38

PHOTONS: LIGHT WAVES BEHAVING AS PARTICLES

VP38.3.1. **IDENTIFY:** This problem involves the energy and momentum of a photon.

SET UP: $E = hc/\lambda$, $\lambda = h/p$.

EXECUTE: **(a)** We want the energy and momentum of the photon. $E = hc/\lambda = hc/(625\text{ nm}) = 3.18 \times 10^{-19}$ J. $p = h/\lambda = h/(625\text{ nm}) = 1.06\times10^{-27}\text{ kg}\cdot\text{m/s}$.

(b) We want the power output of the laser. $\dfrac{E}{t} = \dfrac{\left(2.15\times10^{16}\text{photons}\right)\left(3.18\times10^{-19}\text{ J/photon}\right)}{1.00\text{ s}}$ = 6.77 mW.

EVALUATE: Don't confuse the frequency of the light with the frequency at which photons are emitted. They are *very* different!

VP38.3.2. **IDENTIFY:** This problem is about the photoelectric effect.

SET UP: $hf = K_{\text{max}} + \phi$.

EXECUTE: **(a)** We want the minimum frequency to produced photoelectrons. This is the threshold frequency, at which the kinetic energy is zero, so $hf = \phi$. $f = \phi/h = (4.55\text{ eV})/h = 1.10\times10^{15}$ Hz.

(b) We want the frequency when the maximum kinetic energy is 1.53 eV. Solve $hf = K_{\text{max}} + \phi$ for f, giving

$$f = \frac{K_{\text{max}} + \phi}{h} = \frac{1.53\text{ eV} + 4.55\text{ eV}}{h} = 1.47\times10^{15}\text{ Hz}.$$

EVALUATE: Be prepared to use h in units of either $\text{J}\cdot\text{s}$ or $\text{eV}\cdot\text{s}$, depending on the given units. Doing so can avoid tedious unit conversions.

VP38.3.3. **IDENTIFY:** This problem is about the photoelectric effect.

SET UP: $eV_0 = hc/\lambda - \phi$, $f\lambda = c$.

EXECUTE: **(a)** We want the work function. Solve $eV_0 = hc/\lambda - \phi$ for ϕ using V_0 = 1.37 V and λ = 475 nm, giving ϕ = 1.24 eV.

(b) We want V_0 if λ = 425 nm. Solve $eV_0 = hc/\lambda - \phi$ for V_0 using ϕ = 1.24 eV and λ = 425 nm. This gives V_0 = 1.68 eV.

EVALUATE: Decreasing the photon wavelength increases its energy, so the stopping potential increases. From Table 38.1 we see that the ϕ we found is around half that of sodium, so it is a reasonable value.

VP38.3.4. **IDENTIFY:** This problem is about the photoelectric effect.

SET UP: $E = hc/\lambda$.

EXECUTE: **(a)** We want the kinetic energy in eV. Using $K = \frac{1}{2}mv^2$, with m the electron mass and $v = 6.95 \times 10^5$ m/s gives $K = 2.20 \times 10^{-19}$ J = 1.37 eV.

(b) We want the photon energy. $E = hc/\lambda = hc/(306 \text{ nm}) = 4.05$ eV.
(c) We want ϕ. $E_{photon} = K_{max} + \phi$. $\phi = 4.05 \text{ eV} - 1.37 \text{ eV} = 2.68$ eV.

EVALUATE: The work function is about the same as for sodium, so it is reasonable.

VP38.6.1. **IDENTIFY:** We are dealing with Compton scattering.

SET UP: $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$. We want the increase in the wavelength.

EXECUTE: **(a)** $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi) = \left(2.426\times10^{-12} \text{ m}\right)\left(1-\cos 14.5°\right) = 7.73\times10^{5}$ nm.

(b) $\frac{\lambda' - \lambda}{\lambda} = \frac{7.73\times10^{5} \text{ nm}}{0.251 \text{ nm}} = 0.0308\%$.

EVALUATE: The fractional change in the wavelength is very small. It would be larger if the scattering angle ϕ were closer to 180°.

VP38.6.2. **IDENTIFY:** We are dealing with Compton scattering.

SET UP: $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$, $f\lambda = c$. We want the frequency of the scattered waves. Using $f\lambda = c$ gives

$$\frac{1}{f'} = \frac{1}{f} + \frac{h}{mc^2}(1-\cos\phi).$$

EXECUTE: **(a)** $\phi = 90.00°$: Using the given frequency and scattering angle gives $f' = 1.247\times10^{18}$ Hz.

(b) $\phi = 180.0°$: Working as in (a) but with $\phi = 180.0°$ gives $f' = 1.235\times10^{18}$ Hz.

EVALUATE: For $\phi = 180°$ the x rays are scattered directly back from their original direction, but at $\phi = 90.0°$ they go off perpendicular to their original direction. Note that the frequency of the scattered waves is different in each case.

VP38.6.3. **IDENTIFY:** We are dealing with Compton scattering off of a proton.

SET UP: $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$, where *m* is now the mass of a *proton*. We want the scattering angle ϕ.

Solving for $\cos\phi$ gives $\cos\phi = 1 - \frac{mc}{h}(\lambda' - \lambda)$, with $\lambda = 2.50\times10^{-12}$ m.

EXECUTE: **(a)** $\lambda' = \lambda + 0.000100\lambda$: $\cos\phi = 1 - \frac{mc}{h}(\lambda' - \lambda) = \frac{mc}{h}(\lambda + 0.000100\lambda - \lambda)$

$= 1 - \frac{mc}{h}(0.000100\lambda)$.

Using the *proton* mass gives $\phi = 35.8°$.

(b) $\lambda' = \lambda + 0.000800\lambda$: $\cos\phi = 1 - \frac{mc}{h}(\lambda' - \lambda) = \frac{mc}{h}(\lambda + 0.000100\lambda - \lambda) = 1 - \frac{mc}{h}(0.000800\lambda)$.

$\phi = 121°$.

EVALUATE: Note that the scattering angle has a considerable effect on the wavelength of the scattered wave.

VP38.6.4. **IDENTIFY:** This problem involves the annihilation of a proton and an antiproton.
SET UP: We want the energy and wavelength of the resulting photons. $E = hc/\lambda$.
EXECUTE: **(a)** Initial kinetic energy is negligible. The energy of the two photons is equal to the rest energy of the two protons (which is 938.3 MeV each), and each photon has the same energy due to momentum conservation. Therefore $E_{photon} = 938.3$ MeV. The wavelength of each photon is $\lambda = hc/E$ $= hc/(938.3 \text{ MeV}) = 1.32\times10^{-15}$ m.

(b) Initial kinetic energy of each proton is 545 MeV. In this case, the energy of the photons is equal to the rest energy of the protons plus their kinetic energy, so the energy of each photon is E_{photon} = 938.3 MeV + 545 MeV = 1483.3 MeV. $\lambda = hc/E = hc/(1483.3\text{ MeV}) = 8.36\times10^{-16}$ m.

EVALUATE: The wavelength of the photons is less when the protons have substantial energy. This is a reasonable result because short-wavelength photons have more energy than long-wavelength photons.

VP38.7.1. **IDENTIFY:** This problem involves the uncertainty principle.
SET UP: $\Delta t\Delta E = \hbar/2$, $\Delta x\Delta p_x = \hbar/2$, $\lambda = h/p$, $E = hc/\lambda$.

EXECUTE: **(a)** We want the momentum. $p = h/\lambda = h/(633\text{ nm}) = 1.05\times10^{-27}\text{ kg}\cdot\text{m/s}$.

(b) We want the minimum uncertainty in the momentum. We know that $\Delta x = 700\times10^{-6}$ m.

$$\Delta p_{\min} = \frac{\hbar}{2\Delta x} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2(7.00\times10^{-6}\text{ m})} = 7.54\times10^{-30}\text{ kg}\cdot\text{m/s}.$$

The percent uncertainty is

$$\frac{\Delta p_{\min}}{p_{\min}} = \frac{7.54\times10^{-30}\text{ kg}\cdot\text{m/s}}{1.05\times10^{-27}\text{ kg}\cdot\text{m/s}} = 0.720\%.$$

(c) We want the energy. $E = hc/\lambda = hc/(633\text{ nm}) = 3.14\times10^{-19}$ J.
(d) We want the minimum uncertainty in the energy. We use $\Delta t\Delta E = \hbar/2$ where Δt is the time for the pulse to propagate 7.00×10^{-6} m. Therefore

$$\Delta t = \frac{\Delta x}{c} = \frac{7.00\times10^{-6}\text{ m}}{c} = 2.333\times10^{-14}\text{ s}.$$

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{h}{2\left(2.333\times10^{-14}\text{ s}\right)} = 2.26\times10^{-21}\text{ s}.$$

Dividing the result in (d) by the energy in (c) gives

$$\frac{\Delta E}{E} = 0.720\%.$$

EVALUATE: Note that the uncertainties in p and E are very small percents.

VP38.7.2. **IDENTIFY:** This problem involves the uncertainty principle.
SET UP: $\Delta t\Delta E = h/2$, $E = hf$, $f\lambda = c$.

EXECUTE: **(a)** We want the time Δt of the pulse.

