# 3

# MOTION IN TWO OR THREE DIMENSIONS

**VP3.9.1.** **IDENTIFY:** This is a projectile problem with only vertical acceleration.

**SET UP:** The formulas $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply.

**EXECUTE:** We know the launch speed and launch angle of the projectile, and want to know its maximum height, maximum range, and the time it is in the air.
(a) At the maximum height, $v_y = 0$. We use $v_y = v_0 \sin\alpha_0 - gt$ to find the time.
0 = (25.0 m/s)(sin 36.9°) – (9.80 m/s$^2$)$t$ → $t$ = 1.53 s. Now use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$.
0 = [(25.0 m/s)(sin 36.9°)]$^2$ – 2(9.80 m/s$^2$)($y - y_0$) → $y - y_0$ = 11.5 m.
(b) When the projectile returns to ground level, $v_y = -v_0 \sin\alpha_0$. So
$v_y = v_0 \sin\alpha_0 - gt = -v_0 \sin\alpha_0$
$2v_0 \sin\alpha_0 = gt$
2(25.0 m/s)(sin 36.9°) = (9.80 m/s$^2$)$t$ → $t$ = 3.06 s.
For the horizontal motion, we have
$x = (v_0 \cos\alpha_0)t$ = (25.0 m/s)(cos 36.9°)(3.06 s) = 61.2 m.

**EVALUATE:** (a) Calculate $y$ using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.
$y$ = (25.0 m/s)(sin 36.9°)(1.53 s) – $\frac{1}{2}$(9.80 m/s$^2$)(1.53 s)$^2$ = 11.5 m, which agrees with our result. (b) The acceleration is constant, so the time for the upward motion is equal to the time for the downward motion. Thus $t_{tot}$ = 1.53 s + 1.53 s = 3.06 s, which agrees with our result.

**VP3.9.2.** **IDENTIFY:** This is a projectile problem with only vertical acceleration.

**SET UP:** The formulas $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ apply. At the highest point in the baseball's trajectory, its vertical velocity is zero, but its horizontal velocity is the same as when it left the ground.

**EXECUTE:** (a) At the highest point, $v_y = 0$, so $v_y = v_0 \sin\alpha_0 - gt$ gives
0 = $v_0$ sin(30.0°) – (9.80 m/s$^2$)(1.05 s)$^2$ → $v_0$ = 20.6 m/s.
(b) Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives
0 = [(20.6 m/s)(sin 30.0°)]$^2$ – 2(9.80 m/s$^2$)($y - y_0$) → $y - y_0$ = 5.40 m

**EVALUATE:** We can check by finding $y$ when $t = 1.05$ s, using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.

$y$ = (20.6 m/s)(sin 30.0°)(1.05 s) – $\frac{1}{2}$(9.80 m/s$^2$)(1.05 s)$^2$ = 5.40, which agrees with our result.

**VP3.9.3.** **IDENTIFY:** This is a projectile problem with only vertical acceleration.

**SET UP:** The formulas $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, $x = (v_0 \cos\alpha_0)t$, and $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$ apply. Call the origin the launch point with the +$y$-axis vertically upward.

**EXECUTE:** (a) When the walnut reaches the ground, $y$ = –20.0 m. Use $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.

–20.0 m = (15.0 m/s)(sin 50.0°)$t$ – $\frac{1}{2}$(9.80 m/s$^2$)$t^2$. Using the quadratic formula gives two roots, only one of which is positive: $t$ = 3.51 s.

(b) There is no horizontal acceleration, so we use $x = (v_0 \cos\alpha_0)t$ to find the distance $x$.

$x = (v_0 \cos\alpha_0)t$ = (15.0 m/s)(cos 50.0°)(3.51 s) = 33.8 m.

(c) There is no horizontal acceleration, so $v_x = v_0 \cos\alpha_0$ = (15.0 m/s)(cos 50.0°) = 9.64 m/s. The vertical velocity is

$v_y = v_0 \sin\alpha_0 - gt$ = (15.0 m/s) sin50.0° – (9.80 m/s$^2$)(3.51 s) = –22.9 m/s. The minus sign tells us the walnut is moving downward.

**EVALUATE:** We cannot use the projectile range formula because the landing point is not on the same level as the launch point.

**VP3.9.4.** **IDENTIFY:** This is a projectile problem with only vertical acceleration.

**SET UP:** The formulas $v_x = v_0 \cos\alpha_0$, $v_y = v_0 \sin\alpha_0 - gt$, $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, and $x = (v_0 \cos\alpha_0)t$ apply. Call the origin the launch point with the +$y$-axis vertically upward.

**EXECUTE:** (a) The horizontal and vertical distances are equal, so $y = -x$, so

$-(v_0 \cos\alpha_0)t$ = $(v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$. Since $\alpha_0$ = 0, we have $v_0t = \frac{1}{2}gt^2$, which gives $t = 2v_0/g$. At this time $x = v_0t = v_0(2v_0/g) = 2v_0^2/g$. Since $-x = y$, $y = -2v_0^2/g$.

(b) When the potato is moving at 45° below the horizontal, $v_y = -v_x$. $v_x = v_0$ and $v_y = -v_0 = -gt$. Therefore $t = v_0/g$. At this time, $x = v_0t = v_0(v_0/g) = v_0^2/g$, and $y = -\frac{1}{2}gt^2 = -\frac{1}{2}g(v_0/g)^2 = -v_0^2/2g$.

**EVALUATE:** As a check, solve for $y$ using the time found in part (a).

$y = -\frac{1}{2}gt^2 = -\frac{1}{2}g(2v_0/g)^2 = -2v_0^2/g$, just as we found. Notice in part (b) that $x \neq y$ when the potato is traveling at 45° with the horizontal, but the magnitudes of the velocity are equal. We cannot use the projectile range formula because the landing point is not on the same level as the launch point.

**VP3.12.1.** **IDENTIFY:** This problem involves circular motion at constant speed. The acceleration of the cyclist is toward the center of the circle.

**SET UP:** The radial acceleration of the cyclist is $a_{rad} = \frac{v^2}{R}$ toward the center of the circular track. We know the speed and acceleration and want the radius of the circle.

**EXECUTE:** Solve $a_{rad} = \frac{v^2}{R}$ for $R$, giving $R = v^2/a_{rad}$ = (10.0 m/s)$^2$/(5.00 m/s$^2$) = 20.0 m.

**EVALUATE:** The speed of the cyclist is constant, but not the velocity since it is tangent to the circular path and is always changing direction.

**VP3.12.2.** **IDENTIFY:** This problem involves circular motion at constant speed. The acceleration of the car is toward the center of the circle.

**SET UP:** The radial acceleration of the car is $a_{\text{rad}} = \dfrac{v^2}{R}$ toward the center of the circular track. In terms of the period of motion, the acceleration is $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. We know the speed of the car and the radius of the circle and want to find the period of the motion and centripetal acceleration of the car.

**EXECUTE:** (a) Equate the two expressions for $a_{\text{rad}}$, giving $\dfrac{4\pi^2 R}{T^2} = \dfrac{v^2}{R}$. Now solve for $T$.

$T = \sqrt{\dfrac{4\pi^2 R^2}{v^2}} = 2\pi R/v = 2\pi(265\text{ m})/(40.0\text{ m/s}) = 41.6\text{ s}$.

(b) $a_{\text{rad}} = \dfrac{v^2}{R} = (40.0\text{ m/s})^2/(265\text{ m}) = 6.04\text{ m/s}^2$.

**EVALUATE:** We can find the period by realizing that $vT$ is the circumference of the track, which is $2\pi R$. Therefore $vT = 2\pi R$, so $T = 2\pi R/v = 2\pi(265\text{ m})/(40.0\text{ m/s}) = 41.6\text{ s}$, which is agrees with our answer in (a). The acceleration of the car (and the driver inside) is quite large, over 60% of $g$.

**VP3.12.3.** **IDENTIFY:** This problem involves circular motion at constant speed. The radial acceleration is toward the center of the circle.

**SET UP:** The equation $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ applies and the speed is $v = 2\pi R/T$. The period is the same for all points on the wheel, but the speed (and hence acceleration) is not.

**EXECUTE:** (a) $v_{10} = 2\pi R/T = 2\pi(10.0\text{ cm})/(0.670\text{ s}) = 93.8\text{ cm/s} = 0.938\text{ m/s}$. Since $R$ is twice as great at 20.0 cm, $v_{20} = 2v_{10} = 2(0.938\text{ m/s}) = 1.88\text{ m/s}$.

(b) $a_{10} = 4\pi^2 R/T^2 = 4\pi^2(0.100\text{ m})/(0.670\text{ s})^2 = 8.79\text{ m/s}^2$. As in (a), we see that $a_{20} = 2a_{10} = 2(8.79\text{ m/s}^2) = 17.6\text{ m/s}^2$.

(c) Both the speed and radial acceleration increase as $R$ increases.

**EVALUATE:** As $R$ increases, the points farther from the center must travel a greater distance than points closer to the center. So the speed and acceleration are greater for those distant points.

**VP3.12.4.** **IDENTIFY:** This problem involves circular motion at constant speed. The radial acceleration of a planet is toward the center of the circle, which is essentially the sun.

**SET UP:** The equation $a_{\text{rad}} = \dfrac{v^2}{R}$ applies and the speed of a planet is $v = 2\pi R/T$.

**EXECUTE:** (a) Apply $v = 2\pi R/T$ to each planet using the data given.

$v_V = 2\pi(1.08\times10^{11}\text{ m})/[(225)(86{,}500)\text{s}] = 3.49\times10^4\text{ m/s}$. Similar calculations for the earth and Mars gives $v_E = 2.99.\times.10^4$ m/s and $v_M = 2.41\times10^4$ m/s.

(b) Use $a_{\text{rad}} = \dfrac{v^2}{R}$ with the speeds found in part (a).

$a_V = (3.49\times10^4\text{ m/s})^2/(1.08\times10^{11}\text{ m}) = 1.13\times10^{-2}\text{ m/s}^2$. Likewise we get $a_E = 5.95\times10^{-3}\text{ m/s}^2$ and $a_M = 2.55\times10^{-3}\text{ m/s}^2$.

(c) As the size of the orbit increases, both the orbital speed and the radial acceleration decrease.

**EVALUATE:** Since $R$ increases and $v$ decreases with distance from the sun, it must follow that the radial acceleration also decreases with distance. This is reasonable because the gravitational pull of the sun (to be studied in a later chapter) is weaker for distant planets than for closer ones.

**VP3.12.5.** **IDENTIFY:** This problem involves circular motion at constant speed.

**SET UP:** The equation $a_{\text{rad}} = \dfrac{v^2}{R}$ applies and the speed of an object moving in a circle is $v = 2\pi R/T$.

**EXECUTE:** (a) $v = 2\pi R/T$, so $T = 2\pi R/v$. Take the ratio of the two periods, giving $\dfrac{T_A}{T_B} = \dfrac{2\pi R_A / v_A}{2\pi R_B / v_B} = \dfrac{R_A}{R_B}\dfrac{v_B}{v_A}$. Putting in the given values for the speeds and radii gives $\dfrac{T_A}{T_B} = \dfrac{R(2v)}{(R/2)v} = 4.$

(b) Use $a_{\text{rad}} = \dfrac{v^2}{R}$, take the ratio of the accelerations, and use the given speeds and radii.

$$\frac{a_A}{a_B} = \frac{v_A^2 / R_A}{v_B^2 / R_B} = \left(\frac{v_A}{v_B}\right)^2 \frac{R_B}{R_A} = \left(\frac{v}{2v}\right)^2 \frac{R/2}{R} = \frac{1}{8}.$$

**EVALUATE:** Since $a_{\text{rad}} = \dfrac{v^2}{R}$, doubling the speed will increase $a_{\text{rad}}$ by a factor of 4. Then halving the radius will increase $a_{\text{rad}}$ by another factor of 2, giving a total factor of 8. This tells us that the inner object will have an acceleration 8 times that of the outer object, which is what we found in part (b).

**VP3.15.1.** **IDENTIFY:** This problem is about relative velocities.

**SET UP:** If object $P$ is moving relative to object $B$ and $B$ is moving relative to $A$, then the velocity of $P$ relative to $A$ is given by $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Let subscript $P$ denote the police car, $S$ the SUV, and $E$ the earth.

**EXECUTE:** (a) We have one-dimensional motion, so the relative velocities in the $x$-direction are given by $v_{P/S} = v_{P/E} + v_{E/S}$. Using the given values gives

$v_{P/S} = 35.0$ m/s + 18.0 m/s = 53.0 m/s.

(b) In this case, we want $v_{S/P}$, so

$v_{S/P} = v_{S/E} + v_{E/P} = -18.0$ m/s + (–35.0 m/s) = –53.0 m/s.

**EVALUATE:** A rider in the SUV sees the police car going north at 53.0 m/s, but a rider in the police car sees the SUV going south at 53.0 m/s. Since they are going in opposite directions, their speeds relative to the earth add.

**VP3.15.2.** **IDENTIFY:** This problem is about relative velocities.

**SET UP:** If object $P$ is moving relative to object $B$ and $B$ is moving relative to $A$, then the velocity of $P$ relative to $A$ is given by $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Let subscript $A$ denote the car $A$, $B$ car $B$, and $E$ the earth. The cars have only east-west velocities, so we look at velocity components along those directions.

**EXECUTE:** (a) We want the velocity of car $A$ relative to car $B$.

$v_{A/B} = v_{A/E} + v_{E/B} = 45.0$ m/s + 45.0 m/sk = 90.0 m/s, eastward.

(b) $v_{B/A} = v_{B/E} + v_{E/A} = -45.0$ m/s + (–45.0 m/s) = –90.0 m/s, so the magnitude is 90.0 m/s and the direction is westward.

(c) At the points in this problem, the cars have only east-west velocities, so they have no relative velocity component along the line connecting them. Since both cars are traveling at the same speed in the same clockwise sense in a circle, they always remain the same distance apart. So they are neither approaching nor moving away from each other.

**EVALUATE:** In both cases the cars have east-west velocities in opposite directions, so their velocities relative to the earth add. When $A$ sees $B$ moving eastward at 90.0 m/s, $B$ sees $A$ moving westward at 90.0 m/s.

**VP3.15.3.** **IDENTIFY:** This problem is about relative velocities.

**SET UP:** If object $P$ is moving relative to object $B$ and $B$ is moving relative to $A$, then the velocity of $P$ relative to $A$ is given by $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Let subscript $T$ denote the truck, $S$ the SUV, and $E$ the earth.

**EXECUTE:** For this case, we have $\vec{v}_{T/S} = \vec{v}_{T/E} + \vec{v}_{E/S}$. We want $\vec{v}_{T/S}$. We know that $\vec{v}_{T/E}$ = 16.0 m/s eastbound and $\vec{v}_{S/E}$ = 20.0 m/s southbound. Therefore $\vec{v}_{E/S}$ = 20.0 m/s northbound. Figure VP3.15.3 illustrates the velocity vectors.

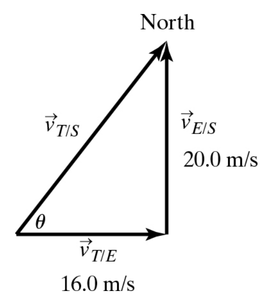



**Figure VP3.15.3**

Applying $A = \sqrt{A_x^2 + A_y^2}$ for the magnitude of a vector, we have

$v_{T/S} = \sqrt{(16.0\text{ m/s})^2 + (20.0\text{ m/s})^2}$ = 25.6 m/s. From the figure, we see that

$\theta$ = arctan[(20.0 m/s)/(16.0 m/s)] = 51.3° north of east.

(b) Using $\vec{v}_{S/T} = -\vec{v}_{T/S}$, the speed is 25.6 m/s, and the direction is 51.3° south of west.

**EVALUATE:** Check (b) by applying the relative velocity formula.

$\vec{v}_{S/T} = \vec{v}_{S/E} + \vec{v}_{E/T} = -20.0\text{ m/s } \hat{i} + (-16.0\text{ m/s})\ \hat{j} = -(16.0\text{ m/s } \hat{i} + 20.0\text{ m/s } \hat{j}) = -\vec{v}_{T/S}$.

**VP3.15.4.** **IDENTIFY:** This problem is about relative velocities.

**SET UP:** If object $P$ is moving relative to object $B$ and $B$ is moving relative to $A$, then the velocity of $P$ relative to $A$ is given by $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$. Let subscript $J$ denote the jet, $A$ the air, and $E$ the earth.

**EXECUTE:** The jet's velocity relative to the earth is $\vec{v}_{J/E} = \vec{v}_{J/A} + \vec{v}_{A/E}$. Figure VP3.15.4 illustrates these vectors. We want to find the magnitude of $\vec{v}_{J/A}$ (the airspeed) and its direction ($\theta$ in the figure). Therefore we first find the components of $\vec{v}_{J/A}$.

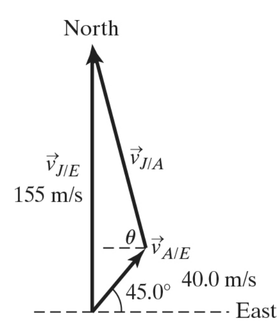



**Figure VP3.15.4**

Since $\vec{v}_{J/E}$ is due north, it has no east-west component. From the figure, we can therefore see that the east-west components of $\vec{v}_{A/E}$ and $\vec{v}_{J/A}$ must have opposite sign and equal magnitudes.

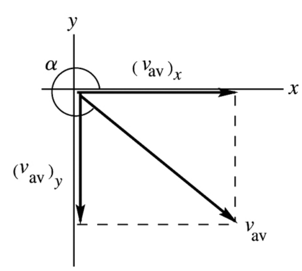
$\vec{v}_{J/A}$ (east component) = $-\vec{v}_{A/E}$ (west component) = –(40.0 m/s) cos 45.0° = –28.28 m/s. From the figure, we also see that
$\vec{v}_{J/A}$ (north component) + $\vec{v}_{A/E}$ (north component) = $\vec{v}_{J/E}$ (north component)
$\vec{v}_{J/A}$ (north component) + (40.0 m/s) cos 45.0° = 155 m/s
$\vec{v}_{J/A}$ (north component) = 126.7 m/s. Now find the magnitude of $\vec{v}_{J/A}$ using its components.
$v_{J/A} = \sqrt{(-28.28\text{ m/s})^2 + (126.7\text{ m/s})^2} = 130$ m/s. From the figure we see that
$\theta$ = arctan[(126.7 m/s)/(28.28 m/s)] = 77.4°. Therefore the airspeed of the jet is 130 m/s and the pilot must point it at 77.4° north of west.
**EVALUATE:** The pilot points the jet in a direction to offset the wind so the plane is flying directly north at 155 m/s as observed by someone standing on the ground.

