

ELECTRIC POTENTIAL

VP23.2.1. IDENTIFY: This problem involves the energy of a system consisting of an electron and a lead nucleus.

SET UP: The electric potential energy (relative to infinity) of two point charges is $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

EXECUTE: (a) We want the work to move the electron. $W = U_2 - U_1$. Use $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$:

$$W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_2} - \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_1} = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_2} - \frac{1}{r_1} \right) = \frac{(-e)(82e)}{4\pi\epsilon_0} \left(\frac{1}{5.00 \times 10^{-10} \text{ m}} - \frac{1}{1.00 \times 10^{-10} \text{ m}} \right).$$

$$W = 1.51 \times 10^{-16} \text{ J}.$$

(b) We want kinetic energy. Use energy conservation with $K_1 = 0$: $U_1 = U_2 + K_2$. Solving for K_2 gives

$$K_2 = U_1 - U_2 = \frac{q_1 q_2}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{(-e)(82e)}{4\pi\epsilon_0} \left(\frac{1}{1.00 \times 10^{-10} \text{ m}} - \frac{1}{8.00 \times 10^{-12} \text{ m}} \right). \quad K_2 = 2.17 \times 10^{-15} \text{ J}.$$

EVALUATE: As the electron gets closer to the nucleus, its potential energy gets more and more negative, so it is losing potential energy and gaining kinetic energy.

VP23.2.2. IDENTIFY: We fire a proton at a nucleus and make use of electric potential energy of point charges.

SET UP: The potential energy is $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

EXECUTE: (a) We want the potential energy. Energy conservation gives $K_1 = U_2$, where 1 is when the proton is very far from the nucleus and 2 is when it is at its closest point and has stopped moving.

$$U_2 = K_1 = \frac{1}{2} m_p v^2 = \frac{1}{2} (1.67 \times 10^{-27} \text{ kg}) (2.50 \times 10^6 \text{ m/s})^2 = 5.22 \times 10^{-15} \text{ J}.$$

(b) Call Q the charge of the nucleus, which is our target variable. $U_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_2} = \frac{1}{4\pi\epsilon_0} \frac{eQ}{r}$. Solve

$$\text{for } Q: Q = \frac{U_2 r 4\pi\epsilon_0}{e} = \frac{(5.22 \times 10^{-15} \text{ J})(5.29 \times 10^{-13} \text{ m})(4\pi\epsilon_0)}{1.60 \times 10^{-19} \text{ C}} = 1.92 \times 10^{-18} \text{ C}.$$

EVALUATE: $Q/e = 12$, so this nucleus has 12 protons, which means it must be magnesium (from Appendix D).

VP23.2.3. IDENTIFY: This problem involves the potential energy of a system of point charges.

SET UP: Fig. VP23.2.3 shows the three charges in their final arrangement. $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

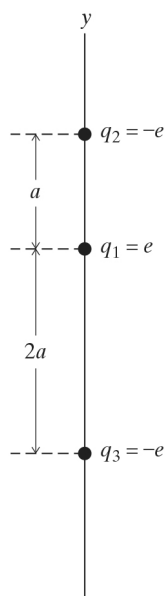


Figure VP23.2.3

EXECUTE: (a) The work needed to bring in the charge q_3 is equal to its final potential energy due to the q_1 and q_2 .

$$W = U_{1,3} + U_{2,3} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}} = \frac{-e}{4\pi\epsilon_0} \left(\frac{e}{2a} + \frac{-e}{3a} \right) = -\frac{e^2}{24\pi\epsilon_0 a}.$$

$$(b) U_{\text{tot}} = U_{1,2} + U_{1,3} + U_{2,3} = \frac{1}{4\pi\epsilon_0} \left(\frac{-e^2}{a} + \frac{-e^2}{3a} + \frac{e^2}{3a} \right) = -\frac{7e^2}{24\pi\epsilon_0 a}.$$

EVALUATE: The total potential energy is negative since q_1 attracts q_2 and q_3 . Only q_2 and q_3 repel each other.

VP23.2.4. IDENTIFY: This problem involves the potential energy of a system of point charges.

SET UP: Fig. VP22.2.4 shows the three charges with q_3 in place. $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

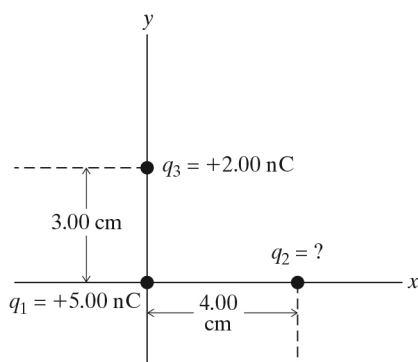


Figure VP23.2.4

EXECUTE: (a) We want q_2 . The work you do is the potential energy of the q_1 - q_2 system.

$$W = U_{1,2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}. \text{ Solve for } q_2: q_2 = \frac{4\pi\epsilon_0 W r_{12}}{q_1} = \frac{4\pi\epsilon_0 (8.10 \times 10^{-6} \text{ J})(0.0400 \text{ m})}{5.00 \text{ nC}} = 7.21 \text{ nC}.$$

(b) The *additional* work is the added potential energy of the system, which is the potential energy of q_3 due to q_1 and q_2 . $W = U_{1,3} + U_{2,3} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}} = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$. Using the result from

part (a) and the given numbers, we get $W = 5.59 \times 10^{-6}$ J.

