Circuits
-
Superposition & Thévenin/Norton equivalence

Spring 2022

Superposition

Linear circuits

We will analyze linear circuits.

One of the most interesting consequence of linearity is **superposition**

Superposition

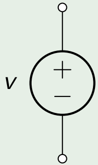
The response of a circuit having more than one independent source can be obtained by **adding the responses** caused by the **independent sources acting alone**

Superposition

Turning off sources

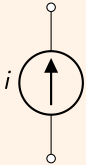
Since the response of the circuit can be obtained by superposition, **all the independent sources but one have to be turned off** at a time

Voltage source turned off (zeroed out)



When turned off $\Longrightarrow$

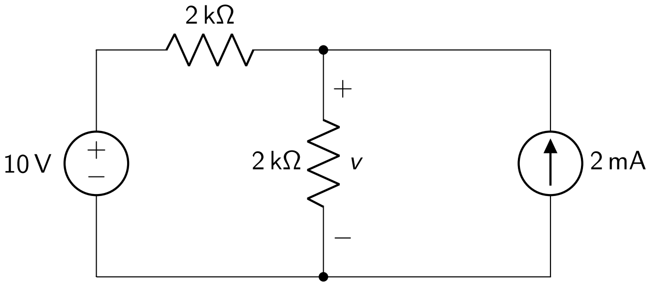
Current source turned off (zeroed out)



When turned off $\Longrightarrow$

Superposition

Determine the voltage v

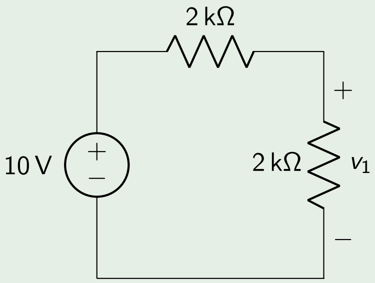


Nodal analysis (for example):

- $2\,\text{mA} + \dfrac{10\,\text{V} - v}{2\,\text{k}\Omega} = \dfrac{v}{2\,\text{k}\Omega}$
- $v = 7\,\text{V}$

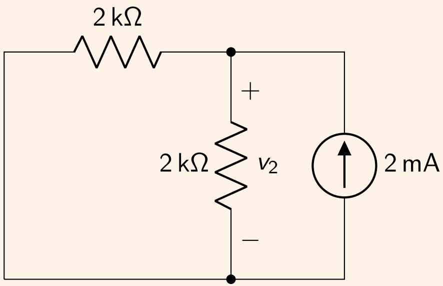
Superposition

Analysis 1



- $v_1 = 5\,\mathrm{V}$

Analysis 2



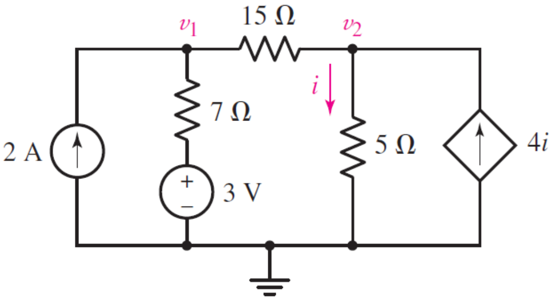
- $v_2 = 2\,\mathrm{V}$

Superposition

- $v = v_1 + v_2 = 7\,\mathrm{V}$

Examples

Determine the voltage across each current source.



Superposition

Summary

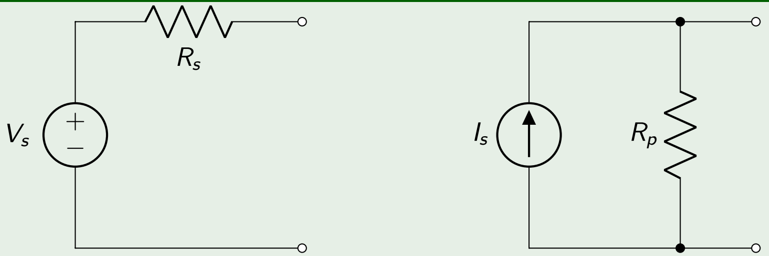
- Superposition relies on linearity
- Analyze as many circuits as there are **independent sources**
- Dependent sources are never zeroed out
- More circuits to analyze, sometimes leading to more equations...
- But the different circuits are also simpler to analyze, with simpler equations

Source transformation

Concept

A voltage source with a resistor in series is **equivalent** to a current source with a resistor in parallel

