15

MECHANICAL WAVES

VP15.3.1. **IDENTIFY:** We are dealing with general characteristics of waves.
SET UP: We know that $f = 1/T$, $v = f\lambda$, $\omega = 2\pi f$, and $k = 2\pi/\lambda$.

EXECUTE: **(a)** $f = 1/T = 1/(5.10\text{ s}) = 0.196$ Hz.
(b) $v = f\lambda = (0.196\text{ Hz})(30.5\text{ m}) = 5.98$ m/s.

(c) $\omega = 2\pi f = 2\pi(0.196\text{ Hz}) = 1.23$ rad/s.

(d) $k = 2\pi/\lambda = 2\pi/(0.206\text{ m}) = 0.206\text{ m}^{-1}$.

EVALUATE: Caution! The wave number k is *not* the number of waves. It is simply defined as $k = 2\pi/\lambda$.

VP15.3.2. **IDENTIFY:** This problem involves the characteristics of a sound wave on Mars.
SET UP: $T = 1/f$, $\omega = 2\pi f$, and $v = f\lambda$.

EXECUTE: **(a)** $T = 1/f = 1/(125\text{ Hz}) = 8.00\times 10^{-3}$ s.
Solve $v = f\lambda$ for λ: $\lambda = v/f = (245\text{ m/s})/(125\text{ Hz}) = 1.96$ m.

(b) Solve $v = f\lambda$ for f: $f = v/\lambda = (245\text{ m/s})/(3.00\text{ m}) = 81.7$ Hz.

$\omega = 2\pi f = 2\pi(81.7\text{ Hz}) = 513$ rad/s.

EVALUATE: On Earth the wavelength would be almost twice as long as on Mars for the same frequency because the speed of sound is about twice as great as on Mars.

VP15.3.3. **IDENTIFY:** We are dealing with a traveling wave on a string.

SET UP: $y(x,t) = A\cos(kx - \omega t)$, $\omega = 2\pi f$, $v = f\lambda$, $k = 2\pi/\lambda$, and $v = \sqrt{\dfrac{F}{\mu}}$. Call the $+x$-axis the direction in which the wave is traveling.

EXECUTE: **(a)** $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{810\text{ N}}{0.0650\text{ kg/m}}} = 112$ m/s.

(b) Solve $v = f\lambda$ for λ: $\lambda = v/f = (112\text{ m/s})/(25.0\text{ Hz}) = 4.47$ m.

(c) Use $y(x,t) = A\cos(kx - \omega t)$ with $x = 2.50$ m, $k = 2\pi/\lambda = 2\pi/(4.47\text{ m}) = 1.41\text{ m}^{-1}$, $A = 5.00$ mm, and $\omega = 2\pi f = 2\pi(25.0\text{ Hz}) = 157$ rad/s. Using these numbers gives

$y(2.50\text{ m},t) = (5.00\text{ mm})\cos[3.52 - (157\text{ rad/s})t]$.

EVALUATE: This wave is traveling in the $+x$ direction. The equation for reflected waves having the same amplitude would be $y(2.50\text{ m},t) = (5.00\text{ mm})\cos[3.52 + (157\text{ rad/s})t]$.

VP15.3.4. **IDENTIFY:** We are dealing with a traveling wave on a string.

SET UP: $v = f\lambda$, $v = \sqrt{\dfrac{F}{\mu}}$. The 0.400 m is the distance from a crest to the adjacent trough, which is one-half of a complete wave. Therefore $\lambda = 2(0.400\text{ m}) = 0.800$ m.

EXECUTE: **(a)** $v = f\lambda$ = (45.0 Hz)(0.800 m) = 36.0 m/s.

(b) Solve $v = \sqrt{\dfrac{F}{\mu}}$ for μ: $v = \sqrt{\dfrac{F}{\mu}}$ = 0.193 kg/m.

EVALUATE: A linear mass density of 0.193 kg/m is a bit large but not unreasonable for a string.

VP15.5.1. **IDENTIFY:** We are dealing with a traveling wave on a rope and the average power it transfers.

SET UP: $P_{\text{av}} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$, $\omega = 2\pi f$, and $f = 1/T$. The target variable is the average power delivered by the wave.

EXECUTE: **(a)** $\omega = 2\pi f$ = $2\pi/T = 2\pi/(0.575\text{ s}) = 10.9$ rad/s.

(b) $P_{\text{av}} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \dfrac{1}{2}\sqrt{\dfrac{m}{L}F}\,\omega^2 A^2 = \dfrac{1}{2}\sqrt{\left(\dfrac{2.50\text{ kg}}{50.0\text{ m}}\right)(600\text{ N})}\ (10.9\text{ rad/s})^2(0.0300\text{ m})^2 = 0.294\text{ W}.$

EVALUATE: Note that the power depends on the *square* of the frequency and the amplitude. This power is much less than that of an ordinary 60 W light bulb.

VP15.5.2. **IDENTIFY:** We are dealing with the maximum power carried by a traveling wave on a piano wire.

SET UP: $P_{\text{max}} = \sqrt{\mu F}\,\omega^2 A^2$, $\omega = 2\pi f$. We know all the quantities except the amplitude, and our target variable is the amplitude of the wave.

EXECUTE: Solve $P_{\text{max}} = \sqrt{\mu F}\,\omega^2 A^2$ for A: $A = \sqrt{\dfrac{P_{\text{max}}}{\omega^2\sqrt{\mu F}}}$. Using P_{max} = 5.20 W, F = 185 N, μ = 5.55×10^{-4} kg/m, and $\omega = 2\pi f$ = 2π(256 Hz) = 512π Hz, we get $A = 2.50\times 10^{-3}$ m = 2.50 mm.

EVALUATE: An amplitude of 2.50 mm on a piano string is large enough to see readily.

VP15.5.3. **IDENTIFY:** We are investigating the power output of a sound speaker.

SET UP: $I = P/A$ where $A = 4\pi r^2$.

EXECUTE: **(a)** $I = P/A$ = (8.00 W)/[4π(2.00 m)^2] = 0.159 W/m^2.

(b) Solve $I = P/(4\pi r^2)$ for r: $r = \sqrt{\dfrac{P}{4\pi I}} = \sqrt{\dfrac{8.00\text{ W}}{4\pi(0.045\text{ W/m}^2)}} = 3.76$ m.

EVALUATE: The distance from the speaker in part (b) is nearly twice as great as in part (a), which is reasonable because the intensity in (b) is considerably less than in (a).

VP15.5.4. **IDENTIFY:** We are dealing with the sonic power received by a frog's "ears."

SET UP: $I = P/4\pi r^2$ and the energy E received by an area A during a time t is $E = IAt$. Our target variable is the amount of energy the frog's membrane receives in one second.

EXECUTE: $I = P/4\pi r^2$ where r is the distance from the source of sound and P is the power the source is emitting. The area A is $A = \pi R^2$, where R is the radius of the membrane. Combining these quantities gives $E = IAt = \left(\dfrac{P}{4\pi r^2}\right)\left(\pi R^2\right)t$. Using $P = 2.00\times 10^{-6}$ W, r = 1.50 m, t = 1.00 s, and $R = 5.00\times 10^{-3}$ m, we get $E = 5.56\times 10^{-12}$ J.

EVALUATE: This is a very small amount of energy, so the frog must have very sensitive "ears"!

VP15.8.1. **IDENTIFY:** We are dealing with a standing wave on a string.

SET UP: The equation for the standing wave is $y(x,t) = A_{\text{SW}} \sin k_n x \cos \omega_n t$, where $\omega = 2\pi f$. The speed of the wave is $v = f\lambda$; it is a constant and directed along the string. The transverse velocity is $v_y(x,t) = \dfrac{\partial y}{\partial t}$; it is different for different points on the string and is directed perpendicular to the string.

EXECUTE: **(a)** The distance of 0.125 m between nodes is one-half a wavelength, so λ = 0.250 m. Therefore $v = f\lambda$ = (256 Hz)(0.250 m) = 64.0 m/s.

(b) $v_y = \dfrac{\partial y}{\partial t} = \dfrac{\partial\left((A_{\text{SW}} \sin k_n x)\sin \omega_n t\right)}{\partial t} = A_{\text{SW}}\omega_n \sin k_n x \cos \omega_n t$. The maximum speed is $A_{\text{SW}}\omega_n$, so $v_{y\text{-max}}$ $= (1.40\times10^{-3}\text{ m})[2\pi(256\text{ Hz})] = 2.25$ m/s.

(c) $a_y = \dfrac{\partial v_y}{\partial t} = -A_{\text{SW}}\omega_n^2 \sin k_n x \sin \omega_n t$. The $= 2L$ maximum acceleration is $A_{\text{SW}}\omega_n^2$, so

$a_{y\text{-max}} = (1.40\times10^{-3}\text{ m})[2\pi(256\text{ Hz})]^2 = 3620\text{ m/s}^2$.

EVALUATE: For a very taut string or wire, the maximum acceleration can be extremely large.

VP15.8.2. **IDENTIFY:** This problem involves a standing wave on the G string of a guitar string.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$, $v = f\lambda$, $\lambda_n = \dfrac{2L}{n}$, and for the fundamental mode $n = 1$ so $\lambda_1 = \dfrac{2L}{1} = 2L$.

EXECUTE: **(a)** We want the wave speed. $v = f_1\lambda_1 = f_1(2L) = (196\text{ Hz})(2)(0.641\text{ m}) = 251$ m/s.

(b) We want the tension in the string, so solve $v = \sqrt{\dfrac{F}{\mu}}$ for F, giving $F = \mu v^2$. Therefore

$F = (2.29\times10^{-3}\text{ kg/m})(251\text{ m/s})^2 = 145$ N.

EVALUATE: This is a very fast wave since the tension is large.

VP15.8.3. **IDENTIFY:** Standing waves form on the cable that is fixed at both ends.

SET UP: $\lambda_n = \dfrac{2L}{n}$, $v = f\lambda$, $v = \sqrt{\dfrac{F}{\mu}}$. Since there are 5 antinodes on the cable, it is vibrating in its 5th harmonic, so $n = 5$. We want to know the wavelength and frequency of the waves and the linear mass density of the cable.

EXECUTE: **(a)** $\lambda_n = \dfrac{2L}{n}$ so $\lambda_5 = \dfrac{2L}{5} = \dfrac{6.00\text{ m}}{5} = 1.20$ m.

(b) Solve $v = f\lambda$ for f, giving $f = \dfrac{v}{\lambda} = \dfrac{96.0\text{ m/s}}{1.20\text{ m}} = 80.0\text{ Hz}$.

(c) Solve $v = \sqrt{\dfrac{F}{\mu}}$ for μ giving $\mu = \dfrac{F}{v^2} = \dfrac{175\text{ N}}{(96.0\text{ m/s})^2} = 1.90\times10^{-2}$ kg/m.

EVALUATE: This density is 19 kg/m, which is not unreasonable for a thin cable.

VP15.8.4. **IDENTIFY:** A string fixed at both ends is vibrating in its second harmonic of a standing wave pattern. This vibration produces a sound wave.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$, $\lambda_n = \dfrac{2L}{n}$, $f_n = nf_1$, $v = f\lambda$. For the second harmonic, $n = 2$. The target variables are the tension in the string and the wavelength of the sound wave the vibrating string produces.

EXECUTE: **(a)** We want the tension in the string. First find the wavelength and use it to find the speed of the wave. Then use $v = \sqrt{\dfrac{F}{\mu}}$ to find the tension. For the string in the second harmonic,

$\lambda_2 = 2L/2 = L = 0.500$ m. The wave speed is $v = f\lambda = (512\text{ Hz})(0.500\text{ m}) = 256$ m/s. Now solve

$v = \sqrt{\dfrac{F}{\mu}}$ for F giving $F = \mu v^2 = (1.17\times10^{-3}\text{ kg/m})(256\text{ m/s})^2 = 76.7$ N.

(b) We want the wavelength of the sound wave this string produces when vibrating in its fundamental harmonic. Each cycle of the string produces a cycle of sound waves, so the frequency of the sound will be the same as the frequency of the string. The string is now vibrating in its *fundamental* frequency, not its second harmonic. Using $f_n = nf_1$, we have $f_2 = 2f_1 = 512$ Hz, so its frequency is $f_1 = (512\text{ Hz})/2 = 256$ Hz. Now solve $v_s = f\lambda$ for λ, where v_s is the speed of sound. This gives $\lambda = \dfrac{v_s}{f} = \dfrac{344\text{ m/s}}{256\text{ Hz}} = 1.34$ m.

EVALUATE: The wavelength of the sound wave is very different from the wavelength of the waves on the string, even though both have the same frequency. This difference is due to the difference in the speeds of the two waves.

15.1. **IDENTIFY:** $v = f\lambda$. $T = 1/f$ is the time for one complete vibration.

SET UP: The frequency of the note one octave higher is 1568 Hz.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{784\text{ Hz}} = 0.439\text{ m}.$ $T = \dfrac{1}{f} = 1.28\text{ ms}.$

(b) $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{1568\text{ Hz}} = 0.219\text{ m}.$

EVALUATE: When f is doubled, λ is halved.

15.2. **IDENTIFY:** $f\lambda = v$.

SET UP: $1.0\text{ mm} = 0.0010\text{ m}$.

EXECUTE: $f = \dfrac{v}{\lambda} = \dfrac{1500\text{ m/s}}{0.0010\text{ m}} = 1.5\times10^6\text{ Hz}.$

EVALUATE: The frequency is much higher than the upper range of human hearing.

15.3. **IDENTIFY:** $v = f\lambda = \lambda/T$.

SET UP: $1.0\text{ h} = 3600\text{ s}$. The crest to crest distance is λ.

EXECUTE: $v = \dfrac{800\times10^3\text{ m}}{3600\text{ s}} = 220\text{ m/s}.$ $v = \dfrac{800\text{ km}}{1.0\text{ h}} = 800\text{ km/h}.$

EVALUATE: Since the wave speed is very high, the wave strikes with very little warning.

15.4. **IDENTIFY:** The fisherman observes the amplitude, wavelength, and period of the waves.
SET UP: The time from the highest displacement to lowest displacement is $T/2$. The distance from highest displacement to lowest displacement is $2A$. The distance between wave crests is λ, and the speed of the waves is $v = f\lambda = \lambda/T$.

EXECUTE: **(a)** $T = 2(2.5\text{ s}) = 5.0\text{ s}.$ $\lambda = 4.8\text{ m}.$ $v = \dfrac{4.8\text{ m}}{5.0\text{ s}} = 0.96\text{ m/s}.$

(b) $A = (0.53\text{ m})/2 = 0.265\text{ m}$ which rounds to 0.27 m.

(c) The amplitude becomes 0.15 m but the wavelength, period and wave speed are unchanged.
EVALUATE: The wavelength, period and wave speed are independent of the amplitude of the wave.

15.5. **IDENTIFY:** We want to relate the wavelength and frequency for various waves.
SET UP: For waves $v = f\lambda$.

EXECUTE: **(a)** $v = 344\text{ m/s}$. For $f = 20{,}000\text{ Hz}$, $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{20{,}000\text{ Hz}} = 1.7\text{ cm}.$ For $f = 20\text{ Hz}$,

$\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{20\text{ Hz}} = 17\text{ m}.$ The range of wavelengths is 1.7 cm to 17 m.

(b) $v = c = 3.00\times10^8\text{ m/s}$. For $\lambda = 700\text{ nm}$, $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{700\times10^{-9}\text{ m}} = 4.3\times10^{14}\text{ Hz}.$ For $\lambda = 400\text{ nm}$,

$f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{400\times10^{-9}\text{ m}} = 7.5\times10^{14}\text{ Hz}.$ The range of frequencies for visible light is $4.3\times10^{14}\text{ Hz}$ to $7.5\times10^{14}\text{ Hz}$.

(c) $v = 344\text{ m/s}.$ $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{23\times10^3\text{ Hz}} = 1.5\text{ cm}.$

(d) $v = 1480\text{ m/s}.$ $\lambda = \dfrac{v}{f} = \dfrac{1480\text{ m/s}}{23\times10^3\text{ Hz}} = 6.4\text{ cm}.$

Evaluate: For a given v, a larger f corresponds to smaller λ. For the same f, λ increases when v increases.

15.6. **Identify:** The string is vibrating in SHM. The maximum force occurs when the acceleration of the bead is a maximum.

Set Up: The equation for the wave is $y(x,t) = A\cos(kx - \omega t)$. The maximum force on the bead occurs when the transverse acceleration is a maximum, so use $\sum F_y = ma_y$, where $a_y = \dfrac{\partial^2 y}{\partial t^2}$. The target variable is the maximum force on the bead.