EVALUATE: Even though two charges are already present, it takes less work to bring in q_3 than it did to bring in q_2 because $q_3 = (2/7.21)q_2 = 0.277q_2$.

VP23.7.1. IDENTIFY: We need to relate the potential to the electric field.

SET UP: Use $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$, $V(0,0,0) = V_0$, $\vec{E} = E_x \hat{i}$, where $E_x = 5.00 \times 10^2$ V/m. The target variable is the potential difference.

EXECUTE: (a) $V_0 - V_P = \int_0^P \vec{E} \cdot d\vec{l} = \int_0^{5.00 \text{ cm}} E_x dx = (5.00 \times 10^2 \text{ V/m})(0.0500 \text{ m}) = +25.0 \text{ V}$.

(b) Since $E_y = E_z = 0$, we have $V_0 - V_P = \int_0^{3.00 \text{ cm}} E_x dx = (5.00 \times 10^2 \text{ V/m})(0.0300 \text{ m}) = +15.0 \text{ V}$.

(c) Since $E_y = E_z = 0$ and $\Delta x = 0$, we have $V_0 - V_P = 0$.

(d) $V_0 - V_P = \int_0^P \vec{E} \cdot d\vec{l} = \int_0^{-5.00 \text{ cm}} E_x dx = (5.00 \times 10^2 \text{ V/m})(-0.0500 \text{ m}) = -25.0 \text{ V}$.

EVALUATE: In parts (a) and (b), $V_P < V_0$, in (c) $V_P = V_0$, and in (d) $V_P > V_0$.

VP23.7.2. IDENTIFY: This problem involves the electric potential and electric field of point charges.

SET UP: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ and $E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$. We want the potential. First sketch the arrangement of charges as in Fig. 23.7.2.

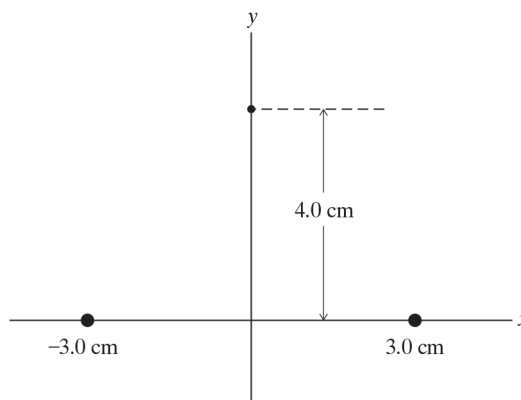


Figure VP23.7.2

EXECUTE: (a) The charges are equal and $r = 3.0 \text{ cm} = 0.030 \text{ m}$ for each of them. So

$$V = V_1 + V_2 = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{r} = \frac{2}{4\pi\epsilon_0} \frac{6.0 \text{ nC}}{0.030 \text{ m}} = 3600 \text{ V} = 3.6 \text{ kV}.$$

(b) Use the same procedure as in part (a) with $r = 5.0 \text{ cm} = 0.050 \text{ m}$, giving $V = 2200 \text{ V} = 2.2 \text{ kV}$.

(c) The point we are interested in lies on the x-axis at $x = 8.0 \text{ cm}$. So $r_1 = 11.0 \text{ cm}$ for the left-hand charge and $r_2 = 5.0 \text{ cm}$ for the right-hand charge. The potential at this point is

$$V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = \frac{(6.0 \text{ nC})}{4\pi\epsilon_0} \left(\frac{1}{0.0500 \text{ m}} + \frac{1}{0.110 \text{ m}} \right) = 160 \text{ V}.$$

(d) The net field can only be zero if the fields due to the two charges have equal magnitudes and are in opposite directions. These conditions are met only at the point (0,0,0), which is the point in part (a).

EVALUATE: Note an important point: In part (d) we saw that if $E = 0$ at a point, it does *not* necessarily follow that $V = 0$ at that point.

VP23.7.3. IDENTIFY: This problem involves the potential due to point charges.

SET UP: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$. Fig. VP23.7.3 shows the charges and the given information.