**3.1.** **IDENTIFY** and **SET UP:** Use $\vec{v}_{\text{av}} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5.3\text{ m} - 1.1\text{ m}}{3.0\text{ s} - 0} = 1.4\text{ m/s}$

$v_{\text{av-}y} = \dfrac{\Delta y}{\Delta t} = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{-0.5\text{ m} - 3.4\text{ m}}{3.0\text{ s} - 0} = -1.3\text{ m/s}$

**(b)**




$\tan\alpha = \dfrac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = \dfrac{-1.3\text{ m/s}}{1.4\text{ m/s}} = -0.9286$

$\alpha = 360° - 42.9° = 317°$

$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2}$

$v_{\text{av}} = \sqrt{(1.4\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 1.9\text{ m/s}$

**Figure 3.1**

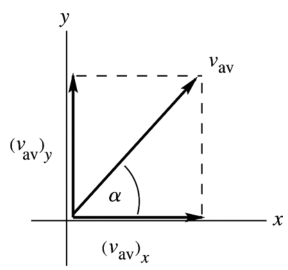
**EVALUATE:** Our calculation gives that $\vec{v}_{\text{av}}$ is in the 4th quadrant. This corresponds to increasing *x* and decreasing *y*.

**3.2.** **IDENTIFY:** Use $\vec{v}_{\text{av}} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form. The distance from the origin is the magnitude of $\vec{r}$.
**SET UP:** At time $t_1$, $x_1 = y_1 = 0$.
**EXECUTE:** **(a)** $x = (v_{\text{av-}x})\Delta t = (-3.8\text{ m/s})(12.0\text{ s}) = -45.6\text{ m}$ and $y = (v_{\text{av-}y})\Delta t = (4.9\text{ m/s})(12.0\text{ s}) = 58.8\text{ m}$.
**(b)** $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\text{ m})^2 + (58.8\text{ m})^2} = 74.4\text{ m}$.
**EVALUATE:** $\Delta\vec{r}$ is in the direction of $\vec{v}_{\text{av}}$. Therefore, $\Delta x$ is negative since $v_{\text{av-}x}$ is negative and $\Delta y$ is positive since $v_{\text{av-}y}$ is positive.

**3.3.** **(a) IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate *x* and *y* for any *t*.
Then use $\vec{v}_{\text{av}} = \dfrac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$ in component form.
**EXECUTE:** $\vec{r} = [4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2]\hat{i} + (5.0\text{ cm/s})t\hat{j}$
At $t = 0$, $\vec{r} = (4.0\text{ cm})\,\hat{i}$.
At $t = 2.0$ s, $\vec{r} = (14.0\text{ cm})\,\hat{i} + (10.0\text{ cm})\,\hat{j}$.

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$

$$v_{\text{av-}y} = \frac{\Delta y}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$




$$v_{\text{av}} = \sqrt{(v_{\text{av}})_x^2 + (v_{\text{av}})_y^2} = 7.1\text{ cm/s}$$

$$\tan\alpha = \frac{(v_{\text{av}})_y}{(v_{\text{av}})_x} = 1.00$$

$$\theta = 45°.$$

**Figure 3.3a**

**EVALUATE:** Both $x$ and $y$ increase, so $\vec{v}_{\text{av}}$ is in the 1st quadrant.

**(b) IDENTIFY and SET UP:** Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

**EXECUTE:** $\vec{v} = \dfrac{d\vec{r}}{dt} = ([5.0\text{ cm/s}^2]t)\hat{i} + (5.0\text{ cm/s})\hat{j}$

$\underline{t=0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t=1.0\text{ s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t=2.0\text{ s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

**(c)** The trajectory is a graph of $y$ versus $x$.

$x = 4.0\text{ cm} + (2.5\text{ cm/s}^2)\,t^2$, $y = (5.0\text{ cm/s})t$

For values of $t$ between 0 and 2.0 s, calculate $x$ and $y$ and plot $y$ versus $x$.

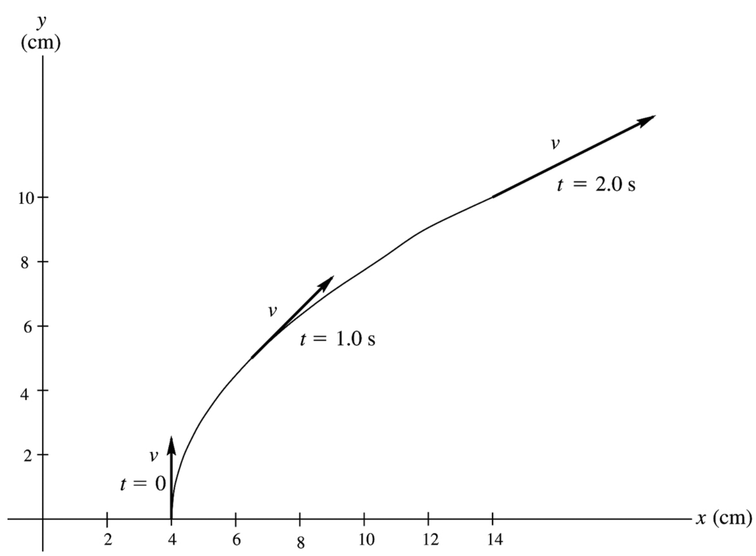



**Figure 3.3b**

**EVALUATE:** The sketch shows that the instantaneous velocity at any $t$ is tangent to the trajectory.

**3.4.** **IDENTIFY:** Given the position vector of a squirrel, find its velocity components in general, and at a specific time find its velocity components and the magnitude and direction of its position vector and velocity.

**SET UP:** $v_x = dx/dt$ and $v_y = dy/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

**EXECUTE:** **(a)** Taking the derivatives gives $v_x(t) = 0.280 \text{ m/s} + (0.0720 \text{ m/s}^2)t$ and $v_y(t) = (0.0570 \text{ m/s}^3)t^2$.

**(b)** Evaluating the position vector at $t = 5.00$ s gives $x = 2.30$ m and $y = 2.375$ m, which gives $r = 3.31$ m.

**(c)** At $t = 5.00$ s, $v_x = +0.64$ m/s, $v_y = 1.425$ m/s, which gives $v = 1.56$ m/s and $\tan\theta = \dfrac{1.425}{0.64}$ so the direction is $\theta = 65.8^\text{o}$ (counterclockwise from $+x$-axis)

**EVALUATE:** The acceleration is not constant, so we cannot use the standard kinematics formulas.

**3.5.** **IDENTIFY** and **SET UP:** Use Eq. $\vec{a}_\text{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form to calculate $a_{\text{av-}x}$ and $a_{\text{av-}y}$.

**EXECUTE:** **(a)** The velocity vectors at $t_1 = 0$ and $t_2 = 30.0$ s are shown in Figure 3.5a.

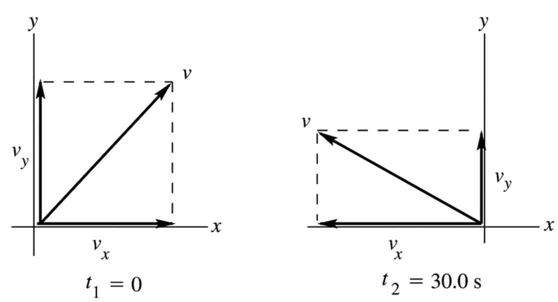
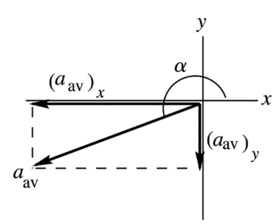



**Figure 3.5a**

**(b)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1} = \dfrac{-170 \text{ m/s} - 90 \text{ m/s}}{30.0 \text{ s}} = -8.67 \text{ m/s}^2$

$a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1} = \dfrac{40 \text{ m/s} - 110 \text{ m/s}}{30.0 \text{ s}} = -2.33 \text{ m/s}^2$

**(c)**




$a = \sqrt{(a_{\text{av-}x})^2 + (a_{\text{av-}y})^2} = 8.98 \text{ m/s}^2$

$\tan\alpha = \dfrac{a_{\text{av-}y}}{a_{\text{av-}x}} = \dfrac{-2.33 \text{ m/s}^2}{-8.67 \text{ m/s}^2} = 0.269$

$\alpha = 15^\circ + 180^\circ = 195^\circ$

**Figure 3.5b**

**EVALUATE:** The changes in $v_x$ and $v_y$ are both in the negative $x$ or $y$ direction, so both components of $\vec{a}_\text{av}$ are in the 3rd quadrant.

**3.6.** **IDENTIFY:** Use $\vec{a}_\text{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ in component form.

**SET UP:** $a_x = (0.45\text{ m/s}^2)\cos 31.0° = 0.39\text{ m/s}^2$, $a_y = (0.45\text{ m/s}^2)\sin 31.0° = 0.23\text{ m/s}^2$

**EXECUTE:** **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6\text{ m/s} + (0.39\text{ m/s}^2)(10.0\text{ s}) = 6.5\text{ m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and $v_y = -1.8\text{ m/s} + (0.23\text{ m/s}^2)(10.0\text{ s}) = 0.52\text{ m/s}$.

**(b)** $v = \sqrt{(6.5\text{ m/s})^2 + (0.52\text{ m/s})^2} = 6.52\text{ m/s}$, at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right) = 4.6°$ counterclockwise from the $+x$-axis.

**(c)** The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

**EVALUATE:** $\vec{v}_1$ is at an angle of $35°$ below the $+x$-axis and has magnitude $v_1 = 3.2\text{ m/s}$, so $v_2 > v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

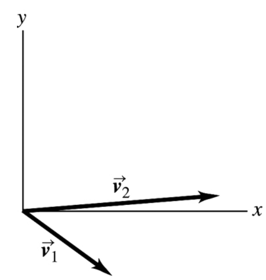



**Figure 3.6**

**3.7.** **IDENTIFY** and **SET UP:** Use $\vec{v} = \dfrac{d\vec{r}}{dt}$ and $\vec{a} = \dfrac{d\vec{v}}{dt}$ to find $v_x$, $v_y$, $a_x$, and $a_y$ as functions of time. The magnitude and direction of $\vec{r}$ and $\vec{a}$ can be found once we know their components.

**EXECUTE:** **(a)** Calculate $x$ and $y$ for $t$ values in the range 0 to 2.0 s and plot $y$ versus $x$. The results are given in Figure 3.7a.

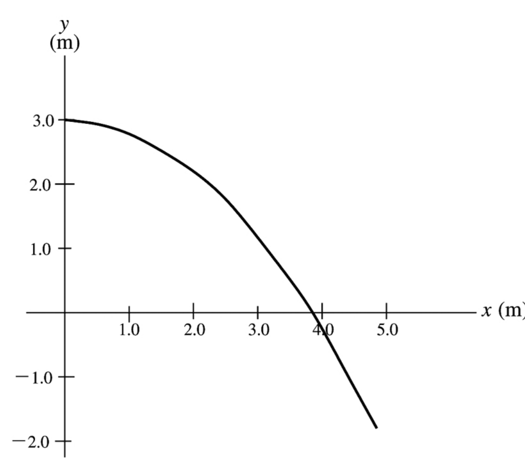


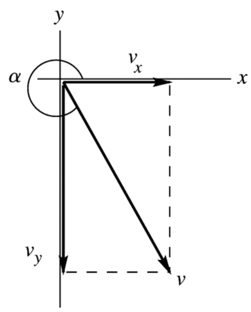

**Figure 3.7a**

**(b)** $v_x = \dfrac{dx}{dt} = \alpha \quad v_y = \dfrac{dy}{dt} = -2\beta t$

$a_x = \dfrac{dv_x}{dt} = 0 \quad a_y = \dfrac{dv_y}{dt} = -2\beta$

Thus $\vec{v} = \alpha\hat{i} - 2\beta t\hat{j}$, $\vec{a} = -2\beta\hat{j}$

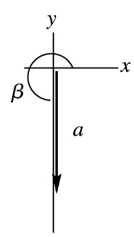
**(c)** Velocity: At $t = 2.0$ s, $v_x = 2.4$ m/s, $v_y = -2(1.2 \text{ m/s}^2)(2.0 \text{ s}) = -4.8$ m/s




$$v = \sqrt{v_x^2 + v_y^2} = 5.4 \text{ m/s}$$
$$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.8 \text{ m/s}}{2.4 \text{ m/s}} = -2.00$$
$$\alpha = -63.4° + 360° = 297°$$

**Figure 3.7b**

Acceleration: At $t = 2.0$ s, $a_x = 0$, $a_y = -2\,(1.2 \text{ m/s}^2) = -2.4 \text{ m/s}^2$




$$a = \sqrt{a_x^2 + a_y^2} = 2.4 \text{ m/s}^2$$
$$\tan\beta = \frac{a_y}{a_x} = \frac{-2.4 \text{ m/s}^2}{0} = -\infty$$
$$\beta = 270°$$

**Figure 3.7c**

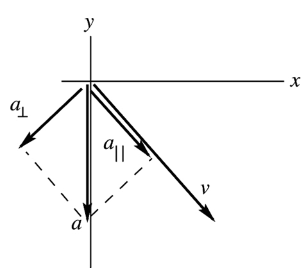



**Figure 3.7d**

**EVALUATE:** **(d)** $\vec{a}$ has a component $a_{\parallel}$ in the same direction as $\vec{v}$, so we know that $v$ is increasing (the bird is speeding up). $\vec{a}$ also has a component $a_\perp$ perpendicular to $\vec{v}$, so that the direction of $\vec{v}$ is changing; the bird is turning toward the $-y$-direction (toward the right)

$\vec{v}$ is always tangent to the path; $\vec{v}$ at $t = 2.0$ s shown in part (c) is tangent to the path at this $t$, conforming to this general rule. $\vec{a}$ is constant and in the $-y$-direction; the direction of $\vec{v}$ is turning toward the $-y$-direction.

**3.8.** **IDENTIFY:** Use the velocity components of a car (given as a function of time) to find the acceleration of the car as a function of time and to find the magnitude and direction of the car's velocity and acceleration at a specific time.

**SET UP:** $a_x = dv_x/dt$ and $a_y = dv_y/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

**EXECUTE:** **(a)** Taking the derivatives gives $a_x(t) = (-0.0360 \text{ m/s}^3)t$ and $a_y(t) = 0.550 \text{ m/s}^2$.

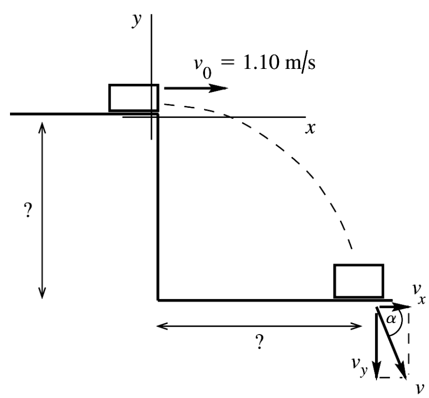
**(b)** Evaluating the velocity components at $t = 8.00$ s gives $v_x = 3.848$ m/s and $v_y = 6.40$ m/s, which gives $v = 7.47$ m/s. The direction is $\tan\theta = \frac{6.40}{3.848}$ so $\theta = 59.0^\circ$ (counterclockwise from $+x$-axis).

**(c)** Evaluating the acceleration components at $t = 8.00$ s gives $a_x = 20.288\ \text{m/s}^2$ and $a_y = 0.550\ \text{m/s}^2$, which gives $a = 0.621\ \text{m/s}^2$. The angle with the $+y$ axis is given by $\tan\theta = \frac{0.288}{0.550}$, so $\theta = 27.6^\circ$. The direction is therefore $118^\circ$ counterclockwise from $+x$-axis.

**EVALUATE:** The acceleration is not constant, so we cannot use the standard kinematics formulas.

**3.9. IDENTIFY:** The book moves in projectile motion once it leaves the tabletop. Its initial velocity is horizontal.

**SET UP:** Take the positive $y$-direction to be upward. Take the origin of coordinates at the initial position of the book, at the point where it leaves the table top.




<u>$x$-component</u>:
$a_x = 0$, $v_{0x} = 1.10$ m/s,
$t = 0.480$ s
<u>$y$-component</u>:
$a_y = -9.80\ \text{m/s}^2$, $v_{0y} = 0$,
$t = 0.480$ s

**Figure 3.9a**

Use constant acceleration equations for the $x$ and $y$ components of the motion, with $a_x = 0$ and $a_y = -g$.

**EXECUTE:** **(a)** $y - y_0 = ?$

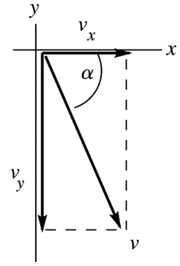
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)(0.480\ \text{s})^2 = -1.129$ m. The tabletop is therefore 1.13 m above the floor.

**(b)** $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10\ \text{m/s})(0.480\ \text{s}) + 0 = 0.528$ m.

**(c)** $v_x = v_{0x} + a_x t = 1.10$ m/s (The $x$-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80\ \text{m/s}^2)(0.480\ \text{s}) = -4.704$ m/s




$v = \sqrt{v_x^2 + v_y^2} = 4.83$ m/s

$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.704\ \text{m/s}}{1.10\ \text{m/s}} = -4.2764$

$\alpha = -76.8^\circ$

Direction of $\vec{v}$ is $76.8^\circ$ below the horizontal

**Figure 3.9b**

**(d)** The graphs are given in Figure 3.9c.

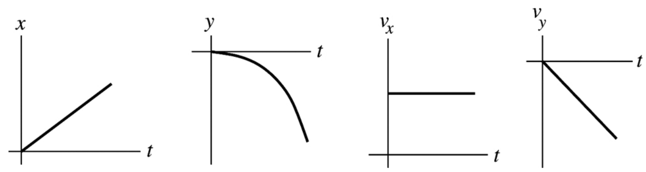



**Figure 3.9c**

**EVALUATE:** In the $x$-direction, $a_x = 0$ and $v_x$ is constant. In the $y$-direction, $a_y = -9.80\ \text{m/s}^2$ and $v_y$ is downward and increasing in magnitude since $a_y$ and $v_y$ are in the same directions. The $x$ and $y$ motions occur independently, connected only by the time. The time it takes the book to fall 1.13 m is the time it travels horizontally.

**3.10.** **IDENTIFY:** The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

**SET UP:** Take $+y$ downward. $a_x = 0$, $a_y = +9.80\ \text{m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

**EXECUTE:** Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(9.00\ \text{m})}{9.80\ \text{m/s}^2}} = 1.36\ \text{s}.$

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{1.75\ \text{m}}{1.36\ \text{s}} = 1.29\ \text{m/s}.$

**EVALUATE:** If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

**3.11.** **IDENTIFY:** Each object moves in projectile motion.

**SET UP:** Take $+y$ to be downward. For each cricket, $a_x = 0$ and $a_y = +9.80\ \text{m/s}^2$. For Chirpy, $v_{0x} = v_{0y} = 0$. For Milada, $v_{0x} = 0.950\ \text{m/s}$, $v_{0y} = 0$.