Execute: Find $a_y = \dfrac{\partial^2 y}{\partial t^2}$ for $y(x,t) = A\cos(kx - \omega t)$. $\dfrac{\partial y}{\partial t} = A\omega \sin(kx - \omega t)$, so $a_y = \dfrac{\partial^2 y}{\partial t^2} = -A\omega^2 \cos(kx - \omega t)$. The maximum magnitude acceleration of the bead is $a_{\max} = A\omega^2$, so the maximum force on the bead is $F_{\max} = ma_{\max} = mA\omega^2$. Putting in the numbers gives

$F_{\max} = (0.00400\text{ kg})(8.00\times10^{-3}\text{ m})[2\pi(20.0\text{ Hz})]^2 = 0.505\text{ N}.$

Evaluate: The weight of the bead is (0.00400 kg)(9.80 m/s^2) = 0.0392 N, so it is small enough to be neglected with reasonable accuracy.

15.7. **Identify:** Use $v = f\lambda$ to calculate v. $T = 1/f$ and k is defined by $k = 2\pi/\lambda$. The general form of the wave function is given by $y(x,\, t) = A\cos 2\pi(x/\lambda + t/T)$, which is the equation for the transverse displacement.

Set Up: $v = 8.00\text{ m/s}$, $A = 0.0700\text{ m}$, $\lambda = 0.320\text{ m}$

Execute: **(a)** $v = f\lambda$ so $f = v/\lambda = (8.00\text{ m/s})/(0.320\text{ m}) = 25.0\text{ Hz}$

$T = 1/f = 1/25.0\text{ Hz} = 0.0400\text{ s}$

$k = 2\pi/\lambda = 2\pi\text{ rad}/0.320\text{ m} = 19.6\text{ rad/m}$

(b) For a wave traveling in the $-x$-direction,

$y(x,\, t) = A\cos 2\pi(x/\lambda + t/T)$

At $x = 0$, $y(0,\, t) = A\cos 2\pi(t/T)$, so $y = A$ at $t = 0$. This equation describes the wave specified in the problem.

Substitute in numerical values:

$y(x,\, t) = (0.0700\text{ m})\cos\left[2\pi\left(x/(0.320\text{ m}) + t/(0.0400\text{ s})\right)\right].$

Or, $y(x,\, t) = (0.0700\text{ m})\cos\left[(19.6\text{ m}^{-1})x + (157\text{ rad/s})t\right].$

(c) From part (b), $y = (0.0700\text{ m})\cos[2\pi(x/0.320\text{ m} + t/0.0400\text{ s})]$.

Plug in $x = 0.360\text{ m}$ and $t = 0.150\text{ s}$:

$y = (0.0700\text{ m})\cos[2\pi(0.360\text{ m}/0.320\text{ m} + 0.150\text{ s}/0.0400\text{ s})]$

$y = (0.0700\text{ m})\cos[2\pi(4.875\text{ rad})] = +0.0495\text{ m} = +4.95\text{ cm}$

(d) In part (c) $t = 0.150\text{ s}$.

$y = A$ means $\cos[2\pi(x/\lambda + t/T)] = 1$

$\cos\theta = 1$ for $\theta = 0,\ 2\pi,\ 4\pi, \ldots = n(2\pi)$ or $n = 0,\ 1,\ 2, \ldots$

So $y = A$ when $2\pi(x/\lambda + t/T) = n(2\pi)$ or $x/\lambda + t/T = n$

$t = T(n - x/\lambda) = (0.0400\text{ s})(n - 0.360\text{ m}/0.320\text{ m}) = (0.0400\text{ s})(n - 1.125)$

For $n = 4$, $t = 0.1150\text{ s}$ (before the instant in part (c))

For $n = 5$, $t = 0.1550\text{ s}$ (the first occurrence of $y = A$ after the instant in part (c)). Thus the elapsed time is $0.1550\text{ s} - 0.1500\text{ s} = 0.0050\text{ s}$.

Evaluate: Part (d) says $y = A$ at 0.115 s and next at 0.155 s; the difference between these two times is 0.040 s, which is the period. At $t = 0.150\text{ s}$ the particle at $x = 0.360\text{ m}$ is at $y = 4.95\text{ cm}$ and traveling upward. It takes $T/4 = 0.0100$ s for it to travel from $y = 0$ to $y = A$, so our answer of 0.0050 s is reasonable.

15.8. **IDENTIFY:** Compare $y(x, t)$ given in the problem to the general form $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$. $f = 1/T$ and $v = f\lambda$

SET UP: The comparison gives $A = 6.50$ mm, $\lambda = 28.0$ cm and $T = 0.0360$ s.

EXECUTE: **(a)** 6.50 mm

(b) 28.0 cm

(c) $f = \frac{1}{0.0360\text{ s}} = 27.8\text{ Hz}$

(d) $v = (0.280\text{ m})(27.8\text{ Hz}) = 7.78\text{ m/s}$

(e) Since there is a minus sign in front of the t/T term, the wave is traveling in the $+x$-direction.

EVALUATE: The speed of propagation does not depend on the amplitude of the wave.

15.9. **IDENTIFY:** Evaluate the partial derivatives and see if the wave equation $\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$ is satisfied.

SET UP: $\frac{\partial}{\partial x}\cos(kx + \omega t) = -k\sin(kx + \omega t)$. $\frac{\partial}{\partial t}\cos(kx + \omega t) = -\omega\sin(kx + \omega t)$. $\frac{\partial}{\partial x}\sin(kx + \omega t) = k\cos(kx + \omega t)$. $\frac{\partial}{\partial t}\sin(kx + \omega t) = \omega\cos(kx + \omega t)$.

EXECUTE: **(a)** $\frac{\partial^2 y}{\partial x^2} = -Ak^2\cos(kx + \omega t)$. $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\cos(kx + \omega t)$. The wave equation is satisfied, if $v = \omega/k$.

(b) $\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx + \omega t)$. $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx + \omega t)$. The wave equation is satisfied, if $v = \omega/k$.

(c) $\frac{\partial y}{\partial x} = -kA\sin(kx)$. $\frac{\partial^2 y}{\partial x^2} = -k^2 A\cos(kx)$. $\frac{\partial y}{\partial t} = -\omega A\sin(\omega t)$. $\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(\omega t)$. The wave equation is not satisfied.

(d) $v_y = \frac{\partial y}{\partial t} = \omega A\cos(kx + \omega t)$. $a_y = \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx + \omega t)$

EVALUATE: The functions $\cos(kx + \omega t)$ and $\sin(kx + \omega t)$ differ only in phase.

15.10. **IDENTIFY:** The general form of the wave function for a wave traveling in the $-x$-direction is given by $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} + \frac{t}{T}\right)$. The time for one complete cycle to pass a point is the period T and the number that pass per second is the frequency f. The speed of a crest is the wave speed v and the maximum speed of a particle in the medium is $v_{\text{max}} = \omega A$.

SET UP: Comparison to $y(x,t) = A\cos(kx + \omega t)$ gives $A = 2.75$ cm, $k = 0.410$ rad/cm and $\omega = 6.20$ rad/s.

EXECUTE: **(a)** $T = \frac{2\pi\text{ rad}}{\omega} = \frac{2\pi\text{ rad}}{6.20\text{ rad/s}} = 1.0134\text{ s}$ which rounds to 1.01 s. In one cycle a wave crest travels a distance $\lambda = \frac{2\pi\text{ rad}}{k} = \frac{2\pi\text{ rad}}{0.410\text{ rad/cm}} = 15.325\text{ cm} = 0.153\text{ m}$.

(b) $k = 0.410$ rad/cm. $f = 1/T = 0.9868\text{ Hz} = 0.987$ waves/second.

(c) $v = f\lambda = (0.9868\text{ Hz})(0.15325\text{ m}) = 0.151\text{ m/s}$.

$v_{\text{max}} = \omega A = (6.20\text{ rad/s})(2.75\text{ cm}) = 17.1\text{ cm/s} = 0.171\text{ m/s}$.

EVALUATE: The transverse velocity of the particles in the medium (water) is not the same as the velocity of the wave.

15.11. **IDENTIFY** and **SET UP:** Read A and T from the graph. Apply $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$ to determine λ and then use $v = f\lambda$ to calculate v.

EXECUTE: **(a)** The maximum y is 4 mm (read from graph).
(b) For either x the time for one full cycle is 0.040 s; this is the period.
(c) Since $y = 0$ for $x = 0$ and $t = 0$ and since the wave is traveling in the $+x$-direction then $y(x,\, t) = A\sin[2\pi(t/T - x/\lambda)]$. (The phase is different from the wave described by $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$; for that wave $y = A$ for $x = 0,\ t = 0$.) From the graph, if the wave is traveling in the $+x$-direction and if $x = 0$ and $x = 0.090$ m are within one wavelength the peak at $t = 0.01$ s for $x = 0$ moves so that it occurs at $t = 0.035$ s (read from graph so is approximate) for $x = 0.090$ m. The peak for $x = 0$ is the first peak past $t = 0$ so corresponds to the first maximum in $\sin[2\pi(t/T - x/\lambda)]$ and hence occurs at $2\pi(t\,/\,T - x/\lambda) = \pi/2$. If this same peak moves to $t_1 = 0.035$ s at $x_1 = 0.090$ m, then

$2\pi(t/T - x/\lambda) = \pi/2.$

Solve for λ: $t_1/T - x_1/\lambda = 1/4$

$x_1/\lambda = t_1/T - 1/4 = 0.035\text{ s}/0.040\text{ s} - 0.25 = 0.625$

$\lambda = x_1/0.625 = 0.090\text{ m}/0.625 = 0.14\text{ m}.$

Then $v = f\lambda = \lambda/T = 0.14\text{ m}/0.040\text{ s} = 3.5\text{ m/s}.$

(d) If the wave is traveling in the $-x$-direction, then $y(x,\, t) = A\sin(2\pi(t/T + x/\lambda))$ and the peak at $t = 0.050$ s for $x = 0$ corresponds to the peak at $t_1 = 0.035$ s for $x_1 = 0.090$ m. This peak at $x = 0$ is the second peak past the origin so corresponds to $2\pi(t/T + x/\lambda) = 5\pi/2$. If this same peak moves to $t_1 = 0.035$ s for $x_1 = 0.090$ m, then $2\pi(t_1/T + x_1/\lambda) = 5\pi/2.$

$t_1/T + x_1/\lambda = 5/4$

$x_1/\lambda = 5/4 - t_1/T = 5/4 - 0.035\text{ s}/0.040\text{ s} = 0.375$

$\lambda = x_1/0.375 = 0.090\text{ m}/0.375 = 0.24\text{ m}.$

Then $v = f\lambda = \lambda/T = 0.24\text{ m}/0.040\text{ s} = 6.0\text{ m/s}.$

EVALUATE: **(e)** No. Wouldn't know which point in the wave at $x = 0$ moved to which point at $x = 0.090$ m.

15.12. **IDENTIFY:** $v_y = \frac{\partial y}{\partial t}$. $v = f\lambda = \lambda/T.$

SET UP: $\frac{\partial}{\partial t} A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right) = +A\left(\frac{2\pi v}{\lambda}\right)\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$

EXECUTE: **(a)** $A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) = +A\cos\frac{2\pi}{\lambda}\left(x - \frac{\lambda}{T}t\right) = +A\cos\frac{2\pi}{\lambda}(x - vt)$ where $\frac{\lambda}{T} = \lambda f = v$ has been used.

(b) $v_y = \frac{\partial y}{\partial t} = \frac{2\pi v}{\lambda} A\sin\frac{2\pi}{\lambda}(x - vt).$

(c) The speed is the greatest when the sine is 1, and that speed is $2\pi vA/\lambda$. This will be equal to v if $A = \lambda/2\pi$, less than v if $A < \lambda/2\pi$ and greater than v if $A > \lambda/2\pi$.

EVALUATE: The propagation speed applies to all points on the string. The transverse speed of a particle of the string depends on both x and t.

15.13. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: For λ and x in cm, v in cm/s and t in s, the argument of the cosine is in radians.

EXECUTE: **(a)** $t = 0$:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.300	0.212	0	–0.212	–0.300	–0.212	0	0.212	0.300

The graph is shown in Figure 15.13a.

(b) (i) $t = 0.400$ s:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	–0.221	–0.0131	0.203	0.300	0.221	0.0131	–0.203	–0.300	–0.221

The graph is shown in Figure 15.13b.

(ii) $t = 0.800$ s:

x(cm)	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
y(cm)	0.0262	–0.193	–0.300	–0.230	–0.0262	0.193	0.300	0.230	0.0262

The graph is shown in Figure 15.13c.

(iii) The graphs show that the wave is traveling in the $+x$-direction.

EVALUATE: We know that $y(x,t) = A\cos 2\pi f\left(\dfrac{x}{v} - t\right)$ is for a wave traveling in the $+x$-direction, and $y(x,t)$ is derived from this. This is consistent with the direction of propagation we deduced from our graph.

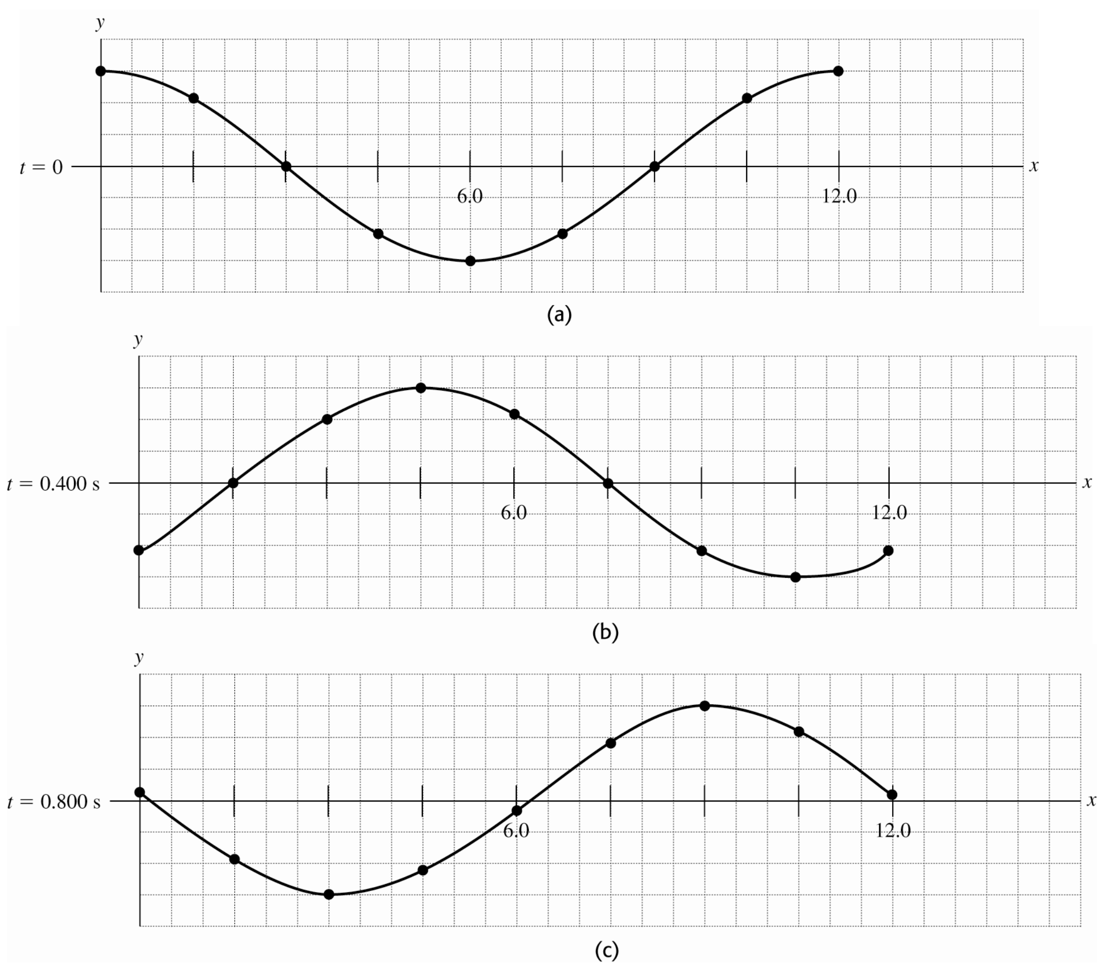


Figure 15.13

15.14. IDENTIFY: We are dealing the tension in a taut vibrating string.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$. We want to know what change in the tension is required to double the frequency of the fundamental mode.

EXECUTE: Combining $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$ gives $f\lambda=\sqrt{\dfrac{F}{\mu}}$. Solving for F gives $F=\mu\lambda^2 f^2$. From this result we see that to double the frequency, F we would have to increase by a factor of 4.

EVALUATE: This is *not* a good idea because such a large increase in tension could damage the instrument or cause the spring to break.

15.15. IDENTIFY and SET UP: Use $v=\sqrt{F/\mu}$ to calculate the wave speed. Then use $v=f\lambda$ to calculate the wavelength.

EXECUTE: **(a)** The tension F in the rope is the weight of the hanging mass:

$F=mg=(1.50\text{ kg})(9.80\text{ m/s}^2)=14.7\text{ N}$

$v=\sqrt{F/\mu}=\sqrt{14.7\text{ N}/(0.0480\text{ kg/m})}=17.5\text{ m/s}.$

(b) $v=f\lambda$ so $\lambda=v/f=(17.5\text{ m/s})/120\text{ Hz}=0.146\text{ m}.$

(c) EVALUATE: $v=\sqrt{F/\mu}$, where $F=mg$. Doubling m increases v by a factor of $\sqrt{2}$. $\lambda=v/f$. f remains 120 Hz and v increases by a factor of $\sqrt{2}$, so λ increases by a factor of $\sqrt{2}$.

