7

POTENTIAL ENERGY AND ENERGY CONSERVATION

VP7.2.1. **IDENTIFY:** We use energy conservation. The ball has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ Call point 1 the place where the ball leaves your hand and point 2 the height where it has the desired speed. In this case, $W_{\text{other}} = 0$.

EXECUTE: **(a)** In this case, $v_2 = v_1/2$ and $U_1 = 0$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$0 + \frac{1}{2}mv_1^2 + 0 = mgy_2 + \frac{1}{2}mv_2^2 = mgy_2 + \frac{1}{2}m\left(\frac{v_1}{2}\right)^2$. Solve for y_2 gives $y_2 = \frac{3v_1^2}{8g} = \frac{3(12.0\text{ m/s})^2}{8(9.80\text{ m/s}^2)} = 5.51$ m.

(b) In this case, $K_2 = K_1/2$ and $U_1 = 0$, so $0 + K_1 + 0 = mgy_2 + K_1/2$. Solving for y_2 gives

$y_2 = \frac{K_1}{2mg} = \frac{\frac{1}{2}mv_1^2}{2mg} = \frac{v_1^2}{4g} = \frac{(12.0\text{ m/s})^2}{4(9.80\text{ m/s}^2)} = 3.67$ m.

EVALUATE: Notice that the ball does *not* have half of its initial kinetic energy when it is half-way to the top. Likewise when it has half of its initial kinetic energy, it is *not* half-way to the top.

VP7.2.2. **IDENTIFY:** We use energy conservation. The rock has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ .Call point 1 the place where the rock leaves your hand and point 2 the height where it has the desired height. In this case, $W_{\text{other}} = 0$ and $U_1 = 0$.

EXECUTE: First find the initial speed in terms of h. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$K_1 = U_2 + K_2 \quad \to \quad \frac{1}{2}mv_1^2 = mgh \quad \to \quad v_1^2 = 2gh$.

(a) In this case, $y_2 = h/4$ and we use , so $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ gives

$\frac{1}{2}mv_1^2 = mg(h/4) + \frac{1}{2}mv_2^2 \quad \to \quad 2(2gh) - gh = 2v_2^2 \quad \to \quad v_2 = \sqrt{\frac{3gh}{2}}$.

(b) Follow the same procedure as in (a) *except* that $y_2 = 3h/4$. Energy conservation gives

$\frac{1}{2}mv_1^2 = mg(3h/4) + \frac{1}{2}mv_2^2$. Using $v_1^2 = 2gh$ gives $v_2 = \sqrt{\frac{gh}{2}}$.

EVALUATE: Our result says that the speed when $y = 3h/4$ is less than when $y_2 = h/4$, which is reasonable because the rock is slowing down as it rises.

VP7.2.3. **IDENTIFY:** We use energy conservation. The ball has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{other} = U_2 + K_2$. Call point 1 the place where the ball leaves your hand and point 2 the height where it reaches its maximum height, so $U_1 = 0$ and $K_2 = 0$. Call E the total mechanical energy of the ball, and use $W = Fs\cos\phi$.

EXECUTE: **(a)** $E_1 = K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0570 \text{ kg})(15.0 \text{ m/s})^2 = 6.4125 \text{ J}$.

$E_2 = U_2 = mgy_2 = (0.0570 \text{ kg})(9.80 \text{ m/s}^2)(8.00 \text{ m}) = 4.4688 \text{ J}$.

$\Delta E = E_2 - E_1 = 4.4688 \text{ J} - 6.4125 \text{ J} = -1.94 \text{ J}$, so the total mechanical energy has *decreased* by 1.94 J.

(b) The loss of mechanical energy is equal to the work done by the friction of air resistance.

$Fs\cos\phi = \Delta E \quad \rightarrow \quad F(8.00 \text{ m})\cos 180° = -1.94 \text{ J} \quad \rightarrow \quad F = 0.243 \text{ N}$.

EVALUATE: From our results, we have $E_2/E_1 = (4.4688 \text{ J})/(6.4125 \text{ J}) = 0.700$. We can also calculate this ratio as $E_2 / E_1 = \frac{U_2}{U_1} = \frac{mgh}{\frac{1}{2}mv_1^2} = \frac{2gh}{v_1^2} = \frac{2(9.80 \text{ m/s}^2)}{(15.0 \text{ m/s})^2} = 0.700$, so $E_2 = 0.700E_1$. The mechanical energy has decreased by 30%.

VP7.2.4. **IDENTIFY:** We use energy conservation. The ball has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{other} = U_2 + K_2$. Call point 1 the place where you catch the ball and point 2 the place where the ball stops, so $U_1 = 0$, $y_2 = -0.150$ m, and $K_2 = 0$. Use $W = Fs\cos\phi$. W_{other} is the work done by your hands in stopping the ball.

EXECUTE: **(a)** Using $U_1 = 0$, $K_2 = 0$, $U_1 + K_1 + W_{other} = U_2 + K_2$ becomes

$\frac{1}{2}mv_1^2 + W_{hands} = mgy_2$

$W_{hands} = m\left(gy_2 - \frac{v_1^2}{2}\right) = (0.270 \text{ kg})\left[(9.80 \text{ m/s}^2)(-0.150 \text{ m}) - \frac{(7.50 \text{ m/s})^2}{2}\right] = -7.99 \text{ J}$.

(b) $W = Fs\cos\phi \quad \rightarrow \quad -7.99 \text{ J} = F(0.150 \text{ m})\cos 180° \quad \rightarrow \quad F = 53.3 \text{ N}$.

EVALUATE: The work done by gravity is $W_g = mgy = (0.270 \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 39.7 \text{ J}$, which is much more than the magnitude of the work your hands do. This is reasonable because your hands also reduce (to zero) the ball's initial kinetic energy.

VP7.5.1. **IDENTIFY** We use energy conservation. The butter has kinetic energy and gravitational potential energy. Use Newton's second law for circular motion.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{other} = U_2 + K_2$. Call point 1 the place at the rim of the bowl and point 2 the bottom of the bowl, so $K_1 = 0$, $y_2 = 0$, and $W_{other} = 0$. Use $\sum F_y = ma_y$ at the bottom of the bowl, where $a_y = \frac{v^2}{R}$. At the top of the bowl, $y_1 = R$ (the radius of the bowl).

EXECUTE: **(a)** Using $K_1 = 0$, $U_2 = 0$, and $W_{other} = 0$, $U_1 + K_1 + W_{other} = U_2 + K_2$ becomes

$mgR = \frac{1}{2}mv_2^2 \quad \rightarrow \quad v_2 = \sqrt{2Rg} = \sqrt{2(0.150 \text{ m})(9.80 \text{ m/s}^2)} = 1.71 \text{ m/s}$.

(b) At the bottom of the bowl, apply Newton's second law for circular motion.

$\sum F_y = ma_y = \frac{v^2}{R} \quad \rightarrow \quad F_{bowl} - mg = mv^2/R \quad \rightarrow \quad F_{bowl} = m(g + v^2/R)$

For the numbers here we get

$F_{bowl} = (5.00\times 10^{-3} \text{ kg})[9.80 \text{ m/s}^2 + (1.71 \text{ m/s})^2/(0.150 \text{ m})] = 0.147 \text{ N}$.

$w = mg = (5.00\times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) = 0.0490 \text{ N}$.

F_{bowl}/w = (0.147 N)/(0.0490 N) = 3.00, so the force due to the bowl is 3 times the weight of the butter.
EVALUATE: The ratio F_{bowl}/w is $m(g + v^2/R)/mg = g + v^2/R$, which is independent of the butter's mass. Therefore *any* object sliding down this bowl under the same conditions would experience a force from the bowl equal to 3 times the weight of the object.

VP7.5.2. **IDENTIFY:** We use energy conservation. The snowboarder has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Call point 1 the bottom of the ditch and point 2 the highest point she reaches, so $K_2 = 0$, $U_1 = 0$.
EXECUTE: **(a)** If there is no friction, $W_{\text{other}} = 0$, so $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes
$K_1 = U_2 \quad \rightarrow \quad \frac{1}{2}mv^2 = mgh \quad \rightarrow \quad h = v^2/2g$ = (9.30 m/s^2)/[2(9.80 m/s^2)] = 4.41 m. This answer does not depend on the shape of the ditch since there is no friction.
(b) The work done by friction is equal to the loss of mechanical energy, so $W_f = U_2 - K_1$.
$W_f = mgh - \frac{1}{2}mv^2 = m(gh - v^2/2)$ = (40.0 kg)[(9.80 m/s^2)(3.50 m) – (9.30 m/s)^2/2] = –358 J.
EVALUATE: The work done by friction is negative because the friction force is opposite to the displacement of the snowboard, which agrees with our result.

VP7.5.3. **IDENTIFY:** We apply energy conservation and Newton's second law to a swinging pendulum. The small sphere (the bob) has kinetic energy and potential energy. Use Newton's second law for circular motion.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Call point 1 the high point of the swing and point 2 its low point, so $K_1 = 0$, $U_2 = 0$, and $W_{\text{other}} = 0$. Use $\sum F_y = ma_y$ at the bottom of the swing, where $a_y = \frac{v^2}{R}$. At the top of the swing, $h = L(1 - \cos\theta)$, where L is the length of the string and θ is the largest angle it makes with the vertical.
EXECUTE: **(a)** For conditions here, we find that $U_1 = K_2 = mgh = mgL(1 - \cos\theta)$, so
K_2 = (0.250 kg)(9.80 m/s^2)(1.20 m)(1 – cos 34.0°) = 0.503 J.
(b) Apply $\sum F_y = ma_y = \frac{v^2}{R}$ at the bottom of the swing: $T - mg = \frac{mv^2}{L} = \frac{2}{L}\left(\frac{1}{2}mv^2\right) = \frac{2K}{L}$.
$T = mg + 2K/L$ = (0.250 kg)(9.80 m/s^2) + 2(0.503 J)/(1.20 m) = 3.29 m.
EVALUATE: As a check, find v^2 from the known kinetic energy, giving v^2 = 4.021 m^2/s^2. Then use $T - mg = mv^2/L$ to find T. The result is the same as we found in (b).

VP7.5.4. **IDENTIFY:** We use energy conservation. The car has kinetic energy and gravitational potential energy.

SET UP: $K = \frac{1}{2}mv^2$ and $U_g = mgy$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Call A point 1 and B point 2, so U_1 = 0. In this case, $K_A = K_i$, $K_B = K_i/4$, $U_B = K_i/2$, and $y_B = 2R$.
EXECUTE: **(a)** $\Delta U_{AB} = mg(2R) = \frac{1}{2}K_i = \frac{1}{2}\left(\frac{1}{2}mv_A^2\right) \quad \rightarrow \quad v_A = \sqrt{8gR}$.
(b) Energy conservation gives $K_A + W_{\text{other}} = K_B + (U_B - U_B)$. Since W_{other} is due to friction, this becomes $K_i + W_f = \frac{1}{4}K_i + \frac{1}{2}K_i = \frac{3}{4}K_i$. Solving for W_f gives
$W_f = -\frac{1}{4}K_i = -\frac{1}{4}\left(\frac{1}{2}mv_A^2\right) = -\frac{1}{8}m(8gR) = -mgR$.

(c) Using $W_f = fs\cos\phi$ and the result from (b) gives $-mgR = f(\pi R)\cos 180° = -\pi Rf$

$f = mg/\pi$.

EVALUATE: From (c) we see that the heavier the roller coaster, the greater the friction force, which is reasonable since friction depends on the normal force at the surface of contact.

VP7.9.1. **IDENTIFY:** We use energy conservation. The system has kinetic energy and elastic potential energy in the spring.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. The mechanical energy is the kinetic energy plus the potential energy: $E = K + U$.

EXECUTE: **(a)** $E = K + U = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2$

$E = \frac{1}{2}(0.240\text{ kg})(0.400\text{ m/s})^2 + \frac{1}{2}(6.00\text{ N/m})(0.100\text{ m})^2 = 0.0492\text{ J}$.

(b) There is no friction, so E is constant. So when the glider stops, $U = 0.0492$ J. Therefore

$\frac{1}{2}kx_1^2 = 0.0492\text{ J} \quad\rightarrow\quad \frac{1}{2}(6.00\text{ N/m})\,x^2 = 0.0492\text{ J} \;\rightarrow\quad x = 0.128\text{ m}$.

EVALUATE: During most of the motion, the glider has kinetic energy and potential energy, but the sum of the two is always equal to 0.0492 J.

VP7.9.2. **IDENTIFY:** We use energy conservation. The system has kinetic energy and elastic potential energy in the spring and there is friction.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. $W = Fs\cos\phi$. Call position 1 when the spring is stretched by 0.100 m and position 2 when the glider has instantaneously stopped, so $K_2 = 0$.

EXECUTE: **(a)** $U_1 + K_1 + W_f = U_2 + K_2$ gives $\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 + W_f = \frac{1}{2}kx_2^2$. Using $k = 6.00$ N/m, $m = 0.240$ kg, $x_1 = 0.100$ m, $x_2 = 0.112$ m, and $v_1 = 0.400$ m/s gives $W_f = -0.0116$ J.

(b) $W_f = fs\cos\phi = \mu_k n(x_2 - x_1)\cos 180° = -\mu_k mg(x_2 - x_1)$. Use the result from (a) for W_f.

$-\mu_k(0.240\text{ kg})(9.80\text{ m/s}^2)(0.112\text{ m} - 0.100\text{ m}) = -0.0116\text{ J} \quad\rightarrow\quad \mu_k = 0.410$.

EVALUATE: From Table 5.1 we see that our result is very reasonable for metal-on-metal coefficients of kinetic friction. For example, $\mu_k = 0.47$ for aluminum on steel.

VP7.9.3. **IDENTIFY:** We use energy conservation. The system has kinetic energy, gravitational potential energy, and elastic potential energy, but there is no friction.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, and $W_{\text{other}} = 0$.

EXECUTE: **(a)** Take point 1 to be the the instant the compressed spring is released and point 2 to be just when the block loses contact with the spring, so $K_1 = 0$ and $U_2 = 0$. This gives

$U_{\text{spring}} + U_g = K_2 \quad\rightarrow\quad \frac{1}{2}kd^2 - mgd = \frac{1}{2}mv_2^2 \quad\rightarrow\quad v_2 = \sqrt{\frac{kd^2}{m} - 2gd}$.

(b) After the block clears the spring, we reapply $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$. Now call point 1 to be the instant it has left the spring and point 2 to be its maximum height. The v_1 in this part is the v_2 we found in part (a). Let maximum height be h.

$\frac{1}{2}mv_1^2 = mgh \quad \rightarrow \quad h = \frac{v_1^2}{2g}$ The *total* vertical distance the block travels is $D = d + h$, so

$D = d + \frac{v_1^2}{2g}$. Using the value of v_2 (which is v_1 for this part) gives

$$D = d + \frac{\frac{kd^2}{m} - 2gd}{2g} = d + \frac{kd^2}{2mg} - d = \frac{kd^2}{2mg}.$$

EVALUATE: Check units to be sure that both answers have the proper dimensions of length.

VP7.9.4. **IDENTIFY:** We use energy conservation. The system has kinetic energy, gravitational potential energy, and elastic potential energy, but there is no friction.

SET UP: $K = \frac{1}{2}mv^2$ and $U = \frac{1}{2}kx^2$. $U_1 + K_1 + W_{other} = U_2 + K_2$, and $W_{other} = W_f$. Call point 1 to be at the place where the cylinder is released with the spring relaxed, and point 2 to be at the maximum elongation of the spring, so $K_1 = 0$, $U_1 = 0$, and $K_2 = 0$. Call x the maximum elongation of the spring. The friction force and gravity both act through a distance x, so $W_f = -fx$ and $U_{2\text{-grav}} = -mgx$. This energy is negative because the cylinder is *below* point 1.

EXECUTE: $U_1 + K_1 + W_{other} = U_2 + K_2$ gives $-fx = \frac{1}{2}kx^2 - mgx$. Solving for x gives $x = 2(mg - f)/k$.

EVALUATE: Check units to be sure that the answer has the proper dimensions of length.

7.1. **IDENTIFY:** $U_{grav} = mgy$ so $\Delta U_{grav} = mg(y_2 - y_1)$

SET UP: $+y$ is upward.

EXECUTE: **(a)** $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(2400\text{ m} - 1500\text{ m}) = +6.6\times10^5\text{ J}$

(b) $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(1350\text{ m} - 2400\text{ m}) = -7.7\times10^5\text{ J}$

EVALUATE: U_{grav} increases when the altitude of the object increases.

7.2. **IDENTIFY:** The change in height of a jumper causes a change in their potential energy.

SET UP: Use $\Delta U_{grav} = mg(y_2 - y_1)$.

EXECUTE: $\Delta U_{grav} = (72\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ m}) = 420\text{ J}$.

EVALUATE: This gravitational potential energy comes from elastic potential energy stored in the jumper's tensed muscles.

7.3. **IDENTIFY:** Use the free-body diagram for the bag and Newton's first law to find the force the worker applies. Since the bag starts and ends at rest, $K_2 - K_1 = 0$ and $W_{tot} = 0$.

