

## RELATIVITY

**VP37.5.1. IDENTIFY:** This problem is about time dilation.

**SET UP:**  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ . We want the mean lifetime of the  $K^+$  particle.

**EXECUTE:** (a) At  $u = 0.800c$ :  $\Delta t = \frac{1.23 \times 10^{-8} \text{ s}}{\sqrt{1-(0.800)^2}} = 2.05 \times 10^{-8} \text{ s}$ .

(b) At  $u = 0.800c$ : To the first  $K^+$ , the second one is moving at  $u = 0.800c$ , so the answer is the same as in part (a):  $\Delta t = 2.05 \times 10^{-8} \text{ s}$ .

**EVALUATE:** The relative speed of the  $K^+$  is the same in both cases, so the lifetime is the same.

**VP37.5.2. IDENTIFY:** This problem is about time dilation.

**SET UP:**  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ . We want the time interval that both Paul and Alia read.

**EXECUTE:** (a) Paul's timer reads the proper time, so solve for  $\Delta t_0$ .

$$\Delta t_0 = \Delta t \sqrt{1-u^2/c^2} = (20.0 \text{ s}) \sqrt{1-(0.600)^2} = 16.0 \text{ s}.$$

(b) Alia's time is the proper time.  $\Delta t = \frac{24.0 \text{ s}}{\sqrt{1-(0.600)^2}} = 30.0 \text{ s}$ .

**EVALUATE:** The proper time is the time measured by the observer at rest in her frame.

**VP37.5.3. IDENTIFY:** This problem is about length contraction.

**SET UP:**  $L = L_0 \sqrt{1-u^2/c^2}$ . We want the distances and times measured.

**EXECUTE:** (a) In your frame, the proper length is  $1.50 \times 10^{11} \text{ m}$ .  $\Delta t = d/u = (1.50 \times 10^{11} \text{ m})/(0.950c) = 526 \text{ s}$ .

(b) For the astronaut, the distance is length contracted.

$L = (1.50 \times 10^{11} \text{ m}) \sqrt{1-(0.950)^2} = 4.68 \times 10^{10} \text{ m}$ . For the astronaut, the sun is moving toward him at  $0.950c$ , so  $\Delta t = L/u = (4.68 \times 10^{10} \text{ m})/(0.950c) = 164 \text{ s}$ .

**EVALUATE:** Check: Use time dilation. To a person on Earth, the trip takes 526 s. The proper time is the astronaut's time, so  $\Delta t_E = \frac{\Delta t_A}{\sqrt{1-(0.950)^2}} = 526 \text{ s}$ .

**VP37.5.4. IDENTIFY:** This problem is about length contraction.

**SET UP:**  $L = L_0 \sqrt{1-u^2/c^2}$ ,  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ . For you, it takes 6.00 s for the rocket clock to read

2.00 s. You measure  $L_{\text{rocket}} = 24.0 \text{ m}$ . We want the distances measured by the astronaut in the spacecraft.

**EXECUTE:** (a) Earth-Moon distance: First find the spacecraft's speed relative to Earth. The proper time is 2.00 s in the rocket.  $6.00 \text{ s} = \frac{2.00 \text{ s}}{\sqrt{1-u^2/c^2}}$ , so  $\sqrt{1-u^2/c^2} = 0.333$ . To the astronaut, the Earth-

Moon distance is length-contracted.  $L = L_0 \sqrt{1-u^2/c^2} = (3.84 \times 10^5 \text{ km})(0.333) = 1.28 \times 10^5 \text{ km}$ .

(b) The proper length of the spacecraft is measured by the astronaut, so  $L_0 = \frac{L}{\sqrt{1-u^2/c^2}} = \frac{24.0 \text{ m}}{0.333} = 72.0 \text{ m}$ .

**EVALUATE:** The Earth observer sees the spacecraft length-contracted and the astronaut sees the Earth-Moon distance length-contracted.

**VP37.7.1. IDENTIFY:** This problem is about the Lorentz transformation equations.

**SET UP:**  $x' = \gamma(x - ut)$ ,  $t' = \gamma(t - xu/c^2)$ .  $\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-(0.750)^2}} = 1.512$ . Gamora is the

primed frame and Nebula is the unprimed frame.

**EXECUTE:** (a) We want the coordinates as measured by Gamora. Using  $x' = \gamma(x - ut)$ , we have  $ut = (0.750c)(2.50 \text{ s}) = 5.625 \times 10^8 \text{ m}$ .  $x' = (1.512)(0 - 5.625 \times 10^8 \text{ m}) = -8.50 \times 10^8 \text{ m}$ . Using  $t' = \gamma(t - xu/c^2)$  gives  $t' = (1.512)(2.50 \text{ s} - 0) = 3.78 \text{ s}$ .

(b) We want the coordinates as measured by Nebula. She is the unprimed frame, so  $x = \gamma(x' + ut')$  and  $t = \gamma(t' + x'u/c^2)$ .  $x'u/c^2 = (4.00 \times 10^8 \text{ m})(0.750c)/c^2 = 1.00 \text{ s}$ . This gives  $x = \gamma(x' + ut') = (1.512)(-4.00 \times 10^8 \text{ m} + 5.625 \times 10^8 \text{ m}) = 1.46 \times 10^9 \text{ m}$ .  $t = \gamma(t' + x'u/c^2) = (1.512)(2.50 \text{ s} + 1.00 \text{ s}) = 5.29 \text{ s}$ .

**EVALUATE:** Be very careful to decide which are the primed and unprimed frames.

**VP37.7.2. IDENTIFY:** This problem is about the Lorentz transformation equations.

**SET UP:**  $x' = \gamma(x - ut)$ ,  $t' = \gamma(t - xu/c^2)$ .  $\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-(0.800)^2}} = 1.6667$ . Doreen is the

primed frame and Kamala is the unprimed frame. In Kamala's frame, the events are simultaneous but occur at different places. In Doreen's frame the events are 0.600 s apart. We want the distance between the two events as measured by Doreen and Kamala.

**EXECUTE:** (a) In Kamala's frame: Use the Lorentz transformation equation for time and solve for  $\Delta x_K$ .  $\Delta t'_D = \gamma(\Delta t_K - x_K u/c^2)$ .  $0.600 \text{ s} = (1.6667)(0 - \Delta x_K(0.800c)/c^2)$ .  $\Delta x_K = 1.35 \times 10^8 \text{ m}$ .

(b) In Doreen's frame:  $\Delta x'_D = \gamma(\Delta x_K - u \Delta t_K) = \Delta x'_D = \gamma(\Delta x_K - 0) = (1.6667)(1.35 \times 10^8 \text{ m}) = 2.25 \times 10^8 \text{ m}$ .

**EVALUATE:** The distances are different because of their relative motion.

**VP37.7.3. IDENTIFY:** This problem is about relative velocity.

**SET UP:**  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ ,  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$ . Fig. 37.7.3 shows the motions involved.

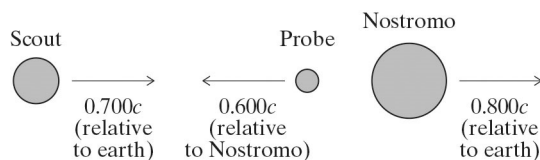


Figure VP37.7.3

**EXECUTE:** (a) We want the velocity of the probe relative to Earth. Let *Nostromo* be the primed frame moving relative to Earth at  $0.800c$  in the  $+x$  direction. Earth is the unprimed frame.

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.600c + 0.800c}{1 + (0.800c)(-0.600c)/c^2} = +0.385c.$$

(b) We want the velocity of the Scout ship relative to *Nostromo*. Let *Nostromo* be the primed frame and Earth the unprimed frame. We want  $v'_x$ . Using  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$  gives

$$v'_x = \frac{0.700c - 0.800c}{1 - (0.800c)(0.700c)/c^2} = -0.227c.$$

**EVALUATE:** The result in (b) is reasonable because *Nostromo* is moving faster than the Scout probe, so relative to *Nostromo*, the Scout is moving in the  $-x$  direction.

**VP37.7.4. IDENTIFY** This problem is about relative velocity.

**SET UP:**  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ . We want the speed of *Yamato* relative to *Macross*.

**EXECUTE:** (a) *Yamato* is flying toward Earth. Let *Macross* be the primed frame and Earth the

unprimed frame. We want  $v'_y$ .  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$  becomes  $v'_y = \frac{v_y - u}{1 - uv_y/c^2}$

$$= \frac{-650c - 0.750c}{1 - (0.750c)(-0.650c)/c^2} = -0.941c. \text{ Yamato's speed is } 0.941c.$$

$$(b) \text{ Yamato is flying away from Earth. } v'_y = \frac{+650c - 0.750c}{1 - (0.750c)(+0.650c)/c^2} = -0.195c.$$

**EVALUATE:** In both cases *Yamato* has a negative velocity component because relative to *Macross*, *Yamato* is getting closer, hence moving in the  $-x$  direction. The relative speed in (b) is less than that in (a), as expected.

**VP37.11.1. IDENTIFY:** This problem deals with the force on a proton and its momentum at high speeds.

**SET UP and EXECUTE:**  $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - (0.950)^2}} = 3.20256$ . (a) We want the momentum.

$$p = m\gamma c = (1.67 \times 10^{-27} \text{ kg})(3.20256)c = 1.52 \times 10^{-18} \text{ kg} \cdot \text{m/s}.$$

(b) We want the acceleration. If the force and velocity are along the same line,  $F = \gamma^3 ma$ . So  $a = \frac{F}{m\gamma^3}$ .

Using the given numbers gives  $a = 1.64 \times 10^{12} \text{ m/s}^2$ .

(c) We want the acceleration. If the force and velocity are perpendicular,  $F = \gamma ma$ . So  $a = \frac{F}{m\gamma}$ . Using

the given numbers we get  $a = 1.68 \times 10^{13} \text{ m/s}^2$ .

**EVALUATE:** Without relativity the acceleration would be  $a = F/m = 5.39 \times 10^{13} \text{ m/s}^2$ , which is greater than when relativity is taken into account.

**VP37.11.2. IDENTIFY:** This problem is about the energy of a moving electron.

**SET UP and EXECUTE:** (a) We want the kinetic energy.  $E = K + mc^2$ , so  $K = E - mc^2$

$$= 4.00 \times 10^{-13} \text{ J} + (9.11 \times 10^{-31} \text{ kg})c^2 = 3.18 \times 10^{-13} \text{ J}.$$

(b) We want  $\gamma$ .  $E = m\gamma c^2$ . Solve for  $\gamma$  and use the given numbers.  $\gamma = \frac{E}{mc^2} = 4.88$ .

(c) We want the speed. Solve  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$  for  $v$ .  $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/4.88^2} = 0.979$ , so

$$v = 0.979c.$$

**EVALUATE:** At speeds near the speed of light, the kinetic energy is much different from  $\frac{1}{2}mv^2$ .

**VP37.11.3. IDENTIFY:** This problem is about the energy of a moving electron.

**SET UP:**  $E^2 = (pc)^2 + (mc^2)^2$ ,  $E = m\gamma c^2$ ,  $v/c = \sqrt{1-1/\gamma^2}$ .

**EXECUTE:** (a) We want the total energy  $E$ . Solve  $E^2 = (pc)^2 + (mc^2)^2$  for  $E$  and use the given numbers for the momentum  $p$  and rest mass  $m$ . The result is  $E = 1.13 \times 10^{-13}$  J.

(b) We want  $\gamma$ . Solve  $E = m\gamma c^2$  for  $\gamma$  and use the given  $m$  and  $E$  we found in (a), giving  $\gamma = 1.38$ .

(c) We want the speed  $v$ .  $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/1.38^2} = 0.689$ , so  $v = 0.689c$ .

**EVALUATE:** Careful!  $E \neq pc + mc^2$ .

**VP37.11.4 IDENTIFY:** This problem is about energy in particle decay.

**SET UP:** The  $\psi(2S)$  is at rest, so by momentum conservation the K mesons have equal but opposite momentum and therefore equal speeds and equal kinetic energies.  $mc^2 = 495$  MeV for each K meson.  $E = K + mc^2$ ,  $K = (\gamma - 1)mc^2$ .

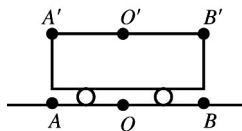
**EXECUTE:** (a) We want the kinetic energy  $K$  of each meson. Using  $E = K + mc^2$  gives  $3686 \text{ MeV} = (K + 495 \text{ MeV}) + (K + 495 \text{ MeV})$ . Solving gives  $K = 1348$  MeV for each meson.

(b) We want  $\gamma$ .  $K = (\gamma - 1)mc^2 = (\gamma - 1)(495 \text{ MeV})$ . Solve for  $\gamma$  and use  $K = 1348$  MeV from part (a), giving  $\gamma = 3.72$ .

