44

PARTICLE PHYSICS AND COSMOLOGY

VP44.1.1. **IDENTIFY:** This problem is about a cyclotron accelerating protons.

SET UP and EXECUTE: **(a)** We want the frequency. Use Eq. (44.7).

$$f = \frac{eB}{2\pi m} = \frac{e(0.600\text{ T})}{2\pi m_p} = 9.15\text{ MHz}.$$

(b) We want the maximum kinetic energy of the proton. Use Eq. (44.8) with $B = 0.600$ T and $R = 0.800$ m.

$$K_{max} = \frac{e^2B^2R^2}{2m_p} = 11.0\text{ MeV}.$$

(c) We want the proton's speed. The rest energy of the proton is 938 MeV which is much greater than the kinetic energy of 11 MeV, so we do not have to use the relativistic equation. Solving $K = \frac{1}{2}mv^2$ for v gives

$$v = \sqrt{2K/m_p} = 4.60\times10^7\text{ m/s} = 0.153c.$$

EVALUATE: Since $v \approx 15\%$ of c, it is reasonable not to use relativity. If we had used relativity, our result would be

$$K = mc^2(\gamma - 1) \rightarrow \gamma = 1.012$$

$$v = c\sqrt{1 - 1/\gamma^2} = 0.152c.$$

The percent difference between the two answers is 0.001/0.152 = 0.66%, which is extremely small.

VP44.1.2. **IDENTIFY:** This problem is about a cyclotron accelerating hydrogen ions (protons).

SET UP and EXECUTE: **(a)** We want the radius R. $R = m_pv/eB$. Using $B = 0.460$ T and the given speed, we get $R = 0.136$ m.

(b) We want the angular frequency. Use Eq. (44.7) with $q = e$ and $B = 0.460$ T, which gives $\omega = eB/m$ $= 4.41 \times 10^7$ rad/s.

EVALUATE: This angular frequency is rather small compared to many modern cyclotrons.

VP44.1.3. **IDENTIFY:** This problem deals with the operation of a cyclotron.

SET UP and EXECUTE: **(a)** We want the magnetic field. The rest energy of the proton is 938 MeV and its kinetic energy here is 80.0 keV, which is much less than its rest energy. Therefore we do not need to use special relativity. We use Eq. (44.8), with R = (11.4 cm)/2 and K_{max} = 80.0 keV, and solve for B, which gives

$$B = \frac{\sqrt{2m_pK_{max}}}{eR} = 0.717\text{ T}.$$

(b) We want the frequency. Use Eq. (44.7) with $B = 0.717$ T and $q = e$.

$$f = \frac{eB}{2\pi m} = \frac{e(0.717\text{ T})}{2\pi m_p} = 10.9\text{ MHz}.$$

EVALUATE: Modern cyclotrons produce much greater kinetic energy, but after all, this was the first one.

VP44.1.4. **IDENTIFY:** This problem deals with the operation of a cyclotron.
SET UP and **EXECUTE:** **(a)** We want the magnetic field and know that the frequency is 5.80 MHz. Use Eq. (44.7) and solve for B using $q = 2e$ and the given mass of an alpha particle.

$$B = \frac{2\pi m_\alpha f}{2e} = 0.756 \text{ T}.$$

(b) We want the kinetic energy when $R = 0.650$ m. Using Eq. (44.8) with $B = 0.756$ T gives

$$K_{\text{max}} = \frac{(2e)^2 B^2 R^2}{2m_\alpha} = 11.6 \text{ MeV}.$$

EVALUATE: A magnetic field of 0.756 T is certainly physically reasonable in a physics laboratory.

VP44.3.1. **IDENTIFY:** This problem deals with the available energy when particles collide.
SET UP and **EXECUTE:** We follow the procedure of Example 44.2.
(a) We want the available energy E_a. The total available energy must be the total rest energy in the center-of-momentum frame.

$$E_a = 2m_p c^2 + m_{n'} c^2 = 2(938 \text{ MeV}) + 958 \text{ MeV} = 2834 \text{ MeV}.$$

(b) We want the kinetic energy K of the incoming proton. First use Eq. (44.10) to find E_m (the total energy of the incoming proton). Then use $E_m = K + m_p c^2$ to find K.

$$E_m = \frac{E_a^2}{2m_p c^2} - m_p c^2 = K + m_p c^2$$

$$K = \frac{E_a^2}{2m_p c^2} - 2m_p c^2$$

Using $E_a = 2834$ MeV and $m_p c^2 = 938$ MeV gives us $K = 2410$ MeV = 2.41 GeV.
EVALUATE: Colliding beams would require far less kinetic energy because nearly all the incoming kinetic energy is available energy.

VP44.3.2. **IDENTIFY:** This problem deals with the available energy during collisions.
SET UP and **EXECUTE:** **(a)** We want E_a. The minimum available energy is the rest energy of the particles after the collision. $E_a = m_p c^2 + m_\Delta c^2 = 938 \text{ MeV} + 1232 \text{ MeV} = 2170 \text{ MeV} = 2.17 \text{ GeV}$.
(b) We want the kinetic energy. Use the same method as in problem VP44.3.1.

$$K = \frac{E_a^2}{2m_p c^2} - 2m_p c^2$$

Using $E_a = 2170$ MeV and $m_p c^2 = 938$ MeV gives $K = 634$ MeV.
EVALUATE: The incoming proton could have more than 634 MeV of kinetic energy. In that case, the products would each have kinetic energy.

VP44.3.3. **IDENTIFY:** This problem deals with the available energy during collisions.
SET UP and **EXECUTE:** **(a)** We want E_a. The minimum available energy is the rest energy of the product (the Δ^0), which is 1232 MeV.
(b) We want the minimum kinetic energy K of the pion. The target and incident particle have different masses, so use Eq. (44.9) to find the available energy. Then use this to find E_m, and finally use $E_m = K + mc^2$ to find K.
The quantities in Eq. (44.9) are:

$$M = \text{target} = \text{proton, so } Mc^2 = 938 \text{ MeV}$$
$$m = \text{incident particle} = \pi^-, \text{ so } mc^2 = 140 \text{ MeV}$$

Using the result of part (a), Eq. (44.9) gives

$$(1232 \text{ MeV})^2 = 2(938 \text{ MeV})E_m + (938 \text{ MeV})^2 + (140 \text{ MeV})^2$$

$$E_m = 329.6 \text{ MeV}.$$

Now use $E_m = K + mc^2$ to find K.

$$329.6 \text{ MeV} = K + 140 \text{ MeV, which gives } K = 190 \text{ MeV}.$$

EVALUATE: Notice that the kinetic energy is a significant fraction of E_m.

VP44.3.4. **IDENTIFY:** This problem deals with the available energy during collisions.

SET UP and **EXECUTE:** **(a)** We want E_a. E_m = 7000 GeV and $m_p c^2$ = 938 MeV, so $E_m >> m_p c^2$. Therefore we use Eq. (44.11). Using the given rest energies gives

$$E_a = \sqrt{2\,mc^2 E_m} = \sqrt{2(938 \text{ MeV})(7000 \text{ GeV})} = 115 \text{ GeV}.$$

(b) We want the minimum mass of particle X. E_a is at *least* equal to the rest energy of the products of the collision, which are 2 protons and particle X. So 115 GeV = 2(938 MeV) + $m_X c^2$. This gives m_X = 113 GeV/c^2.

EVALUATE: Particle X is 120 times heavier than the proton.

VP44.9.1. **IDENTIFY:** This problem concerns the red shift.

SET UP and **EXECUTE:** **(a)** The target variable is z.

$$z = \frac{\Delta\lambda}{\lambda} = \frac{725 \text{ nm} - 656.3 \text{ nm}}{656.3 \text{ nm}} = 0.105.$$

(b) We want the speed. Use Eq. (44.14) with the given wavelengths.

$$v = \frac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}c = \frac{(725/656.3)^2 - 1}{(725/656.3)^2 + 1}c = 0.0992c.$$

EVALUATE: The fact that we have a red shift instead of a blue shift tells us that the galaxy is receding from us at 9.92% the speed of light.

VP44.9.2. **IDENTIFY:** This problem is about the red shift of a galaxy.

SET UP: Eq. (44.13) applies.

EXECUTE: **(a)** We want wavelength that we observe. Use Eq. (44.13).

$$\lambda_0 = \lambda_s\sqrt{\frac{c+v}{c-v}} = (396.9 \text{ nm})\sqrt{\frac{c+0.0711c}{c-0.0711c}} = 426 \text{ nm}.$$

(b) We want z.

$$z = \frac{\Delta\lambda}{\lambda} = \frac{426 \text{ nm} - 396.9 \text{ nm}}{396.9 \text{ nm}} = 0.0738.$$

EVALUATE: Since v is only about 7% the speed of light, the red shift is not very large.

VP44.9.3. **IDENTIFY:** This problem requires the use of Hubble's law.

SET UP: Hubble's law: $v = H_0 r$, where H_0 = (68 km/s)/Mpc. The target variable is the distance to the galaxy.

EXECUTE: **(a)** $v = 0.0992c$. Apply Hubble's law using the measured speed.

$$r = \frac{v}{H_0} = \frac{0.0992c}{68\,\frac{\text{km/s}}{\text{Mpc}}} = 440 \text{ Mpc} = 4.4\times10^8 \text{ pc}.$$

(b) $v = 0.0711c$. Use the same approach as in part (a).

$$r = \frac{v}{H_0} = \frac{0.0711c}{68\,\frac{\text{km/s}}{\text{Mpc}}} = 310 \text{ Mpc} = 3.1\times10^8 \text{ pc}.$$

EVALUATE: Speeds can be determined quite easily using spectral analysis, but H_0 is not so easy to measure. These distances are only as accurate as H_0.

VP44.9.4. **IDENTIFY:** This problem requires the use of Hubble's law and the red shift.

SET UP: Hubble's law: $v = H_0 r$, where H_0 = (68 km/s)/Mpc.

EXECUTE: **(a)** We want the speed of recession of the galaxy. Use Eq. (44.14).

$$v = \frac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}c = \frac{(666/615)^2 - 1}{(666/615)^2 + 1}c = 0.0795c = 2.38\times10^7 \text{ m/s}.$$

(b) We want the distance to the galaxy. Use Hubble's law.

$$r = \frac{v}{H_0} = \frac{2.38\times10^7 \text{ m/s}}{68\,\dfrac{\text{km/s}}{\text{Mpc}}} = 350 \text{ Mpc} = 3.5\times10^8 \text{ pc}.$$

EVALUATE: Using 1 pc = 3.26 ly, we find that this galaxy is 1.1 billion light-years from us!