SET UP: A sketch showing the initial and final positions of the bag is given in Figure 7.3a. $\sin\phi = \frac{2.0\text{ m}}{3.5\text{ m}}$ and $\phi = 34.85°$. The free-body diagram is given in Figure 7.3b. $\vec{F}$ is the horizontal force applied by the worker. In the calculation of U_{grav} take $+y$ upward and $y = 0$ at the initial position of the bag.

EXECUTE: **(a)** $\Sigma F_y = 0$ gives $T\cos\phi = mg$ and $\Sigma F_x = 0$ gives $F = T\sin\phi$. Combining these equations to eliminate T gives $F = mg\tan\phi = (90.0\text{ kg})(9.80\text{ m/s}^2)\tan 34.85° = 610\text{ N}$.

(b) (i) The tension in the rope is radial and the displacement is tangential so there is no component of T in the direction of the displacement during the motion and the tension in the rope does no work.

(ii) $W_{tot} = 0$ so

$$W_{worker} = -W_{grav} = U_{grav,2} - U_{grav,1} = mg(y_2 - y_1) = (90.0\text{ kg})(9.80\text{ m/s}^2)(0.6277\text{ m}) = 550\text{ J}.$$

EVALUATE: The force applied by the worker varies during the motion of the bag and it would be difficult to calculate W_{worker} directly.

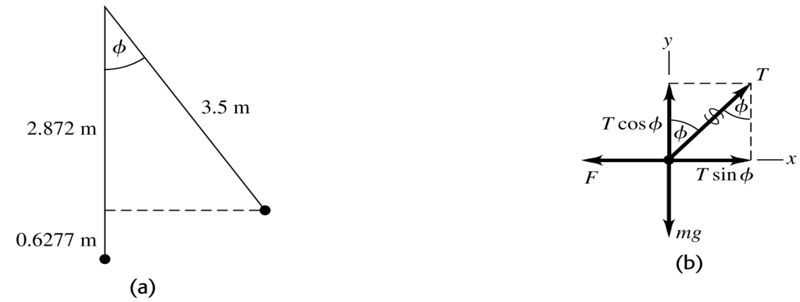


Figure 7.3

7.4. IDENTIFY: The energy from the food goes into the increased gravitational potential energy of the hiker. We must convert food calories to joules.

SET UP: The change in gravitational potential energy is $\Delta U_{\text{grav}} = mg\,(y_f - y_i)$, while the increase in kinetic energy is negligible. Set the food energy, expressed in joules, equal to the mechanical energy developed.

EXECUTE: **(a)** The food energy equals $mg(y_2 - y_1)$, so

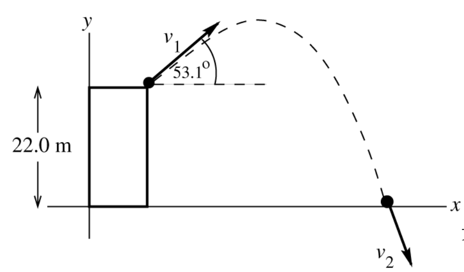
$$y_2 - y_1 = \frac{(140 \text{ food calories})(4186 \text{ J/1 food calorie})}{(65 \text{ kg})(9.80 \text{ m/s}^2)} = 920 \text{ m}.$$

(b) The mechanical energy would be 20% of the results of part (a), so $\Delta y = (0.20)(920 \text{ m}) = 180 \text{ m}$.

EVALUATE: Since only 20% of the food calories go into mechanical energy, the hiker needs much less of a climb to turn off the calories in the bar.

7.5. IDENTIFY and SET UP: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 are shown in Figure 7.5.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Solve for K_2 and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain v_2.



$W_{\text{other}} = 0$ (The only force on the ball while it is in the air is gravity.)

$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1$, $y_1 = 22.0$ m

$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

Figure 7.5

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(22.0 \text{ m})} = 24.0 \text{ m/s}$$

EVALUATE: The projection angle of 53.1° doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

(b) Nothing changes in the calculation. The expression derived in part (a) for v_2 is independent of the angle, so $v_2 = 24.0$ m/s, the same as in part (a).

(c) The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

7.6. **IDENTIFY:** The normal force does no work, so only gravity does work and $K_1 + U_1 = K_2 + U_2$ applies.

SET UP: $K_1 = 0$. The crate's initial point is at a vertical height of $d\sin\alpha$ above the bottom of the ramp.

EXECUTE: **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$. $mgd\sin\alpha = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

(b) $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $0 = K_2 + U_{\text{grav},2}$. $0 = \frac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

(c) The normal force is perpendicular to the displacement and does no work.

EVALUATE: When we use $U_{\text{grav}} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

7.7. **IDENTIFY:** The take-off kinetic energy of the flea goes into gravitational potential energy.

SET UP: Use $K_1 + U_1 = K_2 + U_2$. Let $y_1 = 0$ and $y_2 = h$ and note that $U_1 = 0$ while $K_2 = 0$ at the maximum height. Consequently, conservation of energy becomes $mgh = \frac{1}{2}mv_1^2$.

EXECUTE: **(a)** $v_1 = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(0.20\text{ m})} = 2.0\text{ m/s}$.

(b) $K_1 = mgh = (0.50\times 10^{-6}\text{ kg})(9.80\text{ m/s}^2)(0.20\text{ m}) = 9.8\times 10^{-7}\text{ J}$. The kinetic energy per kilogram is

$$\frac{K_1}{m} = \frac{9.8\times 10^{-7}\text{ J}}{0.50\times 10^{-6}\text{ kg}} = 2.0\text{ J/kg}.$$

(c) The human can jump to a height of $h_\text{h} = h_\text{f}\left(\dfrac{l_\text{h}}{l_\text{f}}\right) = (0.20\text{ m})\left(\dfrac{2.0\text{ m}}{2.0\times 10^{-3}\text{ m}}\right) = 200\text{ m}$. To attain this height, he would require a takeoff speed of: $v_1 = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(200\text{ m})} = 63\text{ m/s}$.

(d) The human's kinetic energy per kilogram is $\dfrac{K_1}{m} = gh = (9.80\text{ m/s}^2)(0.60\text{ m}) = 5.9\text{ J/kg}$.

(e) **EVALUATE:** The flea stores the energy in its tensed legs.

7.8. **IDENTIFY:** This problem involves kinetic energy and gravitational potential energy.

SET UP: Estimates: maximum speed is 2.5 m/s, mass is 70 kg. $U_g = mgh$, $K = \dfrac{1}{2}mv^2$.

EXECUTE: **(a)** $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(70\text{ kg})(2.5\text{ m/s})^2 = 220\text{ J}$.

(b) $U_\text{g} = mgh$, so $h = U_\text{g}/mg = (220\text{ J})/[(70\text{ kg})(9.8\text{ m/s}^2)] = 0.32\text{ m}$.

EVALUATE: These are reasonable values since we put in reasonable estimates.

7.9. **IDENTIFY:** $W_\text{tot} = K_B - K_A$. The forces on the rock are gravity, the normal force and friction.

SET UP: Let $y = 0$ at point B and let $+y$ be upward. $y_A = R = 0.50\text{ m}$. The work done by friction is negative; $W_f = -0.22\text{ J}$. $K_A = 0$. The free-body diagram for the rock at point B is given in Figure 7.9. The acceleration of the rock at this point is $a_\text{rad} = v^2/R$, upward.

EXECUTE: **(a)** (i) The normal force is perpendicular to the displacement and does zero work.

(ii) $W_\text{grav} = U_{\text{grav},A} - U_{\text{grav},B} = mgy_A = (0.20\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m}) = 0.98\text{ J}$.

(b) $W_\text{tot} = W_n + W_f + W_\text{grav} = 0 + (-0.22\text{ J}) + 0.98\text{ J} = 0.76\text{ J}$. $W_\text{tot} = K_B - K_A$ gives $\frac{1}{2}mv_B^2 = W_\text{tot}$.

$$v_B = \sqrt{\frac{2W_\text{tot}}{m}} = \sqrt{\frac{2(0.76\text{ J})}{0.20\text{ kg}}} = 2.8\text{ m/s}.$$

(c) Gravity is constant and equal to mg. n is not constant; it is zero at A and not zero at B. Therefore, $f_\text{k} = \mu_\text{k} n$ is also not constant.

(d) $\Sigma F_y = ma_y$ applied to Figure 7.9 gives $n - mg = ma_{\text{rad}}$.

$$n = m\left(g + \frac{v^2}{R}\right) = (0.20\text{ kg})\left(9.80\text{ m/s}^2 + \frac{[2.8\text{ m/s}]^2}{0.50\text{ m}}\right) = 5.1\text{ N}.$$

EVALUATE: In the absence of friction, the speed of the rock at point B would be $\sqrt{2gR} = 3.1$ m/s. As the rock slides through point B, the normal force is greater than the weight $mg = 2.0$ N of the rock.

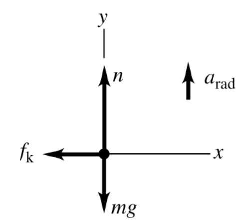


Figure 7.9

7.10. IDENTIFY: The child's energy is transformed from gravitational potential energy to kinetic energy as she swings downward.

SET UP: Let $y_2 = 0$. For part (a), $U_1 = mgy_1$. For part (b) use $K_2 + U_2 = K_1 + U_1$ with $U_2 = K_1 = 0$ and $K_2 = \frac{1}{2}mv_2{}^2$; the result is $\frac{1}{2}mv_2{}^2 = mgy_1$.

EXECUTE: **(a)** Figure 7.10 shows that the difference in potential energy at the top of the swing is proportional to the height difference, $y_1 = (2.20\text{ m})(1 - \cos 42°) = 0.56$ m. The difference in potential energy is thus $U_1 = mgy_1 =$ $(25\text{ kg})(9.80\text{ m/s}^2)(0.56\text{ m}) = 140$ J.

(b) $v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(0.56\text{ m})} = 3.3\text{ m/s}$.

EVALUATE: (c) The tension is radial while the displacement is tangent to the circular path; thus there is no component of the tension along the direction of the displacement and the tension in the ropes does no work on the child.

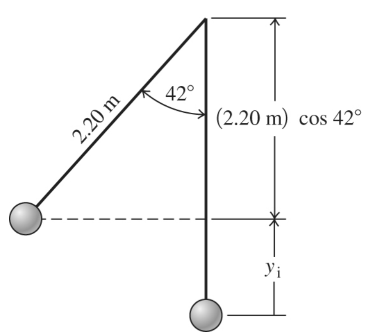


Figure 7.10

7.11. IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the car.

SET UP: Take $y = 0$ at point A. Let point 1 be A and point 2 be B.

EXECUTE: $U_1 = 0$, $U_2 = mg(2R) = 28{,}224$ J, $W_{\text{other}} = W_f$

$K_1 = \frac{1}{2}mv_1^2 = 37{,}500$ J, $K_2 = \frac{1}{2}mv_2^2 = 3840$ J

The work-energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400$ J.

EVALUATE: Friction does negative work. The final mechanical energy $(K_2 + U_2 = 32{,}064\ \text{J})$ is less than the initial mechanical energy $(K_1 + U_1 = 37{,}500\ \text{J})$ because of the energy removed by friction work.

7.12. **IDENTIFY:** Only gravity does work, so apply $K_1 + U_1 = K_2 + U_2$.

SET UP: $v_1 = 0$, so $\frac{1}{2}mv_2^2 = mg(y_1 - y_2)$.

EXECUTE: Tarzan is lower than his original height by a distance $y_1 - y_2 = l(\cos 30° - \cos 45°)$ so his speed is $v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9\ \text{m/s}$, a bit quick for conversation.

EVALUATE: The result is independent of Tarzan's mass.

7.13. **IDENTIFY:** This problem involves kinetic energy, gravitational potential energy, and energy conservation.

SET UP: $U_g = mgh$, $K = \dfrac{1}{2}mv^2$, $U_1 + K_1 = U_2 + K_2$.

EXECUTE: (a) $\Delta U_6 = m_6 g\Delta y$ = (6.00 kg)(9.80 /s^2)(0.200 m) = 11.8 J

$\Delta U_8 = m_8 g\Delta y$ = (8.00 kg)(9.80 m/s^2)(–0.200 m) = –15.7 J.

(b) $W_6 = T\Delta y$ = (0.200 m)T and $W_8 = T\Delta y$ = (–0.200 m)T.

(c) From part (b), we have W_T = (0.200 m)T + (–0.200 m)T = 0.

$\Delta U_g = \Delta U_6 + \Delta U_8$ = 11.8 J – 15.7 J = –3.9 J, and K_1 = 0.

$U_1 + K_1 = U_2 + K_2$ gives $K_2 = U_1 - U_2 = -(U_2 - U_1) = -\Delta U$ = –(–3.9 J) = 3.9 J.

$$K_2 = \frac{1}{2}mv_2^2 = -\Delta U \quad \rightarrow \quad v_2 = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2(-3.9\ \text{J})}{14\ \text{kg}}} = 0.75\ \text{m/s}.$$

EVALUATE: To check, we could find v_2 using Newton's second law. Treating the two masses as a single system gives $m_8 g - m_6 g = (m_8 + m_6)a$. Kinematics gives $a = \dfrac{v_2^2}{2\Delta y}$. Combining these equations and putting in the numbers gives v_2 = 0.75 m/s.

7.14. **IDENTIFY:** Use the information given in the problem with $F = kx$ to find k. Then $U_{el} = \frac{1}{2}kx^2$.

SET UP: x is the amount the spring is stretched. When the weight is hung from the spring, $F = mg$.

EXECUTE: $k = \dfrac{F}{x} = \dfrac{mg}{x} = \dfrac{(3.15\ \text{kg})(9.80\ \text{m/s}^2)}{0.1340\ \text{m} - 0.1200\ \text{m}} = 2205\ \text{N/m}.$

$x = \pm\sqrt{\dfrac{2U_{el}}{k}} = \pm\sqrt{\dfrac{2(10.0\ \text{J})}{2205\ \text{N/m}}} = \pm 0.0952\ \text{m} = \pm 9.52\ \text{cm}.$ The spring could be either stretched 9.52 cm or compressed 9.52 cm. If it were stretched, the total length of the spring would be $12.00\ \text{cm} + 9.52\ \text{cm} = 21.52\ \text{cm}.$ If it were compressed, the total length of the spring would be $12.00\ \text{cm} - 9.52\ \text{cm} = 2.48\ \text{cm}.$

EVALUATE: To stretch or compress the spring 9.52 cm requires a force $F = kx = 210\ \text{N}.$

7.15. **IDENTIFY:** Apply $U_{el} = \frac{1}{2}kx^2$.

SET UP: $kx = F$, so $U_{el} = \frac{1}{2}Fx$, where F is the magnitude of force required to stretch or compress the spring a distance x.

EXECUTE: **(a)** $(1/2)(520\ \text{N})(0.200\ \text{m}) = 52.0\ \text{J}.$

(b) The potential energy is proportional to the square of the compression or extension; $(52.0\ \text{J})\ (0.050\ \text{m}/0.200\ \text{m})^2 = 3.25\ \text{J}.$

EVALUATE: We could have calculated $k = \dfrac{F}{x} = \dfrac{520\text{ N}}{0.200\text{ m}} = 2600\text{ N/m}$ and then used $U_{\text{el}} = \frac{1}{2}kx^2$ directly.

7.16. IDENTIFY: We treat the tendon like a spring and apply Hooke's law to it. Knowing the force stretching the tendon and how much it stretched, we can find its force constant.

SET UP: Use $F_{\text{on tendon}} = kx$. In part (a), $F_{\text{on tendon}}$ equals mg, the weight of the object suspended from it. In part (b), also apply $U_{\text{el}} = \frac{1}{2}kx^2$ to calculate the stored energy.

EXECUTE: **(a)** $k = \dfrac{F_{\text{on tendon}}}{x} = \dfrac{(0.250\text{ kg})(9.80\text{ m/s}^2)}{0.0123\text{ m}} = 199\text{ N/m}.$

(b) $x = \dfrac{F_{\text{on tendon}}}{k} = \dfrac{138\text{ N}}{199\text{ N/m}} = 0.693\text{m} = 69.3\text{ cm};\ \ U_{\text{el}} = \frac{1}{2}(199\text{ N/m})(0.693\text{ m})^2 = 47.8\text{ J}.$

EVALUATE: The 250 g object has a weight of 2.45 N. The 138 N force is much larger than this and stretches the tendon a much greater distance.

7.17. IDENTIFY: Apply $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: $U_0 = \frac{1}{2}kx_0^2$. x is the distance the spring is stretched or compressed.

EXECUTE: **(a)** (i) $x = 2x_0$ gives $U_{\text{el}} = \frac{1}{2}k(2x_0)^2 = 4(\frac{1}{2}kx_0^2) = 4U_0$. (ii) $x = x_0/2$ gives $U_{\text{el}} = \frac{1}{2}k(x_0/2)^2 = \frac{1}{4}(\frac{1}{2}kx_0^2) = U_0/4$.

