11

EQUILIBRIUM AND ELASTICITY

VP11.1.1. **IDENTIFY:** We are dealing with center of gravity. If the center of gravity of the mass-plank system moves beyond the right-hand support point, the plank will tip over.

SET UP: $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. Call the origin the right-hand support point. The center of mass of the system is at that point, so $x_{cm} = 0$. Call m the unknown mass.

EXECUTE: Applying the center of gravity formula $x_{cg} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$ gives

$$0 = \frac{(40.0\text{ kg})(-1.00\text{ m}) + (3.00\text{ m})\,m}{40.0\text{ kg} + m} \quad \rightarrow \quad m = 13.3\text{ kg}.$$

EVALUATE: The added mass of 13.3 kg is much less than the 40.0-kg mass of the plank because this added mass is much farther from the pivot point than the center of mass is.

VP11.1.2. **IDENTIFY:** We are dealing with center of gravity. If the center of gravity of the two-ball system is at the bowling ball, the system will balance.

SET UP: $x_{cg} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. Call the origin the center of the bowling ball and call d the distance between the centers of the two balls.

EXECUTE: With our choice of origin, $x_{cm} = 0.216$ m, so we have

$$0.216\text{ m} = \frac{(7.26\text{ kg})(0) + (0.145\text{ kg})d}{7.26\text{ kg} + 0.145\text{ kg}} \quad \rightarrow \quad d = 11.0\text{ m}.$$

EVALUATE: To check, we can balance torques about the surface of the bowling ball with the rod horizontal. Call M the mass of the bowling ball and R its radius and m the mass of the baseball. This gives $MgR - mg(d - R) = 0 \quad \rightarrow \quad d = R(M + m)/m = 11.0$ m, which agrees with our result.

VP11.1.3. **IDENTIFY:** For balance, the center of gravity of the system of three objects and the rod must be at the support point.

SET UP: Call the center of the rod the origin and call x the distance of the support point from the center of the rod; this is also the distance of the center of gravity from the center. We treat the bar as a point-mass of m located at its center.

EXECUTE: $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \dfrac{m(-L/2) + 2m(0) + 2m(L/2)}{5m} = \dfrac{L}{10}$. The support point should be a distance $L/10$ to the right of the center of the bar, which is the location of the center of gravity of the system.

EVALUATE: To check, substitute the calculated answer back into the center-of-mass formula but using a different origin and use it to calculate the location of the center of mass. For example, using the left end of the bar as the origin, we have $x_{cg} = \dfrac{2m(L/2)+2m(L)}{5m} = 6L/10$, which is $L/10$ to the right of the center of the bar, as we just found.

VP11.1.4. **IDENTIFY:** We know where the center of gravity of the loaded plane should be, and we want to find out how much mass we can have in the baggage compartment.

SET UP: Use $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$. Let the x-axis point horizontally to the right with the nose of the plane on the left end and the origin at the center of gravity of the *loaded* plane. With this choice, $x_{cm} = 0$.

EXECUTE: Let M be the maximum mass in the baggage compartment.

$$0 = \frac{-(1170\text{ kg})(2.76\text{ m} - 2.58\text{ m}) - (75.0\text{ kg})(2.76\text{ m} - 2.67\text{ m}) + M(4.30\text{ m} - 2.76\text{ m})}{m_{total}}$$

$M = 141$ kg.

EVALUATE: Choosing the origin at the center of gravity of the loaded plane makes the algebra rather simple compared to other choices because we do not have to deal with m_{total} in the denominator. Since m_{total} includes M, we would have more work (and more chances for error) with other choices of the origin. It helps to plan ahead!

VP11.4.1. **IDENTIFY:** The truck is in equilibrium, so the torques on it must balance.

SET UP: Apply $\sum\tau_z = 0$. Take torques about the rear wheel. Call d the wheelbase and w the weight of the truck. See Fig. VP11.4.1.

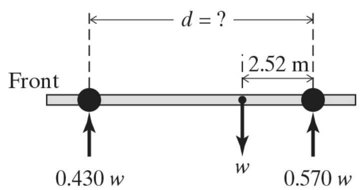


Figure VP11.4.1

EXECUTE: $\sum\tau_z = 0$ gives $(0.430\ w)d = (2.52\text{ m})w \rightarrow \quad d = 5.86$ m.

EVALUATE: To check, calculate the center of gravity of the truck. This gives $(0.570\ w)(2.52\text{ m}) - (0.430\ w)(5.86\text{ m} - 2.52\text{ m}) = 0$, as it should since the truck is in equilibrium.

VP11.4.2. **IDENTIFY:** The plane is in equilibrium, so the torques on it must balance.

SET UP: Apply $\sum\tau_z = 0$. Take torques about the nose wheel. Call F the force due to the main wheel and w the weight of the plane.

EXECUTE: $\sum\tau_z = 0$ gives $-w(2.58\text{ m} - 0.800\text{ m}) + F(3.02\text{ m} - 0.800\text{ m}) = 0$, so $F = 0.802w$. The main wheel supports 80.2% of the weight of the plane, so the nose wheels must support 19.8% of the weight.

EVALUATE: To check, calculate torques about another point.

VP11.4.3. **IDENTIFY:** The sign is in equilibrium, so the forces and torques on the rod must balance.

SET UP: Apply $\sum\tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$.

EXECUTE: **(a)** Apply $\sum\tau_z = 0$ about the right end of the rod, giving

$F_{hinge\text{-}y}L + F_{hinge\text{-}x}(0) + T(0) + w(0) = 0 \quad \rightarrow \quad F_{hinge\text{-}y} = 0$.

(b) $\sum F_y = 0$ gives $F_{hinge\text{-}y} + T\sin\theta - w = 0 \quad \rightarrow \quad 0 + T\sin\theta - w = 0 \quad \rightarrow \quad T = w/\sin\theta$.

(c) $\sum F_x = 0$ gives $F_{hinge\text{-}x} - T\cos\theta = 0$. Using T from part (b) gives

$F_{\text{hinge-}x} = \left(\dfrac{w}{\sin\theta}\right)\cos\theta = w/\tan\theta.$

EVALUATE: Our result in (c) says that if θ is small, the hinge exerts a large horizontal force on the rod. This is reasonable because the cable pulls nearly horizontally against the rod which causes it to push very hard against the hinge.

VP11.4.4. **IDENTIFY:** The sign is in equilibrium, so the forces and torques on the rod must balance.

SET UP: Apply $\sum\tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The rod now also has weight w.

EXECUTE: **(a)** Apply $\sum\tau_z = 0$ about the right end of the rod, giving

$F_{\text{hinge-}y}\,L + F_{\text{hinge-}x}\,(0) + wL/2 + T(0) + w(0) = 0 \quad \rightarrow \quad F_{\text{hinge-}y} = w/2.$

(b) $\sum F_y = 0$ gives $F_{\text{hinge-}y} + T\sin\theta - w/2 - w = 0 \quad \rightarrow \quad T\sin\theta - 3w/2 = 0$

so $T = \dfrac{3w}{2\sin\theta}$.

(c) $\sum F_x = 0$ gives $F_{\text{hinge-}x} - T\cos\theta = 0$. Using T from part (b) gives

$F_{\text{hinge-}x} = \left(\dfrac{3w}{2\sin\theta}\right)\cos\theta = \dfrac{3w}{2\tan\theta}.$

EVALUATE: Our result in (c) says that if θ is small, the hinge exerts a large horizontal force on the rod. This is reasonable because the cable pulls nearly horizontally against the rod, which causes it to push very hard against the hinge. We also find that if the rod weighs as much as the sign, the tension and hinge force each are 1.5 times as great as before.

VP11.7.1. **IDENTIFY:** This problem involves elasticity, tensile stress, tensile stress, and Young's modulus.

SET UP: Young's modulus is $Y = \dfrac{\text{Tensile stress}}{\text{Tensile strain}}$, tensile strain = $\dfrac{\Delta\ell}{\ell_0}$, tensile stress = $\dfrac{F_\perp}{A}$.

EXECUTE: **(a)** Tensile stress = Y (tensile strain). $\dfrac{\Delta\ell}{\ell_0} = \dfrac{5.0\times10^{-3}\,\ell_0}{\ell_0} = 5.0\times10^{-3}$, so

tensile stress = $(11\times10^{10}\text{ Pa})(5.0\times10^{-3}) = 5.5\times10^{8}$ Pa.

(b) Tensile stress = $\dfrac{F_\perp}{A}$, where $A = \pi r^2$. Therefore $F_\perp$ = (tensile strain)(πr^2) so

$F_\perp = (5.5\times10^{8}\text{ Pa})(\pi)(4.5\times10^{-3}\text{ m})^2 = 3.5\times10^{4}$ N.

EVALUATE: The pressure is much less than atmospheric pressure, but the force is about 7600 lb, which is very large. But a wire 4.5 mm in radius is quite thick compared to ordinary electrical copper wires.

VP11.7.2. **IDENTIFY:** This problem involves elasticity, compressive stress and compressive strain, and Young's modulus.

SET UP: Young's modulus is $Y = \dfrac{\text{Compressive stress}}{\text{Compressive strain}}$, compressive strain = $\dfrac{\Delta\ell}{\ell_0}$, compressive stress = $\dfrac{F_\perp}{A}$. From Table 11.1, $Y = 7.0\times10^{10}$ Pa for aluminum.

EXECUTE: **(a)** We want the compressive strain. $Y = \dfrac{\text{Compressive stress}}{\text{Compressive strain}}$ tells us that compressive

strain = $\dfrac{\text{Compressive stress}}{Y} = \dfrac{F_\perp}{AY} = \dfrac{3.2\times10^{4}\text{ N}}{\pi(0.025\text{ m})^2(7.0\times10^{10}\text{ Pa})} = 2.3\times10^{-4}.$

(b) Compressive strain = $\dfrac{\Delta\ell}{\ell_0} \quad \rightarrow \quad \Delta\ell = \ell_0 \times$ compressive strain, which gives

$\Delta\ell = (82\text{ cm})(2.3\times10^{-4}) = 1.9\times10^{-2}\text{ cm} = 0.19$ mm.

EVALUATE: Note that an 82-cm cylinder compresses only 0.19 mm under a force of 32,000 N, which is about 7200 lb. So fractional changes in length are normally quite small.

VP11.7.3. **IDENTIFY:** The increase in pressure compresses the sphere. Using the bulk modulus we can find the change in volume.

SET UP: Bulk modulus is $B = -\dfrac{\Delta p}{\Delta V / V_0}$. To find the *decrease* in volume, we can neglect the minus sign. From Table 11.1, $B = 4.1\times10^{10}$ Pa for lead, and from Table 11.2, $B = 1/k = 2.7\times10^{10}$ Pa for mercury.

EXECUTE: The decrease in volume is $\Delta V = \dfrac{V_0 \Delta p}{B}$.

(a) Lead: $\Delta V = \dfrac{V_0\Delta p}{B} = \dfrac{\frac{4}{3}\pi(0.012\text{ m})^3(2.5\times10^7\text{ Pa})}{4.1\times10\text{ Pa}} = 4.4\times10^{-9}\text{ m}^3.$

(b) Mercury: Table 11.2 gives compressibility (k), so we use $B = 1/k$ to find $B = 2.7\times10^{10}$ Pa. Using the same formula as in (a), but with a different B, gives $\Delta V = 6.7\times10^{-9}\text{ m}^3$.

EVALUATE: If we divide the equations for ΔV, we get $\Delta V_M = \Delta V_L (B_L/B_M)$ $\Delta V_M = (4.4\times10^{-9}\text{ m}^3)(4.1/2.7) = 6.7\times10^{-9}\text{ m}^3$, which agrees with our result. Also note that since $B_L \approx 1.5 B_M$, we would expect ΔV_M to be about 1.5 times as great as ΔV_L (since ΔV is *inversely* proportional to B), which is what we find.

VP11.7.4. **IDENTIFY:** The shear forces distort the cube by a small distance x.

SET UP: The shear modulus is $S = \dfrac{F_{\parallel}h}{Ax}$, so $x = \dfrac{F_{\parallel}h}{AS}$. For brass $S = 3.5\times10^{10}$ Pa from Table 11.1.

EXECUTE: $x = \dfrac{(4.2\times10^4\text{ N})(0.025\text{ m})}{(0.025\text{ m})^2(3.5\times10^{10}\text{ Pa})} = 4.8\times10^{-5}\text{ m}.$

EVALUATE: The length of a side of this cube is 2.5 cm, but the shear displacement is only 4.8×10^{-3} cm, which is 4.8 thousandths of a centimeter. Shear displacements are typically very small.

11.1. **IDENTIFY:** Use $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ to calculate x_{cm}. The center of gravity of the bar is at its center and it can be treated as a point mass at that point.

SET UP: Use coordinates with the origin at the left end of the bar and the $+x$-axis along the bar. $m_1 = 0.120$ kg, $m_2 = 0.055$ kg, $m_3 = 0.110$ kg.

EXECUTE: $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(0.120\text{ kg})(0.250\text{ m}) + 0 + (0.110\text{ kg})(0.500\text{ m})}{0.120\text{ kg} + 0.055\text{ kg} + 0.110\text{ kg}} = 0.298\text{ m}.$ The fulcrum should be placed 29.8 cm to the right of the left-hand end.

EVALUATE: The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

11.2. **IDENTIFY:** Use $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$ to calculate x_{cm} of the composite object.

SET UP: Use coordinates where the origin is at the original center of gravity of the object and $+x$ is to the right. With the 1.50 kg mass added, $x_{\text{cm}} = -2.20$ cm, $m_1 = 5.00$ kg and $m_2 = 1.50$ kg. $x_1 = 0$.

EXECUTE: $x_{\text{cm}} = \dfrac{m_2x_2}{m_1 + m_2}$. $x_2 = \left(\dfrac{m_1 + m_2}{m_2}\right)x_{\text{cm}} = \left(\dfrac{5.00\text{ kg} + 1.50\text{ kg}}{1.50\text{ kg}}\right)(-2.20\text{ cm}) = -9.53\text{ cm}.$

The additional mass should be attached 9.53 cm to the left of the original center of gravity.

EVALUATE: The new center of gravity is somewhere between the added mass and the original center of gravity.

11.3. IDENTIFY: Treat the rod and clamp as point masses. The center of gravity of the rod is at its midpoint, and we know the location of the center of gravity of the rod-clamp system.

SET UP: $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$.

EXECUTE: $1.20\text{ m} = \dfrac{(1.80\text{ kg})(1.00\text{ m}) + (2.40\text{ kg})x_2}{1.80\text{ kg} + 2.40\text{ kg}}$.

$$x_2 = \frac{(1.20\text{ m})(1.80\text{ kg} + 2.40\text{ kg}) - (1.80\text{ kg})(1.00\text{ m})}{2.40\text{ kg}} = 1.35\text{ m}$$

EVALUATE: The clamp is to the right of the center of gravity of the system, so the center of gravity of the system lies between that of the rod and the clamp, which is reasonable.

11.4. IDENTIFY: This problem involves center of gravity and torque.

SET UP: Refer to Fig. 11.9(b) in the textbook. The equation $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$ gives the location of the center of gravity, and torque is equal to the force times the lever arm. In the center of gravity formula, we should use the masses of the objects. However since mass is $m = w/g$, we do not have to divide each weight by g since the g will cancel out because it occurs in each term in the numerator and denominator.

EXECUTE: **(a)** $x_{cg} = \dfrac{(180\text{ N})(1.5\text{ m}) + (800\text{ N})(1.0\text{ m})}{180\text{ N} + 800\text{ N}}$ = 1.1 m to the right of the foot of the ladder.

(b) $\tau = (980\text{ N})(1.1\text{ m}) = 1070\ \text{N}\cdot\text{m}$.

EVALUATE: **(c)** In the text the torque is $\tau = (180\text{ N})(1.5\text{ m}) + (800\text{ N})(1.0\text{ m}) = 1070\ \text{N}\cdot\text{m}$. Our answer in (b) agrees with the text answer.

11.5. IDENTIFY: We need to calculate the center of gravity of a compound object consisting of two spheres and a steel rod. The center of gravity of each of them is at their midpoint since they are all uniform.

SET UP: Use $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$ with the origin at the center of the rod. Let the 0.900-kg sphere be on the left end of the rod and the 0380-kg sphere on the right end.

EXECUTE: $x_{cg} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(0.900\text{ kg})(-28.0\text{ m}) + (0.300\text{ kg})(0\text{ m}) + (0.380\text{ kg})(26.0\text{ cm})}{0.900\text{ kg} + 0.300\text{ kg} + 0.380\text{ kg}}$

$= -9.70$ cm. So the center of gravity is 9.70 cm from the center of the rod toward the 0.900-kg sphere.

EVALUATE: The center of gravity is toward the heavier sphere, which is reasonable.

11.6. IDENTIFY: Apply the first and second conditions for equilibrium to the trap door.

SET UP: For $\sum\tau_z = 0$ take the axis at the hinge. Then the torque due to the applied force must balance the torque due to the weight of the door.

EXECUTE: **(a)** The force is applied at the center of gravity, so the applied force must have the same magnitude as the weight of the door, or 300 N. In this case the hinge exerts no force.

(b) With respect to the hinges, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or 150 N.

The hinges supply an upward force of $300\text{ N} - 150\text{ N} = 150\text{ N}$.

EVALUATE: Less force must be applied when it is applied farther from the hinges.

11.7. IDENTIFY: Apply $\sum\tau_z = 0$ to the ladder.

SET UP: Take the axis to be at point A. The free-body diagram for the ladder is given in Figure 11.7. The torque due to F must balance the torque due to the weight of the ladder.

EXECUTE: $F(8.0\text{ m})\sin 40° = (3400\text{ N})(10.0\text{ m})$, so $F = 6.6$ kN.

EVALUATE: The force required is greater than the weight of the ladder, because the moment arm for F is less than the moment arm for w.

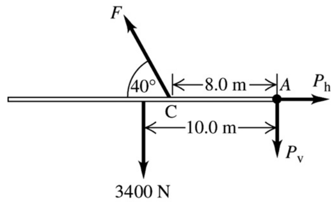


Figure 11.7

11.8. **IDENTIFY:** Apply the first and second conditions of equilibrium to the board.
SET UP: The free-body diagram for the board is given in Figure 11.8. Since the board is uniform its center of gravity is 1.50 m from each end. Apply $\sum F_y = 0,$ with $+y$ upward. Apply $\sum \tau_z = 0$ with the axis at the end where the first person applies a force and with counterclockwise torques positive.
EXECUTE: $\sum F_y = 0$ gives $F_1 + F_2 - w = 0$ and $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}.$ $\sum \tau_z = 0$ gives $F_2 x - w(1.50\text{ m}) = 0$ and $x = \left(\dfrac{w}{F_2}\right)(1.50\text{ m}) = \left(\dfrac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m}.$ The other person lifts with a force of 100 N at a point 2.40 m from the end where the other person lifts.
EVALUATE: By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.

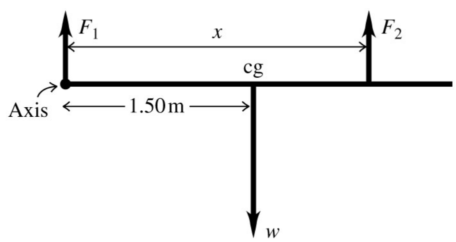


Figure 11.8

11.9. **IDENTIFY:** Apply $\sum F_y = 0$ and $\sum \tau_z = 0$ to the board.
SET UP: Let $+y$ be upward. Let x be the distance of the center of gravity of the motor from the end of the board where the 400 N force is applied.
EXECUTE: **(a)** If the board is taken to be massless, the weight of the motor is the sum of the applied forces, 1000 N. The motor is a distance $\dfrac{(2.00\text{ m})(600\text{ N})}{(1000\text{ N})} = 1.20\text{ m}$ from the end where the 400 N force is applied, and so is 0.800 m from the end where the 600 N force is applied.
(b) The weight of the motor is $400\text{ N} + 600\text{ N} - 200\text{ N} = 800\text{ N}.$ Applying $\sum \tau_z = 0$ with the axis at the end of the board where the 400 N acts gives $(600\text{ N})(2.00\text{ m}) = (200\text{ N})(1.00\text{ m}) + (800\text{ N})x$ and $x = 1.25\text{ m}.$ The center of gravity of the motor is 0.75 m from the end of the board where the 600 N force is applied.
EVALUATE: The motor is closest to the end of the board where the larger force is applied.

