12

FLUID MECHANICS

VP12.4.1. **IDENTIFY:** We want the pressure at a depth in a fluid.
SET UP: $p = p_0 + \rho gh$ gives the absolute pressure.

EXECUTE: $p = p_0 + \rho gh = 1.22\times10^5\text{ Pa} + (455\text{ kg/m}^3)(9.80\text{ m/s}^2)(2.00\text{ m}) = 1.31\times10^5\text{ Pa}$.

EVALUATE: This pressure is about 30% above atmospheric pressure.

VP12.4.2. **IDENTIFY:** We are dealing with the gauge pressure at a depth in a fluid.
SET UP: $p = \rho gh$ gives the gauge pressure which is the pressure above atmospheric pressure.

EXECUTE: **(a)** The gauge pressure is due only to the gasoline.
$p = \rho gh = (740\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.150\text{ m}) = 1.09\times10^3\text{ Pa}$.

(b) The gauge pressure at the top of the water is 1.09×10^3 Pa, so the pressure at the bottom is
$p = p_0 + \rho gh = 1.09\times10^3\text{ Pa} + (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.150\text{ m}) = 2.56\times10^3\text{ Pa}$.

EVALUATE: To find the *absolute* pressure in each case we would have to add 1.01×10^5 Pa to our answers. A pressure gauge would read the gauge pressure.

VP12.4.3. **IDENTIFY:** We are dealing with the pressure at a depth in a fluid in a manometer.
SET UP: $p = p_0 + \rho gh$ gives the absolute pressure. Using Fig. 12.8a in the text, and looking at the right-hand column, the pressure in that column at a level y_1 is p. This is also the pressure due to the column from y_1 to the top which is a depth h in the right-hand column. Therefore the difference in heights h is $p - p_{\text{atm}} = \rho gh$.

EXECUTE: Solve for h: $h = \dfrac{p - p_{\text{atm}}}{\rho g} = \dfrac{2.1\times10^5\text{ Pa} - 1.01\times10^5\text{ Pa}}{(1.36\times10^4\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 0.818\text{ m} = 81.8\text{ cm}$.

EVALUATE: According to our results, if the pressure p in the container were greater, h would be greater. If the density of the fluid in the manometer were less, h would also be greater. Both cases are physically reasonable, which suggests our solution is correct.

VP12.4.4. **IDENTIFY:** We are dealing with the pressure at a depth in a fluid in a manometer.
SET UP: $p = \rho gh$ gives the gauge pressure. The gauge pressure at the bottom of the oil is the same as the gauge pressure at the bottom of the water since both tubes are open to the air.
EXECUTE: **(a)** Equating the gauge pressures gives $\rho_{\text{oil}} g h_{\text{oil}} = \rho_{\text{water}} g h_{\text{water}}$.

$$h_{\text{water}} = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} h_{\text{oil}} = \frac{916\text{ kg/m}^3}{1000\text{ kg/m}^3}\cdot 25.0\text{ cm} = 22.9\text{ cm}.$$

(b) $p_{\text{gauge}} = \rho gh = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.150\text{ m}) = 1.47\times10^3\text{ Pa}$.
(c) Solve $p_{\text{gauge}} = \rho gh$ for h, giving
$h = p_{\text{gauge}}/\rho g = (1.47\times10^3\text{ Pa})/[(916\text{ kg/m}^3)(9.80\text{ m/s}^2)] = 0.164\text{ m} = 16.4\text{ cm}$.

EVALUATE: From (b) and (c), we see that we would have to be 15.0 cm deep in water to have the same pressure as at 16.4 cm in oil. This is reasonable since the density of oil is less than that of water.

VP12.5.1. **IDENTIFY:** This problem involves, density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced.
EXECUTE: **(a)** $m = \rho V$, so $w = \rho Vg = (1150\ \text{kg/m}^3)(7.50\times10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) = 8.45\ \text{N}$.
(b) B equals the weight of the displaced fluid or gas, so $B = \rho Vg$.
(i) $B_{\text{air}} = \rho_{\text{air}} Vg = (1.20\ \text{kg/m}^3)(7.50\times10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) = 8.82\times10^{-3}$ N. The object would sink since the buoyant force on it is less than its weight.
(ii) $B_{\text{water}} = \rho_{\text{water}} Vg = (1000\ \text{kg/m}^3)(7.50\times10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) = 7.35$ N. The object would sink since the buoyant force on it is less than its weight.
(iii) $B_{\text{glycerine}} = \rho_{\text{glycerine}} Vg = (1260\ \text{kg/m}^3)(7.50\times10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) = 9.26$ N. The object would rise since the buoyant force on it is greater than its weight.
EVALUATE: Notice that in each case in (b), the object sinks if its density is greater than the fluid and rises if its density is less than that of the fluid. This is a useful point to keep in mind.

VP12.5.2. **IDENTIFY:** This problem involves density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced. $\sum F_y = 0$ for the sphere.
EXECUTE: **(a)** $B = \rho Vg = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.20\times10^{-3}\ \text{m}^3) = 11.8$ N.
(b) $\sum F_y = 0$ gives $w = T + B = 29.4\ \text{N} + 11.8\ \text{N} = 41.2\ \text{N}$.
(c) $\rho = m/V = (W/g)/V = (41.2\ \text{N})/[(9.80\ \text{m/s}^2)(1.20\times10^{-3}\ \text{m}^3)] = 3.50\times10^3\ \text{kg/m}^3$.
EVALUATE: The sphere is denser than water, so the buoyant force is less than its weight, so there must be a tension in the cable. This agrees with our results.

VP12.5.3. **IDENTIFY:** This problem involves density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced. $\sum F_y = 0$ for the cube. Call V the volume of the cube and ρ_c its density. Let ρ be the density of the fluid.
EXECUTE: Archimedes's principle gives $B = \rho g(V/2)$. $\sum F_y = 0$ for the cube gives

$$T + B - w_{\text{cube}} = 0 \quad \rightarrow \quad T + \rho gV/2 - \rho_c gV = 0 \quad \rightarrow \quad \rho = \frac{2(\rho_c gV - T)}{gV}$$

$$\rho = \frac{2\left[(7.50\times10^3\text{kg/m}^3)(9.80\ \text{m/s}^2)(5.50\times10^{-3}\text{m}^3) - 375\ \text{N}\right]}{(9.80\ \text{m/s}^2)(5.50\times10^{-3}\text{m}^3)} = 1.1\times10^3\ \text{kg/m}^3.$$

EVALUATE: The density of this fluid is slightly greater than that of water.

VP12.5.4. **IDENTIFY:** This problem involves density, buoyancy, and Archimedes's principle.
SET UP: Density is $\rho = m/V$. Archimedes's principle says that the buoyant force B on an immersed object is equal to the weight of the fluid displaced.
EXECUTE: **(a)** $B = \rho_L Vg$ and $V = Ad$, so $B = \rho_L Adg$.
(b) For floating $B = w_{\text{cyl}} \quad \rightarrow \quad \rho_L Vg = \rho_{\text{cyl}} ALg \quad \rightarrow \quad \rho_{\text{cyl}} = \dfrac{d}{L}\rho_L$.
EVALUATE: If $d = L$, our result gives $\rho_{\text{cyl}} = \rho_L$. In that case the cylinder would be fully submerged. Since the cylinder and liquid have the same densities, the volumes of the cylinder and the displaced liquid would have to be equal for the cylinder to float.

VP12.9.1. **IDENTIFY:** This is a problem in fluid flow involving Bernoulli's equation and the continuity equation.
SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$ and the continuity equation is $A_1 v_1 = A_2 v_2$.

Execute: **(a)** Use $A_1v_1 = A_2v_2$ to find the speed of flow at ground level.

$A_1v_1 = A_2v_2 \quad \rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2 \quad \rightarrow \quad v_2 = (r_1/r_2)^2 v_1$

$v_2 = [(1.0\text{ cm})/(0.50\text{ cm})]^2(1.6\text{ m/s}) = 6.4\text{ m/s}$.

(b) Use Bernoulli's equation with point 2 to be floor level. This makes $v_1 = 1.6$ m/s, $v_2 = 6.4$ m/s, $y_1 = 9.0$ m, $y_2 = 0$, $p_1 = 3.0\times 10^5$ Pa, and $\rho = 1000\text{ kg/m}^3$. We want p_2 at ground level. Solving Bernoulli's equation gives $p_2 = 3.7\times 10^5$ Pa.

Evaluate: The pressure in the pipe is about 3.7 times atmospheric pressure. It would be even greater if the pipe were longer than 9.0 m.

VP12.9.2. **Identify:** This problem is about fluid flow. It involves Bernoulli's equation, volume flow rate, and the continuity equation.

Set Up: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, the volume flow rate is $dV/dt = Av$, and the continuity equation is $A_1v_1 = A_2v_2$.

Execute: **(a)** We know the volume flow rate, so use $dV/dt = Av$ to find v_2.

$$v = \frac{dV/dt}{A_2} = \frac{dV/dt}{\pi r_2^2} = \frac{4.4\times 10^{-3}\text{ m}^3/\text{s}}{\pi(0.010\text{ m})^2} = 14\text{ m/s}.$$

(b) Apply Bernoulli's equation. Call point 1 the top of the ethanol and point 2 the level of the out-coming ethanol at the bottom. We want $p_1 - p_{atm}$ (the *gauge* pressure) and we know that $p_2 = p_{atm}$, $y_1 = 3.2$ m, $y_2 = 0$, $\rho = 810\text{ kg/m}^3$, $v_2 = 14$ m/s, and $v_1 \approx 0$ (because $A_1 >> A_2$). Solve for $p_1 - p_{atm}$ gives $p_{gauge} = p_1 - p_{atm} = 5.4\times 10^4$ Pa.

Evaluate: The tank is pressurized because $p_1 > p_{atm}$.

VP12.9.3. **Identify:** This problem deals with a Venturi meter.

Set Up: From Example 12.9, we have $p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$. We want the difference in height h of the liquid levels in the two tubes. We also know from Example 12.9 that $v = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$. First solve for v_1 and then use that result to solve for h. The volume flow rate is $dV/dt = Av$.

Execute: **(a)** First use $p_1 - p_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$ to find v_1.

$$810\text{ Pa} = \frac{1}{2}(1000\text{ kg/m}^3)v_1^2\left[\left(\frac{\pi(2.5\text{ cm})^2}{\pi(1.2\text{ cm})^2}\right)^2 - 1\right] \quad \rightarrow \quad v_1 = 0.3014\text{ m/s}.$$

Now use $v_1 = \sqrt{\frac{2gh}{(A_1/A_2)^2 - 1}}$ to solve for h. Use $v_1 = 0.3014$ m/s and the same areas as above. This gives $h = 5.9\times 10^{-4}\text{ m}^3/\text{s}$.

(b) $dV/dt = A_1v_1 = \pi r_1^2 v_1 = \pi(0.025\text{ m})^2(0.3014\text{ m/s}) = 5.9\times 10^{-4}\text{ m}^3/\text{s}$.

Evaluate: The volume flow rate is the same throughout the pipe, so we calculated it at point 1. But it would also be true at point 2 or any other point in the pipe.

VP12.9.4. **Identify:** This problem is about fluid flow. It involves Bernoulli's equation, volume flow rate, and the continuity equation.

SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, the volume flow rate is $dV/dt = Av$, and the continuity equation is $A_1 v_1 = A_2 v_2$. We follow exactly the same procedure as in problem VP12.9.2 *except* that v_1 cannot be neglected.

EXECUTE: The continuity equation gives v_1: $A_1 v_1 = A_2 v_2 \rightarrow v_1 = v_2(A_2/A_1)$. Now use Bernoulli's equation with $p_1 = p_0$, $p_2 = p_{atm}$, $y_1 = h$, $y_2 = 0$, $v_1 = v_2(A_2/A_1)$. This equation becomes

$p_0 - p_{atm} + \rho g h = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_2^2 \left(\frac{A_2}{A_1}\right)^2$. Solving for v_2 gives $v_2 = \sqrt{\dfrac{2\left(\dfrac{p_0 - p_{atm}}{\rho}\right) + 2gh}{1 - (A_2/A_1)^2}}$.

EVALUATE: To check our result, consider the case of $A_1 >> A_2$. In that case the denominator approaches 1 and $v_2 \rightarrow \sqrt{2\left(\dfrac{p_0 - p_{atm}}{\rho}\right) + 2gh}$. This is the result obtained in Example 2.8 in the text, so our result looks reasonable. Also if $A_1 >> A_2$ and the tank is open at the top, $p_0 = p_{atm}$ and $v_2 \rightarrow \sqrt{2gh}$, which is the as for an object in freefall, as we would expect.

12.1. IDENTIFY: Use $\rho = m/V$ to calculate the mass and then use $w = mg$ to calculate the weight.

SET UP: $\rho = m/V$ so $m = \rho V$ From Table 12.1, $\rho = 7.8 \times 10^3 \text{ kg/m}^3$.

EXECUTE: For a cylinder of length L and radius R,
$V = (\pi R^2)L = \pi(0.01425 \text{ m})^2(0.858 \text{ m}) = 5.474 \times 10^{-4} \text{ m}^3$.

Then $m = \rho V = (7.8 \times 10^3 \text{ kg/m}^3)(5.474 \times 10^{-4} \text{ m}^3) = 4.27 \text{ kg}$, and

$w = mg = (4.27 \text{ kg})(9.80 \text{ m/s}^2) = 41.8 \text{ N}$ (about 9.4 lbs). A cart is not needed.

EVALUATE: The rod is less than 1m long and less than 3 cm in diameter, so a weight of around 10 lbs seems reasonable.

12.2. IDENTIFY: The volume of the remaining object is the volume of a cube minus the volume of a cylinder, and it is this object for which we know the mass. The target variables are the density of the metal of the cube and the original weight of the cube.

SET UP: The volume of a cube with side length L is L^3, the volume of a cylinder of radius r and length L is $\pi r^2 L$, and density is $\rho = m/V$.

EXECUTE: **(a)** The volume of the metal left after the hole is drilled is the volume of the solid cube minus the volume of the cylindrical hole:
$V = L^3 - \pi r^2 L = (5.0 \text{ cm})^3 - \pi(1.0 \text{ cm})^2(5.0 \text{ cm}) = 109 \text{ cm}^3 = 1.09 \times 10^{-4} \text{ m}^3$. The cube with the hole has mass $m = \dfrac{w}{g} = \dfrac{6.30 \text{ N}}{9.80 \text{ m/s}^2} = 0.6429 \text{ kg}$ and density $\rho = \dfrac{m}{V} = \dfrac{0.6429 \text{ kg}}{1.09 \times 10^{-4} \text{ m}^3} = 5.9 \times 10^3 \text{ kg/m}^3$.

(b) The solid cube has volume $V = L^3 = 125 \text{ cm}^3 = 1.25 \times 10^{-4} \text{ m}^3$ and mass
$m = \rho V = (5.9 \times 10^3 \text{ kg/m}^3)(1.25 \times 10^{-4} \text{ m}^3) = 0.7372 \text{ kg}$. The original weight of the cube was $w = mg = 7.2 \text{ N}$.

EVALUATE: As Table 12.1 shows, the density of this metal is about twice that of aluminum and half that of lead, so it is reasonable.

12.3. IDENTIFY: $\rho = m/V$

SET UP: The density of gold is $19.3 \times 10^3 \text{ kg/m}^3$.

EXECUTE: $V = (5.0 \times 10^{-3} \text{ m})(15.0 \times 10^{-3} \text{ m})(30.0 \times 10^{-3} \text{ m}) = 2.25 \times 10^{-6} \text{ m}^3$.

$\rho = \dfrac{m}{V} = \dfrac{0.0158 \text{ kg}}{2.25 \times 10^{-6} \text{ m}^3} = 7.02 \times 10^3 \text{ kg/m}^3$. The metal is not pure gold.

Evaluate: The average density is only 36% that of gold, so at most 36% of the mass is gold.

12.4. Identify: Average density is $\rho = m/V$.

Set Up: For a sphere, $V = \frac{4}{3}\pi R^3$. The sun has mass $M_{\text{sun}} = 1.99\times10^{30}$ kg and radius 6.96×10^8 m.