(c) We want the speed.  $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/3.72^2} = 0.963$ , so  $v = 0.963c$ .

**EVALUATE:** The  $K^+$  and  $K^-$  mesons are highly relativistic with  $v = 0.963c$  and  $\gamma = 3.72$ .

**37.1. IDENTIFY and SET UP:** Consider the distance  $A$  to  $O'$  and  $B$  to  $O'$  as observed by an observer on the ground (Figure 37.1).



**Figure 37.1**

**EXECUTE:** The statement that the events are simultaneous to an observer on the train means that light pulses from  $A'$  and  $B'$  arrive at  $O'$  at the same time. To the observer at  $O$ , light from  $A'$  has a longer distance to travel than light from  $B'$  so  $O$  will conclude that the pulse from  $A(A')$  started before the pulse at  $B(B')$ . To the observer at  $O$ , bolt  $A$  appeared to strike first.

**EVALUATE:** Section 37.2 shows that if the events are simultaneous to the observer on the ground, then an observer on the train measures that the bolt at  $B'$  struck first.

**37.2. IDENTIFY:** Apply  $\Delta t = \gamma \Delta t_0$ .

**SET UP:** The lifetime measured in the muon frame is the proper time  $\Delta t_0$ .  $u = 0.900c$  is the speed of the muon frame relative to the laboratory frame. The distance the particle travels in the lab frame is its speed in that frame times its lifetime in that frame.

**EXECUTE:** (a)  $\gamma = \frac{1}{\sqrt{1-(0.9)^2}} = 2.29$ .  $\Delta t = \gamma \Delta t_0 = (2.29)(2.20 \times 10^{-6} \text{ s}) = 5.05 \times 10^{-6} \text{ s}$ .

(b)  $d = v \Delta t = (0.900)(3.00 \times 10^8 \text{ m/s})(5.05 \times 10^{-6} \text{ s}) = 1.36 \times 10^3 \text{ m} = 1.36 \text{ km}$ .

**EVALUATE:** The lifetime measured in the lab frame is larger than the lifetime measured in the muon frame.

**37.3. IDENTIFY and SET UP:** The problem asks for  $u$  such that  $\Delta t_0 / \Delta t = \frac{1}{2}$ .

**EXECUTE:**  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$  gives  $u = c \sqrt{1 - (\Delta t_0 / \Delta t)^2} = (3.00 \times 10^8 \text{ m/s}) \sqrt{1 - (\frac{1}{2})^2} = 2.60 \times 10^8 \text{ m/s}$ ;

$\frac{u}{c} = 0.867$ .

**EVALUATE:** Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for  $u$ .

**37.4. IDENTIFY:** Time dilation occurs because the rocket is moving relative to Mars.

**SET UP:** The time dilation equation is  $\Delta t = \gamma \Delta t_0$ , where  $t_0$  is the proper time.

**EXECUTE:** (a) The two time measurements are made at the same place on Mars by an observer at rest there, so the observer on Mars measures the proper time.

(b)  $\Delta t = \gamma \Delta t_0 = \frac{1}{\sqrt{1-(0.985)^2}} (75.0 \mu\text{s}) = 435 \mu\text{s}$ .

**EVALUATE:** The pulse lasts for a longer time relative to the rocket than it does relative to the Mars observer.

**37.5. (a) IDENTIFY and SET UP:**  $\Delta t_0 = 2.60 \times 10^{-8} \text{ s}$ ;  $\Delta t = 4.20 \times 10^{-7} \text{ s}$ . In the lab frame the pion is created and decays at different points, so this time is not the proper time.

**EXECUTE:**  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$  says  $1 - \frac{u^2}{c^2} = \left( \frac{\Delta t_0}{\Delta t} \right)^2$ .

$\frac{u}{c} = \sqrt{1 - \left( \frac{\Delta t_0}{\Delta t} \right)^2} = \sqrt{1 - \left( \frac{2.60 \times 10^{-8} \text{ s}}{4.20 \times 10^{-7} \text{ s}} \right)^2} = 0.998$ ;  $u = 0.998c$ .

**EVALUATE:**  $u < c$ , as it must be, but  $u/c$  is close to unity and the time dilation effects are large.

(b) **IDENTIFY and SET UP:** The speed in the laboratory frame is  $u = 0.998c$ ; the time measured in this frame is  $\Delta t$ , so the distance as measured in this frame is  $d = u \Delta t$ .

**EXECUTE:**  $d = (0.998)(2.998 \times 10^8 \text{ m/s})(4.20 \times 10^{-7} \text{ s}) = 126 \text{ m}$ .

**EVALUATE:** The distance measured in the pion's frame will be different because the time measured in the pion's frame is different (shorter).

**37.6. IDENTIFY:** Apply  $\Delta t = \gamma \Delta t_0$ .

**SET UP:** For part (a) the proper time is measured by the race pilot.  $\gamma = 1.667$ .

**EXECUTE:** (a)  $\Delta t = \frac{1.20 \times 10^8 \text{ m}}{(0.800)(3.00 \times 10^8 \text{ m/s})} = 0.500 \text{ s}$ .  $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{0.500 \text{ s}}{1.667} = 0.300 \text{ s}$ .

(b)  $(0.300 \text{ s})(0.800c) = 7.20 \times 10^7 \text{ m}$ .

(c) You read  $\frac{1.20 \times 10^8 \text{ m}}{(0.800)(3 \times 10^8 \text{ m/s})} = 0.500 \text{ s}$ .

**EVALUATE:** The two events are the spaceracer passing you and the spaceracer reaching a point  $1.20 \times 10^8 \text{ m}$  from you. The timer traveling with the spaceracer measures the proper time between these two events.

**37.7. IDENTIFY and SET UP:** The proper time is measured in the frame where the two events occur at the same point.

**EXECUTE:** (a) The time of 12.0 ms measured by the first officer on the craft is the proper time.

$$(b) \Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \text{ gives } u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = c\sqrt{1-(12.0 \times 10^{-3}/0.150)^2} = 0.997c.$$

**EVALUATE:** The observer at rest with respect to the searchlight measures a much shorter duration for the event.

**37.8. IDENTIFY:** This problem involves time dilation.

**SET UP:**  $\Delta t = \gamma \Delta t_0$ . The captain's time is the proper time. We want  $x$  and  $t$  in our frame.

$$\text{EXECUTE: } \Delta t = \gamma \Delta t_0 = (100)(1.00 \text{ s}) = 100 \text{ s. } x = vt \approx ct = c(100 \text{ s}) = 3.00 \times 10^{10} \text{ m.}$$

**EVALUATE:** Note that  $v \approx c$ , not  $v = c$ . But with  $\gamma = 100$ , the speed is *extremely* close to  $c$ .

**37.9. IDENTIFY and SET UP:**  $l = l_0 \sqrt{1-u^2/c^2}$ . The length measured when the spacecraft is moving is  $l = 74.0 \text{ m}$ ;  $l_0$  is the length measured in a frame at rest relative to the spacecraft.

$$\text{EXECUTE: } l_0 = \frac{l}{\sqrt{1-u^2/c^2}} = \frac{74.0 \text{ m}}{\sqrt{1-(0.600c/c)^2}} = 92.5 \text{ m.}$$

**EVALUATE:**  $l_0 > l$ . The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

**37.10. IDENTIFY and SET UP:** When the meterstick is at rest with respect to you, you measure its length to be 1.000 m, and that is its proper length,  $l_0$ .  $l = 0.3048 \text{ m}$ .

$$\text{EXECUTE: } l = l_0 \sqrt{1-u^2/c^2} \text{ gives } u = c\sqrt{1-(l/l_0)^2} = c\sqrt{1-(0.3048/1.00)^2} = 0.9524c = 2.86 \times 10^8 \text{ m/s.}$$

**EVALUATE:** The needed speed is well beyond modern capabilities for any rocket.

**37.11. IDENTIFY and SET UP:** The  $2.2 \mu\text{s}$  lifetime is  $\Delta t_0$  and the observer on earth measures  $\Delta t$ . The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is  $l$  and  $l_0$  is 10 km.

**EXECUTE:** (a) The greatest speed the muon can have is  $c$ , so the greatest distance it can travel in  $2.2 \times 10^{-6} \text{ s}$  is  $d = vt = (3.00 \times 10^8 \text{ m/s})(2.2 \times 10^{-6} \text{ s}) = 660 \text{ m} = 0.66 \text{ km}$ .

$$(b) \Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{2.2 \times 10^{-6} \text{ s}}{\sqrt{1-(0.999)^2}} = 4.9 \times 10^{-5} \text{ s.}$$

$$d = vt = (0.999)(3.00 \times 10^8 \text{ m/s})(4.9 \times 10^{-5} \text{ s}) = 15 \text{ km.}$$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

$$(c) l = l_0 \sqrt{1-u^2/c^2} = (10 \text{ km})\sqrt{1-(0.999)^2} = 0.45 \text{ km.}$$

**EVALUATE:** In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

**37.12. IDENTIFY:** The astronaut lies along the motion of the rocket, so his height will be Lorentz-contracted.

**SET UP:** The doctor in the rocket measures his proper length  $l_0$ .

**EXECUTE:** (a)  $l_0 = 2.00 \text{ m}$ .  $l = l_0 \sqrt{1-u^2/c^2} = (2.00 \text{ m})\sqrt{1-(0.910)^2} = 0.829 \text{ m}$ . The person on Earth would measure his height to be 0.829 m.

$$(b) l = 2.00 \text{ m. } l_0 = \frac{l}{\sqrt{1-u^2/c^2}} = \frac{2.00 \text{ m}}{\sqrt{1-(0.910)^2}} = 4.82 \text{ m. This is not a reasonable height for a human.}$$

(c) There is no length contraction in a direction perpendicular to the motion and both observers measure the same height, 2.00 m.

**EVALUATE:** The length of an object moving with respect to the observer is shortened in the direction of the motion, so in (a) and (b) the observer on Earth measures a shorter height.

**37.13. IDENTIFY:** Apply  $l = l_0 \sqrt{1 - u^2/c^2}$ .

**SET UP:** The proper length  $l_0$  of the runway is its length measured in the Earth's frame. The proper time  $\Delta t_0$  for the time interval for the spacecraft to travel from one end of the runway to the other is the time interval measured in the frame of the spacecraft.

**EXECUTE:** (a)  $l_0 = 3600$  m.

$$l = l_0 \sqrt{1 - \frac{u^2}{c^2}} = (3600 \text{ m}) \sqrt{1 - \frac{(4.00 \times 10^7 \text{ m/s})^2}{(3.00 \times 10^8 \text{ m/s})^2}} = (3600 \text{ m})(0.991) = 3568 \text{ m}.$$

$$(b) \Delta t = \frac{l_0}{u} = \frac{3600 \text{ m}}{4.00 \times 10^7 \text{ m/s}} = 9.00 \times 10^{-5} \text{ s}.$$

$$(c) \Delta t_0 = \frac{l}{u} = \frac{3568 \text{ m}}{4.00 \times 10^7 \text{ m/s}} = 8.92 \times 10^{-5} \text{ s}.$$

**EVALUATE:**  $\frac{1}{\gamma} = 0.991$ , so  $\Delta t = \gamma \Delta t_0$  gives  $\Delta t = \frac{8.92 \times 10^{-5} \text{ s}}{0.991} = 9.00 \times 10^{-5} \text{ s}$ . The result from length contraction is consistent with the result from time dilation.

**37.14. IDENTIFY:** This problem requires use of the Lorentz transformation equations.

**SET UP:**  $t' = \gamma(t - xu/c^2)$ ,  $t = \gamma(t' + x'u/c^2)$ . We want to know for what values  $x$  and  $x'$  will  $t$  be equal to  $t'$ .

**EXECUTE:** Using  $t' = \gamma(t - xu/c^2)$ , let  $t = t'$  and solve for  $x$ .  $t(\gamma - 1) = xu\gamma/c^2$ .

$$x = \frac{c^2 t(\gamma - 1)}{u\gamma} = \frac{c^2 t}{u} \left(1 - \frac{1}{\gamma}\right). \text{ Now find } x'. t' = \gamma(t - xu/c^2) \text{ and } t = \gamma(t' + x'u/c^2). \text{ If } t = t', \text{ then it}$$

follows that  $x'u/c^2 = -xu/c^2$ , so  $x' = -x$ .

**EVALUATE:** The clocks are coordinated since  $t = t'$ .

**37.15. IDENTIFY:** Apply  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$ .

**SET UP:** The velocities  $\vec{v}'$  and  $\vec{v}$  are both in the  $+x$ -direction, so  $v'_x = v'$  and  $v_x = v$ .

$$(a) v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.400c + 0.600c}{1 + (0.400)(0.600)} = 0.806c.$$

$$(b) v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.900c + 0.600c}{1 + (0.900)(0.600)} = 0.974c.$$

$$(c) v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.990c + 0.600c}{1 + (0.990)(0.600)} = 0.997c.$$

**EVALUATE:** Speed  $v$  is always less than  $c$ , even when  $v' + u$  is greater than  $c$ .

**37.16. IDENTIFY:** Apply  $\Delta t = \gamma \Delta t_0$  and the equations for  $x$  and  $t$  that are developed in Example 37.6.