11.10. **IDENTIFY:** Apply the first and second conditions of equilibrium to the shelf.
SET UP: The free-body diagram for the shelf is given in Figure 11.10. Take the axis at the left-hand end of the shelf and let counterclockwise torque be positive. The center of gravity of the uniform shelf is at its center.
EXECUTE: **(a)** $\sum \tau_z = 0$ gives $-w_t(0.200\text{ m}) - w_s(0.300\text{ m}) + T_2(0.400\text{ m}) = 0.$

$$T_2 = \frac{(25.0\text{ N})(0.200\text{ m}) + (50.0\text{ N})(0.300\text{ m})}{0.400\text{ m}} = 50.0\text{ N}$$

$\sum F_y = 0$ gives $T_1 + T_2 - w_t - w_s = 0$ and $T_1 = 25.0$ N. The tension in the left-hand wire is 25.0 N and the tension in the right-hand wire is 50.0 N.

EVALUATE: We can verify that $\sum \tau_z = 0$ is zero for any axis, for example for an axis at the right-hand end of the shelf.

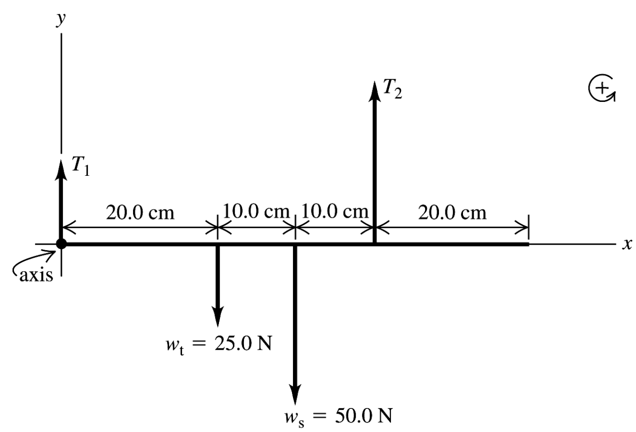


Figure 11.10

11.11. IDENTIFY: Apply the conditions for equilibrium to the bar. Set each tension equal to its maximum value.

SET UP: Let cable A be at the left-hand end. Take the axis to be at the left-hand end of the bar and x be the distance of the weight w from this end. The free-body diagram for the bar is given in Figure 11.11.

EXECUTE: **(a)** $\sum F_y = 0$ gives $T_A + T_B - w - w_{\text{bar}} = 0$ and

$w = T_A + T_B - w_{\text{bar}} = 500.0\text{ N} + 400.0\text{ N} - 350.0\text{ N} = 550\text{ N}.$

(b) $\sum \tau_z = 0$ gives $T_B(1.50\text{ m}) - wx - w_{\text{bar}}(0.750\text{ m}) = 0.$

$x = \dfrac{T_B(1.50\text{ m}) - w_{\text{bar}}(0.750\text{ m})}{w} = \dfrac{(400.0\text{ N})(1.50\text{ m}) - (350\text{ N})(0.750\text{ m})}{550\text{ N}} = 0.614\text{ m}.$ The weight should be placed 0.614 m from the left-hand end of the bar (cable A).

EVALUATE: If the weight is moved to the left, T_A exceeds 500.0 N and if it is moved to the right T_B exceeds 400.0 N.

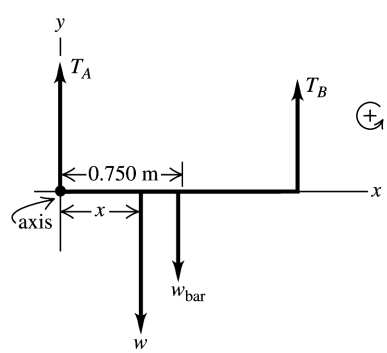


Figure 11.11

11.12. **IDENTIFY:** Apply the first and second conditions for equilibrium to the ladder.

SET UP: Let n_2 be the upward normal force exerted by the ground and let n_1 be the horizontal normal force exerted by the wall. The maximum possible static friction force that can be exerted by the ground is $\mu_s n_2$.

EXECUTE: **(a)** Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force n_2. For the vertical forces to balance,

$n_2 = w_1 + w_m = 160\text{ N} + 740\text{ N} = 900\text{ N}$, and the maximum frictional force is

$\mu_s n_2 = (0.40)(900\text{ N}) = 360\text{ N}$.

(b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground,

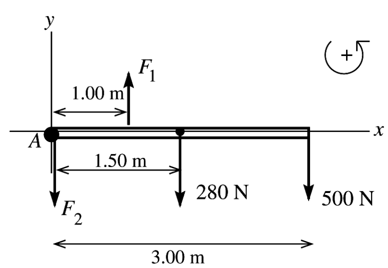
$(4.0\text{ m})n_1 = (1.5\text{ m})(160\text{ N}) + (1.0\text{ m})(3/5)(740\text{ N}) = 684\text{ N}\cdot\text{m}$, so $n_1 = 171.0\text{ N}$. This horizontal force must be balanced by the friction force, which must then be 170 N to two figures.

(c) Setting the friction force, and hence n_1, equal to the maximum of 360 N and solving for the distance x along the ladder, $(4.0\text{ m})(360\text{ N}) = (1.50\text{ m})(160\text{ N}) + x(3/5)(740\text{ N})$, so $x = 2.7\text{ m}$.

EVALUATE: The normal force exerted by the ground doesn't change as the man climbs up the ladder. But the normal force exerted by the wall and the friction force exerted by the ground both increase as he moves up the ladder.

11.13. **IDENTIFY:** The system of the person and diving board is at rest so the two conditions of equilibrium apply.

(a) SET UP: The free-body diagram for the diving board is given in Figure 11.13. Take the origin of coordinates at the left-hand end of the board (point A).



$\vec{F}_1$ is the force applied at the support point and $\vec{F}_2$ is the force at the end that is held down.

Figure 11.13

EXECUTE: $\sum\tau_A = 0$ gives $+F_1(1.0\text{ m}) - (500\text{ N})(3.00\text{ m}) - (280\text{ N})(1.50\text{ m}) = 0$

$$F_1 = \frac{(500\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m})}{1.00\text{ m}} = 1920\text{ N}$$

(b) $\sum F_y = ma_y$

$F_1 - F_2 - 280\text{ N} - 500\text{ N} = 0$

$F_2 = F_1 - 280\text{ N} - 500\text{ N} = 1920\text{ N} - 280\text{ N} - 500\text{ N} = 1140\text{ N}$

EVALUATE: We can check our answers by calculating the net torque about some point and checking that $\sum\tau_z = 0$ for that point also. Net torque about the right-hand end of the board:

$(1140\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m}) - (1920\text{ N})(2.00\text{ m}) = 3420\text{ N}\cdot\text{m} + 420\text{ N}\cdot\text{m} - 3840\text{ N}\cdot\text{m} = 0$, which checks.

11.14. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The boy exerts a downward force on the beam that is equal to his weight.

EXECUTE: **(a)** The graphs are given in Figure 11.14.

(b) $x = 6.25$ m when $F_A = 0,$ which is 1.25 m beyond point *B*.

(c) Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900$ N.

The distance that point *B* must be from the right end is then $\dfrac{(300\text{ N})(4.50\text{ m})}{(900\text{ N})} = 1.50$ m.

EVALUATE: When the beam is on the verge of tipping it starts to lift off the support *A* and the normal force F_A exerted by the support goes to zero.

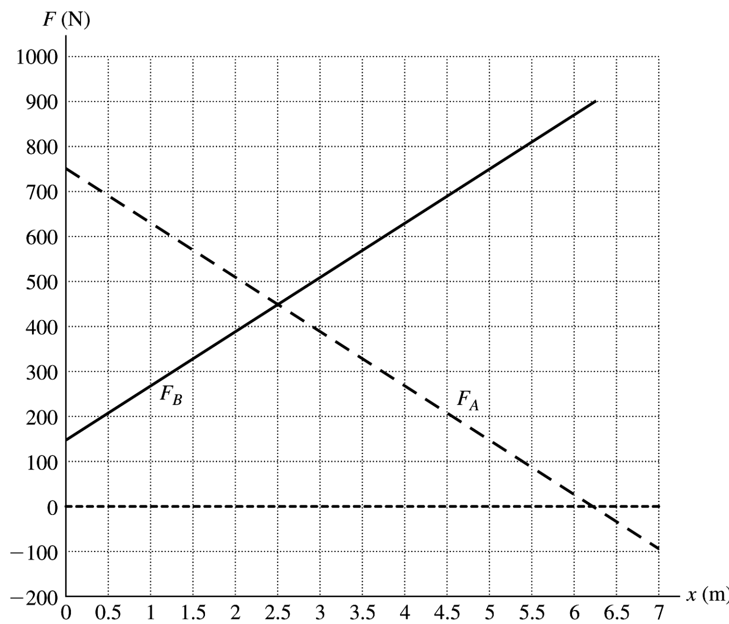
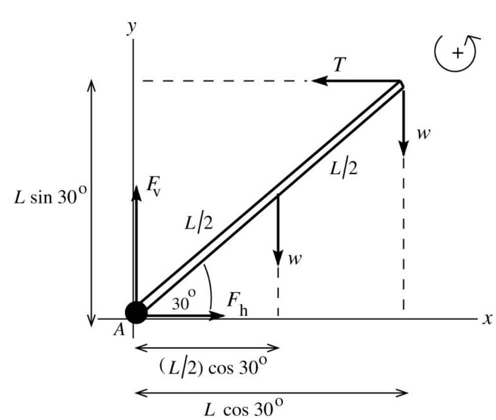


Figure 11.14

11.15. IDENTIFY: Apply the first and second conditions of equilibrium to the strut.

(a) SET UP: The free-body diagram for the strut is given in Figure 11.15a. Take the origin of coordinates at the hinge (point A) and $+y$ upward. Let F_h and F_v be the horizontal and vertical components of the force $\vec{F}$ exerted on the strut by the pivot. The tension in the vertical cable is the weight *w* of the suspended object. The weight *w* of the strut can be taken to act at the center of the strut. Let *L* be the length of the strut.



EXECUTE:

$\sum F_y = ma_y$

$F_v - w - w = 0$

$F_v = 2w$

Figure 11.15a

Sum torques about point A. The pivot force has zero moment arm for this axis and so doesn't enter into the torque equation.

$\tau_A = 0$

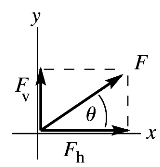
$TL\sin 30.0° - w((L/2)\cos 30.0°) - w(L\cos 30.0°) = 0$

$T\sin 30.0° - (3w/2)\cos 30.0° = 0$

$$T = \frac{3w\cos 30.0°}{2\sin 30.0°} = 2.60w$$

Then $\sum F_x = ma_x$ implies $T - F_h = 0$ and $F_h = 2.60w$.

We now have the components of $\vec{F}$ so can find its magnitude and direction (Figure 11.15b).



$F = \sqrt{F_h^2 + F_v^2}$

$F = \sqrt{(2.60w)^2 + (2.00w)^2}$

$F = 3.28w$

$\tan\theta = \dfrac{F_v}{F_h} = \dfrac{2.00w}{2.60w}$

$\theta = 37.6°$

Figure 11.15b

(b) Set Up: The free-body diagram for the strut is given in Figure 11.15c.

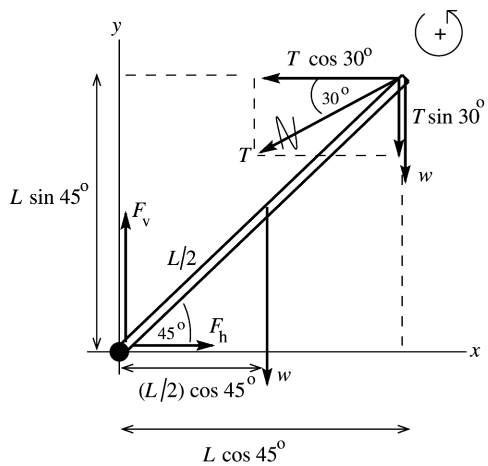


Figure 11.15c

The tension T has been replaced by its x and y components. The torque due to T equals the sum of the torques of its components, and the latter are easier to calculate.

Execute: $\sum \tau_A = 0 + (T\cos 30.0°)(L\sin 45.0°) - (T\sin 30.0°)(L\cos 45.0°) -$

$w[(L/2)\cos 45.0°] - w(L\cos 45.0°) = 0$

The length L divides out of the equation. The equation can also be simplified by noting that $\sin 45.0° = \cos 45.0°$.

Then $T(\cos 30.0° - \sin 30.0°) = 3w/2$.

$$T = \frac{3w}{2(\cos 30.0° - \sin 30.0°)} = 4.10w$$

$$\sum F_x = ma_x$$
$$F_h - T\cos 30.0° = 0$$
$$F_h = T\cos 30.0° = (4.10w)(\cos 30.0°) = 3.55w$$
$$\sum F_y = ma_y$$
$$F_v - w - w - T\sin 30.0° = 0$$
$$F_v = 2w + (4.10w)\sin 30.0° = 4.05w$$

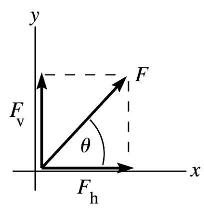


Figure 11.15d

From Figure 11.15d,

$$F = \sqrt{F_h^2 + F_v^2}$$
$$F = \sqrt{(3.55w)^2 + (4.05w)^2} = 5.39w$$
$$\tan\theta = \frac{F_v}{F_h} = \frac{4.05w}{3.55w}$$
$$\theta = 48.8°$$

EVALUATE: In each case the force exerted by the pivot does not act along the strut. Consider the net torque about the upper end of the strut. If the pivot force acted along the strut, it would have zero torque about this point. The two forces acting at this point also have zero torque and there would be one nonzero torque, due to the weight of the strut. The net torque about this point would then not be zero, violating the second condition of equilibrium.

11.16. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.16. H_v and H_h are the vertical and horizontal components of the force exerted on the beam at the wall (by the hinge). Since the beam is uniform, its center of gravity is 2.00 m from each end. The angle θ has $\cos\theta = 0.800$ and $\sin\theta = 0.600$. The tension *T* has been replaced by its *x*- and *y*-components.

EXECUTE: **(a)** H_v, H_h and $T_x = T\cos\theta$ all produce zero torque. $\sum \tau_z = 0$ gives $-w(2.00\text{ m}) - w_{load}(4.00\text{ m}) + T\sin\theta(4.00\text{ m}) = 0$ and

$$T = \frac{(190\text{ N})(2.00\text{ m}) + (300\text{ N})(4.00\text{ m})}{(4.00\text{ m})(0.600)} = 658.3\text{ N},\ \text{which rounds to 658 N.}$$

(b) $\sum F_x = 0$ gives $H_h - T\cos\theta = 0$ and $H_h = (658.3\text{ N})(0.800) = 527\text{ N}$. $\sum F_y = 0$ gives $H_v - w - w_{load} + T\sin\theta = 0$ and $H_v = w + w_{load} - T\sin\theta = 190\text{ N} + 300\text{ N} - (658\text{ N})(0.600) = 95\text{ N}$.

EVALUATE: For an axis at the right-hand end of the beam, only *w* and H_v produce torque. The torque due to *w* is counterclockwise so the torque due to H_v must be clockwise. To produce a clockwise torque, H_v must be upward, in agreement with our result from $\sum F_y = 0$.

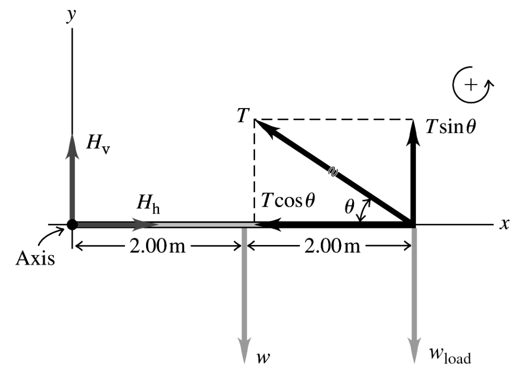


Figure 11.16

11.17. **IDENTIFY:** The boom is at rest, so the forces and torques on it must each balance.

SET UP: $\sum\tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. The free-body is shown in Figure 11.17. Call L the length of the boom.

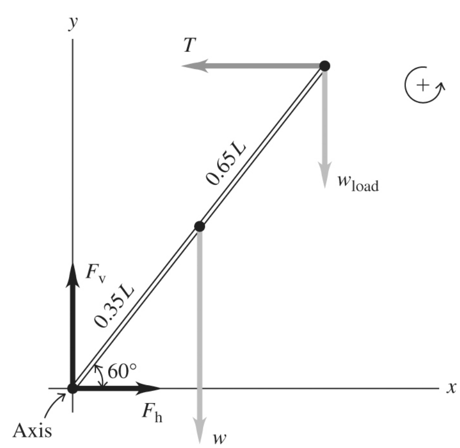


Figure 11.17

EXECUTE: **(a)** $\sum\tau = 0$ gives $T(L\sin 60.0°) - w_{load}(L\cos 60.0°) - w(0.35L\cos 60.0°) = 0$ and

$$T = \frac{w_{load}\cos 60.0° + w(0.35\cos 60.0°)}{\sin 60.0°} = \frac{(5000\text{ N})\cos 60.0° + (2600\text{ N})(0.35\cos 60.0°)}{\sin 60.0°} = 3.41\times 10^3\text{ N}.$$

(b) $\sum F_x = 0$ gives $F_h - T = 0$ and $F_h = 3410$ N.

$\sum F_y = 0$ gives $F_v - w - w_{load} = 0$ and $F_v = 5000\text{ N} + 2600\text{ N} = 7600\text{ N}$

EVALUATE: The bottom of the boom is the best point about which to take torques because only one unknown (the tension) appears in our equation. Using the top (or the center of mass) would give a torque equation with two (or three) unknowns.

11.18. **IDENTIFY:** The wheelbarrow is normally either held at rest or is moving with uniform velocity. Therefore the forces and torques on it must both balance.

SET UP: Estimate: The maximum upward force is 75 lb. Apply $\sum F_y = 0$ and $\sum\tau_z = 0$. Our target variables are the maximum weight of dirt we can life and the force the ground exerts on the wheels with that load in the wheelbarrow. Start with a free-body diagram of the wheelbarrow as shown in Fig. 11.18. Call F the upward force we exert (75 lb), w_{wb} the weight of the wheelbarrow (80.0 N = 18.0 lb), w_d the weight of the load of dirt, and F_{grd} the upward force the ground exerts on the front wheel.

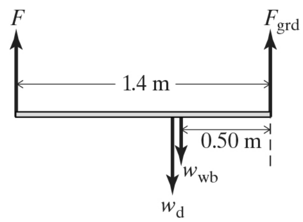


Figure 11.18

EXECUTE: **(a)** Apply $\sum\tau_z = 0$ about the point where the wheel is in contact with the ground. Using the notation in Fig 11.18 gives $-F(1.4 \text{ m}) + (w_{wb} + w_d)(0.50 \text{ m}) = 0$. Putting in the numbers and solving for w_d gives $w_d = \dfrac{(75 \text{ lb})(1.4) - (18.0 \text{ lb})(0.50)}{0.50} = 190 \text{ lb}$.

(b) Now apply $\sum F_y = 0: F + F_{grd} - w_{wb} - w_d = 0$, which gives $F_{grd} = 18.0 \text{ lb} + 190 \text{ lb} - 75 \text{ lb} = 130 \text{ lb}$.

EVALUATE: You can lift such a heavy load of dirt with only 75 lb because of your long lever arm compared to that of the dirt.

11.19. **IDENTIFY:** The beam is at rest so the forces and torques on it must each balance.

