16

SOUND AND HEARING

VP16.9.1. **IDENTIFY:** We want to know the pressure amplitude of a sound wave.

SET UP: $I = \dfrac{p_{\max}^2}{2\rho v}$

EXECUTE: **(a)** Solve for $p_{\max}$ giving $p_{\max} = \sqrt{2\rho v I}$. Putting in the numbers we get

$p_{\max} = \sqrt{2(1.20\ \text{kg/m}^3)(344\ \text{m/s})(5.50\times10^{-8}\ \text{W/m}^2)} = 6.74\times10^{-3}\ \text{Pa}.$

(b) Neither ρ nor v is affected by the frequency change, so $p_{\max}$ remain unchanged.

EVALUATE: Changing the air density would affect the pressure amplitude.

VP16.9.2. **IDENTIFY:** This problem deals with sound intensity and intensity level.

SET UP: $\beta = (10\ \text{dB})\log\dfrac{I}{I_0}$

EXECUTE: **(a)** We know the sound level is 85.0 dB and want the sound intensity. Using

$\beta = (10\ \text{dB})\log\dfrac{I}{I_0}$ gives $85.0\,\text{dB} = (10\ \text{dB})\log\dfrac{I}{I_0}$. Solving for I gives

$I = 10^{8.5} I_0 = 10^{8.5}10^{-12}\ \text{W/m}^2 = 10^{-3.50}\ \text{W/m}^2 = 3.16\times10^{-4}\ \text{W/m}^2.$

(b) Solve $\beta = (10\ \text{dB})\log\dfrac{I}{I_0}$ for I, giving $I = I_0 10^{\beta/(10\ \text{dB})}$. Now take the ratio of the two intensities.

$\dfrac{I_{85}}{I_{67}} = \dfrac{I_0 10^{(85.0\,\text{dB})/(10\ \text{dB})}}{I_0 10^{(67.0\,\text{dB})/(10\ \text{dB})}} = \dfrac{10^{8.50}}{10^{6.70}} = 63.1,$ so I_{85} is 63.1 times greater than I_{67}.

EVALUATE: The ratio of the sound intensity levels is 85/67 = 1.27, but the intensity ratio is *much* greater.

VP16.9.3. **IDENTIFY:** This problem involves sound intensity and intensity level.

SET UP: We know the sound intensity level of the lion's roar is 114 at 1.00 m, and we want to know what it is at 4.00 m and 15.8 m from the lion. The sound intensity obeys an inverse-square law, but the sound intensity level does not. We use $\beta = (10\ \text{dB})\log\dfrac{I}{I_0}$ and $I_2 = I_1\left(\dfrac{r_1}{r_2}\right)^2$.

EXECUTE: **(a)** <u>At 1.00 m</u>: $\beta_1 = (10\ \text{dB})\log\dfrac{I_1}{I_0} = 114$ dB. Solve for I_1: $I_1 = 10^{11.4} I_0$.

<u>At 4.00 m</u>: $I_2 = I_1\left(\dfrac{r_1}{r_2}\right)^2$ gives $I_4 = I_1\left(\dfrac{1.00\ \text{m}}{4.00\ \text{m}}\right)^2 = \dfrac{I_1}{16.0} = \dfrac{10^{11.4} I_0}{16.0}$. Now find the sound intensity level:

$\beta_4 = (10\ \text{dB})\log\dfrac{I_4}{I_0} = (10\ \text{dB})\log\left(\dfrac{10^{11.4} I_0}{16.0 I_0}\right) = 102\ \text{dB}.$

(b) At 15.8 m we follow the same procedure as above, giving

$$\beta_{15.8} = (10\text{ dB})\log\frac{I_{16}}{I_0} = (10\text{ dB})\log\left(\frac{10^{11.4}I_0}{(15.8)^2 I_0}\right) = 90.0\text{ dB}.$$

EVALUATE: The sound intensity obeys an inverse-square law, but sound intensity level does not.

VP16.9.4. **IDENTIFY:** We are investigating the relationship between sound intensity level and the pressure amplitude of a sound wave.

SET UP: We know that $I = \dfrac{p_{\max}^2}{2\rho v}$, $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$, and $I_0 = 10^{-12}$ W/m^2. We want to find the pressure amplitude of the sound wave and then see how changing it would affect the sound intensity level.

EXECUTE: **(a)** First use $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$ to find the intensity I. $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$ = 66.0 dB, so

$I = 10^{6.60}I_0 = 3.9811\times10^{-6}$ W/m^2. Now use I to find $p_{\max}$. Solving $I = \dfrac{p_{\max}^2}{2\rho v}$ for $p_{\max}$ gives

$p_{\max} = \sqrt{2\rho v I}$, which gives $p_{\max} = \sqrt{2(0.920\text{ kg/m}^3)(344\text{ m/s})(3.9811\times10^{-6}\text{ W/m}^2)} = 5.02\times10^{-2}$ Pa.

(b) Since $I \propto p_{\max}^2$, increasing $p_{\max}$ by a factor of 10.0 will increase I by a factor of $10.0^2 = 100$, so $I_2 = 100I_1$. Taking the ratio of the sound intensity levels gives

$\dfrac{\beta_2}{\beta_1} = \dfrac{(10\text{ dB})\log(I_2/I_0)}{(10\text{ dB})\log(I_1/I_0)} = \dfrac{\log(100I_1/I_0)}{\log(I_1/I_0)} = \dfrac{2+\log(I_1/I_0)}{\log(I_1/I_0)}$. But $\beta_1 = (10\text{ dB})\log(I_1/I_0)$, so

$\log(I_1/I_0) = \dfrac{\beta_1}{10} = \dfrac{66.0\text{ dB}}{10} = 6.60$. Therefore $\dfrac{\beta_2}{\beta_1} = \dfrac{2+\frac{\beta_1}{10}}{\frac{\beta_1}{10}} = \dfrac{2+6.60}{6.60} = 1.303$. So we have

$\beta_2 = 1.303\beta_1 = (1.303)(66.0\text{ dB}) = 86.0$ dB.

EVALUATE: Careful in handling logarithms because $\log(100I_1/I_0)$ is *not* equal to $100\log I_1/I_0$.

VP16.12.1. **IDENTIFY:** We are dealing with standing sound waves in open and stopped pipes.

SET UP: For an open pipe, $f_n = nv/2L$ (n = 1, 2, 3, …), and for a stopped pipe $f_n = nv/4L$ (n = 1, 3, 5, …).

EXECUTE: **(a)** We want the length L of this open pipe. For the fundamental frequency, $f_1 = v/2L$, so the length is L = (344 m/s)/[2(220 Hz)] = 0.782 m.

(b) For the open pipe in its 3rd harmonic, n = 3, so $f_3 = \dfrac{3v}{2L_3}$.

For the stopped pipe in its fundamental frequency, $f_1 = \dfrac{v}{4L_1}$.

The two frequencies are the same, so $\dfrac{3v}{2L_3} = \dfrac{v}{4L_1}$, so $L_1 = L_3/6$ = (0.782 m)/6 = 0.130 m.

EVALUATE: Careful on stopped pipes: n must be an *odd* integer, but on open pipes n can be odd and even integers.

VP16.12.2. **IDENTIFY:** We are investigating the harmonics of a stopped and an open organ pipe. We want to find what harmonics of a stopped pipe will resonate at the same frequency as the third harmonic of an open pipe.

SET UP: For an open pipe $f_n = nv/2L$ (n = 1, 2, 3, …), and for a stopped pipe $f_n = nv/4L$ (n = 1, 3, 5, …). We know that $f_3 = 3v/2L_o$ for the open pipe. We want to find values of n for the stopped pipe so that it will have the same frequency as the third harmonic of the open pipe. That is f_n(stopped) = f_3(open), where we want to find n for the stopped pipe. Equating the frequencies gives $\frac{nv}{4L_s} = \frac{3v}{2L_o}$, so $n = 6\frac{L_s}{L_o}$.

EXECUTE: **(a)** In this case $L_s = L_o/6$ so $n = 6\frac{L_s}{L_o} = 6\left(\frac{L_o}{6L_o}\right) = 1$.

(b) In this case, $L_s = L_o/2$, so $n = 6\frac{L_s}{L_o} = 6\left(\frac{L_o}{2L_o}\right) = 3$.

(c) In this case, $L_s = L_o/3$, so $n = 6\frac{L_s}{L_o} = 6\left(\frac{L_o}{3L_o}\right) = 2$. But n is only *odd* for a stopped pipe, so there are no harmonics of the stopped pipe having a frequency that is equal to the 3rd harmonic frequency of the open pipe.

EVALUATE: We cannot always make two pipes resonate at a given frequency.

VP16.12.3. **IDENTIFY:** A string vibrating in its fundamental frequency causes a nearby stopped organ pipe to vibrate in its fundamental frequency. So we are dealing with standing waves on a string and in a stopped pipe.

SET UP: For a string fixed at its ends, $f_n = nv_{str}/2L$ and $\lambda_n = 2L/n$ (n = 1, 2, 3, …), and for a stopped pipe $f_n = nv/4L$ (n = 1, 3, 5, …). For any wave $v = f\lambda$.

EXECUTE: **(a)** For the fundamental mode of the string, $\lambda_1 = 2L$ so $v_{str} = f\lambda = f(2L) =$ (165 Hz)(2)(0.680 m) = 224 m/s.

(b) The organ pipe in its fundamental mode is vibrating at the same frequency as the string. Using $f_n = nv/4L$ and solving for L gives $L = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(165 \text{ Hz})} = 0.521$ m.

EVALUATE: The frequency of the sound wave is the same as the frequency at which the string is vibrating, but the wavelengths of the two waves are *not* the same because they have different speeds.

VP16.12.4. **IDENTIFY:** A stopped pipe 1.00 m long that is filled with helium is vibrating in its third harmonic, which causes a nearby open pipe to vibrate in its fifth harmonic.

SET UP: We want to know the frequency and wavelength of the sound in both pipes and the length of the open pipe. The speed of sound in the helium is v_{He} = 999 m/s, $v = f\lambda$, $\lambda_n = 4L/n$ (n = 1, 3, 4, …) for a stopped pipe, and $f_n = nv/2L$ (n = 1, 2, 3 …) for an open pipe.

EXECUTE: **(a)** In the stopped pipe, $\lambda_3 = 4L/3$ = 4(1.00 m)/3 = 1.33 m.

Using $v = f\lambda$ gives $f_3 = \frac{v_{He}}{\lambda_3} = \frac{999 \text{ m/s}}{1.33 \text{ m}} =$ 749 Hz.

(b) In the open pipe, we know that f_5 = 749 Hz.

Using $v = f\lambda$ gives $\lambda_5 = \frac{v_{air}}{f_5} = \frac{344 \text{ m/s}}{749 \text{ Hz}} =$ 0.459 m.

(c) Using $f_n = nv/2L$ in the open pipe gives $f_5 = \frac{5v_{air}}{2L}$. Solving for L gives

$$L = \frac{5v_{air}}{2f_5} = \frac{5(344 \text{ m/s})}{2(749 \text{ Hz})} = 1.15 \text{ m}.$$

EVALUATE: The speed of sound is different in the helium than it is in air because the density of the helium is not the same as that of air.

VP16.18.1. **IDENTIFY:** You do not hear the same frequency that the siren is emitting due to the motion of the ambulance. This is due to the Doppler effect.

SET UP: We have a moving source of sound and a stationary listener. The target variables are the frequency and wavelength of the sound heard by the listener, so use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ and $v = f\lambda$.

EXECUTE: **(a)** Listener in front of source: $v_S = -26.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives

$$f_L = \left(\frac{340\text{ m/s} + 0}{340\text{ m/s} - 26.0\text{ m/s}}\right)(2.80\times10^3\text{ Hz}) = 3.03\times10^3\text{ Hz}.$$

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{340\text{ m/s}}{3.03\times10^3\text{ Hz}} = 0.112$ m.

(b) Listener behind source: $v_S = +26.0$ m/s. $f_L = \left(\dfrac{340\text{ m/s} + 0}{340\text{ m/s} + 26.0\text{ m/s}}\right)(2.80\times10^3\text{ Hz}) = 2.60\times10^3$ Hz.

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{340\text{ m/s}}{2.60\times10^3\text{ Hz}} = 0.131$ m.

EVALUATE: When the source moves toward the listener, the wave crests are closer together so the frequency is increased. When the source moves away from the listener, the distance between wave crests is greater so the frequency is decreased.

VP16.18.2. **IDENTIFY:** The bike rider hears a sound different from what the bagpiper is emitting due to the bike's motion. This is due to the Doppler effect.

SET UP: The bagpiper is the stationary source and the bike rider is the moving observer. We want the frequency and wavelength of the sound the bike rider hears, so we use $v = f\lambda$ and $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ with $v_S = 0$.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{340\text{ m/s}}{440\text{ Hz}} = 0.773$ m.

(b) <u>Biker approaching source</u>: $v_L = +10.0$ m/s. Using $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives

$$f_L = \left(\frac{340\text{ m/s} + 10.0\text{ m/s}}{340\text{ m/s} + 0}\right)(440\text{ Hz}) = 453\text{ Hz}.$$

Now use $v = f\lambda$, but v is the speed of sound relative to the listener, which is

340 m/s + 10.0 m/s = 350 m/s. Therefore $\lambda = \dfrac{v}{f} = \dfrac{350\text{ m/s}}{453\text{ Hz}} = 0.773$ m.

<u>Biker moving away from source</u>: $v_L = -10.0$ m/s. Now we get

$$f_L = \left(\frac{340\text{ m/s} - 10.0\text{ m/s}}{340\text{ m/s}}\right)(440\text{ Hz}) = 427\text{ Hz}.$$

Now use $v = f\lambda$, where $v = 340$ m/s – 10.0 m/s = 330 m/s. $\lambda = \dfrac{v}{f} = \dfrac{330\text{ m/s}}{453\text{ Hz}} = 0.773$ m.

EVALUATE: The frequency varies due to the speed of the listener because she runs into the waves at a higher (or lower) rate due to her motion. But the wavelength is just the distance between wave crests, so it is not affected by her motion and we get the same answer in both cases.

VP16.18.3. **IDENTIFY:** The police car and the sports car are both moving, so both of their motions affect the frequency that the listener receives. We need to use the Doppler effect.

SET UP: Our target variable is the frequency f_L of sound that the listener receives. We use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. Figure VP16.18.3 shows both situations.

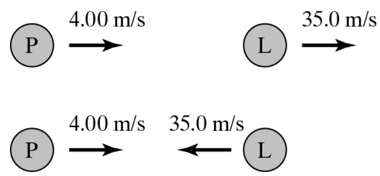


Figure VP16.18.3

EXECUTE: **(a)** The police car is moving in the direction of the listener and the listener is moving away from the police car, so $v_S = -40.0$ m/s and $v_L = -35.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives

$$f_L = \left(\frac{340 \text{ m/s} - 35.0 \text{ m/s}}{340 \text{ m/s} - 40.0 \text{ m/s}}\right)(1200 \text{ Hz}) = 1220 \text{ Hz}.$$

(b) The police car is moving in the direction of the listener and the listener is in the direction of the police car, so $v_S = -40.0$ m/s and $v_L = +35.0$ m/s. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives

$$f_L = \left(\frac{340 \text{ m/s} + 35.0 \text{ m/s}}{340 \text{ m/s} - 40.0 \text{ m/s}}\right)(1200 \text{ Hz}) = 1500 \text{ Hz}.$$

EVALUATE: In part (a), the motion of the police car causes a higher frequency but the motion of the sports car causes a lower frequency, so there is a small change in the frequency heard by the listener compared to the emitted frequency. In part (b) both of their motions increase the frequency, so there is a large frequency change.

VP16.18.4. **IDENTIFY:** The motion of the car increases the sound frequency in front of it, and this motion also increases the frequency of the sound the driver hears reflected from the wall. There are two Doppler effects to consider.

SET UP: We need to break this problem into two steps: (1) the car is a moving source and the wall is a stationary listener and (2) the wall is a stationary source and the car is a moving listener. The wall reflects the same frequency it receives from the car. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. In part (a) we want to find the frequency that the driver receives reflected from the wall. In part (b) we want to find how fast he most drive to hear sound of frequency 495 Hz coming from the wall.

EXECUTE: **(a)** Car as moving source and wall as stationary listener: $v_L = 0$ (the wall), $v_S = -25.0$ m/s. Use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ to find the frequency f_w that the wall receives.

$$f_w = \left(\frac{340 \text{ m/s}}{340 \text{ m/s} - 25.0 \text{ m/s}}\right)(415 \text{ Hz}) = 447.94 \text{ Hz}.$$

Car as moving listener and wall as stationary source: $v_S = 0$ (the wall), $v_L = +25.0$ m/s, and $f_S = 447.94$ Hz. Use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ to find the frequency f_d that the driver receives coming back from the wall.

$$f_d = \left(\frac{340 \text{ m/s} + 25.0 \text{ m/s}}{340 \text{ m/s}}\right)(447.94 \text{ Hz}) = 481 \text{ Hz}.$$

(b) We want to find v_S so that $f_c = 495$ Hz. We follow the same steps as in part (a).