44.1. **IDENTIFY:** The antimatter annihilates with an equal amount of matter.

SET UP: The energy of the matter is $E = (\Delta m)c^2$.

EXECUTE: Putting in the numbers gives

$$E = (\Delta m)c^2 = (400 \text{ kg} + 400 \text{ kg})(3.00\times10^8 \text{ m/s})^2 = 7.2\times10^{19} \text{ J}.$$

This is about 70% of the annual energy use in the U.S.

EVALUATE: If this huge amount of energy were released suddenly, it would blow up the *Enterprise*! Getting useable energy from matter-antimatter annihilation is not so easy to do!

44.2. **IDENTIFY:** The energy (rest mass plus kinetic) of the muons is equal to the energy of the photons.

SET UP: $\gamma + \gamma \to \mu^+ + \mu^-$, $E = hc/\lambda$. $K = (\gamma - 1)mc^2$.

EXECUTE: **(a)** $\gamma + \gamma \to \mu^+ + \mu^-$. Each photon must have energy equal to the rest mass energy of a μ^+ or a μ^-: $\dfrac{hc}{\lambda} = 105.7\times10^6$ eV. $\lambda = \dfrac{(4.136\times10^{-15} \text{ eV}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{105.7\times10^6 \text{ eV}} = 1.17\times10^{-14} \text{ m} = 0.0117 \text{ pm}.$

Conservation of linear momentum requires that the μ^+ and μ^- move in opposite directions with equal speeds.

(b) $\lambda = \dfrac{0.0117 \text{ pm}}{2}$, so each photon has energy $2(105.7 \text{ MeV}) = 211.4 \text{ MeV}$. The energy released in the reaction is $2(211.4 \text{ MeV}) - 2(105.7 \text{ MeV}) = 211.4 \text{ MeV}$. The kinetic energy of each muon is half this, 105.7 MeV. Using $K = (\gamma - 1)mc^2$ gives $\gamma - 1 = \dfrac{K}{mc^2} = \dfrac{105.7 \text{ MeV}}{105.7 \text{ MeV}} = 1$. $\gamma = 2$. $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2}. \quad v = \sqrt{\frac{3}{4}}c = 0.866c = 2.60\times10^8 \text{ m/s}.$$

EVALUATE: The muon speeds are a substantial fraction of the speed of light, so special relativity must be used.

44.3. **IDENTIFY:** The energy released is the energy equivalent of the mass decrease that occurs in the decay.

SET UP: The mass of the pion is $m_{\pi^+} = 270m_e$ and the mass of the muon is $m_{\mu^+} = 207m_e$. The rest energy of an electron is 0.511 MeV.

EXECUTE: **(a)** $\Delta m = m_{\pi+} - m_{\mu+} = 270m_e - 207m_e = 63m_e \Rightarrow E = 63(0.511 \text{ MeV}) = 32 \text{ MeV}.$

EVALUATE: **(b)** A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

44.4. **IDENTIFY:** In the annihilation the total energy of the proton and antiproton is converted to the energy of the two photons.

SET UP: The rest energy of a proton or antiproton is 938.3 MeV. Conservation of linear momentum requires that the two photons have equal energies. The energy of a photon is $E = hf$, and $f\lambda = c$.

EXECUTE: **(a)** The energy will be the proton rest energy, 938.3 MeV, so $hf = 938.3$ MeV. Solving for f gives $f = (938.3\times10^{6}\ \text{eV})/(4.136\times10^{-15}\ \text{eV}\cdot\text{s}) = 2.27\times10^{23}$ Hz. The wavelength is

$$\lambda = c/f = 1.32\times10^{-15}\ \text{m} = 1.32\ \text{fm}.$$

(b) The energy of each photon will be $938.3\ \text{MeV} + 620\ \text{MeV} = 1558\ \text{MeV}$, so $f = (1558\ \text{MeV})/h$ $= 3.77\times10^{23}$ Hz. $\lambda = c/f = 7.96\times10^{-16}\ \text{m} = 0.796$ fm.

EVALUATE: When the initial kinetic energy of the proton and antiproton increases, the wavelength of the photons decreases.

44.5. **IDENTIFY:** The kinetic energy of the alpha particle is due to the mass decrease.

SET UP and **EXECUTE:** ${}^{1}_{0}\text{n} + {}^{10}_{5}\text{B} \to {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$. The mass decrease in the reaction is

$m({}^{1}_{0}\text{n}) + m({}^{10}_{5}\text{B}) - m({}^{7}_{3}\text{Li}) - m({}^{4}_{2}\text{He}) = 1.008665\ \text{u} + 10.012937\ \text{u} - 7.016005\ \text{u} - 4.002603\ \text{u} = 0.002994\ \text{u}$ and the energy released is $E = (0.002994\ \text{u})(931.5\ \text{MeV/u}) = 2.79\ \text{MeV}$. Assuming the initial momentum is zero, $m_{\text{Li}}v_{\text{Li}} = m_{\text{He}}v_{\text{He}}$ and $v_{\text{Li}} = \dfrac{m_{\text{He}}}{m_{\text{Li}}}v_{\text{He}}$. $\frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 = E$ becomes

$$\frac{1}{2}m_{\text{Li}}\left(\frac{m_{\text{He}}}{m_{\text{Li}}}\right)^2 v_{\text{He}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 = E \text{ and } v_{\text{He}} = \sqrt{\frac{2E}{m_{\text{He}}}\left(\frac{m_{\text{Li}}}{m_{\text{Li}} + m_{\text{He}}}\right)}.\ \ E = 4.470\times10^{-13}\ \text{J}.$$

$$m_{\text{He}} = 4.002603\ \text{u} - 2(0.0005486\ \text{u}) = 4.0015\ \text{u} = 6.645\times10^{-27}\ \text{kg}.$$

$$m_{\text{Li}} = 7.016005\ \text{u} - 3(0.0005486\ \text{u}) = 7.0144\ \text{u}. \text{ This gives } v_{\text{He}} = 9.26\times10^{6}\ \text{m/s}.$$

EVALUATE: The speed of the alpha particle is considerably less than the speed of light, so it is not necessary to use the more complicated relativistic formulas.

44.6. **IDENTIFY:** The range is limited by the lifetime of the particle, which itself is limited by the uncertainty principle.

SET UP: $\Delta E\Delta t = \hbar/2$.

EXECUTE: $\Delta t = \dfrac{\hbar}{2\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s}/2\pi)}{2(783\times10^{6}\ \text{eV})} = 4.20\times10^{-25}$ s. The range of the force is

$$c\Delta t = (2.998\times10^{8}\ \text{m/s})(4.20\times10^{-25}\ \text{s}) = 1.26\times10^{-16}\ \text{m} = 0.126\ \text{fm}.$$

EVALUATE: This range is less than the diameter of an atomic nucleus.

44.7. **IDENTIFY:** This problem is about the available energy during a collision of equal-mass particles.

SET UP: We are comparing the available kinetic energy if the target is stationary or if we have colliding beams.

EXECUTE: **(a)** Stationary target. Use Eq. 44.10 and solve for K.

$$E_{\text{a}}^2 = 2mc^2(mc^2 + K)$$

$$K = \frac{E_{\text{a}}^2}{2mc^2} - mc^2 = \frac{(2\ \text{TeV})^2}{2(938\ \text{MeV})} - 938\ \text{MeV} = 2130\ \text{TeV}.$$

(b) Colliding beams. The available energy is the kinetic energy minus the rest energy of the two protons.

$$E_{\text{a}} = 2K - 2m_{\text{p}}c^2$$

$$K = \frac{E_{\text{a}} + 2m_{\text{p}}c^2}{2} = \frac{2.00\ \text{TeV} + 2(938\ \text{MeV})}{2} = 1.00\ \text{TeV}.$$

EVALUATE: By using colliding beams we need less than 1/1000 the kinetic energy than with a stationary target.

44.8. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be leftover kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2(E_m + mc^2)$ for two particles of equal mass when one is initially stationary. In this case, the initial kinetic energy $(30.0\text{ GeV} = 30{,}000\text{ MeV})$ is much more than the rest energy of the electron (0.511 MeV), so the formula for available energy reduces to

$$E_a = \sqrt{2mc^2 E_m}.$$

EXECUTE: **(a)** Using the formula for available energy gives

$$E_a = \sqrt{2mc^2 E_m} = \sqrt{2(0.511\text{ MeV})(30.0\text{ GeV})} = 175\text{ MeV}.$$

(b) For colliding beams of equal mass, each particle has half the available energy, so each has 87.5 MeV. The *total* energy is twice this, or 175 MeV.

EVALUATE: Colliding beams provide considerably more available energy to do experiments than do beams hitting a stationary target. With a stationary electron target in part (a), we had to give the moving electron 30,000 MeV of energy to get the same available energy that we got with only 175 MeV of energy with the colliding beams.

44.9. **IDENTIFY** and **SET UP:** The angular frequency is $\omega = |q|B/m$ so $B = m\omega/|q|$. And since $\omega = 2\pi f$, this becomes $B = 2\pi mf/|q|$.

EXECUTE: **(a)** A deuteron is a deuterium nucleus $({}_1^2\text{H})$. Its charge is $q = +e$. Its mass is the mass of the neutral ${}_1^2\text{H}$ atom (Table 43.2) minus the mass of the one atomic electron:

$$m = 2.014102\text{ u} - 0.0005486\text{ u} = 2.013553\text{ u}(1.66054\times10^{-27}\text{ kg/1 u}) = 3.344\times10^{-27}\text{ kg}.$$

$$B = \frac{2\pi mf}{|q|} = \frac{2\pi(3.344\times10^{-27}\text{ kg})(9.00\times10^{6}\text{ Hz})}{1.602\times10^{-19}\text{ C}} = 1.18\text{ T}.$$

(b) Eq. (44.8): $K = \dfrac{q^2B^2R^2}{2m} = \dfrac{[(1.602\times10^{-19}\text{ C})(1.18\text{ T})(0.320\text{ m})]^2}{2(3.344\times10^{-27}\text{ kg})}.$

$$K = 5.471\times10^{-13}\text{ J} = (5.471\times10^{-13}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 3.42\text{ MeV}.$$

$$K = \tfrac{1}{2}mv^2,\text{ so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.471\times10^{-13}\text{ J})}{3.344\times10^{-27}\text{ kg}}} = 1.81\times10^{7}\text{ m/s}.$$

EVALUATE: $v/c = 0.06$, so it is ok to use the nonrelativistic expression for kinetic energy.

44.10. **IDENTIFY:** We are dealing with the magnetic force on a positron.

