# Circuits

-

# Nodal and Mesh analysis

Spring 2022

# Analysis

## Until now

Each circuit is unique and the analysis was generally a mix between:

- KCL
- KVL
- Ohm's law

## Approach

Usually, we need some **experience** to know when to apply which law...

## Methodical approach

We are gonna see 2 generic approaches:

- Nodal analysis
- Mesh analysis

# Nodal Analysis

上海纽约大学 NYU SHANGHAI

## Principle

Nodal analysis is based on KCL

## Essential node: definition

An essential node is a point at which **3 or more elements** are connected to

## Reference node

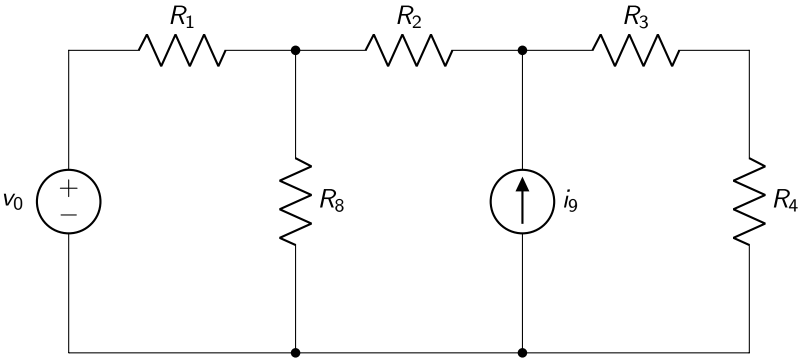
Nodal analysis will introduce a reference node: voltages will be defined from that node

the earth ground

# Nodal Analysis

How many nodes? 5

How many essential nodes? 3.



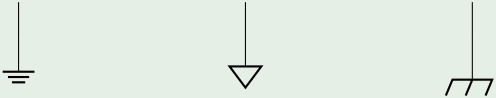
# Nodal Analysis

## Pick a reference node

Usually, the analysis is **easier** if you choose the node that **interconnects the most branches**

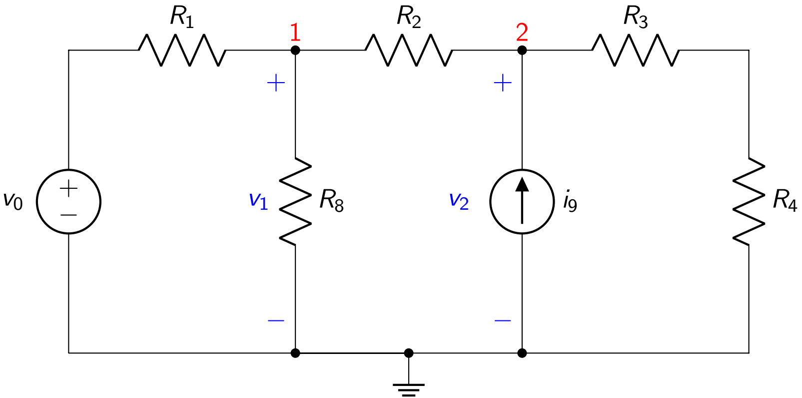
- mandatory to be an essential node
- classically located at the bottom of the circuit
- use a symbol similar to **ground**

## Symbols for ground



# Nodal Analysis

Place the reference node and label the essential nodes



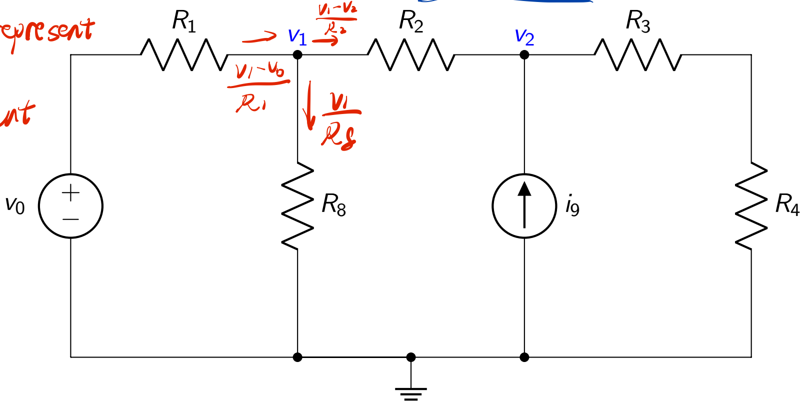
# Nodal Analysis

上海纽约大学 NYU SHANGHAI

when applying Nodal Analysis we should label voltage at the essential nodes instead of currents