Car as listener and wall as source: $v_L = v_c = ?$ (where v_c is the *magnitude* of the car's speed), $f_S = f_w = ?$, $f_L = f_c = 495$ Hz, $v_S = 0$ (wall). Using $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$ gives $495 \text{ Hz} = \left(\frac{340 \text{ m/s} + v_c}{340 \text{ m/s}}\right) f_w$, so

$$f_w = \frac{(495 \text{ Hz})(340 \text{ m/s})}{340 \text{ m/s} + v_c}.$$

Car as source and wall as listener: $v_S = -v_c$, $v_L = 0$, $f_S = 415$ Hz, $f_L = f_w = ?$ so $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$ gives

$f_w = \left(\frac{340 \text{ m/s}}{340 \text{ m/s} - v_c}\right)(415 \text{ Hz})$. Now use the result for f_w that we just found in the previous step.

Equating these two equations for f_w and solving for v_c gives us $\frac{(495 \text{ Hz})(340 \text{ m/s})}{340 \text{ m/s} + v_c} =$

$\left(\frac{340 \text{ m/s}}{340 \text{ m/s} - v_c}\right)(415 \text{ Hz})$, so $v_c = 29.9$ m/s.

EVALUATE: Our result in part (b) gives $v_c = 29.9$ m/s which is greater than the speed of 25.0 m/s in part (a). This result is reasonable because the frequency change in (b) is greater than the one in (a).

16.1. **IDENTIFY** and **SET UP:** $v = f\lambda$ gives the wavelength in terms of the frequency. Use $p_{max} = BkA$ to relate the pressure and displacement amplitudes.

EXECUTE: **(a)** $\lambda = v/f = (344 \text{ m/s})/1000 \text{ Hz} = 0.344$ m.

(b) $p_{max} = BkA$ and Bk is constant gives $p_{max1}/A_1 = p_{max2}/A_2$

$$A_2 = A_1\left(\frac{p_{max2}}{p_{max1}}\right) = 1.2\times10^{-8} \text{ m}\left(\frac{30 \text{ Pa}}{3.0\times10^{-2} \text{ Pa}}\right) = 1.2\times10^{-5} \text{ m}.$$

(c) $p_{max} = BkA = 2\pi BA/\lambda$

$p_{max}\lambda = 2\pi BA = \text{constant}$ so $p_{max1}\lambda_1 = p_{max2}\lambda_2$ and $\lambda_2 = \lambda_1\left(\frac{p_{max1}}{p_{max2}}\right) = (0.344 \text{ m})\left(\frac{3.0\times10^{-2} \text{ Pa}}{1.5\times10^{-3} \text{ Pa}}\right)$

$= 6.9$ m

$f = v/\lambda = (344 \text{ m/s})/6.9 \text{ m} = 50$ Hz.

EVALUATE: The pressure amplitude and displacement amplitude are directly proportional. For the same displacement amplitude, the pressure amplitude decreases when the frequency decreases and the wavelength increases.

16.2. **IDENTIFY:** Apply $p_{max} = BkA$. $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}$, so $p_{max} = \frac{2\pi fBA}{v}$.

SET UP: $v = 344$ m/s

EXECUTE: $f = \frac{vp_{max}}{2\pi BA} = \frac{(344 \text{ m/s})(10.0 \text{ Pa})}{2\pi(1.42\times10^5 \text{ Pa})(1.00\times10^{-6} \text{ m})} = 3.86\times10^3$ Hz

EVALUATE: Audible frequencies range from about 20 Hz to about 20,000 Hz, so this frequency is audible.

16.3. **IDENTIFY:** Use $p_{max} = BkA$ to relate the pressure and displacement amplitudes.

SET UP: As stated in Example 16.1 the adiabatic bulk modulus for air is $B = 1.42\times10^5$ Pa. Use $v = f\lambda$ to calculate λ from f, and then $k = 2\pi/\lambda$.

EXECUTE: **(a)** $f = 150$ Hz

Need to calculate k: $\lambda = v/f$ and $k = 2\pi/\lambda$ so $k = 2\pi f/v = (2\pi \text{ rad})(150 \text{ Hz}) / 344 \text{ m/s} = 2.74 \text{ rad/m}$.

Then $p_{max} = BkA = (1.42\times10^5 \text{ Pa})(2.74 \text{ rad/m})(0.0200\times10^{-3} \text{ m}) = 7.78$ Pa. This is below the pain threshold of 30 Pa.

(b) *f* is larger by a factor of 10 so $k = 2\pi f/v$ is larger by a factor of 10, and $p_{\text{max}} = BkA$ is larger by a factor of 10. $p_{\text{max}} = 77.8\text{ Pa},$ above the pain threshold.

(c) There is again an increase in *f*, *k*, and p_{max} of a factor of 10, so $p_{\text{max}} = 778\text{ Pa},$ far above the pain threshold.

EVALUATE: When *f* increases, λ decreases so *k* increases and the pressure amplitude increases.

16.4. **IDENTIFY** and **SET UP:** Use the relation $v = f\lambda$ to find the wavelength or frequency of various sounds.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{1531\text{ m/s}}{17\text{ Hz}} = 90\text{ m}.$

(b) $f = \dfrac{v}{\lambda} = \dfrac{1531\text{ m/s}}{0.015\text{ m}} = 102\text{ kHz}.$

(c) $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{25\times 10^{3}\text{ Hz}} = 1.4\text{ cm}.$

(d) For $f = 78\text{ kHz},$ $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{78\times 10^{3}\text{ Hz}} = 4.4\text{ mm}.$ For $f = 39\text{ kHz},$ $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{39\times 10^{3}\text{ Hz}} = 8.8\text{ mm}.$

The range of wavelengths is 4.4 mm to 8.8 mm.

(e) $\lambda = 0.25\text{ mm}$ so $f = \dfrac{v}{\lambda} = \dfrac{1550\text{ m/s}}{0.25\times 10^{-3}\text{m}} = 6.2\text{ MHz}.$

EVALUATE: Nonaudible (to human) sounds cover a wide range of frequencies and wavelengths.

16.5. **IDENTIFY** and **SET UP:** Use $t = \text{distance/speed}$. Calculate the time it takes each sound wave to travel the $L = 60.0\text{ m}$ length of the pipe. Use $v = \sqrt{\dfrac{Y}{\rho}}$ to calculate the speed of sound in the brass rod.

EXECUTE: Wave in air: $t = (60.0\text{ m})/(344\text{ m/s}) = 0.1744\text{ s}.$

Wave in the metal: $v = \sqrt{\dfrac{Y}{\rho}} = \sqrt{\dfrac{9.0\times 10^{10}\text{ Pa}}{8600\text{ kg/m}^3}} = 3235\text{ m/s},$ so $t = \dfrac{60.0\text{ m}}{3235\text{ m/s}} = 0.01855\text{ s}.$

The time interval between the two sounds is $\Delta t = 0.1744\text{ s} - 0.01855\text{ s} = 0.156\text{ s}.$

EVALUATE: The restoring forces that propagate the sound waves are much greater in solid brass than in air, so *v* is much larger in brass.

16.6. **IDENTIFY:** $v = f\lambda$. Apply $v = \sqrt{\dfrac{B}{\rho}}$ for the waves in the liquid and $v = \sqrt{\dfrac{Y}{\rho}}$ for the waves in the metal bar.

SET UP: In part (b) the wave speed is $v = \dfrac{d}{t} = \dfrac{1.50\text{ m}}{3.90\times 10^{-4}\text{ s}}.$

EXECUTE: **(a)** Using $v = \sqrt{\dfrac{B}{\rho}},$ we have $B = v^2\rho = (\lambda f)^2\rho,$ so

$B = [(8\text{ m})(400\text{ Hz})]^2(1300\text{ kg/m}^3) = 1.33\times 10^{10}\text{ Pa}.$

(b) Using $v = \sqrt{\dfrac{Y}{\rho}},$ we have

$Y = v^2\rho = (L/t)^2\rho = [(1.50\text{ m})/(3.90\times 10^{-4}\text{ s})]^2(6400\text{ kg / m}^3) = 9.47\times 10^{10}\text{ Pa}.$

EVALUATE: In the liquid, $v = 3200\text{ m/s}$ and in the metal, $v = 3850\text{ m/s}.$ Both these speeds are much greater than the speed of sound in air.

16.7. **IDENTIFY:** $d = vt$ for the sound waves in air and in water.

SET UP: Use $v_{\text{water}} = 1482\text{ m/s}$ at $20°\text{C},$ as given in Table 16.1. In air, $v = 344\text{ m/s}.$

EXECUTE: Since along the path to the diver the sound travels 1.2 m in air, the sound wave travels in water for the same time as the wave travels a distance $22.0\text{ m} - 1.20\text{ m} = 20.8\text{ m}$ in air. The depth of the diver is $(20.8\text{ m})\dfrac{v_{\text{water}}}{v_{\text{air}}} = (20.8\text{ m})\dfrac{1482\text{ m/s}}{344\text{ m/s}} = 89.6\text{ m}.$ This is the depth of the diver; the distance from the horn is $90.8\text{ m}.$

EVALUATE: The time it takes the sound to travel from the horn to the person on shore is $t_1 = \dfrac{22.0\text{ m}}{344\text{ m/s}} = 0.0640\text{ s}.$ The time it takes the sound to travel from the horn to the diver is $t_2 = \dfrac{1.2\text{ m}}{344\text{ m/s}} + \dfrac{89.6\text{ m}}{1482\text{ m/s}} = 0.0035\text{ s} + 0.0605\text{ s} = 0.0640\text{ s}.$ These times are indeed the same. For three figure accuracy the distance of the horn above the water can't be neglected.

16.8. IDENTIFY: Apply $v = \sqrt{\dfrac{\gamma RT}{M}}$ to each gas.

SET UP: In each case, express M in units of kg/mol. For H_2, $\gamma = 1.41.$ For He and Ar, $\gamma = 1.67.$

EXECUTE: **(a)** $v_{H_2} = \sqrt{\dfrac{(1.41)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(2.02\times10^{-3}\text{ kg/mol})}} = 1.32\times10^{3}\text{ m/s}$

(b) . $v_{\text{He}} = \sqrt{\dfrac{(1.67)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(4.00\times10^{-3}\text{ kg/mol})}} = 1.02\times10^{3}\text{ m/s}$.

(c) $v_{\text{Ar}} = \sqrt{\dfrac{(1.67)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(39.9\times10^{-3}\text{ kg/mol})}} = 323\text{ m/s}.$

(d) Repeating the calculation of Example 16.4 at $T = 300.15\text{ K}$ gives $v_{\text{air}} = 348\text{ m/s},$ and so $v_{H_2} = 3.80v_{\text{air}}, v_{\text{He}} = 2.94v_{\text{air}}$ and $v_{\text{Ar}} = 0.928v_{\text{air}}.$

EVALUATE: v is larger for gases with smaller M.

16.9. IDENTIFY: $v = f\lambda.$ The relation of v to gas temperature is given by $v = \sqrt{\dfrac{\gamma RT}{M}}.$

SET UP: Let $T = 22.0°\text{C} = 295.15\text{ K}.$

EXECUTE: At $22.0°\text{C},$ $\lambda = \dfrac{v}{f} = \dfrac{325\text{ m/s}}{1250\text{ Hz}} = 0.260\text{ m} = 26.0\text{ cm}.$ $\lambda = \dfrac{v}{f} = \dfrac{1}{f}\sqrt{\dfrac{\gamma RT}{M}}.$ $\dfrac{\lambda}{\sqrt{T}} = \dfrac{1}{f}\sqrt{\dfrac{\gamma R}{M}},$ which is constant, so $\dfrac{\lambda_1}{\sqrt{T_1}} = \dfrac{\lambda_2}{\sqrt{T_2}}.$ $T_2 = T_1\left(\dfrac{\lambda_2}{\lambda_1}\right)^2 = (295.15\text{ K})\left(\dfrac{28.5\text{ cm}}{26.0\text{ cm}}\right)^2 = 354.6\text{ K} = 81.4°\text{C}.$

EVALUATE: When T increases v increases and for fixed f, λ increases. Note that we did not need to know either γ or M for the gas.

16.10. IDENTIFY: $v = \sqrt{\dfrac{\gamma RT}{M}}.$ Take the derivative of v with respect to T. In part (b) replace dv by Δv and dT by ΔT in the expression derived in part (a).

SET UP: $\dfrac{d(x^{1/2})}{dx} = \tfrac{1}{2}x^{-1/2}.$ In $v = \sqrt{\dfrac{\gamma RT}{M}},$ T must be in kelvins. $20°\text{C} = 293\text{ K}.$ $\Delta T = 1\text{ C}° = 1\text{ K}.$

EXECUTE: **(a)** $\dfrac{dv}{dT} = \sqrt{\dfrac{\gamma R}{M}}\dfrac{dT^{1/2}}{dT} = \sqrt{\dfrac{\gamma R}{M}}\tfrac{1}{2}T^{-1/2} = \dfrac{1}{2T}\sqrt{\dfrac{\gamma RT}{M}} = \dfrac{v}{2T}.$ Rearranging gives $\dfrac{dv}{v} = \dfrac{1}{2}\dfrac{dT}{T},$ the desired result.

(b) $\dfrac{\Delta v}{v} = \dfrac{1}{2}\dfrac{\Delta T}{T}.$ $\Delta v = \dfrac{v}{2}\dfrac{\Delta T}{T} = \left(\dfrac{344\text{ m/s}}{2}\right)\left(\dfrac{1\text{ K}}{293\text{ K}}\right) = 0.59\text{ m/s}.$

EVALUATE: Since $\dfrac{\Delta T}{T} = 3.4\times10^{-3}$ and $\dfrac{\Delta v}{v}$ is one-half this, replacing dT by ΔT and dv by Δv is accurate. Using the result from part (a) is much simpler than calculating v for $20°\text{C}$ and for $21°\text{C}$ and subtracting, and is not subject to round-off errors.

16.11. **IDENTIFY** and **SET UP:** Sound delivers energy (and hence power) to the ear. For a whisper, $I = 1\times10^{-10}\ \text{W/m}^2$. The area of the tympanic membrane is $A = \pi r^2$, with $r = 4.2\times10^{-3}\ \text{m}$. Intensity is energy per unit time per unit area.

EXECUTE: **(a)** $E = IAt = (1\times10^{-10}\ \text{W/m}^2)\pi(4.2\times10^{-3}\ \text{m})^2(1\ \text{s}) = 5.5\times10^{-15}\ \text{J}.$

(b) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.5\times10^{-15}\ \text{J})}{2.0\times10^{-6}\ \text{kg}}} = 7.4\times10^{-5}\ \text{m/s} = 0.074\ \text{mm/s}.$

EVALUATE: Compared to the energy of ordinary objects, it takes only a very small amount of energy for hearing. As part (b) shows, a mosquito carries a lot more energy than is needed for hearing.

16.12. **IDENTIFY:** The sound intensity level decreases by 13.0 dB, and from this we can find the change in the intensity.

SET UP: $\beta = 10\log(I/I_0)$. $\Delta\beta = 13.0$ dB.

EXECUTE: **(a)** $\Delta\beta = \beta_2 - \beta_1 = 10\ \text{dB}\log(I_2/I_0) - 10\ \text{dB}\log(I_1/I_0) = 10\ \text{dB}\log(I_2/I_1) = 13.0$ dB, so $1.3 = \log(I_2/I_1)$ which gives $I_2/I_1 = 20.0$.

(b) EVALUATE: According to the equation in part (a) the difference in two sound intensity levels is determined by the ratio of the sound intensities. So you don't need to know I_1, just the ratio I_2/I_1.

16.13. **IDENTIFY** and **SET UP:** We want the sound intensity level to increase from 20.0 dB to 60.0 dB. The previous problem showed that $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\dfrac{I_2}{I_1}\right)$. We also know that $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$.

EXECUTE: Using $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\dfrac{I_2}{I_1}\right)$, we have $\Delta\beta = +40.0$ dB. Therefore $\log\left(\dfrac{I_2}{I_1}\right) = 4.00$, so $\dfrac{I_2}{I_1} = 1.00\times10^4$. Using $\dfrac{I_2}{I_1} = \dfrac{r_1^2}{r_2^2}$ and solving for r_2, we get $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (15.0\ \text{m})\sqrt{\dfrac{1}{1.00\times10^4}} = 15.0\ \text{cm}.$

EVALUATE: A change of 10^2 in distance gives a change of 10^4 in intensity. Our analysis assumes that the sound spreads from the source uniformly in all directions.

16.14. **IDENTIFY:** This problem deals with sound intensity and intensity level.

SET UP: The sound intensity level is $\beta = (10\ \text{dB})\log\dfrac{I}{I_0}$ and the intensity is $I = \dfrac{P}{A} = \dfrac{P}{4\pi r^2}$. We want to know by what factor the power P should increase so that β will increase by 5.00 dB at 20.0 m from the source. We know that $\beta_2 - \beta_1 = 5.00$ dB and we want to find P_2/P_1.