**EXECUTE:** Milada's horizontal component of velocity has no effect on her vertical motion. She also reaches the ground in 2.70 s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (0.950\ \text{m/s})(2.70\ \text{s}) = 2.57\ \text{m}.$

**EVALUATE:** The $x$ and $y$ components of motion are totally separate and are connected only by the fact that the time is the same for both.

**3.12.** **IDENTIFY:** The football moves in projectile motion.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -g$. At the highest point in the trajectory, $v_y = 0$.

**EXECUTE:** **(a)** $v_y = v_{0y} + a_y t$. The time $t$ is $\dfrac{v_{0y}}{g} = \dfrac{12.0\ \text{m/s}}{9.80\ \text{m/s}^2} = 1.224\ \text{s}$, which we round to 1.22 s.

**(b)** Different constant acceleration equations give different expressions but the same numerical result:

$\frac{1}{2}gt^2 = \frac{1}{2}v_{y0}t = \dfrac{v_{0y}^2}{2g} = 7.35\ \text{m}.$

**(c)** Regardless of how the algebra is done, the time will be twice that found in part (a), which is 2(1.224 s) = 2.45 s.

**(d)** $a_x = 0$, so $x - x_0 = v_{0x}t = (20.0\ \text{m/s})(2.45\ \text{s}) = 49.0\ \text{m}.$

**(e)** The graphs are sketched in Figure 3.12.

**EVALUATE:** When the football returns to its original level, $v_x = 20.0\ \text{m/s}$ and $v_y = -12.0\ \text{m/s}.$

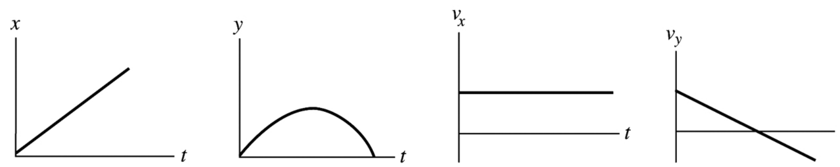



**Figure 3.12**

**3.13.** **IDENTIFY:** The car moves in projectile motion. The car travels $21.3\text{ m} - 1.80\text{ m} = 19.5\text{ m}$ downward during the time it travels 48.0 m horizontally.

**SET UP:** Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

**EXECUTE:** **(a)** Use the vertical motion to find the time in the air:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(19.5\text{ m})}{9.80\text{ m/s}^2}} = 1.995\text{ s}$

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{48.0\text{ m}}{1.995\text{ s}} = 24.1\text{ m/s}$.

**(b)** $v_x = 24.06\text{ m/s}$ since $a_x = 0$. $v_y = v_{0y} + a_y t = -19.55\text{ m/s}$. $v = \sqrt{v_x^2 + v_y^2} = 31.0\text{ m/s}$.

**EVALUATE:** Note that the speed is considerably less than the algebraic sum of the *x*- and *y*-components of the velocity.

**3.14.** **IDENTIFY:** Knowing the maximum reached by the froghopper and its angle of takeoff, we want to find its takeoff speed and the horizontal distance it travels while in the air.

**SET UP:** Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. At the maximum height $v_y = 0$. The constant-acceleration formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ apply.

**EXECUTE:** **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.587\text{ m})} = 3.39\text{ m/s}$. $v_{0y} = v_0 \sin\theta_0$ so

$v_0 = \dfrac{v_{0y}}{\sin\theta_0} = \dfrac{3.39\text{ m/s}}{\sin 58.0°} = 4.00\text{ m/s}$.

**(b)** Use the vertical motion to find the time in the air. When the froghopper has returned to the ground, $y - y_0 = 0$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(3.39\text{ m/s})}{-9.80\text{ m/s}^2} = 0.692\text{ s}$.

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (v_0 \cos\theta_0)t = (4.00\text{ m/s})(\cos 58.0°)(0.692\text{ s}) = 1.47\text{ m}$.

**EVALUATE:** $v_y = 0$ when $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{3.39\text{ m/s}}{-9.80\text{ m/s}^2} = 0.346\text{ s}$. The total time in the air is twice this.

**3.15.** **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude $v_0$. The height *h* of the table and $v_0$ are the same; the acceleration due to gravity changes from $g_{\text{E}} = 9.80\text{ m/s}^2$ on earth to $g_{\text{X}}$ on planet X.

**SET UP:** Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x} = v_0$, $v_{0y} = 0$, $a_x = 0$, $a_y = -g$, where *g* is either $g_{\text{E}}$ or $g_{\text{X}}$.

**EXECUTE:** Use the vertical motion to find the time in the air: $y - y_0 = -h$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2h}{g}}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_{0x}t = v_0\sqrt{\dfrac{2h}{g}}$. $x - x_0 = D$ on earth and $2.76D$ on

Planet X. $(x-x_0)\sqrt{g}=v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_E}=2.76D\sqrt{g_X}$.

$g_X=\dfrac{g_E}{(2.76)^2}=0.131g_E=1.28\text{ m/s}^2$.

**EVALUATE:** On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor and it travels farther horizontally.

**3.16.** **IDENTIFY:** The shell moves in projectile motion.
**SET UP:** Let $+x$ be horizontal, along the direction of the shell's motion, and let $+y$ be upward. $a_x=0$, $a_y=-9.80\text{ m/s}^2$.

**EXECUTE:** **(a)** $v_{0x}=v_0\cos\alpha_0=(40.0\text{ m/s})\cos 60.0°=20.0\text{ m/s}$,
$v_{0y}=v_0\sin\alpha_0=(40.0\text{ m/s})\sin 60.0°=34.6\text{ m/s}$.

**(b)** At the maximum height $v_y=0$. $v_y=v_{0y}+a_yt$ gives $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{0-34.6\text{ m/s}}{-9.80\text{ m/s}^2}=3.53\text{ s}$.

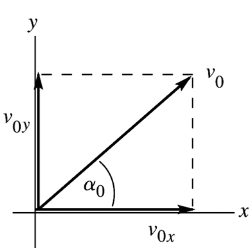
**(c)** $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(34.6\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=61.2\text{ m}$.

**(d)** The total time in the air is twice the time to the maximum height, so
$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2=(20.0\text{ m/s})(2)(3.53\text{ s})=141\text{ m}$.

**(e)** At the maximum height, $v_x=v_{0x}=20.0\text{ m/s}$ and $v_y=0$. At all points in the motion, $a_x=0$ and $a_y=-9.80\text{ m/s}^2$.

**EVALUATE:** The equation for the horizontal range $R$ derived in the text is $R=\dfrac{v_0^2\sin 2\alpha_0}{g}$. This gives $R=\dfrac{(40.0\text{ m/s})^2\sin(120.0°)}{9.80\text{ m/s}^2}=141\text{ m}$, which agrees with our result in part (d).

**3.17.** **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.
**SET UP:** First find the $x$- and $y$-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.




$v_{0x}=v_0\cos\alpha_0=(30.0\text{ m/s})\cos 36.9°=24.0\text{ m/s}$
$v_{0y}=v_0\sin\alpha_0=(30.0\text{ m/s})\sin 36.9°=18.0\text{ m/s}$

**Figure 3.17a**

Use constant acceleration equations for the $x$ and $y$ motions, with $a_x=0$ and $a_y=-g$.

**EXECUTE:** **(a)** $y$-component (vertical motion):
$y-y_0=+10.0\text{ m}$, $v_{0y}=18.0\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$, $t=?$
$y-y_0=v_{0y}+\frac{1}{2}a_yt^2$
$10.0\text{ m}=(18.0\text{ m/s})t-(4.90\text{ m/s}^2)t^2$

$(4.90\ \text{m/s}^2)t^2 - (18.0\ \text{m/s})t + 10.0\ \text{m} = 0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4\,(4.90)(10.0)}\right]\ \text{s} = (1.837 \pm 1.154)\ \text{s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683\ \text{s}$ and at $t_2 = 2.99\ \text{s}$. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

**(b)** $v_x = v_{0x} = +24.0\ \text{m/s}$, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

$\underline{t_1 = 0.683\ \text{s}}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.683\ \text{s}) = +11.3\ \text{m/s}$. ($v_y$ is positive means that the ball is traveling upward at this point.)

$\underline{t_2 = 2.99\ \text{s}}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(2.99\ \text{s}) = -11.3\ \text{m/s}$. ($v_y$ is negative means that the ball is traveling downward at this point.)

**(c)** $v_x = v_{0x} = 24.0\ \text{m/s}$

Solve for $v_y$:

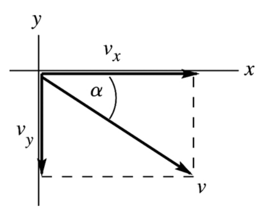
$v_y = ?$, $y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +18.0\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0\ \text{m/s}$ (negative, since the baseball must be traveling downward at this point)

Now solve for the magnitude and direction of $\vec{v}$.




$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{(24.0\ \text{m/s})^2 + (-18.0\ \text{m/s})^2} = 30.0\ \text{m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-18.0\ \text{m/s}}{24.0\ \text{m/s}}$$

$\alpha = -36.9^\circ$, $36.9^\circ$ below the horizontal

**Figure 3.17b**

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9^\circ$ below the horizontal.

**EVALUATE:** The discussion in parts (a) and (b) explains the significance of two values of *t* for which $y - y_0 = +10.0\ \text{m}$. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

**3.18.** **IDENTIFY:** The shot moves in projectile motion.

**SET UP:** Let $+y$ be upward.

**EXECUTE:** **(a)** If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\ \text{m/s}^2$ vertically downward.

**(b)** The *x*-component of velocity is constant at $v_x = (12.0\ \text{m/s})\cos 51.0^\circ = 7.55\ \text{m/s}$. The *y*-component is $v_{0y} = (12.0\ \text{m/s})\sin 51.0^\circ = 9.32\ \text{m/s}$ at release and

$v_y = v_{0y} - gt = (9.32\ \text{m/s}) - (9.80\ \text{m/s})(2.08\ \text{s}) = -11.06\ \text{m/s}$ when the shot hits.

**(c)** $x - x_0 = v_{0x}t = (7.55\ \text{m/s})(2.08\ \text{s}) = 15.7\ \text{m}$.

**(d)** The initial and final heights are not the same.
**(e)** With $y = 0$ and $v_{0y}$ as found above, the equation for $y - y_0$ as a function of time gives $y_0 = 1.81\text{m}$.
**(f)** The graphs are sketched in Figure 3.18.
**EVALUATE:** When the shot returns to its initial height, $v_y = -9.32$ m/s. The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of $v_y$ at the ground is larger than 9.32 m/s.

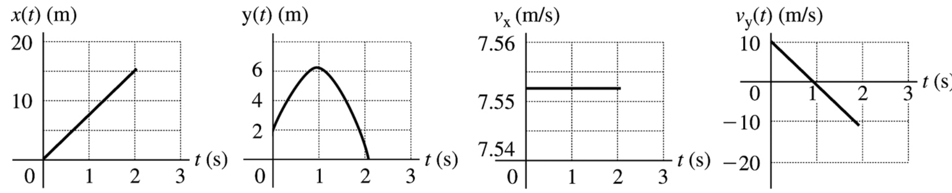
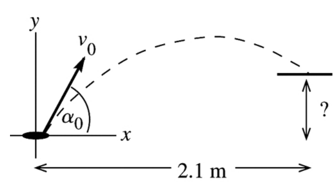



**Figure 3.18**

**3.19.** **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive $y$ to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.




$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s}) \cos 60°$
$v_{0x} = 3.20$ m/s
$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s}) \sin 60°$
$v_{0y} = 5.54$ m/s

**Figure 3.19**

**(a) SET UP:** Use the horizontal ($x$-component) of motion to solve for $t$, the time the quarter travels through the air:
$t = ?,\ x - x_0 = 2.1 \text{ m},\ v_{0x} = 3.2 \text{ m/s},\ a_x = 0$
$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = v_{0x}t$, since $a_x = 0$

**EXECUTE:** $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656 \text{ s}$

**SET UP:** Now find the vertical displacement of the quarter after this time:
$y - y_0 = ?,\ a_y = -9.80 \text{ m/s}^2,\ v_{0y} = +5.54 \text{ m/s},\ t = 0.656 \text{ s}$
$y - y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$

**EXECUTE:** $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

**(b) SET UP:** $v_y = ?,\ t = 0.656 \text{ s},\ a_y = -9.80 \text{ m/s}^2,\ v_{0y} = +5.54 \text{ m/s}\quad v_y = v_{0y} + a_y t$

**EXECUTE:** $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

**EVALUATE:** The minus sign for $v_y$ indicates that the $y$-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1$ m it is on its way down.

**3.20.** **IDENTIFY:** Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. $v_{0x} = v_0\cos\theta_0$, $v_{0y} = v_0\sin\theta_0$.

**EXECUTE:** **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0\text{ m}}{(25.0\text{ m/s})(3.00\text{ s})} = 0.600;$

$\theta_0 = 53.1°$

**(b)** At the highest point $v_x = v_{0x} = (25.0\text{ m/s})\cos 53.1° = 15.0\text{ m/s}$, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0\text{ m/s}$. At all points in the motion, $a = 9.80\text{ m/s}^2$ downward.

**(c)** Find $y - y_0$ when $t = 3.00\text{s}$:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (25.0\text{ m/s})(\sin 53.1°)(3.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s})^2 = 15.9\text{ m}$

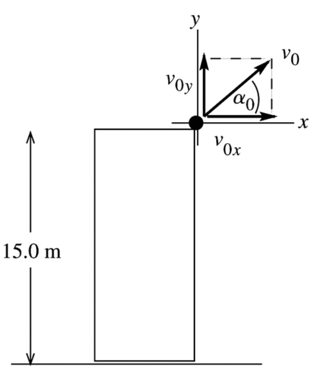
$v_x = v_{0x} = 15.0\text{ m/s}$, $v_y = v_{0y} + a_y t = (25.0\text{ m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\text{ s}) = -9.41\text{ m/s}$, and

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.0\text{ m/s})^2 + (-9.41\text{ m/s})^2} = 17.7\text{ m/s}$

**EVALUATE:** The acceleration is the same at all points of the motion. It takes the water $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04\text{ s}$ to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

**3.21.** **IDENTIFY:** Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the $x$ and $y$ components of the motion.

**SET UP:**




$v_{0x} = v_0\cos\alpha_0 = 25.2\text{ m/s}$

$v_{0y} = v_0\sin\alpha_0 = 16.3\text{ m/s}$

**Figure 3.21a**

**(a)** At the maximum height $v_y = 0$.

$a_y = -9.80\text{ m/s}^2$, . $v_y = 0$, . $v_{0y} = +16.3\text{ m/s}$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y\,(y - y_0)$

**EXECUTE:** $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = +13.6\text{ m}$

**(b) SET UP:** Find the velocity by solving for its $x$ and $y$ components.

$v_x = v_{0x} = 25.2\text{ m/s}$ (since $a_x = 0$)

$v_y = ?$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = -15.0\ \text{m}$ (negative because at the ground the rock is below its initial position), $v_{0y} = 16.3\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ ($v_y$ is negative because at the ground the rock is traveling downward.)

**EXECUTE:** $v_y = -\sqrt{(16.3\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-15.0\ \text{m})} = -23.7\ \text{m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2\ \text{m/s})^2 + (-23.7\ \text{m/s})^2} = 34.6\ \text{m/s}$.

**(c) SET UP:** Use the vertical motion ($y$-component) to find the time the rock is in the air:
$t = ?$, $v_y = -23.7\ \text{m/s}$ (from part (b)), $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +16.3\ \text{m/s}$

**EXECUTE:** $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-23.7\ \text{m/s} - 16.3\ \text{m/s}}{-9.80\ \text{m/s}^2} = +4.08\ \text{s}$

**SET UP:** Can use this $t$ to calculate the horizontal range:
$t = 4.08\ \text{s}$, $v_{0x} = 25.2\ \text{m/s}$, $a_x = 0$, $x - x_0 = ?$

**EXECUTE:** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2\ \text{m/s})(4.08\ \text{s}) + 0 = 103\ \text{m}$

**(d)** Graphs of $x$ versus $t$, $y$ versus $t$, $v_x$ versus $t$ and $v_y$ versus $t$:

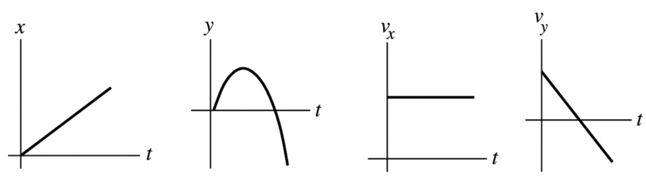



**Figure 3.21b**

**EVALUATE:** The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3\ \text{m/s}$ the time it takes the rock to return to the level of the roof $(y = 0)$ is $t = 2v_{0y}/g = 3.33\ \text{s}$. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

**3.22. IDENTIFY:** This is a problem in projectile motion. The acceleration is $g$ downward with constant horizontal velocity. The constant-acceleration equations apply.

**SET UP:** We use $y = (v_0 \sin\alpha_0)t - \dfrac{1}{2}gt^2$ and $v_y = v_0 \sin\alpha_0 - gt$ for the vertical motion and $v_x = v_0 \cos\alpha_0$ for the horizontal motion. We want the time when $y = 5.00$ m.

**EXECUTE:** Use $y = (v_0 \sin\alpha_0)t - \dfrac{1}{2}gt^2$ to find the times.

$5.00\ \text{m} = (15.0\ \text{m/s})(\sin 53.0°)t - \dfrac{1}{2}(9.80\ \text{m/s}^2)t^2$

Using the quadratic formula to solve this equation gives $t = 0.534$ s and $t = 1.91$ s.
Now look at the horizontal velocity.
$v_x = v_0 \cos\alpha_0 = (15.0\ \text{m/s}) \cos 53.0° = 9.03$ m/s at both times.
At $t = 0.534$ s: $v_y = v_0 \sin\alpha_0 - gt = (15.0\ \text{m/s}) \sin 53.0° - (9.80\ \text{m/s}^2)(0.534\ \text{s}) = 6.75$ m/s upward.
At $t = 1.92$ s: $v_y = (15.0\ \text{m/s}) \sin 53.0° - (9.80\ \text{m/s}^2)(1.92\ \text{s}) = -6.75$ m/s downward.

**EVALUATE:** To check, use $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to find $v_y$ when $y$ = 5.00 m, which gives $v_y^2$ = [(15.0 m/s)(sin 53.0°)]$^2$ – 2(9.80 m/s$^2$)(5.00 m) $\rightarrow$ $v_y$ = ±6.75 m/s, which agrees with our results. We get two answers because the rock is going upward at $t$ = 0.534 s and downward at $t$ = 1.91 s.