SET UP: $\sum\tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. The distance along the beam from the hinge to where the cable is attached is 3.0 m. The angle ϕ that the cable makes with the beam is given by $\sin\phi = \dfrac{4.0 \text{ m}}{5.0 \text{ m}}$, so $\phi = 53.1°$. The center of gravity of the beam is 4.5 m from the hinge. Use coordinates with $+y$ upward and $+x$ to the right. Take the pivot at the hinge and let counterclockwise torque be positive. Express the hinge force as components H_v and H_h. Assume H_v is downward and that H_h is to the right. If one of these components is actually in the opposite direction we will get a negative value for it. Set the tension in the cable equal to its maximum possible value, $T = 1.00$ kN.

EXECUTE: **(a)** The free-body diagram is shown in Figure 11.19, with $\vec{T}$ resolved into its x- and y-components.

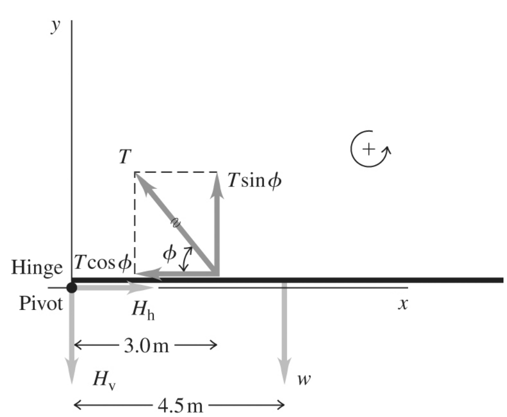


Figure 11.19

(b) $\sum\tau = 0$ gives $(T\sin\phi)(3.0 \text{ m}) - w(4.5 \text{ m}) = 0$

$$w = \frac{(T\sin\phi)(3.00 \text{ m})}{4.50 \text{ m}} = \frac{(1000 \text{ N})(\sin 53.1°)(3.00 \text{ m})}{4.50 \text{ m}} = 533 \text{ N}$$

(c) $\sum F_x = 0$ gives $H_h - T\cos\phi = 0$ and $H_h = (1.00 \text{ kN})(\cos 53.1°) = 600 \text{ N}$

$\sum F_y = 0$ gives $T\sin\phi - \mathrm{H}_v - w = 0$ and $H_v = (1.00 \text{ kN})(\sin 53.1°) - 533 \text{ N} = 267 \text{ N}$.

EVALUATE: $T\cos\phi$, H_v and H_h all have zero moment arms for a pivot at the hinge and therefore produce zero torque. If we consider a pivot at the point where the cable is attached we can see that H_v must be downward to produce a torque that opposes the torque due to w.

11.20. **IDENTIFY:** Apply the conditions for equilibrium to the crane.
SET UP: The free-body diagram for the crane is sketched in Figure 11.20. F_h and F_v are the components of the force exerted by the axle. $\vec{T}$ pulls to the left so F_h is to the right. $\vec{T}$ also pulls downward and the two weights are downward, so F_v is upward.
EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T[(13\text{ m})\sin 25°] - w_c[(7.0\text{ m})\cos 55°] - w_b[(16.0\text{ m})\cos 55°] = 0$.

$$T = \frac{(11{,}000\text{ N})[(16.0\text{ m})\cos 55°] + (15{,}000\text{ N})[(7.0\text{ m})\cos 55°]}{(13.0\text{ m})\sin 25°} = 2.93\times10^4\text{ N}.$$

(b) $\sum F_x = 0$ gives $F_h - T\cos 30° = 0$ and $F_h = 2.54\times10^4$ N.

$\sum F_y = 0$ gives $F_v - T\sin 30° - w_c - w_b = 0$ and $F_v = 4.06\times10^4$ N.

EVALUATE: $\tan\theta = \dfrac{F_v}{F_h} = \dfrac{4.06\times10^4\text{ N}}{2.54\times10^4\text{ N}}$ and $\theta = 58°$. The force exerted by the axle is not directed along the crane.

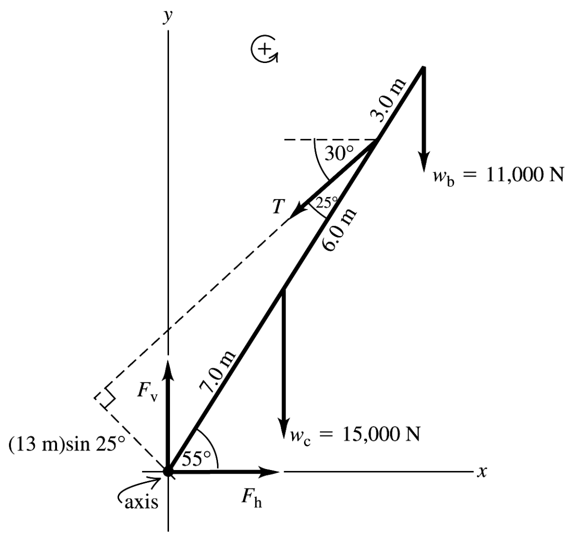


Figure 11.20

11.21. **IDENTIFY:** Apply the first and second conditions of equilibrium to the rod.
SET UP: The force diagram for the rod is given in Figure 11.21.

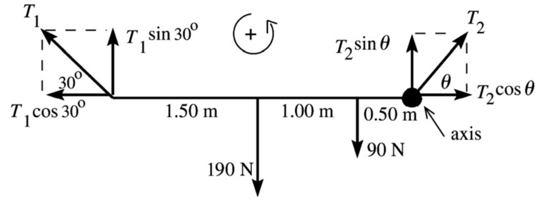


Figure 11.21

EXECUTE: $\sum \tau_z = 0,$ axis at right end of rod, counterclockwise torque is positive

$(190\text{ N})(1.50\text{ m}) + (90\text{ N})(0.50\text{ m}) - (T_1 \sin 30.0°)(3.00\text{ m}) = 0$

$$T_1 = \frac{285\text{ N}\cdot\text{m} + 45\text{ N}\cdot\text{m}}{1.50\text{ m}} = 220\text{ N}$$

$\sum F_x = ma_x$

$T_2 \cos\theta - T_1 \cos 30° = 0$ and $T_2 \cos\theta = (220\text{ N})(\cos 30°) = 190.5\text{ N}$

$\sum F_y = ma_y$

$T_1 \sin 30° + T_2 \sin\theta - 190\text{ N} - 90\text{ N} = 0$

$T_2 \sin\theta = 280\text{ N} - (220\text{ N})\sin 30° = 170\text{ N}$

Then $\dfrac{T_2 \sin\theta}{T_2 \cos\theta} = \dfrac{170\text{ N}}{190.5\text{ N}}$ gives $\tan\theta = 0.89239$ and $\theta = 41.7°$

And $T_2 = \dfrac{170\text{ N}}{\sin 41.7°} = 255\text{ N}.$

EVALUATE: The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction. Since $T_2 > T_1,$ the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

11.22. **IDENTIFY:** Apply the first and second conditions for equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.22.

EXECUTE: The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point; $T(3.00\text{ m}) = (1.40\text{ kN})(2.00\text{ m})\cos 25.0° + (5.00\text{ kN})(4.50\text{ m})\cos 25.0°,$ and $T = 7.64\text{ kN}.$ The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of $T,$ $6.00\text{ kN} - T\cos 25.0° = -0.53\text{ kN}.$ The vertical component is downward. The horizontal force is $T\sin 25.0° = 3.23\text{ kN}.$

EVALUATE: The vertical component of the tension is nearly the same magnitude as the total weight of the object and the vertical component of the force exerted by the pivot is much less than its horizontal component.

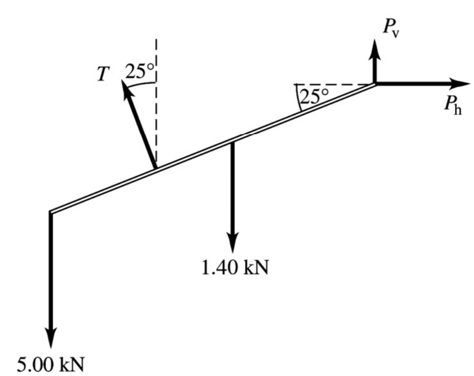


Figure 11.22

11.23. **(a) IDENTIFY** and **SET UP:** Use $\tau = Fl$ to calculate the torque (magnitude and direction) for each force and add the torques as vectors. See Figure 11.23a.

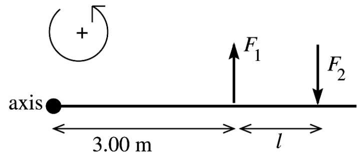


Figure 11.23a

EXECUTE:

$\tau_1 = F_1 l_1 = +(8.00\ \text{N})(3.00\ \text{m})$

$\tau_1 = +24.0\ \text{N}\cdot\text{m}$

$\tau_2 = -F_2 l_2 = -(8.00\ \text{N})(l + 3.00\ \text{m})$

$\tau_2 = -24.0\ \text{N}\cdot\text{m} - (8.00\ \text{N})l$

$\sum\tau_z = \tau_1 + \tau_2 = +24.0\ \text{N}\cdot\text{m} - 24.0\ \text{N}\cdot\text{m} - (8.00\ \text{N})l = -(8.00\ \text{N})l$

Want l that makes $\sum\tau_z = -6.40\ \text{N}\cdot\text{m}$ (net torque must be clockwise)

$-(8.00\ \text{N})l = -6.40\ \text{N}\cdot\text{m}$

$l = (6.40\ \text{N}\cdot\text{m})/8.00\ \text{N} = 0.800\ \text{m}$

(b) $|\tau_2| > |\tau_1|$ since F_2 has a larger moment arm; the net torque is clockwise.

(c) See Figure 11.23b.

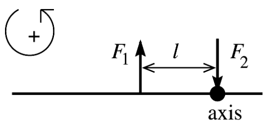


Figure 11.23b

$\tau_1 = -F_1 l_1 = -(8.00\ \text{N})l$

$\tau_2 = 0$ since $\vec{F}_2$ is at the axis

$\sum\tau_z = -6.40\ \text{N}\cdot\text{m}$ gives $-(8.00\ \text{N})l = -6.40\ \text{N}\cdot\text{m}$

$l = 0.800\ \text{m}$, same as in part (a).

EVALUATE: The force couple gives the same magnitude of torque for the pivot at any point.

11.24. IDENTIFY: The person is in equilibrium, so the torques on him must balance. The target variable is the force exerted by the deltoid muscle.

SET UP: The free-body diagram for the arm is given in Figure 11.24. Take the pivot at the shoulder joint and let counterclockwise torques be positive. Use coordinates as shown. Let F be the force exerted by the deltoid muscle. There are also the weight of the arm and forces at the shoulder joint, but none of these forces produce any torque when the arm is in this position. The forces F and T have been replaced by their x- and y-components. $\sum\tau_z = 0$.

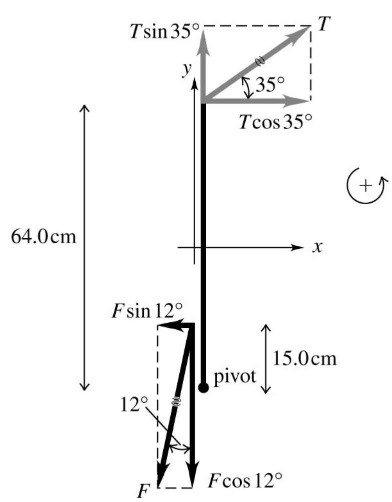


Figure 11.24

EXECUTE: $\sum\tau_z = 0$ gives $(F\sin 12.0°)(15.0\text{ cm}) - (T\cos 35°)(64.0\text{ cm}) = 0.$

$$F = \frac{(36.0\text{ N})(\cos 35°)(64.0\text{ cm})}{(\sin 12.0°)(15.0\text{ cm})} = 605\text{ N}.$$

EVALUATE: The force exerted by the deltoid muscle is much larger than the tension in the cable because the deltoid muscle makes a small angle (only 12.0°) with the humerus.

11.25. IDENTIFY: This problem involves the use of torques and graphical interpretation.

SET UP: In order to interpret the graph, apply $\sum\tau_z = 0$ to the rod and solve for the tension T in terms of the angle θ. Take torques about the hinge and call L the length of the rod. The mass of the rod is the target variable.

EXECUTE: $\sum\tau_z = 0$: $TL\sin\theta - mg\frac{L}{2}\cos\theta$, which gives $T = \frac{mg}{2}\frac{\cos\theta}{\sin\theta} = \left(\frac{mg}{2}\right)\cot\theta$. From this we see that a graph of T versus $\cot\theta$ should be a straight line having slope $mg/2$. Using the given slope we have $m = \frac{2(\text{slope})}{g} = \frac{2(30.0\text{ N})}{9.80\text{ m/s}^2} = 6.12\text{ kg}.$

EVALUATE: A situation like this might occur if you had a very heavy rod that could not easily be removed to weigh it, but could be pivoted about the hinge. The tension could be measured using a strain gauge.

11.26. IDENTIFY: Use $Y = \frac{l_0 F_\perp}{A\Delta l}$.

SET UP: $A = 50.0\text{ cm}^2 = 50.0\times10^{-4}\text{ m}^2.$

EXECUTE: Relaxed: $Y = \frac{(0.200\text{ m})(25.0\text{ N})}{(50.0\times10^{-4}\text{ m}^2)(3.0\times10^{-2}\text{ m})} = 3.33\times10^4\text{ Pa}.$

Maximum tension: $Y = \frac{(0.200\text{ m})(500\text{ N})}{(50.0\times10^{-4}\text{ m}^2)(3.0\times10^{-2}\text{ m})} = 6.67\times10^5\text{ Pa}.$

EVALUATE: The muscle tissue is much more difficult to stretch when it is under maximum tension.

11.27. IDENTIFY and SET UP: Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$ and solve for A and then use $A = \pi r^2$ to get the radius and $d = 2r$ to calculate the diameter.

EXECUTE: $Y = \frac{l_0 F_\perp}{A\Delta l}$ so $A = \frac{l_0 F_\perp}{Y\Delta l}$ (A is the cross-section area of the wire)

For steel, $Y = 2.0\times10^{11}\text{ Pa}$ (Table 11.1)

Thus $A = \frac{(2.00\text{ m})(700\text{ N})}{(2.0\times10^{11}\text{ Pa})(0.25\times10^{-2}\text{ m})} = 2.8\times10^{-6}\text{ m}^2.$

$A = \pi r^2$, so $r = \sqrt{A/\pi} = \sqrt{2.8\times10^{-6}\text{ m}^2/\pi} = 9.44\times10^{-4}\text{ m}$

$d = 2r = 1.9\times10^{-3}\text{ m} = 1.9\text{ mm}.$

EVALUATE: Steel wire of this diameter doesn't stretch much; $\Delta l/l_0 = 0.12\%$.

11.28. IDENTIFY: Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$.

SET UP: From Table 11.1, for steel, $Y = 2.0\times10^{11}\text{ Pa}$ and for copper, $Y = 1.1\times10^{11}\text{ Pa}.$
$A = \pi(d^2/4) = 1.77\times10^{-4}\text{ m}^2.$ $F_\perp = 4000\text{ N}$ for each rod.

EXECUTE: **(a)** The strain is $\frac{\Delta l}{l_0} = \frac{F}{YA}$. For steel $\frac{\Delta l}{l_0} = \frac{(4000\text{ N})}{(2.0\times10^{11}\text{ Pa})(1.77\times10^{-4}\text{ m}^2)} = 1.1\times10^{-4}.$

Similarly, the strain for copper is 2.1×10^{-4}.

(b) Steel: $(1.1\times10^{-4})(0.750\text{ m}) = 8.3\times10^{-5}\text{ m}$. Copper: $(2.1\times10^{-4})(0.750\text{ m}) = 1.6\times10^{-4}\text{ m}$.

EVALUATE: Copper has a smaller Y and therefore a greater elongation.

11.29. **IDENTIFY:** Apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$.

SET UP: $A = 0.50\text{ cm}^2 = 0.50\times10^{-4}\text{ m}^2$

EXECUTE: $Y = \dfrac{(4.00\text{ m})(5000\text{ N})}{(0.50\times10^{-4}\text{ m}^2)(0.20\times10^{-2}\text{ m})} = 2.0\times10^{11}\text{ Pa}$

EVALUATE: Our result is the same as that given for steel in Table 11.1.

11.30. **IDENTIFY:** Apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$.

SET UP: $A = \pi r^2 = \pi(3.5\times10^{-3}\text{ m})^2 = 3.85\times10^{-5}\text{ m}^2$. The force applied to the end of the rope is the weight of the climber: $F_\perp = (65.0\text{ kg})(9.80\text{ m/s}^2) = 637\text{ N}$.

EXECUTE: $Y = \dfrac{(45.0\text{ m})(637\text{ N})}{(3.85\times10^{-5}\text{ m}^2)(1.10\text{ m})} = 6.77\times10^{8}\text{ Pa}$

EVALUATE: Our result is a lot smaller than the values given in Table 11.1. An object made of rope material is much easier to stretch than if the object were made of metal.

11.31. **IDENTIFY:** The increased pressure compresses the lead sphere, so we are dealing with bulk stress and strain.

SET UP: The bulk modulus is $B = -\dfrac{\Delta p}{\Delta V / V_0}$, and for lead it is $B = 4.1\times10^{10}$ Pa. The volume compresses by 0.50%, so $\Delta V = -0.0050V_0$. The target variable is the pressure increase that causes this amount of compression.

EXECUTE: Solve $-\dfrac{\Delta p}{\Delta V / V_0}$ for Δp : $\Delta p = -B\left(\dfrac{\Delta V}{V_0}\right) = -\left(4.1\times10^{10}\text{ Pa}\right)\left(\dfrac{-0.0050V_0}{V_0}\right) = 2.05\times10^{8}\text{ Pa} =$ 2.0×10^{3} atm. The pressure is 2000 atmospheres above atmospheric pressure.

EVALUATE: This is a very large pressure, but it would take a large pressure to compress a lead sphere.

11.32. **IDENTIFY:** Apply $\text{stress} = \dfrac{F_\perp}{A}$, $\text{strain} = \dfrac{\text{stress}}{Y}$, $Y = \dfrac{l_0 F_\perp}{A\Delta l}$.

SET UP: The cross-sectional area of the post is $A = \pi r^2 = \pi(0.125\text{ m})^2 = 0.0491\text{ m}^2$. The force applied to the end of the post is $F_\perp = (8000\text{ kg})(9.80\text{ m/s}^2) = 7.84\times10^{4}\text{ N}$. The Young's modulus of steel is $Y = 2.0\times10^{11}\text{ Pa}$.

EXECUTE: **(a)** $\text{stress} = \dfrac{F_\perp}{A} = -\dfrac{7.84\times10^{4}\text{ N}}{0.0491\text{ m}^2} = -1.60\times10^{6}\text{ Pa}$. The minus sign indicates that the stress is compressive.

(b) $\text{strain} = \dfrac{\text{stress}}{Y} = -\dfrac{1.60\times10^{6}\text{ Pa}}{2.0\times10^{11}\text{ Pa}} = -8.0\times10^{-6}$. The minus sign indicates that the length decreases.

(c) $\Delta l = l_0(\text{strain}) = (2.50\text{ m})(-8.0\times10^{-6}) = -2.0\times10^{-5}\text{ m}$

EVALUATE: The fractional change in length of the post is very small.

11.33. **IDENTIFY:** The amount of compression depends on the bulk modulus of the bone.

SET UP: $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $1\text{ atm} = 1.01\times10^{5}\text{ Pa}$.

EXECUTE: (a) $\Delta p = -B\dfrac{\Delta V}{V_0} = -(15\times10^9\ \text{Pa})(-0.0010) = 1.5\times10^7\ \text{Pa} = 150\ \text{atm}.$

(b) The depth for a pressure increase of 1.5×10^7 Pa is 1.5 km.

EVALUATE: An extremely large pressure increase is needed for just a 0.10% bone compression, so pressure changes do not appreciably affect the bones. Unprotected dives do not approach a depth of 1.5 km, so bone compression is not a concern for divers.