Execute: **(a)** $\rho = \dfrac{M_{\text{sun}}}{V_{\text{sun}}} = \dfrac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(6.96\times10^8\ \text{m})^3} = \dfrac{1.99\times10^{30}\ \text{kg}}{1.412\times10^{27}\ \text{m}^3} = 1.409\times10^3\ \text{kg/m}^3$

(b) $\rho = \dfrac{1.99\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(2.00\times10^4\ \text{m})^3} = \dfrac{1.99\times10^{30}\ \text{kg}}{3.351\times10^{13}\ \text{m}^3} = 5.94\times10^{16}\ \text{kg/m}^3$

Evaluate: For comparison, the average density of the earth is $5.5\times10^3\ \text{kg/m}^3$. A neutron star is extremely dense.

12.5. Identify: Apply $\rho = m/V$ to relate the densities and volumes for the two spheres.

Set Up: For a sphere, $V = \frac{4}{3}\pi r^3$. For lead, $\rho_1 = 11.3\times10^3\ \text{kg/m}^3$ and for aluminum, $\rho_{\text{a}} = 2.7\times10^3\ \text{kg/m}^3$.

Execute: $m = \rho V = \frac{4}{3}\pi r^3\rho$. Same mass means $r_{\text{a}}^3\rho_{\text{a}} = r_1^3\rho_1$. $\dfrac{r_{\text{a}}}{r_1} = \left(\dfrac{\rho_1}{\rho_{\text{a}}}\right)^{1/3} = \left(\dfrac{11.3\times10^3}{2.7\times10^3}\right)^{1/3} = 1.6.$

Evaluate: The aluminum sphere is larger, since its density is less.

12.6. Identify: $w = mg$ and $m = \rho V$. Find the volume V of the pipe.

Set Up: For a hollow cylinder with inner radius R_1, outer radius R_2, and length L the volume is $V = \pi(R_2^2 - R_1^2)L$. $R_1 = 1.25\times10^{-2}$ m and $R_2 = 1.75\times10^{-2}$ m.

Execute: $V = \pi[(0.0175\ \text{m})^2 - (0.0125\ \text{m})^2](1.50\ \text{m}) = 7.07\times10^{-4}\ \text{m}^3$.

$m = \rho V = (8.9\times10^3\ \text{kg/m}^3)(7.07\times10^{-4}\ \text{m}^3) = 6.29$ kg. $w = mg = 61.6$ N.

Evaluate: The pipe weighs about 14 pounds.

12.7. Identify: The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh$.

Set Up: Freshwater has density $1.00\times10^3\ \text{kg/m}^3$ and seawater has density $1.03\times10^3\ \text{kg/m}^3$.

Execute: **(a)** $p - p_0 = (1.00\times10^3\ \text{kg/m}^3)(3.71\ \text{m/s}^2)(500\ \text{m}) = 1.86\times10^6$ Pa.

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^6\ \text{Pa}}{(1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 184\ \text{m}$

Evaluate: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

12.8. Identify: The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

Set Up: For blood, $\rho = 1.06\times10^3\ \text{kg/m}^3$. $y_1 - y_2 = 1.65$ m. The surface area of the segment is πDL, where $D = 1.50\times10^{-3}$ m and $L = 2.00\times10^{-2}$ m.

Execute: **(a)** $p_1 - p_2 = (1.06\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.65\ \text{m}) = 1.71\times10^4$ Pa.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$A = \pi DL = \pi(1.50\times10^{-3}\ \text{m})(2.00\times10^{-2}\ \text{m}) = 9.42\times10^{-5}\ \text{m}^2$.

$\Delta F_\perp = (1.71\times10^4\ \text{Pa})(9.42\times10^{-5}\ \text{m}^2) = 1.61$ N.

Evaluate: The pressure difference is about $\frac{1}{6}$ atm.

12.9. IDENTIFY: Apply $p = p_0 + \rho gh.$

SET UP: Gauge pressure is $p - p_{\text{air}}$.

EXECUTE: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein: $\rho gh = 5980$ Pa and

$$h = \frac{5980 \text{ Pa}}{\rho g} = \frac{5980 \text{ N/m}^2}{(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.581 \text{ m}.$$

EVALUATE: The bag of fluid is typically hung from a vertical pole to achieve this height above the patient's arm.

12.10. IDENTIFY: $p_0 = p_{\text{surface}} + \rho gh$ where p_{surface} is the pressure at the surface of a liquid and p_0 is the pressure at a depth *h* below the surface.

SET UP: The density of water is 1.00×10^3 kg/m^3.

EXECUTE: **(a)** For the oil layer, $p_{\text{surface}} = p_{\text{atm}}$ and p_0 is the pressure at the oil-water interface.

$$p_0 - p_{\text{atm}} = p_{\text{gauge}} = \rho gh = (600 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.120 \text{ m}) = 706 \text{ Pa}$$

(b) For the water layer, $p_{\text{surface}} = 706 \text{ Pa} + p_{\text{atm}}$.

$$p_0 - p_{\text{atm}} = p_{\text{gauge}} = 706 \text{ Pa} + \rho gh = 706 \text{ Pa} + (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.250 \text{ m}) = 3.16\times10^3 \text{ Pa}$$

EVALUATE: The gauge pressure at the bottom of the barrel is due to the combined effects of the oil layer and water layer. The pressure at the bottom of the oil layer is the pressure at the top of the water layer.

12.11. IDENTIFY: The pressure due to the glycerin balances the pressure due to the unknown liquid, so we are dealing with the pressure at a depth in a liquid.

SET UP: Both sides of the U-shaped tube are open to the air, so it is the gauge pressures that balance. Therefore we use $p_g = \rho gh.$ At the level of the bottom of the column of the unknown, the pressure in the right side is equal to the pressure in the left side. (See Fig. 12.11.) On the right side, we have a 35.0-cm column of the unknown, and on the left side we have a column of glycerin whose top is 12.0 cm below that of the unknown, so its height is 35.0 cm – 12.0 cm = 23.0 cm above the bottom of the bottom of the column of the unknown liquid. The target variable is the density of the unknown liquid.

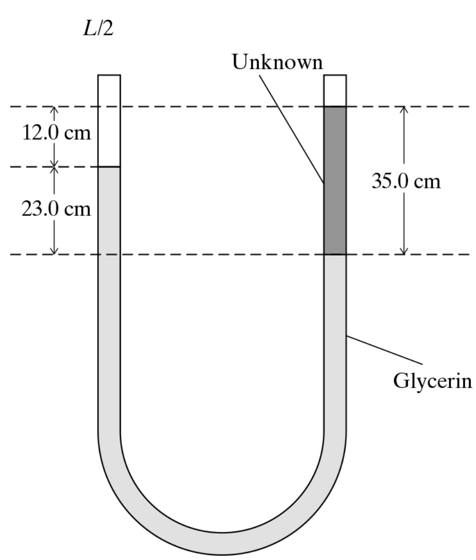


Figure 12.11

EXECUTE: We use $p_g = \rho gh$ to find the gauge pressure at the bottom of each column.

Right-hand column (the unknown): $p = \rho_x g(35.0 \text{ cm})$

Left-hand column (glycerin): $p = \rho_g g(35.0 \text{ cm} - 12.0 \text{ cm}) = \rho_g g(23.0 \text{ cm})$

Equating these pressures and solving for ρ_x gives $\rho_x = \frac{23 \text{ cm}}{35 \text{ cm}}\rho_g$. Using ρ_g = 1260 kg/m^3 from Table 12.1, we have $\rho_x = \left(\frac{23 \text{ cm}}{35 \text{ cm}}\right)(1260 \text{ kg/m}^3) = 828 \text{ kg/m}^3$.

EVALUATE: Our result shows that the unknown is less dense than glycerin. This is reasonable because it takes only a 23-cm column of glycerin to balance a 35-cm column of the unknown.

12.12. **IDENTIFY** and **SET UP:** Use $p_g = \rho gh$ to calculate the gauge pressure at this depth. Use $F = pA$ to calculate the force the inside and outside pressures exert on the window, and combine the forces as vectors to find the net force.

EXECUTE: **(a)** gauge pressure $= p - p_0 = \rho gh$ From Table 12.1 the density of seawater is $1.03\times10^3 \text{ kg/m}^3$, so

$$p - p_0 = \rho gh = (1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(250 \text{ m}) = 2.52\times10^6 \text{ Pa}.$$

(b) The force on each side of the window is $F = pA$. Inside the pressure is x p_0 and outside in the water the pressure is $p = p_0 + \rho gh$. The forces are shown in Figure 12.12.

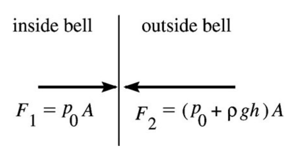


Figure 12.12

The net force is

$$F_2 - F_1 = (p_0 + \rho gh)A - p_0 A = (\rho gh)A$$

$$F_2 - F_1 = (2.52\times10^6 \text{ Pa})\pi(0.150 \text{ m})^2$$

$$F_2 - F_1 = 1.78\times10^5 \text{ N}$$

EVALUATE: The pressure at this depth is very large, over 20 times normal air pressure, and the net force on the window is huge. Diving bells used at such depths must be constructed to withstand these large forces.

12.13. **IDENTIFY:** The external pressure on the eardrum increases with depth in the ocean. This increased pressure could damage the eardrum.

SET UP: The density of seawater is $1.03\times10^3 \text{ kg/m}^3$. The area of the eardrum is $A = \pi r^2$, with $r = 4.1$ mm. The pressure increase with depth is $\Delta p = \rho gh$ and $F = pA$.

EXECUTE: $\Delta F = (\Delta p)A = \rho ghA$. Solving for h gives

$$h = \frac{\Delta F}{\rho gA} = \frac{1.5 \text{ N}}{(1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)\pi(4.1\times10^{-3} \text{ m})^2} = 2.8 \text{ m}.$$

EVALUATE: 2.8 m is less than 10 ft, so it is probably a good idea to wear ear plugs if you scuba dive.

12.14. **IDENTIFY** and **SET UP:** Use $p = p_0 + \rho gh$ to calculate the pressure at the specified depths in the open tube. The pressure is the same at all points the same distance from the bottom of the tubes, so the pressure calculated in part (b) is the pressure in the tank. Gauge pressure is the difference between the absolute pressure and air pressure.

EXECUTE: $p_a = 980 \text{ millibar} = 9.80\times10^4 \text{ Pa}$

(a) Apply $p = p_0 + \rho gh$ to the right-hand tube. The top of this tube is open to the air so $p_0 = p_a$. The density of the liquid (mercury) is $13.6\times10^3 \text{ kg/m}^3$.

Thus $p = 9.80\times10^4 \text{ Pa} + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0700 \text{ m}) = 1.07\times10^5 \text{ Pa}$.

(b) a $p = p_0 + \rho gh = 9.80\times10^4\ \text{Pa} + (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 1.03\times10^5\ \text{Pa}.$

(c) Since $y_2 - y_1 = 4.00\ \text{cm}$ the pressure at the mercury surface in the left-hand end tube equals that calculated in part (b). Thus the absolute pressure of gas in the tank is $1.03\times10^5\ \text{Pa}.$

(d) $p - p_0 = \rho gh = (13.6\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0400\ \text{m}) = 5.33\times10^3\ \text{Pa}.$

EVALUATE: If $p = p_0 + \rho gh$ is evaluated with the density of mercury and $p - p_\text{a} = 1\ \text{atm} = 1.01\times10^5\ \text{Pa},$ then $h = 76\ \text{cm}.$ The mercury columns here are much shorter than 76 cm, so the gauge pressures are much less than $1.0\times10^5\ \text{Pa}.$

12.15. **IDENTIFY:** Apply $p = p_0 + \rho gh.$

SET UP: For water, $\rho = 1.00\times10^3\ \text{kg/m}^3.$

EXECUTE: $p - p_\text{air} = \rho gh = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(6.1\ \text{m}) = 6.0\times10^4\ \text{Pa}.$

EVALUATE: The pressure difference increases linearly with depth.

12.16. **IDENTIFY:** The gauge pressure of the person must be equal to the pressure due to the column of water in the straw.

SET UP: Apply $p = p_0 + \rho gh.$

EXECUTE: **(a)** The gauge pressure is $p_\text{g} = \rho gh = -(1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.1\ \text{m}) = -1.1\times10^4\ \text{Pa}.$

(b) In order for water to go up the straw, the pressure at the top of the straw must be lower than atmospheric pressure. Therefore the gauge pressure, $p - p_\text{atm}$, must be negative.

EVALUATE: The actual pressure is not negative, just the difference between the pressure at the top of the straw and atmospheric pressure.

12.17. **IDENTIFY:** $p = p_0 + \rho gh.$ $F = pA.$

SET UP: For seawater, $\rho = 1.03\times10^3\ \text{kg/m}^3.$

EXECUTE: The force F that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho gh)A - w = (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(30\ \text{m})(0.75\ \text{m}^2) - 300\ \text{N} = 2.27\times10^5\ \text{N}.$$

EVALUATE: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

12.18. **IDENTIFY** and **SET UP:** Apply $p = p_0 + \rho gh$ to the water and mercury columns. The pressure at the bottom of the water column is the pressure at the top of the mercury column.

EXECUTE: With just the mercury, the gauge pressure at the bottom of the cylinder is $p - p_0 = \rho_\text{m} gh_\text{m}.$ With the water to a depth h_w, the gauge pressure at the bottom of the cylinder is $p - p_0 = \rho_\text{m} gh_\text{m} + \rho_\text{w} gh_\text{w}.$ If this is to be double the first value, then $\rho_\text{w} gh_\text{w} = \rho_\text{m} gh_\text{m}.$

$$h_\text{w} = h_\text{m}(\rho_\text{m}/\rho_\text{w}) = (0.0800\ \text{m})(13.6\times10^3/1.00\times10^3) = 1.088\ \text{m}$$

The volume of water is $V = hA = (1.088\ \text{m})(12.0\times10^{-4}\ \text{m}^2) = 1.306\times10^{-3}\ \text{m}^3 = 1310\ \text{cm}^3 = 1.31\ \text{L}.$

EVALUATE: The density of mercury is 13.6 times the density of water and $(13.6)(8\ \text{cm}) = 109\ \text{cm},$ so the pressure increase from the top to the bottom of a 109-cm tall column of water is the same as the pressure increase from top to bottom for an 8-cm tall column of mercury.

12.19. **IDENTIFY:** The gauge pressure at the top of the oil column must produce a force on the disk that is equal to its weight.

SET UP: The area of the bottom of the disk is $A = \pi r^2 = \pi(0.150\ \text{m})^2 = 0.0707\ \text{m}^2.$

EXECUTE: **(a)** $p - p_0 = \dfrac{w}{A} = \dfrac{45.0\text{ N}}{0.0707\text{ m}^2} = 636\text{ Pa}.$

(b) The increase in pressure produces a force on the disk equal to the increase in weight. By Pascal's law the increase in pressure is transmitted to all points in the oil.

(i) $\Delta p = \dfrac{83.0\text{ N}}{0.0707\text{ m}^2} = 1170\text{ Pa}.$ (ii) 1170 Pa

EVALUATE: The absolute pressure at the top of the oil produces an upward force on the disk but this force is partially balanced by the force due to the air pressure at the top of the disk.

12.20. **IDENTIFY:** This problem deals with the absolute pressure at a depth in a fluid.

SET UP: The absolute (or total) pressure is $p = p_0 + \rho gh$. The target variable is the depth at which the absolute pressure is twice and four times the surface pressure. The density of seawater is 1030 kg/m^3.

EXECUTE: **(a)** Using $p = p_0 + \rho gh$ gives $2p_0 = p_0 + \rho gh$, so $h = \dfrac{p_0}{\rho g}$, which gives

$$h = \frac{1.0\times 10^5\text{ Pa}}{(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 10.2\text{ m}.$$

(b) Follow the same procedure as in part (a), giving $4p_0 = p_0 + \rho gh$, so $h = \dfrac{3p_0}{\rho g}$, which gives

$$h = \frac{3p_0}{\rho g} = \frac{3(1.03\times 10^5\text{ Pa})}{(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 30.6\text{ m}.$$

EVALUATE: Notice that by tripling the depth we do not triple the absolute pressure. But we would triple the gauge pressure.

