}\Delta(pV) = \frac{3}{2}(p_2V_2 - p_1V_1)$. We want to find ΔU for each process. We know that $p_1 = 1.20 \times 10^5$ Pa and $V_1 = 0.250$ m³ for each process.

EXECUTE: (a) $V_1 = V_2 = 0.250$ m³ and $p_2 = 2.4 \times 10^5$ Pa. Use $\Delta U = \frac{3}{2}(p_2V_2 - p_1V_1)$ which becomes

$$\Delta U = \frac{3}{2}V_1(p_2 - p_1) = \frac{3}{2}(0.250 \text{ m}^3)(1.20 \times 10^5 \text{ Pa}) = 4.50 \times 10^4 \text{ J}.$$

(b) $p_1 = p_2 = 1.20 \times 10^5 \text{ Pa}$ and $V_2 = 0.125 \text{ m}^3$. Use $\Delta U = \frac{3}{2}(p_2 V_2 - p_1 V_1)$ which becomes $\Delta U = \frac{3}{2} p_1 (V_2 - V_1) = \frac{3}{2} (1.20 \times 10^5 \text{ Pa})(0.125 \text{ m}^3 - 0.250 \text{ m}^3) = -2.25 \times 10^4 \text{ J}$.

(c) $p_2 = 1.80 \times 10^5 \text{ Pa}$ and $V_2 = 0.600 \text{ m}^3$. Use $\Delta U = \frac{3}{2}(p_2 V_2 - p_1 V_1)$ which gives $\Delta U = 1.17 \times 10^5 \text{ J}$.

EVALUATE: Even though only one of the processes was at constant volume, we could use $\Delta U = nC_V \Delta T$ to calculate ΔU because ΔU is the same as if the process were at constant volume. Remember that ΔU is independent of path.

VP19.6.4. IDENTIFY: We increase the temperature of a gas from T to $2T$ by isochoric, isobaric, and adiabatic processes. In each case we want to find the change in the internal energy of the gas, the heat flow into it, and work done by the gas. The first law of thermodynamics applies.

SET UP: $\Delta U = Q - W$, $\Delta U = nC_V \Delta T$, at constant pressure $Q = nC_p \Delta T$, and for a diatomic gas

$$C_V = \frac{5}{2}R \text{ and } C_p = \frac{7}{2}R. \text{ The target variable is } \Delta U \text{ for each process.}$$

EXECUTE: (a) The *volume* is constant for an isochoric process, so $W = 0$. Therefore $\Delta U = Q - W$ gives $\Delta U = Q = nC_V \Delta T$. Since the temperature increases from T to $2T$, $\Delta T = 2T - T = T$. Thus $\Delta U = Q = nC_V \Delta T = n\left(\frac{5}{2}R\right)T = \frac{5}{2}nRT$.

(b) The *pressure* is constant for an isobaric process. $Q = nC_p \Delta T = n\left(\frac{7}{2}R\right)T = \frac{7}{2}nRT$. $\Delta U = nC_V \Delta T = \frac{5}{2}nRT$, the same as in part (a). From $\Delta U = Q - W$ we have $W = Q - \Delta U = \frac{7}{2}nRT - \frac{5}{2}nRT = nRT$.

(c) For an adiabatic process, no heat enters or leaves the gas, so $Q = 0$. From part (a) we know that $\Delta U = nC_V \Delta T = \frac{5}{2}nRT$. $\Delta U = Q - W$ gives $W = Q - \Delta U = 0 - \Delta U = -\frac{5}{2}nRT$.

EVALUATE: As we have seen here, doubling the gas temperature can have different results depending on how the process is carried out.

VP19.7.1. IDENTIFY: We are investigating monatomic argon gas during an adiabatic expansion.

SET UP: pV^γ is constant during an adiabatic process, $\gamma = C_p / C_V$, $W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2)$.

EXECUTE: (a) $\gamma = \frac{C_p}{C_V} = \frac{5/2R}{3/2R} = 5/3$.

(b) We want the final pressure p_2 . Since pV^γ is constant, we have $p_1 V_1^\gamma = p_2 V_2^\gamma$ which gives

$p_2 = p_1 \left(\frac{V_1}{V_2}\right)^\gamma$. Using $p_1 = 4.00 \times 10^5 \text{ Pa}$, $V_1 = 2.00 \times 10^{-3} \text{ m}^3$, $V_2 = 6.00 \times 10^{-3} \text{ m}^3$, and $\gamma = 5/3$, we have $p_2 = 6.41 \times 10^4 \text{ Pa}$.

(c) We want the work W . $W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2) = \frac{3/2R}{R}(p_1 V_1 - p_2 V_2) = \frac{3}{2}(p_1 V_1 - p_2 V_2)$. Using the pressures and volumes above, we get $W = 623 \text{ J}$.

EVALUATE: Careful! An adiabatic process is *not* the same as an isothermal process. The temperature can change during an adiabatic process but not during an isothermal process.

VP19.7.2. IDENTIFY: We are investigating diatomic oxygen O_2 gas during an adiabatic compression.

SET UP: $TV^{\gamma-1}$ and pV^γ are constant during an adiabatic process, $\gamma = C_p / C_v$,

$$W = \frac{1}{\gamma-1}(p_1V_1 - p_2V_2).$$

EXECUTE: (a) $\gamma = \frac{C_p}{C_v} = \frac{7/2R}{5/2R} = 7/5$.

(b) We want the final volume V_2 . Since $TV^{\gamma-1}$ is constant, we have $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ which gives

$$V_2^{\gamma-1} = V_1^{\gamma-1} \frac{T_1}{T_2}. \text{ Taking the } \gamma-1 \text{ root gives } V_2 = V_1 \left(\frac{T_1}{T_2} \right)^{1/(\gamma-1)} = V_1 \left(\frac{T_1}{T_2} \right)^{5/2}.$$

Using $p_1 = 1.00 \times 10^5 \text{ Pa}$, $V_1 = 6.50 \times 10^{-3} \text{ m}^3$, $T_1 = 325 \text{ K}$, and $T_2 = 855 \text{ K}$, we have $V_2 = 5.79 \times 10^{-4} \text{ m}^3$.

(c) We want the final pressure p_2 . pV^γ is constant so $p_1V_1^\gamma = p_2V_2^\gamma$. Solving for p_2 gives

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma. \text{ Using } p_1 = 1.00 \times 10^5 \text{ Pa}, V_1 = 6.50 \times 10^{-3} \text{ m}^3, \text{ and } V_2 = 5.79 \times 10^{-4} \text{ m}^3, \text{ we get } p_2 = 2.95 \times 10^6 \text{ Pa}.$$

(d) We want the work done by the gas during this compression. Use $W = \frac{1}{\gamma-1}(p_1V_1 - p_2V_2)$ and put in

the numbers from above, we get $W = -2650 \text{ J}$. The minus sign tells us that work is done *on* the gas to compress it.

EVALUATE: Regardless of the process, the gas must still obey the ideal gas law, so pV/T should be the same for states 1 and 2. Using the values for the pressure, volume, and temperature of each state, we get $p_1V_1/T_1 = 2.00 \text{ J/K}$ and $p_2V_2/T_2 = 2.00 \text{ J/K}$, so our results are consistent with the ideal gas law.

VP19.7.3. IDENTIFY: Monatomic neon gas expands adiabatically.

SET UP: $TV^{\gamma-1}$ and pV^γ are constant during an adiabatic process and $\gamma = C_p / C_v = 5/3$. We also have $\Delta U = Q - W$, $\Delta U = nC_v\Delta T$, $pV = nRT$. We know $T_1 = 305 \text{ K}$, $V_1 = 0.0400 \text{ m}^3$, and $V_2 = 0.0900 \text{ m}^3$, and $n = 5.00 \text{ mol}$. $Q = 0$ for an adiabatic process.

EXECUTE: (a) We want the initial pressure p_1 . Solving $pV = nRT$ gives $p_1 = nRT_1/V_1$. Using the numbers above gives $p_1 = 3.17 \times 10^5 \text{ Pa}$.

(b) We want the final pressure p_2 . Since pV^γ is constant, $p_1V_1^\gamma = p_2V_2^\gamma$. Solving for p_2 gives

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma. \text{ Using } \gamma = 5/3 \text{ and the values for the volumes and pressure, we get } p_2 = 8.20 \times 10^4 \text{ Pa}.$$

(c) We want the final temperature T_2 . Since $TV^{\gamma-1}$ is constant, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$. Solving for T_2 gives

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}. \text{ Using } T_1 = 305 \text{ K} \text{ and the values for the volumes, we have } T_2 = 178 \text{ K}.$$

(d) We want the work done by the gas. For an adiabatic process, $Q = 0$, so $\Delta U = Q - W$ gives $W =$

$$-\Delta U = -nC_v\Delta T = -n \left(\frac{3}{2}R \right) \Delta T. \text{ Using } \Delta T = 178 \text{ K} - 305 \text{ K} \text{ and } n = 5.00 \text{ mol, we get } W = 7940 \text{ J}.$$

EVALUATE: We can check the answer in part (d) by using $W = \frac{1}{\gamma-1}(p_1V_1 - p_2V_2)$. Putting in the numbers from above gives $W = 7950 \text{ J}$. The slight difference in the last digit is from rounding.

VP19.7.4. IDENTIFY: Diatomic hydrogen gas H_2 expands adiabatically.

SET UP: $TV^{\gamma-1}$ and pV^γ are constant during an adiabatic process and $\gamma = C_p / C_v = 7/5$. We also have $\Delta U = Q - W$, $\Delta U = nC_v\Delta T$, $pV = nRT$. We know that $T_1 = 325 \text{ K}$, $T_2 = 195 \text{ K}$, and $n = 1.25 \text{ mol}$. $Q = 0$ for an adiabatic process.

EXECUTE: (a) We want the work done by the gas. Using $\Delta U = Q - W$ with $Q = 0$ gives $W = -\Delta U =$

$$-nC_v\Delta T = -n\left(\frac{5}{2}R\right)\Delta T = -(1.25 \text{ mol})(5/2)(8.314 \text{ J/mol} \cdot \text{K})(130 \text{ K}), \text{ so } W = -3.38 \times 10^3 \text{ J}.$$

(b) We want V_2/V_1 . Since $TV^{\gamma-1}$ is constant, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, which gives Using the numbers gives

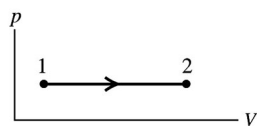
$$\frac{V_2}{V_1} = \left(\frac{325 \text{ K}}{195 \text{ K}}\right)^{5/2} = 3.59.$$

(c) We want p_2/p_1 . Solving $p_1V_1^\gamma = p_2V_2^\gamma$ for p_2/p_1 gives $\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{1}{3.59}\right)^{7/5} = 0.167$.

EVALUATE: The gas must also obey the ideal gas law, so we check the ratio p_2/p_1 ratio. Using $pV = nRT$ this ratio is $\frac{p_2}{p_1} = \frac{nRT_2/V_2}{nRT_1/V_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) = \left(\frac{195 \text{ K}}{325 \text{ K}}\right)\left(\frac{1}{3.59}\right) = 0.167$, which agrees with our result in part

(c).

19.1. (a) IDENTIFY and SET UP: The pressure is constant and the volume increases.



The pV -diagram is sketched in Figure 19.1.

Figure 19.1

(b) $W = \int_{V_1}^{V_2} p dV$. Since p is constant, $W = p \int_{V_1}^{V_2} dV = p(V_2 - V_1)$. The problem gives T rather than p and V , so use the ideal gas law to rewrite the expression for W .

EXECUTE: $pV = nRT$ so $p_1V_1 = nRT_1$, $p_2V_2 = nRT_2$; subtracting the two equations gives

$p(V_2 - V_1) = nR(T_2 - T_1)$. Thus the work is $W = nR(T_2 - T_1)$ during a constant pressure process for an ideal gas. Then $W = nR(T_2 - T_1) = (2.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(107^\circ\text{C} - 27^\circ\text{C}) = +1330 \text{ J}$.

EVALUATE: The gas expands when heated and does positive work.

19.2. IDENTIFY: At constant pressure, $W = p\Delta V = nR\Delta T$. Since the gas is doing work, it must be expanding, so ΔV is positive, which means that ΔT must also be positive.

SET UP: $R = 8.3145 \text{ J/mol} \cdot \text{K}$. ΔT has the same numerical value in kelvins and in $^\circ\text{C}$.

EXECUTE: $\Delta T = \frac{W}{nR} = \frac{2.40 \times 10^3 \text{ J}}{(6 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = 48.1 \text{ K}$. $\Delta T_K = \Delta T_C$ and

$$T_2 = 27.0^\circ\text{C} + 48.1^\circ\text{C} = 75.1^\circ\text{C}.$$

EVALUATE: When $W > 0$ the gas expands. When p is constant and V increases, T increases.

19.3. IDENTIFY: For an isothermal process $W = nRT \ln(p_1/p_2)$. $pV = nRT$ says V decreases when p increases and T is constant.

SET UP: $T = 65.0 + 273.15 = 338.15 \text{ K}$. $p_2 = 3p_1$.

EXECUTE: (a) The pV -diagram is sketched in Figure 19.3.

$$(b) W = (2.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(338.15 \text{ K}) \ln \left(\frac{p_1}{3p_1} \right) = -6180 \text{ J}.$$

EVALUATE: Since V decreases, W is negative.

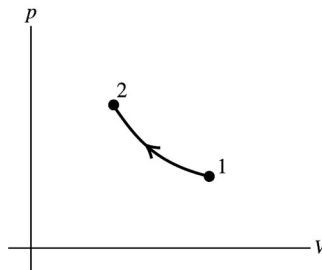


Figure 19.3

19.4. IDENTIFY: The work done in a cycle is the area enclosed by the cycle in a pV diagram.

SET UP: (a) 1 mm of Hg = 133.3 Pa. $p_{\text{gauge}} = p - p_{\text{air}}$. In calculating the enclosed area only changes in pressure enter and you can use gauge pressure. $1 \text{ L} = 10^{-3} \text{ m}^3$.

(b) Since $pV = nRT$ and T is constant, the maximum number of moles of air in the lungs is when pV is a maximum. In the ideal gas law the absolute pressure $p = p_{\text{gauge}} + p_{\text{air}}$ must be used.

$$p_{\text{air}} = 760 \text{ mm of Hg. } 1 \text{ mm of Hg} = 1 \text{ torr.}$$

EXECUTE: (a) By counting squares and noting that the area of 1 square is (1 mm of Hg)(0.1 L), we estimate that the area enclosed by the cycle is about $7.5 \text{ (mm of Hg)} \cdot \text{L} = 1.00 \text{ N} \cdot \text{m}$. The net work done is positive.

(b) The maximum pV is when $p = 11 \text{ torr} + 760 \text{ torr} = 771 \text{ torr} = 1.028 \times 10^5 \text{ Pa}$ and $V = 1.4 \text{ L} = 1.4 \times 10^{-3} \text{ m}^3$. The maximum pV is $(pV)_{\text{max}} = 144 \text{ N} \cdot \text{m}$. $pV = nRT$ so

$$n_{\text{max}} = \frac{(pV)_{\text{max}}}{RT} = \frac{144 \text{ N} \cdot \text{m}}{(8.315 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.059 \text{ mol}.$$

EVALUATE: While inhaling the gas does positive work on the lungs, but while exhaling the lungs do work on the gas, so the net work is positive.

19.5. IDENTIFY: For an isothermal process $W = nRT \ln(p_1/p_2)$. Solve for p_1 .

SET UP: For a compression (V decreases) W is negative, so $W = -392 \text{ J}$. $T = 295.15 \text{ K}$.

$$\text{EXECUTE: (a) } \frac{W}{nRT} = \ln \left(\frac{p_1}{p_2} \right). \quad \frac{p_1}{p_2} = e^{W/nRT}.$$

$$\frac{W}{nRT} = \frac{-392 \text{ J}}{(0.305 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(295.15 \text{ K})} = -0.5238.$$

$$p_1 = p_2 e^{W/nRT} = (1.76 \text{ atm})e^{-0.5238} = 1.04 \text{ atm}.$$

(b) In the process the pressure increases and the volume decreases. The pV -diagram is sketched in Figure 19.5.