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{\hbar}{2\left(5.50\times10^{-29}\text{ J}\right)} = 9.59\times10^{-7}\text{ s}.$$

(b) We want the length of the pulse. $L = c\Delta t = c\left(9.59\times10^{-7}\text{ s}\right) = 288$ m.

(c) We want the energy. $E = hf = h(3.00\text{ GHz}) = 1.99\times10^{-24}$ J.

(d) We want the uncertainty in the frequency. $E = hf$, so $\Delta E = h\Delta f$. Also $\Delta E = \dfrac{h}{2\Delta t}$ $\Delta E = \dfrac{\hbar}{2\Delta t}$.

Combining gives $\dfrac{\hbar}{2\Delta t} = h\Delta f$, so

$$\Delta f = \frac{1}{4\pi\Delta t} = \frac{1}{4\pi\left(9.59\times10^{-7}\text{ s}\right)} = 8.30\times10^{-5}\text{ GHz}.$$

EVALUATE: As we have seen before, the fractional uncertainties arising from the uncertainty principle are usually very small.

VP38.7.3. **IDENTIFY:** This problem involves the uncertainty principle.
SET UP: $\Delta t\Delta E \geq \hbar/2$, $E = hf$, $f\lambda = c$.

EXECUTE: **(a)** We want the wavelength and frequency of the laser light. Using $E = hf$, we see that $\Delta E = h\Delta f$. We know that ΔE = 6.70%E = 0.0670hf and that $\Delta E = 2.33 \times 10^{-20}$ J. Therefore 2.33×10^{-20} J = 0.0670hf, which gives $f = 5.25 \times 10^{14}$ Hz. Using this frequency, the wavelength is $\lambda = c/f$ = 572 nm.
(b) We want the minimum uncertainty in f. $E = hf$, so $\Delta E = h\Delta f$. Thus

$$\Delta f = \frac{\Delta E}{h} = \frac{2.33\times10^{-20}\ \text{J}}{h} = 3.52\times10^{13}\ \text{Hz}.$$

(c) We want the time duration Δt of the pulse. Using $\Delta t\Delta E \geq \hbar/2$ gives

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{\hbar}{2(2.33\times10^{-20}\ \text{J})} = 2.26\times10^{-15}\ \text{s} = 2.26\ \text{fs}.$$

EVALUATE: As before, the fractional uncertainties are very small.

VP38.7.4. **IDENTIFY:** This problem involves the uncertainty principle.
SET UP: $\Delta t\Delta E \geq \hbar/2$, $E = hc/\lambda$. The average photon energy is 0.0155 eV, the energy of a pulse is 2.39 eV, and on the *average* there are 5.00×10^{12} photons per pulse.
EXECUTE: **(a)** We want the time duration of a pulse. As pointed out in Example 7.4, the minimum uncertainty Δt for a photon is the time duration of the pulse. Using $\Delta t\Delta E \geq \hbar/2$ gives

$$\Delta t = \frac{\hbar}{2\Delta E} = \frac{\left(4.136\times10^{-15}\ \text{eV}\cdot\text{s}\right)/2\pi}{2\left(0.0155\ \text{eV}\right)} = 2.12\times10^{-14}\ \text{s}.$$

(b) We want the wavelength. Using $E = hc/\lambda$ gives $\lambda = hc/E = hc/(2.39\ \text{eV})$ = 519 nm.
(c) We want the energy per pulse. This energy is equal to the energy per average photon times the number of photons in a pulse. Therefore E = (2.39 eV/photon)(5.00×10^{12} photons/pulse) = 1.195×10^{13} eV = 1.91 μJ per pulse.
EVALUATE: The energy per pulse is small, but the pulse lasts for a very short time, so the power during the pulse is very large. Note that some of the photons have wavelengths longer than 519 and some have shorter wavelengths.

38.1. **IDENTIFY** and **SET UP:** Apply $c = f\lambda$, $p = h/\lambda$, and $E = pc$.

EXECUTE:
$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\,\text{m/s}}{5.20\times10^{-7}\ \text{m}} = 5.77\times10^{14}\ \text{Hz}.$$

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.20\times10^{-7}\ \text{m}} = 1.28\times10^{-27}\ \text{kg}\cdot\text{m/s}.$$

$$E = pc = (1.28\times10^{-27}\ \text{kg}\cdot\text{m/s})\,(3.00\times10^{8}\,\text{m/s}) = 3.84\times10^{-19}\ \text{J} = 2.40\ \text{eV}.$$

EVALUATE: Visible-light photons have energies of a few eV.

38.2. **IDENTIFY** and **SET UP:** $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00\times10^{8}$ m/s. $1\ \text{eV} = 1.60\times10^{-16}$ J.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{505\times10^{-9}\ \text{m}} = 5.94\times10^{14}\ \text{Hz}.$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(5.94\times10^{14}\ \text{Hz}) = 3.94\times10^{-19}\ \text{J} = 2.46\ \text{eV}.$

(c) $K = \frac{1}{2}mv^2$, so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(3.94\times10^{-19}\ \text{J})}{9.5\times10^{-15}\ \text{kg}}} = 9.1\ \text{mm/s}.$

EVALUATE: Compared to kinetic energies of common objects moving at typical speeds, the energy of a visible-light photon is extremely small.

38.3. **IDENTIFY** and **SET UP:** $c = f\lambda$. The source emits $(0.05)(75\text{ J}) = 3.75\text{ J}$ of energy as visible light each second. $E = hf$, with $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\text{ m/s}}{600\times10^{-9}\text{ m}} = 5.00\times10^{14}\text{ Hz}.$

(b) $E = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(5.00\times10^{14}\text{ Hz}) = 3.32\times10^{-19}\text{ J}$. The number of photons emitted per second is $\dfrac{3.75\text{ J}}{3.32\times10^{-19}\text{ J/photon}} = 1.13\times10^{19}$ photons.

EVALUATE: **(c)** No. The frequency of the light depends on the energy of each photon. The number of photons emitted per second is proportional to the power output of the source.

38.4. **IDENTIFY** and **SET UP:** $P_{\text{av}} = \dfrac{\text{energy}}{t}$. $1\text{ eV} = 1.60\times10^{-19}\text{ J}$. For a photon, $E = hf = \dfrac{hc}{\lambda}$.

$$h = 6.63\times10^{-34}\text{ J}\cdot\text{s}.$$

EXECUTE: **(a)** $\text{energy} = P_{\text{av}}t = (0.600\text{ W})(20.0\times10^{-3}\text{ s}) = 1.20\times10^{-2}\text{ J} = 7.5\times10^{16}\text{ eV}.$

(b) $E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{652\times10^{-9}\text{ m}} = 3.05\times10^{-19}\text{ J} = 1.91\text{ eV}.$

(c) The number of photons is the total energy in a pulse divided by the energy of one photon:

$$\frac{1.20\times10^{-2}\text{ J}}{3.05\times10^{-19}\text{ J/photon}} = 3.93\times10^{16}\text{ photons.}$$

EVALUATE: The number of photons in each pulse is very large.

38.5. **IDENTIFY** and **SET UP:** A photon has zero rest mass, so its energy is $E = pc$ and its momentum is $p = \dfrac{h}{\lambda}$.

EXECUTE: **(a)** $E = pc = (8.24\times10^{-28}\text{ kg}\cdot\text{m/s})(2.998\times10^{8}\text{ m/s}) = 2.47\times10^{-19}\text{ J}$
$= (2.47\times10^{-19}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 1.54\text{ eV}.$

(b) $p = \dfrac{h}{\lambda}$, so $\lambda = \dfrac{h}{p} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{8.24\times10^{-28}\text{ kg}\cdot\text{m/s}} = 8.04\times10^{-7}\text{ m} = 804\text{ nm}.$

EVALUATE: This wavelength is longer than visible wavelengths; it is in the infrared region of the electromagnetic spectrum. To check our result we could verify that the same E is given by $E = hc/\lambda$, using the λ we have calculated.

38.6. **IDENTIFY** and **SET UP:** For the photoelectric effect, the maximum kinetic energy of the photoelectrons is $\tfrac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi$. Take the work function ϕ from Table 38.1. Solve for v_{max}. Note that we wrote f as c/λ.

EXECUTE: $\tfrac{1}{2}mv_{\text{max}}^2 = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{235\times10^{-9}\text{ m}} - (5.1\text{ eV})(1.602\times10^{-19}\text{ J/1 eV}).$

$$\tfrac{1}{2}mv_{\text{max}}^2 = 8.453\times10^{-19}\text{ J} - 8.170\times10^{-19}\text{ J} = 2.83\times10^{-20}\text{ J}.$$

$$v_{\text{max}} = \sqrt{\frac{2(2.83\times10^{-20}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 2.49\times10^{5}\text{ m/s}.$$

EVALUATE: The work function in eV was converted to joules for use in the equation $\tfrac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi$. A photon with $\lambda = 235\text{ nm}$ has energy greater then the work function for the surface.

38.7. **IDENTIFY**: The photoelectric effect occurs. The kinetic energy of the photoelectron is the difference between the initial energy of the photon and the work function of the metal.

SET UP: $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$, $E = hc/\lambda$.