12.21. **IDENTIFY:** $F_2 = \dfrac{A_2}{A_1}F_1$. F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4$. D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 is the diameter at the car.

EXECUTE: $mg = \dfrac{\pi D_2^2/4}{\pi D_1^2/4}F_1$. $\dfrac{D_2}{D_1} = \sqrt{\dfrac{mg}{F_1}} = \sqrt{\dfrac{(1520\text{ kg})(9.80\text{ m/s}^2)}{125\text{ N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

12.22. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the piston, with $+y$ upward. $F = pA$.

SET UP: $1\text{ atm} = 1.013\times 10^5\text{ Pa}$. The force diagram for the piston is given in Figure 12.22. p is the absolute pressure of the hydraulic fluid.

EXECUTE: $pA - w - p_{\text{atm}}A = 0$ and

$$p - p_{\text{atm}} = p_{\text{gauge}} = \frac{w}{A} = \frac{mg}{\pi r^2} = \frac{(1200\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.15\text{ m})^2} = 1.7\times 10^5\text{ Pa} = 1.7\text{ atm}$$

EVALUATE: The larger the diameter of the piston, the smaller the gauge pressure required to lift the car.

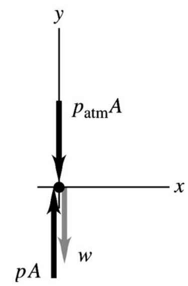


Figure 12.22

12.23. **IDENTIFY:** We are dealing with the pressure at a depth in a fluid.

SET UP: The pressure with the wine is the same as the pressure with mercury, except that the heights of the fluids are different because they have different densities. $p = p_0 + \rho g h$ gives the pressure in each case, with p_0 the same for both cases. The target variable is the height of the wine column.

EXECUTE: $p_0 + \rho_w g h_w = p_0 + \rho_m g h_m$, so $h_w = h_m \dfrac{\rho_m}{\rho_w} = (0.750\text{ m})\dfrac{13.6\times10^3\text{ kg/m}^3}{990\text{ kg/m}^3} = 10.3\text{ m}.$

EVALUATE: A wine barometer over 10 m high is highly unwieldy compared to one around 1 m high for mercury. Besides, there are better uses for wine!

12.24. **IDENTIFY:** This problem deals with buoyant force on your floating body. Archimedes's principle applies.

SET UP: Estimate: About 5% of the body is above water. Weight is 165 lb ≈ 734 N. The buoyant force B is equal to your weight and is equal to the weight of the seawater displaced by your body. The volume of seawater is 95% of your volume. The weight of a volume of material is $w = \rho g V$ and average density is $\rho_{\text{av}} = m/V$. The target variables are the volume of your body and its average density.

EXECUTE: **(a)** Let subscripts w refer to water and quantities without susbscripts refer to you. When floating, $B = w_{\text{w}} = \rho_{\text{w}} g V_{\text{w}}$ and $V_{\text{w}} = 0.95V$. Using $B = w$ gives $\rho_w g(0.95V) = w$. Solving for V gives

$$V = \frac{w}{0.95\rho_{\text{w}} g} = \frac{734\text{ N}}{(0.95)(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 7.7\times10^{-2}\text{ m}^3.$$

(b) $\rho_{\text{av}} = \dfrac{m}{V} = \dfrac{W/g}{V} = \dfrac{(734\text{ N})/(9.80\text{ m/s}^2)}{7.7\times10^{-2}\text{ m}^3} = 970\text{ kg/m}^3 \approx 95\%\ \rho_{\text{seawater}}$.

EVALUATE: To see if your volume is reasonable, assume you are all pure water and calculate your weight. $w_{\text{w}} = \rho_{\text{w}} g V_{\text{w}} = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.077\text{ m}^3) = 755\text{ N}$, which is very close to your weight. So the volume is reasonable.

12.25. **IDENTIFY:** A buoyant force acts on the athlete, so Archimedes's principle applies. He doesn't sink, so the forces on him must balance.

SET UP: Apply $\sum F_y = 0$ to the athlete. The volume of water he displaces is equal to his volume since he is totally submerged, so $V_{\text{w}} = V_{\text{ath}}$. The buoyant force it exerts is equal to the weight of that volume of water. We want to find the volume and average density of the athlete. $\rho_{\text{av}} = m/V$.

EXECUTE: Using $\sum F_y = 0$ gives $B + 20\text{ N} - 900\text{ N} = 0$, so $B = 880$ N. The buoyant force is equal to the weight of the water he displaces, and $V_{\text{w}} = V_{\text{ath}}$, so $B = \rho_{\text{w}} g V_{\text{w}} = \rho_{\text{w}} g V_{\text{ath}}$. Solving for his volume gives $V_{\text{ath}} = \dfrac{B}{\rho_{\text{w}} g} = \dfrac{880\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 8.98\times10^{-2}\text{ m}^3$. His average density is $\rho_{\text{av}} = \dfrac{m}{V} = \dfrac{W/g}{V} = \dfrac{(900\text{ N})/(9.80\text{ m/s}^2)}{8.98\times10^{-2}\text{ m}^3} = 1020\text{ kg/m}^3$.

EVALUATE: The athlete is slightly denser than pure water. This is reasonable because he probably has little fat, which is less dense than muscle.

12.26. **IDENTIFY:** The buoyant force B acts upward on the rock, opposing gravity. Archimedes's principle applies, and the forces must balance.

SET UP: $\rho = m/V$.

EXECUTE: With the rock of mass m in the water: $T + B = mg$. where T is the tension in the string. Call V the volume of the rock and ρ_w the density of water. By Archimedes's principle, $m = \rho_w V$, so we get $T + \rho_w Vg = mg$. Solving for V gives $V = (mg - T)/\rho_w g$. Now look at the rock in the liquid, where ρ is the density of the liquid. For the smallest density liquid, the rock is totally submerged, so the volume of liquid displaced is V. For floating we have $B = mg$, which gives $\rho gV = mg$. Solving for ρ and using the equation for V that we just found, we get

$$\rho = \frac{m}{\dfrac{mg-T}{\rho_w g}} = \frac{\rho_w mg}{mg-T} = \frac{(1000\ \text{kg/m}^3)(1.80\ \text{kg})(9.80\ \text{m/s}^2)}{(1.80\ \text{kg})(9.80\ \text{m/s}^2) - 12.8\ \text{N}} = 3640\ \text{kg/m}^3.$$

EVALUATE: In the water, the buoyant force was not enough to balance the weight of the rock, so there was a tension of 12.8 N in the string. In the new liquid, the buoyant force is equal to the weight. Therefore the liquid must be denser than water, which in fact it is.

12.27. **IDENTIFY:** By Archimedes's principle, the additional buoyant force will be equal to the additional weight (the man).

SET UP: $V = \dfrac{m}{\rho}$ where $dA = V$ and d is the additional distance the buoy will sink.

EXECUTE: With man on buoy must displace additional 80.0 kg of water.

$$V = \frac{m}{\rho} = \frac{80.0\ \text{kg}}{1030\ \text{kg/m}^3} = 0.07767\ \text{m}^3.\ \ dA = V \ \text{so}\ d = \frac{V}{A} = \frac{0.07767\ \text{m}^3}{\pi(0.450\ \text{m})^2} = 0.122\ \text{m}.$$

EVALUATE: We do not need to use the mass of the buoy because it is already floating and hence in balance.

12.28. **IDENTIFY:** Apply Newton's second law to the woman plus slab. The buoyancy force exerted by the water is upward and given by $B = \rho_{\text{water}} V_{\text{displ}} g$, where V_{displ} is the volume of water displaced.

SET UP: The floating object is the slab of ice plus the woman; the buoyant force must support both. The volume of water displaced equals the volume V_{ice} of the ice. The free-body diagram is given in Figure 12.28.

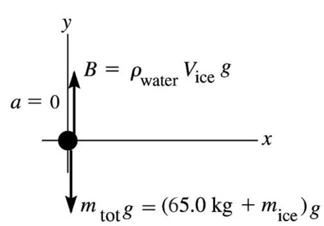


Figure 12.28

EXECUTE: $\Sigma F_y = ma_y$

$B - m_{\text{tot}} g = 0$

$\rho_{\text{water}} V_{\text{ice}} g = (65.0\ \text{kg} + m_{\text{ice}}) g$

But $\rho = m/V$ so $m_{\text{ice}} = \rho_{\text{ice}} V_{\text{ice}}$

$$V_{\text{ice}} = \frac{65.0\ \text{kg}}{\rho_{\text{water}} - \rho_{\text{ice}}} = \frac{65.0\ \text{kg}}{1000\ \text{kg/m}^3 - 920\ \text{kg/m}^3} = 0.81\ \text{m}^3.$$

EVALUATE: The mass of ice is $m_{\text{ice}} = \rho_{\text{ice}} V_{\text{ice}} = 750$ kg.

12.29. **IDENTIFY:** Apply $\Sigma F_y = ma_y$ to the sample, with $+y$ upward. $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: $w = mg = 17.50$ N and $m = 1.79$ kg.

EXECUTE: $T + B - mg = 0$. $a_x = 0$

$$V_{obj} = \frac{B}{\rho_{water} g} = \frac{6.30\text{ N}}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.43\times10^{-4}\text{ m}^3.$$

$$\rho = \frac{m}{V} = \frac{1.79\text{ kg}}{6.43\times10^{-4}\text{ m}^3} = 2.78\times10^3\text{ kg/m}^3.$$

EVALUATE: The density of the sample is greater than that of water and it doesn't float.

12.30. IDENTIFY: The upward buoyant force B exerted by the liquid equals the weight of the fluid displaced by the object. Since the object floats the buoyant force equals its weight.

SET UP: Glycerin has density $\rho_{gly} = 1.26\times10^3\text{ kg/m}^3$ and seawater has density $\rho_{sw} = 1.03\times10^3\text{ kg/m}^3$. Let V_{obj} be the volume of the apparatus. $g_E = 9.80\text{ m/s}^2$; $g_C = 5.40\text{ m/s}^2$. Let V_{sub} be the volume submerged on Caasi.

EXECUTE: On earth $B = \rho_{sw}(0.250V_{obj})g_E = mg_E$. $m = (0.250)\rho_{sw}V_{obj}$. On Caasi, $B = \rho_{gly}V_{sub}g_C = mg_C$. $m = \rho_{gyl}V_{sub}$. The two expressions for m must be equal, so

$$(0.250)V_{obj}\rho_{sw} = \rho_{gly}V_{sub} \text{ and } V_{sub} = \left(\frac{0.250\rho_{sw}}{\rho_{gly}}\right)V_{obj} = \left(\frac{[0.250][1.03\times10^3\text{ kg/m}^3]}{1.26\times10^3\text{ kg/m}^3}\right)V_{obj} = 0.204V_{obj}.$$

20.4% of the volume will be submerged on Caasi.

EVALUATE: Less volume is submerged in glycerin since the density of glycerin is greater than the density of seawater. The value of g on each planet cancels out and has no effect on the answer. The value of g changes the weight of the apparatus and the buoyant force by the same factor.

12.31. IDENTIFY: In air and in the liquid, the forces on the rock must balance. Archimedes's principle applies in the liquid.

SET UP: $B = \rho Vg$, $\rho = m/V$, call m the mass of the rock, V its volume, and ρ its density; T is the tension in the string and ρ_L is the density of the liquid.

EXECUTE: In air: $T = mg = \rho Vg$. $V = T/\rho g = (28.0\text{ N})/[(1200\text{ kg/m}^3)(9.80\text{ m/s}^2)] = 0.00238\text{ m}^3$.

In the liquid: $T + B = mg$, so

$T = mg - B = \rho Vg - \rho_L Vg = gV(\rho - \rho_L) = (9.80\text{ m/s}^2)(0.00238\text{ m}^3)(1200\text{ kg/m}^3 - 750\text{ kg/m}^3) = 10.5$ N.

EVALUATE: When the rock is in the liquid, the tension in the string is less than the tension when the rock is in air since the buoyant force helps balance some of the weight of the rock.

12.32. IDENTIFY: $B = \rho_{water}V_{obj}g$. The net force on the sphere is zero.

SET UP: The density of water is $1.00\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** $B = (1000\text{ kg/m}^3)(0.650\text{ m}^3)(9.80\text{ m/s}^2) = 6.37\times10^3\text{ N}$

(b) $B = T + mg$ and $m = \dfrac{B-T}{g} = \dfrac{6.37\times10^3\text{ N} - 1120\text{ N}}{9.80\text{ m/s}^2} = 536\text{ kg}.$

(c) Now $B = \rho_{water}V_{sub}g$, where V_{sub} is the volume of the sphere that is submerged. $B = mg$.

$$\rho_{water}V_{sub}g = mg \text{ and } V_{sub} = \frac{m}{\rho_{water}} = \frac{536\text{ kg}}{1000\text{ kg/m}^3} = 0.536\text{ m}^3.\quad \frac{V_{sub}}{V_{obj}} = \frac{0.536\text{ m}^3}{0.650\text{ m}^3} = 0.824 = 82.4\%.$$

EVALUATE: The average density of the sphere is $\rho_{sph} = \dfrac{m}{V} = \dfrac{536\text{ kg}}{0.650\text{ m}^3} = 825\text{ kg/m}^3$. $\rho_{sph} < \rho_{water}$, and that is why it floats with 82.4% of its volume submerged.

12.33. IDENTIFY and SET UP: Use $p = p_0 + \rho gh$ to calculate the gauge pressure at the two depths.

(a) The distances are shown in Figure 12.33a.

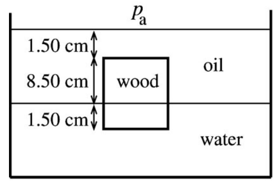


Figure 12.33a

EXECUTE: $p - p_0 = \rho g h$

The upper face is 1.50 cm below the top of the oil, so

$$p - p_0 = (790\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0150$$

$$p - p_0 = 116\ \text{Pa}$$

(b) The pressure at the interface is $p_{\text{interface}} = p_a + \rho_{\text{oil}} g(0.100\ \text{m})$. The lower face of the block is 1.50 cm below the interface, so the pressure there is $p = p_{\text{interface}} + \rho_{\text{water}} g(0.0150\ \text{m})$. Combining these two equations gives

$$p - p_a = \rho_{\text{oil}} g(0.100\ \text{m}) + \rho_{\text{water}} g(0.0150\ \text{m})$$

$$p - p_a = [(790\ \text{kg/m}^3)(0.100\ \text{m}) + (1000\ \text{kg/m}^3)(0.0150\ \text{m})](9.80\ \text{m/s}^2)$$

$$p - p_a = 921\ \text{Pa}$$

(c) IDENTIFY and SET UP: Consider the forces on the block. The area of each face of the block is $A = (0.100\ \text{m})^2 = 0.0100\ \text{m}^2$. Let the absolute pressure at the top face be p_t and the pressure at the bottom face be p_b. In $p = \dfrac{F_\perp}{A}$, use these pressures to calculate the force exerted by the fluids at the top and bottom of the block. The free-body diagram for the block is given in Figure 12.33b.

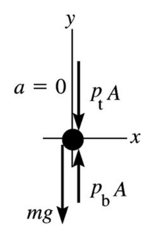


Figure 12.33b

EXECUTE: $\Sigma F_y = m a_y$

$$p_b A - p_t A - mg = 0$$

$$(p_b - p_t)A = mg$$

Note that $(p_b - p_t) = (p_b - p_a) - (p_t - p_a) = 921\ \text{Pa} - 116\ \text{Pa} = 805\ \text{Pa}$; the difference in absolute pressures equals the difference in gauge pressures.

$$m = \frac{(p_b - p_t)A}{g} = \frac{(805\ \text{Pa})(0.0100\ \text{m}^2)}{9.80\ \text{m/s}^2} = 0.821\ \text{kg}.$$

And then $\rho = m/V = 0.821\ \text{kg}/(0.100\ \text{m})^3 = 821\ \text{kg/m}^3$.