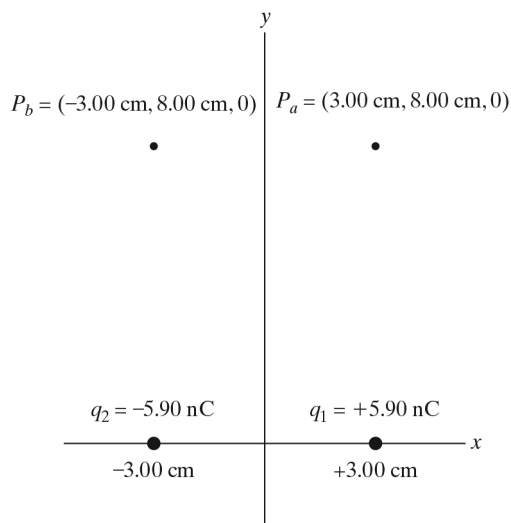


Figure VP23.7.3

EXECUTE: (a) We want V_a . $r_2 = \sqrt{(6.00 \text{ cm})^2 + (8.00 \text{ cm})^2} = 10.0 \text{ cm} = 0.100 \text{ m}$ and $r_1 = 8.00 \text{ cm} =$

$$0.0800 \text{ m}. \quad V_a = V_1 + V_2 = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{5.90 \text{ nC}}{0.0800 \text{ m}} + \frac{-5.90 \text{ nC}}{0.100 \text{ m}} \right) = +133 \text{ V}.$$

$$\text{(b)} \quad V_b = V_1 + V_2 = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{5.90 \text{ nC}}{0.100 \text{ m}} + \frac{-5.90 \text{ nC}}{0.0800 \text{ m}} \right) = -133 \text{ V}.$$

(c) We want the proton's speed at a . The proton starts at b and goes to a , so it is going from a low to a high potential. Therefore it is losing kinetic energy in the process. So $K_a = K_b - q(V_a - V_b)$.

$$\frac{1}{2}mv_a^2 = \frac{1}{2}mv_b^2 - e(V_a - V_b). \quad v_a = \sqrt{v_b^2 - 2e(V_a - V_b)/m}.$$

Using the given quantities and the results of parts (a) and (b), we get $v_a = 1.98 \times 10^5 \text{ m/s}$.

EVALUATE: A positive charge (like a proton) will be accelerated to higher speed by the electric field going from high to low potential (just as a falling object is sped up by the gravitational field as it goes from high potential energy to low potential energy).

VP23.7.4. IDENTIFY: We want to get the potential difference by integrating the electric field. We'll need calculus for this.

SET UP: Use $V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}$. Call V_0 the potential at the center of the sphere and V_R the potential at

the surface when $r = R$. $d\vec{l} = dr \hat{r}$ and $\vec{E} = \frac{Qr}{4\pi\epsilon_0 R^3} \hat{r}$ inside the sphere.

EXECUTE: (a) Using the above quantities gives

$$V_R - V_0 = -\int_0^R \frac{Qr}{4\pi\epsilon_0 R^3} dr = -\left(\frac{Q}{4\pi\epsilon_0 R^3} \right) \frac{r^2}{2} \Big|_0^R = -\frac{Q}{8\pi\epsilon_0 R}.$$

(b) From part (a), we have $V_R = V_0 - \frac{Q}{8\pi\epsilon_0 R}$. This tells us that $V_R < V_0$, so the center is at a

higher potential than the surface. Another way to see this is to note that the electric field does work on a positive charge as it goes from the center to the surface because the field points outward. Thus the center is at a higher potential than the surface.

EVALUATE: Notice that $E = 0$ at the center of the sphere, but the potential is highest there and not zero.

VP23.12.1. IDENTIFY: We want to find the work using electric potential.

SET UP: $W = q\Delta V$ and for a point charge $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$.

EXECUTE: (a) From $r = 5R$ to $r = 3R$: $W = q_0(V_{3R} - V_{5R}) = q_0 \left[\frac{q}{4\pi\epsilon_0} \left(\frac{1}{3R} - \frac{1}{5R} \right) \right] = \frac{qq_0}{30\pi\epsilon_0 R}$.

(b) From $r = 2R$ to $r = 7R$: $W = q_0(V_{7R} - V_{2R}) = \frac{q_0q}{4\pi\epsilon_0} \left(\frac{1}{3R} - \frac{1}{5R} \right) = -\frac{5qq_0}{56\pi\epsilon_0 R}$.

(c) From $r = 4R$ to $r = R/2$: Inside the conducting sphere $E = 0$ so no work is done on q_0 . So the work required is the work to go from $r = 4R$ to $r = R$, which is

$$W = q_0(V_{4R} - V_R) = \frac{q_0q}{4\pi\epsilon_0} \left(\frac{1}{4R} - \frac{1}{R} \right) = \frac{3qq_0}{16\pi\epsilon_0 R}.$$

EVALUATE: In parts (a) and (c) the work we do is positive because we must push on q_0 against the electric field. In (b) the work is negative because we must hold back on q_0 since the electric field pulls it in.