15.16. IDENTIFY: The frequency and wavelength determine the wave speed and the wave speed depends on the tension.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$. $\mu=m/L$. $v=f\lambda$.

EXECUTE: $F=\mu v^2=\mu(f\lambda)^2=\dfrac{0.120\text{ kg}}{2.50\text{ m}}([40.0\text{ Hz}][0.750\text{ m}])^2=43.2\text{ N}$

EVALUATE: If the frequency is held fixed, increasing the tension will increase the wavelength.

15.17. IDENTIFY: The speed of the wave depends on the tension in the wire and its mass density. The target variable is the mass of the wire of known length.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$ and $\mu=m/L$.

EXECUTE: First find the speed of the wave: $v=\dfrac{3.80\text{ m}}{0.0492\text{ s}}=77.24\text{ m/s}$. $v=\sqrt{\dfrac{F}{\mu}}$. $\mu=\dfrac{F}{v^2}=$

$\dfrac{(54.0\text{ kg})(9.8\text{ m/s}^2)}{(77.24\text{ m/s})^2}=0.08870\text{ kg/m}$. The mass of the wire is

$m=\mu L=(0.08870\text{ kg/m})(3.80\text{ m})=0.337\text{ kg}.$

EVALUATE: This mass is 337 g, which is a bit large for a wire 3.80 m long. It must be fairly thick.

15.18. IDENTIFY: For transverse waves on a string, $v=\sqrt{F/\mu}$. The general form of the equation for waves traveling in the $+x$-direction is $y(x,t)=A\cos(kx-\omega t)$. For waves traveling in the $-x$-direction it is $y(x,t)=A\cos(kx+\omega t)$. $v=\omega/k$.

SET UP: Comparison to the general equation gives $A=8.50\text{ mm}$, $k=172\text{ rad/m}$ and $\omega=4830\text{ rad/s}$. The string has mass 0.00128 kg and $\mu=m/L=0.000850\text{ kg/m}$.

EXECUTE: **(a)** $v=\dfrac{\omega}{k}=\dfrac{4830\text{ rad/s}}{172\text{ rad/m}}=28.08\text{ m/s}$. $t=\dfrac{d}{v}=\dfrac{1.50\text{ m}}{28.08\text{ m/s}}=0.0534\text{ s}=53.4\text{ ms}.$

(b) $W=F=\mu v^2=(0.000850\text{ kg/m})(28.08\text{ m/s})^2=0.670\text{ N}.$

(c) $\lambda = \dfrac{2\pi \text{ rad}}{k} = \dfrac{2\pi \text{ rad}}{172 \text{ rad/m}} = 0.0365 \text{ m}.$ The number of wavelengths along the length of the string is $\dfrac{1.50 \text{ m}}{0.0365 \text{ m}} = 41.1.$

(d) For a wave traveling in the opposite direction,
$y(x,t) = (8.50 \text{ mm})\cos([172 \text{ rad/m}]x + [4830 \text{ rad/s}]t).$

EVALUATE: We have assumed that the tension in the string is constant and equal to W. This is reasonable since $W \gg 0.0125$ N, so the weight of the string has a negligible effect on the tension.

15.19. **IDENTIFY:** For transverse waves on a string, $v = \sqrt{F/\mu}$. $v = f\lambda$.

SET UP: The wire has $\mu = m/L = (0.0165 \text{ kg})/(0.750 \text{ m}) = 0.0220 \text{ kg/m}$.

EXECUTE: **(a)** $v = f\lambda = (625 \text{ Hz})(3.33\times10^{-2} \text{ m}) = 20.813 \text{ m/s}$. The tension is

$F = \mu v^2 = (0.0220 \text{ kg/m})(20.813 \text{ m/s})^2 = 9.53 \text{ N}.$

(b) $v = 20.8$ m/s

EVALUATE: If λ is kept fixed, the wave speed and the frequency increase when the tension is increased.

15.20. **IDENTIFY:** The rope is heavy, so the tension at any point in it must support not only the weight attached but the weight of the rope below that point. Assume that the rope is uniform.

SET UP: $v = \sqrt{F/\mu}$ and $\mu = m/L$ = [(29.4 N)/(9.80 m/s^2)]/(6.00 m) = 0.500 kg/m.

EXECUTE: **(a)** At the bottom, the rope supports only the 0.500-kg object, so
$T = mg$ = (0.500 kg)(9.80 m/s^2) = 4.90 N. Now use $v = \sqrt{F/\mu}$ find v.
v = [(4.90 N)/(0.500 kg/m)]$^{1/2}$ = 3.13 m/s.

(b) At the middle, the tension supports the 0.500-kg object plus half the weight of the rope, so
T = (29.4 N)/2 + 4.90 N = 19.6 N. Therefore v = [(19.6 N)/(0.500 kg/m)]$^{1/2}$ = 6.26 m/s.

(c) At the top, the tension supports the entire rope plus the object, so T = 29.4 N + 4.90 N = 34.3 N.
Therefore v = [(34.3 N)/(0.500 kg/m)]$^{1/2}$ = 8.28 m/s.

(d) T_{middle}= 19.6 N. $T_{av} = (T_{top} + T_{bot})/2$ = (34.3 N + 4.90 N)/2 = 19.6 N, which is equal to T_{middle}.
v_{middle} = 6.26 m/s. v_{av} = (8.28 m/s + 3.13 m/s)/2 = 5.71 m/s, which is not equal to v_{middle}.

EVALUATE: The average speed is not equal to the speed at the middle because the speed depends on the square root of the tension. So even though the tension at the middle is the average of the top and bottom tensions, that is not true of the wave speed.

15.21. **IDENTIFY:** We are dealing with the power carried by a wave on a vibrating string.

SET UP: Estimates: The amplitude is the length of an arm ≈ 65 cm = 0.65 m. The maximum tension is T_{max} ≈ 25 lb ≈ 110 N. The time to complete each pulse: do about 2 per second, so the time per pulse is about 0.50 s. We want to know about the power we supply. Use $\omega = 2\pi / T$ and $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$.

EXECUTE: **(a)** $\omega = 2\pi / T$ = 2π/(0.50 s) = 4π rad/s. The average power is given by $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$= \frac{1}{2}\sqrt{(0.500 \text{ kg/m})(110 \text{ N})}(4\pi \text{ rad/s})^2(0.65 \text{ m})^2 = 250 \text{ W}.$

(b) $P_{av} \propto (A\omega)^2$, so if we halve A and double ω, the effects cancel out so there is no change in the average power.

EVALUATE: With this power you could keep around four 60-W light bulbs burning, but it would be difficult to keep it up for very long!

15.22. **IDENTIFY:** Apply $P_{av} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. d

SET UP: $\omega = 2\pi f$. $\mu = m/L$.

EXECUTE: **(a)** Using $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$, we get $P_{\text{av}} = \frac{1}{2}\sqrt{\left(\frac{3.00\times10^{-3}\text{ kg}}{0.80\text{ m}}\right)(25.0\text{ N})}\left[2\pi(120.0\text{ Hz})\right]^2$

$(1.6\times10^{-3}\text{ m})^2 = 0.223\text{ W}$ or 0.22 W to two significant figures.

(b) P_{av} is proportional to A^2, so halving the amplitude quarters the average power, to 0.056 W.

EVALUATE: The average power is also proportional to the square of the frequency.

15.23. **IDENTIFY:** The average power carried by the wave depends on the mass density of the wire and the tension in it, as well as on the square of both the frequency and amplitude of the wave (the target variable).

SET UP: $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$, $v = \sqrt{\frac{F}{\mu}}$.

EXECUTE: Solving $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$ for A gives $A = \left(\frac{2P_{\text{av}}}{\omega^2\sqrt{\mu F}}\right)^{1/2}$. $P_{\text{av}} = 0.365\text{ W}$. $\omega = 2\pi f =$

$2\pi(69.0\text{ Hz}) = 433.5\text{ rad/s}$. The tension is $F = 94.0\text{ N}$ and $v = \sqrt{\frac{F}{\mu}}$ so $\mu = \frac{F}{v^2} = \frac{94.0\text{ N}}{(406\text{ m/s})^2} =$

$5.703\times10^{-4}\text{ kg/m}$. $A = \left(\frac{2(0.365\text{ W})}{(433.5\text{ rad/s})^2\sqrt{(5.703\times10^{-4}\text{ kg/m})(94.0\text{ N})}}\right)^{1/2} = 4.10\times10^{-3}\text{ m} = 4.10\text{ mm}$.

EVALUATE: Vibrations of strings and wires normally have small amplitudes, which this wave does.

15.24. **IDENTIFY:** Apply $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$.

SET UP: $I_1 = 0.11\text{ W/m}^2$. $r_1 = 7.5\text{ m}$. Set $I_2 = 1.0\text{ W/m}^2$ and solve for r_2.

EXECUTE: $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (7.5\text{ m})\sqrt{\frac{0.11\text{ W/m}^2}{1.0\text{ W/m}^2}} = 2.5\text{ m}$, so it is possible to move

$r_1 - r_2 = 7.5\text{ m} - 2.5\text{ m} = 5.0\text{ m}$ closer to the source.

EVALUATE: I increases as the distance r of the observer from the source decreases.

15.25. **IDENTIFY:** For a point source, $I = \frac{P}{4\pi r^2}$ and $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$.

SET UP: $1\,\mu\text{W} = 10^{-6}\text{ W}$

EXECUTE: **(a)** $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (30.0\text{ m})\sqrt{\frac{10.0\text{ W/m}^2}{1\times10^{-6}\text{ W/m}^2}} = 95\text{ km}$

(b) $\frac{I_2}{I_3} = \frac{r_3^2}{r_2^2}$, with $I_2 = 1.0\,\mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\frac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\,\mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\text{ W/m}^2)(4\pi)(30.0\text{ m})^2 = 1.1\times10^5\text{ W}$

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.

15.26. **IDENTIFY:** The tension and mass per unit length of the rope determine the wave speed. Compare $y(x, t)$ given in the problem to the general form given in $y(x,t) = A\cos 2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)$. $v = \omega/k$. The average power is given by $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$.

SET UP: Comparison with $y(x,t) = A\cos(kx - \omega t)$ gives $A = 2.30$ mm, $k = 6.98$ rad/m and $\omega = 742$ rad/s.

EXECUTE: **(a)** $A = 2.30$ mm

(b) $f = \dfrac{\omega}{2\pi} = \dfrac{742\text{ rad/s}}{2\pi} = 118\text{ Hz}$

(c) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{6.98\text{ rad/m}} = 0.90\text{ m}$

(d) $v = \dfrac{\omega}{k} = \dfrac{742\text{ rad/s}}{6.98\text{ rad/m}} = 106\text{ m/s}$

(e) The wave is traveling in the $-x$-direction because the phase of $y(x, t)$ has the form $kx + \omega t$.

(f) The linear mass density is $\mu = (3.38\times10^{-3}\text{ kg})/(1.35\text{ m}) = 2.504\times10^{-3}\text{ kg/m}$, so the tension is $F = \mu v^2 = (2.504\times10^{-3}\text{ kg/m})(106.3\text{ m/s})^2 = 28.3\text{ N}$.

(g) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}\sqrt{(2.50\times10^{-3}\text{ kg/m})(28.3\text{ N})}(742\text{ rad/s})^2(2.30\times10^{-3}\text{ m})^2 = 0.39\text{ W}$

EVALUATE: In part (d) we could also calculate the wave speed as $v = f\lambda$ and we would obtain the same result.

15.27. **IDENTIFY** and **SET UP:** Apply $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$ and $I = \dfrac{P}{4\pi r^2}$ to relate *I* and *r*.

Power is related to intensity at a distance *r* by $P = I(4\pi r^2)$. Energy is power times time.

EXECUTE: **(a)** $I_1 r_1^2 = I_2 r_2^2$

$I_2 = I_1(r_1/r_2)^2 = (0.026\text{ W/m}^2)(4.3\text{ m}/3.1\text{ m})^2 = 0.050\text{ W/m}^2$

(b) $P = 4\pi r^2 I = 4\pi(4.3\text{ m})^2(0.026\text{ W/m}^2) = 6.04\text{ W}$

Energy $= Pt = (6.04\text{ W})(3600\text{ s}) = 2.2\times10^4\text{ J}$

EVALUATE: We could have used $r = 3.1$ m and $I = 0.050\text{ W/m}^2$ in $P = 4\pi r^2 I$ and would have obtained the same *P*. Intensity becomes less as *r* increases because the radiated power spreads over a sphere of larger area.

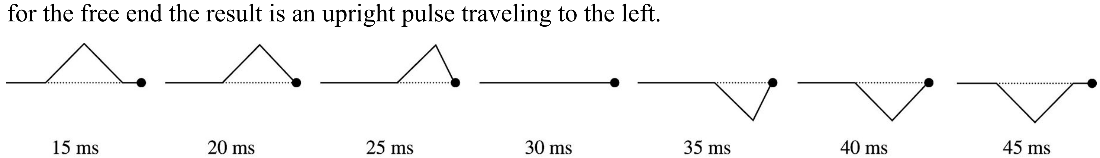
15.28. **IDENTIFY:** The distance the wave shape travels in time *t* is *vt*. The wave pulse reflects at the end of the string, at point *O*.

SET UP: The reflected pulse is inverted when *O* is a fixed end and is not inverted when *O* is a free end.

EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.28(a).

(b) The wave form for the given times, respectively, is shown in Figure 15.28(b).

EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the left and for the free end the result is an upright pulse traveling to the left.



(a)

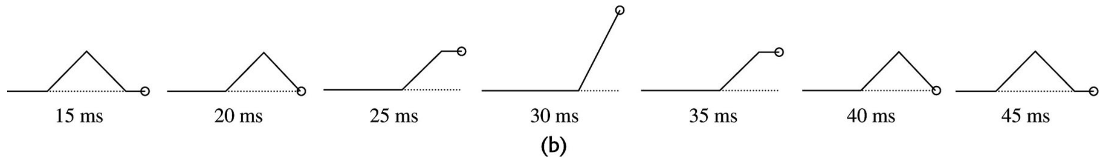


Figure 15.28

15.29. **IDENTIFY:** The distance the wave shape travels in time t is vt. The wave pulse reflects at the end of the string, at point O.

SET UP: The reflected pulse is inverted when O is a fixed end and is not inverted when O is a free end.

EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.29(a).

(b) The wave form for the given times, respectively, is shown in Figure 15.29(b).

EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the right and for the free end the result is an upright pulse traveling to the right.