EVALUATE: We can calculate the buoyant force as $B = (\rho_{\text{oil}} V_{\text{oil}} + \rho_{\text{water}} V_{\text{water}})g$ where $V_{\text{oil}} = (0.0100\ \text{m}^2)(0.0850\ \text{m}) = 8.50\times10^{-4}\ \text{m}^3$ is the volume of oil displaced by the block and $V_{\text{water}} = (0.0100\ \text{m}^2)(0.0150\ \text{m}) = 1.50\times10^{-4}\ \text{m}^3$ is the volume of water displaced by the block. This gives $B = (0.821\ \text{kg})g$. The mass of water displaced equals the mass of the block.

12.34. **IDENTIFY:** The sum of the vertical forces on the ingot is zero. $\rho = m/V$. The buoyant force is $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: The density of aluminum is $2.7\times10^3\ \text{kg/m}^3$. The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7\times10^3\ \text{kg/m}^3} = 3.36\times10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + B - mg = 0$.

$B = \rho_{\text{water}} V_{\text{obj}} g = (1.00\times10^3\ \text{kg/m}^3)(3.36\times10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$.

$T = mg - B = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$.

EVALUATE: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

12.35. **IDENTIFY:** The vertical forces on the rock sum to zero. The buoyant force equals the weight of liquid displaced by the rock. $V = \frac{4}{3}\pi R^3$.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: The rock displaces a volume of water whose weight is $39.2\ \text{N} - 28.4\ \text{N} = 10.8\ \text{N}$. The mass of this much water is thus $10.8\ \text{N}/(9.80\ \text{m/s}^2) = 1.102\ \text{kg}$ and its volume, equal to the rock's volume, is $\dfrac{1.102\ \text{kg}}{1.00\times10^3\ \text{kg/m}^3} = 1.102\times10^{-3}\ \text{m}^3$. The weight of unknown liquid displaced is $39.2\ \text{N} - 21.5\ \text{N} = 17.7\ \text{N}$, and its mass is $(17.7\ \text{N})/(9.80\ \text{m/s}^2) = 1.806\ \text{kg}$. The liquid's density is thus $(1.806\ \text{kg})/(1.102\times10^{-3}\ \text{m}^3) = 1.64\times10^3\ \text{kg/m}^3$.

EVALUATE: The density of the unknown liquid is a little more than 1.5 times the density of water.

12.36. **IDENTIFY:** The block floats in water and then in a second liquid, so we apply Archimedes's principle.
SET UP: In both cases, the buoyant force is equal to the weight of the block and is also equal to the weight of the liquid displaced by the block. Call *V* the volume of the block and use $w = \rho g V$. The target variable is the density of the second liquid.
EXECUTE: <u>In water</u>: $B = \rho_w g V_W = \rho_w g(0.700V)$
<u>In the second liquid</u>: $B = \rho_L g V_L = \rho_L g(0.800V)$

Equate the buoyant forces and solve for ρ_L: $\rho_L = \dfrac{0.700}{0.800}\rho_w = 875\ \text{kg/m}^3$.

EVALUATE: In the second liquid the block has more of its volume submerged than in the water, so the second liquid must be less dense than water. This agrees with our result.

12.37. **IDENTIFY:** The cylinder is partially submerged in water, so we apply Archimedes's principle. The vertical forces on it must balance.
SET UP: The density of this cylinder is 370/1000 = 37% that of water, so it would float with 37% of its volume under the water and 63% above water. But it is partially submerged with 70.0% under water. Therefore the buoyant force *B* on it must be greater than its weight, and this would force the cylinder upward. The tension in the cable prevents this from happening. The target variable is the tension *T* in the cable. We apply $\Sigma F_y = 0$ and $w = \rho g V$.

EXECUTE: Start with $\Sigma F_y = 0$: $B - T - w_c = 0$. We see that we first need to find *B*. Call *V* the volume of the cylinder and ρ_c its density. Therefore $w_c = \rho_c gV$. By Archimedes's principle, the buoyant force *B* is equal to the weight of the water displaced by the cylinder, and we know that the cylinder has 70% of its volume below the water, which is $0.700V$. Therefore $B = \rho_w g(0.700V)$. Dividing *B* by w_c gives

$\dfrac{B}{w_c} = \dfrac{\rho_w g(0.700V)}{\rho_c gV} = \dfrac{\rho_w}{\rho_c} = \dfrac{1000\ \text{kg/m}^3}{370\ \text{kg/m}^3} = 2.7027$, so $B = 2.7027w_c$. Putting this result into $\Sigma F_y = 0$ gives
$2.7027w_c - T - w_c = 0$. Solving for *T* gives
$T = 1.7027w_c = (1.7027)(30.0\ \text{kg})(9.80\ \text{m/s}^2) = 501\ \text{N}$.
EVALUATE: If the cable were to break, the net upward force at that instant on the cylinder would be $B - w_c = 2.7027w_c - w_c = 1.70w_c$, so the cylinder would accelerate upward and eventually float with 63% of its volume above water.

12.38. IDENTIFY: The volume flow rate is Av.

SET UP: $Av = 0.750\ \text{m}^3/\text{s}$. $A = \pi D^2/4$.

EXECUTE: **(a)** $v\pi D^2/4 = 0.750\ \text{m}^3/\text{s}$. $v = \dfrac{4(0.750\ \text{m}^3/\text{s})}{\pi(4.50\times10^{-2}\ \text{m})^2} = 472\ \text{m/s}$.

(b) vD^2 must be constant, so $v_1D_1^2 = v_2D_2^2$. $v_2 = v_1\left(\dfrac{D_1}{D_2}\right)^2 = (472\ \text{m/s})\left(\dfrac{D_1}{3D_1}\right)^2 = 52.4\ \text{m/s}$.

EVALUATE: The larger the hole, the smaller the speed of the fluid as it exits.

12.39. IDENTIFY: Apply the equation of continuity.

SET UP: $A = \pi r^2$, $v_1A_1 = v_2A_2$.

EXECUTE: $v_2 = v_1(A_1/A_2)$.

$A_1 = \pi(0.80\ \text{cm})^2$, $A_2 = 20\pi(0.10\ \text{cm})^2$. $v_2 = (3.0\ \text{m/s})\dfrac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\ \text{m/s}$.

EVALUATE: The total area of the shower head openings is less than the cross-sectional area of the pipe, and the speed of the water in the shower head opening is greater than its speed in the pipe.

12.40. IDENTIFY: Apply the equation of continuity. The volume flow rate is vA.

SET UP: $1.00\ \text{h} = 3600\ \text{s}$. $v_1A_1 = v_2A_2$.

EXECUTE: **(a)** $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.105\ \text{m}^2}\right) = 2.3\ \text{m/s}$

(b) $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.047\ \text{m}^2}\right) = 5.2\ \text{m/s}$

(c) $V = v_1A_1t = (3.50\ \text{m/s})(0.070\ \text{m}^2)(3600\ \text{s}) = 880\ \text{m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

12.41. IDENTIFY and **SET UP:** Apply the continuity equation, $v_1A_1 = v_2A_2$. In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20\ \text{m}^3/\text{s}$

$$v = \frac{1.20\ \text{m}^3/\text{s}}{A} = \frac{1.20\ \text{m}^3/\text{s}}{\pi r^2} = \frac{1.20\ \text{m}^3/\text{s}}{\pi(0.150\ \text{m})^2} = 17.0\ \text{m/s}$$

(b) $vA = 1.20\ \text{m}^3/\text{s}$

$v\pi r^2 = 1.20\ \text{m}^3/\text{s}$

$$r = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{v\pi}} = \sqrt{\frac{1.20\ \text{m}^3/\text{s}}{(3.80\ \text{m/s})\pi}} = 0.317\ \text{m}$$

EVALUATE: The speed is greater where the area and radius are smaller.

12.42. IDENTIFY: This problem is about fluid flow, so we use Bernoulli's equation and the volume flow rate.

SET UP: Apply $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$ and $dv/dt = Av$. Choose $y = 0$ at the base of the tank, so $y_2 = 0$ and $y_1 = h$. Since the tank is large, assume that $v_2 >> v_1$, so we use $v_1 = 0$. The top of the tank and the end of the pipe are open to the atmosphere, so $p_1 = p_2$. We need to find a relationship between the volume flow rate dV/dt and h in order to interpret the graph. The target variable is g on this planet.

EXECUTE: Bernoulli's equation reduces to $\rho g h = \frac{1}{2}\rho v_2^2$, which gives $v_2 = \sqrt{2gh}$. The continuity equation gives $dV/dt = Av_2$. Using our result for v_2 gives $dV/dt = Av_2 = A\sqrt{2gh}$. Squaring gives $(dV/dt)^2 = 2A^2gh$.

From this result we see that a graph of $(dV/dt)^2$ versus h should be a straight line with slope $2A^2g$. This gives $g = \frac{\text{slope}}{2A^2} = g = \frac{1.94\times10^{-5}\ \text{m}^5/\text{s}^2}{2(9.0\times10^{-4}\ \text{m}^2)} = 9.0\ \text{m/s}^2$.

EVALUATE: This is a reasonable value for g on a solid planet.

12.43. **IDENTIFY:** Water is flowing out of the tank and collecting in a bucket, so we use Bernoulli's equation and the volume flow rate.

SET UP: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, $dV/dt = Av$. Choose $y = 0$ at the base of the tank, so $y_2 = 0$ and $y_1 = h$. The top of the tank and the end of the pipe are open to the atmosphere, so $p_1 = p_2$. The target variable is the time it takes to collect a gallon of water. Call R the radius of the tank and r the radius of the small hole at the bottom.

EXECUTE: **(a)** As 1 gal flows out of the tank the change in volume in the tank is 1 gal = $3.788\times10^{-3}\ \text{m}^3$. The change in volume ΔV of water in the tank due to a height change Δh is $\Delta V = \pi R^2 \Delta h$, so Δh = $\Delta V/(\pi R^2) = (3.788\times10^{-3}\ \text{m}^3)/[\pi(1.50\ \text{m})^2] = 5.36\times10^{-4}\ \text{m} = 0.536\ \text{mm}$.

(b) Now use $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. Based upon the result in part (a), we can treat the height of water in the tank as constant while a gallon flows out and treat the speed v_1 of the water at the top of the tank as being essentially zero. The top of the tank and the end of the pipe are open to the atmosphere, so $p_1 = p_2$. Take $y = 0$ at the bottom of the tank. We need to find the speed v_2 of the water as it comes out of the small hole at the bottom of the tank. Bernoulli's equation becomes $\rho g h = \frac{1}{2}\rho v_2^2$, which gives $v_2 = \sqrt{2gh}$. Now use the volume flow rate at the bottom. $dV/dt = Av_2 = \pi r^2\sqrt{2gh}$. This gives $\Delta V = \pi r^2\sqrt{2gh}\,\Delta t$, so $\Delta t = \frac{\Delta V}{\pi r^2\sqrt{2gh}}$. Using $\Delta V = 3.788\times10^{-3}\ \text{m}^3$, $r = 0.250$ cm = 0.0025 m, and $h = 2.00$ m gives $\Delta t = 30.8$ s.

EVALUATE: It is reasonable to neglect the change in depth of the water in the tank as one gallon flows out because $\Delta h \ll h$: 0.536 mm << 2.00 m.

12.44. **IDENTIFY:** $\rho = m/V$. Apply the equation of continuity and Bernoulli's equation to points 1 and 2.

SET UP: The density of water is 1 kg/L.

EXECUTE: **(a)** $\frac{(220)(0.355\ \text{kg})}{60.0\ \text{s}} = 1.30\ \text{kg/s}$.

(b) The density of the liquid is $\frac{0.355\ \text{kg}}{0.355\times10^{-3}\ \text{m}^3} = 1000\ \text{kg/m}^3$, and so the volume flow rate is $\frac{1.30\ \text{kg/s}}{1000\ \text{kg/m}^3} = 1.30\times10^{-3}\ \text{m}^3/\text{s} = 1.30\ \text{L/s}$. This result may also be obtained from $\frac{(220)(0.355\ \text{L})}{60.0\ \text{s}} = 1.30\ \text{L/s}$.

(c) $v_1 = \frac{1.30\times10^{-3}\ \text{m}^3/\text{s}}{2.00\times10^{-4}\ \text{m}^2} = 6.50\ \text{m/s}$. $v_2 = v_1/4 = 1.63\ \text{m/s}$.

(d) $p_1 = p_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$.

$p_1 = 152\ \text{kPa} + (1000\ \text{kg/m}^3)\left(\tfrac{1}{2}\left[(1.63\ \text{m/s})^2 - (6.50\ \text{m/s})^2\right] + (9.80\ \text{m/s}^2)(-1.35\ \text{m})\right)$. $p_1 = 119\ \text{kPa}$.

EVALUATE: The increase in height and the increase in fluid speed at point 1 both cause the pressure at point 1 to be less than the pressure at point 2.

12.45. IDENTIFY and SET UP:

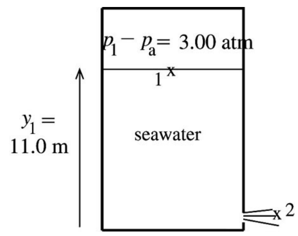


Figure 12.45

Apply Bernoulli's equation with points 1 and 2 chosen as shown in Figure 12.45. Let $y = 0$ at the bottom of the tank so $y_1 = 11.0\text{ m}$ and $y_2 = 0$. The target variable is v_2.

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$A_1 v_1 = A_2 v_2$, so $v_1 = (A_2/A_1)v_2$. But the cross-sectional area of the tank (A_1) is much larger than the cross-sectional area of the hole (A_2), so $v_1 << v_2$ and the $\frac{1}{2}\rho v_1^2$ term can be neglected.

EXECUTE: This gives $\frac{1}{2}\rho v_2^2 = (p_1 - p_2) + \rho g y_1$.

Use $p_2 = p_a$ and solve for v_2:

$$v_2 = \sqrt{2(p_1 - p_a)/\rho + 2gy_1} = \sqrt{\frac{2(3.039\times 10^5\text{ Pa})}{1030\text{ kg/m}^3} + 2(9.80\text{ m/s}^2)(11.0\text{ m})}$$

$v_2 = 28.4\text{ m/s}$

EVALUATE: If the pressure at the top surface of the water were air pressure, then Toricelli's theorem (Example: 12.8) gives $v_2 = \sqrt{2g(y_1 - y_2)} = 14.7\text{ m/s}$. The actual afflux speed is much larger than this due to the excess pressure at the top of the tank.

12.46. IDENTIFY: A change in the speed of the blood indicates that there is a difference in the cross-sectional area of the artery. Bernoulli's equation applies to the fluid.

SET UP: Bernoulli's equation is $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. The two points are close together so we can neglect $\rho g(y_1 - y_2)$. $\rho = 1.06\times 10^3\text{ kg/m}^3$. The continuity equation is $A_1 v_1 = A_2 v_2$.

EXECUTE: Solve $p_1 - p_2 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$ for v_2:

$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho} + v_1^2} = \sqrt{\frac{2(1.20\times 10^4\text{ Pa} - 1.15\times 10^4\text{ Pa})}{1.06\times 10^3\text{ kg/m}^3} + (0.300\text{ m/s})^2} = 1.0\text{ m/s} = 100\text{ cm/s}.$$

$v_2 = 1.0\text{ m/s} = 100\text{ cm/s}$.

The continuity equation gives $\dfrac{A_2}{A_1} = \dfrac{v_1}{v_2} = \dfrac{30\text{ cm/s}}{100\text{ cm/s}} = 0.30$. $A_2 = 0.30A_1$, so 70% of the artery is blocked.

EVALUATE: A 70% blockage reduces the blood speed from 100 cm/s to 30 cm/s, which should easily be detectable.

12.47. IDENTIFY and SET UP:

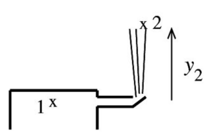


Figure 12.47

Apply Bernoulli's equation to points 1 and 2 as shown in Figure 12.47. Point 1 is in the mains and point 2 is at the maximum height reached by the stream, so $v_2 = 0$.

Solve for p_1 and then convert this absolute pressure to gauge pressure.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Let $y_1 = 0$, $y_2 = 15.0$ m. The mains have large diameter, so $v_1 \approx 0$.

Thus $p_1 = p_2 + \rho g y_2$.