11.34. IDENTIFY: Apply $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$.

SET UP: $\Delta V = -\dfrac{V_0 \Delta p}{B}$. Δp is positive when the pressure increases.

EXECUTE: (a) The volume would increase slightly.
(b) The volume change would be twice as great.
(c) The volume change is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

EVALUATE: For lead, $B = 4.1\times10^{10}$ Pa, so $\Delta p/B$ is very small and the fractional change in volume is very small.

11.35. IDENTIFY and SET UP: Use $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$ and $k = 1/B$ to calculate B and k.

EXECUTE: $B = -\dfrac{\Delta p}{\Delta V/V_0} = -\dfrac{(3.6\times10^6\ \text{Pa})(600\ \text{cm}^3)}{(-0.45\ \text{cm}^3)} = +4.8\times10^9\ \text{Pa}$

$k = 1/B = 1/4.8\times10^9\ \text{Pa} = 2.1\times10^{-10}\ \text{Pa}^{-1}$

EVALUATE: k is the same as for glycerine (Table 11.2).

11.36. IDENTIFY: Apply $\dfrac{\Delta V}{V_0} = -\dfrac{\Delta p}{B}$. Density $= m/V$.

SET UP: At the surface the pressure is 1.0×10^5 Pa, so $\Delta p = 1.16\times10^8$ Pa. $V_0 = 1.00\ \text{m}^3$. At the surface $1.00\ \text{m}^3$ of water has mass 1.03×10^3 kg.

EXECUTE: (a) $B = -\dfrac{(\Delta p)V_0}{\Delta V}$ gives $\Delta V = -\dfrac{(\Delta p)V_0}{B} = -\dfrac{(1.16\times10^8\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times10^9\ \text{Pa}} = -0.0527\ \text{m}^3$

(b) At this depth 1.03×10^3 kg of seawater has volume $V_0 + \Delta V = 0.9473\ \text{m}^3$. The density is

$\dfrac{1.03\times10^3\ \text{kg}}{0.9473\ \text{m}^3} = 1.09\times10^3\ \text{kg/m}^3.$

EVALUATE: The density is increased because the volume is compressed due to the increased pressure.

11.37. IDENTIFY: Apply $S = \dfrac{F_{\parallel}}{A}\dfrac{h}{x}$.

SET UP: $F_{\parallel} = 9.0\times10^5$ N. $A = (0.100\ \text{m})(0.500\times10^{-2}\ \text{m})$. $h = 0.100$ m. From Table 11.1, $S = 7.5\times10^{10}$ Pa for steel.

EXECUTE: (a) Shear strain $= \dfrac{F_{\parallel}}{AS} = \dfrac{(9\times10^5\ \text{N})}{[(0.100\ \text{m})(0.500\times10^{-2}\ \text{m})][7.5\times10^{10}\ \text{Pa}]} = 2.4\times10^{-2}.$

(b) Since shear strain $= x/h$, $x = (\text{Shear strain})\cdot h = (0.024)(0.100\ \text{m}) = 2.4\times10^{-3}$ m.

EVALUATE: This very large force produces a small displacement; $x/h = 2.4\%$.

11.38. **IDENTIFY:** The force components parallel to the face of the cube produce a shear which can deform the cube.

SET UP: $S = \dfrac{F_{\parallel}}{A\phi}$, where $\phi = x/h$. $F_{\parallel}$ is the component of the force tangent to the surface, so $F_{\parallel} = (1375\text{ N})\cos 8.50° = 1360\text{ N}$. ϕ must be in radians, $\phi = 1.24° = 0.0216\text{ rad}$.

EXECUTE: $S = \dfrac{1360\text{ N}}{(0.0925\text{ m})^2(0.0216\text{ rad})} = 7.36\times 10^6\text{ Pa}$.

EVALUATE: The shear modulus of this material is much less than the values for metals given in Table 11.1 in the text.

11.39. **IDENTIFY:** The problem involves the stretching of metal wires, so it makes use of tensile stress and strain and Young's modulus.

SET UP: The steel and aluminum wires have the same fractional change in length. We use $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. We know that $\dfrac{\Delta\ell}{\ell_0}$ is the same for both wires, $r_{al} = 2r_{st}$, and $A = \pi r^2$. Table 11.1 gives us the values of Y for both metals. Our target variable is the tension T_{al} in the aluminum wire in terms of the tension T_{st} in the steel wire. Since $\dfrac{\Delta\ell}{\ell_0}$ is the same for both wires, we can equate this expression for both wires and solve for T_{al}.

EXECUTE: Steel: $\dfrac{\Delta\ell}{\ell_0} = \dfrac{T_{st}}{Y_{st}\pi r_{st}^2}$. Aluminum: $\dfrac{T_{al}}{Y_{al}\pi r_{al}^2}$. Equating gives $\dfrac{T_{st}}{Y_{st}\pi r_{st}^2} = \dfrac{T_{al}}{Y_{al}\pi r_{al}^2}$. Using the fact that $r_{al} = 2r_{st}$ and $A = \pi r^2$, we have $T_{al} = \dfrac{Y_{al}}{Y_{st}}\left(\dfrac{r_{al}}{r_{st}}\right)^2 T_{st} = \dfrac{7.0\times 10^{10}\text{ Pa}}{20\times 10^{10}\text{ Pa}}\left(\dfrac{2r_{st}}{r_{st}}\right)^2 T_{st} = 1.4T_{st}$.

EVALUATE: Although aluminum has a smaller Young's modulus than does steel, the aluminum wire is twice as thick and hence has 4 times the area as the steel wire, so it can support more tension than the steel for the same fractional stretch.

11.40. **IDENTIFY:** The applied force stretches the wire, so are dealing with tensile stress and strain and Young's modulus.

SET UP: Since we graph $\dfrac{\Delta\ell}{\ell_0}$ versus $F_\perp$, we need to discover a relationship between these quantities so we can interpret the slope of the graph. We start with $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. Our target variable is Y for the metal of the wire.

EXECUTE: Solve $\dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$ for $\dfrac{\Delta\ell}{\ell_0}$ in terms of $F_\perp$, which gives $\dfrac{\Delta\ell}{\ell_0} = \dfrac{1}{AY}F_\perp$. Therefore a graph of $\dfrac{\Delta\ell}{\ell_0}$ versus $F_\perp$ should be a straight line having slope $\dfrac{1}{AY}$. Thus $Y = \dfrac{1}{A(\text{slope})}$, which gives

$$Y = \frac{1}{(8.00\times 10^{-6}\text{ m}^2)(8.00\times 10^{-7}\text{ N}^{-1})} = 1.6\times 10^{11}\text{ Pa}.$$

EVALUATE: From Table 11.1, we see that this value is about the same as Y for steel, nickel, iron, and copper, so it is a reasonable result.

11.41. **IDENTIFY:** We are dealing with compression due to increased pressure, so we must use bulk stress and strain and the bulk modulus B.

SET UP: Since we are dealing with liquids, we will need to use Table 11.2. It gives the compressibility k, which is $1/B$, so $k = -\dfrac{\Delta V / V_0}{\Delta p}$. We know that $\Delta V / V_0$ will be negative (for a compression) when Δp is positive, so we can neglect the minus signs. We also know that $\Delta V / V_0$ is the same for both liquids. The target variable is the pressure increase $\Delta p_{\rm a}$ in the alcohol.

EXECUTE: Equating $\Delta V / V_0$ for both liquids gives $k_{\rm a}\Delta p_{\rm a} = k_{\rm g}\Delta p_{\rm g} = k_{\rm g}\Delta p_1$. Solving for $\Delta p_{\rm a}$ gives

$$\Delta p_{\rm a} = \frac{k_{\rm g}}{k_{\rm a}}\Delta p_1 = \frac{21\times10^{-11}\ \text{Pa}^{-1}}{110\times10^{-11}\ \text{Pa}^{-1}}\Delta p_1 = 0.19\Delta p_1.$$

EVALUATE: Since $k_{\rm a} \approx 5k_{\rm g}$, alcohol is 5 times more compressible than glycerin, so it takes only about 1/5 the pressure increase to produce the same compression as for glycerin. This is a reasonable result.

11.42. **IDENTIFY:** The breaking stress of the wire is the value of $F_\perp/A$ at which the wire breaks.

SET UP: From Table 11.3, the breaking stress of brass is 4.7×10^8 Pa. The area A of the wire is related to its diameter by $A = \pi d^2/4$.

EXECUTE: $A = \dfrac{350\ \text{N}}{4.7\times10^8\ \text{Pa}} = 7.45\times10^{-7}\ \text{m}^2$, so $d = \sqrt{4A/\pi} = 0.97$ mm.

EVALUATE: The maximum force a wire can withstand without breaking is proportional to the square of its diameter.

11.43. **IDENTIFY** and **SET UP:** Use stress $= \dfrac{F_\perp}{A}$.

EXECUTE: Tensile stress $= \dfrac{F_\perp}{A} = \dfrac{F_\perp}{\pi r^2} = \dfrac{90.8\ \text{N}}{\pi(0.92\times10^{-3}\ \text{m})^2} = 3.41\times10^7$ Pa

EVALUATE: A modest force produces a very large stress because the cross-sectional area is small.

11.44. **IDENTIFY:** The elastic limit is a value of the stress, $F_\perp/A$. Apply $\sum\vec{F} = m\vec{a}$ to the elevator in order to find the tension in the cable.

SET UP: $\dfrac{F_\perp}{A} = \frac{1}{3}(2.40\times10^8\ \text{Pa}) = 0.80\times10^8$ Pa. The free-body diagram for the elevator is given in Figure 11.44. $F_\perp$ is the tension in the cable.

EXECUTE: $F_\perp = A(0.80\times10^8\ \text{Pa}) = (3.00\times10^{-4}\ \text{m}^2)(0.80\times10^8\ \text{Pa}) = 2.40\times10^4$ N. $\sum F_y = ma_y$ applied to the elevator gives $F_\perp - mg = ma$ and $a = \dfrac{F_\perp}{m} - g = \dfrac{2.40\times10^4\ \text{N}}{1200\ \text{kg}} - 9.80\ \text{m/s}^2 = 10.2\ \text{m/s}^2$

EVALUATE: The tension in the cable is about twice the weight of the elevator.

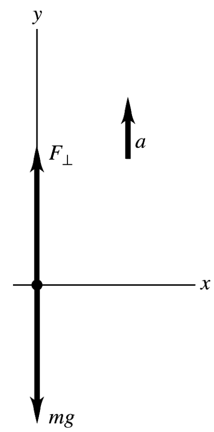


Figure 11.44

11.45. **IDENTIFY:** To pull out the nail, you exert a torque on the handle which in turn produces a torque on the nail.

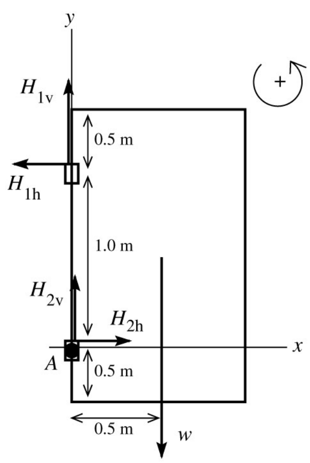
SET UP: Estimate: $F_2 = 25$ lb in Fig. 11.45 in the text. Use $\sum \tau_z = 0$ about contact point A in the figure. The target variable is the force the hammer exerts on the nail (F_1 in the figure).

EXECUTE: Using the figure in the text, applying $\sum \tau_z = 0$ to the hammer gives $F_2(0.300 \text{ m}) - F_1(\sin 60°)(0.080 \text{ m}) = 0.$ Using $F_2 = 25$ lb and solving for F_1 gives $F_1 = 110$ lb. Note that in the figure $\vec{F}_1$ is the force on the nail, but by Newton's third law, the force on the hammer is equal and opposite to this force.

EVALUATE: The long handle allows the force on the nail to be about 4 times your force. A longer handle would increase this factor even more.

11.46. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

SET UP: The free-body diagram for the door is given in Figure 11.46. Let $\vec{H}_1$ and $\vec{H}_2$ be the forces exerted by the upper and lower hinges. Take the origin of coordinates at the bottom hinge (point A) and $+y$ upward.



EXECUTE:
We are given that
$H_{1v} = H_{2v} = w/2 = 165$ N.

$\sum F_x = ma_x$

$H_{2h} - H_{1h} = 0$

$H_{1h} = H_{2h}$

The horizontal components of the hinge forces are equal in magnitude and opposite in direction.

Figure 11.46

Sum torques about point A. H_{1v}, H_{2v}, and H_{2h} all have zero moment arm and hence zero torque about an axis at this point. Thus $\sum \tau_A = 0$ gives $H_{1h}(1.00 \text{ m}) - w(0.50 \text{ m}) = 0$

$$H_{1h} = w\left(\frac{0.50 \text{ m}}{1.00 \text{ m}}\right) = \tfrac{1}{2}(330 \text{ N}) = 165 \text{ N}.$$

The horizontal component of each hinge force is 165 N.

EVALUATE: The horizontal components of the force exerted by each hinge are the only horizontal forces so must be equal in magnitude and opposite in direction. With an axis at A, the torque due to the horizontal force exerted by the upper hinge must be counterclockwise to oppose the clockwise torque exerted by the weight of the door. So, the horizontal force exerted by the upper hinge must be to the left. You can also verify that the net torque is also zero if the axis is at the upper hinge.

11.47. **IDENTIFY:** The center of gravity of the combined object must be at the fulcrum. Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to calculate x_{cm}.

SET UP: The center of gravity of the sand is at the middle of the box. Use coordinates with the origin at the fulcrum and $+x$ to the right. Let $m_1 = 25.0$ kg, so $x_1 = 0.500$ m. Let $m_2 = m_{sand}$, so $x_2 = -0.625$ m. $x_{cm} = 0$.

EXECUTE: $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = 0$ and $m_2 = -m_1\dfrac{x_1}{x_2} = -(25.0 \text{ kg})\left(\dfrac{0.500 \text{ m}}{-0.625 \text{ m}}\right) = 20.0 \text{ kg}.$

EVALUATE: The mass of sand required is less than the mass of the plank since the center of the box is farther from the fulcrum than the center of gravity of the plank is.

11.48. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the bridge.

SET UP: Let the axis of rotation be at the left end of the bridge and let counterclockwise torques be positive.

EXECUTE: If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from

$T(12.0 \text{ m}) = (600 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) + (200 \text{ kg})(9.80 \text{ m/s}^2)(6.0 \text{ m})$, so $T = 6860$ N.

This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance x Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by x and the tension T by $T_{max} = 5.80\times10^3$ N and solve for x;

$$x = \frac{(5.80\times10^3 \text{ N})(12.0 \text{ m}) - (200 \text{ kg})(9.80 \text{ m/s}^2)(6.0 \text{ m})}{(600 \text{ kg})(9.80 \text{ m/s}^2)} = 9.84 \text{ m}.$$

EVALUATE: Before Lancelot goes onto the bridge, the tension in the supporting cable is $T = \dfrac{(6.0 \text{ m})(200 \text{ kg})(9.80 \text{ m/s}^2)}{12.0 \text{ m}} = 980 \text{ N}$, well below the breaking strength of the cable. As he moves along the bridge, the increase in tension is proportional to x, the distance he has moved along the bridge.

11.49. **IDENTIFY:** Apply the conditions of equilibrium to the climber. For the minimum coefficient of friction the static friction force has the value $f_s = \mu_s n$.

SET UP: The free-body diagram for the climber is given in Figure 11.49. f_s and n are the vertical and horizontal components of the force exerted by the cliff face on the climber. The moment arm for the force T is $(1.4 \text{ m})\cos10°$.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T(1.4 \text{ m})\cos10° - w(1.1 \text{ m})\cos35.0° = 0$.

$$T = \frac{(1.1 \text{ m})\cos35.0°}{(1.4 \text{ m})\cos10°}(82.0 \text{ kg})(9.80 \text{ m/s}^2) = 525 \text{ N}$$

(b) $\sum F_x = 0$ gives $n = T\sin 25.0° = 222\ \text{N}$. $\sum F_y = 0$ gives $f_s + T\cos 25° - w = 0$ and $f_s = (82.0\ \text{kg})(9.80\ \text{m/s}^2) - (525\ \text{N})\cos 25° = 328\ \text{N}$.

(c) $\mu_s = \dfrac{f_s}{n} = \dfrac{328\ \text{N}}{222\ \text{N}} = 1.48$

EVALUATE: To achieve this large value of μ_s the climber must wear special rough-soled shoes.

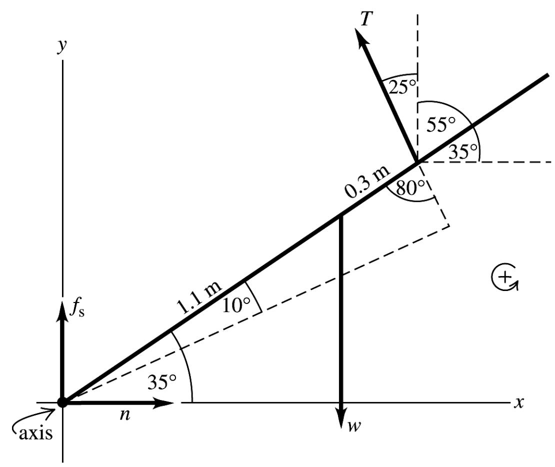


Figure 11.49

11.50. **IDENTIFY:** The beam is at rest, so the forces and torques on it must balance.

SET UP: The weight of the beam acts 4.0 m from each end. Take the pivot at the hinge and let counterclockwise torques be positive. Represent the force exerted by the hinge by its horizontal and vertical components, H_h and H_v. $\sum F_x = 0$, $\sum F_y = 0$ and $\sum \tau_z = 0$.

EXECUTE: **(a)** The free-body diagram for the beam is given in Figure 11.50a.

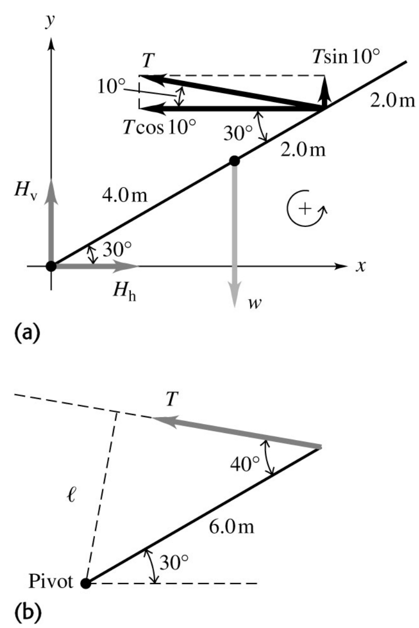


Figure 11.50

(b) The moment arm for T is sketched in Figure 11.50b and is equal to $(6.0\text{ m})\sin 40.0°$. $\sum\tau_z = 0$ gives $T(6.0\text{ m})(\sin 40.0°) - w(4.0\text{ m})(\cos 30.0°) = 0$.

$$T = \frac{(1150\text{ kg})(9.80\text{ m/s}^2)(4.0\text{ m})(\cos 30.0°)}{(6.0\text{ m})(\sin 40.0°)} = 1.01\times 10^4\text{ N}.$$

(c) $\sum F_x = 0$ gives $H_\text{h} - T\cos 10.0° = 0$ and $H_\text{h} = T\cos 10.0° = 9.97\times 10^3\text{ N}$.

EVALUATE: The tension is less than the weight of the beam because it has a larger moment arm than the weight force has.

11.51. IDENTIFY: In each case, to achieve balance the center of gravity of the system must be at the fulcrum. Use $x_\text{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to locate x_cm, with m_i replaced by w_i.

SET UP: Let the origin be at the left-hand end of the rod and take the $+x$-axis to lie along the rod. Let $w_1 = 255\text{ N}$ (the rod) so $x_1 = 1.00\text{ m}$, let $w_2 = 225\text{ N}$ so $x_2 = 2.00\text{ m}$ and let $w_3 = W$. In part (a) $x_3 = 0.500\text{ m}$ and in part (b) $x_3 = 0.750\text{ m}$.