**SET UP:**  $S$  is Stanley's frame and  $S'$  is Mavis's frame. The proper time for the two events is the time interval measured in Mavis's frame.  $\gamma = 1.667$  ( $\gamma = 5/3$  if  $u = (4/5)c$ ).

**EXECUTE:** (a) In Mavis's frame the event "light on" has space-time coordinates  $x' = 0$  and  $t' = 5.00$  s, so from the result of Example 37.6,  $x = \gamma(x' + ut')$  and

$$t = \gamma\left(t' + \frac{ux'}{c^2}\right) \Rightarrow x = \gamma ut' = 2.00 \times 10^9 \text{ m}, t = \gamma t' = 8.33 \text{ s}.$$

(b) The 5.00-s interval in Mavis's frame is the proper time  $\Delta t_0$ , so  $\Delta t = \gamma \Delta t_0 = 8.33$  s, the same as in part (a).

(c)  $(8.33 \text{ s})(0.800c) = 2.00 \times 10^9 \text{ m}$ , which is the distance  $x$  found in part (a).

**EVALUATE:** Mavis would measure that she would be a distance  $(5.00 \text{ s})(0.800c) = 1.20 \times 10^9 \text{ m}$  from Stanley when she turns on her light. In  $l = l_0/\gamma$ ,  $l_0 = 2.00 \times 10^9 \text{ m}$  and  $l = 1.20 \times 10^9 \text{ m}$ .

**37.17. IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

**SET UP:** The relativistic velocity addition formula is  $v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$ .

**EXECUTE:** (a) For the pursuit ship to catch the cruiser, the distance between them must be decreasing, so the velocity of the cruiser relative to the pursuit ship must be directed toward the pursuit ship.

(b) Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity  $v'$  of the cruiser knowing the velocity of the primed frame  $u$  and the velocity of the cruiser  $v$

in the unprimed frame (Tatooine).  $v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} = \frac{0.600c - 0.800c}{1 - (0.600)(0.800)} = -0.385c$ .

The result implies that the cruiser is moving toward the pursuit ship at  $0.385c$ .

**EVALUATE:** The nonrelativistic formula would have given  $-0.200c$ , which is considerably different from the correct result.

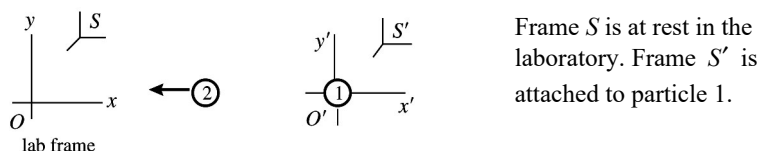
**37.18. IDENTIFY and SET UP:** Let the starfighter's frame be  $S$  and let the enemy spaceship's frame be  $S'$ . Let the positive  $x$ -direction for both frames be from the enemy spaceship toward the starfighter. Then  $u = +0.400c$ .  $v' = +0.700c$ .  $v$  is the velocity of the missile relative to you.

**EXECUTE:** (a)  $v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.700c + 0.400c}{1 + (0.400)(0.700)} = 0.859c$ .

(b) Use the distance it moves as measured in your frame and the speed it has in your frame to calculate the time it takes in your frame.  $t = \frac{8.00 \times 10^9 \text{ m}}{(0.859)(3.00 \times 10^8 \text{ m/s})} = 31.0 \text{ s}$ .

**EVALUATE:** Note that the speed in (a) is not  $1.1c$  as nonrelativistic physics would predict.

**37.19. IDENTIFY and SET UP:** Reference frames  $S$  and  $S'$  are shown in Figure 37.19.



**Figure 37.19**

$u$  is the speed of  $S'$  relative to  $S$ ; this is the speed of particle 1 as measured in the laboratory. Thus  $u = +0.650c$ . The speed of particle 2 in  $S'$  is  $0.950c$ . Also, since the two particles move in opposite directions, 2 moves in the  $-x'$ -direction and  $v'_x = -0.950c$ . We want to calculate  $v_x$ , the speed of

particle 2 in frame  $S$ , so use  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$ .

**EXECUTE:**  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.950c + 0.650c}{1 + (0.650c)(-0.950c)/c^2} = \frac{-0.300c}{1 - 0.6175} = -0.784c$ . The speed of the second particle, as measured in the laboratory, is  $0.784c$ .



**EVALUATE:** The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is  $0.300c$ . The correct relativistic calculation gives a result more than twice this.

- 37.20. IDENTIFY and SET UP:** Let  $S$  be the laboratory frame and let  $S'$  be the frame of one of the particles, as shown in Figure 37.20. Let the positive  $x$ -direction for both frames be from particle 1 to particle 2. In the lab frame particle 1 is moving in the  $+x$ -direction and particle 2 is moving in the  $-x$ -direction. Then  $u = 0.9380c$  and  $v_x = -0.9380c$ .  $v'_x$  is the velocity of particle 2 relative to particle 1.

**EXECUTE:** 
$$v'_x = \frac{v_x - u}{1 - uv_x/c^2} = \frac{-0.9380c - 0.9380c}{1 - (0.9380c)(-0.9380c)/c^2} = -0.9980c.$$
 The speed of particle 2 relative to particle 1 is  $0.9980c$ .

**EVALUATE:**  $v'_x < 0$  shows particle 2 is moving toward particle 1.

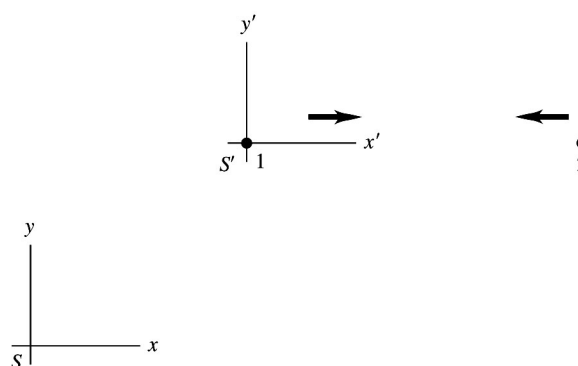


Figure 37.20

- 37.21. IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

**SET UP:** The relativistic velocity addition formula is 
$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}.$$

**EXECUTE:** In the relativistic velocity addition formula for this case,  $v'_x$  is the relative speed of particle 1 with respect to particle 2,  $v$  is the speed of particle 2 measured in the laboratory, and  $u$  is the speed of particle 1 measured in the laboratory,  $u = -v$ .

$$v'_x = \frac{v - (-v)}{1 - (-v)v/c^2} = \frac{2v}{1 + v^2/c^2}. \quad \frac{v'_x}{c^2}v^2 - 2v + v'_x = 0 \quad \text{and} \quad (0.890c)v^2 - 2c^2v + (0.890c^3) = 0.$$

This is a quadratic equation with solution  $v = 0.611c$  ( $v$  must be less than  $c$ ).

**EVALUATE:** The nonrelativistic result would be  $0.445c$ , which is considerably different from this result.

- 37.22. IDENTIFY:** There is a Doppler effect in the frequency of the radiation due to the motion of the star.

**SET UP:** The star is moving away from the earth, so 
$$f = \sqrt{\frac{c - u}{c + u}} f_0.$$

$$\text{EXECUTE: } f = \sqrt{\frac{c - 0.520c}{c + 0.520c}} f_0 = 0.5620 f_0 = (0.5620)(8.64 \times 10^{14} \text{ Hz}) = 4.86 \times 10^{14} \text{ Hz}.$$

**EVALUATE:** The earth observer measures a lower frequency than the star emits because the star is moving away from the earth.

**37.23. IDENTIFY and SET UP:** Source and observer are approaching, so use  $f = \sqrt{\frac{c+u}{c-u}} f_0$ . Solve

for  $u$ , the speed of the light source relative to the observer.

**EXECUTE:** (a)  $f^2 = \left(\frac{c+u}{c-u}\right) f_0^2$ .

$$(c-u)f^2 = (c+u)f_0^2 \text{ and } u = \frac{c(f^2 - f_0^2)}{f^2 + f_0^2} = c \left( \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} \right).$$

$$\lambda_0 = 675 \text{ nm}, \quad \lambda = 575 \text{ nm}.$$

$$u = \left( \frac{(675 \text{ nm}/575 \text{ nm})^2 - 1}{(675 \text{ nm}/575 \text{ nm})^2 + 1} \right) c = 0.159c = (0.159)(2.998 \times 10^8 \text{ m/s}) = 4.77 \times 10^7 \text{ m/s}; \text{ definitely speeding}$$

(b)  $4.77 \times 10^7 \text{ m/s} = (4.77 \times 10^7 \text{ m/s})(1 \text{ km}/1000 \text{ m})(3600 \text{ s}/1 \text{ h}) = 1.72 \times 10^8 \text{ km/h}$ . Your fine would be  $\$1.72 \times 10^8$  (172 million dollars).

**EVALUATE:** The source and observer are approaching, so  $f > f_0$  and  $\lambda < \lambda_0$ . Our result gives  $u < c$ , as it must.

**37.24. IDENTIFY:** There is a Doppler effect in the frequency of the radiation due to the motion of the source.

**SET UP:**  $f > f_0$  so the source is moving toward you.  $f = \sqrt{\frac{c+u}{c-u}} f_0$ .

**EXECUTE:**  $(f/f_0)^2 = \frac{c+u}{c-u}$ .  $c(f/f_0)^2 - (f/f_0)^2 u = c + u$ .

$$u = \frac{c[(f/f_0)^2 - 1]}{(f/f_0)^2 + 1} = \left[ \frac{(1.25)^2 - 1}{(1.25)^2 + 1} \right] c = 0.220c, \text{ toward you}.$$

**EVALUATE:** The difference in frequency is rather large (1.25 times), so the motion of the source must be a substantial fraction of the speed of light (around 20% in this case).

**37.25. IDENTIFY:** The problem involves momentum and the Lorentz factor.

**SET UP:**  $p = m\gamma v$ ,  $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$ ,  $u/c = \sqrt{1-1/\gamma^2}$ , particle 1:  $\gamma_1 = 1.12$ , particle 2:  $u_2 = 2u_1$ .

**EXECUTE:** (a) We want  $\gamma_2$ . First find  $u_1$ .  $u_1 = c\sqrt{1-1/\gamma_1^2} = c\sqrt{1-1/1.12^2} = 0.450343c$ , so  $u_2 = 2u_1 = 0.90068c$ .  $\gamma_2 = \frac{1}{\sqrt{1-(0.90068)^2}} = 2.30$ .

(b) We want  $p_2/p_1$ .  $\frac{p_2}{p_1} = \frac{m\gamma_2 u_2}{m\gamma_1 u_1} = \frac{\gamma_2(2u_1)}{\gamma_1 u_1} = \frac{2\gamma_2}{\gamma_1} = \frac{2(2.30)}{1.12} = 4.11$ .

**EVALUATE:** For speeds much less than that of light, we would have  $p_2 = 2p_1$ , but near the speed of light we find  $p_2 = 4.11p_1$ , a very significant difference.

**37.26. IDENTIFY and SET UP:** The force is found from  $F = \gamma^3 ma$  or  $F = \gamma ma$ , whichever is applicable.

**EXECUTE:** (a) Indistinguishable from  $F = ma = 0.145 \text{ N}$ .

(b)  $\gamma^3 ma = 1.75 \text{ N}$ .

(c)  $\gamma^3 ma = 51.7 \text{ N}$ .

(d)  $\gamma ma = 0.145 \text{ N}$ ,  $0.333 \text{ N}$ ,  $1.03 \text{ N}$ .

**EVALUATE:** When  $v$  is large, much more force is required to produce a given magnitude of acceleration when the force is parallel to the velocity than when the force is perpendicular to the velocity.

**37.27. IDENTIFY:** The speed of the proton is a substantial fraction of the speed of light, so we must use the relativistic formula for momentum.

**SET UP:**  $p = \gamma mv$ .  $p_0 = \gamma_0 mv_0$ .  $\frac{p}{p_0} = \frac{\gamma v}{\gamma_0 v_0}$ .  $v/v_0 = 2.00$ .

**EXECUTE:**  $\gamma_0 = \frac{1}{\sqrt{1-v_0^2/c^2}} = \frac{1}{\sqrt{1-(0.400)^2}} = 1.0911$ .  $\gamma = \frac{1}{\sqrt{1-(0.800)^2}} = 1.667$ .

$p = p_0(2)\left(\frac{1.667}{1.091}\right) = 3.06 p_0$ .