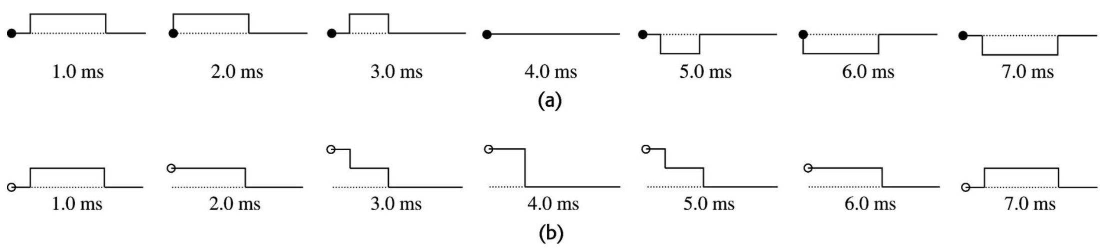


Figure 15.29

15.30. **IDENTIFY:** Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.30.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

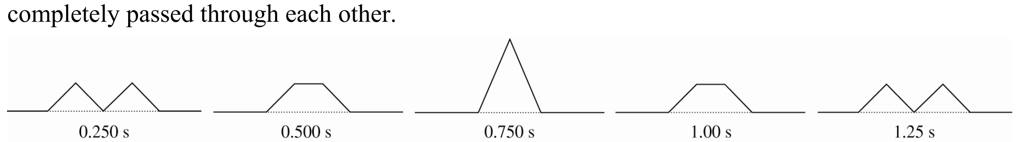


Figure 15.30

15.31. **IDENTIFY:** Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.31.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

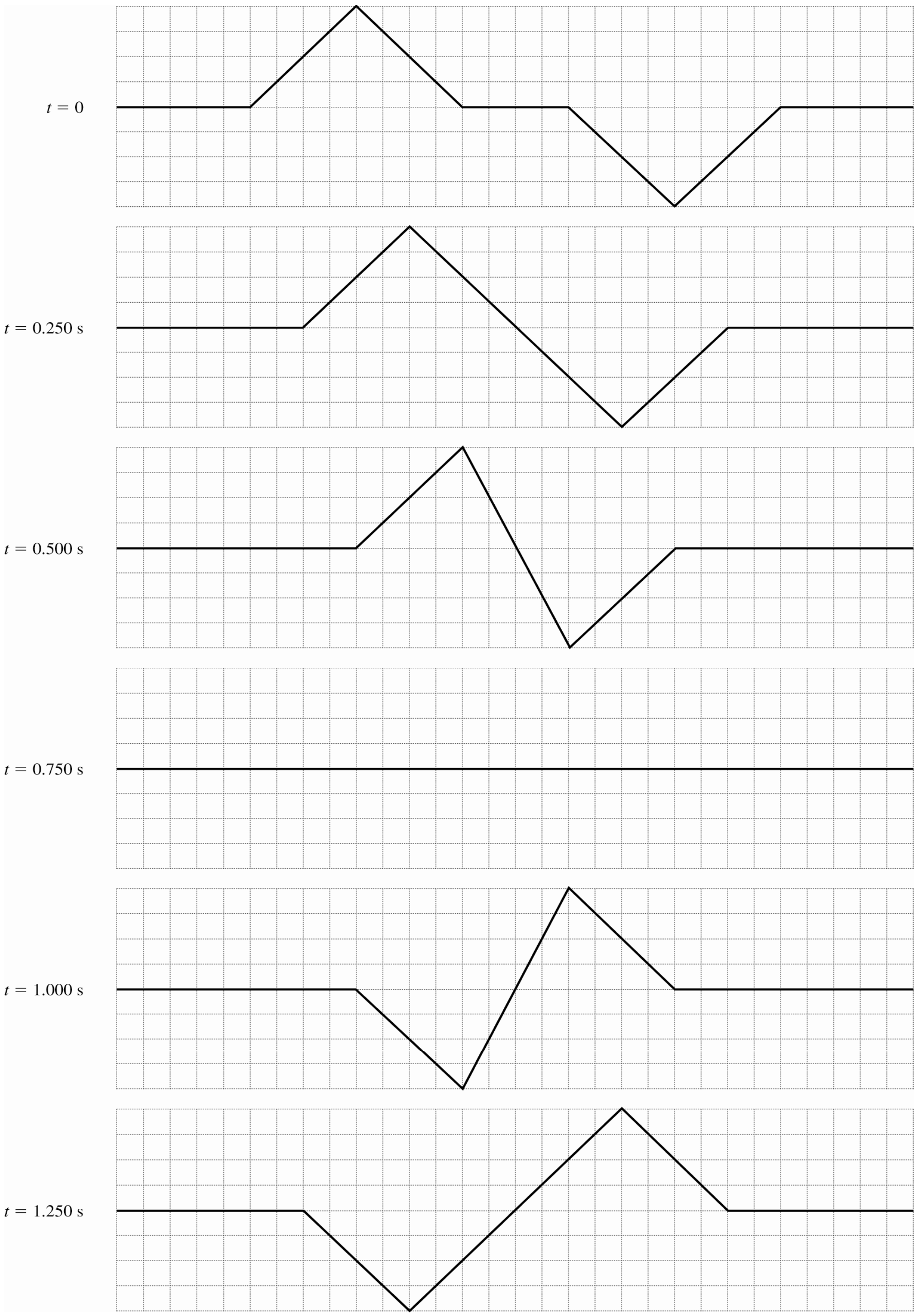


Figure 15.31

15.32. **IDENTIFY:** Apply the principle of superposition.

SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.

EXECUTE: The shape of the string at each specified time is shown in Figure 15.32.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

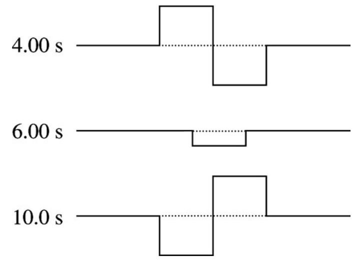


Figure 15.32

15.33. **IDENTIFY:** We are investigating standing waves on a violin string that is fixed at both ends.
SET UP: Estimate: The portion of the violin string that is free to vibrate is about 30 cm = 0.30 m long. For the bass viol, the free portion is 1.0 m long. We use $v = \sqrt{\frac{F}{\mu}}$ and $v = f\lambda$.

EXECUTE: **(a)** We want the speed of the wave, so $v = f\lambda$ = (659 Hz)(0.30 m) = 200 m/s.

(b) We want to compare the wavelength of the wave on the string to the wavelength of the sound wave it produces. Both waves have the same frequency but different speeds. In air we have $\lambda_{air} = v_{air} / f$ = (344 m/s)/(659 Hz) = 0.522 m. On the string λ_{string} = 0.30 m, so $\lambda_{air} > \lambda_{string}$.

(c) $v = f\lambda$ = (98 Hz)(1.0 m) = 98 m/s (on the string).

$\lambda_{air} = v_{air} / f$ = (344 m/s)/(98 Hz) = 3.5 m (in air).

So $\lambda_{air} > \lambda_{string}$.

EVALUATE: Each cycle of vibration of the string produces one cycle of sound, so the frequency is the same for the sound and for the string. But the sound wave travels at a different speed than the wave on the string, so the wavelength of the sound is not the same as the wavelength of the wave on the string.

15.34. **IDENTIFY:** Apply $y(x,t) = (A_{SW} \sin kx)\sin \omega t$ and $v = f\lambda$. At an antinode, $y(t) = A_{SW} \sin \omega t$. k and ω for the standing wave have the same values as for the two traveling waves.
SET UP: $A_{SW} = 0.850$ cm. The antinode to antinode distance is $\lambda/2$, so $\lambda = 30.0$ cm. $v_y = \partial y/\partial t$.

EXECUTE: **(a)** The node to node distance is $\lambda/2 = 15.0$ cm.

(b) λ is the same as for the standing wave, so $\lambda = 30.0$ cm. $A = \frac{1}{2}A_{SW} = 0.425$ cm.

$$v = f\lambda = \frac{\lambda}{T} = \frac{0.300 \text{ m}}{0.0750 \text{ s}} = 4.00 \text{ m/s}.$$

(c) $v_y = \dfrac{\partial y}{\partial t} = A_{SW}\,\omega \sin kx \cos \omega t$. At an antinode $\sin kx = 1$, so $v_y = A_{SW}\,\omega \cos \omega t$. $v_{max} = A_{SW}\omega$.

$$\omega = \frac{2\pi \text{ rad}}{T} = \frac{2\pi \text{ rad}}{0.0750 \text{ s}} = 83.8 \text{ rad/s}. \quad v_{max} = (0.850\times 10^{-2} \text{ m})(83.8 \text{ rad/s}) = 0.712 \text{ m/s}. \quad v_{min} = 0.$$

(d) The distance from a node to an adjacent antinode is $\lambda/4 = 7.50$ cm.

EVALUATE: The maximum transverse speed for a point at an antinode of the standing wave is twice the maximum transverse speed for each traveling wave, since $A_{SW} = 2A$.

15.35. **IDENTIFY** and **SET UP:** Nodes occur where $\sin kx = 0$ and antinodes are where $\sin kx = \pm 1$.

EXECUTE: Use $y = (A_{SW} \sin kx)\sin \omega t$:

(a) At a node $y = 0$ for all t. This requires that $\sin kx = 0$ and this occurs for $kx = n\pi$, $n = 0, 1, 2, \ldots$

$$x = n\pi/k = \frac{n\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})n, n = 0, 1, 2, \ldots$$

(b) At an antinode $\sin kx = \pm 1$ so y will have maximum amplitude. This occurs when $kx = (n+\frac{1}{2})\pi$, $n = 0,\ 1,\ 2,\ldots$

$$x = (n+\tfrac{1}{2})\pi/k = (n+\tfrac{1}{2})\frac{\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})(n+\tfrac{1}{2}),\ n = 0,\ 1,\ 2,\ldots$$

EVALUATE: $\lambda = 2\pi/k = 2.66$ m. Adjacent nodes are separated by $\lambda/2$, adjacent antinodes are separated by $\lambda/2$, and the node to antinode distance is $\lambda/4$.

15.36. **IDENTIFY:** For a string fixed at both ends, $\lambda_n = \dfrac{2L}{n}$ and $f_n = n\left(\dfrac{v}{2L}\right)$.

SET UP: For the fundamental, $n = 1$. For the second overtone, $n = 3$. For the fourth harmonic, $n = 4$.

EXECUTE: **(a)** $\lambda_1 = 2L = 3.00$ m. $f_1 = \dfrac{v}{2L} = \dfrac{(62.0 \text{ m/s})}{2(1.50 \text{ m})} = 20.7$ Hz.

(b) $\lambda_3 = \lambda_1/3 = 1.00$ m. $f_3 = 3f_1 = 62.0$ Hz.

(c) $\lambda_4 = \lambda_1/4 = 0.75$ m. $f_4 = 4f_1 = 82.7$ Hz.

EVALUATE: As n increases, λ decreases and f increases.

15.37. **IDENTIFY:** Use $v = f\lambda$ for v and $v = \sqrt{F/\mu}$ for the tension F. $v_y = \partial y/\partial t$ and $a_y = \partial v_y/\partial t$.

(a) SET UP: The fundamental standing wave is sketched in Figure 15.37.

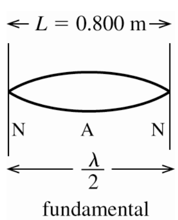


Figure 15.37

$f = 60.0$ Hz

From the sketch,

$\lambda/2 = L$ so

$\lambda = 2L = 1.60$ m

EXECUTE: $v = f\lambda = (60.0 \text{ Hz})(1.60 \text{ m}) = 96.0$ m/s

(b) The tension is related to the wave speed by $v = \sqrt{F/\mu}$:

$v = \sqrt{F/\mu}$ so $F = \mu v^2$.

$\mu = m/L = 0.0400 \text{ kg}/0.800 \text{ m} = 0.0500$ kg/m

$F = \mu v^2 = (0.0500 \text{ kg/m})(96.0 \text{ m/s})^2 = 461$ N.

(c) $\omega = 2\pi f = 377$ rad/s and $y(x,t) = A_{SW} \sin kx \sin \omega t$

$v_y = \omega A_{SW} \sin kx \cos \omega t;\ a_y = -\omega^2 A_{SW} \sin kx \sin \omega t$

$(v_y)_{max} = \omega A_{SW} = (377 \text{ rad/s})(0.300 \text{ cm}) = 1.13$ m/s.

$(a_y)_{max} = \omega^2 A_{SW} = (377 \text{ rad/s})^2 (0.300 \text{ cm}) = 426 \text{ m/s}^2$.

EVALUATE: The transverse velocity is different from the wave velocity. The wave velocity and tension are similar in magnitude to the values in the examples in the text. Note that the transverse acceleration is quite large.

15.38. **IDENTIFY:** The fundamental frequency depends on the wave speed, and that in turn depends on the tension.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$ where $\mu = m/L$. $f_1 = \dfrac{v}{2L}$. The nth harmonic has frequency $f_n = nf_1$.

EXECUTE: **(a)** $v = \sqrt{\dfrac{F}{m/L}} = \sqrt{\dfrac{FL}{m}} = \sqrt{\dfrac{(800 \text{ N})(0.400 \text{ m})}{3.00\times 10^{-3} \text{ kg}}} = 327$ m/s. $f_1 = \dfrac{v}{2L} = \dfrac{327 \text{ m/s}}{2(0.400 \text{ m})} = 409$ Hz.

(b) $n = \dfrac{10{,}000\ \text{Hz}}{f_1} = 24.4$. The 24th harmonic is the highest that could be heard.

EVALUATE: In part (b) we use the fact that a standing wave on the wire produces a sound wave in air of the same frequency.

15.39. **IDENTIFY:** Compare $y(x, t)$ given in the problem to $y(x,t) = (A_{\text{SW}} \sin kx)\sin \omega t$. From the frequency and wavelength for the third harmonic find these values for the eighth harmonic.

(a) SET UP: The third harmonic standing wave pattern is sketched in Figure 15.39.

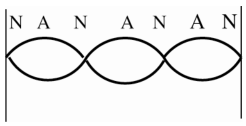


Figure 15.39

EXECUTE: **(b)** Use the general equation for a standing wave on a string:

$y(x, t) = (A_{\text{SW}} \sin kx)\sin \omega t$

$A_{\text{SW}} = 2A$, so $A = A_{\text{SW}}/2 = (5.60\ \text{cm})/2 = 2.80\ \text{cm}$

(c) The sketch in part (a) shows that $L = 3(\lambda/2)$. $k = 2\pi/\lambda$, $\lambda = 2\pi/k$

Comparison of $y(x, t)$ given in the problem to $y(x, t) = (A_{\text{SW}} \sin kx)\sin \omega t$ gives $k = 0.0340$ rad/cm.

So, $\lambda = 2\pi/(0.0340\ \text{rad/cm}) = 184.8\ \text{cm}$

$L = 3(\lambda/2) = 277\ \text{cm}$

(d) $\lambda = 185$ cm, from part (c)

$\omega = 50.0$ rad/s so $f = \omega/2\pi = 7.96$ Hz

period $T = 1/f = 0.126$ s $v = f\lambda = 1470$ cm/s

(e) $v_y = \partial y/\partial t = \omega A_{\text{SW}} \sin kx \cos \omega t$

$v_{y,\,\max} = \omega A_{\text{SW}} = (50.0\ \text{rad/s})(5.60\ \text{cm}) = 280\ \text{cm/s}$

(f) $f_3 = 7.96\ \text{Hz} = 3f_1$, so $f_1 = 2.65$ Hz is the fundamental

$f_8 = 8f_1 = 21.2$ Hz; $\omega_8 = 2\pi f_8 = 133$ rad/s

$\lambda = v/f = (1470\ \text{cm/s})/(21.2\ \text{Hz}) = 69.3$ cm and $k = 2\pi/\lambda = 0.0906$ rad/cm

$y(x, t) = (5.60\ \text{cm})\sin\left[(0.0906\ \text{rad/cm})x\right]\sin[(133\ \text{rad/s})t]$.

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. In the eighth harmonic the frequency and wave number are larger than in the third harmonic.

15.40. **IDENTIFY:** Compare the $y(x, t)$ specified in the problem to the general form of $y(x,t) = (A_{\text{SW}} \sin kx)\sin \omega t$.

SET UP: The comparison gives $A_{\text{SW}} = 4.44$ mm, $k = 32.5$ rad/m and $\omega = 754$ rad/s.

EXECUTE: **(a)** $A = \frac{1}{2}A_{\text{SW}} = \frac{1}{2}(4.44\ \text{mm}) = 2.22\ \text{mm}$.

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{32.5\ \text{rad/m}} = 0.193\ \text{m}$.

(c) $f = \dfrac{\omega}{2\pi} = \dfrac{754\ \text{rad/s}}{2\pi} = 120\ \text{Hz}$.

(d) $v = \dfrac{\omega}{k} = \dfrac{754\ \text{rad/s}}{32.5\ \text{rad/m}} = 23.2\ \text{m/s}$.

(e) If the wave traveling in the $+x$-direction is written as $y_1(x, t) = A\cos(kx - \omega t)$, then the wave traveling in the $-x$-direction is $y_2(x, t) = -A\cos(kx + \omega t)$, where $A = 2.22$ mm from part (a), $k = 32.5$ rad/m and $\omega = 754$ rad/s.

(f) The harmonic cannot be determined because the length of the string is not specified.
EVALUATE: The two traveling waves that produce the standing wave are identical except for their direction of propagation.

15.41. **IDENTIFY:** Standing waves are formed on a string fixed at both ends.
SET UP: The distance between adjacent nodes is one-half of a wavelength. The second overtone is the third harmonic ($n = 3$). $\lambda_n = 2L/n$.
EXECUTE: **(a)** We want the length of the string. In the third harmonic $\lambda_3 = 2L/3$, so $L = 3\lambda_3/2$. The distance between adjacent nodes is $\lambda/2 = 8.00$ cm, so $\lambda = 16.0$ cm. Therefore $L = 3\lambda_3/2 = 24.0$ cm $= 0.240$ m.
(b) For the 4th harmonic, $n = 4$, so $\lambda_4 = 2L/4 = (48.0 \text{ cm})/4 = 12.0 \text{ cm} = 0.120$ m. The node-to-node distance is $\lambda/2 = (12.0 \text{ cm})/2 = 6.00 \text{ cm} = 0.0600$ m.
EVALUATE: The node-to-node distance decreases with higher harmonics because the wavelength decreases, which is consistent with your results.

15.42. **IDENTIFY:** $v = \sqrt{F/\mu}$. $v = f\lambda$. The standing waves have wavelengths $\lambda_n = \dfrac{2L}{n}$ and frequencies $f_n = nf_1$. The standing wave on the string and the sound wave it produces have the same frequency.
SET UP: For the fundamental $n = 1$ and for the second overtone $n = 3$. The string has $\mu = m/L = (8.75 \times 10^{-3} \text{ kg})/(0.750 \text{ m}) = 1.17 \times 10^{-2}$ kg/m.
EXECUTE: **(a)** $\lambda = 2L/3 = 2(0.750 \text{ m})/3 = 0.500$ m. The sound wave has frequency

$f = \dfrac{v}{\lambda} = \dfrac{344 \text{ m/s}}{0.765 \text{ m}} = 449.7$ Hz. For waves on the string,
$v = f\lambda = (449.7 \text{ Hz})(0.500 \text{ m}) = 224.8$ m/s. The tension in the string is
$F = \mu v^2 = (1.17 \times 10^{-2} \text{ kg/m})(224.8 \text{ m/s})^2 = 591$ N.
(b) $f_1 = f_3/3 = (449.7 \text{ Hz})/3 = 150$ Hz.
EVALUATE: The waves on the string have a much longer wavelength than the sound waves in the air because the speed of the waves on the string is much greater than the speed of sound in air.