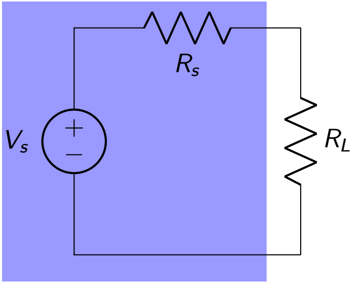
Equivalence

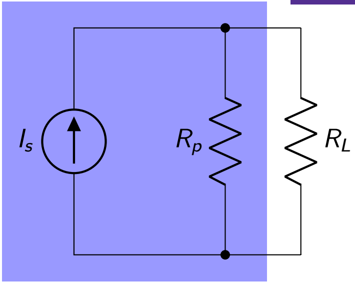


Validity

The equivalence holds if both circuits have the **same voltage-current relationship** at their terminals

Source transformation





Terminal

- $V = \dfrac{R_L}{R_s + R_L} V_s$
- $I = \dfrac{V_s}{R_s + R_L}$

Terminal

- $V = \dfrac{R_L}{R_p + R_L} R_p I_s$
- $I = \dfrac{R_p I_s}{R_p + R_L}$

Source transformation

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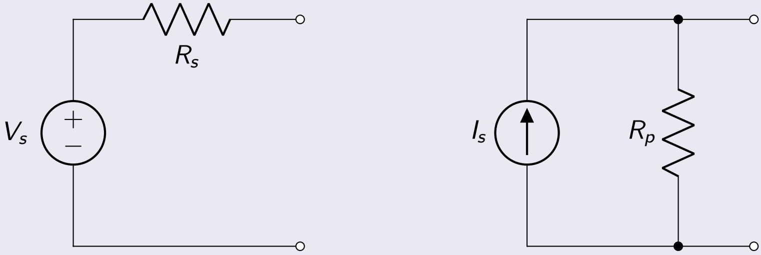
Equivalence

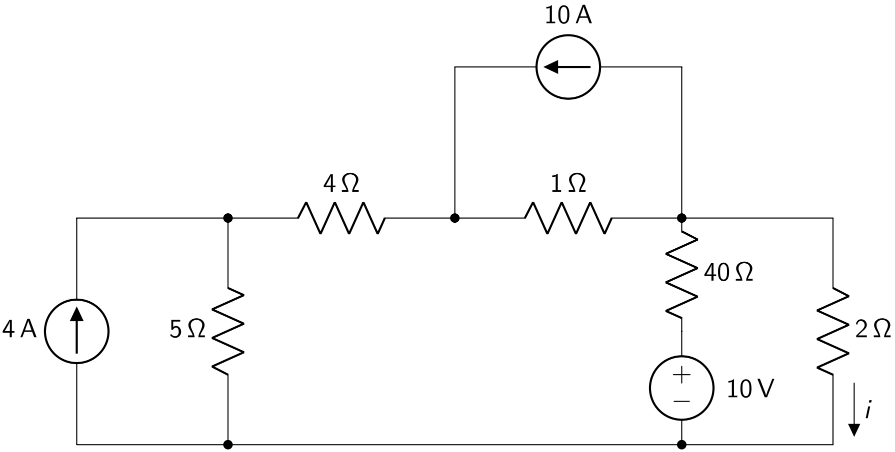


Equivalent when:

- $R_s = R_p = R$
- $V_s = RI_s$

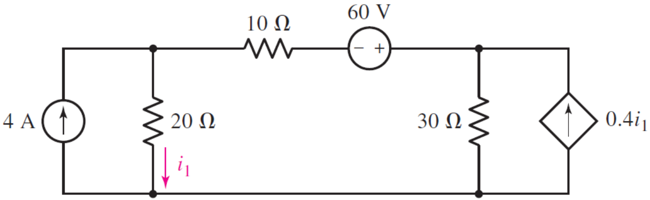
Source transformation

Apply multiple source transformation to determine i



Examples

Determine the source transformation should be performed or not.



Thévenin equivalent circuit

Theorem

A **two-terminal linear circuit**, constituted of sources and resistors, is equivalent to a **voltage source in series with a resistor**

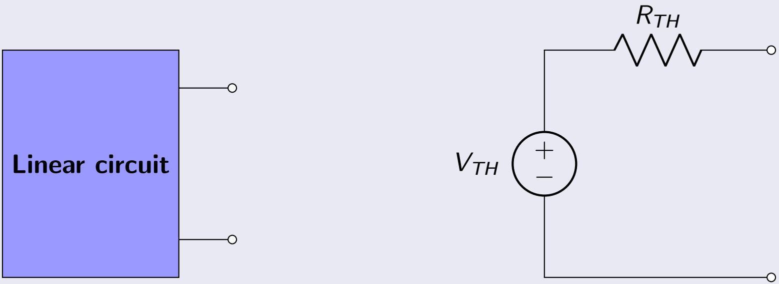
Application

Usually, it allows us to simplify the analysis of a circuit when only a few values need to be determined

for example, previous example. . .