**EVALUATE:** The speed doubles but the momentum more than triples.

**37.28. IDENTIFY:** In this problem we deal with length contraction and momentum.

**SET UP:**  $L = L_0/\gamma$ ,  $p = m\gamma v$ ,  $v/c = \sqrt{1-1/\gamma^2}$ . We measure  $L = 90$  m, but its proper length is 120 m.

**EXECUTE: (a)** The diameter will be 25 m. There is no length contraction perpendicular to the direction of motion.

**(b)** We want the momentum  $p$ . Use length contraction to find  $\gamma$ .  $L = L_0/\gamma$ :  $90 \text{ m} = (120 \text{ m})/\gamma$ , so  $\gamma = 1.33$ . Now use  $\gamma$  to find  $v$ .  $v/c = \sqrt{1-1/\gamma^2} = \sqrt{1-1/1.33^2} = 0.6614$ . Now use  $p = m\gamma v$  to find  $p$ .  $p = m\gamma v = (4000 \text{ kg})(1.33)(0.6614c) = 1.1 \times 10^{12} \text{ kg} \cdot \text{m/s}$ .

**EVALUATE:** Ignoring relativity we would measure a length of 120 m and a momentum of  $p = mv = (4000 \text{ kg})(0.6614c) = 7.9 \times 10^{11} \text{ kg} \cdot \text{m/s}$ . A big difference from the actual momentum!

**37.29. IDENTIFY:** Apply  $p = \frac{mv}{\sqrt{1-v^2/c^2}}$  and  $F = \gamma^3 ma$ .

**SET UP:** For a particle at rest (or with  $v \ll c$ ),  $a = F/m$ .

**EXECUTE: (a)**  $p = \frac{mv}{\sqrt{1-v^2/c^2}} = 2mv$ .

$\Rightarrow 1 = 2\sqrt{1-v^2/c^2} \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.866c$ .

**(b)**  $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$  so  $\frac{1}{1-v^2/c^2} = 2^{2/3} \Rightarrow \frac{v}{c} = \sqrt{1-2^{-2/3}} = 0.608$ .

**EVALUATE:** The momentum of a particle and the force required to give it a given acceleration both increase without bound as the speed of the particle approaches  $c$ .

**37.30. IDENTIFY:** Use  $E = mc^2$  to relate the mass decrease to the energy produced.

**SET UP:** 1 kg is equivalent to 2.2 lbs and 1 ton = 2000 lbs. 1 W = 1 J/s.

**EXECUTE: (a)**  $E = mc^2$ ,  $m = E/c^2 = (3.8 \times 10^{26} \text{ J})/(2.998 \times 10^8 \text{ m/s})^2 = 4.2 \times 10^9 \text{ kg} = 4.6 \times 10^6 \text{ tons}$ .

**(b)** The current mass of the sun is  $1.99 \times 10^{30} \text{ kg}$ , so it would take it  $(1.99 \times 10^{30} \text{ kg})/(4.2 \times 10^9 \text{ kg/s}) = 4.7 \times 10^{20} \text{ s} = 1.5 \times 10^{13} \text{ years}$  to use up all its mass.

**EVALUATE:** The power output of the sun is very large, but only a small fraction of the sun's mass is converted to energy each second.

**37.31. IDENTIFY:** Apply  $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$ .

**SET UP:** The rest energy is  $mc^2$ .

**EXECUTE:** (a)  $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2.$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}}c = 0.866c.$$

(b)  $K = 5mc^2 \Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \frac{1}{36} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{35}{36}}c = 0.986c.$

**EVALUATE:** If  $v \ll c$ , then  $K$  is much less than the rest energy of the particle.

**37.32. IDENTIFY:** At such a high speed, we must use the relativistic formulas for momentum and kinetic energy.

**SET UP:**  $m_\mu = 207m_e = 1.89 \times 10^{-28}$  kg.  $v$  is very close to  $c$  and we must use relativistic expressions.

$$p = \frac{mv}{\sqrt{1-v^2/c^2}}, \quad K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2.$$

**EXECUTE:**  $p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{(1.89 \times 10^{-28} \text{ kg})(0.999)(3.00 \times 10^8 \text{ m/s})}{\sqrt{1-(0.999)^2}} = 1.27 \times 10^{-18} \text{ kg} \cdot \text{m/s}.$

Using  $K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$  gives

$$K = (1.89 \times 10^{-28} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left( \frac{1}{\sqrt{1-(0.999)^2}} - 1 \right) = 3.63 \times 10^{-10} \text{ J}.$$

**EVALUATE:** The nonrelativistic values are  $p_{\text{nr}} = mv = 5.66 \times 10^{-20} \text{ kg} \cdot \text{m/s}$  and  $K_{\text{nr}} = \frac{1}{2}mv^2 = 8.49 \times 10^{-12} \text{ J}$ . Each relativistic result is much larger.

**37.33. IDENTIFY and SET UP:** Use  $E = mc^2 + K$  and  $E^2 = (mc^2)^2 + (pc)^2$ .

**EXECUTE:** (a)  $E = mc^2 + K$ , so  $E = 4.00mc^2$  means  $K = 3.00mc^2 = 4.50 \times 10^{-10} \text{ J}$ .

(b)  $E^2 = (mc^2)^2 + (pc)^2$ ;  $E = 4.00mc^2$ , so  $15.0(mc^2)^2 = (pc)^2$ .

$$p = \sqrt{15}mc = 1.94 \times 10^{-18} \text{ kg} \cdot \text{m/s}.$$

(c)  $E = mc^2/\sqrt{1-v^2/c^2}$ .

$$E = 4.00mc^2 \text{ gives } 1 - v^2/c^2 = 1/16 \text{ and } v = \sqrt{15/16}c = 0.968c.$$

**EVALUATE:** The speed is close to  $c$  since the kinetic energy is greater than the rest energy. Nonrelativistic expressions relating  $E$ ,  $K$ ,  $p$ , and  $v$  will be very inaccurate.

**37.34. IDENTIFY:** Apply the work energy theorem in the form  $W = \Delta K$ .

**SET UP:**  $K$  is given by  $K = (\gamma - 1)mc^2$ . When  $v = 0$ ,  $\gamma = 1$ .

**EXECUTE:** (a)  $W = \Delta K = (\gamma_f - 1)mc^2 = (4.07 \times 10^{-3})mc^2$ .

(b)  $(\gamma_f - \gamma_i)mc^2 = 4.79mc^2$ .

(c) The result of part (b) is far larger than that of part (a).

**EVALUATE:** The amount of work required to produce a given increase in speed (in this case an increase of  $0.090c$ ) increases as the initial speed increases.

- 37.35. IDENTIFY and SET UP:** The total energy is given in terms of the momentum by  $E^2 = (mc^2)^2 + (pc)^2$ . In terms of the total energy  $E$ , the kinetic energy  $K$  is  $K = E - mc^2$ . The rest energy is  $mc^2$ .

**EXECUTE: (a)**

$$E = \sqrt{(mc^2)^2 + (pc)^2} = \sqrt{[(6.64 \times 10^{-27})(2.998 \times 10^8)^2]^2 + [(2.10 \times 10^{-18})(2.998 \times 10^8)]^2} \text{ J.}$$

$$E = 8.67 \times 10^{-10} \text{ J.}$$

$$\text{(b) } mc^2 = (6.64 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 5.97 \times 10^{-10} \text{ J.}$$

$$K = E - mc^2 = 8.67 \times 10^{-10} \text{ J} - 5.97 \times 10^{-10} \text{ J} = 2.70 \times 10^{-10} \text{ J.}$$

$$\text{(c) } \frac{K}{mc^2} = \frac{2.70 \times 10^{-10} \text{ J}}{5.97 \times 10^{-10} \text{ J}} = 0.452.$$

**EVALUATE:** The incorrect nonrelativistic expressions for  $K$  and  $p$  give  $K = p^2/2m = 3.3 \times 10^{-10} \text{ J}$ ; the correct relativistic value is less than this.

- 37.36. IDENTIFY:** With such a large potential difference, the electrons will be accelerated to relativistic speeds, so we must use the relativistic formula for kinetic energy.

**SET UP:**  $K = \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) mc^2$ . The classical expression for kinetic energy is  $K = \frac{1}{2}mv^2$ .

**EXECUTE:** For an electron  $mc^2 = (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 8.20 \times 10^{-14} \text{ J}$ .

$$K = 7.50 \times 10^5 \text{ eV} = 1.20 \times 10^{-13} \text{ J.}$$

$$\text{(a) } \frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1-v^2/c^2}}. \quad \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1.20 \times 10^{-13} \text{ J}}{8.20 \times 10^{-14} \text{ J}} + 1 = 2.46.$$

$$v = c\sqrt{1 - (1/2.46)^2} = 0.914c = 2.74 \times 10^8 \text{ m/s.}$$

$$\text{(b) } K = \frac{1}{2}mv^2 \text{ gives } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.20 \times 10^{-13} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} = 5.13 \times 10^8 \text{ m/s.}$$

**EVALUATE:** At a given speed the relativistic value of the kinetic energy is larger than the nonrelativistic value. Therefore, for a given kinetic energy the relativistic expression for kinetic energy gives a smaller speed than the nonrelativistic expression.

- 37.37. IDENTIFY and SET UP:** The nonrelativistic expression is  $K_{\text{nonrel}} = \frac{1}{2}mv^2$  and the relativistic expression is  $K_{\text{rel}} = (\gamma - 1)mc^2$ .

**EXECUTE: (a)**  $v = 8 \times 10^7 \text{ m/s} \Rightarrow \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 1.0376$ . For  $m = m_p$ ,  $K_{\text{nonrel}} = \frac{1}{2}mv^2 = 5.34 \times 10^{-12} \text{ J}$ .

$$K_{\text{rel}} = (\gamma - 1)mc^2 = 5.65 \times 10^{-12} \text{ J.} \quad \frac{K_{\text{rel}}}{K_{\text{nonrel}}} = 1.06.$$

$$\text{(b) } v = 2.85 \times 10^8 \text{ m/s; } \gamma = 3.203.$$

$$K_{\text{nonrel}} = \frac{1}{2}mv^2 = 6.78 \times 10^{-11} \text{ J; } K_{\text{rel}} = (\gamma - 1)mc^2 = 3.31 \times 10^{-10} \text{ J; } K_{\text{rel}}/K_{\text{nonrel}} = 4.88.$$

**EVALUATE:**  $K_{\text{rel}}/K_{\text{nonrel}}$  increases without bound as  $v$  approaches  $c$ .

- 37.38. IDENTIFY:** The total energy is conserved in the collision.

**SET UP:** Use  $E = mc^2 + K$  for the total energy. Since all three particles are at rest after the collision, the final total energy is  $2Mc^2 + mc^2$ . The initial total energy of the two protons is  $\gamma 2Mc^2$ .

**EXECUTE:** (a)  $2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1 + \frac{m}{2M} = 1 + \frac{9.75}{2(16.7)} = 1.292.$

Note that since  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ , we have that  $\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.292)^2}} = 0.6331.$

(b) According to  $K = (\gamma - 1)mc^2$  the kinetic energy of each proton is

$$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left( \frac{1.00 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}} \right) = 274 \text{ MeV}.$$

(c) The rest energy of  $\eta^0$  is  $mc^2 = (9.75 \times 10^{-28} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left( \frac{1.00 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}} \right) = 548 \text{ MeV}.$

**EVALUATE:** (d) The kinetic energy lost by the protons is the energy that produces the  $\eta^0$ ,  $548 \text{ MeV} = 2(274 \text{ MeV}).$

**37.39. IDENTIFY and SET UP:** Use  $K = q\Delta V = e\Delta V$  and conservation of energy to relate the potential

difference to the kinetic energy gained by the electron. Use  $K = mc^2 \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right)$  to calculate the

kinetic energy from the speed.

**EXECUTE:** (a)  $K = q\Delta V = e\Delta V.$

$$K = mc^2 \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) = 4.025mc^2 = 3.295 \times 10^{-13} \text{ J} = 2.06 \text{ MeV}.$$

$$\Delta V = K/e = 2.06 \times 10^6 \text{ V}.$$

(b) From part (a),  $K = 3.30 \times 10^{-13} \text{ J} = 2.06 \text{ MeV}.$

**EVALUATE:** The speed is close to  $c$  and the kinetic energy is four times the rest mass.

**37.40. IDENTIFY and SET UP:** The astronaut in the spaceship measures the proper time, since the end of a

swing occurs at the same location in his frame.  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}.$

**EXECUTE:** (a)  $\Delta t_0 = 1.80 \text{ s}.$   $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{1.80 \text{ s}}{\sqrt{1-(0.75c/c)^2}} = 2.72 \text{ s}.$

(b)  $\Delta t = 1.80 \text{ s}.$   $\Delta t_0 = \Delta t \sqrt{1-u^2/c^2} = (1.80 \text{ s}) \sqrt{1-(0.75c/c)^2} = 1.19 \text{ s}.$

**EVALUATE:** The motion of the spaceship makes a considerable difference in the measured values for the period of the pendulum!

