Exercises 08
-
RLC circuits

Exercise 1 - Circuit

parallel RLC

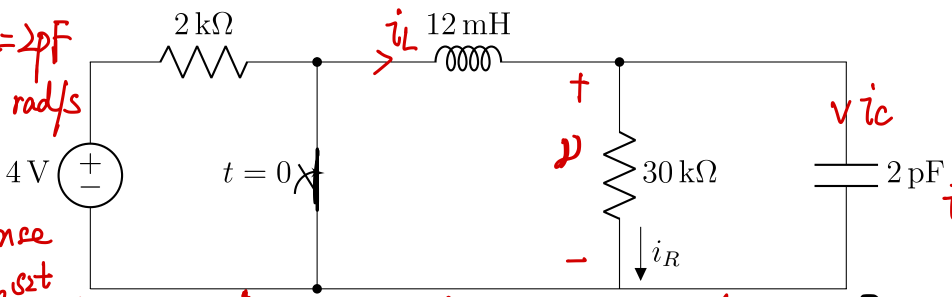
$R = 30k\Omega \quad L = 12mH \quad C = 2pF$

$w_0 = \frac{1}{\sqrt{LC}} = 6.455 \times 10^6 \, rad/s$

$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} = 1.29 > 1$

$\Rightarrow$ over damped response

$v(t) = A_1 \cdot e^{s_1 t} + A_2 \cdot e^{s_2 t}$



Determine i_R.

$s_1 = -w_0 \xi + w_0\sqrt{\xi^2 - 1}$

$s_2 = -w_0 \xi - w_0\sqrt{\xi^2 - 1}$

$i_R(t) = C_1 \cdot e^{s_1 t} + C_2 \cdot e^{s_2 t}$

$v_c(0^-) = v_c(0^+) = 3.75V$

$i_c(0^-) = 0 \quad i_c(0^+) = 0$

$i_L(0^-) = \frac{1}{8} mA = i_R(0^-) \Rightarrow C_1 + C_2 = \frac{1}{8} mA$ (1)

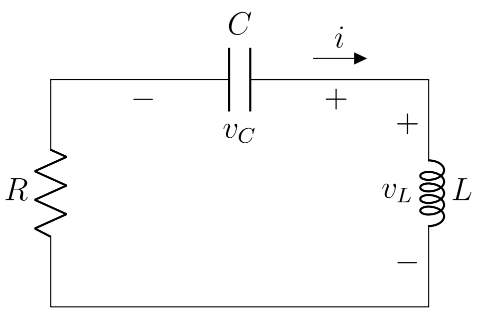
$v_R(0^-) = 4V \cdot \frac{30}{32} = 3.75V \Rightarrow i_c(0^+) = C \cdot \frac{dv_c(t)}{dt} = C \cdot \frac{dv(t)}{dt} = C \cdot \frac{R \cdot di_R(t)}{dt}$

$= C \cdot R \cdot (C_1 \cdot s_1 \cdot e^{s_1 t} + C_2 \cdot s_2 \cdot e^{s_2 t})\Big|_{t=0^+} = 0$ (2)

(1),(2) $\Rightarrow C_1 = 161.3 \mu A \quad C_2 = -36.34 \mu A$

$i_R(t) = 161.3\mu A \cdot e^{s_1 t} - 36.34 \mu A \cdot e^{s_2 t}$

Exercise 2 - Series RCL circuit



$w_0 = \frac{1}{\sqrt{LC}} \quad \alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$

$\alpha > 1$ overdamped

$i(t) = A_1 \cdot e^{s_1 t} + A_2 \cdot e^{s_2 t}$

$\alpha = 1$ critically damped

$i(t) = (A_1 + A_2 t) \cdot e^{-w_0 t}$

Determine overdamped, critically damped, underdamped response.

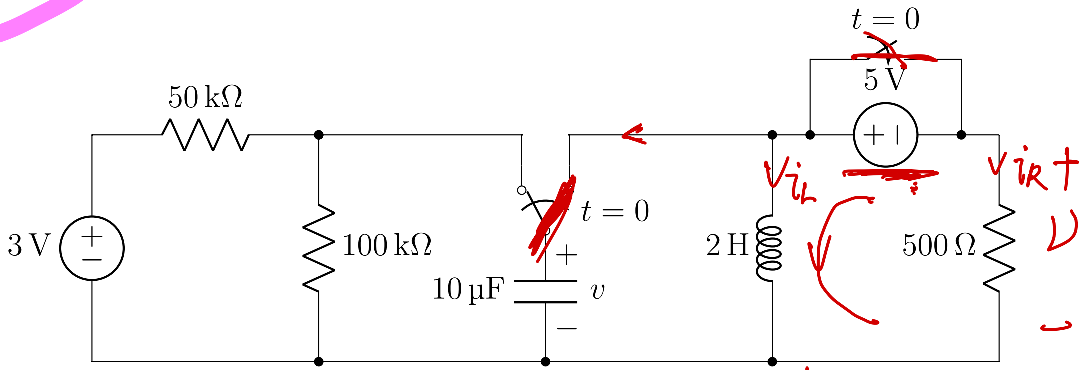
$s = -w_0 \alpha \pm w_0 \sqrt{\alpha^2 - 1}$

