36

DIFFRACTION

VP36.3.1. **IDENTIFY:** This problem is about single-slit diffraction.

SET UP: For small angles, dark spots occur at $y_m = R\dfrac{m\lambda}{a}$. The small-angle approximation is valid.

EXECUTE: **(a)** We want R. Solve $y_m = R\dfrac{m\lambda}{a}$. $R = \dfrac{a\lambda}{my} = \dfrac{(0.250\text{ mm})(14.0\text{ mm})}{2(645\text{ mn})} = 2.71\text{ m}$.

(b) We want the distance 2y. Use $y_m = R\dfrac{m\lambda}{a}$ and divide. $\dfrac{y_{525}}{y_{645}} = \dfrac{\dfrac{Rm(525\text{ nm})}{a}}{\dfrac{Rm(645\text{ nm})}{a}} = 0.81395$. The distance is $2y_{645}$ = (0.81395)(28.0 mm) = 22.8 m.

EVALUATE: Since $y \propto \lambda$, shorter-wavelength dark fringes should be close together, as we have just seen.

VP36.3.2. **IDENTIFY:** We are dealing with single-slit diffraction.

SET UP: For small angles, dark spots occur at $y_m = R\dfrac{m\lambda}{a}$. The small-angle approximation is valid.

EXECUTE: **(a)** We want λ. Solve $y_m = R\dfrac{m\lambda}{a}$. $\lambda = \dfrac{ay_m}{mR} = \dfrac{(0.221\text{ mm})(45.7\text{ mm})}{3(5.00\text{ m})} = 673\text{ nm}$.

(b) We want θ to the second dark fringe. $a\sin\theta_2 = 2\lambda$ gives $\theta_2 = 0.349°$.

EVALUATE: With such a small angle of 0.349°, it is clear that the small-angle approximation is valid.

VP36.3.3. **IDENTIFY:** This problem involves the intensity of a single-slit diffraction pattern.

SET UP: $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$, 17.5 rad = 1002.7°.

EXECUTE: **(a)** We want θ. Using $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$ gives

$$\theta = \arcsin\left(\frac{\lambda\beta}{2\pi a}\right) = \arcsin\left(\frac{(545\text{ nm})(35.0\text{ rad})}{2\pi(0.250\text{ mm})}\right) = 0.696°.$$

(b) We want I. $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2 = I_0\left(\dfrac{\sin 17.5\text{ rad}}{17.5\text{ rad}}\right)^2 = I_0\left(\dfrac{\sin 1002.7°}{17.5\text{ rad}}\right)^2 = 0.00311I_0$.

EVALUATE: It is important to realize that in the intensity equation, the $\beta/2$ in the numerator can be expressed as degrees or radians, but in the numerator it must be in radians.

VP36.3.4. **IDENTIFY:** We are dealing with the intensity in single-slit diffraction.

SET UP: $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$.

EXECUTE: **(a)** We want θ to the first dark fringe. $a\sin\theta_1 = \lambda$.

$\theta_1 = \arcsin(\lambda/a) = \arcsin\left(\dfrac{576\text{ nm}}{0.185\text{ mm}}\right) = 0.178°.$

(b) We want the intensity. $\dfrac{\beta}{2} = \dfrac{2\pi a\sin\theta}{\lambda}$. $\theta = \dfrac{0.178°}{2} = 0.0892°$. First get $\beta/2$.

$\dfrac{\beta}{2} = \dfrac{\pi(0.185\text{ mm})\sin(0.0892°)}{576\text{ nm}} = 1.5708\text{ rad} = 90.0°$. Now use $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$.

$I = I_0\left(\dfrac{\sin 90.0°}{1.5708\text{ rad}}\right)^2 = 0.405I_0.$

EVALUATE: The intensity is close to $I_0/2$, but not equal to it.

VP36.5.1. **IDENTIFY:** We have a diffraction grating.

SET UP: Bright fringes occur at $d\sin\theta_m = m\lambda$, $d = 1/(825\text{ slits/mm})$.

EXECUTE: **(a)** We want λ. Solve $d\sin\theta_m = m\lambda$ for λ, giving

$\lambda = \dfrac{[1/(825\text{ slits/mm})]\sin 41.0°}{2} = 398\text{ nm}.$

(b) We want θ_1 and θ_3. $\theta_1 = \arcsin(\lambda/d) = \arcsin\left(\dfrac{398\text{ nm}}{825\text{ mm}}\right) = 19.1°.$

$\theta_3 = \arcsin(3\lambda/d) = \arcsin\left(\dfrac{3(398\text{ nm})}{825\text{ mm}}\right) = 79.8°.$

EVALUATE: Notice that the bright fringes are *not* evenly spaced.

VP36.5.2. **IDENTIFY:** We have a diffraction grating.

SET UP: Bright fringes occur at $d\sin\theta_m = m\lambda$.

EXECUTE: **(a)** We want the slit density. Solve $d\sin\theta_m = m\lambda$ for d, giving

$d = \dfrac{\lambda}{\sin\theta_1} = \dfrac{625\text{ nm}}{\sin 24.0°} = 1537\text{ nm}$. The slit spacing is $1/d$, so the slit spacing is 651 slits/mm.

(b) We want θ_2 and θ_3. $\theta_2 = \arcsin(2\lambda/d) = \arcsin\left(\dfrac{2(625\text{ nm})}{1537\text{ nm}}\right) = 54.4°.$

$\theta_3 = \arcsin(3\lambda/d) = \arcsin\left(\dfrac{3(625\text{ nm})}{1537\text{ nm}}\right) = \arcsin(1.22)$, so there is no θ_3.

EVALUATE: Once $\sin\theta > 1$ there are no more fringes.

VP36.5.3. **IDENTIFY:** We are dealing with x-ray diffraction by a crystal.

SET UP: The plane spacing is $d = 0.124$ nm. Strong interference maxima occur when $2d\sin\theta_m = m\lambda$.

EXECUTE: **(a)** We want θ for the first strong maximum. Solve $2d\sin\theta_m = m\lambda$.

$\theta_1 = \arcsin(\lambda/2d) = \arcsin\left(\dfrac{0.124\text{ nm}}{2(0.165\text{ nm})}\right) = 22.1°.$

(b) Are there other angles? $\theta_2 = \arcsin(2\lambda/2d) = \arcsin\left(\dfrac{0.124\text{ nm}}{0.165\text{ nm}}\right) = 48.7°$. There are no other angles because $\sin\theta_3 > 1$.

EVALUATE: If $\lambda << d$, there could be many more maxima.

VP36.5.4. **IDENTIFY:** This problem is about x-ray diffraction by a crystal.

SET UP: The plane spacing is d = 0.158 nm. Strong interference maxima occur when $2d\sin\theta_m = m\lambda$. We want λ.

EXECUTE: **(a)** $2d\sin\theta_m = m\lambda = \lambda$, so λ = 2(0.158 nm) sin 36.0° = 0.186 nm.

(b) $2d\sin\theta_3 = 3\lambda$. λ = 2(0.158 nm)(sin 88.0°)/3 = 0.105 nm.

EVALUATE: Since $\sin\theta_m = m\lambda/2d$, to have a large m, λ/d must be small, so $\lambda << d$.

VP36.6.1. **IDENTIFY:** This problem is about the resolving power of a telescope.

SET UP: Rayleigh criterion: $\theta \approx 1.22\dfrac{\lambda}{D}$.

EXECUTE: **(a)** We want D. Solve for D: D = (1.22) (1.70 cm)/(9.00×10^{-5}rad) = 230 m.

(b) We want θ if λ = 21.1 cm. $\theta \approx 1.22\dfrac{\lambda}{D}$ = (1.22)(21.1 cm)/(230 m) = 0.114 mrad = 0.0640°.

EVALUATE: To increase resolution, either increase D or decrease λ, or both.

VP36.6.2. **IDENTIFY:** We are looking at resolving power.

SET UP: For small angles $\theta \approx 1.22\dfrac{\lambda}{D}$, $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$, $m = -s'/s$, f = 0.250 m = 250 mm.

EXECUTE: **(a)** We want the maximum distance. At $f/4$ the diameter of the lens is $D = f/4$, so D = (250 mm)/4 = 62.5 mm. This gives $\theta \approx 1.22\dfrac{\lambda}{D}$ = (1.22)(550 nm)/(62.5 mm) = 1.074×10^{-5}rad. For the two points, $\sin\theta = \dfrac{5.00 \text{ mm}}{x}$, so x = (5.00 mm)/(1.074×10^{-5}rad.) = 466 m.

(b) We want the distance between the two points on the lens's image. Use $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'}$. $\dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s}$ = 1/(0.250 m) – 1/(466 m), so s' = 0.2501 m. $h' = h_0|m| = h_0\dfrac{s'}{s}$. This gives

$$h' = (5.00 \text{ mm})\left(\frac{0.2501 \text{ m}}{466 \text{ m}}\right) = 0.00268 \text{ mm}.$$

EVALUATE: 466 m is around 1500 ft. Most people could not even *see* the two points to focus at that distance.

VP36.6.3. **IDENTIFY:** This problem involves an Airy disk and a camera lens.

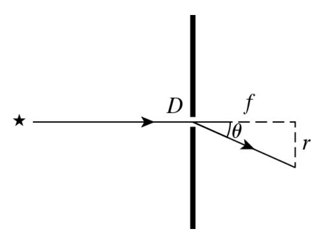

Figure VP36.6.3

SET UP and **EXECUTE:** **(a)** We want D. Refer to Fig. VP36.63. $\sin\theta = 1.22\dfrac{\lambda}{D}$ The image of a very distant object is formed at the focal point of the lens, so it is 135 mm from the lens. $r << f$, so $\theta \approx r/f$. Thus $\dfrac{r}{f} = 1.22\dfrac{\lambda}{D}$, giving $D = \dfrac{1.22\lambda f}{r}$ = (1.22)(575 nm)(135 mm)/(0.0112 mm) = 8.46 mm.

(b) We want the *f*-number. If the *f*-number is *f/x*, it means that $D = f/x$. Therefore $x = \dfrac{f}{D} = \dfrac{135\text{ mm}}{8.46\text{ mm}} = 16.0$, so the *f*-number is *f*/16.0.

EVALUATE: An *f*-number of *f*/16 is considered a fairly small lens aperture.

VP36.6.4. **IDENTIFY:** This problem is about the resolving power of a telescope.

SET UP: Rayleigh criterion: $\theta \approx 1.22\dfrac{\lambda}{D}$. We want the distance to the planet and its star.

EXECUTE: **(a)** λ = 690 nm. $\theta \approx 1.22\dfrac{\lambda}{D}$ = (1.22)(690 nm)/(8.0 m) = 1.052×10^{-7} rad. For planet and star: $\theta = R/x \rightarrow x = R/\theta = 1.5\times10^{11}\text{ m}/1.052\times10^{-7}\text{ rad} = 1.6\times10^{19}$ m. Using the fact that 1 ly $= 9.46\times10^{15}$ m, we get x = 150 ly.

(b) λ = 1400 nm. λ increases by a factor 1400/690, so θ increases by the same factor since $\theta \propto \lambda$. $x \propto 1/\theta$, so x decreases by a factor 690/1400. Therefore x = (690/1400)(150 ly) = 74 ly.

EVALUATE: Clearly it's better to use the shortest possible wavelength.

36.1. **IDENTIFY:** Use $y = x\tan\theta$ to calculate the angular position θ of the first minimum. The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$, $m = \pm1, \pm2, \ldots$. First minimum means $m = 1$ and $\sin\theta_1 = \lambda/a$ and $\lambda = a\sin\theta_1$. Use this equation to calculate λ.

SET UP: The central maximum is sketched in Figure 36.1.

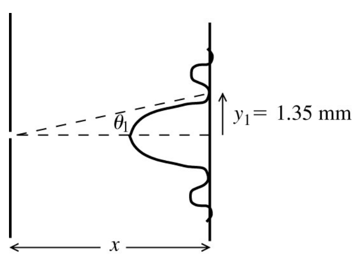

Figure 36.1

EXECUTE: $y_1 = x\tan\theta_1$.

$\tan\theta_1 = \dfrac{y_1}{x} = \dfrac{1.35\times10^{-3}\text{ m}}{2.00\text{ m}} = 0.675\times10^{-3}$.

$\theta_1 = 0.675\times10^{-3}$ rad.

$\lambda = a\sin\theta_1 = (0.750\times10^{-3}\text{ m})\sin(0.675\times10^{-3}\text{ rad}) = 506\text{ nm}.$

EVALUATE: θ_1 is small so the approximation used to obtain $y_m = x\dfrac{m\lambda}{a}$ is valid and this equation could have been used.

36.2. **IDENTIFY:** This problem is about single-slit diffraction.

SET UP: Dark spots occur when $a\sin\theta_m = m\lambda$, $y = R\tan\theta$, for small angles $y_m = R\dfrac{m\lambda}{a}$. We want the width of the central maximum on the screen when λ = 0.125 mm.