15.43. **IDENTIFY** and **SET UP:** Use the information given about the A_4 note to find the wave speed that depends on the linear mass density of the string and the tension. The wave speed isn't affected by the placement of the fingers on the bridge. Then find the wavelength for the D_5 note and relate this to the length of the vibrating portion of the string.
EXECUTE: **(a)** $f = 440$ Hz when a length $L = 0.600$ m vibrates; use this information to calculate the speed v of waves on the string. For the fundamental $\lambda/2 = L$ so $\lambda = 2L = 2(0.600 \text{ m}) = 1.20$ m. Then $v = f\lambda = (440 \text{ Hz})(1.20 \text{ m}) = 528$ m/s. Now find the length $L = x$ of the string that makes $f = 587$ Hz.

$\lambda = \dfrac{v}{f} = \dfrac{528 \text{ m/s}}{587 \text{ Hz}} = 0.900$ m
$L = \lambda/2 = 0.450$ m, so $x = 0.450 \text{ m} = 45.0$ cm.
(b) No retuning means same wave speed as in part (a). Find the length of vibrating string needed to produce $f = 392$ Hz.

$\lambda = \dfrac{v}{f} = \dfrac{528 \text{ m/s}}{392 \text{ Hz}} = 1.35$ m
$L = \lambda/2 = 0.675$ m; string is shorter than this. No, not possible.
EVALUATE: Shortening the length of this vibrating string increases the frequency of the fundamental.

15.44. **IDENTIFY:** $y(x,t) = (A_{SW} \sin kx)\sin \omega t$. $v_y = \partial y/\partial t$. $a_y = \partial^2 y/\partial t^2$.
SET UP: $v_{max} = (A_{SW} \sin kx)\omega$. $a_{max} = (A_{SW} \sin kx)\omega^2$.

EXECUTE: (a) (i) $x = \dfrac{\lambda}{2}$ is a node, and there is no motion.

(ii) $x = \dfrac{\lambda}{4}$ is an antinode, and $v_{\text{max}} = \omega A = 2\pi fA$, $a_{\text{max}} = \omega v_{\text{max}} = (2\pi f)v_{\text{max}} = 4\pi^2 f^2 A$.

(iii) $\cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ and this factor multiplies the results of (ii), so $v_{\text{max}} = \sqrt{2}\pi fA$, $a_{\text{max}} = 2\sqrt{2}\pi^2 f^2 A$.

(b) The amplitude is $2A\sin kx$, or (i) 0, (ii) $2A$, (iii) $2A/\sqrt{2}$.

(c) The time between the extremes of the motion is the same for any point on the string (although the period of the zero motion at a node might be considered indeterminate) and is $1/2f$.

EVALUATE: Any point in a standing wave moves in SHM. All points move with the same frequency but have different amplitude.

15.45. IDENTIFY: We are dealing with a traveling sinusoidal wave on a string.
SET UP: The equation $y(x,t) = A\cos(kx - \omega t)$ describes this wave. We know the maximum transverse speed and acceleration. We use $v_{y\text{-max}} = A\omega$, $a_{y\text{-max}} = A\omega^2$, and $v = f\lambda$. The target variables are the speed v of the wave and its amplitude A.
EXECUTE: Take the ratio of the maximum acceleration to the maximum speed, giving
$\dfrac{a_{y\text{-max}}}{v_{y\text{-max}}} = \dfrac{A\omega^2}{A\omega} = \omega$. Using the known values gives $\omega = \dfrac{8.50\times10^4\ \text{m/s}^2}{3.00\ \text{m/s}} = 2.8333\times10^4$ rad/s. Now use

$$v = f\lambda = (\omega/2\pi)\lambda = \left(\frac{2.8333\times10^4\ \text{rad/s}}{2\pi}\right)(0.400\ \text{m}) = 1.80\times10^3\ \text{m/s}.$$

From the velocity equation $v_{y\text{-max}} = A\omega$ we have $A = \dfrac{v_{y\text{-max}}}{\omega}$, which gives

$$A = \frac{3.00\ \text{m/s}}{2.8333\times10^4\ \text{rad/s}} = 1.06\times10^{-4}\ \text{m}.$$

EVALUATE: The propagation speed v is constant. But the transverse speed $v_y = \dfrac{\partial y}{\partial t}$ is *not* constant; it varies with time and from place to place on the string.

15.46. IDENTIFY: Compare $y(x,t)$ given in the problem to the general form $y(x,t) = A\cos(kx - \omega t)$.
SET UP: The comparison gives $A = 0.750$ cm, $k = 0.400\pi$ rad/cm and $\omega = 250\pi$ rad/s.
EXECUTE: (a) $A = 0.750$ cm, $\lambda = \dfrac{2}{0.400\ \text{rad/cm}} = 5.00$ cm, $f = 125$ Hz, $T = \frac{1}{f} = 0.00800$ s and $v = \lambda f = 6.25$ m/s.

(b) The sketches of the shape of the rope at each time are given in Figure 15.46.
(c) To stay with a wavefront as t increases, x decreases and so the wave is moving in the $-x$-direction.
(d) From $v = \sqrt{F/\mu}$, the tension is $F = \mu v^2 = (0.050\ \text{kg/m})(6.25\ \text{m/s})^2 = 1.95$ N.
(e) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = 5.42$ W.

EVALUATE: The argument of the cosine is $(kx + \omega t)$ for a wave traveling in the $-x$-direction, and that is the case here.

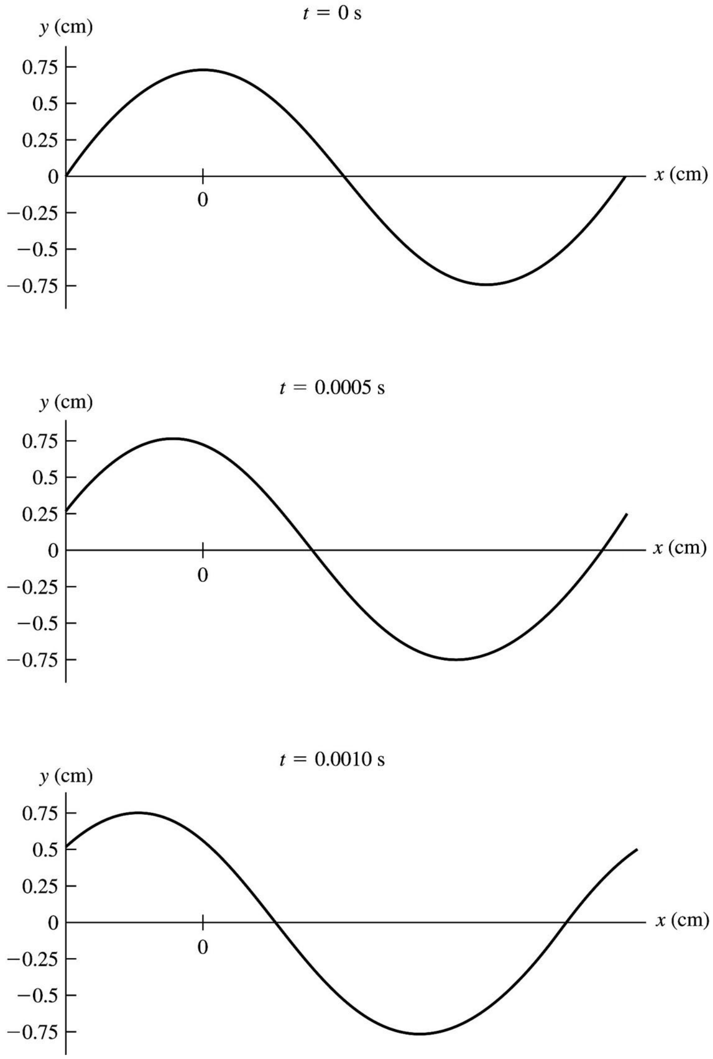


Figure 15.46

15.47. **IDENTIFY** and **SET UP:** Calculate v, ω, and k from $v = f\lambda$, $\omega = vk$, $k = 2\pi/\lambda$. Then apply $y(x,t) = A\cos(kx - \omega t)$ to obtain $y(x,t)$.

$A = 2.50\times10^{-3}$ m, $\lambda = 1.80$ m, $v = 36.0$ m/s

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (36.0\text{ m/s})/1.80\text{ m} = 20.0\text{ Hz}$

$\omega = 2\pi f = 2\pi(20.0\text{ Hz}) = 126\text{ rad/s}$

$k = 2\pi/\lambda = 2\pi\text{ rad}/1.80\text{ m} = 3.49\text{ rad/m}$

(b) For a wave traveling to the right, $y(x,t) = A\cos(kx - \omega t)$. This equation gives that the $x = 0$ end of the string has maximum upward displacement at $t = 0$.

Put in the numbers: $y(x,t) = (2.50\times10^{-3}\text{ m})\cos[(3.49\text{ rad/m})x - (126\text{ rad/s})t]$.

(c) The left-hand end is located at $x = 0$. Put this value into the equation of part (b):

$y(0,t) = +(2.50\times10^{-3}\text{ m})\cos((126\text{ rad/s})t)$.

(d) Put $x = 1.35$ m into the equation of part (b):

$y(1.35\text{ m}, t) = (2.50\times10^{-3}\text{ m})\cos((3.49\text{ rad/m})(1.35\text{ m}) - (126\text{ rad/s})t)$.

$y(1.35\text{ m}, t) = (2.50\times10^{-3}\text{ m})\cos(4.71\text{ rad} - (126\text{ rad/s})t)$

$4.71 \text{ rad} = 3\pi/2$ and $\cos(\theta) = \cos(-\theta)$, so $y(1.35 \text{ m}, t) = (2.50\times10^{-3} \text{ m})\cos((126 \text{ rad/s})t - 3\pi/2 \text{ rad})$

(e) $y = A\cos(kx - \omega t)$ (part (b))

The transverse velocity is given by $v_y = \dfrac{\partial y}{\partial t} = A\dfrac{\partial}{\partial t}\cos(kx - \omega t) = +A\omega\sin(kx - \omega t)$.

The maximum v_y is $A\omega = (2.50\times10^{-3} \text{ m})(126 \text{ rad/s}) = 0.315 \text{ m/s}$.

(f) $y(x, t) = (2.50\times10^{-3} \text{ m})\cos((3.49 \text{ rad/m})x - (126 \text{ rad/s})t)$

$t = 0.0625 \text{ s}$ and $x = 1.35 \text{ m}$ gives

$y = (2.50\times10^{-3} \text{ m})\cos((3.49 \text{ rad/m})(1.35 \text{ m}) - (126 \text{ rad/s})(0.0625 \text{ s})) = -2.50\times10^{-3} \text{ m}$.

$v_y = +A\omega\sin(kx - \omega t) = +(0.315 \text{ m/s})\sin((3.49 \text{ rad/m})x - (126 \text{ rad/s})t)$

$t = 0.0625 \text{ s}$ and $x = 1.35 \text{ m}$ gives

$v_y = (0.315 \text{ m/s})\sin((3.49 \text{ rad/m})(1.35 \text{ m}) - (126 \text{ rad/s})(0.0625 \text{ s})) = 0.0$

EVALUATE: The results of part (f) illustrate that $v_y = 0$ when $y = \pm A$, as we saw from SHM in Chapter 14.

15.48. **IDENTIFY:** Apply $\Sigma\tau_z = 0$ to find the tension in each wire. Use $v = \sqrt{F/\mu}$ to calculate the wave speed for each wire and then $t = L/v$ is the time for each pulse to reach the ceiling, where $L = 1.25 \text{ m}$.

SET UP: The wires have $\mu = \dfrac{m}{L} = \dfrac{0.290 \text{ N}}{(9.80 \text{ m/s}^2)(1.25 \text{ m})} = 0.02367 \text{ kg/m}$. The free-body diagram for the beam is given in Figure 15.48. Take the axis to be at the end of the beam where wire A is attached.

EXECUTE: $\Sigma\tau_z = 0$ gives $T_B L = w(L/3)$ and $T_B = w/3 = 583 \text{ N}$. $T_A + T_B = 1750 \text{ N}$, so $T_A = 1167 \text{ N}$.

$v_A = \sqrt{\dfrac{T_A}{\mu}} = \sqrt{\dfrac{1167 \text{ N}}{0.02367 \text{ kg/m}}} = 222 \text{ m/s}$. $t_A = \dfrac{1.25 \text{ m}}{222 \text{ m/s}} = 0.00563 \text{ s} = 5.63 \text{ ms}$.

$v_B = \sqrt{\dfrac{583 \text{ N}}{0.02367 \text{ kg/m}}} = 156.9 \text{ m/s}$. $t_B = \dfrac{1.25 \text{ m}}{156.9 \text{ m/s}} = 0.007965 \text{ s} = 7.965 \text{ ms}$.

$\Delta t = t_B - t_A = 7.965 \text{ ms} - 5.63 \text{ ms} = 2.34 \text{ ms}$.

EVALUATE: The wave pulse travels faster in wire A, since that wire has the greater tension, so the pulse in wire A arrives first.

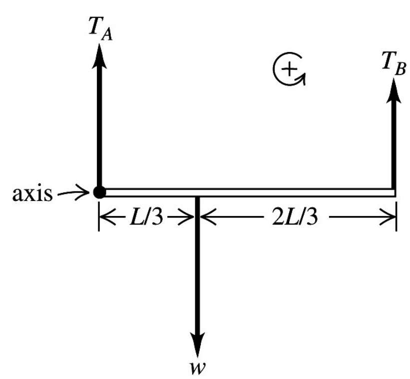


Figure 15.48

15.49. **IDENTIFY:** The block causes tension in the wire supporting the rod. A standing wave pattern exists on the wire, and the torques on the rod balance.

SET UP: The graph is a plot of the square of the frequency f versus the mass m hanging from the rod. The target variable is the mass of the wire. To interpret the graph, we need to find a relationship between f^2 and m. We apply $\sum \tau_z = 0$ about the hinge and use $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$.

EXECUTE: $\sum \tau_z = 0$: $TL\sin\theta - mg\dfrac{L}{2} = 0$ gives $T = \dfrac{mg}{2\sin\theta}$. Now use $v = \sqrt{\dfrac{F}{\mu}}$ and $v = f\lambda$. Equating these velocity equations gives $f\lambda = \sqrt{\dfrac{F}{\mu}}$, so $f^2 = \dfrac{1}{\lambda^2}\left(\dfrac{F}{\mu}\right)$. Using the result for the tension gives $f^2 = \dfrac{1}{\mu\lambda^2}\left(\dfrac{mg}{2\sin\theta}\right) = \left(\dfrac{g}{2\mu\lambda^2\sin\theta}\right)m$. The wire is vibrating in its fundamental mode so $\lambda = 2L$. The linear mass density of the wire is $\mu = \dfrac{m_w}{L}$. Therefore the final equation becomes $f^2 = \left(\dfrac{g}{8m_w L\sin\theta}\right)m$. From this we see that a graph of f^2 versus m should be a straight line with slope equal to $\dfrac{g}{8m_w L\sin\theta}$. Thus $m_w = \dfrac{g}{(\text{slope})8L\sin\theta}$ which gives

$$m_w = \frac{9.80 \text{ m/s}^2}{(20.4 \text{ kg}^{-1}\cdot\text{s}^{-2})(8)(2.00 \text{ m})(\sin 30.0°)} = 0.0600 \text{ kg} = 60.0 \text{ g}.$$

EVALUATE: A mass of 60 g for a 2.00-m wire is not unreasonable.

15.50. IDENTIFY: The maximum vertical acceleration must be at least g.

SET UP: $a_{max} = \omega^2 A$

EXECUTE: $g = \omega^2 A_{min}$ and thus $A_{min} = g/\omega^2$. Using $\omega = 2\pi f = 2\pi v/\lambda$ and $v = \sqrt{F/\mu}$, this becomes $A_{min} = \dfrac{g\lambda^2\mu}{4\pi^2 F}$.

EVALUATE: When the amplitude of the motion increases, the maximum acceleration of a point on the rope increases.

15.51. IDENTIFY: Calculate the speed of the wave and use that to find the length of the wire since we know how long it takes the wave to travel the length of the wire.

SET UP: $v = \sqrt{F/\mu}$, $x = v_x t$, and $\mu = m/L$.

EXECUTE: **(a)** $\mu = m/L$ = $(14.5\times10^{-9}$ kg)/(0.0200 m) = 7.25×10^{-7} kg/m. Now combine $v = \sqrt{F/\mu}$ and $x = v_x t$: $vt = L$, so $L = t\sqrt{F/\mu}$ = $(26.7\times10^{-3}\text{ s})\sqrt{\dfrac{(0.400\text{ kg})(9.80\text{ m/s}^2)}{7.25\times10^{-7}\text{ kg/m}}} = 62.1$ m.

