31

ALTERNATING CURRENT

VP31.3.1. **IDENTIFY:** We are investigating an inductor in an ac circuit.
SET UP: $X_L = \omega L.$
EXECUTE: **(a)** We want the inductive reactance. $X_L = \omega L = 2\pi fL$ = 2π(108 Hz)(2.50 mH) = 1.70 MΩ.
(b) We want the current amplitude. $I_L = \dfrac{V_L}{X_L} =$ (4.20 kV)/(1.70 MΩ) = 2.48 mA.
EVALUATE: The answer in (b) is the current amplitude, but the current does not always have this value since it varies sinusoidally.

VP31.3.2. **IDENTIFY:** This is an ac circuit containing a resistor and a capacitor.
SET UP: The current is of the form $i(t) = I\cos\omega t$, $X_C = 1/\omega C$.
EXECUTE: **(a)** We want $v_R(t)$. $v_R = Ri = R\,I\cos\omega t$ = (125 V)(2.40 mA) cos(1750 rad/s t) = $(0.300\text{ V})\cos(1750\text{ rad/s } t)$.
(b) We want X_C. $X_C = 1/\omega C$ = 1/[(1750 rad/s)(7.00 μF)] = 81.6 Ω.
(c) We want $v_C(t)$. v_C lags the current by $\pi/2$, so
$v_C = V_C\cos(\omega t - \pi/2) = IX_C\cos(\omega t - \pi/2) = (2.40\text{ mA})(81.6\ \Omega)\cos(1750\text{ rad/s } t - \pi/2)$
$= (0.196\text{ V})\cos[(1750\text{ rad/s})t - \pi/2].$
EVALUATE: There is a $\pi/2$ phase different between v_C and i because when i is a maximum the capacitor is uncharged.

VP31.3.3. **IDENTIFY:** We have a resistor and capacitor in an ac circuit.
SET UP: $X_C = 1/\omega C$, $\omega = 2\pi f$, and $X_C = R$ in this case.
EXECUTE: **(a)** We want the frequency. $X_C = R$, so $X_C = 1/\omega C = R$. $\omega = 2\pi f$, so $f = \dfrac{1}{2\pi RC}$. This gives f = 1/[2π(155 Ω)(8.00 μF)] = 128 Hz.
(b) We want the amplitudes. $V_R = RI$ = (155 Ω)(4.00 mA) = 0.620 V. $V_C = IX_C$ = 0.620 V since $X_C = R$
(c) v_C lags v_R by π/2 so when v_R is a maximum, v_C = 0. When v_C is a maximum, v_R = 0 because of the π/2 phase difference.
EVALUATE: Minimum current mean s no potential across R but maximum potential across C.

VP31.3.4. **IDENTIFY:** We have a resistor and capacitor in an ac circuit.
SET UP and **EXECUTE:** **(a)** We want the current amplitude. $I = V/R$ = (2.45 V)/(115 Ω) = 21.3 mA.
(b) What is I at t = 3.50 ms? i is a maximum at t = 0, so $i = I\cos\omega t$. Using I = 21.3 mA and the given quantities gives i = 11.5 mA.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(c) We want V_C. When $i = 0$, $v_R = 0$, so $v_C = V_C$ and is equal to V_R which is 2.45 V.

(d) We want C. $V_C = IX_C = I/C$. $C = \dfrac{I}{\omega V_C}$ = (21.3 mA)/[(5100 rad/s)(2.45 V)] = 1.71 μF.

EVALUATE: When $v_R = 0$, $v_C = V_C$ and when $v_C = 0$, $v_R = V_R$ and $V_R = V_C$ but $v_R \neq v_C$.

VP31.5.1. IDENTIFY: We have an *L-R-C* series ac circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \dfrac{X_L - X_C}{R}$, $\omega = 2\pi f$.

EXECUTE: **(a)** We want f. $X_L = \omega L = 2\pi fL = R$. $f = R/2\pi L$ = (255 Ω)[2π(4.50 mH)] = 9020 Hz.

(b) We want the impedance. Use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with $X_L = R$, $X_C = 1/\omega C$, and $\omega = 2\pi f$ = 56,670 rad/s. This gives X_C = 6.64 Ω. Put these values into the equation for Z giving Z = 356 Ω.

(c) We want the current amplitude. $I = V/Z$ = (55.0 V)/(356 Ω) = 0.154 A.

(d) We want ϕ. $\tan\phi = \dfrac{X_L - X_C}{R}$ = (255 Ω – 6.64 Ω)/(255 Ω), so ϕ = 44.2°.

(e) Since ϕ is positive, the voltage *leads* the current.

EVALUATE: The resistance never changes but the impedance depends on the frequency.

VP31.5.2. IDENTIFY: We have an *L-R-C* series ac circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** We want R. Calculate ωL and $1/\omega C$ from the given quantities, which gives

ωL = 35.1 Ω and $1/\omega C$ = 308.6 Ω. Solve $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ for R using the known Z.

The result is R = 387 Ω.

(b) We want the current amplitude. $I = V/Z$ = (20.0 V)/(474 Ω) = 0.0422 A = 42.2 mA.

(c) We want V_R. $V_R = RI$ = (387 Ω)(0.0422 A) = 16.3 V.

(d) We want V_L. $V_L = IX_L$ = (0.0422 A)(35.1 Ω) = 1.48 V.

(e) We want V_C. $V_C = IX_C$ = (0.0422 A)(308.6 Ω) = 13.0 V.

EVALUATE: Note that the amplitudes do *not* add up to the voltage amplitude. This is not a problem because these amplitudes do not all occur at the same time due to phase differences.

VP31.5.3. IDENTIFY: We are dealing with an *L-R-C* series ac circuit.

SET UP: $\tan\phi = \dfrac{X_L - X_C}{R}$, $X_L = \omega L$, $X_C = 1/\omega C$.

EXECUTE: **(a)** We want $X_L - X_C$. Solve $\tan\phi = \dfrac{X_L - X_C}{R}$. $X_L - X_C$ = (65.0 Ω)tan 15.0° = 17.4 Ω.

(b) We want L. X_L = 17.4 Ω + X_C. Use $X_L = \omega L$ and $X_C = 1/\omega C$ and solve for L, giving

$L = \dfrac{17.4\ \Omega + 1/\omega C}{\omega}$. Using the given values gives L = 8.65 mH.

EVALUATE: If we wanted the current to lead the voltage, we would need $X_L - X_C$ to be negative, so we would need $X_C > X_L$.

VP31.5.4. IDENTIFY: This problem involves an *L-R-C* series ac circuit.

SET UP and **EXECUTE:** **(a)** We want the voltage amplitudes.

$V_R = IR$ = (0.120 A)(95.0 Ω) = 11.4 V.

$V_L = IX_L = I\omega L$ = (0.120 A)(8000 rad/s)(6.50 mH) = 6.24 V.

$V_C = IX_C = I/\omega C$ = (0.120 A)/[(8000 rad/s)(0.440 μF)] = 34.1 V.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) We want the instantaneous voltages at t = 0.305 ms. The current is a maximum at t = 0, so $i(t) = I\cos\omega t$. Use the voltage amplitudes from part (a).

$v_R = Ri = RI\cos\omega t = (11.4\text{ V})$ cos[(8000 rad/s)(0.305 ms)] = –8.71 V.

v_L leads the current by $\pi/2$ so $v_L = V_L\cos(\omega t + \pi/2)$ = (6.24 V) cos[(8000 rad/s)(0.305 s) + π/2)] = –4.03 V.

v_C lags the current by $\pi/2$ so $v_C = V_C\cos(\omega t - \pi/2)$ = 22.0 V.

EVALUATE: Note that the voltage amplitudes add upto 51.7 V while the instantaneous voltages add up to 9.26 V. The instantaneous voltages all occur at the same time (0.305 s), but the voltage amplitudes do not.

VP31.7.1. **IDENTIFY:** We are dealing with power in an ac circuit.

SET UP and **EXECUTE:** The toaster is a pure resistor. **(a)** We want the average power. $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}$ = (120 V)(3.95 A) = 474 W.

(b) We want the maximum power. $P_{\text{max}} = 2P_{\text{av}}$ = 948 W.

(c) We want R. $P_{\text{av}} = I_{\text{rms}}^2 R$, so $R = P_{\text{av}}/I_{\text{rms}}^2$ = (474 W)/(3.95 A)^2 = 30.4 Ω.

EVALUATE: Check: $P_{\text{max}} = I_{\text{max}}V_{\text{max}} = \left(I_{\text{rms}}\sqrt{2}\right)\left(V_{\text{rms}}\sqrt{2}\right) = 2I_{\text{rms}}V_{\text{rms}} = 2P_{\text{av}}$. It's OK.

VP31.7.2. **IDENTIFY:** We have an L-R-C series ac circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $\tan\phi = \dfrac{X_L - X_C}{R}$, power factor = $\cos\phi$.

EXECUTE: **(a)** We want X_L and X_C. $X_L = \omega L$ = (1300 rad/s)(82.3 mH) = 107 Ω.

$X_C = 1/\omega C$ = 1/[(1300 rad/s)(1.10 μF)] = 699 Ω.

(b) We want ϕ. $\phi = \arctan\left(\dfrac{X_L - X_C}{R}\right)$ = (107 Ω – 699 Ω)/(275 Ω) = –65.1°.

(c) We want the power factor. cos ϕ = cos(–65.1°) = 0.421.

EVALUATE: Since ϕ is negative, the voltage lags the current by 65.1°.

VP31.7.3. **IDENTIFY:** We have an L-R-C series ac circuit.

SET UP: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $X_L = \omega L$, $X_C = 1/\omega C$.

EXECUTE: **(a)** We want the impedance. $X_L = \omega L$ = (1300 rad/s)(1.10 μF) = 107 Ω.

$X_C = 1/\omega C$ = 1/[(1300 rad/s)(1.10 μF)] = 699 Ω. Now use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with the above values, giving Z = 653 Ω.

(b) We want the current amplitude. $I = V/Z$ = (35.0 V)/(653 Ω) = 53.6 mA.

(c) We want the average power in the resistor. $P_{\text{av}} = \frac{1}{2}IV_R = \frac{1}{2}I^2R$ = (0.0536 A)^2(275 Ω)/2 = 0.395 W.

EVALUATE: The average power is *not* I^2R because the current is not equal to its maximum value I all the time; it is usually less than this value. That's where the factor of ½ comes from in part (c).

VP31.7.4. **IDENTIFY:** This problem involves power in an ac series circuit.

SET UP: $X_L = \omega L$, $X_C = 1/\omega C$, $\tan\phi = \dfrac{X_L - X_C}{R}$, power factor = $\cos\phi$.

EXECUTE: **(a)** We want the reactances. Using the given quantities, the results are $X_C = 1/\omega C$ = 85.5 Ω, $X_L = \omega L$ = 97.5 Ω.

(b) We want ϕ. $\cos\phi$ = power factor = 0.800, so ϕ = 36.9°.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(c) We want R. Solve $\tan\phi = \dfrac{X_L - X_C}{R}$ for R, giving $R = \dfrac{X_L - X_C}{\tan\phi}$ = (97.8 Ω – 85.5 Ω)/tan 36.9° = 16.0 Ω.

EVALUATE: Since $X_L > X_C$, ϕ is positive.

31.1. IDENTIFY: The maximum current is the current amplitude, and it must not ever exceed 1.50 A.

SET UP: $I_{\text{rms}} = I/\sqrt{2}$. I is the current amplitude, the maximum value of the current.

EXECUTE: $I = 1.50\text{A}$ gives $I_{\text{rms}} = \dfrac{1.50\text{ A}}{\sqrt{2}} = 1.06\text{ A}.$

EVALUATE: The current amplitude is larger than the root-mean-square current.

31.2. IDENTIFY and **SET UP:** Apply $V_{\text{rms}} = \dfrac{V}{\sqrt{2}}.$

EXECUTE: **(a)** $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45.0\text{ V}}{\sqrt{2}} = 31.8\text{ V}.$

(b) Since the voltage is sinusoidal, the average is zero.

EVALUATE: The voltage amplitude is larger than V_{rms}.

31.3. IDENTIFY: We want the phase angle for the source voltage relative to the current, and we want the inductance if we know the current amplitude.

SET UP: $X_L = \dfrac{V}{I}$ and $X_L = 2\pi fL.$

EXECUTE: **(a)** $\phi = +90°$. The source voltage leads the current by $90°$.

(b) $X_L = \dfrac{V}{I} = \dfrac{45.0\text{ V}}{3.90\text{ A}} = 11.54\ \Omega$. Solving $X_L = 2\pi fL$ for f gives

$$f = \frac{X_L}{2\pi L} = \frac{11.54\ \Omega}{2\pi(9.50\times10^{-3}\text{ H})} = 193\text{ Hz}.$$

EVALUATE: The angular frequency is about 1200 rad/s.

