10

DYNAMICS OF ROTATIONAL MOTION

VP10.3.1. **IDENTIFY:** The force of the cable produces a torque on the cylinder, giving it an angular acceleration. We apply the rotational analog of Newton's second law.

SET UP: $\sum\tau_z = I\alpha_z$, $I = \frac{1}{2}MR^2$ for a solid cylinder, $a_{\tan} = r\alpha_z$ apply in this case.

EXECUTE: **(a)** $a_{\tan} = r\alpha_z$ gives $\alpha_z = a_{\tan}/R$ = (0.60 m/s^2)/(0.060 m) = 10 rad/s^2.

(b) Apply $\sum\tau_z = I\alpha_z$ to the cylinder. There is only one torque, so $\tau_z = I\alpha_z = \frac{1}{2}MR^2\alpha_z$.

$\tau_z = \frac{1}{2}$(50 kg)(0.060 m)^2(10 rad/s^2) = 0.90 N $\cdot$ m .

(c) $\tau_z = FR \quad \rightarrow \quad F = \tau_z/R$ = (0.90 N $\cdot$ m)/(0.060 m) = 15 N.

EVALUATE: When using $\sum\tau_z = I\alpha_z$, α_z *must* be in radian measure.

VP10.3.2. **IDENTIFY:** Gravity pulls the block, causing tension in the cable. This tension produces a torque on the cylinder, giving it an angular acceleration. We apply the rotational analog of Newton's second law to the wheel and the linear form to the falling block.

SET UP: $\sum\tau_z = I\alpha_z$, $I = MR^2$ for a hollow cylinder, $a_{\tan} = r\alpha_z$, $\sum F_y = ma_y$ apply in this case. Call m the mass of the block and M the mass of the wheel.

EXECUTE: **(a)** Apply $\sum F_y = ma_y$ to the block. Since the block accelerates downward, it is best to call the y-axis positive downward. This gives $mg - T = ma_y$ (Eq. 1)

Now apply $\sum\tau_z = I\alpha_z$ to the wheel. $TR = MR^2\alpha_z$. Using $a_y = a_{\tan} = R\alpha_z$, the second equation becomes $T = MR(a_y/R) = Ma_y$ (Eq. 2)

Combining Eq. 1 and Eq. 2 gives $mg - Ma_y = ma_y$. Solving for a_y gives

$a_y = \dfrac{mg}{m+M} = \dfrac{g}{1+M/m}$.

(b) Eq. 2 gives $T = Ma_y = \dfrac{Mg}{1+M/m}$, which can also be written as $T = \dfrac{mg}{1+m/M}$.

EVALUATE: Careful! The torque on the wheel is *not* equal to mgR! It is the *tension* that turns the wheel, and $T \neq mg$. Look at our answers in some limiting cases. If $m >> M$, the ratio M/m is very small, so the acceleration approaches g. This is reasonable because the wheel has very little effect on the block, so the block is essentially in freefall. The tension approaches zero because M approaches zero. This is reasonable because the block is essentially in freefall. If $M >> m$, the acceleration approaches zero since the small m produces very little acceleration of the much heavier wheel. The tension approaches mg because $m/M << 1$, so the block is essentially just hanging with almost no acceleration.

VP10.3.3. **IDENTIFY:** Gravity pulls the block, causing tension in the cable. The torque on the cylinder is due to the tension in the cable. We apply the rotational analog of Newton's second law to the cylinder and the linear form to the block. We do not know the moment of inertia of the drum.

SET UP: $\sum\tau_z = I\alpha_z$, $a_{\tan} = r\alpha_z$, $\sum F_y = ma_y$ apply in this case. Call m the mass of the block.

EXECUTE: **(a)** Apply $\sum F_y = ma_y$ to the block. Since the block accelerates downward, it is best to call the y-axis positive downward. This gives $mg - T = ma$. Solving for T gives

$T = m(g - a)$ (Eq. 1)

(b) Applying $\sum\tau_z = I\alpha_z$ to the drum gives $TR = I\alpha_z$ (Eq. 2)

We also have $a_y = a_{\tan} = R\alpha_z$, which gives $\alpha_z = a_y/R$ (Eq. 3)

Combining Equations 1, 2, and 3 and solving for I gives $I = mR^2(g/a - 1)$.

EVALUATE: Check our result for $a \to g$, which gives $I \to 0$. This is reasonable because the block is then essentially in freefall, meaning that the drum had almost no effect on it. This would be the case of the drum's moment of inertia was very very small. You could also interpret our result for $a > g$ to understand its meaning in that case.

VP10.3.4. **IDENTIFY:** Gravity pulls the block, causing tension in the cable. The torque on the cylinder is due to the tension in the cable and the torque produced by the motor. We apply the rotational analog of Newton's second law to the cylinder and the linear form to the block.

SET UP: $\sum\tau_z = I\alpha_z$, $a_{\tan} = r\alpha_z$, $\sum F_y = ma_y$ apply in this case. Call m the mass of the block and M the mass of the cylinder. $I = \frac{1}{2}MR^2$ for a solid uniform cylinder.

EXECUTE: **(a)** Apply $\sum F_y = ma_y$ to the block. Since the block accelerates upward, it is best to call the y-axis positive upward. This gives $T - mg = ma$. Solving for T gives $T = m(g + a)$.

(b) $\tau_z = TR = mR(g + a)$.

(c) Apply $\sum\tau_z = I\alpha_z$ to the cylinder. The torque due to the motor must be opposite in direction to that of the tension and it must have a greater magnitude.

$\tau_{\text{motor}} + \tau_{\text{tension}} = I\alpha_z \quad \to \quad \tau_{\text{motor}} - TR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$. Using the tension from part (a) and solving for τ_{motor}, we get $\tau_{\text{motor}} = mR(g + a) + fc\, MRa$.

EVALUATE: Check if our result is reasonable. If either m or M are large, the torque must be large to produce the given acceleration.

VP10.7.1. **IDENTIFY:** The cylinder is rotating and translating at the same time. The rotational analog of Newton's second law applies to its rotational motion, and the linear form applies to its center of mass motion.

SET UP: $\sum F_y = ma_y$ applies to the vertical motion and $\sum\tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid hollow cylinder, $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$ and $a = r\alpha$.

EXECUTE: **(a)** We want the acceleration, so we apply $\sum F_y = ma_y$: $Mg - T = Ma_y$ (Eq. 1)

Now apply $\sum\tau_z = I\alpha_z$. For this cylinder, $R_1 = R$ and $R_2 = R/2$, so $I = 5MR^2/4$. Using this result and $\alpha = a_y / R$, we have $TR = \frac{1}{2}\left(\frac{5R^2}{4}\right)\left(\frac{a_y}{R}\right)M$, so $T = \frac{5}{8}Ma_y$ (Eq. 2)

Combining Eq. 1 and Eq. 2 and solving for a_y gives $a_y = \frac{8}{13}g$.

(b) From Eq. 2, we have $T = \frac{5}{8}Ma_y = \frac{5}{8}M\left(\frac{8}{13}g\right) = \frac{5}{13}Mg$.

EVALUATE: Note the difference between the answer to this problem and Ex. 10.6. In both cases, the yo-yo has the same mass but a different distribution of that mass. The acceleration is less in this problem than in Example 10.6 because the moment of inertia is greater than in the example.

VP10.7.2. **IDENTIFY:** As the solid sphere rolls down a ramp its linear velocity and its angular velocity both increase. Newton's second law in both its linear and rotational forms applies.

SET UP: $\sum F_x = ma_x$ applies to the linear motion and $\sum \tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid sphere $I = \frac{2}{3}MR^2$, and $a_x = R\alpha$ because the sphere does not slip. Take the x-axis along the surface of the ramp, pointing downward.

EXECUTE: Apply $\sum F_x = ma_x$. The friction force is up the ramp, so we get

$Mg \sin\beta - f = Ma_x$ (Eq. 1)

Apply $\sum \tau_z = I\alpha_z$, which gives $fR = \frac{2}{3}MR^2\ \alpha_z = \frac{2}{3}MR^2\ (a_x/R)$, which simplifies to

$f = \frac{2}{3}Ma_x$ (Eq. 2)

Combining Eq. 1 and Eq. 2 and solving for a_x gives $a_x = \frac{3}{5}g\sin\beta$.

(b) From Eq. 2 and the result from (a), we have $f = \frac{2}{3}Ma_x = \frac{2}{3}M\left(\frac{3}{5}g\sin\beta\right) = \frac{2}{5}Mg\sin\beta$.

(c) $\tau_z = fR = \frac{2}{5}MgR\sin\beta$.

EVALUATE: If there were no friction and the ball just slid down the incline, its acceleration would be $g\sin\beta$. But with rolling, friction is *up* the ramp, so it opposes the component of gravity down the ramp. Therefore the acceleration is *less* than $g\sin\beta$.

VP10.7.3. **IDENTIFY:** The yo-yo is rotating and translating at the same time. The rotational analog of Newton's second law applies to its rotational motion, and the linear form applies to its center of mass motion.

SET UP: $\sum F_y = ma_y$ applies to the vertical motion and $\sum \tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid cylinder, $I = \frac{1}{2}MR^2$. We know that the tension in the string is 2/3 the weight of the yo-yo, and we want to find the acceleration of its center of mass and the angular acceleration about its center of mass. The only torque about the center of mass is due to the tension in the string.

EXECUTE: **(a)** We want the acceleration of the center of mass, so we apply $\sum F_y = ma_y$.

$Mg - T = Ma_y \rightarrow Mg - 2Mg/3 = Ma_y \rightarrow a_y = -g/3$. The magnitude is $g/3$ and the direction is downward.

(b) We want the angular acceleration about the center of mass, so we apply $\sum \tau_z = I\alpha_z$.

$TR = \frac{1}{2}MR^2\alpha_{\text{cm}} \rightarrow \alpha_{\text{cm}} = \frac{4g}{3R}$.

EVALUATE: Notice that a_{cm} is *not* equal to $R\alpha_{\text{cm}}$. The cylinder is turning and moving, but it is not rolling.

VP10.7.4. **IDENTIFY:** The cylinder is rotating and translating at the same time and is rolling without slipping. The rotational analog of Newton's second law applies to its rotational motion, and the linear form applies to its center of mass motion.

SET UP: $\sum F_x = ma_x$ applies to the center of mass motion and $\sum \tau_z = I\alpha_z$ applies to the rotational motion about the center of mass. For a solid cylinder, $I = \frac{1}{2}MR^2$, and for rolling without slipping $a_{\text{cm}} =$

$r\,\alpha_{cm}$. We want to find the friction force on the cylinder and the maximum angle of the ramp for which slipping will not occur. Call the x-axis along the surface of the ramp pointing downward.

EXECUTE: (a) We want the acceleration of the center of mass, so we apply $\sum F_x = ma_x$. The friction force f acts up the ramp to prevent sliding, so $Mg \sin\beta - f = Ma_x$ (Eq. 1)

Applying $\sum \tau_z = I\alpha_z$ about the center of mass of the cylinder gives $fR = \frac{1}{2}MR^2\alpha_{cm}$. For rolling we also have $a_{cm} = r\,\alpha_{cm}$, so this becomes $fR = \frac{1}{2}MR^2(a_{cm}/R)$, which gives $f = \frac{1}{2}Ma_{cm}$ (Eq. 2)

Putting Eq. 2 into Eq. 1 gives $f = \frac{Mg}{3}\sin\beta$.

(b) When the ramp is at the maximum angle β, the cylinder is just ready to slip, so f_s is at its maximum value of $\mu_s n$. Applying $\sum F_y = 0$ to the cylinder gives $n = Mg\cos\beta$. Combining this result with the answer to (a) gives $\frac{Mg}{3}\sin\beta = \mu_s Mg\cos\beta \rightarrow \tan\beta = 3\mu_s$, so

$\beta = \arctan(3\mu_s)$.

EVALUATE: Check a special case: If $\mu_s = 0$ (perfectly smooth ramp), then $\beta = 0$, which means that any elevation at all will cause slipping. This is a reasonable result for a perfectly smooth ramp.

VP10.12.1. IDENTIFY: The two-disk system does not experience any external torque, so its angular momentum is conserved.

SET UP: Angular momentum is $L = I\omega$, $I = \frac{1}{2}MR^2$ for a solid disk, and rotational kinetic energy is $K = \frac{1}{2}I\omega^2$.

EXECUTE: (a) Conservation of angular momentum tells us that $L_1 = L_2$, so $I_A\omega_A + I_B\omega_B = I_{A+B}\omega_2$. We know that $I_B = I_A/4$ and $\omega_B = \omega_A/2$, so this gives $I_A\omega_A + \frac{I_A}{4}\cdot\frac{\omega_A}{2} = \left(I_A + \frac{I_A}{4}\right)\omega_2$. Solving for ω_2 gives

$\omega_2 = \frac{9}{10}\omega_A$.

(b) We want K_2/K_1. $K_1 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}\left(\frac{I_A}{4}\right)\left(\frac{\omega_A}{2}\right)^2 = \frac{17}{32}I_A\omega_A^2$.

Using the result from (a) gives $K_2 = \frac{1}{2}I_{A+B}\omega^2 = \frac{1}{2}\left(I_A + \frac{I_A}{4}\right)\left(\frac{9}{10}\omega_A\right)^2 = \frac{405}{800}I_A\omega_A^2$.

The fraction of the initial rotational kinetic energy that remains is $\frac{K_2}{K_1} = \frac{\frac{405}{800}I_A\omega_A^2}{\frac{17}{32}I_A\omega_A^2} = \frac{81}{85} = 0.953$.

EVALUATE: The disks stick together so kinetic energy is lost, just as when objects collide in an inelastic collision. In this case, 95.3% of the kinetic energy remains, so only 4.7% is lost.

VP10.12.2. IDENTIFY: We follow exactly the same procedure as in VP10.12.1 *except* that the initial angular velocities are in *opposite* directions.

SET UP: The set up is the same as in VP10.12.1 except that $\omega_B = -\omega_A/2$

EXECUTE: **(a)** Conservation of angular momentum tells us that $L_1 = L_2$, so $I_A\omega_A + I_B\omega_B = I_{A+B}\omega_2$. We know that $I_B = I_A/4$ and $\omega_B = -\omega_A/2$, so this gives $I_A\omega_A - \frac{I_A}{4}\cdot\frac{\omega_A}{2} = \left(I_A + \frac{I_A}{4}\right)\omega_2$. Solving for ω_2 gives $\omega_2 = \frac{7}{10}\omega_A$.

(b) We want K_2/K_1. $K_1 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}\left(\frac{I_A}{4}\right)\left(\frac{\omega_A}{2}\right)^2 = \frac{17}{32}I_A\omega_A^2$.

Using the result from (a) gives $K_2 = \frac{1}{2}I_{A+B}\omega^2 = \frac{1}{2}\left(I_A + \frac{I_A}{4}\right)\left(\frac{7}{10}\omega_A\right)^2 = \frac{49}{160}I_A\omega_A^2$.

The fraction of the initial rotational kinetic energy that remains is $\frac{K_2}{K_1} = \frac{\frac{49}{160}I_A\omega_A^2}{\frac{17}{32}I_A\omega_A^2} = \frac{49}{85} = 0.576$.

EVALUATE: The disks stick together so kinetic energy is lost, just as when objects collide in an inelastic collision. In this case, only 57.6% of the kinetic energy remains, so 42.4% is lost.

VP10.12.3. **IDENTIFY:** During the collision, the hinge exerts a force on the door. But if we look at the angular momentum about the hinge, the hinge exerts no torque, so the angular momentum of the bullet-door system is conserved about the hinge.

SET UP: About the hinge $L_{\text{bullet}} = mvd$, $L_{\text{door}} = I_{\text{door}}\,\omega$, and $I_{\text{door}} = \frac{1}{3}Md^2$. $K = \frac{1}{2}I\omega^2$ for rotation and $K = \frac{1}{2}mv^2$ for linear motion. Figure VP10.12.3 shows before and after sketches.

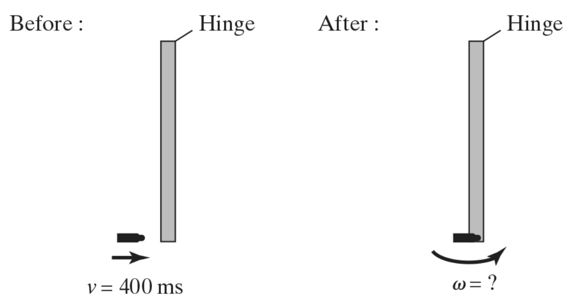


Figure VP10.12.3

EXECUTE: **(a)** We want the angular velocity ω just after the bullet hits the door, so use conservation of angular momentum about the hinge. $L_{\text{bullet}} + L_{\text{door}} = L_{\text{door + bullet}}$.

$$mvd = (I_{\text{door}} + I_{\text{bullet}})\,\omega = \left(\frac{1}{3}Md^2 + md^2\right)\omega$$

$$\omega = \frac{mv}{d\left(\frac{M}{3} + m\right)} = \frac{(0.0100\text{ kg})(400\text{ m/s})}{(1.00\text{ m})\left(\frac{15\text{ kg}}{3} + 0.0100\text{ kg}\right)} = 0.80\text{ rad/s}.$$

(b) $K_1 = K_{\text{bullet}} = \frac{1}{2}mv^2 = \frac{1}{2}(0.0100\text{ kg})(400\text{ m/s})^2 = 800\text{ J}$

$$K_2 = K_{\text{bullet}} + K_{\text{door}} = \frac{1}{2}\left(md^2\omega^2 + \frac{1}{3}Md^2\omega^2\right) = \left(m + \frac{M}{3}\right)\frac{(d\omega)^2}{2}$$

$$K_2 = \left(0.010\text{ kg} + \frac{15\text{ kg}}{3}\right)\frac{[(1.00\text{ m})(0.80\text{ rad/s})]^2}{2} = 1.6\text{ J}$$

$K_2/K_1 = (1.6\text{ J})/(800\text{ J}) = 0.0020 = 1/500$.

EVALUATE: This is a *very* inelastic collision. Only 0.20% of the original kinetic energy remained, so 99.8% was lost.

VP10.12.4. **IDENTIFY:** During the collision, the angular momentum of the clay-sphere system is conserved about the vertical shaft.

SET UP: About the shaft $L_{\text{clay}} = MvR$, $L_{\text{sphere}} = I_{\text{sphere}}\,\omega$, and $I_{\text{sphere}} = \frac{2}{3}MR^2$. $K = \frac{1}{2}I\omega^2$ for rotation and $K = \frac{1}{2}mv^2$ for linear motion.

EXECUTE: **(a)** Before: $L = MvR$. After: $L = M(v/2)R = \frac{1}{2}MvR$.

(b) The clay clay lost half of its angular momentum, so the sphere must have gained that amount by conservation of angular momentum. Therefore $L_{\text{sphere}} = \frac{1}{2}MvR = I_{\text{sphere}}\,\omega$. This gives $\frac{2}{3}MR^2\,\omega = \frac{1}{2}MvR$, so $\omega = \dfrac{3v}{4R}$.

(c) $K_1 = K_{\text{clay}} = \frac{1}{2}Mv^2$

$$K_2 = K_{\text{clay}} + K_{\text{sphere}} = \frac{1}{2}M(v/2)^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}M\left(\frac{v}{2}\right)^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{3v}{4R}\right)^2 = \frac{5}{16}Mv^2$$

$$\frac{K_2}{K_1} = \frac{\frac{5}{16}Mv^2}{\frac{1}{2}Mv^2} = \frac{5}{8}.$$

EVALUATE: The system lost 3/8 of its kinetic energy during this collision, so it was *not* an elastic collision.

10.1. **IDENTIFY:** Use $\tau = Fl$ to calculate the magnitude of the torque and use the right-hand rule illustrated in Section 10.1 in the textbook to calculate the torque direction.

(a) SET UP: Consider Figure 10.1a.

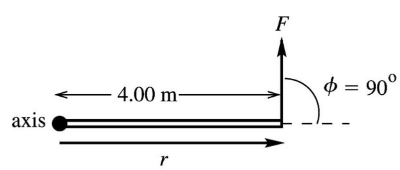


Figure 10.1a

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 90°$

$l = 4.00\text{ m}$

$\tau = (10.0\text{ N})(4.00\text{ m}) = 40.0\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(b) SET UP: Consider Figure 10.1b.