Equivalent representation

we use ohm's law to represent the current



# Nodal Analysis

上海纽约大学
NYU SHANGHAI

## Strategy

- Express the **currents** as functions of the **voltages** at essential nodes
- and then **apply KCL** at each essential node (excluding the reference node)

## System of equations

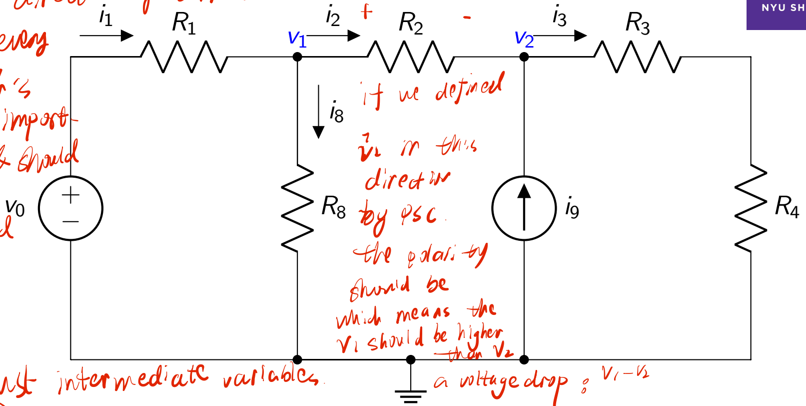
It should lead to a system of $N$ equations with $N$ unknowns

$N$ is the number of essential nodes (excluding the reference node), and the unknowns are the **voltages at these nodes**

# Nodal Analysis

上海纽约大学 NYU SHANGHAI

the direction of current at every branch's very important & should be labelled




if we defined $i_2$ in this direction by psc. the polarity should be which means the $v_1$ should be higher than $v_2$

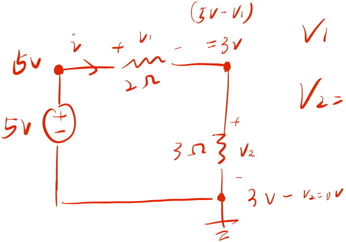
a voltage drop : $v_1 - v_2$

just intermediate variables

$$i_1 = \frac{v_0 - v_1}{R_1} \qquad i_2 = \frac{v_1 - v_2}{R_2} \qquad i_3 = \frac{v_2}{R_3 + R_4} \qquad i_8 = \frac{v_1}{R_8}$$

not unknowns.

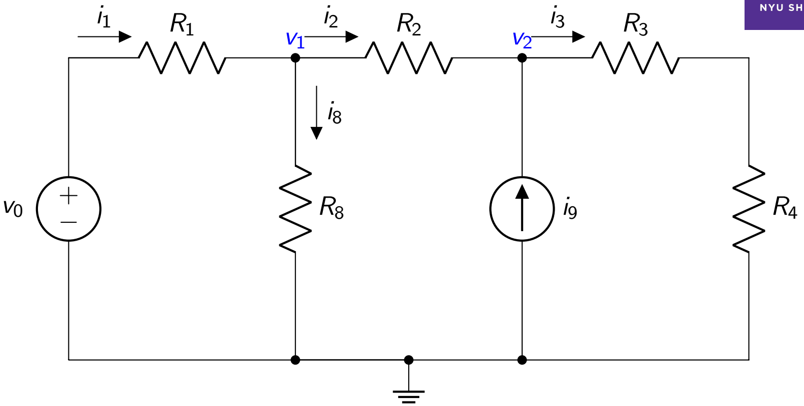
- $i_1 = i_2 + i_8$
- $i_2 + i_9 = i_3$




$$V_1 = 5V \cdot \frac{2\Omega}{5\Omega} = 2V$$

$$V_2 = 3V$$

# Nodal Analysis




- $\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_8}\right)\cdot v_1 - \frac{1}{R_2}\cdot v_2 = \frac{v_0}{R_1}$
- $-\frac{1}{R_2}\cdot v_1 + \left(\frac{1}{R_3+R_4}+\frac{1}{R_2}\right)\cdot v_2 = i_9$