31.4. IDENTIFY: We want the phase angle for the source voltage relative to the current, and we want the capacitance if we know the current amplitude.

SET UP: $X_C = \dfrac{V}{I}$ and $X_C = \dfrac{1}{2\pi fC}.$

EXECUTE: **(a)** $\phi = -90°$. The source voltage lags the current by $90°$.

(b) $X_C = \dfrac{V}{I} = \dfrac{60.0\text{ V}}{5.30\text{ A}} = 11.3\ \Omega$. Solving $X_C = \dfrac{1}{2\pi fC}$ for C gives

$$C = \frac{1}{2\pi fX_C} = \frac{1}{2\pi(80.0\text{ Hz})(11.3\ \Omega)} = 1.76\times10^{-4}\text{ F}.$$

EVALUATE: This is a $176\text{-}\mu\text{F}$ capacitor, which is not unreasonable.

31.5. IDENTIFY and **SET UP:** Use $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}.$

EXECUTE: **(a)** $X_L = \omega L = 2\pi fL = 2\pi(80.0\text{ Hz})(3.00\text{ H}) = 1510\ \Omega.$

(b) $X_L = 2\pi fL$ gives $L = \dfrac{X_L}{2\pi f} = \dfrac{120\ \Omega}{2\pi(80.0\text{ Hz})} = 0.239\text{ H}.$

(c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(80.0\text{ Hz})(4.00\times10^{-6}\text{ F})} = 497\ \Omega.$

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(d) $X_C = \dfrac{1}{2\pi fC}$ gives $C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{2\pi(80.0\ \text{Hz})(120\ \Omega)} = 1.66\times10^{-5}\ \text{F}.$

EVALUATE: X_L increases when L increases; X_C decreases when C increases.

31.6. **IDENTIFY:** The reactance of capacitors and inductors depends on the angular frequency at which they are operated, as well as their capacitance or inductance.

SET UP: The reactances are $X_C = 1/\omega C$ and $X_L = \omega L.$

EXECUTE: **(a)** Equating the reactances gives $\omega L = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}.$

(b) Using the numerical values we get $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(5.00\ \text{mH})(3.50\ \mu\text{F})}} = 7560\ \text{rad/s}.$

$$X_C = X_L = \omega L = (7560\ \text{rad/s})(5.00\ \text{mH}) = 37.8\ \Omega.$$

EVALUATE: At other angular frequencies, the two reactances could be very different.

31.7. **IDENTIFY:** The reactance of an inductor is $X_L = \omega L = 2\pi fL.$ The reactance of a capacitor is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}.$

SET UP: The frequency f is in Hz.

EXECUTE: **(a)** At 60.0 Hz, $X_L = 2\pi(60.0\ \text{Hz})(0.450\ \text{H}) = 170\ \Omega.$ X_L is proportional to f so at 600 Hz, $X_L = 1700\ \Omega.$

(b) At 60.0 Hz, $X_C = \dfrac{1}{2\pi(60.0\ \text{Hz})(2.50\times10^{-6}\ \text{F})} = 1.06\times10^{3}\ \Omega.$ X_C is proportional to $1/f$, so at 600 Hz, $X_C = 106\ \Omega.$

(c) $X_L = X_C$ says $2\pi fL = \dfrac{1}{2\pi fC}$ and $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.450\ \text{H})(2.50\times10^{-6}\ \text{F})}} = 150\ \text{Hz}.$

EVALUATE: X_L increases when f increases. X_C decreases when f increases.

31.8. **IDENTIFY:** $V_L = I\omega L.$

SET UP: ω is the angular frequency, in rad/s. $f = \dfrac{\omega}{2\pi}$ is the frequency in Hz.

EXECUTE: $V_L = I\omega L = 2\pi fIL,$ so $f = \dfrac{V_L}{2\pi IL} = \dfrac{(12.0\ \text{V})}{2\pi(1.80\times10^{-3}\ \text{A})(4.50\times10^{-4}\ \text{H})} = 2.36\times10^{6}\ \text{Hz}$ = 2.36 MHz.

EVALUATE: When f is increased, I decreases.

31.9. **IDENTIFY:** In an L-R ac circuit, we want to find out how the voltage across a resistor varies with time if we know how the voltage varies across the inductor.

SET UP: $v_L = -I\omega L\sin\omega t$ and $v_R = V_R\cos(\omega t).$

EXECUTE: **(a)** $v_L = -I\omega L\sin\omega t.$ $\omega = 480\ \text{rad/s}.$ $I\omega L = 12.0\ \text{V}.$

$I = \dfrac{12.0\ \text{V}}{\omega L} = \dfrac{12.0\ \text{V}}{(480\ \text{rad/s})(0.180\ \text{H})} = 0.1389\ \text{A}.$ $V_R = IR = (0.1389\ \text{A})(90.0\ \Omega) = 12.5\ \text{V}.$

$v_R = V_R\cos(\omega t) = (12.5\ \text{V})\cos[(480\ \text{rad/s})t].$

(b) $v_R = (12.5\ \text{V})\cos[(480\ \text{rad/s})(2.00\times10^{-3}\ \text{s})] = 7.17\ \text{V}.$

EVALUATE: The instantaneous voltage (7.17 V) is less than the voltage amplitude (12.5 V).

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

31.10. **IDENTIFY:** Compare v_C that is given in the problem to the general form $v_C = \frac{I}{\omega C}\sin\omega t$ and determine ω.

SET UP: $X_C = \frac{1}{\omega C}$. $v_R = iR$ and $i = I\cos\omega t$.

EXECUTE: **(a)** $X_C = \frac{1}{\omega C} = \frac{1}{(120\text{ rad/s})(4.80\times10^{-6}\text{ F})} = 1736\ \Omega$.

(b) $I = \frac{V_C}{X_C} = \frac{7.60\text{ V}}{1736\ \Omega} = 4.378\times10^{-3}\text{ A}$ and $i = I\cos\omega t = (4.378\times10^{-3}\text{ A})\cos[(120\text{ rad/s})t]$. Then

$v_R = iR = (4.38\times10^{-3}\text{ A})(250\ \Omega)\cos[(120\text{ rad/s})t] = (1.10\text{ V})\cos[(120\text{ rad/s})t]$.

EVALUATE: The voltage across the resistor has a different phase than the voltage across the capacitor.

31.11. **IDENTIFY** and **SET UP:** The voltage and current for a resistor are related by $v_R = iR$. Deduce the frequency of the voltage and use this in $X_L = \omega L$ to calculate the inductive reactance. The equation $v_L = I\omega L\cos(\omega t + 90°)$ gives the voltage across the inductor.

EXECUTE: **(a)** $v_R = (3.80\text{ V})\cos[(720\text{ rad/s})t]$.

$v_R = iR$, so $i = \frac{v_R}{R} = \left(\frac{3.80\text{ V}}{150\ \Omega}\right)\cos[(720\text{ rad/s})t] = (0.0253\text{ A})\cos[(720\text{ rad/s})t]$.

(b) $X_L = \omega L$.

$\omega = 720$ rad/s, $L = 0.250$ H, so $X_L = \omega L = (720\text{ rad/s})(0.250\text{ H}) = 180\ \Omega$.

(c) If $i = I\cos\omega t$ then $v_L = V_L\cos(\omega t + 90°)$ (from Eq. 31.10).

$V_L = I\omega L = IX_L = (0.02533\text{ A})(180\ \Omega) = 4.56\text{ V}$.

$v_L = (4.56\text{ V})\cos[(720\text{ rad/s})t + 90°]$.

But $\cos(a + 90°) = -\sin a$ (Appendix B), so $v_L = -(4.56\text{ V})\sin[(720\text{ rad/s})t]$.

EVALUATE: The current is the same in the resistor and inductor and the voltages are $90°$ out of phase, with the voltage across the inductor leading.

31.12. **IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no capacitor, $Z = \sqrt{R^2 + X_L^2}$ and $\tan\phi = \frac{X_L}{R}$. $X_L = \omega L$. $I = \frac{V}{Z}$. $V_L = IX_L$ and $V_R = IR$. For an inductor, the voltage leads the current.

EXECUTE: **(a)** $X_L = \omega L = (250\text{ rad/s})(0.400\text{ H}) = 100\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (100\ \Omega)^2} = 224\ \Omega$.

(b) $I = \frac{V}{Z} = \frac{30.0\text{ V}}{224\ \Omega} = 0.134\text{ A}$.

(c) $V_R = IR = (0.134\text{ A})(200\ \Omega) = 26.8\text{ V}$. $V_L = IX_L = (0.134\text{ A})(100\ \Omega) = 13.4\text{ V}$.

(d) $\tan\phi = \frac{X_L}{R} = \frac{100\ \Omega}{200\ \Omega}$ and $\phi = +26.6°$. Since ϕ is positive, the source voltage leads the current.

(e) The phasor diagram is sketched in Figure 31.12.

EVALUATE: Note that $V_R + V_L$ is greater than V. The loop rule is satisfied at each instance of time but the voltages across R and L reach their maxima at different times.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

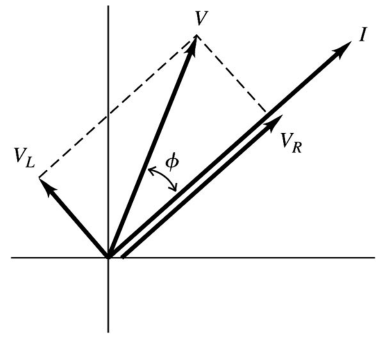


Figure 31.12

31.13. **IDENTIFY:** Apply the equations in Section 31.3.

SET UP: $\omega = 250$ rad/s, $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu$F and $V = 30.0$ V.

EXECUTE: **(a)** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$Z = \sqrt{(200\ \Omega)^2 + ((250\text{ rad/s})(0.400\text{ H}) - 1/((250\text{ rad/s})(6.00\times10^{-6}\text{ F})))^2} = 601\ \Omega.$

(b) $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\ \Omega} = 0.0499$ A.

(c) $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\ \Omega - 667\ \Omega}{200\ \Omega}\right) = -70.6°$, and the voltage lags the current.

(d) $V_R = IR = (0.0499\text{ A})(200\ \Omega) = 9.98$ V; $V_L = I\omega L = (0.0499\text{ A})(250\text{ rad/s})(0.400\text{ H}) = 4.99$ V;

$V_C = \dfrac{I}{\omega C} = \dfrac{(0.0499\text{ A})}{(250\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 33.3$ V.

EVALUATE: **(e)** At any instant, $v = v_R + v_C + v_L$. But v_C and v_L are 180° out of phase, so v_C can be larger than v at a value of t, if $v_L + v_R$ is negative at that t.

31.14. **IDENTIFY:** For an *L-R-C* series ac circuit, we want to find the voltages and voltage amplitudes across all the circuit elements.

SET UP: $X_C = \dfrac{1}{\omega C}$, $X_L = \omega L$, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $I = \dfrac{V}{Z}$ and $\tan\phi = \dfrac{X_L - X_C}{R}$. The instantaneous voltages are $v_R = V_R\cos(\omega t) = IR\cos(\omega t)$, $v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t)$, $v_C = V_C\sin(\omega t) = IX_C\sin(\omega t)$ and $v = V\cos(\omega t + \phi)$.

EXECUTE: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(250\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 666.7\ \Omega.$

$X_L = \omega L = (250\text{ rad/s})(0.900\text{ H}) = 225\ \Omega.$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (225\ \Omega - 666.7\ \Omega)^2} = 484.9\ \Omega.$

$I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{484.9\ \Omega} = 0.06187\text{ A} = 61.87\text{ mA}.$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{225\ \Omega - 666.7\ \Omega}{200\ \Omega} = -2.2085$ and $\phi = -1.146$ rad.

(a) $v_R = V_R\cos(\omega t) = IR\cos(\omega t) = (0.06187\text{ A})(200\ \Omega)\cos[(250\text{ rad/s})(20.0\times10^{-3}\text{ s})] = 3.51$ V.

$v_L = -V_L\sin(\omega t) = -IX_L\sin(\omega t) = -(0.06187\text{ A})(225\ \Omega)\sin[(250\text{ rad/s})(20.0\times10^{-3}\text{ s})] = 13.35$ V.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

$v_C = V_C \sin(\omega t) = IX_C \sin(\omega t) = (0.06187\text{ A})(666.7\ \Omega)\sin[(250\text{ rad/s})(20.0\times10^{-3}\text{ s})] = -39.55\text{ V}.$

$v = V\cos(\omega t + \phi) = (30.0\text{ V})\cos[(250\text{ rad/s})(20.0\times10^{-3}\text{ s}) - 1.146\text{ rad}] = -22.70\text{ V}.$

$v_R + v_L + v_C = 3.51\text{ V} + 13.35\text{ V} + (-39.55\text{ V}) = -22.7\text{ V}.$ $v_R + v_L + v_C$ is equal to v.