EVALUATE: W is the work done by the gas, so when the surroundings do work on the gas, W is negative. The gas was compressed at constant temperature, so its pressure must have increased, which means that $p_1 < p_2$, which is what we found.

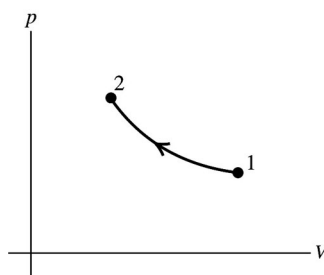


Figure 19.5

19.6. (a) IDENTIFY and SET UP: The pV -diagram is sketched in Figure 19.6.

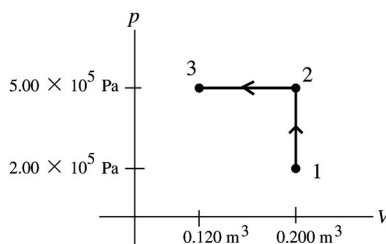


Figure 19.6

(b) Calculate W for each process, using the expression for W that applies to the specific type of process.

EXECUTE: $1 \rightarrow 2$: $\Delta V = 0$, so $W = 0$

$2 \rightarrow 3$: Since p is constant, $W = p \Delta V = (5.00 \times 10^5 \text{ Pa})(0.120 \text{ m}^3 - 0.200 \text{ m}^3) = -4.00 \times 10^4 \text{ J}$ (W is negative since the volume decreases in the process.)

$W_{\text{tot}} = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} = -4.00 \times 10^4 \text{ J}.$

EVALUATE: The volume decreases so the total work done is negative.

19.7. IDENTIFY: Calculate W for each step using the appropriate expression for each type of process.

SET UP: When p is constant, $W = p \Delta V$. When $\Delta V = 0$, $W = 0$.

EXECUTE: (a) $W_{13} = p_1(V_2 - V_1)$, $W_{32} = 0$, $W_{24} = p_2(V_1 - V_2)$ and $W_{41} = 0$. The total work done by the system is $W_{13} + W_{32} + W_{24} + W_{41} = (p_1 - p_2)(V_2 - V_1)$, which is the area in the pV plane enclosed by the loop.

(b) For the process in reverse, the pressures are the same, but the volume changes are all the negatives of those found in part (a), so the total work is negative of the work found in part (a).

EVALUATE: When $\Delta V > 0$, $W > 0$ and when $\Delta V < 0$, $W < 0$.

19.8. IDENTIFY: An ideal gas undergoes a process during which its pressure is directly proportional to its volume.

SET UP: We want the work done by the gas. We know that $p = \alpha V$ where $\alpha > 0$. The work done by (or on) the gas is the area under the curve in a pV -diagram. Start by making such a diagram in Fig. 19.8.

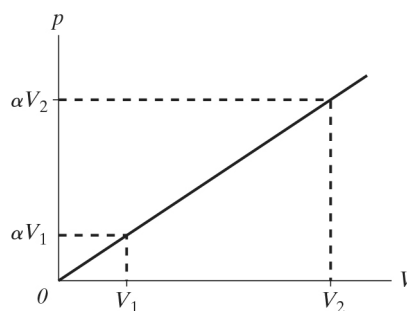


Figure 19.8

EXECUTE: The region from V_1 to V_2 is composed of a triangle plus a rectangle, so the total area is $W =$

$$A = A_{\text{tri}} + A_{\text{rect}} = \frac{1}{2}(V_2 - V_1)(\alpha V_2 - \alpha V_1) + (V_2 - V_1)(\alpha V_1) = \frac{\alpha}{2}(V_2^2 - V_1^2).$$

EVALUATE: Since $V_2 > V_1$, the work is positive so the gas does work.

19.9. IDENTIFY: $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

SET UP: $Q = +1.15 \times 10^5$ J, since heat enters the gas.

EXECUTE: (a) $W = p\Delta V = (1.65 \times 10^5 \text{ Pa})(0.320 \text{ m}^3 - 0.110 \text{ m}^3) = 3.47 \times 10^4$ J.

(b) $\Delta U = Q - W = 1.15 \times 10^5 \text{ J} - 3.47 \times 10^4 \text{ J} = 8.04 \times 10^4$ J.

EVALUATE: (c) $W = p\Delta V$ for a constant pressure process and $\Delta U = Q - W$ both apply to any material. The ideal gas law wasn't used and it doesn't matter if the gas is ideal or not.

19.10. IDENTIFY: The type of process is not specified. We can use $\Delta U = Q - W$ because this applies to all processes. Calculate ΔU and then from it calculate ΔT .

SET UP: Q is positive since heat goes into the gas; $Q = +1500$ J.

W is positive since gas expands; $W = +2100$ J.

EXECUTE: $\Delta U = 1500 \text{ J} - 2100 \text{ J} = -600$ J.

We can also use $\Delta U = n(\frac{3}{2}R)\Delta T$ since this is true for any process for an ideal gas.

$$\Delta T = \frac{2\Delta U}{3nR} = \frac{2(-600 \text{ J})}{3(5.00 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = -9.62 \text{ C}^\circ$$

$$T_2 = T_1 + \Delta T = 127^\circ\text{C} + (-9.62 \text{ C}^\circ) = 117^\circ\text{C}.$$

EVALUATE: More energy leaves the gas in the expansion work than enters as heat. The internal energy therefore decreases, and for an ideal gas this means the temperature decreases. We didn't have to convert ΔT to kelvins since ΔT is the same on the Kelvin and Celsius scales.

19.11. IDENTIFY: Part ab is isochoric, but bc is not any of the familiar processes.

SET UP: $pV = nRT$ determines the Kelvin temperature of the gas. The work done in the process is the area under the curve in the pV diagram. Q is positive since heat goes into the gas.

$$1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}. \quad 1 \text{ L} = 1 \times 10^{-3} \text{ m}^3. \quad \Delta U = Q - W.$$

EXECUTE: (a) The lowest T occurs when pV has its smallest value. This is at point a , and

$$T_a = \frac{p_a V_a}{nR} = \frac{(0.20 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(2.0 \text{ L})(1.0 \times 10^{-3} \text{ m}^3/\text{L})}{(0.0175 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 278 \text{ K}.$$

(b) a to b : $\Delta V = 0$ so $W = 0$.

b to c : The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve so $W = \frac{1}{2}(0.50 \text{ atm} + 0.30 \text{ atm})(6.0 \text{ L} - 2.0 \text{ L})$ and

$$W = (1.6 \text{ L} \cdot \text{atm})(1 \times 10^{-3} \text{ m}^3/\text{L})(1.013 \times 10^5 \text{ Pa/atm}) = 162 \text{ J}.$$

(c) For abc , $W = 162$ J. $\Delta U = Q - W = 215 \text{ J} - 162 \text{ J} = 53$ J.

EVALUATE: 215 J of heat energy went into the gas. 53 J of energy stayed in the gas as increased internal energy and 162 J left the gas as work done by the gas on its surroundings.

- 19.12. IDENTIFY and SET UP:** Calculate W using the equation for a constant pressure process. Then use $\Delta U = Q - W$ to calculate Q .

EXECUTE: (a) $W = \int_{V_1}^{V_2} p \, dV = p(V_2 - V_1)$ for this constant pressure process.

$W = (1.80 \times 10^5 \text{ Pa})(1.20 \text{ m}^3 - 1.70 \text{ m}^3) = -9.00 \times 10^4 \text{ J}$. (The volume decreases in the process, so W is negative.)

(b) $\Delta U = Q - W$. $Q = \Delta U + W = -1.40 \times 10^5 \text{ J} + (-9.00 \times 10^4 \text{ J}) = -2.30 \times 10^5 \text{ J}$. Negative Q means heat flows out of the gas.

(c) **EVALUATE:** $W = \int_{V_1}^{V_2} p \, dV = p(V_2 - V_1)$ (constant pressure) and $\Delta U = Q - W$ apply to *any* system, not just to an ideal gas. We did not use the ideal gas equation, either directly or indirectly, in any of the calculations, so the results are the same whether the gas is ideal or not.

- 19.13. IDENTIFY:** We read values from the pV -diagram and use the ideal gas law, as well as the first law of thermodynamics.

SET UP: Use $pV = nRT$ to calculate T at each point. The work done in a process is the area under the curve in the pV diagram. $\Delta U = Q - W$ for all processes.

EXECUTE: (a) $pV = nRT$ so $T = \frac{pV}{nR}$.

Point a: $T_a = \frac{(2.0 \times 10^5 \text{ Pa})(0.010 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 535 \text{ K}$.

Point b: $T_b = \frac{(5.0 \times 10^5 \text{ Pa})(0.070 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 9350 \text{ K}$.

Point c: $T_c = \frac{(8.0 \times 10^5 \text{ Pa})(0.070 \text{ m}^3)}{(0.450 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 15,000 \text{ K}$.

(b) The work done by the gas is positive since the volume increases. The magnitude of the work is the area under the curve: $W = \frac{1}{2}(2.0 \times 10^5 \text{ Pa} + 5.0 \times 10^5 \text{ Pa})(0.070 \text{ m}^3 - 0.010 \text{ m}^3) = 2.1 \times 10^4 \text{ J}$.

(c) $\Delta U = Q - W$ so $Q = \Delta U + W = 15,000 \text{ J} + 2.1 \times 10^4 \text{ J} = 3.6 \times 10^4 \text{ J}$.

EVALUATE: Q is positive so heat energy goes into the gas.

- 19.14. IDENTIFY:** $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

SET UP: $Q = +2.20 \times 10^6 \text{ J}$; $Q > 0$ since this amount of heat goes into the water.

$p = 2.00 \text{ atm} = 2.03 \times 10^5 \text{ Pa}$.

EXECUTE: (a) $W = p\Delta V = (2.03 \times 10^5 \text{ Pa})(0.824 \text{ m}^3 - 1.00 \times 10^{23} \text{ m}^3) = 1.67 \times 10^5 \text{ J}$

(b) $\Delta U = Q - W = 2.20 \times 10^6 \text{ J} - 1.67 \times 10^5 \text{ J} = 2.03 \times 10^6 \text{ J}$.

EVALUATE: $2.20 \times 10^6 \text{ J}$ of energy enters the water. $1.67 \times 10^5 \text{ J}$ of energy leaves the materials through expansion work and the remainder stays in the material as an increase in internal energy.

- 19.15. IDENTIFY:** For a certain thermodynamics process, $|Q| = 100 \text{ J}$ and $|W| = 300 \text{ J}$. The first law of thermodynamics applies.

SET UP and EXECUTE: We use $\Delta U = Q - W$ and want to find Q and W for each value of ΔU .

(a) $\Delta U = +400 \text{ J}$: $400 \text{ J} = Q - W$, so $Q = +100 \text{ J}$ and $W = -300 \text{ J}$.

(b) $\Delta U = +200 \text{ J}$: $200 \text{ J} = Q - W$, so $Q = -100 \text{ J}$ and $W = -300 \text{ J}$.

(c) $\Delta U = -200 \text{ J}$: $-200 \text{ J} = Q - W$, so $Q = +100 \text{ J}$ and $W = +300 \text{ J}$.

(d) $\Delta U = -400 \text{ J}$: $-400 \text{ J} = Q - W$, so $Q = -100 \text{ J}$ and $W = +300 \text{ J}$.

EVALUATE: All the quantities in $\Delta U = Q - W$ can be positive and negative.

19.16. IDENTIFY: $\Delta U = Q - W$.

SET UP: $Q < 0$ when heat leaves the gas.

EXECUTE: For an isothermal process, $\Delta U = 0$, so $W = Q = -410 \text{ J}$.

EVALUATE: In a compression the volume decreases and $W < 0$.

19.17. IDENTIFY: For a constant pressure process, $W = p\Delta V$, $Q = nC_p\Delta T$, and $\Delta U = nC_V\Delta T$. $\Delta U = Q - W$ and $C_p = C_V + R$. For an ideal gas, $p\Delta V = nR\Delta T$.

SET UP: From Table 19.1, $C_V = 28.46 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) The pV diagram is shown in Figure 19.17 (next page).

(b) $W = pV_2 - pV_1 = nR(T_2 - T_1) = (0.250 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(100.0 \text{ K}) = 208 \text{ J}$.

(c) The work is done on the piston.

(d) Since $\Delta U = nC_V\Delta T$ holds for any process, we have

$$\Delta U = nC_V\Delta T = (0.250 \text{ mol})(28.46 \text{ J/mol} \cdot \text{K})(100.0 \text{ K}) = 712 \text{ J}.$$

(e) Either $Q = nC_p\Delta T$ or $Q = \Delta U + W$ gives $Q = 920 \text{ J}$ to three significant figures.

(f) The lower pressure would mean a correspondingly larger volume, and the net result would be that the work done would be the same as that found in part (b).

EVALUATE: $W = nR\Delta T$, so W , Q and ΔU all depend only on ΔT . When T increases at constant pressure, V increases and $W > 0$. ΔU and Q are also positive when T increases.

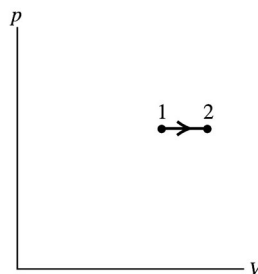


Figure 19.17

19.18. IDENTIFY: For constant volume $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

SET UP: $R = 8.315 \text{ J/mol} \cdot \text{K}$. For helium, $C_V = 12.47 \text{ J/mol} \cdot \text{K}$ and $C_p = 20.78 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) $Q = nC_V\Delta T = (0.0100 \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(40.0 \text{ C}^\circ) = 4.99 \text{ J}$. The pV -diagram is sketched in Figure 19.18a.

(b) $Q = nC_p\Delta T = (0.0100 \text{ mol})(20.78 \text{ J/mol} \cdot \text{K})(40.0 \text{ C}^\circ) = 8.31 \text{ J}$. The pV -diagram is sketched in Figure 19.18b.

(c) More heat is required for the constant pressure process. ΔU is the same in both cases. For constant volume $W = 0$ and for constant pressure $W > 0$. The additional heat energy required for constant pressure goes into expansion work.

(d) $\Delta U = nC_V\Delta T = 4.99 \text{ J}$ for both processes. ΔU is path independent and for an ideal gas depends only on ΔT .

EVALUATE: $C_p = C_V + R$, so $C_p > C_V$.

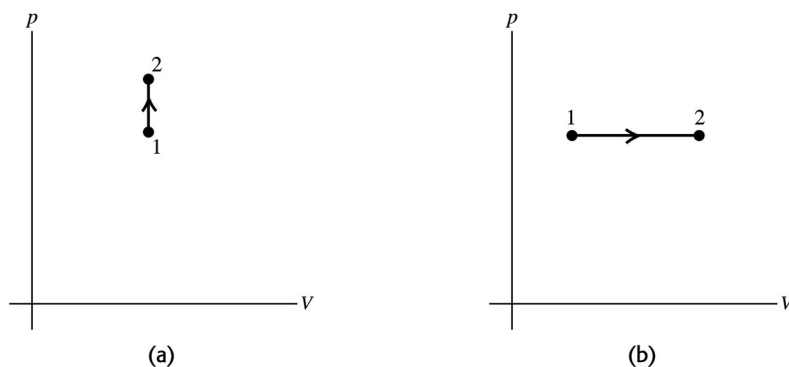


Figure 19.18

19.19. IDENTIFY: For constant volume, $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$.

SET UP: From Table 19.1 in the text, $C_V = 20.76 \text{ J/mol} \cdot \text{K}$ and $C_p = 29.07 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) Using $Q = nC_V\Delta T$, $\Delta T = \frac{Q}{nC_V} = \frac{645 \text{ J}}{(0.185 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})} = 167.9 \text{ K}$ and $T = 948 \text{ K}$.