EXECUTE: **(a)** $x_\text{cm} = 1.25\text{ m}$. $x_\text{cm} = \dfrac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$ gives $w_3 = \dfrac{(w_1 + w_2)x_\text{cm} - w_1x_1 - w_2x_2}{x_3 - x_\text{cm}}$ and

$$W = \frac{(480\text{ N})(1.25\text{ m}) - (255\text{ N})(1.00\text{ m}) - (225\text{ N})(2.00\text{ m})}{0.500\text{ m} - 1.25\text{ m}} = 140\text{ N}.$$

(b) Now $w_3 = W = 140\text{ N}$ and $x_3 = 0.750\text{ m}$.

$x_\text{cm} = \dfrac{(255\text{ N})(1.00\text{ m}) + (225\text{ N})(2.00\text{ m}) + (140\text{ N})(0.750\text{ m})}{255\text{ N} + 225\text{ N} + 140\text{ N}} = 1.31\text{ m}$. W must be moved $1.31\text{ m} - 1.25\text{ m} = 6\text{ cm}$ to the right.

EVALUATE: Moving W to the right means x_cm for the system moves to the right.

11.52. IDENTIFY: Apply $\sum\tau_z = 0$ to the hammer.

SET UP: Take the axis of rotation to be at point A.

EXECUTE: The force $\vec{F}_1$ is directed along the length of the nail, and so has a moment arm of $(0.080\text{ m})\sin 60°$. The moment arm of $\vec{F}_2$ is 0.300 m, so

$$F_2 = F_1\frac{(0.0800\text{ m})\sin 60°}{(0.300\text{ m})} = (400\text{ N})(0.231) = 92.4\text{ N}.$$

EVALUATE: The force F_2 that must be applied to the hammer handle is much less than the force that the hammer applies to the nail, because of the large difference in the lengths of the moment arms.

11.53. IDENTIFY: Apply the conditions of equilibrium to the horizontal beam. Since the two wires are symmetrically placed on either side of the middle of the sign, their tensions are equal and are each equal to $T_\text{w} = mg/2 = 137\text{ N}$.

SET UP: The free-body diagram for the beam is given in Figure 11.53. F_v and F_h are the vertical and horizontal forces exerted by the hinge on the beam. Since the cable is 2.00 m long and the beam is 1.50 m long, $\cos\theta = \dfrac{1.50\text{ m}}{2.00\text{ m}}$ and $\theta = 41.4°$. The tension T_c in the cable has been replaced by its horizontal and vertical components.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T_\text{c}(\sin 41.4°)(1.50\text{ m}) - w_\text{beam}(0.750\text{ m}) - T_\text{w}(1.50\text{ m}) - T_\text{w}(0.60\text{ m}) = 0$.

$T_\text{c} = \dfrac{(16.0\text{ kg})(9.80\text{ m/s}^2)(0.750\text{ m}) + (137\text{ N})(1.50\text{ m} + 0.60\text{ m})}{(1.50\text{ m})(\sin 41.4°)} = 408.6\text{ N}$, which rounds to 409 N.

(b) $\sum F_y = 0$ gives $F_v + T_c \sin 41.4° - w_{beam} - 2T_w = 0$ and

$F_v = 2T_w + w_{beam} - T_c \sin 41.4° = 2(137\text{ N}) + (16.0\text{ kg})(9.80\text{ m/s}^2) - (408.6\text{ N})(\sin 41.4°) = 161\text{ N}$. The hinge must be able to supply a vertical force of 161 N.

EVALUATE: The force from the two wires could be replaced by the weight of the sign acting at a point 0.60 m to the left of the right-hand edge of the sign.

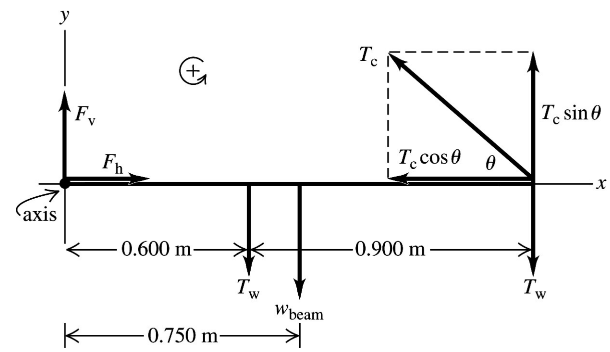


Figure 11.53

11.54. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bar.

SET UP: The free-body diagram for the bar is given in Figure 11.54. n is the normal force exerted on the bar by the surface. There is no friction force at this surface. H_h and H_v are the components of the force exerted on the bar by the hinge. The components of the force of the bar on the hinge will be equal in magnitude and opposite in direction.

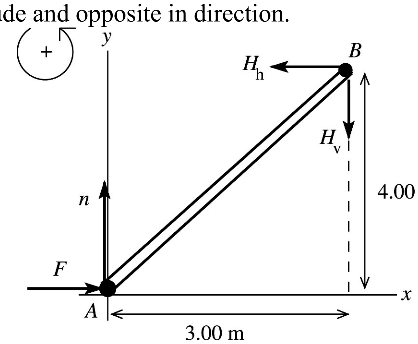


Figure 11.54

EXECUTE:

$\sum F_x = ma_x$

$F = H_h = 220\text{ N}$

$\sum F_y = ma_y$

$n - H_v = 0$

$H_v = n$, but we don't know either of these forces.

$\sum \tau_B = 0$ gives $F(4.00\text{ m}) - n(3.00\text{ m}) = 0$.

$n = (4.00\text{ m}/3.00\text{ m})F = \frac{4}{3}(220\text{ N}) = 293\text{ N}$ and then $H_v = 293\text{ N}$.

Force of bar on hinge:
horizontal component 220 N, to right
vertical component 293 N, upward

EVALUATE: $H_h/H_v = 220/293 = 0.75 = 3.00/4.00$, so the force the hinge exerts on the bar is directed along the bar. $\vec{n}$ and $\vec{F}$ have zero torque about point A, so the line of action of the hinge force $\vec{H}$ must pass through this point also if the net torque is to be zero.

11.55. **IDENTIFY:** We want to locate the center of mass of the leg-cast system. We can treat each segment of the leg and cast as a point-mass located at its center of mass.

SET UP: The force diagram for the leg is given in Figure 11.55. The weight of each piece acts at the center of mass of that piece. The mass of the upper leg is $m_{ul} = (0.215)(37\text{ kg}) = 7.955\text{ kg}$. The mass of the lower leg is $m_{\mathrm{ll}} = (0.140)(37\ \mathrm{kg}) = 5.18\ \mathrm{kg}$. Use the coordinates shown, with the origin at the hip and the x-axis along the leg, and use $x_{\mathrm{cm}} = \dfrac{x_{\mathrm{ul}}m_{\mathrm{ul}} + x_{\mathrm{ll}}m_{\mathrm{ll}} + x_{\mathrm{cast}}m_{\mathrm{cast}}}{m_{\mathrm{ul}} + m_{\mathrm{ll}} + m_{\mathrm{cast}}}$.

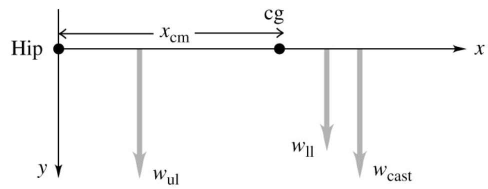


Figure 11.55

EXECUTE: Using $x_{\mathrm{cm}} = \dfrac{x_{\mathrm{ul}}m_{\mathrm{ul}} + x_{\mathrm{ll}}m_{\mathrm{ll}} + x_{\mathrm{cast}}m_{\mathrm{cast}}}{m_{\mathrm{ul}} + m_{\mathrm{ll}} + m_{\mathrm{cast}}}$, we have

$$x_{\mathrm{cm}} = \frac{(18.0\ \mathrm{cm})(7.955\ \mathrm{kg}) + (69.0\ \mathrm{cm})(5.18\ \mathrm{kg}) + (78.0\ \mathrm{cm})(5.50\ \mathrm{kg})}{7.955\ \mathrm{kg} + 5.18\ \mathrm{kg} + 5.50\ \mathrm{kg}} = 49.9\ \mathrm{cm}$$

EVALUATE: The strap is attached to the left of the center of mass of the cast, but it is still supported by the rigid cast since the cast extends beyond its center of mass.

11.56. **IDENTIFY:** Apply the first and second conditions for equilibrium to the bridge.

SET UP: Find torques about the hinge. Use L as the length of the bridge and w_{T} and w_{B} for the weights of the truck and the raised section of the bridge. Take $+y$ to be upward and $+x$ to be to the right.

EXECUTE: **(a)** $TL\sin 70° = w_{\mathrm{T}}(\tfrac{3}{4}L)\cos 30° + w_{\mathrm{B}}(\tfrac{1}{2}L)\cos 30°$, so

$$T = \frac{(\tfrac{3}{4}m_{\mathrm{T}} + \tfrac{1}{2}m_{\mathrm{B}})(9.80\ \mathrm{m/s^2})\cos 30°}{\sin 70°} = 2.84 \times 10^5\ \mathrm{N}.$$

(b) Horizontal: $T\cos(70° - 30°) = 2.18 \times 10^5\ \mathrm{N}$ (to the right).

Vertical: $w_{\mathrm{T}} + w_{\mathrm{B}} - T\sin 40° = 2.88 \times 10^5\ \mathrm{N}$ (upward).

EVALUATE: If ϕ is the angle of the hinge force above the horizontal,

$\tan\phi = \dfrac{2.88 \times 10^5\ \mathrm{N}}{2.18 \times 10^5\ \mathrm{N}}$ and $\phi = 52.9°$. The hinge force is not directed along the bridge.

11.57. **IDENTIFY:** The rod is suspended at rest, so the forces and torques on it must balance. Once the wire breaks, it rotates downward about the hinge, so we can use energy conservation.

SET UP: While the rod is at rest, we use $\sum \tau_z = 0$. After the wire breaks, we apply energy conservation $U_1 + K_1 + W_{\mathrm{other}} = U_2 + K_2$ with $I = \dfrac{1}{3}ML^2$, $K_1 = 0$, $U_2 = 0$, and $W_{\mathrm{other}} = 0$ because the hinge is frictionless. Our target variables are the angle the wire makes with the horizontal and its angular speed after the wire breaks. Begin with a free-body diagram of the rod, as in Fig. 11.57.

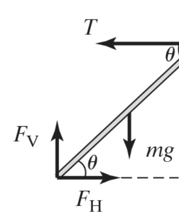


Figure 11.57

EXECUTE: **(a)** $\sum\tau_z = 0$: $TL\sin\theta - mg\dfrac{L}{2}\cos\theta = 0$, which gives $\theta = \arctan\left(\dfrac{mg}{2T}\right)$.

(b) Calling $y = 0$ at the level when the rod is horizontal, $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$mg\dfrac{L}{2}\sin\theta = \dfrac{1}{2}\left(\dfrac{1}{3}mL^2\right)\omega^2$, from which we get $\omega = \sqrt{\dfrac{3g\sin\theta}{L}}$.

EVALUATE: Check is some special cases: If θ is large, ω is large, which is reasonable because a large θ means that the center of gravity of the rod would be higher than for a small θ. If $\theta = 0$, $\omega = 0$, which is reasonable since the rod started from rest horizontally.

11.58. **IDENTIFY:** The arm is stationary, so the forces and torques must each balance.

SET UP: $\sum\tau = 0$, $\sum F_x = 0$, $\sum F_y = 0$. Let the forearm be at an angle ϕ below the horizontal. Take the pivot at the elbow joint and let counterclockwise torques be positive. Let $+y$ be upward and let $+x$ be to the right. Each forearm has mass $m_{\text{arm}} = \frac{1}{2}(0.0600)(72\text{ kg}) = 2.16\text{ kg}$. The weight held in each hand is $w = mg$, with $m = 7.50$ kg. $\vec{T}$ is the force the biceps muscle exerts on the forearm. $\vec{E}$ is the force exerted by the elbow and has components E_{v} and E_{h}.

EXECUTE: **(a)** The free-body diagram is shown in Figure 11.58.

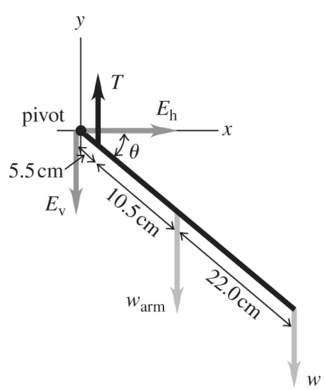


Figure 11.58

(b) $\sum\tau = 0$ gives $T(5.5\text{ cm})(\cos\theta) - w_{\text{arm}}(16.0\text{ cm})(\cos\theta) - w(38.0\text{ cm})(\cos\theta) = 0$

$$T = \frac{16.0w_{\text{arm}} + 38.0w}{5.5} = \frac{16.0(2.16\text{ kg})(9.80\text{ m/s}^2) + 38.0(7.50\text{ kg})(9.80\text{ m/s}^2)}{5.5} = 569\text{ N}$$

(c) $\sum F_x = 0$ gives $E_{\text{h}} = 0$. $\sum F_y = 0$ gives $T - E_{\text{v}} - w_{\text{arm}} - w = 0$, so

$$E_{\text{v}} = T - w_{\text{arm}} - w = 569\text{ N} - (2.16\text{ kg})(9.80\text{ m/s}^2) - (7.50\text{ kg})(9.80\text{ m/s}^2) = 474\text{ N}$$

Since we calculate E_{v} to be positive, we correctly assumed that it was downward when we drew the free-body diagram.

(d) The weight and the pull of the biceps are both always vertical in this situation, so the factor $\cos\theta$ divides out of the $\sum\tau = 0$ equation in part (b). Therefore the force T stays the same as she raises her arm.

EVALUATE: The biceps force must be much greater than the weight of the forearm and the weight in her hand because it has such a small lever arm compared to those two forces.

11.59. **IDENTIFY:** The rod is held in position, so the forces and torques on it must balance.

SET UP: Start with a free-body diagram of the rod, as in Fig. 11.59. Apply $\sum \tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The target variable is the angle β in the figure.

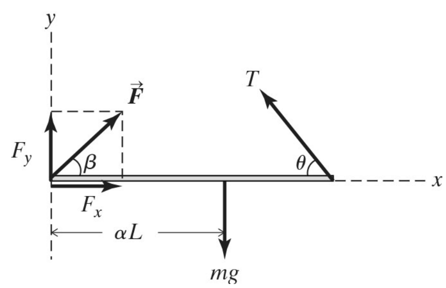


Figure 11.59

EXECUTE: **(a)** If we can find the components F_x and F_y of the hinge force $\vec{F}$, we can use them to find β. Taking torques about the right end of the rod gives $F_y L = mg(L - \alpha L) = mgL(1-\alpha)$, which gives

$F_y = mg(1-\alpha)$ (Eq. 1)

$\sum F_x = 0$: $F_x = T\cos\theta$ (Eq. 2)

$\sum \tau_z = 0$ about the hinge: $TL\sin\theta = mg\alpha L$ (Eq. 3)

Dividing Eq. 3 by Eq. 2 gives $\dfrac{TL\sin\theta}{T\cos\theta} = \dfrac{mg\alpha L}{F_x}$, which gives $F_x = \dfrac{mg\alpha}{\tan\theta}$. Now use this result and Eq. 1 to find β. $\tan\beta = \dfrac{F_y}{F_x} = \dfrac{mg(1-\alpha)}{\dfrac{mg\alpha}{\tan\theta}} = \left(\dfrac{1}{\alpha} - 1\right)\tan\theta$, so $\beta = \arctan\left(\dfrac{1}{\alpha} - 1\right)$.

(b) If $\beta = \theta$ we get $\tan\beta = \left(\dfrac{1}{\alpha} - 1\right)\tan\theta = \tan\theta$, so $\alpha = ½$.

(c) If $\alpha = 1$, we get $\tan\beta = \left(\dfrac{1}{\alpha} - 1\right)\tan\theta = 0$, so $\beta = 0$.

EVALUATE: In part (c), if $\beta = 0$ then $F_y = 0$, so all the weight of m is supported by T_y. Taking torques about the hinge gives $TL\sin\theta - mgL = 0$, so $T\sin\theta = mg$, which agrees with our answer with $\beta = 0$.

11.60. **IDENTIFY:** The rod is held fixed so it is in equilibrium. Therefore the forces and torques on it must balance.

SET UP: Apply $\sum \tau_z = 0$, $\sum F_x = 0$, and $\sum F_y = 0$. The target variable is the friction force *f* at the wall and the maximum angle θ for which slipping will not occur. Fig. 11.60 shows a free-body diagram of the rod.

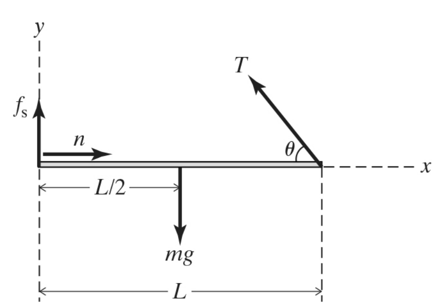


Figure 11.60

EXECUTE: **(a)** Using $\sum \tau_z = 0$ about the right end gives $fL = mg\dfrac{L}{2}$, so $f = \dfrac{mg}{2}$.

(b) $\sum F_x = 0$: $n = T\cos\theta$

$\sum F_y = 0$: $T\sin\theta + f = mg$. Using $f = \dfrac{mg}{2}$ and simplifying gives $T\sin\theta = mg - f = mg - \dfrac{mg}{2} = \dfrac{mg}{2}$,

so $T = \dfrac{mg}{2\sin\theta}$. Using this result, we find n to be $n = T\cos\theta = \left(\dfrac{mg}{2\sin\theta}\right)\cos\theta = \dfrac{mg}{2\tan\theta}$. At the maximum angle θ the rod is just ready to slip, so static friction is at its maximum value of $f_{\max} = \mu_s n$.

Combining this with our results that $f = \dfrac{mg}{2}$ and $n = \dfrac{mg}{2\tan\theta}$, we have $f = \dfrac{mg}{2} = \mu_s n = \dfrac{\mu_s mg}{2\tan\theta}$, which gives $\tan\theta = \mu_s$, so $\theta = \arctan\mu_s$.

EVALUATE: Check in some special cases. As $\theta \to 90°$, $\tan\theta \to \infty$ so $\mu_s \to \infty$. This is reasonable because the normal force $n = \dfrac{mg}{2\tan\theta} \to 0$, so we would need an extremely large μ_s (that is, an extremely rough wall) to hold up the rod. As $\theta \to 0$, $T = \dfrac{mg}{2\sin\theta} \to \infty$. This means that the normal force would get extremely large, so we would need a very small coefficient of friction to hold up the rod. Our results are reasonable.

11.61. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.61.

EXECUTE: $\sum \tau_z = 0$, axis at hinge, gives $T(6.0\text{ m})(\sin 40°) - (6490\text{ N})(3.75\text{ m})(\cos 30°) = 0$ and $T = 5500\text{ N}$.

EVALUATE: The tension in the cable is less than the weight of the beam. $T\sin 40°$ is the component of T that is perpendicular to the beam.

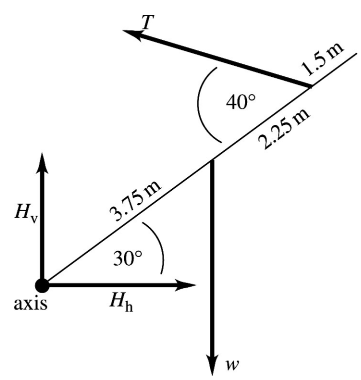


Figure 11.61

11.62. **IDENTIFY:** Apply the first and second conditions of equilibrium to the drawbridge.

SET UP: The free-body diagram for the drawbridge is given in Figure 11.62. H_v and H_h are the components of the force the hinge exerts on the bridge. In part (c), apply $\sum \tau_z = I\alpha$ to the rotating bridge and in part (d) apply energy conservation to the bridge.

