Circuits
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Series and parallel equivalences

NYU 上海 SHANGHAI 纽约大学

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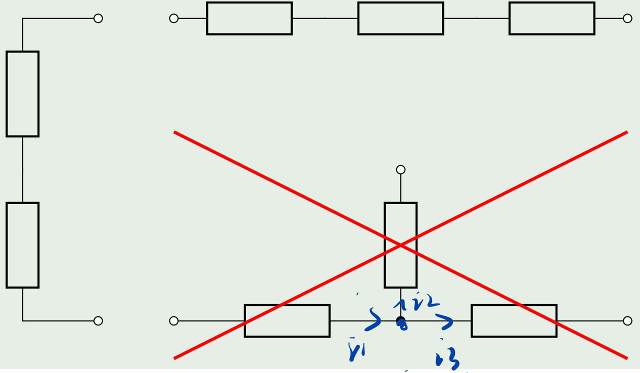
Series connection

Definition

Series connection: when elements carry the same current

Examples



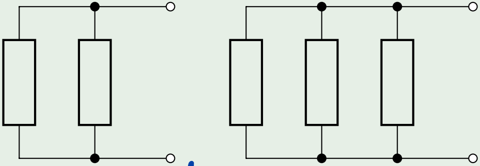
$i_1 = i_2 + i_3$

Parallel connection

Definition

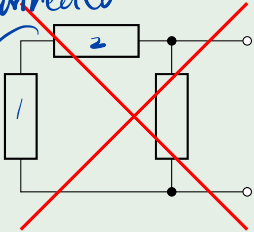
Parallel connection: when elements have a common voltage across them

Examples



1& 2 series connected.

4



3& 4 parallel connected.

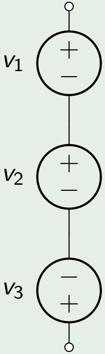
Voltage sources

Voltage sources connected in series

Series-connected voltage sources can be replaced by a **single equivalent voltage** source

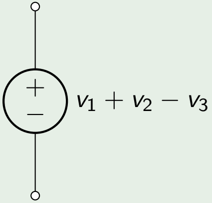
The equivalent voltage is equal to the algebraic sum of individual sources

Example



We do not have parallel connected voltage sources

equivalent to

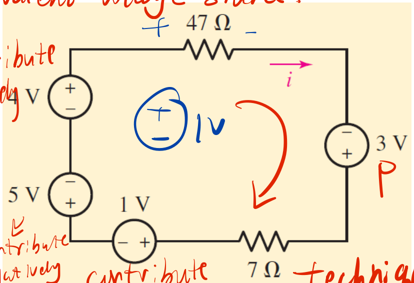


Examples

First we need to define the polarity for the equivalent voltage source.

One technique

Determine the current *i* in the circuit after first replacing the four sources with a single equibalent source.



Only look at voltage sources $-4V-3V+1V+5V$

$= -1V$

that satisfies the Psc.

contribute a negative part of the current in this direction

$4+3-5-1=1V$

technique 2.

② look at how does it contribute to the hypothetical current

$47i+7i-1=0$

$i = \frac{1}{54}$ A

Current sources

Why don't we consider series current sources?

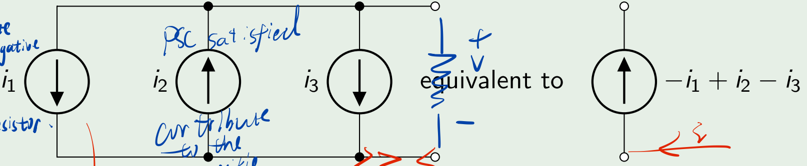
Current sources connected in parallel

Parallel-connected current sources can be replaced by a **single equivalent current** source

The equivalent current is equal to the algebraic sum of individual sources

technique 2)

Example



contribute the negative voltage across the resistor.

KCL satisfied

contribute to the positive voltage across the resistor

+

−

i

technique 1

Contribute

I_3 I_1 I_2

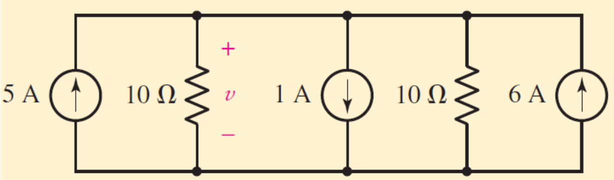
make equivalent current.

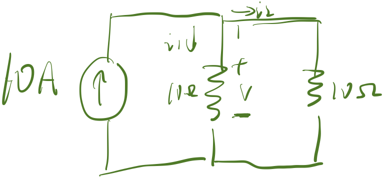
$i = i_2 - i_1 - i_3$

Examples

When we are combining. we only look at independent sources

Determine the voltage v in the circuit after first replacing the three sources with a single equibalent source.