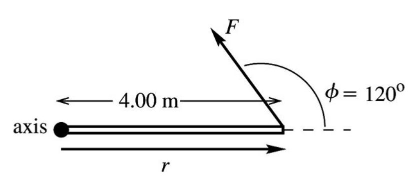


Figure 10.1b

EXECUTE: $\tau = Fl$

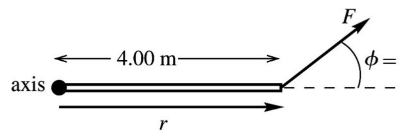
$l = r\sin\phi = (4.00\text{ m})\sin 120°$

$l = 3.464\text{ m}$

$\tau = (10.0\text{ N})(3.464\text{ m}) = 34.6\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(c) SET UP: Consider Figure 10.1c.



EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 30^\circ$

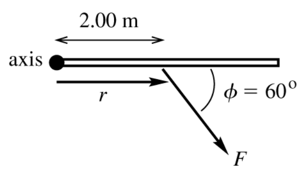
$l = 2.00\text{ m}$

$\tau = (10.0\text{ N})(2.00\text{ m}) = 20.0\text{ N}\cdot\text{m}$

Figure 10.1c

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(d) SET UP: Consider Figure 10.1d.



EXECUTE: $\tau = Fl$

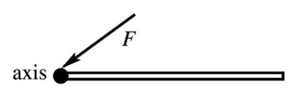
$l = r\sin\phi = (2.00\text{ m})\sin 60^\circ = 1.732\text{ m}$

$\tau = (10.0\text{ N})(1.732\text{ m}) = 17.3\text{ N}\cdot\text{m}$

Figure 10.1d

This force tends to produce a clockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed into the plane of the figure.

(e) SET UP: Consider Figure 10.1e.

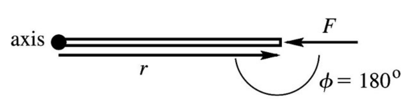


EXECUTE: $\tau = Fl$

$r = 0$ so $l = 0$ and $\tau = 0$

Figure 10.1e

(f) SET UP: Consider Figure 10.1f.



EXECUTE: $\tau = Fl$

$l = r\sin\phi,\ \ \phi = 180^\circ,$

so $l = 0$ and $\tau = 0$

Figure 10.1f

EVALUATE: The torque is zero in parts (e) and (f) because the moment arm is zero; the line of action of the force passes through the axis.

10.2. **IDENTIFY:** $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

SET UP: Let counterclockwise torques be positive.

EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}.$

$\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0^\circ = +12.0\text{ N}\cdot\text{m}.$ $\sum\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}.$ The net torque is $28.0\text{ N}\cdot\text{m},$ clockwise.

EVALUATE: Even though $F_1 < F_2,$ the magnitude of τ_1 is greater than the magnitude of $\tau_2,$ because F_1 has a larger moment arm.

10.3. IDENTIFY and SET UP: Use $\tau = Fl$ to calculate the magnitude of each torque and use the right-hand rule (Figure 10.4 in the textbook) to determine the direction. Consider Figure 10.3.

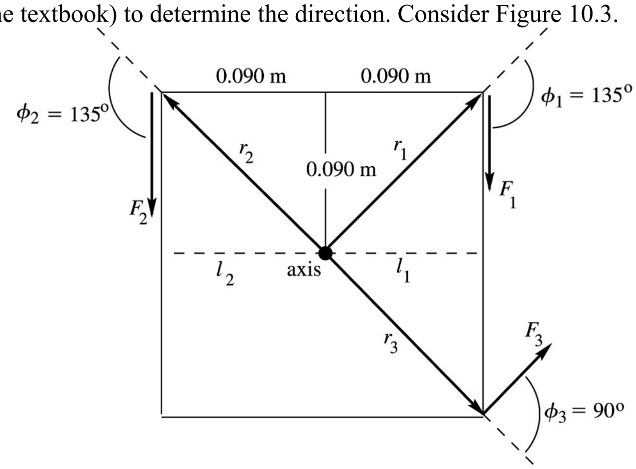


Figure 10.3

Let counterclockwise be the positive sense of rotation.

EXECUTE: $r_1 = r_2 = r_3 = \sqrt{(0.090 \text{ m})^2 + (0.090 \text{ m})^2} = 0.1273 \text{ m}$

$\tau_1 = -F_1 l_1$

$l_1 = r_1 \sin\phi_1 = (0.1273 \text{ m})\sin 135° = 0.0900 \text{ m}$

$\tau_1 = -(18.0 \text{ N})(0.0900 \text{ m}) = -1.62 \text{ N} \cdot \text{m}$

$\vec{\tau}_1$ is directed into paper

$\tau_2 = +F_2 l_2$

$l_2 = r_2 \sin\phi_2 = (0.1273 \text{ m})\sin 135° = 0.0900 \text{ m}$

$\tau_2 = +(26.0 \text{ N})(0.0900 \text{ m}) = +2.34 \text{ N} \cdot \text{m}$

$\vec{\tau}_2$ is directed out of paper

$\tau_3 = +F_3 l_3$

$l_3 = r_3 \sin\phi_3 = (0.1273 \text{ m})\sin 90° = 0.1273 \text{ m}$

$\tau_3 = +(14.0 \text{ N})(0.1273 \text{ m}) = +1.78 \text{ N} \cdot \text{m}$

$\vec{\tau}_3$ is directed out of paper

$\sum\tau = \tau_1 + \tau_2 + \tau_3 = -1.62 \text{ N} \cdot \text{m} + 2.34 \text{ N} \cdot \text{m} + 1.78 \text{ N} \cdot \text{m} = 2.50 \text{ N} \cdot \text{m}$

EVALUATE: The net torque is positive, which means it tends to produce a counterclockwise rotation; the vector torque is directed out of the plane of the paper. In summing the torques it is important to include + or − signs to show direction.

10.4. IDENTIFY: Use $\tau = Fl = rF\sin\phi$ to calculate the magnitude of each torque and use the right-hand rule to determine the direction of each torque. Add the torques to find the net torque.

SET UP: Let counterclockwise torques be positive. For the 11.9 N force (F_1), $r = 0$. For the 14.6 N force (F_2), $r = 0.350 \text{ m}$ and $\phi = 40.0°$. For the 8.50 N force (F_3), $r = 0.350 \text{ m}$ and $\phi = 90.0°$.

EXECUTE: $\tau_1 = 0$. $\tau_2 = -(14.6 \text{ N})(0.350 \text{ m})\sin 40.0° = -3.285 \text{ N} \cdot \text{m}$.

$\tau_3 = +(8.50 \text{ N})(0.350 \text{ m})\sin 90.0° = +2.975 \text{ N} \cdot \text{m}$. $\sum\tau = -3.285 \text{ N} \cdot \text{m} + 2.975 \text{ N} \cdot \text{m} = -0.31 \text{ N} \cdot \text{m}$. The net torque is $0.31 \text{ N} \cdot \text{m}$ and is clockwise.

EVALUATE: If we treat the torques as vectors, $\vec{\tau}_2$ is into the page and $\vec{\tau}_3$ is out of the page.

10.5. **IDENTIFY** and **SET UP:** Calculate the torque using Eq. (10.3) and also determine the direction of the torque using the right-hand rule.

(a) $\vec{r} = (-0.450\text{ m})\hat{i} + (0.150\text{ m})\hat{j}$; $\vec{F} = (-5.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j}$. The sketch is given in Figure 10.5.

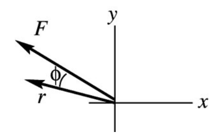


Figure 10.5

EXECUTE: **(b)** When the fingers of your right hand curl from the direction of $\vec{r}$ into the direction of $\vec{F}$ (through the smaller of the two angles, angle ϕ) your thumb points into the page (the direction of $\vec{\tau}$, the $-z$-direction).

(c) $\vec{\tau} = \vec{r} \times \vec{F} = [(-0.450\text{ m})\hat{i} + (0.150\text{ m})\hat{j}] \times [(-5.00\text{ N})\hat{i} + (4.00\text{ N})\hat{j}]$

$\vec{\tau} = +(2.25\text{ N}\cdot\text{m})\hat{i}\times\hat{i} - (1.80\text{ N}\cdot\text{m})\hat{i}\times\hat{j} - (0.750\text{ N}\cdot\text{m})\hat{j}\times\hat{i} + (0.600\text{ N}\cdot\text{m})\hat{j}\times\hat{j}$

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$

$\hat{i}\times\hat{j} = \hat{k},\;\; \hat{j}\times\hat{i} = -\hat{k}$

Thus $\vec{\tau} = -(1.80\text{ N}\cdot\text{m})\hat{k} - (0.750\text{ N}\cdot\text{m})(-\hat{k}) = (-1.05\text{ N}\cdot\text{m})\hat{k}$.

EVALUATE: The calculation gives that $\vec{\tau}$ is in the $-z$-direction. This agrees with what we got from the right-hand rule.

10.6. **IDENTIFY:** Knowing the force on a bar and the point where it acts, we want to find the position vector for the point where the force acts and the torque the force exerts on the bar.

SET UP: The position vector is $\vec{r} = x\hat{i} + y\hat{j}$ and the torque is $\vec{\tau} = \vec{r}\times\vec{F}$.

EXECUTE: **(a)** Using $x = 3.00\text{ m}$ and $y = 4.00\text{ m}$, we have $\vec{r} = (3.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j}$.

(b) $\vec{\tau} = \vec{r}\times\vec{F} = [(3.00\text{ m})\hat{i} + (4.00\text{ m})\hat{j}]\times[(7.00\text{ N})\hat{i} + (-3.00\text{ N})\hat{j}]$.

$\vec{\tau} = (-9.00\text{ N}\cdot\text{m})\hat{k} + (-28.0\text{ N}\cdot\text{m})(-\hat{k}) = (-37.0\text{ N}\cdot\text{m})\hat{k}$. The torque has magnitude $37.0\text{ N}\cdot\text{m}$ and is in the $-z$-direction.

EVALUATE: Applying the right-hand rule for the vector product to $\vec{r}\times\vec{F}$ shows that the torque must be in the $-z$-direction because it is perpendicular to both $\vec{r}$ and $\vec{F}$, which are both in the x-y plane.

10.7. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ for the magnitude of the torque and the right-hand rule for the direction.

SET UP: In part (a), $r = 0.250\text{ m}$ and $\phi = 37°$.

EXECUTE: **(a)** $\tau = (17.0\text{ N})(0.250\text{ m})\sin 37° = 2.56\text{ N}\cdot\text{m}$. The torque is counterclockwise.

(b) The torque is maximum when $\phi = 90°$ and the force is perpendicular to the wrench. This maximum torque is $(17.0\text{ N})(0.250\text{ m}) = 4.25\text{ N}\cdot\text{m}$.

EVALUATE: If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

10.8. **IDENTIFY:** The constant force produces a torque which gives a constant angular acceleration to the disk and a linear acceleration to points on the disk.

SET UP: $\sum\tau_z = I\alpha_z$ applies to the disk, $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ because the angular acceleration is constant. The acceleration components of the rim are $a_{\text{tan}} = r\alpha$ and $a_{\text{rad}} = r\omega^2$, and the magnitude of the acceleration is $a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2}$.

EXECUTE: **(a)** $\sum\tau_z = I\alpha_z$ gives $Fr = I\alpha_z$. For a uniform disk,

$I = \frac{1}{2}MR^2 = \frac{1}{2}(40.0\text{ kg})(0.200\text{ m})^2 = 0.800\text{ kg}\cdot\text{m}^2$. $\alpha_z = \dfrac{Fr}{I} = \dfrac{(30.0\text{ N})(0.200\text{ m})}{0.800\text{ kg}\cdot\text{m}^2} = 7.50\text{ rad/s}^2$.

$\theta - \theta_0 = 0.200\text{ rev} = 1.257\text{ rad}$. $\omega_{0z} = 0$, so $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives

$\omega_z = \sqrt{2(7.50\text{ rad/s}^2)(1.257\text{ rad})} = 4.342\text{ rad/s}$. $v = r\omega = (0.200\text{ m})(4.342\text{ rad/s}) = 0.868\text{ m/s}$.

(b) $a_{\text{tan}} = r\alpha = (0.200\text{ m})(7.50\text{ rad/s}^2) = 1.50\text{ m/s}^2$. $a_{\text{rad}} = r\omega^2 = (0.200\text{ m})(4.342\text{ rad/s})^2 = 3.771\text{ m/s}^2$.

$a = \sqrt{a_{\text{tan}}^2 + a_{\text{rad}}^2} = 4.06\text{ m/s}^2$.

EVALUATE: The net acceleration is neither toward the center nor tangent to the disk.

10.9. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$.

SET UP: $\omega_{0z} = 0$. $\omega_z = (400\text{ rev/min})\left(\dfrac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 41.9\text{ rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = (1.60\text{ kg}\cdot\text{m}^2)\dfrac{41.9\text{ rad/s}}{8.00\text{ s}} = 8.38\text{ N}\cdot\text{m}$.

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in rad/s^2.

10.10. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the wheel. The acceleration a of a point on the cord and the angular acceleration α of the wheel are related by $a = R\alpha$.

SET UP: Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \frac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. P is the pull on the cord and F is the force exerted on the wheel by the axle.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{(40.0\text{ N})(0.250\text{ m})}{\frac{1}{2}(9.20\text{ kg})(0.250\text{ m})^2} = 34.8\text{ rad/s}^2$.

$a = R\alpha = (0.250\text{ m})(34.8\text{ rad/s}^2) = 8.70\text{ m/s}^2$.

(b) $F_x = -P$, $F_y = Mg$. $F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0\text{ N})^2 + ([9.20\text{ kg}][9.80\text{ m/s}^2])^2} = 98.6\text{ N}$.

$\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{Mg}{P} = \dfrac{(9.20\text{ kg})(9.80\text{ m/s}^2)}{40.0\text{ N}}$ and $\phi = 66.1°$. The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.

(c) The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20\text{ kg})(9.80\text{ m/s}^2) - 40.0\text{ N} = 50.2\text{ N}$. The force exerted by the axle has magnitude 50.2 N and is upward.

EVALUATE: The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

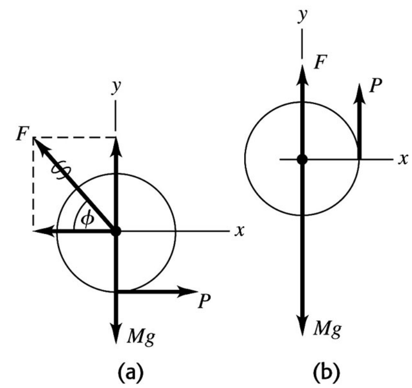


Figure 10.10

10.11. **IDENTIFY:** Use $\sum \tau_z = I\alpha_z$ to calculate α. Use a constant angular acceleration kinematic equation to relate α_z, ω_z, and t.

SET UP: For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150\text{ m}$ and the torque due to this force is negative.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{-(0.0200\text{ N})(0.0150\text{ m})}{\frac{2}{5}(0.225\text{ kg})(0.0150\text{ m})^2} = -14.8\text{ rad/s}^2$

(b) $\omega_z - \omega_{0z} = -22.5\text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{-22.5\text{ rad/s}}{-14.8\text{ rad/s}^2} = 1.52\text{ s}$.

EVALUATE: The fact that α_z is negative means its direction is opposite to the direction of spin. The negative α_z causes ω_z to decrease.

10.12. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the stone and $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find a for the stone.

SET UP: For the motion of the stone take $+y$ to be downward. The pulley has $I = \frac{1}{2}MR^2$. $a = R\alpha$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $12.6\text{ m} = \frac{1}{2}a_y(3.00\text{ s})^2$ and $a_y = 2.80\text{ m/s}^2$.

Then $\sum F_y = ma_y$ applied to the stone gives $mg - T = ma$.

$\sum \tau_z = I\alpha_z$ applied to the pulley gives $TR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2(a/R)$. $T = \frac{1}{2}Ma$.

Combining these two equations to eliminate T gives

$$m = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\text{ kg}}{2}\right)\left(\frac{2.80\text{ m/s}^2}{9.80\text{ m/s}^2 - 2.80\text{ m/s}^2}\right) = 2.00\text{ kg}.$$

(b) $T = \frac{1}{2}Ma = \frac{1}{2}(10.0\text{ kg})(2.80\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: The tension in the wire is less than the weight $mg = 19.6\text{ N}$ of the stone, because the stone has a downward acceleration.

10.13. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each book and apply $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find the common acceleration of the books.

SET UP: $m_1 = 2.00\text{ kg}$, $m_2 = 3.00\text{ kg}$. Let T_1 be the tension in the part of the cord attached to m_1 and T_2 be the tension in the part of the cord attached to m_2. Let the $+x$-direction be in the direction of the acceleration of each book. $a = R\alpha$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(1.20\text{ m})}{(0.800\text{ s})^2} = 3.75\text{ m/s}^2$. $a_1 = 3.75\text{ m/s}^2$ so

$T_1 = m_1 a_1 = 7.50\text{ N}$ and $T_2 = m_2(g - a_1) = 18.2\text{ N}$.

(b) The torque on the pulley is $(T_2 - T_1)R = 0.803\text{ N}\cdot\text{m}$, and the angular acceleration is

$\alpha = a_1/R = 50\text{ rad/s}^2$, so $I = \tau/\alpha = 0.016\text{ kg}\cdot\text{m}^2$.

EVALUATE: The tensions in the two parts of the cord must be different, so there will be a net torque on the pulley.

10.14. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the bucket, with $+y$ downward. Apply $\sum \tau_z = I\alpha_z$ to the cylinder, with the direction the cylinder rotates positive.

SET UP: The free-body diagram for the bucket is given in Figure 10.14a and the free-body diagram for the cylinder is given in Figure 10.14b. $I = \frac{1}{2}MR^2$. $a(\text{bucket}) = R\alpha(\text{cylinder})$

EXECUTE: **(a)** For the bucket, $mg - T = ma$. For the cylinder, $\sum \tau_z = I\alpha_z$ gives $TR = \frac{1}{2}MR^2\alpha$.

$\alpha = a/R$ then gives $T = \frac{1}{2}Ma$. Combining these two equations gives $mg - \frac{1}{2}Ma = ma$ and

$$a = \frac{mg}{m + M/2} = \left(\frac{15.0\text{ kg}}{15.0\text{ kg} + 6.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 7.00\text{ m/s}^2.$$

$T = m(g - a) = (15.0\text{ kg})(9.80\text{ m/s}^2 - 7.00\text{ m/s}^2) = 42.0\text{ N}$.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(7.00\text{ m/s}^2)(10.0\text{ m})} = 11.8\text{ m/s}$.

(c) $a_y = 7.00\text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = 10.0\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}\alpha_y t^2$ gives

$$t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(10.0\text{ m})}{7.00\text{ m/s}^2}} = 1.69\text{ s}$$

(d) $\sum F_y = ma_y$ applied to the cylinder gives $n - T - Mg = 0$ and

$n = T + mg = 42.0\text{ N} + (12.0\text{ kg})(9.80\text{ m/s}^2) = 160\text{ N}$.

EVALUATE: The tension in the rope is less than the weight of the bucket, because the bucket has a downward acceleration. If the rope were cut, so the bucket would be in free fall, the bucket would strike the water in $t = \sqrt{\dfrac{2(10.0\text{ m})}{9.80\text{ m/s}^2}} = 1.43\text{ s}$ and would have a final speed of 14.0 m/s. The presence of the cylinder slows the fall of the bucket.

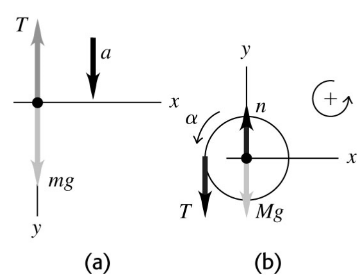


Figure 10.14

10.15. **IDENTIFY:** The constant force produces a torque which gives a constant angular acceleration to the wheel.

SET UP: $\omega_z = \omega_{0z} + \alpha_z t$ because the angular acceleration is constant, and $\sum \tau_z = I\alpha_z$ applies to the wheel.