(b) $V_R = IR = (0.06187\text{ A})(200\ \Omega) = 12.4\text{ V}$, $V_L = IX_L = (0.06187\text{ A})(225\ \Omega) = 13.9\text{ V}$, and $V_C = IX_C = (0.06187\text{ A})(666.7\ \Omega) = 41.2\text{ V}$.

$V_R + V_C + V_L = 12.4\text{ V} + 41.2\text{ V} + 13.9\text{ V} = 67.5\text{ V}.$ $V_R + V_C + V_L$ is not equal to V.

EVALUATE: The instantaneous voltages do add up to v because they all occur at the same time, so they must add to v by Kirchhoff's loop rule. The amplitudes do not add to V because the maxima do not occur at the same time due to phase differences between the inductor, capacitor and resistor.

31.15. **IDENTIFY** and **SET UP:** Use the equation that preceeds Eq. (31.20): $V^2 = V_R^2 + (V_L - V_C)^2$.

EXECUTE: $V = \sqrt{(30.0\text{ V})^2 + (50.0\text{ V} - 90.0\text{ V})^2} = 50.0\text{ V}.$

EVALUATE: The equation follows directly from the phasor diagrams of Fig. 31.13 (b or c) in the textbook. Note that the voltage amplitudes do not simply add to give 170.0 V for the source voltage.

31.16. **IDENTIFY:** This is an L-R-C series ac circuit.

SET UP: The voltage across the resistor is $v_R(t) = V_R\cos\omega t$, so $v_L(t) = -V_L\sin\omega t$.

EXECUTE: Using $v_L(t) = -V_L\sin\omega t$, we get $\sin\omega t = -\dfrac{v_L}{V_L} = -\dfrac{80.0\text{ mV}}{180\text{ mV}} = -0.4444.$ So $\omega t = -26.39°$.

$v_R(t) = V_R\cos\omega t = (160\text{ V})\cos(-26.39°) = 143\text{ V}.$

$v_C(t) = V_C\sin\omega t = (120\text{ V})\sin(26.39°) = 53.3\text{ V}.$

EVALUATE: The instantaneous voltages add to 80.0 V + 143 V + 53.3 V = 276 V, but the voltage amplitudes add to 180 V + 120 V + 160 V = 440 V. At any time the instantaneous voltages all add to the same value. But the voltage amplitudes do not add to that value because they do not all occur at the same time.

31.17. **IDENTIFY:** We are dealing with an L-R-C series ac circuit.

SET UP: $\tan\phi = \dfrac{V_L - V_C}{V_R}$, $V = \sqrt{V_R^2 + (V_L - V_C)^2}$. The target variable is ϕ. Solve $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ for V_R. Then use $\tan\phi = \dfrac{V_L - V_C}{V_R}$ to find ϕ.

EXECUTE: Solve for V_R: $V_R = \sqrt{V^2 - (V_L - V_C)^2} = \sqrt{(240\text{ V})^2 - (310\text{ V} - 180\text{ V})^2} = 201.7\text{ V}.$ Now find ϕ. $\phi = \arctan\left(\dfrac{V_L - V_C}{V_R}\right) = \arctan\left(\dfrac{310\text{ V} - 180\text{ V}}{201.7\text{ V}}\right) = 32.8°.$

EVALUATE: The angle of the voltage with respect to the current is 32.8°.

31.18. **IDENTIFY:** For an L-R ac circuit, we want to use the resistance, voltage amplitude of the source and power in the resistor to find the impedance, the voltage amplitude across the inductor and the power factor.

SET UP: $P_{\text{av}} = \dfrac{1}{2}I^2R$, $Z = \dfrac{V}{I}$, $V_R = IR$, and $\tan\phi = \dfrac{X_L}{R}$.

EXECUTE: **(a)** $P_{\text{av}} = \dfrac{1}{2}I^2R.$ $I = \sqrt{\dfrac{2P_{\text{av}}}{R}} = \sqrt{\dfrac{2(286\text{ W})}{300\ \Omega}} = 1.381\text{ A}.$ $Z = \dfrac{V}{I} = \dfrac{500\text{ V}}{1.381\text{ A}} = 362\ \Omega.$

(b) $V_R = IR = (1.381\text{ A})(300\ \Omega) = 414\text{ V}.$ $V_L = \sqrt{V^2 - V_R^2} = \sqrt{(500\text{ V})^2 - (414\text{ V})^2} = 280\text{ V}.$

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(c) $\tan\phi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{280\text{ V}}{414\text{ V}}$ gives $\phi = 34.1°$. The power factor is $\cos\phi = 0.828$.

EVALUATE: The voltage amplitude across the resistor cannot exceed the voltage amplitude (500 V) of the ac source.

31.19. **IDENTIFY:** For a pure resistance, $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}} = I_{\text{rms}}^2 R.$

SET UP: 20.0 W is the average power P_{av}.

EXECUTE: **(a)** The average power is one-half the maximum power, so the maximum instantaneous power is 40.0 W.

(b) $I_{\text{rms}} = \frac{P_{\text{av}}}{V_{\text{rms}}} = \frac{20.0\text{ W}}{120\text{ V}} = 0.167\text{ A}.$

(c) $R = \frac{P_{\text{av}}}{I_{\text{rms}}^2} = \frac{20.0\text{ W}}{(0.167\text{ A})^2} = 720\ \Omega.$

EVALUATE: We can also calculate the average power as $P_{\text{av}} = \frac{V_{R,\text{rms}}^2}{R} = \frac{V_{\text{rms}}^2}{R} = \frac{(120\text{ V})^2}{720\ \Omega} = 20.0\text{ W}.$

31.20. **IDENTIFY:** The average power supplied by the source is $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$. The power consumed in the resistance is $P_{\text{av}} = I_{\text{rms}}^2 R.$

SET UP: $\omega = 2\pi f = 2\pi(1.25\times10^3\text{ Hz}) = 7.854\times10^3\text{ rad/s}.$ $X_L = \omega L = 157\ \Omega.$ $X_C = \frac{1}{\omega C} = 909\ \Omega.$

EXECUTE: **(a)** First, let us find the phase angle between the voltage and the current:

$\tan\phi = \frac{X_L - X_C}{R} = \frac{157\ \Omega - 909\ \Omega}{350\ \Omega}$ and $\phi = -65.04°$. The impedance of the circuit is

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(350\ \Omega)^2 + (-752\ \Omega)^2} = 830\ \Omega.$ The average power provided by the generator is then $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos(\phi) = \frac{V_{\text{rms}}^2}{Z}\cos(\phi) = \frac{(120\text{ V})^2}{830\ \Omega}\cos(-65.04°) = 7.32\text{ W}.$

(b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\frac{120\text{ V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\text{ W}.$

EVALUATE: Conservation of energy requires that the answers to parts (a) and (b) are equal.

31.21. **IDENTIFY:** We are dealing with power in an *L-R-C* series ac circuit.
SET UP and **EXECUTE:** **(a)** From Fig. 31.13(b) in the textbook, $\cos\phi = V_R/V = IR/V$. Thus $V\cos\phi = IR$. From Eq. (31.31), $P_{\text{av}} = \frac{1}{2}IV\cos\phi = \frac{1}{2}I(IR) = \frac{1}{2}I^2R.$

(b) We want *R*. Use our results from part (a). $P_{\text{av}} = \frac{1}{2}I^2R = \frac{1}{2}\frac{(IR)^2}{R} = \frac{1}{2}\frac{(V\cos\phi)^2}{R}$. Solve for *R*.

$R = \frac{(V\cos\phi)^2}{2P_{\text{av}}} = \frac{[(120\text{ V})(\cos 53.1°)]^2}{2(80.0\text{ W})} = 32.4\ \Omega.$

EVALUATE: The current amplitude is $I = \sqrt{2P_{av}/R} = 2.22\ A.$

31.22. **IDENTIFY:** This is an *R-C* ac circuit.

SET UP and **EXECUTE:** **(a)** We want V_R. Solve $V = \sqrt{V_R^2 + V_C^2}$ for V_R.

$V_R = \sqrt{V^2 - V_C^2} = \sqrt{(900\text{ V})^2 - (500\text{ V})^2} = 748\text{ V}.$

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) We want C. $\frac{V_R}{V_C} = \frac{IR}{IX_C} = \frac{R}{1/\omega C} = R\omega C . C = \frac{V_R/V_C}{R\omega}$. Using the numbers gives $C = 249\ \mu$F.

(c) $\tan\phi = \frac{-1/\omega C}{R} = -\frac{1}{R\omega C}$. Since ϕ is negative, the voltage *lags* the current.

(d) $P_{\text{av}} = \frac{1}{2}IV\cos\phi = \frac{1}{2}V\left(\frac{V_R}{R}\right)\cos\phi = \left(\frac{VV_R}{2R}\right)\frac{R}{\sqrt{R^2 + (1/\omega C)^2}}$. Using the given numbers, this gives

$P_{\text{av}} = 932$ W.

EVALUATE: The power is all used up in the resistor.

31.23. IDENTIFY and SET UP: Use the equations of Section 31.3 to calculate ϕ, Z, and V_{rms}. The average power delivered by the source is given by $P_{\text{av}} = I_{\text{rms}}V_{\text{rms}}\cos\phi$ and the average power dissipated in the resistor is $I_{\text{rms}}^2 R$.

EXECUTE: (a) $X_L = \omega L = 2\pi fL = 2\pi(400\text{ Hz})(0.120\text{ H}) = 301.6\ \Omega$.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\text{ Hz})(7.3\times 10^{-6}\text{ F})} = 54.51\ \Omega.$$

$\tan\phi = \frac{X_L - X_C}{R} = \frac{301.6\ \Omega - 54.41\ \Omega}{240\ \Omega}$, so $\phi = +45.8°$. The power factor is $\cos\phi = +0.697$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240\ \Omega)^2 + (301.6\ \Omega - 54.51\ \Omega)^2} = 344\ \Omega$.

(c) $V_{\text{rms}} = I_{\text{rms}}Z = (0.450\text{ A})(344\ \Omega) = 155$ V.

(d) $P_{\text{av}} = I_{\text{rms}}V_{\text{rms}}\cos\phi = (0.450\text{ A})(155\text{ V})(0.697) = 48.6$ W.

(e) $P_{\text{av}} = I_{\text{rms}}^2 R = (0.450\text{ A})^2(240\ \Omega) = 48.6$ W.

EVALUATE: The average electrical power delivered by the source equals the average electrical power consumed in the resistor.

(f) All the energy stored in the capacitor during one cycle of the current is released back to the circuit in another part of the cycle. There is no net dissipation of energy in the capacitor.

(g) The answer is the same as for the capacitor. Energy is repeatedly being stored and released in the inductor, but no net energy is dissipated there.

31.24. IDENTIFY and SET UP: $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. $\cos\phi = \frac{R}{Z}$.

EXECUTE: $I_{\text{rms}} = \frac{80.0\text{ V}}{105\ \Omega} = 0.762$ A. $\cos\phi = \frac{75.0\ \Omega}{105\ \Omega} = 0.714$.

$P_{\text{av}} = (80.0\text{ V})(0.762\text{ A})(0.714) = 43.5$ W.

EVALUATE: Since the average power consumed by the inductor and by the capacitor is zero, we can also calculate the average power as $P_{\text{av}} = I_{\text{rms}}^2 R = (0.762\text{ A})^2(75.0\ \Omega) = 43.5$ W.

31.25. IDENTIFY: The angular frequency and the capacitance can be used to calculate the reactance X_C of the capacitor. The angular frequency and the inductance can be used to calculate the reactance X_L of the inductor. Calculate the phase angle ϕ and then the power factor is $\cos\phi$. Calculate the impedance of the circuit and then the rms current in the circuit. The average power is $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$. On the average no power is consumed in the capacitor or the inductor, it is all consumed in the resistor.

SET UP: The source has rms voltage $V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{45\text{ V}}{\sqrt{2}} = 31.8$ V.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

EXECUTE: (a) $X_L = \omega L = (360\text{ rad/s})(15\times10^{-3}\text{ H}) = 5.4\ \Omega.$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(360\text{ rad/s})(3.5\times10^{-6}\text{ F})} = 794\ \Omega.$ $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{5.4\ \Omega - 794\ \Omega}{250\ \Omega}$ and $\phi = -72.4°.$

The power factor is $\cos\phi = 0.302.$

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250\ \Omega)^2 + (5.4\ \Omega - 794\ \Omega)^2} = 827\ \Omega.$ $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{31.8\text{ V}}{827\ \Omega} = 0.0385\text{ A}.$

$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = (31.8\text{ V})(0.0385\text{ A})(0.302) = 0.370\text{ W}.$

(c) The average power delivered to the resistor is $P_{\text{av}} = I_{\text{rms}}^2 R = (0.0385\text{ A})^2(250\ \Omega) = 0.370\text{ W}.$ The average power delivered to the capacitor and to the inductor is zero.