EXECUTE: When λ = 500 nm, we can use the small-angle approximation to find the width a of the slit. The width of the central maximum is 8.00 mm, so 8.00 mm = $2y_1 = 2R\dfrac{\lambda}{a} = 2(2.00\text{ m})\dfrac{500\text{ nm}}{a}$, which gives a = 0.250 mm. Now find θ when λ = 0.125 mm. We don't know that we can use the small-angle approximation, so use $a\sin\theta_m = m\lambda$. At the first dark fringe we have $\sin\theta = \dfrac{\lambda}{a} = \dfrac{0.125\text{ mm}}{0.250\text{ mm}} = 0.500$, so θ = 30.0°. (As we see, the small-angle approximation would not be justified.) $y = R\tan\theta$, and the width of the central maximum is $2y$, so we have

$2y = 2R\tan\theta = 2(2.00\text{ m})\tan 30.0° = 2.31\text{ m}.$

Evaluate: The central maximum is much wider with the greater wavelength light. In that case, the small-angle approximation certainly cannot be used.

36.3. Identify: The dark fringes are located at angles θ that satisfy $\sin\theta = \dfrac{m\lambda}{a}$, $m = \pm 1, \pm 2, \ldots$.

Set Up: The largest value of $|\sin\theta|$ is 1.00.

Execute: **(a)** Solve for m that corresponds to $\sin\theta = 1$: $m = \dfrac{a}{\lambda} = \dfrac{0.0666\times 10^{-3}\text{ m}}{585\times 10^{-9}\text{ m}} = 113.8$. The largest value m can have is 113. $m = \pm 1, \pm 2, \ldots, \pm 113$ gives 226 dark fringes.

(b) For $m = \pm 113$, $\sin\theta = \pm 113\left(\dfrac{585\times 10^{-9}\text{ m}}{0.0666\times 10^{-3}\text{ m}}\right) = \pm 0.9926$ and $\theta = \pm 83.0°$.

Evaluate: When the slit width a is decreased, there are fewer dark fringes. When $a < \lambda$ there are no dark fringes and the central maximum completely fills the screen.

36.4. Identify and **Set Up:** λ/a is very small, so the approximate expression $y_m = x\dfrac{m\lambda}{a}$ is accurate. The distance between the two dark fringes on either side of the central maximum is $2y_1$.

Execute: $y_1 = \dfrac{\lambda x}{a} = \dfrac{(633\times 10^{-9}\text{ m})(3.50\text{ m})}{0.750\times 10^{-3}\text{ m}} = 2.95\times 10^{-3}\text{ m} = 2.95\text{ mm}$. $2y_1 = 5.90\text{ mm}$.

Evaluate: When a is decreased, the width $2y_1$ of the central maximum increases.

36.5. Identify: The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$.

Set Up: $a = 12.0$ cm. $x = 8.00$ m.

Execute: The angle to the first minimum is $\theta = \arcsin\left(\dfrac{\lambda}{a}\right) = \arcsin\left(\dfrac{9.00\text{ cm}}{12.00\text{ cm}}\right) = 48.6°$.

So the distance from the central maximum to the first minimum is just $y_1 = x\tan\theta = (8.00\text{ m})\tan(48.6°) = \pm(9.07\text{ m})$.

Evaluate: $2\lambda/a$ is greater than 1, so only the $m = 1$ minimum is heard.

36.6. Identify: The angle that locates the first diffraction minimum on one side of the central maximum is given by $\sin\theta = \dfrac{\lambda}{a}$. The time between crests is the period T. $f = \dfrac{1}{T}$ and $\lambda = \dfrac{v}{f}$.

Set Up: The time between crests is the period, so $T = 1.0$ h.

Execute: **(a)** $f = \dfrac{1}{T} = \dfrac{1}{1.0\text{ h}} = 1.0\text{ h}^{-1}$. $\lambda = \dfrac{v}{f} = \dfrac{800\text{ km/h}}{1.0\text{ h}^{-1}} = 800\text{ km}$.

(b) Africa-Antarctica: $\sin\theta = \dfrac{800\text{ km}}{4500\text{ km}}$ and $\theta = 10.2°$.

Australia-Antarctica: $\sin\theta = \dfrac{800\text{ km}}{3700\text{ km}}$ and $\theta = 12.5°$.

Evaluate: Diffraction effects are observed when the wavelength is about the same order of magnitude as the dimensions of the opening through which the wave passes.

36.7. Identify: We can model the hole in the concrete barrier as a single slit that will produce a single-slit diffraction pattern of the water waves on the shore.

Set Up: For single-slit diffraction, the angles at which destructive interference occurs are given by $\sin\theta_m = m\lambda/a$, where $m = 1, 2, 3, \ldots$.

EXECUTE: (a) The frequency of the water waves is $f = 75.0\ \text{min}^{-1} = 1.25\ \text{s}^{-1} = 1.25\ \text{Hz},$ so their wavelength is $\lambda = v/f = (15.0\ \text{cm/s})/(1.25\ \text{Hz}) = 12.0\ \text{cm}.$

At the first point for which destructive interference occurs, we have $\tan\theta = (0.613\ \text{m})/(3.20\ \text{m}) \Rightarrow \theta = 10.84°.$ $a\sin\theta = \lambda$ and $a = \lambda/\sin\theta = (12.0\ \text{cm})/(\sin 10.84°) = 63.8\ \text{cm}.$

(b) Find the angles at which destructive interference occurs.

$\sin\theta_2 = 2\lambda/a = 2(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_2 = \pm 22.1°.$

$\sin\theta_3 = 3\lambda/a = 3(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_3 = \pm 34.3°.$

$\sin\theta_4 = 4\lambda/a = 4(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_4 = \pm 48.8°.$

$\sin\theta_5 = 5\lambda/a = 5(12.0\ \text{cm})/(63.8\ \text{cm}) \rightarrow \theta_5 = \pm 70.1°.$

EVALUATE: These are large angles, so we cannot use the approximation that $\theta_m \approx m\lambda/a.$

36.8. IDENTIFY: The angle is small, so $y_m = x\dfrac{m\lambda}{a}$ applies.

SET UP: The width of the central maximum is $2y_1,$ so $y_1 = 3.00\ \text{mm}.$

EXECUTE: (a) $y_1 = \dfrac{x\lambda}{a} \Rightarrow a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-7}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-4}\ \text{m}.$

(b) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-5}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-2}\ \text{m} = 4.2\ \text{cm}.$

(c) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-10}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-7}\ \text{m}.$

EVALUATE: The ratio a/λ stays constant, so a is smaller when λ is smaller.

36.9. IDENTIFY and SET UP: $v = f\lambda$ gives $\lambda.$ The person hears no sound at angles corresponding to diffraction minima. The diffraction minima are located by $\sin\theta = m\lambda/a,\ m = \pm1, \pm2, \ldots$ Solve for $\theta.$

EXECUTE: $\lambda = v/f = (344\ \text{m/s})/(1250\ \text{Hz}) = 0.2752\ \text{m};$ $a = 1.00\ \text{m}.$ $m = \pm1,$ $\theta = \pm16.0°;$ $m = \pm2,$ $\theta = \pm33.4°;$ $m = \pm3,$ $\theta = \pm55.6°;$ no solution for larger m.

EVALUATE: $\lambda/a = 0.28$ so for the large wavelength sound waves diffraction by the doorway is a large effect. Diffraction would not be observable for visible light because its wavelength is much smaller and $\lambda/a << 1.$

36.10. IDENTIFY: Compare E_y to the expression $E_y = E_{\text{max}}\sin(kx - \omega t)$ and determine k, and from that calculate $\lambda.$ $f = c/\lambda.$ The dark bands are located by $\sin\theta = \dfrac{m\lambda}{a}.$

SET UP: $c = 3.00\times10^{8}\ \text{m/s}.$ The first dark band corresponds to $m = 1.$

EXECUTE: (a) $E = E_{\text{max}}\sin(kx - \omega t).$ $k = \dfrac{2\pi}{\lambda} \Rightarrow \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.40\times10^{7}\ \text{m}^{-1}} = 4.488\times10^{-7}\ \text{m}.$

$f\lambda = c \Rightarrow f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^{8}\ \text{m/s}}{4.488\times10^{-7}\ \text{m}} = 6.68\times10^{14}\ \text{Hz}.$

(b) $a\sin\theta = \lambda.$ $a = \dfrac{\lambda}{\sin\theta} = \dfrac{4.488\times10^{-7}\ \text{m}}{\sin 28.6°} = 9.38\times10^{-7}\ \text{m}.$

(c) $a\sin\theta = m\lambda (m = 1, 2, 3, \ldots).$ $\sin\theta_2 = \pm2\dfrac{\lambda}{a} = \pm2\dfrac{4.488\times10^{-7}\ \text{m}}{9.38\times10^{-7}\ \text{m}}$ so $\theta_2 = \pm73.2°.$

EVALUATE: For $m = 3,$ $\dfrac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

36.11. IDENTIFY: Calculate the angular positions of the minima and use $y = x\tan\theta$ to calculate the distance on the screen between them.

(a) SET UP: The central bright fringe is shown in Figure 36.11a.

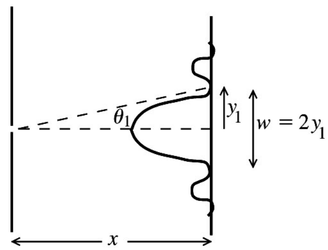

Figure 36.11a

EXECUTE: The first minimum is located by

$$\sin\theta_1 = \frac{\lambda}{a} = \frac{633\times10^{-9}\text{ m}}{0.350\times10^{-3}\text{ m}} = 1.809\times10^{-3}.$$

$$\theta_1 = 1.809\times10^{-3}\text{ rad}.$$

$$y_1 = x\tan\theta_1 = (3.00\text{ m})\tan(1.809\times10^{-3}\text{ rad}) = 5.427\times10^{-3}\text{ m}.$$

$$w = 2y_1 = 2(5.427\times10^{-3}\text{ m}) = 1.09\times10^{-2}\text{ m} = 10.9\text{ mm}.$$

(b) SET UP: The first bright fringe on one side of the central maximum is shown in Figure 36.11b.

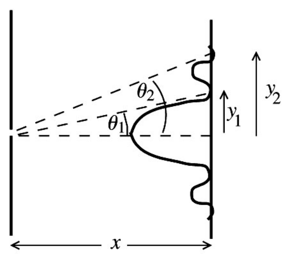

Figure 36.11b

EXECUTE: $w = y_2 - y_1$.

$y_1 = 5.427\times10^{-3}$ m (part (a)).

$$\sin\theta_2 = \frac{2\lambda}{a} = 3.618\times10^{-3}.$$

$$\theta_2 = 3.618\times10^{-3}\text{ rad}.$$

$$y_2 = x\tan\theta_2 = 1.085\times10^{-2}\text{ m}.$$

$$w = y_2 - y_1 = 1.085\times10^{-2}\text{ m} - 5.427\times10^{-3}\text{ m} = 5.4\text{ mm}.$$

EVALUATE: The central bright fringe is twice as wide as the other bright fringes.

36.12. IDENTIFY: We are dealing with single-slit diffraction.

SET UP: We want the wavelength to double the width of the central maximum on a screen. We cannot use the small-angle approximation in this case. The first dark fringe occurs when $a\sin\theta = \lambda$, $y = R\tan\theta$. Call $\lambda_1 = 120\ \mu$m and λ_2 the wavelength we want to find.

EXECUTE: $y_1 = R\tan\theta_1$ gives $\theta_1 = \arctan(\lambda_1/R) = \arctan\dfrac{45.0\text{ cm}}{150\text{ cm}} = 16.7°$. $y_2 = 2y_1$, so $y_2 = R\tan\theta_2$ so $\theta_2 = \arctan(\lambda_2/R) = \arctan\dfrac{90.0\text{ cm}}{150\text{ cm}} = 31.0°$. At the first dark spots $a\sin\theta = \lambda$ so $\dfrac{a\sin\theta_2}{a\sin\theta_1} = \dfrac{\lambda_2}{\lambda_1}$. This gives $\dfrac{\sin 31.0°}{\sin 16.7°} = \dfrac{\lambda_2}{120\ \mu\text{m}}$. So $\lambda_2 = 215\ \mu$m.

EVALUATE: Notice that to double the pattern width on the screen we do *not* double the wavelength or the angle θ.

36.13. IDENTIFY: The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$. For part (b) apply $I = I_0\left\{\dfrac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2$.

SET UP: For the first minimum, $m = 1$. The intensity at $\theta = 0$ is I_0.

EXECUTE: **(a)** $\sin\theta = \dfrac{m\lambda}{a} = \sin 90.0° = 1 = \dfrac{m\lambda}{a} = \dfrac{\lambda}{a}$. Thus $a = \lambda = 580\text{ nm} = 5.80\times10^{-4}$ mm.

(b) Using $I = I_0\left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2$ gives

$$\frac{I}{I_0} = \left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 = \left\{\frac{\sin[\pi(\sin\pi/4)]}{\pi(\sin\pi/4)}\right\}^2 = 0.128.$$

EVALUATE: If $a = \lambda/2$, for example, then at $\theta = 45°$, $\frac{I}{I_0} = \left\{\frac{\sin[(\pi/2)(\sin\pi/4)]}{(\pi/2)(\sin\pi/4)}\right\}^2 = 0.65$. As a/λ decreases, the screen becomes more uniformly illuminated.