(b) The mass of the wire is $m = \mu L$ = $(7.25\times10^{-7}$ kg/m)(62.1 m) = 4.50×10^{-5} kg = 0.0450 g.

EVALUATE: The mass of the wire is negligible compared to the 0.400-kg object hanging from the wire.

15.52. IDENTIFY: The frequencies at which a string vibrates depend on its tension, mass density and length.

SET UP: $f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$, where $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{FL}{m}}$. F is the tension in the string, L is its length and m is its mass.

EXECUTE: **(a)** $f_1 = \dfrac{v}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{FL}{m}} = \dfrac{1}{2}\sqrt{\dfrac{F}{Lm}}$. Solving for F gives

$$F = (2f_1)^2 Lm = 4(262\text{ Hz})^2(0.350\text{ m})(8.00\times10^{-3}\text{ kg}) = 769\text{ N}.$$

(b) $m = \dfrac{F}{L(2f_1)^2} = \dfrac{769\ \text{N}}{(0.350\ \text{m})(4)(466\ \text{Hz})^2} = 2.53\ \text{g}.$

(c) For S_1, $\mu = \dfrac{8.00\times10^{-3}\ \text{kg}}{0.350\ \text{m}} = 0.0229\ \text{kg/m}.$ $F = 769\ \text{N}$ and $v = \sqrt{F/\mu} = 183\ \text{m/s}.$ $f_1 = \dfrac{v}{2L}$ gives $L = \dfrac{v}{2f_1} = \dfrac{183\ \text{m/s}}{2(277\ \text{Hz})} = 33.0\ \text{cm}.$ $x = 35.0\ \text{cm} - 33.0\ \text{cm} = 2.00\ \text{cm}.$

(d) For S_2, $\mu = \dfrac{2.53\times10^{-3}\ \text{kg}}{0.350\ \text{m}} = 7.23\times10^{-3}\ \text{kg/m}.$ $F = 769\ \text{N}$ and $v = \sqrt{F/\mu} = 326\ \text{m/s}.$ $L = 0.330\ \text{m}$ and $f_1 = \dfrac{v}{2L} = \dfrac{326\ \text{m/s}}{2(0.330\ \text{m})} = 494\ \text{Hz}.$

EVALUATE: If the tension is the same in the strings, the mass densities must be different to produce sounds of different pitch.

15.53. **IDENTIFY:** Apply $\Sigma\tau_z = 0$ to one post and calculate the tension in the wire. $v = \sqrt{F/\mu}$ for waves on the wire. $v = f\lambda$. The standing wave on the wire and the sound it produces have the same frequency. For standing waves on the wire, $\lambda_n = \dfrac{2L}{n}$.

SET UP: For the fifth overtone, $n = 6$. The wire has $\mu = m/L = (0.732\ \text{kg})/(5.00\ \text{m}) = 0.146\ \text{kg/m}$. The free-body diagram for one of the posts is given in Figure 15.53. Forces at the pivot aren't shown. We take the rotation axis to be at the pivot, so forces at the pivot produce no torque.

EXECUTE: $\Sigma\tau_z = 0$ gives $w\left(\dfrac{L}{2}\cos57.0^\circ\right) - T(L\sin57.0^\circ) = 0.$ $T = \dfrac{w}{2\tan57.0^\circ} = \dfrac{235\ \text{N}}{2\tan57.0^\circ} = 76.3\ \text{N}.$

For waves on the wire, $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{76.3\ \text{N}}{0.146\ \text{kg/m}}} = 22.9\ \text{m/s}.$ For the fifth overtone standing wave on the wire, $\lambda = \dfrac{2L}{6} = \dfrac{2(5.00\ \text{m})}{6} = 1.67\ \text{m}.$ $f = \dfrac{v}{\lambda} = \dfrac{22.9\ \text{m/s}}{1.67\ \text{m}} = 13.7\ \text{Hz}.$ The sound waves have frequency 13.7 Hz and wavelength $\lambda = \dfrac{344\ \text{m/s}}{13.7\ \text{Hz}} = 25.0\ \text{m}.$

EVALUATE: The frequency of the sound wave is just below the lower limit of audible frequencies. The wavelength of the standing wave on the wire is much less than the wavelength of the sound waves, because the speed of the waves on the wire is much less than the speed of sound in air.

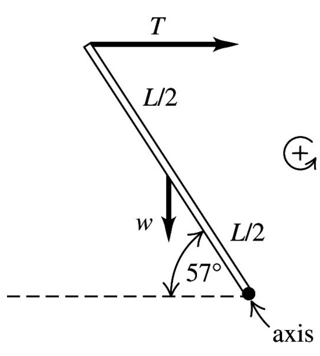


Figure 15.53

15.54. **IDENTIFY:** The mass of the planet (the target variable) determines g at its surface, which in turn determines the weight of the lead object hanging from the string. The weight is the tension in the string, which determines the speed of a wave pulse on that string.

SET UP: At the surface of the planet $g = G\dfrac{m_p}{R_p^2}$. The pulse speed is $v = \sqrt{\dfrac{F}{\mu}}$.

EXECUTE: On earth, $v = \dfrac{4.00\text{ m}}{0.0390\text{ s}} = 1.0256\times10^2\text{ m/s}$. $\mu = \dfrac{0.0280\text{ kg}}{4.00\text{ m}} = 7.00\times10^{-3}\text{ kg/m}^3$. $F = Mg$, so $v = \sqrt{\dfrac{Mg}{\mu}}$ and the mass of the lead weight is

$M = \left(\dfrac{\mu}{g}\right)v^2 = \left(\dfrac{7.00\times10^{-3}\text{ kg/m}}{9.8\text{ m/s}^2}\right)(1.0256\times10^2\text{ m/s})^2 = 7.513\text{ kg}$. On the planet,

$v = \dfrac{4.00\text{ m}}{0.0685\text{ s}} = 58.394\text{ m/s}$. Therefore $g = \left(\dfrac{\mu}{M}\right)v^2 = \left(\dfrac{7.00\times10^{-3}\text{ kg/m}}{7.513\text{ kg}}\right)(58.394\text{ m/s})^2 = 3.1770\text{ m/s}^2$.

$g = G\dfrac{m_p}{R_p^2}$ and $m_p = \dfrac{gR_p^2}{G} = \dfrac{(3.1770\text{ m/s}^2)(7.20\times10^7\text{ m})^2}{6.6743\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 2.47\times10^{26}\text{ kg}$.

EVALUATE: This mass is about 41 times that of earth, but its radius is about 10 times that of earth, so the result is reasonable.

15.55. IDENTIFY: The wavelengths of standing waves depend on the length of the string (the target variable), which in turn determine the frequencies of the waves.

SET UP: $f_n = nf_1$ where $f_1 = \dfrac{v}{2L}$.

EXECUTE: $f_n = nf_1$ and $f_{n+1} = (n+1)f_1$. We know the wavelengths of two adjacent modes, so $f_1 = f_{n+1} - f_n = 630\text{ Hz} - 525\text{ Hz} = 105\text{ Hz}$. Solving $f_1 = \dfrac{v}{2L}$ for L gives $L = \dfrac{v_1}{2f} = \dfrac{384\text{ m/s}}{2(105\text{ Hz})} = 1.83\text{ m}$.

EVALUATE: The observed frequencies are both audible which is reasonable for a string that is about a half meter long.

15.56. IDENTIFY: A standing wave exists on a string fixed at both ends.
SET UP: The graph plots the number n of nodes (*not* antinodes) versus the frequency f_n. Therefore we look for a relationship between f_n and n so we can interpret the slope of the graph. The target variable is the speed v of waves on the string. We use $v = f\lambda$.

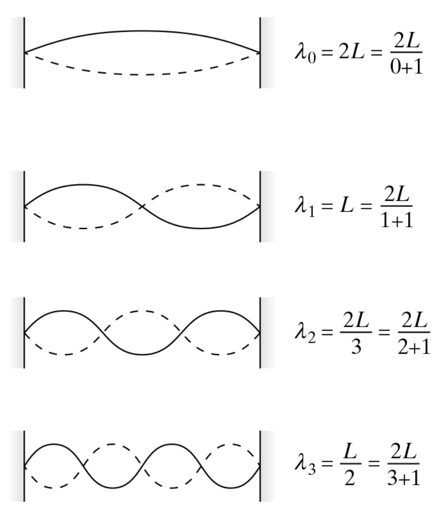


Figure 15.56

EXECUTE: We cannot use $\lambda_n = 2L/n$ because in that formula n is the number of *antinodes* and we need a relationship involving the number of *nodes* (not counting the nodes at the ends of the string). Fig. 15.56 helps to find this relationship. We already know that the wavelengths of the harmonics are $2L$, L, $2L/3$, $L/2$, … . Letting n be the number of antinodes between the ends, from the figure we see that $\lambda_0 = 2L$, $\lambda_1 = L = 2L/2$, $\lambda_2 = 2L/3$, $\lambda_3 = L/2 = 2L/4$, … . Notice that the denominator is 1 more than n, so the general formula is $\lambda_n = \dfrac{2L}{n+1}$, where n = 0, 1, 2, … . Now we need to relate this result to the frequency f_n. Using $v = f\lambda$ gives $f_n\lambda_n = v$. Using $\lambda_n = \dfrac{2L}{n+1}$, we get $f_n\left(\dfrac{2L}{n+1}\right) = v$. Solving for n gives $n = \dfrac{2L}{v}f_n - 1$. From this equation we see that a graph of n versus f_n should be a straight line having slope $2L/v$, so $v = \dfrac{2L}{\text{slope}} = \dfrac{2(0.800\text{ m})}{7.30\times10^{-3}\text{ s}} = 219\text{ m/s}.$

EVALUATE: The method used here would be useful if the string could not be removed to measure its mass and length to use $v = \sqrt{\dfrac{F}{\mu}}$.

15.57. **IDENTIFY:** The tension in the wires along with their lengths determine the fundamental frequency in each one (the target variables). These frequencies are different because the wires have different linear mass densities. The bar is in equilibrium, so the forces and torques on it balance.

SET UP: $T_\text{a} + T_\text{c} = w$, $\Sigma\tau_z = 0$, $v = \sqrt{\dfrac{F}{\mu}}$, $f_1 = v/2L$ and $\mu = \dfrac{m}{L}$, where $m = \rho V = \rho\pi r^2 L$. The densities of copper and aluminum are given in a table in the text.

EXECUTE: Using the subscript "a" for aluminum and "c" for copper, we have $T_\text{a} + T_\text{c} = w = 638\text{ N}$. $\Sigma\tau_z = 0$, with the axis at left-hand end of bar, gives $T_\text{c}(1.40\text{ m}) = w(0.90\text{ m})$, so $T_\text{c} = 410.1\text{ N}$. $T_\text{a} = 638\text{ N} - 410.1\text{ N} = 227.9\text{ N}$. $f_1 = \dfrac{v}{2L}$. $\mu = \dfrac{m}{L} = \dfrac{\rho\pi r^2 L}{L} = \rho\pi r^2$.

<u>For the copper wire</u>: $F = 410.1\text{ N}$ and $\mu = (8.90\times10^3\text{ kg/m}^3)\pi(0.280\times10^{-3}\text{ m})^2 = 2.19\times10^{-3}\text{ kg/m}$, so $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{410.1\text{ N}}{2.19\times10^{-3}\text{ kg/m}}} = 432.7\text{ m/s}$. $f_1 = \dfrac{v}{2L} = \dfrac{432.7\text{ m/s}}{2(0.600\text{ m})} = 361\text{ Hz}$.

<u>For the aluminum wire</u>: $F = 227.9\text{ N}$ and $\mu = (2.70\times10^3\text{ kg/m}^3)\pi(0.280\times10^{-3}\text{ m})^2 = 6.65\times10^{-4}\text{ kg/m}$, so $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{227.9\text{ N}}{6.65\times10^{-4}\text{ kg/m}}} = 585.4\text{ m/s}$, which gives $f_1 = \dfrac{585.4\text{ m/s}}{2(0.600\text{ m})} = 488\text{ Hz}$.

EVALUATE: The wires have different fundamental frequencies because they have different tensions and different linear mass densities.

15.58. **IDENTIFY:** We are dealing with a standing wave on a string fixed at both ends.

SET UP: We want to know the maximum transverse speed at two points on the string. The equation for the displacement $y(x,t)$ of the string is $y(x,t) = A_\text{SW}\sin kx \sin\omega t$. The transverse velocity is $v_y = \dfrac{\partial y}{\partial t}$, $v = f\lambda$, $k = 2\pi/\lambda$, and $\omega = 2\pi f$. The first antinode occurs at x = 0.150 m, which is ¼ of a wavelength from the end, so λ = 0.600 m.

EXECUTE: **(a)** The maximum transverse speed occurs at the antinodes. Using $v_y = \dfrac{\partial y}{\partial t}$, we get $v_y = \omega A_\text{SW}\sin kx\cos\omega t$. The maximum speed is ωA_SW. Now use $v = f\lambda$ and $\omega = 2\pi f$ to find ω:

$v = f\lambda = (\omega/2\pi)\lambda$, so $\omega = 2\pi v/\lambda$. Thus $v_{y,\max} = \omega A_{\text{SW}} = \dfrac{2\pi v A_{\text{SW}}}{\lambda}$. Putting in the numbers gives

$v_{y,\max} = \dfrac{2\pi(260 \text{ m/s})(0.00180 \text{ m})}{0.600 \text{ m}} = 4.90 \text{ m/s}.$

(b) Use $v_y = \omega A_{\text{SW}} \sin kx \cos \omega t$ at $x = 0.075$ m. The maximum speed occurs when $\cos \omega t = \pm 1$, so

$v_{y,\max} = (4.90 \text{ m/s}) \sin\left[(10.472 \text{ m}^{-1})(0.075 \text{ m})\right] = 3.5 \text{ m/s}.$

EVALUATE: Notice that the wave speed (260 m/s) moves much faster than any point on the string.

15.59. IDENTIFY: The distance between adjacent nodes is one-half the wavelength. The second overtone is the third harmonic ($n = 3$).

SET UP: The wavelengths are $\lambda_n = 2L/n$, $f = 1/T$, $v = \sqrt{F/\mu}$, $v = f\lambda$, and $\mu = m/L$.

EXECUTE: **(a)** The node-to-node distance is $\lambda/2$, so $\lambda = 2(6.28 \text{ cm}) = 12.56 \text{ cm} = 0.1256$ m. In the third harmonic, $\lambda_n = 2L/n$ gives $0.1256 \text{ m} = 2L/3$, so $L = 0.1884$ m which rounds to 0.188 m.

(b) The time to go from top to bottom is one-half the period, $T/2$, so $T = 2(8.40 \text{ ms}) = 16.8 \text{ ms} = 0.0168$ s.

$f = 1/T = 1/(0.0168 \text{ s}) = 59.52$ Hz. Combining $v = \sqrt{F/\mu}$, $\mu = m/L$, and $v = f\lambda$ gives

$(f\lambda)^2 = \dfrac{F}{\mu} \rightarrow \mu = \dfrac{F}{(f\lambda)^2} = \dfrac{m}{L} \rightarrow m = \dfrac{FL}{(f\lambda)^2}$. Putting in the numbers gives

$m = (5.00 \text{ N})(0.1884 \text{ m})/[(59.52 \text{ Hz})(0.1256 \text{ m})]^2 = 0.0169 \text{ kg} = 16.9$ g.

EVALUATE: In a standing wave pattern, the nodes, we well as the antinodes, are spaced one-half a wavelength apart.

15.60. IDENTIFY: The wavelengths of the standing waves on the wire are given by $\lambda_n = \dfrac{2L}{n}$. When the ball is changed the wavelength changes because the length of the wire changes; $\Delta l = \dfrac{Fl_0}{AY}$.

SET UP: For the third harmonic, $n = 3$. For copper, $Y = 11 \times 10^{10}$ Pa. The wire has cross-sectional area $A = \pi r^2 = \pi(0.512 \times 10^{-3} \text{ m})^2 = 8.24 \times 10^{-7} \text{ m}^2$.

EXECUTE: **(a)** $\lambda_3 = \dfrac{2(1.20 \text{ m})}{3} = 0.800$ m

(b) The increase in length when the 100.0 N ball is replaced by the 500.0 N ball is given by

$\Delta l = \dfrac{(\Delta F)l_0}{AY}$, where $\Delta F = 400.0$ N is the increase in the force applied to the end of the wire.

$\Delta l = \dfrac{(400.0 \text{ N})(1.20 \text{ m})}{(8.24 \times 10^{-7} \text{ m}^2)(11 \times 10^{10} \text{ Pa})} = 5.30 \times 10^{-3}$ m. The change in wavelength is $\Delta\lambda = \tfrac{2}{3}\Delta l = 3.5$ mm.