# Nodal Analysis

上海纽约大学 NYU SHANGHAI

## Summary

7 steps to follow:

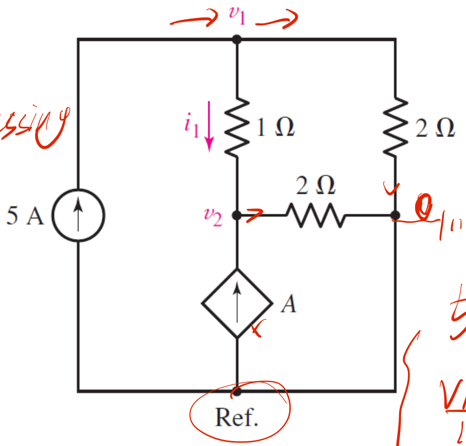
1. Identify essential nodes
2. Pick a reference node (essential. bottom, most branches)
3. Label other essential nodes with voltages
4. Apply KCL to these nodes
5. Determine the currents as functions of nodal voltages
6. Solve linear system of equations for the voltages
7. Post-process any variable of interest

# equations = # essential nodes we label

# Examples

Determine the voltage $v_1$ if $A$ is (a) $2i_1$; (b) $2v_1$.

post -processing




We only need to label two essential nodes

we need 2 equations help us decide the # of equations needed.

$$5A = \frac{V_1 - V_2}{1\,\Omega} + \frac{V_1}{2\,\Omega}$$

$$\frac{V_1 - V_2}{1\,\Omega} + x = \frac{V_2}{2\,\Omega}$$

$$x = \begin{cases} 2i_1 = 2\,\dfrac{V_1 - V_2}{1\,\Omega} \\ 2V_1 \end{cases}$$

# Supernode

## Problem with nodal analysis

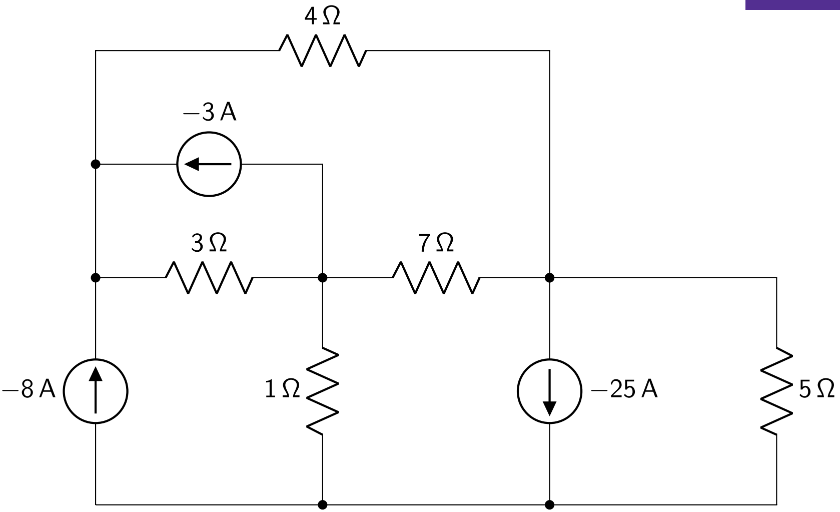
If essential nodes (excluding the reference node) are connected through a branch with a single voltage source (dependent or independent), the current in that branch cannot be expressed simply as a function of node voltages. We can't decide the current on that branch