SET UP and **EXECUTE:** We want the momentum of the positron. Solve $R = mv/|q|B$ for p, which gives $p = ReB = (0.15\text{ m})e(0.40\text{ T}) = 9.60\times10^{-21}\text{ kg}\cdot\text{m/s}$.

EVALUATE: To check if this positron is relativistic, use $p = mv$ to find its speed, which gives $v = p/m$. Using the momentum we just found gives $v = 1.05\times10^{10}$ m/s which is 300 times the speed of light! So we would need to use relativistic momentum to find the speed.

44.11. **IDENTIFY** and **SET UP:** The masses of the target and projectile particles are equal, so we can use the equation $E_a^2 = 2mc^2(E_m + mc^2)$. E_a is specified; solve for the energy E_m of the beam particles.

EXECUTE: **(a)** Solve for E_m: $E_m = \dfrac{E_a^2}{2mc^2} - mc^2.$

The mass for the alpha particle can be calculated by subtracting two electron masses from the ${}_2^4\text{He}$ atomic mass:

$$m = m_\alpha = 4.002603\text{ u} - 2(0.0005486\text{ u}) = 4.001506\text{ u}.$$

Then $mc^2 = (4.001506\text{ u})(931.5\text{ MeV/u}) = 3.727\text{ GeV}.$

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(16.0\text{ GeV})^2}{2(3.727\text{ GeV})} - 3.727\text{ GeV} = 30.6\text{ GeV}.$$

(b) Each beam must have $\frac{1}{2}E_a = 8.0\text{ GeV}$.

EVALUATE: For a stationary target the beam energy is nearly twice the available energy. In a colliding beam experiment all the energy is available and each beam needs to have just half the required available energy.

44.12. **IDENTIFY:** We are dealing with a cyclotron that is accelerating protons.
SET UP: The magnetic field is 1.3 T and the radius of the proton path is 11 cm.
EXECUTE: **(a)** We want the maximum kinetic energy. Use Eq. (44.8) with the known quantities.

$$K_{\text{max}} = \frac{q^2B^2R^2}{2m} = \frac{e^2(1.3\text{ T})^2(0.11\text{ m})^2}{2(1.67\times10^{-27}\text{ kg})} = 0.98\text{ MeV}.$$

The proton rest energy is 938 MeV and K_{max} = 0.98 MeV, so it is accurate to use nonrelativistic expressions.
(b) We want the frequency. Using the known quantities gives $f = eB/2\pi m$ = 20 MHz.
EVALUATE: A kinetic energy of 0.98 MeV is *much* less than present-day accelerators can produce.

44.13. **IDENTIFY:** $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$. The relativistic formula for the angular frequency is

$$\omega = \frac{|q|B}{m\gamma}.$$

SET UP: A proton has rest energy $mc^2 = 938.3\text{ MeV}$.

EXECUTE: **(a)** $\gamma = \frac{E}{mc^2} = \frac{1000\times10^3\text{ MeV}}{938.3\text{ MeV}} = 1065.8$, so $v = 0.999999559c$.

(b) Nonrelativistic: $\omega = \frac{eB}{m} = 3.83\times10^8\text{ rad/s}$.

Relativistic: $\omega = \frac{eB}{m}\frac{1}{\gamma} = 3.59\times10^5\text{ rad/s}$.

EVALUATE: The relativistic expression gives a smaller value for ω.

44.14. **IDENTIFY** and **SET UP:** To create the η^0, the minimum available energy must be equal to the rest mass energy of the products, which in this case is the η^0 plus two protons. In a collider, all of the initial energy is available, so the beam energy is the available energy. Use the particle masses from Table 44.3.
EXECUTE: The minimum amount of available energy must be rest mass energy

$$E_a = 2m_p + m_\eta = 2(938.3\text{ MeV}) + 547.3\text{ MeV} = 2420\text{ MeV}.$$

Each incident proton has half of the rest mass energy, or $1210\text{ MeV} = 1.21\text{ GeV}$.

EVALUATE: We would need much more initial energy if one of the initial protons were stationary. The result here (1.21 GeV) is the *minimum* amount of energy needed; the original protons could have more energy and still trigger this reaction.

44.15. **IDENTIFY** and **SET UP:** For the reaction $\text{p} + \text{p} \rightarrow \text{p} + \text{p} + \text{p} + \overline{\text{p}}$, the two incident protons must have enough kinetic energy to produce a p and a $\overline{\text{p}}$, plus any kinetic energy of the products. If they have the minimum kinetic energy, the products are at rest. The proton and antiproton have equal masses. The available energy for two equal-mass particles is $E_a^2 = 2mc^2(E_m + mc^2)$, where $E_m = K + mc^2$.

EXECUTE: **(a)** In a head-on collision with equal speeds, the laboratory frame is the center-of-momentum frame. For the minimum kinetic energy of the incident protons, the products are all at rest. In that case, the incident protons need only enough kinetic energy to produce a proton and an antiproton. Since the incident protons have equal energy, each one must have kinetic energy equal to the rest energy of a proton, which is 938 MeV.

(b) In this case, the target proton is at rest. Since 4 particles are produced, each of mass m, the available energy E_a must be at least equal to $4mc^2$. Therefore $E_a^2 = 2mc^2(E_m + mc^2) = (4mc^2)^2 = 16m^2c^4$, which gives $E_m = 7mc^2$. Using $E_m = K + mc^2$, we get $K = 6mc^2 = 6(938\text{ MeV}) = 5630\text{ MeV}$.
EVALUATE: When the two protons collide head-on with equal speeds, they need only 938 MeV of kinetic energy each, for a total of 1879 MeV. But when the target is stationary, the kinetic energy needed is 5630 MeV, which is 3 times as much as for a head-on collision.

44.16. **IDENTIFY:** This problem deals with the decay of the omega-minus particle.
SET UP: Use masses (expressed in MeV) from Table 44.3.
EXECUTE: **(a)** We want the energy Q that is released in this decay. From Table 44.3 we have the following masses:
$\mathrm{M}_{\Omega^-} = 1672\text{ MeV}$
$\mathrm{M}_{\Xi^-} = 1322\text{ MeV}$
$\mathrm{M}_{\pi^0} = 135.0\text{ MeV}$
$Q = 1672\text{ MeV} – (1321\text{ MeV} + 135.0\text{ MeV}) = 215\text{ MeV}$.
(b) Use the values for B and S from Table 44.3. The initial B is +1 and the final B is +1, so $\Delta B = 0$. The initial S is –3 and the final S is –2, so $\Delta S = +1$.
EVALUATE: $\Delta B = 0$ agrees with conservation of baryon number, but $\Delta S = +1$ violates strangeness conservation, so this decay is not allowed for the strong interaction. The initial and final lepton numbers are both zero, so this decay is allowed through the weak interaction.

44.17. **IDENTIFY:** The kinetic energy comes from the mass decrease.
SET UP: Table 44.3 gives $m(\mathrm{K}^+) = 493.7\text{ MeV}/c^2$, $m(\pi^0) = 135.0\text{ MeV}/c^2$, and $m(\pi^\pm) = 139.6\text{ MeV}/c^2$.
EXECUTE: **(a)** Charge must be conserved, so $\mathrm{K}^+ \to \pi^0 + \pi^+$ is the only possible decay.
(b) The mass decrease is
$m(\mathrm{K}^+) - m(\pi^0) - m(\pi^+) = 493.7\text{ MeV}/c^2 - 135.0\text{ MeV}/c^2 - 139.6\text{ MeV}/c^2$
$= 219.1\text{ MeV}/c^2$. The energy released is 219.1 MeV.
EVALUATE: The π mesons do not share this energy equally since they do not have equal masses.

44.18. **IDENTIFY:** The energy is due to the mass difference.
SET UP: The energy released is the energy equivalent of the mass decrease. From Table 44.3, the μ^- has mass $105.7\text{ MeV}/c^2$ and the e^- has mass $0.511\text{ MeV}/c^2$.
EXECUTE: The mass decrease is $105.7\text{ MeV}/c^2 - 0.511\text{ MeV}/c^2 = 105.2\text{ MeV}/c^2$ and the energy equivalent is 105.2 MeV.
EVALUATE: The electron does not get all of this energy; the neutrinos also get some of it.

44.19. **IDENTIFY:** Table 44.1 gives the mass in units of GeV/c^2. This is the value of mc^2 for the particle.
SET UP: $m(\mathrm{Z}^0) = 91.2\text{ GeV}/c^2$.
EXECUTE: $E = 91.2\times10^9\text{ eV} = 1.461\times10^{-8}\text{ J}$; $m = E/c^2 = 1.63\times10^{-25}\text{ kg}$; $m(\mathrm{Z}^0)/m(\mathrm{p}) = 97.2$
EVALUATE: The rest energy of a proton is 938 MeV; the rest energy of the Z^0 is 97.2 times as great.

44.20. **IDENTIFY:** If the initial and final rest mass energies were equal, there would be no leftover energy for kinetic energy. Therefore the kinetic energy of the products is the difference between the mass energy of the initial particles and the final particles.
SET UP: The difference in mass is $\Delta m = M_{\Omega^-} - m_{\Lambda^0} - m_{\mathrm{K}^-}$.

EXECUTE: Using Table 44.3, the energy difference is
$E = (\Delta m)c^2 = 1672\text{ MeV} - 1116\text{ MeV} - 494\text{ MeV} = 62\text{ MeV}.$

EVALUATE: There is less rest mass energy after the reaction than before because 62 MeV of the initial energy was converted to kinetic energy of the products.

44.21. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.
EXECUTE: The mass decrease is $m(\Sigma^+) - m(\text{p}) - m(\pi^0)$ and the energy released is
$mc^2(\Sigma^+) - mc^2(\text{p}) - mc^2(\pi^0) = 1189\text{ MeV} - 938.3\text{ MeV} - 135.0\text{ MeV} = 116\text{ MeV}.$ (The mc^2 values for each particle were taken from Table 44.3.)
EVALUATE: The mass of the decay products is less than the mass of the original particle, so the decay is energetically allowed and energy is released.

44.22. **IDENTIFY** and **SET UP:** The p and n have baryon number $+1$ and the antiproton $\overline{\text{p}}$ has baryon number -1. $\text{e}^+, \text{e}^-, \overline{\nu}_e,$ and γ all have baryon number zero. Baryon number is conserved if the total baryon number of the products equals the total baryon number of the reactants.
EXECUTE: **(a)** reactants: $B = 1 + 1 = 2.$ Products: $B = 1 + 0 = 1.$ Not conserved.
(b) reactants: $B = 1 + 1 = 2.$ Products: $B = 0 + 0 = 0.$ Not conserved.
(c) reactants: $B = +1.$ Products: $B = 1 + 0 + 0 = +1.$ Conserved.
(d) reactants: $B = 1 - 1 = 0.$ Products: $B = 0.$ Conserved.
EVALUATE: Even though a reaction obeys conservation of baryon number it may still not occur spontaneously, if it is not energetically allowed or if other conservation laws are violated.