EXECUTE: Use the data for the 400.0-nm light to calculate ϕ. Solving for ϕ gives $\phi = \dfrac{hc}{\lambda} - \frac{1}{2}mv_{\text{max}}^2$ $= \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{400.0\times10^{-9}\ \text{m}} - 1.10\ \text{eV} = 3.10\ \text{eV} - 1.10\ \text{eV} = 2.00\ \text{eV}$. Then for 300.0 nm, we have $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{300.0\times10^{-9}\ \text{m}} - 2.00\ \text{eV}$, which gives $\frac{1}{2}mv_{\text{max}}^2 = 4.14\ \text{eV} - 2.00\ \text{eV} = 2.14\ \text{eV}$.

EVALUATE: When the wavelength decreases the energy of the photons increases and the photoelectrons have a larger minimum kinetic energy.

38.8. **IDENTIFY** and **SET UP:** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, where V_0 is the stopping potential. The stopping potential in volts equals eV_0 in electron volts. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$ and $f = c/\lambda$.

EXECUTE: **(a)** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, so $eV_0 = hf - \phi = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{190\times10^{-9}\ \text{m}} - 2.3\ \text{eV}$ $= 6.53\ \text{eV} - 2.3\ \text{eV} = 4.23\ \text{eV}$, which rounds to 4.2 eV. The stopping potential is 4.2 volts.

(b) $\frac{1}{2}mv_{\text{max}}^2 = 4.2\ \text{eV}$.

(c) $v_{\text{max}} = \sqrt{\dfrac{2(4.23\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\text{kg}}} = 1.2\times10^{6}\ \text{m/s}$.

EVALUATE: If the wavelength of the light is decreased, the maximum kinetic energy of the photoelectrons increases.

38.9. **(b) IDENTIFY:** Solve part (b) first. First use $eV_0 = hf - \phi$ to find the work function ϕ

SET UP: $eV_0 = hf - \phi$, so $\phi = hf - eV_0 = \dfrac{hc}{\lambda} - eV_0$.

EXECUTE: $\phi = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{254\times10^{-9}\ \text{m}} - (1.602\times10^{-19}\ \text{C})(0.181\ \text{V})$.

$\phi = 7.821\times10^{-19}\ \text{J} - 2.900\times10^{-20}\ \text{J} = 7.531\times10^{-19}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 4.70\ \text{eV}$.

(a) IDENTIFY and **SET UP:** The threshold frequency f_{th} is the smallest frequency that still produces photoelectrons. It corresponds to $K_{\text{max}} = 0$ in the equation $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$, so $hf_{\text{th}} = \phi$.

EXECUTE: $f = \dfrac{c}{\lambda}$ says $\dfrac{hc}{\lambda_{\text{th}}} = \phi$.

$$\lambda_{\text{th}} = \frac{hc}{\phi} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{7.531\times10^{-19}\ \text{J}} = 2.64\times10^{-7}\ \text{m} = 264\ \text{nm}.$$

EVALUATE: As calculated in part (b), $\phi = 4.70\ \text{eV}$. This is the value given in Table 38.1 for copper.

38.10. **IDENTIFY:** The acceleration gives energy to the electrons which is then given to the x ray photons.

SET UP: $E = hc/\lambda$, so $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(15.0\times10^{3}\ \text{V})} = 8.29\times10^{-11}\ \text{m} = 0.0829\ \text{nm}$.

EVALUATE: This wavelength certainly is in the x ray region of the electromagnetic spectrum.

38.11. **IDENTIFY:** This problem is about the characteristics of photons.

SET UP: $E = hf$, $\lambda = h/p$, $f\lambda = c$. We want the frequency, wavelength, and momentum of the photon.

EXECUTE: $E_{ph} = 0.700K_{el} = 0.700eV = hf$. So $f = (0.700eV)/h$. Using $V = 50.0$ kV and h in terms of eV, we get $f = 8.46 \times 10^{18}$ Hz. Using this result gives $\lambda = c/f = 0.0355$ nm. Using this wavelength gives $p = h/\lambda = 1.87 \times 10^{-23}\ \text{kg}\cdot\text{m/s}$.

EVALUATE: The frequency of this photon is much greater than that of visible light.

38.12. **IDENTIFY** and **SET UP:** $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: **(a)** $V = \dfrac{hc}{e\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.150\times10^{-9}\ \text{m})} = 8.29\ \text{kV}.$

(b) $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(30.0\times10^{3}\ \text{V})} = 4.14\times10^{-11}\ \text{m} = 0.0414\ \text{nm}.$

EVALUATE: Shorter wavelengths require larger potential differences.

38.13. **IDENTIFY:** Energy is conserved when the x ray collides with the stationary electron.

SET UP: $E = hc/\lambda$, and energy conservation gives $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda'} + K_e$.

EXECUTE: Solving for K_e gives $K_e = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})$ $\left(\dfrac{1}{0.100\times10^{-9}\ \text{m}} - \dfrac{1}{0.110\times10^{-9}\ \text{m}}\right)$. $K_e = 1.81\times10^{-16}\ \text{J} = 1.13\ \text{keV}.$

EVALUATE: The electron does not get all the energy of the incident photon.

38.14. **IDENTIFY** and **SET UP:** The wavelength of the x rays produced by the tube is given by $\dfrac{hc}{\lambda} = eV$. $\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi)$. $\dfrac{h}{mc} = 2.426\times10^{-12}$ m. The energy of the scattered x ray is $\dfrac{hc}{\lambda'}$.

EXECUTE: **(a)** $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(24.0\times10^{3}\ \text{V})} = 5.167\times10^{-11}\ \text{m},$ which rounds to 0.0517 nm = 51.7 pm.

(b) $\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi) = 5.167\times10^{-11}\ \text{m} + (2.426\times10^{-12}\ \text{m})(1-\cos 45.0^\circ) = 5.238\times10^{-11}\ \text{m},$ which rounds to 0.0524 nm = 52.4 pm.

(c) $E = \dfrac{hc}{\lambda'} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{5.238\times10^{-11}\ \text{m}} = 2.37\times10^{4}\ \text{eV} = 23.7\ \text{keV}.$

EVALUATE: The incident x ray has energy 24.0 keV. In the scattering event, the photon loses energy and its wavelength increases.

38.15. **IDENTIFY:** Apply $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi) = \lambda_C(1-\cos\phi)$.

SET UP: Solve for λ': $\lambda' = \lambda + \lambda_C(1-\cos\phi)$.

The largest λ' corresponds to $\phi = 180^\circ$, so $\cos\phi = -1$.

EXECUTE: $\lambda' = \lambda + 2\lambda_C = 0.0665\times10^{-9}\ \text{m} + 2(2.426\times10^{-12}\ \text{m}) = 7.135\times10^{-11}\ \text{m} = 0.0714\ \text{nm}.$ This wavelength occurs at a scattering angle of $\phi = 180^\circ$.

EVALUATE: The incident photon transfers some of its energy and momentum to the electron from which it scatters. Since the photon loses energy its wavelength increases, $\lambda' > \lambda$.

38.16. **IDENTIFY:** Compton scattering occurs. We know speed, and hence the kinetic energy, of the scattered electron. Energy is conserved.

SET UP: $\frac{hc}{\lambda} = \frac{hc}{\lambda'} + E_e$ where $E_e = \frac{1}{2}mv^2$.

EXECUTE: $E_e = \frac{1}{2}mv^2 = \frac{1}{2}(9.108\times10^{-31}\text{ kg})(8.90\times10^6\text{ m/s})^2 = 3.607\times10^{-17}$ J.

$\frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{0.1385\times10^{-9}\text{ m}} = 1.434\times10^{-15}$ J. Therefore, $\frac{hc}{\lambda'} = \frac{hc}{\lambda} - E_e = 1.398\times10^{-15}$ J,

which gives $\lambda' = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{1.398\times10^{-15}\text{ J}} = 0.1421$ nm.

$\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi) = 3.573\times10^{-12}$ m, so $1-\cos\phi = 1.473$, which gives $\phi = 118°$.

EVALUATE: The photon partly backscatters, but not through 180°.

38.17. **IDENTIFY** and **SET UP:** The shift in wavelength of the photon is $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$ where λ' is the wavelength after the scattering and $\frac{h}{mc} = \lambda_C = 2.426\times10^{-12}$ m. The energy of a photon of wavelength λ is $E = \frac{hc}{\lambda} = \frac{1.24\times10^{-6}\text{ eV}\cdot\text{m}}{\lambda}$. Conservation of energy applies to the collision, so the energy lost by the photon equals the energy gained by the electron.

EXECUTE:

(a) $\lambda' - \lambda = \lambda_C(1-\cos\phi) = (2.426\times10^{-12}\text{ m})(1-\cos 35.0°) = 4.39\times10^{-13}\text{ m} = 4.39\times10^{-4}$ nm.

(b) $\lambda' = \lambda + 4.39\times10^{-4}\text{ nm} = 0.04250\text{ nm} + 4.39\times10^{-4}\text{ nm} = 0.04294$ nm.

(c) $\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi) = 0.1050\times10^{-9}\text{ m} + (2.426\times10^{-12}\text{ m})(1-\cos 60.0°) = 0.1062\times10^{-9}$ m and

$E_{\lambda'} = \frac{hc}{\lambda'} = 2.888\times10^4$ eV, so the photon loses 300 eV of energy.