EVALUATE: On average the power delivered to the circuit equals the power consumed in the resistor. The capacitor and inductor store electrical energy during part of the current oscillation but each return the energy to the circuit during another part of the current cycle.

31.26. IDENTIFY: At resonance in an *L-R-C* ac circuit, we know the reactance of the capacitor and the voltage amplitude across it. From this information, we want to find the voltage amplitude of the source.

SET UP: At resonance, $Z = R.$ $V_C = IX_C.$

EXECUTE: $I = \dfrac{V}{X_C} = \dfrac{600\text{ V}}{200\ \Omega} = 3.00\text{ A}.$ $Z = R = 300\ \Omega.$ $V = IZ = (3.00\text{ A})(300\ \Omega) = 900\text{ V}.$

EVALUATE: At resonance, $Z = R,$ but X_C is not zero.

31.27. IDENTIFY and SET UP: The current is largest at the resonance frequency. At resonance, $X_L = X_C$ and $Z = R.$ For part (b), calculate Z and use $I = V/Z.$

EXECUTE: (a) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 113\text{ Hz}.$ $I = V/R = 15.0\text{ mA}.$

(b) $X_C = 1/\omega C = 500\ \Omega.$ $X_L = \omega L = 160\ \Omega.$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega.$ $I = V/Z = 7.61\text{ mA}.$ $X_C > X_L$ so the source voltage lags the current.

EVALUATE: $\omega_0 = 2\pi f_0 = 710\text{ rad/s}.$ $\omega = 400\text{ rad/s}$ and is less than $\omega_0.$ When $\omega < \omega_0,$ $X_C > X_L.$ Note that I in part (b) is less than I in part (a).

31.28. IDENTIFY: The impedance and individual reactances depend on the angular frequency at which the circuit is driven.

SET UP: The impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2},$ the current amplitude is $I = V/Z$ and the instantaneous values of the potential and current are $v = V\cos(\omega t + \phi),$ where $\tan\phi = (X_L - X_C)/R,$ and $i = I\cos\omega t.$

EXECUTE: (a) Z is a minimum when $\omega L = \dfrac{1}{\omega C},$ which gives

$\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(8.00\text{ mH})(12.5\ \mu\text{F})}} = 3162\text{ rad/s},$ which rounds to 3160 rad/s. $Z = R = 175\ \Omega.$

(b) $I = V/Z = (25.0\text{ V})/(175\ \Omega) = 0.143\text{ A}.$

(c) $i = I\cos\omega t = I/2,$ so $\cos\omega t = \frac{1}{2},$ which gives $\omega t = 60° = \pi/3\text{ rad}.$ $v = V\cos(\omega t + \phi),$ where $\tan\phi = (X_L - X_C)/R = 0/R = 0.$ So, $v = (25.0\text{ V})\cos\omega t = (25.0\text{ V})(1/2) = 12.5\text{ V}.$
$v_R = Ri = (175\ \Omega)(1/2)(0.143\text{ A}) = 12.5\text{ V}.$

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

$v_C = V_C\cos(\omega t - 90°) = IX_C\cos(\omega t - 90°) = \dfrac{0.143\text{ A}}{(3162\text{ rad/s})(12.5\ \mu\text{F})}\cos(60° - 90°) = +3.13\text{ V}.$

$v_L = V_L\cos(\omega t + 90°) = IX_L\cos(\omega t + 90°) = (0.143\text{ A})(3162\text{ rad/s})(8.00\text{ mH})\cos(60° + 90°).$

$v_L = -3.13\text{ V}.$

(d) $v_R + v_L + v_C = 12.5\text{ V} + (-3.13\text{ V}) + 3.13\text{ V} = 12.5\text{ V} = v_{\text{source}}.$

EVALUATE: The instantaneous potential differences across all the circuit elements always add up to the value of the source voltage at that instant. In this case (resonance), the potentials across the inductor and capacitor have the same magnitude but are $180°$ out of phase, so they add to zero, leaving all the potential difference across the resistor.

31.29. **IDENTIFY** and **SET UP:** At the resonance frequency, $Z = R$. Use that $V = IZ$, $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$. P_{av} is given by $P_{\text{av}} = \frac{1}{2}VI\cos\phi$.

EXECUTE: **(a)** $V = IZ = IR = (0.500\text{ A})(300\ \Omega) = 150\text{ V}.$

(b) $V_R = IR = 150\text{ V}.$

$X_L = \omega L = L(1/\sqrt{LC}) = \sqrt{L/C} = 2582\ \Omega; V_L = IX_L = 1290\text{ V}.$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582\ \Omega; V_C = IX_C = 1290\text{ V}.$

(c) $P_{\text{av}} = \frac{1}{2}VI\cos\phi = \frac{1}{2}I^2R$, since $V = IR$ and $\cos\phi = 1$ at resonance.

$P_{\text{av}} = \frac{1}{2}(0.500\text{ A})^2(300\ \Omega) = 37.5\text{ W}.$

EVALUATE: At resonance $V_L = V_C$. Note that $V_L + V_C > V$. However, at any instant $v_L + v_C = 0$.

31.30. **IDENTIFY:** The current is maximum at the resonance frequency, so choose C such that $\omega = 50.0$ rad/s is the resonance frequency. At the resonance frequency $Z = R$.

SET UP: $V_L = I\omega L.$

EXECUTE: **(a)** The amplitude of the current is given by $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$. Thus, the current will have a maximum amplitude when $\omega L = \dfrac{1}{\omega C}$. Therefore,

$$C = \frac{1}{\omega^2 L} = \frac{1}{(50.0\text{ rad/s})^2(3.00\text{ H})} = 1.33\times 10^{-4}\text{ F} = 133\ \mu\text{F}.$$

(b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \dfrac{V}{R} = \dfrac{120\text{ V}}{400\ \Omega} = 0.300\text{ A}$. Thus, the amplitude of the voltage across the inductor is

$V_L = I(\omega L) = (0.300\text{ A})(50.0\text{ rad/s})(3.00\text{ H}) = 45.0\text{ V}.$

EVALUATE: For the value of C found in part (a), the resonance angular frequency is 50.0 rad/s.

31.31. **IDENTIFY** and **SET UP:** At resonance $X_L = X_C$, $\phi = 0$ and $Z = R$. $R = 150\ \Omega$, $L = 0.750\text{ H}$, $C = 0.0180\ \mu\text{F}$, $V = 150\text{ V}$

EXECUTE: **(a)** At the resonance frequency $X_L = X_C$ and from $\tan\phi = \dfrac{X_L - X_C}{R}$ we have that $\phi = 0°$ and the power factor is $\cos\phi = 1.00$.

(b) $P_{\text{av}} = \frac{1}{2}VI\cos\phi$ (Eq. 31.31).

At the resonance frequency $Z = R$, so $I = \dfrac{V}{Z} = \dfrac{V}{R}$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist.
No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

$$P_{av} = \frac{1}{2}V\left(\frac{V}{R}\right)\cos\phi = \frac{1}{2}\frac{V^2}{R} = \frac{1}{2}\frac{(150\text{ V})^2}{150\ \Omega} = 75.0\text{ W}.$$

EVALUATE: **(c)** When C and f are changed but the circuit is kept on resonance, nothing changes in $P_{av} = V^2/(2R)$, so the average power is unchanged: $P_{av} = 75.0$ W. The resonance frequency changes but since $Z = R$ at resonance the current doesn't change.

31.32. **IDENTIFY:** This problem is about resonance in an L-R-C series ac circuit.

SET UP and **EXECUTE:** At resonance, $Z = R$ and $P = V^2/2R = (80.0\text{ V})^2/[2(400\ \Omega)] = 8.00$ W.

EVALUATE: At resonance, $\cos\phi = 1$.

31.33. **IDENTIFY:** At resonance $Z = R$ and $X_L = X_C$.

SET UP: $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $V = IZ$. $V_R = IR$, $V_L = IX_L$ and $V_C = V_L$.

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.280\text{ H})(4.00\times 10^{-6}\text{ F})}} = 945$ rad/s.

(b) $I = 1.70$ A at resonance, so $R = Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{1.70\text{ A}} = 70.6\ \Omega$.

(c) At resonance, $V_R = 120$ V, $V_L = V_C = I\omega L = (1.70\text{ A})(945\text{ rad/s})(0.280\text{ H}) = 450$ V.

EVALUATE: At resonance, $V_R = V$ and $V_L - V_C = 0$.

31.34. **IDENTIFY:** Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary. A transformer transforms voltages according to $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance R is $R_{eff} = \dfrac{R}{(N_2/N_1)^2}$. Resistance R is related to P_{av} and V_{rms} by $P_{av} = \dfrac{V_{rms}^2}{R}$. Conservation of energy requires $P_{av,1} = P_{av,2}$ so $V_1 I_1 = V_2 I_2$.

SET UP: Let $V_1 = 240$ V and $V_2 = 120$ V, so $P_{2,av} = 1600$ W. These voltages are rms.

EXECUTE: **(a)** $V_1 = 240$ V and we want $V_2 = 120$ V, so use a step-down transformer with $N_2/N_1 = \frac{1}{2}$.

(b) $P_{av} = V_1 I_1$, so $I_1 = \dfrac{P_{av}}{V_1} = \dfrac{1600\text{ W}}{240\text{ V}} = 6.67$ A.

(c) The resistance R of the blower is $R = \dfrac{V_2^2}{P_{av}} = \dfrac{(120\text{ V})^2}{1600\text{ W}} = 9.00\ \Omega$. The effective resistance of the blower is $R_{eff} = \dfrac{9.00\ \Omega}{(1/2)^2} = 36.0\ \Omega$.

EVALUATE: $I_2 = V_2/R_2 = (120\text{ V})/(9.00\ \Omega) = 13.3$ A, so $I_2 V_2 = (13.3\text{ A})(120\text{ V}) = 1600$ W. Energy is provided to the primary at the same rate that it is consumed in the secondary. Step-down transformers step up resistance and the current in the primary is less than the current in the secondary.

31.35. **IDENTIFY** and **SET UP:** The equation $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ relates the primary and secondary voltages to the number of turns in each. $I = V/R$ and the power consumed in the resistive load is $I_{rms}^2 = V_{rms}^2/R$. Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary.

EXECUTE: **(a)** $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ so $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{120\text{ V}}{12.0\text{ V}} = 10$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) $I_2 = \dfrac{V_2}{R} = \dfrac{12.0\ \text{V}}{5.00\ \Omega} = 2.40\ \text{A}.$

(c) $P_{\text{av}} = I_2^2 R = (2.40\ \text{A})^2(5.00\ \Omega) = 28.8\ \text{W}.$

(d) The power drawn from the line by the transformer is the 28.8 W that is delivered by the load.

$P_{\text{av}} = \dfrac{V_1^2}{R}$ so $R = \dfrac{V_1^2}{P_{\text{av}}} = \dfrac{(120\ \text{V})^2}{28.8\ \text{W}} = 500\ \Omega.$

And $\left(\dfrac{N_1}{N_2}\right)^2 (5.00\ \Omega) = (10)^2(5.00\ \Omega) = 500\ \Omega,$ as was to be shown.

EVALUATE: The resistance is "transformed." A load of resistance R connected to the secondary draws the same power as a resistance $(N_1/N_2)^2 R$ connected directly to the supply line, without using the transformer.

31.36. IDENTIFY: $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}}$ and $P_{\text{av},1} = P_{\text{av},2}$. $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$. Let I_1, V_1 and I_2, V_2 be rms values for the primary and secondary.

SET UP: $V_1 = 120\ \text{V}$. $V_2 = 13{,}000\ \text{V}$.

EXECUTE: **(a)** $\dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{13{,}000\ \text{V}}{120\ \text{V}} = 108.$

(b) $P_{\text{av}} = V_2 I_2 = (13{,}000\ \text{V})(8.50\times 10^{-3}\ \text{A}) = 110\ \text{W}.$

(c) $I_1 = \dfrac{P_{\text{av}}}{V_1} = \dfrac{110\ \text{W}}{120\ \text{V}} = 0.917\ \text{A}.$

EVALUATE: Since the power supplied to the primary must equal the power delivered by the secondary, in a step-up transformer the current in the primary is greater than the current in the secondary.

31.37. IDENTIFY and **SET UP:** Use $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ to relate L and R to ϕ. The voltage across the coil leads the current in it by $52.3°$, so $\phi = +52.3°$.