**37.41. IDENTIFY and SET UP:** There must be a length contraction such that the length  $a$  becomes the same as  $b$ ;  $l_0 = a$ ,  $l = b$ .  $l_0$  is the distance measured by an observer at rest relative to the spacecraft. Use

$$l = l_0 \sqrt{1-u^2/c^2} \text{ and solve for } u.$$

**EXECUTE:**  $\frac{l}{l_0} = \sqrt{1-u^2/c^2}$  so  $\frac{b}{a} = \sqrt{1-u^2/c^2};$

$$a = 1.40b \text{ gives } b/1.40b = \sqrt{1-u^2/c^2}. \text{ and thus } 1-u^2/c^2 = 1/(1.40)^2.$$

$$u = \sqrt{1-1/(1.40)^2}c = 0.700c = 2.10 \times 10^8 \text{ m/s}.$$

**EVALUATE:** A length on the spacecraft in the direction of the motion is shortened. A length perpendicular to the motion is unchanged.

**37.42. IDENTIFY and SET UP:** The proper length of a side is  $l_0 = a$ . The side along the direction of motion is shortened to  $l = l_0 \sqrt{1 - v^2/c^2}$ . The sides in the two directions perpendicular to the motion are unaffected by the motion and still have a length  $a$ .

**EXECUTE:**  $V = a^2 l = a^3 \sqrt{1 - v^2/c^2}$ .

**EVALUATE:** Only the side parallel to the direction of motion is contracted.

**37.43. IDENTIFY and SET UP:** The proper time  $\Delta t_0$  is the time that elapses in the frame of the space probe.  $\Delta t$  is the time that elapses in the frame of the earth. The distance traveled is 42.2 light years, as measured in the earth frame.

**EXECUTE:** Light travels 42.2 light years in 42.2 y, so  $\Delta t = \left( \frac{c}{0.9930c} \right) (42.2 \text{ y}) = 42.5 \text{ y}$ .

$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = (42.5 \text{ y}) \sqrt{1 - (0.9930)^2} = 5.0 \text{ y}$ . She measures her biological age to be 19 y + 5.0 y = 24.0 y.

**EVALUATE:** Her age measured by someone on earth is 19 y + 42.5 y = 61.5 y.

**37.44. IDENTIFY:** This problem involves time dilation, energy, and momentum.

**SET UP:**  $\Delta t = \gamma \Delta t_0$ ,  $v/c = \sqrt{1 - 1/\gamma^2}$ ,  $K = (\gamma - 1)mc^2$ . In the lab:  $t_{\text{mean}} = 403 \text{ ps}$ , in the particle's frame:  $t_{\text{mean}} = 80.2 \text{ ps}$ .

**EXECUTE:** (a) We want the speed. Use time dilation:  $\Delta t = \gamma \Delta t_0$  gives  $403 \text{ ps} = \gamma(80.2 \text{ ps})$ , so

$\gamma = 5.025$ . Now get the speed:  $v/c = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/5.025^2} = 0.980c$ .

(b) We want  $\Delta x$  in the lab.  $\Delta x = v \Delta t = (0.980c)(403 \text{ ps}) = 11.8 \text{ cm}$ .

(c) We want its energies in the lab frame.

Rest energy:  $E_{\text{rest}} = mc^2 = (2.12 \times 10^{-27} \text{ kg})c^2 = 1.91 \times 10^{-10} \text{ J}$ .

Kinetic energy:  $K = (\gamma - 1)mc^2 = (5.025 - 1)(1.91 \times 10^{-10} \text{ J}) = 7.68 \times 10^{-10} \text{ J}$ .

Total energy:  $E_{\text{tot}} = K + E_{\text{rest}} = 7.68 \times 10^{-10} \text{ J} + 1.91 \times 10^{-10} \text{ J} = 9.59 \times 10^{-10} \text{ J}$ .

(d) We want the energies in its rest frame.  $K = 0$ , so  $E_{\text{tot}} = E_{\text{rest}} = 1.91 \times 10^{-10} \text{ J}$ .

**EVALUATE:** Even for nonrelativistic motion,  $K = 0$  in the particle's rest frame.

**37.45. IDENTIFY:** This problem involves relative velocity and mass.

**SET UP:**  $m_{\text{rel}} = m\gamma = \frac{m}{\sqrt{1 - u^2/c^2}}$ ,  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ . We want the mass of the standard kilogram.

**EXECUTE:** (a) In classical physics, mass doesn't change with speed, so both crews would measure 1.0 kg.

(b) For A:  $m_{\text{rel}} = m\gamma_A = \frac{m}{\sqrt{1 - u_A^2/c^2}} = \frac{1.00 \text{ kg}}{\sqrt{1 - 0.80^2}} = 1.7 \text{ kg}$ .

For B: We need to find the speed of B relative to Earth. Use  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ , letting Earth be the primed

frame and A the unprimed frame. We want  $v'$ .  $v'_{B/E} = \frac{0.98c - 0.80c}{1 - (0.98c)(0.80c)/c^2} = 0.833c$ . The mass is

$m_{\text{rel}} = \frac{m}{\sqrt{1 - u_B^2/c^2}} = \frac{1.00 \text{ kg}}{\sqrt{1 - 0.833^2}} = 1.8 \text{ kg}$ .

**EVALUATE:** B measures a greater mass than A measures because B is moving faster relative to Earth than A is. Both of them, however, would measure 1.00 kg as the rest mass of the standard kilogram.

**37.46. IDENTIFY:** We are looking at a world in which  $c = 65$  mph.

**SET UP:** The standard relativity equations all apply except that we use  $c = 65$  mph. You are driving at 60 mph.

**EXECUTE: (a)** We want the time difference between the kitchen clock and the moving wristwatch.

**Kitchen clock time:** You drive 60 mi at 60 mph, so it would take 1.0 h.

**Wristwatch time:** Your time is the proper time. Use  $\Delta t = \gamma \Delta t_0$ .

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-(60/65)^2}} = 2.60$$

$$\Delta t_0 = \Delta t / \gamma = (1.0 \text{ h}) / (2.60) = 0.38 \text{ h} = 23 \text{ min}$$

**(b)** Estimate: 15 ft long.

**(c)** We want its length  $L$  from a roadside observer.  $L_0 = 15$  ft = proper length.

$$L = L_0 / \gamma = (15 \text{ ft}) / (2.60) = 5.8 \text{ ft} = 5 \text{ ft } 9 \text{ in.}$$

**(d)** We want the speed relative to you. Use  $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$ , with your car being the primed frame and

$$\text{earth the unprimed frame. We want } v'. \quad v' = \frac{-60 \text{ mph} - 60 \text{ mph}}{1 - (60)(-60)/(65)^2} = 64.8 \text{ mph.}$$

**(e)** We want the car's length in your frame.  $L = L_0 \sqrt{1-u^2/c^2} = (15 \text{ ft}) \sqrt{1-64.8^2/65^2} = 1.2 \text{ ft.}$

**(f)** We want the work to reach 60 mph.  $c = 65 \text{ mph} = 29.055 \text{ m/s}$ .  $W = K = (\gamma - 1)mc^2$   
 $= (2.60 - 1)(2000 \text{ kg})(29.055 \text{ m/s})^2 = 2.7 \text{ MJ.}$

**(g)** We want the work in the real world. Since  $60 \text{ mph} \ll c$ , we use  $K = \frac{1}{2}mv^2$  with 60 mph  
 $= 26.82 \text{ m/s}$ . This gives  $W = 720 \text{ kJ.}$

**EVALUATE:** Relativity would be obvious to us if we normally moved near the speed of light.

**37.47. IDENTIFY:** Since the speed is very close to the speed of light, we must use the relativistic formula for kinetic energy.

**SET UP:** The relativistic formula for kinetic energy is  $K = mc^2 \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right)$  and the relativistic

$$\text{mass is } m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}}.$$

**EXECUTE: (a)**  $K = 7.0 \times 10^{12} \text{ eV} = 1.12 \times 10^{-6} \text{ J}$ . Using this value in the relativistic kinetic energy

formula and substituting the mass of the proton for  $m$ , we get  $K = mc^2 \left( \frac{1}{\sqrt{1-v^2/c^2}} - 1 \right)$  which gives

$$\frac{1}{\sqrt{1-v^2/c^2}} = 7.45 \times 10^3 \text{ and } 1 - \frac{v^2}{c^2} = \frac{1}{(7.45 \times 10^3)^2}. \text{ Solving for } v \text{ gives } 1 - \frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c},$$

since  $c+v \approx 2c$ . Substituting  $v = (1-\Delta)c$ , we have  $1 - \frac{v^2}{c^2} = \frac{2(c-v)}{c} = \frac{2[c-(1-\Delta)c]}{c} = 2\Delta$ . Solving for

$$\Delta \text{ gives } \Delta = \frac{1-v^2/c^2}{2} = \frac{(7.45 \times 10^3)^2}{2} = 9.0 \times 10^{-9}.$$



(b) Using the relativistic mass formula and the result that  $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45 \times 10^3$ , we have

$$m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}} = m \left( \frac{1}{\sqrt{1-v^2/c^2}} \right) = (7.5 \times 10^3)m.$$

**EVALUATE:** At such high speeds, the proton's mass is 7500 times as great as its rest mass.

**37.48. IDENTIFY and SET UP:** The acceleration parallel to the direction of the force is given by  $F_{\parallel} = \gamma^3 m a_{\parallel}$ , and the acceleration perpendicular to the direction of the force is given by  $F_{\perp} = \gamma m a_{\perp}$ , where

$$\gamma = 1/\sqrt{1-v^2/c^2}.$$

**EXECUTE:** Applying the above formulas to the conditions of this problem, we have

$\gamma = 1/\sqrt{1-(0.700)^2} = 1.400$ ,  $F_x = F \cos(30.0^\circ) = \gamma^3 m a_x$  and  $F_y = F \sin(30.0^\circ) = \gamma m a_y$ . The angle  $\theta$  that the acceleration makes with respect to the  $x$ -axis is given by  $\tan \theta = a_y/a_x$ . Dividing the acceleration given by the two force equations gives

$$\tan \theta = \frac{\frac{F \sin(30.0^\circ)}{\gamma m}}{\frac{F \cos(30.0^\circ)}{\gamma^3 m}} = \gamma^2 \tan(30.0^\circ) = (1.400)^2 \tan(30.0^\circ) = 1.132 \quad \rightarrow \quad \theta = 48.5^\circ.$$

The acceleration makes a counterclockwise angle of  $48.5^\circ$  from the  $+x$ -axis. Therefore it makes an angle of  $18.5^\circ$  counterclockwise from the direction of the force.

**EVALUATE:** Notice that the acceleration is not in the same direction as the force.

**37.49. IDENTIFY and SET UP:** The clock on the plane measures the proper time  $\Delta t_0$ .

$$\Delta t = 4.00 \text{ h} = (4.00 \text{ h})(3600 \text{ s/h}) = 1.44 \times 10^4 \text{ s}.$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \quad \text{and} \quad \Delta t_0 = \Delta t \sqrt{1-u^2/c^2}.$$

**EXECUTE:**  $\frac{u}{c}$  small so  $\sqrt{1-u^2/c^2} = (1-u^2/c^2)^{1/2} \approx 1 - \frac{1}{2} \frac{u^2}{c^2}$ ; thus  $\Delta t_0 = \Delta t \left( 1 - \frac{1}{2} \frac{u^2}{c^2} \right)$ .

The difference in the clock readings is

$$\Delta t - \Delta t_0 = \frac{1}{2} \frac{u^2}{c^2} \Delta t = \frac{1}{2} \left( \frac{250 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}} \right)^2 (1.44 \times 10^4 \text{ s}) = 5.01 \times 10^{-9} \text{ s}.$$

The clock on the plane has the shorter elapsed time.

**EVALUATE:**  $\Delta t_0$  is always less than  $\Delta t$ ; our results agree with this. The speed of the plane is much less than the speed of light, so the difference in the reading of the two clocks is very small.

**37.50. IDENTIFY:** In the rest frame of the spaceship the trip takes  $\Delta t_0 = 3.35$  years.

**SET UP:** As seen from the Earth the trip takes  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ . The distance to the star is 7.11 ly and the speed of light is  $c = 1 \text{ ly/y}$ . Let  $u$  be the speed of the spaceship (in ly/y) as seen from the Earth.