## Solution

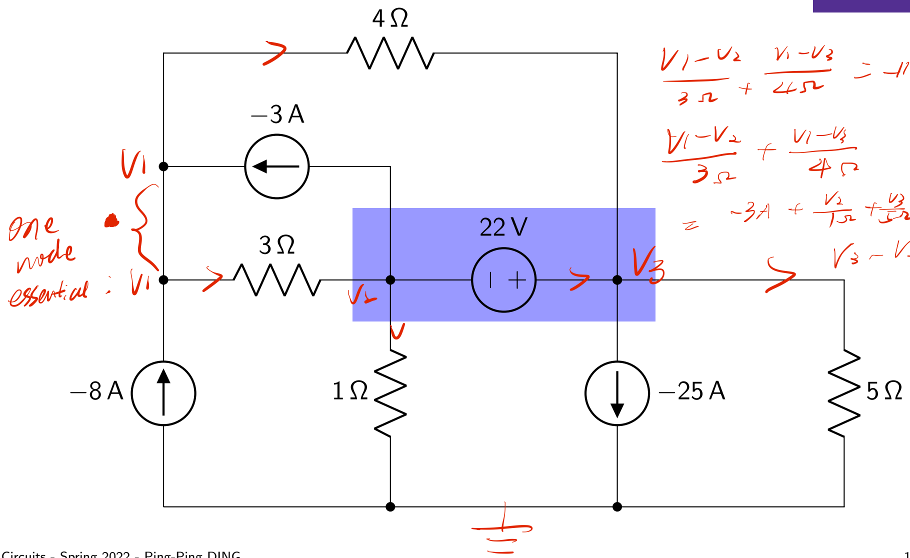
**Get rid of the current variable** in that branch by applying KCL to a **supernode**

An extra equation is then required (voltage equation for that branch)

# Supernode



# Supernode



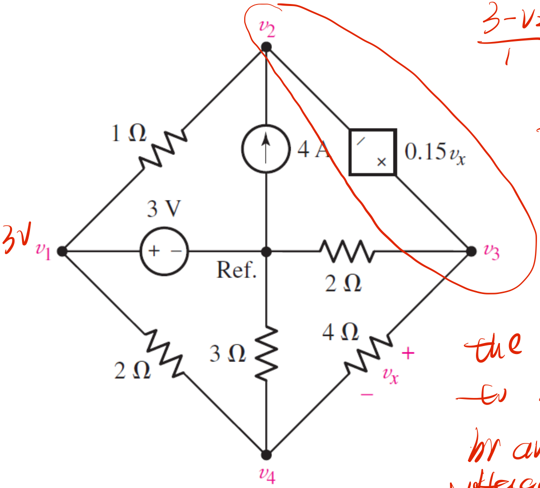

$$\frac{V_1 - V_2}{3\,\Omega} + \frac{V_1 - V_3}{4\,\Omega} = -11\,A$$

$$\frac{V_1 - V_2}{3\,\Omega} + \frac{V_1 - V_3}{4\,\Omega} = -3A + \frac{V_2}{1\,\Omega} + \frac{V_3}{5\,\Omega} - 25A$$

$$V_3 - V_2 = 22\,V$$

# Examples

**Determine the nodal voltages in the circuit.**




$$\frac{3-V_2}{1} + 4 + \frac{-V_3}{2} + \frac{V_4 - V_3}{4} = 0$$

$$\frac{3-V_4}{2} + \frac{-V_4}{3} + \frac{V_3 - V_4}{4} = 0$$

$$V_3 - V_2 = 0.15 V_x$$

$$= (V_3 - V_4) \times 0.15$$

the supernode need to include the total branch: having the voltage sources

# Mesh Analysis

### Principle

Mesh analysis is based on KVL

### Mesh: definition

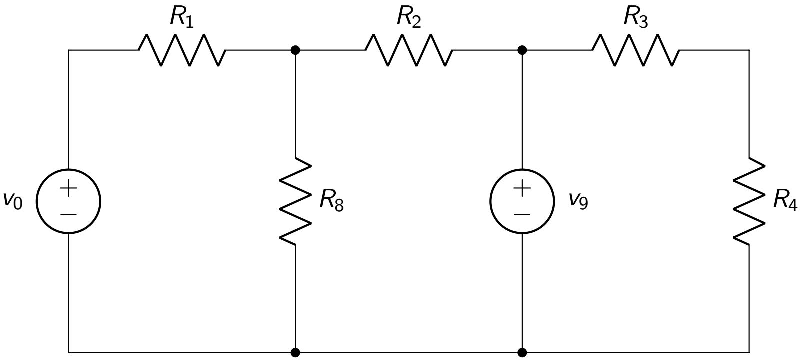
A mesh is a loop that does not enclose another loop