EXECUTE: **(a)** $\sum \tau_z = 0$ with the axis at the hinge gives $-w(7.0\text{ m})(\cos 37°) + T(3.5\text{ m})(\sin 37°) = 0$ and $T = 2w\dfrac{\cos 37°}{\sin 37°} = 2\dfrac{(45{,}000\text{ N})}{\tan 37°} = 1.19\times 10^5\text{ N}$.

(b) $\sum F_x = 0$ gives $H_h = T = 1.19\times 10^5\text{ N}$. $\sum F_y = 0$ gives $H_v = w = 4.50\times 10^4\text{ N}$.

$H = \sqrt{H_h^2 + H_v^2} = 1.27\times10^5$ N. $\tan\theta = \dfrac{H_v}{H_h}$ and $\theta = 20.7°$. The hinge force has magnitude 1.27×10^5 N and is directed at 20.7° above the horizontal.

(c) We can treat the bridge as a uniform bar rotating around one end, so $I = 1/3\, mL^2$. $\sum\tau_z = I\alpha_z$ gives $mg(L/2)\cos37° = 1/3\ mL^2\alpha$. Solving for α gives $\alpha = \dfrac{3g\cos37°}{2L} = \dfrac{3(9.80\text{ m/s}^2)\cos37°}{2(14.0\text{ m})} = 0.839\text{ rad/s}^2$.

(d) Energy conservation gives $U_1 = K_2$, giving $mgh = 1/2\ I\omega^2 = (1/2)(1/3\ mL^2)\omega^2$. Trigonometry gives $h = L/2\ \sin37°$. Canceling *m*, the energy conservation equation gives $g(L/2)\ \sin37° = (1/6)L^2\omega^2$.

Solving for ω gives $\omega = \sqrt{\dfrac{3g\sin37°}{L}} = \sqrt{\dfrac{3(9.80\text{ m/s}^2)\sin37°}{14.0\text{ m}}} = 1.12$ rad/s.

EVALUATE: The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

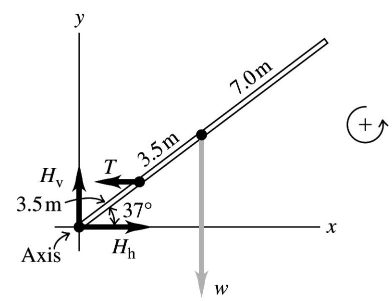


Figure 11.62

11.63. IDENTIFY: The amount the tendon stretches depends on Young's modulus for the tendon material. The foot is in rotational equilibrium, so the torques on it balance.

SET UP: $Y = \dfrac{F_T/A}{\Delta l / l_0}$. The foot is in rotational equilibrium, so $\sum\tau_z = 0$.

EXECUTE: **(a)** The free-body diagram for the foot is given in Figure 11.63. *T* is the tension in the tendon and *A* is the force exerted on the foot by the ankle. $n = (75\text{ kg})g$, the weight of the person.

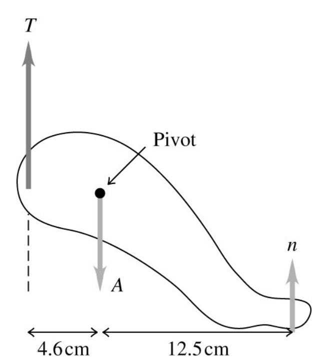


Figure 11.63

(b) Apply $\sum\tau_z = 0$, letting counterclockwise torques be positive and with the pivot at the ankle: $T(4.6\text{ cm}) - n(12.5\text{ cm}) = 0$. $T = \left(\dfrac{12.5\text{ cm}}{4.6\text{ cm}}\right)(75\text{ kg})(9.80\text{ m/s}^2) = 2000$ N, which is 2.72 times his weight.

(c) The foot pulls downward on the tendon with a force of 2000 N.

$$\Delta l = \left(\frac{F_T}{YA}\right) l_0 = \frac{2000\ \text{N}}{(1470\times 10^6\ \text{Pa})(78\times 10^{-6}\ \text{m}^2)}(25\ \text{cm}) = 4.4\ \text{mm}.$$

EVALUATE: The tension is quite large, but the Achilles tendon stretches about 4.4 mm, which is only about 1/6 of an inch, so it must be a strong tendon.

11.64 **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The center of mass of the beam is 1.0 m from the suspension point.

EXECUTE: **(a)** Taking torques about the suspension point,
$w(4.00\ \text{m})\sin 30° + (140.0\ \text{N})(1.00\ \text{m})\sin 30° = (100\ \text{N})(2.00\ \text{m})\sin 30°.$

The common factor of $\sin 30°$ divides out, from which $w = 15.0$ N.

(b) In this case, a common factor of $\sin 45°$ would be factored out, and the result would be the same.

EVALUATE: All the forces are vertical, so the moments are all horizontal and all contain the factor $\sin\theta$, where θ is the angle the beam makes with the horizontal.

11.65. **IDENTIFY:** The rod is held in place, so the torques on it must balance. The added weight of the object causes the wire to stretch slightly, so we need to use tensile stress and strain.

SET UP: We use $\sum \tau_z = 0$ and $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$. The target variable is the distance the aluminum wire stretches due to the added weight.

EXECUTE: **(a)** A little trigonometry gives cos 30.0° = (1.20 m)/L_{wire}, so L_{wire} = 1.39 m.

(b) Fig. 11.65 shows a free-body diagram of the rod with the object attached. The stretching of the wire is due to the *increase* in tension due to the addition of the 90.0-kg object. Therefore in Fig. 11.65 we do not use the weight *mg* of the rod when computing the torque about the hinge.

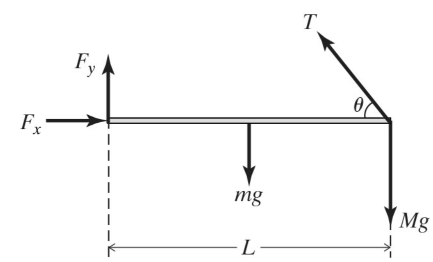


Figure 11.65

$\sum \tau_z = 0$: $TL\sin\theta = MgL \quad \rightarrow \quad T = \dfrac{mg}{\sin\theta} = \dfrac{(90.0\ \text{kg})(9.80\ \text{m/s}^2)}{\sin 30.0°} = 1764\ \text{N}.$ Now use Young's modulus for aluminum (from Table 11.1) to find the increase in the length of the wire. Solving $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$ for $\Delta\ell$ gives $\Delta\ell = \dfrac{F_\perp \ell_0}{AY} = \dfrac{F_\perp \ell_0}{\pi r^2 Y} = \dfrac{(1764\ \text{N})(1.39\ \text{m})}{\pi(0.00250\ \text{m})^2(7.0\times 10^{10}\ \text{Pa})} = 1.8\times 10^{-3}\ \text{m} = 1.8\ \text{mm}.$

EVALUATE: Since $\Delta\ell << \ell_0$ we are justified in treating the rod as being horizontal after object is added. Our result is reasonable since most materials stretch very little under ordinary circumstances.

11.66. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to each object, including the point where *D*, *C*, and *B* are joined. Apply $\sum \tau_z = 0$ to the rod.

SET UP: To find T_C and T_D, use a coordinate system with axes parallel to the cords.

EXECUTE: *A* and *B* are straightforward, the tensions being the weights suspended:
$T_A = (0.0360\ \text{kg})(9.80\ \text{m/s}^2) = 0.353\ \text{N}$ and $T_B = (0.0240\ \text{kg} + 0.0360\ \text{kg})(9.80\ \text{m}/\text{s}^2) = 0.588\ \text{N}.$

Applying $\sum F_x = 0$ and $\sum F_y = 0$ to the point where the cords are joined, $T_C = T_B \cos 36.9° = 0.470$ N and $T_D = T_B \cos 53.1° = 0.353$ N. To find T_E, take torques about the point where string *F* is attached. $T_E(1.00\text{ m}) = T_D \sin 36.9°(0.800\text{ m}) + T_C \sin 53.1°(0.200\text{ m}) + (0.120\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m})$ and $T_E = 0.833$ N.

T_F may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament. $(0.180\text{kg})(9.80\text{m/s}^2) = 1.76$ N, from which $T_F = 0.931$ N.

EVALUATE: The vertical line through the spheres is closer to *F* than to *E*, so we expect $T_F > T_E$, and this is indeed the case.

11.67. **IDENTIFY:** The torques must balance since the person is not rotating.

SET UP: Figure 11.67a shows the distances and angles. $\theta + \phi = 90°$. $\theta = 56.3°$ and $\phi = 33.7°$. The distances x_1 and x_2 are $x_1 = (90\text{ cm})\cos\theta = 50.0$ cm and $x_2 = (135\text{ cm})\cos\phi = 112$ cm. The free-body diagram for the person is given in Figure 11.67b. $w_1 = 277$ N is the weight of his feet and legs, and $w_t = 473$ N is the weight of his trunk. n_f and f_f are the total normal and friction forces exerted on his feet and n_h and f_h are those forces on his hands. The free-body diagram for his legs is given in Figure 11.67c. *F* is the force exerted on his legs by his hip joints. For balance, $\sum \tau_z = 0$.

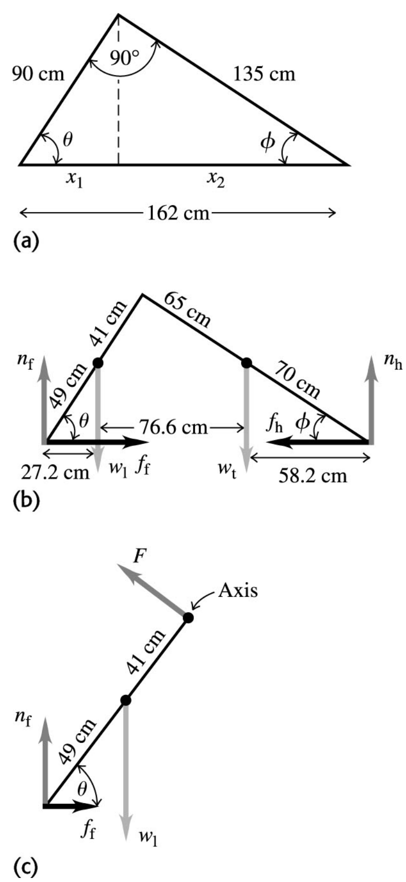


Figure 11.67

EXECUTE: **(a)** Consider the force diagram of Figure 11.67b. $\sum \tau_z = 0$ with the pivot at his feet and counterclockwise torques positive gives $n_h(162\text{ cm}) - (277\text{ N})(27.2\text{ cm}) - (473\text{ N})(103.8\text{ cm}) = 0$.

$n_\text{h} = 350$ N, so there is a normal force of 175 N at each hand. $n_\text{f} + n_\text{h} - w_\text{l} - w_\text{t} = 0$ so $n_\text{f} = w_\text{l} + w_\text{t} - n_\text{h} = 750\text{ N} - 350\text{ N} = 400$ N, so there is a normal force of 200 N at each foot.

(b) Consider the force diagram of Figure 11.67c. $\sum \tau_z = 0$ with the pivot at his hips and counterclockwise torques positive gives $f_\text{f}(74.9\text{ cm}) + w_\text{l}(22.8\text{ cm}) - n_\text{f}(50.0\text{ cm}) = 0.$

$f_\text{f} = \dfrac{(400\text{ N})(50.0\text{ cm}) - (277\text{ N})(22.8\text{ cm})}{74.9\text{ cm}} = 182.7$ N. There is a friction force of 91 N at each foot.

$\sum F_x = 0$ in Figure 11.67b gives $f_\text{h} = f_\text{f}$, so there is a friction force of 91 N at each hand.

EVALUATE: In this position the normal forces at his feet and at his hands don't differ very much.

11.68. **IDENTIFY:** The ball is going in a circle, so it obeys Newton's second law. Since the brass wire stretches slightly, we must use Young's modulus and stress and strain.

SET UP: We know the fractional change in the wire's length. The target variable is the speed of the ball at the bottom of its circular path. Apply $\sum F = m\dfrac{v^2}{R}$ and then use $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$.

EXECUTE: At the lowest point, the ball's acceleration is upward. The fractional change in length of the wire is only 2.0×10^{-5}, so we can use ℓ_0 for the radius of the circle. $\sum F = m\dfrac{v^2}{R}$ gives $T - mg = \dfrac{mv^2}{\ell_0}$, so $T = mg + \dfrac{mv^2}{\ell_0}$. Now use $Y = \dfrac{F_\perp}{A}\dfrac{\ell_0}{\Delta\ell}$, where $F_\perp$ is the tension in the wire. Doing so gives

$Y = \dfrac{mg + \dfrac{mv^2}{\ell_0}}{A\left(\dfrac{\Delta\ell}{\ell_0}\right)}$. Solving for v gives $v = \sqrt{\dfrac{\ell_0}{m}\left[YA\left(\dfrac{\Delta\ell}{\ell_0}\right) - mg\right]}$. Using $\dfrac{\Delta\ell}{\ell_0} = 2.0\times10^{-5}$, $\ell_0 = 1.40$ m, $A = 6.00\text{ mm}^2 = 6.00\times10^{-6}\text{ m}^2$, $m = 0.0800$ kg, and $Y = 9.0\times10^{10}$ Pa for brass (from Table 11.1 in the text), we have $v = 13.2$ m/s.

EVALUATE: A speed of 13.2 m/s is about 30 mph, yet this speed would only produce a fractional length change of 0.000020. The fractional length change would be even less at the top since the ball is moving slower up there than at the bottom.

11.69. **IDENTIFY:** Apply the equilibrium conditions to the crate. When the crate is on the verge of tipping it touches the floor only at its lower left-hand corner and the normal force acts at this point. The minimum coefficient of static friction is given by the equation $f_\text{s} = \mu_\text{s} n$.

SET UP: The free-body diagram for the crate when it is ready to tip is given in Figure 11.69.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $P(1.50\text{ m})\sin 53.0° - w(1.10\text{ m}) = 0.$

$$P = w\left(\frac{1.10\text{ m}}{[1.50\text{ m}][\sin 53.0°]}\right) = 1.15\times10^3\text{ N}$$

(b) $\sum F_y = 0$ gives $n - w - P\cos 53.0° = 0.$

$$n = w + P\cos 53.0° = 1250\text{ N} + (1.15\times10^3\text{ N})\cos 53° = 1.94\times10^3\text{ N}$$

(c) $\sum F_y = 0$ gives $f_\text{s} = P\sin 53.0° = (1.15\times10^3\text{ N})\sin 53.0° = 918$ N.

(d) $\mu_\text{s} = \dfrac{f_\text{s}}{n} = \dfrac{918\text{ N}}{1.94\times10^3\text{ N}} = 0.473$

EVALUATE: The normal force is greater than the weight because P has a downward component.

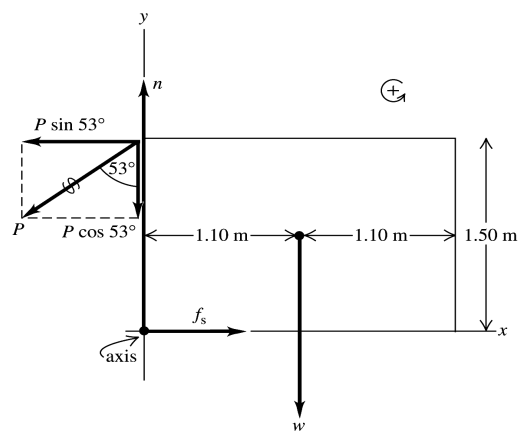


Figure 11.69

11.70. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the meterstick.

SET UP: The wall exerts an upward static friction force f and a horizontal normal force n on the stick. Denote the length of the stick by l. $f = \mu_s n$.

EXECUTE: **(a)** Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = w/2$. Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point), and noting that the moment arm of the normal force is $l \tan\theta$, $n \tan\theta = w/2$. Then, $(f/n) = \tan\theta < 0.40$, so $\theta < \arctan(0.40) = 22°$.

(b) Taking torques as in part (a), $fl = w\dfrac{l}{2} + w(l - x)$ and $nl \tan\theta = w\dfrac{l}{2} + wx$. In terms of the coefficient of friction μ_s, $\mu_s > \dfrac{f}{n} = \dfrac{l/2 + (l - x)}{l/2 + x}\tan\theta = \dfrac{3l - 2x}{l + 2x}\tan\theta$. Solving for x, $x > \dfrac{l}{2}\dfrac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2$ cm.

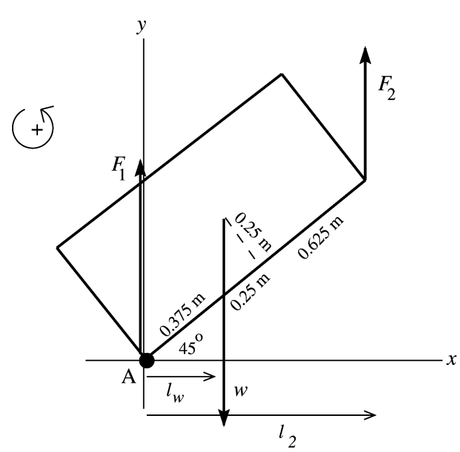
(c) In the above expression, setting $x = 10$ cm and $l = 100$ cm and solving for μ_s gives

$$\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$$

EVALUATE: For $\theta = 15°$ and without the block suspended from the stick, a value of $\mu_s \geq 0.268$ is required to prevent slipping. Hanging the block from the stick increases the value of μ_s that is required.

11.71. **IDENTIFY:** Apply the first and second conditions of equilibrium to the crate.

SET UP: The free-body diagram for the crate is given in Figure 11.71.



$l_w = (0.375 \text{ m})\cos 45°$

$l_2 = (1.25 \text{ m})\cos 45°$

Let $\vec{F}_1$ and $\vec{F}_2$ be the vertical forces exerted by you and your friend. Take the origin at the lower left-hand corner of the crate (point A).

Figure 11.71

EXECUTE: $\sum F_y = ma_y$ gives $F_1 + F_2 - w = 0$

$F_1 + F_2 = w = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

$\sum \tau_A = 0$ gives $F_2 l_2 - w l_w = 0$

$$F_2 = w\left(\frac{l_w}{l_2}\right) = 1960\text{ N}\left(\frac{0.375\text{ m}\cos 45°}{1.25\text{ m}\cos 45°}\right) = 590\text{ N}$$

Then $F_1 = w - F_2 = 1960\text{ N} - 590\text{ N} = 1370\text{ N}.$

EVALUATE: The person below (you) applies a force of 1370 N. The person above (your friend) applies a force of 590 N. It is better to be the person above. As the sketch shows, the moment arm for $\vec{F}_1$ is less than for $\vec{F}_2$, so must have $F_1 > F_2$ to compensate.

11.72. **IDENTIFY:** The beam is at rest, so the forces and torques on it must all balance.

SET UP: The cables could point inward toward each other or outward away from each other. We shall assume they point away from each other. Call *d* the distance of the center of gravity from the left end, call *w* the weight of the beam, and call *T* the tension in the right-hand cable. $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\sum \tau_z = 0$.

EXECUTE: $\Sigma F_x = 0$ gives (620 N)(sin30.0°) – T(sin50.0°) = 0, so T = 404.68 N.

$\Sigma F_y = 0$ gives (620 N)(cos30.0°) + (404.68 N)(cos50.0°) – w = 0, so w = 797 N.

Taking torques about the left end, $\sum \tau_z = 0$ gives (404.68 N)(cos50.0°)(4.00 m) – (797 N)d = 0, so d = 1.31 m from the left end of the beam, or 2.69 m from the right end.

EVALUATE: The center of gravity is closer to the cable having the greater tension. The answer would be no different if we assumed that the cables pointed inward toward each other.

11.73. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the forearm.

SET UP: The free-body diagram for the forearm is given in Figure 11.10 in the textbook.