44.23. **IDENTIFY** and **SET UP:** The lepton numbers for the particles are given in Table 44.2.
EXECUTE: **(a)** $\mu^- \to \text{e}^- + \nu_\text{e} + \overline{\nu}_\mu \Rightarrow L_\mu: +1 \neq -1, L_\text{e}: 0 \neq +1 + 1,$ so lepton numbers are not conserved.

(b) $\tau^- \to \text{e}^- + \overline{\nu}_\text{e} + \nu_\tau \Rightarrow L_\text{e}: 0 = +1 - 1;\ L_\tau: +1 = +1,$ so lepton numbers are conserved.

(c) $\pi^+ \to \text{e}^+ + \gamma.$ Lepton numbers are not conserved since just one lepton is produced from zero original leptons.

(d) $\text{n} \to \text{p} + \text{e}^- + \overline{\nu}_\text{e} \Rightarrow L_\text{e}: 0 = +1 - 1,$ so the lepton numbers are conserved.

EVALUATE: The decays where lepton numbers are conserved are among those listed in Tables 44.2 and 44.3.

44.24. **IDENTIFY** and **SET UP:** Compare the sum of the strangeness quantum numbers for the particles on each side of the decay equation. The strangeness quantum numbers for each particle are given in Table 44.3.
EXECUTE: **(a)** $\text{K}^+ \to \mu^+ + \nu_\mu;\ S_{\text{K}^+} = +1, S_{\mu+} = 0, S_{\nu_\mu} = 0.$
$S = 1$ initially; $S = 0$ for the products; *S* is not conserved.
(b) $\text{n} + \text{K}^+ \to \text{p} + \pi^0;\ S_\text{n} = 0, S_{\text{K}^+} = +1, S_\text{p} = 0, S_{\pi^0} = 0.$
$S = 1$ initially; $S = 0$ for the products; *S* is not conserved.
(c) $\text{K}^+ + \text{K}^- \to \pi^0 + \pi^0;\ S_{\text{K}^+} = +1;\ S_{\text{K}^-} = -1;\ S_{\pi^0} = 0.$
$S = +1 - 1 = 0$ initially; $S = 0$ for the products; *S* is conserved.
(d) $\text{p} + \text{K}^- \to \Lambda^0 + \pi^0;\ S_\text{p} = 0, S_{\text{K}^-} = -1, S_{\Lambda^0} = -1, S_{\pi^0} = 0.$
$S = -1$ initially; $S = -1$ for the products; *S* is conserved.
EVALUATE: Strangeness is not a conserved quantity in weak interactions, and strangeness nonconserving reactions or decays can occur.

44.25. **IDENTIFY** and **SET UP:** Each value for the combination is the sum of the values for each quark. Use Table 44.4.
EXECUTE: **(a)** ***uds:***
$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$S = 0 + 0 - 1 = -1$

$C = 0 + 0 + 0 = 0$

(b) $\underline{c\bar{u}}$**:**

The values for $\bar{u}$ are the negative for those for u.

$Q = \frac{2}{3}e - \frac{2}{3}e = 0$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = +1 + 0 = +1$

(c) $\underline{ddd}$**:**

$Q = -\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$

$S = 0 + 0 + 0 = 0$

$C = 0 + 0 + 0 = 0$

(d) $\underline{d\bar{c}}$**:**

$Q = -\frac{1}{3}e - \frac{2}{3}e = -e$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = 0 - 1 = -1$

EVALUATE: The charge, baryon number, strangeness, and charm quantum numbers of a particle are determined by the particle's quark composition.

44.26. IDENTIFY: Quark combination produce various particles.

SET UP: The properties of the quarks are given in Table 44.5. An antiquark has charge and quantum numbers of opposite sign from the corresponding quark.

EXECUTE: (a) $Q/e = \frac{2}{3} + \frac{2}{3} + (-\frac{1}{3}) = +1$. $B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$. $S = 0 + 0 + (-1) = -1$. $C = 0 + 0 + 0 = 0$.

(b) $Q/e = \frac{2}{3} + \frac{1}{3} = +1$. $B = \frac{1}{3} + (-\frac{1}{3}) = 0$. $S = 0 + 1 = 1$. $C = 1 + 0 = 1$.

(c) $Q/e = \frac{1}{3} + \frac{1}{3} + (-\frac{2}{3}) = 0$. $B = -\frac{1}{3} + (-\frac{1}{3}) + (-\frac{1}{3}) = -1$. $S = 0 + 0 + 0 = 0$. $C = 0 + 0 + 0 = 0$.

(d) $Q/e = -\frac{2}{3} + (-\frac{1}{3}) = -1$. $B = -\frac{1}{3} + \frac{1}{3} = 0$. $S = 0 + 0 = 0$. $C = -1 + 0 = -1$.

EVALUATE: The charge must always come out to be a whole number.

44.27. IDENTIFY: The charge, baryon number, and strangeness of the particles are the sums of these values for their constituent quarks.

SET UP: The properties of the six quarks are given in Table 44.5.

EXECUTE: (a) $S = 1$ indicates the presence of one $\bar{s}$ antiquark and no s quark. To have baryon number 0 there can be only one other quark, and to have net charge $+e$ that quark must be a u, and the quark content is $u\bar{s}$.

(b) The particle has an $\bar{s}$ antiquark, and for a baryon number of -1 the particle must consist of three antiquarks. For a net charge of $-e$, the quark content must be $\overline{dd}\,\bar{s}$.

(c) $S = -2$ means that there are two s quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a u quark and the quark content is uss.

EVALUATE: The particles with baryon number zero are mesons and consist of a quark-antiquark pair. Particles with baryon number 1 consist of three quarks and are baryons. Particles with baryon number -1 consist of three antiquarks and are antibaryons.

44.28. **IDENTIFY:** The decrease in the rest energy of the particles that exist before and after the decay equals the energy that is released.

SET UP: The upsilon has rest energy 9460 MeV and each tau has rest energy 1777 MeV.

EXECUTE: $(m_\Upsilon - 2m_\tau)c^2 = (9460 \text{ MeV} - 2(1777 \text{ MeV})) = 5906 \text{ MeV}.$

EVALUATE: Over half of the rest energy of the upsilon is released in the decay.

44.29. **IDENTIFY** and **SET UP:** The mass of a proton is 938 MeV/c^2, and the mass of the Higgs boson is 125 GeV/c^2 = 125 ×10^3 MeV/c^2.

EXECUTE: $m_{\text{Higgs}}/m_{\text{p}} = (125 \times 10^3 \text{ MeV}/c^2)/(938 \text{ MeV}/c^2) = 133$.

EVALUATE: Since the Higgs particle is 133 times as massive as the proton, it takes a great deal of energy to create one. This is the reason that high-energy particle accelerators are needed to test for the existence of the Higgs boson.

44.30. **(a) IDENTIFY** and **SET UP:** First calculate the speed v. Then use that in Hubble's law to find r.

EXECUTE: $v = \left[\dfrac{(\lambda_0/\lambda_\text{S})^2 - 1}{(\lambda_0/\lambda_\text{S})^2 + 1}\right] c = \left[\dfrac{(658.5 \text{ nm}/590 \text{ nm})^2 - 1}{(658.5 \text{ nm}/590 \text{ nm})^2 + 1}\right] c = 0.1094c$

$$v = (0.1094)(2.998 \times 10^8 \text{ m/s}) = 3.28 \times 10^7 \text{ m/s}. \; v = rH_0.$$

(b) IDENTIFY and **SET UP:** Use Hubble's law to calculate r.

EXECUTE: $r = \dfrac{v}{H_0} = \dfrac{3.28 \times 10^4 \text{ km/s}}{[(67.3 \text{ km/s})/\text{Mpc}](1 \text{ Mpc}/3.26 \text{ Mly})} = 1590 \text{ Mly} = 1.59 \times 10^9 \text{ ly}.$

EVALUATE: The red shift $\lambda_0/\lambda_\text{S} - 1$ for this galaxy is 0.116. It is therefore about twice as far from earth as the galaxy in Examples 44.8 and 44.9, that had a red shift of 0.053.

44.31. **(a) IDENTIFY** and **SET UP:** Hubble's law is $v = H_0 r$, with $H_0 = (67.3 \text{ km/s})/(\text{Mpc})$. $1 \text{ Mpc} = 3.26 \text{ Mly}$.

EXECUTE: $r = 5210$ Mly, so

$v = H_0 r = [(67.3 \text{ km/s})/\text{Mpc}](1 \text{ Mpc}/3.26 \text{ Mly})(5210 \text{ Mly}) = 1.08 \times 10^5 \text{ km/s} = 1.08 \times 10^8 \text{ m/s}.$

(b) IDENTIFY and **SET UP:** Use v from part (a) in $\lambda_0 = \lambda_\text{S}\sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$.

EXECUTE: $\dfrac{\lambda_0}{\lambda_\text{S}} = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$.

$$\frac{v}{c} = \frac{1.08 \times 10^8 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}} = 0.3602, \text{ so } \frac{\lambda_0}{\lambda_\text{S}} = \sqrt{\frac{1+0.3602}{1-0.3602}} = 1.46.$$

EVALUATE: The galaxy in Examples 44.8 and 44.9 is 710 Mly away so has a smaller recession speed and redshift than the galaxy in this problem.

44.32. **IDENTIFY:** In Example 44.8, z is defined as $z = \dfrac{\lambda_0 - \lambda_\text{S}}{\lambda_\text{S}}$. Apply $\lambda_0 = \lambda_\text{S}\sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$ to solve for v. Hubble's law is given by $v = H_0 r$.

SET UP: The Hubble constant has a value of $H_0 = 6.73 \times 10^4 \dfrac{\text{m/s}}{\text{Mpc}}$.

EXECUTE: **(a)** $1 + z = 1 + \dfrac{(\lambda_0 - \lambda_\text{S})}{\lambda_\text{S}} = \dfrac{\lambda_0}{\lambda_\text{S}}$. Now we use $\lambda_0 = \lambda_\text{S}\sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$ to obtain

$$1 + z = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+v/c}{1-v/c}} = \sqrt{\frac{1+\beta}{1-\beta}}.$$

(b) Solving the above equation for β we obtain $\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{1.7^2 - 1}{1.7^2 + 1} = 0.4859.$ Thus,

$$v = 0.4859c = 1.46 \times 10^8 \text{ m/s}.$$

(c) We can use Hubble's law to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.46 \times 10^8 \text{ m/s})}{(6.73 \times 10^4 \text{ m/s})/\text{Mpc}} = 2.17 \times 10^3 \text{ Mpc}.$$

EVALUATE: $1 \text{ pc} = 3.26 \text{ ly},$ so the distance in part (c) is 7.07×10^9 ly.