EXECUTE: $\beta_1 = (10\ \text{dB})\log\dfrac{I_1}{I_0}$ and $\beta_2 = (10\ \text{dB})\log\dfrac{I_2}{I_0}$. We know that $\beta_2 - \beta_1 = 5.00$ dB, so we have $\beta_2 - \beta_1 = (10\ \text{dB})\log(I_2/I_0) - (10\ \text{dB})\log(I_1/I_0)$. Using the properties of logarithms gives $\beta_2 - \beta_1 = (10\ \text{dB})[\log I_2 - \log I_0 - (\log I_1 - \log I_0)] = (10\ \text{dB})\log(I_2/I_1)$. Now use $I = \dfrac{P}{4\pi r^2}$, giving $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\dfrac{P_2/4\pi r^2}{P_1/4\pi r^2}\right) = (10\ \text{dB})\log(P_2/P_1)$. This gives us $5.00\ \text{dB} = (10\ \text{dB})\log(P_2/P_1)$, so $P_2/P_1 = 10^{1/2} = \sqrt{10} = 3.16.$

EVALUATE: Our result does not depend on the distance from the source, since the r divided out. Therefore the sound intensity level at all points would increase by 5.00 dB if the power of the source increased by a factor of 3.16.

16.15. **IDENTIFY** and **SET UP:** Apply $p_{max} = BkA$, $I = \frac{1}{2}B\omega kA^2$, and $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$.

EXECUTE: **(a)** $\omega = 2\pi f = (2\pi \text{ rad})(320\text{ Hz}) = 2011\text{ rad/s}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v} = \frac{2011\text{ rad/s}}{344\text{ m/s}} = 5.84\text{ rad/m}$$

$B = 1.42\times10^5\text{ Pa}$ (Example 16.1)

Then $p_{max} = BkA = (1.42\times10^5\text{ Pa})(5.84\text{ rad/m})(5.00\times10^{-6}\text{ m}) = 4.14\text{ Pa}$.

(b) Using $I = \frac{1}{2}\omega BkA^2$ gives

$I = \frac{1}{2}(2011\text{ rad/s})(1.42\times10^5\text{ Pa})(5.84\text{ rad/m})(5.00\times10^{-6}\text{ m})^2 = 2.08\times10^{-2}\text{ W/m}^2$.

(c) $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$: $\beta = (10\text{ dB})\log(I/I_0)$, with $I_0 = 1\times10^{-12}\text{ W/m}^2$.

$\beta = (10\text{ dB})\log[(2.08\times10^{-2}\text{ W/m}^2)/(1\times10^{-12}\text{ W/m}^2)] = 103\text{ dB}$.

EVALUATE: Even though the displacement amplitude is very small, this is a very intense sound. Compare the sound intensity level to the values in Table 16.2.

16.16. **IDENTIFY:** Changing the sound intensity level will decrease the rate at which energy reaches the ear.

SET UP: Example 16.9 shows that $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$.

EXECUTE: **(a)** $\Delta\beta = -30\text{ dB}$ so $\log\left(\frac{I_2}{I_1}\right) = -3$ and $\frac{I_2}{I_1} = 10^{-3} = 1/1000$.

(b) $I_2/I_1 = \frac{1}{2}$ so $\Delta\beta = 10\log\left(\frac{1}{2}\right) = -3.0\text{ dB}$.

EVALUATE: Because of the logarithmic relationship between the intensity and intensity level of sound, a small change in the intensity level produces a large change in the intensity.

16.17. **IDENTIFY:** Use $I = \frac{vp_{max}^2}{2B}$ to relate I and p_{max}. $\beta = (10\text{ dB})\log(I/I_0)$. The equation $p_{max} = BkA$ says the pressure amplitude and displacement amplitude are related by $p_{max} = BkA = B\left(\frac{2\pi f}{v}\right)A$.

SET UP: At $20°\text{C}$ the bulk modulus for air is $1.42\times10^5\text{ Pa}$ and $v = 344\text{ m/s}$. $I_0 = 1\times10^{-12}\text{ W/m}^2$.

EXECUTE: **(a)** $I = \frac{vp_{max}^2}{2B} = \frac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})^2}{2(1.42\times10^5\text{ Pa})} = 4.4\times10^{-12}\text{ W/m}^2$

(b) $\beta = (10\text{ dB})\log\left(\frac{4.4\times10^{-12}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}\right) = 6.4\text{ dB}$

(c) $A = \frac{vp_{max}}{2\pi fB} = \frac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})}{2\pi(400\text{ Hz})(1.42\times10^5\text{ Pa})} = 5.8\times10^{-11}\text{ m}$

EVALUATE: This is a very faint sound and the displacement and pressure amplitudes are very small. Note that the displacement amplitude depends on the frequency but the pressure amplitude does not.

16.18. **IDENTIFY** and **SET UP:** Apply the relation $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$ that is derived in Example 16.9.

EXECUTE: **(a)** $\Delta\beta = (10\text{ dB})\log\left(\frac{4I}{I}\right) = 6.0\text{ dB}$

(b) The total number of crying babies must be multiplied by four, for an increase of 12 kids.
EVALUATE: For $I_2 = \alpha I_1$, where α is some factor, the increase in sound intensity level is $\Delta\beta = (10\text{ dB})\log\alpha$. For $\alpha = 4$, $\Delta\beta = 6.0\text{ dB}$.

16.19. **IDENTIFY** and **SET UP:** Let 1 refer to the mother and 2 to the father. Use the result derived in Example 16.9 for the difference in sound intensity level for the two sounds. Relate intensity to distance from the source using $I_1/I_2 = r_2^2/r_1^2$.

EXECUTE: From Example 16.9, $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$

Using $I_1/I_2 = r_2^2/r_1^2$ gives us

$\Delta\beta = \beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1) = (10\text{ dB})\log(r_1/r_2)^2 = (20\text{ dB})\log(r_1/r_2)$

$\Delta\beta = (20\text{ dB})\log(1.50\text{ m}/0.30\text{ m}) = 14.0\text{ dB}$.

EVALUATE: The father is 5 times closer so the intensity at his location is 25 times greater.

16.20. **IDENTIFY:** We must use the relationship between intensity and sound level.

SET UP: Example 16.9 shows that $\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$.

EXECUTE: **(a)** $\Delta\beta = 5.00\text{ dB}$ gives $\log\left(\frac{I_2}{I_1}\right) = 0.5$ and $\frac{I_2}{I_1} = 10^{0.5} = 3.16$.

(b) $\frac{I_2}{I_1} = 100$ gives $\Delta\beta = 10\log(100) = 20\text{ dB}$.

(c) $\frac{I_2}{I_1} = 2$ gives $\Delta\beta = 10\log 2 = 3.0\text{ dB}$.

EVALUATE: Every doubling of the intensity increases the decibel level by 3.0 dB.

16.21. **IDENTIFY:** The intensity of sound obeys an inverse square law.

SET UP: $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$. $\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$, with $I_0 = 1\times10^{-12}\text{ W/m}^2$.

EXECUTE: **(a)** $\beta = 53\text{ dB}$ gives $5.3 = \log\left(\frac{I}{I_0}\right)$ and $I = (10^{5.3})I_0 = 2.0\times10^{-7}\text{ W/m}^2$.

(b) $r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (3.0\text{ m})\sqrt{\frac{4}{1}} = 6.0\text{ m}$.

(c) $\beta = \frac{53\text{ dB}}{4} = 13.25\text{ dB}$ gives $1.325 = \log\left(\frac{I}{I_0}\right)$ and $I = 2.1\times10^{-11}\text{ W/m}^2$.

$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (3.0\text{ m})\sqrt{\frac{2.0\times10^{-7}\text{ W/m}^2}{2.1\times10^{-11}\text{ W/m}^2}} = 290\text{ m}$.

EVALUATE: **(d)** Intensity obeys the inverse square law but noise level does not.

16.22. **IDENTIFY:** We are looking at the standing wave pattern in a pipe. The pattern has displacement antinodes at both ends of the pipe.

SET UP: For an open pipe, $\lambda_n = 2L/n$, and for any wave $v = f\lambda$. Table 16.1 tells us that at 20°C v_{air} = 344 m/s and v_{He} = 999 m/s.

EXECUTE: **(a)** The fact that displacement nodes occur at both ends of the pipe tells us that it must be an open pipe.

(b) The pattern shows 5 nodes, so it is the 5th harmonic.

(c) $\lambda_n = 2L/n$, so $\lambda_5 = 2L/5$, which gives $L = 5\lambda_5/2$. Now get λ_5 which we can use to find L.

$\lambda_5 = v/f_5 = (344\text{ m/s})/(1710\text{ Hz}) = 0.2012\text{ m}$. Therefore $L = 5\lambda_5/2 = 5(0.2012\text{ m})/2 = 0.503\text{ m}$.

(d) L remains the same, so $\lambda_1 = 2L$. $f_1\lambda_1 = v_{\text{He}}$, so $f_1 = \frac{v_{\text{He}}}{\lambda_1} = \frac{v_{\text{He}}}{2L} = \frac{999\text{ m/s}}{2(0.503\text{ m})} = 993\text{ Hz}.$

EVALUATE: Replacing the air with helium changes the harmonic frequencies. But it does not change the wavelengths because they are determined by the length of the pipe.

16.23. IDENTIFY and **SET UP:** An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.

EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.23a. The open ends are displacement antinodes.

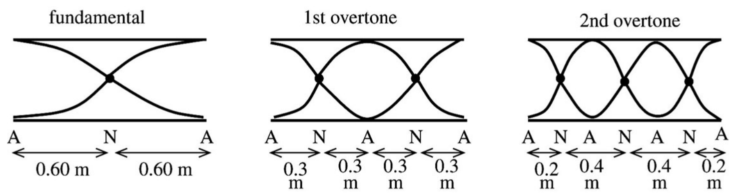


Figure 16.23a

Location of the displacement nodes (N) measured from the left end:
fundamental 0.60 m
1st overtone 0.30 m, 0.90 m
2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:
fundamental 0, 1.20 m
1st overtone 0, 0.60 m, 1.20 m
2nd overtone 0, 0.40 m, 0.80 m, 1.20 m

(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.23b.

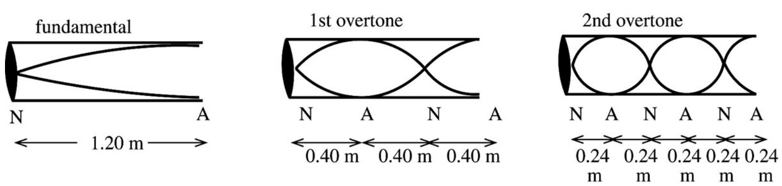


Figure 16.23b

Location of the displacement nodes (N) measured from the closed end:
fundamental 0
1st overtone 0, 0.80 m
2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:
fundamental 1.20 m
1st overtone 0.40 m, 1.20 m
2nd overtone 0.24 m, 0.72 m, 1.20 m

EVALUATE: The node-to-node or antinode-to-antinode distance is $\lambda/2$. For the higher overtones the frequency is higher and the wavelength is smaller.

16.24. **IDENTIFY:** For an open pipe, $f_1 = \dfrac{v}{2L}$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$. $v = f\lambda$.

SET UP: $v = 344$ m/s. For a pipe, there must be a displacement node at a closed end and an antinode at the open end.

EXECUTE: **(a)** $L = \dfrac{v}{2f_1} = \dfrac{344 \text{ m/s}}{2(524 \text{ Hz})} = 0.328 \text{ m}.$

(b) There is a node at one end, an antinode at the other end and no other nodes or antinodes in between, so $\dfrac{\lambda_1}{4} = L$ and $\lambda_1 = 4L = 4(0.328 \text{ m}) = 1.31 \text{ m}.$

(c) $f_1 = \dfrac{v}{4L} = \dfrac{1}{2}\left(\dfrac{v}{2L}\right) = \dfrac{1}{2}(524 \text{ Hz}) = 262 \text{ Hz}.$

EVALUATE: We could also calculate f_1 for the stopped pipe as $f_1 = \dfrac{v}{\lambda_1} = \dfrac{344 \text{ m/s}}{1.31 \text{ m}} = 262 \text{ Hz},$ which agrees with our result in part (c).

16.25. **IDENTIFY:** For a stopped pipe, the standing wave frequencies are given by $f_n = nv/4L$.
SET UP: The first three standing wave frequencies correspond to $n = 1$, 3, and 5.

EXECUTE: $f_1 = \dfrac{(344 \text{ m/s})}{4(0.17 \text{ m})} = 506 \text{ Hz}, \; f_3 = 3f_1 = 1517 \text{ Hz}, \; f_5 = 5f_1 = 2529 \text{ Hz}.$

EVALUATE: All three of these frequencies are in the audible range, which is about 20 Hz to 20,000 Hz.

16.26. **IDENTIFY:** The vocal tract is modeled as a stopped pipe, open at one end and closed at the other end, so we know the wavelength of standing waves in the tract.

SET UP: For a stopped pipe, $\lambda_n = 4L/n \;(n = 1, 3, 5, \ldots)$ and $v = f\lambda$, so $f_1 = \dfrac{v}{4L}$ with $f_1 = 220$ Hz.

EXECUTE: $L = \dfrac{v}{4f_1} = \dfrac{344 \text{ m/s}}{4(220 \text{ Hz})} = 39.1 \text{ cm}.$ This result is a reasonable value for the mouth to diaphragm distance for a typical adult.

EVALUATE: 1244 Hz is not an integer multiple of the fundamental frequency of 220 Hz; it is 5.65 times the fundamental. The production of sung notes is more complicated than harmonics of an air column of fixed length.

16.27. **IDENTIFY:** A pipe open at one end and closed at the other is a stopped pipe.
SET UP: For an open pipe, the fundamental is $f_1 = v/2L$, and for a stopped pipe, it is $f_1 = v/4L$.

EXECUTE: **(a)** For an open pipe, $f_1 = \dfrac{v}{2L} = \dfrac{344 \text{ m/s}}{2(4.88 \text{ m})} = 35.2 \text{ Hz}.$

(b) For a stopped pipe, $f_1 = \dfrac{v}{4L} = \dfrac{35.2 \text{ Hz}}{2} = 17.6 \text{ Hz}.$

EVALUATE: Even though the pipes both have the same length, their fundamental frequencies are very different, depending on whether they are open or closed at their ends.

16.28. **IDENTIFY:** There must be a node at each end of the pipe. For the fundamental there are no additional nodes and each successive overtone has one additional node. $v = f\lambda$.

SET UP: $v = 344$ m/s. The node to node distance is $\lambda/2$.

EXECUTE: **(a)** $\dfrac{\lambda_1}{2} = L$ so $\lambda_1 = 2L$. Each successive overtone adds an additional $\lambda/2$ along the pipe, so $n\left(\dfrac{\lambda_n}{2}\right) = L$ and $\lambda_n = \dfrac{2L}{n}$, where $n = 1, 2, 3, \ldots$ $f_n = \dfrac{v}{\lambda_n} = \dfrac{nv}{2L}$.

(b) $f_1 = \dfrac{v}{2L} = \dfrac{344 \text{ m/s}}{2(2.50 \text{ m})} = 68.8 \text{ Hz}.$ $f_2 = 2f_1 = 138 \text{ Hz}.$ $f_3 = 3f_1 = 206 \text{ Hz}.$ All three of these frequencies are audible.

EVALUATE: A pipe of length L closed at both ends has the same standing wave wavelengths, frequencies and nodal patterns as for a string of length L that is fixed at both ends.

16.29. **IDENTIFY:** We are looking at the standing wave pattern in a pipe. The pattern has a displacement node at one end and a displacement antinode at the other end of the pipe.

SET UP: For a stopped pipe $\lambda_n = 4L/n$ (n an odd integer). $v = f\lambda$

EXECUTE: **(a)** With a displacement node at one end and an antinode at the other, this must be a stopped pipe.

(b) The n^{th} harmonic has $\dfrac{n+1}{2}$ nodes, so for 5 nodes, 5 = $\dfrac{n+1}{2}$, which gives n = 5. This pipe is resonating in its 5^{th} harmonic.

(c) Using $v = f\lambda$ gives $\lambda_9 = \dfrac{v}{f_9} = \dfrac{344 \text{ m/s}}{1710 \text{ Hz}} = 0.20117 \text{ m}.$ Using $\lambda_n = 4L/n$ gives

$L = \dfrac{n\lambda_n}{4} = \dfrac{9(0.20117 \text{ m})}{4} = 0.453 \text{ m}.$

(d) $f_9 = 9f_1$, so f_1 = (1710 Hz)/9 = 190 Hz.

EVALUATE: A stopped pipe does not have even harmonics because it has a displacement node at the closed end and an antinode at the open end.

