4

NEWTON'S LAWS OF MOTION

VP4.1.1. **IDENTIFY:** This is a problem about vector addition. We know the magnitude and direction of three forces and want to find the magnitude and direction of their resultant force.
SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A\cos\theta$ and $A_y = A\sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.
EXECUTE: For the three given vectors, the components of the resultant are
$R_x = 40.0\text{ N} + 0\text{ N} + (60.0\text{ N})\cos 36.9° = 88.0\text{ N}$
$R_y = 0\text{ N} + (–80.0\text{ N}) + (60.0\text{ N})\sin 36.9° = –44.0\text{ N}$
$R = \sqrt{(88.0\text{ N})^2 + (–44.0\text{ N})^2} = 98.4\text{ N}$.
$\theta = \arctan[(–44.0\text{ N})/(88.0\text{ N})] = –26.6°$. The minus sign tells us that θ is clockwise from the $+x$-axis.
EVALUATE: Since R_x is positive and R_y is negative, the resultant should point into the fourth quadrant, which agrees with our result.

VP4.1.2. **IDENTIFY:** This is a problem about vector addition. We know the magnitude and direction of three forces and want to find the magnitude and direction of their resultant force.
SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A\cos\theta$ and $A_y = A\sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.
EXECUTE: The components of the resultant are
$R_x = 45.0\text{ N} + 0\text{ N} + (235\text{ N})\cos 143.1° = –143\text{ N}$
$R_y = 0\text{ N} + 105\text{ N} + 235\text{ N}\sin 143.1° = 246\text{ N}$
$R = \sqrt{(–143\text{ N})^2 + (246\text{ N})^2} = 285\text{ N}$.
$\theta = \arctan[(246\text{ N})/(–143\text{ N})] = –60.0°$, so $\theta = 120°$ counterclockwise from the $+x$-axis.
EVALUATE: The resultant has a negative x-component and a positive y-component, so it should point into the second quadrant, which is what our result shows.

VP4.1.3. **IDENTIFY:** This is a problem about vector addition. We know the magnitude and direction of three forces and want to find the magnitude and direction of their resultant force.
SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A\cos\theta$ and $A_y = A\sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

EXECUTE: The components of the resultant are

$R_x = 0\text{ N} + (60.0\text{ N})\cos 70.0° + (15.0\text{ N})\cos 160.0° = 6.4\text{ N}$

$R_y = -60.0\text{ N} + (60.0\text{ N})\sin 70.0° + (15.0\text{ N})\sin 160.0° = 1.5\text{ N}$

$R = \sqrt{(6.4\text{ N})^2 + (1.5\text{ N})^2} = 6.6\text{ N}$

$\theta = \arctan[(1.5\text{ N})/(64\text{ N})] = 13°$ counterclockwise from the $+x$-axis.

EVALUATE: Since all the components of the resultant are positive, it should point into the first quadrant, which is what we found.

VP4.1.4. **IDENTIFY:** We know the resultant of three vectors, and we know two of them. We want to find the magnitude and direction of the unknown third vector.

SET UP: The components of a vector of magnitude A that make an angle θ with the $+x$-axis are $A_x = A\cos\theta$ and $A_y = A\sin\theta$. The magnitude and direction are $A = \sqrt{A_x^2 + A_y^2}$ and $\theta = \arctan\dfrac{A_y}{A_x}$. The components of the resultant are $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$.

EXECUTE: Let T refer to Ernesto's force, K for Kamala's force, and T for Tsuroku's unknown force. The components of the resultant force are

$R_x = 35.0\text{ N} + 0\text{ N} + T_x = (24.0\text{ N})\cos 210° \rightarrow \quad T_x = -55.8\text{ N}$

$R_y = 0\text{ N} + 50.0\text{ N} + T_y = (24.0\text{ N})\sin 210° \rightarrow \quad T_y = -62.0\text{ N}$

$T = \sqrt{(-55.8\text{ N})^2 + (-62.0\text{ N})^2} = 83.4\text{ N}$

$\theta = \arctan[(-62.0\text{ N})/(-55.8\text{ N})] = 48.0°$

Since both components of Tsuroku's force are negative, its direction is 48.0° below the $-x$-axis, which is 48.0° south of west.

EVALUATE: A graphical sum will confirm this result.

VP4.4.1. **IDENTIFY:** Apply Newton's second law to the box.

SET UP: Take the x-axis along the floor. Use $\sum F_x = ma_x$ to find a_x. The only horizontal force acting on the box is the force due to the worker.

EXECUTE: $\sum F_x = ma_x$ gives $25\text{ N} = (55\text{ kg})\,a_x \rightarrow a_x = 0.45\text{ m/s}^2$. This acceleration is in the direction of the worker's force.

EVALUATE: Other forces act on the box, such as gravity downward and the upward push of the floor. But these do not affect the horizontal acceleration since they have no horizontal components.

VP4.4.2. **IDENTIFY:** Apply Newton's second law to the cheese.

SET UP: Take the x-axis along the surface of the table. Use $\sum F_x = ma_x$ to find a_x. The only horizontal force acting on the box is the 0.50-N force.

EXECUTE: **(a)** The forces are gravity acting vertically downward, the normal force due to the tabletop acting vertically upward, and the 0.50-N force due to your hand acting horizontally.

(b) $\sum F_x = ma_x$ gives $0.50\text{ N} = (2.0\text{ kg})\,a_x \rightarrow a_x = 0.25\text{ m/s}^2$.

EVALUATE: The vertical forces do not affect the horizontal motion, so only the 0.50-N force causes the acceleration.

VP4.4.3. **IDENTIFY:** Apply Newton's second law to the puck.

SET UP: Take the x-axis along the direction of the horizontal hit on the puck. We know the acceleration of the puck, so use $\sum F_x = ma_x$ to find the force of the hit. The only horizontal force acting on the puck is the hit. For the vertical forces, use $\sum F_y = ma_y$.

EXECUTE: (a) With the x-axis horizontal, $\sum F_x = ma_x$ gives

$F_x = (0.16\text{ kg})(75\text{ m/s}^2) = 12\text{ N}$.

(b) The vertical forces are gravity (the weight w) and the normal force n due to the ice. Using $\sum F_y = ma_y$, we have $n - w = 0$ since $a_y = 0$. So the normal force must be equal to the weight of the puck.

EVALUATE: The vertical forces do not affect the horizontal motion.

VP4.4.4. **IDENTIFY:** We apply Newton's second law to the plate. We know its horizontal acceleration and mass and one of the horizontal forces acting on it, so we can find the friction force, which is horizontal.

SET UP: Apply $\sum F_x = ma_x$ with the x-axis horizontal. The two horizontal forces are friction f and the push P.

EXECUTE: (a) $\sum F_x = ma_x$ = (0.800 kg)(12.0 m/s^2) = 9.60 N. This is the net force.

(b) $F_{\text{net}} = P - f$, so 9.60 N = 14.0 N – f, so f = 4.4 N. The direction is opposite to the push.

EVALUATE: Friction is less than the push, which it should be since the plate accelerates in the direction of the push.

VP4.5.1. **IDENTIFY:** The forces on the child are constant, so her acceleration is constant. Thus we can use the constant-acceleration motion equations to find her acceleration. Then apply Newton's second law to find the friction force.

SET UP: The formulas $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ and $\sum F_x = ma_x$ both apply.

EXECUTE: First find the girl's acceleration using $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. Putting in the known numbers gives 0^2 = (3.00 m/s)^2 + $2a_x$(2.25 m), giving a_x = –2.00 m/s^2. The minus sign means that a_x is opposite to v_x. Now use $\sum F_x = ma_x$ to find the friction force. Since friction is the only horizontal force acting on her, it must be in the same direction as her acceleration. This gives $f = ma_x$ = (20.0 kg)(–2.00 m/s^2) = –40.0 N. The magnitude is 40.0 N and the direction is the same as the acceleration, which is opposite to the velocity.

EVALUATE: The other forces (gravity and the normal force due to the ice) are vertical, so they do not affect the horizontal motion.

VP4.5.2. **IDENTIFY:** This problem involves Newton's second law and motion with uniform acceleration. Thus we can use the constant-acceleration motion equations.

SET UP: First use $\sum F_x = ma_x$ to find the plane's acceleration. Then use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find how far it travels while stopping.

EXECUTE: Using $\sum F_x = ma_x$ gives 2.90 N = (1.70×10^5 kg) a_x → a_x = 1.706 m/s^2. Now use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find $x - x_0$. Call the +x-direction to be that of the velocity, so a_x will be negative. Thus

0^2 = (75.0 m/s)^2 + 2(–1.706 m/s^2)($x - x_0$) → $x - x_0$ = 1650 m = 1.65 km.

EVALUATE: This distance is about a mile, which is not so unreasonable for stopping a large plane landing at a fairly high speed.

VP4.5.3. **IDENTIFY:** This problem involves Newton's second law and motion with uniform acceleration. Thus we can use the constant-acceleration motion equations. We know the truck's mass, its initial speed, and the distance it travels while stopping. We want to find how long it takes to stop, its acceleration while stopping, and the braking force while stopping.

SET UP: The braking force is opposite to the truck's velocity but is in the same direction as the truck's acceleration. The equations $v_x = v_{0x} + a_x t$ and $\sum F_x = ma_x$ apply. In addition, the average velocity is $v_{av-x} = \dfrac{\Delta x}{\Delta t}$, and for uniform acceleration, it is also true that $v_{av} = \dfrac{v_0 + v}{2}$.

EXECUTE: **(a)** Combining the two equations for $v_{\text{av-}x}$ gives

$$\Delta t = \frac{\Delta x}{v_{av-x}} = \frac{\Delta x}{\left(\frac{v_0 + v}{2}\right)} = \frac{48.0\text{ m}}{\frac{25.0\text{ m/s} + 0}{2}} = 3.84\text{ s}.$$

(b) Using $v_x = v_{0x} + a_x t$ gives $0 = 25.0$ m/s + a_x (3.84 s)

$a_x = -6.51$ m/s^2. The minus sign means that a_x is opposite to v_x. The magnitude is 6.51 m/s^2.

(c) $\sum F_x = ma_x$ = (2400 kg)(6.51 m/s^2) = 1.56×10^4 N.

EVALUATE: This may seem like a large force, but it is the only force stopping a massive object with a large acceleration.

VP4.5.4. **IDENTIFY:** This problem involves Newton's second law and motion with uniform acceleration. Thus we can use the constant-acceleration motion equations.

SET UP: The equations $\sum F_x = ma_x$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ apply.

EXECUTE: **(a)** Gravity acts downward and the normal force due to the road acts upward. The horizontal forces are the push and the friction force from the road. The friction force is opposite to the push.

(b) Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the acceleration of the car, giving

(1.40 m/s)^2 = 0 + 2a_x(5.00 m) $\rightarrow$ $a_x = 0.1960$ m/s^2. Now apply $\sum F_x = ma_x$.

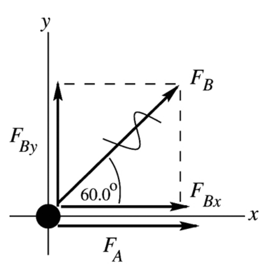
$P - f = ma_x$ $\rightarrow$ 8.00×10^2 N $- f = (1.15\times10^3$ kg)(0.1960 m/s^2) $\rightarrow$ $f = 575$ N.