36.14. **IDENTIFY:** $I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$. $\beta = \frac{2\pi}{\lambda}a\sin\theta$.

SET UP: The angle θ is small, so $\sin\theta \approx \tan\theta \approx y/x$.

EXECUTE: $\beta = \frac{2\pi a}{\lambda}\sin\theta \approx \frac{2\pi a}{\lambda}\frac{y}{x} = \frac{2\pi(4.50\times10^{-4}\text{ m})}{(6.20\times10^{-7}\text{ m})(3.00\text{ m})}y = (1520\text{ m}^{-1})y.$

(a) $y = 1.00\times10^{-3}$ m: $\frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(1.00\times10^{-3}\text{ m})}{2} = 0.760.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(0.760)}{0.760}\right)^2 = 0.822I_0.$$

(b) $y = 3.00\times10^{-3}$ m: $\frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(3.00\times10^{-3}\text{ m})}{2} = 2.28.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(2.28)}{2.28}\right)^2 = 0.111I_0.$$

(c) $y = 5.00\times10^{-3}$ m: $\frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(5.00\times10^{-3}\text{ m})}{2} = 3.80.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(3.80)}{3.80}\right)^2 = 0.0259I_0.$$

EVALUATE: The first minimum occurs at $y_1 = \frac{\lambda x}{a} = \frac{(6.20\times10^{-7}\text{ m})(3.00\text{ m})}{4.50\times10^{-4}\text{ m}} = 4.1\text{ mm}$. The distances in parts (a) and (b) are within the central maximum. $y = 5.00$ mm is within the first secondary maximum.

36.15. **IDENTIFY:** The space between the skyscrapers behaves like a single slit and diffracts the radio waves.
SET UP: Cancellation of the waves occurs when $a\sin\theta = m\lambda$, $m = 1, 2, 3, \ldots$, and the intensity of the waves is given by $I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \frac{\pi a\sin\theta}{\lambda}$.

EXECUTE: **(a)** First find the wavelength of the waves:
$\lambda = c/f = (3.00\times10^8\text{ m/s})/(88.9\text{ MHz}) = 3.375\text{ m}$.
For no signal, $a\sin\theta = m\lambda$.
$m = 1$: $\sin\theta_1 = (1)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_1 = \pm13.0°$.
$m = 2$: $\sin\theta_2 = (2)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_2 = \pm26.7°$.
$m = 3$: $\sin\theta_3 = (3)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_3 = \pm42.4°$.
$m = 4$: $\sin\theta_4 = (4)(3.375\text{ m})/(15.0\text{ m}) \Rightarrow \theta_4 = \pm64.1°$.

(b) $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2=\dfrac{\pi a\sin\theta}{\lambda}=\dfrac{\pi(15.0\text{ m})\sin(5.00°)}{3.375\text{ m}}=1.217$ rad.

$$I=(3.50\text{ W/m}^2)\left[\frac{\sin(1.217\text{ rad})}{1.217\text{ rad}}\right]^2=2.08\text{ W/m}^2.$$

EVALUATE: The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

36.16. **IDENTIFY:** The intensity on the screen varies as the light spreads out (diffracts) after passing through the single slit.

SET UP: $I=I_0\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta=\dfrac{2\pi}{\lambda}a\sin\theta.$

EXECUTE: $\beta=\dfrac{2\pi}{\lambda}a\sin\theta=\left(\dfrac{2\pi}{592\times10^{-9}\text{ m}}\right)(0.0290\times10^{-3}\text{ m})(\sin 1.20^{\text{o}})=6.44589$ rad.

$$I=I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2=(4.00\times10^{-5}\text{ W/m}^2)\left[\frac{\sin(6.44589\text{ rad})}{6.44589\text{ rad}}\right]^2=2.54\times10^{-8}\text{ W/m}^2.$$

EVALUATE: The intensity is less than 1/1500 of the intensity of the light at the center of the central maximum.

36.17. **IDENTIFY:** In this problem we combine double-slit interference with single-slit diffraction.
SET UP: Let D signify a double-slit effect and S signify a single-slit effect. The phases are

$\phi_D=344°$, $\phi_S=172°$. $I=I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2.$

EXECUTE: **(a)** We want to relate the slit width a to the slit spacing d. $\dfrac{\phi_D}{360°}=\dfrac{344°}{360°}=\dfrac{d\sin\theta}{\lambda}$ and

$\dfrac{\phi_S}{360°}=\dfrac{172°}{360°}=\dfrac{a\sin\theta}{\lambda}$. Taking the ratio gives $\dfrac{344°/360°}{172°/360°}=\dfrac{d\sin\theta/\lambda}{a\sin\theta/\lambda}=\dfrac{d}{a}$, so $d=2a$.

(b) We want the intensity at $\theta=22.0°$. Use $I=I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$. ϕ is the phase difference for double-slit interference, so $\phi=344°$. β is the phase difference for single-slit diffraction, so

$\beta=172°=3.00$ rad. Using $I=I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$ gives

$$I=(0.234\text{ W/m}^2)\cos^2(344°/2)\left(\frac{\sin(172°/2)}{(3.00\text{ rad})/2}\right)^2=0.101\text{ W/m}^2.$$

EVALUATE: The overall intensity is due to diffraction by each slit and interference of the light from the two slits.

36.18. **IDENTIFY:** The intensity at the screen is due to a combination of single-slit diffraction and double-slit interference.

SET UP: $I=I_0\left(\cos^2\dfrac{\phi}{2}\right)\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$, where $\phi=\dfrac{2\pi d}{\lambda}\sin\theta$ and $\beta=\dfrac{2\pi}{\lambda}a\sin\theta.$

EXECUTE: $\tan\theta=\dfrac{9.00\times10^{-4}\text{ m}}{0.750\text{ m}}=1.200\times10^{-3}$. θ is small, so $\sin\theta\approx\tan\theta$.

$$\phi=\frac{2\pi d}{\lambda}\sin\theta=\frac{2\pi(0.640\times10^{-3}\text{ m})}{568\times10^{-9}\text{ m}}(1.200\times10^{-3})=8.4956\text{ rad}.$$

$$\beta = \frac{2\pi a}{\lambda}\sin\theta = \frac{2\pi(0.434\times10^{-3}\text{ m})}{568\times10^{-9}\text{ m}}(1.200\times10^{-3}) = 5.7611\text{ rad.}$$

$$I = (5.00\times10^{-4}\text{ W/m}^2)(\cos 4.2478\text{ rad})^2\left[\frac{\sin 2.8805\text{ rad}}{2.8805}\right]^2 = 8.06\times10^{-7}\text{ W/m}^2.$$

EVALUATE: The intensity as decreased by a factor of almost a thousand, so it would be difficult to see the light at the screen.

36.19. **(a) IDENTIFY** and **SET UP:** The interference fringes (maxima) are located by $d\sin\theta = m\lambda$, with $m = 0, \pm1, \pm2, \ldots$. The intensity I in the diffraction pattern is given by $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, with $\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta$. We want $m = \pm3$ in the first equation to give θ that makes $I = 0$ in the second equation.

EXECUTE: $d\sin\theta = m\lambda$ gives $\beta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{3\lambda}{d}\right) = 2\pi(3a/d)$.

$I = 0$ says $\frac{\sin\beta/2}{\beta/2} = 0$ so $\beta = 2\pi$ and then $2\pi = 2\pi(3a/d)$ and $(d/a) = 3$.

(b) IDENTIFY and **SET UP:** Fringes $m = 0, \pm1, \pm2$ are within the central diffraction maximum and the $m = \pm3$ fringes coincide with the first diffraction minimum. Find the value of m for the fringes that coincide with the second diffraction minimum.

EXECUTE: Second minimum implies $\beta = 4\pi$.

$$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = 2\pi(m/3).$$

Then $\beta = 4\pi$ says $4\pi = 2\pi(m/3)$ and $m = 6$. Therefore the $m = \pm4$ and $m = +5$ fringes are contained within the first diffraction maximum on one side of the central maximum; two fringes.

EVALUATE: The central maximum is twice as wide as the other maxima so it contains more fringes.

36.20. **IDENTIFY:** The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

SET UP: The double-slit factor is $I_{DS} = I_0\left(\cos^2\frac{\phi}{2}\right)$ and the single-slit factor is $I_{SS} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2$.

EXECUTE: **(a)** $d\sin\theta = m\lambda \Rightarrow \sin\theta = m\lambda/d$.

$\sin\theta_1 = \lambda/d, \sin\theta_2 = 2\lambda/d, \sin\theta_3 = 3\lambda/d, \sin\theta_4 = 4\lambda/d$.

(b) At the interference bright fringes, $\cos^2\phi/2 = 1$ and $\beta/2 = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(d/3)\sin\theta}{\lambda}$.

At θ_1, $\sin\theta_1 = \lambda/d$, so $\beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3$. The intensity is therefore

$$I_1 = I_0\left(\cos^2\frac{\phi}{2}\right)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0(1)\left(\frac{\sin\pi/3}{\pi/3}\right)^2 = 0.684\,I_0.$$

At θ_2, $\sin\theta_2 = 2\lambda/d$, so $\beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3$. Using the same procedure as for θ_1, we have

$$I_2 = I_0(1)\left(\frac{\sin 2\pi/3}{2\pi/3}\right)^2 = 0.171\,I_0.$$

At θ_3, we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin\,\pi = 0$.

At θ_4, $\sin\,\theta_4 = 4\lambda/d$, so $\beta/2 = 4\pi/3$, which gives $I_4 = I_0\left(\dfrac{\sin 4\pi/3}{4\pi/3}\right)^2 = 0.0427\,I_0.$

(c) Since $d = 3a$, every third interference maximum is missing.

(d) In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a$.

Evaluate: The result in this problem is different from that in Figure 36.12c in the textbook because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the "envelope" of the intensity function decreases more rapidly here than in Figure 36.12c in the text because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4$.

36.21. **(a) Identify** and **Set Up:** If the slits are very narrow then the central maximum of the diffraction pattern for each slit completely fills the screen and the intensity distribution is given solely by the two-slit interference. The maxima are given by $d\sin\theta = m\lambda$ so $\sin\theta = m\lambda/d$. Solve for θ.

Execute: <u>1st order maximum</u>: $m = 1$, so $\sin\theta = \dfrac{\lambda}{d} = \dfrac{580\times10^{-9}\ \text{m}}{0.530\times10^{-3}\ \text{m}} = 1.094\times10^{-3}$; $\theta = 0.0627°$.

<u>2nd order maximum</u>: $m = 2$, so $\sin\theta = \dfrac{2\lambda}{d} = 2.188\times10^{-3}$; $\theta = 0.125°$.

(b) Identify and **Set Up:** The intensity is given by Eq. (36.12): $I = I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2.$

Calculate ϕ and β at each θ from part (a).

Execute: $\phi = \left(\dfrac{2\pi d}{\lambda}\right)\sin\theta = \left(\dfrac{2\pi d}{\lambda}\right)\left(\dfrac{m\lambda}{d}\right) = 2\pi m$, so $\cos^2(\phi/2) = \cos^2(m\pi) = 1.$

(Since the angular positions in part (a) correspond to interference maxima.)

$$\beta = \left(\frac{2\pi a}{\lambda}\right)\sin\theta = \left(\frac{2\pi a}{\lambda}\right)\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = m2\pi\left(\frac{0.320\ \text{mm}}{0.530\ \text{mm}}\right) = m(3.794\ \text{rad}).$$

<u>1st order maximum</u>: $m = 1$, so $I = I_0(1)\left(\dfrac{\sin(3.794/2)\text{rad}}{(3.794/2)\text{rad}}\right)^2 = 0.249 I_0.$

<u>2nd order maximum</u>: $m = 2$, so $I = I_0(1)\left(\dfrac{\sin 3.794\ \text{rad}}{3.794\ \text{rad}}\right)^2 = 0.0256 I_0.$

Evaluate: The first diffraction minimum is at an angle θ given by $\sin\theta = \lambda/a$ so $\theta = 0.104°$. The first order fringe is within the central maximum and the second order fringe is inside the first diffraction maximum on one side of the central maximum. The intensity here at this second fringe is much less than I_0.

36.22. **Identify:** The diffraction minima are located by $\sin\theta = \dfrac{m_\text{d}\lambda}{a}$ and the two-slit interference maxima are located by $\sin\theta = \dfrac{m_\text{i}\lambda}{d}$. The third bright band is missing because the first order single-slit minimum occurs at the same angle as the third order double-slit maximum.

Set Up: The pattern is sketched in Figure 36.22. $\tan\theta = \dfrac{3\ \text{cm}}{90\ \text{cm}}$, so $\theta = 1.91°$.

Execute: <u>Single-slit dark spot</u>: $a\sin\theta = \lambda$ and

$$a = \frac{\lambda}{\sin\theta} = \frac{500\ \text{nm}}{\sin 1.91°} = 1.50\times10^4\ \text{nm} = 15.0\ \mu\text{m (width)}.$$

Double-slit bright fringe: $d\sin\theta = 3\lambda$ and

$$d = \frac{3\lambda}{\sin\theta} = \frac{3(500 \text{ nm})}{\sin 1.91°} = 4.50\times10^4 \text{ nm} = 45.0\ \mu\text{m (separation)}.$$

EVALUATE: Note that $d/a = 3.0$.