16.30. **IDENTIFY:** The wire will vibrate in its second overtone with frequency f_3^{wire} when $f_3^{\text{wire}} = f_1^{\text{pipe}}$. For a stopped pipe, $f_1^{\text{pipe}} = \dfrac{v}{4L_{\text{pipe}}}$. The second overtone standing wave frequency for a wire fixed at both ends is $f_3^{\text{wire}} = 3\left(\dfrac{v_{\text{wire}}}{2L_{\text{wire}}}\right)$. $v_{\text{wire}} = \sqrt{F/\mu}$.

SET UP: The wire has $\mu = \dfrac{m}{L_{\text{wire}}} = \dfrac{7.25\times10^{-3} \text{ kg}}{0.620 \text{ m}} = 1.169\times10^{-2} \text{ kg/m}.$ The speed of sound in air is $v = 344 \text{ m/s}.$

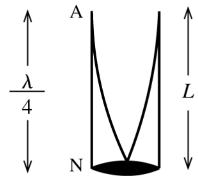
EXECUTE: $v_{\text{wire}} = \sqrt{\dfrac{4110 \text{ N}}{1.169\times10^{-2} \text{ kg/m}}} = 592.85 \text{ m/s}.$ $f_3^{\text{wire}} = f_1^{\text{pipe}}$ gives $3\dfrac{v_{\text{wire}}}{2L_{\text{wire}}} = \dfrac{v}{4L_{\text{pipe}}}.$

$L_{\text{pipe}} = \dfrac{2L_{\text{wire}}v}{12v_{\text{wire}}} = \dfrac{2(0.620 \text{ m})(344 \text{ m/s})}{12(592.85 \text{ m/s})} = 0.0600 \text{ m} = 6.00 \text{ cm}.$

EVALUATE: The fundamental for the pipe has the same frequency as the third harmonic of the wire. But the wave speeds for the two objects are different and the two standing waves have different wavelengths.

16.31. **IDENTIFY** and **SET UP:** Use the standing wave pattern to relate the wavelength of the standing wave to the length of the air column and then use $v = f\lambda$ to calculate f. There is a displacement antinode at the top (open) end of the air column and a node at the bottom (closed) end, as shown in Figure 16.31.

EXECUTE: **(a)**



$\lambda / 4 = L$

$\lambda = 4L = 4(0.140 \text{ m}) = 0.560 \text{ m}$

$f = \dfrac{v}{\lambda} = \dfrac{344 \text{ m/s}}{0.560 \text{ m}} = 614 \text{ Hz}$

Figure 16.31

(b) Now the length L of the air column becomes $\frac{1}{2}(0.140 \text{ m}) = 0.070 \text{ m}$ and $\lambda = 4L = 0.280 \text{ m}$.

$f = \dfrac{v}{\lambda} = \dfrac{344 \text{ m/s}}{0.280 \text{ m}} = 1230 \text{ Hz}$

EVALUATE: Smaller L means smaller λ which in turn corresponds to larger f.

16.32. **IDENTIFY** and **SET UP:** The path difference for the two sources is d. For destructive interference, the path difference is a half-integer number of wavelengths. For constructive interference, the path difference is an integer number of wavelengths. $v = f\lambda$.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{344 \text{ m/s}}{725 \text{ Hz}} = 0.474 \text{ m}$. Destructive interference will first occur when $d = \lambda/2 = 0.237 \text{ m}$.

(b) Destructive interference will next occur when $d = 3\lambda/2 = 0.711 \text{ m}$.

(c) Constructive interference will first occur when $d = \lambda = 0.474 \text{ m}$.

EVALUATE: Constructive interference should first occur midway between the first two points where destructive interference occurs. This midpoint is (0.237 m + 0.711 m)/2 = 0.474 m, which is just what we found in part (c).

16.33. **(a) IDENTIFY** and **SET UP:** Path difference from points A and B to point Q is $3.00 \text{ m} - 1.00 \text{ m} = 2.00 \text{ m}$, as shown in Figure 16.33. Constructive interference implies path difference $= n\lambda$, $n = 1, 2, 3, \ldots$

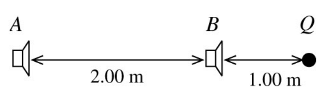


Figure 16.33

EXECUTE: $2.00 \text{ m} = n\lambda$ so $\lambda = 2.00 \text{ m}/n$

$f = \dfrac{v}{\lambda} = \dfrac{nv}{2.00 \text{ m}} = \dfrac{n(344 \text{ m/s})}{2.00 \text{ m}} = n(172 \text{ Hz}), \quad n = 1, 2, 3, \ldots$

The lowest frequency for which constructive interference occurs is 172 Hz.

(b) IDENTIFY and **SET UP:** Destructive interference implies path difference $= (n/2)\lambda$, $n = 1, 3, 5, \ldots$

EXECUTE: $2.00 \text{ m} = (n/2)\lambda$ so $\lambda = 4.00 \text{ m}/n$

$f = \dfrac{v}{\lambda} = \dfrac{nv}{4.00 \text{ m}} = \dfrac{n(344 \text{ m/s})}{(4.00 \text{ m})} = n(86 \text{ Hz}), \quad n = 1, 3, 5, \ldots.$

The lowest frequency for which destructive interference occurs is 86 Hz.

EVALUATE: As the frequency is slowly increased, the intensity at Q will fluctuate, as the interference changes between destructive and constructive.

16.34. **IDENTIFY:** Constructive interference occurs when the difference of the distances of each source from point P is an integer number of wavelengths. The interference is destructive when this difference of path lengths is a half integer number of wavelengths.

SET UP: The wavelength is $\lambda = v/f = (344\ \text{m/s})/(206\ \text{Hz}) = 1.67\ \text{m}$. Since P is between the speakers, x must be in the range 0 to L, where $L = 2.00\ \text{m}$ is the distance between the speakers.

EXECUTE: The difference in path length is $\Delta l = (L - x) - x = L - 2x$, or $x = (L - \Delta l)/2$. For destructive interference, $\Delta l = (n + (1/2))\lambda$, and for constructive interference, $\Delta l = n\lambda$.

(a) Destructive interference: $n = 0$ gives $\Delta l = 0.835\ \text{m}$ and $x = 0.58\ \text{m}$. $n = -1$ gives $\Delta l = -0.835\ \text{m}$ and $x = 1.42\ \text{m}$. No other values of n place P between the speakers.

(b) Constructive interference: $n = 0$ gives $\Delta l = 0$ and $x = 1.00\ \text{m}$. $n = 1$ gives $\Delta l = 1.67\ \text{m}$ and $x = 0.17\ \text{m}$. $n = -1$ gives $\Delta l = -1.67\ \text{m}$ and $x = 1.83\ \text{m}$. No other values of n place P between the speakers.

(c) Treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls, ceiling and floor.

EVALUATE: Points of constructive interference are a distance $\lambda/2$ apart, and the same is true for the points of destructive interference.

16.35. IDENTIFY: For constructive interference the path difference is an integer number of wavelengths and for destructive interference the path difference is a half-integer number of wavelengths.

SET UP: $\lambda = v/f = (344\ \text{m/s})/(688\ \text{Hz}) = 0.500\ \text{m}$

EXECUTE: To move from constructive interference to destructive interference, the path difference must change by $\lambda/2$. If you move a distance x toward speaker B, the distance to B gets shorter by x and the distance to A gets longer by x so the path difference changes by $2x$. $2x = \lambda/2$ and $x = \lambda/4 = 0.125\ \text{m}$.

EVALUATE: If you walk an additional distance of 0.125 m farther, the interference again becomes constructive.

16.36. IDENTIFY: Destructive interference occurs when the path difference is a half integer number of wavelengths.

SET UP: $v = 344\ \text{m/s}$, so $\lambda = v/f = (344\ \text{m/s})/(172\ \text{Hz}) = 2.00\ \text{m}$. If $r_A = 8.00\ \text{m}$ and r_B are the distances of the person from each is $r_B - r_A = \left(n + \tfrac{1}{2}\right)\lambda$, where n is any integer.

EXECUTE: Requiring $r_B = r_A + \left(n + \tfrac{1}{2}\right)\lambda > 0a$ gives $n + \tfrac{1}{2} > -r_A/\lambda = 0 - (8.00\ \text{m})/(2.00\ \text{m}) = -4$, so the smallest value of r_B occurs when $n = -4$, and the closest distance to B is

$r_B = 8.00\ \text{m} + \left(-4 + \tfrac{1}{2}\right)(2.00\ \text{m}) = 1.00\ \text{m}$.

EVALUATE: For $r_B = 1.00\ \text{m}$, the path difference is $r_A - r_B = 7.00\ \text{m}$. This is 3.5λ.

16.37. IDENTIFY: For constructive interference, the path difference is an integer number of wavelengths. For destructive interference, the path difference is a half-integer number of wavelengths.

SET UP: One speaker is 4.50 m from the microphone and the other is 4.92 m from the microphone, so the path difference is 0.42 m. $f = v/\lambda$.

EXECUTE: **(a)** $\lambda = 0.42\ \text{m}$ gives $f = \dfrac{v}{\lambda} = 820\ \text{Hz}$; $2\lambda = 0.42\ \text{m}$ gives $\lambda = 0.21\ \text{m}$ and $f = \dfrac{v}{\lambda} = 1640\ \text{Hz}$; $3\lambda = 0.42\ \text{m}$ gives $\lambda = 0.14\ \text{m}$ and $f = \dfrac{v}{\lambda} = 2460\ \text{Hz}$, and so on. The frequencies for constructive interference are $n(820\ \text{Hz})$, $n = 1, 2, 3, \ldots.$

(b) $\lambda/2 = 0.42\ \text{m}$ gives $\lambda = 0.84\ \text{m}$ and $f = \dfrac{v}{\lambda} = 410\ \text{Hz}$; $3\lambda/2 = 0.42\ \text{m}$ gives $\lambda = 0.28\ \text{m}$ and $f = \dfrac{v}{\lambda} = 1230\ \text{Hz}$; $5\lambda/2 = 0.42\ \text{m}$ gives $\lambda = 0.168\ \text{m}$ and $f = \dfrac{v}{\lambda} = 2050\ \text{Hz}$, and so on. The frequencies for destructive interference are $(2n+1)(410\ \text{Hz})$, $n = 0, 1, 2, \ldots.$

EVALUATE: The frequencies for constructive interference lie midway between the frequencies for destructive interference.

16.38. IDENTIFY: $f_{\text{beat}} = |f_1 - f_2|$. $v = f\lambda$.

SET UP: $v = 344$ m/s. Let $\lambda_1 = 64.8$ cm and $\lambda_2 = 65.2$ cm. $\lambda_2 > \lambda_1$ so $f_1 > f_2$.

EXECUTE: $f_1 - f_2 = v\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right) = \dfrac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} = \dfrac{(344\text{ m/s})(0.04\times10^{-2}\text{ m})}{(0.648\text{ m})(0.652\text{ m})} = 0.33$ beats/s, which rounds to 0.3 beats/s.

EVALUATE: We could have calculated f_1 and f_2 and subtracted, but doing it this way we would have to be careful to retain enough figures in intermediate calculations to avoid round-off errors.

16.39. IDENTIFY: The beat is due to a difference in the frequencies of the two sounds.

SET UP: $f_{\text{beat}} = f_1 - f_2$. Tightening the string increases the wave speed for transverse waves on the string and this in turn increases the frequency.

EXECUTE: **(a)** If the beat frequency increases when she raises her frequency by tightening the string, it must be that her frequency is 433 Hz, 3 Hz above concert *A*.

(b) She needs to lower her frequency by loosening her string.

EVALUATE: The beat would only be audible if the two sounds are quite close in frequency. A musician with a good sense of pitch can come very close to the correct frequency just from hearing the tone.

16.40. IDENTIFY: $f_{\text{beat}} = |f_a - f_b|$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$.

SET UP: $v = 344$ m/s. Let $L_a = 1.14$ m and $L_b = 1.16$ m. $L_b > L_a$ so $f_{1a} > f_{1b}$.

EXECUTE: $f_{1a} - f_{1b} = \dfrac{v}{4}\left(\dfrac{1}{L_a} - \dfrac{1}{L_b}\right) = \dfrac{v(L_b - L_a)}{4L_aL_b} = \dfrac{(344\text{ m/s})(2.00\times10^{-2}\text{ m})}{4(1.14\text{ m})(1.16\text{ m})} = 1.3$ Hz. There are 1.3 beats per second.

EVALUATE: Increasing the length of the pipe increases the wavelength of the fundamental and decreases the frequency.

16.41. IDENTIFY: Apply the Doppler shift equation $f_{\text{L}} = \left(\dfrac{v + v_{\text{L}}}{v + v_{\text{S}}}\right)f_{\text{S}}$.

SET UP: The positive direction is from listener to source. $f_{\text{S}} = 1200$ Hz. $f_{\text{L}} = 1240$ Hz.

EXECUTE: $v_{\text{L}} = 0$. $v_{\text{S}} = -25.0$ m/s. $f_{\text{L}} = \left(\dfrac{v}{v + v_{\text{S}}}\right)f_{\text{S}}$ gives

$$v = \frac{v_{\text{S}}f_{\text{L}}}{f_{\text{S}} - f_{\text{L}}} = \frac{(-25\text{ m/s})(1240\text{ Hz})}{1200\text{ Hz} - 1240\text{ Hz}} = 780\text{ m/s}.$$

EVALUATE: $f_{\text{L}} > f_{\text{S}}$ since the source is approaching the listener.

16.42. IDENTIFY and SET UP: Apply $\lambda = \dfrac{v - v_{\text{S}}}{f_{\text{S}}}$ and $\lambda = \dfrac{v + v_{\text{S}}}{f_{\text{S}}}$ for the wavelengths in front of and behind the source. Then $f = v/\lambda$. When the source is at rest $\lambda = \dfrac{v}{f_{\text{S}}} = \dfrac{344\text{ m/s}}{400\text{ Hz}} = 0.860$ m.

EXECUTE: **(a)** $\lambda = \dfrac{v - v_{\text{S}}}{f_{\text{S}}} = \dfrac{344\text{ m/s} - 25.0\text{ m/s}}{400\text{ Hz}} = 0.798$ m

(b) $\lambda = \dfrac{v + v_{\text{S}}}{f_{\text{S}}} = \dfrac{344\text{ m/s} + 25.0\text{ m/s}}{400\text{ Hz}} = 0.922$ m

(c) $f_{\text{L}} = v/\lambda$ (since $v_{\text{L}} = 0$), so $f_{\text{L}} = (344\text{ m/s})/0.798\text{ m} = 431$ Hz

(d) $f_{\text{L}} = v/\lambda = (344\text{ m/s})/0.922\text{ m} = 373$ Hz

EVALUATE: In front of the source (source moving toward listener) the wavelength is decreased and the frequency is increased. Behind the source (source moving away from listener) the wavelength is increased and the frequency is decreased.

16.43. **IDENTIFY:** Apply the Doppler shift equation $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$.

SET UP: The positive direction is from listener to source. $f_S = 392$ Hz.

EXECUTE: **(a)** $v_S = 0$. $v_L = -15.0$ m/s. $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} - 15.0 \text{ m/s}}{344 \text{ m/s}}\right)(392 \text{ Hz}) = 375 \text{ Hz}$

(b) $v_S = +35.0$ m/s. $v_L = +15.0$ m/s. $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} + 35.0 \text{ m/s}}\right)(392 \text{ Hz}) = 371 \text{ Hz}$

(c) $f_{\text{beat}} = f_1 - f_2 = 4$ Hz

EVALUATE: The distance between whistle *A* and the listener is increasing, and for whistle *A* $f_L < f_S$. The distance between whistle *B* and the listener is also increasing, and for whistle *B* $f_L < f_S$.

16.44. **IDENTIFY:** Apply $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$.

SET UP: $f_S = 1000$ Hz. The positive direction is from the listener to the source. $v = 344$ m/s.

EXECUTE: **(a)** $v_S = -(344 \text{ m/s})/2 = -172$ m/s, $v_L = 0$.

$$f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s}}{344 \text{ m/s} - 172 \text{ m/s}}\right)(1000 \text{ Hz}) = 2000 \text{ Hz}$$

(b) $v_S = 0$, $v_L = +172$ m/s. $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} + 172 \text{ m/s}}{344 \text{ m/s}}\right)(1000 \text{ Hz}) = 1500 \text{ Hz}$

EVALUATE: The answer in (b) is much less than the answer in (a). It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

16.45. **IDENTIFY:** The distance between crests is λ. In front of the source $\lambda = \dfrac{v - v_S}{f_S}$ and behind the source $\lambda = \dfrac{v + v_S}{f_S}$. $f_S = 1/T$.

SET UP: $T = 1.6$ s. $v = 0.32$ m/s. The crest to crest distance is the wavelength, so $\lambda = 0.12$ m.