EXECUTE: $\tan\phi = \dfrac{X_L - X_C}{R}$. But there is no capacitance in the circuit so $X_C = 0$. Thus

$\tan\phi = \dfrac{X_L}{R}$ and $X_L = R\tan\phi = (48.0\ \Omega)\tan 52.3° = 62.1\ \Omega.$

$X_L = \omega L = 2\pi f L$ so $L = \dfrac{X_L}{2\pi f} = \dfrac{62.1\ \Omega}{2\pi(80.0\ \text{Hz})} = 0.124\ \text{H}.$

EVALUATE: $\phi > 45°$ when $(X_L - X_C) > R$, which is the case here.

31.38. IDENTIFY: We have an L-R-C series ac circuit.

SET UP: $\tan\phi = \dfrac{X_L - X_C}{R}$, $P_{\text{av}} = \dfrac{1}{2} I^2 R$, $P_{\text{av}} = \dfrac{1}{2} IV\cos\phi.$

EXECUTE: **(a)** We want X_L. Use $\tan\phi = \dfrac{X_L - X_C}{R}$. $X_L = R\tan\phi + X_C$. Using the numbers gives

$X_L = (300\ \Omega)\tan(-53.0°) + 500\ \Omega = 102\ \Omega.$

(b) We want I. Use $P_{\text{av}} = \dfrac{1}{2} I^2 R$ and solve for I. $I = \sqrt{2P_{\text{av}}/R}$ gives $I = 0.730$ A.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(c) We want V. Use $P_{\text{av}} = \frac{1}{2} IV\cos\phi$ and solve for V. $V = \frac{2P_{\text{av}}}{I\cos\phi} = 364\text{ V}$.

EVALUATE: The circuit is not close to resonance because X_L is very different from X_C (102 Ω compared to 500 Ω).

31.39. **IDENTIFY:** An L-R-C ac circuit operates at resonance. We know L, C, and V and want to find R.

SET UP: At resonance, $Z = R$ and $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$. $X_C = \frac{1}{\omega C}$, $I = V/Z$.

EXECUTE: $\omega = \frac{1}{\sqrt{LC}} = 633.0\text{ rad/s}$ $X_C = \frac{1}{\omega C} = \frac{1}{(633\text{ rad/s})(4.80\times10^{-6}\text{ F})} = 329.1\,\Omega$.

$I = \frac{V_C}{X_C} = \frac{80.0\text{ V}}{329.1\,\Omega} = 0.2431\text{ A}$. At resonance $Z = R$, so $I = \frac{V}{R}$. $R = \frac{V}{I} = \frac{56.0\text{ V}}{0.2431\text{ A}} = 230\,\Omega$.

EVALUATE: At resonance, the impedance is a minimum.

31.40. **IDENTIFY:** $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$. $V_{\text{rms}} = I_{\text{rms}}R$. $V_{C,\text{rms}} = I_{\text{rms}}X_C$. $V_{L,\text{rms}} = I_{\text{rms}}X_L$.

SET UP: $V_{\text{rms}} = \frac{V}{\sqrt{2}} = \frac{30.0\text{ V}}{\sqrt{2}} = 21.2\text{ V}$.

EXECUTE: **(a)** $\omega = 200\text{ rad/s}$, so $X_L = \omega L = (200\text{ rad/s})(0.400\text{ H}) = 80.0\,\Omega$ and

$X_C = \frac{1}{\omega C} = \frac{1}{(200\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 833\,\Omega$. $Z = \sqrt{(200\,\Omega)^2 + (80.0\,\Omega - 833\,\Omega)^2} = 779\,\Omega$.

$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{21.2\text{ V}}{779\,\Omega} = 0.0272\text{ A}$. V_1 reads $V_{R,\text{rms}} = I_{\text{rms}}R = (0.0272\text{ A})(200\,\Omega) = 5.44\text{ V}$.

V_2 reads $V_{L,\text{rms}} = I_{\text{rms}}X_L = (0.0272\text{ A})(80.0\,\Omega) = 2.18\text{ V}$.

V_3 reads $V_{C,\text{rms}} = I_{\text{rms}}X_C = (0.0272\text{ A})(833\,\Omega) = 22.7\text{ V}$.

V_4 reads $\left|\frac{V_L - V_C}{\sqrt{2}}\right| = \left|V_{L,\text{rms}} - V_{C,\text{rms}}\right| = \left|2.18\text{ V} - 22.7\text{ V}\right| = 20.5\text{ V}$.

V_5 reads $V_{\text{rms}} = 21.2\text{ V}$.

(b) $\omega = 1000\text{ rad/s}$ so $X_L = \omega L = (5)(80.0\,\Omega) = 400\,\Omega$ and $X_C = \frac{1}{\omega C} = \frac{833\,\Omega}{5} = 167\,\Omega$.

$Z = \sqrt{(200\,\Omega)^2 + (400\,\Omega - 167\,\Omega)^2} = 307\,\Omega$. $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{21.2\text{ V}}{307\,\Omega} = 0.0691\text{ A}$.

V_1 reads $V_{R,\text{rms}} = 13.8\text{ V}$. V_2 reads $V_{L,\text{rms}} = 27.6\text{ V}$. V_3 reads $V_{C,\text{rms}} = 11.5\text{ V}$.

V_4 reads $\left|V_{L,\text{rms}} - V_{C,\text{rms}}\right| = \left|27.6\text{ V} - 11.5\text{ V}\right| = 16.1\text{ V}$. V_5 reads $V_{\text{rms}} = 21.2\text{ V}$.

EVALUATE: The resonance frequency for this circuit is $\omega_0 = \frac{1}{\sqrt{LC}} = 645\text{ rad/s}$. 200 rad/s is less than the resonance frequency and $X_C > X_L$. 1000 rad/s is greater than the resonance frequency and $X_L > X_C$.

31.41. **IDENTIFY:** We can use geometry to calculate the capacitance and inductance, and then use these results to calculate the resonance angular frequency.

SET UP: The capacitance of an air-filled parallel plate capacitor is $C = \frac{\epsilon_0 A}{d}$. The inductance of a long solenoid is $L = \frac{\mu_0 A N^2}{l}$. The inductor has $N = (125\text{ coils/cm})(9.00\text{ cm}) = 1125$ coils. The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$. $\epsilon_0 = 8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$. $\mu_0 = 4\pi\times10^{-7}\text{ T}\cdot\text{m/A}$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

EXECUTE: $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.50\times10^{-2}\ \text{m})^2}{8.00\times10^{-3}\ \text{m}} = 2.24\times10^{-12}\ \text{F}.$

$$L = \frac{\mu_0 AN^2}{l} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.250\times10^{-2}\ \text{m})^2(1125)^2}{9.00\times10^{-2}\ \text{m}} = 3.47\times10^{-4}\ \text{H}.$$

$$\omega_0 = \frac{1}{\sqrt{(3.47\times10^{-4}\ \text{H})(2.24\times10^{-12}\ \text{F})}} = 3.59\times10^7\ \text{rad/s}.$$

EVALUATE: The result is a rather high angular frequency.

31.42. **IDENTIFY:** Use geometry to calculate the self-inductance of the toroidal solenoid. Then find its reactance and use this to find the impedance, and finally the current amplitude, of the circuit.

SET UP: $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, $X_L = 2\pi fL$, $Z = \sqrt{R^2 + X_L^2}$, and $I = V/Z$.

EXECUTE: $L = \dfrac{\mu_0 N^2 A}{2\pi r} = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(2900)^2(0.450\times10^{-4}\ \text{m}^2)}{9.00\times10^{-2}\ \text{m}} = 8.41\times10^{-4}\ \text{H}.$

$X_L = 2\pi fL = (2\pi)(495\ \text{Hz})(8.41\times10^{-4}\ \text{H}) = 2.616\ \Omega.$ $Z = \sqrt{R^2 + X_L^2} = 3.832\ \Omega.$

$$I = \frac{V}{Z} = \frac{24.0\ \text{V}}{3.832\ \Omega} = 6.26\ \text{A}.$$

EVALUATE: The inductance is physically reasonable.

31.43. **IDENTIFY** and **SET UP:** Source voltage lags current so it must be that $X_C > X_L$.

EXECUTE: **(a)** We must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional L needed to raise X_L to equal X_C.

(b) Power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more X_L is needed: $R = Z\cos\phi = (60.0\ \Omega)(0.720) = 43.2\ \Omega$

$\tan\phi = \dfrac{X_L - X_C}{R}$ so $X_L - X_C = R\tan\phi.$

$\cos\phi = 0.720$ gives $\phi = -43.95°$ (ϕ is negative since the voltage lags the current).

Then $X_L - X_C = R\tan\phi = (43.2\ \Omega)\tan(-43.95°) = -41.64\ \Omega.$

Therefore need to add $41.64\ \Omega$ of X_L.

$X_L = \omega L = 2\pi fL$ and $L = \dfrac{X_L}{2\pi f} = \dfrac{41.64\ \Omega}{2\pi(50.0\ \text{Hz})} = 0.133\ \text{H}$, amount of inductance to add.

EVALUATE: From the information given we can't calculate the original value of L in the circuit, just how much to add. When this L is added the current in the circuit will increase.

31.44. **IDENTIFY:** We are dealing with a transformer as an ac adapter.

SET UP and **EXECUTE:** **(a)** Voltage = 19.5 V, current = 6.7 A.

(b) Power = 130 W. IV = (19.5 V)(6.7 A) = 131 W, so $P = IV$.

(c) Primary: 120 V rms, 200 turns. The full-wave rectifier following the secondary coil maintains a voltage amplitude V = 19.5 V. We want the number of turns in the secondary. $V_1 = V_{\text{rms}}\sqrt{2}$. The secondary output should be V_2 = 19.5 V. $N_2 = \dfrac{V_2}{V_1}N_1 = \dfrac{19.5\ \text{V}}{120\sqrt{2}\ \text{V}}(200) = 23$ turns.

(d) We want I_1. $P = IV$ gives 130 W = I_1(120 V), so I_1 = 1.1 A.

(e) Estimate the size: Outside: 5 cm by 5 cm by 1.5 cm. Inside: 3 cm by 3 cm by 1 cm. One coil: 4 cm. 200 coils: 800 cm = 8.0 m.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(f) We want B inside the core. Apply Ampere's law. Use permeability μ instead of μ_0, where $\mu = K_m\mu_0$, with K_m– being the relative permeability. $\oint \vec{B} \cdot d\hat{l} = K_m\mu_0 I_{\text{encl}}$. Use a rectangular path with one side of length l = 3 cm inside the core enclosing all the loops. $Bl = K\mu_0 N_1 I_{\text{encl}}$.

$B = K\mu_0 N_1 I_{\text{encl}}/l$ = (5000) μ_0 (200)(1.1 A)/(0.030 m) = 46 T.

EVALUATE: The very large field in the core is due to the large permeability of the metal. If it were air-filled, B would be only about 9 mT.

31.45. IDENTIFY: We know the impedances and the average power consumed. From these we want to find the power factor and the rms voltage of the source.

SET UP: $P = I_{\text{rms}}^2 R$. $\cos\phi = \dfrac{R}{Z}$. $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $V_{\text{rms}} = I_{\text{rms}} Z$.

EXECUTE: **(a)** $I_{\text{rms}} = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{60.0\ \text{W}}{300\ \Omega}} = 0.447\ \text{A}$. $Z = \sqrt{(300\ \Omega)^2 + (500\ \Omega - 300\ \Omega)^2} = 361\ \Omega$.

$\cos\phi = \dfrac{R}{Z} = \dfrac{300\ \Omega}{361\ \Omega} = 0.831$.

(b) $V_{\text{rms}} = I_{\text{rms}} Z = (0.447\ \text{A})(361\ \Omega) = 161\ \text{V}$.

EVALUATE: The voltage amplitude of the source is $V_{\text{rms}}\sqrt{2} = 228\ \text{V}$.

31.46. IDENTIFY and **SET UP:** $X_C = \dfrac{1}{\omega C}$. $X_L = \omega L$.

EXECUTE: **(a)** $\dfrac{1}{\omega_1 C} = \omega_1 L$ and $LC = \dfrac{1}{\omega_1^2}$. At angular frequency ω_2,

$\dfrac{X_L}{X_C} = \dfrac{\omega_2 L}{1/\omega_2 C} = \omega_2^2 LC = (2\omega_1)^2 \dfrac{1}{\omega_1^2} = 4$. $X_L > X_C$.

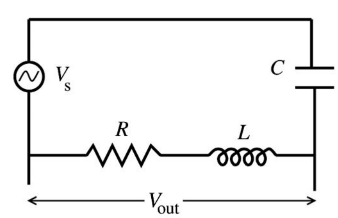
(b) At angular frequency ω_3, $\dfrac{X_L}{X_C} = \omega_3^2 LC = \left(\dfrac{\omega_1}{3}\right)^2 \left(\dfrac{1}{\omega_1^2}\right) = \dfrac{1}{9}$. $X_C > X_L$.