(b) (i) $U = 2U_0$ gives $\frac{1}{2}kx^2 = 2(\frac{1}{2}kx_0^2)$ and $x = x_0\sqrt{2}$. (ii) $U = U_0/2$ gives $\frac{1}{2}kx^2 = \frac{1}{2}(\frac{1}{2}kx_0^2)$ and $x = x_0/\sqrt{2}$.

EVALUATE: U is proportional to x^2 and x is proportional to $\sqrt{U}$.

7.18. IDENTIFY: This problem involves energy conservation, elastic potential energy, and kinetic energy.

SET UP: $K = \dfrac{1}{2}mv^2$ and $U_{\text{spring}} = \dfrac{1}{2}kx^2$. The elastic potential energy in the spring is transferred to the block as kinetic energy.

EXECUTE: $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kx^2 \quad \rightarrow \quad v = x\sqrt{k/m} = d\sqrt{k/m}.$

EVALUATE: Our result says that for a given mass and compression distance, the larger that k is the greater that v will be. A large k means a stiff spring, so our result is reasonable.

7.19. IDENTIFY and SET UP: Use energy methods. There are changes in both elastic and gravitational potential energy; elastic; $U = \frac{1}{2}kx^2$, gravitational: $kx - mg = ma$,

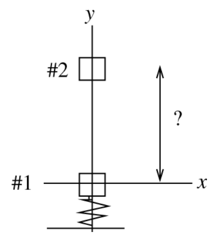
EXECUTE: **(a)** $U_{\text{el}} = \frac{1}{2}kx^2$ so $x = \sqrt{\dfrac{2U_{\text{el}}}{k}} = \sqrt{\dfrac{2(1.20\text{ J})}{800\text{ N/m}}} = 0.0548\text{ m} = 5.48\text{ cm}.$

(b) The work done by gravity is equal to the gain in elastic potential energy: $W_{\text{grav}} = U_{\text{el}}$.
$mgx = ½\, kx^2$, so $x = 2mg/k = 2(1.60\text{ kg})(9.80\text{ m/s}^2)/(800\text{ N/m}) = 0.0392\text{ m} = 3.92\text{ cm}$.

EVALUATE: When the spring is compressed 3.92 cm, it exerts an upward force of 31.4 N on the book, which is greater than the weight of the book (15.6 N). The book will be accelerated upward from this position.

7.20. IDENTIFY: Use energy methods. There are changes in both elastic and gravitational potential energy.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.20.



The spring force and gravity are the only forces doing work on the cheese, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}} + U_{\text{el}}$.

Figure 7.20

EXECUTE: Cheese released from rest implies $K_1 = 0$.
At the maximum height $v_2 = 0$ so $K_2 = 0$. $U_1 = U_{1,\text{el}} + U_{1,\text{grav}}$
$y_1 = 0$ implies $U_{1,\text{grav}} = 0$
$U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(1800\text{ N/m})(0.15\text{ m})^2 = 20.25\text{ J}$
(Here x_1 refers to the amount the spring is stretched or compressed when the cheese is at position 1; it is *not* the *x*-coordinate of the cheese in the coordinate system shown in the sketch.)
$U_2 = U_{2,\text{el}} + U_{2,\text{grav}}$ $U_{2,\text{grav}} = mgy_2$, where y_2 is the height we are solving for. $U_{2,\text{el}} = 0$ since now the spring is no longer compressed. Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_{1,\text{el}} = U_{2,\text{grav}}$

$$y_2 = \frac{20.25\text{ J}}{mg} = \frac{20.25\text{ J}}{(1.20\text{ kg})(9.80\text{ m/s}^2)} = 1.72\text{ m}$$

EVALUATE: The description in terms of energy is very simple; the elastic potential energy originally stored in the spring is converted into gravitational potential energy of the system.

7.21. **IDENTIFY:** The energy of the book-spring system is conserved. There are changes in both elastic and gravitational potential energy.
SET UP: $U_{\text{el}} = \frac{1}{2}kx^2$, $U_{\text{grav}} = mgy$, $W_{\text{other}} = 0$.

EXECUTE: **(a)** $U = \frac{1}{2}kx^2$ so $x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(3.20\text{ J})}{1600\text{ N/m}}} = 0.0632\text{ m} = 6.32\text{ cm}$

(b) Points 1 and 2 in the motion are sketched in Figure 7.21. We have $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}} = 0$ (only work is that done by gravity and spring force), $K_1 = 0$, $K_2 = 0$, and $y = 0$ at final position of book. Using $U_1 = mg(h+d)$ and $U_2 = \frac{1}{2}kd^2$ we obtain $0 + mg(h+d) + 0 = \frac{1}{2}kd^2$. The original gravitational potential energy of the system is converted into potential energy of the compressed spring. Finally, we use the quadratic formula to solve for *d*: $\frac{1}{2}kd^2 - mgd - mgh = 0$, which gives $d = \frac{1}{k}\left(mg \pm \sqrt{(mg)^2 + 4\left(\frac{1}{2}k\right)(mgh)}\right)$. In our analysis we have assumed that *d* is positive, so we get

$$d = \frac{(1.20\text{ kg})(9.80\text{ m/s}^2) + \sqrt{\left[(1.20\text{ kg})(9.80\text{ m/s}^2)\right]^2 + 2(1600\text{ N/m})(1.20\text{ kg})(9.80\text{ m/s}^2)(0.80\text{ m})}}{1600\text{ N/m}},$$

which gives $d = 0.12$ m = 12 cm.
EVALUATE: It was important to recognize that the total displacement was $h + d$; gravity continues to do work as the book moves against the spring. Also note that with the spring compressed 0.12 m it exerts an upward force (192 N) greater than the weight of the book (11.8 N). The book will be accelerated upward from this position.

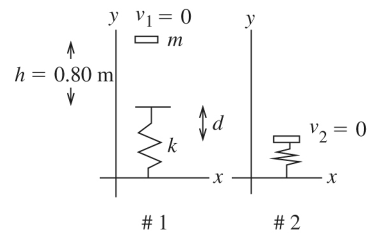


Figure 7.21

7.22. **(a) IDENTIFY and SET UP:** Use energy methods. Both elastic and gravitational potential energy changes. Work is done by friction.

Choose point 1 and let that be the origin, so $y_1 = 0$. Let point 2 be 1.00 m below point 1, so $y_2 = -1.00$ m.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000 \text{ kg})(4.0 \text{ m/s})^2 = 16{,}000 \text{ J}, \ \ U_1 = 0$

$W_{\text{other}} = -f\,|y_2| = -(17{,}000 \text{ N})(1.00 \text{ m}) = -17{,}000 \text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$U_2 = U_{2,\text{grav}} + U_{2,\text{el}} = mgy_2 + \frac{1}{2}ky_2^2$

$U_2 = (2000 \text{ kg})(9.80 \text{ m/s}^2)(-1.00 \text{ m}) + \frac{1}{2}(1.06\times10^4 \text{ N/m})(1.00 \text{ m})^2$

$U_2 = -19{,}600 \text{ J} + 5300 \text{ J} = -14{,}300 \text{ J}$

Thus $16{,}000 \text{ J} - 17{,}000 \text{ J} = \frac{1}{2}mv_2^2 - 14{,}300 \text{ J}$

$\frac{1}{2}mv_2^2 = 13{,}300 \text{ J}$

$$v_2 = \sqrt{\frac{2(13{,}300 \text{ J})}{2000 \text{ kg}}} = 3.65 \text{ m/s}$$

EVALUATE: The elevator stops after descending 3.00 m. After descending 1.00 m it is still moving but has slowed down.

(b) IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the elevator. We know the forces and can solve for $\vec{a}$.

SET UP: The free-body diagram for the elevator is given in Figure 7.22.

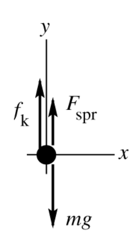


Figure 7.22

EXECUTE: $F_{\text{spr}} = kd$, where *d* is the distance the spring is compressed

$\Sigma F_y = ma_y$

$f_k + F_{\text{spr}} - mg = ma$

$f_k + kd - mg = ma$

$$a = \frac{f_k + kd - mg}{m} = \frac{17{,}000 \text{ N} + (1.06\times10^4 \text{ N/m})(1.00 \text{ m}) - (2000 \text{ kg})(9.80 \text{ m/s}^2)}{2000 \text{ kg}} = 4.00 \text{ m/s}^2$$

We calculate that *a* is positive, so the acceleration is upward.

EVALUATE: The velocity is downward and the acceleration is upward, so the elevator is slowing down at this point.

7.23. **IDENTIFY:** Only the spring does work and $K_1 + U_1 = K_2 + U_2$ applies. $a = \frac{F}{m} = \frac{-kx}{m}$, where F is the force the spring exerts on the mass.

SET UP: Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5$ J. Let point 2 be where the mass leaves the spring, so $U_{el,2} = 0$.

EXECUTE: **(a)** $K_1 + U_{el,1} = K_2 + U_{el,2}$ gives $U_{el,1} = K_2$. $\frac{1}{2}mv_2^2 = U_{el,1}$ and

$$v_2 = \sqrt{\frac{2U_{el,1}}{m}} = \sqrt{\frac{2(11.5\text{ J})}{2.50\text{ kg}}} = 3.03\text{ m/s}.$$

K is largest when U_{el} is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

(b) The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{el} = \frac{1}{2}kx^2$ so $x = -\sqrt{\frac{2U_{el}}{k}} = -\sqrt{\frac{2(11.5\text{ J})}{2500\text{ N/m}}} = -0.0959$ m. The minus sign indicates compression. $F = -kx = ma_x$ and $a_x = -\frac{kx}{m} = -\frac{(2500\text{ N/m})(-0.0959\text{ m})}{2.50\text{ kg}} = 95.9\text{ m/s}^2$.

EVALUATE: If the end of the spring is displaced to the left when the spring is compressed, then a_x in part (b) is to the right, and vice versa.

7.24. **IDENTIFY:** The spring force is conservative but the force of friction is nonconservative. Energy is conserved during the process. Initially all the energy is stored in the spring, but part of this goes to kinetic energy, part remains as elastic potential energy, and the rest does work against friction.

SET UP: Energy conservation: $K_1 + U_1 + W_{other} = K_2 + U_2$, the elastic energy in the spring is $U = \frac{1}{2}kx^2$, and the work done by friction is $W_f = -f_k s = -\mu_k mgs$.

EXECUTE: The initial and final elastic potential energies are
$U_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(840\text{ N/m})(0.0300\text{ m})^2 = 0.378$ J and $U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}(840\text{ N/m})(0.0100\text{ m})^2 = 0.0420$ J.
The initial and final kinetic energies are $K_1 = 0$ and $K_2 = \frac{1}{2}mv_2^2$. The work done by friction is
$W_{other} = W_{f_k} = -f_k s = -\mu_k mgs = -(0.40)(2.50\text{ kg})(9.8\text{ m/s}^2)(0.0200\text{ m}) = -0.196$ J. Energy conservation gives $K_2 = \frac{1}{2}mv_2^2 = K_1 + U_1 + W_{other} - U_2 = 0.378\text{ J} + (-0.196\text{ J}) - 0.0420\text{ J} = 0.140$ J. Solving for v_2 gives $v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.140\text{ J})}{2.50\text{ kg}}} = 0.335$ m/s.

EVALUATE: Mechanical energy is not conserved due to friction.

7.25. **IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ and $F = ma$.

SET UP: $W_{other} = 0$. There is no change in U_{grav}. $K_1 = 0$, $U_2 = 0$.

EXECUTE: $\frac{1}{2}kx^2 = \frac{1}{2}mv_x^2$. The relations for m, v_x, k and x are $kx^2 = mv_x^2$ and $kx = 5mg$.

Dividing the first equation by the second gives $x = \frac{v_x^2}{5g}$, and substituting this into the second gives

$$k = 25\frac{mg^2}{v_x^2}.$$

(a) $k = 25\frac{(1160\text{ kg})(9.80\text{ m/s}^2)^2}{(2.50\text{ m/s})^2} = 4.46\times 10^5$ N/m

(b) $x = \frac{(2.50\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 0.128$ m

EVALUATE: Our results for k and x do give the required values for a_x and v_x:

$$a_x = \frac{kx}{m} = \frac{(4.46\times10^5\ \text{N/m})(0.128\ \text{m})}{1160\ \text{kg}} = 49.2\ \text{m/s}^2 = 5.0g \ \text{ and } \ v_x = x\sqrt{\frac{k}{m}} = 2.5\ \text{m/s}.$$

7.26. IDENTIFY: This problem involves the work to stretch a spring and the energy stored in the spring.

SET UP: $U_{\text{spring}} = \frac{1}{2}kx^2$ and $F = kx$ (Hooke's law).

EXECUTE: **(a)** At a 2.00-cm stretch: $F = kx$ becomes $5.00\ \text{N} = k(2.00\ \text{cm})$. At the additional stretch, the spring is now stretched 6.00 cm, so $F = k(6.00\ \text{cm})$. This gives $\frac{F}{5.00\ \text{N}} = \frac{k(6.00\ \text{cm})}{k(2.00\ \text{cm})}$, which gives $F = 15.0$ N.

(b) At 2.00-cm stretch, $U_2 = \frac{1}{2}kx^2 = \frac{1}{2}k(2.00\ \text{cm})^2$ and $U_6 = \frac{1}{2}k(6.00\ \text{cm})^2$. Therefore

$$\frac{U_6}{U_2} = \frac{\frac{1}{2}k(6.00\ \text{cm})^2}{\frac{1}{2}k(2.00\ \text{cm})^2} = 9.00.$$

EVALUATE: The force increases by a factor of 3 but the stored energy increases by a factor of 9 because the energy is proportional to the *square* of x while the force is only proportional to x. We could have solved this problem by first finding k and then using the force and energy formulas. However by taking ratios we never needed to know k and we did not have to convert any units.

7.27. IDENTIFY: Since the force is constant, use $W = Fs\cos\phi$.

SET UP: For both displacements, the direction of the friction force is opposite to the displacement and $\phi = 180°$.

EXECUTE: **(a)** When the book moves to the left, the friction force is to the right, and the work is $-(1.8\ \text{N})(3.0\ \text{m}) = -5.4\ \text{J}$.

(b) The friction force is now to the left, and the work is again -5.4 J.

(c) The total work is sum of the work in both directions, which is –10.8 J.

(d) The net work done by friction for the round trip is not zero, so friction is not a conservative force.

EVALUATE: The direction of the friction force depends on the motion of the object. For the gravity force, which is conservative, the force does not depend on the motion of the object.

7.28. IDENTIFY and SET UP: The force is not constant so we must integrate to calculate the work. $W = \int_1^2 \vec{F}\cdot d\vec{l}$, $\vec{F} = -\alpha x^2\hat{i}$.

EXECUTE: **(a)** $d\vec{l} = dy\hat{j}$ (x is constant; the displacement is in the $+y$-direction)

$\vec{F}\cdot d\vec{l} = 0$ (since $\hat{i}\cdot\hat{j} = 0$) and thus $W = 0$.

(b) $d\vec{l} = dx\hat{i}$

$\vec{F}\cdot d\vec{l} = (-\alpha x^2\hat{i})\cdot(dx\hat{i}) = -\alpha x^2\,dx$

$$W = \int_{x_1}^{x_2}(-\alpha x^2)\,dx = -\tfrac{1}{3}\alpha x^3\Big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha\,(x_2^3 - x_1^3) = -\frac{12\ \text{N/m}^2}{3}\left[(0.300\ \text{m})^3 - (0.10\ \text{m})^3\right] = -0.10\ \text{J}$$

(c) $d\vec{l} = dx\hat{i}$ as in part (b), but now $x_1 = 0.30$ m and $x_2 = 0.10$ m, so $W = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10$ J.

(d) EVALUATE: The total work for the displacement along the x-axis from 0.10 m to 0.30 m and then back to 0.10 m is the sum of the results of parts (b) and (c), which is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.

EXECUTE: $W_{x_1 \to x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = \tfrac{1}{3}\alpha x_1^3 - \tfrac{1}{3}\alpha x_2^3$

The definition of the potential energy function is $W_{x_1 \to x_2} = U_1 - U_2$. Comparison of the two expressions for *W* gives $U = \tfrac{1}{3}\alpha x^3$. This does correspond to $U = 0$ when $x = 0$.

EVALUATE: In part (a) the work done is zero because the force and displacement are perpendicular. In part (b) the force is directed opposite to the displacement and the work done is negative. In part (c) the force and displacement are in the same direction and the work done is positive.

7.29. **IDENTIFY:** Some of the mechanical energy of the skier is converted to internal energy by the nonconservative force of friction on the rough patch. Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

SET UP: For part (a) use $E_{\text{mech, 2}} = E_{\text{mech, 1}} - f_k s$ where $f_k = \mu_k mg$. Let $y_2 = 0$ at the bottom of the hill; then $y_1 = 2.50\text{ m}$ along the rough patch. The energy equation is $\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}mv_1^2 + mgy_1 - \mu_k mgs$. Solving for her final speed gives $v_2 = \sqrt{v_1^2 + 2gy_1 - 2\mu_k gs}$. For part (b), the internal energy is calculated as the negative of the work done by friction: $-W_f = +f_k s = +\mu_k mgs$.