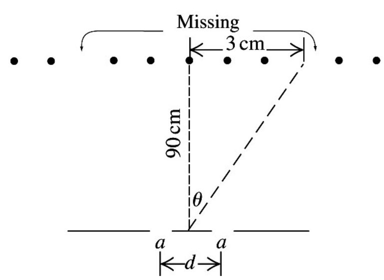

Figure 36.22

36.23. IDENTIFY: We combine double-slit interference with single-slit diffraction.

SET UP: Single-slit dark spots: $a\sin\theta_m = m\lambda$, double-slit bright spots: $d\sin\theta_m = m\lambda$. We want to find the slit width a.

EXECUTE: **(a)** The m = 3 double-slit bright spot is missing because it lies where the single-slit intensity is zero. For single slit dark spot $a\sin\theta_k = k\lambda$ and for double slit bright spot $d\sin\theta_m = m\lambda$. The minimum a occurs for the minimum k, which is 1. Thus $a\sin\theta = \lambda$ (single slit) and $d\sin\theta = 3\lambda$ (double slit). Dividing gives $a/d = 1/3$, so $a = d/3 = (9.00 \text{ mm})/3 = 3.00$ mm.

(b) For the next larger a, we use k = 2, so $a\sin\theta = 2\lambda$ and $d\sin\theta = 3\lambda$. Dividing gives $a/d = 2/3$, which gives $a = 2/3\ d = 6.00$ mm.

EVALUATE: It is also possible for a double-slit minimum to cancel a single-slit maximum.

36.24. IDENTIFY: This problem involves the intensity of a single-slit and double-slit pattern.

SET UP: $I = I_0\cos^2(\phi/2)\left(\frac{\sin\beta/2}{\beta/2}\right)^2$, $\beta = \frac{2\pi a\sin\theta}{\lambda}$, $\phi = \frac{2\pi d\sin\theta}{\lambda}$, $a\sin\theta_m = m\lambda$ (single-slit minima), $d\sin\theta_m = m\lambda$ (double-slit maxima). We want I_1/I_2 for the maxima.

EXECUTE: The double-slit m = 5 maximum is missing because it lies at a minimum for the single-slit diffraction pattern. Since the double-slit m = 0 through m = 4 maxima are present, the single-slit minimum must be the first one. Thus $a\sin\theta = \lambda$ and $d\sin\theta = 5\lambda$. Dividing gives $a/d = 1/5$, so $a = d/5 = 0.200$ mm.

At $m = 1$: $d\sin\theta = \lambda$, so $\frac{\phi_1}{2} = \frac{\pi d\sin\theta}{\lambda} = \pi.$ $\frac{\beta_1}{2} = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(d/5)\sin\theta}{\lambda} = \pi/5 = 36.0°.$ Therefore

$$I_1 = I_0\cos^2(\phi/2)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0\cos^2(\pi)\left(\frac{\sin 36.0°}{\pi/5}\right)^2 = 0.8751 I_0.$$

At $m = 2$: $d\sin\theta = 2\lambda$, so $\frac{\phi_2}{2} = \frac{\pi d\sin\theta}{\lambda} = 2\pi.$ $\frac{\beta_2}{2} = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(d/5)\sin\theta}{\lambda} = 2\pi/5 = 72.0°.$ Thus

$$I_2 = I_0\cos^2(2\pi)\left(\frac{\sin 72.0°}{2\pi/5}\right)^2 = 0.5728 I_0.$$

$$\frac{I_1}{I_2} = \frac{0.8751 I_0}{0.5728 I_0} = 1.53.$$

EVALUATE: $I_1 > I_2$ because the maxima get dimmer as we move away from the center.

36.25. **IDENTIFY:** Knowing the wavelength of the light and the location of the first interference maxima, we can calculate the line density of the grating.

SET UP: The line density in lines/cm is $1/d$, with d in cm. The bright spots are located by $d\sin\theta = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots.$

EXECUTE: **(a)** $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{(1)(632.8\times10^{-9}\text{ m})}{\sin 17.8°} = 2.07\times10^{-6}\text{ m} = 2.07\times10^{-4}\text{ cm}.\ \dfrac{1}{d} = 4830\text{ lines/cm}.$

(b) $\sin\theta = \dfrac{m\lambda}{d} = m\left(\dfrac{632.8\times10^{-9}\text{ m}}{2.07\times10^{-6}\text{ m}}\right) = m(0.3057).$ For $m = \pm 2,\ \theta = \pm 37.7°.$ For $m = \pm 3,\ \theta = \pm 66.5°.$

EVALUATE: The angles are large, so they are not equally spaced; $37.7° \neq 2(17.8°)$ and $66.5° \neq 3(17.8°)$.

36.26. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda.$

SET UP: The order corresponds to the values of m.

EXECUTE: First-order: $d\sin\theta_1 = \lambda.$ Fourth-order: $d\sin\theta_4 = 4\lambda.$

$\dfrac{d\sin\theta_4}{d\sin\theta_1} = \dfrac{4\lambda}{\lambda},\ \sin\theta_4 = 4\sin\theta_1 = 4\sin 11.3°$ and $\theta_4 = 51.6°.$

EVALUATE: We did not have to solve for d.

36.27. **IDENTIFY:** This problem is about a diffraction grating.

SET UP: Bright fringes occur when $d\sin\theta_m = m\lambda$. Dark fringes occur when $a\sin\theta_m = m\lambda$.

EXECUTE: **(a)** We want d. The 4th order bright spot should occur at 66.6°, so $d\sin\theta_m = m\lambda$ gives $d\sin 66.6° = 4(550\text{ nm})$. So $d = 2.40\ \mu$m.

(b) We want the line density. The density is $1/d = 1/(2.40\ \mu\text{m}) = 417$ slits/mm.

(c) We want the width a of each slit. $a\sin\theta_m = m\lambda$ gives $a\sin 66.6° = 3(550\text{ nm})$, so $a = 1.80\ \mu$m.

EVALUATE: The width of the slits is 1.80/2.40 = 0.750 times the slit spacing, so single-slit diffraction certainly plays a role.

36.28. **IDENTIFY:** The bright spots are located by $d\sin\theta = m\lambda.$

SET UP: Third-order means $m = 3$ and second-order means $m = 2.$

EXECUTE: $\dfrac{m\lambda}{\sin\theta} = d = \text{constant},$ so $\dfrac{m_r\lambda_r}{\sin\theta_r} = \dfrac{m_v\lambda_v}{\sin\theta_v}.$

$\sin\theta_v = \sin\theta_r\left(\dfrac{m_v}{m_r}\right)\left(\dfrac{\lambda_v}{\lambda_r}\right) = (\sin 65.0°)\left(\dfrac{2}{3}\right)\left(\dfrac{400\text{ nm}}{700\text{ nm}}\right) = 0.345$ and $\theta_v = 20.2°.$

EVALUATE: The third-order line for a particular λ occurs at a larger angle than the second-order line. In a given order, the line for violet light (400 nm) occurs at a smaller angle than the line for red light (700 nm).

36.29. **IDENTIFY:** This problem involves the resolving power of a diffraction grating.

SET UP: $R = \dfrac{\lambda}{\Delta\lambda} = mN$.

EXECUTE: **(a)** We want $R_{\min}$. $R_{\min} = \dfrac{\lambda}{\Delta\lambda} = \dfrac{430.790\text{ nm} - 430.774\text{ nm}}{430.774\text{ nm}} = 2.69\times10^4.$

(b) We want the lowest order needed. $R = Nm$ gives $m = R/N = 26{,}900/12{,}800 = 2.10$. So $m = 2$ is not quite enough to resolve these lines, but $m = 3$ will do it. So the third-order is the lowest.

EVALUATE: More lines would allow a lower order. In this case, 13,500 lines would allow $m = 2$ because 26,900/13,600 = 1.98, which is less than 2.

36.30. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.

SET UP: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^{5}\ \text{m}^{-1}} = 2.857\times10^{-6}$ m. The visible spectrum is between approximately 380 nm and 750 nm.

EXECUTE: **(a)** $m=1$: $\theta_{380} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{3.80\times10^{-7}\ \text{m}}{2.857\times10^{-6}\ \text{m}}\right) = 6.643°.$

$\theta_{750} = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{7.50\times10^{-7}\ \text{m}}{2.857\times10^{-6}\ \text{m}}\right) = 15.2185°.$ $\Delta\theta_1 = 15.2185° - 6.643° = 8.58°.$

(b) $m=3$: $\theta_{380} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(3.80\times10^{-7}\ \text{m})}{2.857\times10^{-6}\ \text{m}}\right) = 23.516°.$

$\theta_{750} = \arcsin\left(\dfrac{3\lambda}{d}\right) = \arcsin\left(\dfrac{3(7.50\times10^{-7}\ \text{m})}{2.857\times10^{-6}\ \text{m}}\right) = 51.952°.$ $\Delta\theta_1 = 51.952° - 23.516° = 28.4°.$

EVALUATE: $\Delta\theta$ is larger in third order.

36.31. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.

SET UP: 5000 slits/cm $\Rightarrow d = \dfrac{1}{5.00\times10^{5}\ \text{m}^{-1}} = 2.00\times10^{-6}$ m.

EXECUTE: **(a)** $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(2.00\times10^{-6}\ \text{m})\sin 13.5°}{1} = 4.67\times10^{-7}$ m.

(b) $m=2$: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{2(4.67\times10^{-7}\ \text{m})}{2.00\times10^{-6}\ \text{m}}\right) = 27.8°.$

EVALUATE: Since the angles are fairly small, the second-order deviation is approximately twice the first-order deviation.

36.32. **IDENTIFY:** The grooves in a CD and DVD form a reflection diffraction grating. Constructive interference when $d\sin\theta = m\lambda$.

SET UP: The maxima are located by $d\sin\theta = m\lambda$, where $d = 1.60\times10^{-6}$ m for a CD and $d = 0.740\times10^{-6}$ m for a DVD.

EXECUTE: **(a)** For a CD we have: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m(6.328\times10^{-7}\ \text{m})}{1.60\times10^{-6}\ \text{m}}\right) = \arcsin(0.396m).$

For $m = 1$ we have $\theta_1 = 23.3°$. For $m = 2$ we have $\theta_2 = 52.3°$. There are no other maxima.

(b) For a DVD we have: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m(6.328\times10^{-7}\ \text{m})}{0.740\times10^{-6}\ \text{m}}\right) = \arcsin(0.855m)$. For $m = 1$ we have $\theta_1 = 58.8°$. There are no other maxima.

EVALUATE: The reflective surface produces the same interference pattern as a grating with slit separation *d*.

36.33. **IDENTIFY:** The resolution is described by $R = \dfrac{\lambda}{\Delta\lambda} = Nm$. Maxima are located by $d\sin\theta = m\lambda$.

SET UP: For 500 slits/mm, $d = (500\ \text{slits/mm})^{-1} = (500{,}000\ \text{slits/m})^{-1}$.

EXECUTE: **(a)** $N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{6.5645\times10^{-7}\ \text{m}}{2(6.5645\times10^{-7}\ \text{m} - 6.5627\times10^{-7}\ \text{m})} = 1820$ slits.

(b) $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) \Rightarrow \theta_1 = \sin^{-1}((2)(6.5645\times10^{-7}\text{ m})(500{,}000\text{ m}^{-1})) = 41.0297°$ and

$\theta_2 = \sin^{-1}((2)(6.5627\times10^{-7}\text{ m})(500{,}000\text{ m}^{-1})) = 41.0160°$. $\Delta\theta = 0.0137°$.

EVALUATE: $d\cos\theta\, d\theta = \lambda/N$, so for 1820 slits the angular interval $\Delta\theta$ between each of these maxima and the first adjacent minimum is $\Delta\theta = \frac{\lambda}{Nd\cos\theta} = \frac{6.56\times10^{-7}\text{ m}}{(1820)(2.0\times10^{-6}\text{ m})\cos 41°} = 0.0137°$. This is the same as the angular separation of the maxima for the two wavelengths and 1820 slits is just sufficient to resolve these two wavelengths in second order.

36.34. **IDENTIFY:** The maxima are given by $2d\sin\theta = m\lambda$, $m = 1, 2, \ldots$.

SET UP: $d = 3.50\times10^{-10}$ m.

EXECUTE: **(a)** Using $m = 1$ gives

$\lambda = \frac{2d\sin\theta}{m} = 2(3.50\times10^{-10}\text{ m})\sin 22.0° = 2.62\times10^{-10}\text{ m} = 0.262\text{ nm} = 262\text{ pm}$. This is an x ray.

(b) $\sin\theta = m\left(\frac{\lambda}{2d}\right) = m\left(\frac{2.62\times10^{-10}\text{ m}}{2(3.50\times10^{-10}\text{ m})}\right) = 0.3743m$. $m = 2$: $\theta = 48.5°$. The equation doesn't have any solutions for $m > 2$.

EVALUATE: In this problem $\lambda/d = 0.75$.

36.35. **IDENTIFY** and **SET UP:** The maxima occur at angles θ given by $2d\sin\theta = m\lambda$, where *d* is the spacing between adjacent atomic planes. Solve for *d*.