EVALUATE: The push is 800 N and the opposing friction force is 575 N, so the car accelerates in the direction of the push.

4.1. **IDENTIFY:** Vector addition.

SET UP: Use a coordinate system where the $+x$-axis is in the direction of $\vec{F}_A$, the force applied by dog A. The forces are sketched in Figure 4.1.

EXECUTE:



$F_{Ax} = +270\text{ N},\ F_{Ay} = 0$

$F_{Bx} = F_B\cos60.0° = (300\text{ N})\cos60.0° = +150\text{ N}$

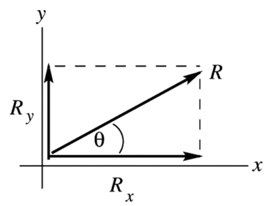
$F_{By} = F_B\sin60.0° = (300\text{ N})\sin60.0° = +260\text{ N}$

Figure 4.1a

$\vec{R} = \vec{F}_A + \vec{F}_B$

$R_x = F_{Ax} + F_{Bx} = +270\text{ N} + 150\text{ N} = +420\text{ N}$

$R_y = F_{Ay} + F_{By} = 0 + 260\text{ N} = +260\text{ N}$



$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(420\text{ N})^2 + (260\text{ N})^2} = 494\text{ N}$$
$$\tan\theta = \frac{R_y}{R_x} = 0.619$$
$$\theta = 31.8°$$

Figure 4.1b

EVALUATE: The forces must be added as vectors. The magnitude of the resultant force is less than the sum of the magnitudes of the two forces and depends on the angle between the two forces.

4.2. **IDENTIFY:** We know the magnitudes and directions of three vectors and want to use them to find their components, and then to use the components to find the magnitude and direction of the resultant vector.
SET UP: Let $F_1 = 985\text{ N}$, $F_2 = 788\text{ N}$, and $F_3 = 411\text{ N}$. The angles θ that each force makes with the $+x$ axis are $\theta_1 = 31°$, $\theta_2 = 122°$, and $\theta_3 = 233°$. The components of a force vector are $F_x = F\cos\theta$ and $F_y = F\sin\theta$, and $R = \sqrt{R_x^2 + R_y^2}$ and $\tan\theta = \dfrac{R_y}{R_x}$.

EXECUTE: **(a)** $F_{1x} = F_1\cos\theta_1 = 844\text{ N}$, $F_{1y} = F_1\sin\theta_1 = 507\text{ N}$, $F_{2x} = F_2\cos\theta_2 = -418\text{ N}$, $F_{2y} = F_2\sin\theta_2 = 668\text{ N}$, $F_{3x} = F_3\cos\theta_3 = -247\text{ N}$, and $F_{3y} = F_3\sin\theta_3 = -328\text{ N}$.

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 179\text{ N}$; $R_y = F_{1y} + F_{2y} + F_{3y} = 847\text{ N}$. $R = \sqrt{R_x^2 + R_y^2} = 886\text{ N}$; $\tan\theta = \dfrac{R_y}{R_x}$ so $\theta = 78.1°$. $\vec{\boldsymbol{R}}$ and its components are shown in Figure 4.2.

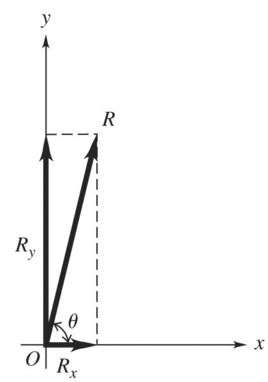


Figure 4.2

EVALUATE: A graphical sketch of the vector sum should agree with the results found in (b). Adding the forces as vectors gives a very different result from adding their magnitudes.

4.3. **IDENTIFY:** We know the resultant of two vectors of equal magnitude and want to find their magnitudes. They make the same angle with the vertical.

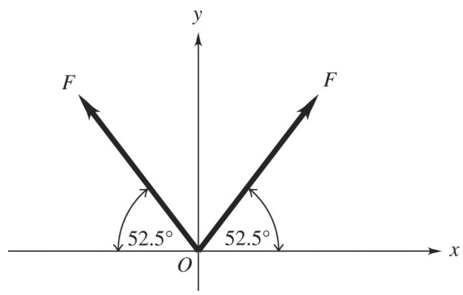


Figure 4.3

SET UP: Take $+y$ to be upward, so $\sum F_y = 5.00$ N. The strap on each side of the jaw exerts a force F directed at an angle of 52.5° above the horizontal, as shown in Figure 4.3.

EXECUTE: $\sum F_y = 2F \sin 52.5° = 5.00$ N, so $F = 3.15$ N.

EVALUATE: The resultant force has magnitude 5.00 N which is *not* the same as the sum of the magnitudes of the two vectors, which would be 6.30 N.

4.4. IDENTIFY: $F_x = F\cos\theta$, $F_y = F\sin\theta$.

SET UP: Let $+x$ be parallel to the ramp and directed up the ramp. Let $+y$ be perpendicular to the ramp and directed away from it. Then $\theta = 30.0°$.

EXECUTE: **(a)** $F = \dfrac{F_x}{\cos\theta} = \dfrac{90.0\text{ N}}{\cos 30°} = 104$ N.

(b) $F_y = F\sin\theta = F_x \tan\theta = (90\text{ N})(\tan 30°) = 52.0$ N.

EVALUATE: We can verify that $F_x^2 + F_y^2 = F^2$. The signs of F_x and F_y show their direction.

4.5. IDENTIFY: Add the two forces using components.

SET UP: $F_x = F\cos\theta$, $F_y = F\sin\theta$, where θ is the angle $\vec{F}$ makes with the $+x$ axis.

EXECUTE: **(a)** $F_{1x} + F_{2x} = (9.00\text{ N})\cos 120° + (6.00\text{ N})\cos(233.1°) = -8.10$ N

$F_{1y} + F_{2y} = (9.00\text{ N})\sin 120° + (6.00\text{ N})\sin(233.1°) = +3.00$ N.

(b) $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(8.10\text{ N})^2 + (3.00\text{ N})^2} = 8.64$ N.

EVALUATE: Since $F_x < 0$ and $F_y > 0$, $\vec{F}$ is in the second quadrant.

4.6. IDENTIFY: Use constant acceleration equations to calculate a_x and t. Then use $\sum \vec{F} = m\vec{a}$ to calculate the net force.

SET UP: Let $+x$ be in the direction of motion of the electron.

EXECUTE: **(a)** $v_{0x} = 0$, $(x - x_0) = 1.80\times10^{-2}$ m, $v_x = 3.00\times10^6$ m/s. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(3.00\times10^6\text{ m/s})^2 - 0}{2(1.80\times10^{-2}\text{ m})} = 2.50\times10^{14}\text{ m/s}^2$$

(b) $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{3.00\times10^6\text{ m/s} - 0}{2.50\times10^{14}\text{ m/s}^2} = 1.2\times10^{-8}$ s

(c) $\sum F_x = ma_x = (9.11\times10^{-31}\text{ kg})(2.50\times10^{14}\text{ m/s}^2) = 2.28\times10^{-16}$ N.

EVALUATE: The acceleration is in the direction of motion since the speed is increasing, and the net force is in the direction of the acceleration.

4.7. IDENTIFY: Friction is the only horizontal force acting on the skater, so it must be the one causing the acceleration. Newton's second law applies.

SET UP: Take $+x$ to be the direction in which the skater is moving initially. The final velocity is $v_x = 0$, since the skater comes to rest. First use the kinematics formula $v_x = v_{0x} + a_x t$ to find the acceleration, then apply $\sum \vec{F} = m\vec{a}$ to the skater.

EXECUTE: $v_x = v_{0x} + a_x t$ so $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{0 - 2.40 \text{ m/s}}{3.52 \text{ s}} = -0.682 \text{ m/s}^2$. The only horizontal force on the skater is the friction force, so $f_x = ma_x = (68.5 \text{ kg})(-0.682 \text{ m/s}^2) = -46.7 \text{ N}$. The force is 46.7 N, directed opposite to the motion of the skater.

EVALUATE: Although other forces are acting on the skater (gravity and the upward force of the ice), they are vertical and therefore do not affect the horizontal motion.

4.8. **IDENTIFY:** The elevator and everything in it are accelerating upward, so we apply Newton's second law in the vertical direction.

SET UP: Your mass is $m = w/g = 63.8 \text{ kg}$. Both you and the package have the same acceleration as the elevator. Take $+y$ to be upward, in the direction of the acceleration of the elevator, and apply $\sum F_y = ma_y$.

EXECUTE: **(a)** Your free-body diagram is shown in Figure 4.8a, where *n* is the scale reading. $\sum F_y = ma_y$ gives $n - w = ma$. Solving for *n* gives

$n = w + ma = 625 \text{ N} + (63.8 \text{ kg})(2.50 \text{ m/s}^2) = 784 \text{ N}$.

(b) The free-body diagram for the package is given in Figure 4.8b. $\sum F_y = ma_y$ gives $T - w = ma$, so $T = w + ma = (3.85 \text{ kg})(9.80 \text{ m/s}^2 + 2.50 \text{ m/s}^2) = 47.4 \text{ N}$.

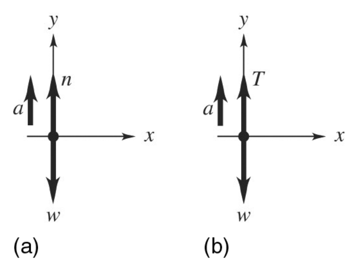


Figure 4.8

EVALUATE: The objects accelerate upward so for each of them the upward force is greater than the downward force.

4.9. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: Let $+x$ be the direction of the force and acceleration. $\sum F_x = 48.0 \text{ N}$.

EXECUTE: $\sum F_x = ma_x$ gives $m = \dfrac{\sum F_x}{a_x} = \dfrac{48.0 \text{ N}}{2.20 \text{ m/s}^2} = 21.8 \text{ kg}$.

EVALUATE: The vertical forces sum to zero and there is no motion in that direction.

4.10. **IDENTIFY:** Use the information about the motion to find the acceleration and then use $\sum F_x = ma_x$ to calculate *m*.

SET UP: Let $+x$ be the direction of the force. $\sum F_x = 80.0 \text{ N}$.

EXECUTE: **(a)** $x - x_0 = 11.0 \text{ m}$, $t = 5.00 \text{ s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(11.0 \text{ m})}{(5.00 \text{ s})^2} = 0.880 \text{ m/s}^2$. $m = \dfrac{\sum F_x}{a_x} = \dfrac{80.0 \text{ N}}{0.880 \text{ m/s}^2} = 90.9 \text{ kg}$.

(b) $a_x = 0$ and v_x is constant. After the first 5.0 s, $v_x = v_{0x} + a_x t =$ (0.880 m/s^2) (5.00 s) = 4.40 m/s.
$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 =$ (4.40 m/s)(5.00 s) = 22.0 m.

EVALUATE: The mass determines the amount of acceleration produced by a given force. The block moves farther in the second 5.00 s than in the first 5.00 s.

4.11. IDENTIFY and **SET UP:** Use Newton's second law in component form to calculate the acceleration produced by the force. Use constant acceleration equations to calculate the effect of the acceleration on the motion.