The pV -diagram is sketched in Figure 19.19a.

(b) Using $Q = nC_p\Delta T$, $\Delta T = \frac{Q}{nC_p} = \frac{645 \text{ J}}{(0.185 \text{ mol})(29.07 \text{ J/mol} \cdot \text{K})} = 119.9 \text{ K}$ and $T = 900 \text{ K}$.

The pV -diagram is sketched in Figure 19.19b.

EVALUATE: At constant pressure some of the heat energy added to the gas leaves the gas as expansion work and the internal energy change is less than if the same amount of heat energy is added at constant volume. ΔT is proportional to ΔU .

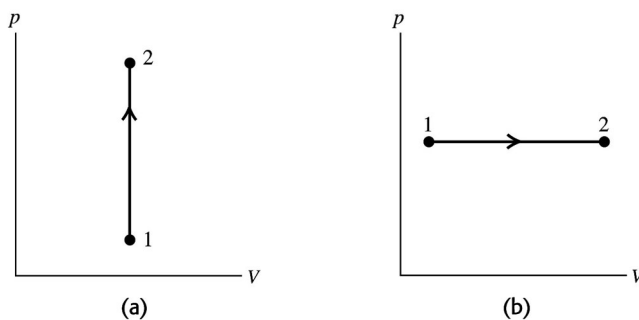


Figure 19.19

19.20. IDENTIFY: A monatomic ideal gas goes between points a and b by two different paths.

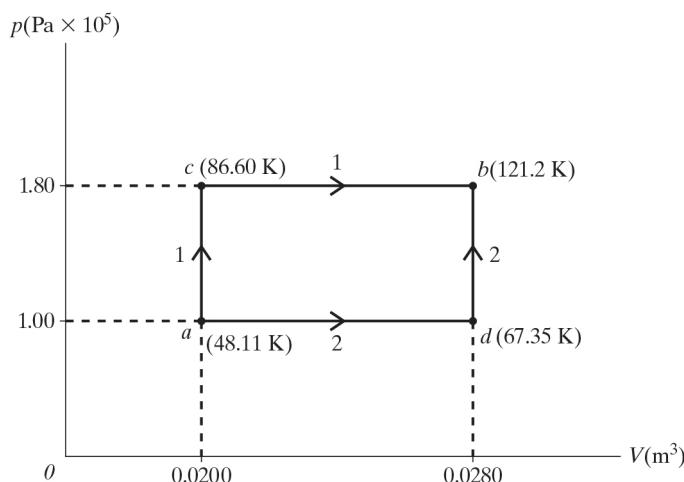


Figure 19.20

SET UP: We want to calculate $Q - W$ for each of the two paths shown in the figure. For a monatomic ideal gas $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. We also have $Q_V = nC_V\Delta T$, $Q_p = nC_p\Delta T$, $\Delta U = Q - W$, and $pV = nRT$. All of the processes in these paths are either at constant volume or constant pressure, so we first find the temperature at each of the four points and add them to the pV -diagram.

EXECUTE: From $pV = nRT$ we get $T = pV/nR$. Using the values on the pV -diagram for point a , we have $p_a = 1.00 \times 10^5$ Pa, $V_a = 0.0200$ m³ and $n = 5.00$ mol. This gives $T_a = 48.11$ K. We could do the same thing for the other three points. Or we could use the fact that, for example, during segment ac the

volume is constant, so T/p is constant. This gives us $\frac{T_c}{T_a} = \frac{p_c}{p_a} = \frac{1.80 \times 10^5 \text{ Pa}}{1.00 \times 10^5 \text{ Pa}} = 1.8$, so

$T_c = 1.8T_a = (1.8)(48.11 \text{ K}) = 86.60$ K. We could apply this type of procedure to the other path segments. The final result is $T_b = 121.2$ K and $T_d = 67.35$ K.

(a) Segment ac : Volume is constant, so $Q_{ac} = nC_V\Delta T_{ac} = n\left(\frac{3}{2}R\right)\Delta T_{ac}$. Using $n = 5.00$ mol and

$\Delta T = T_c - T_a = 86.60 \text{ K} - 48.11 \text{ K} = 38.49 \text{ K}$ gives $Q_{ac} = 2400$ J.

$W_{ac} = 0$ because volume is constant.

Segment cb : Pressure is constant, so $Q_{cb} = nC_p\Delta T_{cb} = n\left(\frac{5}{2}R\right)\Delta T_{cb}$. Using the same value for n and

$\Delta T = 121.2 \text{ K} - 86.60 \text{ K} = 34.6 \text{ K}$ gives $Q_{cb} = 3595.8$ J.

W_{cb} is the area under the curve in the pV -diagram, which gives us

$W_{cb} = (0.0280 \text{ m}^3 - 0.0200 \text{ m}^3)(1.80 \times 10^5 \text{ Pa}) = 1440$ J.

Path 1: $Q_1 = Q_{ac} + Q_{cb} = 2400 \text{ J} + 3595.8 \text{ J} = 6000$ J.

(b) Follow the same procedure as for path 1. The results are

Segment ad : Pressure is constant. $Q_{ad} = 2000$ J. W_{ad} = area under the curve = 800 J.

Segment db : Volume is constant. $Q_{db} = 3358$ J. $W_{db} = 0$ because the volume is constant.

Path 2: $Q_2 = 2000 \text{ J} + 3358 \text{ J} = 5360$ J. $W_2 = 800$ J.

(c) Path 1: $Q_1 - W_1 = 6000 \text{ J} - 1440 \text{ J} = 4560$ J.

Path 2: $Q_2 - W_1 = 5360 \text{ J} - 800 \text{ J} = 4560$ J.

The quantity $Q - W$ is the same for both paths.

EVALUATE: Recognize from the first law of thermodynamics that $Q - W$ is equal to ΔU . Since ΔU for an ideal gas depends only on ΔT , and ΔT is the same for paths 1 and 2 since it is $T_b - T_a$ for both of them, it follows that ΔU is the same for both paths. Therefore $Q - W$ must be the same.

- 19.21. IDENTIFY:** $\Delta U = Q - W$. For an ideal gas, $\Delta U = C_V \Delta T$, and at constant pressure, $W = p \Delta V = nR \Delta T$.

SET UP: $C_V = \frac{3}{2}R$ for a monatomic gas.

EXECUTE: $\Delta U = n(\frac{3}{2}R)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}W$. Then $Q = \Delta U + W = \frac{5}{2}W$, so $W/Q = \frac{2}{5}$.

EVALUATE: For diatomic or polyatomic gases, C_V is a different multiple of R and the fraction of Q that is used for expansion work is different.

- 19.22. IDENTIFY:** Apply $pV = nRT$ to calculate T . For this constant pressure process, $W = p\Delta V$.

$Q = nC_p\Delta T$. Use $\Delta U = Q - W$ to relate Q , W , and ΔU .

SET UP: $2.50 \text{ atm} = 2.53 \times 10^5 \text{ Pa}$. For a monatomic ideal gas, $C_V = 12.47 \text{ J/mol} \cdot \text{K}$ and $C_p = 20.78 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) $T_1 = \frac{pV_1}{nR} = \frac{(2.53 \times 10^5 \text{ Pa})(3.20 \times 10^{-2} \text{ m}^3)}{(3.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 325 \text{ K}$.

$T_2 = \frac{pV_2}{nR} = \frac{(2.53 \times 10^5 \text{ Pa})(4.50 \times 10^{-2} \text{ m}^3)}{(3.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 456 \text{ K}$.

(b) $W = p\Delta V = (2.53 \times 10^5 \text{ Pa})(4.50 \times 10^{-2} \text{ m}^3 - 3.20 \times 10^{-2} \text{ m}^3) = 3.29 \times 10^3 \text{ J}$.

(c) $Q = nC_p\Delta T = (3.00 \text{ mol})(20.78 \text{ J/mol} \cdot \text{K})(456 \text{ K} - 325 \text{ K}) = 8.17 \times 10^3 \text{ J}$.

(d) $\Delta U = Q - W = 4.88 \times 10^3 \text{ J}$.

EVALUATE: We could also calculate ΔU as

$\Delta U = nC_V\Delta T = (3.00 \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(456 \text{ K} - 325 \text{ K}) = 4.90 \times 10^3 \text{ J}$, which agrees with the value we calculated in part (d).

- 19.23. IDENTIFY:** $\Delta U = Q - W$. Apply $Q = nC_p\Delta T$ to calculate C_p . Apply $\Delta U = nC_V\Delta T$ to calculate C_V .

$\gamma = C_p/C_V$.

SET UP: $\Delta T = 15.0 \text{ C}^\circ = 15.0 \text{ K}$. Since heat is added, $Q = +970 \text{ J}$.

EXECUTE: (a) $\Delta U = Q - W = 1970 \text{ J} - 223 \text{ J} = 747 \text{ J}$.

(b) $C_p = \frac{Q}{n\Delta T} = \frac{970 \text{ J}}{(1.75 \text{ mol})(15.0 \text{ K})} = 37.0 \text{ J/mol} \cdot \text{K}$. $C_V = \frac{\Delta U}{n\Delta T} = \frac{747 \text{ J}}{(1.75 \text{ mol})(15.0 \text{ K})} = 28.5 \text{ J/mol} \cdot \text{K}$.

$\gamma = \frac{C_p}{C_V} = \frac{37.0 \text{ J/mol} \cdot \text{K}}{28.5 \text{ J/mol} \cdot \text{K}} = 1.30$.

EVALUATE: The value of γ we calculated is similar to the values given in Tables 19.1 for polyatomic gases.

- 19.24. IDENTIFY:** A monatomic gas is compressed at constant pressure, so this is an isobaric compression.

SET UP: We want to find the work W , heat flow Q , and internal energy change ΔU . We know that $pV = nRT$, $\Delta U = Q - W$, $W = p\Delta V$ when pressure is constant, $Q_p = nC_p\Delta T$, and $\Delta U = nC_V\Delta T$. For an

ideal monatomic gas $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$.

EXECUTE: (a) We want the work done by (or on) the gas. $W = p\Delta V = p(V_2 - V_1)$, which gives

$W = (1.80 \times 10^4 \text{ Pa})(0.0500 \text{ m}^3 - 0.0800 \text{ m}^3) = -540 \text{ J}$. The minus sign tells us that work is done *on* the gas.

(b) At constant pressure we have $Q_p = nC_p\Delta T = n\left(\frac{5}{2}R\right)\Delta T$. At constant pressure, $pV = nRT$ tells us

that $p\Delta V = nR\Delta T$, so $\Delta T = \frac{p\Delta V}{nR} = \frac{W}{nR}$. Therefore $Q_p = n\left(\frac{5}{2}R\right)\left(\frac{W}{nR}\right) = \frac{5}{2}W = \frac{5}{2}(-540 \text{ J}) = -1350 \text{ J}$.

The minus sign tells us that heat *leaves* the gas.

(c) $\Delta U = Q - W = -1350 \text{ J} - (-540 \text{ J}) = -810 \text{ J}$. The internal energy *decreases*.

EVALUATE: As a check in part (c), use $\Delta U = nC_V\Delta T = Q_p = n\left(\frac{3}{2}R\right)\left(\frac{W}{nR}\right) = \frac{3}{2}W$, so

$Q_p = \frac{3}{2}(-540 \text{ J}) = -810 \text{ J}$, which agrees with our result.

19.25. IDENTIFY: Calculate W and ΔU and then use the first law to calculate Q .

(a) SET UP: $W = \int_{V_1}^{V_2} p dV$

$pV = nRT$ so $p = nRT/V$

$W = \int_{V_1}^{V_2} (nRT/V) dV = nRT \int_{V_1}^{V_2} dV/V = nRT \ln(V_2/V_1)$ (work done during an isothermal process).

EXECUTE: $W = (0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(350 \text{ K}) \ln(0.25V_1/V_1) = (436.5 \text{ J}) \ln(0.25) = -605 \text{ J}$.

EVALUATE: W for the gas is negative, since the volume decreases.

(b) SET UP: $\Delta U = nC_V\Delta T$ for any ideal gas process.

EXECUTE: $\Delta T = 0$ (isothermal) so $\Delta U = 0$.

EVALUATE: $\Delta U = 0$ for any ideal gas process in which T doesn't change.

(c) SET UP: $\Delta U = Q - W$

EXECUTE: $\Delta U = 0$ so $Q = W = -605 \text{ J}$. (Q is negative; the gas liberates 605 J of heat to the surroundings.)

EVALUATE: $Q = nC_V\Delta T$ is only for a constant volume process, so it doesn't apply here.

$Q = nC_p\Delta T$ is only for a constant pressure process, so it doesn't apply here.

19.26. IDENTIFY: We want to investigate how much heat input goes into work for the expansion of liquid ethanol.

SET UP: We are looking at the work W by expanding ethanol due to heat input Q . We use $W = p\Delta V$ when pressure is constant, $\rho = m/V$, $Q = mc\Delta T$ and $\Delta V = \beta V_0\Delta T$.

EXECUTE: (a) $m = (46.1 \text{ g/mol})(5.00 \text{ mol}) = 0.231 \text{ kg}$.

(b) We want the heat Q that enters the ethanol. $Q = mc\Delta T = (0.231 \text{ kg})(2428 \text{ J/kg} \cdot \text{K})(50 \text{ K}) = 2.80 \times 10^4 \text{ J}$.

(c) We want the work done by the ethanol $W = p\Delta V$ and $\Delta V = \beta V_0\Delta T$. Combining these gives $\Delta V = p\beta V_0\Delta T$. We know the mass and density of the ethanol, so use $\rho = m/V$, which gives

$V_0 = m/\rho$. Therefore $W = p\beta(m/\rho)\Delta T$. Using $p = 1.01 \times 10^5 \text{ Pa}$, $\beta = 75 \times 10^{-5} \text{ K}^{-1}$, $\Delta T = 50 \text{ K}$, $\rho = 810 \text{ kg/m}^3$, and $m = 0.231 \text{ kg}$ gives $W = 1.08 \text{ J}$ which rounds to $W = 1.1 \text{ J}$.

(d) $\frac{W}{Q} = \frac{1.08 \text{ J}}{2.80 \times 10^4 \text{ J}} = 3.8 \times 10^{-5}$.

EVALUATE: The work is a very small fraction of the heat input. Most of that heat goes into increasing the internal energy and therefore the temperature of the ethanol.

19.27. IDENTIFY: For an adiabatic process of an ideal gas, $p_1 V_1^\gamma = p_2 V_2^\gamma$, $W = \frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2)$, and

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}.$$

SET UP: For a monatomic ideal gas $\gamma = 5/3$.

EXECUTE: (a) $p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma = (1.50 \times 10^5 \text{ Pa}) \left(\frac{0.0800 \text{ m}^3}{0.0400 \text{ m}^3} \right)^{5/3} = 4.76 \times 10^5 \text{ Pa}.$

(b) This result may be substituted into $W = \frac{1}{\gamma-1} p_1 V_1 (1 - (V_1/V_2)^{\gamma-1})$, or, substituting the above form for

$$p_2, \quad W = \frac{1}{\gamma-1} p_1 V_1 (1 - (V_1/V_2)^{\gamma-1}) = \frac{3}{2} (1.50 \times 10^5 \text{ Pa}) (0.0800 \text{ m}^3) \left(1 - \left(\frac{0.0800}{0.0400} \right)^{2/3} \right) = -1.06 \times 10^4 \text{ J}.$$

(c) From $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$, $(T_2/T_1) = (V_2/V_1)^{\gamma-1} = (0.0800/0.0400)^{2/3} = 1.59$, and since the final temperature is higher than the initial temperature, the gas is heated.

EVALUATE: In an adiabatic compression $W < 0$ since $\Delta V < 0$. $Q = 0$ so $\Delta U = -W$. $\Delta U > 0$ and the temperature increases.