**3.23. IDENTIFY:** This is a problem in projectile motion. The acceleration is $g$ downward with constant horizontal velocity. The constant-acceleration equations apply.

**SET UP:** Estimate: 75 ft which is about 25 m. The range of a projectile is $R = \frac{v_0^2}{g}\sin 2\alpha_0$. For the maximum range, $2\alpha_0$ = 90°, so $\alpha_0$ = 45°, so we assume that we throw it at 45° above the horizontal.

**EXECUTE:** **(a)** Solve the range formula for $v_0$: $v_0 = \sqrt{Rg} = \sqrt{(25\text{ m})(9.80\text{ m/s}^2)}$ = 15.65 m/s, which rounds to 16 m/s.

**(b)** At the maximum height, $v_y$ = 0. Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $y - y_0 = h$ gives 0 = [(15.65 m/s)(sin 45°)]$^2$ – 2(9.80 m/s$^2$)$h$ $\rightarrow$ $h$ = 6.5 m.

**(c)** Apply the range formula on Mars: $R = \frac{(15.65\text{ m/s})^2}{3.7\text{ m/s}^2}\sin 90°$ = 66 m.

**EVALUATE:** The range is inversely proportional to $g$, so $R_M/R_E = g_E/g_M$ = 9.8/3.7 = 2.65. So the range of Mars would be 2.65 times the range on earth, which is (2.65)(25 m) = 66 m, in agreement with our result.

**3.24. IDENTIFY:** The merry-go-round is rotating at a constant rate.
**SET UP:** Estimate: $T$ = 5.0 s for one rotation. Convert units: $R$ = 6.0 ft = 1.829 m. The radial acceleration is $a_{rad} = \frac{v^2}{R}$ and $v = 2\pi R/T$.
**EXECUTE:** **(a)** $v = 2\pi R/T = 2\pi$(1.829 m)/(5.0 s) = 2.3 m/s.
**(b)** $a_{rad} = \frac{v^2}{R}$ = (2.3 m/s)$^2$/(1.829 m) = 2.9 m/s$^2$.
**(c)** If $T \rightarrow T/2$, $v \rightarrow 2v$ so $v^2 \rightarrow 4v^2$. Since $a_{rad} \propto v^2$, $a_{rad} \rightarrow 4a_{rad}$ = 4(2.9 m/s$^2$) = 12 m/s$^2$.
**EVALUATE:** The acceleration in part (b) is 2.9/9.8 $g$ = 0.30$g$, which seems a bit large for a merry-go-round for young children. Perhaps the estimate of 5.0 s is too small. If we revise the estimate to 10 s, $v$ will become 1.15 m/s and $a_{rad}$ will become 0.72 m/s$^2$, which is 0.72/9.8 $g$ = 0.074$g$. This is about 7.4% of $g$, which seems a bit more reasonable.

**3.25. IDENTIFY:** This problem deals with circular motion.
**SET UP:** Apply the equation $a_{rad} = 4\pi^2 R/T^2$, where $T$ = 24 h.
**EXECUTE:** **(a)** $a_{rad} = \frac{4\pi^2(6.38\times10^6\text{ m})}{[(24\text{ h})(3600\text{ s/h})]^2} = 0.034\text{ m/s}^2 = 3.4\times10^{-3}g$.
**(b)** Solving the equation $a_{rad} = 4\pi^2 R/T^2$ for the period $T$ with $a_{rad} = g$,

$$T = \sqrt{\frac{4\pi^2(6.38\times10^6\text{m})}{9.80\text{ m/s}^2}} = 5070\text{ s} = 1.4\text{ h}.$$

**EVALUATE:** $a_{rad}$ is proportional to $1/T^2$, so to increase $a_{rad}$ by a factor of $\frac{1}{3.4\times10^{-3}} = 294$ requires that $T$ be multiplied by a factor of $\frac{1}{\sqrt{294}}$. $\frac{24\text{ h}}{\sqrt{294}} = 1.4$ h.

**3.26. IDENTIFY:** We want to find the acceleration of the inner ear of a dancer, knowing the rate at which she spins.

**SET UP:** $R = 0.070$ m. For 3.0 rev/s, the period $T$ (time for one revolution) is $T = \dfrac{1.0 \text{ s}}{3.0 \text{ rev}} = 0.333$ s.

The speed is $v = d/T = (2\pi R)/T$, and $a_{\text{rad}} = v^2/R$.

**EXECUTE:** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R} = \dfrac{4\pi^2 R}{T^2} = \dfrac{4\pi^2(0.070 \text{ m})}{(0.333 \text{ s})^2} = 24.92 \text{ m/s}^2$, which rounds to 25 m/s$^2$ with two significant figures. As a fraction of $g$, this acceleration is (24.92 m/s$^2$)/(9.80 m/s$^2$) = 2.54, which rounds to 2.5 to two significant figures.

**EVALUATE:** The acceleration is large and the force on the fluid must be 2.5 times its weight.

**3.27.** **IDENTIFY:** The ball has no vertical acceleration, but it has a horizontal acceleration toward the center of the circle. This acceleration is the radial acceleration.

**SET UP:** $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$, $R = L\sin\theta$.

**EXECUTE:** Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ with $R = L\sin\theta$. $a_{\text{rad}} = \dfrac{4\pi^2 L\sin\theta}{T^2}$. Using $L = 0.800$ m, $\theta = 37.0°$, and $T = 0.600$ s, this gives $a_{\text{rad}} = 52.8$ m/s$^2$.

**EVALUATE:** Compare this acceleration to $g$: $a_{\text{rad}}/g = 52.8/9.80 = 5.39$, so $a_{\text{rad}} = 5.39g$, which is fairly large.

**3.28.** **IDENTIFY:** Each blade tip moves in a circle of radius $R = 3.40$ m and therefore has radial acceleration $a_{\text{rad}} = v^2/R$.

**SET UP:** 550 rev/min = 9.17 rev/s, corresponding to a period of $T = \dfrac{1}{9.17 \text{ rev/s}} = 0.109$ s.

**EXECUTE:** **(a)** $v = \dfrac{2\pi R}{T} = 196$ m/s.

**(b)** $a_{\text{rad}} = \dfrac{v^2}{R} = 1.13\times10^4 \text{ m/s}^2 = 1.15\times10^3 g$.

**EVALUATE:** $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives the same results for $a_{\text{rad}}$ as in part (b).

**3.29.** **IDENTIFY:** For the curved lowest part of the dive, the pilot's motion is approximately circular. We know the pilot's acceleration and the radius of curvature, and from this we want to find the pilot's speed.

**SET UP:** $a_{\text{rad}} = 5.5g = 53.9 \text{ m/s}^2$. 1 mph = 0.4470 m/s. $a_{\text{rad}} = \dfrac{v^2}{R}$.

**EXECUTE:** $a_{\text{rad}} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(280 \text{ m})(53.9 \text{ m/s}^2)} = 122.8 \text{ m/s} = 274.8$ mph. Rounding these answers to 2 significant figures (because of $5.5g$), gives $v = 120$ m/s = 270 mph.

**EVALUATE:** This speed is reasonable for the type of plane flown by a test pilot.

**3.30.** **IDENTIFY:** The object has constant horizontal speed (but *not* constant velocity), so its acceleration points toward the center of the circle. This is the radial acceleration.

**SET UP:** We know $v$ and $T$ and we want $a_{\text{rad}}$ in terms of $v$ and $T$. $a_{\text{rad}} = \dfrac{v^2}{R}$ and $v = 2\pi R/T$.

**EXECUTE:** We want to express $a_{\text{rad}}$ in terms of $v$ and $T$. From $v = 2\pi R/T$ we get $R = vT/2\pi$, so

$$a_{\text{rad}} = \frac{v^2}{(vT/2\pi)} = 2\pi v/T.$$

**(a)** Using $a_{\text{rad}} = 2\pi v/T$, we see that if $v$ doubles, $T$ must also double to keep $a_{\text{rad}}$ the same.

**(b)** First express $a_{\text{rad}}$ in terms of $R$ and $T$ as $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. If $R$ doubles, the $T^2$ must also double to keep $a_{\text{rad}}$ the same. Therefore $T$ must increase by a factor of $\sqrt{2}$.

**EVALUATE:** Using $a_{\text{rad}} = 2\pi v/T$ would not be helpful in part (b) because the equation does not contain $R$.

**3.31. IDENTIFY:** Uniform circular motion.

**SET UP:** Since the magnitude of $\vec{v}$ is constant, $v_{\tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by $v^2/R$.

**EXECUTE:** **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(6.00\text{ m/s})^2}{14.0\text{ m}} = 2.57\text{ m/s}^2$, upward.

**(b)** The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is $2.57\text{ m/s}^2$, downward.

**(c) SET UP:** The time to make one rotation is the period $T$, and the speed $v$ is the distance for one revolution divided by $T$.

**EXECUTE:** $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi\,(14.0\text{ m})}{6.00\text{ m/s}} = 14.7\text{ s}$.

**EVALUATE:** The radial acceleration is constant in magnitude since $v$ is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

**3.32. IDENTIFY:** The roller coaster car is going in a circle. Its radial acceleration is always toward the center of the circle.

**SET UP:** $a_{\text{rad}} = \dfrac{v^2}{R}$

**EXECUTE:** **(a)** The radial acceleration is toward the center of the circle, which is directly downward when the car is at the top.

**(b)** By the same reasoning as in (a), the direction is vertically upward when the car is at the bottom.

**(c)** Use $a_{\text{rad}} = \dfrac{v^2}{R}$ and take the ratio of the accelerations: $\dfrac{a_{\text{rad,bottom}}}{a_{\text{rad,top}}} = \dfrac{v_2^2 / R}{v_1^2 / R} = (v_2 / v_1)^2$.

**EVALUATE:** Our result shows that the acceleration is greater at the bottom since $v_2 > v_1$.

**3.33. IDENTIFY:** Each part of his body moves in uniform circular motion, with $a_{\text{rad}} = \dfrac{v^2}{R}$. The speed in rev/s is $1/T$, where $T$ is the period in seconds (time for 1 revolution). The speed $v$ increases with $R$ along the length of his body but all of him rotates with the same period $T$.

**SET UP:** For his head $R = 8.84\text{ m}$ and for his feet $R = 6.84\text{ m}$.

**EXECUTE:** **(a)** $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(8.84\text{ m})(12.5)(9.80\text{ m/s}^2)} = 32.9\text{ m/s}$

**(b)** Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. Since his head has $a_{\text{rad}} = 12.5g$ and $R = 8.84\text{ m}$,

$T = 2\pi\sqrt{\dfrac{R}{a_{\text{rad}}}} = 2\pi\sqrt{\dfrac{8.84\text{ m}}{12.5(9.80\text{ m/s}^2)}} = 1.688\text{ s}$. Then his feet have $a_{\text{rad}} = \dfrac{R}{T^2} = \dfrac{4\pi^2(6.84\text{ m})}{(1.688\text{ s})^2} = 94.8\text{ m/s}^2 =$ 9.67 g. The difference between the acceleration of his head and his feet is $12.5g - 9.67g = 2.83g = 27.7\text{ m/s}^2$.

**(c)** $\frac{1}{T} = \frac{1}{1.69\text{ s}} = 0.592\text{ rev/s} = 35.5\text{ rpm}$

**EVALUATE:** His feet have speed $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(6.84\text{ m})(94.8\text{ m/s}^2)} = 25.5\text{ m/s}$.

**3.34.** **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2 / R$.

**SET UP:** The radius of the earth's orbit is $r = 1.50 \times 10^{11}\text{ m}$ and its orbital period is $T = 365\text{ days} = 3.16 \times 10^7\text{ s}$. For Mercury, $r = 5.79 \times 10^{10}\text{ m}$ and $T = 88.0\text{ days} = 7.60 \times 10^6\text{ s}$.

**EXECUTE:** **(a)** $v = \frac{2\pi r}{T} = 2.98 \times 10^4\text{ m/s}$

**(b)** $a_{\text{rad}} = \frac{v^2}{r} = 5.91 \times 10^{-3}\text{ m/s}^2$.

**(c)** $v = 4.79 \times 10^4\text{ m/s}$, and $a_{\text{rad}} = 3.96 \times 10^{-2}\text{ m/s}^2$.

**EVALUATE:** Mercury has a larger orbital velocity and a larger radial acceleration than earth.

**3.35.** **IDENTIFY:** Relative velocity problem. The time to walk the length of the moving sidewalk is the length divided by the velocity of the woman relative to the ground.

**SET UP:** Let W stand for the woman, G for the ground and S for the sidewalk. Take the positive direction to be the direction in which the sidewalk is moving.

The velocities are $v_{\text{W/G}}$ (woman relative to the ground), $v_{\text{W/S}}$ (woman relative to the sidewalk), and $v_{\text{S/G}}$ (sidewalk relative to the ground).

The equation for relative velocity becomes $v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}}$.

The time to reach the other end is given by $t = \frac{\text{distance traveled relative to ground}}{v_{\text{W/G}}}$

**EXECUTE:** **(a)** $v_{\text{S/G}} = 1.0\text{ m/s}$

$v_{\text{W/S}} = +1.5\text{ m/s}$

$v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}} = 1.5\text{ m/s} + 1.0\text{ m/s} = 2.5\text{ m/s}$.

$t = \frac{35.0\text{ m}}{v_{\text{W/G}}} = \frac{35.0\text{ m}}{2.5\text{ m/s}} = 14\text{ s}$.

**(b)** $v_{\text{S/G}} = 1.0\text{ m/s}$

$v_{\text{W/S}} = -1.5\text{ m/s}$

$v_{\text{W/G}} = v_{\text{W/S}} + v_{\text{S/G}} = -1.5\text{ m/s} + 1.0\text{ m/s} = -0.5\text{ m/s}$. (Since $v_{\text{W/G}}$ now is negative, she must get on the moving sidewalk at the opposite end from in part (a).)

$t = \frac{-35.0\text{ m}}{v_{\text{W/G}}} = \frac{-35.0\text{ m}}{-0.5\text{ m/s}} = 70\text{ s}$.

**EVALUATE:** Her speed relative to the ground is much greater in part (a) when she walks with the motion of the sidewalk.

**3.36.** **IDENTIFY:** The relative velocities are $\vec{v}_{\text{S/F}}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{\text{S/G}}$, the scooter relative to the ground and $\vec{v}_{\text{F/G}}$, the flatcar relative to the ground. $\vec{v}_{\text{S/G}} = \vec{v}_{\text{S/F}} + \vec{v}_{\text{F/G}}$. Carry out the vector addition by drawing a vector addition diagram.

**SET UP:** $\vec{v}_{\text{S/F}} = \vec{v}_{\text{S/G}} - \vec{v}_{\text{F/G}}$. $\vec{v}_{\text{F/G}}$ is to the right, so $-\vec{v}_{\text{F/G}}$ is to the left.

**EXECUTE:** In each case the vector addition diagram gives

**(a)** 5.0 m/s to the right

**(b)** 16.0 m/s to the left

**(c)** 13.0 m/s to the left.

**EVALUATE:** The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{S/F}$ and $\vec{v}_{F/G}$ are in the same direction and their magnitudes add.

**3.37.** **IDENTIFY:** Apply the relative velocity relation.
**SET UP:** The relative velocities are $\vec{v}_{C/E}$, the canoe relative to the earth, $\vec{v}_{R/E}$, the velocity of the river relative to the earth and $\vec{v}_{C/R}$, the velocity of the canoe relative to the river.
**EXECUTE:** $\vec{v}_{C/E} = \vec{v}_{C/R} + \vec{v}_{R/E}$ and therefore $\vec{v}_{C/R} = \vec{v}_{C/E} - \vec{v}_{R/E}$. The velocity components of $\vec{v}_{C/R}$ are $-0.50 \text{ m/s} + (0.40 \text{ m/s})/\sqrt{2}$, east and $(0.40 \text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.
**EVALUATE:** The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

**3.38.** **IDENTIFY:** Calculate the rower's speed relative to the shore for each segment of the round trip.
**SET UP:** The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.
**EXECUTE:** The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).
The total time the rower takes is $\dfrac{1.5 \text{ km}}{6.8 \text{ km/h}} + \dfrac{1.5 \text{ km}}{1.2 \text{ km/h}} = 1.47 \text{ h} = 88.2 \text{ min}.$
**EVALUATE:** It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

**3.39.** **IDENTIFY:** The resultant velocity, relative to the ground, is directly southward. This velocity is the sum of the velocity of the bird relative to the air and the velocity of the air relative to the ground.
**SET UP:** $v_{B/A} = 100$ km/h. $\vec{v}_{A/G} = 40$ km/h, east. $\vec{v}_{B/G} = \vec{v}_{B/A} + \vec{v}_{A/G}$.
**EXECUTE:** We want $\vec{v}_{B/G}$ to be due south. The relative velocity addition diagram is shown in Figure 3.39.

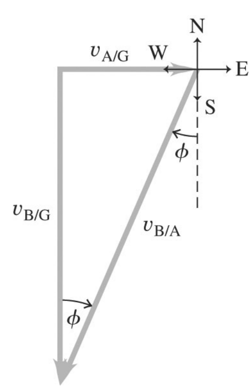



**Figure 3.39**

**(a)** $\sin\phi = \dfrac{v_{A/G}}{v_{B/A}} = \dfrac{40 \text{ km/h}}{100 \text{ km/h}}$, $\phi = 24°$, west of south.

**(b)** $v_{B/G} = \sqrt{v_{B/A}^2 - v_{A/G}^2} = 91.7$ km/h. $t = \dfrac{d}{v_{B/G}} = \dfrac{500 \text{ km}}{91.7 \text{ km/h}} = 5.5$ h.

**EVALUATE:** The speed of the bird relative to the ground is less than its speed relative to the air. Part of its velocity relative to the air is directed to oppose the effect of the wind.

**3.40.** **IDENTIFY:** Relative velocity problem in two dimensions.

**(a) SET UP:** $\vec{v}_{P/A}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{P/A}$ has magnitude 35 m/s and direction south.

$\vec{v}_{A/E}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{A/E}$ is to the southwest ($45°$ S of W) and has magnitude 10 m/s.