EXECUTE: $\omega_{0z} = 0$ and $\omega_z = 12.0$ rev/s $= 75.40$ rad/s. $\omega_z = \omega_{0z} + \alpha_z t$, so

$\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{75.40 \text{ rad/s}}{2.00 \text{ s}} = 37.70 \text{ rad/s}^2$. $\sum\tau_z = I\alpha_z$ gives

$I = \dfrac{Fr}{\alpha_z} = \dfrac{(80.0 \text{ N})(0.120 \text{ m})}{37.70 \text{ rad/s}^2} = 0.255 \text{ kg}\cdot\text{m}^2$.

EVALUATE: The units of the answer are the proper ones for moment of inertia.

10.16. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each box and $\sum\tau_z = I\alpha_z$ to the pulley. The magnitude a of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha$.

SET UP: The free-body diagrams for each object are shown in Figure 10.16. For the pulley, $R = 0.250$ m and $I = \frac{1}{2}MR^2$. T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0$ kg, $m_2 = 5.00$ kg and $M = 2.00$ kg. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum\tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$. Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$a = \left(\dfrac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\dfrac{5.00 \text{ kg}}{12.0 \text{ kg} + 5.00 \text{ kg} + 1.00 \text{ kg}}\right)(9.80 \text{ m/s}^2) = 2.72 \text{ m/s}^2$. Then

$T_1 = m_1 a = (12.0 \text{ kg})(2.72 \text{ m/s}^2) = 32.6$ N. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00 \text{ kg})(9.80 \text{ m/s}^2 - 2.72 \text{ m/s}^2) = 35.4$ N. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72 \text{ m/s}^2$

(c) For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00 \text{ kg})(9.80 \text{ m/s}^2) + 35.4 \text{ N} = 55.0$ N.

EVALUATE: The equation $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force $m_2 g$ must accelerate all three objects.

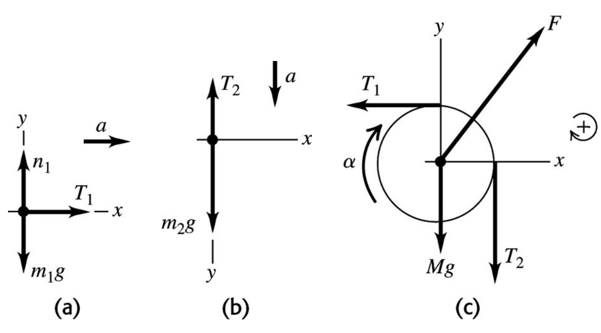


Figure 10.16

10.17. **IDENTIFY:** The rotational form of Newton's second law applies to the cylinder. Interpretation of graphical data is necessary.

SET UP: Since $\theta - \theta_0$ is proportional to t^2, the equation $\theta - \theta_0 = \omega_{0z}t + \dfrac{1}{2}\alpha_z t^2$ applies to the rotational motion. Since the rotation starts from rest, $\omega_{0z} = 0$, so we have $\theta - \theta_0 = \dfrac{1}{2}\alpha_z t^2$. Therefore a graph of

$\theta - \theta_0$ versus t^2 should be a straight line having slope $\frac{1}{2}\alpha_z$. Our target variable is the moment of inertia of the cylinder. Once we know α_z, we can apply $\sum \tau_z = I\alpha_z$ to find I.

EXECUTE: Use the slope of the graph to find α_z. As we discussed above, slope = $\frac{1}{2}\alpha_z$, so α_z = 2(slope) = 2(16.0 rad/s^2) = 32.0 rad/s^2. Now use $\sum \tau_z = I\alpha_z$ to find I. $FR = I\alpha_z$, so

$$I = \frac{FR}{\alpha_z} = \frac{(3.00\text{ N})(0.140\text{ m})}{32.0\text{ rad/s}^2} = 0.0131\ \text{kg}\cdot\text{m}^2.$$

EVALUATE: Using the slope to find α_z is more accurate than using individual data points because individual measurements vary, but finding the slope essentially "averages out" the data points.

10.18. **IDENTIFY:** The spheres have kinetic energy due to the motion of their center of mass and the rotation about the center of mass.

SET UP: $K_{\text{total}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$ for a solid sphere, and $v_{\text{cm}} = r\omega$ when there is no slipping. The work done on each sphere is equal to its loss of kinetic energy. The work is the target variable.

EXECUTE: $W_A = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$.

$W_A = \frac{7}{10}mv_{\text{cm}}^2 = \frac{7}{10}(5.00\text{ kg})(4.00\text{ m/s})^2 = 56.0$ J. A similar calculation for sphere B, except using $I = \frac{2}{3}MR^2$, gives $W_B = \frac{5}{6}mv_{\text{cm}}^2 = \frac{5}{6}(5.00\text{ kg})(4.00\text{ m/s})^2 = 66.7$ J.

EVALUATE: Sphere B requires more work because it has a larger moment of inertia than A. Even though both spheres have the same size, mass, and linear speed, B's mass is farther from the rotation axis so its moment of inertia is greater than that of A.

10.19. **IDENTIFY:** Since there is rolling without slipping, $v_{\text{cm}} = R\omega$. The kinetic energy is given by $K_{\text{tot}} = K_{\text{cm}} + K_{\text{rot}}$ where $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$ and $K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2$. The velocity of any point on the rim of the hoop is the vector sum of the tangential velocity of the rim and the velocity of the center of mass of the hoop.

SET UP: $\omega = 2.60$ rad/s and $R = 0.600$ m. For a hoop rotating about an axis at its center, $I = MR^2$.

EXECUTE: **(a)** $v_{\text{cm}} = R\omega = (0.600\text{ m})(2.60\text{ rad/s}) = 1.56$ m/s.

(b) $K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}(MR^2)(v_{\text{cm}}/R^2) = Mv_{\text{cm}}^2 = (2.20\text{ kg})(1.56\text{ m/s})^2 = 5.35$ J

(c) (i) $v = 2v_{\text{cm}} = 3.12$ m/s. $\vec{v}$ is to the right. (ii) $v = 0$

(iii) $v = \sqrt{v_{\text{cm}}^2 + v_{\tan}^2} = \sqrt{v_{\text{cm}}^2 + (R\omega)^2} = \sqrt{2}v_{\text{cm}} = 2.21$ m/s. $\vec{v}$ at this point is at $45°$ below the horizontal.

(d) To someone moving to the right at $v = v_{\text{cm}}$, the hoop appears to rotate about a stationary axis at its center. (i) $v = R\omega = 1.56$ m/s, to the right. (ii) $v = 1.56$ m/s, to the left. (iii) $v = 1.56$ m/s, downward.

EVALUATE: For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass. In the rest frame of the ground, different points on the hoop have different speed.

10.20. **IDENTIFY:** Newton's second law applies to the sphere rolling down the incline.

SET UP: From Example 10.7, we have $f_s = \frac{2}{7}Mg\sin\beta$. For maximum static friction, $f_s = \mu_s n$. The target variable is μ_s.

EXECUTE: **(a)** Balancing forces perpendicular to the surface of the incline gives $n = Mg\cos\beta$. Using this in the result from Example 10.7 gives $\mu_s Mg\cos\beta = \frac{2}{7}Mg\sin\beta$. Solving for μ_s we get $\mu_s = \frac{2}{7}\tan\beta$.

(b) As $\beta \to 0°$, $\mu_s \to 0$. As $\beta \to 90°$, $\mu_s \to \infty$.

EVALUATE: The results in part (a) are reasonable. As $\beta \to 0°$, the ball will roll on a smooth horizontal surface. If it is already moving and rotating, it will continue to do so. If it is not already rotating, it will not start to do so. As $\beta \to 90°$, $\mu_s \to \infty$. As $\beta \to 0°$, the normal force gets smaller and smaller, so we would need a larger and larger value of μ_s to prevent slipping.

10.21. **IDENTIFY:** Apply $K = K_{cm} + K_{rot}$.

SET UP: For an object that is rolling without slipping, $v_{cm} = R\omega$.

EXECUTE: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1 + (M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1 + (MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is 1/3.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is $2/7$.

(c) $I_{cm} = (2/3)MR^2$ so the ratio is $2/5$.

(d) $I_{cm} = (5/8)MR^2$ so the ratio is $5/13$.

EVALUATE: The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\frac{1}{1+1/\beta} = \frac{\beta}{1+\beta}$. The ratio increases as β increases.

10.22. **IDENTIFY:** Only gravity does work, so $W_{other} = 0$ and conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$.

SET UP: Let $y_2 = 0$, so $U_2 = 0$ and $y_1 = 0.750$ m. The hoop is released from rest so $K_1 = 0$. $v_{cm} = R\omega$. For a hoop with an axis at its center, $I_{cm} = MR^2$.

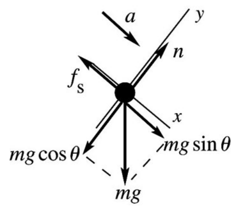
EXECUTE: **(a)** Conservation of energy gives $U_1 = K_2$. $K_2 = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_1$. $\omega = \frac{\sqrt{gy_1}}{R} = \frac{\sqrt{(9.80\text{ m/s}^2)(0.750\text{ m})}}{0.0800\text{ m}} = 33.9\text{ rad/s}$.

(b) $v = R\omega = (0.0800\text{ m})(33.9\text{ rad/s}) = 2.71\text{ m/s}$

EVALUATE: An object released from rest and falling in free fall for 0.750 m attains a speed of $\sqrt{2g(0.750\text{ m})} = 3.83\text{ m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.23. **IDENTIFY:** Apply $\sum \vec{F}_{ext} = m\vec{a}_{cm}$ and $\sum \tau_z = I_{cm}\alpha_z$ to the motion of the ball.

(a) SET UP: The free-body diagram is given in Figure 10.23a.



EXECUTE: $\sum F_y = ma_y$

$n = mg\cos\theta$ and $f_s = \mu_s mg\cos\theta$

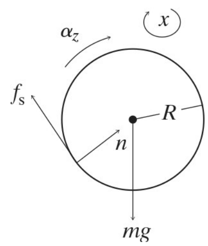
$\sum F_x = ma_x$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (Eq. 1)

Figure 10.23a

SET UP: Consider Figure 10.23b.



The normal force n is directed through the center of the ball and mg acts at the center of the ball, so neither of them produces a torque about the center.

Figure 10.23b

EXECUTE: $\sum\tau = \tau_f = \mu_s mg(\cos\theta)R$; $I = \frac{2}{5}mR^2$

$\sum\tau_z = I_{cm}\alpha_z$ gives $\mu_s\, mg(\cos\theta)R = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (Eq. 2)

We have two equations in the two unknowns a and μ_s. Solving gives $a = \frac{5}{7}g\sin\theta$ and

$\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613.$

(b) Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$. $a = \frac{3}{5}g\sin\theta$ and

$\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of μ_s calculated in part (a) is not large enough to prevent slipping for the hollow ball.

(c) EVALUATE: There is no slipping at the point of contact. More friction is required for a hollow ball since for a given m and R it has a larger I and more torque is needed to provide the same α. Note that the required μ_s is independent of the mass or radius of the ball and only depends on how that mass is distributed.

10.24. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the translational motion of the center of mass and $\sum\tau_z = I\alpha_z$ to the rotation about the center of mass.

SET UP: Let $+x$ be down the incline and let the shell be turning in the positive direction. The free-body diagram for the shell is given in Figure 10.24. From Table 9.2, $I_{cm} = \frac{2}{3}mR^2$.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{cm}$. $\sum\tau_z = I\alpha_z$ gives $fR = \left(\frac{2}{3}mR^2\right)\alpha$. With $\alpha = a_{cm}/R$ this becomes $f = \frac{2}{3}ma_{cm}$. Combining the equations gives $mg\sin\beta - \frac{2}{3}ma_{cm} = ma_{cm}$ and

$$a_{cm} = \frac{3g\sin\beta}{5} = \frac{3(9.80\text{ m/s}^2)(\sin 38.0°)}{5} = 3.62\text{ m/s}^2.\quad f = \tfrac{2}{3}ma_{cm} = \tfrac{2}{3}(2.00\text{ kg})(3.62\text{ m/s}^2) = 4.83\text{ N}.$$

The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45$ N.

$$\mu_s = \frac{f}{n} = \frac{4.83\text{ N}}{15.45\text{ N}} = 0.313.$$

(b) The acceleration is independent of m and doesn't change. The friction force is proportional to m so will double; $f = 9.66$ N. The normal force will also double, so the minimum μ_s required for no slipping wouldn't change.

EVALUATE: If there is no friction and the object slides without rolling, the acceleration is $g \sin\beta$. Friction and rolling without slipping reduce a to 0.60 times this value.

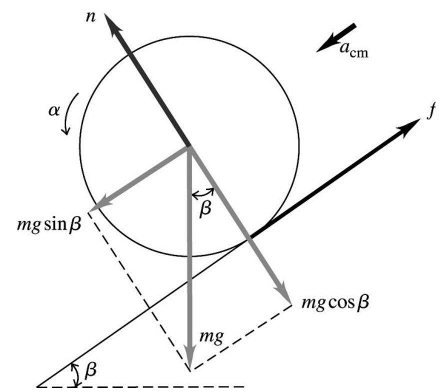
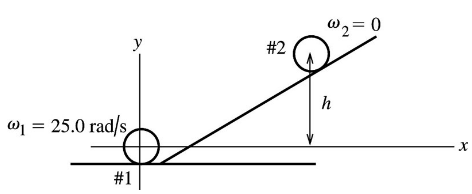


Figure 10.24

10.25. **IDENTIFY:** Apply conservation of energy to the motion of the wheel.

SET UP: The wheel at points 1 and 2 of its motion is shown in Figure 10.25.



Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

Figure 10.25

The wheel has both translational and rotational motion so its kinetic energy is $K = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}Mv_{cm}^2$.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

$W_{other} = W_{fric} = -2600$ J (the friction work is negative)

$K_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}Mv_1^2$; $v = R\omega$ and $I = 0.800MR^2$ so

$K_1 = \frac{1}{2}(0.800)MR^2\omega_1^2 + \frac{1}{2}MR^2\omega_1^2 = 0.900MR^2\omega_1^2$

$K_2 = 0$, $U_1 = 0$, $U_2 = Mgh$

Thus $0.900MR^2\omega_1^2 + W_{fric} = Mgh$

$M = w/g = 392\text{ N}/(9.80\text{ m/s}^2) = 40.0$ kg

$$h = \frac{0.900MR^2\omega_1^2 + W_{fric}}{Mg}$$

$$h = \frac{(0.900)(40.0\text{ kg})(0.600\text{ m})^2(25.0\text{ rad/s})^2 - 2600\text{ J}}{(40.0\text{ kg})(9.80\text{ m/s}^2)} = 14.0\text{ m}.$$

EVALUATE: Friction does negative work and reduces h.

10.26. **IDENTIFY:** Apply conservation of energy to the motion of the marble.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}MR^2$. $v_{cm} = R\omega$ for no slipping.

Let $y = 0$ at the bottom of the bowl. The marble at its initial and final locations is sketched in Figure 10.26.

EXECUTE: **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$ and $v = \sqrt{\frac{10}{7}gh}$.

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{rot} = mgh' + K_{rot}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

(b) $mgh = mgh'$ so $h' = h$.

EVALUATE: **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.

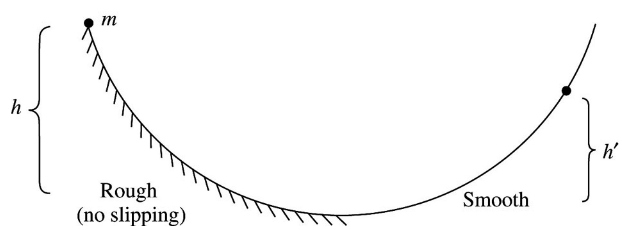


Figure 10.26

10.27. **IDENTIFY:** We want to investigate the kinetic energy of a bowling ball as it rolls down the bowling lane.

SET UP: $K_{total} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$, $I = \frac{2}{5}MR^2$ for a solid sphere, and $v_{cm} = r\omega$ when there is no slipping. Estimate: It takes 5.0 s to travel the 60 ft.

EXECUTE: **(a)** The target variable is the rotation rate of the ball. v_{cm} = (60 ft)/(5.0 s) = 12 ft/s. With no slipping $v_{cm} = r\omega$, so $\omega = \frac{v_{cm}}{R} = \frac{12\text{ ft/s}}{\left(\frac{8.5}{2}\right)\left(\frac{1}{12}\right)\text{ft}}$ = 34 rad/s = 5.4 rev/s.

(b) We want to find out what fraction of the ball's kinetic energy is rotational.

$K_{tot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2$, which gives

$K_{tot} = \frac{7}{10}mv_{cm}^2 = \frac{7}{10}\left(\frac{12\text{ lb}}{32.2\text{ ft/s}^2}\right)(12\text{ ft/s})^2 = 37.6\text{ ft}\cdot\text{lb}$. The fraction that is rotational is

$$\frac{K_{rot}}{K_{tot}} = \frac{\frac{1}{2}I\omega^2}{\frac{7}{10}mv_{cm}^2} = \frac{\frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2}{\frac{7}{10}mv_{cm}^2} = \frac{2}{7}.$$ Therefore 2/7 of the kinetic energy is rotational.

EVALUATE: The distribution of the kinetic between rotational and translational forms depends only on the *geometry* of the ball, not on its mass or size. If the ball were hollow, its moment of inertia would be different which would give a different result for the fraction we just found.

10.28. **IDENTIFY:** We want to compare the kinetic energy of two rolling balls.

SET UP: $K_{\text{tot}} = \dfrac{1}{2}mv_{\text{cm}}^2 + \dfrac{1}{2}I\omega^2$, $I = \dfrac{2}{5}MR^2$ for a solid sphere, and $v_{\text{cm}} = r\omega$ when there is no slipping. We know the kinetic energy of ball 2 is 27.0 J, and our target variable is the kinetic energy of ball 1.

EXECUTE: $K_{\text{tot}} = \dfrac{1}{2}mv_{\text{cm}}^2 + \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}\left(\dfrac{2}{5}mR^2\right)\omega^2 = \dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}\left(\dfrac{2}{5}mR^2\right)\left(\dfrac{v_{\text{cm}}}{R}\right)^2$, which gives $K_1 = \dfrac{7}{10}m_1v_1^2$. We get a similar result for ball 2: $K_2 = \dfrac{7}{10}m_2v_2^2$. Taking the ratio gives

$$\frac{K_1}{K_2} = \frac{\frac{7}{10}m_1v_1^2}{\frac{7}{10}m_2v_2^2} = \left(\frac{m_1}{m_2}\right)\left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\frac{1}{2}m_2}{m_2}\right)\left(\frac{\frac{1}{3}v_2}{v_2}\right)^2 = \frac{1}{18}.$$ So $K_1 = K_2/18 = (27.0\text{ J})/18 = 1.50$ J.

EVALUATE: The result does *not* depend on the relative size of the balls since R cancels out.

10.29. **IDENTIFY:** As the cylinder falls, its potential energy is transformed into both translational and rotational kinetic energy. Its mechanical energy is conserved.

SET UP: The hollow cylinder has $I = \frac{1}{2}m(R_a^2 + R_b^2)$, where $R_a = 0.200$ m and $R_b = 0.350$ m. Use coordinates where $+y$ is upward and $y = 0$ at the initial position of the cylinder. Then $y_1 = 0$ and $y_2 = -d$, where d is the distance it has fallen. $v_{\text{cm}} = R\omega$. $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$ and $\omega_{0z} = 10.47$ rad/s,

EXECUTE: **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$, $U_1 = 0$. $0 = U_2 + K_2$ and $0 = -mgd + \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$. $\frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{2}m(R_a^2 + R_b^2)\right](v_{\text{cm}}/R_b)^2 = \frac{1}{4}m\left[1 + (R_a/R_b)^2\right]v_{\text{cm}}^2$, so

$\frac{1}{2}\left\{1 + \frac{1}{2}\left[1 + (R_a/R_b)^2\right]\right\}v_{\text{cm}}^2 = gd$ and

$$d = \frac{\left\{1 + \frac{1}{2}\left[1 + (R_a/R_b)^2\right]\right\}v_{\text{cm}}^2}{2g} = \frac{(1 + 0.663)(6.66\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 3.76\text{ m}.$$

(b) $K_2 = \frac{1}{2}mv_{\text{cm}}^2$ since there is no rotation. So $mgd = \frac{1}{2}mv_{\text{cm}}^2$ which gives

$v_{\text{cm}} = \sqrt{2gd} = \sqrt{2(9.80\text{ m/s}^2)(3.76\text{ m})} = 8.58\text{ m/s}.$

(c) In part (a) the cylinder has rotational as well as translational kinetic energy and therefore less translational speed at a given kinetic energy. The kinetic energy comes from a decrease in gravitational potential energy and that is the same, so in (a) the translational speed is less.