EXECUTE: **(a)** $f_S = 1/T = 0.625$ Hz. $\lambda = \dfrac{v - v_S}{f_S}$ gives

$$v_S = v - \lambda f_S = 0.32 \text{ m/s} - (0.12 \text{ m})(0.625 \text{ Hz}) = 0.25 \text{ m/s}.$$

(b) $\lambda = \dfrac{v + v_S}{f_S} = \dfrac{0.32 \text{ m/s} + 0.25 \text{ m/s}}{0.625 \text{ Hz}} = 0.91 \text{ m}$

EVALUATE: If the duck was held at rest but still paddled its feet, it would produce waves of wavelength $\lambda = \dfrac{0.32 \text{ m/s}}{0.625 \text{ Hz}} = 0.51$ m. In front of the duck the wavelength is decreased and behind the duck the wavelength is increased. The speed of the duck is 78% of the wave speed, so the Doppler effects are large.

16.46. **IDENTIFY:** Apply the Doppler effect formula $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S.$

(a) SET UP: The positive direction is from the listener toward the source, as shown in Figure 16.46a.

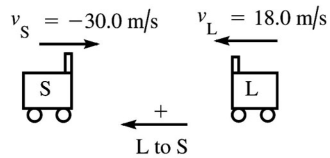


Figure 16.46a

EXECUTE: $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344 \text{ m/s} + 18.0 \text{ m/s}}{344 \text{ m/s} - 30.0 \text{ m/s}}\right)(352 \text{ Hz}) = 406 \text{ Hz}.$

EVALUATE: Listener and source are approaching and $f_L > f_S$.

(b) SET UP: See Figure 16.46b.

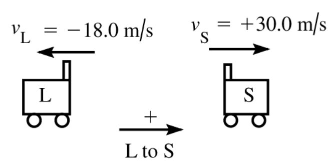


Figure 16.46b

EXECUTE: $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{344 \text{ m/s} - 18.0 \text{ m/s}}{344 \text{ m/s} + 30.3 \text{ m/s}}\right)(352 \text{ Hz}) = 307 \text{ Hz}.$

EVALUATE: Listener and source are moving away from each other and $f_L < f_S$.

16.47. **IDENTIFY:** Apply $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S.$

SET UP: The positive direction is from the motorcycle toward the car. The car is stationary, so $v_S = 0$.

EXECUTE: $f_L = \frac{v+v_L}{v+v_S}f_S = (1 + v_L/v)f_S,$ which gives

$v_L = v\left(\frac{f_L}{f_S} - 1\right) = (344 \text{ m/s})\left(\frac{490 \text{ Hz}}{520 \text{ Hz}} - 1\right) = -19.8 \text{ m/s}.$ You must be traveling at 19.8 m/s.

EVALUATE: $v_L < 0$ means that the listener is moving away from the source.

16.48. **IDENTIFY:** We have a moving source and a stationary observer. The beat frequency is due to interference between the Doppler-shifted sound from the horn in the moving car and the horn in the stationary car. The beat frequency is equal to the difference between these two frequencies.

SET UP: Apply $f_L = \left(\frac{v+v_L}{v+v_S}\right)f_S = \left(\frac{v}{v+v_S}\right)f_S,$ where $f_S = 260 \text{ Hz}.$ Since the source is moving toward you, you will hear the moving car horn at a higher pitch than your horn, and the beat frequency will be given by $f_{beat} = f_L - f_S.$

EXECUTE: We can determine f_L from the beat frequency: $f_{beat} = 6.0\text{ Hz} = f_L - f_S = f_L - 260\text{ Hz}$; thus, $f_L = 266\text{ Hz}$. Assuming that $v = 344\text{ m/s}$, we obtain $266\text{ Hz} = \left(\dfrac{344\text{ m/s}}{344\text{ m/s} + v_S}\right)(260\text{ Hz})$.

Solving for v_S we obtain $v_S = \left(\dfrac{260\text{ Hz}}{266\text{ Hz}} - 1\right)(344\text{ m/s}) = -7.8\text{ m/s}$. Thus, your friend is moving at 7.8 m/s toward you.

EVALUATE: What frequency will your friend hear? In this case, you have a stationary source (your horn) and a moving observer (your friend). The positive direction points from listener to source. Thus, we have $f_L = \left(\dfrac{344\text{ m/s} + 7.8\text{ m/s}}{344\text{ m/s}}\right)(260\text{ Hz}) = 265.8\text{ Hz} \approx 266\text{ Hz}$. At low speeds, there is little difference in the Doppler shift of a moving source or that of a moving observer.

16.49. **IDENTIFY:** The source of sound is moving and so is the listener, so we are dealing with the Doppler effect.

SET UP: We use $v = f\lambda$, where v is the speed relative to the listener, and $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$.

EXECUTE: **(a)** You and the sound waves are moving toward each other, so the speed of sound that you measure is $v = 340\text{ m/s} + 15\text{ m/s} = 355\text{ m/s}$.

(b) The wavelength of the sound that you receive is the same as the wavelength that the police car would measure behind the car. The speed of those waves relative to the police car is $v = 340\text{ m/s} + 15\text{ m/s} = 355\text{ m/s}$ because the sound and police car are moving away from each other. The wavelength of those waves is $\lambda = \dfrac{v}{f} = \dfrac{355\text{ m/s}}{500\text{ Hz}} = 0.710\text{ m}$, which is also the wavelength you observe.

(c) The speed of the sound waves relative to you is $v = 340\text{ m/s} + 15\text{ m/s} = 355\text{ m/s}$ since you are moving toward the waves, and the wavelength you measure is 0.710 m. So the frequency you measure is $f = \dfrac{v}{\lambda} = \dfrac{355\text{ m/s}}{0.710\text{ m}} = 500\text{ Hz}$. This is the same frequency the police siren is emitting.

EVALUATE: We can check using the Doppler effect formula $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$, where $v_L = +15\text{ m/s}$ and $v_S = +15\text{ m/s}$. Doing so gives $f_L = f_S = 500\text{ Hz}$.

16.50. **IDENTIFY:** There is a Doppler shift due to the motion of the fire engine as well as due to the motion of the truck, which reflects the sound waves.

SET UP: We use the Doppler shift equation $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S$.

EXECUTE: **(a)** First consider the truck as the listener, as shown in Figure 16.50(a).

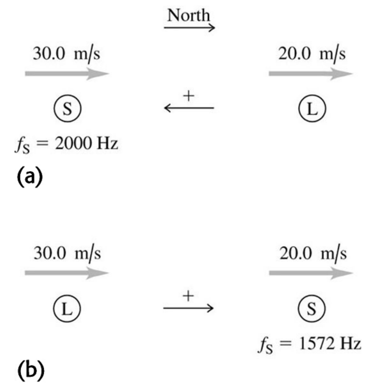


Figure 16.50

$f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s} - 20.0\text{ m/s}}{344\text{ m/s} - 30.0\text{ m/s}}\right)(2000\text{ Hz}) = 2064\text{ Hz}.$ Now consider the truck as a source, with $f_S = 2064\text{ Hz},$ and the fire engine driver as the listener, as shown in Figure 16.50(b). $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S = \left(\dfrac{344\text{ m/s} + 30.0\text{ m/s}}{344\text{ m/s} + 20.0\text{ m/s}}\right)(2064\text{ Hz}) = 2120\text{ Hz}.$ The objects are getting closer together so the frequency is increased.

(b) The driver detects a frequency of 2120 Hz and the waves returning from the truck move past him at 344 m/s + 30.0 m/s, so the wavelength he measures is $\lambda = \dfrac{344\text{ m/s} + 30\text{ m/s}}{2120\text{ Hz}} = 0.176\text{ m}.$ The wavelength of waves emitted by the fire engine when it is stationary is $\lambda = \dfrac{344\text{ m/s}}{2000\text{ Hz}} = 0.172\text{ m}.$

EVALUATE: In (a) the objects are getting closer together so the frequency is increased. In (b), the quantities to use in the equation $v = f\lambda$ are measured *relative to the observer*.

16.51. IDENTIFY: Apply the Doppler shift formulas. We first treat the stationary police car as the source and then as the observer as he receives his own sound reflected from the on-coming car.

SET UP: $f_L = \left(\dfrac{v + v_L}{v + v_S}\right)f_S.$

EXECUTE: (a) Since the frequency is increased the moving car must be approaching the police car. Let v_c be the speed of the moving car. The speed v_p of the police car is zero. First consider the moving car as the listener, as shown in Figure 16.51(a).

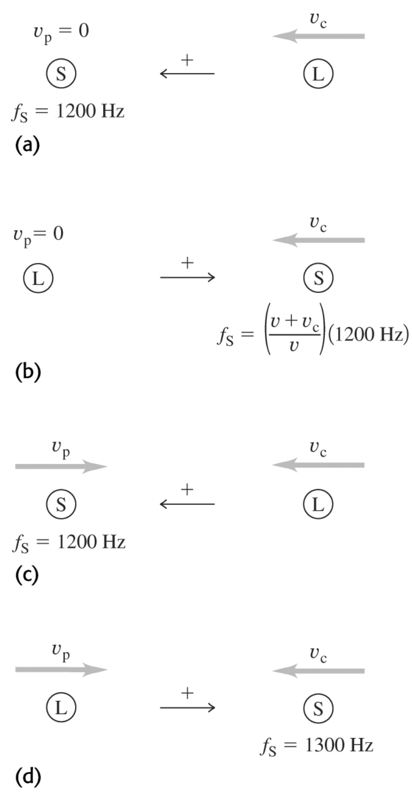


Figure 16.51

$$f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{v + v_c}{v}\right)(1200 \text{ Hz})$$

Then consider the moving car as the source and the police car as the listener, as shown in Figure 16.51(b):

$$f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S \text{ gives } 1250 \text{ Hz} = \left(\frac{v}{v - v_c}\right)\left(\frac{v + v_c}{v}\right)(1200 \text{ Hz}).$$

Solving for v_c gives

$$v_c = \left(\frac{50}{2450}\right) v = \left(\frac{50}{2450}\right)(344 \text{ m/s}) = 7.02 \text{ m/s}$$

(**b**) Repeat the calculation of part (a), but now $v_p = 20.0$ m/s, toward the other car.

Waves received by the car (Figure 16.51(c)):

$$f_L = \left(\frac{v + v_c}{v - v_p}\right) f_S = \left(\frac{344 \text{ m/s} + 7 \text{ m/s}}{344 \text{ m/s} - 20 \text{ m/s}}\right)(1200 \text{ Hz}) = 1300 \text{ Hz}$$

Waves reflected by the car and received by the police car (Figure 16.51(d)):

$$f_L = \left(\frac{v + v_p}{v - v_c}\right) f_S = \left(\frac{344 \text{ m/s} + 20 \text{ m/s}}{344 \text{ m/s} - 7 \text{ m/s}}\right)(1300 \text{ Hz}) = 1404 \text{ Hz}$$

EVALUATE: The cars move toward each other with a greater relative speed in (b) and the increase in frequency is much larger there.

16.52. **IDENTIFY:** A sound detector is moving toward a stationary source, so we are dealing with the Doppler effect.

SET UP: The graph we must interpret plots f_L versus v_L, so we need to find a relationship between those quantities to be able to interpret the graph. We use $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$.

EXECUTE: Solve $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ for f_L explicitly in terms of v_L, giving $f_L = \dfrac{f_S v_L}{v+v_S} + \dfrac{v f_S}{v+v_S}$. From this equation, we see that the graph should be a straight line having slope $\dfrac{f_S}{v+v_S}$ and y-intercept $\dfrac{v f_S}{v+v_S}$.

From this we see that (y-intercept) = (slope)v. Solving for v gives

$$v = \frac{y\text{-intercept}}{\text{slope}} = \frac{600.0\text{ Hz}}{1.75\text{ m}^{-1}} = 3.43\,\text{m/s}.$$

EVALUATE: This result is very close to the value $v = 344$ m/s in Table 16.1.

16.53. **IDENTIFY:** Apply $\sin\alpha = v/v_S$ to calculate α. Use the method of Example 16.19 to calculate t.

SET UP: Mach 1.70 means $v_S/v = 1.70$.

EXECUTE: **(a)** In $\sin\alpha = v/v_S$, $v/v_S = 1/1.70 = 0.588$ and $\alpha = \arcsin(0.588) = 36.0°$.

(b) As in Example 16.19, $t = \dfrac{1250\text{ m}}{(1.70)(344\text{ m/s})(\tan\ 36.0°)} = 2.94\text{ s}.$

EVALUATE: The angle α decreases when the speed v_S of the plane increases.

16.54. **IDENTIFY:** Apply $\sin\alpha = v/v_S$.

SET UP: The Mach number is the value of v_S/v, where v_S is the speed of the shuttle and v is the speed of sound at the altitude of the shuttle.

EXECUTE: **(a)** $\dfrac{v}{v_S} = \sin\alpha = \sin 58.0° = 0.848$. The Mach number is $\dfrac{v_S}{v} = \dfrac{1}{0.848} = 1.18$.

(b) $v_S = \dfrac{v}{\sin\alpha} = \dfrac{331\text{ m/s}}{\sin 58.0°} = 390\text{ m/s}$

(c) $\dfrac{v_S}{v} = \dfrac{390\text{ m/s}}{344\text{ m/s}} = 1.13$. The Mach number would be 1.13. $\sin\alpha = \dfrac{v}{v_S} = \dfrac{344\text{ m/s}}{390\text{ m/s}}$ and $\alpha = 61.9°$.

EVALUATE: The smaller the Mach number, the larger the angle of the shock-wave cone.

16.55. **IDENTIFY:** This problem involves the speed of sound in a gas.

SET UP: Eq. 16.10: $v = \sqrt{\dfrac{\gamma RT}{M}}$. Information from Example 16.4: $M = 0.0288$ kg/mol, $\gamma = 1.40$. If T is in K, $T_C = T - 273$, so $T = T_C + 273$. We want to determine the speed of sound at 20°C.

EXECUTE: **(a)** $v = \sqrt{\dfrac{\gamma RT}{M}} = \sqrt{\dfrac{\gamma R(T_C+273)}{M}} = \sqrt{\dfrac{\gamma R}{M}}\sqrt{T_C+273} = \sqrt{\dfrac{273\gamma R}{M}}\sqrt{\dfrac{T_C}{273}+1}$. If T_C is much less than 273°C, $T_C/273 << 1$, so we can use the approximation $\sqrt{1+x} \approx 1+\dfrac{x}{2}$, which gives

$v \approx \sqrt{\dfrac{273\gamma R}{M}}\left[1+\dfrac{T_C}{2(273)}\right] = \sqrt{\dfrac{273\gamma R}{M}}\left[1+\dfrac{T_C}{546}\right]$. Putting in $M = 0.0288$ kg/mol, $\gamma = 1.40$, and $R = 8.314$ J/mol$\cdot$K gives $v = (332\text{ m/s})(1 + T_C/546)$.

(b) At 20°C we have $v = (332\text{ m/s})(1 + 20/546) = 344$ m/s.

(c) When $T_C = 120°C$, $T_C/273 = 0.440$ is not much much greater than 1, so the approximation is less valid. So the answer is no.

EVALUATE: For part (c), compare the second and third terms of the power series. The second term is $\frac{1}{2}\left(\frac{120}{273}\right)=0.220$. The third term is $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}\left(\frac{120}{273}\right)^2=-0.024$, which is large enough to affect the sum.

16.56. **IDENTIFY:** Use the equations that relate intensity level and intensity, intensity and pressure amplitude, pressure amplitude and displacement amplitude, and intensity and distance.

(a) SET UP: Use the intensity level β to calculate I at this distance. $\beta=(10\text{ dB})\log(I/I_0)$

EXECUTE: $52.0\text{ dB}=(10\text{ dB})\log[I/(10^{-12}\text{ W/m}^2)]$

$\log[I/(10^{-12}\text{ W/m}^2)]=5.20$ implies $I=1.585\times10^{-7}\text{ W/m}^2$

SET UP: Then use $I=\frac{p_{\max}^2}{2\rho v}$ to calculate $p_{\max}$:

$I=\frac{p_{\max}^2}{2\rho v}$ so $p_{\max}=\sqrt{2\rho vI}$

From Example 16.5, $\rho=1.20\text{ kg/m}^3$ for air at 20°C.

EXECUTE: $p_{\max}=\sqrt{2\rho vI}=\sqrt{2(1.20\text{ kg/m}^3)(344\text{ m/s})(1.585\times10^{-7}\text{ W/m}^2)}=0.0114\text{ Pa}$

(b) SET UP: Use $p_{\max}=BkA$ so $A=\frac{p_{\max}}{Bk}$

For air $B=1.42\times10^5$ Pa (Example 16.1).

EXECUTE: $k=\frac{2\pi}{\lambda}=\frac{2\pi f}{v}=\frac{(2\pi\text{ rad})(587\text{ Hz})}{344\text{ m/s}}=10.72\text{ rad/m}$

$$A=\frac{p_{\max}}{Bk}=\frac{0.0114\text{ Pa}}{(1.42\times10^5\text{ Pa})(10.72\text{ rad/m})}=7.49\times10^{-9}\text{ m}$$

(c) SET UP: $\beta_2-\beta_1=(10\text{ dB})\log(I_2/I_1)$ (Example 16.9).