$\alpha < 1$ underdamped

$i(t) = e^{-w_0 \alpha t}(B_1 \cos w_d t + B_2 \sin w_d t)$

$w_d = w_0 \sqrt{1 - \alpha^2}$

Exercise 3 - Circuit



(a) Determine $\frac{dv}{dt}$ at $t = 0^+$;

(b) Determine v at $t = 1\text{ms}$;

(c) Determine t_0, the first value of t greater than zero at which $v = 0$.

$\frac{dv(t)}{dt}\Big|_{t=0^+} = -\omega_0\xi \cdot B_1 + \omega_d \cdot B_2$

$= -1.4\ kV/s$ ②

①, ② ⇒ $B_1 = 2V$, $B_2 = -6V$

$v|_{t=1ms} = 0.693\ V$

a) $v(0^+) = v(0^-) = 3V \cdot \frac{100k\Omega}{150k\Omega} = 2V$

$i_L(0^+) = i_L(0^-) = \frac{5V}{500\Omega} = 10mA$

$i_R = \frac{2V}{500\Omega} = 4mA$

$\frac{dv}{dt}\Big|_{t=0^+} \cdot C = -i_L - i_R = -14mA$

$\frac{dv}{dt}\Big|_{t=0^+} = \frac{-14mA}{10\mu F} = -1.4\ kV/s$

b) $R = 500\Omega$ $L = 2H$ $C = 10\mu F$

$\omega_0 = 1/\sqrt{LC} \approx 223.6\ rad/s$

$\xi = \frac{1}{2R} \cdot \sqrt{\frac{L}{C}} = 0.447 < 1$ underdamped

$v(t) = e^{-\omega_0\xi t} \cdot (B_1 \cos\omega_d t + B_2 \sin\omega_d t)$

$\omega_d = \omega_0\sqrt{1-\xi^2} = 200\ rad/s$

$v(t=0^+) = 2V = B_1$ ①

c) $v = 0$

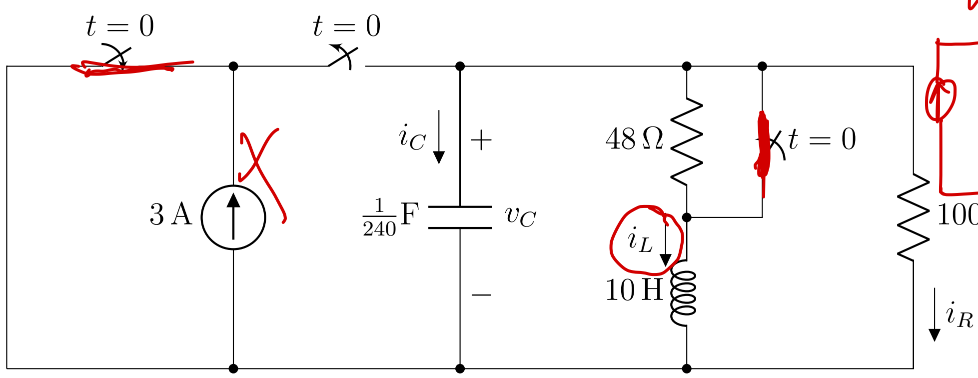
⇒ $B_1 \cdot \cos\omega_d t + B_2 \sin\omega_d t = 0$

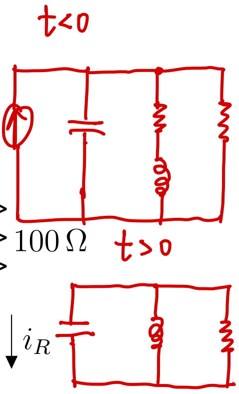
⇒ $\frac{\sin\omega_d t}{\cos\omega_d t} = \tan\omega_d t = -\frac{1}{3}$

⇒ $\omega_d t = \arctan(-\frac{1}{3}) = \frac{18.435}{180} \cdot \pi$

⇒ $t \approx 1.609\ ms$

Exercise 4 - Circuit





Determine $i_L(t)$.