EXECUTE: **(a)** During this time interval the acceleration is constant and equal to

$$a_x = \frac{F_x}{m} = \frac{0.250 \text{ N}}{0.160 \text{ kg}} = 1.562 \text{ m/s}^2$$

We can use the constant acceleration kinematic equations from Chapter 2.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(1.562 \text{ m/s}^2)(2.00 \text{ s})^2 = 3.12 \text{ m},$ so the puck is at $x = 3.12$ m.

$v_x = v_{0x} + a_x t = 0 + (1.562 \text{ m/s}^2)(2.00 \text{ s}) = 3.12 \text{ m/s}.$

(b) In the time interval from $t = 2.00$ s to 5.00 s the force has been removed so the acceleration is zero. The speed stays constant at $v_x = 3.12$ m/s. The distance the puck travels is

$x - x_0 = v_{0x}t = (3.12 \text{ m/s})(5.00 \text{ s} - 2.00 \text{ s}) = 9.36 \text{ m}.$ At the end of the interval it is at

$x = x_0 + 9.36 \text{ m} = 12.5 \text{ m}.$

In the time interval from $t = 5.00$ s to 7.00 s the acceleration is again $a_x = 1.562 \text{ m/s}^2$. At the start of this interval $v_{0x} = 3.12$ m/s and $x_0 = 12.5$ m.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (3.12 \text{ m/s})(2.00 \text{ s}) + \frac{1}{2}(1.562 \text{ m/s}^2)(2.00 \text{ s})^2.$

$x - x_0 = 6.24 \text{ m} + 3.12 \text{ m} = 9.36 \text{ m}.$

Therefore, at $t = 7.00$ s the puck is at $x = x_0 + 9.36 \text{ m} = 12.5 \text{ m} + 9.36 \text{ m} = 21.9 \text{ m}.$

$v_x = v_{0x} = a_x t = 3.12 \text{ m/s} + (1.562 \text{ m/s}^2)(2.00 \text{ s}) = 6.24 \text{ m/s}.$

EVALUATE: The acceleration says the puck gains 1.56 m/s of velocity for every second the force acts. The force acts a total of 4.00 s so the final velocity is (1.56 m/s)(4.0 s) = 6.24 m/s.

4.12. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$. Then use a constant acceleration equation to relate the kinematic quantities.

SET UP: Let $+x$ be in the direction of the force.

EXECUTE: **(a)** $a_x = F_x / m$ = (14.0 N)/(32.5 kg) = 0.4308 m/s^2, which rounds to 0.431 m/s^2 for the final answer.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. With $v_{0x} = 0$, $x = \frac{1}{2}a_x t^2 = \frac{1}{2}(0.4308 \text{ m/s}^2)(10.0 \text{ s})^2 = 21.5 \text{ m}.$

(c) $v_x = v_{0x} + a_x t$. With $v_{0x} = 0$, $v_x = a_x t$ = (0.4308 m/s^2)(10.0 s) = 4.31 m/s.

EVALUATE: The acceleration connects the motion to the forces.

4.13. IDENTIFY: The force and acceleration are related by Newton's second law.

SET UP: $\sum F_x = ma_x$, where $\sum F_x$ is the net force. $m = 4.50$ kg.

EXECUTE: **(a)** The maximum net force occurs when the acceleration has its maximum value.

$\sum F_x = ma_x = (4.50 \text{ kg})(10.0 \text{ m/s}^2) = 45.0 \text{ N}.$ This maximum force occurs between 2.0 s and 4.0 s.

(b) The net force is constant when the acceleration is constant. This is between 2.0 s and 4.0 s.

(c) The net force is zero when the acceleration is zero. This is the case at $t = 0$ and $t = 6.0$ s.

EVALUATE: A graph of $\sum F_x$ versus t would have the same shape as the graph of a_x versus t.

4.14. **IDENTIFY:** The force and acceleration are related by Newton's second law. $a_x = \dfrac{dv_x}{dt}$, so a_x is the slope of the graph of v_x versus t.

SET UP: The graph of v_x versus t consists of straight-line segments. For $t = 0$ to $t = 2.00$ s, $a_x = 4.00\ \text{m/s}^2$. For $t = 2.00$ s to 6.00 s, $a_x = 0$. For $t = 6.00$ s to 10.0 s, $a_x = 1.00\ \text{m/s}^2$. $\sum F_x = ma_x$, with $m = 2.75$ kg. $\sum F_x$ is the net force.

EXECUTE: **(a)** The maximum net force occurs when the acceleration has its maximum value. $\sum F_x = ma_x = (2.75\ \text{kg})(4.00\ \text{m/s}^2) = 11.0\ \text{N}$. This maximum occurs in the interval $t = 0$ to $t = 2.00$ s.

(b) The net force is zero when the acceleration is zero. This is between 2.00 s and 6.00 s.

(c) Between 6.00 s and 10.0 s, $a_x = 1.00\ \text{m/s}^2$, so $\sum F_x = (2.75\ \text{kg})(1.00\ \text{m/s}^2) = 2.75\ \text{N}$.

EVALUATE: The net force is largest when the velocity is changing most rapidly.

4.15. **IDENTIFY:** The net force and the acceleration are related by Newton's second law. When the rocket is near the surface of the earth the forces on it are the upward force $\vec{F}$ exerted on it because of the burning fuel and the downward force $\vec{F}_{\text{grav}}$ of gravity. $F_{\text{grav}} = mg$.

SET UP: Let $+y$ be upward. The weight of the rocket is $F_{\text{grav}} = (8.00\ \text{kg})(9.80\ \text{m/s}^2) = 78.4\ \text{N}$.

EXECUTE: **(a)** At $t = 0$, $F = A = 100.0\ \text{N}$. At $t = 2.00$ s, $F = A + (4.00\ \text{s}^2)B = 150.0\ \text{N}$ and $B = \dfrac{150.0\ \text{N} - 100.0\ \text{N}}{4.00\ \text{s}^2} = 12.5\ \text{N/s}^2$.

(b) (i) At $t = 0$, $F = A = 100.0\ \text{N}$. The net force is $\sum F_y = F - F_{\text{grav}} = 100.0\ \text{N} - 78.4\ \text{N} = 21.6\ \text{N}$. $a_y = \dfrac{\sum F_y}{m} = \dfrac{21.6\ \text{N}}{8.00\ \text{kg}} = 2.70\ \text{m/s}^2$. (ii) At $t = 3.00$ s, $F = A + B(3.00\ \text{s})^2 = 212.5\ \text{N}$. $\sum F_y = 212.5\ \text{N} - 78.4\ \text{N} = 134.1\ \text{N}$. $a_y = \dfrac{\sum F_y}{m} = \dfrac{134.1\ \text{N}}{8.00\ \text{kg}} = 16.8\ \text{m/s}^2$.

(c) Now $F_{\text{grav}} = 0$ and $\sum F_y = F = 212.5\ \text{N}$. $a_y = \dfrac{212.5\ \text{N}}{8.00\ \text{kg}} = 26.6\ \text{m/s}^2$.

EVALUATE: The acceleration increases as F increases.

4.16. **IDENTIFY:** Weight and mass are related by $w = mg$. The mass is constant but g and w depend on location.

SET UP: On earth, $g = 9.80\ \text{m/s}^2$.

EXECUTE: **(a)** $\dfrac{w}{g} = m$, which is constant, so $\dfrac{w_{\text{E}}}{g_{\text{E}}} = \dfrac{w_{\text{A}}}{g_{\text{A}}}$. $w_{\text{E}} = 17.5\ \text{N}$, $g_{\text{E}} = 9.80\ \text{m/s}^2$, and $w_{\text{M}} = 3.24\ \text{N}$. $g_{\text{M}} = \left(\dfrac{w_{\text{A}}}{w_{\text{E}}}\right)g_{\text{E}} = \left(\dfrac{3.24\ \text{N}}{17.5\ \text{N}}\right)(9.80\ \text{m/s}^2) = 1.81\ \text{m/s}^2$.

(b) $m = \dfrac{w_{\text{E}}}{g_{\text{E}}} = \dfrac{17.5\ \text{N}}{9.80\ \text{m/s}^2} = 1.79\ \text{kg}$.

EVALUATE: The weight at a location and the acceleration due to gravity at that location are directly proportional.

4.17. **IDENTIFY** and **SET UP:** $F = ma$. We must use $w = mg$ to find the mass of the boulder.

EXECUTE: $m = \dfrac{w}{g} = \dfrac{2400\ \text{N}}{9.80\ \text{m/s}^2} = 244.9\ \text{kg}$

Then $F = ma = (244.9\text{ kg})(12.0\text{ m/s}^2) = 2940\text{ N}.$

EVALUATE: We must use mass in Newton's second law. Mass and weight are proportional.

4.18. IDENTIFY: Find weight from mass and vice versa.

SET UP: Equivalencies we'll need are: $1\,\mu\text{g} = 10^{-6}\text{ g} = 10^{-9}\text{ kg},\ 1\text{ mg} = 10^{-3}\text{ g} = 10^{-6}\text{ kg},$ $1\text{ N} = 0.2248\text{ lb},$ and $g = 9.80\text{ m/s}^2 = 32.2\text{ ft/s}^2.$

EXECUTE: **(a)** $m = 210\,\mu\text{g} = 2.10\times10^{-7}\text{ kg}.\ \ w = mg = (2.10\times10^{-7}\text{ kg})(9.80\text{ m/s}^2) = 2.06\times10^{-6}\text{ N}.$

(b) $m = 12.3\text{ mg} = 1.23\times10^{-5}\text{ kg}.\ \ w = mg = (1.23\times10^{-5}\text{ kg})(9.80\text{ m/s}^2) = 1.21\times10^{-4}\text{ N}.$

(c) $(45\text{ N})\left(\dfrac{0.2248\text{ lb}}{1\text{ N}}\right) = 10.1\text{ lb}.\ \ m = \dfrac{w}{g} = \dfrac{45\text{ N}}{9.80\text{ m/s}^2} = 4.6\text{ kg}.$

EVALUATE: We are not converting mass to weight (or vice versa) since they are different types of quantities. We are finding what a given mass will weigh and how much mass a given weight contains.

4.19. IDENTIFY and **SET UP:** $w = mg$. The mass of the watermelon is constant, independent of its location. Its weight differs on earth and Jupiter's moon. Use the information about the watermelon's weight on earth to calculate its mass:

EXECUTE: **(a)** $w = mg$ gives that $m = \dfrac{w}{g} = \dfrac{44.0\text{ N}}{9.80\text{ m/s}^2} = 4.49\text{ kg}.$

(b) On Jupiter's moon, $m = 4.49\text{ kg},$ the same as on earth. Thus the weight on Jupiter's moon is $w = mg = (4.49\text{ kg})(1.81\text{ m/s}^2) = 8.13\text{ N}.$

EVALUATE: The weight of the watermelon is less on Io, since g is smaller there.

4.20. IDENTIFY and SET UP: This problem requires some estimation and a web search.

EXECUTE: Web search: The mass of a Sumo wrestler is approximately 148 kg, which is about 326 lb. Estimate: The average student in class weighs about 165 lb which is about 75 kg.

EVALUATE: These are average values.