VP23.12.2. IDENTIFY: This problem involves two oppositely charged parallel plates.

SET UP: The electric field between the plates is uniform everywhere.

EXECUTE: (a) We want the potential at 3.0 mm above the lower plate. The electric field is uniform, so $V_{ab} = Ed$. At a distance x above the lower plate, the potential difference over that distance is Ex . Since E

is uniform $\frac{V_x}{V_{ab}} = \frac{Ex}{Ed} = \frac{x}{d} = \frac{3.00 \text{ mm}}{4.50 \text{ mm}} = \frac{2}{3}$, so $V_x = \frac{2}{3}V_{ab} = \frac{2}{3}(+24.0 \text{ V}) = +16.0 \text{ V}$.

(b) $U = qV$, where V is relative to b . So $U = (2.00 \text{ nC})(16.0 \text{ V}) = 32 \text{ nJ} = 3.20 \times 10^{-8} \text{ J}$.

(c) Since it is positive, the particle will move in the direction of the electric field, which is toward the lower plate. Using $U_1 = K_2$ gives $\frac{1}{2}mv^2 = U$, so $v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(3.20 \times 10^{-8} \text{ J})}{5.00 \times 10^{-9} \text{ kg}}} = 3.58 \text{ m/s}$.

EVALUATE: Note in part (c) that the positive particle moved from higher to lower potential.

VP23.12.3. IDENTIFY: We are dealing with the potential of a uniformly charged ring.

SET UP: We know that $U = qV$. Example 23.11 showed that for a ring $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$. We want

the potential energy of the point charge, its kinetic energy, and its speed.

EXECUTE: (a) We want the potential energy of the charge at 8.00 cm from the center of the ring.

$$U_8 = qV = \frac{q}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + a^2}} = \frac{(3.00 \text{ nC})}{4\pi\epsilon_0} \frac{5.00 \text{ nC}}{\sqrt{(0.0800 \text{ m})^2 + (0.0250 \text{ m})^2}} = 1.61 \times 10^{-6} \text{ J}.$$

(b) At the center of the ring, $x = 0$. Using the above equations gives $U_0 = 5.39 \times 10^{-6} \text{ J}$.

(c) Energy conservation: $K_8 + U_8 = K_0 + U_0 \rightarrow K_0 = K_8 + U_8 - U_0$. From this we get

$$K_0 = \frac{1}{2}mv_8^2 + U_8 - U_0. \text{ Using the results from parts (a) and (b) as well as the given quantities, we get } K_0 = 3.42 \times 10^{-6} \text{ J. Solving } K_0 = \frac{1}{2}mv_8^2 \text{ for } v_8 \text{ gives } v_8 = \sqrt{\frac{2K_0}{m}}. \text{ Using } K_0 \text{ and the given quantities gives } v_8 = 41.3 \text{ m/s.}$$

EVALUATE: The point charge loses kinetic energy (and hence speed) because the electric field of the ring does negative work on it as it approaches the center of the ring.

VP23.12.4. IDENTIFY: We want to calculate the potential due to a uniformly charged rod.

SET UP: Break the rod into infinitesimal segments of length dx carrying charge dq . Applying

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \text{ to a typical segment, the potential at the origin is } dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{x}. \text{ Fig. VP23.12.4 shows the arrangement.}$$

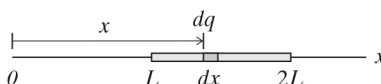


Figure VP23.12.4

EXECUTE: (a) $dq = \lambda dx = (Q/L)dx$.

(b) Using Fig. VP23.12.4 and $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{x}$, and integrating, we have

$$V = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{dq}{x} = \int_0^L \frac{Q/L}{4\pi\epsilon_0} \frac{dx}{x} = \frac{Q}{4\pi\epsilon_0 L} \ln x \Big|_L^{2L} = \frac{Q}{4\pi\epsilon_0 L} \ln 2.$$

EVALUATE: We cannot treat the rod as being equivalent to a point charge at its center. If that were the case, the potential due to the rod at the origin would be $\frac{Q}{4\pi\epsilon_0 (3L/2)} = \frac{Q}{6\pi\epsilon_0 L}$, which is *not* what we calculated.

23.1. IDENTIFY: Apply $W_{a \rightarrow b} = U_a - U_b$ to calculate the work. The electric potential energy of a pair of point charges is given by $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

SET UP: Let the initial position of q_2 be point a and the final position be point b , as shown in Figure 23.1.