### Virtual currents

The strategy for mesh analysis is to consider that some **virtual mesh currents** are flowing through each mesh.
The **unknowns** for Mesh Analysis are these virtual mesh currents

# Mesh Analysis

How many loops? 6
How many meshes? 3

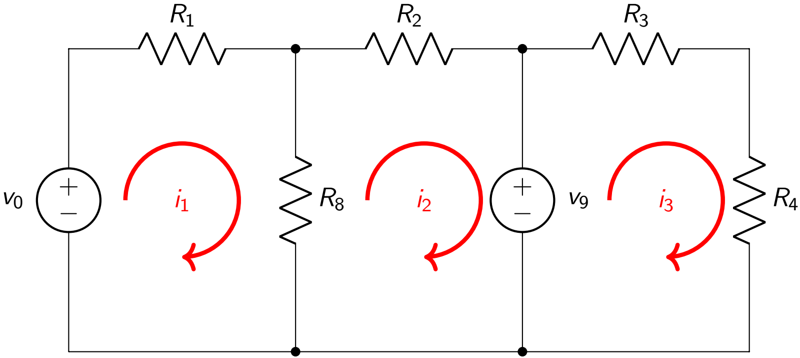


# Mesh Analysis

virtual currents

Labeling the circuit with mesh currents

# meshes = min # equations of Analyzing by KVL.



# Mesh Analysis

## Applying KVL for each mesh

- $v_0 - R_1 \cdot i_1 - R_8 \cdot (i_1 - i_2) = 0$
- $R_8 \cdot (i_1 - i_2) - R_2 \cdot i_2 - v_9 = 0$
- $v_9 - (R_3 + R_4) \cdot i_3 = 0$

## System of equations

- $(R_1 + R_8) \cdot i_1 - R_8 \cdot i_2 = v_0$ the unknowns are virtual currents.
- $R_8 \cdot i_1 - (R_2 + R_8) \cdot i_2 = v_9$
- $(R_3 + R_4) \cdot i_3 = v_9$

# Mesh Analysis

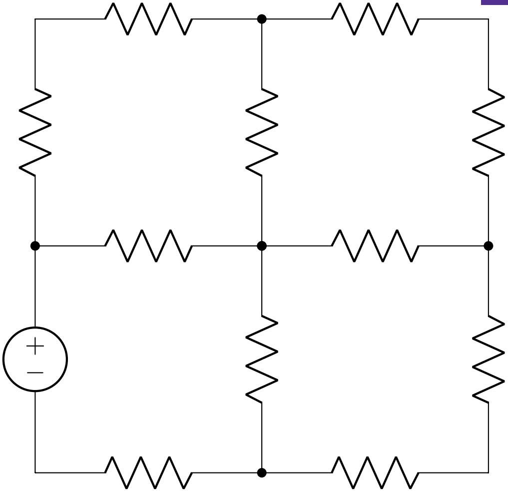
## Summary

6 steps to follow:

1. Check if you have a **planar circuit**
2. Identify and label the virtual currents for each mesh
3. Apply KVL to these meshes
4. Determine the voltages as functions of mesh currents
5. Solve linear system of equations for the currents
6. Post-process any variable of interest

# Mesh Analysis

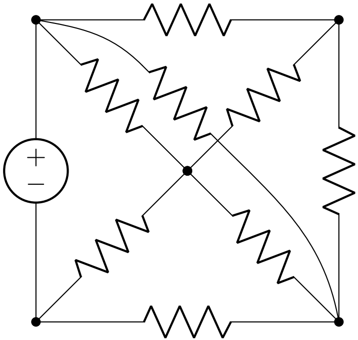
Planar



# Mesh Analysis

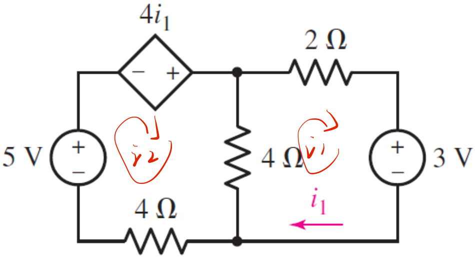
Mesh's a property of planar circuit, and is undefined for a non-planar circuit.