**EXECUTE: (a)** The time for the trip as seen from the earth will be  $\Delta t = \frac{7.11 \text{ ly}}{u} = \frac{3.35 \text{ y}}{\sqrt{1-u^2/c^2}}$ . Solving

for  $u$  we obtain  $\left( \frac{7.11 \text{ ly}}{3.35 \text{ y}} \right)^2 (1-u^2/c^2) = u^2$ , which reduces to

$$u = \left( \frac{7.11 \text{ ly}}{3.35 \text{ y}} \right) \cdot \frac{1}{\sqrt{1 + \left( \frac{7.11 \text{ ly}}{(3.35 \text{ y})(1 \text{ ly/y})} \right)^2}} = 0.905 \text{ ly/y} = 0.905c. \text{ Thus, as seen from the Earth, the trip}$$

$$\text{takes } \frac{7.11 \text{ ly}}{u} = \frac{7.11 \text{ ly}}{0.905 \text{ ly/y}} = 7.86 \text{ years.}$$

(b) According to the passengers the distance is given by  $x' = u\Delta t_0 = (0.905 \text{ ly/y})(3.35 \text{ y}) = 3.03 \text{ ly}$ .

**EVALUATE:** The distance to the star as seen by the passengers could also be calculated by using the length contraction formula:  $l = l_0 \sqrt{1 - u^2/c^2}$ .

**37.51. IDENTIFY and SET UP:** In crown glass the speed of light is  $v = \frac{c}{n}$ . Calculate the kinetic energy of an electron that has this speed.

$$\text{EXECUTE: } v = \frac{2.998 \times 10^8 \text{ m/s}}{1.52} = 1.972 \times 10^8 \text{ m/s.}$$

$$K = mc^2(\gamma - 1).$$

$$mc^2 = (9.109 \times 10^{-31} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2 = 8.187 \times 10^{-14} \text{ J} (1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 0.5111 \text{ MeV.}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - ((1.972 \times 10^8 \text{ m/s})/(2.998 \times 10^8 \text{ m/s}))^2}} = 1.328.$$

$$K = mc^2(\gamma - 1) = (0.5111 \text{ MeV})(1.328 - 1) = 0.168 \text{ MeV.}$$

**EVALUATE:** No object can travel faster than the speed of light in vacuum but there is nothing that prohibits an object from traveling faster than the speed of light in some material.

**37.52. IDENTIFY:** We use relativistic energy to investigate the quark model of the proton.

$$\text{SET UP: } K = (\gamma - 1)mc^2, \quad v/c = \sqrt{1 - 1/\gamma^2}, \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}.$$

$$\text{EXECUTE: (a) We want } \gamma_u. \quad m_p c^2 = 2K_u + K_d = 3K_u. \quad K_u = \frac{1}{3}m_p c^2 \text{ and } K_u = m_u c^2(\gamma_u - 1). \text{ So}$$

$$\frac{1}{3}m_p c^2 = m_u c^2(\gamma_u - 1). \text{ Solve for } \gamma_u \text{ using the given masses: } \gamma_u = 136.$$

(b) We want  $\gamma_d$ . Use the same procedure as in (a), giving  $\gamma_u = 65.8$ .

(c) We want the speeds.  $v_u/c = \sqrt{1 - 1/\gamma_u^2} = 0.99997$ , so the answer is yes.  $v_d/c = \sqrt{1 - 1/\gamma_d^2} = 0.9999$ , so the answer is yes.

(d) We want to know what % of the proton mass is from gluons.  $m_p c^2 = 2K_u + K_d + E_g$ .

$$\frac{E_g}{m_p c^2} = \frac{m_p c^2 - (2K_u + K_d)}{m_p c^2} = \frac{m_p - 2m_u(\gamma_u - 1) - m_d(\gamma_d - 1)}{m_p}. \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - 0.90^2}} = 2.294. \text{ Using}$$

$$\text{this } \gamma \text{ and the given masses gives } \frac{E_g}{m_p c^2} = 0.987 \approx 99\%.$$

(e) We want to estimate the frequency  $f$ . If  $T$  is the time for one cycle of amplitude  $A$  at speed  $v$ ,

$$\text{then } vT = 4A. \quad f = 1/T = 1/(4A/v) = v/4A. \quad A = \frac{1}{2}(1.7 \times 10^{-15} \text{ m}) = 8.5 \times 10^{-16} \text{ m. Using this value with } v$$

$$= 0.90c \text{ gives our estimate } f = 8 \times 10^{22} \text{ Hz} \approx 10^{23} \text{ Hz.}$$

**EVALUATE:** Rough models such as this one do not give precise results, but they do provide approximate estimations that can be compared to experimental results.

**37.53. IDENTIFY and SET UP:** The energy released is  $E = (\Delta m)c^2$ .  $\Delta m = \left(\frac{1}{10^4}\right)(12.0 \text{ kg})$ .  $P_{\text{av}} = \frac{E}{t}$ .

The change in gravitational potential energy is  $mg\Delta y$ .

**EXECUTE:** (a)  $E = (\Delta m)c^2 = \left(\frac{1}{10^4}\right)(12.0 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = 1.08 \times 10^{14} \text{ J}$ .

(b)  $P_{\text{av}} = \frac{E}{t} = \frac{1.08 \times 10^{14} \text{ J}}{4.00 \times 10^{-6} \text{ s}} = 2.70 \times 10^{19} \text{ W}$ .

(c)  $E = \Delta U = mg\Delta y$ .  $m = \frac{E}{g\Delta y} = \frac{1.08 \times 10^{14} \text{ J}}{(9.80 \text{ m/s}^2)(1.00 \times 10^3 \text{ m})} = 1.10 \times 10^{10} \text{ kg}$ .

**EVALUATE:** The mass decrease is only 1.2 grams, but the energy released is very large.

**37.54. IDENTIFY:** The protons are moving at speeds that are comparable to the speed of light, so we must use the relativistic velocity addition formula.

**SET UP:**  $S$  is lab frame and  $S'$  is frame of proton moving in  $+x$ -direction.  $v_x = -0.700c$ . In lab frame

each proton has speed  $\alpha c$ .  $u = +\alpha c$ .  $v_x = -\alpha c$ .  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2} = \frac{-0.700c + \alpha c}{1 - 0.700\alpha} = -\alpha c$ .

**EXECUTE:**  $(1 - 0.700\alpha)(-\alpha) = -0.700 + \alpha$ .  $0.700\alpha^2 - 2\alpha + 0.700 = 0$ . The quadratic formula gives  $\alpha = 2.45$  or  $\alpha = 0.408$ . We cannot have  $v > c$  so  $\alpha = 0.408$ . Each proton has speed  $0.408c$  in the earth frame.

**EVALUATE:** To the earth observer, the protons are separating at  $2(0.408c) = 0.816c$ , but to the protons they are separating at  $0.700c$ .

**37.55. IDENTIFY:** We are investigating the effect of motion on a magnetic field.

**SET UP and EXECUTE:** (a) We want the magnetic moment.  $\vec{\mu} = I\vec{A}\hat{j} = IHL\hat{j}$ .

(b) We want the torque.  $\vec{\tau} = \vec{\mu} \times \vec{B} = IHLB\hat{i}$

(c) We want  $\vec{\mu}'$ .  $\vec{\mu}' = IHL\hat{j}$ .  $L' = L/\gamma$ , so  $\vec{\mu}' = (IHL/\gamma)\hat{j} = \vec{\mu}/\gamma$ .

(d) We want  $\vec{B}'$ . Given that the torques are equal in both frames, we have  $\vec{\mu}' \times \vec{B}' = \vec{\mu} \times \vec{B}$ . Using our result from part (c) gives  $(\vec{\mu}/\gamma) \times \vec{B}' = \vec{\mu} \times \vec{B}$ , so  $\vec{B}' = \vec{B}\gamma$ , which we can express as  $\vec{B}' = B\gamma\hat{k}$ .

(e) Generalize from  $\vec{B}' = \vec{B}\gamma$ , which gives  $\vec{B}'_{\perp} = \vec{B}_{\perp}\gamma$ .

**EVALUATE:** Note that the fields are different when observed from moving reference frames.

**37.56. IDENTIFY:** We are investigating the effect of motion on an electric field.

**SET UP and EXECUTE:** (a) The net force is zero because there is no motion.

(b) We want  $\vec{B}'$ . Using the result of problem 37.55, but with  $\vec{B} = -B\hat{j}$  instead of  $\vec{B} = B\hat{k}$ . This gives  $\vec{B}' = \vec{B}\gamma = -\gamma B\hat{j}$ .

(c) We want  $\vec{F}'$ . Use  $\vec{F}' = Q\vec{v}' \times \vec{B}'$  with  $\vec{v}' = -v\hat{i}$  and  $\vec{B}' = -\gamma B\hat{j}$ , giving  $\vec{F}' = Q\gamma vB\hat{k}$ .

(d) Generalize:  $\vec{F}' = Q\vec{E}' = Q\gamma\vec{v} \times \vec{B}$ , so  $\vec{E}'_{\perp} = \gamma\vec{v} \times \vec{B}$ .

**EVALUATE:** Note that there is no electric field in the lab, but there is one in the moving frame.

**37.57. IDENTIFY and SET UP:** An increase in wavelength corresponds to a decrease in frequency ( $f = c/\lambda$ ),

so the atoms are moving away from the earth. The galaxy is receding, so we use  $f = \sqrt{\frac{c-u}{c+u}}f_0$ .

**EXECUTE:** Solve for  $u$ :  $(f/f_0)^2(c+u) = c-u$  and  $u = c\left(\frac{1-(f/f_0)^2}{1+(f/f_0)^2}\right)$ .

$f = c/\lambda$ ,  $f_0 = c/\lambda_0$  so  $f/f_0 = \lambda_0/\lambda$ .

$$u = c \left( \frac{1 - (\lambda_0/\lambda)^2}{1 + (\lambda_0/\lambda)^2} \right) = c \left( \frac{1 - (656.3/953.4)^2}{1 + (656.3/953.4)^2} \right) = 0.357c = 1.07 \times 10^8 \text{ m/s}.$$

**EVALUATE:** The relative speed is large, 36% of  $c$ . The cosmological implication of such observations will be discussed in Chapter 44.

**37.58. IDENTIFY:** Apply the Lorentz coordinate transformation.

**SET UP:** Let  $t$  and  $t'$  be time intervals between the events as measured in the two frames and let  $x$  and  $x'$  be the difference in the positions of the two events as measured in the two frames.

**EXECUTE:** Setting  $x = 0$  in the Lorentz transformation equations, the first equation becomes  $x' = -\gamma ut$  and the last, upon multiplication by  $c$ , becomes  $ct' = \gamma ct$ . Squaring and subtracting gives

$$c^2 t'^2 - x'^2 = \gamma^2 t^2 (c^2 - u^2). \text{ But } \gamma^2 = c^2 / (c^2 - v^2), \text{ so } \gamma^2 t^2 (c^2 - v^2) = c^2 t^2. \text{ Therefore, } c^2 t'^2 - x'^2 = c^2 t^2$$

$$\text{which gives } x' = c \sqrt{t'^2 - t^2} = c \sqrt{(2.15 \text{ s})^2 - (1.80 \text{ s})^2} = 3.53 \times 10^8 \text{ m}.$$

**EVALUATE:** We did not have to calculate the speed  $u$  of frame  $S'$  relative to frame  $S$ .

**37.59. IDENTIFY:** The baseball is moving toward the radar gun, so apply the Doppler effect equation

$$f = \sqrt{\frac{c+u}{c-u}} f_0.$$

**SET UP:** The baseball had better be moving nonrelativistically, so the Doppler shift formula becomes  $f \cong f_0(1 + (u/c))$ . In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at  $f$ . But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is  $f(1 + (u/c)) = f_0(1 + (u/c))^2 \approx f_0(1 + 2(u/c))$ .

**EXECUTE:**  $\Delta f = 2f_0(u/c)$  and the fractional frequency shift is  $\frac{\Delta f}{f_0} = 2(u/c)$ .

$$u = \frac{\Delta f}{2f_0} c = \frac{(2.86 \times 10^{-7})}{2} (3.00 \times 10^8 \text{ m/s}) = 42.9 \text{ m/s} = 154 \text{ km/h} = 92.5 \text{ mi/h}.$$

**EVALUATE:**  $u \ll c$ , so using the approximate expression in place of  $f = \sqrt{\frac{c+u}{c-u}} f_0$  is very accurate.

**37.60. IDENTIFY:** Apply the relativistic expressions for kinetic energy, velocity transformation, length contraction and time dilation.

**SET UP:** In part (c) let  $S'$  be the earth frame and let  $S$  be the frame of the ball. Let the direction from Einstein to Lorentz be positive, so  $u = -1.80 \times 10^8 \text{ m/s}$ . In part (d) the proper length is  $l_0 = 20.0 \text{ m}$  and in part (f) the proper time is measured by the rabbit.