Inverse-square law: $I_1/I_2=r_2^2/r_1^2$ so $I_2/I_1=r_1^2/r_2^2$

EXECUTE: $\beta_2-\beta_1=(10\text{ dB})\log(r_1/r_2)^2=(20\text{ dB})\log(r_1/r_2)$.

$\beta_2=52.0$ dB and $r_2=5.00$ m. Then $\beta_1=30.0$ dB and we need to calculate r_1.

$52.0\text{ dB}-30.0\text{ dB}=(20\text{ dB})\log(r_1/r_2)$

$22.0\text{ dB}=(20\text{ dB})\log(r_1/r_2)$

$\log(r_1/r_2)=1.10$ so $r_1=12.6r_2=63.0$ m.

EVALUATE: The decrease in intensity level corresponds to a decrease in intensity, and this means an increase in distance. The intensity level uses a logarithmic scale, so simple proportionality between r and β doesn't apply.

16.57. **IDENTIFY:** We are investigating how the speed of sound depends on the air temperature as we look at higher and higher altitudes.

SET UP: Eq. 16.10: $v=\sqrt{\frac{\gamma RT}{M}}$, where M = 0.0288 kg/mol and γ = 1.40. First use the given information to sketch the graph of T versus y, as in Fig. 16.57. We use the slope-intercept equation of a straight line $T = my + b$. For this graph, $m=-\frac{20.0\text{ K}}{300.0\text{ m}}=-0.06667$ K/m and b = 278 K. Using these results, the speed v of sound as a function of altitude y is $v=\sqrt{\frac{\gamma R}{M}(my+b)}$. We want to find the time for the sound to travel 300.0 m straight upward, and we know $v_y = dy/dt$.

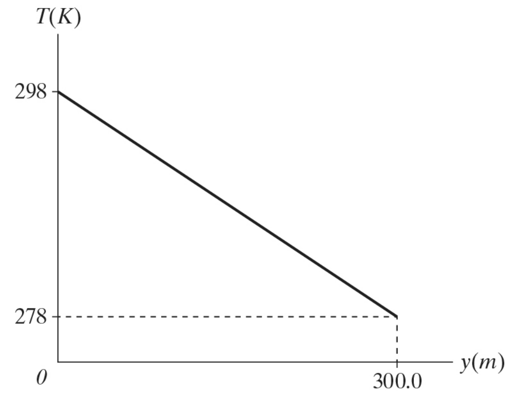


Figure 16.57

EXECUTE: **(a)** $v_y = dy/dt$ gives $dt = \frac{dy}{v}$, so we integrate, giving $\int_0^{t_{300}} dt = \int_0^{300\text{ m}} \frac{dy}{v}$. The time integral gives simply t_{300}, but the y integral requires a bit more effort. Using our equation for v from above, the y integral becomes $\int_0^{300\text{ m}} \frac{dy}{v} = \int_0^{300\text{ m}} \frac{dy}{\sqrt{\frac{\gamma R}{M}}\sqrt{my+b}} = \sqrt{\frac{M}{\gamma R}}\int_0^{300\text{ m}} \frac{dy}{(my+b)^{1/2}}$. To do the integral, let $u = my + b$, so $du = m\,dy$. The integral now becomes $\int \frac{(1/m)du}{u^{1/2}} = \frac{2}{m}u^{1/2}$. Returning to the original variables, the integral is $\sqrt{\frac{M}{\gamma R}}\int_0^{300\text{ m}} \frac{dy}{(my+b)^{1/2}} = \sqrt{\frac{M}{\gamma R}}\frac{2}{m}(my+b)^{1/2}\Big|_0^{300\text{ m}} = t_{300}$. Putting in the numbers for m, b, M, R, and γ gives $t_{300} = 0.879$ s.

(b) At 300.0 m, $T = 5.00°\text{C} = 298$ K, so using these numbers gives $v = \sqrt{\frac{\gamma RT}{M}} = 335.2$ m/s. The horizontal distance the sound travels is $x = vt_{300} = (335.2 \text{ m/s})(0.879 \text{ s}) = 295$ m.

EVALUATE: Since $v \propto \sqrt{T}$ (with T in K units), there is not much difference between the surface temperature and the temperature at 300 m, so v at 300 m is not very much different from v at the surface, as we see.

16.58. IDENTIFY: $f_{\text{beat}} = |f_A - f_B|$. $f_1 = \frac{v}{2L}$ and $v = \sqrt{\frac{FL}{m}}$ gives $f_1 = \frac{1}{2}\sqrt{\frac{F}{mL}}$. Apply $\Sigma\tau_z = 0$ to the bar to find the tension in each wire.

SET UP: For $\Sigma\tau_z = 0$ take the pivot at wire A and let counterclockwise torques be positive. The free-body diagram for the bar is given in Figure 16.58. Let L be the length of the bar.

EXECUTE: $\Sigma\tau_z = 0$ gives $F_B L - w_{\text{lead}}(3L/4) - w_{\text{bar}}(L/2) = 0$.

$F_B = 3w_{\text{lead}}/4 + w_{\text{bar}}/2 = 3(185 \text{ N})/4 + (165 \text{ N})/2 = 221 \text{ N}$. $F_A + F_B = w_{\text{bar}} + w_{\text{lead}}$ so

$F_A = w_{\text{bar}} + w_{\text{lead}} - F_B = 165 \text{ N} + 185 \text{ N} - 221 \text{ N} = 129 \text{ N}$.

$f_{1A} = \frac{1}{2}\sqrt{\frac{129 \text{ N}}{(5.50\times10^{-3} \text{ kg})(0.750 \text{ m})}} = 88.4 \text{ Hz}$. $f_{1B} = f_{1A}\sqrt{\frac{221 \text{ N}}{129 \text{ N}}} = 115.7 \text{ Hz}$.

$f_{\text{beat}} = f_{1B} - f_{1A} = 27.3 \text{ Hz}$.

EVALUATE: The frequency increases when the tension in the wire increases.

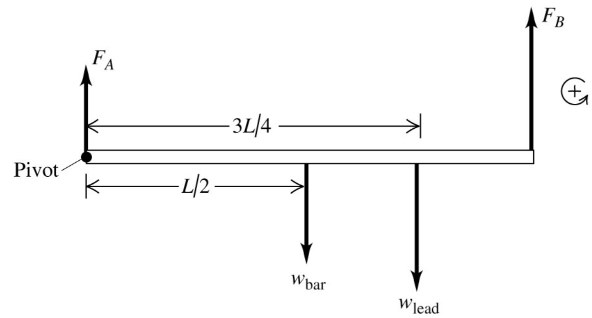


Figure 16.58

16.59. **IDENTIFY** and **SET UP:** The frequency of any harmonic is an integer multiple of the fundamental. For a stopped pipe only odd harmonics are present. For an open pipe, all harmonics are present. See which pattern of harmonics fits to the observed values in order to determine which type of pipe it is. Then solve for the fundamental frequency and relate that to the length of the pipe.

EXECUTE: **(a)** For an open pipe the successive harmonics are $f_n = nf_1,\ n = 1, 2, 3, \ldots$ For a stopped pipe the successive harmonics are $f_n = nf_1,\ n = 1, 3, 5, \ldots.$ If the pipe is open and these harmonics are successive, then $f_n = nf_1 = 1372\text{ Hz}$ and $f_{n+1} = (n+1)f_1 = 1764\text{ Hz}.$ Subtract the first equation from the second: $(n+1)f_1 - nf_1 = 1764\text{ Hz} - 1372\text{ Hz}.$ This gives $f_1 = 392\text{ Hz}.$ Then $n = \dfrac{1372\text{ Hz}}{392\text{ Hz}} = 3.5.$ But n must be an integer, so the pipe can't be open. If the pipe is stopped and these harmonics are successive, then $f_n = nf_1 = 1372\text{ Hz}$ and $f_{n+2} = (n+2)f_1 = 1764\text{ Hz}$ (in this case successive harmonics differ in n by 2). Subtracting one equation from the other gives $2f_1 = 392\text{ Hz}$ and $f_1 = 196\text{ Hz}.$ Then $n = 1372\text{ Hz}/f_1 = 7$ so $1372\text{ Hz} = 7f_1$ and $1764\text{ Hz} = 9f_1.$ The solution gives integer n as it should; the pipe is stopped.

(b) From part (a) these are the seventh and ninth harmonics.

(c) From part (a) $f_1 = 196\text{ Hz}.$

For a stopped pipe $f_1 = \dfrac{v}{4L}$ and $L = \dfrac{v}{4f_1} = \dfrac{344\text{ m/s}}{4(196\text{ Hz})} = 0.439\text{ m}.$

EVALUATE: It is essential to know that these are successive harmonics and to realize that 1372 Hz is not the fundamental. There are other lower frequency standing waves; these are just two successive ones.

16.60. **IDENTIFY:** This problem involves standing waves in a tube and the dependence of the speed of sound on temperature.

SET UP: A Kundt's tube is a stopped pipe, and the node-to-node distance is $\lambda/2,$ so $\dfrac{\lambda}{2} = 47.0\text{ cm}$ which gives $\lambda = 0.940$ m. We know that $v = f\lambda$, $f = 1200$ Hz, and $v = \sqrt{\dfrac{\gamma RT}{M}}.$ At 20°C the speed of sound in helium is 999 m/s. Our target variable is the temperature T of the helium in the tube.

EXECUTE: First find v_T in the tube at temperature T: $v_T = f\lambda$ = (1200 Hz)(0.940 m) = 1128 m/s. Now solve $v_T = \sqrt{\dfrac{\gamma RT}{M}}$ for T, which gives $T = \dfrac{Mv_T^2}{\gamma R}$. We don't know M and γ, but we do know that at 20°C (which is 293 K), v_{293} = 999 m/s for helium. Using $v = \sqrt{\dfrac{\gamma RT}{M}}$ and solving for T_{293} gives $T_{293} = \dfrac{Mv_{293}^2}{\gamma R}.$

Now take the ratio T/T_{293}, which gives $\dfrac{T}{T_{293}} = \dfrac{\dfrac{Mv_T^2}{\gamma R}}{\dfrac{Mv_{293}^2}{\gamma R}} = \left(\dfrac{v_T}{v_{293}}\right)^2 = \left(\dfrac{1128 \text{ m/s}}{999 \text{ m/s}}\right)^2 = 1.275.$ This gives T = (293 K)(1.275) = 374 K = 101°C.

EVALUATE: This type of experiment relies on very simple measurements: the frequency of the tuning fork (or electronic oscillator) and the visible node-to-node distance of the standing wave pattern in the tube. We do not to know anything about the gas since γ and M divide out.

16.61. **IDENTIFY:** Destructive interference occurs when the path difference is a half-integer number of wavelengths. Constructive interference occurs when the path difference is an integer number of wavelengths.

SET UP: $\lambda = \dfrac{v}{f} = \dfrac{344 \text{ m/s}}{784 \text{ Hz}} = 0.439 \text{ m}$

EXECUTE: **(a)** If the separation of the speakers is denoted h, the condition for destructive interference is $\sqrt{x^2 + h^2} - x = \beta\lambda,$ where β is an odd multiple of one-half. Adding x to both sides, squaring, canceling the x^2 term from both sides, and solving for x gives $x = \dfrac{h^2}{2\beta\lambda} - \dfrac{\beta}{2}\lambda.$ Using $\lambda = 0.439$ m and $h = 2.00$ m yields 9.01 m for $\beta = \frac{1}{2},$ 2.71 m for $\beta = \frac{3}{2},$ 1.27 m for $\beta = \frac{5}{2},$ 0.53 m for $\beta = \frac{7}{2},$ and 0.026 m for $\beta = \frac{9}{2}.$ These are the only allowable values of β that give positive solutions for x.

(b) Repeating the above for integral values of $\beta,$ constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a).

(c) If $h = \lambda/2,$ there will be destructive interference at speaker B. If $\lambda/2 > h,$ the path difference can never be as large as $\lambda/2.$ (This is also obtained from the above expression for $x,$ with $x = 0$ and $\beta = \frac{1}{2}$.) The minimum frequency is then $v/2h = (344 \text{ m/s})/(4.0 \text{ m}) = 86 \text{ Hz}.$

EVALUATE: When f increases, λ is smaller and there are more occurrences of points of constructive and destructive interference.

16.62. **IDENTIFY:** Apply $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S.$ The wall first acts as a listener and then as a source.

SET UP: The positive direction is from listener to source. The bat is moving toward the wall so the Doppler effect increases the frequency and the final frequency received, $f_{L2},$ is greater than the original source frequency, $f_{S1}.$ $f_{S1} = 1700$ Hz. $f_{L2} - f_{S1} = 8.00$ Hz.

EXECUTE: The wall receives the sound: $f_S = f_{S1}.$ $f_L = f_{L1}.$ $v_S = -v_{bat}$ and $v_L = 0.$ $f_L = \left(\dfrac{v + v_L}{v + v_S}\right) f_S$ gives $f_{L1} = \left(\dfrac{v}{v - v_{bat}}\right) f_{S1}.$ The wall receives the sound: $f_{S2} = f_{L1}.$ $v_S = 0$ and $v_L = +v_{bat}.$

$$f_{L2} = \left(\frac{v + v_{bat}}{v}\right) f_{S2} = \left(\frac{v + v_{bat}}{v}\right)\left(\frac{v}{v - v_{bat}}\right) f_{S1} = \left(\frac{v + v_{bat}}{v - v_{bat}}\right) f_{S1}.$$

$$f_{L2} - f_{S1} = \Delta f = \left(\frac{v + v_{bat}}{v - v_{bat}} - 1\right) f_{S1} = \left(\frac{2v_{bat}}{v - v_{bat}}\right) f_{S1}.$$

$$v_{bat} = \frac{v\Delta f}{2 f_{S1} + \Delta f} = \frac{(344 \text{ m/s})(8.00 \text{ Hz})}{2(1700 \text{ Hz}) + 8.00 \text{ Hz}} = 0.808 \text{ m/s}.$$

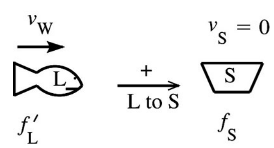
EVALUATE: $f_{S1} < \Delta f$, so we can write our result as the approximate but accurate expression $\Delta f = \left(\dfrac{2v_{\text{bat}}}{v}\right) f_{S1}$.

16.63. **(a) IDENTIFY** and **SET UP:** Use $v = f\lambda$ to calculate λ.

EXECUTE: $\lambda = \dfrac{v}{f} = \dfrac{1482\ \text{m/s}}{18.0\times 10^3\ \text{Hz}} = 0.0823\ \text{m}.$

(b) IDENTIFY: Apply the Doppler effect equation, $f_L = \left(\dfrac{v+v_L}{v}\right) f_S = \left(1+\dfrac{v_L}{v}\right) f_S$. The frequency of the directly radiated waves is $f_S = 18{,}000$ Hz. The moving whale first plays the role of a moving listener, receiving waves with frequency f_L'. The whale then acts as a moving source, emitting waves with the same frequency, $f_S' = f_L'$ with which they are received. Let the speed of the whale be v_W.

SET UP: <u>Whale receives waves</u>: (Figure 16.63a)

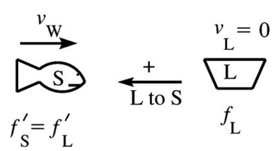


EXECUTE: $v_L = +v_W$

$$f_L' = f_S\left(\frac{v+v_L}{v+v_S}\right) = f_S\left(\frac{v+v_W}{v}\right)$$

Figure 16.63a

SET UP: <u>Whale re-emits the waves</u>: (Figure 16.63b)



EXECUTE: $v_S = -v_W$

$$f_L = f_S\left(\frac{v+v_L}{v+v_S}\right) = f_S'\left(\frac{v}{v-v_W}\right)$$

Figure 16.63b

But $f_S' = f_L'$ so $f_L = f_S\left(\dfrac{v+v_W}{v}\right)\left(\dfrac{v}{v-v_W}\right) = f_S\left(\dfrac{v+v_W}{v-v_W}\right)$.

Then $\Delta f = f_S - f_L = f_S\left(1-\dfrac{v+v_W}{v-v_W}\right) = f_S\left(\dfrac{v-v_W-v-v_W}{v-v_W}\right) = \dfrac{-2f_S v_W}{v-v_W}$.

$$\Delta f = \frac{-2(1.80\times 10^4\ \text{Hz})(4.95\ \text{m/s})}{1482\ \text{m/s} - 4.95\ \text{m/s}} = -120\ \text{Hz}.$$

EVALUATE: Δf is negative, which means that $f_L > f_S$. This is reasonable because the listener and source are moving toward each other so the frequency is raised.

16.64. **IDENTIFY:** This problem deals with sound intensity and sound intensity level.

SET UP and EXECUTE: **(a)** Estimate: 5.0 s delay between the flash and sound of thunder.

(b) $x = vt = (344\ \text{m/s})(5.0\ \text{s}) = 1700$ m.

(c) Estimate: 70 dB. $\beta = (10\ \text{dB})\log\dfrac{I}{I_0} = 70$ dB, so $I = 10^7 I_0 = 10^7\ 10^{-12}\ \text{W/m}^2 = 10^{-5}\ \text{W/m}^2$.