The relative velocity equation is $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

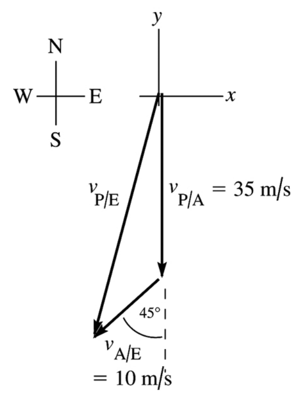



**Figure 3.40a**

**EXECUTE:** **(b)** $(v_{P/A})_x = 0$, $(v_{P/A})_y = -35$ m/s

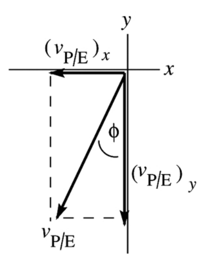
$(v_{A/E})_x = -(10 \text{ m/s})\cos 45° = -7.07$ m/s,

$(v_{A/E})_y = -(10 \text{ m/s})\sin 45° = -7.07$ m/s

$(v_{P/E})_x = (v_{P/A})_x + (v_{A/E})_x = 0 - 7.07 \text{ m/s} = -7.1$ m/s

$(v_{P/E})_y = (v_{P/A})_y + (v_{A/E})_y = -35 \text{ m/s} - 7.07 \text{ m/s} = -42$ m/s

**(c)**




$$v_{P/E} = \sqrt{(v_{P/E})_x^2 + (v_{P/E})_y^2}$$

$$v_{P/E} = \sqrt{(-7.1 \text{ m/s})^2 + (-42 \text{ m/s})^2} = 43 \text{ m/s}$$

$$\tan\phi = \frac{(v_{P/E})_x}{(v_{P/E})_y} = \frac{-7.1}{-42} = 0.169$$

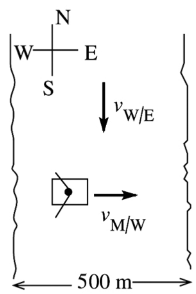
$\phi = 9.6°$; ($9.6°$ west of south)

**Figure 3.40b**

**EVALUATE:** The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

**3.41.** **IDENTIFY:** Relative velocity problem in two dimensions. His motion relative to the earth (time displacement) depends on his velocity relative to the earth so we must solve for this velocity.

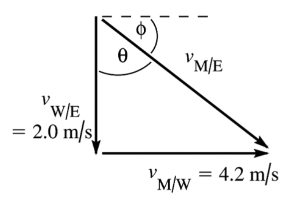
**(a) SET UP:** View the motion from above.




The velocity vectors in the problem are:
$\vec{v}_{M/E}$, the velocity of the man relative to the earth
$\vec{v}_{W/E}$, the velocity of the water relative to the earth
$\vec{v}_{M/W}$, the velocity of the man relative to the water
The rule for adding these velocities is
$\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$

**Figure 3.41a**

The problem tells us that $\vec{v}_{W/E}$ has magnitude 2.0 m/s and direction due south. It also tells us that $\vec{v}_{M/W}$ has magnitude 4.2 m/s and direction due east. The vector addition diagram is then as shown in Figure 3.41b.




This diagram shows the vector addition $\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$ and also has $\vec{v}_{M/W}$ and $\vec{v}_{W/E}$ in their specified directions. Note that the vector diagram forms a right triangle.

**Figure 3.41b**

The Pythagorean theorem applied to the vector addition diagram gives $v_{M/E}^2 = v_{M/W}^2 + v_{W/E}^2$.

**EXECUTE:** $v_{M/E} = \sqrt{v_{M/W}^2 + v_{W/E}^2} = \sqrt{(4.2\text{ m/s})^2 + (2.0\text{ m/s})^2} = 4.7\text{ m/s}$; $\tan\theta = \dfrac{v_{M/W}}{v_{W/E}} = \dfrac{4.2\text{ m/s}}{2.0\text{ m/s}} = 2.10$; $\theta = 65°$; or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction $25°$ S of E.

**(b)** This requires careful thought. To cross the river the man must travel 500 m due east relative to the earth. The man's velocity relative to the earth is $\vec{v}_{M/E}$. But, from the vector addition diagram the eastward component of $v_{M/E}$ equals $v_{M/W} = 4.2$ m/s.

Thus $t = \dfrac{x - x_0}{v_x} = \dfrac{500\text{ m}}{4.2\text{ m/s}} = 119\text{ s}$, which we round to 120 s.

**(c)** The southward component of $\vec{v}_{M/E}$ equals $v_{W/E} = 2.0$ m/s. Therefore, in the 120 s it takes him to cross the river, the distance south the man travels relative to the earth is

$$y - y_0 = v_y t = (2.0\text{ m/s})(119\text{ s}) = 240\text{ m}.$$

**EVALUATE:** If there were no current he would cross in the same time, $(500\text{ m})/(4.2\text{ m/s}) = 120$ s. The current carries him downstream but doesn't affect his motion in the perpendicular direction, from bank to bank.

**3.42.** **IDENTIFY:** Use the relation that relates the relative velocities.
**SET UP:** The relative velocities are the water relative to the earth, $\vec{v}_{W/E}$, the boat relative to the water, $\vec{v}_{B/W}$, and the boat relative to the earth, $\vec{v}_{B/E}$. $\vec{v}_{B/E}$ is due east, $\vec{v}_{W/E}$ is due south and has magnitude 2.0 m/s. $v_{B/W} = 4.2$ m/s. $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$. The velocity addition diagram is given in Figure 3.42.

**EXECUTE:** **(a)** Find the direction of $\vec{v}_{\text{B/W}}$. $\sin\theta = \dfrac{v_{\text{W/E}}}{v_{\text{B/W}}} = \dfrac{2.0 \text{ m/s}}{4.2 \text{ m/s}}$. $\theta = 28.4°$, north of east.

**(b)** $v_{\text{B/E}} = \sqrt{v_{\text{B/W}}^2 - v_{\text{W/E}}^2} = \sqrt{(4.2 \text{ m/s})^2 - (2.0 \text{ m/s})^2} = 3.7 \text{ m/s}$

**(c)** $t = \dfrac{800 \text{ m}}{v_{\text{B/E}}} = \dfrac{800 \text{ m}}{3.7 \text{ m/s}} = 216 \text{ s}.$

**EVALUATE:** It takes longer to cross the river in this problem than it did in Problem 3.41. In the direction straight across the river (east) the component of his velocity relative to the earth is less than 4.2 m/s.

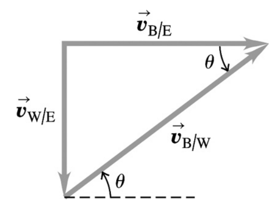



**Figure 3.42**

**3.43.** **IDENTIFY:** Use the relation that relates the relative velocities.

**SET UP:** The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{\text{P/G}}$, the velocity of the plane relative to the air, $\vec{v}_{\text{P/A}}$, and the velocity of the air relative to the ground, $\vec{v}_{\text{A/G}}$. $\vec{v}_{\text{P/G}}$ must be due west and $\vec{v}_{\text{A/G}}$ must be south. $v_{\text{A/G}} = 80 \text{ km/h}$ and $v_{\text{P/A}} = 320 \text{ km/h}$. $\vec{v}_{\text{P/G}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/G}}$. The relative velocity addition diagram is given in Figure 3.43.

**EXECUTE:** **(a)** $\sin\theta = \dfrac{v_{\text{A/G}}}{v_{\text{P/A}}} = \dfrac{80 \text{ km/h}}{320 \text{ km/h}}$ and $\theta = 14°$, north of west.

**(b)** $v_{\text{P/G}} = \sqrt{v_{\text{P/A}}^2 - v_{\text{A/G}}^2} = \sqrt{(320 \text{ km/h})^2 - (80.0 \text{ km/h})^2} = 310 \text{ km/h}.$

**EVALUATE:** To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

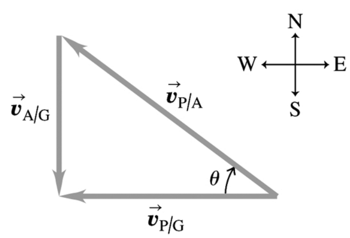



**Figure 3.43**

**3.44.** **IDENTIFY:** The dog runs away from a tree with a non-constant acceleration, so the constant-acceleration equations do not apply. We need to go to the basic definitions of $a$ and $v$ in terms of calculus.

**SET UP:** The dog starts from rest at the tree when $t = 0$, so $x_0 = 0$ and $v_0 = 0$. The dog's acceleration is given by $\vec{a} = (0.400\text{ m/s}^2)\hat{i} - (0.180\text{ m/s}^3)t\,\hat{j}$, so we see that $a_x = 0.400\text{ m/s}^2$ and $a_y = -0.180\text{ m/s}^3\ t$. The basic definitions are $a_x = dv_x/dt$ and $v_x = dx/dt$, and likewise for the $y$-components. We want to know how far the dog is from the tree at the end of 8.00 s of running. By integration of the acceleration we can find the $x$ and $y$ components of the dog's position vector at 8.00 s. From these components we can find the magnitude of the dog's position vector.

**EXECUTE:** First find the components of the dog's velocity using $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

<u>$x$-coordinate</u>: We can save some time because $a_x = 0.400\text{ m/s}^2$ is a constant. So we can use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, which gives $x = \frac{1}{2}a_x t^2 = \frac{1}{2}(0.400\text{ m/s}^2)(8.00\text{ s})^2 = 12.80\text{ m}$.

<u>$y$-coordinate</u>: First find $v_y$ using $a_y = dv_y/dt$. This gives $v_y = \int a_y dt = \int kt\,dt = \frac{kt^2}{2}$, where we let $k = -0.180\text{ m/s}^3$ for convenience. We have used $v_y = 0$ when $t = 0$. Now use $v_y = dy/dt$ to find $y(t)$.

$y = \int v_y dt = \int \frac{kt^2}{2} dt = \frac{kt^3}{6}$, where we have used $y = 0$ when $t = 0$. At 8.00 s, we have

$$y = \frac{(-0.180\text{ m/s}^3)(8.00\text{ s})^3}{6} = -15.36\text{ m.}$$

Now use $A = \sqrt{A_x^2 + A_y^2}$ to find the dog's distance $d$ from the tree. $d = \sqrt{(12.80\text{ m})^2 + (-15.36\text{ m})^2} = 20.0\text{ m}$.

**EVALUATE:** The use of components makes the solution to this problem quite straightforward.

**3.45.** **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$ angle with both axes when its $x$- and $y$-components are equal.

**SET UP:** $\dfrac{d(t^n)}{dt} = nt^{n-1}$.

**EXECUTE:** $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

**EVALUATE:** Both components of $\vec{v}$ change with $t$.

**3.46.** **IDENTIFY:** The acceleration is not constant but is known as a function of time.

**SET UP:** Integrate the acceleration to get the velocity and the velocity to get the position. At the maximum height $v_y = 0$.

**EXECUTE:** **(a)** $v_x = v_{0x} + \frac{\alpha}{3}t^3,\ v_y = v_{0y} + \beta t - \frac{\gamma}{2}t^2,$ and $x = v_{0x}t + \frac{\alpha}{12}t^4,\ y = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3.$

**(b)** Setting $v_y = 0$ yields a quadratic in $t$, $0 = v_{0y} + \beta t - \frac{\gamma}{2}t^2$. Using the numerical values given in the problem, this equation has as the positive solution $t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_{0y}\gamma}\right] = 13.59\text{ s}$. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

**(c)** $y = 0$ gives $0 = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$ and $\frac{\gamma}{6}t^2 - \frac{\beta}{2}t - v_{0y} = 0$. Using the numbers given in the problem, the positive solution is $t = 20.73$ s. For this $t$, $x = 3.85\times10^4$ m.

**EVALUATE:** We cannot use the constant-acceleration kinematics formulas, but calculus provides the solution.

**3.47.** **IDENTIFY:** Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion.

**SET UP:** For motion along the incline let $+x$ be directed up the incline. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = \sqrt{2(1.90\text{ m/s}^2)(200\text{ m})} = 27.57\text{ m/s}$. When the projectile motion begins the rocket has $v_0 = 27.57\text{ m/s}$ at $35.0°$ above the horizontal and is at a vertical height of $(200.0\text{ m})\sin 35.0° = 114.7\text{ m}$. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7\text{ m}$, $v_{0x} = v_0\cos 35.0° = 22.57\text{ m/s}$, $v_{0y} = v_0\sin 35.0° = 15.81\text{ m/s}$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. Let $x = 0$ at point *A*, so $x_0 = (200.0\text{ m})\cos 35.0° = 163.8\text{ m}$.

**EXECUTE:** **(a)** At the maximum height $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (15.81\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 12.77\text{ m}$$ and $y = 114.7\text{ m} + 12.77\text{ m} = 128\text{ m}$. The maximum height above ground is 128 m.

**(b)** The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7\text{ m}$, $v_{0y} = 15.81\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $(4.90\text{ m/s}^2)t^2 - (15.81\text{ m/s})t - 114.7\text{ m}$. The quadratic formula gives $t = 6.713\text{ s}$ for the positive root. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2 = (22.57\text{ m/s})(6.713\text{ s}) = 151.6\text{ m}$ and $x = 163.8\text{ m} + 151.6\text{ m} = 315\text{ m}$. The horizontal range of the rocket is 315 m.

**EVALUATE:** The expressions for *h* and *R* derived in the range formula do not apply here. They are only for a projectile fired on level ground.

**3.48.** **IDENTIFY:** Use the position vector of a dragonfly to determine information about its velocity vector and acceleration vector.

**SET UP:** Use the definitions $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$, and $a_y = dv_y/dt$.

**EXECUTE:** **(a)** Taking derivatives of the position vector gives the components of the velocity vector: $v_x(t) = (0.180\text{ m/s}^2)t$, $v_y(t) = (-0.0450\text{ m/s}^3)t^2$. Use these components and the given direction:

$$\tan 30.0° = \frac{(0.0450\text{ m/s}^3)t^2}{(0.180\text{ m/s}^2)t}$$, which gives $t = 2.31\text{ s}$.

**(b)** Taking derivatives of the velocity components gives the acceleration components: $a_x = 0.180\text{ m/s}^2$, $a_y(t) = -(0.0900\text{ m/s}^3)t$. At $t = 2.31\text{ s}$, $a_x = 0.180\text{ m/s}^2$ and $a_y = -0.208\text{ m/s}^2$, giving $a = 0.275\text{ m/s}^2$. The direction is $\tan\theta = \frac{0.208}{0.180}$, so $\theta = 49.1°$ clockwise from $+x$-axis.

**EVALUATE:** The acceleration is not constant, so we cannot use the standard kinematics formulas.

**3.49.** **IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the *x* and *y* components of motion.

**SET UP:**

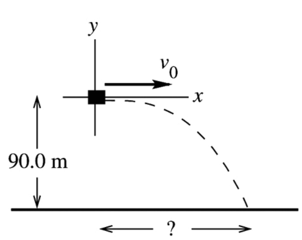



**Figure 3.49**

Take the origin of coordinates at the point where the cannister is released. Take $+y$ to be upward. The initial velocity of the cannister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ where $t = ?$, $v_{0y} = 0$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = -90.0\ \text{m}$ (When the cannister reaches the ground it is 90.0 m *below* the origin.)

**EXECUTE:** Since $v_{0y} = 0$, $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-90.0\ \text{m})}{-9.80\ \text{m/s}^2}} = 4.286\ \text{s}.$

**SET UP:** Then use the horizontal component of the motion to calculate how far the cannister falls in this time: $x - x_0 = ?$, $a_x - 0$, $v_{0x} = 64.0\ \text{m/s}$ .

**EXECUTE:** $x - x_0 = v_0 t + \frac{1}{2}at^2 = (64.0\ \text{m/s})(4.286\ \text{s}) + 0 = 274\ \text{m}.$

**EVALUATE:** The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

**3.50.** **IDENTIFY:** $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)dt$ and $\boldsymbol{a} = \dfrac{d\vec{v}}{dt}.$

**SET UP:** At $t = 0$, $x_0 = 0$ and $y_0 = 0$.

**EXECUTE:** **(a)** Integrating, $\vec{r} = \left(\alpha t - \dfrac{\beta}{3}t^3\right)\hat{\boldsymbol{i}} + \left(\dfrac{\gamma}{2}t^2\right)\hat{\boldsymbol{j}}$. Differentiating, $\vec{a} = (-2\beta t)\hat{\boldsymbol{i}} + \gamma\hat{\boldsymbol{j}}$.

**(b)** The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the *y*-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\ \text{m/s})(4.0\ \text{m/s}^2)}{2(1.6\ \text{m/s}^3)} = 9.0\ \text{m}.$$

**EVALUATE:** The acceleration is not constant.

**3.51.** **IDENTIFY:** The person moves in projectile motion. Her vertical motion determines her time in the air.

**SET UP:** Take $+y$ upward. $v_{0x} = 15.0\ \text{m/s}$, $v_{0y} = +10.0\ \text{m/s}$, $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$.

**EXECUTE:** **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = -30.0\ \text{m}$ gives $-30.0\ \text{m} = (10.0\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$. The quadratic formula gives $t = \dfrac{1}{2(4.9)}\left(+10.0 \pm \sqrt{(-10.0)^2 - 4(4.9)(-30)}\right)$ s. The positive solution is $t = 3.70$ s. During this time she travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (15.0\ \text{m/s})(3.70\ \text{s}) = 55.5\ \text{m}$. She will land 55.5 m south of the point where she drops from the helicopter and this is where the mats should have been placed.

**(b)** The *x*-*t*, *y*-*t*, $v_x$-*t* and $v_y$-*t* graphs are sketched in Figure 3.51.

**EVALUATE:** If she had dropped from rest at a height of 30.0 m it would have taken her $t = \sqrt{\dfrac{2(30.0\ \text{m})}{9.80\ \text{m/s}^2}} = 2.47\ \text{s}$. She is in the air longer than this because she has an initial vertical component of velocity that is upward.

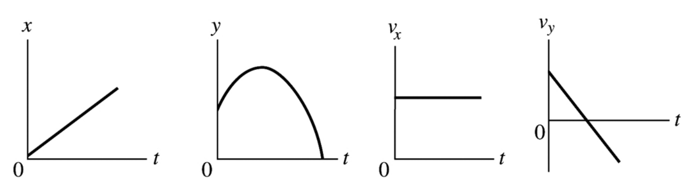



**Figure 3.51**

**3.52.** **IDENTIFY:** The shell moves as a projectile. To just clear the top of the cliff, the shell must have $y - y_0 = 25.0$ m when it has $x - x_0 = 60.0$ m.

**SET UP:** Let $+y$ be upward. $a_x = 0$, $a_y = -g$. $v_{0x} = v_0 \cos 43°$, $v_{0y} = v_0 \sin 43°$.

**EXECUTE:** **(a)** horizontal motion: $x - x_0 = v_{0x} t$ so $t = \dfrac{60.0 \text{ m}}{(v_0 \cos 43°)}$.

vertical motion: $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives $25.0 \text{ m} = (v_0 \sin 43.0°)\, t + \frac{1}{2}(-9.80 \text{ m/s}^2)\, t^2$.

Solving these two simultaneous equations for $v_0$ and $t$ gives $v_0 = 32.6$ m/s and $t = 2.51$ s.