EXECUTE: **(a)** $v_2 = \sqrt{(6.50\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(2.50\text{ m}) - 2(0.300)(9.80\text{ m/s}^2)(4.20\text{ m})} = 8.16\text{ m/s}.$

(b) Internal energy $= \mu_k mgs = (0.300)(62.0\text{ kg})(9.80\text{ m/s}^2)(4.20\text{ m}) = 766\text{ J}.$

EVALUATE: Without friction the skier would be moving faster at the bottom of the hill than at the top, but in this case she is moving *slower* because friction converted some of her initial kinetic energy into internal energy.

7.30. **IDENTIFY:** Some of the initial gravitational potential energy is converted to kinetic energy, but some of it is lost due to work by the nonconservative friction force.

SET UP: The energy of the box at the edge of the roof is given by: $E_{\text{mech, f}} = E_{\text{mech, i}} - f_k s$. Setting $y_f = 0$ at this point, $y_i = (4.25\text{ m})\sin 36^\circ = 2.50\text{ m}$. Furthermore, by substituting $K_i = 0$ and $K_f = \tfrac{1}{2}mv_f^2$ into the conservation equation, $\tfrac{1}{2}mv_f^2 = mgy_i - f_k s$ or $v_f = \sqrt{2gy_i - 2f_k sg/w} = \sqrt{2g(y_i - f_k s/w)}$.

EXECUTE: $v_f = \sqrt{2(9.80\text{ m/s}^2)\left[(2.50\text{ m}) - (22.0\text{ N})(4.25\text{ m})/(85.0\text{ N})\right]} = 5.24\text{ m/s}.$

EVALUATE: Friction does negative work and removes mechanical energy from the system. In the absence of friction the final speed of the toolbox would be 7.00 m/s.

7.31. **IDENTIFY:** We know the potential energy function and want to find the force causing this energy.

SET UP: $F_x = -\dfrac{dU}{dx}$. The sign of F_x indicates its direction.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -4\alpha x^3 = -4(0.630\text{ J/m}^4)x^3.$

$F_x(-0.800\text{ m}) = -4(0.630\text{ J/m}^4)(-0.80\text{ m})^3 = 1.29\text{ N}$. The force is in the $+x$-direction.

EVALUATE: $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$, so the force is always directed toward the origin.

7.32. **IDENTIFY** and **SET UP:** Use $F_x = -\dfrac{dU}{dx}$ to calculate the force from $U(x)$. Use coordinates where the origin is at one atom. The other atom then has coordinate *x*.

EXECUTE:

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{C_6}{x^6}\right) = +C_6\frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}$$

The minus sign means that F_x is directed in the $-x$-direction, toward the origin. The force has magnitude $6C_6/x^7$ and is attractive.

EVALUATE: U depends only on x so $\vec{F}$ is along the x-axis; it has no y- or z-components.

7.33. IDENTIFY: From the potential energy function of the block, we can find the force on it, and from the force we can use Newton's second law to find its acceleration.

SET UP: The force components are $F_x = -\dfrac{\partial U}{\partial x}$ and $F_y = -\dfrac{\partial U}{\partial y}$. The acceleration components are $a_x = F_x/m$ and $a_y = F_y/m$. The magnitude of the acceleration is $a = \sqrt{a_x^2 + a_y^2}$ and we can find its angle with the $+x$ axis using $\tan\theta = a_y/a_x$.

EXECUTE: $F_x = -\dfrac{\partial U}{\partial x} = -(11.6\ \text{J/m}^2)x$ and $F_y = -\dfrac{\partial U}{\partial y} = (10.8\ \text{J/m}^3)y^2$. At the point ($x = 0.300$ m, $y = 0.600$ m), $F_x = -(11.6\ \text{J/m}^2)(0.300\ \text{m}) = -3.48$ N and $F_y = (10.8\ \text{J/m}^3)(0.600\ \text{m})^2 = 3.89$ N. Therefore $a_x = \dfrac{F_x}{m} = -87.0\ \text{m/s}^2$ and $a_y = \dfrac{F_y}{m} = 97.2\ \text{m/s}^2$, giving $a = \sqrt{a_x^2 + a_y^2} = 130\ \text{m/s}^2$ and $\tan\theta = \dfrac{97.2}{87.0}$, so $\theta = 48.2°$. The direction is $132°$ counterclockwise from the $+x$-axis.

EVALUATE: The force is not constant, so the acceleration will not be the same at other points.

7.34. IDENTIFY: Apply $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j}$.

SET UP: $\dfrac{d}{dx}\left(\dfrac{1}{x^2}\right) = -\dfrac{2}{x^3}$ and $\dfrac{d}{dy}\left(\dfrac{1}{y^2}\right) = -\dfrac{2}{y^3}$.

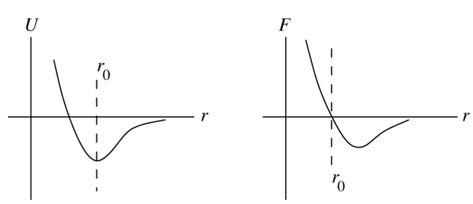
EXECUTE: $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j}$ since U has no z-dependence. $\dfrac{\partial U}{\partial x} = \dfrac{-2\alpha}{x^3}$ and $\dfrac{\partial U}{\partial y} = \dfrac{-2\alpha}{y^3}$, so

$$\vec{F} = -\alpha\left(\frac{-2}{x^3}\hat{i} + \frac{-2}{y^3}\hat{j}\right) = 2\alpha\left(\frac{\vec{i}}{x^3} + \frac{\vec{j}}{y^3}\right).$$

EVALUATE: F_x and x have the same sign and F_y and y have the same sign. When $x > 0$, F_x is in the $+x$-direction, and so forth.

7.35. IDENTIFY and SET UP: Use $F = -dU/dr$ to calculate the force from U. At equilibrium $F = 0$.

(a) EXECUTE: The graphs are sketched in Figure 7.35.



$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

$$F = -\frac{dU}{dr} = +\frac{12a}{r^{13}} - \frac{6b}{r^7}$$

Figure 7.35

(b) At equilibrium $F = 0$, so $\dfrac{dU}{dr} = 0$

$$F = 0 \text{ implies } \frac{+12a}{r^{13}} - \frac{6b}{r^7} = 0$$

$6br^6 = 12a$; solution is the equilibrium distance $r_0 = (2a/b)^{1/6}$

U is a minimum at this r; the equilibrium is stable.

(c) At $r = (2a/b)^{1/6}$, $U = a/r^{12} - b/r^6 = a(b/2a)^2 - b(b/2a) = -b^2/4a$.

At $r \to \infty$, $U = 0$. The energy that must be added is $-\Delta U = b^2/4a$.

(d) $r_0 = (2a/b)^{1/6} = 1.13\times10^{-10}$ m gives that

$2a/b = 2.082\times10^{-60}\ \text{m}^6$ and $b/4a = 2.402\times10^{59}\ \text{m}^{-6}$

$b^2/4a = b(b/4a) = 1.54\times10^{-18}$ J

$b(2.402\times10^{59}\ \text{m}^{-6}) = 1.54\times10^{-18}$ J and $b = 6.41\times10^{-78}\ \text{J}\cdot\text{m}^6$.

Then $2a/b = 2.082\times10^{-60}\ \text{m}^6$ gives $a = (b/2)(2.082\times10^{-60}\ \text{m}^6) =$

$\frac{1}{2}(6.41\times10^{-78}\ \text{J}\cdot\text{m}^6)\ (2.082\times10^{-60}\ \text{m}^6)\ = 6.67\times10^{-138}\ \text{J}\cdot\text{m}^{12}$

Evaluate: As the graphs in part (a) show, $F(r)$ is the slope of $U(r)$ at each r. $U(r)$ has a minimum where $F = 0$.

7.36. Identify: Apply $F_x = -\dfrac{dU}{dx}$.

Set Up: $\dfrac{dU}{dx}$ is the slope of the U versus x graph.

Execute: **(a)** Considering only forces in the x-direction, $F_x = -\dfrac{dU}{dx}$ and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d.

(b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable.

(c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

Evaluate: At point b, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point d, a small displacement in either direction produces a force directed away from d and the equilibrium is unstable.

7.37. Identify: Apply $\Sigma\vec{F} = m\vec{a}$ to the bag and to the box. Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the system of the box and bucket after the bag is removed.

Set Up: Let $y = 0$ at the final height of the bucket, so $y_1 = 2.00$ m and $y_2 = 0$. $K_1 = 0$. The box and the bucket move with the same speed v, so $K_2 = \frac{1}{2}(m_{\text{box}} + m_{\text{bucket}})v^2$. $W_{\text{other}} = -f_k d$, with $d = 2.00$ m and $f_k = \mu_k m_{\text{box}} g$. Before the bag is removed, the maximum possible friction force the roof can exert on the box is $(0.700)(80.0\ \text{kg} + 50.0\ \text{kg})(9.80\ \text{m/s}^2) = 892$ N. This is larger than the weight of the bucket (637 N), so before the bag is removed the system is at rest.

Execute: **(a)** The friction force on the bag of gravel is zero, since there is no other horizontal force on the bag for friction to oppose. The static friction force on the box equals the weight of the bucket, 637 N.

(b) Applying $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $m_{\text{bucket}} g y_1 - f_k d = \frac{1}{2} m_{\text{tot}} v^2$, with $m_{\text{tot}} = 145.0$ kg.

$$v = \sqrt{\frac{2}{m_{\text{tot}}}(m_{\text{bucket}} g y_1 - \mu_k m_{\text{box}} g d)}.$$

$$v = \sqrt{\frac{2}{145.0\ \text{kg}}\left[(65.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) - (0.400)(80.0\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m})\right]} = 2.99\ \text{m/s}.$$

EVALUATE: If we apply $\Sigma\vec{F} = m\vec{a}$ to the box and to the bucket we can calculate their common acceleration a. Then a constant acceleration equation applied to either object gives $v = 2.99$ m/s, in agreement with our result obtained using energy methods.

7.38. IDENTIFY: This problem involves projectile motion and energy conservation.

SET UP: Estimate: horizontal range is about 75 ft, which is about 25 m. The range of a projectile is $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$, energy conservation says $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, $K = \dfrac{1}{2}mv^2$, $U_g = mgy$.

EXECUTE: **(a)** Use range formula to find v_0. At 45° $R = \dfrac{v_0^2}{g}\sin 2\alpha_0$ gives $v_0 = \sqrt{Rg} =$ $\sqrt{(25\text{ m})(9.80\text{ m/s}^2)}$ = 15.65 m/s. $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(0.145\text{ kg})(15.65\text{ m/s})^2 = 18$ J.

(b) At the highest point, $v = v_x = v_0\cos\alpha_0$ = (15.65 m/s) cos 45° = 11.1 m/s, so $\dfrac{1}{2}(0.145\text{ kg})(11.1\text{ m/s})^2 = 8.88$ J. At the maximum height, $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $0 = (v_0\sin\alpha_0)^2 - 2gh$. This gives $h = \dfrac{(v_0\sin\alpha_0)^2}{2g}$ = 6.248 m. The gravitational potential energy is $U_g = mgh = (0.145\text{ kg})(9.80\text{ m/s}^2)(6.248\text{ m}) = 8.88$ J. As we have just found, at the maximum height $U_g = K = 8.88$ J. So with α_0 = 45°, half of the mechanical energy is kinetic energy and half is potential energy.

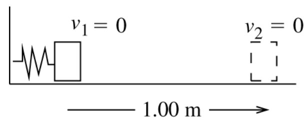
(c) At the highest point $K = \dfrac{1}{2}mv_x^2 = \dfrac{1}{2}m(v_0\cos\alpha_0)^2 = \dfrac{1}{2}mv_0^2\cos^2\alpha_0$. In part (b) we saw that the maximum height is $h = \dfrac{(v_0\sin\alpha_0)^2}{2g}$, so $U_g = mgh = mg\dfrac{(v_0\sin\alpha_0)^2}{2g} = \dfrac{1}{2}mv_0^2\sin^2\alpha_0$. The total energy is $E = K + U_g = \dfrac{1}{2}mv_0^2\cos^2\alpha_0 + \dfrac{1}{2}mv_0^2\sin^2\alpha_0 = \dfrac{1}{2}mv_0^2$. The fraction that is kinetic energy is $\dfrac{K}{E} = \dfrac{\frac{1}{2}mv_0^2\cos^2\alpha_0}{\frac{1}{2}mv_0^2} = \cos^2\alpha_0 = \cos^2 60° = \dfrac{1}{4}$. The fraction of the total energy that is potential energy is $\dfrac{U_g}{E} = \dfrac{\frac{1}{2}mv_0^2\sin^2\alpha_0}{\frac{1}{2}mv_0^2} = \sin^2\alpha_0 = \sin^2 60° = \dfrac{3}{4}$. So ¼ of the energy is kinetic energy and ¾ of the energy is potential energy.

EVALUATE: **(d)** If α_0 = 0: The ball is thrown horizontally. In this case, $K/E = \cos^2 0° = 1$ and $U_g/E = \sin^2 0° = 0$. All of the energy is kinetic and none is potential. This is reasonable because the ball never leaves the ground. If α_0 = 90°: The ball is thrown vertically upward. In this case, $K/E = \cos^2 90° = 0$ and $U_g/E = \sin^2 90° = 1$. All of the energy is potential and none is kinetic. This is reasonable because at its highest point the ball has stopped moving vertically and has no horizontal velocity, so its kinetic energy is zero.

7.39. IDENTIFY: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. The target variable μ_k will be a factor in the work done by friction.

SET UP: Let point 1 be where the block is released and let point 2 be where the block stops, as shown in Figure 7.39.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$



Work is done on the block by the spring and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}}$.

Figure 7.39

EXECUTE: $K_1 = K_2 = 0$

$U_1 = U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(100 \text{ N/m})(0.200 \text{ m})^2 = 2.00 \text{ J}$

$U_2 = U_{2,\text{el}} = 0,$ since after the block leaves the spring has given up all its stored energy

$W_{\text{other}} = W_f = (f_\text{k} \cos\phi)s = \mu_\text{k} mg(\cos\phi)s = -\mu_\text{k} mgs,$ since $\phi = 180°$ (The friction force is directed opposite to the displacement and does negative work.)

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$U_{1,\text{el}} + W_f = 0$

$\mu_\text{k} mgs = U_{1,\text{el}}$

$$\mu_\text{k} = \frac{U_{1,\text{el}}}{mgs} = \frac{2.00 \text{ J}}{(0.50 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \text{ m})} = 0.41.$$

EVALUATE: $U_{1,\text{el}} + W_f = 0$ says that the potential energy originally stored in the spring is taken out of the system by the negative work done by friction.

7.40. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0$. Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy,

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, or $v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400 \text{ N/m}}{2.00 \text{ kg}}}(0.220 \text{ m}) = 3.11 \text{ m/s}.$

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or $L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400 \text{ N/m})(0.220 \text{ m})^2}{(2.00 \text{ kg})(9.80 \text{ m/s}^2)\sin 37.0°} = 0.821 \text{ m}.$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.41. **IDENTIFY:** The mechanical energy of the roller coaster is conserved since there is no friction with the track. We must also apply Newton's second law for the circular motion.

SET UP: For part (a), apply conservation of energy to the motion from point A to point B: $K_B + U_{\text{grav},B} = K_A + U_{\text{grav},A}$ with $K_A = 0$. Defining $y_B = 0$ and $y_A = 13.0$ m, conservation of energy becomes $\frac{1}{2}mv_B{}^2 = mgy_A$ or $v_B = \sqrt{2gy_A}$. In part (b), the free-body diagram for the roller coaster car at point B is shown in Figure 7.41. $\Sigma F_y = ma_y$ gives $mg + n = ma_{\text{rad}}$, where $a_{\text{rad}} = v^2/r$. Solving for the normal force gives $n = m\left(\frac{v^2}{r} - g\right)$.

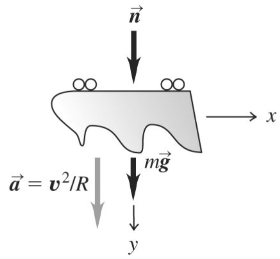


Figure 7.41

EXECUTE: **(a)** $v_B = \sqrt{2(9.80 \text{ m/s}^2)(13.0 \text{ m})} = 16.0 \text{ m/s}.$

(b) $n = (350 \text{ kg})\left[\dfrac{(16.0 \text{ m/s})^2}{6.0 \text{ m}} - 9.80 \text{ m/s}^2\right] = 1.15 \times 10^4 \text{ N}.$

EVALUATE: The normal force n is the force that the tracks exert on the roller coaster car. The car exerts a force of equal magnitude and opposite direction on the tracks.

7.42. **IDENTIFY:** In this problem, we need to use Newton's second law and energy conservation.