EVALUATE: The change in tension changes the wave speed and that in turn changes the frequency of the standing wave, but the problem asks only about the wavelength.

15.61. IDENTIFY and **SET UP:** The average power is given by $P_{\text{av}} = \tfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. Rewrite this expression in terms of v and λ in place of F and ω.

EXECUTE: **(a)** $P_{\text{av}} = \tfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$

$v = \sqrt{F/\mu}$ so $\sqrt{F} = v\sqrt{\mu}$

$\omega = 2\pi f = 2\pi(v/\lambda)$

Using these two expressions to replace $\sqrt{F}$ and ω gives $P_{\text{av}} = 2\mu\pi^2 v^3 A^2/\lambda^2$;

$\mu = (6.00 \times 10^{-3} \text{ kg})/(8.00 \text{ m})$

$$A = \left(\frac{2\lambda^2 P_{\text{av}}}{4\pi^2 v^3 \mu}\right)^{1/2} = 7.07 \text{ cm}$$

(b) EVALUATE: $P_{\text{av}} \sim v^3$ so doubling v increases P_{av} by a factor of 8.

$P_{\text{av}} = 8(50.0 \text{ W}) = 400.0 \text{ W}$

15.62. IDENTIFY: The time between positions 1 and 5 is equal to $T/2$. $v = f\lambda$. The velocity of points on the string is given by $v_y(x,t) = \omega A \sin(kx - \omega t)$.

SET UP: Four flashes occur from position 1 to position 5, so the elapsed time is $4\left(\frac{60 \text{ s}}{5000}\right) = 0.048 \text{ s}$.

The figure in the problem shows that $\lambda = L = 0.500$ m. At point P the amplitude of the standing wave is 1.5 cm.

EXECUTE: (a) $T/2 = 0.048$ s and $T = 0.096$ s. $f = 1/T = 10.4$ Hz. $\lambda = 0.500$ m.

(b) The fundamental standing wave has nodes at each end and no nodes in between. This standing wave has one additional node. This is the first overtone and second harmonic.

(c) $v = f\lambda = (10.4 \text{ Hz})(0.500 \text{ m}) = 5.20 \text{ m/s}$.

(d) In position 1, point P is at its maximum displacement and its speed is zero. In position 3, point P is passing through its equilibrium position and its speed is

$v_{\text{max}} = \omega A = 2\pi f A = 2\pi(10.4 \text{ Hz})(0.015 \text{ m}) = 0.980 \text{ m/s}$.

(e) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$ and $m = \frac{FL}{v^2} = \frac{(1.00 \text{ N})(0.500 \text{ m})}{(5.20 \text{ m/s})^2} = 18.5 \text{ g}$.

EVALUATE: The standing wave is produced by traveling waves moving in opposite directions. Each point on the string moves in SHM, and the amplitude of this motion varies with position along the string.

15.63. IDENTIFY: The chain of beads vibrates in a standing wave pattern.

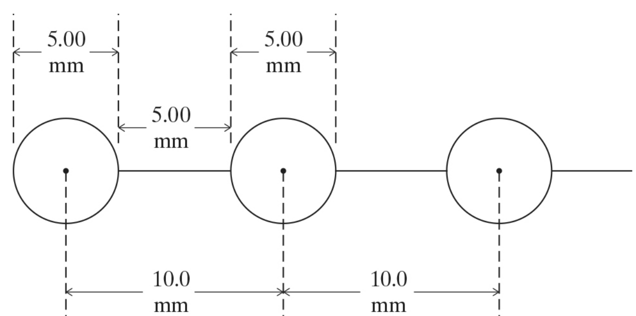


Figure 15.63

SET UP: First find the number of beads on a string 1.005 m long. Fig. 15.63 shows the first few beads. Starting at the left end, the string is made up of units each consisting of one bead of diameter 5.00 mm plus a space of 5.00 mm, so each unit is 10.0 mm long. To make a chain 1.005 m long, we need 100 of these units plus 5.00 mm for the end bead on the right. So the total mass of the 101 beads is 101 g = 0.101 kg. We use $y(x,t) = A_{\text{SW}} \sin kx \sin \omega t$, $v = \sqrt{\frac{F}{\mu}}$, $v = f\lambda$, $v_y = \frac{\partial y}{\partial t}$, $\lambda_n = 2L/n$, $k = 2\pi/\lambda$, $\omega = 2\pi f$, and $F = kx$ (this k is the force constant).

EXECUTE: (a) $\mu = m/L$ = (0.101 kg)/(1.50 m) = 0.0673 kg/m.

(b) $T = kx$ = (28.8 N/m)(1.50 m – 1.005 m) = 14.3 N.

(c) $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{14.3\text{ N}}{0.0673\text{ kg/m}}} = 14.6\text{ m/s}.$

(d) We want the wave speed v. The wave pattern has 4 antinodes, so the chain is vibrating in its 4th harmonic, so $n = 4$. $\lambda_n = 2L/n$ = (3.00 m)/4 = 0.750 m. Using $v = f\lambda$ gives $f = v/\lambda$ = (14.6 m/s)/(0.750 m) = 19.5 Hz.

(e) The motionless beads are at the nodes. In the 4th harmonic, nodes occur at the ends of the chain, in the middle, and ¼ and ¾ of the way along the chain. The beads at these locations are #1, 26, 51, 76, and 101.

(f) Bead #26 is at a node, so bead #13 is at an antinode, so its maximum speed is $v_{y,\max} = A\omega$. Using $\omega = 2\pi f$, we get $A = \dfrac{v_{y,\max}}{\omega} = \dfrac{v_{y,\max}}{2\pi f} = \dfrac{7.54\text{ m/s}}{2\pi(19.5\text{ Hz})} = 0.0615\text{ m} = 6.15\text{ cm}.$

(g) When the chain is stretched, the beads are 15.0 mm apart center-to-center, with the origin at the center of the first bead. The first few coordinates are:

x_1 = 0 mm = (1 – 1)(15.0 mm)

x_2 = 10.0 mm = (2 – 1)(15.0 mm)

x_3 = 20.0 mm = (3 – 1)(15.0 mm)

x_4 = 30.0 mm = (4 – 1)(15.0 mm)

From this pattern we recognize that the nth bead is at $x_n = (n - 1)(15.0\text{ mm})$, where n = 1, 2, … .

(h) The 30th bead is at x_{30} = (30 – 1)(15.0 mm) = 435 mm = 0.435 m. The standing wave equation is $y(x,t) = A_{\text{SW}} \sin kx \sin \omega t$. Thus $v_y = \dfrac{\partial y}{\partial t} = \omega A_{\text{SW}} \sin kx \cos \omega t$, so at any position x, $v_{y\text{-max}} = \omega A_{\text{SW}} \sin kx = 2\pi f A_{\text{SW}} \sin kx$. From part (f) A_{SW} = 0.0615 m, and $k = 2\pi/\lambda$ = 2π/(0.750 m) = 8.378 m^{-1}. Therefore $v_{y,\max}$ = 2π(19.5 Hz)(0.0615 m) sin[(8.3778 m^{-1})(0.435 m)] = –3.63 m/s, so the maximum *speed* is 3.63 m/s.

EVALUATE: The 30th bead has a smaller maximum speed than the 13th bead. This is reasonable since the 13th bead is at an antinode but the 30th bead is not.

15.64. **IDENTIFY** and **SET UP:** $v = \sqrt{F/\mu}$ is the wave speed and $v_y = \dfrac{\partial y}{\partial t}$ is the transverse speed of a point on the string. $v = f\lambda$, $\lambda_n = 2L/n$, $\mu = m/L$, $v_{\max} = \omega A$ (maximum v_y), $a_{\max} = \omega^2 A$ (maximum a_y).

EXECUTE: **(a)** $\mu = m/L$ = (0.00300 kg)/(2.20 m) = 0.0013636 kg/m. In the fundamental mode, n = 1, so $\lambda_n = 2L/n$ = 2L. Combining $v = f\lambda$ and $v = \sqrt{F/\mu}$, we get $f\lambda = \sqrt{F/\mu}$. Putting in the numbers get f(2)(2.20 m) = [(330 N)/(0.0013636 kg/m)]$^{1/2}$, which gives f = 111.8 Hz. Now use $v_{\max} = \omega A$ to get $A = v_{\max}/(2\pi f)$ = (9.00 m/s)/[2π(111.8 Hz)] = 0.0128 m = 1.28 cm.

(b) $a_{\max} = \omega^2 A = (2\pi f)^2 A$ = [2π(111.8 Hz)]^2(0.0128 m) = 6320 m/s^2.

EVALUATE: It is important to distinguish between the transverse velocity of a point on the string, $v_y = \dfrac{\partial y}{\partial t}$, and speed of the wave, $v = \sqrt{F/\mu}$. The wave speed is constant in time, but the transverse speed is not. The maximum acceleration of a point on the string is 645g!

15.65. **IDENTIFY** and **SET UP:** $v = \sqrt{F/\mu}$ is the wave speed and $v_y = \dfrac{\partial y}{\partial t}$ is the transverse speed of a point on the string. $v = f\lambda$, $\lambda_n = 2L/n$, $\mu = m/L$, $v_{\max} = \omega A$ (maximum v_y), $a_{\max} = \omega^2 A$ (maximum a_y). The first overtone is the second harmonic (n = 2).

EXECUTE: **(a)** $v_{\max} = \omega A = 2\pi f A$, which gives 28.0 m/s = (0.0350 m)(2πf), so f = 127.32 Hz. $\lambda_n = 2L/n$ = 2L/2 = L = 2.50 m. Using these results to get v gives $v = f\lambda$ = (127.32 Hz)(2.50 m) = 318.3 m/s. Now combine $v = \sqrt{F/\mu}$ and $\mu = m/L$ to find m, giving

$m = LF/v^2 = (2.50\text{ m})(90.0\text{ N})/(318.3\text{ m/s})^2 = 0.00222\text{ kg} = 2.22\text{ g}$.

(b) $a_{\max} = \omega^2 A = A(2\pi f)^2 = (0.0350\text{ m})[2\pi(127.32\text{ Hz})]^2 = 22{,}400\text{ m/s}^2$.

EVALUATE: It is important to distinguish between the transverse velocity of a point on the string, $v_y = \dfrac{\partial y}{\partial t}$, and speed of the wave, $v = \sqrt{F/\mu}$. The wave speed is constant in time, but the transverse speed is not. The maximum acceleration of a point on the string is about 2300g!

15.66. **IDENTIFY:** The displacement of the string at any point is $y(x,t) = (A_{SW}\sin kx)\sin\omega t$. For the fundamental mode $\lambda = 2L$, so at the midpoint of the string $\sin kx = \sin(2\pi/\lambda)(L/2) = 1$, and $y = A_{SW}\sin\omega t$. The transverse velocity is $v_y = \partial y/\partial t$ and the transverse acceleration is $a_y = \partial v_y/\partial t$.

SET UP: Taking derivatives gives $v_y = \dfrac{\partial y}{\partial t} = \omega A_{SW}\cos\omega t$, with maximum value $v_{y,\max} = \omega A_{SW}$, and $a_y = \dfrac{\partial v_y}{\partial t} = -\omega^2 A_{SW}\sin\omega t$, with maximum value $a_{y,\max} = \omega^2 A_{SW}$.

EXECUTE: **(a)** $\omega = a_{y,\max}/v_{y,\max} = (8.40\times10^3\text{ m/s}^2)/(3.80\text{ m/s}) = 2.21\times10^3\text{ rad/s}$, and then $A_{SW} = v_{y,\max}/\omega = (3.80\text{ m/s})/(2.21\times10^3\text{ rad/s}) = 1.72\times10^{-3}\text{ m}$.

(b) $v = \lambda f = (2L)(\omega/2\pi) = L\omega/\pi = (0.386\text{ m})(2.21\times10^3\text{ rad/s})/\pi = 272\text{ m/s}$.

EVALUATE: The maximum transverse velocity and acceleration will have different (smaller) values at other points on the string.

15.67. **IDENTIFY:** The standing wave frequencies are given by $f_n = n\left(\dfrac{v}{2L}\right)$. $v = \sqrt{F/\mu}$. Use the density of steel to calculate μ for the wire.

SET UP: For steel, $\rho = 7.8\times10^3\text{ kg/m}^3$. For the first overtone standing wave, $n = 2$.

EXECUTE: $v = \dfrac{2Lf_2}{2} = (0.550\text{ m})(311\text{ Hz}) = 171\text{ m/s}$. The volume of the wire is $V = (\pi r^2)L$. $m = \rho V$ so $\mu = \dfrac{m}{L} = \dfrac{\rho V}{L} = \rho\pi r^2 = (7.8\times10^3\text{ kg/m}^3)\pi(0.57\times10^{-3}\text{ m})^2 = 7.96\times10^{-3}\text{ kg/m}$. The tension is $F = \mu v^2 = (7.96\times10^{-3}\text{ kg/m})(171\text{ m/s})^2 = 233\text{ N}$.

EVALUATE: The tension is not large enough to cause much change in length of the wire.

15.68. **IDENTIFY:** The standing wave is given by $y(x,t) = (A_{SW}\sin kx)\sin\omega t$.

SET UP: At an antinode, $\sin kx = 1$. $v_{y,\max} = \omega A$. $a_{y,\max} = \omega^2 A$.

EXECUTE: **(a)** $\lambda = v/f = (192.0\text{ m/s})/(240.0\text{ Hz}) = 0.800\text{ m}$, and the wave amplitude is $A_{SW} = 0.400\text{ cm}$. The amplitude of the motion at the given points is

(i) $(0.400\text{ cm})\sin(\pi) = 0$ (a node) (ii) $(0.400\text{ cm})\sin(\pi/2) = 0.400\text{ cm}$ (an antinode)

(iii) $(0.400\text{ cm})\sin(\pi/4) = 0.283\text{ cm}$

(b) The time is half of the period, or $1/(2f) = 2.08\times10^{-3}\text{ s}$.

(c) In each case, the maximum velocity is the amplitude multiplied by $\omega = 2\pi f$ and the maximum acceleration is the amplitude multiplied by $\omega^2 = 4\pi^2 f^2$:

(i) 0, 0; (ii) 6.03 m/s, $9.10\times10^3\text{ m/s}^2$; (iii) 4.27 m/s, $6.43\times10^3\text{ m/s}^2$.

EVALUATE: The amplitude, maximum transverse velocity, and maximum transverse acceleration vary along the length of the string. But the period of the simple harmonic motion of particles of the string is the same at all points on the string.

15.69. **IDENTIFY:** When the rock is submerged in the liquid, the buoyant force on it reduces the tension in the wire supporting it. This in turn changes the frequency of the fundamental frequency of the vibrations of the wire. The buoyant force depends on the density of the liquid (the target variable). The vertical forces on the rock balance in both cases, and the buoyant force is equal to the weight of the liquid displaced by the rock (Archimedes's principle).

SET UP: The wave speed is $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$. $B=\rho_{\text{liq}}V_{\text{rock}}g$. $\Sigma F_y=0$.

EXECUTE: $\lambda=2L=6.00$ m. In air, $v=f\lambda=(42.0\text{ Hz})(6.00\text{ m})=252$ m/s. $v=\sqrt{\dfrac{F}{\mu}}$ so

$\mu=\dfrac{F}{v^2}=\dfrac{164.0\text{ N}}{(252\text{ m/s})^2}=0.002583$ kg/m. In the liquid, $v=f\lambda=(28.0\text{ Hz})(6.00\text{ m})=168$ m/s.

$F=\mu v^2=(0.002583\text{ kg/m})(168\text{ m/s})^2=72.90$ N. $F+B-mg=0$.

$B=mg-F=164.0\text{ N}-72.9\text{ N}=91.10$ N. For the rock,

$V=\dfrac{m}{\rho}=\dfrac{(164.0\text{ N}/9.8\text{ m/s}^2)}{3200\text{ kg/m}^3}=5.230\times10^{-3}\text{ m}^3$. $B=\rho_{\text{liq}}V_{\text{rock}}g$ and

$\rho_{\text{liq}}=\dfrac{B}{V_{\text{rock}}g}=\dfrac{91.10\text{ N}}{(5.230\times10^{-3}\text{ m}^3)(9.8\text{ m/s}^2)}=1.78\times10^3\text{ kg/m}^3$.

EVALUATE: This liquid has a density 1.78 times that of water, which is rather dense but not impossible.

15.70. **IDENTIFY:** We model a vibrating stretched rubber band as a standing wave pattern, fixed at both ends.

SET UP: **(a)** Estimate: Tension is about 1 lb ≈ 4.5 N. We use $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$. Our target variable is the frequency of vibration.

EXECUTE: **(b)** $\mu=m/L=(0.10\text{ g})/10\text{ cm})=1.0\times10^{-3}$ kg/m.

(c) If we simply pluck the rubber band, it vibrates in its fundamental mode, so $\lambda=2L=0.20$ m.

Combining $v=\sqrt{\dfrac{F}{\mu}}$ and $v=f\lambda$ gives $f=\dfrac{1}{\lambda}\sqrt{\dfrac{F}{\mu}}=\dfrac{1}{0.20\text{ m}}\sqrt{\dfrac{4.5\text{ N}}{1.0\times10^{-3}\text{ kg/m}}}=340$ Hz.