44.33. IDENTIFY: The reaction energy Q is defined in Chapter 43 as $Q = (M_A + M_B - M_C - M_D)c^2$ and is the energy equivalent of the mass change in the reaction. When Q is negative the reaction is endoergic. When Q is positive the reaction is exoergic.

SET UP: Use the particle masses given in Section 43.1. 1 u is equivalent to 931.5 MeV.

EXECUTE: $\Delta m = m_e + m_p - m_n - m_{\nu_e}$ so assuming $m_{\nu_e} \approx 0,$

$\Delta m = 0.0005486 \text{ u} + 1.007276 \text{ u} - 1.008665 \text{ u} = -8.40 \times 10^{-4} \text{ u}$

$\Rightarrow E = (\Delta m)c^2 = (-8.40 \times 10^{-4} \text{ u})(931.5 \text{ MeV/u}) = -0.783 \text{ MeV}$ and is endoergic.

EVALUATE: The energy consumed in the reaction would have to come from the initial kinetic energy of the reactants.

44.34. IDENTIFY: The energy released in the reaction is the energy equivalent of the mass decrease that occurs in the reaction.

SET UP: 1 u is equivalent to 931.5 MeV. The neutral atom masses are given in Table 43.2.

EXECUTE: $3m(^4\text{He}) - m(^{12}\text{C}) = 7.80 \times 10^{-3} \text{ u},$ or 7.27 MeV.

EVALUATE: The neutral atom masses include 6 electrons on each side of the reaction equation. The electron masses cancel and we obtain the same mass change as would be calculated using nuclear masses.

44.35. IDENTIFY: The reaction energy Q is defined in Chapter 43 as $Q = (M_A + M_B - M_C - M_D)c^2$ and is the energy equivalent of the mass change in the reaction. When Q is negative the reaction is endoergic. When Q is positive the reaction is exoergic.

SET UP: 1 u is equivalent to 931.5 MeV. Use the neutral atom masses that are given in Table 43.2.

EXECUTE: $m_{^{12}_{6}\text{C}} + m_{^{4}_{2}\text{He}} - m_{^{16}_{8}\text{O}} = 7.69 \times 10^{-3}\text{u},$ or 7.16 MeV, an exoergic reaction.

EVALUATE: 7.16 MeV of energy is released in the reaction.

44.36. IDENTIFY: We are using Hooke's law to model the strong nuclear force.

SET UP and **EXECUTE:** **(a)** Use Coulomb's law modeling the quarks as a point-like particles each having charge $2e/3$ and separated from each other by $r = 0.5$ fm. This gives

$$F = \frac{1}{4\pi\epsilon_0}\frac{(2e/3)^2}{r^2} \approx 400 \text{ N}.$$

(b) We want the spring constant. $F_{spr} = ks$, so $k = F_{spr}/s = (400 \text{ N})/(0.5 \text{ fm}) = 8.2 \times 10^{17}$ N/m $\approx 8 \times 10^{17}$ N/m.

(c) Convert k to units of MeV/fm^2.

$$k = 8.2 \times 10^{17} \frac{\text{N}}{\text{m}} = \left(8.2 \times 10^{17} \frac{\text{N} \cdot \text{m}}{\text{m}^2}\right)\left(\frac{\text{J}}{\text{N} \cdot \text{m}}\right)\left(\frac{1 \text{ m}}{10^{15} \text{ fm}}\right)^2\left(\frac{1 \text{ MeV}}{1.6 \times 10^{-13} \text{ J}}\right) = 5.1 \text{ MeV/fm}^2 \approx 5 \text{ MeV/fm}^2.$$

(d) We want the stored energy.

$$U = \frac{1}{2}ks^2 = \frac{1}{2}(5.1 \text{ MeV/fm}^2)(0.5 \text{ fm})^2 = 0.64 \text{ MeV} = 640 \text{ KeV}.$$

(e) We want the energy to produce an up and anti-up pair of quarks. The mass of each up quark is 2.3 MeV/c^2, so to produce a pair we need 4.6 MeV.
(f) We want the distance between the quarks. Use the spring constant from part (c) with the energy from part (e).

$$E = \frac{1}{2}ks^2 \to s = \sqrt{2E/k} = \sqrt{\frac{2(4.6\text{ MeV})}{5.1\text{ MeV/fm}^2}} = 1.3\text{ fm}.$$

EVALUATE: The result in part (f) is comparable to the size of an atomic nucleus, so our model is of some use.

44.37. **IDENTIFY:** The energy comes from a mass decrease.
SET UP: A charged pion decays into a muon plus a neutrino. The muon in turn decays into an electron or positron plus two neutrinos.
EXECUTE: **(a)** $\pi^- \to \mu^- + \text{neutrino} \to \text{e}^- + \text{three neutrinos}.$
(b) If we neglect the mass of the neutrinos, the mass decrease is

$$m(\pi^-) - m(\text{e}^-) = 273m_\text{e} - m_\text{e} = 272m_\text{e} = 2.480\times10^{-28}\text{ kg}.$$
$$E = mc^2 = 2.23\times10^{-11}\text{ J} = 139\text{ MeV}.$$

(c) The total energy delivered to the tissue is $(50.0\text{ J/kg})(10.0\times10^{-3}\text{ kg}) = 0.500\text{ J}.$ The number of

$$\pi^- \text{ mesons required is } \frac{0.500\text{ J}}{2.23\times10^{-11}\text{ J}} = 2.24\times10^{10}.$$

(d) The RBE for the electrons that are produced is 1.0, so the equivalent dose is

$$1.0(50.0\text{ Gy}) = 50.0\text{ Sv} = 5.0\times10^3\text{ rem}.$$

EVALUATE: The π are heavier than electrons and therefore behave differently as they hit the tissue.

44.38. **IDENTIFY:** The initial total energy of the colliding proton and antiproton equals the total energy of the two photons.
SET UP: For a particle with mass, $E = K + mc^2.$ For a proton, $m_\text{p}c^2 = 938\text{ MeV}.$

EXECUTE: $K + m_\text{p}c^2 = \dfrac{hc}{\lambda},\ K = \dfrac{hc}{\lambda} - m_\text{p}c^2.$ Using $\lambda = 0.720\text{ fm} = 0.720\times10^{-15}\text{ m}$,
we get K = 784 MeV.
EVALUATE: If the kinetic energies of the colliding particles increase, then the wavelength of each photon decreases.

44.39. **IDENTIFY:** We are investigating colliding proton beams in the Large Hadron Collider.
SET UP and **EXECUTE:** **(a)** With one bunch per 25 ns, the number of bunches per second is 1/(25 ns) = 40 million.
(b) The fraction of protons that collide is 20/(115 billion) = 1.7×10^{-10}.
(c) We want the number of collisions that occur each second. Using the given information and the result of part (a) gives (40 million bunches/s)(20 collisions/bunch) = 800 million collisions.
(d) We want the proton density ρ in a bunch, with N protons in a cylinder of length L and radius R.

$$\rho = \frac{N}{\pi R^2 L} = \frac{115\text{ billion}}{\pi(10\ \mu\text{m})^2(0.300\text{ m})} = 1.2\times10^{12}\text{ protons/mm}^3.$$

(e) We want the density of hadrons in ordinary matter. Follow the hint. Mass is 75 kg. The body is a cylinder 30 cm in diameter of length 1.75 m tall. $V = \pi R^2 L = \pi(0.15\text{ m})^2(1.75\text{ m}) = 0.124\text{ m}^3$. The number N of hadrons is $N = (75\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 4.5\times10^{28}$ hadrons. The density is $\rho = N/V = (4.5\times10^{28})/(0.124\text{ m}^3) = 3.6\times10^{29}\text{ hadrons/m}^3 \approx 4\times10^{20}\text{ hadrons/mm}^3$.
EVALUATE: As a check for part (e) we can use the density of water, which is 1000 kg/m^3. This gives

$$\frac{1000\text{ kg/m}^3}{1.67\times10^{-27}\text{ kg}} \approx 6\times10^{29}\text{ hadrons/m}^3 = 6\times10^{20}\text{ hadrons/mm}^3.$$

This result is quite close to our calculation above. Note that the concentration of hadrons in ordinary matter is around 300 million times as great as in the bunches in the Large Hadron Collider.

44.40. **IDENTIFY:** Apply Eq. (44.9).

SET UP: In Eq. (44.9), $E_a = (m_{\Sigma^0} + m_{K^0})c^2$, and with $M = m_p, m = m_{\pi^-}$ and $E_m = (m_{\pi^-})c^2 + K$,

$$K = \frac{E_a^2 - (m_{\pi^-}c^2)^2 - (m_p c^2)^2}{2m_p c^2} - (m_{\pi^-})c^2.$$

EXECUTE: $K = \dfrac{(1193\text{ MeV} + 497.7\text{ MeV})^2 - (139.6\text{ MeV})^2 - (938.3\text{ MeV})^2}{2(938.3\text{ MeV})} - 139.6\text{ MeV} = 904\text{ MeV}.$

EVALUATE: The increase in rest energy is

$(m_{\Sigma^0} + m_{K^0} - m_{\pi^-} - m_p)c^2 = 1193\text{ MeV} + 497.7\text{ MeV} - 139.6\text{ MeV} - 938.3\text{ MeV} = 613\text{ MeV}.$ The threshold kinetic energy is larger than this because not all the kinetic energy of the beam is available to form new particle states.

44.41. **IDENTIFY:** Baryon number, charge, strangeness, and lepton numbers are all conserved in the reactions.

SET UP: Use Table 44.3 to identify the missing particle, once its properties have been determined.

EXECUTE: **(a)** The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+.

(b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero, and the particle is π^-.

(c) The baryon number is -1, the charge is 0, the strangeness is zero, all lepton numbers are 0, and the particle is an antineutron.

(d) The baryon number is 0 the charge is $+e$, the strangeness is 0, the muonic lepton number is -1, all other lepton numbers are 0, and the particle is μ^+.

EVALUATE: Rest energy considerations would determine if each reaction is endoergic or exoergic.

44.42. **IDENTIFY:** Charge must be conserved. The energy released equals the decrease in rest energy that occurs in the decay.

SET UP: The rest energies are given in Table 44.3.