EXECUTE: Second order says $m = 2$.

$d = \frac{m\lambda}{2\sin\theta} = \frac{2(0.0850\times10^{-9}\text{ m})}{2\sin 21.5°} = 2.32\times10^{-10}\text{ m} = 0.232\text{ nm}.$

EVALUATE: Our result is similar to *d* calculated in Example 36.5.

36.36. **IDENTIFY:** The crystal behaves like a diffraction grating.

SET UP: The maxima are at angles θ given by $2d\sin\theta = m\lambda$, where $d = 0.440$ nm.

EXECUTE: $m = 1$. $\lambda = \frac{2d\sin\theta}{1} = 2(0.440\text{ nm})\sin 39.4° = 0.559\text{ nm}.$

EVALUATE: The result is a reasonable x-ray wavelength.

36.37. **IDENTIFY** and **SET UP:** The angular size of the first dark ring is given by $\sin\theta_1 = 1.22\lambda/D$. Calculate θ_1, and then the diameter of the ring on the screen is $2(4.5\text{ m})\tan\theta_1$.

EXECUTE: $\sin\theta_1 = 1.22\left(\frac{620\times10^{-9}\text{ m}}{7.4\times10^{-6}\text{ m}}\right) = 0.1022$; $\theta_1 = 0.1024$ rad.

The radius of the Airy disk (central bright spot) is $r = (4.5\text{ m})\tan\theta_1 = 0.462$ m. The diameter is $2r = 0.92\text{ m} = 92\text{ cm}$.

EVALUATE: $\lambda/D = 0.084$. For this small *D* the central diffraction maximum is broad.

36.38. **IDENTIFY** and **SET UP:** For the first dark ring, $\sin\theta_1 = 1.22\frac{\lambda}{D}$ and for the second dark ring $\sin\theta_2 = 2.23\frac{\lambda}{D}$. If *y* is the distance from the center to the ring, we can use the small-angle approximation because *x* << *y*, where *x* is the distance from the aperture to the screen. Therefore $\sin\theta \approx \tan\theta = y/x$. The difference in radii of the first two dark rings is 1.65 mm.

EXECUTE: The radius of the first dark ring is $y_1 = x\tan\theta_1 = x(1.22\,\lambda/D)$. The radius of the second ring is $y_2 = x\tan\theta_2 = x(1.22\,\lambda/D)$. Dividing these two equations gives $y_2/y_1 = 2.23/1.22 = 1.8279$, so $y_2 = 1.8279y_1$. We are told that the difference in radii is 1.65 mm, so $y_2 - y_1 = 1.65$ mm. Substituting for y_2 gives $1.8279y_1 - y_1 = 1.65$ mm, so $y_1 = 1.9931$ mm. Now find D using $y_1 = x(1.22\,\lambda/D)$.

1.9931×10^{-3} m = (1.20 m)(1.22)(490 nm)/D, so $D = 3.60\times10^5$ nm = 3.60×10^{-4} m = 0.360 mm.

EVALUATE: This is about the diameter of a good-sized pinhole.

36.39. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: $\theta = \dfrac{W}{h}$, where $W = 28\text{ km}$ and $h = 1200\text{ km}$. θ is small, so $\sin\theta \approx \theta$.

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = 1.22\lambda\dfrac{h}{W} = 1.22(0.036\text{ m})\dfrac{1.2\times10^6\text{ m}}{2.8\times10^4\text{ m}} = 1.88\text{ m}.$

EVALUATE: D must be significantly larger than the wavelength, so a much larger diameter is needed for microwaves than for visible wavelengths.

36.40. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: $\theta = (1/60)°$.

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = \dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(1/60)°} = 2.31\times10^{-3}\text{ m} = 2.3\text{ mm}.$

EVALUATE: The larger the diameter the smaller the angle that can be resolved.

36.41. **IDENTIFY:** The diameter D of the mirror determines the resolution.

SET UP: The resolving power is $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}$.

EXECUTE: The same θ_{res} means that $\dfrac{\lambda_1}{D_1} = \dfrac{\lambda_2}{D_2}$. $D_2 = D_1\dfrac{\lambda_2}{\lambda_1} = (8000\times10^3\text{ m})\left(\dfrac{550\times10^{-9}\text{ m}}{2.0\times10^{-2}\text{ m}}\right) = 220\text{ m}.$

EVALUATE: The Hubble telescope has an aperture of 2.4 m, so this would have to be an *enormous* optical telescope!

36.42. **IDENTIFY:** Rayleigh's criterion limits the angular resolution.

SET UP: Rayleigh's criterion is $\sin\theta \approx \theta = 1.22\lambda/D$. Call R = 11.5 m = the distance from the bear to the lens.

EXECUTE: **(a)** Using Rayleigh's criterion

$\sin\theta \approx \theta = 1.22\lambda/D = (1.22)(550\text{ nm})/(135/4\text{ mm}) = 1.99\times10^{-5}\text{ rad}.$

On the bear this angle subtends a distance x. Using $\theta = x/R$ gives

$x = R\theta = (11.5\text{ m})(1.99\times10^{-5}\text{ rad}) = 2.29\times10^{-4}\text{ m} = 0.23\text{ mm}.$

(b) At $f/22$, D is $4/22$ times as large as at $f/4$. Since θ is proportional to $1/D$, and x is proportional to θ, x is $1/(4/22) = 22/4$ times as large as it was at $f/4$. $x = (0.229\text{ mm})(22/4) = 1.3\text{ mm}.$

EVALUATE: A wide-angle lens, such as one having a focal length of 28 mm, would have a much smaller opening at $f/2$ and hence would have an even less resolving ability.

36.43. **IDENTIFY and SET UP:** Let y be the separation between the two points being resolved and let s be their distance from the telescope. Then the limit of resolution corresponds to $1.22\dfrac{\lambda}{D} = \dfrac{y}{s}$.

EXECUTE: **(a)** Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8\times10^8\text{ m}$. $y = 1.22\lambda s/D$.

Hubble: $D = 2.4$ m and $\lambda = 380$ nm gives the maximum resolution, so $y = 73$ m.

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1\times10^6$ m.

(b) $s = \dfrac{yD}{1.22\lambda}$. Let $y \approx 0.30$ m (the size of a license plate).

$s = (0.30\text{ m})(2.4\text{ m})/[(1.22)(380\times10^{-9}\text{ m})] = 1600\text{ km}.$

EVALUATE: D/λ is much larger for the optical telescope and it has a much larger resolution even though the diameter of the radio telescope is much larger.

36.44. **IDENTIFY** and **SET UP:** Resolved by Rayleigh's criterion means angular separation θ of the objects equals $1.22\lambda/D$. The angular separation θ of the objects is their linear separation divided by their distance from the telescope.

EXECUTE: $\theta = \dfrac{250\times10^3\text{ m}}{5.93\times10^{11}\text{ m}}$, where 5.93×10^{11} m is the distance from the earth to Jupiter. Thus $\theta = 4.216\times10^{-7}$ rad.

Then $\theta = 1.22\dfrac{\lambda}{D}$ and $D = \dfrac{1.22\lambda}{\theta} = \dfrac{1.22(500\times10^{-9}\text{ m})}{4.216\times10^{-7}\text{ rad}} = 1.45\text{ m}.$

EVALUATE: This is a very large telescope mirror. The greater the angular resolution the greater the diameter the lens or mirror must be.

36.44. **IDENTIFY:** We can apply the equation for single-slit diffraction to the hair, with the thickness of the hair replacing the thickness of the slit.

SET UP: The dark fringes are located by $\sin\theta = m\dfrac{\lambda}{a}$. The first dark fringes are for $m = \pm1$. $y = x\tan\theta$ is the distance from the center of the screen, where x = 125 cm is the distance from the hair to the screen. The distance y from the center to one minimum is 2.61 cm.

EXECUTE: $\tan\theta = \dfrac{y}{x} = \dfrac{2.61\text{ cm}}{125\text{ cm}} = 0.02088$ so $\theta = 1.20°$. $a = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8\times10^{-9}\text{ m}}{\sin 1.20°} = 30.2\ \mu\text{m}.$

EVALUATE: Although the thickness of human hairs can vary considerably, 30 μm is a reasonable thickness.

36.46. **IDENTIFY:** The two holes behave like double slits and cause the sound waves to interfere after they pass through the holes. The motion of the speakers causes a Doppler shift in the wavelength of the sound.

SET UP: The wavelength of the sound that strikes the wall is $\lambda = \lambda_0 - v_s T_s$, and destructive interference first occurs where $\sin\theta = \lambda/2$.

EXECUTE: **(a)** First find the wavelength of the sound that strikes the openings in the wall.
$\lambda = \lambda_0 - v_s T_s = v/f_s - v_s/f_s = (v - v_s)/f_s = (344\text{ m/s} - 80.0\text{ m/s})/(960\text{ Hz}) = 0.275\text{ m}.$ Destructive interference first occurs where $d\sin\theta = \lambda/2$, which gives
$d = \lambda/(2\sin\theta) = (0.275\text{ m})/(2\sin 11.4°) = 0.636\text{ m}.$

(b) $\lambda = v/f = (344\text{ m/s})/(960\text{ Hz}) = 0.3583\text{ m}.$

$\sin\theta = \lambda/2d = (0.3583\text{ m})/[2(0.696\text{ m})] \rightarrow \theta = \pm14.9°.$

EVALUATE: The moving source produces sound of shorter wavelength than the stationary source, so the angles at which destructive interference occurs are smaller for the moving source than for the stationary source.

36.47. **IDENTIFY:** In the single-slit diffraction pattern, the intensity is a maximum at the center and zero at the dark spots. At other points, it depends on the angle at which one is observing the light.

SET UP: Dark fringes occur when $\sin\,\theta_m = m\lambda/a$, where $m = 1,\ 2,\ 3,\ \ldots,$ and the intensity is given by $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a \sin\theta}{\lambda}$.

EXECUTE: **(a)** At the maximum possible angle, $\theta = 90°$, so $m_{max} = (a\sin 90°)/\lambda = (0.0250\text{ mm})/(632.8\text{ nm}) = 39.5$.

Since m must be an integer and $\sin\theta$ must be ≤ 1, $m_{max} = 39$. The total number of dark fringes is 39 on each side of the central maximum for a total of 78.

(b) The farthest dark fringe is for $m = 39$, giving $\sin\theta_{39} = (39)(632.8\text{ nm})/(0.0250\text{ mm}) \Rightarrow \theta_{39} = \pm 80.8°$.

(c) The next closer dark fringe occurs at $\sin\theta_{38} = (38)(632.8\text{ nm})/(0.0250\text{ mm}) \Rightarrow \theta_{38} = 74.1°$.

The angle midway these two extreme fringes is $(80.8° + 74.1°)/2 = 77.45°$, and the intensity at this angle is $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(0.0250\text{ mm})\sin(77.45°)}{632.8\text{ nm}} = 121.15\text{ rad}$, which gives

$$I = (8.50\,\text{W/m}^2)\left[\frac{\sin(121.15\text{ rad})}{121.15\text{ rad}}\right]^2 = 5.55\times 10^{-4}\text{ W/m}^2.$$

EVALUATE: At the angle in part (c), the intensity is so low that the light would be barely perceptible.

36.48. **IDENTIFY:** $d = \dfrac{1}{N}$, so the bright fringes are located by $\dfrac{1}{N}\sin\theta = \lambda$.

SET UP: Red: $\dfrac{1}{N}\sin\theta_R = 750\text{ nm}$. Violet: $\dfrac{1}{N}\sin\theta_V = 380\text{ nm}$.

EXECUTE: **(a)** $\dfrac{\sin\theta_R}{\sin\theta_V} = \dfrac{75}{38}$. $\theta_R - \theta_V = 27.0°$, so $\theta_R = \theta_V + 27.0°$. $\dfrac{\sin(\theta_V + 27.0°)}{\sin\theta_V} = \dfrac{75}{38}$. Using the trigonometry identity from Appendix B for the sin of the sum of two angles gives

$$\frac{\sin\theta_V\,\cos 27.0° + \cos\theta_V\,\sin 27.0°}{\sin\theta_V} = \frac{75}{38} \quad\rightarrow\quad \cos 27.0° + \cot\theta_V\sin 27.0° = 75/38.$$

$\tan\theta_V = 0.4193 \Rightarrow \theta_V = 22.75°$ and $\theta_R = \theta_V + 27.0° = 22.75° + 27.0° = 49.75°$.

Using $\dfrac{1}{N}\sin\theta_R = 750\text{ nm}$ gives

$$N = \frac{\sin\theta_R}{750\text{ nm}} = \frac{\sin 49.75°}{750\times 10^{-9}\text{ m}} = 1.02\times 10^6\text{ lines/m} = 1.02\times 10^4\text{ lines/cm}.$$

(b) The spectrum begins at 22.7 ° and ends at 49.7°.

EVALUATE: As N is increased, the angular range of the visible spectrum increases.

36.49. **IDENTIFY** and **SET UP:** $\sin\theta = \lambda/a$ locates the first dark band. In the liquid the wavelength changes and this changes the angular position of the first diffraction minimum.