Thévenin equivalent circuit

Equivalence



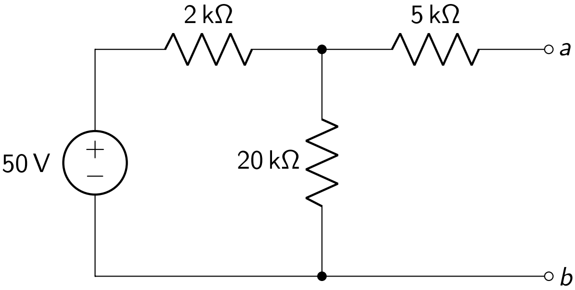
How to determine the Thv́enin equivalence?

Several methods:

1. Evaluate the **open-circuit voltage**
2. Evaluate the **short-circuit current**
3. Evaluate the **resistor value** by turning off every independent source

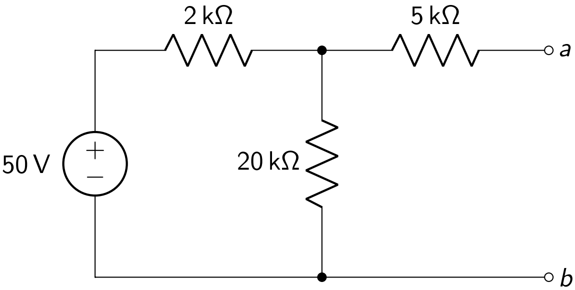
Thévenin equivalent circuit

Find the Thv́enin equivalence (terminal *ab*)



Thévenin equivalent circuit

1. Evaluate the **open-circuit voltage**

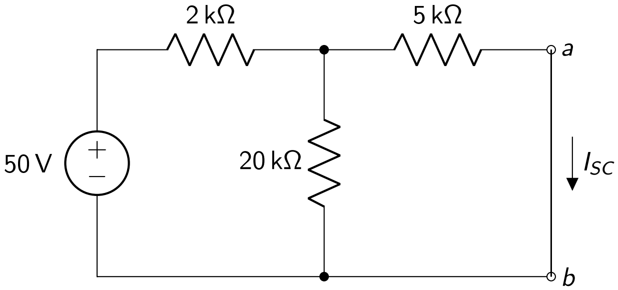


(no current in 5 kΩ resistor)

$$V_{OC} = \frac{20}{22} 50\,\mathrm{V} = 45.45\,\mathrm{V}$$

Thévenin equivalent circuit

2. Evaluate the **short-circuit current**

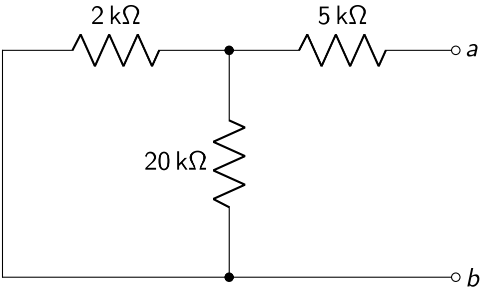


(current $I_{SC}/4$ in the 20 kΩ resistor)

$$50\,\text{V} = \left(\frac{5}{4}2\,\text{k}\Omega + 5\,\text{k}\Omega\right)I_{SC} \quad \Longrightarrow \quad I_{SC} = 6.67\,\text{mA}$$

Thévenin equivalent circuit

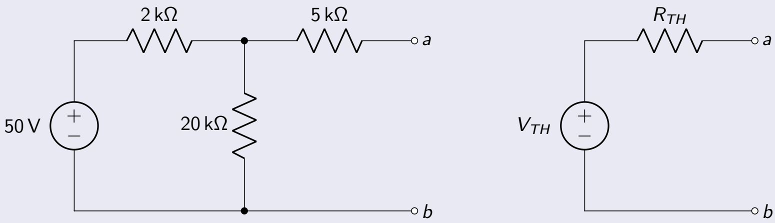
3. Evaluate the **resistor value** by turning off every independent source



$$R_{eq} = 5\,\mathrm{k\Omega} + \frac{2\,\mathrm{k\Omega} \cdot 20\,\mathrm{k\Omega}}{22\,\mathrm{k\Omega}} = 6.82\,\mathrm{k\Omega}$$

Thévenin equivalent circuit

Equivalence



Values

Actually, only 2 experiments are necessary:

- $V_{TH} = V_{OC} = 45.45\,\mathrm{V}$
- $R_{TH} = R_{eq} = 6.82\,\mathrm{k\Omega}$
- $V_{TH} = R_{TH} \cdot I_{SC}$

Thévenin equivalent circuit

Presence of dependent sources

It seems that determining the **open-circuit voltage** and the **equivalent resistance** (when independent sources are turned off) is the most straightforward technique to find Thévenin equivalence