SET UP: $\sum F = m\dfrac{v^2}{R}$ and $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where W_{other} is the work done by friction. At the bottom of the bowl, the normal force on the rock is equal to twice its weight, so $n = 2mg$.

EXECUTE: First use $\sum F = m\dfrac{v^2}{R}$ to find the speed at B. $n - mg = m\dfrac{v_B^2}{R} \rightarrow 2mg - mg = m\dfrac{v_B^2}{R}$, which gives $v_B^2 = Rg$. Now use energy conservation. $U_A + K_A + W_{\text{f}} = U_B + K_B$ gives us

$W_{\text{f}} = -(U_A - U_B) + K_B = -mgR + \dfrac{1}{2}mRg = -\dfrac{mgR}{2}.$

EVALUATE: If there were no friction, energy conservation gives $mgR = \dfrac{1}{2}mv_B^2$, so $v_B^2 = 2Rg$.

Newton's second law gives $n - mg = m\dfrac{v_B^2}{R} = m\left(\dfrac{2Rg}{R}\right)$, so $n = 3mg$. With friction the normal force was $2mg$, which is reasonable because friction slowed down the rock.

7.43. **(a) IDENTIFY:** Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the kinetic energy of the wood as it enters the rough bottom.

SET UP: Let point 1 be where the piece of wood is released and point 2 be just before it enters the rough bottom. Let $y = 0$ be at point 2.

EXECUTE: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4 \text{ J}$.

IDENTIFY: Now apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion along the rough bottom.

SET UP: Let point 1 be where it enters the rough bottom and point 2 be where it stops.
$K_1 + U_1 + W_{\text{other}} = K_2 + U_2.$

EXECUTE: $W_{\text{other}} = W_f = -\mu_k mgs$, $K_2 = U_1 = U_2 = 0$; $K_1 = 78.4 \text{ J}$

$78.4 \text{ J} - \mu_k mgs = 0$; solving for s gives $s = 20.0 \text{ m}$.

The wood stops after traveling 20.0 m along the rough bottom.

(b) Friction does -78.4 J of work.

EVALUATE: The piece of wood stops before it makes one trip across the rough bottom. The final mechanical energy is zero. The negative friction work takes away all the mechanical energy initially in the system.

7.44. **IDENTIFY:** In this problem, we need to use Newton's second law and energy conservation.

SET UP: $\sum F = m\dfrac{v^2}{R}$ and $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where $W_{\text{other}} = 0$. In order that the block will not fall off the track at the top, it must be moving; otherwise it would simply drop down. First find the minimum speed at the top and then use that result to find the speed at the bottom.

EXECUTE: At the top n and mg are both downward, so $\sum F = m\dfrac{v^2}{R}$ gives $n + mg = m\dfrac{v^2}{R}$. The smallest that n can be is zero, so the minimum speed at the top is $v_{\min}^2 = Rg$. Now use energy conservation to find the speed v_B at the bottom. $U_T + K_T = U_B + K_B a$ gives $\dfrac{1}{2}mv_B^2 = \dfrac{1}{2}mv_T^2 + 2mgR$, which gives $v_B = \sqrt{v_T^2 + 4Rg} = \sqrt{Rg + 4Rg} = \sqrt{5Rg}$.

EVALUATE: The larger R, the greater the speed at the bottom.

7.45. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the stone.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Let point 1 be point A and point 2 be point B. Take $y = 0$ at B.

EXECUTE: $mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0\text{ m}$ and $v_1 = 10.0\text{ m/s}$, so $v_2 = \sqrt{v_1^2 + 2gh} = 22.2\text{ m/s}$.

EVALUATE: The loss of gravitational potential energy equals the gain of kinetic energy.

(b) IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the stone from point B to where it comes to rest against the spring.

SET UP: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, with point 1 at B and point 2 where the spring has its maximum compression x.

EXECUTE: $U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2\text{ m/s}$. $W_{\text{other}} = W_f + W_{\text{el}} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\text{ m} + x$. The work-energy relation gives $K_1 + W_{\text{other}} = 0$. $\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$.

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4\text{ m}$.

EVALUATE: Part of the initial mechanical (kinetic) energy is removed by friction work and the rest goes into the potential energy stored in the spring.

(c) IDENTIFY and **SET UP:** Consider the forces.

EXECUTE: When the spring is compressed $x = 16.4\text{ m}$ the force it exerts on the stone is $F_{\text{el}} = kx = 32.8\text{ N}$. The maximum possible static friction force is

$\max f_s = \mu_s mg = (0.80)(15.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}$.

EVALUATE: The spring force is less than the maximum possible static friction force so the stone remains at rest.

7.46. **IDENTIFY:** Once the block leaves the top of the hill it moves in projectile motion. Use $K_1 + U_1 = K_2 + U_2$ to relate the speed v_{B} at the bottom of the hill to the speed v_{Top} at the top and the 70 m height of the hill.

SET UP: For the projectile motion, take $+y$ to be downward. $a_x = 0$, $a_y = g$. $v_{0x} = v_{\text{Top}}$, $v_{0y} = 0$. For the motion up the hill only gravity does work. Take $y = 0$ at the base of the hill.

EXECUTE: First get speed at the top of the hill for the block to clear the pit. $y = \dfrac{1}{2}gt^2$.

$20\text{ m} = \dfrac{1}{2}(9.8\text{ m/s}^2)t^2$. $t = 2.0\text{ s}$. Then $v_{\text{Top}}t = 40\text{ m}$ gives $v_{\text{Top}} = \dfrac{40\text{ m}}{2.0\text{ s}} = 20\text{ m/s}$.

Energy conservation applied to the motion up the hill: $K_{\text{Bottom}} = U_{\text{Top}} + K_{\text{Top}}$ gives

$\frac{1}{2}mv_B^2 = mgh + \frac{1}{2}mv_{Top}^2$. $v_B = \sqrt{v_{Top}^2 + 2gh} = \sqrt{(20\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(70\text{ m})} = 42\text{ m/s}$.

EVALUATE: The result does not depend on the mass of the block.

7.47. **IDENTIFY:** We must use energy conservation. This system has elastic potential energy and gravitational potential energy, and friction does work on it.

SET UP: $U_1 + K_1 + W_{other} = U_2 + K_2$, $U_g = mgy$, $U_{spring} = \frac{1}{2}kx^2$. For the minimum elastic potential energy, the box just stops at the top of the incline. So $K_1 = K_2$ in this case. We want to find the energy stored in the spring.

EXECUTE: $U_1 + K_1 + W_{other} = U_2 + K_2$ gives $U_{1,spring} + W_f = U_{2,g}$, so $U_{1,spring} = U_{2,g} - W_f$. Therefore $U_{1,spring} = mgh - (-f_k)s = mgh + \mu_k ns = mgh + \mu_k mgs\cos\theta = mg(h + \mu_k s\cos\theta)$. Using m = 0.600 kg, s = 2.00 m, θ = 37.0°, h = (2.00 m) sin 37.0°, and μ_k = 0.400 gives $U_{1,spring}$ = 10.8 J.

EVALUATE: The elastic potential energy in the spring must be greater than the gravitational potential energy because friction is doing negative work on the box. $U_g = mgh$ = (0.600 kg)(9.80 m/s^2)(2.00 m)(sin 37.0°) = 7.08 J, which agrees with our expectation.

7.48. **IDENTIFY:** To be at equilibrium at the bottom, with the spring compressed a distance x_0, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0 = w\sin\theta + f$. Apply energy conservation to the motion down the ramp.

SET UP: $K_2 = 0$, $K_1 = \frac{1}{2}mv^2$, where v is the speed at the top of the ramp. Let $U_2 = 0$, so $U_1 = wL\sin\theta$, where L is the total length traveled down the ramp.

EXECUTE: Energy conservation gives $\frac{1}{2}kx_0^2 = (w\sin\theta - f)L + \frac{1}{2}mv^2$. With the given parameters, $\frac{1}{2}kx_0^2 = 421\text{ J}$ and $kx_0 = 1.066\times10^3\text{ N}$. Solving for k gives $k = 1350\text{ N/m}$.

EVALUATE: $x_0 = 0.790\text{ m}$. $w\sin\theta = 551\text{ N}$. The decrease in gravitational potential energy is larger than the amount of mechanical energy removed by the negative work done by friction. $\frac{1}{2}mv^2 = 243\text{ J}$. The energy stored in the spring is larger than the initial kinetic energy of the crate at the top of the ramp.

7.49. **IDENTIFY:** Use $K_1 + U_1 + W_{other} = K_2 + U_2$. Solve for K_2 and then for v_2.

SET UP: Let point 1 be at his initial position against the compressed spring and let point 2 be at the end of the barrel, as shown in Figure 7.49. Use $F = kx$ to find the amount the spring is initially compressed by the 4400 N force.

$K_1 + U_1 + W_{other} = K_2 + U_2$

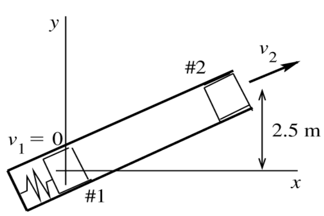


Figure 7.49

Take $y = 0$ at his initial position.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$W_{other} = W_{fric} = -fs$

$W_{other} = -(40\text{ N})(4.0\text{ m}) = -160\text{ J}$

Figure 7.49

$U_{1,grav} = 0$, $U_{1,el} = \frac{1}{2}kd^2$, where d is the distance the spring is initially compressed.

$F = kd$ so $d = \frac{F}{k} = \frac{4400\text{ N}}{1100\text{ N/m}} = 4.00\text{ m}$

and $U_{1,\text{el}} = \frac{1}{2}(1100\text{ N/m})(4.00\text{ m})^2 = 8800\text{ J}$

$U_{2,\text{grav}} = mgy_2 = (60\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m}) = 1470\text{ J},\ \ U_{2,\text{el}} = 0$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$8800\text{ J} - 160\text{ J} = \frac{1}{2}mv_2^2 + 1470\text{ J}$

$\frac{1}{2}mv_2^2 = 7170\text{ J}$ and $v_2 = \sqrt{\dfrac{2(7170\text{ J})}{60\text{ kg}}} = 15.5\text{ m/s}.$

EVALUATE: Some of the potential energy stored in the compressed spring is taken away by the work done by friction. The rest goes partly into gravitational potential energy and partly into kinetic energy.

7.50. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the rocket from the starting point to the base of the ramp. W_{other} is the work done by the thrust and by friction.

SET UP: Let point 1 be at the starting point and let point 2 be at the base of the ramp. $v_1 = 0$, $v_2 = 50.0\text{ m/s}$. Let $y = 0$ at the base and take $+y$ upward. Then $y_2 = 0$ and $y_1 = d\sin 53°$, where d is the distance along the ramp from the base to the starting point. Friction does negative work.

EXECUTE: $K_1 = 0$, $U_2 = 0$. $U_1 + W_{\text{other}} = K_2$. $W_{\text{other}} = (2000\text{ N})d - (500\text{ N})d = (1500\text{ N})d$.

$mgd\sin 53° + (1500\text{ N})d = \frac{1}{2}mv_2^2.$

$$d = \frac{mv_2^2}{2[mg\sin 53° + 1500\text{ N}]} = \frac{(1500\text{ kg})(50.0\text{ m/s})^2}{2[(1500\text{ kg})(9.80\text{ m/s}^2)\sin 53° + 1500\text{ N}]} = 142\text{ m}.$$

EVALUATE: The initial height is $y_1 = (142\text{ m})\sin 53° = 113\text{ m}$. An object free-falling from this distance attains a speed $v = \sqrt{2gy_1} = 47.1\text{ m/s}$. The rocket attains a greater speed than this because the forward thrust is greater than the friction force.

7.51. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the system consisting of the two buckets. If we ignore the inertia of the pulley we ignore the kinetic energy it has.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.51.

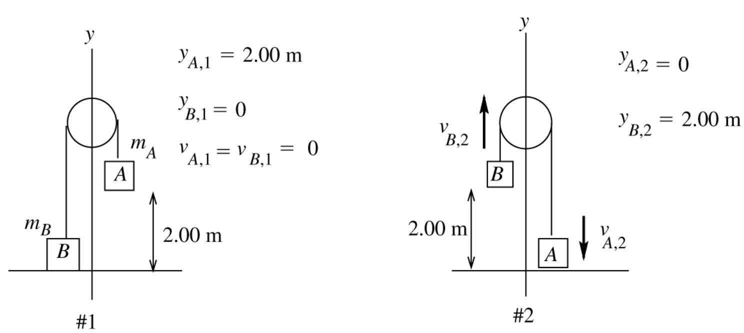


Figure 7.51

The tension force does positive work on the 4.0 kg bucket and an equal amount of negative work on the 12.0 kg bucket, so the net work done by the tension is zero.
Work is done on the system only by gravity, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}}$.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}m_A v_{A,2}^2 + \frac{1}{2}m_B v_{B,2}^2$. But since the two buckets are connected by a rope they move together and have the same speed: $v_{A,2} = v_{B,2} = v_2$. Thus $K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = (8.00\text{ kg})v_2^2$.

$U_1 = m_A g y_{A,1} = (12.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 235.2\text{ J}.$

$U_2 = m_B g y_{B,2} = (4.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 78.4\text{ J}.$

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_1 = K_2 + U_2$.

$235.2\text{ J} = (8.00\text{ kg})v_2^2 + 78.4\text{ J}.$ $v_2 = \sqrt{\dfrac{235.2\text{ J} - 78.4\text{ J}}{8.00\text{ kg}}} = 4.4\text{ m/s}$

EVALUATE: The gravitational potential energy decreases and the kinetic energy increases by the same amount. We could apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to one bucket, but then we would have to include in W_{other} the work done on the bucket by the tension T.

7.52. IDENTIFY: We use energy conservation.

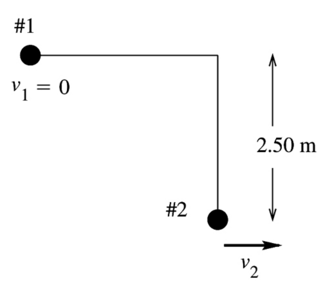
SET UP: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, $U_g = mgy$, $K = \frac{1}{2}mv^2$. Call point 1 the instant the block is released against the spring and point 2 when its speed is 4.00 m/s. We want to find the magnitude of the friction force f.

EXECUTE: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes $U_{\text{spr}} + W_f = K_2 + U_g$. This gives

$U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, so $f = \dfrac{U_{\text{spr}} - m\left(\dfrac{v_2^2}{2} + gh\right)}{s}$. Using $m = 0.200$ kg, $v_2 = 4.00$ m/s, $s = 3.00$ m, $h = s \sin 53.0° = 2.40$ m, and $U_{\text{spr}} = 8.00$ J, this gives $U_{\text{spr}} = 0.568$ N.

EVALUATE: The coefficient of kinetic friction for this value of kinetic friction would be $\mu_k = f/n = f/(mg\cos\theta) = (0.568\text{ N})/[(0.200\text{ kg})(9.80\text{ m/s}^2)(\cos 53.0°)] = 0.482$, which is a reasonable value according to Table 5.1.

7.53. (a) IDENTIFY and SET UP: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the potato. Let point 1 be where the potato is released and point 2 be at the lowest point in its motion, as shown in Figure 7.53a.



$y_1 = 2.50$ m

$y_2 = 0$

The tension in the string is at all points in the motion perpendicular to the displacement, so $W_r = 0$

The only force that does work on the potato is gravity, so $W_{\text{other}} = 0$.

Figure 7.53a

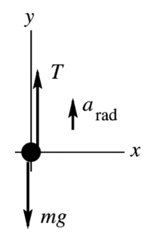
EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$, $U_1 = mgy_1$, $U_2 = 0$. Thus $U_1 = K_2$. $mgy_1 = \frac{1}{2}mv_2^2$, which gives

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(2.50\text{ m})} = 7.00\text{ m/s}.$

EVALUATE: The speed v_2 is the same as if the potato fell through 2.50 m.

(b) IDENTIFY: Apply $\Sigma\vec{F} = m\vec{a}$ to the potato. The potato moves in an arc of a circle so its acceleration is $\vec{a}_{\text{rad}}$, where $a_{\text{rad}} = v^2/R$ and is directed toward the center of the circle. Solve for one of the forces, the tension T in the string.

SET UP: The free-body diagram for the potato as it swings through its lowest point is given in Figure 7.53b.



The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, so at this point it is upward.

Figure 7.53b

EXECUTE: $\Sigma F_y = ma_y$ gives $T - mg = ma_{\text{rad}}$. Solving for T gives $T = m(g + a_{\text{rad}}) = m\left(g + \frac{v_2^2}{R}\right)$, where the radius R for the circular motion is the length L of the string. It is instructive to use the algebraic expression for v_2 from part (a) rather than just putting in the numerical value: $v_2 = \sqrt{2gy_1} = \sqrt{2gL}$, so $v_2^2 = 2gL$. Then $T = m\left(g + \frac{v_2^2}{L}\right) = m\left(g + \frac{2gL}{L}\right) = 3mg$. The tension at this point is three times the weight of the potato, so $T = 3mg = 3(0.300\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$.