(c) The resonance angular frequency ω_0 is the value of ω for which $X_C = X_L$, so $\omega_0 = \omega_1$.

EVALUATE: When ω increases, X_L increases and X_C decreases. When ω decreases, X_L decreases and X_C increases.

31.47. IDENTIFY and **SET UP:** Express Z and I in terms of ω, L, C, and R. The voltages across the resistor and the inductor are 90° out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

EXECUTE: The circuit is sketched in Figure 31.47.



$$X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Figure 31.47

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

$$V_{\text{out}} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_{\text{s}}\sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_{\text{s}}} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$\underline{\omega \text{ small}}$:

As ω gets small, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to \frac{1}{\omega^2 C^2}$, $R^2 + \omega^2 L^2 \to R^2$.

Therefore, $\frac{V_{\text{out}}}{V_{\text{s}}} \to \sqrt{\frac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as ω becomes small.

$\underline{\omega \text{ large}}$:

As ω gets large, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to R^2 + \omega^2 L^2 \to \omega^2 L^2$, $R^2 + \omega^2 L^2 \to \omega^2 L^2$.

Therefore, $\frac{V_{\text{out}}}{V_{\text{s}}} \to \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as ω becomes large.

EVALUATE: $V_{\text{out}}/V_{\text{s}} \to 0$ as ω becomes small, so there is V_{out} only when the frequency ω of V_{s} is large. If the source voltage contains a number of frequency components, only the high frequency ones are passed by this filter.

31.48. **IDENTIFY:** $V = V_C = IX_C$. $I = V/Z$.

SET UP: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.

EXECUTE: $V_{\text{out}} = V_C = \frac{I}{\omega C} \Rightarrow \frac{V_{\text{out}}}{V_{\text{s}}} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If ω is large: $\frac{V_{\text{out}}}{V_{\text{s}}} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \frac{1}{\omega C\sqrt{(\omega L)^2}} = \frac{1}{(LC)\omega^2}$.

If ω is small: $\frac{V_{\text{out}}}{V_{\text{s}}} \approx \frac{1}{\omega C\sqrt{(1/\omega C)^2}} = \frac{\omega C}{\omega C} = 1$.

EVALUATE: When ω is large, X_C is small and X_L is large so Z is large and the current is small. Both factors in $V_C = IX_C$ are small. When ω is small, X_C is large and the voltage amplitude across the capacitor is much larger than the voltage amplitudes across the resistor and the inductor.

31.49. **IDENTIFY:** $I = V/Z$ and $P_{\text{av}} = \frac{1}{2}I^2 R$.

SET UP: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

EXECUTE: **(a)** $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist.
No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) $P_{av} = \frac{1}{2}I^2 R = \frac{1}{2}\left(\frac{V}{Z}\right)^2 R = \frac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}.$

(c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \frac{1}{\omega_0 C}$, so $\omega_0 = \frac{1}{\sqrt{LC}}$.

(d) The average power is

$$P_{av} = \frac{(100\text{ V})^2(200\ \Omega)/2}{(200\ \Omega)^2 + \left[\omega(2.00\text{ H}) - 1/[\omega(0.500\times 10^{-6}\text{ F})]\right]^2} = \frac{1{,}000{,}000\,\omega^2}{40{,}000\omega^2 + (2\omega^2 - 2{,}000{,}000\text{ s}^{-2})^2}\text{ W}.$$

The graph of P_{av} versus ω is sketched in Figure 31.49.

EVALUATE: Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light purple curve in Figure 31.19 in the textbook.

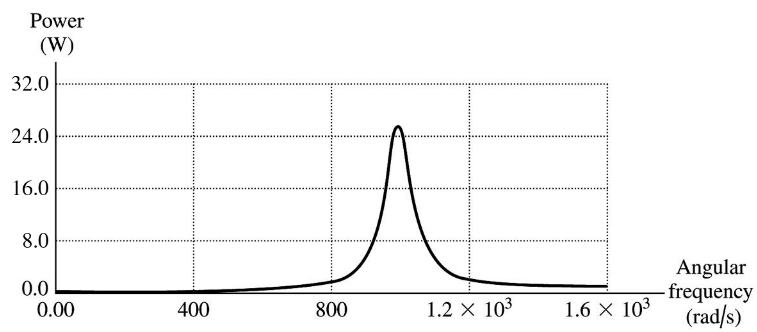


Figure 31.49

31.50. **IDENTIFY:** $V_L = I\omega L$ and $V_C = \frac{I}{\omega C}$.

SET UP: Problem 31.49 shows that $I = \frac{V}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

EXECUTE: **(a)** $V_L = I\omega L = \frac{V\omega L}{\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

(b) $V_C = \frac{I}{\omega C} = \frac{V}{\omega C\sqrt{R^2 + [\omega L - 1/(\omega C)]^2}}$.

(c) The graphs are given in Figure 31.50.

EVALUATE: **(d)** When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} = 1000$ rad/s, the two voltages are equal, and are a maximum, 1000 V.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

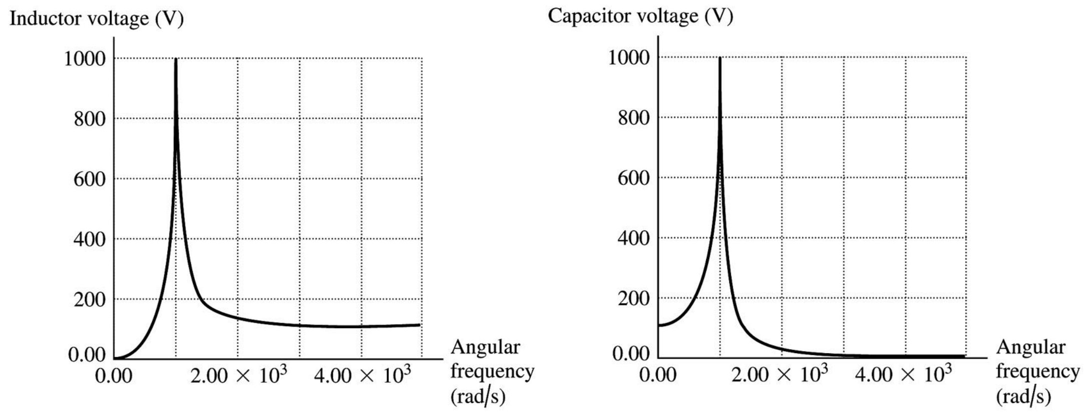


Figure 31.50

31.51. **IDENTIFY:** We know R, X_C, and ϕ so $\tan\phi = \dfrac{X_L - X_C}{R}$ tells us X_L. Use $P_{av} = I_{rms}^2 R$ to calculate I_{rms}. Then calculate Z and use $V_{rms} = I_{rms}Z$ to calculate V_{rms} for the source.

SET UP: Source voltage lags current so $\phi = -54.0°$. $X_C = 350\ \Omega$, $R = 180\ \Omega$, $P_{av} = 140$ W.

EXECUTE: **(a)** $\tan\phi = \dfrac{X_L - X_C}{R}$.

$X_L = R\tan\phi + X_C = (180\ \Omega)\tan(-54.0°) + 350\ \Omega = -248\ \Omega + 350\ \Omega = 102\ \Omega.$

(b) $P_{av} = V_{rms}I_{rms}\cos\phi = I_{rms}^2 R$ (Exercise 31.22). $I_{rms} = \sqrt{\dfrac{P_{av}}{R}} = \sqrt{\dfrac{140\text{ W}}{180\ \Omega}} = 0.882$ A.

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 306\ \Omega.$

$V_{rms} = I_{rms}Z = (0.882\text{ A})(306\ \Omega) = 270$ V.

EVALUATE: We could also use $P_{av} = V_{rms}I_{rms}\cos\phi$.

$V_{rms} = \dfrac{P_{av}}{I_{rms}\cos\phi} = \dfrac{140\text{ W}}{(0.882\text{ A})\cos(-54.0°)} = 270$ V, which agrees. The source voltage lags the current when $X_C > X_L$, and this agrees with what we found.

31.52. **IDENTIFY:** We have an L-R-C series ac circuit.

SET UP and **EXECUTE:** **(a)** To detect a 4.0n GHz signal, a relevant characteristic is the resonance angular frequency. $\omega_0 = 1/\sqrt{LC}$. Solve for LC. $LC = 1/\omega_0^2 = 1/(2\pi f_0)^2 = 1/[2\pi(4\text{ GHz})]^2$ $= 1.6\times10^{-21}\text{ s}^2$.

(b) Use the result from (a). $L(1.0\times10^{-15}\text{ F}) = 1.6\times10^{-21}\text{ s}^2$. $L = 1.6\ \mu$H.

(c) Estimate: Thickness of phone = 2 mm; area = 4 mm^2.

(d) Use $L = \dfrac{\mu_0 N^2 A}{2\pi r}$, solve for N, and use the given numbers. $N = \sqrt{\dfrac{2\pi r L}{\mu_0 A}}$ = 160 turns.

EVALUATE: I would be difficult for a do-it-yourselfer to actually make such a small inductor.

31.53. **IDENTIFY:** We have a variable capacitor in an L-R-C series ac circuit.

SET UP and **EXECUTE:** **(a)** We want C. Use $C = \dfrac{\epsilon_0 A}{d}$. $d = g/2$ and A is the area in common. The maximum area in common is $A_{max} = \pi a^2/2$ when $\theta = \pi$. So $A = A_{max}(\theta/\pi) = (\pi a^2/2)(\theta/\pi) = a^2\theta/2$. The

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

capacitance at any θ is $C = \dfrac{\epsilon_0 \left(a^2\theta/2\right)}{g/2} = \epsilon_0\, a^2\theta/g$. There are 5 sets of capacitors, all in parallel, so the total capacitance is $C = 5\epsilon_0\, a^2\theta/g$.

(b) We want θ to receive a 1180 kHz signal. This frequency should be the resonance frequency f_0 of the circuit. Solve $1/\sqrt{LC} = 2\pi f_0$ for C and use f_0 = 1180 kHz and L = 100 μH. This gives $C = 1/L(2\pi f_0)^2 =$ 182 pF. Now find the angle θ to get that capacitance. Solve $C = 5\epsilon_0\, a^2\theta/g$ for θ, giving $\theta = \dfrac{gC}{5\epsilon_0\, a^2}$ = 2.29 rad = 131°.

(c) We want the amplitude of the output voltage. The output voltage is the amplitude V_C, so $V_{\text{out}} = V_C = IX_C$. At resonance $I = V/R$, so $V_{\text{out}} = IX_C = \left(\dfrac{V}{R}\right)X_C = \left(\dfrac{V}{R}\right)\left(\dfrac{1}{\omega C}\right) = \dfrac{V}{R\omega C} = 7.41$ V.

(d) We want f when θ = 120°. $\omega_0 = 1/\sqrt{LC}$ where $C = 5\epsilon_0\, a^2\theta/g$ and θ = 120° = $2\pi/3$ rad. First find C with these quantities, which gives $C = 1.668\times10^{-10}$ F. Now use $1/\sqrt{LC} = 2\pi f_0$ to find f_0. With the numbers this gives $f_0 = \dfrac{1}{2\pi\sqrt{LC}} =$ 1230 kHz.

(e) We want the amplitude of the output voltage at this frequency. $V_{\text{out}} = \dfrac{V}{R\omega C} = 7.74$ V.

EVALUATE: A device like this allows continuous tuning to signals within its range. Simply turn a dial to rotate the capacitor plates. This design was actually used for tuning equipment.

31.54. **IDENTIFY:** At any instant of time the same rules apply to the parallel ac circuit as to the parallel dc circuit: the voltages are the same and the currents add.

SET UP: For a resistor the current and voltage in phase. For an inductor the voltage leads the current by 90° and for a capacitor the voltage lags the current by 90°.

EXECUTE: **(a)** The parallel *L-R-C* circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

(b) $i_R = \dfrac{v}{R}$ is always in phase with the voltage. $i_L = \dfrac{v}{\omega L}$ lags the voltage by 90°, and $i_C = v\omega C$ leads the voltage by 90°. The phasor diagram is sketched in Figure 31.54.

(c) From the diagram, $I^2 = I_R^2 + (I_C - I_L)^2 = \left(\dfrac{V}{R}\right)^2 + \left(V\omega C - \dfrac{V}{\omega L}\right)^2$.

(d) From part (c): $I = V\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$. But $I = \dfrac{V}{Z}$, so $\dfrac{1}{Z} = \sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$.