EVALUATE: This result seems somewhat high, but our model is just a rough approximation.

15.71. **IDENTIFY:** Compute the wavelength from the length of the string. Use $v=f\lambda$ to calculate the wave speed and then apply $v=\sqrt{F/\mu}$ to relate this to the tension.

(a) SET UP: The tension F is related to the wave speed by $v=\sqrt{F/\mu}$, so use the information given to calculate v.

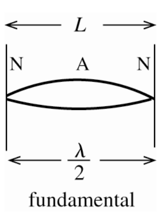


Figure 15.71

EXECUTE: $\lambda/2=L$

$\lambda=2L=2(0.600\text{ m})=1.20$ m

$v=f\lambda=(65.4\text{ Hz})(1.20\text{ m})=78.5$ m/s

$\mu=m/L=14.4\times10^{-3}\text{ kg}/0.600\text{ m}=0.024$ kg/m

Then $F=\mu v^2=(0.024\text{ kg/m})(78.5\text{ m/s})^2=148$ N.

(b) SET UP: $F = \mu v^2$ and $v = f\lambda$ give $F = \mu f^2 \lambda^2$.

μ is a property of the string so is constant.

λ is determined by the length of the string so stays constant.

$\mu,\ \lambda$ constant implies $F/f^2 = \mu\lambda^2 = \text{constant}$, so $F_1/f_1^2 = F_2/f_2^2$.

EXECUTE: $F_2 = F_1\left(\dfrac{f_2}{f_1}\right)^2 = (148\text{ N})\left(\dfrac{73.4\text{ Hz}}{65.4\text{ Hz}}\right)^2 = 186\text{ N}.$

The percent change in F is $\dfrac{F_2 - F_1}{F_1} = \dfrac{186\text{ N} - 148\text{ N}}{148\text{ N}} = 0.26 = 26\%.$

EVALUATE: The wave speed and tension we calculated are similar in magnitude to values in the examples. Since the frequency is proportional to $\sqrt{F}$, a 26% increase in tension is required to produce a 13% increase in the frequency.

15.72. **IDENTIFY:** The mass and breaking stress determine the length and radius of the string. $f_1 = \dfrac{v}{2L}$, with $v = \sqrt{\dfrac{F}{\mu}}.$

SET UP: The tensile stress is $F/\pi r^2$.

EXECUTE: **(a)** The breaking stress is $\dfrac{F}{\pi r^2} = 7.0\times 10^8\text{ N/m}^2$ and the maximum tension is $F = 900$ N, so solving for r gives the minimum radius $r = \sqrt{\dfrac{900\text{ N}}{\pi(7.0\times 10^8\text{ N/m}^2)}} = 6.4\times 10^{-4}$ m. The mass and density are fixed, $\rho = \dfrac{M}{\pi r^2 L}$. so the minimum radius gives the maximum length

$$L = \frac{M}{\pi r^2 \rho} = \frac{4.0\times 10^{-3}\text{ kg}}{\pi(6.4\times 10^{-4}\text{ m})^2(7800\text{ kg/m}^3)} = 0.40\text{ m}.$$

(b) The fundamental frequency is $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{F}{\mu}} = \dfrac{1}{2L}\sqrt{\dfrac{F}{M/L}} = \dfrac{1}{2}\sqrt{\dfrac{F}{ML}}$. Assuming the maximum length of the string is free to vibrate, the highest fundamental frequency occurs when $F = 900$ N and

$$f_1 = \frac{1}{2}\sqrt{\frac{900\text{ N}}{(4.0\times 10^{-3}\text{ kg})(0.40\text{ m})}} = 375\text{ Hz}.$$

EVALUATE: If the radius was any smaller the breaking stress would be exceeded. If the radius were greater, so the stress was less than the maximum value, then the length would be less to achieve the same total mass.

15.73. **IDENTIFY** and **SET UP:** Assume that the mass M is large enough so that there no appreciable motion of the string at the pulley or at the oscillator. For a string fixed at both ends, $\lambda_n = 2L/n$. The node-to-node distance d is $\lambda/2$, so $d = \lambda/2$. $v = f\lambda = \sqrt{F/\mu}$.

EXECUTE: **(a)** Because it is essentially fixed at its ends, the string can vibrate in only wavelengths for which $\lambda_n = 2L/n$, so $d = \lambda/2 = L/n$, where $n = 1, 2, 3, \ldots$.

(b) $f\lambda = \sqrt{F/\mu}$ and $\lambda = 2d$. Combining these two conditions and squaring gives $f^2(4d^2) = T/\mu = Mg/\mu$. Solving for μd^2 gives $\mu d^2 = \left(\dfrac{g}{4f^2}\right)M$. Therefore the graph of μd^2 versus M should be a straight line having slope equal to $g/4f^2$. Figure 15.73 shows this graph.

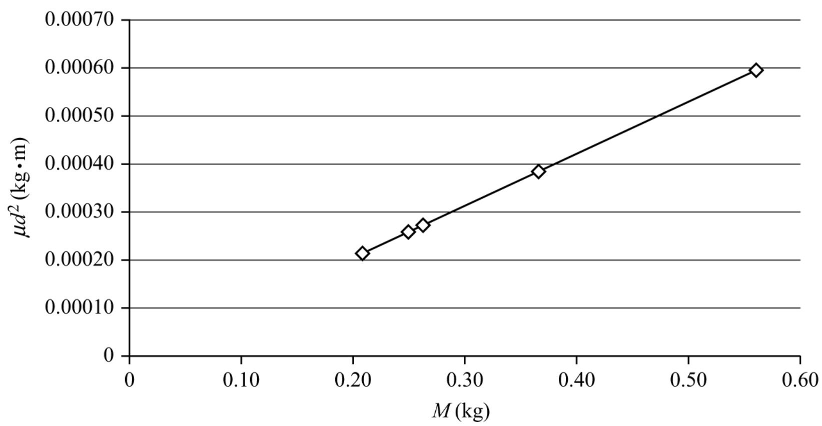


Figure 15.73

(c) The best fit straight line for the data has the equation μd^2 = (0.001088 m)M – 0.00009074 kg·m. The slope is $g/4f^2$, so $g/4f^2$ = 0.001088 m. Solving for f gives f = 47.5 Hz.
(d) For string A, μ = 0.0260 g/cm = 0.00260 kg/m. We want the mass M for λ = 48.0 cm. Using $f\lambda = \sqrt{F/\mu}$ where $F = Mg$, squaring and solving for M, we get $M = \dfrac{\mu(f\lambda)^2}{g}$. Putting in the numbers gives M = (0.00260 kg/m)[(47.5 Hz)(0.480 m)]^2/(9.80 m/s^2) = 0.138 kg = 138 g.
EVALUATE: In part (d), if the string is vibrating in its fundamental mode, n = 1, so $d = L$ = 48.0 cm. The mass of the string in that case would be $m = \mu L$ = (0.00260 kg/m)(0.48 m) = 0.00125 kg = 1.25 g, so the string would be much lighter than the 138-g weight attached to it.

15.74. **IDENTIFY** and **SET UP:** We have a standing wave in its fundamental mode on each string of the guitar. $v = \sqrt{F/\mu}$, $v = f\lambda$, $\lambda_n = 2L/n$, 25.5 in. = 64.77 cm = 0.6477 m.

EXECUTE: **(a)** Each string has the same length, so the wavelength in the fundamental mode is $\lambda_n = 2L/n = 2L$ = 2(0.6477 m) = 1.2954 m.

Combining $v = \sqrt{F/\mu}$ and $v = f\lambda$ and solving for μ gives $\mu = F/(f\lambda)^2$.

For the E2 string, we have μ_{E2} = (78.0 N)/[(82.4 Hz)(1.2954 m)]^2 = 0.006846 kg/m = 0.0685 g/cm. Using similar calculations for the other strings, we get μ_{G3} = 0.0121 g/cm and μ_{E4} = 0.00428 g/cm.
(b) From $f\lambda = \sqrt{F/\mu}$, we get $F = \mu(f\lambda)^2$ = (0.006846 kg/m)[(196.0 Hz)(1.2954 m)]^2 = 441 N.

EVALUATE: A tension of 441 N is nearly 100 lb, which is why a string will snap violently if it happens to break.

15.75. **IDENTIFY** and **SET UP:** $P_{av} = \dfrac{1}{2}\sqrt{\mu F}\omega^2 A^2$, $v = \sqrt{F/\mu}$, and $\omega = 2\pi f$.

EXECUTE: Combining $P_{av} = \dfrac{1}{2}\sqrt{\mu F}\omega^2 A^2$ and $\omega = 2\pi f$ gives $P_{av} = 2\pi^2\sqrt{\mu F}A^2 f^2$. Therefore a graph of P_{av} versus f^2 should be a straight line having slope $2\pi^2\sqrt{\mu F}A^2$.
(b) Calculate the slope from the graph shown in the problem. Estimating the points (40,000 Hz^2, 12.2 W) and (10,000 Hz^2, 3 W), we get a slope of (9.2 W)/(30,000 Hz^2) = 3.07×10^{-4} W/Hz^2. (Answers here will vary, depending on the accuracy in reading the graph.) We can get F from the slope of the graph and then use $v = \sqrt{F/\mu}$ to calculate v. Using our measured slope, we have $2\pi^2\sqrt{\mu F}A^2$ = slope = 3.07×10^{-4} W/Hz^2. Solving for F gives $F = \dfrac{(\text{slope})^2}{4\pi^4 A^4 \mu}$. Putting this result into $v = \sqrt{F/\mu}$ gives us

$v=\sqrt{\dfrac{F}{\mu}}=\sqrt{\dfrac{(\text{slope})^2}{4\pi^4A^4\mu^2}}=\dfrac{\text{slope}}{2\pi^2A^2\mu}$. This gives $v = (3.07\times10^{-4}\ \text{W/Hz}^2)/[2\pi^2(0.0040\ \text{m})^2(0.0035\ \text{kg/m})] =$ 280 m/s.

(c) From the graph, $P = 10.0$ W corresponds to $f^2 = 33{,}000\ \text{Hz}^2$, so

$\omega = 2\pi f = 2\pi\sqrt{33{,}000\ \text{Hz}^2} = 1100$ rad/s.

EVALUATE: At 280 m/s and with an angular frequency of 1100 rad/s, the string is moving too fast to follow individual waves.

15.76. **IDENTIFY:** The membrane is stretched and caused to vibrate in a standing wave pattern. We can visualize this membrane as n tiny strings, each of length L, vibrating together.

SET UP: We want to investigate the characteristics of its vibrational motion. We use $v=\sqrt{\dfrac{F}{\mu}}$, $v = f\lambda$, $k = 2\pi/\lambda$, $P=\sqrt{\mu F}\,\omega^2A^2\sin^2(kx-\omega t)$, and $P_{\text{av}}=\dfrac{1}{2}\sqrt{\mu F}\,\omega^2A^2$.

EXECUTE: **(a)** The tension F is equally divided among the n tiny strings, as is the mass M. Therefore the speed of the wave on each string is $v=\sqrt{\dfrac{F/n}{\mu/n}}=\sqrt{\dfrac{F}{\mu}}$. Using $\mu = M/L$, we get $v=\sqrt{\dfrac{FL}{M}}$. For the values here, we have $v=\sqrt{\dfrac{(81.0\ \text{N})(4.00\ \text{m})}{4.00\ \text{kg}}}=9.00$ m/s.

(b) We want k, so first find λ. Solving $v = f\lambda$ for λ gives $\lambda = v/f$. Therefore we get

$k=\dfrac{2\pi}{\lambda}=\dfrac{2\pi}{v/f}=\dfrac{2\pi f}{v}=\dfrac{2\pi(1.00\ \text{Hz})}{9.00\ \text{m/s}}=0.698\ \text{m}^{-1}$.

(c) $P=\sqrt{\mu F}\,\omega^2A^2\sin^2(kx-\omega t) = \sqrt{(M/L)F}\,(2\pi f)^2A^2\sin^2(kx-\omega t)$, which we can write as

$P=\sqrt{\dfrac{MF}{L}}4\pi^2f^2A^2\sin^2(kx-\omega t).$

(d) $E=P_{\text{av}}t=\dfrac{1}{2}\sqrt{\dfrac{MF}{L}}4\pi^2f^2A^2t=2\pi^2f^2A^2\sqrt{\dfrac{MF}{L}}t.$ For the numbers here, we get

$E=2\pi^2(1.0\ \text{Hz})^2(0.100\ \text{m})^2\sqrt{\dfrac{(4.00\ \text{kg})(81.0\ \text{N})}{4.00\ \text{m}}}(1.00\ \text{s}) = 1.78$ J.

EVALUATE: Think of the membrane as a very wide string.

15.77. **IDENTIFY:** Apply $\Sigma F_y = 0$ to segments of the cable. The forces are the weight of the diver, the weight of the segment of the cable, the tension in the cable and the buoyant force on the segment of the cable and on the diver.

SET UP: The buoyant force on an object of volume V that is completely submerged in water is $B = \rho_{\text{water}}Vg$.

EXECUTE: **(a)** The tension is the difference between the diver's weight and the buoyant force,

$F=(m-\rho_{\text{water}}V)g=\left[120\ \text{kg}-(1000\ \text{kg/m}^3)(0.0800\ \text{m}^3)\right](9.80\ \text{m/s}^2)=392\ \text{N}.$

(b) The increase in tension will be the weight of the cable between the diver and the point at x, minus the buoyant force. This increase in tension is then

$\left[\mu x-\rho(Ax)\right]g=\left[1.10\ \text{kg/m}-(1000\ \text{kg/m}^3)\pi(1.00\times10^{-2}\ \text{m})^2\right](9.80\ \text{m/s}^2)x=(7.70\ \text{N/m})x.$ The tension as a function of x is then $F(x)=(392\ \text{N})+(7.70\ \text{N/m})x.$

(c) Denote the tension as $F(x) = F_0 + ax$, where $F_0 = 392\text{ N}$ and $a = 7.70\text{ N/m}$. Then the speed of transverse waves as a function of *x* is $v = \frac{dx}{dt} = \sqrt{(F_0 + ax)/\mu}$ and the time *t* needed for a wave to reach the surface is found from $t = \int dt = \int \frac{dx}{dx/dt} = \int \frac{\sqrt{\mu}}{\sqrt{F_0 + ax}}\, dx.$

Let the length of the cable be *L*, so $t = \sqrt{\mu}\int_0^L \frac{dx}{\sqrt{F_0 + ax}} = \sqrt{\mu}\frac{2}{a}\sqrt{F_0 + ax}\Big|_0^L = \frac{2\sqrt{\mu}}{a}\left(\sqrt{F_0 + aL} - \sqrt{F_0}\right).$

$$t = \frac{2\sqrt{1.10\text{ kg/m}}}{7.70\text{ N/m}}(\sqrt{392\text{ N} + (7.70\text{ N/m})(100\text{ m})} - \sqrt{392\text{ N}}) = 3.89\text{ s}.$$

EVALUATE: If the weight of the cable and the buoyant force on the cable are neglected, then the tension would have the constant value calculated in part (a). Then $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{392\text{ N}}{1.10\text{ kg/m}}} = 18.9\text{ m/s}$ and $t = \frac{L}{v} = 5.29\text{ s}$. The weight of the cable increases the tension along the cable and the time is reduced from this value.

15.78. **IDENTIFY** and **SET UP:** Apply $v = f\lambda$.

EXECUTE: $v = f\lambda$ gives (125 Hz) λ = 3.75 m/s, so λ = 0.030 m = 3 cm, which is choice (d).

EVALUATE: These are waves on the vocal cords, not sound waves in air.

15.79. **IDENTIFY** and **SET UP:** Using the figure shown with the problem in the text, the wave is traveling in the +*z*-direction, so it must be of the form $A\sin(kz - \omega t)$.

EXECUTE: The required wave is of the form $A\sin(kz - \omega t)$. Putting this in terms of *f* and *v* gives $A\sin(kz - \omega t) = \sin[-(\omega t - kz)] = \sin[-(2\pi ft - 2\pi z/\lambda)] = \sin[-2\pi f(t - z/f\lambda)] = -\sin[2\pi f(t - z/v)]$, which is of the form of choice (b).

EVALUATE: A wave traveling in the opposite direction would be of the form $\sin[2\pi f(t + z/v)]$.

15.80. **IDENTIFY** and **SET UP:** The graph of *v* versus *f* is a straight line that appears to pass through the origin, so *v* is directly proportional to *v*. $v = f\lambda$.

EXECUTE: $v = f\lambda$ gives λ = *v/f*. From the graph of *v* versus *f*, *v* is proportional to *f*, so *v* = *Kf*, where *K* is a constant. Thus λ = *v/f* = *Kf/f* = *K* = constant, which is choice (c).

EVALUATE: Normally we expect the wavelength to decrease as the frequency increases, but that is only true if the wave speed is consant. In this case the speed depends on the frequency, so it is possible for the wavelength to remain constant.