EVALUATE: The tension is greater than the weight; the acceleration is upward so the net force must be upward.

7.54. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to each stage of the motion.

SET UP: Let $y = 0$ at the bottom of the slope. In part (a), W_{other} is the work done by friction. In part (b), W_{other} is the work done by friction and the air resistance force. In part (c), W_{other} is the work done by the force exerted by the snowdrift.

EXECUTE: **(a)** The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction, $K_1 = mgh - W_f = (60.0\text{ kg})(9.8\text{ N/kg})(65.0\text{ m}) - 10{,}500\text{ J}$, or $K_1 = 38{,}200\text{ J} - 10{,}500\text{ J} = 27{,}720\text{ J}$. Then $v_1 = \sqrt{\frac{2K_1}{m}} = \sqrt{\frac{2(27{,}720\text{ J})}{60\text{ kg}}} = 30.4\text{ m/s}$.

(b) $K_2 = K_1 - (W_f + W_{\text{air}}) = 27{,}720\text{ J} - (\mu_k mgd + f_{\text{air}}d)$.

$K_2 = 27{,}720\text{ J} - [(0.2)(588\text{ N})(82\text{ m}) + (160\text{ N})(82\text{ m})]$ or $K_2 = 27{,}720\text{ J} - 22{,}763\text{ J} = 4957\text{ J}$. Then,

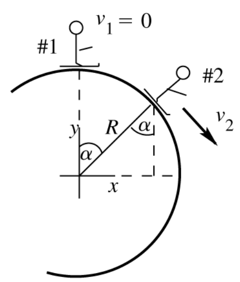
$v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4957\text{ J})}{60\text{ kg}}} = 12.9\text{ m/s}$.

(c) Use the work-energy theorem to find the force. $W = \Delta K$, $F = K/d = (4957\text{ J})/(2.5\text{ m}) = 2000\text{ N}$.

EVALUATE: In each case, W_{other} is negative and removes mechanical energy from the system.

7.55. **IDENTIFY** and **SET UP:** First apply $\Sigma\vec{F} = m\vec{a}$ to the skier.

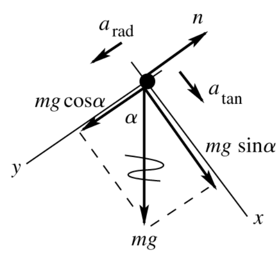
Find the angle α where the normal force becomes zero, in terms of the speed v_2 at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates v_2 and α. Solve these two equations for α.



Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.55a
Loses contact implies $n \to 0$.
$y_1 = R,\ \ y_2 = R\cos\alpha$

Figure 7.55a

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.55b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R$, directed in toward the center of the snowball.



EXECUTE: $\Sigma F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

Figure 7.55b

Now use conservation of energy to get another equation relating v_2 to α:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0$.

$K_1 = 0,\ \ K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR,\ \ U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1-\cos\alpha)$

Combine this with the $\Sigma F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1-\cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

EVALUATE: She speeds up and her a_{rad} increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that α where she loses contact does not depend on her mass or on the radius of the snowball.

7.56. **IDENTIFY:** We use energy conservation.

SET UP: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ where $W_{\text{other}} = 0$, $U_{\text{spr}} = \frac{1}{2}kx^2$, $K = \frac{1}{2}mv^2$. Call point 1 when the block's speed is 3.00 m/s and point 2 at maximum compression of the spring. This makes $v_1 =$ 3.00 m/s, $x_1 = 0.160$ m, and $K_2 = 0$.

EXECUTE: **(a)** $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes $\frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_{\max}^2$. Now solve for $v_{\max}$:

$v_{\max} = \sqrt{\dfrac{kx_1^2 + mv_1^2}{m}}$. Using $x_1 = 0.160$ m, $v_1 = 3.00$ m/s, and $k = 200$ N/m gives $v_{\max} = 4.67$ m/s.

(b) The maximum compression occurs when the kinetic energy is zero, so $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_{\max}^2$,

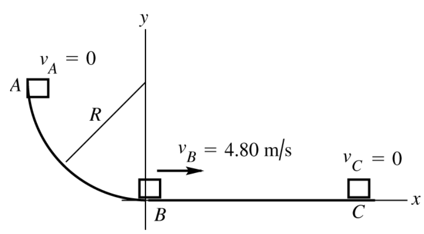
which gives $x_{\max} = v_{\max}\sqrt{m/k} = (4.67 \text{ m/s})\sqrt{\dfrac{0.400 \text{ kg}}{200 \text{ N/m}}} = 0.209$ m.

(c) At maximum compression $v = 0$ and a is a maximum. So $\sum F_x = ma_x$ gives $kx_{\max} = ma$, so

$a = \dfrac{kx_{\max}}{m} = \dfrac{(200 \text{ N/m})(0.209 \text{ m})}{0.400 \text{ kg}} = 104 \text{ m/s}^2$.

EVALUATE: At maximum compression, the spring force is a maximum because x is a maximum, so the acceleration is a maximum. But at this instant the block has stopped, so $v = 0$ at maximum acceleration.

7.57. **IDENTIFY** and **SET UP:**



$y_A = R$

$y_B = y_C = 0$

Figure 7.57

(a) Apply conservation of energy to the motion from B to C:
$K_B + U_B + W_{\text{other}} = K_C + U_C$. The motion is described in Figure 7.57.

EXECUTE: The only force that does work on the package during this part of the motion is friction, so $W_{\text{other}} = W_f = f_k(\cos\phi)s = \mu_k mg(\cos 180°)s = \mu_k mgs$

$K_B = \frac{1}{2}mv_B^2, \quad K_C = 0$

$U_B = 0, \quad U_C = 0$

Thus $K_B + W_f = 0$

$\frac{1}{2}mv_B^2 - \mu_k mgs = 0$

$\mu_k = \dfrac{v_B^2}{2gs} = \dfrac{(4.80 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(3.00 \text{ m})} = 0.392.$

EVALUATE: The negative friction work takes away all the kinetic energy.

(b) IDENTIFY and **SET UP:** Apply conservation of energy to the motion from A to B:

$$K_A + U_A + W_{\text{other}} = K_B + U_B$$

EXECUTE: Work is done by gravity and by friction, so $W_{\text{other}} = W_f$.

$K_A = 0, \quad K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200 \text{ kg})(4.80 \text{ m/s})^2 = 2.304 \text{ J}$

$U_A = mgy_A = mgR = (0.200 \text{ kg})(9.80 \text{ m/s}^2)(1.60 \text{ m}) = 3.136 \text{ J}, \quad U_B = 0$

Thus $U_A + W_f = K_B$

$W_f = K_B - U_A = 2.304 \text{ J} - 3.136 \text{ J} = \text{-}0.83 \text{ J}$

EVALUATE: W_f is negative as expected; the friction force does negative work since it is directed opposite to the displacement.

7.58. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the initial and final positions of the truck.

SET UP: Let $y = 0$ at the lowest point of the path of the truck. W_{other} is the work done by friction. $f_r = \mu_r n = \mu_r mg\cos\beta$.

EXECUTE: Denote the distance the truck moves up the ramp by *x*. $K_1 = \frac{1}{2}mv_0^2$, $U_1 = mgL\sin\alpha$, $K_2 = 0$, $U_2 = mgx\sin\beta$ and $W_{\text{other}} = -\mu_r mgx\cos\beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for *x*, we get $x = \dfrac{K_1 + mgL\sin\alpha}{mg(\sin\beta + \mu_r\cos\beta)} = \dfrac{(v_0^2/2g) + L\sin\alpha}{\sin\beta + \mu_r\cos\beta}$.

EVALUATE: *x* increases when v_0 increases and decreases when μ_r increases.

7.59. **(a) IDENTIFY:** We are given that $F_x = -\alpha x - \beta x^2$, $\alpha = 60.0$ N/m and $\beta = 18.0$ N/m^2. Use $W_{F_x} = \int_{x_1}^{x_2} F_x(x)\,dx$ to calculate *W* and then use $W = -\Delta U$ to identify the potential energy function $U(x)$.

SET UP: $W_{F_x} = U_1 - U_2 = \int_{x_1}^{x_2} F_x(x)\,dx$

Let $x_1 = 0$ and $U_1 = 0$. Let x_2 be some arbitrary point *x*, so $U_2 = U(x)$.

EXECUTE: $U(x) = -\int_0^x F_x(x)\,dx = -\int_0^x (-\alpha x - \beta x^2)\,dx = \int_0^x (\alpha x + \beta x^2)\,dx = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3$.

EVALUATE: If $\beta = 0$, the spring does obey Hooke's law, with $k = \alpha$, and our result reduces to $\frac{1}{2}kx^2$.

(b) IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the object.

SET UP: The system at points 1 and 2 is sketched in Figure 7.59.

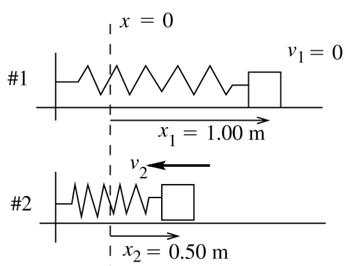


Figure 7.59

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the object is the spring force, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$U_1 = U(x_1) = \frac{1}{2}\alpha x_1^2 + \frac{1}{3}\beta x_1^3 = \frac{1}{2}(60.0\text{ N/m})(1.00\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(1.00\text{ m})^3 = 36.0\text{ J}$

$U_2 = U(x_2) = \frac{1}{2}\alpha x_2^2 + \frac{1}{3}\beta x_2^3 = \frac{1}{2}(60.0\text{ N/m})(0.500\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(0.500\text{ m})^3 = 8.25\text{ J}$

Thus $36.0\text{ J} = \frac{1}{2}mv_2^2 + 8.25\text{ J}$, which gives $v_2 = \sqrt{\dfrac{2(36.0\text{ J} - 8.25\text{ J})}{0.900\text{ kg}}} = 7.85\text{ m/s}$.

EVALUATE: The elastic potential energy stored in the spring decreases and the kinetic energy of the object increases.

7.60. **IDENTIFY:** Mechanical energy is conserved on the hill, which gives us the speed of the sled at the top. After it leaves the cliff, we must use projectile motion.

SET UP: Use conservation of energy to find the speed of the sled at the edge of the cliff. Let $y_{\text{i}} = 0$ so $y_{\text{f}} = h = 11.0$ m. $K_{\text{f}} + U_{\text{f}} = K_{\text{i}} + U_{\text{i}}$ gives $\frac{1}{2}mv_{\text{f}}^2 + mgh = \frac{1}{2}mv_{\text{i}}^2$ or $v_{\text{f}} = \sqrt{v_{\text{i}}^2 - 2gh}$. Then analyze the projectile motion of the sled: use the vertical component of motion to find the time t that the sled is in the air; then use the horizontal component of the motion with $a_x = 0$ to find the horizontal displacement.

EXECUTE: $v_{\text{f}} = \sqrt{(22.5\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(11.0\text{ m})} = 17.1$ m/s. $y_{\text{f}} = v_{\text{i},y}t + \frac{1}{2}a_y t^2$ gives

$$t = \sqrt{\frac{2y_{\text{f}}}{a_y}} = \sqrt{\frac{2(-11.0\text{ m})}{-9.80\text{ m/s}^2}} = 1.50\text{ s}.\quad x_{\text{f}} = v_{\text{i},x}t + \tfrac{1}{2}a_x t^2 \text{ gives } x_{\text{f}} = v_{\text{i},x}t = (17.1\text{ m/s})(1.50\text{ s}) = 25.6\text{ m}.$$

EVALUATE: Conservation of energy can be used to find the speed of the sled at any point of the motion but does not specify how far the sled travels while it is in the air.

7.61. **IDENTIFY:** We have a conservative force, so we can relate the force and the potential energy function. Energy conservation applies.

SET UP: $F_x = -dU/dx$, U goes to 0 as x goes to infinity, and $F(x) = \dfrac{\alpha}{(x+x_0)^2}$.

EXECUTE: **(a)** Using $dU = -F_x dx$, we get $U_x - U_\infty = -\displaystyle\int_\infty^x \frac{\alpha}{(x+x_0)^2}dx = \frac{\alpha}{x+x_0}$.

(b) Energy conservation tells us that $U_1 = K_2 + U_2$. Therefore $\dfrac{\alpha}{x_1+x_0} = \dfrac{1}{2}mv_x^2 + \dfrac{\alpha}{x_2+x_0}$. Putting in m = 0.500 kg, $\alpha = 0.800\text{ N}\cdot\text{m}$, x_0 = 0.200 m, x_1 = 0, and x_2 = 0.400 m, solving for v gives v = 3.27 m/s.

EVALUATE: The potential energy is not infinite even though the integral in (a) is taken over an infinite distance because the force rapidly gets smaller with increasing distance x.

7.62. **IDENTIFY:** We need to use energy conservation and apply Newton's second law.

SET UP: $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ where $W_{\text{other}} = 0$, $U_{\text{g}} = mgy$, and $K = \dfrac{1}{2}mv^2$. $\sum F_{\text{rad}} = ma_{\text{rad}}$, where $a_{\text{rad}} = \dfrac{v^2}{R}$. The ball has momentarily stopped swinging when θ = 37.0°, but it is swinging when θ = 25.0°. Since the ball is swinging in a circular arc, it has radial acceleration toward the center of the circle. The forces causing that acceleration are the tension and the component of gravity toward (or away from) the center of the circle. Fig. 7.62 shows a free-body diagram of the ball. We want to find the tension in the rope at two different angles.

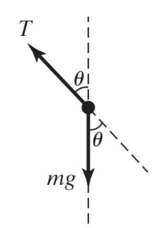


Figure 7.62

EXECUTE: **(a)** At the maximum angle of swing, the ball has stopped, so its radial acceleration (v^2/R) is zero at that instant. This means that the net radial force must be zero. From Fig. 7.62 we can see the force components. Thus $T - mg\cos\theta = 0$, and solving for T gives

$T = mg\cos\theta =$ (0.200 kg)(9.80 m/s^2)(cos 37.0°) = 1.57 N.

(b) The ball is now moving, so it has radial acceleration. Therefore $\sum F_{\text{rad}} = ma_{\text{rad}}$ gives $T - mg\cos\theta = m\dfrac{v^2}{R}$. Solving for T and using $R = L$, we get $T = mg\cos\theta + m\dfrac{v^2}{L}$. To find T, we need to find v^2 when θ = 25.0°. We use $U_1 + K_1 = U_2 + K_2$. Call point 1 when θ = 37.0° and point 2 when θ = 25.0°. In that case we have $U_1 = K_2 + U_2$, which gives $K_2 =$ $-(U_2 - U_1) = -mg(y_2 - y_1) = -mgL(\cos\theta_1 - \cos\theta_2)$. In terms of v_2, this is $\frac{1}{2}mv_2^2 = -mgL(\cos\theta_1 - \cos\theta_2)$, which simplifies to $v_2^2 = -2gL(\cos\theta_1 - \cos\theta_2)$, so $v_2^2 = -2g(1.40\text{ m})(\cos 37.0° - \cos 25.0°)$ = 2.95 m^2/s^2. Now return to the tension. From earlier work, we have $T = mg\cos\theta_2 + m\dfrac{v^2}{L} = m\left(g\cos\theta_2 + \dfrac{v^2}{L}\right)$. Using v_2^2 = 2.95 m^2/s^2, m = 0.200 kg, θ = 25.0°, and L = 1.40 m, we find that T = 2.20 N.

EVALUATE: We have found that the tension T is greater at 25° than at 37° because it helps to accelerate the ball toward the center at 25°.

7.63. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block.

SET UP: Let $y = 0$ at the floor. Let point 1 be the initial position of the block against the compressed spring and let point 2 be just before the block strikes the floor.

EXECUTE: With $U_2 = 0, K_1 = 0,\ K_2 = U_1.\ \frac{1}{2}mv_2^2 = \frac{1}{2}kx^2 + mgh.$ Solving for v_2,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900\text{ N/m})(0.045\text{ m})^2}{(0.150\text{ kg})} + 2(9.80\text{ m/s}^2)(1.20\text{ m})} = 7.01\text{ m/s}.$$

EVALUATE: The potential energy stored in the spring and the initial gravitational potential energy all go into the final kinetic energy of the block.

7.64. **IDENTIFY:** At equilibrium the upward spring force equals the weight mg of the object. Apply conservation of energy to the motion of the fish.

SET UP: The distance that the mass descends equals the distance the spring is stretched. $K_1 = K_2 = 0$, so $U_1(\text{gravitational}) = U_2(\text{spring})$

EXECUTE: Following the hint, the force constant k is found from $mg = kd$, or $k = mg/d$. When the fish falls from rest, its gravitational potential energy decreases by mgy; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}(mg/d)y^2$. Equating these, $\frac{1}{2}\frac{mg}{d}y^2 = mgy$, or $y = 2d$.