(d) $I = P/A$, so $P_{\text{av}} = IA = I(4\pi r^2) = (10^{-5}\ \text{W/m}^2)(4\pi)(1700\ \text{m})^2 = 370$ W.

(e) Estimate: 4.0 s duration. $E = P_{\text{av}}t = (370\ \text{W})(4.0\ \text{s}) = 1500$ J.

EVALUATE: The power due to a lightning strike is much greater than 370 W because the strike lasts only a fraction of a second and can create a great deal of heat. We are looking only at the *sound* energy.

16.65. IDENTIFY: This problem deals with sound intensity and sound intensity level.

SET UP and EXECUTE: (a) Estimate: Somewhere between busy traffic and an elevated train, so use 85 dB. $\beta = (10\text{ dB})\log\dfrac{I}{I_0} = 85$ dB, so $I = 10^{8.5}I_0 = 10^{8.5}\,10^{-12}\text{ W/m}^2 = 3.2\times10^{-4}\text{ W/m}^2$.

(b) $I = P/A$, so $P_{av} = IA = I(4\pi r^2) = (3.2\times10^{-4}\text{ W/m}^2)(4\pi)(18\text{ m})^2 = 1.3$ W.

(c) Estimate: 2.0 s for the crashing sound. $E = P_{av}t = (1.3\text{ W})(2.0\text{ s}) = 2.6$ J.

(d) Estimate: 4.0 s to travel down the alley. $v = x/t = (18\text{ m})/(4.0\text{ s}) = 4.5$ m/s.

(e) $K = \dfrac{1}{2}mv_{cm}^2 + \dfrac{1}{2}I\omega^2$, with $\omega = v_{cm}/R$ for rolling and $I = \dfrac{2}{5}MR^2$. Using these gives

$$K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_{cm}}{R}\right)^2 = \frac{7}{10}mv_{cm}^2 = \frac{7}{10}(6.4\text{ kg})(4.5\text{ m/s})^2 = 91\text{ J}.$$

(f) $\dfrac{2.6\text{ J}}{91\text{ J}} = 0.029 \approx 3\%$.

EVALUATE: Just from watching the heavy pins scatter, it is clear that most of the ball's kinetic energy is transferred to them, with little left over for sound energy. The 3% is not unreasonable.

16.66. IDENTIFY: Apply $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. The heart wall first acts as the listener and then as the source.

SET UP: The positive direction is from listener to source. The heart wall is moving toward the receiver so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the source frequency, f_{S1}. $f_{L2} - f_{S1} = 72$ Hz.

EXECUTE: Heart wall receives the sound: $f_S = f_{S1}$. $f_L = f_{L1}$. $v_S = 0$. $v_L = -v_{wall}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$ gives $f_{L1} = \left(\dfrac{v-v_{wall}}{v}\right)f_{S1}$.

Heart wall emits the sound: $f_{S2} = f_{L1}$. $v_S = +v_{wall}$. $v_L = 0$.

$$f_{L2} = \left(\frac{v}{v+v_{wall}}\right)f_{S2} = \left(\frac{v}{v+v_{wall}}\right)\left(\frac{v-v_{wall}}{v}\right)f_{S1} = \left(\frac{v-v_{wall}}{v+v_{wall}}\right)f_{S1}.$$

$f_{L2} - f_{S1} = \left(1 - \dfrac{v-v_{wall}}{v+v_{wall}}\right)f_{S1} = \left(\dfrac{2v_{wall}}{v+v_{wall}}\right)f_{S1}$. $v_{wall} = \dfrac{(f_{L2}-f_{S1})v}{2f_{S1} - (f_{L2}-f_{S1})}$. $f_{S1} \gg f_{L2} - f_{S1}$ and

$$v_{wall} = \frac{(f_{L2}-f_{S1})v}{2f_{S1}} = \frac{(72\text{ Hz})(1500\text{ m/s})}{2(2.00\times10^6\text{ Hz})} = 0.0270\text{ m/s} = 2.70\text{ cm/s}.$$

EVALUATE: $f_{S1} = 2.00\times10^6$ Hz and $f_{L2} - f_{S1} = 72$ Hz, so the approximation we made is very accurate. Within this approximation, the frequency difference between the reflected and transmitted waves is directly proportional to the speed of the heart wall.

16.67. IDENTIFY: Follow the method of Example 16.18 and apply the Doppler shift formula twice, once with the insect as the listener and again with the insect as the source.

SET UP: Let v_{bat} be the speed of the bat, v_{insect} be the speed of the insect, and f_i be the frequency with which the sound waves both strike and are reflected from the insect. The positive direction in each application of the Doppler shift formula is from the listener to the source.

EXECUTE: The frequencies at which the bat sends and receives the signals are related by

$f_L = f_i\left(\dfrac{v+v_{bat}}{v-v_{insect}}\right) = f_S\left(\dfrac{v+v_{insect}}{v-v_{bat}}\right)\left(\dfrac{v+v_{bat}}{v-v_{insect}}\right)$. Solving for v_{insect},

$$v_{\text{insect}} = v\left[\frac{1-\dfrac{f_S}{f_L}\left(\dfrac{v+v_{\text{bat}}}{v-v_{\text{bat}}}\right)}{1+\dfrac{f_S}{f_L}\left(\dfrac{v+v_{\text{bat}}}{v-v_{\text{bat}}}\right)}\right] = v\left[\frac{f_L(v-v_{\text{bat}})-f_S(v+v_{\text{bat}})}{f_L(v-v_{\text{bat}})+f_S(v+v_{\text{bat}})}\right].$$

Letting $f_L = f_{\text{refl}}$ and $f_S = f_{\text{bat}}$ gives the result.

(b) If $f_{\text{bat}} = 80.7$ kHz, $f_{\text{refl}} = 83.5$ kHz, and $v_{\text{bat}} = 3.9$ m/s, then $v_{\text{insect}} = 2.0$ m/s.

EVALUATE: $f_{\text{refl}} > f_{\text{bat}}$ because the bat and insect are approaching each other.

16.68. **IDENTIFY:** Apply the Doppler effect formula $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. In the SHM the source moves toward and away from the listener, with maximum speed $\omega_p A_p$.

SET UP: The direction from listener to source is positive.

EXECUTE: **(a)** The maximum velocity of the siren is $\omega_P A_P = 2\pi f_P A_P$. You hear a sound with frequency $f_L = f_{\text{siren}} v/(v+v_S)$, where v_S varies between $+2\pi f_P A_P$ and $-2\pi f_P A_P$.

$f_{L-\max} = f_{\text{siren}} v/(v-2\pi f_P A_P)$ and $f_{L-\min} = f_{\text{siren}} v/(v+2\pi f_P A_P)$.

(b) The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

EVALUATE: When the platform is moving upward the frequency you hear is greater than f_{siren} and when it is moving downward the frequency you hear is less than f_{siren}. When the platform is at its maximum displacement from equilibrium its speed is zero and the frequency you hear is f_{siren}.

16.69. **IDENTIFY:** The sound from the speaker moving toward the listener will have an increased frequency, while the sound from the speaker moving away from the listener will have a decreased frequency. The difference in these frequencies will produce a beat.

SET UP: The greatest frequency shift from the Doppler effect occurs when one speaker is moving away and one is moving toward the person. The speakers have speed $v_0 = r\omega$, where $r = 0.75$ m.

$f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$, with the positive direction from the listener to the source. $v = 344$ m/s.

EXECUTE: **(a)** $f = \dfrac{v}{\lambda} = \dfrac{344 \text{ m/s}}{0.313 \text{ m}} = 1100$ Hz. $\omega = (75 \text{ rpm})\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = 7.85$ rad/s and

$v_0 = (0.75 \text{ m})(7.85 \text{ rad/s}) = 5.89$ m/s.

For speaker A, moving toward the listener: $f_{LA} = \left(\dfrac{v}{v-5.89 \text{ m/s}}\right)(1100 \text{ Hz}) = 1119$ Hz.

For speaker B, moving toward the listener: $f_{LB} = \left(\dfrac{v}{v+5.89 \text{ m/s}}\right)(1100 \text{ Hz}) = 1081$ Hz.

$f_{\text{beat}} = f_1 - f_2 = 1119 \text{ Hz} - 1081 \text{ Hz} = 38$ Hz.

(b) A person can hear individual beats only up to about 7 Hz and this beat frequency is much larger than that.

EVALUATE: As the turntable rotates faster the beat frequency at this position of the speakers increases.

16.70. **IDENTIFY** and **SET UP:** Assuming that the gas is nearly ideal, the speed of sound in it is given by $v = \sqrt{\dfrac{\gamma RT}{M}}$, where T is in absolute (Kelvin) units and M is the molar mass of the gas.

EXECUTE: **(a)** Squaring $v=\sqrt{\dfrac{\gamma RT}{M}}$, gives $v^2=\left(\dfrac{\gamma R}{M}\right)T$. On a graph of v^2 versus T, we would expect a straight line with slope equal to $\dfrac{\gamma R}{M}$. Figure 16.70 shows the graph of the data given in the problem.

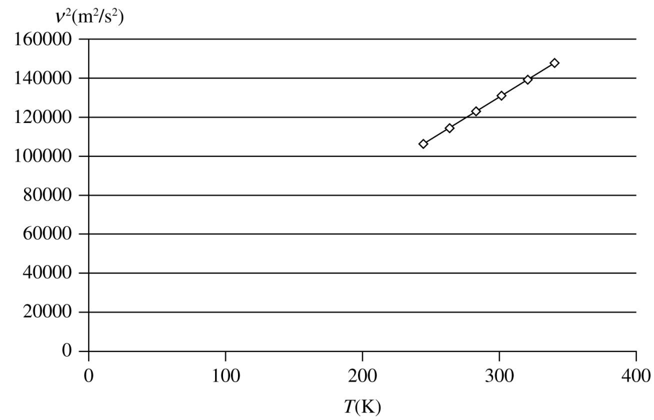


Figure 16.70

(b) The best-fit equation for the graph in Figure 16.70 is $v^2 = (416.47\ \text{m}^2/\text{K}\cdot\text{s}^2)\,T + 296.65\ \text{m}^2/\text{s}^2$. Solving our expression for the slope for M gives $M = \gamma R$ /slope. Putting in the numbers gives $M = (1.40)(8.3145\ \text{J/mol}\cdot\text{K})/(416.47\ \text{m}^2/\text{K}\cdot\text{s}^2) = 0.0279$ kg/mol = 27.9 g/mol.

EVALUATE: Nitrogen N_2 is a diatomic gas with a molecular mass of 28.0 g/mol, so the gas is probably nitrogen.

16.71. **IDENTIFY** and **SET UP:** There is a node at the piston, so the distance the piston moves is the node to node distance, $\lambda/2$. Use $v=f\lambda$ to calculate v and $v=\sqrt{\dfrac{\gamma RT}{M}}$ to calculate γ from v.

EXECUTE: **(a)** $\lambda/2=37.5$ cm, so $\lambda=2(37.5\text{ cm})=75.0\text{ cm}=0.750$ m.
$v=f\lambda=(500\text{ Hz})(0.750\text{ m})=375$ m/s

(b) Solve $v=\sqrt{\gamma RT/M}$ for γ: $\gamma=\dfrac{Mv^2}{RT}=\dfrac{(28.8\times10^{-3}\text{ kg/mol})(375\text{ m/s})^2}{(8.3145\text{ J/mol}\cdot\text{K})(350\text{ K})}=1.39.$

(c) EVALUATE: There is a node at the piston so when the piston is 18.0 cm from the open end the node is inside the pipe, 18.0 cm from the open end. The node to antinode distance is $\lambda/4=18.8$ cm, so the antinode is 0.8 cm beyond the open end of the pipe.
The value of γ we calculated agrees with the value given for air in Example 16.4.

16.72. **IDENTIFY** and **SET UP:** We know from the problem that $f_R = f_S\left(1-\dfrac{v}{c}\right)^{1/2}\left(1+\dfrac{v}{c}\right)^{1/2}$. The radius of the nebula is $R=vt$, where t is the time since the supernova explosion. When the source and receiver are moving toward each other, v is negative and $f_R > f_S$. The light from the explosion reached earth 960 years ago, so that is the amount of time the nebula has expanded. $1\text{ ly}=9.461\times10^{15}$ m.

EXECUTE: **(a)** According to the binomial theorem, $(1 \pm x)^n \approx 1 \pm nx$ if $|x| << 1$. Applying this to the two square roots, where $n=\pm\dfrac{1}{2}$ and $x = v/c$, the equation $f_R = f_S\left(1-\dfrac{v}{c}\right)^{1/2}\left(1+\dfrac{v}{c}\right)^{1/2}$ becomes

$$f_R \approx f_S\left(1-\frac{1}{2}\frac{v}{c}\right)\left(1-\frac{1}{2}\frac{v}{c}\right) \approx f_S\left(1-\frac{1}{2}\frac{v}{c}\right)^2 \approx f_S\left[1-2\left(\frac{1}{2}\frac{v}{c}\right)\right] \approx f_S\left(1-\frac{v}{c}\right).$$

(b) Solving the equation we derived in part (a) for v gives

$v = c\dfrac{f_S - f_R}{f_S} = (3.00\times10^8\text{ m/s})\dfrac{-0.018\times10^{14}\text{ Hz}}{4.568\times10^{14}\text{ Hz}} = -1.2\times10^6\text{ m/s}$, with the minus sign indicating that the gas is approaching the earth, as is expected since $f_R > f_S$.

(c) As of 2014, the supernova occurred 960 years ago. The diameter D is therefore

$D = 2(960\text{ y})(3.156\times10^7\text{ s/y})(1.2\times10^6\text{ m/s}) = 7.15\times10^{16}\text{ m} = 7.6\text{ ly}.$

(d) The ratio of the width of the nebula to 2π times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is 60×360 arc minutes. The distance to the nebula is then $\left(\dfrac{2}{2\pi}\right)(3.75\text{ ly})\dfrac{(60)(360)}{5} = 5.2\times10^3$ ly. The time it takes light to travel this distance is 5200 yr, so the explosion actually took place 5200 yr before 1054 C.E., or about 4100 B.C.E.

EVALUATE: $\left|\dfrac{v}{c}\right| = 4.0\times10^{-3}$, so even though $|v|$ is very large the approximation required for $v = c\dfrac{\Delta f}{f}$ is accurate.

16.73. **IDENTIFY:** The wire vibrating in its fundamental causes the tube to resonate with the same frequency in its fundamental. We are dealing with standing waves on a string and in an open pipe.

SET UP: Fig. 16.73a illustrates the information in the problem. The information we have is as follows:
wire: μ = 1.40 g/m = 0.00140 kg/m, vibrating in its fundamental f_1.
pole: M = 8.00 kg, L = 1.56 m
tube: 39.0 cm long, m = 4.00 kg, hollow (open pipe), vibrating in its fundamental f_1
We want to find the frequency f_1 at which the wire and tube are vibrating and the height h in Fig. 16.73a.
The equations we use for the tube are $f_n = nv_s/2L_t$, $\lambda_n = 2L_t/n$, $\lambda_1 = 2L_t$ and for the wire $f_n = nv/2L_w$, $\lambda_n = 2L_w/n$, $\lambda_1 = 2L_w$. We also use $v = f\lambda$.

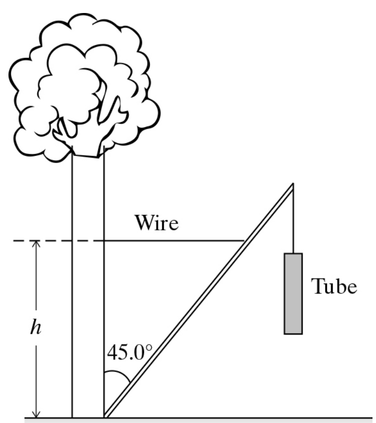


Figure 16.73a

EXECUTE: **(a)** For the tube in its fundamental mode, we have $\lambda_1 = 2L_t$ = 2(0.390 m) = 0.780 m. The frequency is $f_1 = \dfrac{v_s}{\lambda_t} = \dfrac{344\text{ m/s}}{0.780\text{ m}} = 441\text{ Hz}$. This is the frequency at which the wire and the tube are resonating.