EXECUTE: **(a)** The decay products must be neutral, so the only possible combinations are $\pi^0\pi^0\pi^0$ *or* $\pi^0\pi^+\pi^-$.

(b) $m_{n^0} - 3m_{\pi^0} = 142.9\text{ MeV}/c^2$, so the kinetic energy of the π^0 mesons is 142.9 MeV. For the other reaction, $m_{n^0} - m_{\pi^0} - m_{\pi^-} - m_{\pi^+} = 133.7\text{ MeV}$.

EVALUATE: The total momentum of the decay products must be zero. This imposes a correlation between the directions of the velocities of the decay products.

44.43. **IDENTIFY** and **SET UP:** Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \approx \hbar/2$. Let ΔE be the energy width and let Δt be the lifetime.

EXECUTE: $\dfrac{\hbar}{2\Delta E} = \dfrac{(1.054\times10^{-34}\text{ J}\cdot\text{s})}{2(4.4\times10^{6}\text{ eV})(1.6\times10^{-19}\text{ J/eV})} = 7.5\times10^{-23}\text{ s}.$

EVALUATE: The shorter the lifetime, the greater the energy width.

44.44. **IDENTIFY:** Apply the Heisenberg uncertainty principle in the form $\Delta E \Delta t \approx \hbar/2$. Let Δt be the mean lifetime.

SET UP: The rest energy of the ψ is 3097 MeV.

EXECUTE: $\Delta t = 7.6\times10^{-21}\text{ s} \Rightarrow \Delta E = \dfrac{\hbar}{2\Delta t} = \dfrac{1.054\times10^{-34}\text{ J}\cdot\text{s}}{2(7.6\times10^{-21}\text{ s})} = 6.93\times10^{-15}\text{ J} = 43\text{ keV}.$

$$\frac{\Delta E}{m_\psi c^2} = \frac{0.043\text{ MeV}}{3097\text{ MeV}} = 1.4\times 10^{-5}.$$

EVALUATE: The energy width due to the lifetime of the particle is a small fraction of its rest energy.

44.45. **IDENTIFY:** Apply $\left|\frac{dN}{dt}\right| = \lambda N$ to find the number of decays in one year.

SET UP: Water has a molecular mass of 18.0×10^{-3} kg/mol.

EXECUTE: **(a)** The number of protons in a kilogram is

$(1.00\text{ kg})\left(\frac{6.022\times 10^{23}\text{ molecules/mol}}{18.0\times 10^{-3}\text{ kg/mol}}\right)(2\text{ protons/molecule}) = 6.7\times 10^{25}.$ Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_\text{p}c^2 = 938.3\text{ MeV} = 1.503\times 10^{-10}\text{ J},$ and so the energy deposited in a year, per kilogram, is

$$(6.7\times 10^{25})\left(\frac{\ln 2}{1.0\times 10^{18}\text{ y}}\right)(1\text{ y})(1.50\times 10^{-10}\text{ J}) = 7.0\times 10^{-3}\text{ Gy} = 0.70\text{ rad}.$$

(b) For an RBE of unity, the equivalent dose is $(1)(0.70\text{ rad}) = 0.70\text{ rem}.$

EVALUATE: The equivalent dose is much larger than that due to the natural background. It is not feasible for the proton lifetime to be as short as 1.0×10^{18} y.

44.46. **IDENTIFY** and **SET UP:** $\phi \to \text{K}^+ + \text{K}^-.$ The total energy released is the energy equivalent of the mass decrease.

EXECUTE: **(a)** The mass decrease is $m(\phi) - m(\text{K}^+) - m(\text{K}^-).$ The energy equivalent of the mass decrease is $mc^2(\phi) - mc^2(\text{K}^+) - mc^2(\text{K}^-).$ The rest mass energy mc^2 for the ϕ meson is given in Problem 44.43, and the values for K^+ and K^- are given in Table 44.3. The energy released then is $1019.4\text{ MeV} - 2(493.7\text{ MeV}) = 32.0\text{ MeV}.$ The K^+ gets half this, 16.0 Mev.

EVALUATE: **(b)** Does the decay $\phi \to \text{K}^+ + \text{K}^- + \pi^0$ occur? The energy equivalent of the $\text{K}^+ + \text{K}^- + \pi^0$ mass is $493.7\text{ MeV} + 493.7\text{ MeV} + 135.0\text{ MeV} = 1122\text{ MeV}.$ This is greater than the energy equivalent of the ϕ mass. The mass of the decay products would be greater than the mass of the parent particle; the decay is energetically forbidden.

(c) Does the decay $\phi \to \text{K}^+ + \pi^-$ occur? The reaction $\phi \to \text{K}^+ + \text{K}^-$ is observed. K^+ has strangeness $+1$ and K^- has strangeness $-1,$ so the total strangeness of the decay products is zero. If strangeness must be conserved we deduce that the ϕ particle has strangeness zero. π^- has strangeness 0, so the product $\text{K}^+ + \pi^-$ has strangeness $-1.$ The decay $\phi \to \text{K}^+ + \pi^-$ violates conservation of strangeness. Does the decay $\phi \to K^+ + \mu^-$ occur? μ^- has strangeness 0, so this decay would also violate conservation of strangeness.

44.47. **IDENTIFY:** We are dealing with muons in cosmic rays. The energies involved are much greater than the rest energy of muons, so we must use the relativistic equations.

SET UP and **EXECUTE:** **(a)** We want the muon's speed. First find γ and then use it to find the speed.

$$E = m\gamma c^2 \quad \to \quad 6.000\text{ GeV} = \gamma\,(105.7\text{ MeV}) \quad \to \quad \gamma = 56.76443$$

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(56.76443)^2} = 0.99969c = 2.997\times 10^8\text{ m/s}.$$

(b) We want the distance the muon travels in one lifetime. $x = vt = (2.997\times 10^8\text{ m/s})(2.197\ \mu\text{s}) = 658.4\text{ m}.$

(c) We want the distance to the earth's surface. To the muon, the 15.00 km is Lorentz contracted, so the distance it sees itself traveling is $L = L_0/\gamma$ = (15.00 km)/(56.76443) = 0.264 km = 264 m.
(d) We want the time in the earth's frame. The muon's frame is the proper frame.

$$\Delta t = \gamma \Delta t_0 = (56.76443)(2.197\ \mu\text{s}) = 124.7\ \mu\text{s}.$$

(e) We want the distance traveled as seen from the earth frame. $x = vt = (2.997 \times 10^8\ \text{m/s})(124.7\ \mu\text{s})$ = 37.4 km.
(f) As observed in the earth frame, the muon is created 15.00 km above the surface and travels 37.4 km, so it survives its trip through 15.00 km of atmosphere and travels downward into the surface of the earth. At such high energy, the muon can penetrate deeply into the surface without hitting anything, so the depth it reaches is 37.4 km – 15.00 km = 22.4 km.
EVALUATE: Without relativistic effects we would have far fewer cosmic rays striking the earth's surface.

44.48. **IDENTIFY:** The energy comes from the mass difference.
SET UP: $\Xi^- \to \Lambda^0 + \pi^-$. $p_\Lambda = p_\pi = p$. $E_\Xi = E_\Lambda + E_\pi$. $m_{\Xi^-}c^2 = 1322\ \text{MeV}$. $m_\pi c^2 = 139.6\ \text{MeV}$.

$$m_\Xi c^2 = \sqrt{m_\Lambda^2 c^4 + p^2c^2} + \sqrt{m_\pi^2 c^4 + p^2 c^2}.$$

EXECUTE: **(a)** The total energy released is

$$(m_{\Xi^-} - m_\pi - m_\Lambda)c^2 = 1322\ \text{MeV} - 139.6\ \text{MeV} - 1116\ \text{MeV} = 66.4\ \text{MeV}.$$

(b) $m_\Xi c^2 = \sqrt{m_\Lambda^2 c^4 + p^2c^2} + \sqrt{m_\pi^2 c^4 + p^2 c^2}$. $m_\Xi c^2 - \sqrt{m_\Lambda^2 c^4 + p^2c^2} = \sqrt{m_\pi^2 c^4 + p^2 c^2}$.
Square both sides:

$$m_\Xi^2 c^4 + m_\Lambda^2 c^4 + p^2c^2 - 2m_\Xi c^2 E_\Lambda = m_\pi^2 c^4 + p^2c^2.\quad E_\Lambda = \frac{m_\Xi^2c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2}.$$

$$K_\Lambda = \frac{m_\Xi^2c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2} - m_\Lambda c^2.\quad E_\pi = E_\Xi - E_\Lambda = m_\Xi c^2 - \frac{m_\Xi^2c^4 + m_\Lambda^2 c^4 - m_\pi^2 c^4}{2m_\Xi c^2}.$$

$$E_\pi = \frac{m_\Xi^2c^4 - m_\Lambda^2 c^4 + m_\pi^2 c^4}{2m_\Xi c^2}.\quad K_\pi = \frac{m_\Xi^2c^4 - m_\Lambda^2 c^4 + m_\pi^2 c^4}{2m_\Xi c^2} - m_\pi c^2.$$

Putting in numbers gives

$$K_\Lambda = \frac{(1322\ \text{MeV})^2 + (1116\ \text{MeV})^2 - (139.6\ \text{MeV})^2}{2(1322\ \text{MeV})^2} - 1116\ \text{MeV} = 8.7\ \text{MeV (13\% of total)}.$$

$$K_\pi = \frac{(1322\ \text{MeV})^2 - (1116\ \text{MeV})^2 + (139.6\ \text{MeV})^2}{2(1322\ \text{MeV})^2} - 139.6\ \text{MeV} = 57.7\ \text{MeV (87\% of total)}.$$

EVALUATE: The two particles do not have equal kinetic energies because they have different masses.