EXECUTE: **(a)** $\sum \tau_z = 0$, axis at elbow gives

$$wL - (T\sin\theta)D = 0.\ \sin\theta = \frac{h}{\sqrt{h^2 + D^2}} \text{ so } w = T\frac{hD}{L\sqrt{h^2 + D^2}}.$$

$$w_{\max} = T_{\max}\frac{hD}{L\sqrt{h^2 + D^2}}.$$

(b) $\dfrac{dw_{\max}}{dD} = \dfrac{T_{\max} h}{L\sqrt{h^2 + D^2}}\left(1 - \dfrac{D^2}{h^2 + D^2}\right)$; the derivative is positive.

EVALUATE: **(c)** The result of part (b) shows that $w_{\max}$ increases when *D* increases, since the derivative is positive. $w_{\max}$ is larger for a chimp since *D* is larger.

11.72. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the wheel.

SET UP: Take torques about the upper corner of the curb.

EXECUTE: The force $\vec{F}$ acts at a perpendicular distance $R - h$ and the weight acts at a perpendicular distance $\sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$. Setting the torques equal for the minimum necessary force,

$$F = mg\frac{\sqrt{2Rh - h^2}}{R - h}.$$

(b) The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R - h$, so the minimum force is $(mg)\sqrt{2Rh - h^2}/(2R - h)$.

EVALUATE: **(c)** Less force is required when the force is applied at the top of the wheel, since in this case $\vec{F}$ has a larger moment arm.

11.75. IDENTIFY: Apply the first and second conditions of equilibrium to the gate.

SET UP: The free-body diagram for the gate is given in Figure 11.75.

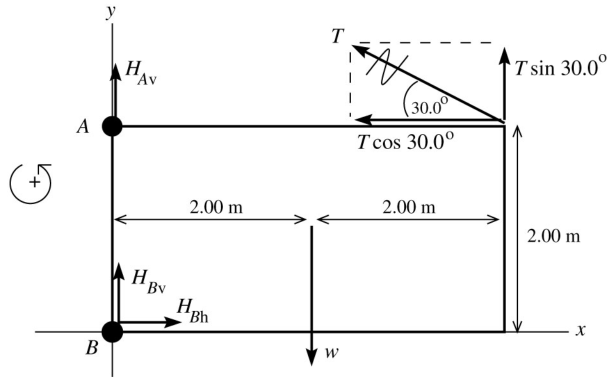


Figure 11.75

Use coordinates with the origin at B. Let $\vec{H}_A$ and $\vec{H}_B$ be the forces exerted by the hinges at A and B. The problem states that $\vec{H}_A$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

EXECUTE: **(a)** $\sum\tau_B = 0$ gives $+(T\sin 30.0°)(4.00\text{ m}) + (T\cos 30.0°)(2.00\text{ m}) - w(2.00\text{ m}) = 0$

$T(2\sin 30.0° + \cos 30.0°) = w$

$$T = \frac{w}{2\sin 30.0° + \cos 30.0°} = \frac{700\text{ N}}{2\sin 30.0° + \cos 30.0°} = 375\text{ N}.$$

(b) $\sum F_x = ma_x$ says $H_{Bh} - T\cos 30.0° = 0$

$H_{Bh} = T\cos 30.0° = (375\text{ N})\cos 30.0° = 325\text{ N}.$

(c) $\sum F_y = ma_y$ says $H_{Av} + H_{Bv} + T\sin 30.0° - w = 0$

$H_{Av} + H_{Bv} = w - T\sin 30.0° = 700\text{ N} - (375\text{ N})\sin 30.0° = 512\text{ N}.$

EVALUATE: T has a horizontal component to the left so H_{Bh} must be to the right, as these are the only two horizontal forces. Note that we cannot determine H_{Av} and H_{Bv} separately, only their sum.

11.76. IDENTIFY: Use $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ to locate the x-coordinate of the center of gravity of the block combinations.

SET UP: The center of mass and the center of gravity are the same point. For two identical blocks, the center of gravity is midway between the center of the two blocks.

EXECUTE: **(a)** The center of gravity of the top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ to the left of the right edge of the upper block, so the overhang is $3L/4$.

(b) Take the two-block combination from part (a), and place it on top of the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4) + (L/6) = 11L/12$. Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8$, for a total of $25L/24$.

(c) As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

EVALUATE: The sequence of maximum overhangs is $\dfrac{18L}{24}, \dfrac{22L}{24}, \dfrac{25L}{24}, \ldots$. The increase of overhang when one more block is added is decreasing.

11.77. **IDENTIFY:** Apply the first and second conditions of equilibrium, first to both marbles considered as a composite object and then to the bottom marble.

(a) SET UP: The forces on each marble are shown in Figure 11.77.

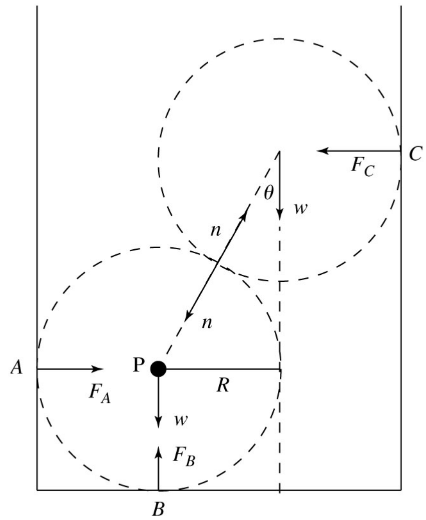


Figure 11.77

EXECUTE:

$F_B = 2w = 1.47$ N

$\sin\theta = R/2R$ so $\theta = 30°$

$\sum\tau_z = 0$, axis at P

$F_C(2R\cos\theta) - wR = 0$

$$F_C = \frac{mg}{2\cos 30°} = 0.424 \text{ N}$$

$F_A = F_C = 0.424$ N

(b) Consider the forces on the bottom marble. The horizontal forces must sum to zero, so $F_A = n\sin\theta$.

$$n = \frac{F_A}{\sin 30°} = 0.848 \text{ N}$$

Could use instead that the vertical forces sum to zero

$F_B - mg - n\cos\theta = 0$

$$n = \frac{F_B - mg}{\cos 30°} = 0.848 \text{ N, which checks.}$$

EVALUATE: If we consider each marble separately, the line of action of every force passes through the center of the marble so there is clearly no torque about that point for each marble. We can use the results we obtained to show that $\sum F_x = 0$ and $\sum F_y = 0$ for the top marble.

11.78. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the right-hand beam.

SET UP: Use the hinge as the axis of rotation and take counterclockwise rotation as positive. If F_{wire} is the tension in each wire and $w = 260$ N is the weight of each beam, $2F_{\text{wire}} - 2w = 0$ and $F_{\text{wire}} = w$. Let L be the length of each beam.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $F_{\text{wire}}L\sin\frac{\theta}{2} - F_c\frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0$, where θ is the angle between the beams and F_c is the force exerted by the cross bar. The length drops out, and all other quantities except F_c are known, so $F_c = \dfrac{F_{\text{wire}}\sin(\theta/2) - \frac{1}{2}w\sin(\theta/2)}{\frac{1}{2}\cos(\theta/2)} = (2F_{\text{wire}} - w)\tan\dfrac{\theta}{2}$. Therefore

$$F_c = (260 \text{ N})\tan\frac{53°}{2} = 130 \text{ N}.$$

(b) The crossbar is under compression, as can be seen by imagining the behavior of the two beams if the crossbar were removed. It is the crossbar that holds them apart.

(c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmetry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the crossbar. The hinge exerts a force 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

EVALUATE: The force exerted on each beam increases as θ increases and exceeds the weight of the beam for $\theta \geq 90°$.

11.79. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bale.

(a) SET UP: Find the angle where the bale starts to tip. When it starts to tip only the lower left-hand corner of the bale makes contact with the conveyor belt. Therefore the line of action of the normal force n passes through the left-hand edge of the bale. Consider $\Sigma\tau_z = 0$ with point A at the lower left-hand corner. Then $\tau_n = 0$ and $\tau_f = 0,$ so it must be that $\tau_{mg} = 0$ also. This means that the line of action of the gravity must pass through point A. Thus the free-body diagram must be as shown in Figure 11.79a.

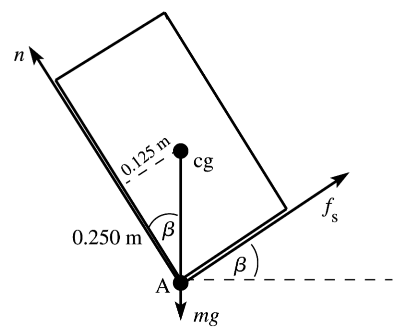


Figure 11.79a

EXECUTE: $\tan\beta = \dfrac{0.125 \text{ m}}{0.250 \text{ m}}$

$\beta = 27°,$ angle where tips

SET UP: At the angle where the bale is ready to slip down the incline f_s has its maximum possible value, $f_s = \mu_s n.$ The free-body diagram for the bale, with the origin of coordinates at the cg is given in Figure 11.79b.

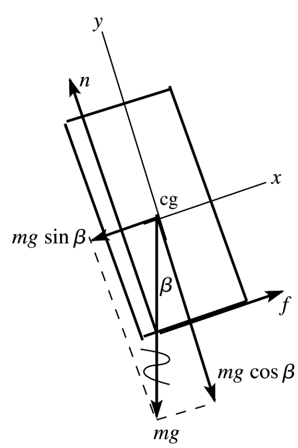


Figure 11.79b

EXECUTE:

$\sum F_y = ma_y$

$n - mg\cos\beta = 0$

$n = mg\cos\beta$

$f_s = \mu_s mg\cos\beta$

(f_s has maximum value when bale ready to slip)

$\sum F_x = ma_x$

$f_s - mg\sin\beta = 0$

$\mu_s mg\cos\beta - mg\sin\beta = 0$

$\tan\beta = \mu_s$

$\mu_s = 0.60$ gives that $\beta = 31°$

$\beta = 27°$ to tip; $\beta = 31°$ to slip, so tips first

(b) The magnitude of the friction force didn't enter into the calculation of the tipping angle; still tips at $\beta = 27°.$ For $\mu_s = 0.40$ slips at $\beta = \arctan(0.40) = 22°.$

Now the bale will start to slide down the incline before it tips.

EVALUATE: With a smaller μ_s the slope angle β where the bale slips is smaller.

11.80. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the slab.

SET UP: The free-body diagram is given in Figure 11.80a. $\tan\beta = \dfrac{3.75 \text{ m}}{1.75 \text{ m}}$ so $\beta = 65.0°$. $20.0° + \beta + \alpha = 90°$ so $\alpha = 5.0°$. The distance from the axis to the center of the block is $\sqrt{\left(\dfrac{3.75 \text{ m}}{2}\right)^2 + \left(\dfrac{1.75 \text{ m}}{2}\right)^2} = 2.07 \text{ m}.$

EXECUTE: **(a)** $w(2.07 \text{ m})\sin 5.0° - T(3.75 \text{ m})\sin 52.0° = 0$. $T = 0.061w$. Each worker must exert a force of $0.012w$, where w is the weight of the slab.

(b) As θ increases, the moment arm for w decreases and the moment arm for T increases, so the worker needs to exert less force.

(c) $T \to 0$ when w passes through the support point. This situation is sketched in Figure 11.80b. $\tan\theta = \dfrac{(1.75 \text{ m})/2}{(3.75 \text{ m})/2}$ and $\theta = 25.0°$. If θ exceeds this value the gravity torque causes the slab to tip over.

EVALUATE: The moment arm for T is much greater than the moment arm for w, so the force the workers apply is much less than the weight of the slab.

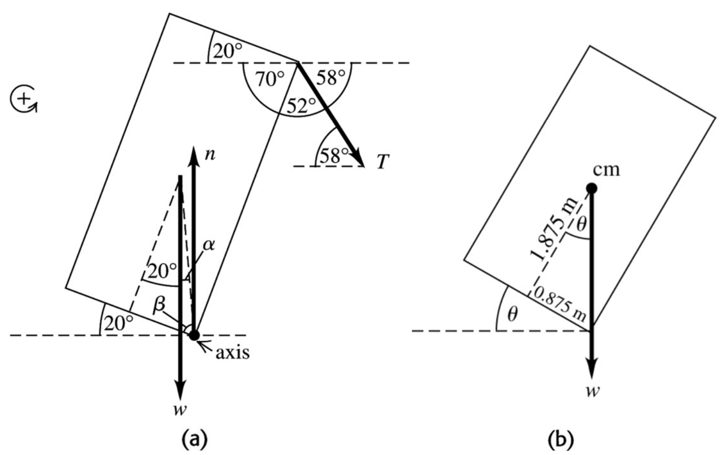


Figure 11.80

11.81. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

(a) SET UP: The free-body diagram for the door is given in Figure 11.81.

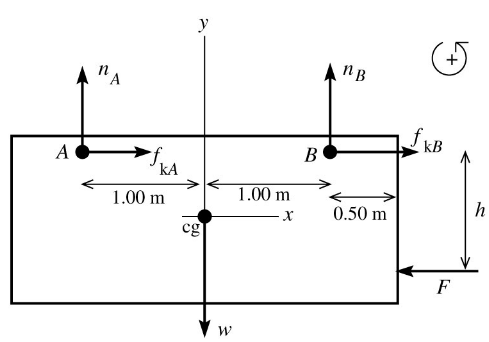


Figure 11.81

Take the origin of coordinates at the center of the door (at the cg). Let n_A, f_{kA}, n_B, and f_{kB} be the normal and friction forces exerted on the door at each wheel.

EXECUTE: $\sum F_y = ma_y$

$n_A + n_B - w = 0$

$n_A + n_B = w = 950\text{ N}$

$\sum F_x = ma_x$

$f_{kA} + f_{kB} - F = 0$

$F = f_{kA} + f_{kB}$

$f_{kA} = \mu_k n_A,\ \ f_{kB} = \mu_k n_B,$ so $F = \mu_k(n_A + n_B) = \mu_k w = (0.52)(950\text{ N}) = 494\text{ N}$

$\sum \tau_B = 0$

$n_B,\ \ f_{kA},$ and f_{kB} all have zero moment arms and hence zero torque about this point.

Thus $+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$n_A = \frac{w(1.00\text{ m}) - F(h)}{2.00\text{ m}} = \frac{(950\text{ N})(1.00\text{ m}) - (494\text{ N})(1.60\text{ m})}{2.00\text{ m}} = 80\text{ N}$$

And then $n_B = 950\text{ N} - n_A = 950\text{ N} - 80\text{ N} = 870\text{ N}.$

(b) SET UP: If h is too large the torque of F will cause wheel A to leave the track. When wheel A just starts to lift off the track n_A and f_{kA} both go to zero.

EXECUTE: The equations in part (a) still apply.

$n_A + n_B - w = 0$ gives $n_B = w = 950\text{ N}$

Then $f_{kB} = \mu_k n_B = 0.52(950\text{ N}) = 494\text{ N}$

$F = f_{kA} + f_{kB} = 494\text{ N}$

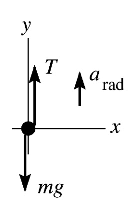
$+w(1.00\text{ m}) - n_A(2.00\text{ m}) - F(h) = 0$

$$h = \frac{w(1.00\text{ m})}{F} = \frac{(950\text{ N})(1.00\text{ m})}{494\text{ N}} = 1.92\text{ m}$$

EVALUATE: The result in part (b) is larger than the value of h in part (a). Increasing h increases the clockwise torque about B due to F and therefore decreases the clockwise torque that n_A must apply.

11.82. IDENTIFY: Apply Newton's second law to the mass to find the tension in the wire. Then apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ to the wire to find the elongation this tensile force produces.

(a) SET UP: Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.82a.



The mass moves in an arc of a circle with radius $R = 0.70\text{ m}.$ It has acceleration $\vec{a}_{\text{rad}}$ directed in toward the center of the circle, so at this point $\vec{a}_{\text{rad}}$ is upward.

Figure 11.82a

EXECUTE: $\sum F_y = ma_y$

$T - mg = mR\omega^2$ so that $T = m(g + R\omega^2).$

But ω must be in rad/s:

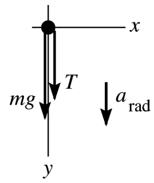
$\omega = (120\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 12.57\text{ rad/s}.$

Then $T = (12.0\text{ kg})\left[9.80\text{ m/s}^2 + (0.70\text{ m})(12.57\text{ rad/s})^2\right] = 1445\text{ N}.$

Now calculate the elongation Δl of the wire that this tensile force produces:

$$Y = \frac{F_\perp l_0}{A\Delta l} \text{ so } \Delta l = \frac{F_\perp l_0}{YA} = \frac{(1445\text{ N})(0.70\text{ m})}{(7.0\times10^{10}\text{ Pa})(0.014\times10^{-4}\text{ m}^2)} = 0.0103\text{ m} = 1.0\text{ cm}.$$

(b) SET UP: The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.82b.



EXECUTE:

$\sum F_y = ma_y$

$mg + T = mR\omega^2$

$T = m(R\omega^2 - g)$

Figure 11.82b

$$T = (12.0\text{ kg})\left[(0.70\text{ m})(12.57\text{ rad/s})^2 - 9.80\text{ m/s}^2\right] = 1210\text{ N}.$$

$$\Delta l = \frac{F_\perp l_0}{YA} = \frac{(1210\text{ N})(0.70\text{ m})}{(7.0\times10^{10}\text{ Pa})(0.014\times10^{-4}\text{ m}^2)} = 8.6\times10^{-3}\text{ m} = 0.86\text{ cm}.$$

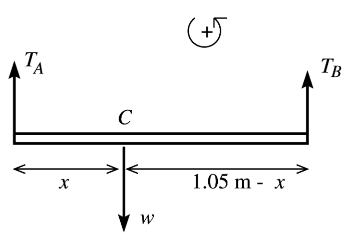
EVALUATE: At the lowest point T and w are in opposite directions and at the highest point they are in the same direction, so T is greater at the lowest point and the elongation is greatest there. The elongation is at most 1.4% of the length.

11.83. IDENTIFY: Use the second condition of equilibrium to relate the tension in the two wires to the distance w is from the left end. Use $\text{stress} = \frac{F_\perp}{A}$ and $Y = \frac{l_0 F_\perp}{A\Delta l}$ to relate the tension in each wire to its stress and strain.

(a) SET UP: $\text{stress} = F_\perp/A$, so equal stress implies T/A same for each wire.

$T_A/2.00\text{ mm}^2 = T_B/4.00\text{ mm}^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point C, a distance x to the right of wire A. The free-body diagram for the rod is given in Figure 11.83.



EXECUTE:

$\sum \tau_C = 0$

$+T_B(1.05\text{ m} - x) - T_A x = 0$

Figure 11.83

But $T_B = 2.00T_A$ so $2.00T_A(1.05\text{ m} - x) - T_A x = 0$

$2.10\text{ m} - 2.00x = x$ and $x = 2.10\text{ m}/3.00 = 0.70\text{ m}$ (measured from A).

(b) SET UP: $Y = \text{stress/strain}$ gives that $\text{strain} = \text{stress}/Y = F_\perp/AY$.

EXECUTE: Equal strain thus implies

$$\frac{T_A}{(2.00\text{ mm}^2)(1.80\times10^{11}\text{ Pa})} = \frac{T_B}{(4.00\text{ mm}^2)(1.20\times10^{11}\text{ Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A.$$

The $\sum \tau_C = 0$ equation still gives $T_B(1.05\text{ m} - x) - T_A x = 0.$

But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\text{ m} - x) - T_A x = 0.$

$1.40\text{ m} = 2.33x$ and $x = 1.40\text{ m}/2.33 = 0.60\text{ m}$ (measured from A).

EVALUATE: Wire B has twice the diameter so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller Y for B partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

11.84. **IDENTIFY:** Apply $Y = \dfrac{l_0 F_\perp}{A\Delta l}$ and calculate Δl.