(d) Energy conservation says the electron gains 300 eV of energy.

EVALUATE: The photon transfers energy to the electron. Since the photon loses energy, its wavelength increases.

38.18. **IDENTIFY:** The change in wavelength of the scattered photon is given by the equation

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{mc\lambda}(1-\cos\phi) \Rightarrow \lambda = \frac{h}{mc\left(\frac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

SET UP: For backward scattering, $\phi = 180°$. Since the photon scatters from a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: $\lambda = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(1.67\times10^{-27}\text{ kg})(3.00\times10^8\text{ m/s})(0.100)}(1+1) = 2.65\times10^{-14}$ m.

EVALUATE: The maximum change in wavelength, $2h/mc$, is much smaller for scattering from a proton than from an electron.

38.19. **IDENTIFY:** We are dealing with Compton scattering.

SET UP: $\lambda = h/p$, $E = hc/\lambda$, $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$.

EXECUTE: **(a)** The initial photon lost energy during Compton scattering. Before this it had just enough energy to create electron-positron pairs, so after the collision it has less energy. Therefore it cannot create these pairs, so the answer is no.

(b) We want the momentum of the scattered photon. First use find λ' using

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

and then use $\lambda = h/p$ to find the momentum. Before scattering $E = 2mc^2 = hc/\lambda$, so $\lambda = h/2mc$.

$$\lambda' = \frac{h}{mc}(1-\cos\phi) + \frac{h}{2mc} = \frac{h}{mc}\left(\frac{3}{2} - \cos\phi\right).$$

Using this result gives with $\phi = 20.0°$ gives

$$p = \frac{h}{\lambda'} = \frac{h}{\dfrac{h}{mc}\left(\dfrac{3}{2} - \cos\phi\right)} = \frac{mc}{\dfrac{3}{2} - \cos\phi} = 4.88\times10^{-22}\ \text{kg}\cdot\text{m/s}.$$

EVALUATE: The momentum of the scattered photon depends on the scattering angle ϕ. The smallest that the momentum can be is when $\cos\phi = -1$ ($\phi = 180°$, which is backscatter), for which $p_{\text{min}} = 2mc/5$.

38.20. **(a) IDENTIFY** and **SET UP:** Use the relativistic equation $K = (\gamma - 1)mc^2$ to calculate the kinetic energy K.

EXECUTE: $K = mc^2\left(\dfrac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = 0.1547\ mc^2.$ $m = 9.109\times10^{-31}$ kg, so $K = 1.27\times10^{-14}$ J.

(b) IDENTIFY and **SET UP:** The total energy of the particles equals the sum of the energies of the two photons. Linear momentum must also be conserved.

EXECUTE: The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 = 9.46\times10^{-13}$ J. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy 9.46×10^{-14} J.

(c) IDENTIFY and **SET UP:** Use $E = hc/\lambda$ to relate the photon energy to the photon wavelength.

EXECUTE: $E = hc/\lambda$, so $\lambda = hc/E = hc/(9.46\times10^{-14}\ \text{J}) = 2.10$ pm.

EVALUATE: When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

38.21. **IDENTIFY:** The wavelength of the pulse tells us the momentum of the photon. The uncertainty in the momentum is determined by the uncertainty principle.

SET UP: $p = \dfrac{h}{\lambda}$ and $\Delta x \Delta p_x = \dfrac{\hbar}{2}$.

EXECUTE: $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{556\times10^{-9}\ \text{m}} = 1.19\times10^{-27}\ \text{kg}\cdot\text{m/s}.$ The spatial length of the pulse is $\Delta x = c\Delta t = (2.998\times10^8\ \text{m/s})(9.00\times10^{-15}\ \text{s}) = 2.698\times10^{-6}$ m. The uncertainty principle gives $\Delta x\Delta p_x = \dfrac{\hbar}{2}$. Solving for the uncertainty in the momentum, we have $\Delta p_x = \dfrac{\hbar}{2\Delta x} = \dfrac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2(2.698\times10^{-6}\ \text{m})}$ $= 1.96\times10^{-29}\ \text{kg}\cdot\text{m/s}.$

EVALUATE: This is 1.6% of the average momentum.

38.22. **IDENTIFY:** We know the beam went through the slit, so the uncertainty in its vertical position is the width of the slit.

SET UP: $\Delta y \Delta p_y = \dfrac{\hbar}{2}$ and $p_x = \dfrac{h}{\lambda}$. Call the x-axis horizontal and the y-axis vertical.

EXECUTE: **(a)** Let $\Delta y = a = 6.20\times 10^{-5}\text{ m}$. Solving $\Delta y \Delta p_y = \dfrac{\hbar}{2}$ for the uncertainty in momentum gives

$$\Delta p_y = \frac{\hbar}{2\Delta y} = \frac{1.055\times 10^{-34}\text{ J}\cdot\text{s}}{2(6.20\times 10^{-5}\text{ m})} = 8.51\times 10^{-31}\text{ kg}\cdot\text{m/s}.$$

(b) $p_x = \dfrac{h}{\lambda} = \dfrac{6.626\times 10^{-34}\text{ J}\cdot\text{s}}{585\times 10^{-9}\text{ m}} = 1.13\times 10^{-27}\text{ kg}\cdot\text{m/s}.$ $\theta = \dfrac{\Delta p_y}{p_x} = \dfrac{8.51\times 10^{-31}}{1.13\times 10^{-27}} = 7.53\times 10^{-4}\text{ rad}.$

The width is $(2.00\text{ m})(7.53\times 10^{-4}) = 1.51\times 10^{-3}\text{ m} = 1.51\text{ mm}.$

EVALUATE: We must be especially careful not to confuse the x- and y-components of the momentum.

38.23. **IDENTIFY:** The uncertainty principle relates the uncertainty in the duration time of the pulse and the uncertainty in its energy, which we know.

SET UP: $E = hc/\lambda$ and $\Delta E \Delta t = \hbar/2$.

EXECUTE: $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(2.998\times 10^{8}\text{ m/s})}{625\times 10^{-9}\text{ m}} = 3.178\times 10^{-19}\text{ J}.$ The uncertainty in the energy is 1.0% of this amount, so $\Delta E = 3.178\times 10^{-21}\text{ J}.$ We now use the uncertainty principle. Solving $\Delta E \Delta t = \dfrac{\hbar}{2}$ for the time interval gives $\Delta t = \dfrac{\hbar}{2\Delta E} = \dfrac{1.055\times 10^{-34}\text{ J}\cdot\text{s}}{2(3.178\times 10^{-21}\text{ J})} = 1.66\times 10^{-14}\text{ s} = 16.6\text{ fs}.$

EVALUATE: The uncertainty in the energy limits the duration of the pulse. The more precisely we know the energy, the longer the duration must be.

38.24. **IDENTIFY:** The number N of visible photons emitted per second is the visible power divided by the energy hf of one photon.

SET UP: At a distance r from the source, the photons are evenly spread over a sphere of area $A = 4\pi r^2$.

EXECUTE: **(a)** $N = \dfrac{P}{hf} = \dfrac{(120\text{ W})(0.10)}{h(5.00\times 10^{14}\text{ Hz})} = 3.62\times 10^{19}\text{ photons/s}.$

(b) $\dfrac{N}{4\pi r^2} = 1.00\times 10^{11}\text{ photons/s}\cdot\text{cm}^2$ gives

$$r = \left(\frac{3.62\times 10^{19}\text{ photons/s}}{4\pi(1.00\times 10^{11}\text{ photons/s}\cdot\text{cm}^2)}\right)^{1/2} = 5370\text{ cm} = 53.7\text{ m}.$$

EVALUATE: The number of photons emitted per second by an ordinary household source is very large.

38.25. **IDENTIFY** and **SET UP:** The energy added to mass m of the blood to heat it to $T_{\text{f}} = 100°\text{C}$ and to vaporize it is $Q = mc(T_{\text{f}} - T_{\text{i}}) + mL_{\text{v}}$, with $c = 4190\text{ J/kg}\cdot\text{K}$ and $L_{\text{v}} = 2.256\times 10^{6}\text{ J/kg}$. The energy of one photon is $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times 10^{-25}\text{ J}\cdot\text{m}}{\lambda}$.

EXECUTE: **(a)** $Q = (2.0\times 10^{-9}\text{ kg})(4190\text{ J/kg}\cdot\text{K})(100°\text{C} - 33°\text{C}) + (2.0\times 10^{-9}\text{ kg})(2.256\times 10^{6}\text{ J/kg}) =$ $5.07\times 10^{-3}\text{ J}.$ The pulse must deliver 5.07 mJ of energy.

(b) $P = \dfrac{\text{energy}}{t} = \dfrac{5.07\times 10^{-3}\text{ J}}{450\times 10^{-6}\text{ s}} = 11.3\text{ W}.$

(c) One photon has energy $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{585\times10^{-9}\ \text{m}} = 3.40\times10^{-19}$ J. The number N of photons per pulse is the energy per pulse divided by the energy of one photon:

$$N = \frac{5.07\times10^{-3}\ \text{J}}{3.40\times10^{-19}\ \text{J/photon}} = 1.49\times10^{16}\ \text{photons.}$$

EVALUATE: The power output of the laser is small but it is focused on a small area, so the laser intensity is large.