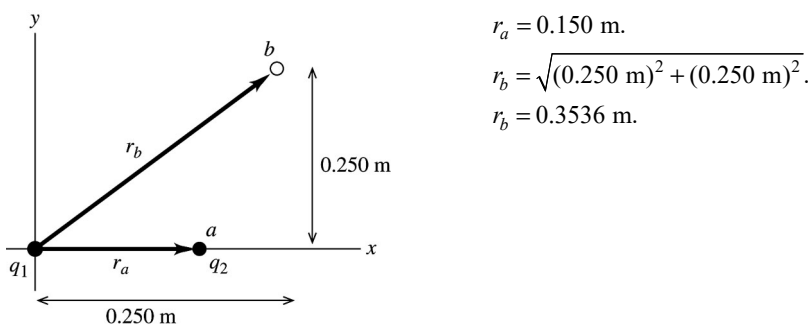


Figure 23.1

EXECUTE: $W_{a \rightarrow b} = U_a - U_b.$

$$U_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_a} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(+2.40 \times 10^{-6} \text{ C})(-4.30 \times 10^{-6} \text{ C})}{0.150 \text{ m}}.$$

$$U_a = -0.6184 \text{ J}.$$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_b} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(+2.40 \times 10^{-6} \text{ C})(-4.30 \times 10^{-6} \text{ C})}{0.3536 \text{ m}}.$$

$$U_b = -0.2623 \text{ J}.$$

$$W_{a \rightarrow b} = U_a - U_b = -0.6184 \text{ J} - (-0.2623 \text{ J}) = -0.356 \text{ J}.$$

EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger r .

23.2. IDENTIFY: Apply $W_{a \rightarrow b} = U_a - U_b.$

SET UP: $U_a = +5.4 \times 10^{-8} \text{ J}.$ Solve for $U_b.$

EXECUTE:

$$W_{a \rightarrow b} = -1.9 \times 10^{-8} \text{ J} = U_a - U_b. \quad U_b = U_a - W_{a \rightarrow b} = +5.4 \times 10^{-8} \text{ J} - (-1.9 \times 10^{-8} \text{ J}) = 7.3 \times 10^{-8} \text{ J}.$$

EVALUATE: When the electric force does negative work the electrical potential energy increases.

23.3. IDENTIFY: The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is e and the charges are equidistant from each

other, so the total potential energy is $U = \frac{1}{4\pi\epsilon_0} \left(\frac{e^2}{r} + \frac{e^2}{r} + \frac{e^2}{r} \right) = \frac{3e^2}{4\pi\epsilon_0 r}.$

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60 \times 10^{-19} \text{ C})^2 (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}{2.00 \times 10^{-15} \text{ m}} = 3.46 \times 10^{-13} \text{ J} = 2.16 \text{ MeV}.$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.4. IDENTIFY: The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity,

$$U = (1/4\pi\epsilon_0)(qq_0/r), \text{ we have } W = \Delta U = U_2 - U_1 = \frac{1}{4\pi\epsilon_0} \left(\frac{e^2}{r_2} - \frac{e^2}{r_1} \right).$$

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2 \left(\frac{1}{3.00 \times 10^{-15} \text{ m}} - \frac{1}{2.00 \times 10^{-10} \text{ m}} \right) = 7.68 \times 10^{-14} \text{ J}$$

(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\frac{1}{2}mv^2\right) = mv^2$.

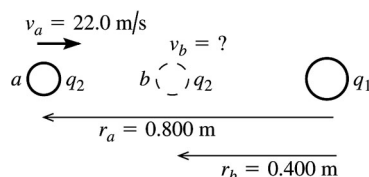
EXECUTE: Solving for v gives $v = \sqrt{\frac{\Delta U}{m}} = \sqrt{\frac{7.68 \times 10^{-14} \text{ J}}{1.67 \times 10^{-27} \text{ kg}}} = 6.78 \times 10^6 \text{ m/s}.$

EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.5. (a) IDENTIFY: Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$. U for the pair of point charges

is given by $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

SET UP:



Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1 , as shown in Figure 23.5a.

Figure 23.5a

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(1.50 \times 10^{-3} \text{ kg})(22.0 \text{ m/s})^2 = 0.3630 \text{ J}.$$

$$U_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_a} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{0.800 \text{ m}} = +0.2454 \text{ J}.$$

$$K_b = \frac{1}{2}mv_b^2.$$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_b} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{0.400 \text{ m}} = +0.4907 \text{ J}.$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$.

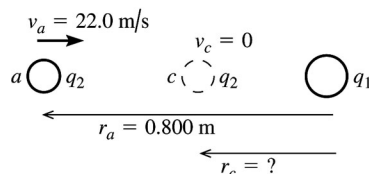
$$\frac{1}{2}mv_b^2 = +0.3630 \text{ J} + (0.2454 \text{ J} - 0.4907 \text{ J}) = 0.1177 \text{ J}.$$

$$v_b = \sqrt{\frac{2(0.1177 \text{ J})}{1.50 \times 10^{-3} \text{ kg}}} = 12.5 \text{ m/s}.$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c : $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.