44.49. **IDENTIFY:** The kinetic energy comes from the mass difference.
SET UP and **EXECUTE:** $K_\Sigma = 180\ \text{MeV}$. $m_\Sigma c^2 = 1197\ \text{MeV}$. $m_\text{n}c^2 = 939.6\ \text{MeV}$. $m_\pi c^2 = 139.6\ \text{MeV}$.
$E_\Sigma = K_\Sigma + m_\Sigma c^2 = 180\ \text{MeV} + 1197\ \text{MeV} = 1377\ \text{MeV}$. Conservation of the x-component of momentum gives $p_\Sigma = p_{\text{n}x}$. Then $p_{\text{n}x}^2c^2 = p_\Sigma^2 c^2 = E_\Sigma^2 - (m_\Sigma c)^2 = (1377\ \text{MeV})^2 - (1197\ \text{MeV})^2$ $= 4.633 \times 10^5\ (\text{MeV})^2$. Conservation of energy gives $E_\Sigma = E_\pi + E_\text{n}$.
$E_\Sigma = \sqrt{m_\pi^2c^4 + p_\pi^2c^2} + \sqrt{m_\text{n}^2c^4 + p_\text{n}^2c^2}$. $E_\Sigma - \sqrt{m_\text{n}^2c^4 + p_\text{n}^2c^2} = \sqrt{m_\pi^2c^4 + p_\pi^2c^2}$. Square both sides:
$E_\Sigma^2 + m_\text{n}^2c^4 + p_{\text{n}x}^2c^2 + p_{\text{n}y}^2c^2 - 2E_\Sigma E_\text{n} = m_\pi^2c^4 + p_\pi^2c^2$. $p_\pi = p_{\text{n}y}$, so

$$E_\Sigma^2 + m_\text{n}^2c^4 + p_{\text{n}x}^2c^2 - 2E_\Sigma E_\text{n} = m_\pi^2c^4 \text{ and } E_\text{n} = \frac{E_\Sigma^2 + m_\text{n}^2c^4 - m_\pi^2c^4 + p_{\text{n}x}^2c^2}{2E_\Sigma}.$$

$$E_{\mathrm{n}}=\frac{(1377\text{ MeV})^2+(939.6\text{ MeV})^2-(139.6\text{ MeV})^2+4.633\times10^5\ (\text{MeV})^2}{2(1377\text{ MeV})}=1170\text{ MeV}.$$

$$K_{\mathrm{n}}=E_{\mathrm{n}}-m_{\mathrm{n}}c^2=1170\text{ MeV}-939.6\text{ MeV}=230\text{ MeV}.$$

$$E_\pi=E_\Sigma-E_{\mathrm{n}}=1377\text{ MeV}-1170\text{ MeV}=207\text{ MeV}.$$

$$K_\pi=E_\pi-m_\pi c^2=207\text{ MeV}-139.6\text{ MeV}=67\text{ MeV}.$$

$$p_{\mathrm{n}}^2c^2=E_{\mathrm{n}}^2-m_{\mathrm{n}}^2c^2=(1170\text{ MeV})^2-(939.6\text{ MeV})^2=4.861\times10^5\ (\text{MeV})^2.$$

The angle θ the velocity of the neutron makes with the $+x$-axis is given by $\cos\theta=\dfrac{p_{\mathrm{n}x}}{p_{\mathrm{n}}}=\sqrt{\dfrac{4.633\times10^5}{4.861\times10^5}}$ and $\theta=12.5^{\circ}$ below the $+x$-axis.

EVALUATE: The decay particles do not have equal energy because they have different masses.

44.50. **IDENTIFY:** The kinetic energy comes from the mass difference, and momentum is conserved.

SET UP: $\left|p_{\pi^+y}\right|=\left|p_{\pi^-y}\right|$. $p_{\pi^+}\sin\theta=p_{\pi^-}\sin\theta$ and $p_{\pi^+}=p_{\pi^-}=p_\pi$. $m_{\mathrm{K}}c^2=497.7\text{ MeV}$.

$$m_\pi c^2=139.6\text{ MeV}.$$

EXECUTE: Conservation of momentum for the decay gives $p_{\mathrm{K}}=2p_{\pi x}$ and $p_{\mathrm{K}}^2=4p_{\pi x}^2$.

$p_{\mathrm{K}}^2c^2=E_{\mathrm{K}}^2-m_{\mathrm{K}}^2c^2$. $E_K=497.7\text{ MeV}+225\text{ MeV}=722.7\text{ MeV}$, so

$p_{\mathrm{K}}^2c^2=(722.7\text{ MeV})^2-(497.7\text{ MeV})^2=2.746\times10^5\ (\text{MeV})^2$ and

$p_{\pi x}^2c^2=[2.746\times10^5\ (\text{MeV})^2]/4=6.865\times10^4\ (\text{MeV})^2$. Conservation of energy says $E_{\mathrm{K}}=2E_\pi$.

$E_\pi=\dfrac{E_{\mathrm{K}}}{2}=361.4\text{ MeV}$. $K_\pi=E_\pi-m_\pi c^2=361.4\text{ MeV}-139.6\text{ MeV}=222\text{ MeV}$.

$p_\pi^2c^2=E_\pi^2-(m_\pi c^2)^2=(361.4\text{ MeV})^2-(139.6\text{ MeV})^2=1.11\times10^5\ (\text{MeV})^2$. The angle θ that the velocity of the π^+ particle makes with the $+x$-axis is given by $\cos\theta=\sqrt{\dfrac{p_{\pi x}^2c^2}{p_\pi^2c^2}}=\sqrt{\dfrac{6.865\times10^4}{1.11\times10^5}}$, which gives $\theta=38.2^{\circ}$.

EVALUATE: The pions have the same energy and go off at the same angle because they have equal masses.

44.51. **IDENTIFY** and **SET UP:** For nonrelativistic motion, the maximum kinetic energy in a cyclotron is $K_{\max}=\dfrac{q^2R^2}{2m}B^2$. The angular frequency is $\omega=|q|B/m$.

EXECUTE: **(a)** The rest energy of a proton is 938 MeV, and the kinetic energies in the data table in the problem are around 1 MeV or less, so there is no need to use relativistic expressions.

(b) Figure 44.51 shows the graph of $K_{\max}$ versus B^2 for the data in the problem. The graph is clearly a straight line and has slope equal to 6.748 MeV/T^2 = 1.081×10^{-12} J/T^2. The formula for $K_{\max}$ is $K_{\max}=\dfrac{q^2R^2}{2m}B^2$, so a graph of $K_{\max}$ versus B^2 should be a straight line with slope equal to $q^2R^2/2m$.

Solving $q^2R^2/2m$ = slope for R gives

$$R=\sqrt{\frac{2m(\text{slope})}{q^2}}=\sqrt{\frac{2(1.673\times10^{-27}\text{ kg})(1.081\times10^{-12}\text{ J/T}^2)}{(1.602\times10^{-19}\text{ C})^2}}=0.375\text{ m}=37.5\text{ cm}.$$

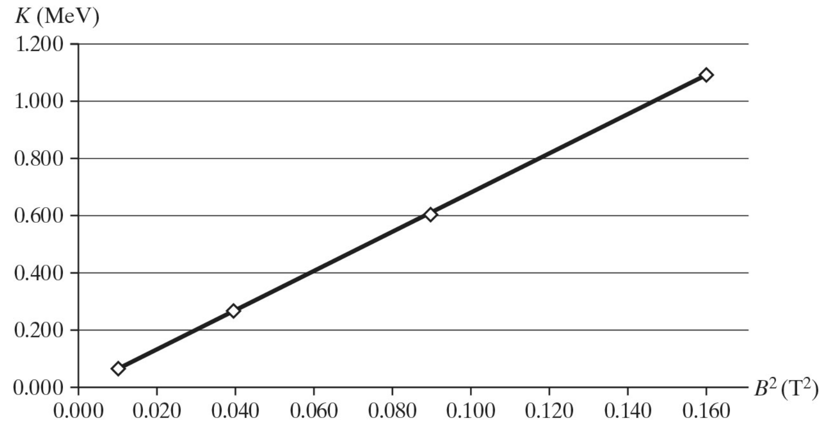

Figure 44.51

(c) Using the result from our graph, we get $K_{\max} = (\text{slope})B^2 = (6.748\ \text{MeV/T}^2)(0.25\ \text{T})^2 = 0.42\ \text{MeV}$.
(d) The angular speed is $\omega = |q|B/m = (1.602\times10^{-19}\ \text{C})(0.40\ \text{T})/(1.67\times10^{-27}\ \text{kg}) = 3.8\times10^{7}\ \text{rad/s}$.

EVALUATE: In part (c) we can check by using $K_{\max} = q^2R^2B^2/2m = (qRB)^2/2m$. Using $B = 0.25$ T and the standard values for the other quantities gives $K_{\max} = 6.75\times10^{-14}\ \text{J} = 0.42\ \text{MeV}$, which agrees with our result.

44.52. **IDENTIFY:** Use Table 44.3 for data on the given particles. Apply conservation of energy in part (b).
SET UP: For any decay, conservation of energy tells us that $E_i = E_f$. If the decaying particle is at rest (or in its rest frame), this gives $m_i c^2 = m_{\text{products}}c^2 + K$.
EXECUTE: **(a)** The masses from Table 44.3 are:
Σ^-: 1197 MeV/c^2
Ξ^0: 1315 MeV/c^2
Δ^{++}: 1232 MeV/c^2
Ω^-: 1672 MeV/c^2
We see that Ω^- has the largest mass and Σ^- has the smallest mass.
(b) Solving $m_i c^2 = m_{\text{products}}c^2 + K$ for K gives $K = (m_i - m_{\text{products}})c^2$. Therefore the greater the difference between the mass of the decaying particle and the mass of decay products, the greater the kinetic energy. We show the decays and the mass differences below.
$\Sigma^- \to n + \pi^-$: $K = (1197 - 939.6 - 139.6)\ \text{MeV}/c^2 = 117.8\ \text{MeV}/c^2$
$\Xi^0 \to \Lambda^0 + \pi^0$: $K = (1315 - 1116 - 135)\ \text{MeV}/c^2 = 64\ \text{MeV}/c^2$
$\Delta^{++} \to p + \pi^+$: $K = (1232 - 938.3 - 139.6)\ \text{MeV}/c^2 = 154.1\ \text{MeV}/c^2$
$\Omega^- \to \Lambda^0 + K^-$: $K = (1672 - 1116 - 493.7)\ \text{MeV}/c^2 = 62.3\ \text{MeV}/c^2$
The kinetic energy is largest for the Δ^{++} decay and smallest for the Ω^- decay.
EVALUATE: A large-mass particle does not necessarily result in the release of more kinetic energy. For example, the Ω^- particle has more mass than the Δ^{++}, yet the decay products of the Ω^- have less kinetic energy than those of the Δ^{++} decay.

44.53. **IDENTIFY** and **SET UP:** Construct the diagram as specified in the problem. In part (b), use quark charges $\boldsymbol{u} = +\frac{2}{3}$, $\boldsymbol{d} = \frac{-1}{3}$, and $\boldsymbol{s} = \frac{-1}{3}$ as a guide.

EXECUTE: **(a)** The diagram is given in Figure 44.53. The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a "hole" in an otherwise regular lattice in the $S - Q$ plane.
(b) The quark composition of each particle is shown in the figure.

EVALUATE: The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this "hole" and mass regularity that led to an accurate prediction of the properties of the Ω^-!