38.26. **IDENTIFY:** The photoelectric effect occurs, so the energy of the photon is used to eject an electron, with any excess energy going into kinetic energy of the electron.
SET UP: Conservation of energy gives $hf = hc/\lambda = K_{\text{max}} + \phi.$

EXECUTE: **(a)** Using $hc/\lambda = K_{\text{max}} + \phi,$ we solve for the work function:

$$\phi = hc/\lambda - K_{\text{max}} = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(124\ \text{nm}) - 4.16\ \text{eV} = 5.85\ \text{eV.}$$

(b) The number N of photoelectrons per second is equal to the number of photons that strike the metal per second. $N\times(\text{energy of a photon}) = 2.50$ W. $N(hc/\lambda) = 2.50$ W.

$$N = (2.50\ \text{W})(124\ \text{nm})/[(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})] = 1.56\times10^{18}\ \text{electrons/s.}$$

(c) N is proportional to the power, so if the power is cut in half, so is N, which gives

$$N = (1.56\times10^{18}\ \text{el/s})/2 = 7.80\times10^{17}\ \text{el/s.}$$

(d) If we cut the wavelength by half, the energy of each photon is doubled since $E = hc/\lambda.$ To maintain the same power, the number of photons must be half of what they were in part (b), so N is cut in half to 7.80×10^{17} el/s. We could also see this from part (b), where N is proportional to $\lambda.$ So if the wavelength is cut in half, so is N.
EVALUATE: In part (c), reducing the power does not reduce the maximum kinetic energy of the photons; it only reduces the number of ejected electrons. In part (d), reducing the wavelength *does* change the maximum kinetic energy of the photoelectrons because we have increased the energy of each photon.

38.27. **IDENTIFY:** We are dealing with the characteristics of a photon.
SET UP and **EXECUTE:** Using $E = hc/\lambda,$ we want to estimate the number of photons in a typical room.
(a) Estimate: On a bright day the light intensity in a room is about 1/3 of what it is outside, so

$$I = (1000\ \text{W/m}^2)/3 = 330\ \text{W/m}^2.$$

(b) Estimate: Floor area is $A \approx 15$ ft by 30 ft $\approx$ 5 m by 10 m $\approx 50\ \text{m}^2$. Height is $H \approx 9$ ft $\approx$ 3 m.
(c) $t = H/c = (3\ \text{m})/c = 10^{-8}\ \text{s} = 10$ ns.
(d) $P = IA = (330\ \text{W/m}^2)(50\ \text{m}^2) = 16{,}500$ W.
(e) $E_{\text{room}} = Pt = (16{,}500\ \text{W})(10^{-8}\ \text{s}) = 165\ \mu\text{J}$.
(f) $E = hc/\lambda = hc/(500\ \text{nm}) = 3.98\times10^{-19}$ J.
(g) Using the answers from parts (e) and (f) gives $N = E_{\text{room}}/E = 4\times10^{14}$.
EVALUATE: These numbers give only rough approximations, not precise calculations.

38.28. **IDENTIFY:** Compton scattering occurs. For backscattering, the scattering angle of the photon is 180°. Momentum is conserved during the collision.
SET UP: Let $+x$ be in the direction of propagation of the incident photon. The momentum of a photon is $p = h/\lambda.$ The change in wavelength of the light during Compton scattering is given by

$\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi),$ where $\phi = 180°$ in this case.

EXECUTE: $\lambda' = \lambda + 2\dfrac{h}{mc} = 0.0980\times10^{-9}\ \text{m} + 4.852\times10^{-12}\ \text{m} = 0.1029\times10^{-9}\ \text{m}$. Momentum conservation gives $\dfrac{h}{\lambda} = -\dfrac{h}{\lambda'} + p_\text{e}$. Solving for p_e gives $p_\text{e} = \dfrac{h}{\lambda} + \dfrac{h}{\lambda'} = h\left(\dfrac{\lambda+\lambda'}{\lambda\lambda'}\right) = (6.626\times10^{-34}\ \text{J}\cdot\text{s})$

$$\frac{9.80\times10^{-11}\ \text{m} + 10.29\times10^{-11}\ \text{m}}{(9.80\times10^{-11}\ \text{m})(10.29\times10^{-11}\ \text{m})} = 1.32\times10^{-23}\ \text{kg}\cdot\text{m/s}.$$

EVALUATE: The electron gains the most amount of momentum when backscattering occurs.

38.29. **IDENTIFY:** Compton scattering occurs, and we know the angle of scattering and the initial wavelength (and hence momentum) of the incident photon.

SET UP: $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ and $p = h/\lambda$. Let $+x$ be the direction of propagation of the incident photon and let the scattered photon be moving at $30.0°$ clockwise from the $+y$-axis.

EXECUTE:

$$\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi) = 0.1050\times10^{-9}\ \text{m} + (2.426\times10^{-12}\ \text{m})(1-\cos 60.0°) = 0.1062\times10^{-9}\ \text{m}.$$

$P_{ix} = P_{fx}$. $\dfrac{h}{\lambda} = \dfrac{h}{\lambda'}\cos 60.0° + p_{ex}$.

$$p_{ex} = \frac{h}{\lambda} - \frac{h}{2\lambda'} = h\frac{2\lambda' - \lambda}{(2\lambda')(\lambda)} = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\frac{2.1243\times10^{-10}\ \text{m} - 1.050\times10^{-10}\ \text{m}}{(2.1243\times10^{-10}\ \text{m})(1.050\times10^{-10}\ \text{m})}.$$

$p_{ex} = 3.191\times10^{-24}\ \text{kg}\cdot\text{m/s}$. $P_{iy} = P_{fy}$. $0 = \dfrac{h}{\lambda'}\sin 60.0° + p_{ey}$.

$$p_{ey} = -\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\sin 60.0°}{0.1062\times10^{-9}\ \text{m}} = -5.403\times10^{-24}\ \text{kg}\cdot\text{m/s}. \quad p_\text{e} = \sqrt{p_{ex}^2 + p_{ey}^2} = 6.28\times10^{-24}\ \text{kg}\cdot\text{m/s}.$$

$\tan\theta = \dfrac{p_{ey}}{p_{ex}} = \dfrac{-5.403}{3.191}$ and $\theta = -59.4°$.

EVALUATE: The incident photon does not give all of its momentum to the electron, since the scattered photon also has momentum.

38.30. **IDENTIFY:** This problem deals with photons and the nutritional energy in food.

SET UP and **EXECUTE:** **(a)** Estimate: One medium tomato (123 g) has 22 kcal.

(22 kcal)(4186 J/kcal) = 93,000 J.

(b) Estimate: 50 leaves of area 20 cm^2 each, so $A \approx 1000\ \text{cm}^2 = 0.10\ \text{m}^2$.

(c) $P = IA = (800\ \text{W/m}^2)(0.10\ \text{m}^2) = 80\ \text{W}$.

(d) $E = hc/\lambda = hc/(600\ \text{nm}) = 3.31\times10^{-19}$ J per photon. The total energy each second for photosynthesis is 5% of 80 J = 4.0 J. The number N of photons needed to deliver this energy is given by $N(3.31\times10^{-19}\ \text{J}) = 4.0$ J, so $N = 1.2\times10^{19}$ photons.

(e) The number N' of photons for a single tomato is $N' = (1/10)(N/2) = 6.0\times10^{17}$ photons.

(f) Each tomato needs 93,000 J from part (a). If N is the number of photons needed, then $N(3.31\times10^{-19}\ \text{J}) = 93{,}000$ J, which gives $N = 2.82\times10^{23}$ photons.

(g) From part (e): 6.0×10^{27} photons/s supply energy to the tomato.

From part (f): The tomato needs 2.82×10^{23} photons.

Calling t the time to get these photons gives $(6.0\times10^{17}\ \text{photons/s})t = 2.82\times10^{23}$ photons, which gives $t = 4.7\times10^5$ s = 131 h. At 12 h/day, we get t = 11 days.

EVALUATE: Increasing the leaf area exposed to sunlight would help capture more energy and lead to faster ripening.

38.31. **IDENTIFY** and **SET UP:** Find the average change in wavelength for one scattering and use that in $\Delta\lambda$ in $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ to calculate the average scattering angle ϕ.

EXECUTE: **(a)** The wavelength of a 1 MeV photon is

$\lambda = \dfrac{hc}{E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{1\times10^{6}\ \text{eV}} = 1\times10^{-12}\ \text{m}.$

The total change in wavelength therefore is $500\times10^{-9}\ \text{m} - 1\times10^{-12}\ \text{m} = 500\times10^{-9}\ \text{m}.$

If this shift is produced in 10^{26} Compton scattering events, the wavelength shift in each scattering event is $\Delta\lambda = \dfrac{500\times10^{-9}\ \text{m}}{1\times10^{26}} = 5\times10^{-33}\ \text{m}.$

(b) Use this $\Delta\lambda$ in $\Delta\lambda = \dfrac{h}{mc}(1-\cos\phi)$ and solve for ϕ. We anticipate that ϕ will be very small, since $\Delta\lambda$ is much less than h/mc, so we can use $\cos\phi \approx 1-\phi^2/2.$

$$\Delta\lambda = \frac{h}{mc}\left[1-(1-\phi^2/2)\right] = \frac{h}{2mc}\phi^2.$$

$$\phi = \sqrt{\frac{2\Delta\lambda}{(h/mc)}} = \sqrt{\frac{2(5\times10^{-33}\ \text{m})}{2.426\times10^{-12}\ \text{m}}} = 6.4\times10^{-11}\ \text{rad} = (4\times10^{-9})°.$$

ϕ in radians is much less than 1, so the approximation we used is valid.