EXECUTE: $K_a = +0.3630 \text{ J}$ (from part (a)).

$U_a = +0.2454 \text{ J}$ (from part (a)).

Figure 23.5b

$K_c = 0$ (at distance of closest approach the speed is zero).

$$U_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_c}.$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_c} = +0.3630 \text{ J} + 0.2454 \text{ J} = 0.6084 \text{ J}$.

$$r_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{0.6084 \text{ J}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{+0.6084 \text{ J}} = 0.323 \text{ m}.$$

EVALUATE: $U \rightarrow \infty$ as $r \rightarrow 0$ so q_2 will stop no matter what its initial speed is.

23.6. IDENTIFY: The total potential energy is the scalar sum of the individual potential energies of each pair of charges.

SET UP: For a pair of point charges the electrical potential energy is $U = k \frac{qq'}{r}$. In the O-H-N

combination the O^- is 0.170 nm from the H^+ and 0.280 nm from the N^- . In the N-H-N combination the N^- is 0.190 nm from the H^+ and 0.300 nm from the other N^- . U is positive for like charges and negative for unlike charges.

EXECUTE: (a) O-H-N:

$$\text{O}^- - \text{H}^+ : U = -(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{0.170 \times 10^{-9} \text{ m}} = -1.35 \times 10^{-18} \text{ J}.$$

$$\text{O}^- - \text{N}^- : U = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{0.280 \times 10^{-9} \text{ m}} = +8.22 \times 10^{-19} \text{ J}.$$

N-H-N:

$$\text{N}^- - \text{H}^+ : U = -(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{0.190 \times 10^{-9} \text{ m}} = -1.21 \times 10^{-18} \text{ J}.$$

$$\text{N}^- - \text{N}^- : U = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{0.300 \times 10^{-9} \text{ m}} = +7.67 \times 10^{-19} \text{ J}.$$

The total potential energy is

$$U_{\text{tot}} = -1.35 \times 10^{-18} \text{ J} + 8.22 \times 10^{-19} \text{ J} - 1.21 \times 10^{-18} \text{ J} + 7.67 \times 10^{-19} \text{ J} = -9.71 \times 10^{-19} \text{ J}.$$

(b) In the hydrogen atom the electron is 0.0529 nm from the proton.

$$U = -(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{0.0529 \times 10^{-9} \text{ m}} = -4.35 \times 10^{-18} \text{ J}.$$

EVALUATE: The magnitude of the potential energy in the hydrogen atom is about a factor of 4 larger than what it is for the adenine-thymine bond.

23.7. IDENTIFY: Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b .

The maximum force is at the distance of closest approach, $F = k \frac{|q_1 q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass $1.67 \times 10^{-27} \text{ kg}$ and charge $q = +e = +1.60 \times 10^{-19} \text{ C}$.

EXECUTE: $K_a = U_b$. $2(\frac{1}{2}mv_a^2) = k \frac{q_1 q_2}{r_b}$. $mv_a^2 = k \frac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^5 \text{ m/s})^2} = 3.45 \times 10^{-12} \text{ m}.$$

$$F = k \frac{e^2}{r_b^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{(3.45 \times 10^{-12} \text{ m})^2} = 1.94 \times 10^{-5} \text{ N}.$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

23.8. IDENTIFY: Call the three charges 1, 2, and 3. $U = U_{12} + U_{13} + U_{23}$.

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.400 m.

$$\text{EXECUTE: } U = \frac{3kq^2}{0.400 \text{ m}} = \frac{3k(1.2 \times 10^{-6} \text{ C})^2}{0.400 \text{ m}} = 0.0971 \text{ J}.$$

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.9. IDENTIFY: The protons repel each other and therefore accelerate away from one another. As they get farther and farther away, their kinetic energy gets greater and greater but their acceleration keeps decreasing. Conservation of energy and Newton's laws apply to these protons.

SET UP: Let a be the point when they are 0.750 nm apart and b be the point when they are very far apart. A proton has charge $+e$ and mass $1.67 \times 10^{-27} \text{ kg}$. As they move apart the protons have equal kinetic energies and speeds. Their potential energy is $U = ke^2/r$ and $K = \frac{1}{2}mv^2$. $K_a + U_a = K_b + U_b$.

EXECUTE: (a) They have maximum speed when they are far apart and all their initial electrical potential energy has been converted to kinetic energy. $K_a + U_a = K_b + U_b$.