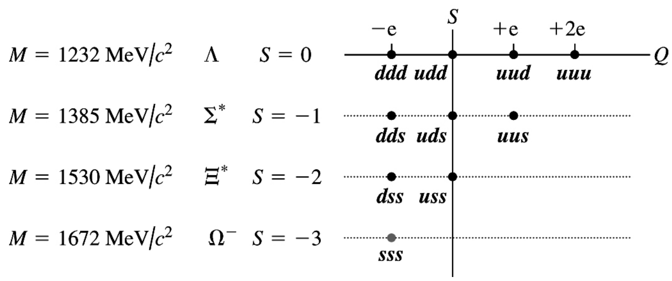

Figure 44.53

44.54. **IDENTIFY:** This problem deals with the decay of a kaon, K^+.

SET UP: We use relativistic equations and treat the neutrino as massless. The structure and decay of a K^+ are $K^+ = u\bar{d}$, $K^+ \to \bar{\mu}^+ + \nu_\mu$.

EXECUTE: **(a)** Energy conservation: $M_K c^2 = \gamma M_\mu c^2 + E_\nu$.

(b) Momentum conservation: The initial momentum is zero. The neutrino is treated as massless, so $p_\nu = E_\nu/c$. Momentum conservation gives $\gamma M_\mu v = E_\nu/c$.

(c) We want the speed of the muon. Combine the energy and momentum equations from (a) and (b). This leads to $M_K c^2 = \gamma M_\mu c^2 + \gamma M_\mu vc$.

Next substitute for σ and then for γ, and finally solve for v.

$$\sigma = \gamma(1 + v/c)$$

$$\sigma = \frac{1}{\sqrt{1 - v^2/c^2}}(1 + v/c)$$

$$v = c\frac{\sigma^2 - 1}{\sigma^2 + 1}.$$

(d) $\sigma = M_K/M_\mu = (493.7\ \text{MeV}/c^2)/(105.7\ \text{MeV}/c^2) = 4.671$.

(e) We want E_μ. First use the results of (c) and (d) to find v. $v = c(4.671^2 - 1)/(4.671^2 + 1) = 0.91234c$. Now use this result to find γ, giving $\gamma = 2.4424$. Now use $E_\mu = \gamma M_\mu c^2$ to find E_μ.

$E_\mu = (2.4424)(105.7\ \text{MeV}/c^2)c^2 = 258.2$ MeV.

(f) We want the energy of the neutrino. Combine $\gamma M_\mu v = E_\nu/c$ from part (b) and $v/c = 0.91234$ from part (e). This gives $E_\nu = \gamma M_\mu vc = \gamma M_\mu c^2(v/c) = (258.2\ \text{MeV})(0.91234) = 235.5\ \text{MeV}$.

(g) Adding the energy of the muon and neutrino gives 258.2 MeV + 235.5 MeV = 493.7 MeV, which is the rest energy of the kaon. So the answer is *yes*.

EVALUATE: The result in part (g) is consistent with the conservation of energy.

44.55. **IDENTIFY:** We are dealing with an expanding curved space.

SET UP: Follow the directions with each part.

EXECUTE: **(a)** We want D. If θ is in radians, then $D = R\theta$.

(b) We want V. Use $V = dD/dt$ and the result from part (a).

$$V = \frac{dD}{dt} = \frac{d(R\theta)}{dt} = \theta\frac{dR}{dt} = \frac{D}{R}\frac{dR}{dt}.$$

(c) If $V = BD$, find $B(t)$. Use the result from part (b).

$$V = \frac{D}{R}\frac{dR}{dt} = \frac{1}{R}\frac{dR}{dt}D = BD \rightarrow B = \frac{1}{R}\frac{dR}{dt}.$$

(d) We want B. R is increasing at a constant rate of 1.00 μm/s, so $R = R_0 + v_R t$. At 4 years R = 500.0 m + (1.00 μm/s)(4)(3.156 × 10^7 s) = 626 m.

$$B = \frac{1}{R}\frac{dR}{dt} = \left(\frac{1}{626\text{ m}}\right)(1.00\ \mu\text{m/s}) = 1.60\times10^{-9}\ \text{s}^{-1}.$$

(e) We want D. $D = R\theta$ = (626 m)(π/3 rad) = 656 m.

(f) We want the separation speed V. Use the result from part (b).

$$V = \frac{dD}{dt} = \frac{D}{R}\frac{dR}{dt} = \left(\frac{656\text{ m}}{626\text{ m}}\right)(1.00\ \mu\text{m/s}) = 1.05\ \mu\text{m/s}.$$

(g) We want the time to reach Xibalba. In this universe, c = 6.35 μm/s and v_R = 1.00 μm/s. The waves travel along the circular arc, but this arc is increasing in length due to the expansion of space. For an infinitesimal time interval dt, the wave travels through an arc distance cdt and an angle $d\theta = (cdt)/R$. Using $R = R_0 + v_R t$, we have

$$d\theta = \frac{cdt}{R_0 + v_R t}$$

Integrating will give a relationship between θ and t.

$$\theta(t) = \int_0^t \frac{cdt'}{R_0 + v_R t'} = \frac{c}{v_R}\left[\ln(R_0 + v_R t) - \ln R_0\right] = \frac{c}{v_R}\ln\left(1 + \frac{v_R t}{R_0}\right).$$

Now solve for t when $\theta = \pi/3$.

$$t = \frac{R_0}{v_R}\left(e^{\pi v_R/3c} - 1\right).$$

From part (d) we know that when the ripple waves are sent, R = 626 m, so R_0 = 626 m. Putting in the numbers gives

$$t = \left(\frac{626\text{ m}}{1.00\ \mu\text{m/s}}\right)\left[e^{\pi(1.00\ \mu\text{m/s})[3(6.35\ \mu\text{m/s})]} - 1\right] = 1.122\times10^8\ \text{s} = 3.56\ \text{y}.$$

(h) We want the wavelength that the Xibalbans observe. Use Eq. (44.16) with v = 1.05 μm/s at the instant the ripple waves are sent, from part (f).

$$\lambda_0 = \lambda_s\sqrt{\frac{c+v}{c-v}} = (1.00\text{ nm})\sqrt{\frac{(6.35+1.05)\ \mu\text{m/s}}{(6.35+1.05)\ \mu\text{m/s}}} = 1.18\text{ nm}.$$

EVALUATE: The received wavelength is longer than the emitted wavelength. This result is reasonable because this "universe" is expanding.

44.56. IDENTIFY: Follow the steps specified in the problem. The Lorentz velocity transformation is given by

$$v_x = \frac{v_x' + u}{1 + uv_x'/c^2}.$$

SET UP: Let the +x-direction be the direction of the initial velocity of the bombarding particle.

EXECUTE: **(a)** For mass m, in $v_x = \dfrac{v_x' + u}{1 + uv_x'/c^2}$, $u = -v_{\text{cm}}$, $v' = v_0$, and so $v_m = \dfrac{v_0 - v_{\text{cm}}}{1 - v_0\, v_{\text{cm}}/c^2}$. For mass M, $u = -v_{\text{cm}}$, $v' = 0$, so $v_M = -v_{\text{cm}}$.

(b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0$, where γ_m and γ_M correspond to the velocities found in part (a). The algebra reduces to

$\beta_m\gamma_m = (\beta_0 - \beta')\gamma_0\gamma_M$, where $\beta_0 = \dfrac{v_0}{c}$, $\beta' = \dfrac{v_{\text{cm}}}{c}$, and the condition for no net momentum becomes

$m(\beta_0 - \beta')\gamma_0\gamma_M = M\beta'\gamma_M$, or $\beta' = \dfrac{\beta_0}{1+\dfrac{M}{m\gamma_0}} = \beta_0\dfrac{m}{m+M\sqrt{1-\beta_0^2}}$. $v_{\text{cm}} = \dfrac{mv_0}{m+M\sqrt{1-(v_0/c)^2}}$.

(c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms $v_m = v_0\gamma_0\dfrac{M}{m+M\gamma_0}$, $v_M = -v_0\gamma_0\dfrac{m}{m\gamma_0+M}$. After some more algebra,

$\gamma_m = \dfrac{m+M\gamma_0}{\sqrt{m^2+M^2+2mM\gamma_0}}$, $\gamma_M = \dfrac{M+m\gamma_0}{\sqrt{m^2+M^2+2mM\gamma_0}}$, from which

$m\gamma_m + M\gamma_M = \sqrt{m^2+M^2+2mM\gamma_0}$. This last expression, multiplied by c^2, is the available energy E_{a} in the center of mass frame, so that

$E_{\text{a}}^2 = (m^2+M^2+2mM\gamma_0)c^4 = (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0c^2) = (mc^2)^2 + (Mc^2)^2 + 2Mc^2E_m$, which is Eq. (44.9).

EVALUATE: The energy E_{a} in the center-of-momentum frame is the energy that is available to form new particle states.

44.57. **IDENTIFY** and **SET UP:** Energy and momentum are conserved.

EXECUTE: The positron is moving slowly, so its only appreciable energy is its rest energy m_ec^2. The total energy released by the annihilation is $2m_ec^2$, but the two photons share it equally to conserve momentum. Therefore they also have equal energy, so each photon has energy m_ec^2, which is choice (d).

EVALUATE: If the positron had significant kinetic energy, the two photons would not have the same momentum and hence would not have the same energy.

44.58. **IDENTIFY** and **SET UP:** One photon travels 3 cm longer than the other one. If $2L$ is the distance between the detectors, one photon travels a distance L + 3 cm and other a distance L – 3 cm. The time for a photon to travel a distance x is $t = x/c$.

EXECUTE: $t_1 = (L + 3\text{ cm})/c$ and $t_2 = (L - 3\text{ cm})/c$. The time interval Δt between the arrival of the two photons is $\Delta t = t_1 - t_2 = (L + 3\text{ cm})/c - (L - 3\text{ cm})/c = (6\text{ cm})/c = (0.06\text{ m})/c = 0.2 \times 10^{-9}\text{ s} = 0.2$ ns. This is within the 10-ns window, so the two photons will be counted as simultaneous. Thus choice (d) is correct.

EVALUATE: 0.2 ns is well within the 10-ns window for simultaneity. The annihilation would have to occur over 1.5 m from the center for the photons not to be counted as simultaneous.

44.59. **IDENTIFY** and **SET UP:** The absorption of photons obeys the equation $N = N_0e^{-\mu x}$, where $\mu = 0.1\text{ cm}^{-1}$.

EXECUTE: $N/N_0 = e^{-\mu x} = e^{-(0.1\text{ cm}^{-1})(20\text{ cm})} = 0.14 = 14\%$. Choice (c) is correct.

EVALUATE: If 14% of the photons exit the body, 86% were absorbed within 20 cm of tissue.