EVALUATE: At its lowest point the fish is not in equilibrium. The upward spring force at this point is $ky = 2kd$, and this is equal to twice the weight. At this point the net force is mg, upward, and the fish has an upward acceleration equal to g.

7.65. **IDENTIFY:** The spring does positive work on the box but friction does negative work.

SET UP: $U_{\text{el}} = \frac{1}{2}kx^2$ and $W_{\text{other}} = W_f = -\mu_k mgx$.

EXECUTE: **(a)** $U_{\text{el}} + W_{\text{other}} = K$ gives $\frac{1}{2}kx^2 + (-\mu_k mgx) = \frac{1}{2}mv^2$. Using the numbers for the problem, k = 45.0 N/m, x = 0.280 m, μ_k = 0.300, and m = 1.60 kg, solving for v gives v = 0.747 m/s.

(b) Call x the distance the spring is compressed when the speed of the box is a maximum and x_0 the initial compression distance of the spring. Using an approach similar to that in part (a) gives $\frac{1}{2}kx_0^2 - \mu_k mg(x_0 - x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. Rearranging gives $mv^2 = kx_0^2 - kx^2 - 2\mu_k mg(x_0 - x)$. For the maximum speed, $d(v^2)/dx = 0$, which gives $-2kx + 2\mu_k mg = 0$. Solving for x_{max}, the compression distance at maximum speed, gives $x_{\text{max}} = \mu_k mg/k$. Now substitute this result into the expression above for mv^2, put in the numbers, and solve for v, giving v = 0.931 m/s.

EVALUATE: Another way to find the result in (b) is to realize that the spring force decreases as x decreases, but the friction force remains constant. Eventually these two forces will be equal in magnitude. After that the friction force will be greater than the spring force, and friction will begin to slow down the box. So the maximum box speed occurs when the spring force is equal to the friction force. At that instant, $kx = f_k$, which gives $x = 0.105$ m. Then energy conservation can be used to find v with this value of x.

7.66. **IDENTIFY:** The spring obeys Hooke's law. Gravity and the spring provide the vertical forces on the brick. The mechanical energy of the system is conserved.
SET UP: Use $K_f + U_f = K_i + U_i$. In part (a), setting $y_f = 0$, we have $y_i = x$, the amount the spring will stretch. Also, since $K_i = K_f = 0$, $\frac{1}{2}kx^2 = mgx$. In part (b), $y_i = h + x$, where $h = 1.0$ m.

EXECUTE: **(a)** $x = \dfrac{2mg}{k} = \dfrac{2(3.0\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N/m}} = 0.039\text{ m} = 3.9\text{ cm}.$

(b) $\frac{1}{2}kx^2 = mg(h + x)$, $kx^2 - 2mgx - 2mgh = 0$ and $x = \dfrac{mg}{k}\left(1 \pm \sqrt{1 + \dfrac{2hk}{mg}}\right)$. Since x must be positive,

we have $x = \dfrac{mg}{k}\left(1 + \sqrt{1 + \dfrac{2hk}{mg}}\right) = \dfrac{(3.0\text{ kg})(9.80\text{ m/s}^2)}{1500\text{ N/m}}\left(1 + \sqrt{1 + \dfrac{2(1.0\text{ m})(1500\text{ N/m})}{3.0\text{ kg}(9.80\text{ m/s}^2)}}\right) = 0.22\text{ m} = 22\text{ cm}.$

EVALUATE: In part (b) there is additional initial energy (from gravity), so the spring is stretched more.

7.67. **IDENTIFY:** Only conservative forces (gravity and the spring) act on the fish, so its mechanical energy is conserved.
SET UP: Energy conservation tells us $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $W_{\text{other}} = 0$. $U_g = mgy$, $K = \frac{1}{2}mv^2$, and $U_{spring} = \frac{1}{2}ky^2$.
EXECUTE: **(a)** $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Let y be the distance the fish has descended, so $y = 0.0500$ m. $K_1 = 0$, $W_{\text{other}} = 0$, $U_1 = mgy$, $K_2 = \dfrac{1}{2}mv_2^2$, and $U_2 = \dfrac{1}{2}ky^2$. Solving for K_2 gives

$K_2 = U_1 - U_2 = mgy - \dfrac{1}{2}ky^2 = (3.00\text{ kg})(9.8\text{ m/s}^2)(0.0500\text{ m}) - \dfrac{1}{2}(900\text{ N/m})(0.0500\text{ m})^2$

$K_2 = 1.47\text{ J} - 1.125\text{ J} = 0.345\text{ J}$. Solving for v_2 gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(0.345\text{ J})}{3.00\text{ kg}}} = 0.480\text{ m/s}.$

(b) The maximum speed is when K_2 is maximum, which is when $dK_2/dy = 0$. Using $K_2 = mgy - \dfrac{1}{2}ky^2$ gives $\dfrac{dK_2}{dy} = mg - ky = 0$. Solving for y gives

$y = \dfrac{mg}{k} = \dfrac{(3.00\text{ kg})(9.8\text{ m/s}^2)}{900\text{ N/m}} = 0.03267\text{ m}$. At this y,

$K_2 = (3.00\text{ kg})(9.8\text{ m/s}^2)(0.03267\text{ m}) - \dfrac{1}{2}(900\text{ N/m})(0.03267\text{ m})^2.$

$K_2 = 0.9604\text{ J} - 0.4803\text{ J} = 0.4801\text{ J}$, so $v_2 = \sqrt{\dfrac{2K_2}{m}} = 0.566\text{ m/s}.$

EVALUATE: The speed in part (b) is greater than the speed in part (a), as it should be since it is the maximum speed.

7.68. **IDENTIFY:** We need to use projectile motion and energy conservation.
SET UP: We work "backwards" in a sense. First use projectile motion to find the horizontal velocity of the wood at the base of the slide. Then use energy conservation to find how much work friction did as the wood slid down the slide. Use $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$, where W_{other} is the

work done by friction on the slide, which is what we want to find. The sketch in Fig. 7.68 organizes the quantities in the problem.

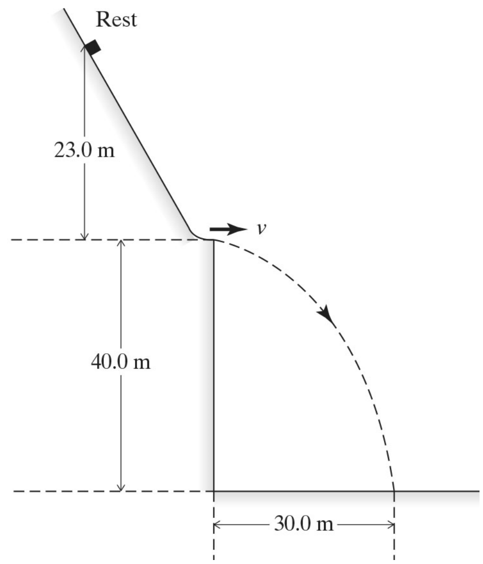


Figure 7.68

EXECUTE: Once the wood leaves the slide, it falls 40.0 m from rest while traveling 30.0 m horizontally at constant horizontal velocity. Using $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $y = \frac{1}{2}gt^2$, so

$t = \sqrt{\dfrac{2y}{g}} = \sqrt{\dfrac{80.0\text{ m}}{9.80\text{ m/s}^2}} = 2.857$ s. The constant speed v needed to travel 30.0 m horizontally in 2.857 s is $v = x/t = (30.0\text{ m})/(2.857\text{ s}) = 10.5$ m/s. This is the speed at the bottom of the slide. Now use energy conservation, calling point 1 at the top of the slide and point 2 the bottom of the slide. $U_1 + K_1 + W_{\text{other}} = U_2 + K_2$ becomes $mgh + W_{\text{f}} = \frac{1}{2}mv^2$. Solve for W_{f}: $W_{\text{f}} = m\left(\dfrac{v^2}{2} - gh\right)$. Using m = 3.00 kg, v = 10.5 m/s, and h = 23.0 m gives $W_{\text{f}} = -511$ J.

EVALUATE: The work must be negative because friction works *against* the block's motion and therefore takes away mechanical energy.

7.69. **(a) IDENTIFY** and **SET UP:** Apply $K_A + U_A + W_{\text{other}} = K_B + U_B$ to the motion from A to B.

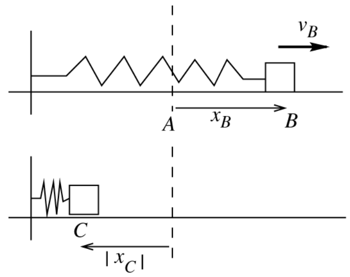
EXECUTE: $K_A = 0,\ K_B = \frac{1}{2}mv_B^2,\ U_A = 0,\ U_B = U_{\text{el},B} = \frac{1}{2}kx_B^2,$ where $x_B = 0.25$ m, and $W_{\text{other}} = W_F = Fx_B$. Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by F goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\dfrac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}.$

(b) IDENTIFY: Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block. Let point C be where the block is closest to the wall. When the block is at point C the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between B and C is $x_B + |x_C|$.

SET UP: The motion from A to B to C is described in Figure 7.69.



$K_B + U_B + W_{\text{other}} = K_C + U_C$

EXECUTE: $W_{\text{other}} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\text{ J} - 1.25\text{ J} = 3.75\text{ J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\text{ J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

Figure 7.69

Thus $3.75\text{ J} + 1.25\text{ J} = \frac{1}{2}k|x_C|^2$, giving $|x_c| = \sqrt{\dfrac{2(5.0\text{ J})}{40.0\text{ N/m}}} = 0.50\text{ m}$. The distance of the block from the wall is $0.60\text{ m} - 0.50\text{ m} = 0.10\text{ m}$.

EVALUATE: The work $(20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ done by F puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points B and C is 5.0 J.

7.70. **IDENTIFY:** Applying Newton's second law, we can use the known normal forces to find the speeds of the block at the top and bottom of the circle. We can then use energy conservation to find the work done by friction, which is the target variable.

SET UP: For circular motion $\Sigma F = m\dfrac{v^2}{R}$. Energy conservation tells us that $K_A + U_A + W_{\text{other}} = K_B + U_B$, where W_{other} is the work done by friction. $U_g = mgy$ and $K = \frac{1}{2}mv^2$.

EXECUTE: Use the given values for the normal force to find the block's speed at points A and B. At point A, Newton's second law gives $n_A - mg = m\dfrac{v_A^2}{R}$. So

$v_A = \sqrt{\dfrac{R}{m}(n_A - mg)} = \sqrt{\dfrac{0.500\text{ m}}{0.0400\text{ kg}}(3.95\text{ N} - 0.392\text{ N})} = 6.669\text{ m/s}$. Similarly at point B,

$n_B + mg = m\dfrac{v_B^2}{R}$. Solving for v_B gives

$v_B = \sqrt{\dfrac{R}{m}(n_B + mg)} = \sqrt{\dfrac{0.500\text{ m}}{0.0400\text{ kg}}(0.680\text{ N} + 0.392\text{ N})} = 3.660\text{ m/s}$. Now apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to find the work done by friction. $K_A + U_A + W_{\text{other}} = K_B + U_B$.

$W_{\text{other}} = K_B + U_B - K_A$.

$W_{\text{other}} = \dfrac{1}{2}(0.040\text{ kg})(3.66\text{ m/s})^2 + (0.04\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) - \dfrac{1}{2}(0.04\text{ kg})(6.669\text{ m/s})^2$.

$W_{\text{other}} = 0.2679\text{ J} + 0.392\text{ J} - 0.8895\text{ J} = -0.230\text{ J}$.

EVALUATE: The work done by friction is negative, as it should be. This work is equal to the loss of mechanical energy between the top and bottom of the circle.

7.71. **IDENTIFY:** We can apply Newton's second law to the block. The only forces acting on the block are gravity downward and the normal force from the track pointing toward the center of the circle. The mechanical energy of the block is conserved since only gravity does work on it. The normal force does no work since it is perpendicular to the displacement of the block. The target variable is the normal force at the top of the track.

SET UP: For circular motion $\Sigma F = m\frac{v^2}{R}$. Energy conservation tells us that $K_A + U_A + W_{\text{other}} = K_B + U_B$, where $W_{\text{other}} = 0$. $U_g = mgy$ and $K = \frac{1}{2}mv^2$.

EXECUTE: Let point A be at the bottom of the path and point B be at the top of the path. At the bottom of the path, $n_A - mg = m\frac{v^2}{R}$ (from Newton's second law).

$v_A = \sqrt{\frac{R}{m}(n_A - mg)} = \sqrt{\frac{0.800\text{ m}}{0.0500\text{ kg}}(3.40\text{ N} - 0.49\text{ N})} = 6.82\text{ m/s}$. Use energy conservation to find the speed at point B. $K_A + U_A + W_{\text{other}} = K_B + U_B$, giving $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mg(2R)$. Solving for v_B gives $v_B = \sqrt{v_A^2 - 4Rg} = \sqrt{(6.82\text{ m/s})^2 - 4(0.800\text{ m})(9.8\text{ m/s}^2)} = 3.89\text{ m/s}$. Then at point B, Newton's second law gives $n_B + mg = m\frac{v_B^2}{R}$. Solving for n_B gives $n_B = m\frac{v_B^2}{R} - mg =$

$(0.0500\text{ kg})\left(\frac{(3.89\text{ m/s})^2}{0.800\text{ m}} - 9.8\text{ m/s}^2\right) = 0.456\text{ N}$.

EVALUATE: The normal force at the top is considerably less than it is at the bottom for two reasons: the block is moving slower at the top and the downward force of gravity at the top aids the normal force in keeping the block moving in a circle.

7.72. **IDENTIFY:** Only gravity does work, so apply $K_1 + U_1 = K_2 + U_2$. Use $\Sigma\vec{F} = m\vec{a}$ to calculate the tension.

SET UP: Let $y = 0$ at the bottom of the arc. Let point 1 be when the string makes a $45°$ angle with the vertical and point 2 be where the string is vertical. The rock moves in an arc of a circle, so it has radial acceleration $a_{\text{rad}} = v^2/r$.

EXECUTE: **(a)** At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, which gives

$v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(0.80\text{ m})(1-\cos 45°)} = 2.1\text{ m/s}$.

(b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\text{ kg})(9.80\text{ m/s}^2)\cos 45° = 0.83\text{ N}$.

(c) At the bottom of the circle, the tension is the sum of the weight and the mass times the radial acceleration, $mg + mv_2^2/l = mg(1 + 2(1-\cos 45°)) = 1.9\text{ N}$.

EVALUATE: When the string passes through the vertical, the tension is greater than the weight because the acceleration is upward.

7.73. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the block.

SET UP: The motion from A to B is described in Figure 7.73.

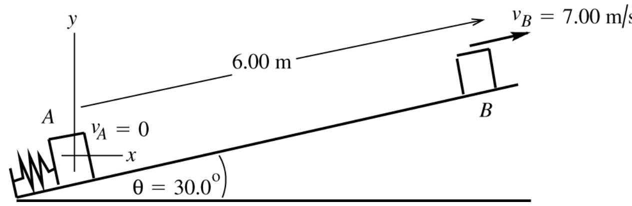


Figure 7.73

The normal force is $n = mg\cos\theta$, so $f_k = \mu_k n = \mu_k mg\cos\theta$. $y_A = 0$; $y_B = (6.00\text{ m})\sin 30.0° = 3.00$ m.
$K_A + U_A + W_{\text{other}} = K_B + U_B$

EXECUTE: Work is done by gravity, by the spring force, and by friction, so $W_{\text{other}} = W_f$ and
$U = U_{\text{el}} + U_{\text{grav}}$

$K_A = 0$, $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.50\text{ kg})(7.00\text{ m/s})^2 = 36.75$ J

$U_A = U_{\text{el},A} + U_{\text{grav},A} = U_{\text{el},A}$, since $U_{\text{grav},A} = 0$

$U_B = U_{\text{el},B} + U_{\text{grav},B} = 0 + mgy_B = (1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 44.1$ J

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg\cos\theta(\cos 180°)s = -\mu_k mg\cos\theta s$

$W_{\text{other}} = -(0.50)(1.50\text{ kg})(9.80\text{ m/s}^2)(\cos 30.0°)(6.00\text{ m}) = -38.19$ J

Thus $U_{\text{el},A} - 38.19\text{ J} = 36.75\text{ J} + 44.10\text{ J}$, giving $U_{\text{el},A} = 38.19\text{ J} + 36.75\text{ J} + 44.10\text{ J} = 119$ J.

EVALUATE: U_{el} must always be positive. Part of the energy initially stored in the spring was taken away by friction work; the rest went partly into kinetic energy and partly into an increase in gravitational potential energy.

7.74. IDENTIFY: We know the potential energy function for a conservative force. Mechanical energy is conserved.

SET UP: $F_x = -dU/dx$ and $U(x) = -\alpha x^2 + \beta x^3$.

EXECUTE: **(a)** $U_1 + K_1 = U_2 + K_2$ gives $0 + 0 = U_2 + K_2$, so $K_2 = -U_2 = -(-\alpha x_2^2 + \beta x_2^3) = \frac{1}{2}mv^2$. Using $m = 0.0900$ kg, $x = 4.00$ m, $\alpha = 2.00\text{ J/m}^2$, and $\beta = 0.300\text{ J/m}^3$, solving for v gives $v = 16.9$ m/s.