EVALUATE: If part (a) were repeated for a solid cylinder, $R_a = 0$ and $d = 3.39$ m. For a thin-walled hollow cylinder, $R_a = R_b$ and $d = 4.52$ cm. Note that all of these answers are independent of the mass m of the cylinder.

10.30. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and $\sum\vec{F} = m\vec{a}$ to the motion of the bowling ball.

SET UP: $a_{\text{cm}} = R\alpha$. $f_{\text{s}} = \mu_{\text{s}}n$. Let $+x$ be directed down the incline.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.30.

The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.

(b) The friction force results in an angular acceleration, given by $I\alpha = fR$. $\sum\vec{F} = m\vec{a}$ applied to the motion of the center of mass gives $mg\sin\beta - f = ma_{\text{cm}}$, and the acceleration and angular acceleration are related by $a_{\text{cm}} = R\alpha$.

Combining, $mg\sin\beta = ma_{\text{cm}}\left(1+\dfrac{I}{mR^2}\right) = ma_{\text{cm}}(7/5).$ $a_{\text{cm}} = (5/7)g\sin\beta.$

(c) From either of the above relations between f and a_{cm}, $f = \dfrac{2}{5}ma_{\text{cm}} = \dfrac{2}{7}mg\sin\beta \le \mu_s n = \mu_s mg\cos\beta.$

$\mu_s \ge (2/7)\tan\beta.$

EVALUATE: If $\mu_s = 0$, $a_{\text{cm}} = mg\sin\beta$. a_{cm} is less when friction is present. The ball rolls farther uphill when friction is present, because the friction removes the rotational kinetic energy and converts it to gravitational potential energy. In the absence of friction the ball retains the rotational kinetic energy that is has initially.

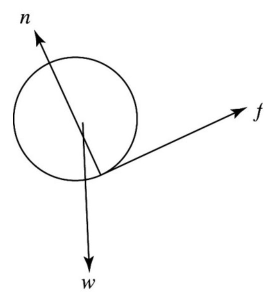


Figure 10.30

10.31. **IDENTIFY:** As the ball rolls up the hill, its kinetic energy (translational and rotational) is transformed into gravitational potential energy. Since there is no slipping, its mechanical energy is conserved.

SET UP: The ball has moment of inertia $I_{\text{cm}} = \frac{2}{3}mR^2$. Rolling without slipping means $v_{\text{cm}} = R\omega$. Use coordinates where $+y$ is upward and $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = h = 5.00$ m. The ball's kinetic energy is $K = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$ and its potential energy is $U = mgh$.

EXECUTE: **(a)** Conservation of energy gives $K_1 + U_1 = K_2 + U_2$. $U_1 = 0$, $K_2 = 0$ (the ball stops).

Therefore $K_1 = U_2$ and $\frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 = mgh$. $\frac{1}{2}I_{\text{cm}}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\dfrac{v_{\text{cm}}}{R}\right)^2 = \frac{1}{3}mv_{\text{cm}}^2$, so

$\frac{5}{6}mv_{\text{cm}}^2 = mgh$. Therefore $v_{\text{cm}} = \sqrt{\dfrac{6gh}{5}} = \sqrt{\dfrac{6(9.80\text{ m/s}^2)(5.00\text{ m})}{5}} = 7.67$ m/s and

$\omega = \dfrac{v_{\text{cm}}}{R} = \dfrac{7.67\text{ m/s}}{0.113\text{ m}} = 67.9$ rad/s.

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{3}mv_{\text{cm}}^2 = \frac{1}{3}(0.426\text{ kg})(7.67\text{ m/s})^2 = 8.35$ J.

EVALUATE: Its translational kinetic energy at the base of the hill is $\frac{1}{2}mv_{\text{cm}}^2 = \frac{3}{2}K_{\text{rot}} = 12.52$ J. Its total kinetic energy is 20.9 J, which equals its final potential energy:

$mgh = (0.426\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 20.9$ J.

10.32. **IDENTIFY:** Apply $P = \tau\omega$ and $W = \tau\Delta\theta$.

SET UP: P must be in watts, $\Delta\theta$ must be in radians, and ω must be in rad/s. 1 rev $= 2\pi$ rad. 1 hp $= 746$ W. π rad/s $= 30$ rev/min.

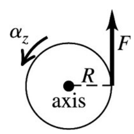
EXECUTE: **(a)** $\tau = \dfrac{P}{\omega} = \dfrac{(175\text{ hp})(746\text{ W/hp})}{(2400\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)} = 519\text{ N}\cdot\text{m}.$

(b) $W = \tau\Delta\theta = (519\ \text{N}\cdot\text{m})(2\pi\ \text{rad}) = 3260\ \text{J}$

EVALUATE: $\omega = 40$ rev/s, so the time for one revolution is 0.025 s. $P = 1.306\times10^5$ W, so in one revolution, $W = Pt = 3260$ J, which agrees with our result.

10.33. **(a) IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to find α_z and then use a constant angular acceleration equation to find ω_z.

SET UP: The free-body diagram is given in Figure 10.33.



EXECUTE: Apply $\sum\tau_z = I\alpha_z$ to find the angular acceleration:

$FR = I\alpha_z$

$$\alpha_z = \frac{FR}{I} = \frac{(18.0\ \text{N})(2.40\ \text{m})}{2100\ \text{kg}\cdot\text{m}^2} = 0.02057\ \text{rad/s}^2$$

Figure 10.33

SET UP: Use the constant α_z kinematic equations to find ω_z.

$\omega_z = ?$; ω_{0z} (initially at rest); $\alpha_z = 0.02057\ \text{rad/s}^2$; $t = 15.0$ s

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.02057\ \text{rad/s}^2)(15.0\ \text{s}) = 0.309\ \text{rad/s}$

(b) IDENTIFY and **SET UP:** Calculate the work from $W = \tau_z\Delta\theta$, using a constant angular acceleration equation to calculate $\theta - \theta_0$, or use the work-energy theorem. We will do it both ways.

EXECUTE: (1) $W = \tau_z\Delta\theta$

$\Delta\theta = \theta - \theta_0 = \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2 = 0 + \tfrac{1}{2}(0.02057\ \text{rad/s}^2)(15.0\ \text{s})^2 = 2.314\ \text{rad}$

$\tau_z = FR = (18.0\ \text{N})(2.40\ \text{m}) = 43.2\ \text{N}\cdot\text{m}$

Then $W = \tau_z\Delta\theta = (43.2\ \text{N}\cdot\text{m})(2.314\ \text{rad}) = 100\ \text{J}$.

or

(2) $W_{\text{tot}} = K_2 - K_1$

$W_{\text{tot}} = W$, the work done by the child

$K_1 = 0$; $K_2 = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(2100\ \text{kg}\cdot\text{m}^2)(0.309\ \text{rad/s})^2 = 100\ \text{J}$

Thus $W = 100$ J, the same as before.

EVALUATE: Either method yields the same result for W.

(c) IDENTIFY and **SET UP:** Use $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$ to calculate P_{av}.

EXECUTE: $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{100\ \text{J}}{15.0\ \text{s}} = 6.67\ \text{W}.$

EVALUATE: Work is in joules, power is in watts.

10.34. **IDENTIFY:** The power output of the motor is related to the torque it produces and to its angular velocity by $P = \tau_z\omega_z$, where ω_z must be in rad/s.

SET UP: The work output of the motor in 60.0 s is $\dfrac{2}{3}(9.00\ \text{kJ}) = 6.00\ \text{kJ}$, so $P = \dfrac{6.00\ \text{kJ}}{60.0\ \text{s}} = 100\ \text{W}.$

$\omega_z = 2500\ \text{rev/min} = 262\ \text{rad/s}.$

EXECUTE: $\tau_z = \dfrac{P}{\omega_z} = \dfrac{100\ \text{W}}{262\ \text{rad/s}} = 0.382\ \text{N}\cdot\text{m}.$

EVALUATE: For a constant power output, the torque developed decreases when the rotation speed of the motor increases.

10.35. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ and constant angular acceleration equations to the motion of the wheel.

SET UP: 1 rev $= 2\pi$ rad. π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

$$\tau_z = \frac{\left[(1/2)(2.80\text{ kg})(0.100\text{ m})^2\right](1200\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)}{2.5\text{ s}} = 0.704\text{ N}\cdot\text{m}.$$

(b) $\omega_{\text{av}}\Delta t = \dfrac{(600\text{ rev/min})(2.5\text{ s})}{60\text{ s/min}} = 25.0\text{ rev} = 157\text{ rad}.$

(c) $W = \tau\Delta\theta = (0.704\text{ N}\cdot\text{m})(157\text{ rad}) = 111\text{ J}.$

(d) $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[(1/2)(2.80\text{ kg})(0.100\text{ m})^2\right]\left[(1200\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)\right]^2 = 111\text{ J}.$

the same as in part (c).

EVALUATE: The agreement between the results of parts (c) and (d) illustrates the work-energy theorem.

10.36. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the motion of the propeller and then use constant acceleration equations to analyze the motion. $W = \tau\Delta\theta.$

SET UP: $I = \frac{1}{12}mL^2 = \frac{1}{12}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2.$

EXECUTE: **(a)** $\alpha = \dfrac{\tau}{I} = \dfrac{1950\text{ N}\cdot\text{m}}{42.2\text{ kg}\cdot\text{m}^2} = 46.2\text{ rad/s}^2.$

(b) $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\text{ rad/s}^2)(5.0\text{ rev})(2\pi\text{ rad/rev})} = 53.9\text{ rad/s}.$

(c) $W = \tau\theta = (1950\text{ N}\cdot\text{m})(5.00\text{ rev})(2\pi\text{ rad/rev}) = 6.13\times10^4\text{ J}.$

(d) $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{53.9\text{ rad/s}}{46.2\text{ rad/s}^2} = 1.17\text{ s}.$ $P_{\text{av}} = \dfrac{W}{\Delta t} = \dfrac{6.13\times10^4\text{ J}}{1.17\text{ s}} = 52.5\text{ kW}.$

(e) $P = \tau\omega = (1950\text{ N}\cdot\text{m})(53.9\text{ rad/s}) = 105\text{ kW}.$

EVALUATE: $P = \tau\omega.$ τ is constant and ω is linear in t, so P_{av} is half the instantaneous power at the end of the 5.00 revolutions. We could also calculate W from
$W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(53.9\text{ rad/s})^2 = 6.13\times10^4\text{ J}.$

10.37. (a) IDENTIFY: Use $L = mvr\sin\phi.$

SET UP: Consider Figure 10.37.

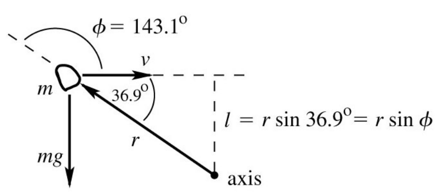


Figure 10.37

EXECUTE: $L = mvr\sin\phi =$
$(2.00\text{ kg})(12.0\text{ m/s})(8.00\text{ m})\sin 143.1°$
$L = 115\text{ kg}\cdot\text{m}^2/\text{s}$

To find the direction of $\vec{L}$ apply the right-hand rule by turning $\vec{r}$ into the direction of $\vec{v}$ by pushing on it with the fingers of your right hand. Your thumb points into the page, in the direction of $\vec{L}$.

(b) IDENTIFY and SET UP: By $\vec{\tau} = \dfrac{d\vec{L}}{dt}$ the rate of change of the angular momentum of the rock equals the torque of the net force acting on it.

EXECUTE: $\tau = mg(8.00\text{ m})\cos 36.9° = 125\text{ kg}\cdot\text{m}^2/\text{s}^2$

To find the direction of $\vec{\tau}$ and hence of $d\vec{L}/dt$, apply the right-hand rule by turning $\vec{r}$ into the direction of the gravity force by pushing on it with the fingers of your right hand. Your thumb points out of the page, in the direction of $d\vec{L}/dt$.

EVALUATE: $\vec{L}$ and $d\vec{L}/dt$ are in opposite directions, so L is decreasing. The gravity force is accelerating the rock downward, toward the axis. Its horizontal velocity is constant but the distance l is decreasing and hence L is decreasing.

10.38. IDENTIFY: $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

SET UP: $\omega = 0.80\text{ rev/s} = 5.026\text{ rad/s}$. $I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}R^2$ and $I_{\text{woman}} = m_{\text{woman}}R^2$.

EXECUTE: $I = (55\text{ kg} + 50.0\text{ kg})(4.0\text{ m})^2 = 1680\text{ kg}\cdot\text{m}^2$.

$L = (1680\text{ kg}\cdot\text{m}^2)(5.026\text{ rad/s}) = 8.4\times10^3\text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: The disk and the woman have similar values of I, even though the disk has twice the mass.

10.39. IDENTIFY and **SET UP:** Use $L = I\omega$.

EXECUTE: The second hand makes 1 revolution in 1 minute, so
$\omega = (1.00\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 0.1047\text{ rad/s}$.

For a slender rod, with the axis about one end,
$I = \frac{1}{3}ML^2 = \frac{1}{3}(6.00\times10^{-3}\text{ kg})(0.150\text{ m})^2 = 4.50\times10^{-5}\text{ kg}\cdot\text{m}^2$.

Then $L = I\omega = (4.50\times10^{-5}\text{ kg}\cdot\text{m}^2)(0.1047\text{ rad/s}) = 4.71\times10^{-6}\text{ kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: $\vec{L}$ is clockwise.

10.40. IDENTIFY: $L_z = I\omega_z$

SET UP: For a particle of mass m moving in a circular path at a distance r from the axis, $I = mr^2$ and $v = r\omega$. For a uniform sphere of mass M and radius R and an axis through its center, $I = \frac{2}{5}MR^2$. The earth has mass $m_{\text{E}} = 5.97\times10^{24}\text{ kg}$, radius $R_{\text{E}} = 6.37\times10^6\text{ m}$ and orbit radius $r = 1.50\times10^{11}\text{ m}$. The earth completes one rotation on its axis in $24\text{ h} = 86{,}400\text{ s}$ and one orbit in $1\text{ y} = 3.156\times10^7\text{ s}$.

EXECUTE: (a)

$$L_z = I\omega_z = mr^2\omega_z = (5.97\times10^{24}\text{ kg})(1.50\times10^{11}\text{ m})^2\left(\frac{2\pi\text{ rad}}{3.156\times10^7\text{ s}}\right) = 2.67\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}.$$

The radius of the earth is much less than its orbit radius, so it is very reasonable to model it as a particle for this calculation.

(b) $L_z = I\omega_z = (\frac{2}{5}MR^2)\omega = \frac{2}{5}(5.97\times10^{24}\text{ kg})(6.38\times10^6\text{ m})^2\left(\frac{2\pi\text{ rad}}{86{,}400\text{ s}}\right) = 7.07\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The angular momentum associated with each of these motions is very large.

10.41. IDENTIFY: $\omega_z = d\theta/dt$. $L_z = I\omega_z$ and $\tau_z = dL_z/dt$.

SET UP: For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2$.
$R = 0.240\text{ m}$.

EXECUTE: (a) $A = 1.50\text{ rad/s}^2$ and $B = 1.10\text{ rad/s}^4$, so that $\theta(t)$ will have units of radians.

(b) (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3$. At $t = 3.00\text{ s}$,

$\omega_z = 2(1.50\text{ rad/s}^2)(3.00\text{ s}) + 4(1.10\text{ rad/s}^4)(3.00\text{ s})^3 = 128\text{ rad/s}$.

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0\text{ kg})(0.240\text{ m})^2(128\text{ rad/s}) = 59.0\text{ kg}\cdot\text{m}^2/\text{s}$.

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0\text{ kg})(0.240\text{ m})^2\left[2(1.50\text{ rad/s}^2) + 12(1.10\text{ rad/s}^4)(3.00\text{ s})^2\right] = 56.1\text{ N}\cdot\text{m}.$

EVALUATE: The angular speed of rotation is increasing. This increase is due to an acceleration α_z that is produced by the torque on the sphere. When *I* is constant, as it is here, $\tau_z = dL_z/dt = Id\omega_z/dt = I\alpha_z$.

10.42. **IDENTIFY** and **SET UP:** $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where *r* is the distance of the block from the hole.

$mr_1^2\omega_1 = mr_2^2\omega_2$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1 = \left(\frac{0.300\text{ m}}{0.150\text{ m}}\right)^2 (2.85\text{ rad/s}) = 11.4\text{ rad/s}$$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$v_1 = r_1\omega_1 = (0.300\text{ m})(2.85\text{ rad/s}) = 0.855\text{ m/s}$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0250\text{ kg})(0.855\text{ m/s})^2 = 0.00914\text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$v_2 = r_2\omega_2 = (0.150\text{ m})(11.4\text{ rad/s}) = 1.71\text{ m/s}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0250\text{ kg})(1.71\text{ m/s}^2)^2 = 0.03655\text{ J}$

$\Delta K = K_2 - K_1 = 0.03655\text{ J} - 0.00914\text{ J} = 0.0274\text{ J} = 27.4\text{ mJ}.$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0274$ J.

EVALUATE: Smaller *r* means smaller *I*. $L = I\omega$ is constant so ω increases and *K* increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

10.43. **IDENTIFY:** Apply conservation of angular momentum.

SET UP: For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi\text{ rad}}{(30\text{ d})(86{,}400\text{ s/d})}\right)\left(\frac{7.0\times10^5\text{ km}}{16\text{ km}}\right)^2 = 4.6\times10^3\text{ rad/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. *L* is constant and ω increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

10.44. **IDENTIFY** and **SET UP:** Apply conservation of angular momentum to the diver.

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\text{ kg}\cdot\text{m}^2)/(18\text{ kg}\cdot\text{m}^2)$.

EXECUTE: If she had not tucked, she would have made $(2\text{ rev})(3.6\text{ kg}\cdot\text{m}^2)/(18\text{ kg}\cdot\text{m}^2) = 0.40\text{ rev}$ in the last 1.0 s, so she would have made $(0.40\text{ rev})(1.5/1.0) = 0.60\text{ rev}$ in the total 1.5 s.

EVALUATE: Untucked she rotates slower and completes fewer revolutions.

10.45. **IDENTIFY:** Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

EXECUTE: $L_\text{i} = L_\text{f}$ so $I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f}$.

$I_\text{i} = 0.40\text{ kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\text{ kg})(1.8\text{ m})^2 = 2.56\text{ kg}\cdot\text{m}^2.$

$I_\text{f} = 0.40\text{ kg}\cdot\text{m}^2 + (8.0\text{ kg})(0.25\text{ m})^2 = 0.90\text{ kg}\cdot\text{m}^2.$

$$\omega_\text{f} = \left(\frac{I_\text{i}}{I_\text{f}}\right)\omega_\text{i} = \left(\frac{2.56\text{ kg}\cdot\text{m}^2}{0.90\text{ kg}\cdot\text{m}^2}\right)(0.40\text{ rev/s}) = 1.14\text{ rev/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega.$ ω increases and L is constant, so K increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

10.46. **IDENTIFY:** Apply conservation of angular momentum to the collision.

SET UP: Let the width of the door be l. The initial angular momentum of the mud is $mv(l/2)$, since it strikes the door at its center. For the axis at the hinge, $I_\text{door} = \frac{1}{3}Ml^2$ and $I_\text{mud} = m(l/2)^2$.

EXECUTE: $\omega = \dfrac{L}{I} = \dfrac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}.$

$$\omega = \frac{(0.500\text{ kg})(12.0\text{ m/s})(0.500\text{ m})}{(1/3)(40.0\text{ kg})(1.00\text{ m})^2 + (0.500\text{ kg})(0.500\text{ m})^2} = 0.223\text{ rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\text{ rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

EVALUATE: Angular momentum is conserved but there is a large decrease in the kinetic energy of the system.