(b) We want to find h in Fig. 16.73a. Now look at the wire. $f_w\lambda_w = v_w = \sqrt{\dfrac{F}{\mu}}$. In its fundamental mode $\lambda_1 = 2L_w$. From Fig. 16.73a we see that $L_w = h\tan 45.0° = h$, so $\lambda_1 = 2h$. From part (a) we know that f_1 = 441 Hz, so $f_w\lambda_w = v_w = \sqrt{\dfrac{F}{\mu}}$ gives $f_1(2h)$ = (441 Hz)(2h) = $\sqrt{\dfrac{F}{\mu}}$, which becomes

$$(882\text{ Hz})h = \sqrt{\frac{F}{\mu}}. \qquad \text{(Eq. 1)}$$

We need to find F in order to find h. Make a free-body diagram of the pole as in Fig. 16.73b and apply $\sum\tau_z = 0$ about the hinge. Using Fig. 16.73 as a guide, we get

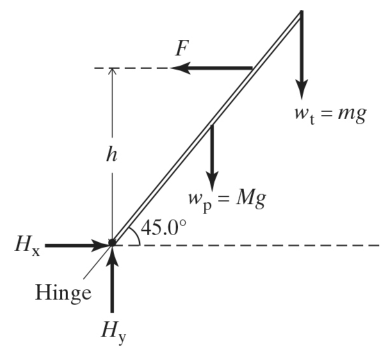


Figure 16.73b

$Fh - Mg\dfrac{L}{2}\cos 45.0° - mgL\cos 45.0° = 0,$ which simplifies to $Fh = \left(\dfrac{M}{2} + m\right)gL\cos 45.0°.$ Putting in M =8.00 kg, m = 4.00 kg, and L = 1.56 m, and solving for F, we get $F = \dfrac{86.48\text{ N}\cdot\text{m}}{h}.$ Now return to Eq. 1. Square the equation and substitute for F, giving $(882\text{ Hz})^2h^2 = \dfrac{F}{\mu} = \dfrac{86.48\text{ N}\cdot\text{m}}{h(0.00140\text{ kg/m})}$. Solving for h gives h = 0.430 m.

EVALUATE: Changing h would change the length of the wire as well as the tension in it. These changes would affect the fundamental frequency of the wire.

16.74. **IDENTIFY:** The sound from the two speakers travels different distances to reach the aisle. Therefore interference occurs along the aisle.

SET UP: Start with a figure to illustrate the information given, as in Fig. 16.74. The path difference d between sound from speakers A and B is $d = r - x$. For destructive interference to occur, the path difference must be $d = \dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, \dfrac{5\lambda}{2}, \cdots = (2n-1)\dfrac{\lambda}{2}$ $(n = 1,2,3,...)$. From the figure, we can see that $d = \sqrt{x^2 + (15.0\text{ m})^2} - x$, so $d = \sqrt{x^2 + (15.0\text{ m})^2} - x = (2n-1)\dfrac{\lambda}{2}$ $(n = 1,2,3,...)$. The wavelength of the sound from both speakers is $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{440\text{ Hz}} = 0.7818\text{ m}.$

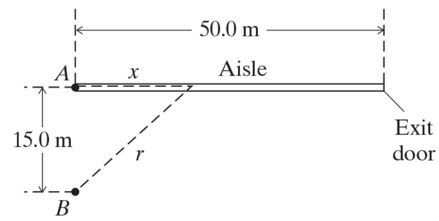


Figure 16.74

EXECUTE: **(a)** The path difference d increases as we get closer to speaker A in the figure. The limit would be right at the speaker, in which case $x = 0$, $r = 15.0$ m, and $d = 15.0$ m. Therefore we have $d = 15.0\text{ m} = (2n-1)\dfrac{\lambda}{2} = (2n-1)\dfrac{0.7818\text{ m}}{2}$. Solving for n gives $n = 19.69$. But n must be an integer and cannot exceed 19.69, so $n = 19$. This is the total number of points along the aisle where the waves from the two speakers would cancel each other. But aisle is only 50.0 m long, so the theater may not be deep enough for all these points to occur. The smallest path difference would occur at the back door 50.0 m from the stage. The path difference at that point is d_{min} which is $d_{\text{min}} = \sqrt{(50.0\text{ m})^2 + (15.0\text{ m})^2} - 50.0\text{ m} = 2.2015\text{ m}$. Now use this distance to find out how may points of cancellation will fit inside the theater. $2.2015\text{ m} = (2n-1)\left(\dfrac{0.7818\text{ m}}{2}\right) = (0.3909\text{ m})(2n-1)$. Solving for n gives $n = 3.316$, which suggests $n = 3$. However for $n = 3$, $d = (2n-1)\dfrac{\lambda}{2} = \dfrac{5\lambda}{2} = \dfrac{5(0.7818\text{ m})}{2} = 1.955\text{ m}$. But this distance is shorter than the minimum path difference of 2.2015 m that we calculated earlier. So we would have to be farther than 50.0 m from the stage to experience it, which is outside the theater. Thus the $n = 3$ point does not occur, and neither do the $n = 2$ and $n = 1$ points. Therefore we have found that a total of 19 cancellation points are possible, but 3 of them would occur outside the theater, so the number that occur inside the theater are 19 – 3 = 16.

(b) We have just seen that the $n = 3$ interference point does not occur, so the farthest point at which cancellation does occur would be for the $n = 4$ point for which $d = \dfrac{7}{2}\lambda = \dfrac{7}{2}(0.7818\text{ m}) = 2.7363\text{ m}$.

Using this value of d and solving for the distance x from the stage gives $\sqrt{x^2 + (15.0\text{ m})^2} - x = 2.7363\text{ m}$. Squaring and solving for x gives $x = 39.746$ m. The distance from the exit door is 50.0 m – 39.746 m = 10.3 m.

(c) The second frequency must be such that it cancels at the same points as the original frequency (and it may cancel at other points also). This frequency must be some multiple of the original frequency. Call the original wavelength λ and the second frequency λ', and realize that λ' must be less than λ, so we can write it as $\lambda' = \lambda / k$. For cancellation of the sound at both frequencies to occur, d must be odd multiples of a half wavelength for each frequency f and f'. That is, $d = N(\lambda/2)$ and $d = N'(\lambda'/2)$, where both N and N' must be odd integers. Substituting $\lambda' = \lambda / k$ into $d = N(\lambda/2)$ gives $d = N\left(\dfrac{k\lambda'}{2}\right) = Nk\dfrac{\lambda'}{2}$. This result tells us that Nk must be an odd integer. Since N is already an odd integer, k must also be an odd integer for the product Nk to be odd. The frequency f' of the second sound is $f' = \dfrac{v}{\lambda'} = \dfrac{v}{\lambda/k} = k(v/\lambda) = kf$, where k is an odd integer. We want the smallest possible frequency for f', so k must have its smallest possible value. The smallest is $k = 1$, but in that case the second frequency is the same as the first one, which is not what we want. So k must be 3, which makes the second wavelength $\lambda' = \frac{\lambda}{3} = \frac{0.7818\text{ m}}{3} = 0.2606$ m and the second frequency $f' = 3f = 3(440\text{ Hz}) = 1320$ Hz.

(d) We follow the same procedure as in part (a), but this time for f' and λ'. For destructive interference, the path difference must be $d = (2N-1)\frac{\lambda'}{2}$, N = 1, 2, 3, … . The greatest path difference occurs closest to speaker A, with the limit being d = 15.0 m. In this case we have 15.0 m = $(2N-1)\frac{\lambda'}{2} = (2N-1)\frac{0.2606\text{ m}}{2}$. Solving for N gives N = 58. This means that destructive interference can occur at a maximum of 58 points. However the hall may not be deep enough for all of them to occur. As we saw in part (a), the minimum path difference d_{min} that can occur in the hall is at the back door, and at that point d_{min} = 2.2015 m. Using this information to find the minimum value of N for points inside the hall gives 2.2015 m = $(2N-1)\frac{\lambda'}{2} = (2N-1)\frac{0.2606\text{ m}}{2}$. Solving for N we get N = 8.95, so we use N = 8. But first check to see if d is shorter than d_{min} for this result. Doing so gives $d = (2N-1)\frac{\lambda'}{2} = (15)\frac{0.2606\text{ m}}{2} = 1.955$ m, which is less than 2.2015 m. Therefore the N = 8 point does not occur within the hall, and neither do all the others for N = 7, 6, etc. So the total number of points within the hall at which destructive interference occurs is 58 – 8 = 50. But 16 of those points occur at the same places as the 440-Hz sound, so the *additional* number of points is 50 – 16 = 34 points.

(e) At the point closest to the speaker, N = 58 from part (d). The path difference at this point is $d = (2N-1)\frac{\lambda'}{2} = (115)\frac{0.2606\text{ m}}{2} = 14.9845$ m. Using $\sqrt{x^2 + (15.0\text{ m})^2} - x = 14.9845$ m, we solve for x, giving x = 0.0155 m = 1.55 cm.

EVALUATE: The sound cannot cancel completely at the points of destructive interference because the intensity from the two speakers is different due to the difference in distance from them. But the loudness will definitely reach a minimum at the points of destructive interference.

16.75. IDENTIFY: The phase of the wave is determined by the value of $x - vt$, so t increasing is equivalent to x decreasing with t constant. The pressure fluctuation and displacement are related by the equation $p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$.

SET UP: $y(x,t) = -\frac{1}{B}\int p(x,t)dx$. If $p(x,t)$ versus x is a straight line, then $y(x,t)$ versus x is a parabola. For air, $B = 1.42\times10^5$ Pa.

EXECUTE: **(a)** The graph is sketched in Figure 16.75a.

(b) From $p(x,t) = BkA\sin(kx - \omega t)$, the function that has the given $p(x,0)$ at $t = 0$ is given graphically in Figure 16.75b. Each section is a parabola, not a portion of a sine curve. The period is $\lambda/v = (0.200\text{ m})/(344\text{ m/s}) = 5.81\times10^{-4}$ s and the amplitude is equal to the area under the p versus x curve between $x = 0$ and $x = 0.0500$ m divided by B, or 7.04×10^{-6} m.

(c) Assuming a wave moving in the $+x$-direction, $y(0,t)$ is as shown in Figure 16.75c.

(d) The maximum velocity of a particle occurs when a particle is moving through the origin, and the particle speed is $v_y = -\frac{\partial y}{\partial x}v = \frac{pv}{B}$. The maximum velocity is found from the maximum pressure, and $v_{y\max} = (40\text{ Pa})(344\text{ m/s})/(1.42\times10^5\text{ Pa}) = 9.69$ cm/s. The maximum acceleration is the maximum pressure gradient divided by the density,

$$a_{\max} = \frac{(80.0\text{ Pa})/(0.100\text{ m})}{(1.20\text{ kg/m}^3)} = 6.67\times10^2\text{ m/s}^2.$$

(e) The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign). The acceleration as a function of time is a square wave with amplitude $667\ \text{m/s}^2$ and frequency $f = v/\lambda = (344\ \text{m/s})/(0.200\ \text{m}) = 1.72\ \text{kHz}$.

EVALUATE: We can verify that $p(x,t)$ versus x has a shape proportional to the slope of the graph of $y(x,t)$ versus x. The same is also true of the graphs versus t.

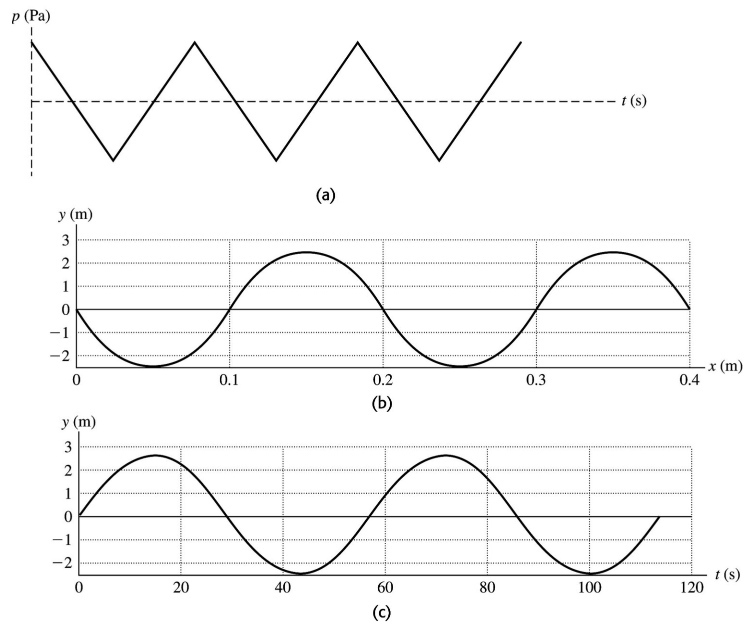


Figure 16.75

16.76. **IDENTIFY and SET UP:** Consider the derivation of the speed of a longitudinal wave in Section 16.2.

EXECUTE: **(a)** The quantity of interest is the change in force per fractional length change. The force constant k' is the change in force per length change, so the force change per fractional length change is $k'L$, the applied force at one end is $F = (k'L)(v_y/v)$ and the longitudinal impulse when this force is applied for a time *t is* $k'Ltv_y/v$. The change in longitudinal momentum is $((vt)m/L)v_y$ and equating the expressions, canceling a factor of *t* and solving for *v* gives $v^2 = L^2k'/m$.

(b) $v = (2.00\ \text{m})\sqrt{(1.50\ \text{N/m})/(0.250\ \text{kg})} = 4.90\ \text{m/s}$

EVALUATE: *A* larger k' corresponds to a stiffer spring and for a stiffer spring the wave speed is greater.

16.77. **IDENTIFY and SET UP:** The time between pulses is limited by the time for the wave to travel from the transducer to the structure and then back again. Use $x = v_x t$ and $f = 1/T$.

EXECUTE: **(a)** The wave travels 10 cm in and 10 cm out, so $t = x/v_x = (0.20\ \text{m})/(1540\ \text{m/s}) = 0.13 \times 10^{-3}\ \text{s} = 0.13\ \text{ms}$. The period can be no shorter than this, so the highest pulse frequency is $f = 1/t = 1/(0.13\ \text{ms}) = 7700\ \text{Hz}$, which is choice (b).

EVALUATE: The pulse frequency is not the same thing as the frequency of the ultrasound waves, which is around 1.0 MHz.

16.78. **IDENTIFY and SET UP:** Call *S* the intensity level of the beam. The beam attenuates by 100 dB per meter, so in 10 cm (0.10 m) it would attenuate by 1/10 of this amount. Therefore $\Delta S = -10$ dB. $S = 10\ \text{dB}\log(I/I_0)$.

EXECUTE: **(a)** $\Delta S = S_2 - S_1 = 10 \text{ dB} \log(I_2/I_0) - 10 \text{ dB} \log(I_1/I_0) = 10 \text{ dB} \log(I_2/I_0)$ since $I_1 = I_0$. Therefore $-10 \text{ dB} = 10 \text{ dB} \log(I_2/I_0)$, which gives $I_2/I_0 = 10^{-1}$, so $I_2 = 1/10\ I_0$, which is choice (d).
EVALUATE: In the next 10 cm, the beam would attenuate by another factor of 1/10, so it would be 1/100 of the initial intensity.

16.79. **IDENTIFY** and **SET UP:** The beam goes through 5.0 cm of tissue and 2.0 cm of bone. Use $d = vt$ to calculate the total time in this case and compare it with the time to travel 7.0 cm through only tissue.
EXECUTE: $d = vt$ gives $t = x/v$. Calculate the time to go through 2.0 cm of bone and 5.0 cm of tissue and then get the total time t_{tot}. $t_T = x_T/v_T$ and $t_B = x_B/v_B$, so $t_{tot} = x_T/v_T + x_B/v_B$. Putting in the numbers gives
$t_{tot} = (0.050 \text{ m})/(1540 \text{ m/s}) + (0.020 \text{ m})/(3080 \text{ m/s}) = 3.896 \times 10^{-5}$ s. If the wave went through only tissue during this time, it would have traveled $x = v_T t_{tot} = (1540 \text{ m/s})(3.896 \times 10^{-5} \text{ s}) = 6.0 \times 10^{-2} \text{ m} =$ 6.0 cm. So the beam traveled 7.0 cm, but you think it traveled 6.0 cm, so the structure is actually 1.0 cm deeper than you think, which makes choice (a) the correct one.
EVALUATE: A difference of 1.0 cm when a structure is 7.0 below the surface can be very significant.

16.80. **IDENTIFY** and **SET UP:** In a standing wave pattern, the distance between antinodes is one-half the wavelength of the waves. Use $v = f\lambda$ to find the wavelength.

EXECUTE: $\lambda = v/f = (1540 \text{ m/s})/(1.00 \text{ MHz}) = 1.54 \times 10^{-3} \text{ m} = 1.54$ mm. The distance D between antinodes is $D = \lambda/2 = (1.54 \text{ mm})/2 = 0.77 \text{ mm} \approx 0.75$ mm, which is choice (b).

EVALUATE: Decreasing the frequency could reduce the distance between antinodes if this is desired.

16.81. **IDENTIFY** and **SET UP:** The antinode spacing is $\lambda/2$. Use $v = f\lambda$.

EXECUTE: **(a)** The antinode spacing d is $\lambda/2$. Using $v = f\lambda$, we have $d = \lambda/2 = v/2f$. For the numbers in this problem, we have $d = (1540 \text{ m/s})/[2(1.0 \text{ kHz})] = 0.77 \text{ m} = 77$ cm. The cranium is much smaller than 77 cm, so there will be no standing waves within it at 1.0 kHz, which is choice (b).
EVALUATE: Using 1.0 MHz waves, the distance between antinodes would be 1000 times smaller, or 0.077 cm, so there could certainly be standing waves within the cranium.