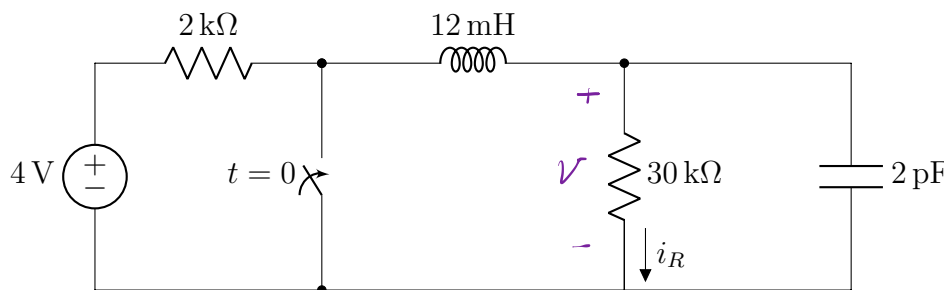


Exercises 06

- RLC circuits

Exercise 1 - Circuit



$$= CR(C_1 s_1 e^{s_1 t} + C_2 s_2 e^{s_2 t}) \Big|_{t=0} \Rightarrow$$

$$\Rightarrow C_1 = 161.3 \mu A, C_2 = -36.34 \mu A$$

$$i_R(t) = 161.3 \mu A e^{s_1 t} - 36.34 \mu A e^{s_2 t}$$

Determine i_R .

Parallel RLC

$$R = 30 k\Omega, L = 12 mH, C = 2 pF$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 6.455 \times 10^6 \text{ rad/s}$$

$$\xi = \frac{1}{2R} \sqrt{\frac{L}{C}} = 1.29 > 1$$

\Rightarrow overdamped response.

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\omega_0 \xi \pm \omega_0 \sqrt{\xi^2 - 1}$$

$$v_R = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$v_R(0^-) = \frac{1}{8} \text{ mA}$$

$$v_R(0^+) = 4V \cdot \frac{30}{30} = 3.75V$$

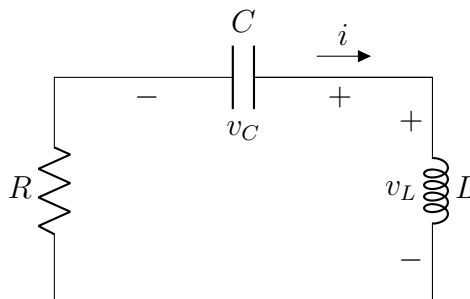
$$\Rightarrow C_1 + C_2 = \frac{1}{8} \text{ mA}$$

$$v_C(0^-) = 0, v_C(0^+) = 0$$

$$v_C(0^+) = C \cdot \frac{dv_C(t)}{dt}$$

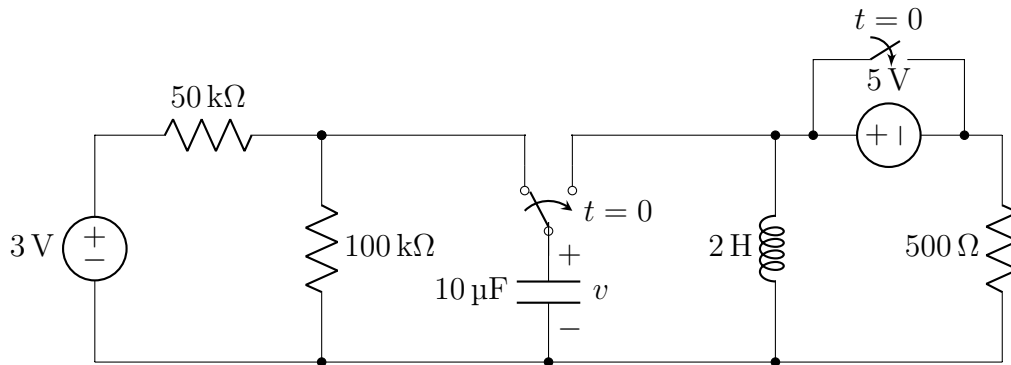
$$= C \cdot \frac{dV(t)}{dt} = C \cdot \frac{R \cdot di_R(t)}{dt}$$

Exercise 2 - Series RCL circuit



Determine overdamped, critically damped, underdamped response.

Exercise 3 - Circuit



- (a) Determine $\frac{dv}{dt}$ at $t = 0^+$;
 (b) Determine v at $t = 1\text{ms}$;
 (c) Determine t_0 , the first value of t greater than zero at which $v = 0$.

a). $v(0^+) = v(0^-) = 3V \cdot \frac{100k\Omega}{100k\Omega + 50k\Omega} = 2V$
 $i_L(0^+) = i_L(0^-) = \frac{5V}{500\Omega} = 10\text{mA}$
 $i_R = \frac{2V}{500\Omega} = 4\text{mA}$

$$\frac{dv}{dt}\bigg|_{t=0^+} C = -i_L - i_R = -14\text{mA}$$

$$\frac{dv}{dt}\bigg|_{t=0^+} = \frac{-14\text{mA}}{10\mu\text{F}} = -1.4\text{V/s}$$