(c) IDENTIFY and **SET UP:** We know the total transit time and the total number of scatterings, so we can calculate the average time between scatterings.

EXECUTE: The total time to travel from the core to the surface is $(10^6\ \text{y})(3.156\times10^7\ \text{s/y}) = 3.2\times10^{13}\ \text{s}.$

There are 10^{26} scatterings during this time, so the average time between scatterings is

$t = \dfrac{3.2\times10^{13}\ \text{s}}{10^{26}} = 3.2\times10^{-13}\ \text{s}.$

The distance light travels in this time is $d = ct = (3.0\times10^8\ \text{m/s})(3.2\times10^{-13}\ \text{s}) = 0.1\ \text{mm}.$

EVALUATE: The photons are on the average scattered through a very small angle in each scattering event. The average distance a photon travels between scatterings is very small.

38.32. **IDENTIFY:** In this problem, a positron with speed v and kinetic energy K collides with a stationary electron. They annihilate and produce to two photons each of wavelength λ.

SET UP: $E = hc/\lambda,\ K = mc^2(\gamma-1),\ p = m\gamma v,\ v_x' = \dfrac{v_x - u}{1-uv_x/c^2}.$

EXECUTE: **(a)** Energy conservation: $K + mc^2 + mc^2 = 2hc/\lambda.$ Using $K = mc^2(\gamma-1)$ gives

$$(\gamma-1)mc^2 + 2mc^2 = 2hc/\lambda.$$

Rearranging gives

$$(\gamma+1)mc^2 = 2hc/\lambda.$$

(b) Momentum conservation: Using $p = m\gamma v$ for the positron gives

$$m\gamma v = 2\frac{h}{\lambda}\cos\phi.$$

(c) We want the energy of the photon and the angle ϕ.

From part (a): $(\gamma+1)mc^2 = 2hc/\lambda = 2E_{\text{photon}}.$

This gives $E_{\text{photon}} = \dfrac{mc^2(\gamma+1)}{2}.$

From part (b): $m\gamma v = 2\dfrac{h}{\lambda}\cos\phi$. This gives $\cos\phi = \dfrac{m\gamma v\lambda}{2h}$.

Combine the results from (a) and (b) by eliminating λ and solve for $\cos\phi$.

$$\cos\phi = \frac{\gamma v}{c(1+\gamma)}.$$

Using $v = c\sqrt{1-1/\gamma^2}$ and doing some algebra gives

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}}.$$

(d) We want the energy of the photon and the angle z if K = 5.11 MeV. Using $K = mc^2(\gamma-1)$ with mc^2 = 0.511 MeV gives γ = 11. From part (c)

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}} = 3.07\text{ MeV}.$$

Also from part (c) we get

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}} = \arccos\sqrt{\frac{10}{12}} = 24.1°.$$

(e) We want v if the photons are perpendicular to each other. In this case, ϕ = 45.0°. Solving

$$\phi = \arccos\sqrt{\frac{\gamma-1}{\gamma+1}}\text{ gives }\gamma = 2.\text{ Using }v = c\sqrt{1-1/\gamma^2}\text{ gives }v = 0.866c.$$

(f) We want to transform to the center-of-momentum frame where the total momentum is zero. Call u the speed of this reference frame. In this frame, the electron and positron have equal but opposite velocities. Use the relativistic velocity addition equation $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$, with R the lab frame and R' the center-of-momentum frame. We want to find u.

For the positron: $v_{\text{px}}' = \dfrac{v-u}{1-uv/c^2}$

For the electron: $v_{ex}' = \dfrac{0-u}{1-0} = -u$

In the center-of-momentum frame, $v_{\text{px}}' = -v_{\text{ex}}'$, so $v_{\text{px}}' = \dfrac{v-u}{1-uv/c^2} = -(-u) = u$. This leads to the equation $u^2(v/c^2) - 2u + v = 0$. Solve this equation for u, use the positive root, and use $v = c\sqrt{1-1/\gamma^2}$. The result is

$$u = v\frac{\gamma}{\gamma+1} = c\sqrt{\frac{\gamma-1}{\gamma+1}}.$$

EVALUATE: Check: If $v \ll c$, c= 1, so $u = v\dfrac{\gamma}{\gamma+1} = v\dfrac{1}{1+1} = \dfrac{v}{2}$. This result is reasonable because in the center-of-momentum frame without using special relativity the electron and positron are coming toward each other each with speed $v/2$.

38.33. IDENTIFY and **SET UP:** Conservation of energy applied to the collision gives $E_\lambda = E_{\lambda'} + E_{\text{e}}$, where E_{e} is the kinetic energy of the electron after the collision and E_λ and $E_{\lambda'}$ are the energies of the photon before and after the collision. The energy of a photon is related to its wavelength according to $E = hf = hc/\lambda$.

EXECUTE: **(a)** $E_e = hc\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right) = hc\left(\frac{\lambda'-\lambda}{\lambda\lambda'}\right).$

$E_e = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})\left(\frac{0.0032\times10^{-9}\ \text{m}}{(0.1100\times10^{-9}\ \text{m})(0.1132\times10^{-9}\ \text{m})}\right).$

$E_e = 5.105\times10^{-17}\ \text{J} = 319\ \text{eV}.$

$E_e = \frac{1}{2}mv^2$, so $v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{2(5.105\times10^{-17}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.06\times10^{7}\ \text{m/s}.$

(b) The wavelength λ of a photon with energy E_e is given by $E_e = hc/\lambda$, so

$$\lambda = \frac{hc}{E_e} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{5.105\times10^{-17}\ \text{J}} = 3.89\ \text{nm}.$$

EVALUATE: Only a small portion of the incident photon's energy is transferred to the struck electron; this is why the wavelength calculated in part (b) is much larger than the wavelength of the incident photon in the Compton scattering.

38.34. **IDENTIFY:** The equation $\lambda'-\lambda = \left(\frac{h}{mc}\right)(1-\cos\phi)$ relates λ and λ' to ϕ. Apply conservation of energy to obtain an expression that relates λ and v to λ'.

SET UP: The kinetic energy of the electron is $K = (\gamma-1)mc^2$. The energy of a photon is $E = \frac{hc}{\lambda}.$

EXECUTE: **(a)** The final energy of the photon is $E' = \frac{hc}{\lambda'}$, and $E = E' + K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda = \frac{hc}{E'+K} = \frac{hc}{(hc/\lambda')+K} = \frac{hc}{(hc/\lambda')+(\gamma-1)mc^2} = \frac{\lambda'}{1+\frac{\lambda' mc}{h}\left[\frac{1}{(1-v^2/c^2)^{1/2}}-1\right]}.$$

($K = mc^2(\gamma-1)$ since the relativistic expression must be used for three-figure accuracy).

(b) $\phi = \arccos[1-\Delta\lambda/(h/mc)].$

(c) $\gamma - 1 = \frac{1}{\left(1-\left(\frac{1.80}{3.00}\right)^2\right)^{1/2}} - 1 = 1.25 - 1 = 0.250,\ \frac{h}{mc} = 2.43\times10^{-12}\ \text{m}$

$$\Rightarrow \lambda = \frac{5.10\times10^{-3}\ \text{nm}}{1+\frac{(5.10\times10^{-12}\ \text{m})(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})(0.250)}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}} = 3.34\times10^{-3}\ \text{nm}.$$

$$\phi = \arccos\left(1-\frac{(5.10\times10^{-12}\ \text{m}-3.34\times10^{-12}\ \text{m})}{2.43\times10^{-12}\ \text{m}}\right) = 74.0^\circ.$$

EVALUATE: For this final electron speed, $v/c = 0.600$ and $K = \frac{1}{2}mv^2$ is not accurate.

38.35. **IDENTIFY** and **SET UP:** Apply the photoelectric effect. $eV_0 = hf - \phi$. For a photon, $f\lambda = c$.

EXECUTE: **(a)** Using $eV_0 = hf - \phi$ and $f\lambda = c$, we get $eV_0 = hc/\lambda - \phi$. Solving for V_0 gives $V_0 = \frac{hc}{e}\cdot\frac{1}{\lambda} - \frac{\phi}{e}$. Therefore a graph of V_0 versus $1/\lambda$ should be a straight line with slope equal to hc/e and y-intercept equal to $-\phi/e$. Figure 38.35 shows this graph for the data given in the problem. The best-fit equation for this graph is $V_0 = (1230\ \text{V}\cdot\text{nm})\cdot\frac{1}{\lambda} - 4.76\ \text{V}$. The slope is equal to $1230\ \text{V}\cdot\text{nm}$, which is equal to $1.23\times10^{-6}\ \text{V}\cdot\text{m}$, and the y-intercept is –4.76 V.