4.21. IDENTIFY: Apply $\sum F_x = ma_x$ to find the resultant horizontal force.

SET UP: Let the acceleration be in the $+x$ direction.

EXECUTE: $\sum F_x = ma_x = (55\text{ kg})(15\text{ m/s}^2) = 825\text{ N}.$ The force is exerted by the blocks. The blocks push on the sprinter because the sprinter pushes on the blocks.

EVALUATE: The force the blocks exert on the sprinter has the same magnitude as the force the sprinter exerts on the blocks. The harder the sprinter pushes, the greater the force on her.

4.22. IDENTIFY: Newton's third law applies.

SET UP: The car exerts a force on the truck and the truck exerts a force on the car.

EXECUTE: The force and the reaction force are always exactly the same in magnitude, so the force that the truck exerts on the car is 1600 N, by Newton's third law.

EVALUATE: Even though the truck is much larger and more massive than the car, it cannot exert a larger force on the car than the car exerts on it.

4.23. IDENTIFY: The system is accelerating so we use Newton's second law.

SET UP: The acceleration of the entire system is due to the 250-N force, but the acceleration of box B is due to the force that box A exerts on it. $\sum F = ma$ applies to the two-box system and to each box individually.

EXECUTE: For the two-box system: $a_x = \dfrac{250\text{ N}}{25.0\text{ kg}} = 10.0\text{ m/s}^2.$ Then for box B, where F_A is the force exerted on B by A, $F_A = m_B a = (5.0\text{ kg})(10.0\text{ m/s}^2) = 50\text{ N}.$

EVALUATE: The force on B is less than the force on A.

4.24. **IDENTIFY:** The reaction forces in Newton's third law are always between a pair of objects. In Newton's second law all the forces act on a single object.
SET UP: Let $+y$ be downward. $m = w/g$.

EXECUTE: The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the
gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N. $\dfrac{\sum F_y}{m} = a_y$ gives $a_y = \dfrac{650\text{ N} - 620\text{ N}}{(650\text{ N})/(9.80\text{ m/s}^2)} = 0.452\text{ m/s}^2$. The passenger's acceleration is 0.452 m/s^2, downward.
EVALUATE: There is a net downward force on the passenger, and the passenger has a downward acceleration.

4.25. **IDENTIFY:** Apply Newton's second law to the earth.
SET UP: The force of gravity that the earth exerts on her is her weight, $w = mg = (45\text{ kg})(9.8\text{ m/s}^2) = 441\text{ N}$. By Newton's third law, she exerts an equal and opposite force on the earth.
Apply $\sum \vec{F} = m\vec{a}$ to the earth, with $\left|\sum \vec{F}\right| = w = 441\text{ N}$, but must use the mass of the earth for m.

EXECUTE: $a = \dfrac{w}{m} = \dfrac{441\text{ N}}{6.0\times10^{24}\text{ kg}} = 7.4\times10^{-23}\text{ m/s}^2$.

EVALUATE: This is *much* smaller than her acceleration of 9.8 m/s^2. The force she exerts on the earth equals in magnitude the force the earth exerts on her, but the acceleration the force produces depends on the mass of the object and her mass is *much* less than the mass of the earth.

4.26. **IDENTIFY:** The surface of block B can exert both a friction force and a normal force on block A. The friction force is directed so as to oppose relative motion between blocks B and A. Gravity exerts a downward force w on block A.
SET UP: The pull is a force on B not on A.
EXECUTE: **(a)** If the table is frictionless there is a net horizontal force on the combined object of the two blocks, and block B accelerates in the direction of the pull. The friction force that B exerts on A is to the right, to try to prevent A from slipping relative to B as B accelerates to the right. The free-body diagram is sketched in Figure 4.26a (next page). f is the friction force that B exerts on A and n is the normal force that B exerts on A.
(b) The pull and the friction force exerted on B by the table cancel and the net force on the system of two blocks is zero. The blocks move with the same constant speed and B exerts no friction force on A. The free-body diagram is sketched in Figure 4.26b (next page).
EVALUATE: If in part (b) the pull force is decreased, block B will slow down, with an acceleration directed to the left. In this case the friction force on A would be to the left, to prevent relative motion between the two blocks by giving A an acceleration equal to that of B.

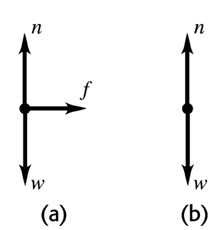


Figure 4.26

4.27. **IDENTIFY:** Identify the forces on each object.

SET UP: In each case the forces are the noncontact force of gravity (the weight) and the forces applied by objects that are in contact with each crate. Each crate touches the floor and the other crate, and some object applies $\vec{F}$ to crate A.

EXECUTE: **(a)** The free-body diagrams for each crate are given in Figure 4.27.

F_{AB} (the force on m_A due to m_B) and F_{BA} (the force on m_B due to m_A) form an action-reaction pair.

(b) Since there is no horizontal force opposing *F*, any value of *F*, no matter how small, will cause the crates to accelerate to the right. The weight of the two crates acts at a right angle to the horizontal, and is in any case balanced by the upward force of the surface on them.

EVALUATE: Crate *B* is accelerated by F_{BA} and crate *A* is accelerated by the net force $F - F_{AB}$. The greater the total weight of the two crates, the greater their total mass and the smaller will be their acceleration.

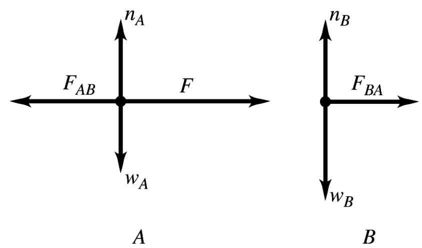


Figure 4.27

4.28. **IDENTIFY:** Use a constant acceleration equation to find the stopping time and acceleration. Then use $\sum\vec{F} = m\vec{a}$ to calculate the force.

SET UP: Let $+x$ be in the direction the bullet is traveling. $\vec{F}$ is the force the wood exerts on the bullet.

EXECUTE: **(a)** $v_{0x} = 350$ m/s, $v_x = 0$ and $(x - x_0) = 0.130$ m. $(x - x_0) = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives

$$t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(0.130\text{ m})}{350\text{ m/s}} = 7.43\times10^{-4}\text{ s.}$$

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{0 - (350\text{ m/s})^2}{2(0.130\text{ m})} = -4.71\times10^5\text{ m/s}^2$

$\sum F_x = ma_x$ gives $-F = ma_x$ and $F = -ma_x = -(1.80\times10^{-3}\text{ kg})(-4.71\times10^5\text{ m/s}^2) = 848$ N.

EVALUATE: The acceleration and net force are opposite to the direction of motion of the bullet.

4.29. **IDENTIFY:** Since the observer in the train sees the ball hang motionless, the ball must have the same acceleration as the train car. By Newton's second law, there must be a net force on the ball in the same direction as its acceleration.

SET UP: The forces on the ball are gravity, which is *w*, downward, and the tension $\vec{T}$ in the string, which is directed along the string.

EXECUTE: **(a)** The acceleration of the train is zero, so the acceleration of the ball is zero. There is no net horizontal force on the ball and the string must hang vertically. The free-body diagram is sketched in Figure 4.29a.

(b) The train has a constant acceleration directed east so the ball must have a constant eastward acceleration. There must be a net horizontal force on the ball, directed to the east. This net force must come from an eastward component of $\vec{T}$ and the ball hangs with the string displaced west of vertical. The free-body diagram is sketched in Figure 4.29b.

EVALUATE: When the motion of an object is described in an inertial frame, there must be a net force in the direction of the acceleration.

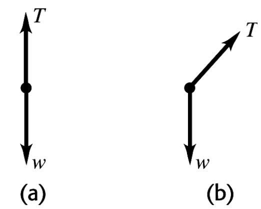


Figure 4.29

4.30. **IDENTIFY:** Identify the forces on the chair. The floor exerts a normal force and a friction force.
SET UP: Let $+y$ be upward and let $+x$ be in the direction of the motion of the chair.

EXECUTE: **(a)** The free-body diagram for the chair is given in Figure 4.30.
(b) For the chair, $a_y = 0$ so $\sum F_y = ma_y$ gives $n - mg - F\sin 37° = 0$ and $n = 142$ N.

EVALUATE: n is larger than the weight because $\vec{F}$ has a downward component.

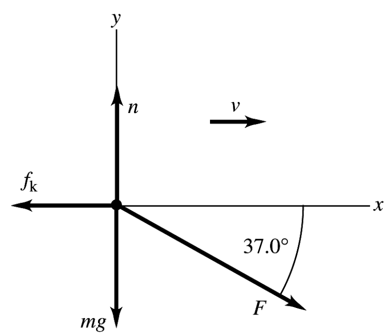


Figure 4.30

4.31. **IDENTIFY:** This problem requires some estimation and a web search.
SET UP: We want to find the force the pitcher exerts on a baseball while pitching a fastball. Estimate: The distance a pitcher moves the ball during a pitch is about twice an arm length, which is about 3.0 ft.
Web search: A major league baseball weighs between 5.00 oz and 5.25 oz, so use an average of 5.125 oz, which is 0.320 lb with a mass of 1.0×10^{-2} slugs. The average speed of a pitched fastball is 92 mph which is 135 ft/s.
Assumptions: The ball moves in a straight horizontal line with constant acceleration during the pitch. We use the information above to calculate the acceleration of the ball during the pitch. Then apply Newton's second law to find the average force on the ball. $\sum F_x = ma_x$ applies to the ball and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ for constant acceleration.

EXECUTE: Use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to find the acceleration: $(135 \text{ ft/s})^2 = 0 + 2a_x(3.0 \text{ ft})$, which gives $a_x = 3.0 \times 10^3 \text{ ft/s}^2$. Now apply $\sum F_x = ma_x$ to the ball. The only force accelerating it is the push P of the pitcher, so $P = ma = (1.0 \times 10^{-2} \text{ slugs})(3.0 \times 10^3 \text{ ft/s}^2) = 30$ lb. This push is about 130 N.
EVALUATE: A 30-lb push may not seem like much, but the ball has a rather small mass, so this push can produce a large acceleration. In this case, $a_x/g = (3000 \text{ ft/s}^2)/(32.2 \text{ ft/s}^2) = 93$, so a_x is nearly $100g$!

4.32. **IDENTIFY:** Use the motion of the ball to calculate g, the acceleration of gravity on the planet. Then $w = mg$.

SET UP: Let $+y$ be downward and take $y_0 = 0$. $v_{0y} = 0$ since the ball is released from rest.

EXECUTE: Get g on X: $y = \frac{1}{2}gt^2$ gives $10.0 \text{ m} = \frac{1}{2}g(3.40 \text{ s})^2$. $g = 1.73 \text{ m/s}^2$ and then

$w_X = mg_X = (0.100 \text{ kg})(1.73 \text{ m/s}^2) = 0.173 \text{ N}.$

EVALUATE: g on Planet X is smaller than on earth and the object weighs less than it would on earth.