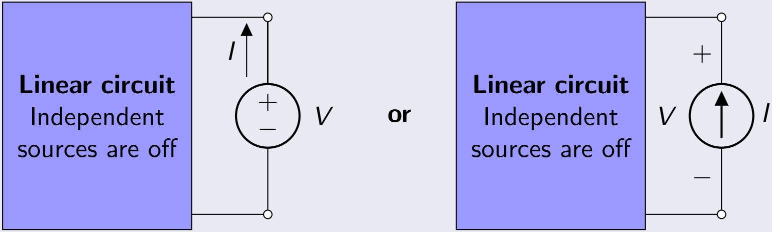
But, what happens when we have dependent sources?

Answer: connect a source to the terminals

- connect a voltage source (or a current source) to the terminals
- the source value can be **any value**
- $R_{eq} = \dfrac{V}{I}$

Thévenin equivalent circuit

Determining R_{eq} with dependent sources

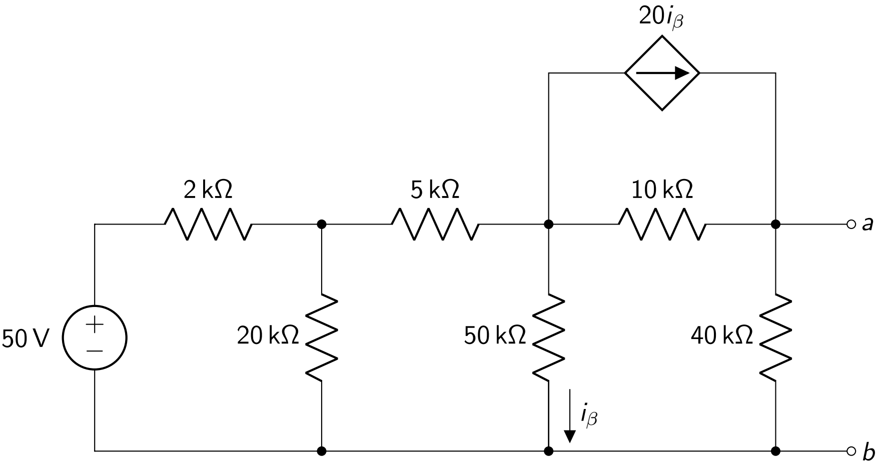


R_{eq} with dependent sources

Beware!
With dependent sources, it is possible to find **negative values** for R_{eq}!!!

Thévenin equivalent circuit

Find the Thévenin equivalence (terminal *ab*)



Norton equivalent circuit

Theorem

A **two-terminal linear circuit**, constituted of sources and resistors, is equivalent to a **current source in parallel with a resistor**

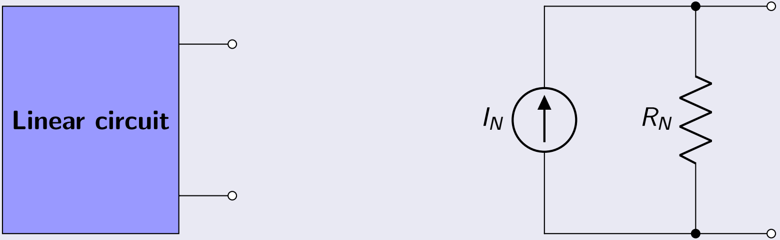
Application

Usually, it allows us to simplify the analysis of a circuit when only a few values need to be determined

for example, previous example. . .

Norton equivalent circuit

Equivalence



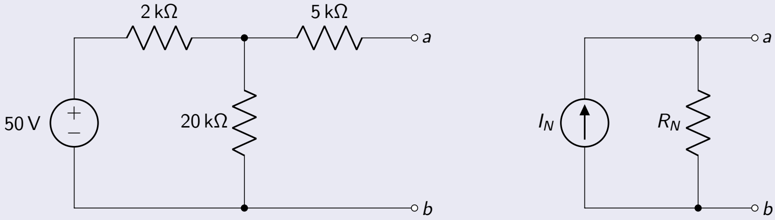
How to determine the Norton equivalence?

Several methods:

1. Evaluate the **open-circuit voltage**
2. Evaluate the **short-circuit current**
3. Evaluate the **resistor value** by turning off every independent source

Norton equivalent circuit

Equivalence



Values

Actually, only 2 experiments are necessary:

- $I_N = I_{SC} = 6.67\,\text{mA}$
- $R_N = R_{eq} = 6.82\,\text{k}\Omega$
- $I_N = \dfrac{V_{OC}}{R_N}$

Norton equivalent circuit

Find the Norton equivalence (terminal *ab*)