EVALUATE: For large ω, $Z \to \dfrac{1}{\omega C}$. The current in the capacitor branch is much larger than the current in the other branches. For small ω, $Z \to \omega L$. The current in the inductive branch is much larger than the current in the other branches.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

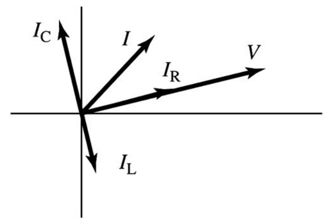


Figure 31.54

31.55. IDENTIFY: Apply the expression for $1/Z$ from Problem 31.54.

SET UP: From Problem 31.54, $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$.

EXECUTE: **(a)** Using $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$, we see that the impedance Z is a maximum when the square root is a minimum, and that occurs when $\omega C - \frac{1}{\omega L} = 0$. But that occurs when $\omega = \frac{1}{\sqrt{LC}}$, which is the resonance angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$. Since $I = V/Z$, the current is then a minimum when Z is a maximum.

(b) Using the result from part (a) gives $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.300\text{ H})(0.100\times10^{-6}\text{ F})}} = 5770\text{ rad/s}$.

(c) At resonance, $Z = R = 100\ \Omega$, so $I = V/R = (240\text{ V})/(100\ \Omega) = 2.40\text{ A}$.

(d) At resonance, the amplitude of the current in the resistor is $I = V/R = (240\text{ V})/(100\ \Omega) = 2.40\text{ A}$.

(e) At resonance, $X_L = \omega L = (5770\text{ rad/s})(0.300\text{ H}) = 1730\ \Omega$, which is also X_C. The amplitude of the maximum current through the inductor is $I = V/X_L = (240\text{ V})/(1730\ \Omega) = 0.139\text{ A}$.

(f) Since we are at resonance, $X_L = X_C = 1730\text{ A}$. Therefore $I = V/X_C = (240\text{ V})/(1730\ \Omega) = 0.139\text{ A}$.

EVALUATE: The parallel circuit is sketched in Figure 31.55. At resonance, $|i_C| = |i_L|$ and at any instant of time these two currents are in opposite directions. Therefore, the net current between a and b is always zero. If the inductor and capacitor each have some resistance, and these resistances aren't the same, then it is no longer true that $i_C + i_L = 0$. The result in part (a) for a parallel L-R-C circuit at resonance that the impedance is a maximum and the current is a minimum is the opposite of the behavior of a series L-R-C circuit at resonance.

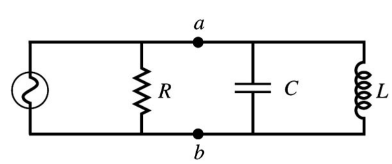


Figure 31.55

31.56. IDENTIFY: Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

SET UP: $V = \sqrt{2}V_{\text{rms}} = 254.6\text{ V}$.

EXECUTE: **(a)** $I_R = \frac{V}{R} = \frac{254.6\text{ V}}{400\ \Omega} = 0.636\text{ A}$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) $I_C = V\omega C = (254.6\text{ V})(360\text{ rad/s})(6.00\times10^{-6}\text{ F}) = 0.550\text{ A}.$

(c) $\phi = \arctan\left(\dfrac{I_C}{I_R}\right) = \arctan\left(\dfrac{0.550\text{ A}}{0.636\text{ A}}\right) = 40.8°.$

(d) $I = \sqrt{I_R^2 + I_C^2} = \sqrt{(0.636\text{ A})^2 + (0.550\text{ A})^2} = 0.841\text{ A}.$

(e) Leads since $\phi > 0$.

EVALUATE: The phasor diagram shows that the current in the capacitor always leads the source voltage.

31.57. IDENTIFY: The average power depends on the phase angle ϕ.

SET UP: The average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$, and the impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = \frac{1}{2}(V_{\text{rms}} I_{\text{rms}})$, which gives $\cos\phi = \frac{1}{2}$, so $\phi = \pi/3 = 60°$. $\tan\phi = (X_L - X_C)/R$, which gives $\tan 60° = (\omega L - 1/\omega C)/R$. Using $R = 75.0\ \Omega$, $L = 5.00\text{ mH}$ and $C = 2.50\ \mu\text{F}$ and solving for ω we get $\omega = 28760\text{ rad/s} = 28{,}800\text{ rad/s}$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L = (28{,}760\text{ rad/s})(5.00\text{ mH}) = 144\ \Omega$ and $X_C = 1/\omega C = 1/[(28{,}760\text{ rad/s})(2.50\ \mu\text{F})] = 13.9\ \Omega$, giving $Z = \sqrt{(75\ \Omega)^2 + (144\ \Omega - 13.9\ \Omega)^2} = 150\ \Omega$; $I = V/Z = (15.0\text{ V})/(150\ \Omega) = 0.100\text{ A}$ and $P_{\text{av}} = \frac{1}{2} VI\cos\phi = \frac{1}{2}(15.0\text{ V})(0.100\text{ A})(1/2) = 0.375\text{ W}$.

EVALUATE: All this power is dissipated in the resistor because the average power delivered to the inductor and capacitor is zero.

31.58. IDENTIFY and **SET UP:** The maximum energy in the inductor depends on the current amplitude in the inductor. $U_L = \frac{1}{2}LI^2$ and $U_C = \frac{1}{2}CV^2$. The impedance of a series *L-R-C* circuit is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2},\quad X_C = \frac{1}{\omega C}.$$

EXECUTE: **(a)** Use $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ to find the impedance of the circuit.

$$Z = \sqrt{(60.0\ \Omega)^2 + \left[(120\text{ rad/s})(0.800\text{ H}) - \frac{1}{(120\text{ rad/s})(3.00\times10^{-4}\text{ F})}\right]^2} = 90.85\,\Omega.$$

The amplitude of the current is therefore $I = V/Z = (90.0\text{ V})/(90.85\ \Omega) = 0.9906\text{ A}$, so the maximum energy stored in the inductor is $U_L = \frac{1}{2}LI^2 = (1/2)(0.800\text{ H})(0.9906\text{ A})^2 = 0.393\text{ J}$.

(b) The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2$, but the capacitor voltage is 90° out of phase with the current. Thus when the current is a maximum, the voltage across the capacitor is zero, so the energy stored in the capacitor is also zero.

(c) The capacitor stores its maximum energy when it is at maximum voltage, which is $V_C = IX_C = I\dfrac{1}{\omega C} = (0.9906\text{ A})\left[\dfrac{1}{(120\text{ rad/s})(3.00\times10^{-4}\text{ F})}\right] = 27.52\text{ V}$. The maximum energy in the capacitor at this time is $U_C = \frac{1}{2}CV^2 = (1/2)(3.00\times10^{-4}\text{ F})(27.52\text{ V})^2 = 0.114\text{ J}$.

EVALUATE: The maximum energy stored in the inductor is not the same as in the capacitor due to the presence of resistance.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

31.59. **IDENTIFY** and **SET UP:** The equation $V_C = IX_C$ allows us to calculate I and then $V = IZ$ gives Z. Solve $Z = \sqrt{R^2 + (X_L - X_C)^2}$ for X_L.

EXECUTE: **(a)** $V_C = IX_C$ so $I = \dfrac{V_C}{X_C} = \dfrac{360\text{ V}}{480\ \Omega} = 0.750\text{ A}.$

(b) $V = IZ$ so $Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{0.750\text{ A}} = 160\ \Omega.$

(c) $Z^2 = R^2 + (X_L - X_C)^2.$

$X_L - X_C = \pm\sqrt{Z^2 - R^2}$, so

$X_L = X_C \pm \sqrt{Z^2 - R^2} = 480\ \Omega \pm \sqrt{(160\ \Omega)^2 - (80.0\ \Omega)^2} = 480\ \Omega \pm 139\ \Omega.$

$X_L = 619\ \Omega$ or $341\ \Omega.$

EVALUATE: **(d)** $X_C = \dfrac{1}{\omega C}$ and $X_L = \omega L.$ At resonance, $X_C = X_L.$ As the frequency is lowered below the resonance frequency X_C increases and X_L decreases. Therefore, for $\omega < \omega_0,\ X_L < X_C.$ So for $X_L = 341\ \Omega$ the angular frequency is less than the resonance angular frequency. ω is greater than ω_0 when $X_L = 619\ \Omega.$ But at these two values of $X_L,$ the magnitude of $X_L - X_C$ is the same so Z and I are the same. In one case $(X_L = 691\ \Omega)$ the source voltage leads the current and in the other $(X_L = 341\ \Omega)$ the source voltage lags the current.

31.60. **IDENTIFY** and **SET UP:** The capacitive reactance is $X_C = \dfrac{1}{\omega C}$, the inductive reactance is $X_L = \omega L,$ and the impedance of an *L-R-C* series circuit is $Z = \sqrt{R^2 + (X_L - X_C)^2}.$

EXECUTE: **(a)** The current amplitude is $I = V/R = (135\text{ V})/(90.0\ \Omega) = 1.50\text{ A}.$
(b) The voltage amplitude across the inductor is $V_L = IX_L = (1.50\ \Omega)(320\ \Omega) = 480\text{ V}.$
(c) The impedance is $Z = V/I = (240\text{ V})/(1.50\text{ A}) = 160\ \Omega$. We also know that the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}.$ We know that $X_{-L} = 320\ \Omega$, so we can find X_C.

$160\ \Omega = \sqrt{(90.0\ \Omega)^2 + (320\ \Omega - X_C)^2}.$ Squaring and solving for X_C gives two values, $X_C = 188\ \Omega$ and $X_C = 452\ \Omega$.

(d) At resonance, $\omega L = \dfrac{1}{\omega C}.$ $X_C < X_L$ for $\omega > \omega_{\text{res}}$ and $X_C > X_L$ for $\omega < \omega_{\text{res}}$. In this circuit, $X_L = 320\ \Omega$, so $\omega < \omega_{\text{res}}$ for $X_C = 452\ \Omega$.

EVALUATE: Due to the square of $(X_L - X_C)$ in the impedance, we get two possibilities in (c).

31.61. **IDENTIFY:** At resonance, $Z = R.$ $I = V/R.$ $V_R = IR,$ $V_C = IX_C$ and $V_L = IX_L.$ $U_C = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2.$

SET UP: The amplitudes of each time-dependent quantity correspond to the maximum values of those quantities.

EXECUTE: **(a)** $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$ At resonance $\omega L = \dfrac{1}{\omega C}$ and $I_{\max} = \dfrac{V}{R}.$

(b) $V_C = IX_C = \dfrac{V}{R\omega_0 C} = \dfrac{V}{R}\sqrt{\dfrac{L}{C}}.$

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist.
No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(c) $V_L = IX_L = \frac{V}{R}\omega_0 L = \frac{V}{R}\sqrt{\frac{L}{C}}.$

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}C\frac{V^2}{R^2}\frac{L}{C} = \frac{1}{2}L\frac{V^2}{R^2}.$

(e) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\frac{V^2}{R^2}.$

EVALUATE: At resonance $V_C = V_L$ and the maximum energy stored in the inductor equals the maximum energy stored in the capacitor.

31.62. IDENTIFY and **SET UP:** We use $I = V/Z$, $X_L = \omega L$, and $Z = \sqrt{R^2 + X_L^2}$.

EXECUTE: $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}}$. Squaring and rearranging gives $\frac{1}{I^2} = \left(\frac{L}{V}\right)^2 \omega^2 + \left(\frac{R}{V}\right)^2$. Therefore a graph of $1/I^2$ versus ω^2 should be a straight line having slope equal to $(L/V)^2$ and y-intercept equal to $(R/V)^2$. We can find the slope using two convenient points on the graph, giving

slope = $\frac{(21.0 - 9.0)\text{ A}^{-2}}{(3500 - 500)\text{ rad}^2/\text{s}^2} = 4.00 \times 10^{-3}\text{ s}^2/\text{rad}^2 \cdot \text{A}^2.$

Solving for L gives $L = V\sqrt{\text{slope}} = (12.0\text{ V})\sqrt{4.00 \times 10^{-3}\text{ s}^2/\text{rad}^2 \cdot \text{A}^2} = 0.759\text{ H}.$

Extending the line, we find the y-intercept is 7.0 A^{-2}. Using this value to solve for R gives

$R = V\sqrt{y\text{-intercept}} = (12.0\text{ V})\sqrt{7.00\text{ A}^{-2}} = 32\ \Omega.$

EVALUATE: These are reasonable values for L and R for a large solenoid, so we're confident in the results.