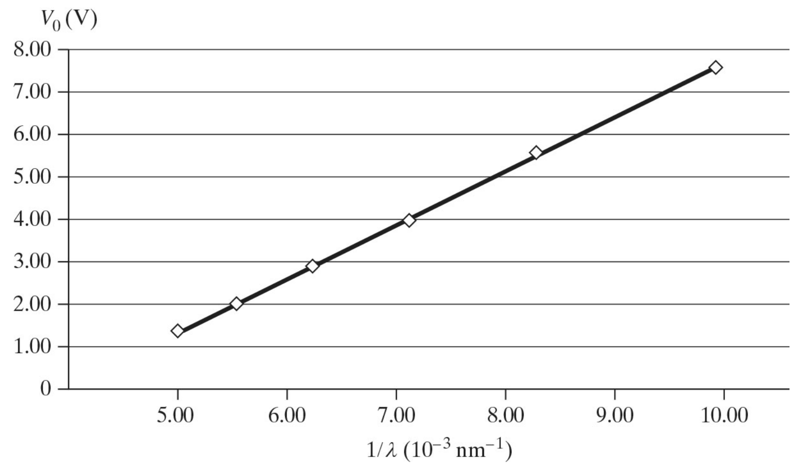

Figure 38.35

(b) Using the slope we have hc/e = slope, so
$h = e(\text{slope})/c = (1.602\times10^{-19}\ \text{C})(1.23\times10^{-6}\ \text{V}\cdot\text{m})/(2.998\times10^{8}\ \text{m/s}) = 6.58\times10^{-34}\ \text{J}\cdot\text{s}.$
The y-intercept is equal to $-\phi/e$, so

$$\phi = -e(y\text{-intercept}) = -(1.602\times10^{-19}\ \text{C})(-4.76\ \text{V}) = 7.63\times10^{-19}\ \text{J} = 4.76\ \text{eV}.$$

(c) For the longest wavelength light, the energy of a photon is equal to the work function of the metal, so $hc/\lambda = \phi$. Solving for λ gives $\lambda = hc/\phi$. Our calculation of h was just a test of the data, so we use the accepted value for h in the calculation.
$\lambda = hc/\phi = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})/(7.63\times10^{-19}\ \text{J}) = 2.60\times10^{-7}\ \text{m} = 260\ \text{nm}.$

(d) The energy of the photon is equal to the sum of the kinetic energy of the photoelectron and the work function, so $hc/\lambda = K + \phi$. This gives $(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})/\lambda = 10.0\ \text{eV} + 4.76\ \text{eV} = 14.76\ \text{eV}$, which gives $\lambda = 8.40\times10^{-8}\ \text{m} = 84.0\ \text{nm}$.

EVALUATE: As we know from Table 38.1, typical metal work functions are several eV, so our results are plausible.

38.36. **IDENTIFY** and **SET UP:** For the photoelectric effect, $eV_0 = hf - \phi$, and the energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: **(a)** The energy of the UV photons is

$$E = hc/\lambda = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})/(270\times10^{-9}\ \text{m}) = 4.59\ \text{eV}.$$

The photon energy must be at least as great as the work function to produce photoelectrons. From Table 38.1, we see that this is the case for aluminum, silver, and sodium.
(b) The maximum kinetic energy of a photoelectron is $K = hf - \phi$, so the smallest work function gives the largest kinetic energy of the electron. This is the case for sodium.
$K = E_{\text{photon}} - \phi = 4.59\ \text{eV} - 2.7\ \text{eV} = 1.89\ \text{eV}$. This is much less than the rest energy (0.511 MeV) of an electron, so we do not need to use the relativistic formula for kinetic energy. Solving $K = \frac{1}{2}mv^2$ for v gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.89\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = 8.2\times10^{5}\ \text{m/s}.$$

(c) The energy of the photon is equal to the work function of the gold, so $hc/\lambda = \phi$. This gives $(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})/\lambda = 5.1\ \text{eV} \rightarrow \lambda = 2.4\times10^{-7}\ \text{m} = 240\ \text{nm}.$

(d) In part (c), the energy of the photon is equal to the work function for gold, so

$$K_{\text{max}} = E_{\text{photon}} - \phi_{\text{sodium}} = \phi_{\text{gold}} - \phi_{\text{sodium}} = 5.1 \text{ eV} - 2.7 \text{ eV} = 2.4 \text{ eV}.$$

EVALUATE: Of the three possible metals in Table 38.1, aluminum would be the most practical to use for the smoke detector. Silver is probably too expensive, and sodium is too reactive with water.

38.37. IDENTIFY and **SET UP:** We have Compton scattering, so $\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi)$, which can also be expressed as $\lambda' - \lambda = \lambda_C(1-\cos\phi)$, where λ_C is the Compton wavelength.

EXECUTE: **(a)** Figure 38.37 shows the graph of λ' versus $1 - \cos\phi$ for the data included in the problem. The best-fit equation of the line is λ' = 5.21 pm + (2.40 pm)(1 – cos ϕ). The slope is 2.40 pm and the y-intercept is 5.21 pm.

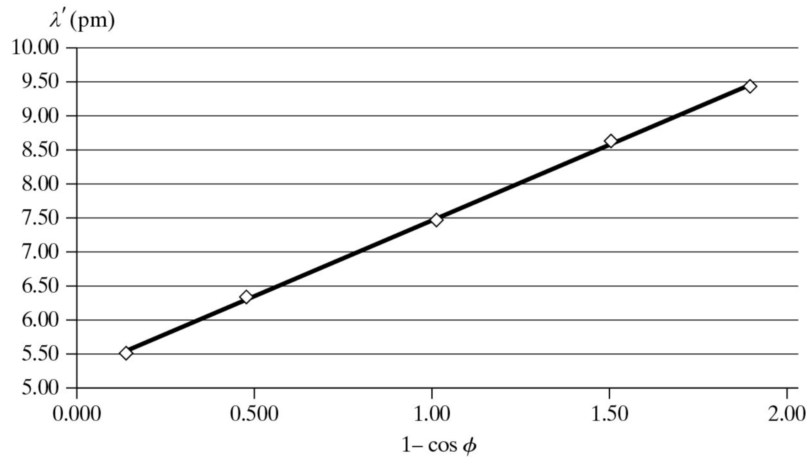

Figure 38.37

(b) Solving $\lambda' - \lambda = \lambda_C(1-\cos\phi)$ for λ' gives $\lambda' = \lambda + \lambda_C(1-\cos\phi)$. The graph of λ' versus $1 - \cos\phi$ should be a straight line with slope equal to λ_C and y-intercept equal to λ. From the slope, we get λ_C = slope = 2.40 pm.

(c) From the y-intercept we get $\lambda = y\text{-intercept} = 5.21$ pm.

EVALUATE: For backscatter, the photon wavelength would be 5.21 pm + 2(2.40 pm) = 10.01 pm.

38.38. IDENTIFY: In this problem we are dealing with the photoelectric effect and must also use Kirchhoff's rules.

SET UP: $eV_0 = hf - \phi = hc/\lambda - \phi.$

EXECUTE: **(a)** We want V_{AC} if R = 3.20 kΩ.

$$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{15.0 \text{ V}}{4.20 \text{ k}\Omega} = 3.5714 \text{ mA}.$$

$$V_R = RI = (3.20 \text{ k}\Omega)(3.5714 \text{ mA}) = 11.4 \text{ V}.$$

The cathode is at a higher potential than the anode, so $V_{AC} = V_A - V_C = -11.4$ V.

(b) We want V_{AC} if R = 334 Ω. Follow the same procedure as in part (a), which gives

$$V_R = (334\ \Omega)\frac{15.0 \text{ V}}{1.334 \text{ K}\Omega} = 3.76 \text{ V}.$$

Therefore $V_{AC} = -3.76$ V.

(c) We want the work function. From part (b), the stopping potential is 3.76 V. Solving $eV_0 = hc/\lambda - \phi$ for ϕ gives

$$\phi = \frac{hc}{\lambda} - eV_0 = \frac{hc}{140\text{ nm}} - e(3.76\text{ V}) = 5.10\text{ eV}.$$

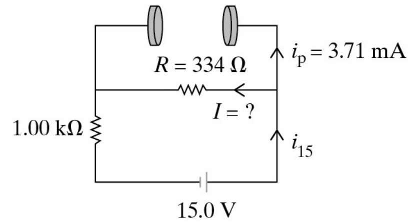

Figure 38.38

(d) We want I. Apply Kirchhoff's rules using the currents shown in Figure 38.38. $i_{15} = I + i$. A counterclockwise path through the lower part of the circuit gives $RI - (1.00\text{ k}\Omega)i_{15} + 15.0\text{ V} = 0$. Solve for I using $i = 3.71$ mA and $R = 334\ \Omega$, which gives $I = 8.46$ mA.
(e) We want V_{AC}. $V_{AC} = V_R = RI = (334\ \Omega)(8.46\text{ mA}) = 2.893$ V.
(f) We want to find R, so that $i = 0$ with $\lambda = 65.0$ nm. If $i = 0$, then V_R is the stopping potential V_0.

$$RI = \frac{hc/\lambda - \phi}{e} \text{ and } I = \frac{\mathcal{E}}{R + 1.00\text{ k}\Omega}$$

$$\frac{hc/\lambda - \phi}{e} = \frac{R\mathcal{E}}{R + 1.00\text{ k}\Omega}$$

Using $\mathcal{E} = 15.0$ V, $\phi = 5.10$ eV, and $\lambda = 65.0$ nm, we get $R = 13.8$ kΩ.