EXECUTE: $\sin\theta_{air} = \dfrac{\lambda_{air}}{a}$; $\sin\theta_{liquid} = \dfrac{\lambda_{liquid}}{a}$. $\lambda_{liquid} = \lambda_{air}\left(\dfrac{\sin\theta_{liquid}}{\sin\theta_{air}}\right) = \lambda_{air}\dfrac{\sin 21.6°}{\sin 38.2°} = 0.5953\lambda_{air}$.

$\lambda_{liquid} = \lambda_{air}/n$, so $n = \lambda_{air}/\lambda_{liquid} = \dfrac{\lambda_{air}}{0.5953\lambda_{air}} = 1.68$.

EVALUATE: Light travels faster in air and n must be > 1.00. The smaller λ in the liquid reduces θ that located the first dark band.

36.50. IDENTIFY: The wavelength of the light is smaller under water than it is in air, which will affect the resolving power of the lens, by Rayleigh's criterion.

SET UP: The wavelength under water is $\lambda = \lambda_0/n,$ and for small angles Rayleigh's criterion is $\theta = 1.22\lambda/D.$

EXECUTE: **(a)** In air the wavelength is $\lambda_0 = c/f = (3.00\times10^8\text{ m/s})/(6.00\times10^{14}\text{ Hz}) = 5.00\times10^{-7}\text{ m}.$ In water the wavelength is $\lambda = \lambda_0/n = (5.00\times10^{-7}\text{ m})/1.33 = 3.76\times10^{-7}\text{ m}.$ With the lens open all the way, we have $D = f/2.8 = (35.0\text{ mm})/2.80 = (0.0350\text{ m})/2.80.$ In the water, we have

$\sin\theta \approx \theta = 1.22\lambda/D = (1.22)(3.76\times10^{-7}\text{ m})[(0.0350\text{ m})/2.80] = 3.67\times10^{-5}\text{ rad}.$

Calling w the width of the resolvable detail, we have

$\theta = w/x \rightarrow w = x\theta = (2750\text{ mm})(3.67\times10^{-5}\text{ rad}) = 0.101\text{ mm}.$

(b) $\theta = 1.22\lambda/D = (1.22)(5.00\times10^{-7}\text{ m})/[(0.0350\text{ m})/2.80] = 4.88\times10^{-5}\text{ rad}.$

$w = x\theta = (2750\text{ mm})(4.88\times10^{-5}\text{ rad}) = 0.134\text{ mm}.$

EVALUATE: Due to the reduced wavelength underwater, the resolution of the lens is better under water than in air.

36.51. IDENTIFY: $I = I_0\left(\dfrac{\sin\gamma}{\gamma}\right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

SET UP: $\dfrac{d\sin\gamma}{d\gamma} = \cos\gamma.$ $\dfrac{d}{d\gamma}\left(\dfrac{1}{\gamma}\right) = -\dfrac{1}{\gamma^2}.$

EXECUTE: **(a)**

$$\frac{dI}{d\gamma} = I_0\frac{d}{d\gamma}\left(\frac{\sin\gamma}{\gamma}\right)^2 = 2\left(\frac{\sin\gamma}{\gamma}\right)\left(\frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2}\right) = 0.\ \frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2} \Rightarrow \gamma\cos\gamma = \sin\gamma \Rightarrow \gamma = \tan\gamma.$$

(b) The graph in Figure 36.51 is a plot of $f(\gamma) = \gamma - \tan\gamma.$ When $f(\gamma)$ equals zero, there is an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the two smallest values of γ are $\gamma = 4.49$ rad and 7.73 rad.

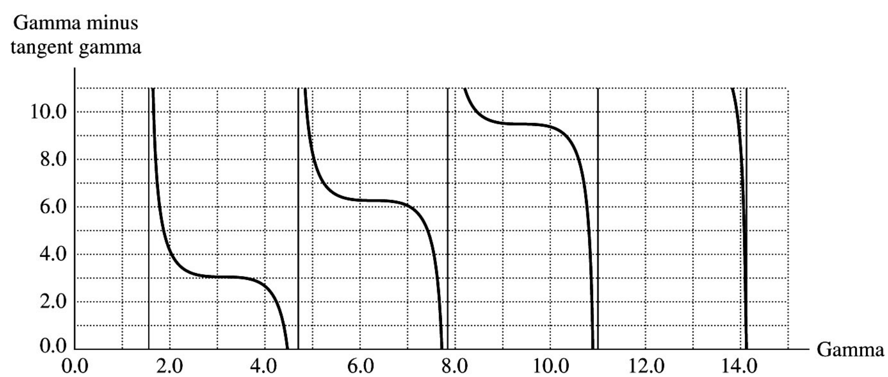

Figure 36.51

(c) The minima occur when $\gamma = \pm m\pi$, where $m = 1, 2, 3, \ldots$, so the first three positive ones are $\gamma = \pi$, 2π, and 3π. From part (b), the first maximum is at $\gamma = 4.49$ rad. Midway between the first and second minima is $\gamma = \pi + \pi/2 = 3\pi/2 = 4.71$ rad. Clearly 4.49 rad $\neq$ 4.71 rad, so this maximum is not midway between the adjacent minima. The second maximum is at $\gamma = 7.73$ rad. Midway between the second and third minima is $\gamma = 2\pi + \pi/2 = 5\pi/2 = 7.85$ rad. Since $7.73 \neq 7.85$, the second maximum is not midway between the adjacent minima.

(d) Using $a = 12\lambda$, we have $\gamma = \dfrac{\pi a \sin\theta}{\lambda} = \dfrac{\pi(12\lambda)\sin\theta}{\lambda} = 12\pi\sin\theta$. Using this we get the following:

<u>First minimum</u>: $\gamma = \pi$, so $\pi = 12\pi\sin\theta \quad \to \quad \theta = 4.78°$.

<u>First maximum</u>: $\gamma = 2\pi$, so $2\pi = 12\pi\sin\theta \quad \to \quad \theta = 6.84°$.

<u>Second minimum</u>: $\gamma = 3\pi$, so $3\pi = 12\pi\sin\theta \quad \to \quad \theta = 9.59°$.

Midway between these minima is $\theta = (9.59° + 4.78°)/2 = 7.19°$. The first maximum is at $\theta = 6.84°$, and since $6.84° \neq 7.19°$, the first minimum is *not* midway between the adjacent minima.

EVALUATE: $\gamma = 0$ is the central maximum. The three values of γ we found are the locations of the first two secondary maxima. The first three minima are at $\gamma = 3.14$ rad, 6.28 rad, and 9.42 rad. The maxima are between adjacent minima, but not precisely midway between them.

36.52. **(a) IDENTIFY** and **SET UP:** The angular position of the first minimum is given by $a\sin\theta = m\lambda$, with $m = 1$. The distance of the minimum from the center of the pattern is given by $y = x\tan\theta$.

EXECUTE: $\sin\theta = \dfrac{\lambda}{a} = \dfrac{540\times10^{-9}\ \text{m}}{0.360\times10^{-3}\ \text{m}} = 1.50\times10^{-3}$; $\theta = 1.50\times10^{-3}$ rad.

$y_1 = x\tan\theta = (1.20\ \text{m})\tan(1.50\times10^{-3}\ \text{rad}) = 1.80\times10^{-3}\ \text{m} = 1.80\ \text{mm}.$

(Note that θ is small enough for $\theta \approx \sin\theta \approx \tan\theta$, and $y_m = x\dfrac{m\lambda}{a}$ applies.)

(b) IDENTIFY and **SET UP:** Find the phase angle β where $I = I_0/2$. Then use $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$ to solve for θ and $y = x\tan\theta$ to find the distance.

EXECUTE: $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$ gives that $I = \tfrac{1}{2}I_0$ when $\beta = 2.78$ rad.

$\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$, so $\sin\theta = \dfrac{\beta\lambda}{2\pi a}$.

$$y = x\tan\theta \approx x\sin\theta \approx \frac{\beta\lambda x}{2\pi a} = \frac{(2.78\ \text{rad})(540\times10^{-9}\ \text{m})(1.20\ \text{m})}{2\pi(0.360\times10^{-3}\ \text{m})} = 7.96\times10^{-4}\ \text{m} = 0.796\ \text{mm}.$$

EVALUATE: The point where $I = I_0/2$ is not midway between the center of the central maximum and the first minimum.

36.53. **IDENTIFY:** Heating the plate causes it to expand, which widens the slit. The increased slit width changes the angles at which destructive interference occurs.

SET UP: First minimum is at angle θ given by $\tan\theta = \dfrac{(2.75\times10^{-3}/2)}{0.620}$. Therefore, θ is small and the equation $y_m = x\dfrac{m\lambda}{a}$ is accurate. The width of the central maximum is $w = \dfrac{2x\lambda}{a}$. The change in slit width is $\Delta a = a\alpha\Delta T$.

EXECUTE: $dw = 2x\lambda\left(-\dfrac{da}{a^2}\right) = -\dfrac{2x\lambda}{a^2}da = -\dfrac{w}{a}da$. Therefore, $\Delta w = -\dfrac{w}{a}\Delta a$. The equation for thermal expansion says $\Delta a = a\alpha\Delta T$, so $\Delta w = -w\alpha\Delta T = -(2.75\ \text{mm})(2.4\times10^{-5}\ \text{K}^{-1})(500\ \text{K}) = -0.033\ \text{mm}$.

When the temperature of the plate increases, the width of the slit increases and the width of the central maximum decreases.

EVALUATE: The fractional change in the width of the central maximum is $\dfrac{0.033\ \text{mm}}{2.75\ \text{mm}} = 1.2\%$. This is small, but observable.

36.54. **IDENTIFY:** The wavelength of the helium spectral line from the receding galaxy will be different from the spectral line on earth due to the Doppler shift in the light from the galaxy.

SET UP: $d\sin\theta = m\lambda.$ $\sin\theta_{\text{lab}} = \dfrac{2\lambda_{\text{lab}}}{d}.$ $\sin\theta_{\text{galaxy}} = \dfrac{2\lambda_{\text{galaxy}}}{d}.$ $\sin\theta_{\text{galaxy}} = \sin\theta_{\text{lab}}\left(\dfrac{\lambda_{\text{lab}}}{\lambda_{\text{galaxy}}}\right).$ The Doppler formula says $f_{\text{R}} = \sqrt{\dfrac{c-v}{c+v}}f_{\text{S}}.$ Using $f = \dfrac{c}{\lambda},$ we have $\dfrac{1}{\lambda_{\text{R}}} = \dfrac{1}{\lambda_{\text{S}}}\sqrt{\dfrac{c-v}{c+v}}.$ Since the lab is the receiver R and the galaxy is the source S, this becomes $\lambda_{\text{lab}} = \lambda_{\text{galaxy}}\sqrt{\dfrac{c+v}{c-v}}.$

EXECUTE: $\sin\theta_{\text{galaxy}} = \sin\theta_{\text{lab}}\sqrt{\dfrac{c+v}{c-v}} = \sin(18.9^\circ)\sqrt{\dfrac{2.998\times10^8\ \text{m/s} + 2.65\times10^7\ \text{m/s}}{2.998\times10^8\ \text{m/s} - 2.65\times10^7\ \text{m/s}}}$ which gives $\theta_{\text{galaxy}} = 20.7^\circ.$

EVALUATE: The galaxy is moving away, so the wavelength of its light will be lengthened, which means that the angle should be increased compared to the angle from light on earth, as we have found.

36.55. **IDENTIFY** and **SET UP:** The condition for an intensity maximum is $d\sin\theta = m\lambda,\ m = 0,\ \pm1,\ \pm2,\ldots.$ Third order means $m = 3.$ The longest observable wavelength is the one that gives $\theta = 90^\circ$ and hence $\sin\theta = 1.$

EXECUTE: 9200 lines/cm so 9.2×10^5 lines/m and $d = \dfrac{1}{9.2\times10^5}\ \text{m} = 1.087\times10^{-6}\ \text{m}.$

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.087\times10^{-6}\ \text{m})(1)}{3} = 3.6\times10^{-7}\ \text{m} = 360\ \text{nm}.$$

EVALUATE: The longest wavelength that can be obtained decreases as the order increases.

36.56. **IDENTIFY:** We are dealing with single-slit diffraction for sound waves.
SET UP and **EXECUTE:** **(a)** Estimate: Door width = 75 cm.
(b) We want *f*. $f = v/\lambda$ = (344 m/s)/(0.75 m) = 460 Hz.

(c) We want *f* for diffraction at ±20°. $a\sin\theta = \lambda = v/f$, so $f = \dfrac{v}{a\sin\theta} = \dfrac{344\ \text{m/s}}{(0.75\ \text{m})\sin 20^\circ} = 1.3\ \text{kHz}.$

(d) It is fairly good. A frequency of 1.3 kHz is typical of many ordinary sounds and much music, so not very much sound would bend around the edge of a doorway.
(e) Since $a\sin\theta = \lambda,$ a small wavelength would undergo less bending, so higher frequency sounds would bend less than lower frequency sounds.
EVALUATE: **(f)** It is somewhat observable. Inside the house there are reflections from nearby walls. Outside has more open space with fewer reflections.

36.57. **IDENTIFY:** The maxima are given by $d\sin\theta = m\lambda.$ We need $\sin\theta = \dfrac{m\lambda}{d} \le 1$ in order for all the visible wavelengths to be seen.