19.28. IDENTIFY: For an adiabatic process of an ideal gas, $p_1 V_1^\gamma = p_2 V_2^\gamma$, no heat enters or leaves the gas. The ideal gas law still applies.

SET UP: $p_1 V_1^\gamma = p_2 V_2^\gamma$, $pV = nRT$, $W = \int_{V_1}^{V_2} p dV$. For an ideal monatomic gas, $\gamma = 5/3$.

EXECUTE: Solving $p_1 V_1^\gamma = p_2 V_2^\gamma$ for p_2 and rearranging gives $p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma$, so we need to find V_2 .

Applying $W = \int_{V_1}^{V_2} p dV$ to an adiabatic process, we use the fact that $pV^\gamma = \text{constant}$. In this case, the constant is $p_1 V_1^\gamma$ (since we know p_1 and V_1), and we'll call it K for the time being. This tells us that

$$p = K/V^\gamma. \text{ Using this in the integral, we get } W = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} K/V^\gamma dV = \frac{K}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma}).$$

$K = p_1 V_1^\gamma = (2500 \text{ Pa})(2.10 \text{ m}^3)^{5/3} = 8609 \text{ N} \cdot \text{m}^3$, $W = 1480 \text{ J}$, and $\gamma = 5/3$. Putting in these numbers

and solving for V_2 gives $V_2 = 2.8697 \text{ m}^3$. Putting this value into $p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma$ gives

$$p_2 = (2500 \text{ Pa}) \left(\frac{2.10 \text{ m}^3}{2.8697 \text{ m}^3} \right)^{5/3} = 1490 \text{ Pa} = 1.49 \text{ kPa}.$$

EVALUATE: The pressure dropped because the gas expanded adiabatically and did work, so our result is reasonable. An alternative approach is the following: We know that $Q = 0$ and $\Delta U = n \left(\frac{3}{2} R \right) \Delta T$. We

have $W = -\Delta U$, so $\Delta U = -1480 \text{ J}$. Therefore $-1480 \text{ J} = n \left(\frac{3}{2} R \right) \Delta T$, which gives $\Delta T = -23.73 \text{ K}$.

The ideal gas law gives $T_1 = p_1 V_1 / nR = (2500 \text{ Pa})(2.10 \text{ m}^3) / [(5.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})] = 126.3 \text{ K}$.

Therefore $T_2 = T_1 + \Delta T = +102.6 \text{ K}$. Using $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives

$$V_2^{2/3} = V_1^{2/3} \left(\frac{T_1}{T_2} \right) = (2.10 \text{ m}^3) \left(\frac{126.3 \text{ K}}{102.6 \text{ K}} \right) = 2.019 \text{ m}^2, \text{ so } V_2 = (2.019 \text{ m}^2)^{3/2} = 2.869 \text{ m}^3. \text{ Therefore}$$

$$p_2 = nRT_2/V_2 = (5.00 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})/(102.6 \text{ K})/(2.869 \text{ m}^3) = 1490 \text{ Pa} = 1.49 \text{ kPa}.$$

- 19.29. (a) IDENTIFY and SET UP:** In the expansion the pressure decreases and the volume increases. The pV -diagram is sketched in Figure 19.29.

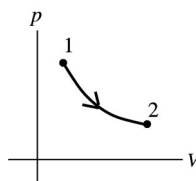


Figure 19.29

(b) Adiabatic means $Q = 0$.

Then $\Delta U = Q - W$ gives $W = -\Delta U = -nC_V \Delta T = nC_V (T_1 - T_2)$.

$C_V = 12.47 \text{ J/mol} \cdot \text{K}$ (Table 19.1).

EXECUTE: $W = (0.450 \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(66.0^\circ\text{C} - 10.0^\circ\text{C}) = +314 \text{ J}$.

W positive for $\Delta V > 0$ (expansion)

(c) $\Delta U = -W = -314 \text{ J}$.

EVALUATE: There is no heat energy input. The energy for doing the expansion work comes from the internal energy of the gas, which therefore decreases. For an ideal gas, when T decreases, U decreases.

- 19.30. IDENTIFY:** Assume the expansion is adiabatic. $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ relates V and T . Assume the air behaves as an ideal gas, so $\Delta U = nC_V \Delta T$. Use $pV = nRT$ to calculate n .

SET UP: For air, $C_V = 29.76 \text{ J/mol} \cdot \text{K}$ and $\gamma = 1.40$. $V_2 = 0.800V_1$. $T_1 = 293.15 \text{ K}$.

$p_1 = 2.026 \times 10^5 \text{ Pa}$. For a sphere, $V = \frac{4}{3}\pi r^3$.

EXECUTE: (a) $T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (293.15 \text{ K}) \left(\frac{V_1}{0.800V_1} \right)^{0.40} = 320.5 \text{ K} = 47.4^\circ\text{C}$.

(b) $V_1 = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} (0.1195 \text{ m})^3 = 7.15 \times 10^{-3} \text{ m}^3$.

$n = \frac{p_1 V_1}{RT_1} = \frac{(2.026 \times 10^5 \text{ Pa})(7.15 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293.15 \text{ K})} = 0.594 \text{ mol}$.

$\Delta U = nC_V \Delta T = (0.594 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(321 \text{ K} - 293 \text{ K}) = 345 \text{ J}$.

EVALUATE: We could also use $\Delta U = -W = -\frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2)$ to calculate ΔU , if we first found p_2 from $pV = nRT$.

- 19.31. IDENTIFY:** Combine $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ with $pV = nRT$ to obtain an expression relating T and p for an adiabatic process of an ideal gas.

SET UP: $T_1 = 299.15 \text{ K}$.

EXECUTE: $V = \frac{nRT}{p}$ so $T_1 \left(\frac{nRT_1}{p_1} \right)^{\gamma-1} = T_2 \left(\frac{nRT_2}{p_2} \right)^{\gamma-1}$ and $\frac{T_1^\gamma}{p_1} = \frac{T_2^\gamma}{p_2}$.

$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} = (299.15 \text{ K}) \left(\frac{0.850 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right)^{0.4/1.4} = 284.8 \text{ K} = 11.6^\circ\text{C}$.

EVALUATE: For an adiabatic process of an ideal gas, when the pressure decreases the temperature decreases.

19.32. IDENTIFY: $pV = nRT$ For an adiabatic process, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For an ideal monatomic gas, $\gamma = 5/3$.

EXECUTE: (a) $T = \frac{pV}{nR} = \frac{(1.00 \times 10^5 \text{ Pa})(2.50 \times 10^{-3} \text{ m}^3)}{(0.1 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})} = 301 \text{ K}.$

(b) (i) Isothermal: If the expansion is *isothermal*, the process occurs at constant temperature and the final temperature is the same as the initial temperature, namely 301 K.

$$p_2 = p_1(V_1/V_2) = \frac{1}{2}p_1 = 5.00 \times 10^4 \text{ Pa}.$$

(ii) Isobaric: $\Delta p = 0$ so $p_2 = 1.00 \times 10^5 \text{ Pa}$. $T_2 = T_1(V_2/V_1) = 2T_1 = 602 \text{ K}.$

(iii) Adiabatic: Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, $T_2 = \frac{T_1V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{(301 \text{ K})(V_1)^{0.67}}{(2V_1)^{0.67}} = (301 \text{ K})(\frac{1}{2})^{0.67} = 189 \text{ K}.$ Then

$$pV = nRT \text{ gives } p_2 = 3.14 \times 10^4 \text{ Pa}.$$

EVALUATE: In an isobaric expansion, T increases. In an adiabatic expansion, T decreases.

19.33. IDENTIFY: As helium undergoes an adiabatic process its Kelvin temperature doubles.

SET UP: Treat helium as an ideal monatomic gas, so $\gamma = 5/3$. We also have $pV = nRT$ and $p_1V_1^\gamma = p_2V_2^\gamma$, and we know that $T_2 = 2T_1$. We want the factor by which p changes during this process, so we want p_2/p_1 .

EXECUTE: From $pV = nRT$ we have $V = nRT/p$. Taking the ratio of the volumes gives

$$\frac{V_1}{V_2} = \frac{nRT_1/p_1}{nRT_2/p_2} = \left(\frac{T_1}{T_2}\right)\left(\frac{p_2}{p_1}\right). \text{ Now use } p_1V_1^\gamma = p_2V_2^\gamma, \text{ which gives } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^\gamma. \text{ Use the previous result}$$

for the volume ratio to get $\frac{p_2}{p_1} = \left(\frac{T_1}{T_2} \cdot \frac{p_2}{p_1}\right)^\gamma$, which can be arranged to get $\frac{p_2}{p_1} \left(\frac{p_2}{p_1}\right)^{-\gamma} = \left(\frac{T_1}{T_2}\right)^\gamma$. Using

$$\gamma = 5/3 \text{ and } T_1/T_2 = 1/2, \text{ this becomes } \frac{p_2}{p_1} = \left(\frac{1}{2}\right)^{-5/2} = 4\sqrt{2}.$$

EVALUATE: Look at the volume ratio V_2/V_1 , which is $\frac{V_2}{V_1} = \frac{nRT_2/p_2}{nRT_1/p_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right)$, so

$$\frac{V_2}{V_1} = (2)\left(\frac{1}{4\sqrt{2}}\right) = \frac{\sqrt{2}}{4} \approx 0.354, \text{ which means that the volume decreases. This is reasonable because the}$$

only way to increase the gas temperature adiabatically is to compress it.

19.34. IDENTIFY: We are looking at an adiabatic compression of an ideal diatomic gas in which the temperature increases.

SET UP: We want to find the change ΔU in the internal energy of the gas. $\Delta U = nC_V\Delta T$ for any process, and $C_V = \frac{5}{2}R$ for an ideal diatomic gas.

EXECUTE: $\Delta U = n\left(\frac{5}{2}R\right)\Delta T$. We need n . We know that 1 mol of a gas contains N_A molecules so the

$$\text{number } N \text{ of molecules is } nN_A. \text{ Therefore } n = \frac{N}{N_A} = \frac{3.01 \times 10^{20} \text{ molecules}}{6.022 \times 10^{23} \text{ molecules/mol}} = 4.99 \times 10^{-4} \text{ mol}.$$

Now we use $\Delta U = n\left(\frac{5}{2}R\right)\Delta T$ with $\Delta T = 35.0 \text{ K}$ to get $\Delta U = 0.364 \text{ J}.$

EVALUATE: For an adiabatic process, $\Delta U = Q - W = 0 - W$, which tells us that $W = -0.364 \text{ J}$. Work was done *on* the gas which compressed it and increased its internal energy and therefore its temperature.

19.35. IDENTIFY and SET UP: For an ideal gas, $pV = nRT$. The work done is the area under the path in the pV -diagram.

EXECUTE: (a) The product pV increases and this indicates a temperature increase.

(b) The work is the area in the pV plane bounded by the blue line representing the process and the verticals at V_a and V_b . The area of this trapezoid is

$$\frac{1}{2}(p_b + p_a)(V_b - V_a) = \frac{1}{2}(2.40 \times 10^5 \text{ Pa})(0.0400 \text{ m}^3) = 4800 \text{ J}.$$

EVALUATE: The work done is the average pressure, $\frac{1}{2}(p_1 + p_2)$, times the volume increase.

19.36. IDENTIFY: Steam from boiling water slightly lifts the lid of a pot. This does work on the lid and therefore cools down the steam slightly. We want to find out the decrease in temperature of the steam.

SET UP and EXECUTE: For an adiabatic process, $Q = 0$ so $\Delta U = Q - W = -W$. $\Delta U = nC_V \Delta T$ where

$$C_V = \frac{5}{2}R \text{ for an ideal diatomic gas. Treat the air as } \text{N}_2, \text{ which is diatomic.}$$

(a) Estimate: Mass of the lid is $250 \text{ g} = 0.250 \text{ kg}$.

(b) Estimate: Lid rises $0.50 \text{ cm} = 0.0050 \text{ m}$.

(c) $W = mgh = (0.250 \text{ kg})(9.80 \text{ m/s}^2)(0.0050 \text{ m}) = 0.12 \text{ J}$.

(d) The expansion is adiabatic so $W = -\Delta U = -nC_V \Delta T = -n\left(\frac{5}{2}R\right)\Delta T$. Therefore

$$\Delta T = -\frac{2W}{5nR} = -\frac{2(0.12 \text{ J})}{5\left(\frac{1}{11.2} \text{ mol}\right)(8.314 \text{ J/mol} \cdot \text{K})} = -0.06 \text{ K. The minus sign tells us that the temperature}$$

has dropped by $0.06 \text{ K} = 0.06 \text{ C}^\circ \approx 0.1 \text{ F}$.

EVALUATE: This is a very small temperature change, but based upon experience it seems reasonable.

19.37. IDENTIFY: We can read the values from the pV -diagram and apply the ideal gas law and the first law of thermodynamics.

SET UP: At each point $pV = nRT$, with $T = 85 \text{ K} + 273 \text{ K} = 358 \text{ K}$. For an isothermal process of an ideal gas, $W = nRT \ln(V_2/V_1)$. $\Delta U = nC_V \Delta T$ for any ideal gas process.

EXECUTE: (a) At point b , $p = 0.200 \text{ atm} = 2.026 \times 10^4 \text{ Pa}$ and $V = 0.100 \text{ m}^3$.

$$n = \frac{pV}{RT} = \frac{(2.026 \times 10^4 \text{ Pa})(0.100 \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(358 \text{ K})} = 0.681 \text{ moles}.$$

(b) n , R , and T are constant so $p_a V_a = p_b V_b$.

$$V_a = V_b \left(\frac{p_b}{p_a} \right) = (0.100 \text{ m}^3) \left(\frac{0.200 \text{ atm}}{0.600 \text{ atm}} \right) = 0.0333 \text{ m}^3.$$

(c) $W = nRT \ln(V_b/V_a) = (0.681 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(358 \text{ K}) \ln\left(\frac{0.100 \text{ m}^3}{0.0333 \text{ m}^3}\right) = 2230 \text{ J} = 2.23 \text{ kJ}$.

W is positive and corresponds to work done by the gas.

(d) $\Delta U = nC_V \Delta T$ so for an isothermal process ($\Delta T = 0$), $\Delta U = 0$.

EVALUATE: W is positive when the volume increases, so the area under the curve is positive. For any isothermal process, $\Delta U = 0$.

19.38. IDENTIFY: Segment ab is isobaric, bc is isochoric, and ca is isothermal.

SET UP: He is a monatomic gas so $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$. For any process of an ideal gas,

$\Delta U = nC_V \Delta T$. For an isothermal process of an ideal gas, $\Delta U = 0$ so $Q = W = nRT \ln(V_2/V_1)$.

EXECUTE: (a) Apply $pV = nRT$ to states a and c . $T_a = T_c$ so nRT is constant and $p_a V_a = p_c V_c$.

$$p_a = p_c \left(\frac{V_c}{V_a} \right) = (2.0 \times 10^5 \text{ Pa}) \left(\frac{0.040 \text{ m}^3}{0.010 \text{ m}^3} \right) = 8.0 \times 10^5 \text{ Pa}.$$

$$(b) T_a = \frac{p_a V_a}{nR} = \frac{(8.0 \times 10^5 \text{ Pa})(0.010 \text{ m}^3)}{(3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 296 \text{ K};$$

$$T_b = \frac{p_b V_b}{nR} = \frac{(8.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3)}{(3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 1184 \text{ K};$$

$$T_c = \frac{p_c V_c}{nR} = \frac{(2.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3)}{(3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 296 \text{ K} = T_a.$$

(c) ab: $Q = nC_p \Delta T = (3.25 \text{ mol}) \left(\frac{5}{2} \right) (8.315 \text{ J/mol} \cdot \text{K}) (1184 \text{ K} - 296 \text{ K}) = 6.00 \times 10^4 \text{ J}$; heat enters the gas.

bc: $Q = nC_V \Delta T = (3.25 \text{ mol}) \left(\frac{3}{2} \right) (8.315 \text{ J/mol} \cdot \text{K}) (296 \text{ K} - 1184 \text{ K}) = -3.60 \times 10^4 \text{ J}$; heat leaves the gas.

ca: $Q = nRT \ln \left(\frac{V_a}{V_c} \right) = (3.25 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(296 \text{ K}) \ln \left(\frac{0.010 \text{ m}^3}{0.040 \text{ m}^3} \right) = -1.11 \times 10^4 \text{ J}$; heat leaves the gas.