EVALUATE: A work function of 5.1 eV is the same as gold and nickel, so it is reasonable.

38.39. IDENTIFY: This problem involves pair production of an electron-positron pair.

SET UP and **EXECUTE:** $\lambda = h/p,\ \ E = hc/\lambda,\ \ p = m\gamma v,\ \ E = m\gamma c^2.$

(a) Energy conservation: $hc/\lambda + Mc^2 = 2myc^2 + M\gamma_M c^2.$

(b) Momentum conservation: $h/\lambda = 2m\gamma v\cos\phi + M\gamma_M V_M$

(c) Eliminate h/λ between the equations in parts (a) and (b) and rearrange to obtain the following:

$$\frac{2m\gamma}{M}\left(1 - \frac{v}{c}\cos\phi\right) + \gamma_M\left(1 - \frac{V_M}{c}\right) = 1.$$

(d) We want V_M if $V_M \ll c$. If $x \ll 1$, $(1 + x)^n \approx 1 + nx$. Using $n = -½$ and $x = -(V_M/c)^2$ gives

$$\gamma_M \approx 1 + \left(-\frac{1}{2}\right)\left[-\left(\frac{V_M}{c}\right)^2\right] \approx 1 + \frac{V_M^2}{2c^2} \approx 1.$$

Using this to rewrite the result from part (c) gives

$$\frac{2m\gamma}{M}\left(1 - \frac{v}{c}\cos\phi\right) + 1\frac{V_M}{c} = 1.$$

Solving for V_M gives

$$V_M = \frac{2m\gamma}{M}(c - v\cos\phi).$$

(e) For V_M to be zero, $c - v\cos\phi$ would have to be zero, which is not possible with $v < c$.

(f) We want V_M if M is a proton and $v = 0$. Using the result in part (d) gives

$$V_M = \frac{2m\gamma c}{M} = 327\text{ Km/s}.$$

(g) We want V_M. Use the result from (d). First use $E = m\gamma c^2$ to find γ.

$$5.00\text{ MeV} = (0.511\text{ MeV})\gamma \to \gamma = 9.785.$$

Now use $v = c\sqrt{1-1/\gamma^2}$ to find v, giving v = 2.984 × 10^8 m/s. Now find V_M for ϕ = 60.0° and using the values we just found for v and γ, which gives

$$V_M = \frac{2m\gamma}{M}(c - v\cos\phi) = 1610 \text{ Km/s}.$$

(h) We want the energy E of the incident photon. This energy is equal to the sum of the energy of the electron and positron and the kinetic energy of the proton. The proton is not relativistic, so we have

$$E = 2E_{\text{electron}} + \frac{1}{2}Mv^2.$$

Using E_{electron} = 5.00 MeV and v = 1620 km/s gives E = 2(5.0 MeV) + 0.014 MeV = 10.0 MeV.

EVALUATE: Most of the energy goes to the electron-positron pair.

38.40. IDENTIFY: Follow the derivation of $\lambda' - \lambda = \left(\frac{h}{mc}\right)(1-\cos\phi)$. Apply conservation of energy and conservation of momentum to the collision.

SET UP: Use the coordinate direction specified in the problem.

EXECUTE: (a) Momentum: $\vec{\boldsymbol{p}} + \vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}' + \vec{\boldsymbol{P}}' \Rightarrow p - P = -p' - P' \Rightarrow p' = P - (p + P')$.

Energy: $pc + E = p'c + E' = p'c + \sqrt{(P'c)^2 + (mc^2)^2}$

$$\Rightarrow (pc - p'c + E)^2 = (P'c)^2 + (mc^2)^2 = (Pc)^2 + ((p+p')c)^2 - 2P(p+p')c^2 + (mc^2)^2.$$

$$(pc - p'c)^2 + E^2 = E^2 + (pc + p'c)^2 - 2(Pc^2)(p+p') + 2Ec(p-p') - 4pp'c^2 + 2Ec(p-p')$$

$$+2(Pc^2)(p+p') = 0$$

$$\Rightarrow p'(Pc^2 - 2pc^2 - Ec) = p(-Ec - Pc^2)$$

$$\Rightarrow p' = p\frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p\frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda\left(\frac{2hc/\lambda + (E - Pc)}{E + Pc}\right) = \lambda\left(\frac{E - Pc}{E + Pc}\right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{\lambda(E - Pc) + 2hc}{E + Pc}$$

If $E \gg mc^2$, $Pc = \sqrt{E^2 - (mc^2)^2} = E\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \approx E\left(1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2 + \ldots\right)$

$$\Rightarrow E - Pc \approx \tfrac{1}{2}\frac{(mc^2)^2}{E} \Rightarrow \lambda' \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right).$$

(b) If $\lambda = 10.6\times10^{-6}$ m, $E = 1.00\times10^{10}$ eV $= 1.60\times10^{-9}$ J

$$\Rightarrow \lambda' \approx \frac{hc}{1.60\times10^{-9}\text{ J}}\left(1 + \frac{(9.11\times10^{-31}\text{ kg})^2c^4(10.6\times10^{-6}\text{ m})}{4hc\,(1.6\times10^{-9}\text{ J})}\right) = (1.24\times10^{-16}\text{ m})(1 + 56.0)$$

$$= 7.08\times10^{-15}\text{ m}.$$

(c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.

EVALUATE: The photon has gained energy from the initial kinetic energy of the electron. Since the photon gains energy, its wavelength decreases.

38.41. **IDENTIFY and SET UP:** The specific gravity of the tumor is 1, so it has the same density as water, 1000 kg/m^3. If 70 Gy are given in 35 days, the daily treatment is 2 Gy.

EXECUTE: The energy E per cell is

$$E/\text{cell} = \frac{(2\ \text{J/kg})\left(\dfrac{1000\ \text{kg}}{(100\ \text{cm})^3}\right)}{10^8\ \text{cells/cm}^3} = (2\times10^{-11}\ \text{J/cell})(6\times10^{18}\ \text{eV/J}) = 1.2\times10^8\ \text{eV/cell} = 120\ \text{MeV/cell}.$$

Choice (c) is correct.

EVALUATE: For 35 treatments the total dose would be 120 MeV $\times 35$ = 4200 MeV = 4.2 GeV per cell.

38.42. **IDENTIFY and SET UP:** Assume that the photon eventually loses all of its energy. Call N the number of ionizations.

EXECUTE: (40 eV)N = 4 MeV = 4 $\times 10^6$ eV $\rightarrow$ $N = 10^5$, so choice (d) is correct.

EVALUATE: This result is an average, since not every ionization would necessarily take 40 eV.

38.43. **IDENTIFY and SET UP:** For Compton scattering $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$. The energy of a photon is $E = hf = hc/\lambda$. The energy gained by the electron is equal to the energy lost by the photon.

EXECUTE: For backscatter, ϕ = 180°, so $\lambda' - \lambda = \left(\dfrac{h}{mc}\right)(1-\cos\phi)$ gives $\lambda' = \lambda + \dfrac{2h}{mc}$.

$E = hc/\lambda = 4$ MeV, so $\lambda = hc/(4\ \text{MeV})$. Therefore

$$\lambda' = \lambda + \frac{2h}{mc} = hc/(4\ \text{MeV}) + 2h/mc = hc[2/(0.511\ \text{MeV}) + 1/(4\ \text{MeV})] = 4.165hc\ \text{MeV}^{-1}.$$

E_{el} = loss of energy of photon.

$$E_{el} = hc/\lambda - hc/\lambda' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left[\frac{1}{hc/(4\ \text{MeV})} - \frac{1}{0.4164hc\ \text{MeV}^{-1}}\right] = 3.8\ \text{MeV}.$$ Therefore choice (a) is the correct one.

EVALUATE: Not all electrons would get this much energy because not all the photons would backscatter.

38.44. **IDENTIFY and SET UP:** The energy of a photon determines whether it is more likely to interact via the photoelectric effect or the Compton effect. The graph in Figure P38.44 shows that at high energies a photon is more likely to interact via the Compton effect, but at low energies it is more likely to interact via the photoelectric effect.

EXECUTE: From the graph we see that a 4-MeV photon has much higher probability of interacting via the Compton effect. But as it loses energy through repeated interactions, it will be more likely to interact via the photoelectric effect. Therefore choice (c) is the best one.

EVALUATE: In Problem 38.42 we saw that a photon can undergo around 10^5 ionization events, and during each of these it loses about 40 eV. Therefore it is reasonable that it would lose significant energy due to these interactions.

38.45. **IDENTIFY and SET UP:** For bremsstrahlung we have $eV_{AC} = hf_{max}$.

EXECUTE: If the accelerating potential V_{AC} is high, the maximum energy hf_{max} of the emitted photons will be high compared to a low accelerating potential. Thus choice (b) is correct.

EVALUATE: Not all the photons will have this energy, since f_{max} is the largest that the frequency can be.