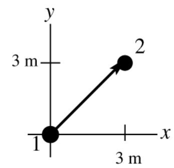
(b) $F_x = -dU/dx = -(-2\alpha x + 3\beta x^2)$. In addition, $F_x = ma_x$, so $a_x = F_x/m$. Using the numbers from (a), gives $a = 17.8\text{ m/s}^2$.

(c) The maximum x will occur when $U = 0$ since the total energy is zero. Therefore $-\alpha x^2 + \beta x^3 = 0$, so $x_{\max} = \alpha/\beta = (2.00\text{ J/m}^2)/(0.300\text{ J/m}^3) = 6.67$ m.

EVALUATE: The object is released from rest but at a small (but not zero) x. Therefore F_x is small but not zero initially, so it will start the object moving.

7.75. IDENTIFY: We are given that $\vec{F} = -\alpha xy^2\hat{j}$, $\alpha = 2.50\text{ N/m}^3$. $\vec{F}$ is not constant so use $W = \int_1^2 \vec{F}\cdot d\vec{l}$ to calculate the work. $\vec{F}$ must be evaluated along the path.

(a) SET UP: The path is sketched in Figure 7.75a.



$d\vec{l} = dx\hat{i} + dy\hat{j}$

$\vec{F} \cdot d\vec{l} = -\alpha xy^2\,dy$

On the path, $x = y$ so $\vec{F} \cdot d\vec{l} = -\alpha y^3\,dy$

Figure 7.75a

EXECUTE: $W = \int_1^2 \vec{F} \cdot d\vec{l} = \int_{y_1}^{y_2} (-\alpha y^3)\,dy = -(\alpha/4)(y_2^4 - y_1^4)$

$y_1 = 0$, $y_2 = 3.00$ m, so $W = -\frac{1}{4}(2.50\ \text{N/m}^3)(3.00\ \text{m})^4 = -50.6$ J

(b) SET UP: The path is sketched in Figure 7.75b.

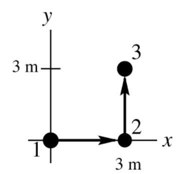


Figure 7.75b

For the displacement from point 1 to point 2, $d\vec{l} = dx\hat{i}$, so $\vec{F} \cdot d\vec{l} = 0$ and $W = 0$. (The force is perpendicular to the displacement at each point along the path, so $W = 0$.)

For the displacement from point 2 to point 3, $d\vec{l} = dy\hat{j}$, so $\vec{F} \cdot d\vec{l} = -\alpha xy^2\,dy$. On this path, $x = 3.00$ m, so

$$\vec{F} \cdot d\vec{l} = -(2.50\ \text{N/m}^3)(3.00\ \text{m})y^2\,dy = -(7.50\ \text{N/m}^2)y^2\,dy.$$

EXECUTE: $W = \int_2^3 \vec{F} \cdot d\vec{l} = -(7.50\ \text{N/m}^2)\int_{y_2}^{y_3} y^2\,dy = -(7.50\ \text{N/m}^2)\frac{1}{3}(y_3^3 - y_2^3)$

$W = -(7.50\ \text{N/m}^2)\left(\frac{1}{3}\right)(3.00\ \text{m})^3 = -67.5$ J.

(c) EVALUATE: For these two paths between the same starting and ending points the work is different, so the force is nonconservative.

7.76. IDENTIFY: Use $F_x = -\dfrac{dU}{dx}$ to relate F_x and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

SET UP: dU/dx is the slope of the graph of U versus x. $K = E - U$, so K is a maximum when U is a minimum. The maximum x is where $E = U$.

EXECUTE: **(a)** The slope of the U *vs*. x curve is negative at point A, so F_x is positive because $F_x = -dU/dx$.

(b) The slope of the curve at point B is positive, so the force is negative.

(c) The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m.

(d) The curve at point C looks pretty close to flat, so the force is zero.

(e) The object had zero kinetic energy at point A, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m.

(f) The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m.
(g) The only potential maximum, and hence the only point of unstable equilibrium, is at point C.
EVALUATE: If E is less than U at point C, the particle is trapped in one or the other of the potential "wells" and cannot move from one allowed region of x to the other.

7.77. **IDENTIFY:** The mechanical energy of the system is conserved, and Newton's second law applies. As the pendulum swings, gravitational potential energy gets transformed to kinetic energy.
SET UP: For circular motion, $F = mv^2/r$. $U_{grav} = mgh$.
EXECUTE: **(a)** Conservation of mechanical energy gives $mgh = ½ mv^2 + mgh_0$, where $h_0 = 0.800$ m. Applying Newton's second law at the bottom of the swing gives $T = mv^2/L + mg$. Combining these two equations and solving for T as a function of h gives $T = (2mg/L)h + mg(1 - 2h_0/L)$. In a graph of T versus h, the slope is $2mg/L$. Graphing the data given in the problem, we get the graph shown in Figure 7.77. Using the best-fit equation, we get $T = (9.293\text{ N/m})h + 257.3$ N. Therefore $2mg/L = 9.293$ N/m. Using $mg = 265$ N and solving for L, we get $L = 2(265\text{ N})/(9.293\text{ N/m}) = 57.0$ m.

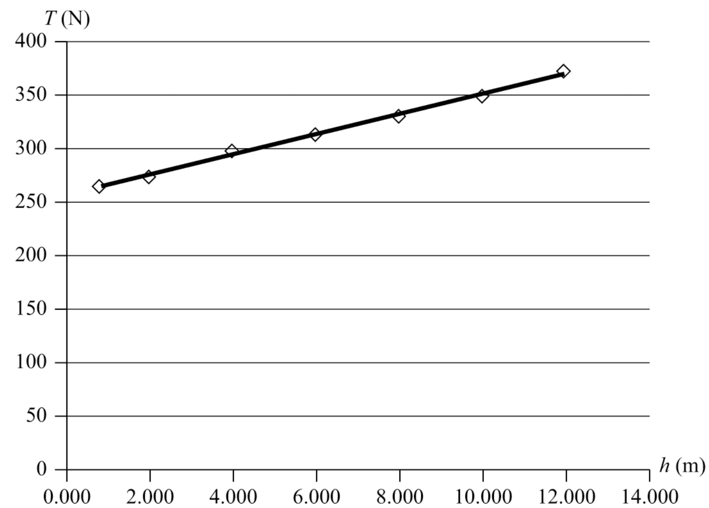


Figure 7.77

(b) $T_{max} = 822$ N, so $T = T_{max}/2 = 411$ N. We use the equation for the graph with $T = 411$ N and solve for h. $411\text{ N} = (9.293\text{ N/m})h + 257.3$ N, which gives $h = 16.5$ m.
(c) The pendulum is losing energy because negative work is being done on it by friction with the air and at the point of contact where it swings.
EVALUATE: The length of this pendulum may seem extremely large, but it is not unreasonable for a museum exhibit, which can cover a height of several floor levels.

7.78. **IDENTIFY:** Friction does negative work, and we can use $K_1 + U_1 + W_{other} = K_2 + U_2$.
SET UP: $U_1 + W_{other} = K_2$
EXECUTE: **(a)** Using $K_2 = U_1 + W_{other}$ gives $\frac{1}{2}mv^2 = mgh - (\mu_k mg\cos\theta)s$ and geometry gives $s = \dfrac{h}{\sin\theta}$. Combining these equations and solving for h gives $h = \dfrac{v^2}{2g\left(1 - \dfrac{\mu_k}{\tan\theta}\right)}$. For each material, $\theta = 52.0°$ and $v = 4.00$ m/s. Using the coefficients of sliding friction from the table in the problem, this formula gives the following results for h. (i) 0.92 m (ii) 1.1 m (iii) 2.4 m.
(b) The mass divides out, so h is unchanged and remains at 1.1 m.

(c) In the formula for h in part (a), we solve for v^2 giving $v^2 = 2gh\left(1 - \frac{\mu_k}{\tan\theta}\right)$. As θ increases (but h remains the same), $\tan\theta$ increases, so the quantity in parentheses increases since $\tan\theta$ is in the denominator. Therefore v increases.

EVALUATE: The answer in (c) makes physical sense because with h constant, a larger value for θ means that the normal force decreases so the magnitude of the friction force also decreases, and therefore friction is less able to oppose the motion of the block as it slides down the slope.

7.79. IDENTIFY: For a conservative force, mechanical energy is conserved and we can relate the force to its potential energy function.

SET UP: $F_x = -dU/dx$.

EXECUTE: **(a)** $U + K = E$ = constant. If two points have the same kinetic energy, they must have the same potential energy since the sum of U and K is constant. Since the kinetic energy curve symmetric, the potential energy curve must also be symmetric.

(b) At $x = 0$ we can see from the graph with the problem that $E = K + 0 = 0.14$ J. Since E is constant, if $K = 0$ at $x = -1.5$ m, then U must be equal to 0.14 J at that point.

(c) $U(x) = E - K(x) = 0.14\ \text{J} - K(x)$, so the graph of $U(x)$ is like the sketch in Figure 7.79.

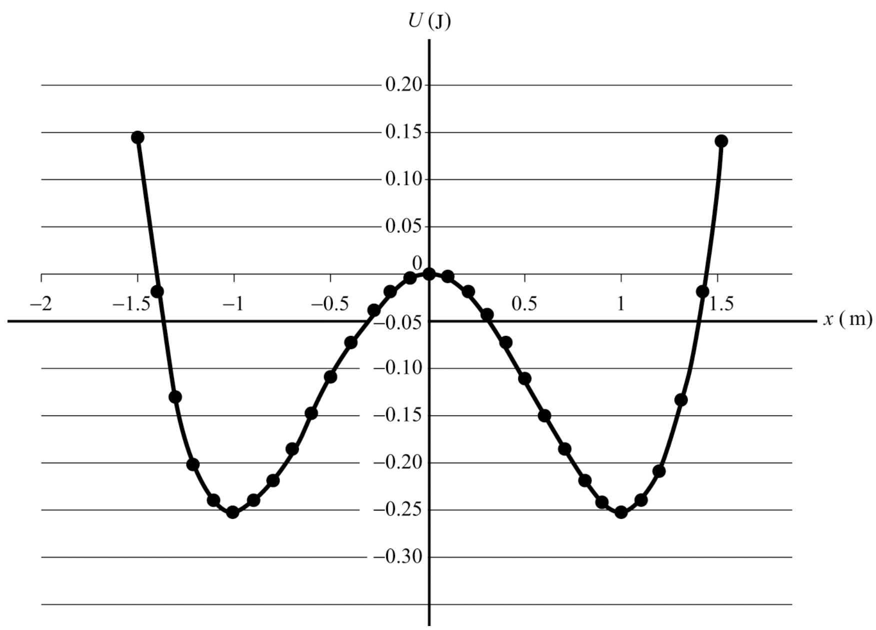


Figure 1.79

(d) Since $F_x = -dU/dx$, $F(x) = 0$ at $x = 0$, +1.0 m, and –1.0 m.

(e) $F(x)$ is positive when the slope of the $U(x)$ curve is negative, and $F(x)$ is negative when the slope of the $U(x)$ curve is positive. Therefore $F(x)$ is positive between $x = -1.5$ m and $x = -1.0$ m and between $x = 0$ and $x = 1.0$ m. $F(x)$ is negative between $x = -1.0$ m and 0 and between $x = 1.0$ m and $x = 1.5$ m.

(f) When released from $x = -1.30$ m, the sphere will move to the right until it reaches $x = -0.55$ m, at which point it has 0.12 J of potential energy, the same as at is original point of release.

EVALUATE: Even though we do not have the equation of the kinetic energy function, we can still learn much about the behavior of the system by studying its graph.

7.80. IDENTIFY: $K = E - U$ determines $v(x)$.

SET UP: v is a maximum when U is a minimum and v is a minimum when U is a maximum. $F_x = -dU/dx$. The extreme values of x are where $E = U(x)$.

EXECUTE: (a) Eliminating β in favor of α and $x_0 (\beta = \alpha/x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \left(\frac{\alpha}{x_0^2}\right)(1-1) = 0.$ $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ (α and β must be taken as positive). The graph of $U(x)$ is sketched in Figure 7.80a.

(b) $v(x) = \sqrt{-\frac{2}{m}U} = \sqrt{\left(\frac{2\alpha}{mx_0^2}\right)\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2\right)}$. The proton moves in the positive x-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$. The graph of $v(x)$ is sketched in Figure 7.80b.

(c) The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\frac{dU}{dx} = 0$, or $\frac{dU}{dx} = \frac{\alpha}{x_0}\left[-2\left(\frac{x_0}{x}\right)^3 + \left(\frac{x_0}{x}\right)^2\right] = 0,$

which has the solution $x = 2x_0$. $U(2x_0) = -\frac{\alpha}{4x_0^2}$, so $v = \sqrt{\frac{\alpha}{2mx_0^2}}$.

(d) The maximum speed occurs at a point where $\frac{dU}{dx} = 0$, and since $F_x = -\frac{dU}{dx}$, the force at this point is zero.

(e) $x_1 = 3x_0$, and $U(3x_0) = -\frac{2\alpha}{9x_0^2}$.

$$v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} = \sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - \frac{2}{9}\right)}.$$

The particle is confined to the region where $U(x) < U(x_1)$. The maximum speed still occurs at $x = 2x_0$, but now the particle will oscillate between x_1 and some minimum value (see part (f)).

(f) Note that $U(x) - U(x_1)$ can be written as

$$\frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right) + \left(\frac{2}{9}\right)\right] = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right) - \frac{1}{3}\right]\left[\left(\frac{x_0}{x}\right) - \frac{2}{3}\right],$$

which is zero (and hence the kinetic energy is zero) at $x = 3x_0 = x_1$ and $x = \frac{3}{2}x_0$. Thus, when the particle is released from x_0, it goes on to infinity, and doesn't reach any maximum distance. When released from x_1, it oscillates between $\frac{3}{2}x_0$ and $3x_0$.

EVALUATE: In each case the proton is released from rest and $E = U(x_i)$, where x_i is the point where it is released. When $x_i = x_0$ the total energy is zero. When $x_i = x_1$ the total energy is negative. $U(x) \to 0$ as $x \to \infty$, so for this case the proton can't reach $x \to \infty$ and the maximum x it can have is limited.

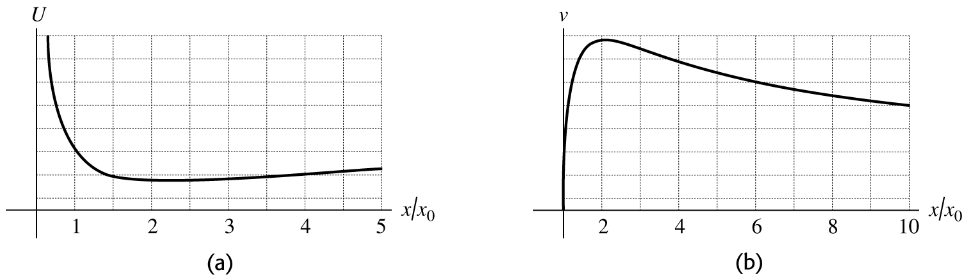


Figure 7.80

7.81. **IDENTIFY:** We model the DNA molecule as an ideal spring.
SET UP: Hooke's law is $F = kx$.
EXECUTE: Since F is proportional to x, if a 3.0-pN force causes a 0.10-nm deflection, a 6.0-pN force, which is twice as great, should use twice as much deflection, or 0.2 nm. This makes choice (c) correct.
EVALUATE: A simple model can give rough but often meaningful insight into the behavior of a complicated system.

7.82. **IDENTIFY** and **SET UP:** If a system obeys Hooke's law, a graph of force versus displacement will be a straight line through the origin having positive slope equal to the force constant.
EXECUTE: The graph is a straight line. Reading its slope from the graph gives (2.0 pN)/(20 nm) = 0.1 pN/nm, which makes choice (b) correct.
EVALUATE: The molecule would obey Hooke's law only over a restricted range of displacements.

7.83. **IDENTIFY** and **SET UP:** The energy is the area under the force-displacement curve.
EXECUTE: Using the area under the triangular section from 0 to 50 nm, we have
$A = \frac{1}{2}(5.0\text{ pN})(50\text{ nm}) = 1.25\times10^{-19}\text{ J} \approx 1.2\times10^{-19}\text{ J}$, which makes choice (b) correct.
EVALUATE: This amount of energy is quite small, but recall that this is the energy of a microscopic molecule.

7.84. **IDENTIFY** and **SET UP:** $P = Fv$ and at constant speed $x = vt$. The DNA follows Hooke's law, so $F = kx$.
EXECUTE: $P = Fv = kxv = k(vt)v = kv^2t$. Since k and v are constant, the power is proportional to the time, so the graph of power versus time should be a straight line through the origin, which fits choice (a).
EVALUATE: The power increases with time because the force increases with x and x increases with t.