(d) ab: $\Delta U = nC_V \Delta T = (3.25 \text{ mol}) \left(\frac{3}{2} \right) (8.315 \text{ J/mol} \cdot \text{K}) (1184 \text{ K} - 296 \text{ K}) = 3.60 \times 10^4 \text{ J}$; the internal energy increased.

bc: $\Delta U = nC_V \Delta T = (3.25 \text{ mol}) \left(\frac{3}{2} \right) (8.315 \text{ J/mol} \cdot \text{K}) (296 \text{ K} - 1184 \text{ K}) = -3.60 \times 10^4 \text{ J}$; the internal energy decreased.

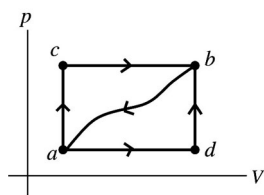
ca: $\Delta T = 0$ so $\Delta U = 0$.

EVALUATE: As we saw in (d), for any closed path on a pV diagram, $\Delta U = 0$ because we are back at the same values of P , V , and T .

19.39. IDENTIFY: Use $\Delta U = Q - W$ and the fact that ΔU is path independent.

$W > 0$ when the volume increases, $W < 0$ when the volume decreases, and $W = 0$ when the volume is constant. $Q > 0$ if heat flows into the system.

SET UP: The paths are sketched in Figure 19.39.



$$Q_{acb} = +90.0 \text{ J (positive since heat flows in)}$$

$$W_{acb} = +60.0 \text{ J (positive since } \Delta V > 0)$$

Figure 19.39

EXECUTE: (a) $\Delta U = Q - W$

ΔU is path independent; Q and W depend on the path.

$$\Delta U = U_b - U_a$$

This can be calculated for any path from a to b , in particular for path acb :

$$\Delta U_{a \rightarrow b} = Q_{acb} - W_{acb} = 90.0 \text{ J} - 60.0 \text{ J} = 30.0 \text{ J}.$$

Now apply $\Delta U = Q - W$ to path adb ; $\Delta U = 30.0 \text{ J}$ for this path also.

$$W_{adb} = +15.0 \text{ J (positive since } \Delta V > 0)$$

$$\Delta U_{a \rightarrow b} = Q_{adb} - W_{adb} \text{ so } Q_{adb} = \Delta U_{a \rightarrow b} + W_{adb} = 30.0 \text{ J} + 15.0 \text{ J} = +45.0 \text{ J}.$$

(b) Apply $\Delta U = Q - W$ to path ba : $\Delta U_{b \rightarrow a} = Q_{ba} - W_{ba}$

$W_{ba} = -35.0 \text{ J}$ (negative since $\Delta V < 0$)

$$\Delta U_{b \rightarrow a} = U_a - U_b = -(U_b - U_a) = -\Delta U_{a \rightarrow b} = -30.0 \text{ J}$$

$$\text{Then } Q_{ba} = \Delta U_{b \rightarrow a} + W_{ba} = -30.0 \text{ J} - 35.0 \text{ J} = -65.0 \text{ J}.$$

($Q_{ba} < 0$; the system liberates heat.)

$$\text{(c) } U_a = 0, \quad U_d = 8.0 \text{ J}$$

$$\Delta U_{a \rightarrow b} = U_b - U_a = +30.0 \text{ J, so } U_b = +30.0 \text{ J}.$$

Process $a \rightarrow d$:

$$\Delta U_{a \rightarrow d} = Q_{ad} - W_{ad}$$

$$\Delta U_{a \rightarrow d} = U_d - U_a = +8.0 \text{ J}$$

$W_{adb} = +15.0 \text{ J}$ and $W_{adb} = W_{ad} + W_{db}$. But the work W_{db} for the process $d \rightarrow b$ is zero since $\Delta V = 0$ for that process. Therefore $W_{ad} = W_{adb} = +15.0 \text{ J}$.

$$\text{Then } Q_{ad} = \Delta U_{a \rightarrow d} + W_{ad} = +8.0 \text{ J} + 15.0 \text{ J} = +23.0 \text{ J} \text{ (positive implies heat absorbed).}$$

Process $d \rightarrow b$:

$$\Delta U_{d \rightarrow b} = Q_{db} - W_{db}$$

$$W_{db} = 0, \text{ as already noted.}$$

$$\Delta U_{d \rightarrow b} = U_b - U_d = 30.0 \text{ J} - 8.0 \text{ J} = +22.0 \text{ J}.$$

$$\text{Then } Q_{db} = \Delta U_{d \rightarrow b} + W_{db} = +22.0 \text{ J} \text{ (positive; heat absorbed).}$$

EVALUATE: The signs of our calculated Q_{ad} and Q_{db} agree with the problem statement that heat is absorbed in these processes.

19.40. IDENTIFY: $\Delta U = Q - W$.

SET UP: $W = 0$ when $\Delta V = 0$.

EXECUTE: For each process, $Q = \Delta U + W$. No work is done in the processes ab and dc , and so

$W_{bc} = W_{abc} = 450 \text{ J}$ and $W_{ad} = W_{adc} = 120 \text{ J}$. The heat flow for each process is: for ab , $Q = 90 \text{ J}$. For bc , $Q = 440 \text{ J} + 450 \text{ J} = 890 \text{ J}$. For ad , $Q = 180 \text{ J} + 120 \text{ J} = 300 \text{ J}$. For dc , $Q = 350 \text{ J}$. Heat is absorbed in each process. Note that the arrows representing the processes all point in the direction of increasing temperature (increasing U).

EVALUATE: ΔU is path independent so is the same for paths adc and abc .

$Q_{adc} = 300 \text{ J} + 350 \text{ J} = 650 \text{ J}$. $Q_{abc} = 90 \text{ J} + 890 \text{ J} = 980 \text{ J}$. Q and W are path dependent and are different for these two paths.

19.41. IDENTIFY: Use $pV = nRT$ to calculate T_c/T_a . Calculate ΔU and W and use $\Delta U = Q - W$ to obtain Q .

SET UP: For path ac , the work done is the area under the line representing the process in the pV -diagram.

$$\text{EXECUTE: (a) } \frac{T_c}{T_a} = \frac{p_c V_c}{p_a V_a} = \frac{(1.0 \times 10^5 \text{ J})(0.060 \text{ m}^3)}{(3.0 \times 10^5 \text{ J})(0.020 \text{ m}^3)} = 1.00. \quad T_c = T_a.$$

(b) Since $T_c = T_a$, $\Delta U = 0$ for process abc . For ab , $\Delta V = 0$ and $W_{ab} = 0$. For bc , p is constant and $W_{bc} = p\Delta V = (1.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3) = 4.0 \times 10^3 \text{ J}$. Therefore, $W_{abc} = +4.0 \times 10^3 \text{ J}$. Since $\Delta U = 0$, $Q = W = +4.0 \times 10^3 \text{ J}$. $4.0 \times 10^3 \text{ J}$ of heat flows into the gas during process abc .

$$\text{(c) } W = \frac{1}{2}(3.0 \times 10^5 \text{ Pa} + 1.0 \times 10^5 \text{ Pa})(0.040 \text{ m}^3) = +8.0 \times 10^3 \text{ J}. \quad Q_{ac} = W_{ac} = +8.0 \times 10^3 \text{ J}.$$

EVALUATE: The work done is path dependent and is greater for process ac than for process abc , even though the initial and final states are the same.

19.42. IDENTIFY: Apply the appropriate expression for W for each type of process. $pV = nRT$ and

$$C_p = C_v + R.$$

SET UP: $R = 8.315 \text{ J/mol} \cdot \text{K}$.

EXECUTE: Path ac has constant pressure, so $W_{ac} = p\Delta V = nR\Delta T$, and

$$W_{ac} = nR(T_c - T_a) = (3 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(492 \text{ K} - 300 \text{ K}) = 4.789 \times 10^3 \text{ J}.$$

Path cb is adiabatic ($Q = 0$), so $W_{cb} = Q - \Delta U = -\Delta U = -nC_v\Delta T$, and using $C_v = C_p - R$,

$$W_{cb} = -n(C_p - R)(T_b - T_c) = -(3 \text{ mol})(29.1 \text{ J/mol} \cdot \text{K} - 8.315 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 492 \text{ K}) = -6.735 \times 10^3 \text{ J}.$$

Path ba has constant volume, so $W_{ba} = 0$. So the total work done is

$$W = W_{ac} + W_{cb} + W_{ba} = 4.789 \times 10^3 \text{ J} - 6.735 \times 10^3 \text{ J} + 0 = -1.95 \times 10^3 \text{ J}.$$

EVALUATE: $W > 0$ when $\Delta V > 0$, $W < 0$ when $\Delta V < 0$ and $W = 0$ when $\Delta V = 0$.

19.43. IDENTIFY: Segment ab is isochoric, bc is isothermal, and ca is isobaric.

SET UP: For bc , $\Delta T = 0$, $\Delta U = 0$, and $Q = W = nRT \ln(V_c/V_b)$. For ideal H_2 (diatomic), $C_v = \frac{5}{2}R$

and $C_p = \frac{7}{2}R$. $\Delta U = nC_v \Delta T$ for any process of an ideal gas.

EXECUTE: (a) $T_b = T_c$. For states b and c , $pV = nRT = \text{constant}$ so $p_b V_b = p_c V_c$ and

$$V_c = V_b \left(\frac{p_b}{p_c} \right) = (0.20 \text{ L}) \left(\frac{2.0 \text{ atm}}{0.50 \text{ atm}} \right) = 0.80 \text{ L}.$$

$$\text{(b) } T_a = \frac{p_a V_a}{nR} = \frac{(0.50 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(0.20 \times 10^{-3} \text{ m}^3)}{(0.0040 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 305 \text{ K. } V_a = V_b \text{ so for states } a \text{ and } b,$$

$$\frac{T}{p} = \frac{V}{nR} = \text{constant so } \frac{T_a}{p_a} = \frac{T_b}{p_b}. T_b = T_c = T_a \left(\frac{p_b}{p_a} \right) = (305 \text{ K}) \left(\frac{2.0 \text{ atm}}{0.50 \text{ atm}} \right) = 1220 \text{ K; } T_c = 1220 \text{ K}.$$

(c) ab : $Q = nC_v \Delta T = n(\frac{5}{2}R) \Delta T$, which gives

$$Q = (0.0040 \text{ mol})(\frac{5}{2})(8.315 \text{ J/mol} \cdot \text{K})(1220 \text{ K} - 305 \text{ K}) = +76 \text{ J. } Q \text{ is positive and heat goes into the gas.}$$

ca : $Q = nC_p \Delta T = n(\frac{7}{2}R) \Delta T$, which gives

$$Q = (0.0040 \text{ mol})(\frac{7}{2})(8.315 \text{ J/mol} \cdot \text{K})(305 \text{ K} - 1220 \text{ K}) = -107 \text{ J. } Q \text{ is negative and heat comes out of the gas.}$$

bc : $Q = W = nRT \ln(V_c/V_b)$, which gives

$$Q = (0.0040 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(1220 \text{ K}) \ln(0.80 \text{ L}/0.20 \text{ L}) = 56 \text{ J. } Q \text{ is positive and heat goes into the gas.}$$

(d) ab : $\Delta U = nC_v \Delta T = n(\frac{5}{2}R) \Delta T$, which gives

$$\Delta U = (0.0040 \text{ mol})(\frac{5}{2})(8.315 \text{ J/mol} \cdot \text{K})(1220 \text{ K} - 305 \text{ K}) = +76 \text{ J. The internal energy increased.}$$

bc : $\Delta T = 0$ so $\Delta U = 0$. The internal energy does not change.

ca : $\Delta U = nC_v \Delta T = n(\frac{5}{2}R) \Delta T$, which gives

$$\Delta U = (0.0040 \text{ mol})(\frac{5}{2})(8.315 \text{ J/mol} \cdot \text{K})(305 \text{ K} - 1220 \text{ K}) = -76 \text{ J. The internal energy decreased.}$$

EVALUATE: The net internal energy change for the complete cycle $a \rightarrow b \rightarrow c \rightarrow a$ is

$$\Delta U_{\text{tot}} = +76 \text{ J} + 0 + (-76 \text{ J}) = 0. \text{ For any complete cycle the final state is the same as the initial state and the net internal energy change is zero. For the cycle the net heat flow is}$$

$$Q_{\text{tot}} = +76 \text{ J} + (-107 \text{ J}) + 56 \text{ J} = +25 \text{ J. } \Delta U_{\text{tot}} = 0 \text{ so } Q_{\text{tot}} = W_{\text{tot}}. \text{ The net work done in the cycle is positive and this agrees with our result that the net heat flow is positive.}$$

19.44 IDENTIFY: The segments ab and bc are not any of the familiar ones, such as isothermal, isobaric, or isochoric, but ac is isobaric.

SET UP: For helium, $C_V = 12.47 \text{ J/mol} \cdot \text{K}$ and $C_p = 20.78 \text{ J/mol} \cdot \text{K}$. $\Delta U = Q - W$. W is the area under the p versus V curve. $\Delta U = nC_V\Delta T$ for any process of an ideal gas.

EXECUTE: (a) $W = \frac{1}{2}(1.0 \times 10^5 \text{ Pa} + 3.5 \times 10^5 \text{ Pa})(0.0060 \text{ m}^3 - 0.0020 \text{ m}^3) + \frac{1}{2}(1.0 \times 10^5 \text{ Pa} + 3.5 \times 10^5 \text{ Pa})(0.0100 \text{ m}^3 - 0.0060 \text{ m}^3) = 1800 \text{ J}$.

Find $\Delta T = T_c - T_a$. p is constant so $\Delta T = \frac{p\Delta V}{nR} = \frac{(1.0 \times 10^5 \text{ Pa})(0.0100 \text{ m}^3 - 0.0020 \text{ m}^3)}{(\frac{1}{3} \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})} = 289 \text{ K}$.

Then

$$\Delta U = nC_V\Delta T = (\frac{1}{3} \text{ mol})(12.47 \text{ J/mol} \cdot \text{K})(289 \text{ K}) = 1.20 \times 10^3 \text{ J}.$$

$Q = \Delta U + W = 1.20 \times 10^3 \text{ J} + 1800 \text{ J} = 3.00 \times 10^3 \text{ J}$. $Q > 0$, so this heat is transferred into the gas.

(b) This process is isobaric, so $Q = nC_p\Delta T = (\frac{1}{3} \text{ mol})(20.78 \text{ J/mol} \cdot \text{K})(289 \text{ K}) = 2.00 \times 10^3 \text{ J}$. $Q > 0$, so this heat is transferred into the gas.

(c) Q is larger in part (a).

EVALUATE: ΔU is the same in parts (a) and (b) because the initial and final states are the same, but in (a) more work is done.

19.45. IDENTIFY and SET UP: We have information on the pressure and volume of the gas during the process, but we know almost nothing else about the gas. We do know that the first law of thermodynamics must apply to the gas during this process, so $Q = \Delta U + W$, and the work done by the gas is $W = \int_{V_1}^{V_2} p dV$. If

W is positive, the gas does work, but if W is negative, work is done on the gas.

EXECUTE: (a) Figure 19.45 shows the pV -diagram for this process. On the pV -diagram, we see that the graph is a closed figure; the gas begins and ends in the same state.