$i_L(t) = e^{-\xi\omega_0 t}(C_1\cos\omega_d t + C_2 \sin\omega_d t)$

⇒ C_1, C_2

$i_L(0^-) = 3A \cdot \frac{100\Omega}{100\Omega + 48\Omega} = 2.027A$

$t>0$ $R = 100\Omega$ $L = 10H$ $C = \frac{1}{240}F$

$\omega_0 = \frac{1}{\sqrt{LC}} = 4.899\ rad/s$

$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}} = 0.245 < 1$ underdamped

$v(t) = e^{-\xi\omega_0 t} \cdot (B_1 \cos\omega_d t + B_2 \sin\omega_d t)$

$\omega_d = \omega_0\sqrt{1-\xi^2} = 4.75\ rad/s$

$v_C(0^+) = v_C(0^-) = i_L(0^-) \cdot 48\Omega = 97.296V$

⇒ $B_1 = 97.296V$ ①

$i_C(0^+) = -i_L(0^+) - i_R(0^+)$

$= -i_L(0^+) - \frac{v_C(0^+)}{100\Omega}$

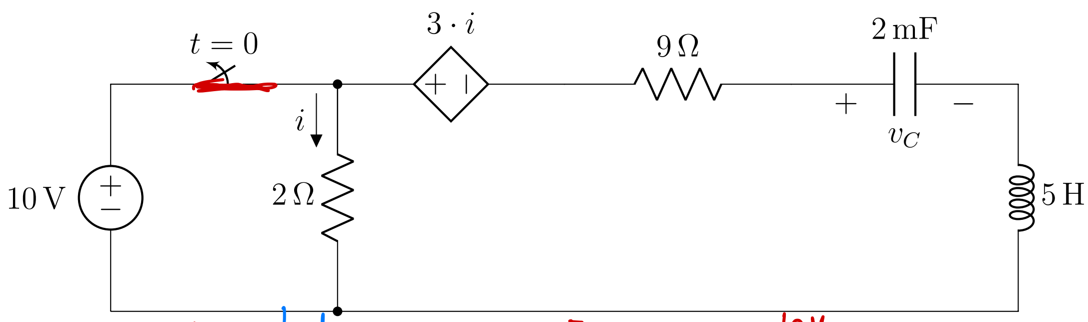
$C \cdot \frac{dv_C(t)}{dt}\Big|_{t=0^+} = -2.027A - \frac{v_C(0^+)}{100\Omega}$ ②

⇒ ①, ② $B_1 = 97.296V$ $B_2 = ???$

$i_L(t) = -i_R(t) - i_C(t)$

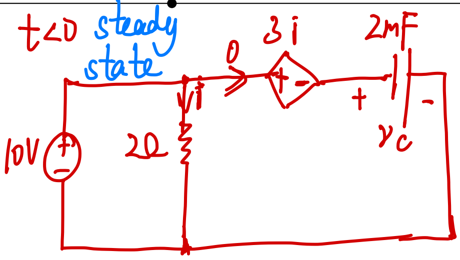
$= -\frac{v_C(t)}{100\Omega} - C \cdot \frac{dv_C(t)}{dt}$

Exercise 5 - Circuit



Determine $v_C(t)$.

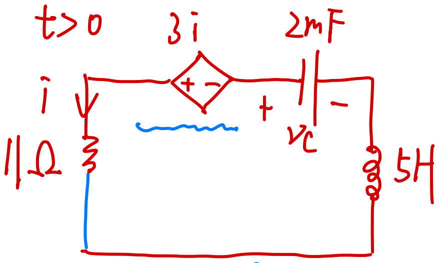
t<0 steady state

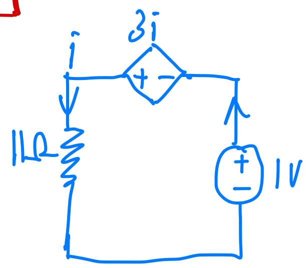


$$i = \frac{10V}{2\Omega} = 5A$$

$$10\,V = 3\times 5\,V + v_c(0^-) \Rightarrow v_c(0^-) = -5V$$

t>0

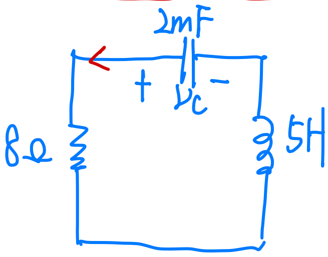




$$1i - 1V - 3i = 0$$

$$i = \frac{1V}{8\Omega}$$

$$R_{eq} = \frac{1V}{i} = 8\Omega$$



$$w_0 = \frac{1}{\sqrt{LC}} = 10 \text{ rad/s}$$

$$\xi = \frac{R}{2}\cdot\sqrt{\frac{C}{L}} = 0.08 < 1 \quad \text{underdamped}$$

$$i(t) = e^{-w_0\xi t}\cdot(B_1 \cos w_d t + B_2 \sin w_d t)$$

$$w_d = w_0\sqrt{1-\xi^2} = 9.968 \text{ rad/s}$$

$$v_c(t) = e^{-w_0\xi t}\cdot(C_1 \cos w_d t + C_2\cdot \sin w_d t)$$

$$v_c(0^+) = v_c(0^-) = -5V = C_1 \qquad ①$$

$$\left.\frac{dv_c(t)}{dt}\right|_{t=0} = \ldots\ldots\ldots = 0 \qquad ②$$

$$①,② \Rightarrow \quad C_1 = -5V \quad C_2 = -0.4013\,V$$

$$\Rightarrow v_c(t) = \ldots\ldots$$