4.33. **IDENTIFY:** Apply Newton's second law to the bucket and constant-acceleration kinematics.
SET UP: The minimum time to raise the bucket will be when the tension in the cord is a maximum since this will produce the greatest acceleration of the bucket.
EXECUTE: Apply Newton's second law to the bucket: $T - mg = ma$. For the maximum acceleration, the tension is greatest, so $a = \frac{T - mg}{m} = \frac{75.0 \text{ N} - (5.60 \text{ kg})(9.8 \text{ m/s}^2)}{5.60 \text{ kg}} = 3.593 \text{ m/s}^2.$

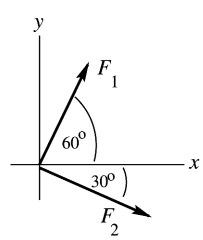
The kinematics equation for $y(t)$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(12.0 \text{ m})}{3.593 \text{ m/s}^2}} = 2.58 \text{ s}.$

EVALUATE: A shorter time would require a greater acceleration and hence a stronger pull, which would break the cord.

4.34. **IDENTIFY:** The pull accelerates both blocks, so we apply Newton's second law to each one.
SET UP: Apply $\sum F_x = ma_x$ to each block. Using B we can find the friction force on each block, which is also the friction force on A, by Newton's third law. Then using A we can find its acceleration. The target variable is the acceleration of block A.
EXECUTE: Block B: The friction force on B due to A opposes the pull. So $\sum F_x = ma_x$ gives $P - f = m_B a_B$, so $f = P - m_B a_B = 12.0 \text{ N} - (6.00 \text{ kg})(1.80 \text{ m/s}^2) = 1.20$ N. Now apply $\sum F_x = ma_x$ to block A. Only friction accelerates this block forward, and it must be 1.20 N. Thus $f = m_A a_A$, which becomes $1.20 \text{ N} = (2.00 \text{ kg})a_A$, so $a_A = 0.600 \text{ m/s}^2$.

EVALUATE: Block B accelerates forward at 1.80 m/s^2 but block A accelerates forward at only 0.600 m/s^2. Thus A is not keeping up with B, which means it is sliding *backward* as observed by a person moving with B.

4.35. **IDENTIFY:** If the box moves in the $+x$-direction it must have $a_y = 0$, so $\sum F_y = 0$.



The smallest force the child can exert and still produce such motion is a force that makes the y-components of all three forces sum to zero, but that doesn't have any x-component.

Figure 4.35

SET UP: $\vec{F}_1$ and $\vec{F}_2$ are sketched in Figure 4.35. Let $\vec{F}_3$ be the force exerted by the child.
$\sum F_y = ma_y$ implies $F_{1y} + F_{2y} + F_{3y} = 0$, so $F_{3y} = -(F_{1y} + F_{2y})$.

EXECUTE: $F_{1y} = +F_1 \sin 60° = (100 \text{ N}) \sin 60° = 86.6 \text{ N}$

$F_{2y} = +F_2 \sin(-30°) = -F_2 \sin 30° = -(140 \text{ N}) \sin 30° = -70.0 \text{ N}$

Then $F_{3y} = -(F_{1y} + F_{2y}) = -(86.6 \text{ N} - 70.0 \text{ N}) = -16.6 \text{ N}$; $F_{3x} = 0$

The smallest force the child can exert has magnitude 17 N and is directed at $90°$ clockwise from the $+x$-axis shown in the figure.

(b) IDENTIFY and **SET UP:** Apply $\sum F_x = ma_x$. We know the forces and a_x so can solve for m. The force exerted by the child is in the $-y$-direction and has no x-component.

EXECUTE: $F_{1x} = F_1 \cos 60° = 50\text{ N}$

$F_{2x} = F_2 \cos 30° = 121.2\text{ N}$

$\sum F_x = F_{1x} + F_{2x} = 50\text{ N} + 121.2\text{ N} = 171.2\text{ N}$

$$m = \frac{\sum F_x}{a_x} = \frac{171.2\text{ N}}{2.00\text{ m/s}^2} = 85.6\text{ kg}$$

Then $w = mg = 840\text{ N}$.

EVALUATE: In part (b) we don't need to consider the y-component of Newton's second law. $a_y = 0$ so the mass doesn't appear in the $\sum F_y = ma_y$ equation.

4.36. **IDENTIFY:** Use constant acceleration equations to calculate the acceleration a_x that would be required. Then use $\sum F_x = ma_x$ to find the necessary force.

SET UP: Let $+x$ be the direction of the initial motion of the auto.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_x = 0$ gives $a_x = -\dfrac{v_{0x}^2}{2(x - x_0)}$. The force F is directed opposite

to

the motion and $a_x = -\dfrac{F}{m}$. Equating these two expressions for a_x gives

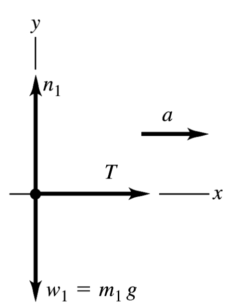
$$F = m\frac{v_{0x}^2}{2(x - x_0)} = (850\text{ kg})\frac{(12.5\text{ m/s})^2}{2(1.8 \times 10^{-2}\text{ m})} = 3.7 \times 10^6\text{ N}.$$

EVALUATE: A very large force is required to stop such a massive object in such a short distance.

4.37. **IDENTIFY:** Use Newton's second law to relate the acceleration and forces for each crate.

(a) SET UP: Since the crates are connected by a rope, they both have the same acceleration, 2.50 m/s^2.

(b) The forces on the 4.00 kg crate are shown in Figure 4.37a.



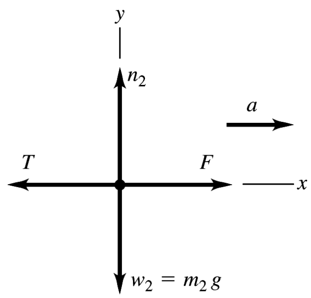
EXECUTE:

$\sum F_x = ma_x$

$T = m_1 a = (4.00\text{ kg})(2.50\text{ m/s}^2) = 10.0\text{ N}.$

Figure 4.37a

(c) SET UP: Forces on the 6.00 kg crate are shown in Figure 4.37b.



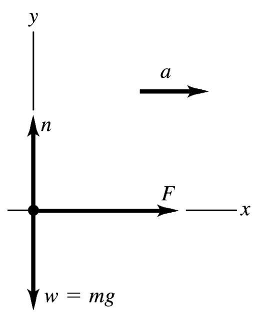
The crate The crate accelerates to the right, so the net force is to the right. F must be larger than T.

Figure 4.37b

(d) EXECUTE: $\sum F_x = ma_x$ gives $F - T = m_2 a$

$F = T + m_2 a = 10.0\text{ N} + (6.00\text{ kg})(2.50\text{ m/s}^2) = 10.0\text{ N} + 15.0\text{ N} = 25.0\text{ N}$

EVALUATE: We can also consider the two crates and the rope connecting them as a single object of mass $m = m_1 + m_2 = 10.0$ kg. The free-body diagram is sketched in Figure 4.37c.



$\sum F_x = ma_x$

$F = ma = (10.0\text{ kg})(2.50\text{ m/s}^2) = 25.0\text{ N}$

This agrees with our answer in part (d).

Figure 4.37c

4.38. **IDENTIFY:** Use kinematics to find the acceleration and then apply Newton's second law.
SET UP: The 60.0-N force accelerates both blocks, but only the tension in the rope accelerates block B. The force F is constant, so the acceleration is constant, which means that the standard kinematics formulas apply. There is no friction.
EXECUTE: **(a)** First use kinematics to find the acceleration of the system. Using $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ with $x - x_0 = 18.0$ m, $v_{0x} = 0$, and $t = 5.00$ s, we get $a_x = 1.44\text{ m/s}^2$. Now apply Newton's second law to the horizontal motion of block A, which gives $F - T = m_A a$. $T = 60.0\text{ N} - (15.0\text{ kg})(1.44\text{ m/s}^2) = 38.4$ N.
(b) Apply Newton's second law to block B, giving $T = m_B a$. $m_B = T/a = (38.4\text{ N})/(1.44\text{ m/s}^2) = 26.7$ kg.
EVALUATE: As an alternative approach, consider the two blocks as a single system, which makes the tension an internal force. Newton's second law gives $F = (m_A + m_B)a$. Putting in numbers gives 60.0 N $= (15.0\text{ kg} + m_B)(1.44\text{ m/s}^2)$, and solving for m_B gives 26.7 kg. Now apply Newton's second law to either block A or block B and find the tension.

4.39. **IDENTIFY** and **SET UP:** Take derivatives of $x(t)$ to find v_x and a_x. Use Newton's second law to relate the acceleration to the net force on the object.
EXECUTE:
(a) $x = (9.0\times10^3\text{ m/s}^2)t^2 - (8.0\times10^4\text{m/s}^3)t^3$

$x = 0$ at $t = 0$

When $t = 0.025$ s, $x = (9.0\times10^3\text{ m/s}^2)(0.025\text{ s})^2 - (8.0\times10^4\text{ m/s}^3)(0.025\text{ s})^3 = 4.4$ m.

The length of the barrel must be 4.4 m.

(b) $v_x = \dfrac{dx}{dt} = (18.0\times10^3\ \text{m/s}^2)t - (24.0\times10^4\ \text{m/s}^3)t^2$

At $t = 0$, $v_x = 0$ (object starts from rest).

At $t = 0.025$ s, when the object reaches the end of the barrel,

$v_x = (18.0\times10^3\ \text{m/s}^2)(0.025\ \text{s}) - (24.0\times10^4\ \text{m/s}^3)(0.025\ \text{s})^2 = 300\ \text{m/s}$

(c) $\sum F_x = ma_x$, so must find a_x.

$\text{a}_\text{x} = \dfrac{\text{dv}_\text{x}}{\text{dt}} = 18.0\times10^3\ \text{m/s}^2 - (48.0\times10^4\ \text{m/s}^3)\,\text{t}$

(i) At $t = 0$, $a_x = 18.0\times10^3\ \text{m/s}^2$ and $\sum F_x = (1.50\ \text{kg})(18.0\times10^3\ \text{m/s}^2) = 2.7\times10^4\ \text{N}$.

(ii) At $t = 0.025$ s, $a_x = 18\times10^3\ \text{m/s}^2 - (48.0\times10^4\ \text{m/s}^3)(0.025\ \text{s}) = 6.0\times10^3\ \text{m/s}^2$ and

$\sum F_x = (1.50\ \text{kg})(6.0\times10^3\ \text{m/s}^2) = 9.0\times10^3\ \text{N}$.

EVALUATE: The acceleration and net force decrease as the object moves along the barrel.

4.40. **IDENTIFY:** This problem involves Newton's second law. The rocket's motion occurs in two stages: during the first stage, its engines produce a constant upward force, and during the second stage the engines turn off. Gravity acts during both stages.

SET UP: Apply $\sum F_y = ma_y$ during both stages. During the first stage, the engine force F acts upward and gravity mg acts downward. During the second stage, only gravity acts on the rocket. The constant-acceleration formulas apply during both stages, but with different acceleration in each stage. Fig. 4.40 shows the information. The rocket stops moving at its highest point.

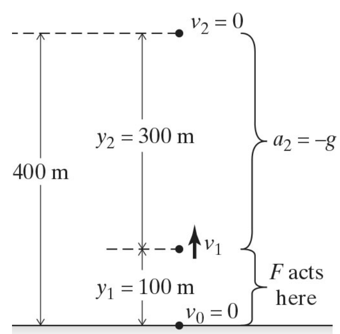


Figure 4.40

EXECUTE: Look at the stages one at a time.