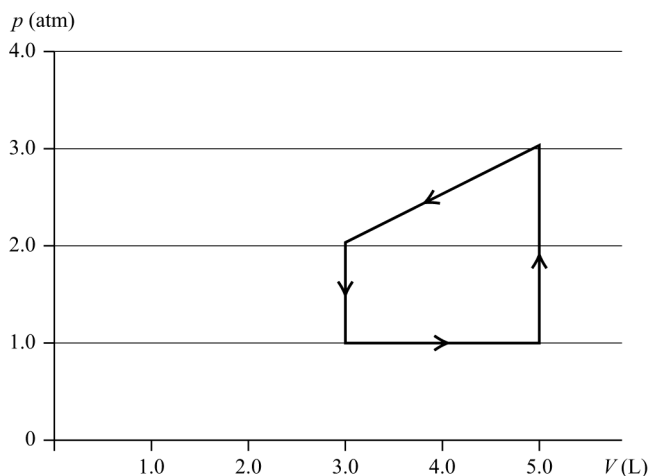


Figure 19.45

(b) Applying $Q = \Delta U + W$, we see that $\Delta U = 0$ because the gas ends up at the same state from which it began. Therefore $Q = W$. $W = \int_{V_1}^{V_2} p dV$, so the work is the area under the curve on a pV -diagram. For a closed cycle such as this one, the work is the area enclosed within the diagram. We calculate this work geometrically: $|W| = \text{area (rectangle)} + \text{area (triangle)} = (2.0 \text{ L})(1.0 \text{ atm}) + \frac{1}{2} (2.0 \text{ L})(1.0 \text{ atm}) = 3.0$

$\text{L} \cdot \text{atm} = 300 \text{ J}$. But the net work is negative, so $Q = -3.0 \text{ L} \cdot \text{atm} = -300 \text{ J}$. Since Q is negative, heat flows out of the gas.

EVALUATE: We know that the work is negative because in the upper part of the diagram, the volume is decreasing, which means that the gas is being compressed.

- 19.46. IDENTIFY:** For a constant pressure process, $Q = nC_p\Delta T$. $\Delta U = Q - W$. $\Delta U = nC_V\Delta T$ for any ideal gas process.

SET UP: For N_2 , $C_V = 20.76 \text{ J/mol} \cdot \text{K}$ and $C_p = 29.07 \text{ J/mol} \cdot \text{K}$. $Q < 0$ if heat comes out of the gas.

EXECUTE: (a) $n = \frac{Q}{C_p\Delta T} = \frac{-2.5 \times 10^4 \text{ J}}{(29.07 \text{ J/mol} \cdot \text{K})(-40.0 \text{ K})} = 21.5 \text{ mol}$.

(b) $\Delta U = nC_V\Delta T = Q(C_V/C_p) = (-2.5 \times 10^4 \text{ J})(20.76/29.07) = -1.79 \times 10^4 \text{ J}$.

(c) $W = Q - \Delta U = -7.15 \times 10^3 \text{ J}$.

(d) ΔU is the same for both processes, and if $\Delta V = 0$, $W = 0$ and $Q = \Delta U = -1.79 \times 10^4 \text{ J}$.

EVALUATE: For a given ΔT , Q is larger in magnitude when the pressure is constant than when the volume is constant.

- 19.47. IDENTIFY:** $pV = nRT$. For an isothermal process $W = nRT \ln(V_2/V_1)$. For a constant pressure process, $W = p\Delta V$.

SET UP: $1 \text{ L} = 10^{-3} \text{ m}^3$.

EXECUTE: (a) The pV -diagram is sketched in Figure 19.47.

(b) At constant temperature, the product pV is constant, so

$$V_2 = V_1(p_1/p_2) = (1.5 \text{ L}) \left(\frac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}} \right) = 6.00 \text{ L. The final pressure is given as being the same as}$$

$$p_3 = p_2 = 2.5 \times 10^4 \text{ Pa. The final volume is the same as the initial volume, so } T_3 = T_1(p_3/p_1) = 75.0 \text{ K.}$$

(c) Treating the gas as ideal, the work done in the first process is $W = nRT \ln(V_2/V_1) = p_1V_1 \ln(p_1/p_2)$.

$$W = (1.00 \times 10^5 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3) \ln \left(\frac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}} \right) = 208 \text{ J.}$$

$$\text{For the second process, } W = p_2(V_3 - V_2) = p_2(V_1 - V_2) = p_2V_1[1 - (p_1/p_2)].$$

$$W = (2.50 \times 10^4 \text{ Pa})(1.5 \times 10^{-3} \text{ m}^3) \left(1 - \frac{1.00 \times 10^5 \text{ Pa}}{2.50 \times 10^4 \text{ Pa}} \right) = -113 \text{ J.}$$

The total work done is $208 \text{ J} - 113 \text{ J} = 95 \text{ J}$.

(d) Heat at constant volume. No work would be done by the gas or on the gas during this process.

EVALUATE: When the volume increases, $W > 0$. When the volume decreases, $W < 0$.

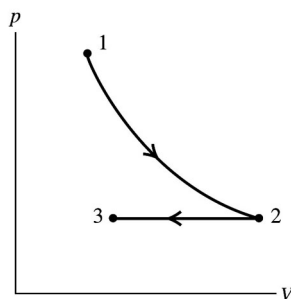


Figure 19.47

19.48. IDENTIFY: We are looking at the effect of gas compression on the buoyant force as a gas container descends in the ocean.

SET UP: As the inverted drum descends in the ocean, the water pressure increases and compresses the air in the drum. The pressure at a depth y in the ocean is $p = p_0 + \rho gy$. The temperature decreases linearly with depth from 23.0°C at the water surface to 3.0°C at a depth of 1000 m. The graph in Fig. 19.48 shows the temperature T as a function of depth y . This graph shows a line with a slope of $-(20.0^\circ\text{C})/(1000\text{ m}) = -0.0200^\circ\text{C}/\text{m}$. The line intercepts the T axis at 23.0°C ($y = 0$ at the surface). Using the slope-intercept form of the equation, the equation is $T = T_0 - By$, where $T_0 = 23.0^\circ\text{C} = 296\text{ K}$ and $B = 0.0200^\circ\text{C}/\text{m}$.

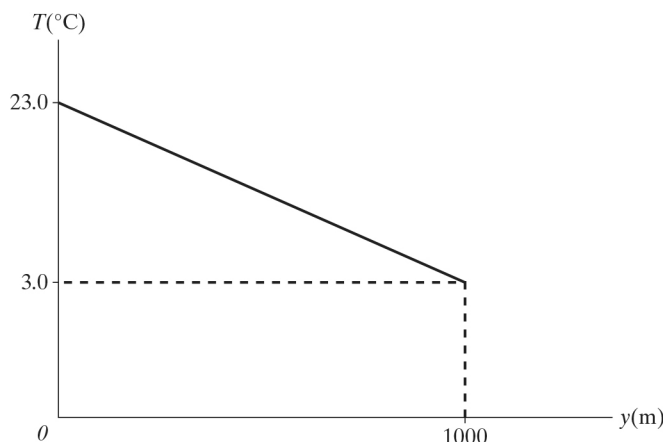


Figure 19.48

EXECUTE: (a) and (b) We want to find the depth for neutral buoyancy of the drum and the volume of the air in the drum at that depth. For neutral buoyancy, the buoyant force B on the drum must be equal to its weight mg , so $B = mg$. By Archimedes's principle, the buoyant force is equal to the weight of water displaced by the drum. At depth y the air in the drum has volume V_y , so $\rho g V_y = mg$. In this equation ρ is the density of seawater (1025 kg/m^3) and V_y is the volume of water displaced by the drum at depth y , which is the same as the volume of air in the drum. From $\rho g V_y = mg$ we get

$$V_y = \frac{m}{\rho} = \frac{17.3\text{ kg}}{1025\text{ kg/m}^3} = 0.0169\text{ m}^3.$$

We now want to find the depth y at which the air volume in the drum is 0.0169 m^3 . At the water surface the volume of the air is the volume of the drum, which is $V_0 = \pi r^2 L$, which gives

$$V_0 = \pi(0.305\text{ m})^2(0.880\text{ m}) = 0.2572\text{ m}^3. \text{ Therefore at this depth } y \text{ we know}$$

$$\frac{V_y}{V_0} = \frac{0.0169\text{ m}^3}{0.2572\text{ m}^3} = 0.06571. \text{ We now look at the gas pressure at depth } y. \text{ We do this because that}$$

pressure must be equal to the water pressure at the same depth. Using $pV = nRT$ gives

$$\frac{p_y}{p_0} = \left(\frac{T_y}{T_0}\right)\left(\frac{V_0}{V_y}\right) = \left(\frac{T_0 - By}{T_0}\right)\left(\frac{1}{0.06571}\right) = 15.219\left(\frac{T_0 - By}{T_0}\right), \text{ from which we get}$$

$$p_y = (15.219)p_0\left(\frac{T_0 - By}{T_0}\right). \text{ Now equate this pressure to the water pressure at depth } y, \text{ which is}$$

$$p = p_0 + \rho gy, \text{ giving } p_0 + \rho gy = (15.219)p_0\left(\frac{T_0 - By}{T_0}\right). \text{ This gives } y = \frac{14.219p_0}{\rho g + 15.219p_0 B / T_0}. \text{ Using}$$

$$p_0 = 101 \times 10^3\text{ Pa}, T_0 = 296\text{ K}, B = 0.0200\text{ K/m} \text{ and } \rho = 1025\text{ kg/m}^3, \text{ we get } y = 142\text{ m}.$$

EVALUATE: The compression of the air in the drum is *not* adiabatic because the air is in contact with the water so heat can flow out of the air as it moves to colder water at depth. In this process, none of the variables (p , V , and T) remained constant.

- 19.49. IDENTIFY:** For an adiabatic process of an ideal gas, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$. $pV = nRT$.

SET UP: For air, $\gamma = 1.40 = \frac{7}{5}$.

EXECUTE: (a) As the air moves to lower altitude its density increases; under an adiabatic compression, the temperature rises. If the wind is fast-moving, Q is not as likely to be significant, and modeling the process as adiabatic (no heat loss to the surroundings) is more accurate.

(b) $V = \frac{nRT}{p}$, so $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives $T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$. The temperature at the higher pressure is

$T_2 = T_1 (p_1/p_2)^{(\gamma-1)/\gamma} = (258.15 \text{ K}) \left[(8.12 \times 10^4 \text{ Pa}) / (5.60 \times 10^4 \text{ Pa}) \right]^{2/7} = 287.1 \text{ K} = 13.9^\circ\text{C}$ so the temperature would rise by 11.9°C .

EVALUATE: In an adiabatic compression, $Q = 0$ but the temperature rises because of the work done on the gas.

- 19.50. IDENTIFY:** The process is adiabatic. Apply $p_1 V_1^\gamma = p_2 V_2^\gamma$ and $pV = nRT$. $Q = 0$ so

$$\Delta U = -W = -\frac{1}{\gamma-1}(p_1 V_1 - p_2 V_2).$$

SET UP: For ideal monatomic helium, $\gamma = 5/3 = 1.667$. $p_1 = 1.00 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$.

$V_1 = 2.00 \times 10^3 \text{ m}^3$. $p_2 = 0.900 \text{ atm} = 9.117 \times 10^4 \text{ Pa}$. $T_1 = 288.15 \text{ K}$.

EXECUTE: (a) $V_2^\gamma = V_1^\gamma \left(\frac{p_1}{p_2} \right)$. $V_2 = V_1 \left(\frac{p_1}{p_2} \right)^{1/\gamma} = (2.00 \times 10^3 \text{ m}^3) \left(\frac{1.00 \text{ atm}}{0.900 \text{ atm}} \right)^{1/1.67} = 2.13 \times 10^3 \text{ m}^3$.

(b) $pV = nRT$ gives $\frac{T_1}{p_1 V_1} = \frac{T_2}{p_2 V_2}$.

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right) \left(\frac{V_2}{V_1} \right) = (288.15 \text{ K}) \left(\frac{0.900 \text{ atm}}{1.00 \text{ atm}} \right) \left(\frac{2.13 \times 10^3 \text{ m}^3}{2.00 \times 10^3 \text{ m}^3} \right) = 276.2 \text{ K} = 3.0^\circ\text{C}.$$

$$\text{(c) } \Delta U = -\frac{(1.013 \times 10^5 \text{ Pa})(2.00 \times 10^3 \text{ m}^3) - (9.117 \times 10^4 \text{ Pa})(2.13 \times 10^3 \text{ m}^3)}{1.667 - 1} = -1.25 \times 10^7 \text{ J}.$$

EVALUATE: The internal energy decreases when the temperature decreases.

- 19.51. IDENTIFY:** Assume that the gas is ideal and that the process is adiabatic. Apply $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ and $p_1 V_1^\gamma = p_2 V_2^\gamma$ to relate pressure and volume and temperature and volume. The distance the piston moves is related to the volume of the gas. Use $W = nC_V(T_1 - T_2)$ to calculate W .

(a) SET UP: $\gamma = C_p/C_V = (C_V + R)/C_V = 1 + R/C_V = 1.40$. The two positions of the piston are shown in Figure 19.51.

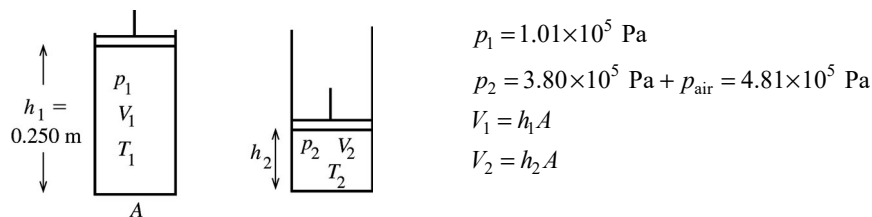


Figure 19.51

EXECUTE: For an adiabatic process of an ideal gas, $p_1 V_1^\gamma = p_2 V_2^\gamma$.

$$p_1 h_1^\gamma A^\gamma = p_2 h_2^\gamma A^\gamma$$

$$h_2 = h_1 \left(\frac{p_1}{p_2} \right)^{1/\gamma} = (0.250 \text{ m}) \left(\frac{1.01 \times 10^5 \text{ Pa}}{4.81 \times 10^5 \text{ Pa}} \right)^{1/1.40} = 0.08199 \text{ m}$$

The piston has moved a distance $h_1 - h_2 = 0.250 \text{ m} - 0.08199 \text{ m} = 0.168 \text{ m}$.

(b) SET UP: $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$T_1 h_1^{\gamma-1} A^{\gamma-1} = T_2 h_2^{\gamma-1} A^{\gamma-1}$$

$$\text{EXECUTE: } T_2 = T_1 \left(\frac{h_1}{h_2} \right)^{\gamma-1} = 300.1 \text{ K} \left(\frac{0.250 \text{ m}}{0.08199 \text{ m}} \right)^{0.40} = 468.7 \text{ K} = 196^\circ\text{C}.$$

(c) SET UP and EXECUTE: $W = nC_V(T_1 - T_2)$ gives

$W = (20.0 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(300.1 \text{ K} - 468.7 \text{ K}) = -7.01 \times 10^4 \text{ J} = -70.1 \text{ kJ}$. This is the work done *by* the gas. The work done *on* the gas by the pump is $+70.1 \text{ kJ}$.

EVALUATE: In an adiabatic compression of an ideal gas the temperature increases. In any compression the work W done by the gas is negative.

19.52. IDENTIFY: For constant pressure, $W = p\Delta V$. For an adiabatic process of an ideal gas,

$$W = \frac{C_V}{R}(p_1 V_1 - p_2 V_2) \text{ and } p_1 V_1^\gamma = p_2 V_2^\gamma.$$

$$\text{SET UP: } \gamma = \frac{C_p}{C_V} = \frac{C_p + C_V}{C_V} = 1 + \frac{R}{C_V}.$$

EXECUTE: (a) The pV -diagram is sketched in Figure 19.52.