$K_a = 0$ and $U_b = 0$, so

$$K_b = U_a = k \frac{e^2}{r_a} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(1.60 \times 10^{-19} \text{ C})^2}{0.750 \times 10^{-9} \text{ m}} = 3.07 \times 10^{-19} \text{ J}.$$

$$K_b = \frac{1}{2}mv_b^2 + \frac{1}{2}mv_b^2, \text{ so } K_b = mv_b^2 \text{ and } v_b = \sqrt{\frac{K_b}{m}} = \sqrt{\frac{3.07 \times 10^{-19} \text{ J}}{1.67 \times 10^{-27} \text{ kg}}} = 1.36 \times 10^4 \text{ m/s}.$$

(b) Their acceleration is largest when the force between them is largest and this occurs at $r = 0.750$ nm, when they are closest.

$$F = k \frac{e^2}{r^2} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{1.60 \times 10^{-19} \text{ C}}{0.750 \times 10^{-9} \text{ m}} \right)^2 = 4.09 \times 10^{-10} \text{ N}.$$

$$a = \frac{F}{m} = \frac{4.09 \times 10^{-10} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 2.45 \times 10^{17} \text{ m/s}^2.$$

EVALUATE: The acceleration of the protons decreases as they move farther apart, but the force between them is repulsive so they continue to increase their speeds and hence their kinetic energies.

23.10. IDENTIFY: The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a \rightarrow b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center} \rightarrow \text{side}} = U_{\text{center}} - U_{\text{side}}$ is the work done by the Coulomb force. There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00$ nm from the two nearest electrons and a distance $r_3 = \sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\epsilon_0)(qq_0/r)$, the total work done by the Coulomb force is

$$W_{\text{center} \rightarrow \text{side}} = U_{\text{center}} - U_{\text{side}} = 4 \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_e}{r_1} - \left(2 \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_e}{r_2} + 2 \frac{1}{4\pi\epsilon_0} \frac{q_\alpha q_e}{r_3} \right).$$

Substituting $q_e = -e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center} \rightarrow \text{side}} = -4e^2 \frac{1}{4\pi\epsilon_0} \left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3} \right) \right].$$

EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4(1.60 \times 10^{-19} \text{ C})^2 (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[\frac{2}{\sqrt{50} \text{ nm}} - \left(\frac{1}{5.00 \text{ nm}} + \frac{1}{\sqrt{125} \text{ nm}} \right) \right] = 6.08 \times 10^{-21} \text{ J}.$$

EVALUATE: Since the work done by the Coulomb force is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side. To move the alpha particle to the midpoint of a side and leave it there at rest an external force must do -6.08×10^{-21} J of work.

23.11. IDENTIFY: We want to find the potential difference by using the electric field.

SET UP and EXECUTE: Start with a sketch as shown in Fig. 23.11.

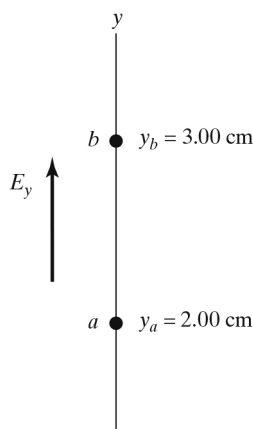


Figure 23.11

$$V_a - V_b = \int_a^b E_y dy = \int_a^b (\alpha + \beta / y^2) dy = (\alpha y - \beta / y) \Big|_{y_a}^{y_b}. \text{ Simplifying gives}$$

$$V_a - V_b = \alpha(y_b - y_a) + \beta \left(\frac{1}{y_a} - \frac{1}{y_b} \right). \text{ Using } y_a = 2.00 \text{ cm, } y_b = 3.00 \text{ cm, and the other values gives } V_a -$$

$V_b = 89.3 \text{ V}$. $V_a = V_b + 89.3 \text{ V}$, so V_a is higher than V_b .

EVALUATE: The electric field points from a to b , so a positive charge would gain kinetic energy as it was accelerated by the field from a to b , so the potential at a must be higher than at b , as we found.

- 23.12. IDENTIFY:** Work is done on the object by the electric field, and this changes its kinetic energy, so we can use the work-energy theorem.

SET UP: $W_{A \rightarrow B} = \Delta K$ and $W_{A \rightarrow B} = q(V_A - V_B)$.

EXECUTE: (a) Applying the two equations above gives $W_{A \rightarrow B} = q(V_A - V_B) = K_B - 0 = K_B$.

$$V_B = V_A - K_B/q = 30.0 \text{ V} - (3.00 \times 10^{-7} \text{ J})/(-6.00 \times 10^{-9} \text{ C}) = 80.0 \text{ V}.$$

(b) The negative charge accelerates from A to B , so the electric field must point from B toward A . Since the field is uniform, we have $E = \frac{\Delta V}{\Delta x} = (50.0 \text{ V})/(0.500 \text{ m}) = 100 \text{ V/m}$.

EVALUATE: A positive charge is accelerated from high to low potential, but a negative charge (as we have here) is accelerated from low to high potential.