31.63. IDENTIFY and **SET UP:** For an L-R-C series circuit, the maximum current occurs at resonance, and the resonance angular frequency is $\omega_{\text{res}} = \frac{1}{\sqrt{LC}}.$

EXECUTE: At resonance, the angular frequency is $\omega_{\text{res}} = \frac{1}{\sqrt{LC}}$. Squaring gives $\omega_{\text{res}}^2 = \frac{1}{L} \cdot \frac{1}{C}$, so a graph of ω_{res}^2 versus $1/C$ should be a straight line with a slope equal to $1/L$. Using two convenient points on the graph, we find the slope to be $\frac{(25.0 - 1.00) \times 10^4\text{ rad}^2/\text{s}^2}{(4.50 - 1.75)\ \times 10^3\text{ F}^{-1}} = 5.455\text{ F/s}^2$. Solving for L gives $L = (\text{slope})^{-1} = (5.455\text{ F/s}^2)^{-1} = 0.183$ H, which rounds to 0.18 H, since we cannot determine the slope of the graph in the text with anything better than 2 significant figures. To find R, we realize that at resonance $Z = R$, so $R = V/I = (90.0\text{ V})/(4.50\text{ A}) = 20.0\ \Omega$.

EVALUATE: These are reasonable values for L and R for a laboratory solenoid.

31.64. IDENTIFY and **SET UP:** For an L-R-C series circuit, $\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$ and the power factor is $\cos\phi = R/Z$.

EXECUTE: **(a)** $\cos\phi = R/Z$, so $R = Z\cos\phi$.

At 80 Hz: $R = (15\ \Omega)\cos(-71°) = 4.88\ \Omega$.

At 160 Hz: $R = (13\ \Omega)\cos(67°) = 5.08\ \Omega$.

The average resistance is $(4.88\ \Omega + 5.08\ \Omega)/2 = 5.0\ \Omega$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) We use $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$ with R = 5.0 Ω from part (a).

At 80 Hz: $\tan(-71°) = \dfrac{2\pi(80\text{ Hz})L - \dfrac{1}{2\pi(80\text{ Hz})C}}{5.0\ \Omega}$.

–14.52 = 160π Hz L – 1/[(160π Hz)C]. Eq. (1)

At 160 Hz: $\tan(67°) = \dfrac{2\pi(160\text{ Hz})L - \dfrac{1}{2\pi(160\text{ Hz})C}}{5.0\ \Omega}$.

11.78 = 320π Hz L – 1/[(320π Hz)C]. Eq. (2)

Multiply Eq. (1) by –2 and add it to Eq. (2), giving

2(14.52) + 11.78 = (1/C)(1/80π – 1/320π).

$C = 7.31\times10^{-5}$ F, which rounds to C = 73 μF.

Substituting this result into either Eq. (1) or Eq. (2) gives L = 25.3 mH, which rounds to L = 25 mH.

(c) The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$, so the resonance frequency is

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.53\times10^{-2}\text{ H})(73.1\times10^{-6}\text{ F})}} = 117\text{ Hz.}$$

At resonance, $Z = R$ = 5.0 Ω and ϕ = 0.

EVALUATE: It is only at resonance that $Z = R$, not at the other frequencies.

31.65. **IDENTIFY:** We have an L-R ac circuit.

SET UP: $v_{in} = V_{in}\cos\omega t$, $G = (20\text{ dB})\ln(V_{out}/V_{in})$, $I = V_{in}/Z$, $Z = \sqrt{R^2 + (\omega L)^2}$, $\phi = \arctan\left(\dfrac{\omega L}{R}\right)$.

EXECUTE: **(a)** We want the current. $I = \dfrac{V_{in}}{Z} = \dfrac{V_{in}}{\sqrt{R^2+(\omega L)^2}}$.

(b) We want ϕ. $\phi = \arctan\left(\dfrac{\omega L}{R}\right)$.

(c) We want V_{out}/V_{in}. $\dfrac{V_{out}}{V_{in}} = \dfrac{V_L}{V_{in}} = \dfrac{IX_L}{V_{in}} = \dfrac{\dfrac{V_{in}}{\sqrt{R^2+(\omega L)^2}}}{V_{in}}\cdot\omega L = \dfrac{\omega L}{\sqrt{R^2+(\omega L)^2}} = \dfrac{1}{\sqrt{1+(R/\omega L)^2}}$.

(d) We want f so G = –3.0 dB. Use $G = (20\text{ dB})\ln(V_{out}/V_{in})$. $-3.0\text{ dB} = (20\text{ dB})\ln(V_{out}/V_{in})$.

$-3.0/20 = \ln(V_{out}/V_{in})$. Write this result in terms of exponents and use the result from (c).

$10^{-3.0/20} = 0.708 = \dfrac{V_{out}}{V_{in}} = \dfrac{1}{\sqrt{1+(R/\omega L)^2}}$. Use $\omega = 2\pi f$ and solve for f, giving $f = \dfrac{R}{2.00\pi L}$.

(e) We want L. Solve the result in (d) when f = 10.0 kHz and R = 100 Ω, giving L = 1.6 mH.

EVALUATE: By varying the ration R/L we can tune to any desired frequency.

31.66. **IDENTIFY:** We have an L-R-C series ac circuit.

SET UP: The output is across the capacitor. $v_{in} = V_{in}\cos\omega t$, $v_{out} = V_{out}\cos(\omega t+\theta)$,

$Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$, $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right)$, $\omega_0 = 1/\sqrt{LC}$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

EXECUTE: **(a)** We want V_{out}. $V_{\text{out}} = V_C = IX_C = \dfrac{V_{\text{in}}}{Z} X_C = \dfrac{V_{\text{in}}}{\omega C\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$, which we can express as $V_{\text{out}} = \dfrac{V_{\text{in}}}{\sqrt{(R\omega C)^2 + \left(\omega^2 LC - 1\right)^2}}$.

(b) We want the phase angle for v_{out}. The output voltage is across the capacitor, so the phase angle θ is the same as for the capacitor. The capacitor voltage lags the current by $\pi/2$, so the phase angle is $\theta = \phi - \pi/2$. $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{\omega^2 LC - 1}{\omega RC}\right)$. So $\theta = \arctan\left(\dfrac{\omega^2 LC - 1}{\omega RC}\right) - \dfrac{\pi}{2}$.

(c) We want θ at resonance. At resonance $\omega_0 = 1/\sqrt{LC}$, so $\phi = 0$. Thus $\theta = -\pi/2$.

(d) We want C. Solving $\omega_0 = 1/\sqrt{LC}$ for C and using the given values, we get

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi f_0)^2 L} = 2.53\ \mu\text{F}.$$

(e) We want V_{out}. $V_{\text{out}} = V_C$. Use the result from part (a) at resonance. $V_{\text{out}} = \dfrac{V_{\text{in}}}{R\omega C} = \dfrac{V_{\text{in}}}{2\pi fRC}$. Using the numbers gives $V_{\text{out}} = 62.9$ mV.

EVALUATE: Simply by varying the frequency the output voltage amplitude changes.

31.67. **IDENTIFY:** $p_R = i^2 R$. $p_L = iL\dfrac{di}{dt}$. $p_C = \dfrac{q}{C}i$.

SET UP: $i = I\cos\omega t$.

EXECUTE: **(a)** $p_R = i^2 R = I^2\cos^2(\omega t)R = V_R I\cos^2(\omega t) = \dfrac{1}{2}V_R I(1+\cos(2\omega t))$.

$$P_{\text{av}}(R) = \frac{1}{T}\int_0^T p_R dt = \frac{V_R I}{2T}\int_0^T [1+\cos(2\omega t)]dt = \frac{V_R I}{2T}[t]_0^T = \tfrac{1}{2}V_R I.$$

(b) $p_L = Li\dfrac{di}{dt} = -\omega L I^2\cos(\omega t)\sin(\omega t) = -\tfrac{1}{2}V_L I\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{\text{av}}(L) = 0$.

(c) $p_C = \dfrac{q}{C}i = v_C i = V_C I\sin(\omega t)\cos(\omega t) = \tfrac{1}{2}V_C I\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{\text{av}}(C) = 0$.

(d) $p = p_R + p_L + p_c = V_R I\cos^2(\omega t) - \tfrac{1}{2}V_L I\sin(2\omega t) + \tfrac{1}{2}V_C I\sin(2\omega t)$ and

$p = I\cos\omega t(V_R\cos\omega t - V_L\sin\omega t + V_C\sin\omega t)$. But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V}$, so

$p = VI\cos\omega t(\cos\phi\cos\omega t - \sin\phi\sin\omega t)$, at any instant of time.

EVALUATE: At an instant of time the energy stored in the capacitor and inductor can be changing, but there is no net consumption of electrical energy in these components.

31.68. **IDENTIFY:** $V_L = IX_L$. $\dfrac{dV_L}{d\omega} = 0$ at the ω where V_L is a maximum. $V_C = IX_C$. $\dfrac{dV_C}{d\omega} = 0$ at the ω where V_C is a maximum.

SET UP: Problem 31.49 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

EXECUTE: **(a)** $V_R = \text{maximum}$ when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(b) $V_L = \text{maximum}$ when $\dfrac{dV_L}{d\omega} = 0$. Therefore: $\dfrac{dV_L}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$.

$$0 = \frac{VL}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{\left[R^2 + (\omega L - 1/\omega C)^2\right]^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = \omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = -\frac{1}{\omega^2 C^2}.\quad \frac{1}{\omega^2} = LC - \frac{R^2 C^2}{2} \text{ and } \omega = \frac{1}{\sqrt{LC - R^2 C^2/2}}.$$

(c) $V_C = \text{maximum}$ when $\dfrac{dV_C}{d\omega} = 0$. Therefore: $\dfrac{dV_C}{d\omega} = 0 = \dfrac{d}{d\omega}\left(\dfrac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$.

$$0 = -\frac{V}{\omega^2 C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}.\quad R^2 + (\omega L - 1/\omega C)^2 = -\omega^2(L^2 - 1/\omega^4 C^2).$$

$$R^2 + \omega^2 L^2 - \frac{2L}{C} = -\omega^2 L^2 \text{ and } \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

EVALUATE: V_L is maximum at a frequency greater than the resonance frequency and V_C is a maximum at a frequency less than the resonance frequency. These frequencies depend on R, as well as on L and on C.

31.69. **IDENTIFY** and **SET UP:** We are told that the platinum electrode behaves like an ideal capacitor in series with the resistance of the fluid. The impedance of an *R-C* circuit is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}$.

EXECUTE: For a dc signal we have $\omega = 2\pi f = 0$. Using $X_C = \dfrac{1}{\omega C}$ we see that as $\omega \to 0$ we have $X_C \to \infty$, and so $Z \to \infty$. The correct choice is (b).

EVALUATE: The oscillation period of such a circuit is $T = 1/f$, so $T \to \infty$ as $\omega \to 0$.

31.70. **IDENTIFY** and **SET UP:** We are told that the platinum electrode behaves like an ideal capacitor in series with the resistance of the fluid, which is given by $R_A = \rho/(10a)$, where $\rho = 100\ \Omega\cdot\text{cm} = 1\ \Omega\cdot\text{m}$ and $d = 2a = 20\ \mu\text{m}$. We know that $X_C = \dfrac{1}{\omega C}$, where we are given $C = 10\ \text{nF} = 10^{-8}\ \text{F}$ and $\omega = 2\pi f = 2\pi[(5000/\pi)\text{Hz}] = 10^4\ \text{rad/s}$. For an *R-C* circuit we know that the impedance is given by s

EXECUTE: $R_A = \rho/(10a) = (1\ \Omega\cdot\text{m})/[10(10^{-5}\ \text{m})] = 10^4\ \Omega$. The capacitive reactance is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(10^4\ \text{rad/s})(10^{-8}\ \text{F})} = 10^4\ \Omega$. Thus the impedance is $Z = \sqrt{R^2 + X_C^2} = \sqrt{(10^4\ \Omega)^2 + (10^4\ \Omega)^2} = \sqrt{2}\times(10^4\ \Omega)$, so the correct choice is (c).

EVALUATE: In this case, the capacitance contributes as much to the impedance as the resistance does.

31.71. **IDENTIFY** and **SET UP:** We know that $V_{\text{rms}} = \dfrac{V}{\sqrt{2}}$, where V is the amplitude (peak value) of the voltage. According to the problem, the peak-to-peak voltage V_{pp} is the difference between the two extreme values of voltage.

EXECUTE: Since the voltage oscillates between $+V$ and $-V$ the peak-to-peak voltage is $V_{\text{pp}} = V - (-V) = 2V = 2\sqrt{2}V_{\text{rms}}$. Thus, the correct answer is (d).

EVALUATE: The voltage amplitude is half the peak-to-peak voltage.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

31.72. **IDENTIFY** and **SET UP:** The impedance of an *R-C* circuit is $Z = \sqrt{R^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}$.

EXECUTE: As the frequency of oscillation gets very large, X_C gets very small, so the impedance approaches the access resistance R. So the impedance approaches a constant but nonzero value, which is choice (c).

EVALUATE: For high oscillation frequency, the access resistance has more effect on the circuit than the capacitance does.

© Copyright 2020 Pearson Education, Inc. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.