**(b)** $v_y$ when shell reaches cliff:

$$v_y = v_{0y} + a_y t = (32.6 \text{ m/s}) \sin 43.0° - (9.80 \text{ m/s}^2)(2.51 \text{ s}) = -2.4 \text{ m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

**EVALUATE:** The shell reaches its maximum height at $t = -\dfrac{v_{0y}}{a_y} = 2.27$ s, which confirms that at $t = 2.51$ s it has passed its maximum height and is on its way down when it strikes the edge of the cliff.

**3.53.** **IDENTIFY:** Find the horizontal distance a rocket moves if it has a non-constant horizontal acceleration but a constant vertical acceleration of *g* downward.

**SET UP:** The vertical motion is *g* downward, so we can use the constant acceleration formulas for that component of the motion. We must use integration for the horizontal motion because the acceleration is not constant. Solving for *t* in the kinematics formula for *y* gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}}$. In the horizontal direction we must use $v_x(t) = v_{0x} + \int_0^t a_x(t')dt'$ and $x - x_0 = \int_0^t v_x(t')dt'$.

**EXECUTE:** Use vertical motion to find *t*. $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(30.0 \text{ m})}{9.80 \text{ m/s}^2}} = 2.474$ s.

In the horizontal direction we have

$v_x(t) = v_{0x} + \int_0^t a_x(t')dt' = v_{0x} + (0.800 \text{ m/s}^3)t^2 = 12.0 \text{ m/s} + (0.800 \text{ m/s}^3)t^2$. Integrating $v_x(t)$ gives $x - x_0 = (12.0 \text{ m/s})t + (0.2667 \text{ m/s}^3)t^3$. At $t = 2.474$ s, $x - x_0 = 29.69 \text{ m} + 4.04 \text{ m} = 33.7$ m.

**EVALUATE:** The vertical part of the motion is familiar projectile motion, but the horizontal part is not.

**3.54.** **IDENTIFY:** The equipment moves in projectile motion. The distance *D* is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

**SET UP:** For the motion of the equipment take $+x$ to be to the right and $+y$ to be upward. Then $a_x = 0$, $a_y = -9.80 \text{ m/s}^2$, $v_{0x} = v_0 \cos\alpha_0 = 7.50$ m/s and $v_{0y} = v_0 \sin\alpha_0 = 13.0$ m/s. When the equipment lands in the front of the ship, $y - y_0 = -8.75$ m.

**EXECUTE:** Use the vertical motion of the equipment to find its time in the air: $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives $t = \dfrac{1}{9.80}\left(13.0 \pm \sqrt{(-13.0)^2 + 4(4.90)(8.75)}\right)$ s. The positive root is $t = 3.21$ s. The horizontal range of the equipment is $x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 = (7.50 \text{ m/s})(3.21 \text{ s}) = 24.1$ m. In 3.21 s the ship moves a horizontal distance $(0.450 \text{ m/s})(3.21 \text{ s}) = 1.44$ m, so $D = 24.1 \text{ m} + 1.44 \text{ m} = 25.5$ m.

**EVALUATE:** The range equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ cannot be used here because the starting and ending points of the projectile motion are at different heights.

**3.55.** **IDENTIFY:** Two-dimensional projectile motion.

**SET UP:** Let $+y$ be upward. $a_x = 0,\ a_y = -9.80\ \text{m/s}^2$. With $x_0 = y_0 = 0$, algebraic manipulation of the equations for the horizontal and vertical motion shows that $x$ and $y$ are related by

$$y = (\tan\theta_0)x - \frac{g}{2v_0{}^2\cos^2\theta_0}x^2.$$

$\theta_0 = 60.0°$. $y = 8.00\ \text{m}$ when $x = 18.0\ \text{m}$.

**EXECUTE:** **(a)** Solving for $v_0$ gives $v_0 = \sqrt{\dfrac{gx^2}{2(\cos^2\theta_0)(x\tan\theta_0 - y)}} = 16.6\ \text{m/s}$.

**(b)** We find the horizontal and vertical velocity components:

$v_x = v_{0x} = v_0\cos\theta_0 = 8.3\ \text{m/s}$.

$v_y{}^2 = v_{0y}{}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(v_0\sin\theta_0)^2 + 2a_y(y - y_0)} = -\sqrt{(14.4\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(8.00\ \text{m})} = -7.1\ \text{m/s}$

$v = \sqrt{v_x{}^2 + v_y{}^2} = 10.9\ \text{m/s}$. $\tan\theta = \dfrac{|v_y|}{|v_x|} = \dfrac{7.1}{8.3}$ and $\theta = 40.5°$, below the horizontal.

**EVALUATE:** We can check our calculated $v_0$.

$$t = \frac{x - x_0}{v_{0x}} = \frac{18.0\ \text{m}}{8.3\ \text{m/s}} = 2.17\ \text{s}.$$

Then $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = (14.4\ \text{m/s})(2.17\ \text{s}) - (4.9\ \text{m/s}^2)(2.17\ \text{s})^2 = 8\ \text{m}$, which checks.

**3.56.** **IDENTIFY:** While the hay falls 150 m with an initial upward velocity and with a downward acceleration of $g$, it must travel a horizontal distance (the target variable) with constant horizontal velocity.

**SET UP:** Use coordinates with $+y$ upward and $+x$ horizontal. The bale has initial velocity components $v_{0x} = v_0\cos\alpha_0 = (75\ \text{m/s})\cos 55° = 43.0\ \text{m/s}$ and $v_{0y} = v_0\sin\alpha_0 = (75\ \text{m/s})\sin 55° = 61.4\ \text{m/s}$. $y_0 = 150\ \text{m}$ and $y = 0$. The equation $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ applies to the vertical motion and a similar equation to the horizontal motion.

**EXECUTE:** Use the vertical motion to find $t$: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $-150\ \text{m} = (61.4\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$. The quadratic formula gives $t = 6.27 \pm 8.36\ \text{s}$. The physical value is the positive one, and $t = 14.6\ \text{s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2 = (43.0\ \text{m/s})(14.6\ \text{s}) = 630\ \text{m}$.

**EVALUATE:** If the airplane maintains constant velocity after it releases the bales, it will also travel horizontally 630 m during the time it takes the bales to fall to the ground, so the airplane will be directly over the impact spot when the bales land.

**3.57.** **IDENTIFY:** From the figure in the text, we can read off the maximum height and maximum horizontal distance reached by the grasshopper. Knowing its acceleration is $g$ downward, we can find its initial speed and the height of the cliff (the target variables).

**SET UP:** Use coordinates with the origin at the ground and $+y$ upward. $a_x = 0,\ a_y = -9.80\ \text{m/s}^2$. The constant-acceleration kinematics formulas $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ apply.

**EXECUTE:** **(a)** $v_y = 0$ when $y - y_0 = 0.0674\ \text{m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.0674\ \text{m})} = 1.15\ \text{m/s}$. $v_{0y} = v_0\sin\alpha_0$ so

$v_0 = \dfrac{v_{0y}}{\sin\alpha_0} = \dfrac{1.15\ \text{m/s}}{\sin 50.0°} = 1.50\ \text{m/s}$.

**(b)** Use the horizontal motion to find the time in the air. The grasshopper travels horizontally $x - x_0 = 1.06$ m. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos 50.0°} = 1.10$ s. Find the vertical displacement of the grasshopper at $t = 1.10$ s:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.15 \text{ m/s})(1.10 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.10 \text{ s})^2 = -4.66$ m. The height of the cliff is 4.66 m.

**EVALUATE:** The grasshopper's maximum height (6.74 cm) is physically reasonable, so its takeoff speed of 1.50 m/s must also be reasonable. Note that the equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ does *not* apply here since the launch point is not at the same level as the landing point.

**3.58. IDENTIFY:** The water moves in projectile motion.

**SET UP:** Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0,$ $a_y = -g.$

**EXECUTE:** The equations of motions are $y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$ and $x = (v_0 \cos \alpha)t.$ When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D.$ When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D.$ In both cases, $\sin \alpha = \cos \alpha = \sqrt{2}/2.$

To reach the *minimum* distance: $6D = \dfrac{\sqrt{2}}{2}v_0 t,$ and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2.$ Solving the first equation for $t$ gives $t = \dfrac{6D\sqrt{2}}{v_0}.$ Substituting this into the second equation gives $2D = 6D - \frac{1}{2}g\left(\dfrac{6D\sqrt{2}}{v_0}\right)^2.$ Solving this for $v_0$ gives $v_0 = 3\sqrt{gD}.$

To reach the *maximum* distance: $7D = \dfrac{\sqrt{2}}{2}v_0 t,$ and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2.$ Solving the first equation for $t$ gives $t = \dfrac{7D\sqrt{2}}{v_0}.$ Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\dfrac{7D\sqrt{2}}{v_0}\right)^2.$ Solving this for $v_0$ gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD},$ which, as expected, is larger than the previous result.

**EVALUATE:** A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of $6D$. The minimum speed required is greater than this, because the water must be at a height of at least $2D$ when it reaches the front of the tank.

**3.59. IDENTIFY:** This is a projectile motion problem. The vertical acceleration is $g$ downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

**SET UP:** Apply the constant-acceleration formulas.

**EXECUTE:** **(a)** The object has only vertical motion, so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v = \sqrt{v_0^2 + 2gH}.$

**(b)** The procedure is exactly the as in part (a) except that $v_{0y} = -v_0$, so the result is the same.

**(c)** Use the same approach as in part (a). $v_x = v_0 \cos \alpha_0$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y^2 = (v_0 \sin \alpha_0)^2 + 2(-g)(-H)$, so $v_y = \sqrt{v_0^2 \sin^2 \alpha_0 + 2gH}.$ The speed is the magnitude of the velocity vector, so $v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(v_0^2 \cos^2 \alpha_0\right) + \left(v_0^2 \sin^2 \alpha_0 + 2gH\right)} = \sqrt{v_0^2 + 2gH}.$

**EVALUATE:** **(d)** From part (c), we see that $v$ does not depend on $\alpha_0$, so $v$ stays the same as $\alpha_0$ is changed.

**3.60.** **IDENTIFY:** To clear the bar the ball must have a height of 10.0 ft when it has a horizontal displacement of 36.0 ft. The ball moves as a projectile. When $v_0$ is very large, the ball reaches the goal posts in a very short time and the acceleration due to gravity causes negligible downward displacement.

**SET UP:** $36.0 \text{ ft} = 10.97 \text{ m}$; $10.0 \text{ ft} = 3.048 \text{ m}$. Let $+x$ be to the right and $+y$ be upward, so $a_x = 0$, $a_y = -g$, $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$.

**EXECUTE:** **(a)** The ball cannot be aimed lower than directly at the bar. $\tan\alpha_0 = \dfrac{10.0 \text{ ft}}{36.0 \text{ ft}}$ and $\alpha_0 = 15.5°$.

**(b)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{x - x_0}{v_0 \cos\alpha_0}$. Then $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$y - y_0 = (v_0 \sin\alpha_0)\left(\frac{x - x_0}{v_0 \cos\alpha_0}\right) - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0} = (x - x_0)\tan\alpha_0 - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2\alpha_0}.$$

$$v_0 = \frac{(x - x_0)}{\cos\alpha_0}\sqrt{\frac{g}{2[(x - x_0)\tan\alpha_0 - (y - y_0)]}} = \frac{10.97 \text{ m}}{\cos 45.0°}\sqrt{\frac{9.80 \text{ m/s}^2}{2[10.97 \text{ m} - 3.048 \text{ m}]}} = 12.2 \text{ m/s} = 43.9 \text{ km/h}.$$

**EVALUATE:** With the $v_0$ and 45° launch angle in part (b), the horizontal range of the ball is $R = \dfrac{v_0^2 \sin 2\alpha_0}{g} = 15.2 \text{ m} = 49.9 \text{ ft}$. The ball reaches the highest point in its trajectory when $x - x_0 = R/2$, which is 25 ft, so when it reaches the goal posts it is on its way down.

**3.61.** **IDENTIFY:** The snowball moves in projectile motion. In part (a) the vertical motion determines the time in the air. In part (c), find the height of the snowball above the ground after it has traveled horizontally 4.0 m.

**SET UP:** Let $+y$ be downward. $a_x = 0$, $a_y = +9.80 \text{ m/s}^2$. $v_{0x} = v_0 \cos\theta_0 = 5.36 \text{ m/s}$, $v_{0y} = v_0 \sin\theta_0 = 4.50 \text{ m/s}$.

**EXECUTE:** **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 14.0 \text{ m}$ gives $14.0 \text{ m} = (4.50 \text{ m/s})\,t + (4.9 \text{ m/s}^2)\,t^2$. The quadratic formula gives $t = \dfrac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4\,(4.9)(-14.0)}\right)$ s. The positive root is $t = 1.29$ s. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (5.36 \text{ m/s})(1.29 \text{ s}) = 6.91 \text{ m}$.

**(b)** The $x$-$t$, $y$-$t$, $v_x$-$t$ and $v_y$-$t$ graphs are sketched in Figure 3.61.

**(c)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{4.0 \text{ m}}{5.36 \text{ m/s}} = 0.746 \text{ s}$. In this time the snowball travels downward a distance $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 6.08 \text{ m}$ and is therefore $14.0 \text{ m} - 6.08 \text{ m} = 7.9 \text{ m}$ above the ground. The snowball passes well above the man and doesn't hit him.

**EVALUATE:** If the snowball had been released from rest at a height of 14.0 m it would have reached the ground in $t = \sqrt{\dfrac{2(14.0 \text{ m})}{9.80 \text{ m/s}^2}} = 1.69 \text{ s}$. The snowball reaches the ground in a shorter time than this because of its initial downward component of velocity.

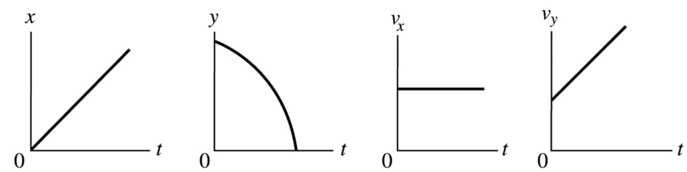



**Figure 3.61**

**3.62.** **IDENTIFY:** The ball moves in projectile motion.

**SET UP:** The woman and ball travel for the same time and must travel the same horizontal distance, so for the ball $v_{0x} = 6.00$ m/s.

**EXECUTE:** **(a)** $v_{0x} = v_0 \cos\theta_0$. $\cos\theta_0 = \dfrac{v_{0x}}{v_0} = \dfrac{6.00 \text{ m/s}}{20.0 \text{ m/s}}$ and $\theta_0 = 72.5°$. The ball is in the air for 5.55s and she runs a distance of $(6.00 \text{ m/s})(5.55 \text{ s}) = 33.3$ m.

**(b)** Relative to the ground the ball moves in a parabola. The ball and the runner have the same horizontal component of velocity, so relative to the runner the ball has only vertical motion. The trajectories as seen by each observer are sketched in Figure 3.62.

**EVALUATE:** The ball could be thrown with a different speed, so long as the angle at which it was thrown was adjusted to keep $v_{0x} = 6.00$ m/s.

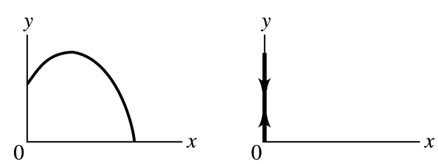
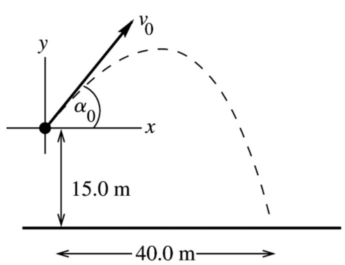



**Figure 3.62**

**3.63.** **(a) IDENTIFY:** Projectile motion.




Take the origin of coordinates at the top of the ramp and take $+y$ to be upward.

The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

**Figure 3.63**

We don't know *t*, the time in the air, and we don't know $v_0$. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

**SET UP:** <u>*y*-component</u>:

$y - y_0 = -15.0$ m, $a_y = -9.80$ m/s$^2$, $v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

**EXECUTE:** $-15.0 \text{ m} = (v_0 \sin 53.0°)\, t - (4.90 \text{ m/s}^2)\, t^2$

**SET UP:** <u>*x*-component</u>:

$x - x_0 = 40.0\text{ m},\ \ a_x = 0,\ \ v_{0x} = v_0\cos 53.0°$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$

**EXECUTE:** $40.0\text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0\text{ m}}{\cos 53.0°} = 66.47\text{ m}.$

Use this to replace $v_0 t$ in the first equation:

$-15.0\text{ m} = (66.47\text{ m})\sin 53° - (4.90\text{ m/s}^2)\,t^2$

$$t = \sqrt{\frac{(66.47\text{ m})\sin 53° + 15.0\text{ m}}{4.90\text{ m/s}^2}} = \sqrt{\frac{68.08\text{ m}}{4.90\text{ m/s}^2}} = 3.727\text{ s}.$$

Now that we have *t* we can use the *x*-component equation to solve for $v_0$:

$$v_0 = \frac{40.0\text{ m}}{t\cos 53.0°} = \frac{40.0\text{ m}}{(3.727\text{ s})\cos 53.0°} = 17.8\text{ m/s}.$$

**EVALUATE:** Using these values of $v_0$ and *t* in the $y = y_0 = v_{0y} + \frac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0\text{ m}.$

**(b) IDENTIFY:** $v_0 = (17.8\text{ m/s})/2 = 8.9\text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:

**SET UP:** $y - y_0 = -100\text{ m};\ \ v_{0y} = +v_0\sin 53.0° = 7.11\text{ m/s};\ \ a_y = -9.80\text{ m/s}^2$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$

**EXECUTE:** $4.90t^2 - 7.11t - 100 = 0$ and $t = \frac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4\,(4.90)(-100)}\right)$

$t = 0.726\text{ s} \pm 4.57\text{ s}$ so $t = 5.30\text{ s}.$

The horizontal distance he travels in this time is
$x - x_0 = v_{0x}t = (v_0\cos 53.0°)\,t = (5.36\text{ m/s})(5.30\text{ s}) = 28.4\text{ m}.$

He lands in the river a horizontal distance of 28.4 m from his launch point.

**EVALUATE:** He has half the minimum speed and makes it only about halfway across.

**3.64.** **IDENTIFY:** The bagels move in projectile motion. Find Henrietta's location when the bagels reach the ground, and require the bagels to have this horizontal range.