10.47. **IDENTIFY** and **SET UP:** There is no net external torque about the rotation axis so the angular momentum $L = I\omega$ is conserved.

EXECUTE: **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$I_1 = I_\text{tt} = \frac{1}{2}MR^2 = \frac{1}{2}(120\text{ kg})(2.00\text{ m})^2 = 240\text{ kg}\cdot\text{m}^2$

$I_2 = I_\text{tt} + I_\text{p} = 240\text{ kg}\cdot\text{m}^2 + mR^2 = 240\text{ kg}\cdot\text{m}^2 + (70\text{ kg})(2.00\text{ m})^2 = 520\text{ kg}\cdot\text{m}^2$

$\omega_2 = (I_1/I_2)\omega_1 = (240\text{ kg}\cdot\text{m}^2/520\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s}) = 1.38\text{ rad/s}$

(b) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(240\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s})^2 = 1080\text{ J}$

$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(520\text{ kg}\cdot\text{m}^2)(1.38\text{ rad/s})^2 = 495\text{ J}$

EVALUATE: The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.
The angular speed decreases because I increases when the parachutist is added to the system.

10.48. **IDENTIFY:** Apply conservation of angular momentum to the system of earth plus asteroid.

SET UP: Take the axis to be the earth's rotation axis. The asteroid may be treated as a point mass and it has zero angular momentum before the collision, since it is headed toward the center of the earth. For the earth, $L_z = I\omega_z$ and $I = \frac{2}{5}MR^2$, where M is the mass of the earth and R is its radius. The length of a day is $T = \dfrac{2\pi\text{ rad}}{\omega}$, where ω is the earth's angular rotation rate.

EXECUTE: Conservation of angular momentum applied to the collision between the earth and asteroid gives $\frac{2}{5}MR^2\omega_1 = (mR^2 + \frac{2}{5}MR^2)\omega_2$ and $m = \frac{2}{5}M\left(\dfrac{\omega_1 - \omega_2}{\omega_2}\right)$. $T_2 = 1.250T_1$ gives $\dfrac{1}{\omega_2} = \dfrac{1.250}{\omega_1}$ and

$\omega_1 = 1.250\omega_2$. $\dfrac{\omega_1 - \omega_2}{\omega_2} = 0.250$. $m = \frac{2}{5}(0.250)M = 0.100M$.

EVALUATE: If the asteroid hit the surface of the earth tangentially it could have some angular momentum with respect to the earth's rotation axis, and could either speed up or slow down the earth's rotation rate.

10.49. **(a) IDENTIFY** and **SET UP:** Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object A be the bug and object B be the bar. Initially, all objects are at rest and $L_1 = 0$. Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity ω_B in the opposite direction.

EXECUTE: $L_2 = m_A v_A r - I_B\omega_B$ where $r = 1.00$ m and $I_B = \frac{1}{3}m_B r^2$

$L_1 = L_2$ gives $m_A v_A r = \frac{1}{3}m_B r^2\omega_B$

$$\omega_B = \frac{3m_A v_A}{m_B r} = 0.120 \text{ rad/s}$$

(b) $K_1 = 0$;

$K_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}(0.0100 \text{ kg})(0.200 \text{ m/s})^2 + \frac{1}{2}(\frac{1}{3}(0.0500 \text{ kg})(1.00 \text{ m})^2)(0.120 \text{ rad/s})^2 = 3.2\times10^{-4}$ J.

(c) The increase in kinetic energy comes from work done by the bug when it pushes against the bar in order to jump.

EVALUATE: There is no external torque applied to the system and the total angular momentum of the system is constant. There are internal forces, forces the bug and bar exert on each other. The forces exert torques and change the angular momentum of the bug and the bar, but these changes are equal in magnitude and opposite in direction. These internal forces do positive work on the two objects and the kinetic energy of each object and of the system increases.

10.50. **IDENTIFY:** As the bug moves outward, it increases the moment of inertia of the rod-bug system. The angular momentum of this system is conserved because no unbalanced external torques act on it.

SET UP: The moment of inertia of the rod is $I = \dfrac{1}{3}ML^2$, and conservation of angular momentum gives $I_1\omega_1 = I_2\omega_2$.

EXECUTE: **(a)** $I = \dfrac{1}{3}ML^2$ gives $M = \dfrac{3I}{L^2} = \dfrac{3(3.00\times10^{-3} \text{ kg}\cdot\text{m}^2)}{(0.500 \text{ m})^2} = 0.0360$ kg.

(b) $L_1 = L_2$, so $I_1\omega_1 = I_2\omega_2$. $\omega_2 = \dfrac{v}{r} = \dfrac{0.160 \text{ m/s}}{0.500 \text{ m}} = 0.320$ rad/s, so

$(3.00\times10^{-3} \text{ kg}\cdot\text{m}^2)(0.400 \text{ rad/s}) = (3.00\times10^{-3} \text{ kg}\cdot\text{m}^2 + m_{\text{bug}}(0.500 \text{ m})^2)(0.320 \text{ rad/s})$.

$$m_{\text{bug}} = \frac{(3.00\times10^{-3} \text{ kg}\cdot\text{m}^2)(0.400 \text{ rad/s} - 0.320 \text{ rad/s})}{(0.320 \text{ rad/s})(0.500 \text{ m})^2} = 3.00\times10^{-3} \text{ kg}.$$

EVALUATE: This is a 3.00 mg bug, which is not unreasonable.

10.51. **IDENTIFY:** Energy is conserved as the cylinder rolls down the incline without slipping.

SET UP: The total energy of the cylinder is $K_{\text{tot}} = \dfrac{1}{2}mv_{\text{cm}}^2 + \dfrac{1}{2}I\omega^2$, $I_{\text{cyl}} = \dfrac{1}{2}MR^2$. As the cylinder rolls down the incline, gravitational potential energy is transformed into kinetic energy, so we use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Our target variable is the acceleration due to gravity on the planet.

EXECUTE: Energy conservation gives $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2$, which, for a solid cylinder, gives $mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2$. Solving for v_{cm}^2 gives $v_{cm}^2 = \left(\frac{4}{3}g\right)h.$ The graph of v_{cm}^2 versus h should be a straight line having slope equal to $4g/3$. Thus $g = \frac{3}{4}(\text{slope}) = \frac{3}{4}(6.42\ \text{m/s}^2) = 4.82\ \text{m/s}^2$.

EVALUATE: On this planet, g is about half of what it is on the earth.

10.52. IDENTIFY: If we take the raven and the gate as a system, the torque about the pivot is zero, so the angular momentum of the system about the pivot is conserved.

SET UP: The system before and after the collision is sketched in Figure 10.52. The gate has $I = \frac{1}{3}ML^2$. Take counterclockwise torques to be positive.

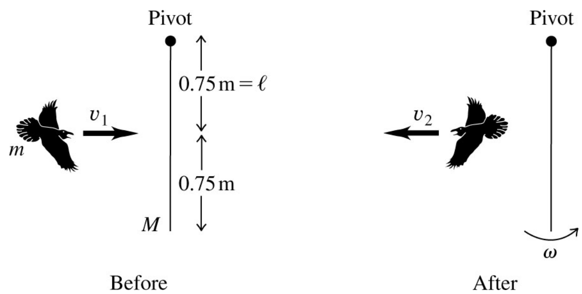


Figure 10.52

EXECUTE: (a) The gravity forces exert no torque at the moment of collision and angular momentum is conserved. $L_1 = L_2$. $mv_1l = -mv_2l + I_{gate}\omega$ with $l = L/2$.

$$\omega = \frac{m(v_1 + v_2)l}{\frac{1}{3}ML^2} = \frac{3m(v_1 + v_2)}{2ML} = \frac{3(1.1\ \text{kg})(5.0\ \text{m/s} + 2.0\ \text{m/s})}{2(4.5\ \text{kg})(1.5\ \text{m})} = 1.71\ \text{rad/s}.$$

(b) Linear momentum is not conserved; there is an external force exerted by the pivot. But the force on the pivot has zero torque. There is no external torque and angular momentum is conserved.

EVALUATE: $K_1 = \frac{1}{2}(1.1\ \text{kg})(5.0\ \text{m/s})^2 = 13.8\ \text{J}.$

$K_2 = \frac{1}{2}(1.1\ \text{kg})(2.0\ \text{m/s})^2 + \frac{1}{2}(\frac{1}{3}[4.5\ \text{kg}][1.5\ \text{m/s}]^2)(1.71\ \text{rad/s})^2 = 7.1\ \text{J}.$ This is an inelastic collision and $K_2 < K_1$.

10.53. IDENTIFY: When the teenager throws the rock, it causes the wooden disk to spin. Because no external torques act on the system due to the throw, the angular momentum of the system is conserved.

SET UP: $L = I\omega$ for a rotating object, and $L = mvr$ for a small object. $I_{disk} = \frac{1}{2}MR^2$. The target variable is the angular speed of the disk just after the rock is thrown.

EXECUTE: The initial angular momentum is zero since nothing is turning. Conservation of angular momentum tells us that $L_1 = L_2$, so $0 = L_{teen} + L_{disk} + L_{rock}$. The rock's motion is opposite to that of the teen and disk. This gives us $0 = mR^2\omega + \frac{1}{2}MR^2\omega - m_{rock}vR$. Solving for ω gives $\omega = \dfrac{m_{rock}v}{R\left(m + \frac{M}{2}\right)}$.

EVALUATE: Check some special cases. If $m_{rock} \to 0$, then $\omega \to 0$, which means that throwing the very light rock had no effect on the disk. If m or M are very large, $\omega \to 0$, which means that the teen or the disk were too massive to be moved by the light rock. If the teen and the rock are both much more massive than the disk, then $\omega \to v/R$, which means that the teen and rock have the same speed but in opposite directions.

10.54. **IDENTIFY:** The bullet collides with (and embeds itself in) the wooden disk, causing the disk and bullet to rotate. The angular momentum of the system is conserved because the collision did not cause any external torques on it.

SET UP: $L = I\omega$ for a rotating object, and $L = mvr$ for a small object. $I_{\text{disk}} = \frac{1}{2}MR^2$. The target variable is the speed of the bullet just before it hit the disk.

EXECUTE: Angular momentum conservation tells us that $L_{\text{bullet}} = L_{\text{bullet+disk}}$. Calling m the bullet mass and M the disk mass gives $mvR = (I_b + I_d)\omega = \left(mR^2 + \frac{1}{2}MR^2\right)\omega.$ Solving for v gives

$$v = R\omega\left(1+\frac{M}{2m}\right) = (0.600\text{ m})(4.00\text{ rad/s})\left(1+\frac{1.60\text{ kg}}{0.0400\text{ kg}}\right) = 98.4\text{ m/s}.$$

EVALUATE: Our result tells us that if m is small, v will need to be large to give the disk angular speed. If m is large, v can be small to give the disk angular speed.

10.55. **IDENTIFY:** An external torque will cause precession of the telescope.

SET UP: $I = MR^2$, with $R = 2.5\times10^{-2}$ m. 1.0×10^{-6} degree $= 1.745\times10^{-8}$ rad.

$\omega = 19{,}200\text{ rpm} = 2.01\times10^{3}\text{ rad/s}.$ $t = 5.0\text{ h} = 1.8\times10^{4}\text{ s}.$

EXECUTE: $\Omega = \dfrac{\Delta\phi}{\Delta t} = \dfrac{1.745\times10^{-8}\text{ rad}}{1.8\times10^{4}\text{ s}} = 9.694\times10^{-13}\text{ rad/s}.$ $\Omega = \dfrac{\tau}{I\omega}$ so $\tau = \Omega I\omega = \Omega MR^2\omega.$ Putting in the numbers gives

$\tau = (9.694\times10^{-13}\text{ rad/s})(2.0\text{ kg})(2.5\times10^{-2}\text{ m})^2(2.01\times10^{3}\text{ rad/s}) = 2.4\times10^{-12}\text{ N}\cdot\text{m}.$

EVALUATE: The external torque must be very small for this degree of stability.

10.56. **IDENTIFY:** The precession angular speed is related to the acceleration due to gravity by $\Omega = \dfrac{mgr}{I\omega}$, with $w = mg$.

SET UP: $\Omega_E = 0.50$ rad/s, $g_E = g$ and $g_M = 0.165g$. For the gyroscope, m, r, I, and ω are the same on the moon as on the earth.

EXECUTE: $\Omega = \dfrac{mgr}{I\omega}$. $\dfrac{\Omega}{g} = \dfrac{mr}{I\omega} = \text{constant},$ so $\dfrac{\Omega_E}{g_E} = \dfrac{\Omega_M}{g_M}.$

$$\Omega_M = \Omega_E\left(\frac{g_M}{g_E}\right) = 0.165\Omega_E = (0.165)(0.50\text{ rad/s}) = 0.0825\text{ rad/s}.$$

EVALUATE: In the limit that $g \to 0$ the precession rate $\to 0$.

10.57. **IDENTIFY:** As you pedal your bike, you turn a large sprocket which then turns a smaller sprocket on the wheel, and this causes the wheel to turn. Fig. 10.57 illustrates this arrangement.

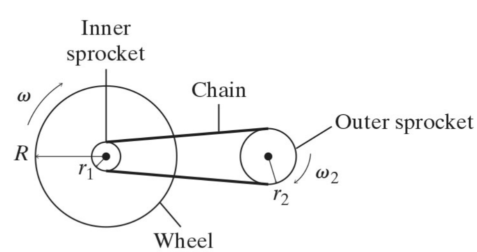


Figure 10.57

SET UP: Estimates: Bicycle wheel is 30 in. in diameter, large sprocket is 8.0 in. in diameter, and small sprocket is 4.0 in. in diameter. This means that in Fig. 10.57, R = 15 in., r_1 = 2.0 in., and r_2 = 4.0 in. We assume that the bike's wheel does not slip on the pavement, so $v_{cm} = R\omega$. The target variable is the angular speed of the large sprocket while the bike is traveling at 30 mph.

EXECUTE: Wheel: $v_{cm} = R\omega$ = 30 mph = 44 ft/s, so ω = (44 ft/s)/(15/12 ft) = 35.2 rad/s.

Inner sprocket: It turns at the same angular speed as the wheel, so its speed is $v_1 = r_1\omega$.

Outer sprocket: It is connected by a chain to the inner sprocket, so its tangential speed is the same as that of the inner sprocket, which means that $v_1 = v_2$ and $v_2 = r_2\omega_2$. Equating these two speeds gives $r_1\omega = r_2\omega_2$, so $\omega_2 = \frac{r_1}{r_2}\omega = \frac{2.0\text{ in.}}{4.0\text{ in.}}(35.2\text{ rad/s}) = 17.6\text{ rad/s}$. Converting to rpm gives

$$\omega_2 = \left(17.6\frac{\text{rad}}{\text{s}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 170\text{ rev/min}.$$

EVALUATE: By contrast, the bike's wheel turns at ω = 35 rad/s = 330 rpm.

10.58. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ and constant acceleration equations to the motion of the grindstone.

SET UP: Let the direction of rotation of the grindstone be positive. The friction force is $f = \mu_k n$ and produces torque fR. $\omega = (120\text{rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 4\pi\text{ rad/s}$. $I = \frac{1}{2}MR^2 = 1.69\text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** The net torque must be

$$\tau = I\alpha = I\frac{\omega_z - \omega_{0z}}{t} = (1.69\text{ kg}\cdot\text{m}^2)\frac{4\pi\text{ rad/s}}{9.00\text{ s}} = 2.36\text{ N}\cdot\text{m}.$$

This torque must be the sum of the applied force FR and the opposing frictional torques τ_f at the axle and $fR = \mu_k nR$ due to the knife. $F = \frac{1}{R}(\tau + \tau_f + \mu_k nR)$.

$$F = \frac{1}{0.500\text{ m}}\left[(2.36\text{ N}\cdot\text{m}) + (6.50\text{ N}\cdot\text{m}) + (0.60)(160\text{ N})(0.260\text{ m})\right] = 67.6\text{ N}.$$

(b) To maintain a constant angular velocity, the net torque τ is zero, and the force F' is

$$F' = \frac{1}{0.500\text{ m}}(6.50\text{ N}\cdot\text{m} + 24.96\text{ N}\cdot\text{m}) = 62.9\text{ N}.$$

(c) The time t needed to come to a stop is found by taking the magnitudes in $\vec{\tau} = \frac{d\vec{L}}{dt}$, with $\tau = \tau_f$ constant; $t = \frac{L}{\tau_f} = \frac{\omega I}{\tau_f} = \frac{(4\pi\text{ rad/s})(1.69\text{ kg}\cdot\text{m}^2)}{6.50\text{ N}\cdot\text{m}} = 3.27\text{ s}.$

EVALUATE: The time for a given change in ω is proportional to α, which is in turn proportional to the net torque, so the time in part (c) can also be found as $t = (9.00\text{ s})\frac{2.36\text{ N}\cdot\text{m}}{6.50\text{ N}\cdot\text{m}}$.

10.59. IDENTIFY: Use the kinematic information to solve for the angular acceleration of the grindstone. Assume that the grindstone is rotating counterclockwise and let that be the positive sense of rotation. Then apply $\sum\tau_z = I\alpha_z$ to calculate the friction force and use $f_k = \mu_k n$ to calculate μ_k.

SET UP: $\omega_{0z} = 850\text{ rev/min}(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 89.0\text{ rad/s}$

$t = 7.50$ s; $\omega_z = 0$ (comes to rest); $\alpha_z = ?$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$

$$\alpha_z = \frac{0 - 89.0\text{ rad/s}}{7.50\text{ s}} = -11.9\text{ rad/s}^2$$

SET UP: Apply $\sum\tau_z = I\alpha_z$ to the grindstone. The free-body diagram is given in Figure 10.59.

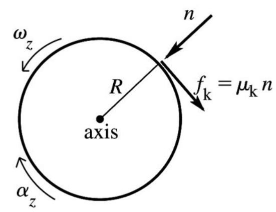


Figure 10.59

The normal force has zero moment arm for rotation about an axis at the center of the grindstone, and therefore zero torque. The only torque on the grindstone is that due to the friction force f_k exerted by the ax; for this force the moment arm is $l = R$ and the torque is negative.

EXECUTE: $\sum\tau_z = -f_k R = -\mu_k nR$

$I = \frac{1}{2}MR^2$ (solid disk, axis through center)

Thus $\sum\tau_z = I\alpha_z$ gives $-\mu_k nR = (\frac{1}{2}MR^2)\alpha_z$

$$\mu_k = -\frac{MR\alpha_z}{2n} = -\frac{(50.0\text{ kg})(0.260\text{ m})(-11.9\text{ rad/s}^2)}{2(160\text{ N})} = 0.483$$

EVALUATE: The friction torque is clockwise and slows down the counterclockwise rotation of the grindstone.

10.60. IDENTIFY: This problem involves moving blocks and a turning pulley, so we need to use Newton's second law in its linear and rotational forms. The constant-acceleration formulas also apply.

SET UP: Apply $\sum F_x = ma_x$ and $\sum F_y = ma_y$ to the blocks and $\sum\tau_z = I\alpha_z$ to the pulley. We also apply $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to block *B*. The target variables are the tension in the rope on both sides of the pulley and the moment of inertia of the pulley.

EXECUTE: **(a)** First use $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to find the acceleration of the blocks, giving $\Delta y = \frac{1}{2}at^2$. $a = 2(1.80\text{ m})/(2.00\text{ s})^2 = 0.900\text{ m/s}^2$. Now apply $\sum F_y = ma_y$ to block *B*. Choose +*y* downward since that is the direction of the acceleration. This gives $m_B g - T_B = m_B a$, and solving for T_B gives $T_B = m_B(g-a) = (6.00\text{ kg})(9.80\text{ m/s}^2 - 0.900\text{ m/s}^2) = 53.4\text{ N}$.

(b) Apply $\sum F_x = ma_x$ to block *A*: $T_A = m_A a = (2.50\text{ kg})(0.900\text{ m/s}^2) = 2.25\text{ N}$.