First stage: Apply $\sum F_y = ma_y$. $T - mg = ma_1$ gives $a_1 = (F - mg)/m$. Now apply $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to relate v_1 to F, using the a_1 we just found. This gives $v_1^2 = 0 + 2\left(\dfrac{F - mg}{m}\right)y_1$.

Second stage: Apply $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. The v_{0y} for the second stage is v_1 from the first stage. Using the result for v_1^2 from the first stage, this equation gives $0 = 2\left(\dfrac{F - mg}{m}\right)y_1 - 2gy_2$.

Solving for F gives $F = \dfrac{mg}{y_1}(y_1 + y_2) = \dfrac{(400\ \text{kg})(9.80\ \text{m/s})}{100\ \text{m}}(400\ \text{m}) = 1.57\times10^4\ \text{N}$.

EVALUATE: We cannot do this problem in a single step because the acceleration is different in the two stages. We cannot treat this as a single process using the average acceleration because the accelerations last for different times.

4.41. **IDENTIFY:** You observe that your weight is different from your normal weight in an elevator, so you must have acceleration. Apply $\sum \vec{F} = m\vec{a}$ to your body inside the elevator.

SET UP: The quantity $w = 683\text{ N}$ is the force of gravity exerted on you, independent of your motion. Your mass is $m = w/g = 69.7\text{ kg}$. Use coordinates with $+y$ upward. Your free-body diagram is shown in Figure 4.41, where n is the scale reading, which is the force the scale exerts on you. You and the elevator have the same acceleration.

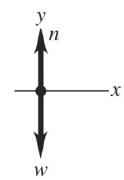


Figure 4.41

EXECUTE: $\sum F_y = ma_y$ gives $n - w = ma_y$ so $a_y = \dfrac{n-w}{m}$.

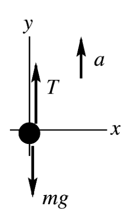
(a) $n = 725\text{ N}$, so $a_y = \dfrac{725\text{ N} - 683\text{ N}}{69.7\text{ kg}} = 0.603\text{ m/s}^2$. a_y is positive so the acceleration is upward.

(b) $n = 595\text{ N}$, so $a_y = \dfrac{595\text{ N} - 683\text{ N}}{69.7\text{ kg}} = -1.26\text{ m/s}^2$. a_y is negative so the acceleration is downward.

EVALUATE: If you appear to weigh less than your normal weight, you must be accelerating downward, but not necessarily *moving* downward. Likewise if you appear to weigh more than your normal weight, you must be acceleration upward, but you could be *moving* downward.

4.42. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the elevator to relate the forces on it to the acceleration.

(a) SET UP: The free-body diagram for the elevator is sketched in Figure 4.42.



The net force is $T - mg$ (upward).

Figure 4.42

Take the $+y$-direction to be upward since that is the direction of the acceleration. The maximum upward acceleration is obtained from the maximum possible tension in the cables.

EXECUTE: $\sum F_y = ma_y$ gives $T - mg = ma$

$$a = \frac{T - mg}{m} = \frac{28{,}000\text{ N} - (2200\text{ kg})(9.80\text{ m/s}^2)}{2200\text{ kg}} = 2.93\text{ m/s}^2.$$

(b) What changes is the weight mg of the elevator.

$$a = \frac{T - mg}{m} = \frac{28{,}000\text{ N} - (2200\text{ kg})(1.62\text{ m/s}^2)}{2200\text{ kg}} = 11.1\text{ m/s}^2.$$

EVALUATE: The cables can give the elevator a greater acceleration on the moon since the downward force of gravity is less there and the same T then gives a greater net force.

4.43. **IDENTIFY:** The ball changes velocity, so it has acceleration. Therefore Newton's second law applies to it.

SET UP: Apply $\sum F_x = ma_x$ to the ball. Assume that the acceleration is constant, so we can use the constant-acceleration equation $v_x = v_{0x} + a_x t$. Call the x-axis horizontal with $+x$ in the direction of the ball's original velocity.

EXECUTE: First use $v_x = v_{0x} + a_x t$ to find the acceleration.

-50.0 m/s = 40.0 m/s + $a_x(8.00 \times 10^{-3}$ s) $\rightarrow$ $a_x = -1.125 \times 10^4$ m/s^2.

Now apply $\sum F_x = ma_x$ to the ball. Only the bat exerts a horizontal force on the ball.

$F_{\text{bat}} = ma_x = (0.145 \text{ kg})(-1.125 \times 10^4 \text{ m/s}^2) = -1630$ N. The minus sign tells us that the force and the acceleration are directed opposite to the original velocity of the ball, which is away from the batter.

EVALUATE: Compare the acceleration to g: $a/g = (1.125 \times 10^4 \text{ m/s}^2)/(9.80 \text{ m/s}^2) = 1150$, so the acceleration is $1150g$ – a huge acceleration but for only a very brief time.

4.44. **IDENTIFY:** The object has acceleration, so Newton's second law applies to it. We need to find the acceleration using the equation for its position as a function of time.

SET UP: First find the acceleration using $v_x = dx/dt$ and $a_x = dv_x/dt$. Then apply $\sum F_x = ma_x$. We know that $x(t) = \alpha t^2 - 2\beta t$.

EXECUTE: $v_x = d(\alpha t^2 - 2\beta t)/dt = 2\alpha t - 2\beta$, $a_x = d($a $2\alpha t - 2\beta)/dt = 2\alpha$. Now apply $\sum F_x = ma_x$, giving $F_{\text{net}} = m(2\alpha) = 2\alpha m$.

EVALUATE: Check units: αt^2 must have SI units of m, so α has units of m/s^2. Thus the units of $2\alpha m$ are (m/s^2)(kg) = $\text{kg} \cdot \text{m/s}^2$ = N, so our answer has the proper units.

4.45. **IDENTIFY:** The system is accelerating, so we apply Newton's second law to each box and can use the constant acceleration kinematics for formulas to find the acceleration.

SET UP: First use the constant acceleration kinematics for formulas to find the acceleration of the system. Then apply $\sum F = ma$ to each box.

EXECUTE: **(a)** The kinematics formula $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(12.0 \text{ m})}{(4.0 \text{ s})^2} = 1.5 \text{ m/s}^2$. For box B, $mg - T = ma$ and

$m = \dfrac{T}{g - a} = \dfrac{36.0 \text{ N}}{9.8 \text{ m/s}^2 - 1.5 \text{ m/s}^2} = 4.34$ kg.

(b) For box A, $T + mg - F = ma$ and $m = \dfrac{F - T}{g - a} = \dfrac{80.0 \text{ N} - 36.0 \text{ N}}{9.8 \text{ m/s}^2 - 1.5 \text{ m/s}^2} = 5.30$ kg.

EVALUATE: The boxes have the same acceleration but experience different forces because they have different masses.

4.46. **IDENTIFY:** Note that in this problem the mass of the rope is given, and that it is not negligible compared to the other masses. Apply $\sum \vec{F} = m\vec{a}$ to each object to relate the forces to the acceleration.

(a) SET UP: The free-body diagrams for each block and for the rope are given in Figure 4.46a.

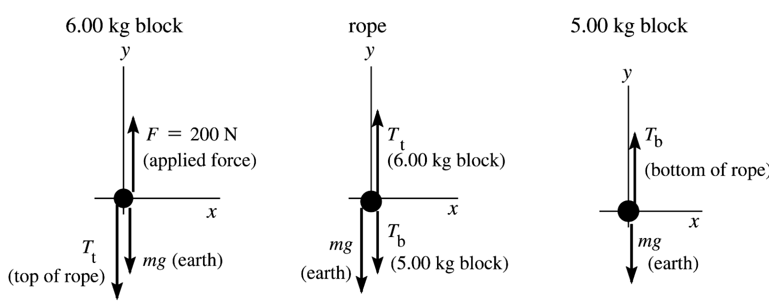
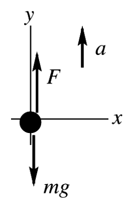


Figure 4.46a

T_t is the tension at the top of the rope and T_b is the tension at the bottom of the rope.

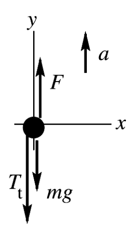
EXECUTE: **(b)** Treat the rope and the two blocks together as a single object, with mass $m = 6.00\text{ kg} + 4.00\text{ kg} + 5.00\text{ kg} = 15.0\text{ kg}$. Take $+y$ upward, since the acceleration is upward. The free-body diagram is given in Figure 4.46b.



$$\sum F_y = ma_y$$
$$F - mg = ma$$
$$a = \frac{F - mg}{m}$$
$$a = \frac{200\text{ N} - (15.0\text{ kg})(9.80\text{ m/s}^2)}{15.0\text{ kg}} = 3.53\text{ m/s}^2$$

Figure 4.46b

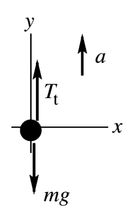
(c) Consider the forces on the top block $(m = 6.00\text{ kg})$, since the tension at the top of the rope (T_t) will be one of these forces.



$$\sum F_y = ma_y$$
$$F - mg - T_t = ma$$
$$T_t = F - m(g + a)$$
$$T_t = 200\text{ N} - (6.00\text{ kg})(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120\text{ N.}$$

Figure 4.46c

Alternatively, you can consider the forces on the combined object rope plus bottom block $(m = 9.00\text{ kg})$:

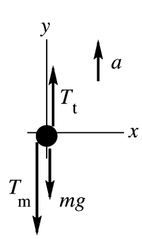


$$\sum F_y = ma_y$$
$$T_t - mg = ma$$
$$T_t = m(g + a) = 9.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120$$

which checks

Figure 4.46d

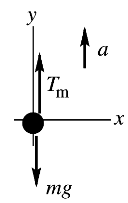
(d) One way to do this is to consider the forces on the top half of the rope $(m = 2.00\text{ kg})$. Let T_m be the tension at the midpoint of the rope.



$$\sum F_y = ma_y$$
$$T_t - T_m - mg = ma$$
$$T_m = T_t - m(g + a) = 120\text{ N} - 2.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2)$$

Figure 4.46e

To check this answer we can alternatively consider the forces on the bottom half of the rope plus the lower block taken together as a combined object $(m = 2.00\text{ kg} + 5.00\text{ kg} = 7.00\text{ kg})$:



$$\sum F_y = ma_y$$
$$T_m - mg = ma$$
$$T_m = m(g+a) = 7.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 93.$$

which checks

Figure 4.46f

EVALUATE: The tension in the rope is not constant but increases from the bottom of the rope to the top. The tension at the top of the rope must accelerate the rope as well the 5.00-kg block. The tension at the top of the rope is less than F; there must be a net upward force on the 6.00-kg block.

4.47. **IDENTIFY:** The rocket engines reduce the speed of the rocket, so Newton's second law applies to the rocket. Two vertical forces act on the rocket: gravity and the force F of the engines. The constant-acceleration equations apply because both F and gravity are constant, which makes the acceleration constant.