**SET UP:** Let $+y$ be downward and let $x_0 = y_0 = 0.$ $a_x = 0,$ $a_y = +g.$ When the bagels reach the ground, $y = 38.0\text{ m}.$

**EXECUTE:** **(a)** When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 38.0 m from rest. Get the time to fall: $y = \dfrac{1}{2}gt^2,$ $38.0\text{ m} = \dfrac{1}{2}(9.80\text{ m/s}^2)\,t^2$ and $t = 2.78\text{ s}.$

So, she has been jogging for $9.00\text{ s} + 2.78\text{ s} = 11.78\text{ s}.$ During this time she has gone $x = vt = (3.05\text{ m/s})(11.78\text{ s}) = 35.9\text{ m}.$ Bruce must throw the bagels so they travel 35.9 m horizontally in 2.78 s. This gives $x = vt.$ $35.9\text{ m} = v\,(2.78\text{ s})$ and $v = 12.9\text{ m/s}.$

**(b)** 35.9 m from the building.

**EVALUATE:** If $v > 12.9\text{ m/s}$ the bagels land in front of her and if $v < 12.9\text{ m/s}$ they land behind her. There is a range of velocities greater than 12.9 m/s for which she would catch the bagels in the air, at some height above the sidewalk.

**3.65.** **IDENTIFY:** The boulder moves in projectile motion.

**SET UP:** Take $+y$ downward. $v_{0x} = v_0,$ $a_x = 0,$ $a_x = 0,$ $a_y = +9.80\text{ m/s}^2.$

**EXECUTE:** **(a)** Use the vertical motion to find the time for the boulder to reach the level of the lake: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ with $y-y_0=+20\text{ m}$ gives $t=\sqrt{\frac{2(y-y_0)}{a_y}}=\sqrt{\frac{2(20\text{ m})}{9.80\text{ m/s}^2}}=2.02\text{ s}$. The rock must travel horizontally 100 m during this time. $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ gives

$$v_0=v_{0x}=\frac{x-x_0}{t}=\frac{100\text{ m}}{2.02\text{ s}}=49.5\text{ m/s}$$

**(b)** In going from the edge of the cliff to the plain, the boulder travels downward a distance of $y-y_0=45\text{ m}$. $t=\sqrt{\frac{2(y-y_0)}{a_y}}=\sqrt{\frac{2(45\text{ m})}{9.80\text{ m/s}^2}}=3.03\text{ s}$ and $x-x_0=v_{0x}t=(49.5\text{ m/s})(3.03\text{ s})=150\text{ m}$.

The rock lands $150\text{ m}-100\text{ m}=50\text{ m}$ beyond the foot of the dam.

**EVALUATE:** The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

**3.66.** **IDENTIFY:** The water follows a parabolic trajectory since it is affected only by gravity, so we apply the principles of projectile motion to it.

**SET UP:** Use coordinates with $+y$ upward. Once the water leaves the cannon it is in free-fall and has $a_x=0$ and $a_y=-9.80\text{ m/s}^2$. The water has $v_{0x}=v_0\cos\theta_0=15.0\text{ m/s}$ and $v_{0y}=v_0\sin\theta_0=20.0\text{ m/s}$.

**EXECUTE:** Use the vertical motion to find $t$ that gives $y-y_0=10.0\text{ m}$: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $10.0\text{ m}=(20.0\text{ m/s})t-(4.90\text{ m/s}^2)t^2$.

The quadratic formula gives $t=2.04\pm1.45\text{ s}$, and $t=0.59\text{ s}$ or $t=3.49\text{ s}$. Both answers are physical.

**For** $t=0.59\text{ s}$: $x-x_0=v_{0x}t=(15.0\text{ m/s})(0.59\text{ s})=8.8\text{ m}$.

**For** $t=3.49\text{ s}$: $x-x_0=v_{0x}t=(15.0\text{ m/s})(3.49\text{ s})=52.4\text{ m}$.

When the cannon is 8.8 m from the building, the water hits this spot on the wall on its way up to its maximum height. When is it 52.4 m from the building it hits this spot after it has passed through its maximum height.

**EVALUATE:** The fact that we have two possible answers means that the firefighters have some choice on where to stand. If the fire is extremely fierce, they would no doubt prefer to stand at the more distant location.

**3.67.** **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is $g$ downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

**SET UP:** Apply the constant-acceleration formulas. We know that the ball travels 50.0 m horizontally and has a speed of 8.0 m/s at its maximum height. We want to know how long the ball is in the air.

**EXECUTE:** The horizontal velocity is constant, so $v_x = 8.0$ m/s. The ball moves 50.0 m at this velocity, so $x = v_xt$ gives $50.0\text{ m} = (8.0\text{ m/s})t \quad \rightarrow \quad t = 6.3\text{ s}$.

**EVALUATE:** The ball's vertical velocity keeps changing, but its horizontal velocity remains constant. At its highest point, the ball does *not stop*. Only its vertical velocity becomes zero there.

**3.68.** **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is $g$ downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

**SET UP:** The box leaves the top of the ramp horizontally with speed $v_0$ and is in freefall until it hits the ramp. Fig. 3.68 illustrates the motion. When it hits the ramp, the box has been moving horizontally at speed $v_0$ for the same time that it has been falling vertically from rest. We want to know how far the box falls before hitting the ramp.

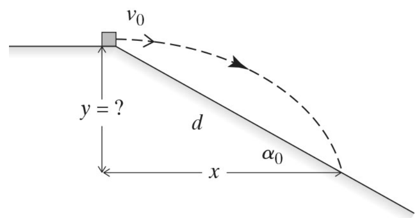



**Figure 3.68**

**EXECUTE:** When the box hits the ramp, the constant-acceleration equations give us $x = v_0t$ and $y = \frac{1}{2}gt^2$. From the figure, we can see that $x = d\cos\alpha_0$ and $y = d\sin\alpha_0$. Therefore $v_0t = d\cos\alpha_0$ and $\frac{1}{2}gt^2 = d\sin\alpha_0$. Combining the last two equations gives $t = (d\cos\alpha_0)/v_0$ so $\frac{1}{2}g\left(\frac{d\cos\alpha_0}{v_0}\right)^2 = d\sin\alpha_0$.

Solving for $d$ gives $d = \dfrac{2v_0^2\sin\alpha_0}{g\cos^2\alpha_0}$. We want to find $y$, so we use the fact that $y = d\sin\alpha_0$ and use the value of $d$ we just found, which gives

$$y = \frac{2v_0^2\sin\alpha_0}{g\cos^2\alpha_0}\cdot\sin\alpha_0 = \frac{2v_0^2\tan^2\alpha_0}{g}.$$

**EVALUATE:** Check some special cases. If $\alpha_0 \to$ 0, the ramp is nearly vertical, so $y$ gets very large. That is reasonable because the box would fall a very long distance before it hit the ramp. If $\alpha_0 \to 0°$, our result gives $y = 0$. This is reasonable because the box would already be on the ramp when it slid off the loading dock, so it would not have to fall any distance to reach the ramp.

**3.69.** **IDENTIFY:** The rock is in free fall once it is in the air, so it has only a downward acceleration of 9.80 m/s$^2$, and we apply the principles of two-dimensional projectile motion to it. The constant-acceleration kinematics formulas apply.

**SET UP:** The vertical displacement must be $\Delta y = y - y_0 = 5.00$ m – 1.60 m = 3.40 m at the instant that the horizontal displacement $\Delta x = x - x_0 = 14.0$ m, and $a_y = -9.80$ m/s$^2$ with $+y$ upward.

**EXECUTE:** **(a)** There is no horizontal acceleration, so 14.0 m = $v_0\cos(56.0°)t$, which gives $t = \dfrac{14.0\text{ m}}{v_0\cos 56.0°}$. Putting this quantity, along with the numerical quantities, into the equation $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ and solving for $v_0$ we get $v_0 = 13.3$ m/s.

**(b)** The initial horizontal velocity of the rock is (13.3 m/s)(cos 56.0°), and when it lands on the ground, $y - y_0 = -1.60$ m. Putting these quantities into the equation $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ leads to a quadratic equation in $t$. Using the positive square root, we get $t = 2.388$ s when the rock lands. The horizontal position at that instant is $x - x_0 =$ (13.3 m/s)(cos 56.0°)(2.388 s) = 17.8 m from the launch point. So the distance beyond the fence is 17.8 m – 14.0 m = 3.8 m.

**EVALUATE:** We cannot use the range formula to find the distance in (b) because the rock's motion does not start and end at the same height.

**3.70.** **IDENTIFY:** This is a projectile motion problem. The vertical acceleration is $g$ downward and the horizontal acceleration is zero. The constant-acceleration equations apply.

**SET UP:** The horizontal range is $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$.

**EXECUTE:** **(a)** We want to know how far the object has moved horizontally when it reaches its maximum height. At the maximum height, $v_y = 0$, so $v_y = v_0\sin\alpha_0 - gt$ gives

$t = \dfrac{v_0 \sin\alpha_0}{g}$ = (16.0 m/s)(sin 60.0°)/(9.80 m/s$^2$) = 1.414 s. This is the time to reach its maximum height.

The horizontal motion during that time gives
$x = (v_0 \cos\alpha_0)t$ = (16.0 m/s)(cos 60.0°)(1.414 s) = 11.3 m.

The horizontal range is $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$ = (16.0 m/s)$^2$(sin 120°)/(9.80 m/s$^2$) = 22.6 m. So we see that at the highest point, the object's horizontal displacement is one-half of its horizontal range.

**(b)** In this case, $x = 0.800R$ = (0.800)(22.6 m) = 18.1 m. We want to know the vertical position of the object when it has traveled this far horizontally. To find the time to reach 18.1 m horizontally, apply $x = (v_0 \cos\alpha_0)t$ and solve for $t$, giving $t$ = (18.1 m)/[(16.0 m/s)(cos 60.0°)] = 2.262 s. Now find the value of $y$ at this time using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$. This gives

$y$ = (16.0 m/s)(sin 60.0°)(2.262 s) – $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$ = 6.27 m.

In part (a) we saw that the object reaches its maximum height in 1.414 s. Using $y = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$, we get

$y_{max}$ = (16.0 m/s)(sin 60.0°)(1.414 s) – $\frac{1}{2}$(9.80 m/s$^2$)(1.414 s)$^2$ = 9.80 m.

Comparing the height at 2.262 s to its maximum height gives $h/h_{max}$ = (6.27 m)/(9.80 m) = 0.640, so it is at 64.0% of its maximum height.

**(c)** When $x - x_0 = \alpha R$, we want to find $\dfrac{y-y_0}{h_{max}}$. We use the same general approach as in part (b). First find $h_{max}$ using the fact that $v_y = 0$ at that point. $v_y^2 = v_{0y}^2 + 2a_y(y-y_0)$ gives $0 = (v_0\sin\alpha_0)^2 - 2gh_{max}$, so $h_{max} = \dfrac{v_0^2\sin^2\alpha_0}{2g}$. Now use the horizontal motion to find $t$ when $x - x_0 = \alpha R$, giving $v_0 \cos\alpha_0\, t = \alpha R$, so $t = t = \dfrac{\alpha R}{v_0\cos\alpha_0}$. Using the range formula $R = \dfrac{2v_0^2\sin\alpha_0\cos\alpha_0}{g}$, the time becomes

$t = \dfrac{\alpha}{v_0\cos\alpha_0}\left(\dfrac{2v_0^2\sin\alpha_0\cos\alpha_0}{g}\right) = \dfrac{2v_0\alpha\sin\alpha_0}{g}$. Now get $y - y_0$ for this time. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 =$ $v_0\sin\alpha_0\left(\dfrac{2v_0\alpha\sin\alpha_0}{g}\right) - \dfrac{1}{2}g\left(\dfrac{2v_0\sin\alpha_0}{g}\right)^2$. Simplifying gives $y - y_0 = \dfrac{2(v_0\sin\alpha_0)^2}{g}\alpha(1-\alpha)$. Now find the ratio $\dfrac{y-y_0}{h_{max}}$ using our results above. $\dfrac{y-y_0}{h_{max}} = \dfrac{\dfrac{2(v_0\sin\alpha_0)^2}{g}\alpha(1-\alpha)}{\dfrac{(v_0\sin\alpha_0)^2}{2g}} = 4\alpha(\alpha-1)$.

**EVALUATE:** Check our result in some special cases.

$\alpha$ = ½: $\dfrac{y-y_0}{h_{max}}$ = 4(½)(1 – ½) = 1. This is correct because $x$ is half-way to $R$, so the object is at its highest point, which makes $y - y_0 = h_{max}$.

$\alpha$ = 0: $\dfrac{y-y_0}{h_{max}}$ = 0. This is reasonable because the object is at its take-off point at ground level.

$\alpha$ = 1: $\dfrac{y-y_0}{h_{max}}$ = 0. This is reasonable because the object is at $x = R$, so it has returned to the ground.

$\alpha = 0.800$: $\dfrac{y - y_0}{h_{\max}} = 4(0.800)(1 - 0.800) = 0.640$. This agrees with our result in part (b).

**3.71.** **IDENTIFY:** Relative velocity problem. The plane's motion relative to the earth is determined by its velocity relative to the earth.

**SET UP:** Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

$\vec{v}_{P/E}$, the velocity of the plane relative to the earth.

$\vec{v}_{P/A}$, the velocity of the plane relative to the air (the magnitude $v_{P/A}$ is the airspeed of the plane and the direction of $\vec{v}_{P/A}$ is the compass course set by the pilot).

$\vec{v}_{A/E}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

**(a)** We are given the following information about the relative velocities:

$\vec{v}_{P/A}$ has magnitude 220 km/h and its direction is west. In our coordinates it has components $(v_{P/A})_x = -220$ km/h and $(v_{P/A})_y = 0$.

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{P/E}$ has components in our coordinate system of

$$(v_{P/E})_x = -\frac{120\text{ km}}{0.500\text{ h}} = -240\text{ km/h (west)}$$

$$(v_{P/E})_y = -\frac{20\text{ km}}{0.500\text{ h}} = -40\text{ km/h (south)}$$

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.71a.

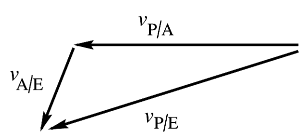



**Figure 3.71a**

We are asked to find $\vec{v}_{A/E}$, so solve for this vector:

$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ gives $\vec{v}_{A/E} = \vec{v}_{P/E} - \vec{v}_{P/A}$.

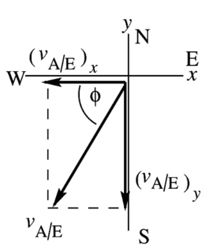
**EXECUTE:** The $x$-component of this equation gives

$(v_{A/E})_x = (v_{P/E})_x - (v_{P/A})_x = -240\text{ km/h} - (-220\text{ km/h}) = -20$ km/h.

The $y$-component of this equation gives

$(v_{A/E})_y = (v_{P/E})_y - (v_{P/A})_y = -40$ km/h.

Now that we have the components of $\vec{v}_{A/E}$ we can find its magnitude and direction.




$$v_{A/E} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2}$$

$$v_{A/E} = \sqrt{(-20\text{ km/h})^2 + (-40\text{ km/h})^2} = 44.7\text{ km/h}$$

$$\tan\phi = \frac{40\text{ km/h}}{20\text{ km/h}} = 2.00;\quad \phi = 63.4^\circ$$

The direction of the wind velocity is $63.4^\circ$ S of W, or $26.6^\circ$ W of S.

**Figure 3.71b**

**EVALUATE:** The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

**(b) SET UP:** The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a).

$\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found.

$\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south.

The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given.

The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.71c.

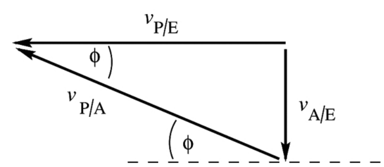



**Figure 3.71c**

The vector addition diagram forms a right triangle.

**EXECUTE:** $\sin\phi = \dfrac{v_{A/E}}{v_{P/A}} = \dfrac{40\text{ km/h}}{220\text{ km/h}} = 0.1818;\ \ \phi = 10.5°.$

The pilot should set her course $10.5°$ north of west.

**EVALUATE:** The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.

**3.72. IDENTIFY:** Use the relation that relates the relative velocities.

**SET UP:** The relative velocities are the raindrop relative to the earth, $\vec{v}_{R/E}$, the raindrop relative to the train, $\vec{v}_{R/T}$, and the train relative to the earth, $\vec{v}_{T/E}$. $\vec{v}_{R/E} = \vec{v}_{R/T} + \vec{v}_{T/E}$. $\vec{v}_{T/E}$ is due east and has magnitude 12.0 m/s. $\vec{v}_{R/T}$ is $30.0°$ west of vertical. $\vec{v}_{R/E}$ is vertical. The relative velocity addition diagram is given in Figure 3.72.

**EXECUTE:** **(a)** $\vec{v}_{R/E}$ is vertical and has zero horizontal component. The horizontal component of $\vec{v}_{R/T}$ is $-\vec{v}_{T/E}$, so is 12.0 m/s westward.

**(b)** $v_{R/E} = \dfrac{v_{T/E}}{\tan 30.0°} = \dfrac{12.0\text{ m/s}}{\tan 30.0°} = 20.8\text{ m/s}.$ $v_{R/T} = \dfrac{v_{T/E}}{\sin 30.0°} = \dfrac{12.0\text{ m/s}}{\sin 30.0°} = 24.0\text{ m/s}.$

**EVALUATE:** The speed of the raindrop relative to the train is greater than its speed relative to the earth, because of the motion of the train.

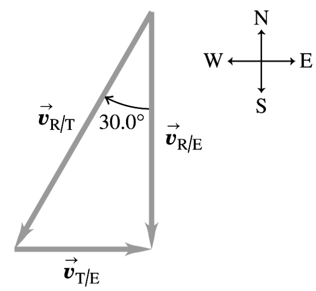



**Figure 3.72**

**3.73. IDENTIFY:** This is a relative velocity problem.

**SET UP:** The three relative velocities are:

$\vec{v}_{J/G}$ : Juan relative to the ground. This velocity is due north and has magnitude $v_{J/G} = 8.00$ m/s.

$\vec{v}_{B/G}$ : the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude $v_{B/G} = 12.00$ m/s.

$\vec{v}_{B/J}$ : the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{B/G} = \vec{v}_{B/J} + \vec{v}_{J/G}$, so $\vec{v}_{B/J} = \vec{v}_{B/G} - \vec{v}_{J/G}$.

The relative velocity addition diagram does not form a right triangle so we must do the vector addition using components.

Take $+y$ to be north and $+x$ to be east.

**EXECUTE:** $v_{B/Jx} = +v_{B/G} \sin 37.0° = 7.222$ m/s

$v_{B/Jy} = +v_{B/G} \cos 37.0° - v_{J/G} = 1.584$ m/s

These two components give $v_{B/J} = 7.39$ m/s at $12.4°$ north of east.

**EVALUATE:** Since Juan is running due north, the ball's eastward component of velocity relative to him is the same as its eastward component relative to the earth. The northward component of velocity for Juan and the ball are in the same direction, so the component for the ball relative to Juan is the difference in their components of velocity relative to the ground.