But $p_2 = p_a$, so $p_1 - p_a = \rho g y_2 = (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(15.0 \text{ m}) = 1.47 \times 10^5$ Pa.

EVALUATE: This is the gauge pressure at the bottom of a column of water 15.0 m high.

12.48. **IDENTIFY:** Toricelli's theorem says the speed of efflux is $v = \sqrt{2gh}$, where h is the distance of the small hole below the surface of the water in the tank. The volume flow rate is vA.

SET UP: $A = \pi D^2/4$, with $D = 6.00 \times 10^{-3}$ m.

EXECUTE: **(a)** $v = \sqrt{2(9.80 \text{ m/s}^2)(14.0 \text{ m})} = 16.6$ m/s

(b) $vA = (16.6 \text{ m/s})\pi(6.00 \times 10^{-3} \text{ m})^2/4 = 4.69 \times 10^{-4} \text{ m}^3\text{/s}$. A volume of $4.69 \times 10^{-4} \text{ m}^3 = 0.469$ L is discharged each second.

EVALUATE: We have assumed that the diameter of the hole is much less than the diameter of the tank.

12.49. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: $y_1 = y_2$. $v_1 A_1 = v_2 A_2$. $A_2 = 2A_1$.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (2.50 \text{ m/s})\left(\dfrac{A_1}{2A_1}\right) = 1.25$ m/s.

$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 1.80 \times 10^4 \text{ Pa} + \frac{1}{2}(1000 \text{ kg/m}^3)[(2.50 \text{ m/s})^2 - (1.25 \text{ m/s})^2] = 2.03 \times 10^4$ Pa.

EVALUATE: The gauge pressure is higher at the second point because the water speed is less there.

12.50. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: The continuity equation says $v_1 A_1 = v_2 A_2$. In Bernoulli's equation, either absolute or gauge pressures can be used at both points. $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

EXECUTE: Using $v_2 = \frac{1}{4}v_1$,

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$

$$p_2 = 5.00 \times 10^4 \text{ Pa} + (1.00 \times 10^3 \text{ kg/m}^3)\left(\frac{15}{32}(3.00 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(11.0 \text{ m})\right) = 1.62 \times 10^5 \text{ Pa}.$$

EVALUATE: The decrease in speed and the decrease in height at point 2 both cause the pressure at point 2 to be greater than the pressure at point 1.

12.51. **IDENTIFY** and **SET UP:** Let point 1 be where $r_1 = 4.00$ cm and point 2 be where $r_2 = 2.00$ cm. The volume flow rate vA has the value $7200 \text{ cm}^3\text{/s}$ at all points in the pipe. Apply $v_1 A_1 = v_2 A_2$ to find the fluid speed at points 1 and 2 and then use Bernoulli's equation for these two points to find p_2.

EXECUTE: $v_1 A_1 = v_1 \pi r_1^2 = 7200 \text{ cm}^3$, so $v_1 = 1.43$ m/s

$v_2 A_2 = v_2 \pi r_2^2 = 7200 \text{ cm}^3\text{/s}$, so $v_2 = 5.73$ m/s

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$y_1 = y_2$ and $p_1 = 2.40 \times 10^5$ Pa, so $p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) = 2.25 \times 10^5$ Pa.

EVALUATE: Where the area decreases the speed increases and the pressure decreases.

12.52. IDENTIFY: Since a pressure difference is needed to keep the fluid flowing, there must be viscosity in the fluid.

SET UP: From Section 12.6, the pressure difference Δp over a length L of cylindrical pipe of radius R is proportional to L/R^4. In this problem, the length L is the same in both cases, so $R^4\,\Delta p$ must be constant. The target variable is the pressure difference.

EXECUTE: Since $R^4\,\Delta p$ is constant, we have $\Delta p_1 R_1^4 = \Delta p_2 R_2^4$.

$$\Delta p_2 = \Delta p_1\left(\frac{R_1}{R_2}\right)^4 = (6.00\times10^4\ \text{Pa})\left(\frac{0.21\ \text{m}}{0.0700\ \text{m}}\right)^4 = 4.86\times10^6\ \text{Pa}.$$

EVALUATE: The pipe is narrower, so the pressure difference must be greater.

12.53. IDENTIFY: Increasing the cross-sectional area of the artery will increase the amount of blood that flows through it per second.

SET UP: The flow rate, $\dfrac{\Delta V}{\Delta t}$, is related to the radius R or diameter D of the artery by Poiseuille's law: $\dfrac{\Delta V}{\Delta t} = \dfrac{\pi R^4}{8\eta}\left(\dfrac{p_1 - p_2}{L}\right) = \dfrac{\pi D^4}{128\eta}\left(\dfrac{p_1 - p_2}{L}\right)$. Assume the pressure gradient $(p_1 - p_2)/L$ in the artery remains the same.

EXECUTE: $(\Delta V/\Delta t)/D^4 = \dfrac{\pi}{128\eta}\left(\dfrac{p_1 - p_2}{L}\right) = \text{constant}$, so $(\Delta V/\Delta t)_{\text{old}}/D_{\text{old}}{}^4 = (\Delta V/\Delta t)_{\text{new}}/D_{\text{new}}{}^4$.

$(\Delta V/\Delta t)_{\text{new}} = 2(\Delta V/\Delta t)_{\text{old}}$ and $D_{\text{old}} = D$. This gives $D_{\text{new}} = D_{\text{old}}\left[\dfrac{(\Delta V/\Delta t)_{\text{new}}}{(\Delta V/\Delta t)_{\text{old}}}\right]^{1/4} = 2^{1/4}D = 1.19D.$

EVALUATE: Since the flow rate is proportional to D^4, a 19% increase in D doubles the flow rate.

12.54. IDENTIFY: Apply $p = p_0 + \rho g h$ and $\Delta V = -\dfrac{(\Delta p)V_0}{B}$, where B is the bulk modulus.

SET UP: Seawater has density $\rho = 1.03\times10^3\ \text{kg/m}^3$. The bulk modulus of water is $B = 2.2\times10^9$ Pa. $p_{\text{air}} = 1.01\times10^5$ Pa.

EXECUTE: (a)

$$p_0 = p_{\text{air}} + \rho g h = 1.01\times10^5\ \text{Pa} + (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(10.92\times10^3\ \text{m}) = 1.10\times10^8\ \text{Pa}$$

(b) At the surface $1.00\ \text{m}^3$ of seawater has mass 1.03×10^3 kg. At a depth of 10.92 km the change in volume is $\Delta V = -\dfrac{(\Delta p)V_0}{B} - \dfrac{(1.10\times10^8\ \text{Pa})(1.00\ \text{m}^3)}{2.2\times10^9\ \text{Pa}} = -0.050\ \text{m}^3$. The volume of this mass of water at this depth therefore is $V = V_0 + \Delta V = 0.950\ \text{m}^3$. $\rho = \dfrac{m}{V} = \dfrac{1.03\times10^3\ \text{kg}}{0.950\ \text{m}^3} = 1.08\times10^3\ \text{kg/m}^3$. The density is 5% larger than at the surface.

EVALUATE: For water B is small and a very large increase in pressure corresponds to a small fractional change in volume.

12.55. IDENTIFY: In part (a), the force is the weight of the water. In part (b), the pressure due to the water at a depth h is $\rho g h$. $F = pA$ and $m = \rho V$.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: (a) The weight of the water is

$$\rho g V = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)((5.00\ \text{m})(4.0\ \text{m})(3.0\ \text{m})) = 5.9\times10^5\ \text{N}.$$

(b) Integration gives the expected result that the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint. If d is the depth of the pool and A is the area of one end of the pool, then $F = \rho g A \dfrac{d}{2} = (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)((4.0\ \text{m})(3.0\ \text{m}))(1.50\ \text{m}) = 1.76\times10^5\ \text{N}$.

EVALUATE: The answer to part (a) can be obtained as $F = pA$, where $p = \rho g d$ is the gauge pressure at the bottom of the pool and $A = (5.0\ \text{m})(4.0\ \text{m})$ is the area of the bottom of the pool.

12.56. **IDENTIFY:** A buoyant force acts on the rock when it is suspended in the liquids, so we apply Archimedes's principle. The vertical forces on the rock must balance.

SET UP: Apply $\sum F_y = 0$ and $w = \rho g V$. The target variable is the weight of the rock. In both cases, the rock is totally immersed, so the volume of fluid displaced is equal to the volume V of the rock. We know the densities of the fluids from Table 12.1.

EXECUTE: Call T the tension in the string. $\sum F_y = 0$ gives $T + B = w$, where $B = \rho g V$.

In water: $T_\text{w} + \rho_\text{w} g V = w$ (Eq. 1)

In ethanol: $T_\text{e} + \rho_\text{e} g V = w$ (Eq. 2)

Combine Eq. (1) and Eq. (2): $\dfrac{\rho_\text{w} g V}{\rho_\text{e} g V} = \dfrac{w - T_\text{w}}{w - T_\text{e}}$. Solving for w gives $w = \dfrac{(\rho_\text{w}/\rho_\text{e})T_\text{e} - T_\text{w}}{(\rho_\text{w}/\rho_\text{e}) - 1}$. Using ρ_w = 1000 kg/m^3, ρ_e = 810 kg/m^3, T_w = 1.20 N, and T_e = 1.60 N, we get w = 3.3 N.

EVALUATE: The tension in ethanol is greater than the tension in water because ethanol is less than water and therefore produces a smaller buoyant force.

12.57. **IDENTIFY:** Use $p = p_0 + \rho g h$ to find the gauge pressure versus depth, use $p = \dfrac{F_\perp}{A}$ to relate the pressure to the force on a strip of the gate, calculate the torque as force times moment arm, and follow the procedure outlined in the hint to calculate the total torque.

SET UP: The gate is sketched in Figure 12.57a.

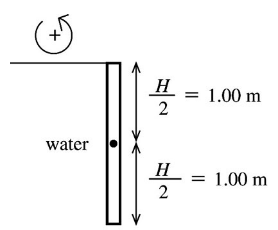


Figure 12.57a

Let τ_u be the torque due to the net force of the water on the upper half of the gate, and τ_l be the torque due to the force on the lower half.

With the indicated sign convention, τ_l is positive and τ_u is negative, so the net torque about the hinge is $\tau = \tau_\text{l} - \tau_\text{u}$. Let H be the height of the gate.

Upper half of gate:

Calculate the torque due to the force on a narrow strip of height dy located a distance y below the top of the gate, as shown in Figure 12.57b. Then integrate to get the total torque.

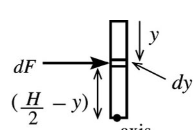


Figure 12.57b

The net force on the strip is $dF = p(y)\,dA$, where $p(y) = \rho g y$ is the pressure at this depth and $dA = W\,dy$ with $W = 4.00$ m.

$dF = \rho g y W\,dy$

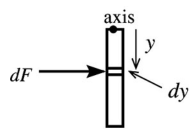
The moment arm is $(H/2-y)$, so $d\tau = \rho g W(H/2-y)y\,dy$.

$\tau_u = \int_0^{H/2} d\tau = \rho g W \int_0^{H/2} (H/2-y)y\,dy = \rho g W((H/4)y^2 - y^3/3)\Big|_0^{H/2}$

$\tau_u = \rho g W(H^3/16 - H^3/24) = \rho g W(H^3/48)$

$\tau_u = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(4.00\text{ m})(2.00\text{ m})^3/48 = 6.533\times 10^3\text{ N}\cdot\text{m}$

Lower half of gate:



Consider the narrow strip shown in Figure 12.57c. The depth of the strip is $(H/2+y)$ so the force dF is $dF = p(y)\,dA = \rho g(H/2+y)W\,dy$.

Figure 12.57c

The moment arm is y, so $d\tau = \rho g W(H/2+y)y\,dy$.

$\tau_l = \int_0^{H/2} d\tau = \rho g W \int_0^{H/2} (H/2+y)y\,dy = \rho g W((H/4)y^2 + y^3/3)\Big|_0^{H/2}$

$\tau_l = \rho g W(H^3/16 + H^3/24) = \rho g W(5H^3/48)$

$\tau_l = (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(4.00\text{ m})5(2.00\text{ m})^3/48 = 3.267\times 10^4\text{ N}\cdot\text{m}$

Then $\tau = \tau_l - \tau_u = 3.267\times 10^4\text{ N}\cdot\text{m} - 6.533\times 10^3\text{ N}\cdot\text{m} = 2.61\times 10^4\text{ N}\cdot\text{m}$.

EVALUATE: The forces and torques on the upper and lower halves of the gate are in opposite directions so find the net value by subtracting the magnitudes. The torque on the lower half is larger than the torque on the upper half since pressure increases with depth.

12.58. **IDENTIFY:** A dense cube floats in a tank containing water and glycerin, so Archimedes's principle applies and the vertical forces on it must balance.

SET UP: The buoyant force B is equal to the weight of the fluid displaced, and $w = \rho g V$. We want to find out what fraction of the cube's volume is below the surface of the glycerin. The water layer is on top of the glycerin layer, so we should first find out if *any* of the cube gets into the glycerin. Calling m the mass of the cube and V its volume, its density is $\rho_c = m/V = \dfrac{9.20\times 10^{-3}\text{ kg}}{(0.0200\text{ m})^3} = 1150\text{ kg/m}^3$. This is greater than the density of water (1000 kg/m^3) but less than the density of glycerin (1260 kg/m^3). Therefore water alone cannot support the cube, so some of it must be in the glycerin. The cube is floating, so $\sum F_y = 0$.

EXECUTE: With V the total volume of the cube and x the volume that is in the glycerin, $V-x$ is the volume in water. By Archimedes's principle, there are *two* buoyant forces on it: one due to the displaced water and the other due to the displaced glycerin. Therefore $\sum F_y = 0$ tells us that $B_w + B_g = mg$. Using $w = \rho g V$, this gives $\rho_w g V_w + \rho_g g V_g = mg$, so $\rho_w g(V-x) + \rho_g g x = mg$. Rearranging gives $x(\rho_g - \rho_w) = m - \rho_w V$. We want the fraction of the cube's volume that is in glycerin, so we want x/V. Solving for this gives $\dfrac{x}{V} = \dfrac{m/V - \rho_w}{\rho_g - \rho_w} = \dfrac{\rho_c - \rho_w}{\rho_g - \rho_w}$. Using the given numbers gives $\dfrac{x}{V} = \dfrac{(1150-1000)\text{ kg/m}^3}{(1260-1000)\text{ kg/m}^3} = 0.577$, so 57.7% of its volume is in glycerin.

EVALUATE: It might seem strange that the water can exert any buoyant force on the cube since the only horizontal surface of the cube that is exposed the water is the *top*, and the force there is *downward*. However, let us look at the buoyant force in terms of the pressure in the fluids. This pressure is

$p = p_0 + \rho gh$. Fig. 12.58 shows the pressure on the upper and lower surfaces of the cube, with p_0 the atmospheric pressure.

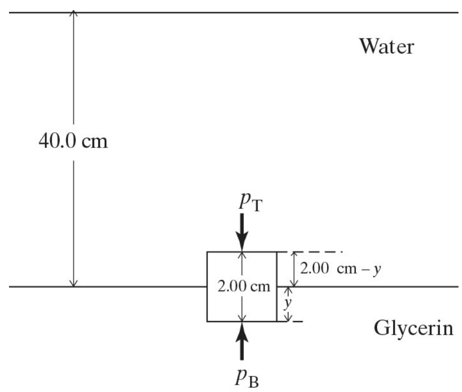


Figure 12.58

The upper surface of the cube is a distance 40 cm – (2 cm – y) = 38 cm + y below the top of the water. The pressure at that depth is the pressure at the top of the cube $p_T = p_0 + \rho_w g(38\text{ cm} + y)$. The bottom of the cube is a distance y below the surface of the glycerin. The pressure at the surface of the glycerin layer is due to atmospheric pressure and the pressure from 40 cm of water, which is $p_0 + p_w =$ $p_0 + \rho_w g(40\text{ cm})$. The pressure at a depth y in the glycerin is the pressure at the bottom face of the cube and is $p_B = p_0 + \rho_w g(40\text{ cm}) + \rho_w gy$. The force on an area A is pA, so the buoyant force on the cube is $B = p_B A - p_T A = (p_B - p_T)A$. Using the expressions we just found for p_B and p_T gives $B = \{p_0 + \rho_w g(40\text{ cm}) + \rho_w gy - [p_0 + \rho_w g(38\text{ cm} + y)]\}A$. Simplifying gives $B = [\rho_w(2\text{ cm} - y) + \rho_g y]Ag$. The buoyant force supports the cube, so $B = \rho_c g V_c$, which gives $\rho_c g V_c = [\rho_w(2\text{ cm} - y) + \rho_g y]Ag$. In terms of the surface area, $V_c = A(2\text{ cm})$. Using this we get $\rho_c(2\text{cm}) = \rho_w(2\text{cm} - y) + \rho_g y$. Solving for y gives $y = \dfrac{(\rho_c - \rho_w)(2\text{cm})}{\rho_g - \rho_w}$. The fraction of the volume in the glycerin is $\dfrac{y}{2\text{cm}}$, so we get $\dfrac{y}{2\text{cm}} = \dfrac{\rho_c - \rho_w}{\rho_g - \rho_w}$, which is the same result we got above, so our result checks.