SET UP: For 650 slits/mm $\Rightarrow d = \dfrac{1}{6.50\times10^5\ \text{m}^{-1}} = 1.53\times10^{-6}$ m. The visible spectrum is approximately between 380 nm and 750 nm.

EXECUTE: $\lambda_1 = 3.8\times10^{-7}\text{m}: m = 1: \dfrac{\lambda_1}{d} = 0.247;\ m = 2: \dfrac{2\lambda_1}{d} = 0.494;\ m = 3: \dfrac{3\lambda_1}{d} = 0.741.$

$\lambda_2 = 7.5\times10^{-7}\text{m}: m = 1: \dfrac{\lambda_2}{d} = 0.4875;\ m = 2: \dfrac{2\lambda_2}{d} = 0.975;\ m = 3: \dfrac{3\lambda_2}{d} = 1.46.$ So, the third order does not contain the violet end of the spectrum, and therefore only the first- and second-order diffraction patterns contain all colors of the spectrum.

EVALUATE: θ for each maximum is larger for longer wavelengths.

36.58. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}.$

SET UP: θ is small, so $\sin\theta \approx \dfrac{\Delta x}{R},$ where Δx is the size of the detail and $R = 7.2\times10^{8}$ ly.

$1\text{ ly} = 9.41\times10^{12}$ km. $\lambda = c/f$.

EXECUTE:

$$\sin\theta = 1.22\frac{\lambda}{D} \approx \frac{\Delta x}{R} \Rightarrow \Delta x = \frac{1.22\lambda R}{D} = \frac{(1.22)cR}{Df} = \frac{(1.22)(3.00\times10^{5}\text{ km/s})(7.2\times10^{8}\text{ ly})}{(77.000\times10^{3}\text{ km})(1.665\times10^{9}\text{ Hz})} = 2.06\text{ ly}.$$

$(9.41\times10^{12}\text{ km/ly})(2.06\text{ ly}) = 1.94\times10^{13}$ km.

EVALUATE: $\lambda = 18$ cm. λ/D is very small, so $\dfrac{\Delta x}{R}$ is very small. Still, R is very large and Δx is many orders of magnitude larger than the diameter of the sun.

36.59. **IDENTIFY:** This problem is about x-ray diffraction applied to electrons.

SET UP: Eq. (36.16): $2d\sin\theta_m = m\lambda$ for constructive interference.

EXECUTE: **(a)** Using Fig. 36.22a in the textbook, we see that $2\theta + 50° = 180°,$ so $\theta = 65°.$

(b) We want λ for electrons. $2d\sin\theta = \lambda$ = 2(0.091 nm)(sin 65°) = 0.16 nm.

(c) We want the electron speed. $K = \dfrac{1}{2}mv^2$ = 54 eV = 8.64×10^{-18} J. $v = \sqrt{2K/m}$. Using our K gives $v = 4.4\times10^{6}$ m/s.

(d) We want the frequency of the electron. $f = v/\lambda$. Using the results of (b) and (c), we have $f = 2.6\times10^{16}$ Hz.

(e) We want h. $E = hf$ gives $h = E/f$. Using E from part (c) and f from part (d) gives $h = 3.3\times10^{-34}$ J·s.

(f) We want h. $v = \dfrac{1}{2}v_{\text{cl}}$, so $f = \dfrac{\frac{1}{2}v_{\text{cl}}}{\lambda}$. Thus $h = \dfrac{E}{f} = \dfrac{E}{\frac{1}{2}v_{\text{cl}}} = 2\dfrac{E}{v_{\text{cl}}/\lambda} = 2(3.3\times10^{-34}\text{ J}\cdot\text{s}) =$

6.6×10^{-34} J·s.

EVALUATE: **(g)** The established value is $h = 6.6\times10^{-34}$ J·s so our result agrees well.

36.60. **IDENTIFY:** The resolution of the eye is limited because light diffracts as it passes through the pupil. The size of the pupil determines the resolution.

SET UP: The smallest angular separation that can be resolved is $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}.$ The angular size of the object is its height divided by its distance from the eye.

EXECUTE: **(a)** The angular size of the object is $\theta = \dfrac{50\times10^{-6}\text{ m}}{25\times10^{-2}\text{ m}} = 2.0\times10^{-4}$ rad.

$\theta_{\text{res}} = 1.22\dfrac{\lambda}{D} = 1.22\left(\dfrac{550\times10^{-9}\text{ m}}{2.0\times10^{-3}\text{ m}}\right) = 3.4\times10^{-4}$ rad. $\theta < \theta_{\text{res}}$ so the object cannot be resolved.

(b) $\theta_{\text{res}} = \dfrac{y}{s}$ and $y = s\theta_{\text{res}} = (25\text{ cm})(3.4\times10^{-4}\text{ rad}) = 8.5\times10^{-3}\text{ cm} = 85\ \mu\text{m}.$

(c) $\theta = \theta_{\text{res}} = 3.4\times10^{-4}\text{ rad} = 0.019° = 1.1\text{ min}.$ This is very close to the experimental value of 1 min.

(d) Diffraction is more important.

EVALUATE: We could not see any clearer if our retinal cells were much smaller than they are now because diffraction is more important in limiting the resolution of our vision.

36.61. **IDENTIFY:** A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.

SET UP: Single-slit diffraction dark fringes occur when $a\sin\theta = m\lambda$, and double-slit interference bright

fringes occur when $d\ \sin\theta = m'\lambda$.

EXECUTE: **(a)** The angle at which the first bright fringe occurs is given by

$\tan\theta_1 = (1.53\text{ mm})/(2500\text{ mm}) \Rightarrow \theta_1 = 0.03507°$. $d\sin\theta_1 = \lambda$ and

$d = \lambda/(\sin\theta_1) = (632.8\text{ nm})/\sin(0.03507°) = 0.00103\text{ m} = 1.03\text{ mm}$.

(b) The 7^{th} double-slit interference bright fringe is just cancelled by the 1^{st} diffraction dark fringe, so $\sin\theta_{\text{diff}} = \lambda/a$ and $\sin\theta_{\text{interf}} = 7\lambda/d$.

The angles are equal, so $\lambda/a = 7\lambda/d \rightarrow a = d/7 = (1.03\text{ mm})/7 = 0.148\text{ mm}$.

EVALUATE: We can generalize that if $d = na$, where n is a positive integer, then every n^{th} double-slit bright fringe will be missing in the pattern.

36.62. **IDENTIFY:** Light from the slit produces a diffraction pattern on the screen.

SET UP: The first dark fringe occurs when $a\ \sin\ \theta_1 = \lambda$, and $\tan\theta_1 = (w/2)/R = w/2R$, where R is the distance from the slit to the screen.

EXECUTE: **(a)** If θ_1 is small, we can use the small-angle approximation $\sin\theta_1 \approx \tan\theta_1 = w/2R$. In that case, $aw = 2R\lambda$, which is constant. But if θ_1 is large, we cannot use $\sin\theta_1 \approx \tan\theta_1$, and wa is not constant. Figure 36.62 shows the graph of aw versus a for the data in the problem, and its curvature for small a shows that wa is not constant for small values of a.

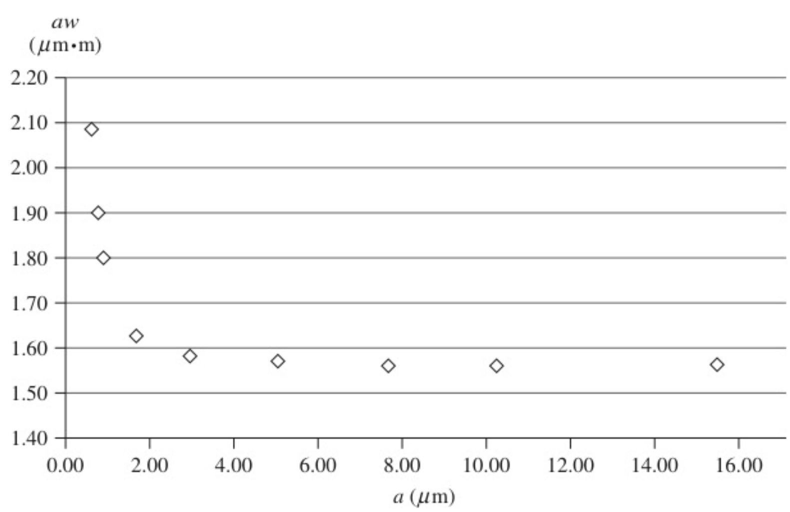

Figure 36.62

(b) We see that our graph flattens out for large values of a and $wa \rightarrow 1.56 \times 10^{-6}\text{ m}^2$. This means we can apply the small-angle approximation. In that case, $aw = 2R\lambda$, so

$\lambda = aw/2R = (1.56 \times 10^{-6}\text{ m}^2)/[2(1.50\text{ m})] = 5.20 \times 10^{-7}\text{ m} = 520\text{ nm}$.

(c) (i) Use $a\sin\theta_1 = \lambda$ with $a = 0.78\ \mu\text{m} = 780\text{ nm}$.

$(780\text{ nm})\sin\theta_1 = 520\text{ nm} \quad \rightarrow \quad \theta_1 = 42°$.

(ii) Now use $a = 15.60\ \mu\text{m} = 15{,}600\text{ nm}$.

$(15{,}600\text{ nm})\sin\theta_1 = 520\text{ nm} \quad \rightarrow \quad \theta_1 = 1.91°$.

EVALUATE: In part (c), we could use the small-angle approximation in (ii) but not in (i).

36.63. **IDENTIFY:** The hole produces a diffraction pattern on the screen.

SET UP: The first dark ring occurs when $D\sin\theta_1 = 1.22\,\lambda$, and $\tan\theta_1 = r/x$.

EXECUTE: **(a)** Solving for D gives $D = (1.22\,\lambda)/(\sin\theta_1)$. First use $\tan\theta_1 = r/x$ to find θ_1, and then calculate D.

<u>First set</u>: $\tan\theta_1 = r/x = (5.6\text{ cm})/(100\text{ cm}) \rightarrow \theta_1 = 3.205°$.

$D = (1.22)(562\text{ nm})/[\sin(3.205°)] = 1.226\times10^4\text{ nm}$.

<u>Second set</u>: $\tan\theta_1 = r/x = (8.5\text{ cm})/(150\text{ cm}) \rightarrow \theta_1 = 3.2433°$.

$D = (1.22)(562\text{ nm})/[\sin(3.2433°)] = 1.212\times10^4\text{ nm}$.

<u>Third set</u>: $\tan\theta_1 = r/x = (11.6\text{ cm})/(200\text{ cm}) \rightarrow \theta_1 = 3.3194°$.

$D = (1.22)(562\text{ nm})/[\sin(3.3194°)] = 1.184\times10^4\text{ nm}$.

<u>Fourth set</u>: $\tan\theta_1 = r/x = (14.1\text{ cm})/(250\text{ cm}) \rightarrow \theta_1 = 3.2281°$.

$D = (1.22)(562\text{ nm})/[\sin(3.2281°)] = 1.218\times10^4\text{ nm}$.

Taking the average gives

$D_{av} = [(1.226 + 1.212 + 1.184 + 1.218)\times10^4\text{ nm}]/4 = 1.21\times10^4\text{ nm} = 12.1\,\mu\text{m}$.

(b) $\sin\theta_2 = 2.23\,\lambda/D = (2.23)(562\text{ nm})/(1.21\times10^4\text{ nm}) \rightarrow \theta_2 = 5.945°$.

$r_2 = x_2\tan\theta_2 = (1.00\text{ m})\tan(5.945°) = 0.104\text{ m} = 10.4\text{ cm}$.

$\sin\theta_3 = 3.24\,\lambda/D = (3.24)(562\text{ nm})/(1.21\times10^4\text{ nm}) \rightarrow \theta_3 = 8.6551°$.

$r_3 = x_3\tan\theta_3 = (1.00\text{ m})\tan(8.6551°) = 0.152\text{ m} = 15.2\text{ cm}$.

EVALUATE: This is a very small hole, but its diameter is still over 20 times the wavelength of the light passing through it.

36.64. **IDENTIFY:** The liquid reduces the wavlength of the light (compared to its value in air), and the scratch causes light passing through it to undergo single-slit diffraction.

SET UP: $\sin\theta = \dfrac{\lambda}{a}$, where λ is the wavelength in the liquid. $n = \dfrac{\lambda_{\text{air}}}{\lambda}$.

EXECUTE: $\tan\theta = \dfrac{(22.4/2)\text{ cm}}{30.0\text{ cm}}$ and $\theta = 20.47°$.

$\lambda = a\sin\theta = (1.25\times10^{-6}\text{ m})\sin 20.47° = 4.372\times10^{-7}\text{ m} = 437.2\text{ nm}$. $n = \dfrac{\lambda_{\text{air}}}{\lambda} = \dfrac{612\text{ nm}}{437.2\text{ nm}} = 1.40$.

EVALUATE: $n > 1$, as it must be, and $n = 1.40$ is reasonable for many transparent films.

36.65. **IDENTIFY:** This problem involves single slit-diffraction and double-slit interference. As the membranes move along, the thickness of the slits varies because the membranes overlap. So we are dealing with a double slit having a variable slit width.