(c) Apply $\sum\tau_z = I\alpha_z$ to the pulley. It is important to realize that the tensions on the pulley are *not* the same because the pulley has mass. The acceleration of the blocks is the same as the tangential acceleration of the rope on the pulley, so $\alpha_z = a/R$, where *R* is the pulley radius. Therefore we get

$$T_B R - T_A R = I(a/R),\ \text{so}\ I = \frac{R^2}{a}(T_B - T_A) = \frac{(0.0800\text{ m})^2}{0.900\text{ m/s}^2}(53.4\text{ N} - 2.25\text{ N}) = 0.364\text{ kg}\cdot\text{m}^2.$$

EVALUATE: We found that $T_B > T_A$ which is reasonable since their torques are what give the pulley its angular acceleration causing it to turn in a direction that allows *B* to go downward.

10.61. IDENTIFY: Use $\sum\tau_z = I\alpha_z$ to find the angular acceleration just after the ball falls off and use conservation of energy to find the angular velocity of the bar as it swings through the vertical position.

SET UP: The axis of rotation is at the axle. For this axis the bar has $I = \frac{1}{12}m_{\text{bar}}L^2$, where $m_{\text{bar}} = 3.80\text{ kg}$ and $L = 0.800\text{ m}$. Energy conservation gives $K_1 + U_1 = K_2 + U_2$. The gravitational potential energy of the bar doesn't change. Let $mg(\sin 36.9° - \mu_k \cos 36.9°) - T = ma$ so $y_2 = -L/2$.

EXECUTE: **(a)** $\tau_z = m_{\text{ball}} g(L/2)$ and $I = I_{\text{ball}} + I_{\text{bar}} = \frac{1}{12} m_{\text{bar}} L^2 + m_{\text{ball}}(L/2)^2$. $\sum \tau_z = I\alpha_z$ gives

$$\alpha_z = \frac{m_{\text{ball}} g(L/2)}{\frac{1}{12} m_{\text{bar}} L^2 + m_{\text{ball}}(L/2)^2} = \frac{2g}{L}\left(\frac{m_{\text{ball}}}{m_{\text{ball}} + m_{\text{bar}}/3}\right) \text{ and}$$

$$\alpha_z = \frac{2(9.80\ \text{m/s}^2)}{0.800\ \text{m}}\left(\frac{2.50\ \text{kg}}{2.50\ \text{kg} + [3.80\ \text{kg}]/3}\right) = 16.3\ \text{rad/s}^2.$$

(b) As the bar rotates, the moment arm for the weight of the ball decreases and the angular acceleration of the bar decreases.

(c) $K_1 + U_1 = K_2 + U_2$. $0 = K_2 + U_2$. $\frac{1}{2}(I_{\text{bar}} + I_{\text{ball}})\omega^2 = -m_{\text{ball}} g(-L/2)$.

$$\omega = \sqrt{\frac{m_{\text{ball}} gL}{m_{\text{ball}} L^2/4 + m_{\text{bar}} L^2/12}} = \sqrt{\frac{g}{L}\left(\frac{4m_{\text{ball}}}{m_{\text{ball}} + m_{\text{bar}}/3}\right)} = \sqrt{\frac{9.80\ \text{m/s}^2}{0.800\ \text{m}}\left(\frac{4(2.50\ \text{kg})}{2.50\ \text{kg} + (3.80\ \text{kg})/3}\right)}$$

$\omega = 5.70$ rad/s.

EVALUATE: As the bar swings through the vertical, the linear speed of the ball that is still attached to the bar is $v = (0.400\ \text{m})(5.70\ \text{rad/s}) = 2.28$ m/s. A point mass in free-fall acquires a speed of 2.80 m/s after falling 0.400 m; the ball on the bar acquires a speed less than this.

10.62. **IDENTIFY:** A solid sphere rolls up a ramp without slipping. Newton's second law in its linear and angular forms applies to the sphere.

SET UP: Apply $\sum \tau_z = I\alpha_z$ and $\sum F_x = ma_x$ to the sphere. Choose the *x*-axis to be parallel to the surface of the ramp with +*x* down the ramp since that is the direction of the acceleration. $I_{\text{sphere}} = \frac{2}{5}MR^2$. Our target variables are the linear acceleration of the sphere and the friction force on it.

EXECUTE: **(a)** $\sum F_x = ma_x$: $mg\sin\beta - f = ma_{\text{cm}}$ (Eq. 1)

$\sum \tau_z = I\alpha_z$: $fR = \frac{2}{5}mR^2\alpha$. There is no slipping, so $\alpha = a_{\text{cm}}/R$, which gives $fR = \frac{2}{5}mR^2\left(\frac{a_{\text{cm}}}{R}\right)$.

Solving for *f* gives $f = \frac{2}{5}ma_{\text{cm}}$ (Eq. 2)

Combining Eq. 1 and Eq. 2 gives $mg\sin\beta - \frac{2}{5}ma_{\text{cm}} = ma_{\text{cm}}$, from which we get $a_{\text{cm}} = \frac{5}{7}g\sin\beta$. This is the same result as in Example 10.7.

(b) Eq. 2 gives $f = \frac{2}{5}ma_{\text{cm}} = \frac{2}{5}m\left(\frac{5}{7}g\sin\beta\right) = \frac{2}{7}mg\sin\beta$. This is the same as required to prevent the sphere to slip while rolling down the ramp.

EVALUATE: Whether rolling up the ramp or down the ramp, friction is up the ramp and gravity has a component down the ramp, so we get the same answers in Example 10.7.

10.63. **IDENTIFY:** Blocks *A* and *B* have linear acceleration and therefore obey the linear form of Newton's second law $\sum F_y = ma_y$. The wheel *C* has angular acceleration, so it obeys the rotational form of Newton's second law $\sum \tau_z = I\alpha_z$.

SET UP: *A* accelerates downward, *B* accelerates upward and the wheel turns clockwise. Apply $\sum F_y = ma_y$ to blocks *A* and *B*. Let +*y* be downward for *A* and +*y* be upward for *B*. Apply $\sum \tau_z = I\alpha_z$ to the wheel, with the clockwise sense of rotation positive. Each block has the same magnitude of acceleration, *a*, and $a = R\alpha$. Call the T_A the tension in the cord between *C* and *A* and T_B the tension between *C* and *B*.

EXECUTE: For A, $\sum F_y = ma_y$ gives $m_A g - T_A = m_A a$. For B, $\sum F_y = ma_y$ gives $T_B - m_B g = m_B a$. For the wheel, $\sum \tau_z = I\alpha_z$ gives $T_A R - T_B R = I\alpha = I(a/R)w$ and $T_A - T_B = \left(\frac{I}{R^2}\right)a$. Adding these three equations gives $(m_A - m_B)g = \left(m_A + m_B + \frac{I}{R^2}\right)a$. Solving for a, we have

$$a = \left(\frac{m_A - m_B}{m_A + m_B + I/R^2}\right)g = \left(\frac{4.00\text{ kg} - 2.00\text{ kg}}{4.00\text{ kg} + 2.00\text{ kg} + (0.220\text{ kg}\cdot\text{m}^2)/(0.120\text{ m})^2}\right)(9.80\text{ m/s}^2) = 0.921\text{ m/s}^2.$$

$$\alpha = \frac{a}{R} = \frac{0.921\text{ m/s}^2}{0.120\text{ m}} = 7.68\text{ rad/s}^2.$$

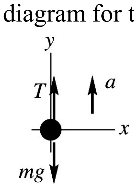
$$T_A = m_A(g - a) = (4.00\text{ kg})(9.80\text{ m/s}^2 - 0.921\text{ m/s}^2) = 35.5\text{ N}.$$

$$T_B = m_B(g + a) = (2.00\text{ kg})(9.80\text{ m/s}^2 + 0.921\text{ m/s}^2) = 21.4\text{ N}.$$

EVALUATE: The tensions must be different in order to produce a torque that accelerates the wheel when the blocks accelerate.

10.64. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the crate and $\sum \tau_z = I\alpha_z$ to the cylinder. The motions are connected by $a(\text{crate}) = R\alpha(\text{cylinder})$.

SET UP: The force diagram for the crate is given in Figure 10.64a.

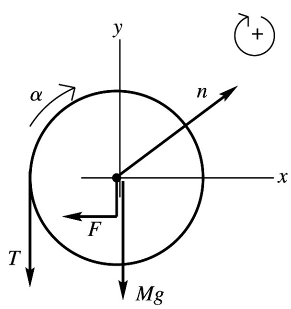


EXECUTE: Applying $\sum F_y = ma_y$ gives $T - mg = ma$. Solving for T gives

$$T = m(g + a) = (50\text{ kg})(9.80\text{ m/s}^2 + 1.40\text{ m/s}^2) = 5$$

Figure 10.64a

SET UP: The force diagram for the cylinder is given in Figure 10.64b.



EXECUTE: $\sum \tau_z = I\alpha_z$ gives $Fl - TR = I\alpha_z$, where $l = 0.12$ m and $R = 0.25$ m. $a = R\alpha$ so $\alpha_z = a/R$. Therefore $Fl = TR + Ia/R$.

Figure 10.64b

$$F = T\left(\frac{R}{l}\right) + \frac{Ia}{Rl} = (560\text{ N})\left(\frac{0.25\text{ m}}{0.12\text{ m}}\right) + \frac{(2.9\text{ kg}\cdot\text{m}^2)(1.40\text{ m/s}^2)}{(0.25\text{ m})(0.12\text{ m})} = 1300\text{ N}.$$

EVALUATE: The tension in the rope is greater than the weight of the crate since the crate accelerates upward. If F were applied to the rim of the cylinder ($l = 0.25$ m), it would have the value $F = 625$ N. This is greater than T because it must accelerate the cylinder as well as the crate. And F is larger than this because it is applied closer to the axis than R so has a smaller moment arm and must be larger to give the same torque.

10.65. IDENTIFY: A hollow sphere and a solid sphere roll up a ramp without slipping starting with the same speed at the base. Energy conservation applies to both of them.

SET UP: We use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, with $K_2 = 0$, $U_1 = 0$, $U_2 = mgh$, and $W_{\text{other}} = 0$. The total kinetic energy is $K_{\text{tot}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$, where $I_{\text{solid}} = \frac{2}{5}MR^2$ and $I_{\text{hollow}} = \frac{2}{3}MR^2$. We want to know which sphere reaches the greater height on the ramp.

EXECUTE: Apply energy conservation to each sphere to find h in each case.

Solid sphere: $mgh_{\text{s}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$, so $h_{\text{s}} = \frac{7}{10}\frac{v_{\text{cm}}^2}{g} = 0.700\frac{v_{\text{cm}}^2}{g}$.

Hollow sphere: $mgh_{\text{H}} = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$, so $h_{\text{H}} = \frac{5}{6}\frac{v_{\text{cm}}^2}{g} = 0.833\frac{v_{\text{cm}}^2}{g}$.

The hollow sphere reaches a greater height than the solid sphere.

EVALUATE: Even though the two spheres have the same size, mass, linear speed, and angular speed at the bottom of the ramp, they do not have the same kinetic energy because the hollow sphere has a greater moment of inertia. Therefore the hollow sphere goes higher up the ramp.

10.66. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the flywheel and $\sum\vec{F} = m\vec{a}$ to the block. The target variables are the tension in the string and the acceleration of the block.

(a) SET UP: Apply $\sum\tau_z = I\alpha_z$ to the rotation of the flywheel about the axis. The free-body diagram for the flywheel is given in Figure 10.66a.

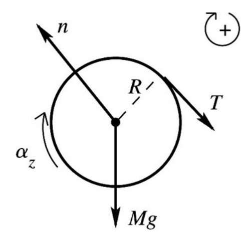


Figure 10.66a

EXECUTE: The forces n and Mg act at the axis so have zero torque.

$\sum\tau_z = TR$

$TR = I\alpha_z$

SET UP: Apply $\sum\vec{F} = m\vec{a}$ to the translational motion of the block. The free-body diagram for the block is given in Figure 10.66b.

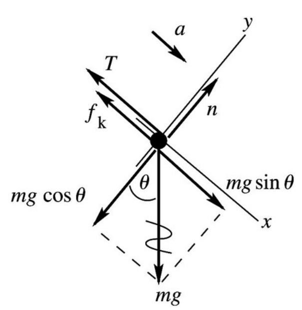


Figure 10.66b

EXECUTE: $\Sigma F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_{\text{k}} = \mu_{\text{k}}n = \mu_{\text{k}}mg\cos 36.9°$

$\sum F_x = ma_x$

$mg\sin 36.9° - T - \mu_{\text{k}}mg\cos 36.9° = ma$

$mg(\sin 36.9° - \mu_{\text{k}}\cos 36.9°) - T = ma$

But we also know that $a_{\text{block}} = R\alpha_{\text{wheel}}$, so $\alpha = a/R$. Using this in the $\sum\tau_z = I\alpha_z$ equation gives $TR = Ia/R$ and $T = (I/R^2)a$. Use this to replace T in the $\sum F_x = ma_x$ equation:

$$mg(\sin 36.9° - \mu_k \cos 36.9°) - (I/R^2)a = ma$$

$$a = \frac{mg(\sin 36.9° - \mu_k \cos 36.9°)}{m + I/R^2}$$

$$a = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)\left[\sin 36.9° - (0.25)\cos 36.9°\right]}{5.00\text{ kg} + 0.500\text{ kg}\cdot\text{m}^2/(0.200\text{ m})^2} = 1.12\text{ m/s}^2.$$

(b) $T = \dfrac{0.500\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2}(1.12\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: If the string is cut the block will slide down the incline with $a = g\sin 36.9° - \mu_k g\cos 36.9° = 3.92\text{ m/s}^2$. The actual acceleration is less than this because $mg\sin 36.9°$ must also accelerate the flywheel. $mg\sin 36.9° - f_k = 19.6$ N. T is less than this; there must be more force on the block directed down the incline than up the incline since the block accelerates down the incline.

10.67. **IDENTIFY:** A force produces a torque on a wheel, giving it an angular acceleration. But the force is not constant, so the angular acceleration is not constant.

SET UP: The force is $F = kt$, where $k = 5.00$ N/s. Our target variable is the magnitude of the force at the instant the wheel has turned through 8.00 rev (which is 16.0π rad). We can apply $\sum\tau_z = I\alpha_z$ to the wheel but we cannot use the constant-acceleration equations. We must return to the basic definitions $\omega_z = d\theta/dt$ and $\alpha_z = d\omega_z/dt$ and integrate.

EXECUTE: First apply $\sum\tau_z = I\alpha_z$ to get α_z, calling R the wheel radius. $FR = ktR = I\alpha_z$, so $\alpha_z = Rkt/I = Bt$, where $B = Rk/I$. Since α_z is a function of time, we must integrate to get the angle turned through $\Delta\theta$. $\omega_z = \int \alpha_z dt = \int Bt\,dt = \dfrac{Bt^2}{2}$, where we have used $\omega_z = 0$ at $t = 0$. Now integrate ω_z to find $\Delta\theta$. $\Delta\theta = \int \omega dt = \int \dfrac{Bt^2}{2}dt = \dfrac{Bt^3}{6}$, so $t = \left(\dfrac{6\Delta\theta}{B}\right)^{1/3}$. The force at this time is

$$F = kt = k\left(\frac{6\Delta\theta}{B}\right)^{1/3} = k\left(\frac{96\pi I}{kR}\right)^{1/3} = (5.00\text{ N/s})\left[\frac{96\pi(2.50\text{ kg}\cdot\text{m}^2)}{(5.00\text{ N/s})(0.0600\text{ m})}\right]^{1/3} = 68.0\text{ N}.$$

EVALUATE: When the acceleration (linear or angular) is not constant, we have little choice but to return to basic definitions and use calculus.

10.68. **IDENTIFY:** Apply both $\sum\vec{F} = m\vec{a}$ and $\sum\tau_z = I\alpha_z$ to the motion of the roller. Rolling without slipping means $a_{\text{cm}} = R\alpha$. Target variables are a_{cm} and f.

SET UP: The free-body diagram for the roller is given in Figure 10.68.

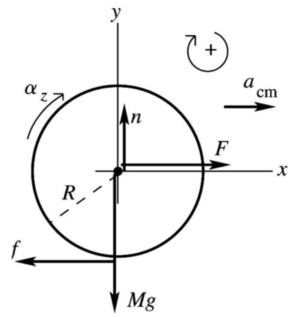


Figure 10.68

EXECUTE: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass:

$\sum F_x = ma_x$

$F - f = Ma_{cm}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$\sum \tau_z = fR$

thin-walled hollow cylinder: $I = MR^2$

Then $\sum \tau_z = I\alpha_z$ implies $fR = MR^2\alpha$.

But $\alpha_{cm} = R\alpha$, so $f = Ma_{cm}$.

Using this in the $\sum F_x = ma_x$ equation gives $F - Ma_{cm} = Ma_{cm}$.

$a_{cm} = F/2M,$ and then $f = Ma_{cm} = M(F/2M) = F/2$.

EVALUATE: If the surface were frictionless the object would slide without rolling and the acceleration would be $a_{cm} = F/M$. The acceleration is less when the object rolls.

10.69. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each object and apply $\sum \tau_z = I\alpha_z$ to the pulley.

SET UP: Call the 75.0 N weight *A* and the 125 N weight *B*. Let T_A and T_B be the tensions in the cord to the left and to the right of the pulley. For the pulley, $I = \frac{1}{2}MR^2$, where $Mg = 80.0$ N and $R = 0.300$ m. The 125 N weight accelerates downward with acceleration *a*, the 75.0 N weight accelerates upward with acceleration *a* and the pulley rotates clockwise with angular acceleration α, where $a = R\alpha$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ applied to the 75.0 N weight gives $T_A - w_A = m_A a$. $\sum \vec{F} = m\vec{a}$ applied to the 125.0 N weight gives $w_B - T_B = m_B a$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $(T_B - T_A)R = (\frac{1}{2}MR^2)\alpha_z$ and $T_B - T_A = \frac{1}{2}Ma$. Combining these three equations gives $w_B - w_A = (m_A + m_B + M/2)a$ and

$$a = \left(\frac{w_B - w_A}{w_A + w_B + w_{\text{pulley}}/2}\right)g = \left(\frac{125\text{ N} - 75.0\text{ N}}{75.0\text{ N} + 125\text{ N} + 40.0\text{ N}}\right)g = 0.2083g.$$

$T_A = w_A(1 + a/g) = 1.2083w_A = 90.62$ N. $T_B = w_B(1 - a/g) = 0.792w_B = 98.96$ N. $\sum \vec{F} = m\vec{a}$ applied to the pulley gives that the force *F* applied by the hook to the pulley is $F = T_A + T_B + w_{\text{pulley}} = 270$ N. The force the ceiling applies to the hook is 270 N.

EVALUATE: The force the hook exerts on the pulley is less than the total weight of the system, since the net effect of the motion of the system is a downward acceleration of mass.

10.70. **IDENTIFY:** Dropping the object on the rotating turntable slows down the turntable and speeds up the object, but it does not change the total angular momentum of the object-turntable system.

SET UP: Angular momentum is conserved, so $L_{\text{table}} = L_{\text{table+object}}$, where $L = I\omega$. Our target variable is the moment of inertia *I* of the table.

EXECUTE: Using so $L_{\text{table}} = L_{\text{table+object}}$ gives $I\omega = \left(I + I_{object}\right)\omega_f = \left(I + mR^2\right)\omega_f$. Now solve for $\dfrac{\omega - \omega_f}{\omega_f}$, which gives $\dfrac{\omega - \omega_f}{\omega_f} = \left(\dfrac{R^2}{I}\right)m$. From this we see that a graph of $\dfrac{\omega - \omega_f}{\omega_f}$ versus m should give a straight line having slope equal to R^2/I. Thus $I = \dfrac{R^2}{\text{slope}} = \dfrac{(3.00\ \text{m})^2}{0.250\ \text{kg}^{-1}} = 36.0\ \text{kg}\cdot\text{m}^2$.

EVALUATE: It would be quite difficult to measure directly the moment of a large heavy turntable, especially if it was not uniform throughout. But the measurements described here would be fairly easy to make.

10.71. IDENTIFY: Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ to the motion of the center of mass and apply $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the rotation about the center of mass.

SET UP: $I = 2(\frac{1}{2}mR^2) = mR^2$. The moment arm for T is b.