12.59. IDENTIFY: This problem deals with the pressure at a depth in a fluid and requires graphical interpretation.

SET UP: The gauge pressure on the bottom of the block supports the weight of the block and the coins. The graph is a plot of the depth h of the bottom of the block versus the mass m of the coins, so we need to look for a relationship between these quantities. The target variable is the mass M of the block. The gauge pressure is $p = \rho gh$ and the force on an area is $F = pA$.

EXECUTE: The net force on the block is $p_g A$, and that force must be equal to the weight of the box plus the coins. This gives $p_g A = mg + Mg$. Using $p_g = \rho gh$, this becomes $\rho ghA = mg + Mg$ so $h = \dfrac{1}{\rho A}m + \dfrac{M}{\rho A}$. Therefore a graph of h versus m should have a slope of $\dfrac{1}{\rho A}$ and a y-intercept of $\dfrac{M}{\rho A}$. From the slope we get $A = \dfrac{1}{\rho(\text{slope})}$. The y-intercept gives $y\text{-int} = \dfrac{M}{\rho A}$, so $M = (y\text{-int})\rho A$. Putting the result for A into the equation for M gives $M = (y\text{-int})\rho A = (y\text{-int})\rho\left(\dfrac{1}{\rho(\text{slope})}\right) = \dfrac{y\text{-int}}{\text{slope}} =$

$\dfrac{0.0312\text{ m}}{0.0390\text{ m/kg}} = 0.800\text{ kg}.$

EVALUATE: Careful graphing is important because the answer depends on *both* the slope and y-intercept of the graph.

12.60. **IDENTIFY:** The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B = \rho_{\text{air}}Vg$. Let g_{M} be the value of g for Mars. For a sphere $V = \frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A = 4\pi R^2$. The mass of the balloon is $(5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B = mg_{\text{M}}$. $\rho_{\text{air}}Vg_{\text{M}} = mg_{\text{M}}$. $\rho_{\text{air}}\frac{4}{3}\pi R^3 = (5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2)$.

$R = \dfrac{3(5.00\times10^{-3}\text{ kg/m}^2)}{\rho_{\text{air}}} = 0.974\text{ m}.$ $m = (5.00\times10^{-3}\text{ kg/m}^2)(4\pi R^2) = 0.0596\text{ kg}.$

(b) $F_{\text{net}} = B - mg = ma$. $B = \rho_{\text{air}}Vg = \rho_{\text{air}}\frac{4}{3}\pi R^3 g = (1.20\text{ kg/m}^3)\left(\dfrac{4\pi}{3}\right)(0.974\text{ m})^3(9.80\text{ m/s}^2) = 45.5\text{ N}.$

$a = \dfrac{B - mg}{m} = \dfrac{45.5\text{ N} - (0.0596\text{ kg})(9.80\text{ m/s}^2)}{0.0596\text{ m}} = 754\text{ m/s}^2,$ upward.

(c) $B = m_{\text{tot}}g$. $\rho_{\text{air}}Vg = (m_{\text{balloon}} + m_{\text{load}})g$. $m_{\text{load}} = \rho_{\text{air}}\frac{4}{3}\pi R^3 - (5.00\times10^{-3}\text{ kg/m}^2)4\pi R^2$.

$m_{\text{load}} = (0.0154\text{ kg/m}^3)\left(\dfrac{4\pi}{3}\right)(5[0.974\text{ m}])^3 - (5.00\times10^{-3}\text{ kg/m}^2)(4\pi)(5[0.974\text{ m}])^2$

$m_{\text{load}} = 7.45\text{ kg} - 1.49\text{ kg} = 5.96\text{ kg}$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2, so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of g for Mars.

12.61. **IDENTIFY:** The buoyant force on an object in a liquid is equal to the weight of the liquid it displaces.

SET UP: $V = \dfrac{m}{\rho}$.

EXECUTE: When it is floating, the ice displaces an amount of glycerin equal to its weight. From Table 12.1, the density of glycerin is 1260 kg/m^3. The volume of this amount of glycerin is

$V = \dfrac{m}{\rho} = \dfrac{0.180\text{ kg}}{1260\text{ kg/m}^3} = 1.429\times10^{-4}\text{ m}^3$. The ice cube produces 0.180 kg of water. The volume of this mass of water is $V = \dfrac{m}{\rho} = \dfrac{0.180\text{ kg}}{1000\text{ kg/m}^3} = 1.80\times10^{-4}\text{ m}^3$. The volume of water from the melted ice is greater than the volume of glycerin displaced by the floating cube and the level of liquid in the cylinder rises. The distance the level rises is $\dfrac{1.80\times10^{-4}\text{ m}^3 - 1.429\times10^{-4}\text{ m}^3}{\pi(0.0350\text{ m})^2} = 9.64\times10^{-3}\text{ m} = 0.964\text{ cm}.$

EVALUATE: The melted ice has the same mass as the solid ice, but a different density.

12.62. **IDENTIFY:** The pressure must be the same at the bottom of the tube. Therefore since the liquids have different densities, they must have difference heights.

SET UP: After the barrier is removed the top of the water moves downward a distance x and the top of the oil moves up a distance x, as shown in Figure 12.62. After the heights have changed, the gauge pressure at the bottom of each of the tubes is the same. The gauge pressure p at a depth h is $p - p_{\text{atm}} = \rho gh$.

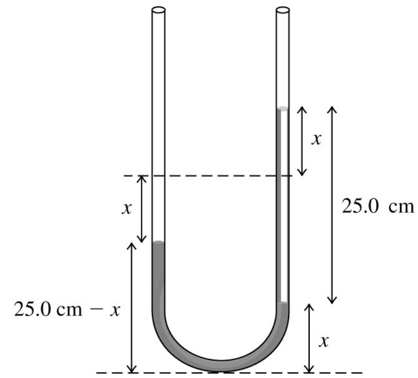


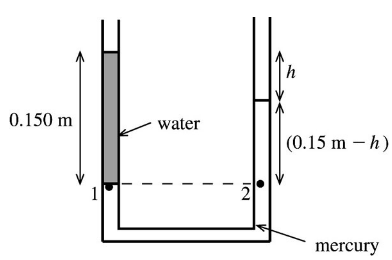
Figure 12.62

EXECUTE: The gauge pressure at the bottom of arm *A* of the tube is $p - p_{\text{atm}} = \rho_{\text{w}} g(25.0\ \text{cm} - x)$. The gauge pressure at the bottom of arm *B* of the tube is $p - p_{\text{atm}} = \rho_{\text{oil}} g(25.0\ \text{cm}) + \rho_{\text{w}} gx$. The gauge pressures must be equal, so $\rho_{\text{w}} g(25.0\ \text{cm} - x) = \rho_{\text{oil}} g(25.0\ \text{cm}) + \rho_{\text{w}} gx$. Dividing out *g* and using $\rho_{\text{oil}} = 0.80\rho_{\text{w}}$, we have $\rho_{\text{w}}(25.0\ \text{cm} - x) = 0.80\rho_{\text{w}}(25.0\ \text{cm}) + \rho_{\text{w}} x$. ρ_{w} divides out and leaves $25.0\ \text{cm} - x = 20.0\ \text{cm} + x$, so $x = 2.5\ \text{cm}$. The height of fluid in arm *A* is $25.0\ \text{cm} - x = 22.5\ \text{cm}$ and the height in arm *B* is $25.0\ \text{cm} + x = 27.5\ \text{cm}$.

(b) (i) If the densities were the same there would be no reason for a difference in height and the height would be 25.0 cm on each side. (ii) The pressure exerted by the column of oil would be very small and the water would divide equally on both sides. The height in arm *A* would be 12.5 cm and the height in arm *B* would be $25.0\ \text{cm} + 12.5\ \text{cm} = 37.5\ \text{cm}$.

EVALUATE: The less dense fluid rises to a higher height, which is physically reasonable.

12.63. **(a) IDENTIFY** and **SET UP:**



Apply $p = p_0 + \rho gh$ to the water in the left-hand arm of the tube.
See Figure 12.63.

Figure 12.63

EXECUTE: $p_0 = p_{\text{a}}$, so the gauge pressure at the interface (point 1) is

$p - p_{\text{a}} = \rho gh = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.150\ \text{m}) = 1470\ \text{Pa}$.

(b) IDENTIFY and **SET UP:** The pressure at point 1 equals the pressure at point 2. Apply $p = p_0 + \rho gh$ to the right-hand arm of the tube and solve for *h*.

EXECUTE: $p_1 = p_{\text{a}} + \rho_{\text{w}} g(0.150\ \text{m})$ and $p_2 = p_{\text{a}} + \rho_{\text{Hg}} g(0.150\ \text{m} - h)$

$p_1 = p_2$ implies $\rho_{\text{w}} g(0.150\ \text{m}) = \rho_{\text{Hg}} g(0.150\ \text{m} - h)$

$$0.150\ \text{m} - h = \frac{\rho_{\text{w}}(0.150\ \text{m})}{\rho_{\text{Hg}}} = \frac{(1000\ \text{kg/m}^3)(0.150\ \text{m})}{13.6\times 10^3\ \text{kg/m}^3} = 0.011\ \text{m}$$

$h = 0.150\ \text{m} - 0.011\ \text{m} = 0.139\ \text{m} = 13.9\ \text{cm}$

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

12.64. IDENTIFY: Follow the procedure outlined in the hint. $F = pA$.

SET UP: The circular ring has area $dA = (2\pi R)dy$. The pressure due to the molasses at depth y is ρgy.

EXECUTE: $F = \int_0^h (\rho gy)(2\pi R)dy = \rho g\pi Rh^2$ where R and h are the radius and height of the tank.

Using the given numerical values gives $F = 2.11\times 10^8$ N.

EVALUATE: The net outward force is the area of the wall of the tank, $A = 2\pi Rh$, times the average pressure, the pressure $\rho gh/2$ at depth $h/2$.

12.65. IDENTIFY: Archimedes's principle applies.

SET UP: $\rho = m/V$, the buoyant force B is equal to the weight of the liquid displaced. Call m the mass of the block.

EXECUTE: **(a)** The volume of water displaced by the block is 80.0% of the volume of the block. Using $B = mg$: $\rho_w V_w g = mg$ gives $\rho_w (0.800 V_{block}) = m$. Therefore $V_{block} = m/(0.800\,\rho_w)$, so

$V_{block} = (40.0\text{ kg})/[(0.800)(1000\text{ kg/m}^3)] = 0.0500\text{ m}^3$.

(b) With the maximum amount of bricks added on, the block is completely submerged but the bricks are not under water. Therefore $m_{bricks}g + mg = \rho_w V_{block}g$. Solving for m_{bricks} and putting in the numbers gives

$m_{bricks} = \rho_w V_{block} - m = (1000\text{ kg/m}^3)(0.0500\text{ m}^3) - 40.0\text{ kg} = 10.0\text{ kg}$.

EVALUATE: If the bricks were to go under water, the buoyant force would increase because a greater volume of water would be displaced.

12.66. IDENTIFY: The buoyant force on the balloon must equal the total weight of the balloon fabric, the basket and its contents and the gas inside the balloon. $m_{gas} = \rho_{gas}V$. $B = \rho_{air}Vg$.

SET UP: The total weight, exclusive of the gas inside the balloon, is $900\text{ N} + 1700\text{ N} + 3200\text{ N} = 5800\text{ N}$.

EXECUTE: $5800\text{ N} + \rho_{gas}Vg = \rho_{air}Vg$ and $\rho_{gas} = 1.23\text{ kg/m}^3 - \dfrac{(5800\text{ N})}{(9.80\text{ m/s}^2)(2200\text{ m}^3)} = 0.96\text{ kg/m}^3$.

EVALUATE: The volume of a given mass of gas increases when the gas is heated, and the density of the gas therefore decreases.

12.67. IDENTIFY: Apply Newton's second law to the barge plus its contents. Apply Archimedes's principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 12.67.

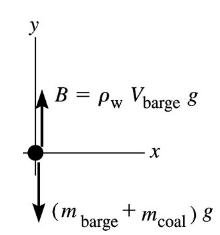


Figure 12.67

EXECUTE: $\sum F_y = ma_y$

$B - (m_{barge} + m_{coal})g = 0$

$\rho_w V_{barge} g = (m_{barge} + m_{coal})g$

$m_{coal} = \rho_w V_{barge} - m_{barge}$

$V_{barge} = (22\text{ m})(12\text{ m})(40\text{ m}) = 1.056\times 10^4\text{ m}^3$

The mass of the barge is $m_{barge} = \rho_s V_s$, where s refers to steel.

From Table 12.1, $\rho_s = 7800\ \text{kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge

$$V_s = (0.040\ \text{m})[2(22\ \text{m})(12\ \text{m}) + 2(40\ \text{m})(12\ \text{m}) + (22\ \text{m})(40\ \text{m})] = 94.7\ \text{m}^3.$$

Therefore, $m_{\text{barge}} = \rho_s V_s = (7800\ \text{kg/m}^3)(94.7\ \text{m}^3) = 7.39\times10^5\ \text{kg}$.

Then $m_{\text{coal}} = \rho_w V_{\text{barge}} - m_{\text{barge}} = (1000\ \text{kg/m}^3)(1.056\times10^4\ \text{m}^3) - 7.39\times10^5\ \text{kg} = 9.8\times10^6\ \text{kg}$.

The volume of this mass of coal is $V_{\text{coal}} = m_{\text{coal}}/\rho_{\text{coal}} = 9.8\times10^6\ \text{kg}/1500\ \text{kg/m}^3 = 6500\ \text{m}^3$; this is less than V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force B must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

12.68. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. The buoyant force when the wood sinks is $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the volume of the wood plus the volume of the lead. $\rho = m/V$.

SET UP: The density of lead is $11.3\times10^3\ \text{kg/m}^3$.

EXECUTE: $V_{\text{wood}} = (0.600\ \text{m})(0.250\ \text{m})(0.080\ \text{m}) = 0.0120\ \text{m}^3$.

$m_{\text{wood}} = \rho_{\text{wood}} V_{\text{wood}} = (700\ \text{kg/m}^3)(0.0120\ \text{m}^3) = 8.40\ \text{kg}$.

$B = (m_{\text{wood}} + m_{\text{lead}})g$. Using $B = \rho_{\text{water}} V_{\text{tot}} g$ and $V_{\text{tot}} = V_{\text{wood}} + V_{\text{lead}}$ gives

$\rho_{\text{water}}(V_{\text{wood}} + V_{\text{lead}})g = (m_{\text{wood}} + m_{\text{lead}})g$. $m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}}$ then gives

$\rho_{\text{water}} V_{\text{wood}} + \rho_{\text{water}} V_{\text{lead}} = m_{\text{wood}} + \rho_{\text{lead}} V_{\text{lead}}$.

$$V_{\text{lead}} = \frac{\rho_{\text{water}} V_{\text{wood}} - m_{\text{wood}}}{\rho_{\text{lead}} - \rho_{\text{water}}} = \frac{(1000\ \text{kg/m}^3)(0.0120\ \text{m}^3) - 8.40\ \text{kg}}{11.3\times10^3\ \text{kg/m}^3 - 1000\ \text{kg/m}^3} = 3.50\times10^{-4}\ \text{m}^3.$$

$m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}} = 3.95\ \text{kg}$.