(b) $R = 500\Omega$

$L = 2\text{H}$

$C = 10\mu\text{F}$

$\omega_0 = \frac{1}{\sqrt{LC}} = 223.6\text{ rad/s}$

$\xi = \frac{1}{2R} \sqrt{\frac{L}{C}} = 0.447 < 1$

Underdamped:

$v(t) = e^{-\xi\omega_0 t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$\omega_d = \omega_0 \sqrt{1 - \xi^2}$

$v(t=0^+) = 2V = C$

$i_L = C \frac{dv}{dt}$

$\frac{dv}{dt} = -\omega_d B_1 + \omega_d B_2$

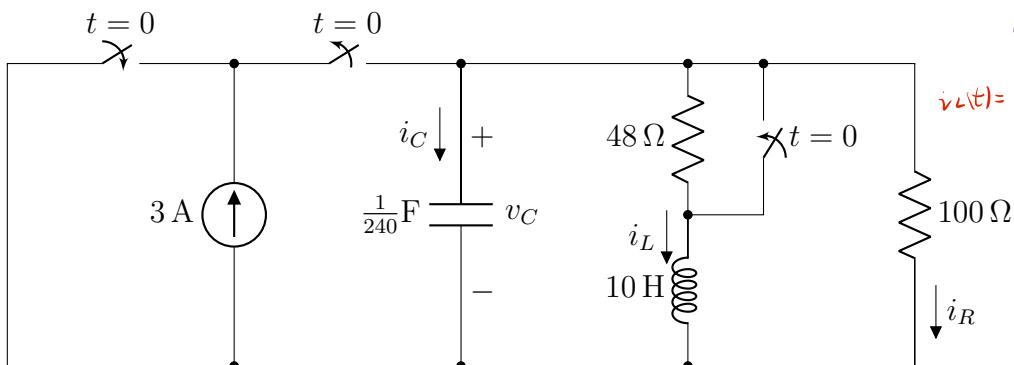
$B_1 = 2V$
 $B_2 = -6V$
 $V = 0.693V$

(c) $B_1 \cos \omega_d t + B_2 \sin \omega_d t = 0$
 $\tan \omega_d t = \frac{B_1}{B_2} = -\frac{1}{3}$

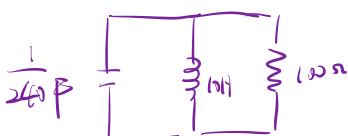
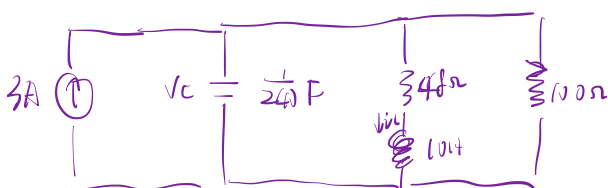
$\omega_d t = \arctan(-\frac{1}{3}) = -\frac{1}{3} \text{ rad}$

$t = 1.69\text{ms}$

Exercise 4 - Circuit



Determine $i_L(t)$.



$i_L(0^+) = 3A \cdot \frac{100\Omega}{100\Omega + 48\Omega} = 2.027A$

$\omega_0 = \frac{1}{\sqrt{LC}} = 4899\text{ rad/s}$

$\xi = \frac{1}{2R} \sqrt{\frac{L}{C}} = 0.245 < 1$

underdamped

$i_C = e^{-\xi\omega_0 t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$\omega_d = \omega_0 \sqrt{1 - \xi^2} = 475\text{ rad/s}$

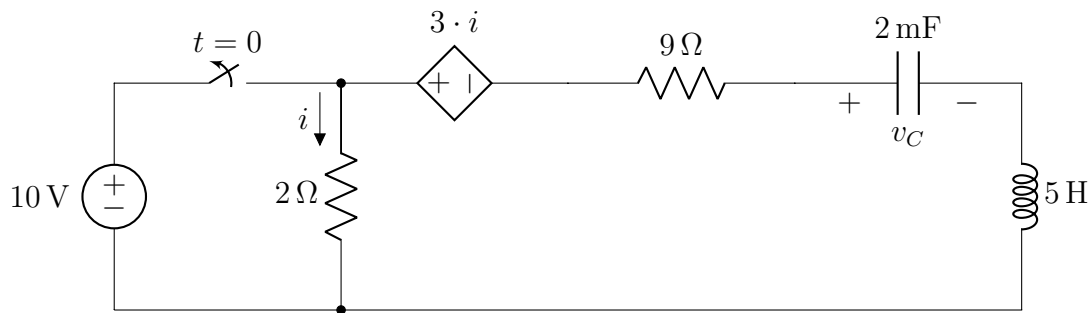
$B_1 = 97.286V$

$v_C(0^+) = v_C(0^-) = v_C$

$i_C(t) = e^{-\xi\omega_0 t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t)$

$\Rightarrow C, B_2$

Exercise 5 - Circuit

Determine $v_C(t)$.