SET UP: The rocket needs to reduce its speed from 30.0 m/s to zero while traveling 80.0 m. We can find the acceleration from this information using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. Then we can apply $\sum F_y = ma_y$ to find the force F. Choose the $+y$-axis upward with the origin at the ground.

EXECUTE: We know that $v_y = 0$ as the rocket reaches the ground. Using $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ we get $0 = (-30.0\text{ m/s})^2 + 2a_y(0 - 80.0\text{ m})$ $\rightarrow$ $a_y = 5.625\text{ m/s}^2$. Now apply $\sum F_y = ma_y$.

$F - mg = ma_y$ $\rightarrow$ $F - (20.0\text{ kg})(9.80\text{ m/s}^2) = (20.0\text{ kg})(5.625\text{ m/s}^2)$ $\rightarrow$ $F = 309\text{ N}$.

EVALUATE: The weight of this rocket is $w = mg = (20.0\text{ kg})(9.80\text{ m/s}^2) = 196\text{ N}$, so we see that $F > w$. This is reasonable because the engine force must oppose gravity to reduce the rocket's speed. Notice that the rocket is moving downward but its acceleration is upward. This is reasonable because the rocket is slowing down, so its acceleration must be opposite to its velocity.

4.48. **IDENTIFY:** On the planet Newtonia, you make measurements on a tool by pushing on it and by dropping it. You want to use those results to find the weight of the object on that planet and on earth.

SET UP: Using $w = mg$, you could find the weight if you could calculate the mass of the tool and the acceleration due to gravity on Newtonia. Newton's laws of motion are applicable on Newtonia, as is your knowledge of falling objects. Let m be the mass of the tool. There is no appreciable friction. Use coordinates where $+x$ is horizontal, in the direction of the 12.0 N force, and let $+y$ be downward.

EXECUTE: First find the mass m: $x - x_0 = 16.0\text{ m}$, $t = 2.00\text{ s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(16.0\text{ m})}{(2.00\text{ s})^2} = 8.00\text{ m/s}^2$. Now apply Newton's second law to the tool. $\sum F_x = ma_x$ gives $F = ma_x$ and $m = \dfrac{F}{a_x} = \dfrac{12.0\text{ N}}{8.00\text{ m/s}^2} = 1.50\text{ kg}$. Find g_N, the acceleration due to gravity on Newtonia. $y - y_0 = 10.0\text{ m}$, $v_{0y} = 0$, $t = 2.58\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(10.0\text{ m})}{(2.58\text{ s})^2} = 3.00\text{ m/s}^2$; $g_N = 3.00\text{ m/s}^2$. The weight on Newtonia is $w_N = mg_N = (1.50\text{ kg})(3.00\text{ m/s}^2) = 4.50\text{ N}$. The weight on earth is $w_E = mg_E = (1.50\text{ kg})(9.80\text{ m/s}^2) = 14.7\text{ N}$.

EVALUATE: The tool weighs about 1/3 on Newtonia of what it weighs on earth since the acceleration due to gravity on Newtonia is about 1/3 what it is on earth.

4.49. **IDENTIFY:** The rocket accelerates due to a variable force, so we apply Newton's second law. But the acceleration will not be constant because the force is not constant.

SET UP: We can use $a_x = F_x/m$ to find the acceleration, but must integrate to find the velocity and then the distance the rocket travels.

EXECUTE: Using $a_x = F_x/m$ gives $a_x(t) = \dfrac{(16.8\text{ N/s})t}{45.0\text{ kg}} = (0.3733\text{ m/s}^3)t$. Now integrate the acceleration to get the velocity, and then integrate the velocity to get the distance moved. $v_x(t) = v_{0x} + \int_0^t a_x(t')dt' = (0.1867\text{ m/s}^3)t^2$ and $x - x_0 = \int_0^t v_x(t')dt' = (0.06222\text{ m/s}^3)t^3$. At $t = 5.00$ s, $x - x_0 = 7.78$ m.

EVALUATE: The distance moved during the next 5.0 s would be considerably greater because the acceleration is increasing with time.

4.50. **IDENTIFY:** A constant force is applied to an object, which gives it a constant acceleration. We apply Newton's second law and can use the constant-acceleration equations.

SET UP: The object starts from rest reaches a speed v_1 due to force F_1 which gives it an acceleration $a_1 = F_1/m$. $\sum F_x = ma_x$ applies to the object.

EXECUTE: **(a)** With F_1 acting, the object travels a distance d. Now with $F_2 = 2F_1$ acting, the object still travels a distance d but reaches speed v_2. We want to find v_2 in terms of v_1. In this case, twice the force acts over the same distance d. We know that $a_1 = F_1/m$ and $a_2 = F_2/m = 2F_1/m = 2a_1$. Applying $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ to both situations gives $v_1^2 = 2a_1 d$ and $v_2^2 = 2a_2 d$. Since $a_2 = 2a_1$, the last equation gives $v_2^2 = 2(2a_1)d = 2(2a_1 d)$, which tells us that $v_2^2 = 2v_1^2$, so $v_2 = \sqrt{2}v_1$.

(b) In this case, twice the force acts for the same amount of time T. As we saw above, $a_2 = 2a_1$. Applying $v_x = v_{0x} + a_x t$ to both situations gives $v_1 = a_1T$ and $v_2 = a_2T = (2a_1)T = 2(a_1T) = 2v_2$, so we find that $v_2 = 2v_1$.

EVALUATE: We have seen in (a) that doubling the force over the same distance increases the speed by a factor of $\sqrt{2}$, but in (b) we saw that doubling the force for the same amount of time increases the speed by a factor of 2.

4.51. **IDENTIFY:** Kinematics will give us the average acceleration of each car, and Newton's second law will give us the average force that is accelerating each car.

SET UP: The cars start from rest and all reach a final velocity of 60 mph (26.8 m/s). We first use kinematics to find the average acceleration of each car, and then use Newton's second law to find the average force on each car.

EXECUTE: **(a)** We know the initial and final velocities of each car and the time during which this change in velocity occurs. The definition of average acceleration gives $a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$. Then $F = ma$ gives the force on each car. For the Alpha Romeo, the calculations are $a_{\text{av}} = (26.8\text{ m/s})/(4.4\text{ s}) = 6.09\text{ m/s}^2$. The force is $F = ma = (895\text{ kg})(6.09\text{ m/s}^2) = 5.451 \times 10^3\text{ N} = 5.451$ kN, which we should round to 5.5 kN for 2 significant figures. Repeating this calculation for the other cars and rounding the force to 2 significant figures gives:

Alpha Romeo: $a = 6.09\text{ m/s}^2$, $F = 5.5$ kN
Honda Civic: $a = 4.19\text{ m/s}^2$, $F = 5.5$ kN
Ferrari: $a = 6.88\text{ m/s}^2$, $F = 9.9$ kN
Ford Focus: $a = 4.97\text{ m/s}^2$, $F = 7.3$ kN
Volvo: $a = 3.72\text{ m/s}^2$, $F = 6.1$ kN

The smallest net force is on the Alpha Romeo and Honda Civic, to two-figure accuracy. The largest net force is on the Ferrari.
(b) The largest force would occur for the largest acceleration, which would be in the Ferrari. The smallest force would occur for the smallest acceleration, which would be in the Volvo.
(c) We use the same approach as in part (a), but now the final velocity is 100 mph (44.7 m/s).
a_{av} = (44.7 m/s)/(8.6 s) = 5.20 m/s^2, and $F = ma$ = (1435 kg)(5.20 m/s^2) = 7.5 kN. The average force is considerably smaller in this case. This is because air resistance increases with speed.
(d) As the speed increases, so does the air resistance. Eventually the air resistance will be equal to the force from the roadway, so the new force will be zero and the acceleration will also be zero, so the speed will remain constant.
EVALUATE: The actual forces and accelerations involved with auto dynamics can be quite complicated because the forces (and hence the accelerations) are not constant but depend on the speed of the car.

4.52. **IDENTIFY:** Calculate $\vec{a}$ from $\vec{a} = d^2\vec{r}/dt^2$. Then $\vec{F}_{\text{net}} = m\vec{a}$.
SET UP: $w = mg$
EXECUTE: Differentiating twice, the acceleration of the helicopter as a function of time is $\vec{a} = (0.120\ \text{m/s}^3)t\hat{\boldsymbol{i}} - (0.12\ \text{m/s}^2)\hat{\boldsymbol{k}}$ and at $t = 5.0$ s, the acceleration is $\vec{a} = (0.60\ \text{m/s}^2)\hat{\boldsymbol{i}} - (0.12\ \text{m/s}^2)\hat{\boldsymbol{k}}$.
The force is then

$$\vec{F} = m\vec{a} = \frac{w}{g}\vec{a} = \frac{(2.75\times10^5\ \text{N})}{(9.80\ \text{m/s}^2)}\left[(0.60\ \text{m/s}^2)\hat{\boldsymbol{i}} - (0.12\ \text{m/s}^2)\hat{\boldsymbol{k}}\right] = (1.7\times10^4\ \text{N})\hat{\boldsymbol{i}} - (3.4\times10^3\ \text{N})\hat{\boldsymbol{k}}$$

EVALUATE: The force and acceleration are in the same direction. They are both time dependent.

4.53. **IDENTIFY:** A block is accelerated upward by a force of magnitude F. For various forces, we know the time for the block to move upward a distance of 8.00 m starting from rest. Since the upward force is constant, so is the acceleration. Newton's second law applies to the accelerating block.
SET UP: The acceleration is constant, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ applies, and $\sum F_y = ma_y$ also applies to the block.
EXECUTE: **(a)** Using the above formula with $v_{0y} = 0$ and $y - y_0 = 8.00$ m, we get $a_y = (16.0\ \text{m})/t^2$. We use this formula to calculate the acceleration for each value of the force F. For example, when F= 250 N, we have a = (16.0 m)/(3.3 s)^2 = 1.47 m/s^2. We make similar calculations for all six values of F and then graph F versus a. We can do this graph by hand or using graphing software. The result is shown in Figure 4.53.

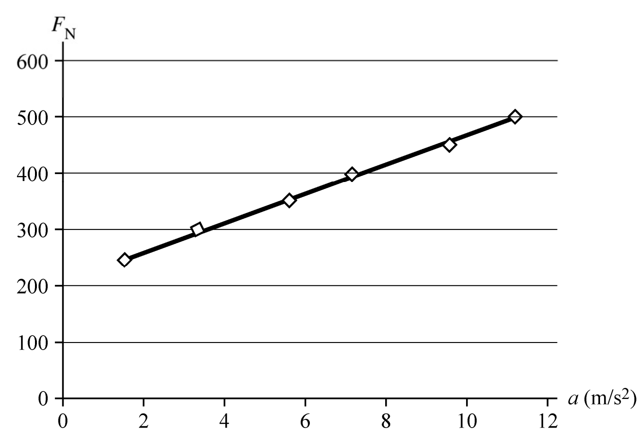


Figure 4.53

(b) Applying Newton's second law to the block gives $F - mg = ma$, so $F = mg + ma$. The equation of our best-fit graph in part (a) is $F = (25.58\text{ kg})a + 213.0\text{ N}$. The slope of the graph is the mass m, so the mass of the block is $m = 26$ kg. The y intercept is mg, so $mg = 213$ N, which gives $g = (213\text{ N})/(25.58\text{ kg}) = 8.3\text{ m/s}^2$ on the distant planet.