**EXECUTE:** (a)  $80.0 \text{ m/s}$  is nonrelativistic, and  $K = \frac{1}{2}mv^2 = 186 \text{ J}$ .

$$\text{(b) } K = (\gamma - 1)mc^2 = 1.31 \times 10^{15} \text{ J}.$$

(c) In Eq. (37.23),  $v' = 2.20 \times 10^8 \text{ m/s}$ ,  $u = -1.80 \times 10^8 \text{ m/s}$ , and so  $v = 7.14 \times 10^7 \text{ m/s}$ .

$$\text{(d) } l = \frac{l_0}{\gamma} = \frac{20.0 \text{ m}}{\gamma} = 13.6 \text{ m}.$$

$$\text{(e) } \frac{20.0 \text{ m}}{2.20 \times 10^8 \text{ m/s}} = 9.09 \times 10^{-8} \text{ s}.$$

$$\text{(f) } \Delta t_0 = \frac{\Delta t}{\gamma} = 6.18 \times 10^{-8} \text{ s}.$$

**EVALUATE:** In part (f) we could also calculate  $\Delta t_0$  as  $\Delta t_0 = \frac{13.6 \text{ m}}{2.20 \times 10^8 \text{ m/s}} = 6.18 \times 10^{-8} \text{ s}$ .

**37.61. IDENTIFY:** We need to use the relativistic form of Newton's second law because the speed of the proton is close to the speed of light.

**SET UP:**  $\vec{F}$  and  $\vec{v}$  are perpendicular, so  $F = \gamma m a = \gamma m \frac{v^2}{R}$ .  $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(0.750)^2}} = 1.512$ .

**EXECUTE:**  $F = (1.512)(1.67 \times 10^{-27} \text{ kg}) \frac{[(0.750)(3.00 \times 10^8 \text{ m/s})]^2}{628 \text{ m}} = 2.04 \times 10^{-13} \text{ N}$ .

**EVALUATE:** If we ignored relativity, the force would be

$F_{\text{rel}}/\gamma = \frac{2.04 \times 10^{-13} \text{ N}}{1.512} = 1.35 \times 10^{-13} \text{ N}$ , which is substantially less than the relativistic force.

**37.62. IDENTIFY and SET UP:** For part (a) follow the procedure specified in the hint. For part (b) apply

$f = \sqrt{\frac{c+u}{c-u}} f_0$  and  $f = \sqrt{\frac{c-u}{c+u}} f_0$ .

**EXECUTE: (a)** As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time  $T = 1/f$  as the proper time, with the result that  $f = \gamma f_0 > f_0$ .

**(b) Toward:**  $f_1 = f_0 \sqrt{\frac{c+u}{c-u}} = 345 \text{ MHz} \left( \frac{1+0.758}{1-0.758} \right)^{1/2} = 930 \text{ MHz}$  and

$f_1 - f_0 = 930 \text{ MHz} - 345 \text{ MHz} = 585 \text{ MHz}$ .

**Away:**  $f_2 = f_0 \sqrt{\frac{c-u}{c+u}} = 345 \text{ MHz} \left( \frac{1-0.758}{1+0.758} \right)^{1/2} = 128 \text{ MHz}$  and  $f_2 - f_0 = -217 \text{ MHz}$ .

**(c)**  $f_3 = \gamma f_0 = 1.53 f_0 = 528 \text{ MHz}$ ,  $f_3 - f_0 = 183 \text{ MHz}$ .

**EVALUATE:** The frequency in part (c) is the average of the two frequencies in part (b). A little algebra shows that  $f_3$  is precisely equal to  $(f_1 + f_2)/2$ .

**37.63. IDENTIFY and SET UP:** The equation  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$  relates the time interval in the laboratory ( $\Delta t$ ) to the time interval ( $\Delta t_0$ ) in the rest frame of the particle.

**EXECUTE: (a)** Solve the above equation for  $(\Delta t)^2$ . Squaring gives  $(\Delta t)^2 = (\Delta t_0)^2 (1-u^2/c^2)^{-1}$ .

Therefore a graph of  $(\Delta t)^2$  versus  $(1-u^2/c^2)^{-1}$  should be a straight line with slope equal to  $(\Delta t_0)^2$ .

Figure 37.63 shows this graph for the data in the problem. The slope of the best-fit straight line is  $0.6709 \times 10^{-15} \text{ s}^2$ , so  $\Delta t_0 = \sqrt{0.6709 \times 10^{-15} \text{ s}^2} = 2.6 \times 10^{-8} \text{ s} = 26 \text{ ns}$ .

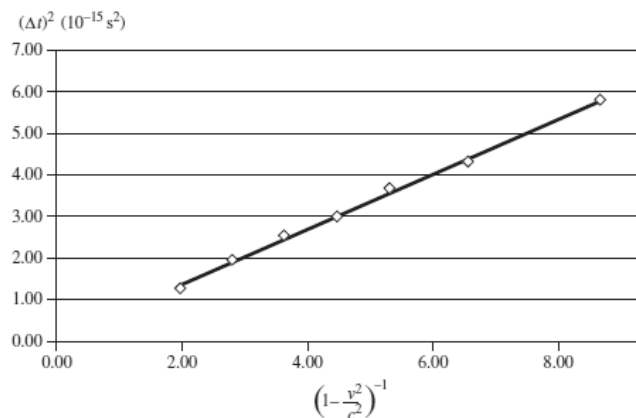


Figure 37.63

(b) Using  $\Delta t = 4 \Delta t_0$ , the equation  $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$  gives  $4\Delta t_0 = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ . Solving for  $u/c$  gives  $u/c = 0.97$ .

**EVALUATE:** At speeds near the speed of light, there is a very large difference between the lifetime measured in the laboratory frame compared to the lifetime in the rest frame of the particle.

**37.64. IDENTIFY:** Apply the Lorentz velocity transformation.

**SET UP:** Let the tank and the light both be traveling in the  $+x$ -direction. Let  $S$  be the lab frame and let  $S'$  be the frame of the tank of water.

**EXECUTE:** In the equation  $v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$ ,  $u = V$ ,  $v' = (c/n)$ .  $v = \frac{(c/n) + V}{1 + \frac{cV}{nc^2}} = \frac{(c/n) + V}{1 + (V/nc)}$ . For  $V \ll c$ ,

$(1 + V/nc)^{-1} \approx (1 - V/nc)$ . This gives

$v \approx [(c/n) + V][1 - (V/nc)] = c/n + V - (V^2/n^2) - (V^2/nc) \approx \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)V$ , so  $k = \left(1 - \frac{1}{n^2}\right)$ . For water,  $n = 1.333$  and  $k = 0.437$ .

**EVALUATE:** The Lorentz transformation predicts a value of  $k$  in excellent agreement with the value that is measured experimentally.

**37.65. IDENTIFY and SET UP:** When the force on a particle is along the same line as its velocity, the force and acceleration are related by  $F = \gamma^3 ma$ , where  $\gamma = 1/\sqrt{1-v^2/c^2}$ .

**EXECUTE: (a)** Solve the equation  $F = \gamma^3 ma$  for  $a^2$ .

$$a = \frac{F}{m\gamma^3} = \frac{F}{m} \left(1 - \frac{v^2}{c^2}\right)^{3/2} \quad \rightarrow \quad a^2 = \left(\frac{F}{m}\right)^2 \left(1 - \frac{v^2}{c^2}\right)^3.$$

From this result we see that a graph of  $a^2$  versus  $\left(1 - \frac{v^2}{c^2}\right)^3$  should be a straight line with slope equal

to  $(F/m)^2$ . Figure 37.65 shows the graph of the data in the table with the problem. It is well fit by a straight line having slope equal to  $1.608 \times 10^9 \text{ m}^2/\text{s}^4$ . Therefore the mass is

$$(F/m)^2 = \text{slope} \quad \rightarrow \quad m = \frac{F}{\sqrt{\text{slope}}} = \frac{8.00 \times 10^{-14} \text{ N}}{\sqrt{1.608 \times 10^9 \text{ m}^2/\text{s}^4}} = 2.0 \times 10^{-18} \text{ kg}.$$

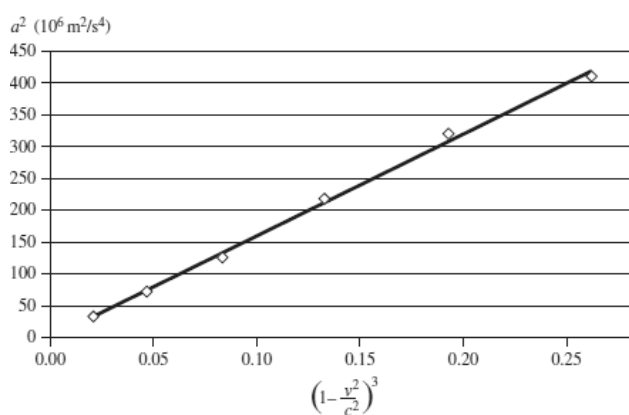


Figure 37.65

(b) In this case,  $v \ll c$ , so  $\gamma$  is essentially equal to 1. Therefore we can use the familiar form of Newton's second law,  $F = ma$ .

$$a = F/m = (8.00 \times 10^{-14} \text{ N}) / (2.0 \times 10^{-18} \text{ kg}) = 4.0 \times 10^4 \text{ m/s}^2.$$

**EVALUATE:** When  $v$  is close to  $c$ ,  $a = F/m$  does not give the correct result. For example, using data from the table in the problem, when  $v/c = 0.85$ , the acceleration is measured to be  $5900 \text{ m/s}^2$ . But using the familiar Newtonian formula we get  $a = F/m = (8.00 \times 10^{-14} \text{ N}) / (2.0 \times 10^{-18} \text{ kg}) = 4.0 \times 10^4 \text{ m/s}^2 = 40,000 \text{ m/s}^2$ , which is *very* different from the relativistic result of  $5900 \text{ m/s}^2$ .

- 37.66. IDENTIFY:** Relative motion between the observer and the source of electromagnetic waves affects the frequency received by the observer due to the Doppler effect. If the source is moving toward the observer with speed  $u$  and emitting frequency  $f_0$ , the frequency  $f$  that the observer receives is given by

$$f = f_0 \sqrt{\frac{c+u}{c-u}}, \text{ and if it is moving away the formula is } f = f_0 \sqrt{\frac{c-u}{c+u}}. \text{ Notice that } f > f_0.$$

**SET UP:** In this case, we know  $f$  and  $f_0$  and want to find  $u$ . Looking at the data given in the problem, we see that  $f < f_0$ , which means that the source must be moving *away from* the observer, so we use

$$f = f_0 \sqrt{\frac{c-u}{c+u}}. \text{ Solve the equation } f = f_0 \sqrt{\frac{c-u}{c+u}} \text{ for } u, \text{ since we know the frequencies. This gives}$$

$$u = \left( \frac{f_0^2 - f^2}{f_0^2 + f^2} \right) c.$$

**EXECUTE: (a)** To see which source is moving fastest, look at  $f_0/f$ ; the larger this ratio, the greater the speed  $u$ .

$$\text{For A: } f_0/f = 9.2/7.1 = 1.30.$$

$$\text{For B: } f_0/f = 8.6/5.4 = 1.59.$$

$$\text{For C: } f_0/f = 7.9/6.1 = 1.30.$$

$$\text{For D: } f_0/f = 8.9/8.1 = 1.10.$$

Therefore source B is moving fastest and source D is moving slowest. The speed of B is

$$u = \left( \frac{(8.6 \text{ THz})^2 - (5.4 \text{ THz})^2}{(8.6 \text{ THz})^2 + (5.4 \text{ THz})^2} \right) c = 0.434c, \text{ which rounds to } 0.43c, \text{ away from the detector.}$$

$$\text{(b) The speed for source D is } u = \left( \frac{(8.9 \text{ THz})^2 - (8.1 \text{ THz})^2}{(8.9 \text{ THz})^2 + (8.1 \text{ THz})^2} \right) c = 0.094c, \text{ away from the detector.}$$

(c) Since B is now approaching the detector, we use  $f = f_0 \sqrt{\frac{c+u}{c-u}}$  with  $u = 0.434c$  and  $f_0 = 8.6$  THz.

This gives  $f = f_0 \sqrt{\frac{c+u}{c-u}} = (8.6 \text{ THz}) \sqrt{\frac{1+0.434}{1-0.434}} = 14 \text{ THz} = 1.4 \times 10^{13} \text{ Hz}$ .

**EVALUATE:** The change in observed frequency is small for low speeds but increases dramatically as  $u \rightarrow c$ .

- 37.67. IDENTIFY:** We are using the Lorentz transformation equations to investigate space-time diagrams.  
**SET UP:**  $S$  is the reference frame of the train station and  $S'$  is the moving frame of the rocket train.  
 $x' = \gamma(x - vt)$ ,  $t' = \gamma(t - xv/c^2)$ .