(b) The work done is $W = p_0(2V_0 - V_0) + \frac{C_V}{R}(p_0(2V_0) - p_3(4V_0))$. $p_3 = p_0(2V_0/4V_0)^\gamma$ and so

$$W = p_0 V_0 \left[1 + \frac{C_V}{R}(2 - 2^{2-\gamma}) \right]. \text{ Note that } p_0 \text{ is the absolute pressure.}$$

(c) The most direct way to find the temperature is to find the ratio of the final pressure and volume to the original and treat the air as an ideal gas. $p_3 = p_2 \left(\frac{V_2}{V_3} \right)^\gamma = p_1 \left(\frac{V_2}{V_3} \right)^\gamma$, since $p_1 = p_2$. Then

$$T_3 = T_0 \frac{p_3 V_3}{p_1 V_1} = T_0 \left(\frac{V_2}{V_3} \right)^\gamma \left(\frac{V_3}{V_1} \right) = T_0 \left(\frac{1}{2} \right)^\gamma 4 = T_0 (2)^{2-\gamma}.$$

(d) Since $n = \frac{p_0 V_0}{RT_0}$, $Q = \frac{p_0 V_0}{RT_0} (C_V + R)(2T_0 - T_0) = p_0 V_0 \left(\frac{C_V}{R} + 1 \right)$. This amount of heat flows into the gas, since $Q > 0$.

EVALUATE: In the isobaric expansion the temperature doubles and in the adiabatic expansion the temperature decreases. If the gas is diatomic, with $\gamma = \frac{7}{5}$, $2 - \gamma = \frac{3}{5}$ and $T_3 = 1.52T_0$, $W = 2.21p_0 V_0$, and $Q = 3.50p_0 V_0$. $\Delta U = 1.29p_0 V_0$. $\Delta U > 0$ and this is consistent with an increase in temperature.

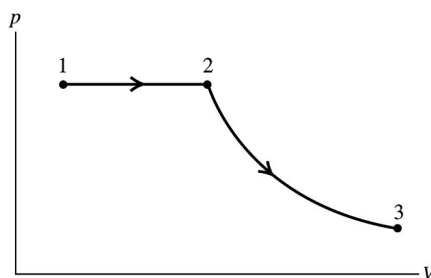


Figure 19.52

19.53. IDENTIFY: In each case calculate either ΔU or Q for the specific type of process and then apply the first law.

(a) SET UP: Isothermal: ($\Delta T = 0$) $\Delta U = Q - W$; $W = +450$ J. For any process of an ideal gas,
 $\Delta U = nC_V\Delta T$.

EXECUTE: Therefore, for an ideal gas, if $\Delta T = 0$ then $\Delta U = 0$ and $Q = W = +450$ J.

(b) SET UP: Adiabatic: ($Q = 0$)

$\Delta U = Q - W$; $W = +450$ J.

EXECUTE: $Q = 0$ says $\Delta U = -W = -450$ J.

(c) SET UP: Isobaric: $\Delta p = 0$

Use W to calculate ΔT and then calculate Q .

EXECUTE: $W = p\Delta V = nR\Delta T$; $\Delta T = W/nR$

$Q = nC_p\Delta T$ and for a monatomic ideal gas $C_p = \frac{5}{2}R$.

Thus $Q = n\frac{5}{2}R\Delta T = (5Rn/2)(W/nR) = 5W/2 = +1125$ J.

$\Delta U = nC_V\Delta T$ for any ideal gas process and $C_V = C_p - R = \frac{3}{2}R$.

Thus $\Delta U = 3W/2 = +675$ J.

EVALUATE: 450 J of energy leaves the gas when it performs expansion work. In the isothermal process this energy is replaced by heat flow into the gas and the internal energy remains the same. In the adiabatic process the energy used in doing the work decreases the internal energy. In the isobaric process 1125 J of heat energy enters the gas, 450 J leaves as the work done and 675 J remains in the gas as increased internal energy.

19.54. IDENTIFY: $pV = nRT$. For the isobaric process, $W = p\Delta V = nR\Delta T$. For the isothermal process,

$$W = nRT \ln\left(\frac{V_2}{V_1}\right).$$

SET UP: $R = 8.315$ J/mol \cdot K.

EXECUTE: **(a)** The pV diagram for these processes is sketched in Figure 19.54.

(b) Find T_2 . For process $1 \rightarrow 2$, n , R and p are constant so $\frac{T}{V} = \frac{p}{nR} = \text{constant}$. $\frac{T_1}{V_1} = \frac{T_2}{V_2}$ and

$$T_2 = T_1 \left(\frac{V_2}{V_1}\right) = (355 \text{ K})(2) = 710 \text{ K}.$$

(c) The maximum pressure is for state 3. For process $2 \rightarrow 3$, n , R and T are constant. $p_2V_2 = p_3V_3$ and

$$p_3 = p_2 \left(\frac{V_2}{V_3}\right) = (2.40 \times 10^5 \text{ Pa})(2) = 4.80 \times 10^5 \text{ Pa}.$$

(d) Proces $1 \rightarrow 2$: $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = 738 \text{ J}$.

Proces $2 \rightarrow 3$: $W = nRT \ln\left(\frac{V_3}{V_2}\right) = (0.250 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(710 \text{ K}) \ln\left(\frac{1}{2}\right) = -1023 \text{ J}.$

Proces $3 \rightarrow 1$: $\Delta V = 0$ and $W = 0.$

The total work done is $738 \text{ J} + (-1023 \text{ J}) = -285 \text{ J}$. This is the work done by the gas. The work done on the gas is 285 J .

EVALUATE: The final pressure and volume are the same as the initial pressure and volume, so the final state is the same as the initial state. For the cycle, $\Delta U = 0$ and $Q = W = -285 \text{ J}$. During the cycle, 285 J of heat energy must leave the gas.

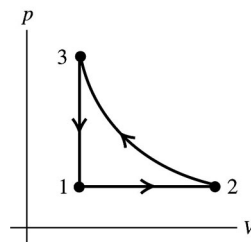


Figure 19.54

- 19.55. IDENTIFY and SET UP:** Use the ideal gas law, the first law of thermodynamics, and expressions for Q and W for specific types of processes.

EXECUTE: (a) initial expansion (state 1 \rightarrow state 2)

$$p_1 = 2.40 \times 10^5 \text{ Pa}, \quad T_1 = 355 \text{ K}, \quad p_2 = 2.40 \times 10^5 \text{ Pa}, \quad V_2 = 2V_1$$

$$pV = nRT; \quad T/V = p/nR = \text{constant, so } T_1/V_1 = T_2/V_2 \text{ and } T_2 = T_1(V_2/V_1) = 355 \text{ K}(2V_1/V_1) = 710 \text{ K}$$

$$\Delta p = 0 \text{ so } W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = +738 \text{ J}$$

$$Q = nC_p\Delta T = (0.250 \text{ mol})(29.17 \text{ J/mol} \cdot \text{K})(710 \text{ K} - 355 \text{ K}) = +2590 \text{ J}$$

$$\Delta U = Q - W = 2590 \text{ J} - 738 \text{ J} = 1850 \text{ J}$$

(b) At the beginning of the final cooling process (cooling at constant volume), $T = 710 \text{ K}$. The gas returns to its original volume and pressure, so also to its original temperature of 355 K .

$$\Delta V = 0 \text{ so } W = 0$$

$$Q = nC_v\Delta T = (0.250 \text{ mol})(20.85 \text{ J/mol} \cdot \text{K})(355 \text{ K} - 710 \text{ K}) = -1850 \text{ J}$$

$$\Delta U = Q - W = -1850 \text{ J}.$$

(c) For any ideal gas process $\Delta U = nC_v\Delta T$. For an isothermal process $\Delta T = 0$, so $\Delta U = 0$.

EVALUATE: The three processes return the gas to its initial state, so $\Delta U_{\text{total}} = 0$; our results agree with this.

- 19.56. IDENTIFY:** $pV = nRT$. For an adiabatic process of an ideal gas, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For N_2 , $\gamma = 1.40$.

EXECUTE: (a) The pV -diagram is sketched in Figure 19.56.

(b) At constant pressure, halving the volume halves the Kelvin temperature, and the temperature at the beginning of the adiabatic expansion is 150 K . The volume doubles during the adiabatic expansion, and from Eq. (19.22), the temperature at the end of the expansion is $(150 \text{ K})(1/2)^{0.40} = 114 \text{ K}$.

(c) The minimum pressure occurs at the end of the adiabatic expansion (state 3). During the final heating the volume is held constant, so the minimum pressure is proportional to the Kelvin temperature,

$$p_{\min} = (1.80 \times 10^5 \text{ Pa})(114 \text{ K}/300 \text{ K}) = 6.82 \times 10^4 \text{ Pa}.$$

EVALUATE: In the adiabatic expansion the temperature decreases.

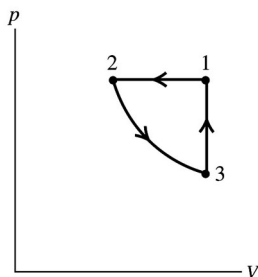


Figure 19.56

- 19.57. IDENTIFY:** Use the appropriate expressions for Q , W , and ΔU for each type of process. $\Delta U = Q - W$ can also be used.

SET UP: For N_2 , $C_V = 20.76 \text{ J/mol} \cdot \text{K}$ and $C_p = 29.07 \text{ J/mol} \cdot \text{K}$.

EXECUTE: (a) $W = p\Delta V = nR\Delta T = (0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(-150 \text{ K}) = -187 \text{ J}$,

$Q = nC_p\Delta T = (0.150 \text{ mol})(29.07 \text{ J/mol} \cdot \text{K})(-150 \text{ K}) = -654 \text{ J}$, $\Delta U = Q - W = -467 \text{ J}$.

(b) From Eq. (19.26), using the expression for the temperature found in Problem 19.56,

$W = \frac{1}{0.40}(0.150 \text{ mol})(8.3145 \text{ J/mol} \cdot \text{K})(150 \text{ K})[1 - (1/2)^{0.40}] = 113 \text{ J}$. $Q = 0$ for an adiabatic process,

and $\Delta U = Q - W = -W = -113 \text{ J}$.

(c) $\Delta V = 0$, so $W = 0$. Using the temperature change as found in Problem 19.66 part (b),

$Q = nC_V\Delta T = (0.150 \text{ mol})(20.76 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 113.7 \text{ K}) = 580 \text{ J}$ and $\Delta U = Q - W = Q = 580 \text{ J}$.

EVALUATE: For each process we could also use $\Delta U = nC_V\Delta T$ to calculate ΔU .

- 19.58. IDENTIFY:** Use the appropriate expression for W for each type of process.

SET UP: For a monatomic ideal gas, $\gamma = 5/3$ and $C_V = 3R/2$.

EXECUTE: (a) $W = nRT \ln(V_2/V_1) = nRT \ln(3) = 3.29 \times 10^3 \text{ J}$.

(b) $Q = 0$ so $W = -\Delta U = -nC_V\Delta T$. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_2 = T_1(1/3)^{2/3}$. Then

$W = nC_VT_1(1 - (1/3)^{2/3}) = 2.33 \times 10^3 \text{ J}$.

(c) $V_2 = 3V_1$, so $W = p\Delta V = 2pV_1 = 2nRT_1 = 6.00 \times 10^3 \text{ J}$.

(d) Each process is shown in Figure 19.58. The most work done is in the isobaric process, as the pressure is maintained at its original value. The least work is done in the adiabatic process.

(e) The isobaric process involves the most work and the largest temperature increase, and so requires the most heat. Adiabatic processes involve no heat transfer, and so the magnitude is zero.

(f) The isobaric process doubles the Kelvin temperature, and so has the largest change in internal energy. The isothermal process necessarily involves no change in internal energy.

EVALUATE: The work done is the area under the path for the process in the pV -diagram. Figure 19.58 shows that the work done is greatest in the isobaric process and least in the adiabatic process.

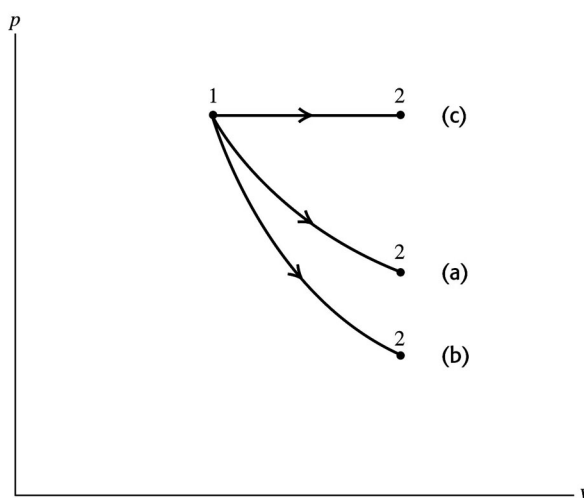


Figure 19.58

19.59. IDENTIFY: For an adiabatic process, no heat enters or leaves the gas. An isochoric process takes place at constant volume, and an isobaric process takes place at constant pressure. The first law of thermodynamics applies.

SET UP: For any process, including an isochoric process, $Q = nC_V \Delta T$, and for an isobaric process, $Q = nC_p \Delta T$. $Q = \Delta U + W$.

EXECUTE: (a) Process a is adiabatic since no heat goes into or out of the system. In processes b and c , the temperature change is the same, but more heat goes into the gas for process c . Since the change in internal energy is the same for both b and c , some of the heat in c must be doing work, but not in b . Therefore b is isochoric and c is isobaric. To summarize: a is adiabatic, b is isochoric, c is isobaric.

(b) $Q_b = nC_V \Delta T$ and $Q_c = nC_p \Delta T$. Subtracting gives

$Q_c - Q_b = nC_p \Delta T - nC_V \Delta T = n(C_p - C_V) \Delta T = nR \Delta T = 20 \text{ J}$. Solving for ΔT gives
 $\Delta T = (20 \text{ J})/nR = (20 \text{ J})/[(0.300 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})] = 8.0^\circ \text{C}$, so $T_2 = 20.0^\circ \text{C} + 8.0^\circ \text{C} = 28.0^\circ \text{C}$.

(c) $\frac{Q_c}{Q_b} = \frac{nC_p \Delta T}{nC_V \Delta T} = \frac{C_p}{C_V} = \gamma = \frac{50 \text{ J}}{30 \text{ J}} = \frac{5}{3}$. Since $\gamma = 5/3$, the gas must be monatomic, in which case we

have $C_V = 3/2 R$ and $C_p = 5/2 R$. Therefore

Process a : $Q = \Delta U + W$ gives $0 = nC_V \Delta T + W$.

$W = -n(3/2 R) \Delta T = -(0.300 \text{ mol})(3/2)(8.314 \text{ J/mol} \cdot \text{K})(8.0 \text{ K}) = -30 \text{ J}$.

Process b : The volume is constant, so $W = 0$.

Process c : $Q = \Delta U + W$. ΔU is the same as for process a because ΔT is the same, so we have
 $50 \text{ J} = 30 \text{ J} + W$, which gives $W = 20 \text{ J}$.

(d) The greatest work has the greatest volume change. Using the results of part (c), process a has the greatest amount of work and hence the greatest volume change.

(e) The volume is increasing if W is positive. Therefore

Process a : W is negative, so the volume decreases.

Process b : $W = 0$ so the volume stays the same.

Process c : W is positive, so the volume increases.

EVALUATE: In Process a , no heat enters the gas, yet its temperature increases. This means that work must have been done on the gas, as we found.

19.60. IDENTIFY and SET UP: The cylinder is insulated, so no heat can go into or out of the gas, which makes this an adiabatic process. For an adiabatic process, $pV^\gamma = \text{constant}$, and the ideal gas law, $pV = nRT$, also applies.

EXECUTE: (a) The graph of $\log p$ versus $\log V$ is shown in Figure 19.60.