EVALUATE: The acceleration due to gravity on this planet is not too different from what it is on earth.

4.54. **IDENTIFY:** The box comes to a stop, so it must have acceleration, so Newton's second law applies. For constant acceleration, the standard kinematics formulas apply.

SET UP: For constant acceleration, $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ and $v_x = v_{0x} + a_xt$ apply. For any motion, $\vec{F}_{\text{net}} = m\vec{a}$.

EXECUTE: **(a)** If the box comes to rest with constant acceleration, its final velocity is zero so $v_{0x} = -a_xt$. And if during this time it travels a distance $x - x_0 = d$, the distance formula above can be put into the form
$d = (-a_xt) + ½\, a_xt^2 = -½\, a_xt^2$. This gives $a_x = -2d/t^2$. For the first push on the box, this gives
$a_x = -2(8.22\text{ m})/(2.8\text{ s})^2 = -2.1\text{ m/s}^2$. If the acceleration is constant, the distance the box should travel after the second push is $d = -½\, a_xt^2 = -(½)(-2.1\text{ m/s}^2)(2.0\text{ s})^2 = 4.2$ m, which is in fact the distance the box did travel. Therefore the acceleration was constant.

(b) The total mass m_T of the box is the initial mass (8.00 kg) plus the added mass. Since $v_x = 0$ and $a_x = 2d/t^2$ as shown in part (a), the magnitude of the initial speed v_{0x} is $v_{0x} = a_xt = (2d/t^2)t = 2d/t$. For no added mass, this calculation gives $v_{0x} = 2(8.22\text{ m})/(2.8\text{ s}) = 5.87$ m/s. Similar calculations with added mass give

$m_T = 8.00$ kg, $v_{0x} = 5.87\text{ m/s} \approx 5.9$ m/s
$m_T = 11.00$ kg, $v_{0x} = 6.72\text{ m/s} \approx 6.7$ m/s
$m_T = 15.00$ kg, $v_{0x} = 6.30\text{ m/s} \approx 6.3$ m/s
$m_T = 20.00$ kg, $v_{0x} = 5.46\text{ m/s} \approx 5.5$ m/s

where all answers have been rounded to 2 significant figures. It is obvious that the initial speed was *not* the same in each case. The ratio of maximum speed to minimum speed is
$v_{0,\max}/v_{0,\min} = (6.72\text{ m/s})/(5.46\text{ m/s}) = 1.2$

(c) We calculate the magnitude of the force f using $f = ma$, getting a using $a = -2d/t^2$, as we showed in part (a). In each case the acceleration is 2.1 m/s^2. So for example, when $m = 11.00$ kg, the force is $f = (11.00\text{ kg})(2.1\text{ m/s}^2) = 23$ N. Similar calculations produce a set of values for f and m. These can be graphed by hand or using graphing software. The resulting graph is shown in Figure 4.54. The slope of this straight-line graph is 2.1 m/s^2 and it passes through the origin, so the slope-y intercept equation of the line is $f = (2.1\text{ m/s}^2)m$.

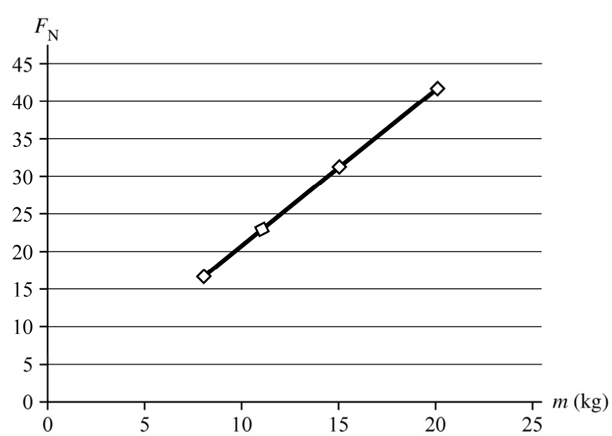


Figure 4.54

EVALUATE: The results of the graph certainly agree with Newton's second law. A graph of F versus m should have slope equal to the acceleration a. This is in fact just what we get, since the acceleration is 2.1 m/s^2 which is the same as the slope of the graph.

4.55. IDENTIFY: The force on the block is a function of time, so the acceleration will also be a function of time. Therefore we cannot use the constant-acceleration formulas, but instead must use the basic velocity and acceleration definitions. We also need to apply Newton's second law.

SET UP: Using $\sum F_x = ma_x$ we can determine the acceleration and use it to find the velocity and position of the block. We use $a_x = dv_x/dt$ and $v_x = dx/dt$. We know that $F(t) = \beta - \alpha t$ and that the object starts from rest.

EXECUTE: **(a)** We want the largest positive value of x. Using $\sum F_x = ma_x$ we can find $a_x(t)$, and from that we can find $v_x(t)$ and then find $x(t)$. First find a_x: $a_x(t) = F_x/m = (\beta - \alpha t)/m$.

Now use $a_x = dv_x/dt$ to find $v_x(t)$. $v_x = \int a_x dt = \int \frac{\beta - \alpha t}{m} dt = \frac{\beta t - \frac{\alpha t^2}{2}}{m}$, where we have used $v_x = 0$ when $t = 0$. Now use $v_x = dx/dt$ to find $x(t)$. $x(t) = \int v_x dt = \int \frac{\beta t - \frac{\alpha t^2}{2}}{m} dt = \frac{\frac{\beta t^2}{2} - \frac{\alpha t^3}{6}}{m}$, where we have used $x = 0$ when $t = 0$. The largest value of x will occur when $v_x = 0$ because the block will go no further.

Equating v_x to zero gives $v_x = \frac{\beta t - \frac{\alpha t^2}{2}}{m} = 0$. Solving for t and calling it t_{max} gives $t_{max} = 2\beta/\alpha$.

Now use t_{max} in our equation for $x(t)$ to find the largest value of x. This gives

$$x(t_{max}) = \frac{\frac{\beta(2\beta/\alpha)^2}{2} - \frac{\alpha(2\beta/\alpha)^3}{6}}{m} = \frac{2\beta^3}{3m\alpha^2}.$$

The force at t_{max} is $F_x = \beta - \alpha t_{max} = \beta - \alpha\left(\frac{2\beta}{\alpha}\right) = -\beta$.

Putting in the values for α and β gives the following results:

$t_{max} = 2\beta/\alpha = 2(4.00\text{ N})/(6.00\text{ N/s}) = 1.33\text{ s}$.

$x_{max} = \frac{2\beta^3}{3m\alpha^2} = \frac{2(4.00\text{ N})^3}{3(2.00\text{ kg})(6.00\text{ N/s})^2} = 0.593\text{ m}$.

$F_{max} = -\beta = -4.00$ N, but we only want the magnitude, so $F_{max} = 4.00$ N.

(b) When $x = 0$, the block has returned to where it started. Using our equation for $x(t)$, we get

$x(t) = \frac{\frac{\beta t^2}{2} - \frac{\alpha t^3}{6}}{m} = 0$, which gives $t = 3\beta/\alpha = 3(4.00\text{ N})/(6.00\text{ N/s}) = 2.00\text{ s}$.

Using our equation for $v_x(t)$ and evaluating it at $t = 2.00$ s gives

$v_x = \frac{\beta t - \frac{\alpha t^2}{2}}{m} = \frac{(4.00\text{ N})(2.00\text{ s}) - \frac{(6.00\text{ N/s})(2.00\text{ s})^2}{2}}{(2.00\text{ kg})} = -2.00$ m/s. Its speed is 2.00 m/s.

EVALUATE: The constant-acceleration equations would be of no help with this type of time-dependent force.

4.56. IDENTIFY: $x = \int_0^t v_x dt$ and $v_x = \int_0^t a_x dt$, and similar equations apply to the y-component.

SET UP: In this situation, the x-component of force depends explicitly on the y-component of position. As the y-component of force is given as an explicit function of time, v_y and y can be found as functions of time and used in the expression for $a_x(t)$.

EXECUTE: $a_y = (k_3/m)t$, so $v_y = (k_3/2m)t^2$ and $y = (k_3/6m)t^3$, where the initial conditions $v_{0y} = 0, y_0 = 0$ have been used. Then, the expressions for a_x, v_x, and x are obtained as functions of time: $a_x = \dfrac{k_1}{m} + \dfrac{k_2 k_3}{6m^2}t^3$, $v_x = \dfrac{k_1}{m}t + \dfrac{k_2 k_3}{24m^2}t^4$ and $x = \dfrac{k_1}{2m}t^2 + \dfrac{k_2 k_3}{120m^2}t^5$.

In vector form, $\vec{\boldsymbol{r}} = \left(\dfrac{k_1}{2m}t^2 + \dfrac{k_2 k_3}{120m^2}t^5\right)\hat{\boldsymbol{i}} + \left(\dfrac{k_3}{6m}t^3\right)\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{v}} = \left(\dfrac{k_1}{m}t + \dfrac{k_2 k_3}{24m^2}t^4\right)\hat{\boldsymbol{i}} + \left(\dfrac{k_3}{2m}t^2\right)\hat{\boldsymbol{j}}$.

EVALUATE: a_x depends on time because it depends on y, and y is a function of time.

4.57. **IDENTIFY:** Newton's second law applies to the dancer's head.

SET UP: We use $a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$ and $\vec{\boldsymbol{F}}_{\text{net}} = m\vec{\boldsymbol{a}}$.

EXECUTE: First find the average acceleration: a_{av} = (4.0 m/s)/(0.20 s) = 20 m/s^2. Now apply Newton's second law to the dancer's head. Two vertical forces act on the head: $F_{\text{neck}} - mg = ma$, so $F_{\text{neck}} = m(g + a)$, which gives F_{neck} = (0.094)(65 kg)(9.80 m/s^2 + 20 m/s^2) = 180 N, which is choice (d).

EVALUATE: The neck force is not simply *ma* because the neck must balance her head against gravity, even if the head were not accelerating. That error would lead one to incorrectly select choice (c).

4.58. **IDENTIFY:** Newton's third law of motion applies.

SET UP: The force the neck exerts on her head is the same as the force the head exerts on the neck.

EXECUTE: Choice (a) is correct.

EVALUATE: These two forces form an action-reaction pair.

4.59. **IDENTIFY:** The dancer is in the air and holding a pose, so she is in free fall.

SET UP: The dancer, including all parts of her body, are in free fall, so they all have the same downward acceleration of 9.80 m/s^2.

EXECUTE: Since her head and her neck have the same downward acceleration, and that is produced by gravity, her neck does not exert any force on her head, so choice (a) 0 N is correct.

EVALUATE: During falling motion such as this, a person (including her head) is often described as being "weightless."

4.60. **IDENTIFY:** The graph shows the vertical force that a force plate exerts on her body.

SET UP and **EXECUTE:** When the dancer is not moving, the force that the force plate exerts on her will be her weight, which appears to be about 650 N. Between 0.0 s and 0.4 s, the force on her is less than her weight and is decreasing, so she must be accelerating downward. At 0.4 s, the graph reaches a relative minimum of around 300 N and then begins to increase after that. Only choice (a) is consistent with this part of the graph.

EVALUATE: At the high points in the graph, the force on her is over twice her weight.