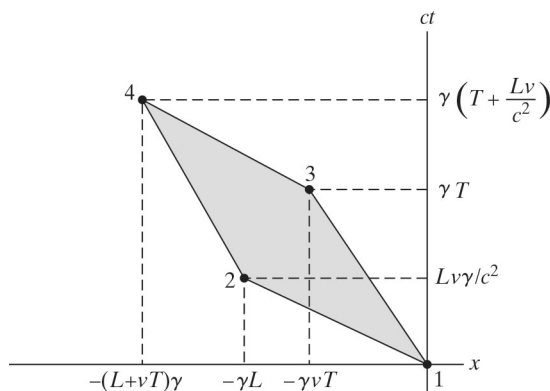
**EXECUTE:** (a) **Event 1:**  $x_1 = 0$ ,  $t_1 = 0$ , so  $x'_1 = 0$ ,  $t'_1 = 0$ . So  $(x'_1, t'_1) = (0, 0)$ .

**Event 2:**  $x_2 = -L$ ,  $t_2 = 0$ .  $x'_2 = -L\gamma$ ,  $t'_2 = [(-L)v/c^2]\gamma = Lv\gamma/c^2$ . So  $(x'_2, t'_2) = (-L\gamma, Lv\gamma/c^2)$ .

**Event 3:**  $x_3 = 0$ ,  $t_3 = T$ .  $x'_3 = -vT\gamma$ ,  $t'_3 = T\gamma$ . So  $(x'_3, t'_3) = (-vT\gamma, T\gamma)$ .

**Event 4:**  $x_4 = -L$ ,  $t_4 = T$ .  $x'_4 = -(L + vT)\gamma$ ,  $t'_4 = (T + Lv/c^2)\gamma$ . So  $(x'_4, t'_4) = (-(L + vT)\gamma, (T + Lv/c^2)\gamma)$ .

(b) Fig. 37.67 shows the spacetime diagram.



**Figure 37.67**

(c) We want the area in frame  $S$ .  $A = LcT = cLT$ .

(d) We want the area in frame  $S'$ . Use the hint in the problem. For two vectors in the  $xy$ -plane, the cross product has a  $z$  component given by  $A_x B_y - B_x A_y$ , so the magnitude of this quantity is the area of the parallelogram. Apply this approach to the diagram in Fig. 37.67.  $\vec{A}$  is the vector from 1 to 3 and  $\vec{B}$  is the vector from 1 to 2. Their components are  $A_x = -\gamma vt$ ,  $A_y = \gamma T$ ,  $B_x = -\gamma L$ , and  $B_y = Lv\gamma/c^2$ . The area is  $A = |A_x B_y - B_x A_y| = LT$ . But recall that we replace  $t$  with  $ct$ , so  $T \rightarrow cT$ . Therefore the area is  $A = cLT$ .

**EVALUATE:** (e) The area is  $cLT$  in both reference frames. So it is independent of the reference frame, or "Lorentz invariant."

- 37.68. IDENTIFY:** Apply the Doppler effect equation.

**SET UP:** At the two positions shown in the figure given in the problem, the velocities of the star relative to the earth are  $u + v$  and  $u - v$ , where  $u$  is the velocity of the center of mass and  $v$  is the orbital velocity.

**EXECUTE:** (a)  $f_0 = 4.568110 \times 10^{14} \text{ Hz}$ ;  $f_+ = 4.568910 \times 10^{14} \text{ Hz}$ ;  $f_- = 4.567710 \times 10^{14} \text{ Hz}$ .



$$\left. \begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}} f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}} f_0 \end{aligned} \right\} \Rightarrow \begin{aligned} f_+^2(c-(u+v)) &= f_0^2(c+(u+v)) \\ f_-^2(c-(u-v)) &= f_0^2(c+(u-v)) \end{aligned}$$

$$(u+v) = \frac{(f_+/f_0)^2 - 1}{(f_+/f_0)^2 + 1} c \quad \text{and} \quad (u-v) = \frac{(f_-/f_0)^2 - 1}{(f_-/f_0)^2 + 1} c. \quad u+v = 5.25 \times 10^4 \text{ m/s} \quad \text{and} \quad u-v = -2.63 \times 10^4 \text{ m/s}.$$

This gives  $u = +1.31 \times 10^4 \text{ m/s}$  (moving toward at 13.1 km/s) and  $v = 3.94 \times 10^4 \text{ m/s}$ .

(b)  $v = 3.94 \times 10^4 \text{ m/s}$ ;  $T = 11.0 \text{ days}$ .  $2\pi R = vt \Rightarrow$

$$R = \frac{(3.94 \times 10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} = 5.96 \times 10^9 \text{ m}. \quad \text{This is about}$$

0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of  $2R$ ) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} = \frac{4(5.96 \times 10^9 \text{ m})(3.94 \times 10^4 \text{ m/s})^2}{6.672 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2} = 5.55 \times 10^{29} \text{ kg} = 0.279 m_{\text{sun}}.$$

**EVALUATE:**  $u$  and  $v$  are both much less than  $c$ , so we could have used the approximate expression  $\Delta f = \pm f_0 v_{\text{rev}}/c$ , where  $v_{\text{rev}}$  is the speed of the source relative to the observer.

**37.69. IDENTIFY:** Apply conservation of total energy, in the frame in which the total momentum is zero (the center of momentum frame).

**SET UP:** In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons.

**EXECUTE:** (a)  $2(\gamma_{\text{cm}} - 1)m_p c^2 = 2m_k c^2 \Rightarrow \gamma_{\text{cm}} = 1 + \frac{m_k}{m_p} = 1.526$ . The velocity of a proton in the center

$$\text{of momentum frame is then } v_{\text{cm}} = c \sqrt{\frac{\gamma_{\text{cm}}^2 - 1}{\gamma_{\text{cm}}^2}} = 0.7554c.$$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as “hopping” into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left,  $u = v_{\text{cm}}$  and  $v' = v_{\text{cm}}$  (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v' + u}{1 + \frac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1 + \frac{v_{\text{cm}}^2}{c^2}} = 0.9619c \Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1 - \frac{v_{\text{lab}}^2}{c^2}}} = 3.658 \Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}} - 1)m_p c^2 = 2494 \text{ MeV}.$$

$$(b) \frac{K_{\text{lab}}}{2m_k} = \frac{2494 \text{ MeV}}{2(493.7 \text{ MeV})} = 2.526.$$

(c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required is just twice the rest mass energy of the kaons.  $K_{\text{cm}} = 2(493.7 \text{ MeV}) = 987.4 \text{ MeV}$ .

**EVALUATE:** The colliding beam situation of part (c) offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.

**37.70. IDENTIFY and SET UP:** Apply the procedures specified in the problem.

**EXECUTE:** For any function  $f = f(x, t)$  and  $x = x(x', t')$ ,  $t = t(x', t')$ , let  $F(x', t') = f(x(x', t'), t(x', t'))$  and use the standard (but mathematically improper) notation  $F(x', t') = f(x', t')$ . The chain rule is then

$$\frac{\partial f(x', t')}{\partial x} = \frac{\partial f(x, t)}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial f(x', t')}{\partial t'} \frac{\partial t'}{\partial x},$$

$$\frac{\partial f(x', t')}{\partial t} = \frac{\partial f(x, t)}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial f(x', t')}{\partial t'} \frac{\partial t'}{\partial t}.$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is

suppressed, and the above relations are then  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'} \frac{\partial t'}{\partial x}$ ,  $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial f}{\partial t'} \frac{\partial t'}{\partial t}$ .

(a)  $\frac{\partial x'}{\partial x} = 1$ ,  $\frac{\partial x'}{\partial t} = -v$ ,  $\frac{\partial t'}{\partial x} = 0$  and  $\frac{\partial t'}{\partial t} = 1$ . Then,  $\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}$ , and  $\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$ . For the time derivative,

$\frac{\partial E}{\partial t} = -v \frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}$ . To find the second time derivative, the chain rule must be applied to both terms; that

$$\text{is, } \frac{\partial}{\partial t} \frac{\partial E}{\partial x'} = -v \frac{\partial^2 E}{\partial x'^2} + \frac{\partial^2 E}{\partial t' \partial x'}, \quad \frac{\partial}{\partial t} \frac{\partial E}{\partial t'} = -v \frac{\partial^2 E}{\partial x' \partial t'} + \frac{\partial^2 E}{\partial t'^2}.$$

Using these in  $\frac{\partial^2 E}{\partial t^2}$ , collecting terms and equating the mixed partial derivatives gives

$$\frac{\partial^2 E}{\partial t^2} = v^2 \frac{\partial^2 E}{\partial x'^2} - 2v \frac{\partial^2 E}{\partial x' \partial t'} + \frac{\partial^2 E}{\partial t'^2}, \text{ and using this and the above expression for } \frac{\partial^2 E}{\partial x'^2} \text{ gives the result.}$$

(b) For the Lorentz transformation,  $\frac{\partial x'}{\partial x} = \gamma$ ,  $\frac{\partial x'}{\partial t} = \gamma v$ ,  $\frac{\partial t'}{\partial x} = \gamma v/c^2$  and  $\frac{\partial t'}{\partial t} = \gamma$ .

The first partials are then  $\frac{\partial E}{\partial x} = \gamma \frac{\partial E}{\partial x'} - \gamma \frac{v}{c^2} \frac{\partial E}{\partial t'}$ ,  $\frac{\partial E}{\partial t} = -\gamma v \frac{\partial E}{\partial x'} + \gamma \frac{\partial E}{\partial t'}$

and the second partials are (again equating the mixed partials)

$$\frac{\partial^2 E}{\partial x^2} = \gamma^2 \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \frac{v^2}{c^4} \frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 \frac{v}{c^2} \frac{\partial^2 E}{\partial x' \partial t'}$$

$$\frac{\partial^2 E}{\partial t^2} = \gamma^2 v^2 \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v \frac{\partial^2 E}{\partial x' \partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \gamma^2 \left( 1 - \frac{v^2}{c^2} \right) \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \left( \frac{v^2}{c^4} - \frac{1}{c^2} \right) \frac{\partial^2 E}{\partial t'^2} = \frac{\partial^2 E}{\partial x'^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t'^2} = 0.$$

**EVALUATE:** The general form of the wave equation is given by Eq. (32.1). The coefficient of the  $\partial^2/\partial t^2$  term is the inverse of the square of the wave speed. This coefficient is the same in both frames, so the wave speed is the same in both frames.

**37.71. IDENTIFY and SET UP:** The relativity formulas apply, but  $c = 300$  m/s in this universe.

$$\gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-(180 \text{ m/s})^2/(300 \text{ m/s})^2} = 1.25. \text{ The length is } l = l_0/\gamma.$$

**EXECUTE:**  $l = l_0/\gamma = (60 \text{ m})/(1.25) = 48 \text{ m}$ , choice (c).

**EVALUATE:** Relativistic effects would be “common sense” in this universe!

**37.72. IDENTIFY and SET UP:** The relativity formulas apply, but  $c = 300$  m/s in this universe.

$$\gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-(180 \text{ m/s})^2/(300 \text{ m/s})^2} = 1.40.$$

**EXECUTE:** The relativistic mass is  $m\gamma = (20,000 \text{ kg})(1.25) = 25,000 \text{ kg}$ , choice (d).

**EVALUATE:** The relativistic mass of the passengers and their luggage would also be greater by the factor 1.25, so excess baggage fees could be rather common.

**37.73. IDENTIFY:** The rest energy is  $E_0 = mc^2$ .

**SET UP:** In our universe,  $E_0 = mc^2$  and in the alternative universe the same formula would apply, except that  $c$  would be different, call it  $c'$ , so  $E'_0 = mc'^2$ . Therefore  $E'_0 = \left(\frac{E_0}{c^2}\right)c'^2$ .

**EXECUTE:**  $E'_0 = \left(\frac{E_0}{c^2}\right)c'^2 = \left(\frac{8.2 \times 10^{-14} \text{ J}}{(3.0 \times 10^8 \text{ m/s})^2}\right)(300 \text{ m/s})^2 = 8.2 \times 10^{-26} \text{ J}$ , choice (b).

**EVALUATE:** The rest energy of all other particles would be reduced by the same fraction in this alternate universe.

**37.74. IDENTIFY and SET UP:** The kinetic energy is  $K = (\gamma - 1)mc^2$ .

**EXECUTE:** The kinetic energy is now equal to the rest energy, so  $K = (\gamma - 1)mc^2 = mc^2$ , which means that  $\gamma = 2$ . Therefore  $2 = 1/\sqrt{1 - v^2/c^2}$ . Solving for  $v$  gives  $v = c\sqrt{1 - (1/2)^2} = (300 \text{ m/s})\sqrt{3/4} = 260 \text{ m/s}$ , choice (b).

**EVALUATE:** Airplanes and rockets would certainly need to take relativistic effects into consideration in this alternative universe!