$10A = \frac{V}{10\Omega} + \frac{V}{10\Omega}$

$i_1 + i_2 = 10A$

$\therefore i_1 = i_2 = 5A.$

$10\Omega\, i_1 = 10\Omega\, i_2$

$V = i_1 \times 10\Omega = 50V$

Resistors in series

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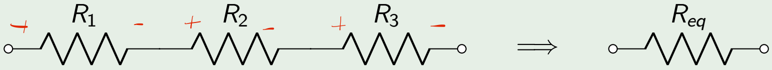
Equivalence

Series-connected resistors can be replaced by a **single resistor**

The equivalent resistance is equal to the sum of individual resistances

doesn't have algebric sum

Formula



$$R_{eq} = R_1 + R_2 + R_3$$

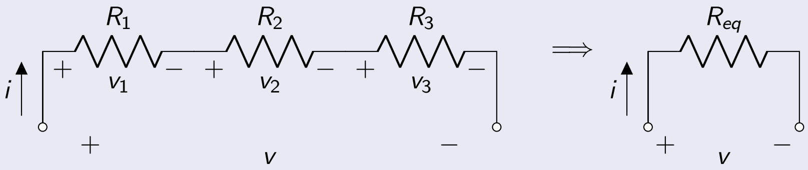
$$i\,R_{eq} = i\,R_1 + i\,R_2 + i\,R_3$$

$$= i\,(R_1 + R_2 + R_3)$$

Resistors in series

Demonstration

By applying KVL



$$
\begin{aligned}
v &= v_1 + v_2 + v_3 \\
&= R_1 \cdot i + R_2 \cdot i + R_3 \cdot i \\
&= (R_1 + R_2 + R_3) \cdot i \\
&= R_{eq} \cdot i
\end{aligned}
$$

$$\Longrightarrow R_{eq} = R_1 + R_2 + R_3$$

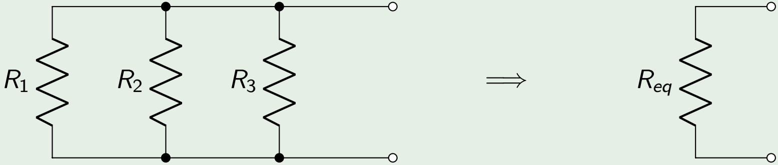
Resistors in parallel

Equivalence

Parallel-connected resistors can be replaced by a **single resistor**

The equivalent inverse of the resistance is equal to the sum of inverse of individual resistances

Formula

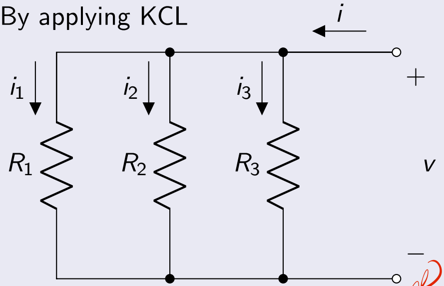


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \Longrightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Resistors in parallel

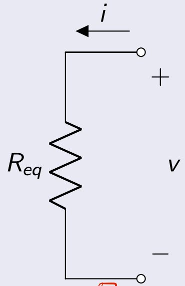
Demonstration

By applying KCL



Derivation: KCL

$\Longrightarrow$



$R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2} = R_1 \parallel R_2.$

$$
\begin{aligned}
i &= i_1 + i_2 + i_3 \\
&= \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} \\
&= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \cdot v \\
&= \frac{v}{R_{eq}}
\end{aligned}
$$

Similar equation doesn't hold

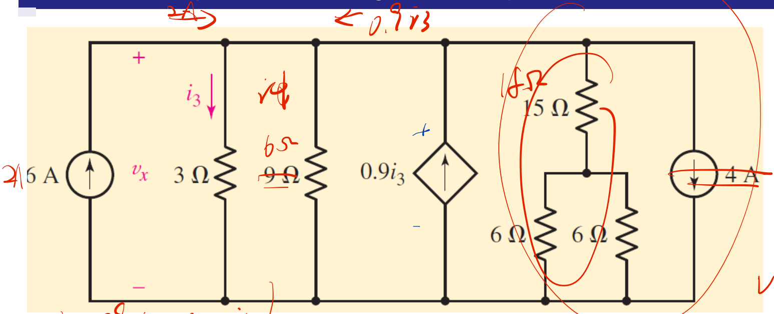
$$\Longrightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

for more parallel resistors.

Examples

$P_{dep} = 0.9 i_3 \times V_x \quad Psc.$

Determine the power and voltage of the dependent source.



KCL: $2A + 0.9 i_3 = i_3 + i_4$

$i_3 \cdot 3\Omega = i_4 \cdot 6\Omega$

Ohm's law

$\frac{6\times 6}{12} = \frac{36}{12} = 3\,\Omega$

$\frac{18 \times 9}{18+9} = 6\Omega$

$i_3 = 3.33A$

$V_x = 10V$

$\frac{3}{2} i_3 = 2 + 0.9 i_3$

$\frac{V_x}{3} + \frac{V_x}{6} = 2 + 0.9 i_3$

$\frac{V_x}{3} = i_3 \qquad \frac{V_x}{6} = \frac{i_3}{2}$

Planar circuit

Planar circuit

It is possible to draw the diagram of a circuit on a plane surface that no branch passes over or under any other branch.