**3.74. IDENTIFY:** This problem involves relative velocities and vector addition.

**SET UP:** The catcher is standing still, so the velocity of the ball relative to the catcher is the same as its velocity relative to the ground. We use $\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$ . Fig. 3.74 shows the vector sum, where B refers to the baseball, S refers to the shortstop, and G is the ground.

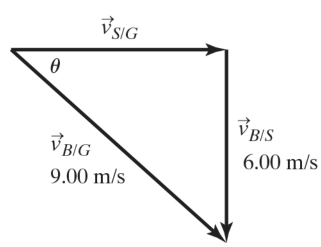



**Figure 3.74**

**EXECUTE:** From the figure, we see that $\sin\theta = \dfrac{6.00 \text{ m/s}}{9.00 \text{ m/s}} = 0.6667 \quad \rightarrow \quad \theta = 41.81°$. We also see that $v_{S/G} = (9.00 \text{ m/s}) \cos\theta = (9.00 \text{ m/s}) \cos 41.81° = 6.71$ m/s.

**EVALUATE:** As seen by the catcher, the ball is moving eastward at 6.71 m/s and southward at 6.00 m/s.

**3.75. IDENTIFY:** We need to use relative velocities.

**SET UP:** If B is moving relative to M and M is moving relative to E, the velocity of B relative to E is $\vec{v}_{B/E} = \vec{v}_{B/M} + \vec{v}_{M/E}$.

**EXECUTE:** Let $+x$ be east and $+y$ be north. We have $v_{B/M,x} = 2.50$ m/s, $v_{B/M,y} = -4.33$ m/s, $v_{M/E,x} = 0$, and $v_{M/E,y} = 6.00$ m/s. Therefore $v_{B/E,x} = v_{B/M,x} + v_{M/E,x} = 2.50$ m/s and $v_{B/E,y} = v_{B/M,y} + v_{M/E,y} = -4.33 \text{ m/s} + 6.00 \text{ m/s} = +1.67$ m/s. The magnitude is $v_{B/E} = \sqrt{(2.50 \text{ m/s})^2 + (1.67 \text{ m/s})^2} = 3.01$ m/s, and the direction is $\tan\theta = \frac{1.67}{2.50}$, which gives $\theta = 33.7^\circ$ north of east.

**EVALUATE:** Since Mia is moving, the velocity of the ball relative to her is different from its velocity relative to the ground or relative to Alice.

**3.76. IDENTIFY:** You have a graph showing the horizontal range of the rock as a function of the angle at which it was launched and want to find its initial velocity. Because air resistance is negligible, the rock is in free fall. The range formula applies since the rock rock was launced from the ground and lands at the ground.

**SET UP:** **(a)** The range formula is $R = \frac{v_0^2 \sin(2\theta)}{g}$, so a plot of $R$ versus $\sin(2\theta_0)$ will give a straight line having slope equal to $v_0^2/g$. We can use that data in the graph in the problem to construct our graph by hand, or we can use graphing software. The resulting graph is shown in Figure 3.76.

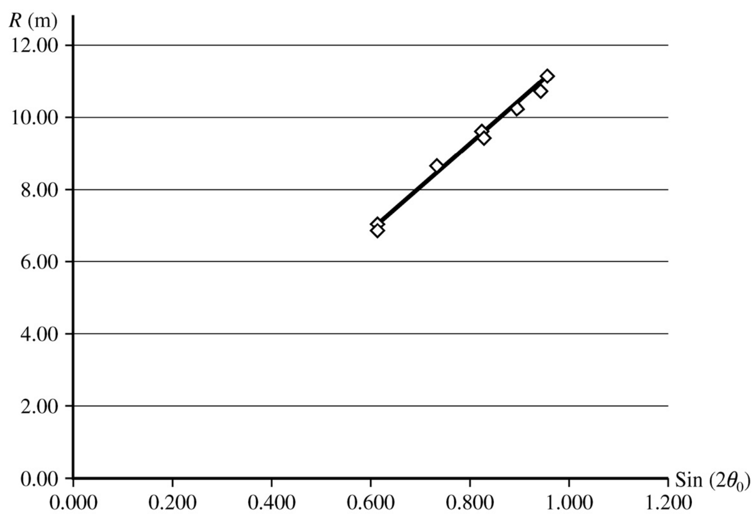



**Figure 3.76**

**(b)** The slope of the graph is 10.95 m, so 10.95 m = $v_0^2/g$. Solving for $v_0$ we get $v_0$ = 10.4 m/s.

**(c)** Solving the formula $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for $y - y_0$ with $v_y = 0$ at the highest point, we get $y - y_0$ = 1.99 m.

**EVALUATE:** This approach to finding the launch speed $v_0$ requires only simple measurements: the range and the launch angle. It would be difficult and would require special equipment to measure $v_0$ directly.

**3.77.** **IDENTIFY:** The table gives data showing the horizontal range of the potato for various launch heights. You want to use this information to determine the launch speed of the potato, assuming negligible air resistance.

**SET UP:** The potatoes are launched horizontally, so $v_{0y} = 0$, and they are in free fall, so $a_y = 9.80\ \text{m/s}^2$ downward and $a_x = 0$. The time a potato is in the air is just the time it takes for it to fall vertically from the launch point to the ground, a distance $h$.

**EXECUTE:** **(a)** For the vertical motion of a potato, we have $h = \frac{1}{2} gt^2$, so $t = \sqrt{2h/g}$. The horizontal range $R$ is given by $R = v_0 t = v_0\sqrt{2h/g}$. Squaring gives $R^2 = \left(\frac{2v_0^2}{g}\right)h$. Graphing $R^2$ versus $h$ will give a straight line with slope $2v_0^2/g$. We can graph the data from the table in the text by hand, or we could use graphing software. The result is shown in Figure 3.77.

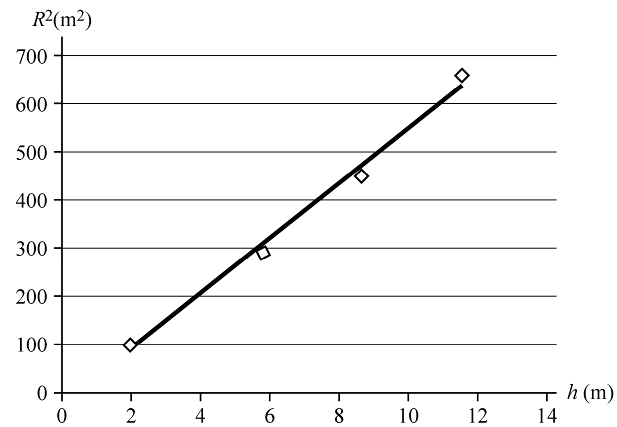



**Figure 3.77**

**(b)** The slope of the graph is 55.2 m, so $v_0 = \sqrt{\frac{(9.80\ \text{m/s}^2)(55.2\ \text{m})}{2}} = 16.4\ \text{m/s}.$

**(c)** In this case, the potatoes are launched and land at ground level, so we can use the range formula with $\theta = 30.0°$ and $v_0 = 16.4$ m/s. The result is $R = \frac{v_0^2 \sin(2\theta)}{g} = 23.8\ \text{m}.$

**EVALUATE:** This approach to finding the launch speed $v_0$ requires only simple measurements: the range and the launch height. It would be difficult and would require special equipment to measure $v_0$ directly.

**3.78.** **IDENTIFY:** This is a vector addition problem. The boat moves relative to the water and the water moves relative to the earth. We know the speed of the boat relative to the water and the times for the boat to go directly across the river, and from these things we want to find out how fast the water is moving and the width of the river.

**SET UP:** For both trips of the boat, $\vec{v}_{\text{B/E}} = \vec{v}_{\text{B/W}} + \vec{v}_{\text{W/E}}$, where the subscripts refer to the boat, earth, and water. The speed of the boat relative to the earth is $v_{\text{B/E}} = d/t$, where $d$ is the width of the river and $t$ is the time to cross the river, which is different in the two crossings.

**EXECUTE:** Figure 3.78 shows a vector sum for the first trip and for the return trip.

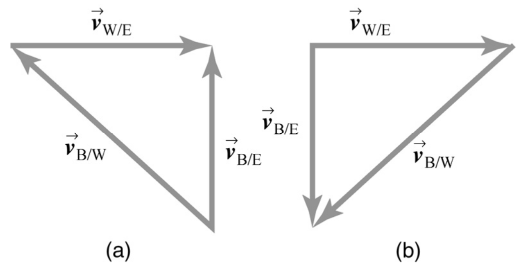



**Figure 3.78a-b**

**(a)** For both trips, the vectors in Figures 3.78 a & b form right triangles, so we can apply the Pythagorean theorem. $v_{B/E}^2 = v_{B/W}^2 - v_{W/E}^2$ and $v_{B/E} = d/t$. For the first trip, $v_{B/W} = 6.00$ m/s and $t = 20.1$ s, giving
$d^2/(20.1\text{ s})^2 = (60.00\text{ m/s}^2) - (v_{W/E})^2$. For the return trip, $v_{B/W} = 9.0$ m/s and $t = 11.2$ s, which gives $d^2/(11.2\text{ s})^2 = (9.00\text{ m/s}^2) - (v_{W/E})^2$. Solving these two equations together gives $d = 90.48$ m, which rounds to 90.5 m (the width of the river) and $v_{W/E} = 3.967$ m/s which rounds to 3.97 m/s (the speed of the current).
**(b)** The shortest time is when the boat heads perpendicular to the current, which is due north. Figure 3.78c illustrates this situation. The time to cross is $t = d/v_{B/W} = (90.48\text{ m})/(6.00\text{ m/s}) = 15.1$ s. The distance $x$ east (down river) that you travel is $x = v_{W/E}t = (3.967\text{ m/s})(15.1\text{ s}) = 59.9$ m east of your starting point.

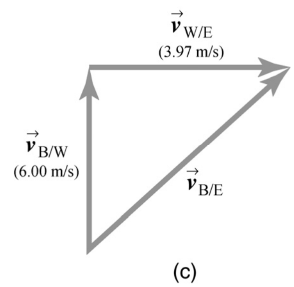



**Figure 3.78c**

**EVALUATE:** In part (a), the boat must have a velocity component up river to cancel out the current velocity. In part (b), velocity of the current has no effect on the crossing time, but it does affect the landing position of the boat.

**3.79.** **IDENTIFY:** Write an expression for the square of the distance $(D^2)$ from the origin to the particle, expressed as a function of time. Then take the derivative of $D^2$ with respect to $t$, and solve for the value of $t$ when this derivative is zero. If the discriminant is zero or negative, the distance $D$ will never decrease.
**SET UP:** $D^2 = x^2 + y^2$, with $x(t)$ and $y(t)$ given by Eqs. (3.19) and (3.20).
**EXECUTE:** Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.
**EVALUATE:** We know that if the object is thrown straight up it moves away from $P$ and then returns, so we are not surprised that the projectile angle must be less than some maximum value for the distance to always increase with time.

**3.80.** **IDENTIFY:** Apply the relative velocity relation.

**SET UP:** Let $v_{\text{C/W}}$ be the speed of the canoe relative to water and $v_{\text{W/G}}$ be the speed of the water relative to the ground.

**EXECUTE:** **(a)** Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{\text{C/W}} - v_{\text{W/G}} = 2.$ We know that heading downstream for a time $t$, $(v_{\text{C/W}} + v_{\text{W/G}})t = 5.$ We also know that for the bottle $v_{\text{W/G}}(t+1) = 3.$ Solving these three equations for $v_{\text{W/G}} = x$, $v_{\text{C/W}} = 2 + x$, therefore $(2 + x + x)t = 5$ or $(2 + 2x)t = 5.$ Also $t = 3/x - 1$, so $(2+2x)\left(\dfrac{3}{x} - 1\right) = 5$ or $2x^2 + x - 6 = 0.$

The positive solution is $x = v_{\text{W/G}} = 1.5\ \text{km/h}.$

**(b)** $v_{\text{C/W}} = 2\ \text{km/h} + v_{\text{W/G}} = 3.5\ \text{km/h}.$

**EVALUATE:** When they head upstream, their speed relative to the ground is $3.5\ \text{km/h} - 1.5\ \text{km/h} = 2.0\ \text{km/h}.$ When they head downstream, their speed relative to the ground is $3.5\ \text{km/h} + 1.5\ \text{km/h} = 5.0\ \text{km/h}.$ The bottle is moving downstream at 1.5 km/s relative to the earth, so they are able to overtake it.

**3.81.** **IDENTIFY:** The rocket has two periods of constant acceleration motion.

**SET UP:** Let $+y$ be upward. During the free-fall phase, $a_x = 0$ and $a_y = -g.$ After the engines turn on, $a_x = (3.00g)\cos 30.0°$ and $a_y = (3.00g)\sin 30.0°.$ Let *t* be the total time since the rocket was dropped and let *T* be the time the rocket falls before the engine starts.

**EXECUTE:** (i) The diagram is given in Figure 3.81 a.

(ii) The *x*-position of the plane is $(236\ \text{m/s})t$ and the *x*-position of the rocket is $(236\ \text{m/s})t + (1/2)(3.00)(9.80\ \text{m/s}^2)\cos 30°(t-T)^2.$ The graphs of these two equations are sketched in Figure 3.81 b.

(iii) If we take $y = 0$ to be the altitude of the airliner, then $y(t) = -1/2gT^2 - gT(t-T) + 1/2(3.00)(9.80\ \text{m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. The airliner has constant *y*. The graphs are sketched in Figure 3.81b.

In each of the Figures 3.81a–c, the rocket is dropped at $t = 0$ and the time *T* when the motor is turned on is indicated.

By setting $y = 0$ for the rocket, we can solve for *t* in terms of *T*:

$0 = -(4.90\ \text{m/s}^2)T^2 - (9.80\ \text{m/s}^2)T(t-T) + (7.35\ \text{m/s}^2)(t-T)^2.$ Using the quadratic formula for the variable $x = t - T$ we find $x = t - T = \dfrac{(9.80\ \text{m/s}^2)T + \sqrt{(9.80\ \text{m/s}^2 T)^2 + (4)(7.35\ \text{m/s}^2)(4.9)T^2}}{2(7.35\ \text{m/s}^2)}$, or $t = 2.72\,T.$ Now, using the condition that $x_{\text{rocket}} - x_{\text{plane}} = 1000\ \text{m}$, we find

$(236\ \text{m/s})t + (12.7\ \text{m/s}^2)(t-T)^2 - (236\ \text{m/s})t = 1000\ \text{m}$, or $(1.72T)^2 = 78.6\ \text{s}^2.$ Therefore $T = 5.15\ \text{s}.$

**EVALUATE:** During the free-fall phase the rocket and airliner have the same *x* coordinate but the rocket moves downward from the airliner. After the engines fire, the rocket starts to move upward and its horizontal component of velocity starts to exceed that of the airliner.

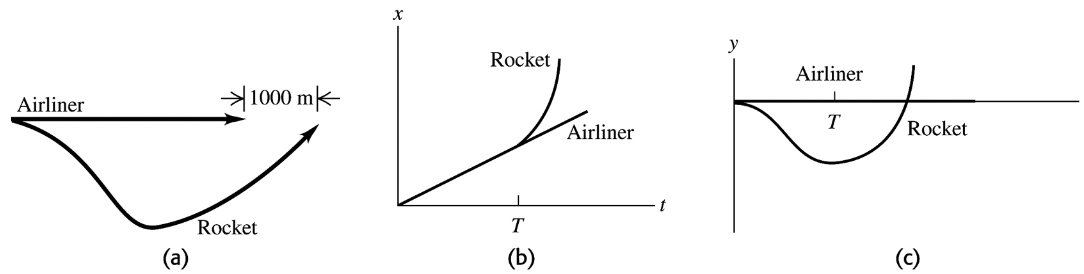



**Figure 3. 81**

**3.82.** **IDENTIFY:** We know the speed of the seeds and the distance they travel.
**SET UP:** We can treat the speed as constant over a very short distance, so $v = d/t$. The minimum frame rate is determined by the maximum speed of the seeds, so we use $v = 4.6$ m/s.
**EXECUTE:** Solving for $t$ gives $t = d/v = (0.20 \times 10^{-3}\text{ s})/(4.6\text{ m/s}) = 4.3 \times 10^{-5}$ s per frame. The frame rate is $1/(4.3 \times 10^{-5}$ s per frame$)$ = 23,000 frames/second. Choice (c) 25,000 frames per second is closest to this result, so choice (c) is the best one.
**EVALUATE:** This experiment would clearly require high-speed photography.

**3.83.** **IDENTIFY:** A seed launched at 90° goes straight up. Since we are ignoring air resistance, its acceleration is 9.80 m/s$^2$ downward.
**SET UP:** For the highest possible speed $v_{0y} = 4.6$ m/s, and $v_y = 0$ at the highest point.
**EXECUTE:** $v_y = v_{0y} - gt$ gives $t = v_{0y}/g = (4.6\text{ m/s})/(9.80\text{ m/s}^2) = 0.47$ s, which is choice (b).
**EVALUATE:** Seeds are rather light and 4.6 m/s is fairly fast, so it might not be such a good idea to ignore air resistance. But doing so is acceptable to get a first approximation to the time.

**3.84.** **IDENTIFY:** A seed launched at 0° starts out traveling horizontally from a height of 20 cm above the ground. Since we are ignoring air resistance, its acceleration is 9.80 m/s$^2$ downward.
**SET UP:** Its horizontal distance is determined by the time it takes the seed to fall 20 cm, starting from rest vertically.
**EXECUTE:** The time to fall 20 cm is $0.20\text{ m} = \frac{1}{2}gt^2$, which gives $t = 0.202$ s. The horizontal distance traveled during this time is $x = (4.6\text{ m/s})(0.202\text{ s}) = 0.93\text{ m} = 93$ cm, which is choice (b).
**EVALUATE:** In reality the seed would travel a bit less distance due to air resistance.

**3.85.** **IDENTIFY:** About 2/3 of the seeds are launched between 6° and 56° above the horizontal, and the average for all the seeds is 31°. So clearly most of the seeds are launched above the horizontal.
**SET UP and EXECUTE:** For choice (a) to be correct, the seeds would need to cluster around 90°, which they do not. For choice (b), most seeds would need to launch below the horizontal, which is not the case. For choice (c), the launch angle should be around +45°. Since 31° is not far from 45°, this is the best choice. For choice (d), the seeds should go straight downward. This would require a launch angle of –90°, which is not the case.
**EVALUATE:** Evolutionarily it would be an advantage for the seeds to get as far from the parent plant as possible so the young plants would not compete with the parent for water and soil nutrients, so 45° is a biologically plausible result. Natural selection would tend to favor plants that launched their seeds at this angle over those that did not.