EVALUATE: The volume of the lead is only 2.9% of the volume of the wood. If the contribution of the volume of the lead to F_B is neglected, the calculation is simplified: $\rho_{\text{water}} V_{\text{wood}} g = (m_{\text{wood}} + m_{\text{lead}})g$ and $m_{\text{lead}} = 3.6\ \text{kg}$. The result of this calculation is in error by about 9%.

12.69. **IDENTIFY:** The water shoots out of the hole at the bottom of the jug and is then in free-fall downward to the floor. We will need to use Bernoulli's equation as well as projectile motion after the milk leaves the jug.

SET UP: Estimates: The milk is 10 in. (25.4 cm = 0.254 m) high in the jug and the table height is 30 in. (0.762 m). We use $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, and $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$. The target variable is the horizontal distance the milk travels before hitting the floor. Call h the height of the milk in the jug and H the height of the table.

EXECUTE: Using $p_1 = p_2 = p_{\text{atm}}$, $v_1 = 0$ ($A_{-1} >> A_2$), $y_1 = h$ = 10 in. = 0.54 m, $y_2 = 0$, we get $\rho g h = \frac{1}{2}\rho v_2^2$, which gives $v_2 = \sqrt{2gh}$. Now use projectile motion. Find the time to fall a distance H from rest vertically. $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2H}{g}}$. The horizontal distance traveled in this time is

$$x = v_2 t = \sqrt{2gh}\sqrt{\frac{2H}{g}} = 2\sqrt{hH} = 2\sqrt{(0.254\ \text{m})(0.762\ \text{m})} = 0.88\ \text{m} = 88\ \text{cm} = 35\ \text{in.}$$

EVALUATE: As the jug drains, h will decrease so the distance x will also decrease.

12.70. **IDENTIFY:** The ethanol is flowing through a pipe, so we need to use Bernoulli's equation, the continuity equation, and volume flow rate.

SET UP: Use $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, $A_1 v_1 = A_2 v_2$ (continuity equation) and $dV/dt = Av$. We want to find the pressure at A so that the volume flow rate at B is 0.0800 m^3/s.

EXECUTE: $p_1 = p$ (unknown), $y_1 = 0$, $y_2 = 3.00$ m, $p_2 = p_{atm}$, v_1 and v_2 are unknown. Use the volume flow rate to find v_2. Using $dV/dt = A_B v_2$ gives $v_2 = \frac{dV/dt}{A_B} = \frac{0.0800\ \text{m}^3/\text{s}}{0.0200\ \text{m}^2} = 4.00\ \text{m/s}$. Now use the continuity equation to find v_1. Solving $A_1 v_1 = A_2 v_2$ for v_1 gives $v_1 = \frac{A_B}{A_A} v_2$. For the numbers here we have $v_1 = \frac{0.0200\ \text{m}^2}{0.0500\ \text{m}^2}(4.00\ \text{m/s}) = 1.60\ \text{m/s}$. Now return to Bernoulli's equation. We want the gauge pressure at A so we want $p = p_2 - p_{atm}$. The Bernoulli equation now becomes $p + \frac{1}{2}\rho v_1^2 = \rho g y_2 + \frac{1}{2}\rho v_2^2$. Solving for p gives $p = \rho\left[g y_2 + \frac{1}{2}\left(v_2^2 - v_1^2\right)\right]$. Using (from Table 12.1) ρ = 810 kg/m^3 for ethanol, $v_1 = 1.60$ m/s, $v_2 = 4.00$ m/s, and $y_2 = 3.00$ m, we get $p = 2.9 \times 10^4$ Pa.

EVALUATE: This is about 30% of atmospheric pressure. The absolute pressure would be about 1.3 atm.

12.71. **IDENTIFY:** After the water leaves the hose the only force on it is gravity. Use conservation of energy to relate the initial speed to the height the water reaches. The volume flow rate is Av.

SET UP: $A = \pi D^2/4$

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ gives $v = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(28.0\ \text{m})} = 23.4\ \text{m/s}$.

$(\pi D^2/4)v = 0.500\ \text{m/s}^3$. $D = \sqrt{\frac{4(0.500\ \text{m/s}^3)}{\pi v}} = \sqrt{\frac{4(0.500\ \text{m/s}^3)}{\pi(23.4\ \text{m/s})}} = 0.165\ \text{m} = 16.5\ \text{cm}$.

(b) $D^2 v$ is constant so if D is twice as great, then v is decreased by a factor of 4. h is proportional to v^2, so h is decreased by a factor of 16. $h = \frac{28.0\ \text{m}}{16} = 1.75\ \text{m}$.

EVALUATE: The larger the diameter of the nozzle the smaller the speed with which the water leaves the hose and the smaller the maximum height.

12.72. **IDENTIFY:** $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$. D and E read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale D; a similar statement applies to m_E.

$m_A = 3.50\ \text{kg} + 7.50\ \text{kg} - 1.00\ \text{kg} - 1.80\ \text{kg} = 8.20\ \text{kg}$.

Forces on A: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\ \text{kg} - 3.50\ \text{kg}}{3.80 \times 10^{-3}\ \text{m}^3} = 1.24 \times 10^3\ \text{kg/m}^3$$

(b) D reads the mass of A: 8.20 kg. E reads the total mass of B and C: 2.80 kg.

EVALUATE: The sum of the readings of the two scales remains the same.

12.73. IDENTIFY: As water flows from the tank, the water level changes. This affects the speed with which the water flows out of the tank and the pressure at the bottom of the tank.

SET UP: Bernoulli's equation, $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$, and the continuity equation, $A_1 v_1 = A_2 v_2$, both apply.

EXECUTE: **(a)** Let point 1 be at the surface of the water in the tank and let point 2 be in the stream of water that is emerging from the tank. $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$. $v_1 = \dfrac{\pi d_2^2}{\pi d_1^2} v_2$, with $d_2 = 0.0200 \text{ m}$ and $d_1 = 2.00 \text{ m}$. $v_1 << v_2$ so the $\frac{1}{2}\rho v_1^2$ term can be neglected. $v_2 = \sqrt{\dfrac{2p_0}{\rho} + 2gh}$, where $h = y_1 - y_2$ and $p_0 = p_1 - p_2 = 5.00 \times 10^3 \text{ Pa}$. Initially $h = h_0 = 0.800 \text{ m}$ and when the tank has drained $h = 0$. At $t = 0$, $v_2 = \sqrt{\dfrac{2(5.00\times 10^3 \text{ Pa})}{1000 \text{ kg/m}^3} + 2(9.8 \text{ m/s}^2)(0.800 \text{ m})} = \sqrt{10 + 15.68} \text{ m/s} = 5.07 \text{ m/s}$.

If the tank is open to the air, $p_0 = 0$ and $v_2 = 3.96 \text{ m/s}$. The ratio is 1.28.

(b) $v_1 = -\dfrac{dh}{dt} = \dfrac{A_2}{A_1} v_2 = \left(\dfrac{d_2}{d_1}\right)^2 \sqrt{\dfrac{2p_0}{\rho} + 2gh} = \left(\dfrac{d_2}{d_1}\right)^2 \sqrt{2g}\sqrt{\dfrac{p_0}{g\rho} + h}$. Separating variables gives $\dfrac{dh}{\sqrt{\dfrac{p_0}{g\rho} + h}} = -\left(\dfrac{d_2}{d_1}\right)^2 \sqrt{2g}\, dt$. We now must integrate $\displaystyle\int_{h_0}^{0} \frac{dh'}{\sqrt{\dfrac{p_0}{g\rho} + h'}} = -\left(\frac{d_2}{d_1}\right)^2 \sqrt{2g} \int_0^t dt'$. To do the left-hand side integral, make the substitution $u = \dfrac{p_0}{g\rho} + h'$, which makes $du = dh'$. The integral is then of the form $\displaystyle\int \frac{du}{u^{1/2}}$, which can be readily integrated using $\displaystyle\int u^n\, du = \frac{u^{n+1}}{n+1}$. The result is

$$2\left(\sqrt{\frac{p_0}{g\rho}} - \sqrt{\frac{p_0}{g\rho} + h_0}\right) = -\left(\frac{d_2}{d_1}\right)^2 \sqrt{2g}\, t.$$ Solving for t gives $t = \left(\dfrac{d_1}{d_2}\right)^2 \sqrt{\dfrac{2}{g}}\left(\sqrt{\dfrac{p_0}{g\rho} + h_0} - \sqrt{\dfrac{p_0}{g\rho}}\right)$. Since

$\dfrac{p_0}{g\rho} = \dfrac{5.00\times 10^3 \text{ Pa}}{(9.8 \text{ m/s}^2)(1000 \text{ kg/m}^3)} = 0.5102 \text{ m}$, we get

$t = \left(\dfrac{2.00}{0.0200}\right)^2 \sqrt{\dfrac{2}{9.8 \text{ m/s}^2}}\left(\sqrt{0.5102 \text{ m} + 0.800 \text{ m}} - \sqrt{0.5102} \text{ m}\right) = 1.944\times 10^3 \text{ s} = 32.4 \text{ min}$. When $p_0 = 0$,

$t = \left(\dfrac{2.00}{0.0200}\right)^2 \sqrt{\dfrac{2}{9.8 \text{ m/s}^2}}\left(\sqrt{0.800 \text{ m}}\right) = 4.04\times 10^3 \text{ s} = 67.3 \text{ min}$. The ratio is 2.08.

EVALUATE: Both ratios are greater than one because a surface pressure greater than atmospheric pressure causes the water to drain with a greater speed and in a shorter time than if the surface were open to the atmosphere with a pressure of one atmosphere.

12.74. IDENTIFY: Apply $\sum F_y = ma_y$ to the ball, with $+y$ upward. The buoyant force is given by Archimedes's principle.

SET UP: The ball's volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.0 \text{ cm})^3 = 7238 \text{ cm}^3$. As it floats, it displaces a weight of water equal to its weight.

EXECUTE: **(a)** By pushing the ball under water, you displace an additional amount of water equal to 76.0% of the ball's volume or $(0.760)(7238 \text{ cm}^3) = 5501 \text{ cm}^3$. This much water has a mass of

$5501\ g = 5.501\,\text{kg}$ and weighs $(5.501\ \text{kg})(9.80\ \text{m/s}^2) = 53.9\ \text{N}$, which is how hard you'll have to push to submerge the ball.

(b) The upward force on the ball in excess of its own weight was found in part (a): 53.9 N. The ball's mass is equal to the mass of water displaced when the ball is floating:

$$(0.240)(7238\ \text{cm}^3)(1.00\ \text{g/cm}^3) = 1737\ \text{g} = 1.737\ \text{kg},$$

and its acceleration upon release is thus $a = \dfrac{F_{\text{net}}}{m} = \dfrac{53.9\ \text{N}}{1.737\ \text{kg}} = 31.0\ \text{m/s}^2.$

EVALUATE: When the ball is totally immersed the upward buoyant force on it is much larger than its weight.

12.75. IDENTIFY: Apply Newton's second law to the block. In part (a), use Archimedes's principle for the buoyancy force. In part (b), use $p = p_0 + \rho g h$ to find the pressure at the lower face of the block and then use $p = \dfrac{F_\perp}{A}$ to calculate the force the fluid exerts.

(a) SET UP: The free-body diagram for the block is given in Figure 12.75a.

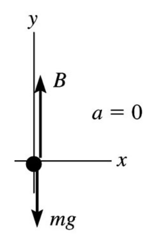


Figure 12.75a

EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_L V_{\text{sub}} g = \rho_B V_{\text{obj}} g$

The fraction of the volume that is submerged is $V_{\text{sub}}/V_{\text{obj}} = \rho_B/\rho_L$.

Thus the fraction that is *above* the surface is $V_{\text{above}}/V_{\text{obj}} = 1 - \rho_B/\rho_L$.

EVALUATE: If $\rho_B = \rho_L$ the block is totally submerged as it floats.

(b) SET UP: Let the water layer have depth *d*, as shown in Figure 12.75b.

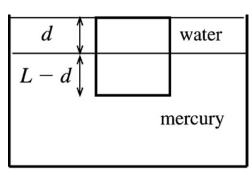


Figure 12.75b

EXECUTE: $p = p_0 + \rho_w g d + \rho_L g(L - d)$

Applying $\sum F_y = ma_y$ to the block gives

$$(p - p_0)A - mg = 0.$$

$[\rho_w g d + \rho_L g(L-d)]A = \rho_B L A g$

A and g divide out and $\rho_w d + \rho_L (L - d) = \rho_B L$

$d(\rho_w - \rho_L) = (\rho_B - \rho_L)L$

$$d = \left(\frac{\rho_L - \rho_B}{\rho_L - \rho_w}\right)L$$

(c) $d = \left(\dfrac{13.6\times10^3\ \text{kg/m}^3 - 7.8\times10^3\ \text{kg/m}^3}{13.6\times10^3\ \text{kg/m}^3 - 1000\ \text{kg/m}^3}\right)(0.100\ \text{m}) = 0.0460\ \text{m} = 4.60\ \text{cm}$

EVALUATE: In the expression derived in part (b), if $\rho_B = \rho_L$ the block floats in the liquid totally submerged and no water needs to be added. If $\rho_L \to \rho_w$ the block continues to float with a fraction $1-\rho_B/\rho_w$ above the water as water is added, and the water never reaches the top of the block $(d \to \infty)$.

12.76. **IDENTIFY:** For the floating tanker, the buoyant force equals its total weight. The buoyant force is given by Archimedes's principle.

SET UP: When the metal is in the tanker, it displaces its weight of water and after it has been pushed overboard it displaces its volume of water.

EXECUTE: **(a)** The change in height Δy is related to the displaced volume ΔV by $\Delta y = \dfrac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so

$$\Delta y = \frac{\Delta V}{A} = \frac{w/(\rho_{\text{water}} g)}{A} = \frac{w}{\rho_{\text{water}} g A} = \frac{(2.50\times10^6\ \text{N})}{(1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)[(60.0\ \text{m})(20.0\ \text{m})]} = 0.213\ \text{m}.$$

(b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 7.20\rho_{\text{water}}$, which gives , $\Delta y' = \dfrac{\Delta y}{7.2}$, so

$$\Delta y - \Delta y' = \Delta y - \frac{\Delta y}{7.20} = \Delta y\left(1-\frac{1}{7.20}\right) = (0.213\ \text{m})\left(1-\frac{1}{7.20}\right) = 0.183\ \text{m};$$ the water level falls this amount.

EVALUATE: The density of the metal is greater than the density of water, so the volume of water that has the same weight as the steel is greater than the volume of water that has the same volume as the steel.

12.77. **IDENTIFY:** After leaving the tank, the water is in free fall, with $a_x = 0$ and $a_y = +g$.

SET UP: The speed of efflux is $\sqrt{2gh}$.

EXECUTE: **(a)** The time it takes any portion of the water to reach the ground is $t = \sqrt{\dfrac{2(H-h)}{g}}$, in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}$.

(b) Note that if $h' = H-h,\ h'(H-h') = (H-h)h$, and so $h' = H-h$ gives the same range. A hole $H-h$ below the water surface is a distance h above the bottom of the tank.

EVALUATE: For the special case of $h = H/2,\ h = h'$ and the two points coincide. For the upper hole the speed of efflux is less but the time in the air during the free fall is greater.

12.78. **IDENTIFY:** Bernoulli's equation applies to the water.

SET UP: First use Bernoulli's equation, $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$, to find the speed of the water as it enters the hold. Then use $dV/dt = Av$ to find the rate at which water flows into the hole, and then solve for the time for 10.0 L to flow in. The pressure at the top of the water is the same as the pressure of the cabin into which the water flows (atmospheric pressure), and the speed of the water at the surface is zero. 1 m^3 = 1000 L.