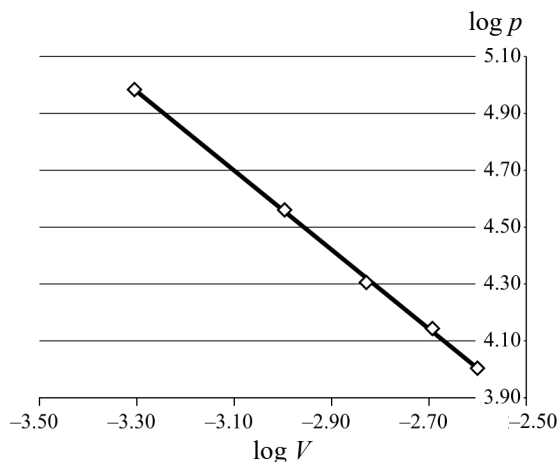


Figure 19.60

For an adiabatic process, $pV^\gamma = \text{constant}$. Taking logs of both sides of this equation gives

$\log(pV^\gamma) = \log p + \log V^\gamma = \log p + \gamma \log V = \log(\text{constant})$. Solving for $\log p$ gives

$\log p = -\gamma \log V + \log(\text{constant})$. Therefore a graph of $\log p$ versus $\log V$ should be a straight line having a slope equal to $-\gamma$.

(b) The equation of the best-fit line in the graph is $\log p = -1.3946 \log V + 0.381$, so $-\gamma = -1.3946$, so $\gamma = 1.4$. This is the adiabatic constant for a diatomic gas, so this gas must be diatomic.

(c) The ideal gas law gives $pV = nRT$, so $nR = pV/T$. Using the first set of data points in the table gives $nR = (0.101 \text{ atm})(2.50 \text{ L})/(293.15 \text{ K}) = 8.61 \times 10^{-4} \text{ L} \cdot \text{atm/K}$, so $1/nR = 1160 \text{ K/L} \cdot \text{atm}$. Using this number, we can calculate T for the rest of the pairs of points in the table. For example, for the next set of points, we have $T = pV/nR = (1160 \text{ K/L} \cdot \text{atm})(2.02 \text{ L})(0.139 \text{ atm}) = 326 \text{ K}$. We do likewise for the other pairs of points. The results are:

For 1.48 L, 0.202 atm: $T = 347 \text{ K}$.

For 1.01 L, 0.361 atm: $T = 423 \text{ K}$.

For 0.50 L, 0.952 atm: $T = 553 \text{ K}$.

As the volume decreases, the temperature is increasing, so the temperature is increasing during compression.

EVALUATE: This result confirms that during an adiabatic compression, the gas temperature increases because work is being done on the gas.

- 19.61. IDENTIFY:** The air in a cylinder can be compressed by a moveable piston at one end. It goes through a cycle that ends in the same state at which it began. The compression ratio v is defined as $V_{\text{max}}/V_{\text{min}} = v$.
SET UP: Summarize the steps of the cycle and make a pV -diagram of the process, shown in Fig. 19.61.

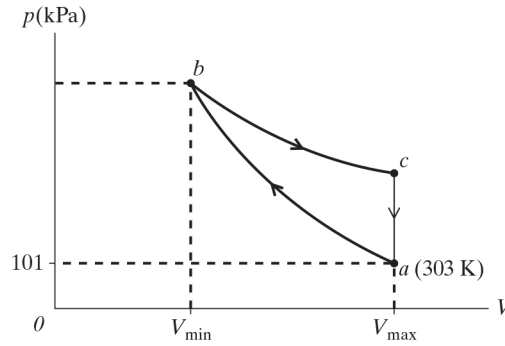


Figure 19.61

Segment ab : Beginning at ambient temperature, an adiabatic compression quickly increases the air temperature.

Segment bc : Isothermal expansion to maximum volume

Segment ca : Isochoric cooling back to ambient temperature

We know the following things about this system: For diatomic air $C_V = 20.8 \text{ J/mol} \cdot \text{K}$ and $\gamma = 1.40$.

The volume of the air in the cylinder is $V = AL$, where A is the area of the faces and L is the length of the gas-containing part of the cylinder. $L_{\max} = 30.0 \text{ cm} = 0.300 \text{ m}$ and $L_{\min} = L_{\max} / v = (0.300 \text{ m}) / v$.

The cycle starts at point a with $T_a = 30.0^\circ\text{C} = 303 \text{ K}$, $p_a = 101 \text{ kPa}$, and $V_a = L_{\max}A = (0.300 \text{ m})A$, where $A = \pi r^2$.

EXECUTE: (a) We want the work done by the air for a complete cycle abc . We need to break this process up into its three segments, so $W_{\text{tot}} = W_{ab} + W_{bc} + W_{ca}$. Segment ab is an adiabatic compression,

so $W_{ab} = \frac{1}{\gamma-1}(p_a V_a - p_b V_b)$. Using $p_a V_a^\gamma = p_b V_b^\gamma$ with $V_b = V_a/v$, this becomes

$$W_{ab} = \frac{1}{\gamma-1} \left[p_a V_a - p_a V_b \left(\frac{V_a}{V_b} \right)^\gamma \right] = \frac{p_a}{\gamma-1} \left[V_a - v^\gamma (V_a / v) \right] = \frac{p_a V_a}{\gamma-1} (1 - v^{\gamma-1}) = \frac{p_a V_a}{\gamma-1} (1 - v^{\gamma-1}).$$

Segment bc is isothermal, so $T = \text{constant} = T_b$. Using $pV = nRT$ and $W_{bc} = \int_b^c p dV$, we have

$$W_{bc} = \int_b^c \frac{nRT_b}{V} dV = nRT_b \ln(V_c / V_b). \text{ But } V_c = V_a \text{ and } V_b = V_a/v, \text{ so } V_c/V_b = v, \text{ so } W_{bc} \text{ is}$$

$$W_{bc} = nRT_b \ln v = p_b V_b \ln v. \text{ From } p_a V_a^\gamma = p_b V_b^\gamma \text{ we have } p_b = p_a \left(\frac{V_a}{V_b} \right)^\gamma, \text{ and we also know that } V_b =$$

$$V_a/v, \text{ so } W_{bc} = p_a \left(\frac{V_a}{V_b} \right)^\gamma \frac{V_a}{v} \ln v = p_a V_a v^{\gamma-1} \ln v = p_a V_a v^{\gamma-1} \ln v. \text{ The work during segment } ca \text{ is 0 because}$$

the volume is constant. $V_a = (0.300 \text{ m})A = (0.300 \text{ m})\pi(0.0150 \text{ m})^2 = 2.1205 \times 10^{-4} \text{ m}^3$. Using the values for p_a , V_a , and γ we get $W_{ab} = (53.5 \text{ J})(1 - v^{0.40})$ and $W_{bc} = (21.4 \text{ J})v^{0.40} \ln v$.

$$W_{\text{tot}} = 53.5 \text{ J} + v^{0.40} [(21.42 \text{ J}) \ln v - 53.5 \text{ J}].$$

(b) We want the maximum integer value of v so that maximum temperature is no greater than 400°C .

The maximum temperature occurs along segment bc , so the maximum that T_b can be is $400^\circ\text{C} = 673 \text{ K}$.

$$\text{Using } T_b V_b^{\gamma-1} = T_a V_a^{\gamma-1} \text{ gives } T_b = T_a \left(\frac{V_a}{V_b} \right)^{\gamma-1} = T_a \left(\frac{v V_b}{V_b} \right)^{\gamma-1} = T_a v^{\gamma-1}. \text{ This gives } v^{0.40} = 673 / 303, \text{ so } v =$$

$(673/303)^{1/0.40} = 7.35$. Since v must be an integer, $v_{\max} = 7$. The temperature at b for this value of v is $T_b = T_a v^{\gamma-1} = (303 \text{ K})7^{0.40} = 660 \text{ K} = 387^\circ\text{C}$. (Note that this is *not* 400°C , but the requirement was that the maximum temperature be *no greater than* 400°C , not that it be *equal to* 400°C .)

(c) We want the minimum value of v so that the gas does at least 25.0 J of work per cycle. Using $W_{\text{tot}} = 53.5 \text{ J} + v^{0.40} [(21.42 \text{ J}) \ln v - 53.5 \text{ J}] = 25.0 \text{ J}$ gives $-28.55 \text{ J} = v^{0.40} [(21.42 \text{ J}) \ln v - 53.5 \text{ J}]$. The LHS of this equation is negative, so the RHS must also be negative. The only way this is possible is for $[(21.42 \text{ J}) \ln v - 53.5 \text{ J}]$ to be negative. This tells us that $\ln v < 53.5/21.42$, so $v < 12.18$. Since v must be an integer, $v < 12$. But in part (b) we saw that $v \leq 7$. We try $v = 7$ to see how much work is done. Using $v = 7$ in $W_{\text{tot}} = 53.5 \text{ J} + v^{0.40} [(21.42 \text{ J}) \ln v - 53.5 \text{ J}]$ gives $W_{\text{tot}} = 27.7 \text{ J}$, which is acceptable since it is more than 25.0 J. Now try $v = 6$, which gives $W_{\text{tot}} = 22.5 \text{ J}$. This is not acceptable since it is less than 25.0 J, and no smaller integers would be acceptable either. Therefore $v_{\text{min}} = 7$.

(d) $v \geq 7$ and $v \leq 7$, so $v = 7$.

(e) We want the heat that leaves the gas during the isochoric segment, which is ca . The work is zero, so $\Delta U = Q - W = Q$. Therefore $Q = \Delta U = nC_V \Delta T$. First we need n . Using $pV = nRT$ gives $n = p_a V_a / RT_a$, which gives $n = 8.50 \times 10^{-3} \text{ mol}$. Now we can use $Q = \Delta U = nC_V \Delta T$. This gives $Q = (8.50 \times 10^{-3} \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(303 \text{ K} - 660 \text{ K}) = -63.1 \text{ J}$. The minus sign tells us that the heat comes *out of* the air.

EVALUATE: This device does 27.7 J of work per cycle operating between a high temperature of 660 K (387°C) and low temperature of 303 K (30°C).

19.62. IDENTIFY: $m = \rho V$. The density of air is given by $\rho = \frac{pM}{RT}$. For an adiabatic process,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}. \quad pV = nRT.$$

SET UP: Using $V = \frac{nRT}{p}$ in $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ gives $T_1 p_1^{1-\gamma} = T_2 p_2^{1-\gamma}$.

EXECUTE: (a) The pV -diagram is sketched in Figure 19.62.

(b) The final temperature is the same as the initial temperature, and the density is proportional to the absolute pressure. The mass needed to fill the cylinder is then

$$m = \rho_0 V \frac{p}{p_{\text{air}}} = (1.23 \text{ kg/m}^3)(575 \times 10^{-6} \text{ m}^3) \frac{1.45 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} = 1.02 \times 10^{-3} \text{ kg}.$$

Without the turbocharger or intercooler the mass of air at $T = 15.0^\circ\text{C}$ and $p = 1.01 \times 10^5 \text{ Pa}$ in a cylinder is $m = \rho_0 V = 7.07 \times 10^{-4} \text{ kg}$. The increase in power is proportional to the increase in mass of air in the cylinder; the percentage increase is $\frac{1.02 \times 10^{-3} \text{ kg}}{7.07 \times 10^{-4} \text{ kg}} - 1 = 0.44 = 44\%$.

(c) The temperature after the adiabatic process is $T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma}$. The density becomes

$$\rho = \rho_0 \left(\frac{T_1}{T_2} \right) \left(\frac{p_2}{p_1} \right) = \rho_0 \left(\frac{p_2}{p_1} \right)^{(1-\gamma)/\gamma} \left(\frac{p_2}{p_1} \right) = \rho_0 \left(\frac{p_2}{p_1} \right)^{1/\gamma}. \quad \text{The mass of air in the cylinder is}$$

$$m = (1.23 \text{ kg/m}^3)(575 \times 10^{-6} \text{ m}^3) \left(\frac{1.45 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \right)^{1/1.40} = 9.16 \times 10^{-4} \text{ kg},$$

The percentage increase in power is $\frac{9.16 \times 10^{-4} \text{ kg}}{7.07 \times 10^{-4} \text{ kg}} - 1 = 0.30 = 30\%$.

EVALUATE: The turbocharger and intercooler each have an appreciable effect on the engine power.

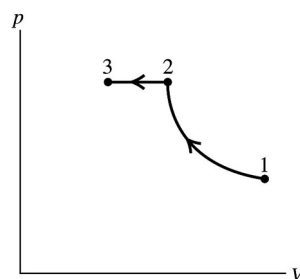


Figure 19.62

- 19.63. IDENTIFY and SET UP:** The gas is cooled at constant volume. The ideal gas law applies, so $pV = nRT$.

At constant volume, this becomes $\frac{p_1}{T_1} = \frac{p_2}{T_2}$.

EXECUTE: Solving $\frac{p_1}{T_1} = \frac{p_2}{T_2}$ for p_2 gives $p_2 = p_1 \frac{T_2}{T_1} = (2000 \text{ psi}) \frac{268 \text{ K}}{293 \text{ K}} = 1830 \text{ psi}$, which is choice

(c).

EVALUATE: As the temperature decreases, the pressure decreases, as expected.

- 19.64. IDENTIFY and SET UP:** The rapid expansion of the gas is an adiabatic process.

EXECUTE: The work the gas does comes from its internal energy, so its temperature decreases, causing some of it to condense. Therefore choice (d) is correct.

EVALUATE: This is the same principle used by snow-making machines.

- 19.65. IDENTIFY and SET UP:** The gas is initially a gauge pressure of 2000 psi (absolute pressure of 2014.7 psi). It will continue to flow out until it is at the same absolute pressure as the outside air, which is 1.0 atm, or 14.7 psi. So we need to find the volume the gas would occupy at 1.0 atm of absolute pressure. The ideal gas law, $pV = nRT$, applies to the gas, and the temperature is constant during this process.

EXECUTE: For an isothermal process, T is constant, so $pV = nRT$ can be put into the form

$$V_2 = V_1 \frac{p_1}{p_2} = (500 \text{ L}) \frac{2014.7 \text{ psi}}{14.7 \text{ psi}} = 6.85 \times 10^4 \text{ L. The volume of gas lost is therefore}$$

$6.85 \times 10^4 \text{ L} - 500 \text{ L} = 6.80 \times 10^4 \text{ L}$. The gas flows at a constant rate of 8.2 L/min, so
 $(8.2 \text{ L/min})t = 6.80 \times 10^4 \text{ L}$, which gives $t = 8300 \text{ min} = 140 \text{ h}$, which is choice (d).

EVALUATE: The rate of flow might not be uniform as the gas approaches 1.0 atm, but for most of the time under high pressure, it should be reasonable to assume that the flow rate can be held constant.

- 19.66. IDENTIFY and SET UP:** The oxygen and the N_2O are at the same temperature in the same container. Therefore to have a 50%/50% mixture by volume, they should have equal numbers of moles. $pV = nRT$ applies.

EXECUTE: The molecular mass of N_2O is $(28 + 16) \text{ g/mol} = 44 \text{ g/mol}$. The amount present is $1.7 \text{ kg} = 1700 \text{ g}$, which is $1700/44 = 38.64 \text{ mol}$. Therefore the O_2 must also contain 38.64 mol. The temperature is $20^\circ\text{C} = 293 \text{ K}$. The pressure is $50 \text{ psi} + 14.7 \text{ psi} = 64.7 \text{ psi}$. Since $1.0 \text{ atm} = 14.7 \text{ psi} = 1.01 \times 10^5 \text{ Pa}$, converting gives $64.7 \text{ psi} = 4.445 \times 10^5 \text{ Pa}$. Using $pV = nRT$, we have

$$V = nRT/p = (38.64 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})/(4.445 \times 10^5 \text{ Pa}) = 0.21 \text{ m}^3, \text{ which is choice (a).}$$

EVALUATE: The mixture is 50%/50% by volume, but not by weight, since N_2O is heavier than O_2 .