SET UP: For single-slit dark spots, $a\sin\theta_m = m\lambda\ (m = \pm1, \pm2, \cdots)$ and for double-slit bright spots $d\sin\theta_m = m\lambda\ (m = 0, \pm1, \pm2, \cdots)$.

EXECUTE: **(a)** We want the slit spacing d. At t = 1.00 s the edges of the screen are totally dark but the rest of the screen contains a series of 19 spots of nearly equal brightness. These are the double-slit interference spots. There is one in the middle of the screen and 9 on each side of the center. The spots that would be at the edges of the screen are missing, and they are the $m = \pm10$ spots. Therefore $\theta_{10} = \pm30.0°$ from the center. For double-slit bright spots, we have $d\sin\theta_{10} = 10\lambda$. Solving for d gives

$$d = \frac{10\lambda}{\sin\theta_{10}} = \frac{10(532\text{ nm})}{\sin(30.0°)} = 10.6\,\mu\text{m}.$$

(b) We want the speed v of the membranes. The width of the slits at 1.00 s is given by $a_1\sin\theta_1 = \lambda$. We use m = 1 because this is the first single-slit dark spot. This dark spot occurs at the angle for which the m = 10 double-slit bright spot would have occurred, so $\theta_1 = 30°$. Solving for a_1 gives

$a_1 = \dfrac{\lambda}{\sin\theta_{10}} = \dfrac{532\text{ nm}}{\sin(30.0°)} = 1064\text{ nm}$. The slit width increased from 0 nm to 1064 nm in 1.00 s. The membranes move with equal speed in opposite directions, so each one traveled half this distance in that time. Therefore $vt_1 = a/2$, which gives $v = a_1/2t_1 = (1064\text{ nm})/[2(1.00\text{ s})] = 532\text{ nm/s}$.

(c) We want the maximum slit width a. At $t = 0$ the slits are closed and at $t = 3.00$ s they are closed again. So at $t = 1.50$ s they reach their maximum width. During this time each membrane travels a distance of $a/2$. Thus $vt = a/2$, which gives $a = 2vt = 2(532\text{ nm/s})(1.50\text{ s}) = 1.60\ \mu\text{m}$.

(d) We want the time at which the $m = \pm 9$ spots suddenly disappear. The slits are now thicker than they were at $t = 1.00$ s so the $m = 1$ single-slit dark spot has moved closer to the center and now cancels the $m = \pm 9$ double-slit bright spots. To find this slit width a_2, consider the double-slit pattern. The $m = 9$ bright spot would be at θ_9 which is given by $d\sin\theta_9 = 9\lambda$. From this we get

$\sin\theta_9 = \dfrac{9\lambda}{d} = \dfrac{9(532\text{ nm})}{10.6\ \mu\text{m}} = 0.4517$. θ_9 is now the angle of the first single-slit dark spot, so $a_2\sin\theta_9 = \lambda$, which gives $a_2 = \dfrac{\lambda}{\sin\theta_9} = \dfrac{532\text{ nm}}{0.4517} = 1178\text{ nm}$. The slit width started at 0 at $t = 0$ and is now equal to 1178 nm. As in part (b), $vt_2 = a_2/2$, so $t_2 = a_2/2v = (1178\text{ nm})/[2(532\text{ nm/s})] = 1.11$ s.

(e) We want the intensity I at the first double-slit bright spot. At 1.50 s the slits are at their maximum width of 1.60 μm. The intensity is $I = I_0\cos^2(\phi/2)\left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$. At the first bright spot $\cos(\phi/2) = 1$ and $d\sin\theta = \lambda$, so $\sin\theta = \lambda/d$. Using this to find $\beta/2$ gives

$\beta/2 = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi a(\lambda/d)}{\lambda} = \dfrac{\pi a}{d} = \dfrac{\pi(1.60\ \mu\text{m})}{10.6\ \mu\text{m}} = 0.4742\text{ rad} = 27.17°$. The intensity is

$I = I_0\left[\dfrac{\sin(27.19°)}{0.4742\text{ rad}}\right]^2 = 0.927I_0 = 92.7\%$ of I_0.

(f) We want the angle at which the first double-slit bright spot occurs. This is the first such spot, so $d\sin\theta = \lambda/d$ = (532 nm)/(10.6 μm) = 0.0519, so θ = 2.88°.

EVALUATE: A device like this would make an interesting demonstration in a physics lecture or at a science fair.

36.66. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** Each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0. However, each successive source will pick up an extra phase from its respective pathlength to point P. $\phi = 2\pi\left(\dfrac{d\sin\theta}{\lambda}\right)$ which is just 2π, the maximum phase, scaled by whatever fraction the path difference, $d\sin\theta$, is of the wavelength, λ. By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

(b) $e^{i(kR-\omega t+n\phi)} = \cos(kR-\omega t+n\phi) + i\sin(kR-\omega t+n\phi)$. The real part is just $\cos(kR-\omega t+n\phi)$. So, $\text{Re}\left[\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega x+n\phi)}\right] = \sum_{n=0}^{N-1} E_0\cos(kR-\omega x+n\phi)$. (Note: Re means "the real part of…."). But this is just $E_0\cos(kR-\omega t) + E_0\cos(kR-\omega t+\phi) + E_0\cos(kR-\omega t+2\phi) + \cdots + E_0\cos(kR-\omega t+(N-1)\phi)$.

(c) $\sum_{n=0}^{N-1} E_0\, e^{i(kR-\omega t+n\phi)} = E_0\sum_{n=0}^{N-1} e^{-i\omega t}\, e^{+ikR} e^{in\phi} = E_0 e^{i(kR-\omega t)}\sum_{n=0}^{N-1} e^{in\phi}$. $\sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1}(e^{i\phi})^n$. But recall $\sum_{n=0}^{N-1} x^n = \dfrac{x^N-1}{x-1}$. Putting everything together:

$$\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)} = E_0 e^{i(kR-\omega t+(N-1)\phi/2)}\frac{(e^{iN\phi/2}-e^{-iN\phi/2})}{(e^{i\phi/2}-e^{-i\phi/2})}$$

$$= E_0[\cos(kR - \omega t + (N-1)\phi/2) + i\sin(kR - \omega t + (N-1)\phi/2)]\left[\frac{\cos N\phi/2 + \sin N\phi/2 - \cos N\phi/2 + i\sin N\phi/2}{\cos\phi/2 + i\sin\phi/2 - \cos\phi/2 + i\sin\phi/2}\right].$$

Taking only the real part gives $\Rightarrow E_0\cos(kR - \omega t + (N-1)\phi/2)\dfrac{\sin(N\phi/2)}{\sin\phi/2} = E.$

(d) $I = |E|^2_{\text{av}} = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$. (The $\cos^2$ term goes to $\frac{1}{2}$ in the time average and is included in the definition of I_0.) $I_0 \propto \dfrac{E_0^2}{2}$.

EVALUATE: **(e)** $N = 2$. $I = I_0\dfrac{\sin^2(2\phi/2)}{\sin^2\phi/2} = \dfrac{I_0(2\sin\phi/2\cos\phi/2)^2}{\sin^2\phi/2} = 4I_0\cos^2\dfrac{\phi}{2}$. Looking at Eq. (35.9), $I'_0 \propto 2E_0^2$ but for us $I_0 \propto \dfrac{E_0^2}{2} = \dfrac{I'_0}{4}$.

36.67. **IDENTIFY** and **SET UP:** From Problem 36.66, $I = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2}$. Use this result to obtain each result specified in the problem.

EXECUTE: **(a)** $\lim\limits_{\phi\to 0} I \to \dfrac{0}{0}$. Use 1'Hôpital's rule: $\lim\limits_{\phi\to 0}\dfrac{\sin(N\phi/2)}{\sin\phi/2} = \lim\limits_{\phi\to 0}\left(\dfrac{N/2}{1/2}\right)\dfrac{\cos(N\phi/2)}{\cos(\phi/2)} = N$. So $\lim\limits_{\phi\to 0} I = N^2 I_0$.

(b) The location of the first minimum is when the numerator first goes to zero at $\dfrac{N}{2}\phi_{\min} = \pi$ or $\phi_{\min} = \dfrac{2\pi}{N}$. The width of the central maximum goes like $2\phi_{\min}$, so it is proportional to $\dfrac{1}{N}$.

(c) Whenever $\dfrac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity. That is, I is a minimum wherever $\phi = \dfrac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which occurs when $\dfrac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a maximum. That is, if $\dfrac{n}{N}$ is an integer, there will be a maximum.

(d) From part (c), if $\dfrac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\dfrac{n}{N}$ is not an integer for fixed N and integer n.) For example, $n = 0$ will be a maximum, but $n = 1, 2\ldots, N-1$ will be minima with another maximum at $n = N$.

(e) Between maxima $\dfrac{\phi}{2}$ is a half-integer multiple of $\pi\left(\text{i.e.,}\dfrac{\pi}{2},\dfrac{3\pi}{2}\text{etc.}\right)$ and if N is odd then $\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2} \to 1$, so $I \to I_0$.

EVALUATE: These results show that the principal maxima become sharper as the number of slits is increased.

36.68. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: **(a)** From the segment dy', the fraction of the amplitude of E_0 that gets through is

$$E_0\left(\frac{dy'}{a}\right) \Rightarrow dE = E_0\left(\frac{dy'}{a}\right)\sin(kx-\omega t).$$

(b) The path difference between each little piece is

$y'\sin\theta \Rightarrow kx = k(D - y'\sin\theta) \Rightarrow dE = \dfrac{E_0 dy'}{a}\sin(k(D-y'\sin\theta)-\omega t).$ This can be rewritten as

$$dE = \frac{E_0 dy'}{a}(\sin(kD-\omega t)\cos(ky'\sin\theta)+\sin(ky'\sin\theta)\cos(kD-\omega t)).$$

(c) So the total amplitude is given by the integral over the slit of the above.

$$\Rightarrow E = \int_{-a/2}^{a/2} dE = \frac{E_0}{a}\int_{-a/2}^{a/2} dy'\,(\sin(kD-\omega t)\cos(ky'\sin\theta)+\sin(ky'\sin\theta)\cos(kD-\omega t)).$$

But the second term integrates to zero, so we have:

$$E = \frac{E_0}{a}\sin(kD-\omega t)\int_{-a/2}^{a/2} dy'(\cos(ky'\sin\theta)) = E_0\sin(kD-\omega t)\left[\left(\frac{\sin(ky'\sin\theta)}{ka\sin\theta/2}\right)\right]_{-a/2}^{a/2}$$

$$\Rightarrow E = E_0\sin(kD-\omega x)\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right) = E_0\sin(kD-\omega x)\left(\frac{\sin(\pi a(\sin\theta)/\lambda)}{\pi a(\sin\theta)/\lambda}\right).$$

At $\theta = 0, \dfrac{\sin[\ldots]}{[\ldots]} = 1 \Rightarrow E = E_0\sin(kD-\omega x).$

(d) Since $I \propto E^2 \Rightarrow I = I_0\left(\dfrac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right)^2 = I_0\left(\dfrac{\sin(\beta/2)}{\beta/2}\right)^2$, where we have used

$I_0 = E_0^2\sin^2(kx-\omega t).$

EVALUATE: The same result for $I(\theta)$ is obtained as was obtained using phasors.

36.69. IDENTIFY and SET UP: For an interference maxima, $2d\sin\theta = m\lambda_0/n.$ The microspheres are in water, so we use n = 1.3. Do a rough calculation to estimate the distance between microspheres.

EXECUTE: Use the numbers in the problem and $n = 1.3.$ Solving $2d\sin\theta = m\lambda_0/n$ for d gives

$d = \dfrac{m\lambda_0}{2n\sin\theta} = \dfrac{(1)(650\text{ nm})}{2(1.3)\sin 39°} \approx 400$ nm. This distance is much greater than the spacing of atoms in a crystal and is comparable to the wavelength of visible light. Therefore choice (d) is correct.

EVALUATE: The atoms in a crystal are tightly bound by the electric force between them and are therefore very close together.

36.70. IDENTIFY and SET UP: For an interference maxima, $2d\sin\theta = m\lambda_0/n.$ The microspheres are in water, so we use n = 1.33.

EXECUTE: $d = \dfrac{m\lambda_0}{2n\sin\theta} = \dfrac{(1)(650\text{ nm})}{2(1.33)\sin 39°} \approx 390$ nm. Choice (a) is the correct one.

EVALUATE: We are assuming that the microspheres do not affect the index of refraction of the water.

36.71. IDENTIFY and SET UP: For an interference maxima, $2d\sin\theta = m\lambda_0/n,$ so $\sin\theta \propto \dfrac{1}{d}.$ If d is smaller, $\sin\theta,$ and hence $\theta,$ is larger.

EXECUTE: At the top $\theta_{\text{top}} = 37°$, and at the bottom $\theta_{\text{bottom}} = 41°.$ Since $\theta_{\text{top}} < \theta_{\text{bottom}}, <$ it follows that $d_{\text{top}} > d_{\text{bottom}}$. Therefore the microspheres are closer together at the bottom than at the top, which makes choice (a) the correct one.

EVALUATE: Gravity tends to pull the spheres downward so they bunch up at the bottom, making them closer together.