EXECUTE: The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\dfrac{a}{b}$. Dividing the second equation by b and adding to the first to eliminate T yields $a = g\dfrac{2m}{(2m + I/b^2)} = g\dfrac{2}{2 + (R/b)^2}$, $\alpha = g\dfrac{2}{2b + R^2/b}$. The tension is found by substitution into either of the two equations:

$$T = (2m)(g - a) = (2mg)\left(1 - \frac{2}{2 + (R/b)^2}\right) = 2mg\frac{(R/b)^2}{2 + (R/b)^2} = \frac{2mg}{(2(b/R)^2 + 1)}.$$

EVALUATE: $a \to 0$ when $b \to 0$. As $b \to R$, $a \to 2g/3$.

10.72. IDENTIFY: Apply conservation of energy to the motion of the shell, to find its linear speed v at points A and B. Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the shell in the circular part of the track to find the normal force exerted by the track at each point. Since $r << R$ the shell can be treated as a point mass moving in a circle of radius R when applying $\sum \vec{F} = m\vec{a}$. But as the shell rolls along the track, it has both translational and rotational kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$. Let 1 be at the starting point and take $y = 0$ to be at the bottom of the track, so $y_1 = h_0$. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = \frac{2}{3}mr^2$ and $\omega = v/r$, so $K = \frac{5}{6}mv^2$. During the circular motion, $a_{\text{rad}} = v^2/R$.

EXECUTE: **(a)** $\sum \vec{F} = m\vec{a}$ at point A gives $n + mg = m\dfrac{v^2}{R}$. The minimum speed for the shell not to fall off the track is when $n \to 0$ and $v^2 = gR$. Let point 2 be A, so $y_2 = 2R$ and $v_2^2 = gR$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = 2mgR + \frac{5}{6}m(gR)$. $h_0 = (2 + \frac{5}{6})R = \frac{17}{6}R$.

(b) Let point 2 be B, so $y_2 = R$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = mgR + \frac{5}{6}mv_2^2$. With $h = \frac{17}{6}R$ this gives $v^2 = \frac{11}{5}gR$. Then $\sum \vec{F} = m\vec{a}$ at B gives $n = m\dfrac{v^2}{R} = \frac{11}{5}mg$.

(c) Now $K = \frac{1}{2}mv^2$ instead of $\frac{5}{6}mv^2$. The shell would be moving faster at A than with friction and would still make the complete loop.

(d) In part (c): $mgh_0 = mg(2R) + \frac{1}{2}mv^2$. $h_0 = \frac{17}{6}R$ gives $v^2 = \frac{5}{3}gR$. $\sum \vec{F} = m\vec{a}$ at point A gives $mg + n = m\dfrac{v^2}{R}$ and $n = m\left(\dfrac{v^2}{R} - g\right) = \frac{2}{3}mg$. In part (a), $n = 0$, since at this point gravity alone supplies the net downward force that is required for the circular motion.

EVALUATE: The normal force at A is greater when friction is absent because the speed of the shell at A is greater when friction is absent than when there is rolling without slipping.

10.73. **IDENTIFY:** As it rolls down the rough slope, the basketball gains rotational kinetic energy as well as translational kinetic energy. But as it moves up the smooth slope, its rotational kinetic energy does not change since there is no friction.

SET UP: $I_{cm} = \frac{2}{3}mR^2$. When it rolls without slipping, $v_{cm} = R\omega$. When there is no friction the angular speed of rotation is constant. Take $+y$ upward and let $y = 0$ in the valley.

EXECUTE: **(a)** Find the speed v_{cm} in the level valley: $K_1 + U_1 = K_2 + U_2$. $y_1 = H_0$, $y_2 = 0$. $K_1 = 0$, $U_2 = 0$. Therefore, $U_1 = K_2$. $mgH_0 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$. $\frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}(\frac{2}{3}mR^2)\left(\dfrac{v_{cm}}{R}\right)^2 = \frac{1}{3}mv_{cm}^2$, so $mgH_0 = \frac{5}{6}mv_{cm}^2$ and $v_{cm}^2 = \dfrac{6gH_0}{5}$. Find the height H it goes up the other side. Its rotational kinetic energy stays constant as it rolls on the frictionless surface. $\frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}I_{cm}\omega^2 + mgH$.

$H = \dfrac{v_{cm}^2}{2g} = \frac{3}{5}H_0$.

(b) Some of the initial potential energy has been converted into rotational kinetic energy so there is less potential energy at the second height H than at the first height H_0.

EVALUATE: Mechanical energy is conserved throughout this motion. But the initial gravitational potential energy on the rough slope is not all transformed into potential energy on the smooth slope because some of that energy remains as rotational kinetic energy at the highest point on the smooth slope.

10.74. **IDENTIFY:** Apply conservation of energy to the motion of the ball as it rolls up the hill. After the ball leaves the edge of the cliff it moves in projectile motion and constant acceleration equations can be used.

(a) SET UP: Use conservation of energy to find the speed v_2 of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y = 0$ at the bottom of the hill, so $y_1 = 0$ and $y_2 = 28.0$ m.

EXECUTE: $K_1 + U_1 = K_2 + U_2$

$\frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 = mgy_2 + \frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2$

Rolling without slipping means $\omega = v/r$ and $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{2}{5}mr^2)(v/r)^2 = \frac{1}{5}mv^2$.

$\frac{7}{10}mv_1^2 = mgy_2 + \frac{7}{10}mv_2^2$

$v_2 = \sqrt{v_1^2 - \frac{10}{7}gy_2} = 15.26$ m/s

SET UP: Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward. Use the vertical motion to find the time in the air:

$v_{0y} = 0$, $a_y = 9.80\ \text{m/s}^2$, $y - y_0 = 28.0$ m, $t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.39$ s

During this time the ball travels horizontally
$x - x_0 = v_{0x}t = (15.26 \text{ m/s})(2.39 \text{ s}) = 36.5 \text{ m}.$

Just before it lands, $v_y = v_{0y} + a_y t = 23.4$ m/s and $v_x = v_{0x} = 15.3$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 28.0$ m/s

(b) EVALUATE: At the bottom of the hill, $\omega = v/r = (25.0 \text{ m/s})/r$. The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega = (15.3 \text{ m/s})/r$. The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

10.75. IDENTIFY: Apply conservation of energy to the motion of the boulder.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$ when there is rolling without slipping. $I = \frac{2}{5}mR^2$.

EXECUTE: Break into two parts, the rough and smooth sections.

Rough: $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$. $v^2 = \frac{10}{7}gh_1$.

Smooth: Rotational kinetic energy does not change. $mgh_2 + \frac{1}{2}mv^2 + K_{\text{rot}} = \frac{1}{2}mv_{\text{Bottom}}^2 + K_{\text{rot}}$.

$$gh_2 + \frac{1}{2}\left(\frac{10}{7}gh_1\right) = \frac{1}{2}v_{\text{Bottom}}^2.\ v_{\text{Bottom}} = \sqrt{\frac{10}{7}gh_1 + 2gh_2} = \sqrt{\frac{10}{7}(9.80 \text{ m/s}^2)(25 \text{ m}) + 2(9.80 \text{ m/s}^2)(25 \text{ m})} = 29.0 \text{ m/s}.$$

EVALUATE: If all the hill was rough enough to cause rolling without slipping,
$v_{\text{Bottom}} = \sqrt{\frac{10}{7}g(50 \text{ m})} = 26.5$ m/s. A smaller fraction of the initial gravitational potential energy goes into translational kinetic energy of the center of mass than if part of the hill is smooth. If the entire hill is smooth and the boulder slides without slipping, $v_{\text{Bottom}} = \sqrt{2g(50 \text{ m})} = 31.3$ m/s. In this case all the initial gravitational potential energy goes into the kinetic energy of the translational motion.

10.76. IDENTIFY: Apply Newton's second law in its linear and rotational form to the cylinder. The cylinder does not slip on the surface of the ramp.

SET UP: $\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$, $\Sigma\tau_z = I\alpha_z$, $I = \frac{1}{2}mR^2$, and $a_{\text{cm}} = R\alpha$ for no slipping. Take the x-axis parallel to the surface of the ramp; call up the ramp positive since that is the direction in which the cylinders must accelerate. Take the y-axis perpendicular to the surface. For uniform acceleration
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$.

EXECUTE: (a) The forces balance in the y-direction, so the normal force n is $n = mg\cos\theta$. In the x-direction, $\Sigma F_x = ma_x$ gives

$F - f_s - mg\sin\theta = ma$.

Now apply $\Sigma\tau_z = I\alpha_z$.

$f_s R = (\frac{1}{2}mR^2)(a/R)$, which gives $a = 2f_s/m$. Putting this result into the previous result gives

$F - f_s - mg\sin\theta = m(2f_s/m) = 2f_s$.

Solving for F gives

$F = 3f_s + mg\sin\theta = 3\mu_s n + mg\sin\theta = 3\mu_s mg\cos\theta + mg\sin\theta = mg(3\mu_s\cos\theta + \sin\theta)$

$F = (460 \text{ kg})(9.80 \text{ m/s}^2)[3(0.120)\cos 37° + \sin 37°] = 4010$ N.

(b) From part (a) we have
$a = 2f_s/m = (2\mu_s mg\cos\theta)/m = 2\mu_s g\cos\theta.$
Linear kinematics using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$t = \sqrt{\frac{2(x-x_0)}{a}} = \sqrt{\frac{2(x-x_0)}{2\mu_s g\cos\theta}} = \sqrt{\frac{6.00\text{ m}}{(0.120)(9.80\text{ m/s}^2)\cos 37°}} = 2.53\text{ s.}$$

EVALUATE: Just lifting the 460-kg vertically would require a force of $mg = 4510$ N, so we don't do very much better by rolling them up the slope since friction opposes the linear motion.

10.77. **IDENTIFY:** Apply conservation of energy to the motion of the wheel.
SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. No slipping means that $\omega = v/R$. Uniform density means $m_r = \lambda 2\pi R$ and $m_s = \lambda R$, where m_r is the mass of the rim and m_s is the mass of each spoke. For the wheel, $I = I_{\text{rim}} + I_{\text{spokes}}$. For each spoke, $I = \frac{1}{3}m_s R^2$.
EXECUTE: **(a)** $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = I_{\text{rim}} + I_{\text{spokes}} = m_r R^2 + 6(\frac{1}{3}m_s R^2)$
Also, $m = m_r + m_s = 2\pi R\lambda + 6R\lambda = 2R\lambda(\pi + 3)$. Substituting into the conservation of energy equation gives $2R\lambda(\pi+3)gh = \frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2 + \frac{1}{2}\left[2\pi R\lambda R^2 + 6(\frac{1}{3}\lambda R R^2)\right]\omega^2$.

$$\omega = \sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}} = \sqrt{\frac{(\pi+3)(9.80\text{ m/s}^2)(58.0\text{ m})}{(0.210\text{ m})^2(\pi+2)}} = 124\text{ rad/s} \quad \text{and } v = R\omega = 26.0\text{ m/s}$$

(b) Doubling the density would have no effect because it does not appear in the answer. ω is inversely proportional to R so doubling the diameter would double the radius which would reduce ω by half, but $v = R\omega$ would be unchanged.
EVALUATE: Changing the masses of the rim and spokes by different amounts would alter the speed v at the bottom of the hill.

10.78. **IDENTIFY:** The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant.
SET UP: For the rod, $I = \frac{1}{12}ML^2$. For each ring, $I = mr^2$, where r is their distance from the axis.
EXECUTE: **(a)** As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by $\omega_2 = \omega_1\frac{I_1}{I_2} = \omega_1\left[\frac{\frac{1}{12}ML^2 + 2mr_1^2}{\frac{1}{12}ML^2 + 2mr_2^2}\right] = \omega_1\left(\frac{5.00\times10^{-4}\text{ kg}\cdot\text{m}^2}{2.00\times10^{-3}\text{ kg}\cdot\text{m}^2}\right) = \frac{\omega_1}{4}$, so $\omega_2 = 12.0$ rev/min.
(b) The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 12.0 rev/min.
EVALUATE: Note that conversion from rev/min to rad/s was not necessary. The angular velocity of the rod decreases as the rings move away from the rotation axis.

10.79. **IDENTIFY:** Use conservation of energy to relate the speed of the block to the distance it has descended. Then use a constant acceleration equation to relate these quantities to the acceleration.
SET UP: For the cylinder, $I = \frac{1}{2}M(2R)^2$, and for the pulley, $I = \frac{1}{2}MR^2$.
EXECUTE: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed v, the pulley has angular velocity v/R and the cylinder has angular velocity $v/2R$, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance y, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

EVALUATE: If the pulley were massless and the cylinder slid without rolling, $Mg = 2Ma$ and $a = g/2$. The rotation of the objects reduces the acceleration of the block.

10.80. **IDENTIFY:** Apply conservation of energy to the motion of the first ball before the collision and to the motion of the second ball after the collision. Apply conservation of angular momentum to the collision between the first ball and the bar.

SET UP: The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34$ m/s. Use conservation of angular momentum to find the angular velocity ω of the bar just after the collision. Take the axis at the center of the bar.

EXECUTE: $L_1 = mvr = (5.00 \text{ kg})(15.34 \text{ m/s})(2.00 \text{ m}) = 153.4 \text{ kg} \cdot \text{m}^2/\text{s}$

Immediately after the collision the bar and both balls are rotating together.

$L_2 = I_{\text{tot}}\omega$

$I_{\text{tot}} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00 \text{ kg})(4.00 \text{ m})^2 + 2(5.00 \text{ kg})(2.00 \text{ m})^2 = 50.67 \text{ kg} \cdot \text{m}^2$

$L_2 = L_1 = 153.4 \text{ kg} \cdot \text{m}^2/\text{s}$

$\omega = L_2/I_{\text{tot}} = 3.027$ rad/s

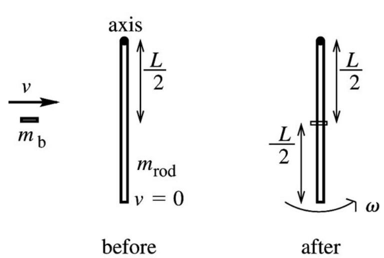
Just after the collision the second ball has linear speed $v = r\omega = (2.00 \text{ m})(3.027 \text{ rad/s}) = 6.055$ m/s and is moving upward. $\frac{1}{2}mv^2 = mgy$ gives $y = 1.87$ m for the height the second ball goes.

EVALUATE: Mechanical energy is lost in the inelastic collision and some of the final energy is in the rotation of the bar with the first ball stuck to it. As a result, the second ball does not reach the height from which the first ball was dropped.

10.81. **IDENTIFY:** Apply conservation of angular momentum to the collision. Linear momentum is not conserved because of the force applied to the rod at the axis. But since this external force acts at the axis, it produces no torque and angular momentum is conserved.

SET UP: The system before and after the collision is sketched in Figure 10.81.

EXECUTE: **(a)** $m_\text{b} = \frac{1}{4}m_{\text{rod}}$



EXECUTE: $L_1 = m_\text{b}vr = \frac{1}{4}m_{\text{rod}}v(L/2)$

$L_1 = \frac{1}{8}m_{\text{rod}}vL$

$L_2 = (I_{\text{rod}} + I_\text{b})\omega$

$I_{\text{rod}} = \frac{1}{3}m_{\text{rod}}L^2$

$I_\text{b} = m_\text{b}r^2 = \frac{1}{4}m_{\text{rod}}(L/2)^2$

$I_\text{b} = \frac{1}{16}m_{\text{rod}}L^2$

Figure 10.81

Thus $L_1 = L_2$ gives $\frac{1}{8}m_{\text{rod}}vL = (\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2)\omega$

$\frac{1}{8}v = \frac{19}{48}L\omega$

$\omega = \frac{6}{19}v/L$

(b) $K_1 = \frac{1}{2}mv^2 = \frac{1}{8}m_{\text{rod}}v^2$

$K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_{\text{rod}} + I_{\text{b}})\omega^2 = \frac{1}{2}(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2)(6v/19L)^2$

$K_2 = \frac{1}{2}(\frac{19}{48})(\frac{6}{19})^2 m_{\text{rod}}v^2 = \frac{3}{152}m_{\text{rod}}v^2$

Then $\dfrac{K_2}{K_1} = \dfrac{\frac{3}{152}m_{\text{rod}}v^2}{\frac{1}{8}m_{\text{rod}}v^2} = 3/19.$

EVALUATE: The collision is inelastic and $K_2 < K_1$.

10.82. **IDENTIFY:** As you walk toward the center of the turntable, the angular momentum of the system (you plus turntable) is conserved. By getting closer to the center, you are decreasing the moment of inertia of the system. Newton's second law applies to you, and static friction provides the centripetal force on you.

SET UP: $I_0\omega_0 = I_2\omega_2$, $I = mr^2$ for a point mass, $a_{\text{rad}} = r\omega^2$, $f_{\text{s}}^{\text{max}} = \mu_{\text{s}}n$, and $\Sigma\vec{F} = m\vec{a}$.

EXECUTE: At the closest distance, the friction force is

$f_{\text{s}} = \mu_{\text{s}}n = \mu_{\text{s}}mg$

Newton's second law gives

$f_{\text{s}} = ma = mr\omega^2$

Combining these two equations gives

$\mu_{\text{s}}mg = mr\omega^2$

Conservation of angular momentum gives $\omega = \dfrac{I_0}{I}\omega_0 = \left(\dfrac{I_{\text{t}} + mr_0^2}{I_{\text{t}} + mr^2}\right)\omega_0$. Solving the earlier equation for μ_{s} and using the previous result gives $\mu_{\text{s}} = \dfrac{\omega^2 r}{g} = \left(\dfrac{I_{\text{t}} + mr_0^2}{I_{\text{t}} + mr^2}\right)^2 \dfrac{\omega_0^2 r}{g}$. Putting in $m = 70.0$ kg, $r = 3.00$ m, and $I_{\text{t}} = 1200\ \text{kg}\cdot\text{m}^2$, and using $\omega_0 = 2\pi/(8.0\ \text{s})$, we get $\mu_{\text{s}} = 0.780$.

EVALUATE: This coefficient of static friction is physically reasonable.

10.83. **IDENTIFY:** As the disks are connected, their angular momentum is conserved, but some of their initial kinetic energy is converted to thermal energy. The 2400 J of thermal energy is equal to the loss of rotational kinetic energy.

SET UP: $I_1\omega_1 = I_2\omega_2$, $K = \frac{1}{2}I\omega^2$.

EXECUTE: Angular momentum conservation gives $I_A\omega_A = (I_A + I_B)\omega \rightarrow \omega = \dfrac{I_A\omega_A}{I_A + I_B}$. The loss of kinetic energy is $\Delta K = K_1 - K_2 = \frac{1}{2}I_A\omega_0^2 - \frac{1}{2}(I_A + I_B)\omega^2$. Combining these two equations gives $\Delta K = \dfrac{I_A\omega_0^2}{2}\left(1 - \dfrac{I_A}{I_A + I_B}\right)$. The loss of kinetic energy should be no more than 2400 J, so $\dfrac{I_A\omega_0^2}{2}\left(1 - \dfrac{I_A}{I_A + I_B}\right) \le 2400\ \text{J}$. The quantity $\dfrac{I_A\omega_0^2}{2}$ is the kinetic energy of A, K_A. Therefore we can solve the inequality for K_A, giving $K_A \le (2400\ \text{J})\left(\dfrac{I_A + I_B}{I_B}\right)$. Since $I_A = I_B/3$, the maximum kinetic energy of A is 3200 J.

EVALUATE: This situation is the rotational analog to a collision in which one object is initially at rest and they stick together. As in that situation, the momentum (angular in this case) is conserved but the kinetic energy is not.

10.84. **IDENTIFY:** This is a collision in which one object is initially stationary and they stick together. The rod is pivoted at one end, so it can only rotate after it is struck. The puck has angular momentum, some of which is transferred to the rod, but the angular momentum of the puck-rod system is conserved.

SET UP: The initial angular momentum of the puck is mvr, the final angular momentum of the rod is $I\omega$, and $I_{\text{rod}} = \frac{1}{3}ML^2$.