Planar



# Examples

Determine the current $i_1$ in the circuit.




$i_1 = -\frac{1}{4}$ A
$i_2 = \frac{3}{8}$ A

$$\begin{cases} 3V + 4(i_1 - i_2) + 2i_1 = 0 \\ -5V - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0 \end{cases}$$

# Supermesh

## Problem with mesh analysis

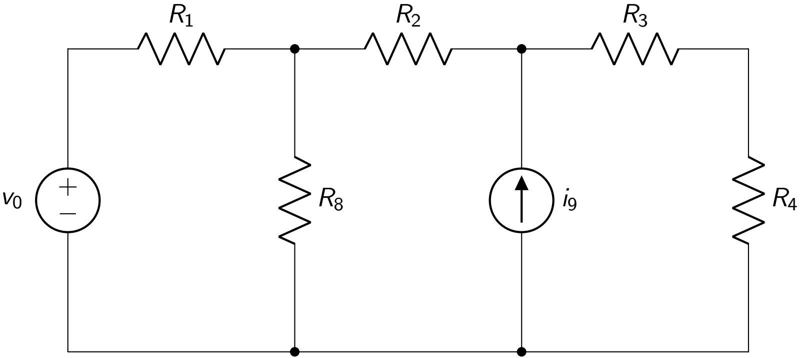
If some mesh has some current source (dependent or independent), the voltage across the source cannot be expressed simply as a function of virtual currents.

## Solution

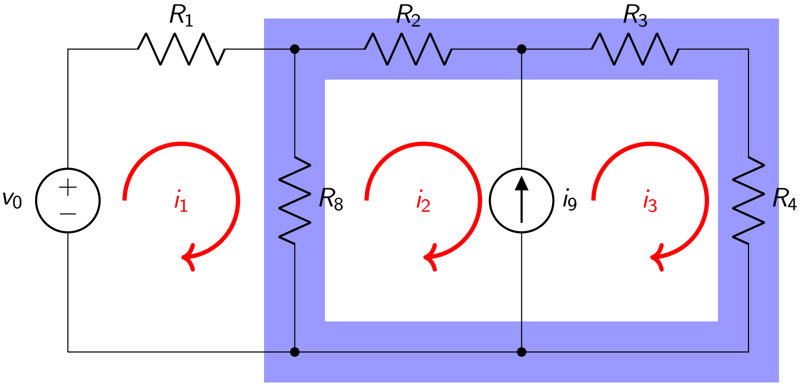
**Get rid of the voltage variable** for the source by applying KVL to a **supermesh**

An extra equation is then required (source current expressed as a function of virtual currents)

# Supermesh



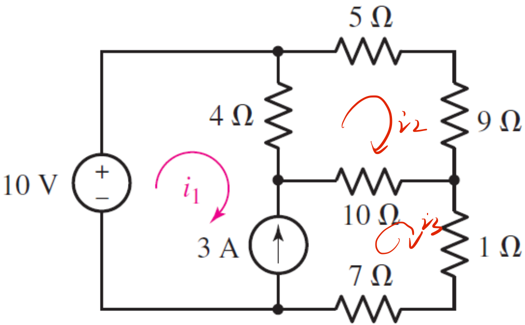
# Supermesh




- KVL on supermesh: $R_8 \cdot (i_1 - i_2) - R_2 \cdot i_2 - (R_3 + R_4) \cdot i_3 = 0$
- Extra equation: $i_9 = -i_2 + i_3$

# Examples

**Determine the current $i_1$ in the circuit.**




$$\begin{cases} -10 + 4\Omega\, i_1 + 10\Omega(i_3 - i_2) + 8\Omega\, i_3 = 0 \\ 14\Omega \cdot i_2 + 10\Omega\,(i_2 - i_3) + 4\Omega \cdot (i_2 - i_1) = 0 \end{cases}$$

与

$i_3 - i_1 = 3A$