EXECUTE: **(a)** Using the above conditions, Bernoulli's equation gives

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_0 + 0 + 0 = p_0 + \tfrac{1}{2}\rho v^2 + \rho g y$$

Solving for v gives $v = \sqrt{-2gy}$. The rate at which water enters ship is $V/t = Av = A\sqrt{-2gy}$. Thus

$$t = V/Av = \frac{V}{A\sqrt{-2gy}} = \frac{10\times10^{-3}\ \text{m}^3}{(1.20\times10^{-4}\ \text{m}^2)\sqrt{-2(9.80\ \text{m/s}^2)(-0.900\ \text{m})}} = 19.8\ \text{s}.$$

(b) The mass of 10.0 L of water is 10 kg, which will have no appreciable effect on the weight of a ship.
EVALUATE: The speed at which the water enters the hole is the same as if it had just fallen a distance of 0.900 m.

12.79. **IDENTIFY:** As you constrict the hose, you decrease its area, but the equation of continuity applies to the water.
SET UP: $A_1v_1 = A_2v_2$. The distance traveled by a projectile that is fired from a height h with an initial horizontal velocity v is $x = vt$ where $t = \sqrt{\dfrac{2h}{g}}$.
EXECUTE: Since h is fixed, t does not change as we constrict the nozzle. Looking at the ratio of distances we obtain $\dfrac{x_1}{x_2} = \dfrac{v_1t}{v_2t} = \dfrac{A_2}{A_1} = \dfrac{\pi r_2^2}{\pi r_1^2}$, which gives

$$x_1 = x_2\left(\frac{r_2}{r_1}\right)^2 = (0.950\text{ m})\left(\frac{1.80\text{ cm}}{0.750\text{ cm}}\right)^2 = 5.47\text{ m}.$$

EVALUATE: A smaller constriction results in a higher exit velocity, which results in a greater range, so our result is plausible.

12.80. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.
SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40\times10^{-4}\text{ m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.

Let points 1 and 2 be chosen as in Figure 12.80.

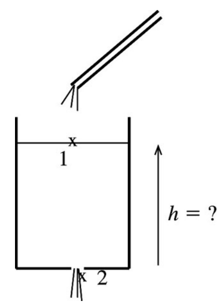


Figure 12.80

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2A_2 = 2.40\times10^{-4}\text{ m}^3/\text{s}$ and

$$v_2 = \frac{2.40\times10^{-4}\text{ m}^3/\text{s}}{1.50\times10^{-4}\text{ m}^2} = 1.60\text{ m/s}$$

$A_1 \gg A_2$ and $v_1A_1 = v_2A_2$ says that $\tfrac{1}{2}\rho v_1^2 \ll \tfrac{1}{2}\rho v_2^2$; neglect the $\tfrac{1}{2}\rho v_1^2$ term.
Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.
$p_1 = p_2 = p_a$ (air pressure)
Then $p_a + \rho g h = p_a + \tfrac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60\text{ m/s})^2/2(9.80\text{ m/s}^2) = 0.131\text{ m} = 13.1\text{ cm}$
EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

12.81. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.
SET UP: The speed of efflux is $\sqrt{2gh}$, where h is the distance of the hole below the surface of the fluid.

EXECUTE: **(a)** $v_3A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80\text{ m/s}^2)(8.00\text{ m})}(0.0160\text{ m}^2) = 0.200\text{ m}^3/\text{s}$.

(b) Since p_3 is atmospheric pressure, the gauge pressure at point 2 is

$p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3)$, using the expression for v_3 found above.

Substitution of numerical values gives $p_2 = 6.97\times10^4$ Pa.

EVALUATE: We could also calculate p_2 by applying Bernoulli's equation to points 1 and 2.

12.82 **IDENTIFY:** Apply Bernoulli's equation to the air in the hurricane.
SET UP: For a particle a distance *r* from the axis, the angular momentum is $L = mvr$.
EXECUTE: **(a)** Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200\text{ km/h})\left(\frac{30}{350}\right) = 17\text{ km/h}$.

(b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2\text{ kg/m}^3)((200\text{ km/h})^2 - (17\text{ km/h})^2)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 = 1.8\times10^3\text{ Pa}.$$

(c) $\frac{v^2}{2g} = 160\text{ m}$.

(d) The pressure difference at higher altitudes is even greater.
EVALUATE: According to Bernoulli's equation, the pressure decreases when the fluid velocity increases.

12.83. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.
SET UP: The speed of efflux at point *D* is $\sqrt{2gh_1}$.

EXECUTE: Applying the equation of continuity to points at *C* and *D* gives that the fluid speed is $\sqrt{8gh_1}$ at *C*. Applying Bernoulli's equation to points *A* and *C* gives that the gauge pressure at *C* is $\rho gh_1 - 4\rho gh_1 = -3\rho gh_1$, and this is the gauge pressure at the surface of the fluid at *E*. The height of the fluid in the column is $h_2 = 3h_1$.

EVALUATE: The gauge pressure at *C* is less than the gauge pressure ρgh_1 at the bottom of tank *A* because of the speed of the fluid at *C*.

12.84. **(a) IDENTIFY:** Apply constant acceleration equations to the falling liquid to find its speed as a function of the distance below the outlet. Then apply $v_1A_1 = v_2A_2$ to relate the speed to the radius of the stream.
SET UP:

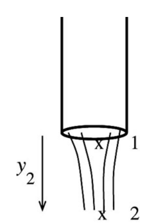


Figure 12.84

Let point 1 be at the end of the pipe and let point 2 be in the stream of liquid at a distance y_2 below the end of the tube, as shown in Figure 12.84.

Consider the free fall of the liquid. Take $+y$ to be downward.

Free fall implies $a_y = g$. v_y is positive, so replace it by the speed *v*.

EXECUTE: $v_2^2 = v_1^2 + 2a(y - y_0)$ gives $v_2^2 = v_1^2 + 2gy_2$ and $v_2 = \sqrt{v_1^2 + 2gy_2}$.

Equation of continuity says $v_1 A_1 = v_2 A_2$

And since $A = \pi r^2$ this becomes $v_1 \pi r_1^2 = v_2 \pi r_2^2$ and $v_2 = v_1 (r_1/r_2)^2$.

Use this in the above to eliminate v_2: $v_1(r_1^2/r_2^2) = \sqrt{v_1^2 + 2gy_2}$

$r_2 = r_1\sqrt{v_1}/(v_1^2 + 2gy_2)^{1/4}$

To correspond to the notation in the problem, let $v_1 = v_0$ and $r_1 = r_0$, since point 1 is where the liquid first leaves the pipe, and let r_2 be *r* and y_2 be *y*. The equation we have derived then becomes

$r = r_0\sqrt{v_0}/(v_0^2 + 2gy)^{1/4}$.

(b) $v_0 = 1.20$ m/s

We want the value of *y* that gives $r = \frac{1}{2} r_0$, or $r_0 = 2r$.

The result obtained in part (a) says $r^4(v_0^2 + 2gy) = r_0^4 v_0^2$.

Solving for *y* gives $y = \dfrac{[(r_0/r)^4 - 1]v_0^2}{2g} = \dfrac{(16-1)(1.20\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.10\text{ m}.$

EVALUATE: The equation derived in part (a) says that *r* decreases with distance below the end of the pipe.

12.85. **IDENTIFY** and **SET UP:** We are given the densities of elements in the table and look up their atomic masses in Appendix D.

EXECUTE: For example, for aluminum, the density is 2.7 g/cm^3 and the atomic mass is 26.98 g/mol.

(a) Figure 12.85 shows the graph of density versus atomic mass.

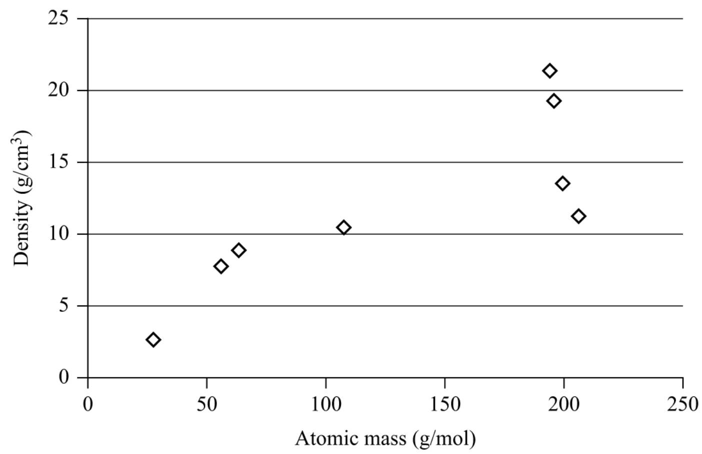


Figure 12.85

(b) From the graph, we see that there is no obvious mathematical relation between the two variables. No straight line or simple curve can be fitted to the data points.

(c) Density depends not only on atomic mass, but also on how tightly atoms are packed together. This packing is determined by the electrical interactions between atoms.

EVALUATE: Not all data can be reduced to straight-line graphs!

12.86. **IDENTIFY:** Archimedes's principle applies. For a floating object, the buoyant force balances the weight.

SET UP: Call m_b the mass of the block and m_n the mass of *n* quarters, so $m_n = nM$, where $M = 5.670$ g is the mass of a single quarter. $m_b + m_n$. The submerged volume is $(L - h)L^2$, where $L = 8.0$ cm.

EXECUTE: The forces balance, so $(m_b + m_n)g = \rho_{\text{liq}} L^2(L - h)g$. Solving for *h* gives

$h = \dfrac{\rho_{\text{liq}} L^3 - m_b}{\rho_{\text{liq}} L^2} - \left(\dfrac{M}{\rho_{\text{liq}} L^2}\right) n$. If we plot h versus n, the slope is $-\dfrac{M}{\rho_{\text{liq}} L^2}$ and the y-intercept is $\dfrac{\rho_{\text{liq}} L^3 - m_b}{\rho_{\text{liq}} L^2}$. For the graph in the problem, we can calculate the slope by reading points from the graph. The top and bottom points seem easiest to read, so they give a slope of (1.2 cm – 3.0 cm)/25 = 0.072 cm. (Due to uncertainties in reading the graph, your answers may differ slightly from the ones here.) Using this slope, we get ρ_{liq} = –(5.670 g)/[(–0.072 cm)(8.0 cm)^2] = 1.24 g/cm^3, which rounds to 1.2 g/cm^3 = 1200 kg/m^3.

(b) Simplifying the y-intercept gives y-intercept = $L - m_b/[(\rho_{\text{liq}})(L^2)]$. From the graph, the y-intercept is 3.0 cm, so we have 3.0 cm = 8.0 cm – m_b/[(1.24 g/cm^3)(8.0 cm)^2], which gives m_b = 400 g = 0.40 kg.

EVALUATE: The unknown liquid is 20% denser than water.

12.87. IDENTIFY: Bernoulli's equation applies. We have free-fall projectile motion after the liquid leaves the tank. The pressure at the hole where the liquid exits is atmospheric pressure p_0. The absolute pressure at the top of the liquid is $p_g + p_0$.

SET UP: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$.

EXECUTE: **(a)** The graph of R^2 versus p_g is shown in Figure 12.87.

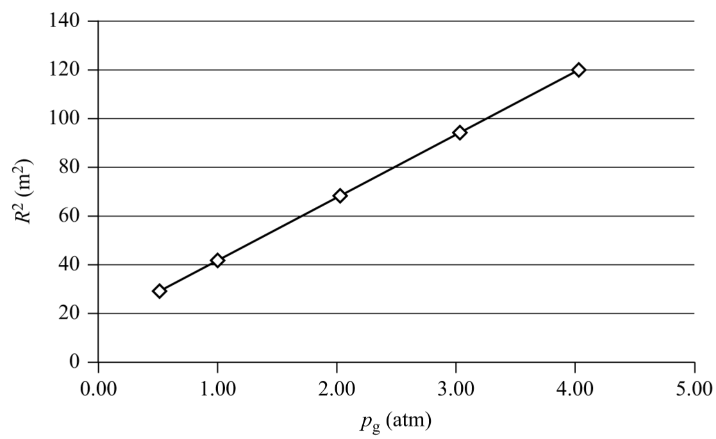


Figure 12.87

Applying Bernoulli's equation between the top and bottom of the liquid in the tank gives $p_0 + p_g + \rho g h = \frac{1}{2}\rho v^2 + p_0$, which simplifies to $p_g + \rho g h = \frac{1}{2}\rho v^2$.

The free-fall motion after leaving the tank gives $vt = R$ and $y = \frac{1}{2} g t^2$, where y = 50.0 cm. Eliminating t between these two equations gives $v^2 = (\rho g/4y)R^2$. Putting this into the result from Bernoulli's equation gives $p_g + \rho g h = (\rho g/4y)R^2$. Solving for R^2 in terms of h gives $R^2 = (4y/\rho g)\ p_g + 4yh$. This is the equation of a straight line of slope $4y/\rho g$, which gives $\rho = 4y/[g(\text{slope})]$ and y-intercept $4yh$. The best-fit equation is R^2 = (25.679 m^2/atm) p_g + 16.385 m^2. The y-intercept gives us h: $4yh$ = y-intercept, so h = (y-intercept)/(4y) = (16.385 m^2)/[4(0.500 m)] = 8.2 m. And the density is ρ = 4y/[g(slope)] = 4(0.500 m)/[(9.80 m/s^2)(25.679 m^2/atm)(1 atm/1.01 × 10^5 Pa)] = 803 kg/m^3.

EVALUATE: The liquid is about 80% as dense as water, and h = 8.2 m which is about 25 ft, so this is a rather large tank.

12.88. **IDENTIFY:** Apply Bernoulli's equation to the fluid in the siphon.

SET UP: The efflux speed from a small hole a distance h below the surface of fluid in a large open tank is $\sqrt{2gh}$.

EXECUTE: **(a)** The fact that the water first moves upward before leaving the siphon does not change the efflux speed, $\sqrt{2gh}$.

(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H+h)$, where the assumption that the cross-sectional area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p=0$ and solving for H gives $H=(p_a/\rho g)-h$.

EVALUATE: The analysis shows that $H+h<\dfrac{p_a}{\rho g}$, so there is also a limitation on $H+h$. For water and normal atmospheric pressure, $\dfrac{p_a}{\rho g}=10.3\text{ m}$.

12.89. **IDENTIFY** and **SET UP:** One atmosphere of pressure is 760 mm Hg. The gauge pressure is $p_g=\rho gh$.

EXECUTE: Since 1 atm is 760 mm Hg, the pressure is (150 mm/750 mm)P_{atm}. Solving for the depth h gives $h=\dfrac{P}{\rho g}=\dfrac{\left(\dfrac{150\text{ mm}}{760\text{ mm}}\right)(1.01\times10^5\text{ Pa})}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)}=2.0\text{ m}$, which is choice (b).

EVALUATE: This result is reasonable since an elephant can have its chest several meters under water.

12.90. **IDENTIFY** and **SET UP:** Use the definition of pressure. $\Delta p = F/A$.

EXECUTE: $\Delta p = F/A$ = (24,000 N)/[π(0.60 m)^2] = 21,200 Pa. Converting to mm Hg gives 21,200 Pa $[(760\text{ mm Hg})/(1.01\times10^5\text{ Pa})]=160\text{ mm Hg}$, which is choice (a).

EVALUATE: The force (24,000 N) is large, but the pressure cannot be too large since the area of the diaphragm is quite large.

12.91. **IDENTIFY** and **SET UP:** The gauge pressure increases with depth, and since the force is proportional to the pressure, so does the force. $p_g=\rho gh$ and $p = F/A$.

EXECUTE: The force is $F = pA = (\rho gh)A$, which tells us that the force increases linearly with distance, as in choice (a).

EVALUATE: The diaphragm experiences greater force as the elephant goes into deeper water.

12.92. **IDENTIFY** and **SET UP:** The gauge pressure depends on the density of the liquid, $p_g=\rho gh$.

EXECUTE: Since $p_g=\rho gh$, a denser liquid will exert a greater pressure. Since salt water is denser than fresh water, the gauge pressure at a given depth will be greater in salt water than in freshwater. Therefore the maximum depth in salt water would be less than in freshwater, which is choice (b).

EVALUATE: Although the pressure in salt water would be greater than in freshwater, it would not be much greater since the density of seawater (1030 kg/m^3) is only slightly greater than that of freshwater (1000 kg/m^3).