EXECUTE: After the collision, $\omega = 2\pi/T$, where $T = 0.736$ s, $r = L$, and $I = I_{\text{rod}} + I_{\text{puck}}$. Conservation of angular momentum gives $mvr = (\frac{1}{3}ML^2 + mL^2)\omega$. Solving for v gives $v = \dfrac{(\frac{1}{3}ML^2 + mL^2)\left(\frac{2\pi}{T}\right)}{mL}$. Putting in $m = 0.163$ kg, $M = 0.800$ kg, $L = 2.00$ m, $T = 0.736$ s gives $v = 45.0$ m/s.

EVALUATE: This situation is the rotational analog to a collision in which one object is initially at rest and they stick together. As in that situation, the momentum (angular in this case) is conserved but the kinetic energy is not.

10.85. **IDENTIFY:** Apply conservation of angular momentum to the collision between the bird and the bar and apply conservation of energy to the motion of the bar after the collision.

SET UP: For conservation of angular momentum take the axis at the hinge. For this axis the initial angular momentum of the bird is $m_{\text{bird}}(0.500\text{ m})v$, where $m_{\text{bird}} = 0.500$ kg and $v = 2.25$ m/s. For this axis the moment of inertia is $I = \frac{1}{3}m_{\text{bar}}L^2 = \frac{1}{3}(1.50\text{ kg})(0.750\text{ m})^2 = 0.281\text{ kg}\cdot\text{m}^2$. For conservation of energy, the gravitational potential energy of the bar is $U = m_{\text{bar}}gy_{\text{cm}}$, where y_{cm} is the height of the center of the bar. Take $y_{\text{cm},1} = 0$, so $y_{\text{cm},2} = -0.375$ m.

EXECUTE: **(a)** $L_1 = L_2$ gives $m_{\text{bird}}(0.500\text{ m})v = (\frac{1}{3}m_{\text{bar}}L^2)\omega$.

$$\omega = \frac{3m_{\text{bird}}(0.500\text{ m})v}{m_{\text{bar}}L^2} = \frac{3(0.500\text{ kg})(0.500\text{ m})(2.25\text{ m/s})}{(1.50\text{ kg})(0.750\text{ m})^2} = 2.00\text{ rad/s}.$$

(b) $U_1 + K_1 = U_2 + K_2$ applied to the motion of the bar after the collision gives

$$\tfrac{1}{2}I\omega_1^2 = m_{\text{bar}}g(-0.375\text{ m}) + \tfrac{1}{2}I\omega_2^2. \quad \omega_2 = \sqrt{\omega_1^2 + \frac{2}{I}m_{\text{bar}}g(0.375\text{ m})}.$$

$$\omega_2 = \sqrt{(2.00\text{ rad/s})^2 + \frac{2}{0.281\text{ kg}\cdot\text{m}^2}(1.50\text{ kg})(9.80\text{ m/s}^2)(0.375\text{ m})} = 6.58\text{ rad/s}.$$

EVALUATE: Mechanical energy is not conserved in the collision. The kinetic energy of the bar just after the collision is less than the kinetic energy of the bird just before the collision.

10.86. **IDENTIFY:** Angular momentum is conserved, since the tension in the string is in the radial direction and therefore produces no torque. Apply $\sum\vec{F} = m\vec{a}$ to the block, with $a = a_{\text{rad}} = v^2/r$.

SET UP: The block's angular momentum with respect to the hole is $L = mvr$.

EXECUTE: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\dfrac{v^2}{r}$. $T = mv^2\dfrac{1}{r} = \dfrac{m^2v^2}{m}\dfrac{r^2}{r^3} = \dfrac{(mvr)^2}{mr^3} = \dfrac{L^2}{mr^3}$. The radius at which the string breaks is

$$r^3 = \frac{L^2}{mT_{\max}} = \frac{(mv_1r_1)^2}{mT_{\max}} = \frac{[(0.130\text{ kg})(4.00\text{ m/s})(0.800\text{ m})]^2}{(0.130\text{ kg})(30.0\text{ N})},$$ from which $r = 0.354$ m.

EVALUATE: Just before the string breaks, the speed of the rock is $(4.00\text{ m/s})\left(\dfrac{0.800\text{ m}}{0.354\text{ m}}\right) = 9.04$ m/s.

We can verify that using $T = mv^2/R$ that $v = 9.04$ m/s and $r = 0.354$ m do give $T = 30.0$ N.

10.87. **IDENTIFY:** Apply conservation of momentum to the system of the runner and turntable.
SET UP: Let the positive sense of rotation be the direction the turntable is rotating initially.
EXECUTE: The initial angular momentum is $I\omega_1 - mRv_1$, with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(I + mR^2)$, so $\omega_2 = \dfrac{I\omega_1 - mRv_1}{I + mR^2}$.

$$\omega_2 = \frac{(80\ \text{kg}\cdot\text{m}^2)(0.200\ \text{rad/s}) - (55.0\ \text{kg})(3.00\ \text{m})(2.8\ \text{m/s})}{(80\ \text{kg}\cdot\text{m}^2) + (55.0\ \text{kg})(3.00\ \text{m})^2} = -0.776\ \text{rad/s}.$$

EVALUATE: The minus sign indicates that the turntable has reversed its direction of motion. This happened because the man had the larger magnitude of angular momentum initially.

10.88. **IDENTIFY:** We use the power and angular velocity to calculate the torque.
SET UP: $P = \tau\omega$, 1 hp = 746 W.
EXECUTE: **(a)** First make the necessary conversions: $1\ \text{ft}\cdot\text{lb} = (0.3048\ \text{m})(4.448\ \text{N}) = 1.356\ \text{N}\cdot\text{m}$
1 rpm = 1 rev/min = (2π rad)/(60 s) = 0.1047 rad/s.
Solve for torque and use the above conversions:
$\tau = P/\omega$ = [(285 hp)/(5300 rpm)]{(746 W/hp)/[(0.1047 rad/s)/rpm]} = 383 N·m = 283 ft·lb.
As we can see, 283 ft·lb is less than the maximum 305 ft·lb.
(b) $P = \tau\omega$ = (305 ft·lb) (3900 rpm)(1.356 N·m / ft·lb) [(0.1047 rad/s)/rpm] = 169 kW = 226 hp.
The power of 226 hp is smaller than the maximum of 285 hp.
(c) Make the following conversions:
hp = $\tau(\text{ft}\cdot\text{lb})\omega(\text{rpm})\left(\dfrac{1.356\ \text{N}\cdot\text{m}}{1\ \text{ft}\cdot\text{lb}}\right)\left(\dfrac{0.1047\ \text{rad/s}}{1\ \text{rpm}}\right)\left(\dfrac{1\ \text{hp}}{746\ \text{W}}\right) = 1.9031\times10^{-4}\ \tau(\text{ft}\cdot\text{lb})\omega(\text{rpm})$, so $1/c$ = 1.9031×10^{-4}, which gives c = 5254.
(d) From (c), $P = \tau\omega$ gives 580 hp = τ(6000 rpm)/5254, so τ = 508 ft·lb.
EVALUATE: Torque, power, and angular velocity are often expressed in diverse units, so conversions are frequently necessary.

10.89. **IDENTIFY:** All the objects have the same mass and start from rest at the same height h. They roll without slipping, so their mechanical energy is conserved. Newton's second law, in its linear and rotational forms, applies to each object. Since the objects have different mass distributions, they will take different times to reach the bottom of the ramp.
SET UP: $K_1 + U_1 = K_2 + U_2$, $\sum\vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$, $\Sigma\tau = I\alpha$, $K_{\text{tot}} = K_{\text{cm}} + K_{\text{rot}}$, $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$, $K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2$.
EXECUTE: **(a)** We can express the moment of inertia of a round object as $I = cmR^2$, where c depends on the shape and mass distribution. Energy conservation gives $K_1 + U_1 = K_2 + U_2$, so
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}cmR^2\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}cmR^2\left(\dfrac{v}{R}\right)^2 = \frac{1}{2}v^2(1+c)$. Solving for v^2 gives
$v^2 = \dfrac{2gh}{1+c}$. This v is the speed at the bottom of the ramp. The object with the greatest speed v will also have the greatest average speed down the ramp and will therefore take the shortest time to reach the bottom. Thus the object with the smallest c will have the greatest v and therefore the shortest time in the bar graph shown with the problem. For a solid cylinder, $I = \frac{1}{2}mR^2$ so c = ½, for a hollow cylinder, $I = mR^2$, so c = 1, and likewise we get c = 2/5 for a solid sphere and c = 2/3 for a hollow sphere. The smallest value of c is 2/5 for a solid sphere, so that object must take the shortest time, which makes it

object A. The largest value of c is 1 for a hollow cylinder, so that object takes the longest time, which makes it object D. The hollow sphere has a larger c than the solid cylinder, so it takes longer than the solid cylinder, so C must be the hollow sphere and B the solid cylinder. Summarizing these results, we have

A: solid sphere, $c = 2/5$
B: solid cylinder, $c = 1/2$
C: hollow sphere, $c = 2/3$
D: hollow cylinder, $c = 1$

(b) All the objects start from rest at the same initial height and roll without slipping, so they all have the same kinetic energy at the bottom of the ramp.

(c) Using $K_{\text{rot}} = \frac{1}{2} I_{\text{cm}}\omega^2$, we have $K_{\text{rot}} = ½ (cmR^2)(v/R)^2 = ½\, mcv^2$. Using our result for v^2 from (a) gives $K_{\text{rot}} = \frac{1}{2}mc\left(\frac{2gh}{1+c}\right) = mgh\left(\dfrac{1}{1+\dfrac{1}{c}}\right)$. From this result, we see that the object with the largest c has the largest rotational kinetic energy because the denominator in the parentheses is the smallest. Therefore the hollow cylinder, with $c = 1$, has the largest rotational kinetic energy.

(d) Apply Newton's second law. Perpendicular to the ramp surface, we get $n = mg \cos\theta$ for the normal force. Parallel to the surface, with down the ramp as positive, we get $mg \sin\theta - f_s = ma$. Taking torques about the center of the rolling object gives $fR = I\alpha = (mcR^2)(a/R)$, which gives $f_s = mca$, so $ma = f_s/c$. Putting this into the previous equation gives $mg \sin\theta - f_s = f_s/c$, which can be written as $mg \sin\theta = f_s(1 + 1/c)$. We want the minimum coefficient of friction to prevent slipping, so $f_s = \mu_s n = \mu_s mg \cos\theta$. Putting this into the previous equation gives $mg \sin\theta = (\mu_s mg \cos\theta)(1 + 1/c)$. Solving for μ_s gives $\mu_s = \dfrac{\tan\theta}{1+\dfrac{1}{c}}$. We want μ_s such that none of the objects will slip, so we must find the maximum μ_s. That will occur when c has its largest value since that will make the denominator smallest, and that is for the hollow cylinder for which $c = 1$. This gives $\mu_s = (\tan 35.0°)/2 = 0.350$.

EVALUATE: As a check, part (a) could be solved using Newton's second law, as we did in part (d). As a check in part (d), find μ_s for the solid sphere which has the smallest value of c. This gives $\mu_s = \dfrac{\tan 35.0°}{1+\dfrac{1}{2/5}} = \dfrac{\tan 35.0°}{3.5} = 0.200$. This is less than the 0.350 we found in (d), so a coefficient of friction of 0.350 is more than enough to prevent slipping of the solid sphere.

10.90. IDENTIFY: The work done by the force F is equal to the kinetic energy gained by the flywheel. This work is the area under the curve in a F-versus-d graph.

SET UP: $W = Fd$, $K = \frac{1}{2}I\omega^2$, $v = r\omega$.

EXECUTE: **(a)** The pull is constant, so the linear and angular accelerations are constant. Therefore $v = 2v_{\text{av}} = 2(d/t)$, so $\omega = v/R = 2d/tR$. The work done is equal to the kinetic energy of the flywheel, so $Fd = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\frac{2d}{tR}\right)^2$. Solving for I gives

$I = Ft^2R^2/2d = (25.0\text{ N})(2.00\text{ s})^2(0.166\text{ m})^2/[2(8.35\text{ m})] = 0.165\ \text{kg}\cdot\text{m}^2$.

(b) The kinetic energy gained is equal to the work done which is equal to the area under the curve on the F-d graph. This gives

$K = (60.0\text{ N})(3.00\text{ m}) + ½\,(60.0\text{ N})(3.00\text{ m}) = 270\text{ J}$.

(c) $K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{\dfrac{2K}{I}} = \sqrt{\dfrac{2(270\text{ J})}{0.165\text{ kg}\cdot\text{m}^2}} = 57.2$ rad/s. Converting to rpm gives

(57.2 rad/s)[(60 s)/(1 min)][(1 rev)/(2π rad)] = 546 rpm.

EVALUATE: In this case, we could have deduced the equation for F as a function of d from the graph and integrated to find the work. But for a more complicated F-d dependence, that would have been impossible, but we could still estimate the area quite accurately from the graph.

10.91. **IDENTIFY:** The answer to part (a) can be taken from the solution to Problem 10.86. The work-energy theorem says $W = \Delta K$.

SET UP: Problem 10.86 uses conservation of angular momentum to show that $r_1 v_1 = r_2 v_2$.

EXECUTE: **(a)** $T = mv_1^2 r_1^2 / r^3$.

(b) $\vec{\boldsymbol{T}}$ and $d\vec{\boldsymbol{r}}$ are always antiparallel. $\vec{\boldsymbol{T}} \cdot d\vec{\boldsymbol{r}} = -T dr$.

$$W = -\int_{r_1}^{r_2} T\,dr = mv_1^2 r_1^2 \int_{r_2}^{r_1} \frac{dr}{r^3} = \frac{mv_1^2}{2} r_1^2 \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

(c) $v_2 = v_1(r_1/r_2)$, so $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \dfrac{mv_1^2}{2}\left[(r_1/r_2)^2 - 1\right]$, which is equal to the work found in part (b).

EVALUATE: The work done by T is positive, since $\vec{\boldsymbol{T}}$ is toward the hole in the surface and the block moves toward the hole. Positive work means the kinetic energy of the object increases.

10.92. **IDENTIFY:** Apply $\sum \vec{\boldsymbol{F}}_{\text{ext}} = m\vec{\boldsymbol{a}}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the motion of the cylinder. Use constant acceleration equations to relate a_x to the distance the object travels. Use the work-energy theorem to find the work done by friction.

SET UP: The cylinder has $I_{\text{cm}} = \frac{1}{2}MR^2$.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.92. The friction force is $f = \mu_k n = \mu_k Mg$, so $a = \mu_k g$. The magnitude of the angular acceleration is $\dfrac{fR}{I} = \dfrac{\mu_k MgR}{(1/2)MR^2} = \dfrac{2\mu_k g}{R}$.

(b) Setting $v = at = \omega R = (\omega_0 - \alpha t)R$ and solving for t gives $t = \dfrac{R\omega_0}{a + R\alpha} = \dfrac{R\omega_0}{\mu_k g + 2\mu_k g} = \dfrac{R\omega_0}{3\mu_k g}$,

and $d = \frac{1}{2}at^2 = \frac{1}{2}(\mu_k g)\left(\dfrac{R\omega_0}{3\mu_k g}\right)^2 = \dfrac{R^2\omega_0^2}{18\mu_k g}$.

(c) The final kinetic energy is $(3/4)Mv^2 = (3/4)M(at)^2$, so the change in kinetic energy is

$$\Delta K = \frac{3}{4}M\left(\mu_k g \frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = -\frac{1}{6}MR^2\omega_0^2.$$

EVALUATE: The fraction of the initial kinetic energy that is removed by friction work is $\dfrac{\frac{1}{6}MR\omega_0^2}{\frac{1}{4}MR\omega_0^2} = \dfrac{2}{3}$.

This fraction is independent of the initial angular speed ω_0.

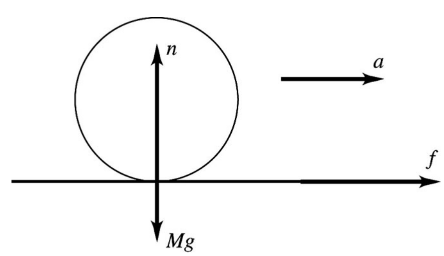


Figure 10.92

10.93. **IDENTIFY:** The vertical forces must sum to zero. Apply $\Omega = \dfrac{\tau_z}{L_z} = \dfrac{wr}{I\omega}.$

SET UP: Denote the upward forces that the hands exert as F_L and F_R. $\tau = (F_L - F_R)r,$ where $r = 0.200$ m.

EXECUTE: The conditions that F_L and F_R must satisfy are $F_L + F_R = w$ and $F_L - F_R = \Omega \dfrac{I\omega}{r},$ where the second equation is $\tau = \Omega L,$ divided by r. These two equations can be solved for the forces by first adding and then subtracting, yielding $F_L = \frac{1}{2}\left(w + \Omega \dfrac{I\omega}{r}\right)$ and $F_R = \frac{1}{2}\left(w - \Omega \dfrac{I\omega}{r}\right)$. Using the values

$w = mg = (8.00 \text{ kg})(9.80 \text{ m/s}^2) = 78.4 \text{ N}$ and

$\dfrac{I\omega}{r} = \dfrac{(8.00 \text{ kg})(0.325 \text{ m})^2(5.00 \text{ rev/s} \times 2\pi \text{ rad/rev})}{(0.200 \text{ m})} = 132.7 \text{ kg} \cdot \text{m/s}$ gives

$F_L = 39.2 \text{ N} + \Omega(66.4 \text{ N} \cdot \text{s}),\ F_R = 39.2 \text{ N} - \Omega(66.4 \text{ N} \cdot \text{s}).$

(a) $\Omega = 0,\ F_L = F_R = 39.2$ N.

(b) $\Omega = 0.05 \text{ rev/s} = 0.314 \text{ rad/s},\ F_L = 60.0 \text{ N},\ F_R = 18.4$ N.

(c) $\Omega = 0.3 \text{ rev/s} = 1.89 \text{ rad/s},\ F_L = 165 \text{ N},\ F_R = -86.2 \text{ N},$ with the minus sign indicating a downward force.

(d) $F_R = 0$ gives $\Omega = \dfrac{39.2 \text{ N}}{66.4 \text{ N} \cdot \text{s}} = 0.590$ rad/s, which is 0.0940 rev/s.

EVALUATE: The larger the precession rate Ω, the greater the torque on the wheel and the greater the difference between the forces exerted by the two hands.

10.94. **IDENTIFY:** The rotational form of Newton's second law applies.
SET UP: $\Sigma\tau = I\alpha$ and $\omega_z = \omega_{0z} + \alpha_z t.$
EXECUTE: $\Sigma\tau = I\alpha = \Delta\omega/\Delta t,$ where $I = I_{\text{person}} + I_0$. Solving for I_{person} gives $I_{\text{person}} = \tau/\alpha - I_0$.
$I_{\text{person}} = \dfrac{2.5 \text{ N} \times \text{m}}{\left(\dfrac{1.0 \text{ rad/s}}{3.0 \text{ s}}\right)} - 1.5 \text{ kg} \times \text{m}^2 = 6.0 \text{ kg} \times \text{m}^2,$ which is choice (b).

EVALUATE: The moment of inertia of the turntable is considerably less than that of the person, which is a good thing. If the moment of inertia of the table were much greater than that of the person, the person's body would have a small effect on the angular acceleration of the table, making it hard to get an accurate measurement.

10.95. **IDENTIFY** and **SET UP:** Moment of inertia depends on the distribution of mass.
EXECUTE: Extending her legs increases the person's moment of inertia to increase. With a constant torque on the turntable, this would decrease her angular acceleration, which is choice (c).
EVALUATE: The person being studied should be told to lie still during the procedure.

10.96. **IDENTIFY** and **SET UP:** The torque is the product of the force times the lever arm, and $\Sigma\tau = I\alpha.$
EXECUTE: Doubling the lever arm with a constant force doubles the torque, which then doubles the angular acceleration, so choice (b) is correct.
EVALUATE: Doubling the diameter of the pulley would also allow the tension to be decreased by a factor of 2 and still keep the same original angular acceleration.

10.97. **IDENTIFY** and **SET UP:** The parallel-axis theorem, $I = I_{\text{cm}} + md^2$, applies to the person.
EXECUTE: The measured moment of inertia would be I, but this would be greater than I_{cm}, so the measured value would be too large, choice (a).
EVALUATE: Care is essential to position the person properly on the turntable.