SET UP: When the ride is at rest the tension $F_\perp$ in the rod is the weight 1900 N of the car and occupants. When the ride is operating, the tension $F_\perp$ in the rod is obtained by applying $\sum \vec{F} = m\vec{a}$ to a car and its occupants. The free-body diagram is shown in Figure 11.84. The car travels in a circle of radius $r = l\sin\theta$, where l is the length of the rod and θ is the angle the rod makes with the vertical. For steel, $Y = 2.0\times10^{11}$ Pa. $\omega = 12.0\text{ rev/min} = 1.2566\text{ rad/s}$.

EXECUTE: **(a)** $\Delta l = \dfrac{l_0 F_\perp}{YA} = \dfrac{(15.0\text{ m})(1900\text{ N})}{(2.0\times10^{11}\text{ Pa})(8.00\times10^{-4}\text{ m}^2)} = 1.78\times10^{-4}\text{ m} = 0.18\text{ mm}$

(b) $\sum F_x = ma_x$ gives $F_\perp \sin\theta = mr\omega^2 = ml\sin\theta\omega^2$ and

$$F_\perp = ml\omega^2 = \left(\frac{1900\text{ N}}{9.80\text{ m/s}^2}\right)(15.0\text{ m})(1.2566\text{ rad/s})^2 = 4.592\times10^3\text{ N}.$$

$$\Delta l = \left(\frac{4.592\times10^3\text{ N}}{1900\text{ N}}\right)(0.18\text{ mm}) = 0.44\text{ mm}.$$

EVALUATE: $\sum F_y = ma_y$ gives $F_\perp\cos\theta = mg$ and $\cos\theta = mg/F_\perp$. As ω increases $F_\perp$ increases and $\cos\theta$ becomes small. Smaller $\cos\theta$ means θ increases, so the rods move toward the horizontal as ω increases.

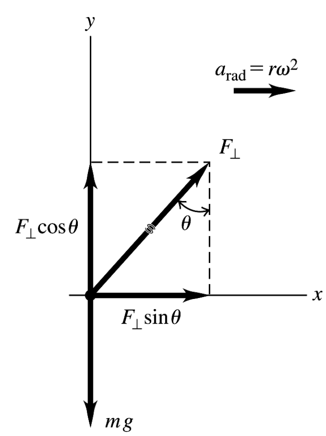


Figure 11.84

11.85. **IDENTIFY:** Apply $\dfrac{F_\perp}{A} = Y\left(\dfrac{\Delta l}{l_0}\right)$. The height from which he jumps determines his speed at the ground. The acceleration as he stops depends on the force exerted on his legs by the ground.

SET UP: In considering his motion take $+y$ downward. Assume constant acceleration as he is stopped by the floor.

EXECUTE: **(a)** $F_\perp = YA\left(\dfrac{\Delta l}{l_0}\right) = (3.0\times10^{-4}\text{ m}^2)(14\times10^9\text{ Pa})(0.010) = 4.2\times10^4\text{ N}$

(b) As he is stopped by the ground, the net force on him is $F_{net} = F_{\perp} - mg$, where $F_{\perp}$ is the force exerted on him by the ground. From part (a), $F_{\perp} = 2(4.2\times10^4 \text{ N}) = 8.4\times10^4 \text{ N}$ and $F = 8.4\times10^4 \text{ N} - (70 \text{ kg})(9.80 \text{ m/s}^2) = 8.33\times10^4 \text{ N}$. $F_{net} = ma$ gives $a = 1.19\times10^3 \text{ m/s}^2$. $a_y = -1.19\times10^3 \text{ m/s}^2$ since the acceleration is upward. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = (-1.19\times10^3 \text{ m/s}^2)(0.030 \text{ s}) = 35.7 \text{ m/s}$. His speed at the ground therefore is $v = 35.7 \text{ m/s}$. This speed is related to his initial height *h* above the floor by $\frac{1}{2}mv^2 = mgh$ and

$$h = \frac{v^2}{2g} = \frac{(35.7 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 65 \text{ m}.$$

EVALUATE: Our estimate is based solely on compressive stress; other injuries are likely at a much lower height.

11.86. **IDENTIFY:** The graph gives the change in length of the wire as a function of the weight hanging from it, which is equal to the tension in the wire. Young's modulus *Y* applies to the stretching of the wire. Energy conservation and Newton's second law apply to the swinging sphere.

SET UP: $Y = \dfrac{l_0 F_{\perp}}{A\Delta l}$, $K_1 + U_1 = K_2 + U_2$, $\Sigma\vec{F} = m\vec{a}$, $a_{rad} = v^2/R$.

EXECUTE: **(a)** Solve $Y = \dfrac{l_0 F_{\perp}}{A\Delta l}$ for Δl and realize that $F_{\perp} = mg$: $\Delta l = \dfrac{gl_0}{AY}m$. Therefore, in the graph of Δl versus *m*, the slope is equal to gl_0/AY. The equation of the graph is given in the problem as Δl = (0.422 mm/kg)*m*, so the slope is 0.422 mm/kg, so gl_0/AY = 0.422 mm/kg = 4.22×10^{-4} m/kg. Solving for *Y* gives $Y = \dfrac{gl_0}{A(4.22\times10^{-4} \text{ m/kg})}$. Using $A = \pi r^2$ and putting in the given numbers gives

$$Y = \frac{(9.80 \text{ m/s})(22.0 \text{ m})}{\pi(4.30\times10^{-4} \text{ m})^2(4.22\times10^{-4} \text{ m/kg})} = 8.80\times10^{11} \text{ Pa}.$$

(b) Use energy conservation to find the speed of the sphere. $K_1 + U_1 = K_2 + U_2$ gives $mgL(1-\cos\theta) = \frac{1}{2}mv^2$. Solving for *v* using θ = 36.0° and *L* = 22.0 m gives *v* = 9.075 m/s.

Now apply Newton's second law to the sphere at the bottom of the swing. $\Sigma\vec{F} = m\vec{a}$ and $a_{rad} = v^2/R$ give

$T - mg = mv^2/L$, so $T = mv^2/L + mg$ = (9.50 kg)(9.075 m/s)^2/(22.0 m) + (9.50 kg)(9.80 m/s^2) = 129 N. Using the value of *Y* found in part (a), we have

$$\Delta l = \frac{F_{\perp} l_0}{AY} = (129 \text{ N})(22.0 \text{ m})/[\pi(4.30\times10^{-4} \text{ m})^2(8.80\times10^{11} \text{ Pa})] = 0.00554 \text{ m} = 5.54 \text{ mm}.$$

EVALUATE: For a wire 22 m long, 5.5 mm is a very small stretch, 0.0055/22 = 0.025%.

11.87. **IDENTIFY:** The bar is at rest, so the forces and torques on it must all balance.

SET UP: $\Sigma F_y = 0$, $\Sigma\tau_z = 0$.

EXECUTE: **(a)** The free-body diagram is shown in Figure 11.87a, where F_p is the force due to the knife-edge pivot.

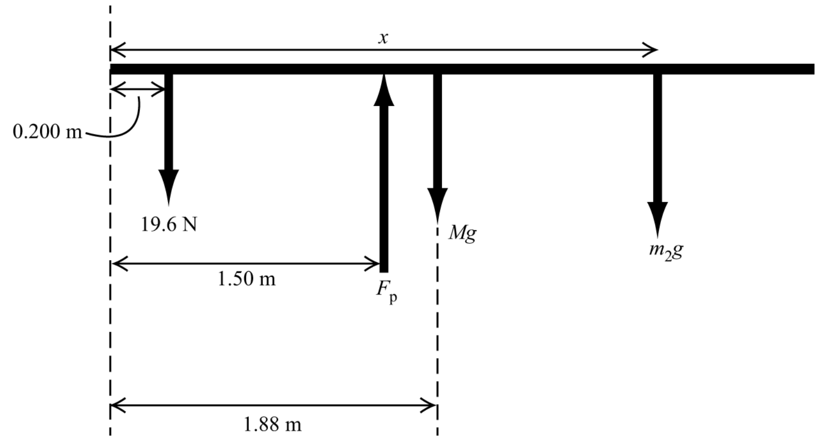


Figure 11.87a

(b) $\sum \tau_z = 0$, with torques taken about the location of the knife-edge pivot, gives

$(2.00\text{ kg})g(1.30\text{ m}) - Mg(0.38\text{ m}) - m_2g(x - 1.50\text{ m}) = 0$

Solving for x gives

$x = [(2.00\text{ kg})(1.30\text{ m}) - M(0.38\text{ m})](1/m_2) + 1.50\text{ m}$

The graph of this equation (x versus $1/m_2$) is a straight line of slope $[(2.00\text{ kg})(1.30\text{ m}) - M(0.38\text{ m})]$.

(c) The plot of x versus $1/m_2$ is shown in Figure 11.87b. The equation of the best-fit line is $x = (1.9955\text{ m}\cdot\text{kg})/m_2 + 1.504\text{ m}$. The slope of the best-fit line is $1.9955\text{ m}\cdot\text{kg}$, so $[(2.00\text{ kg})(1.30\text{ m}) - M(0.38\text{ m})] = 1.9955\text{ m}\cdot\text{kg}$, which gives $M = 1.59$ kg.

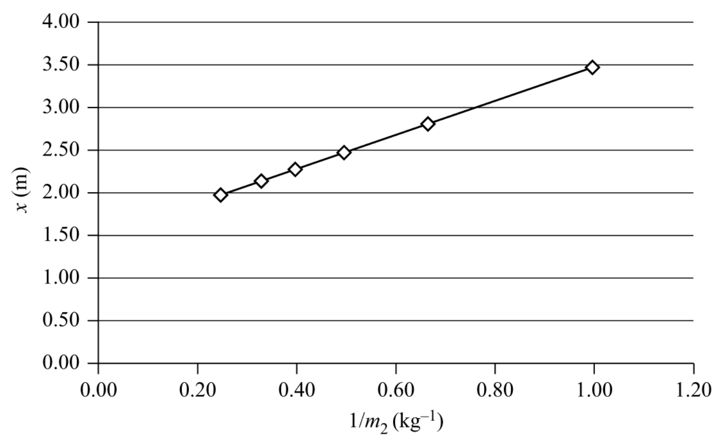


Figure 11.87b

(d) The y-intercept of the best-fit line is 1.50 m. This is plausible. As the graph approaches the y-axis, $1/m_2$ approaches zero, which means that m_2 is getting extremely large. In that case, it would be much larger than any other masses involved, so to balance the system, m_2 would have to be at the knife-point pivot, which is at $x = 1.50$ m.

EVALUATE: The fact that the graph gave a physically plausible result in part (d) suggests that this graphical analysis is reasonable.

11.88. IDENTIFY: The bar is at rest, so the forces and torques on it must all balance.

SET UP: $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\sum \tau_z = 0$.

EXECUTE: **(a)** Take torques about the hinge, calling m the mass of the bar and L its length. $\sum\tau_z = 0$ gives $xT\sin\theta = mg\frac{L}{2}$. Solving for T gives $T = \frac{mgL/2}{x\sin\theta}$. Therefore the alternative having the largest value of $x\sin\theta$ will have the smallest tension, and the one with the smallest value of $x\sin\theta$ will have the greatest tension. Calculating $x\sin\theta$ for each alternative gives the following values. A: 1.00 m, B: 1.30 m, C: 0.451 m, D: 0.483 m. Therefore alternative B gives the smallest tension and C produces the largest tension

(b) Calling H the magnitude of the hinge force, $\Sigma F_x = 0$ gives $H_x = T\cos\theta$. Using the value of T from part (a), we get $H_x = \frac{mg\,L/2}{x\sin\theta}.\cos\theta = \frac{mg\,L/2}{x\tan\theta}$. From this result, we can see that H_x is greatest when $x\tan\theta$ is the smallest, and H_x is least when $x\tan\theta$ is greatest. Calculating $x\tan\theta$ for each alternative gives A: 1.15 m, B: 2.60 m, C: 0.565 m, D: 1.87 m. Therefore alternative C gives the greatest H_x and B gives the smallest H_x.

(c) Taking torques about the point where the cable is connected to the bar, $\sum\tau_z = 0$ gives $H_y x = mg(x - L/2)$. Solving for H_y gives $H_y = mg(1 - L/2x)$. Since H_y could be positive or negative, we should calculate all four possibilities. For alternative A, we have $H_y = mg\left(1 - \frac{2.00\text{ m}}{4.00\text{ m}}\right) = 0.500mg$. For B we have $H_y = mg\left(1 - \frac{2.00\text{ m}}{3.00\text{ m}}\right) = 0.333mg$, and likewise we get $H_y = -0.333mg$ for C and $H_y = -1.00mg$ for D. Therefore alternative D gives the largest H_y and B and C both give the smallest value.

(d) Alternative B is clearly optimal because it results in the smallest values for T, H_x, and H_y. It might be a good idea to avoid alternative C because it has the greatest T and H_x.

EVALUATE: As a check, part (c) could be solved by using $\Sigma F_y = 0$.

11.89. **IDENTIFY:** Apply the equilibrium conditions to the ladder combination and also to each ladder.

SET UP: The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by F_L and F_R (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless.

EXECUTE: **(a)** Taking torques about the right end, $F_L(5.00\text{ m}) = (480\text{ N})(3.40\text{ m}) + (360\text{ N})(0.90\text{ m})$, so $F_L = 391\text{ N}$. F_R may be found in a similar manner, or from $F_R = 840\text{ N} - F_L = 449\text{ N}$.

(b) The tension in the rope may be found by finding the torque on each ladder, using the point A as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50\text{ m}) = F_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})$, so $T = 322.1\text{ N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50\text{ m}) = F_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})$ gives the same result.

(c) The horizontal component of the force at A must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, $480\text{ N} - 391\text{ N} = 449\text{ N} - 360\text{ N} = 89\text{ N}$. The magnitude of the force at A is then $\sqrt{(322.1\text{ N})^2 + (89\text{ N})^2} = 334\text{ N}$.

(d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F_L' = F_L + (0.36)(800\text{ N}) = 679\text{ N}$, and $F_R' = F_R + (0.64)(800\text{ N}) = 961\text{ N}$. Using these forces in the form for the tension found in part (b) gives

$$T = \frac{F'_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})}{(1.50\text{ m})} = \frac{F'_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})}{(1.50\text{ m})} = 937\text{ N}.$$

EVALUATE: The presence of the painter increases the tension in the rope, even though his weight is vertical and the tension force is horizontal.

11.90. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the post, for various choices of the location of the rotation axis.

SET UP: When the post is on the verge of slipping, f_s has its largest possible value, $f_s = \mu_s n.$

EXECUTE: **(a)** Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so

$$F\frac{h}{2} = (n - w)h\tan\theta.$$

Taking torques about the upper point (where the rope is attached to the post), $fh = F\frac{h}{2}$. Using $f \le \mu_s n$ and solving for F, $F \le 2w\left(\frac{1}{\mu_s} - \frac{1}{\tan\theta}\right)^{-1} = 2(400\text{ N})\left(\frac{1}{0.30} - \frac{1}{\tan 36.9°}\right)^{-1} = 400\text{ N}.$

(b) The above relations between F, n and f become $F\frac{3}{5}h = (n - w)h\tan\theta, f = \frac{2}{5}F$, and eliminating f and n and solving for F gives $F \le w\left(\frac{2/5}{\mu_s} - \frac{3/5}{\tan\theta}\right)^{-1}$, and substitution of numerical values gives 750 N to two figures.

(c) If the force is applied a distance y above the ground, the above relations become $Fy = (n - w)h\tan\theta$, $F(h - y) = fh$, which become, on eliminating n and f, $w \ge F\left[\frac{(1 - y/h)}{\mu_s} - \frac{(y/h)}{\tan\theta}\right].$

As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of y is found by setting the term in square brackets equal to zero. Solving for y gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

EVALUATE: For the post to slip, for an axis at the top of the post the torque due to F must balance the torque due to the friction force. As the point of application of F approaches the top of the post, its moment arm for this axis approaches zero.

11.91. **IDENTIFY:** Apply $Y = \frac{l_0 F_\perp}{A\Delta l}$ to calculate $\Delta l.$

SET UP: For steel, $Y = 2.0\times10^{11}$ Pa.

EXECUTE: **(a)** From $Y = \frac{l_0 F_\perp}{A\Delta l}$, $\Delta l = \frac{(4.50\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{(20\times10^{10}\text{ Pa})(5.00\times10^{-7}\text{ m}^2)} = 6.62\times10^{-4}$ m, or 0.66 mm to two figures.

(b) $(4.50\text{ kg})(9.80\text{ m/s}^2)(0.0500\times10^{-2}\text{ m}) = 0.022\text{ J}.$

(c) The magnitude F will vary with distance; the average force is $YA(0.0250\text{ cm}/l_0) = 16.7\text{ N}$, and so the work done by the applied force is $(16.7\text{ N})(0.0500\times10^{-2}\text{ m}) = 8.35\times10^{-3}$ J.

(d) The average force the wire exerts is $(4.50\text{ kg})g + 16.7\text{ N} = 60.8\text{ N}.$ The work done is negative, and equal to $-(60.8\text{ N})(0.0500\times10^{-2}\text{ m}) = -3.04\times10^{-2}$ J.

(e) The equation $Y = \frac{l_0 F_\perp}{A\Delta l}$ can be put into the form of Hooke's law, with $k = \frac{YA}{l_0}$. $U_{el} = \frac{1}{2}kx^2$, so $\Delta U_{el} = \frac{1}{2}k(x_2^2 - x_1^2)$. $x_1 = 6.62\times10^{-4}$ m and $x_2 = 0.500\times10^{-3}\text{ m} + x_1 = 11.62\times10^{-4}$ m. The change in elastic potential energy is

$$\frac{(20\times10^{10}\text{ Pa})(5.00\times10^{-7}\text{ m}^2)}{2(1.50\text{ m})}\left[(11.62\times10^{-4}\text{ m})^2 - (6.62\times10^{-4}\text{ m})^2\right] = 3.04\times10^{-2}\text{ J},$$ the negative of the result of part (d).

EVALUATE: The tensile force in the wire is conservative and obeys the relation $W = -\Delta U$.

11.92. IDENTIFY and SET UP: The forces and torques on the competitor must balance, so $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\sum \tau_z = 0$.

EXECUTE: Take torques about his feet, giving $(T_1 - T_2)(1.5\text{ m})(\cos 30°) = mg(1.0\text{ m})(\sin 30°)$. Solving for T_2 gives $T_2 = 1160\text{ N} - [(80.0\text{ kg})(9.80\text{ m/s}^2)/(1.5\text{ m})]\tan 30° = 858\text{ N} \approx 860\text{ N}$, which is choice (c).

EVALUATE: We find $T_2 < T_1$ as expected.

11.93. IDENTIFY and SET UP: The forces and torques on the competitor must balance, so $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\sum \tau_z = 0$.

EXECUTE: As in the previous problem, $T_1 - T_2$ is proportional to $\tan\theta$, so as θ increases, so does $\tan\theta$ and so does $T_1 - T_2$, which makes choice (a) correct.

EVALUATE: The result is physically reasonable. As he leans back, the ropes get lower, which reduces their moment arm, and his weight also gets lower, which increases its moment arm. Therefore to keep balance, the diffrerence in the tensions must be greater than before.

11.94. IDENTIFY and SET UP: Apply $\tau = Fl$.

EXECUTE: The moment arm for T_1 has increased, so T_1 can be smaller and still produce the same torque needed to balance the torque due to gravity, so choice (c) is correct.

EVALUATE: If the rope is held too high, it will be hard for the competitor to hold it, so there is a limit on how much the holding height can be effectively increased.

11.95. IDENTIFY and SET UP: The competitor will slip if the static friction force would need to be greater than its maximum possible value. $f_s^{max} = \mu_s n$.

EXECUTE: From earlier work, we know that $T_1 - T_2 = 1160\text{ N} - 858\text{ N} = 302\text{ N}$. The maximum static friction force is $f_s^{max} = \mu_s n = (0.50)(80.0\text{ kg})(9.80\text{ m/s}^2) = 392\text{ N}$. He needs only 302 N to balance the tension difference, yet the static friction force could be as great as 392 N, so he is not even ready to slip. Therefore he will not move, choice (d).

EVALUATE: The friction force is 302 N, not 392 N, because he is not just ready to slip.