- 23.13. IDENTIFY and SET UP:** Apply conservation of energy to points A and B .

EXECUTE: $K_A + U_A = K_B + U_B$.

$$U = qV, \text{ so } K_A + qV_A = K_B + qV_B.$$

$$K_B = K_A + q(V_A - V_B) = 0.00250 \text{ J} + (-5.00 \times 10^{-6} \text{ C})(200 \text{ V} - 800 \text{ V}) = 0.00550 \text{ J}.$$

$$v_B = \sqrt{2K_B/m} = 7.42 \text{ m/s}.$$

EVALUATE: It is faster at B ; a negative charge gains speed when it moves to higher potential.

- 23.14. IDENTIFY:** The work-energy theorem says $W_{a \rightarrow b} = K_b - K_a$. $\frac{W_{a \rightarrow b}}{q} = V_a - V_b$.

SET UP: Point a is the starting point and point b is the ending point. Since the field is uniform, $W_{a \rightarrow b} = Fs \cos \phi = E|q|s \cos \phi$. The field is to the left so the force on the positive charge is to the left.

The particle moves to the left so $\phi = 0^\circ$ and the work $W_{a \rightarrow b}$ is positive.

EXECUTE: (a) $W_{a \rightarrow b} = K_b - K_a = 2.20 \times 10^{-6} \text{ J} - 0 = 2.20 \times 10^{-6} \text{ J}$.

(b) $V_a - V_b = \frac{W_{a \rightarrow b}}{q} = \frac{2.20 \times 10^{-6} \text{ J}}{4.20 \times 10^{-9} \text{ C}} = 524 \text{ V}$. Point a is at higher potential than point b .

(c) $E|q|s = W_{a \rightarrow b}$, so $E = \frac{W_{a \rightarrow b}}{|q|s} = \frac{V_a - V_b}{s} = \frac{524 \text{ V}}{6.00 \times 10^{-2} \text{ m}} = 8.73 \times 10^3 \text{ V/m}$.

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.15. IDENTIFY: Apply $W_{a \rightarrow b} = q \int_a^b \vec{E} \cdot d\vec{l}$. Use coordinates where $+y$ is upward and $+x$ is to the right.

Then $\vec{E} = E\hat{j}$ with $E = 4.00 \times 10^4 \text{ N/C}$.

SET UP: (a) The path is sketched in Figure 23.15a.

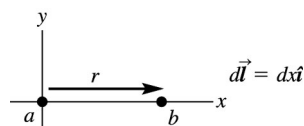


Figure 23.15a

EXECUTE: $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i}) = 0$ so $W_{a \rightarrow b} = q \int_a^b \vec{E} \cdot d\vec{l} = 0$.

EVALUATE: The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) **SET UP:** The path is sketched in Figure 23.15b.

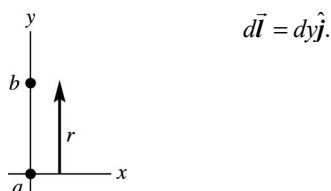


Figure 23.15b

EXECUTE: $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dy\hat{j}) = E dy$.

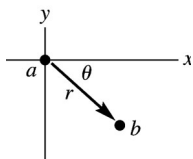
$$W_{a \rightarrow b} = q \int_a^b \vec{E} \cdot d\vec{l} = q' E \int_a^b dy = q' E (y_b - y_a).$$

$y_b - y_a = +0.670 \text{ m}$; it is positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a \rightarrow b} = q' E (y_b - y_a) = (+28.0 \times 10^{-9} \text{ C})(4.00 \times 10^4 \text{ N/C})(+0.670 \text{ m}) = +7.50 \times 10^{-4} \text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) **SET UP:** The path is sketched in Figure 23.15c.



$$y_a = 0.$$

$$y_b = -r \sin \theta = -(2.60 \text{ m}) \sin 45^\circ = -1.838 \text{ m}.$$

The vertical component of the 2.60 m displacement is 1.838 m downward.

Figure 23.15c

EXECUTE: $d\vec{l} = dx\hat{i} + dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = E dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q' E \int_a^b dy = q' E (y_b - y_a).$$

$$W_{a \rightarrow b} = q' E (y_b - y_a) = (+28.0 \times 10^{-9} \text{ C})(4.00 \times 10^4 \text{ N/C})(-1.838 \text{ m}) = -2.06 \times 10^{-3} \text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.16. IDENTIFY: Apply $K_a + U_a = K_b + U_b$.

SET UP: Let $q_1 = +3.00 \text{ nC}$ and $q_2 = +2.00 \text{ nC}$. At point a , $r_{1a} = r_{2a} = 0.250 \text{ m}$. At point b , $r_{1b} = 0.100 \text{ m}$ and $r_{2b} = 0.400 \text{ m}$. The electron has $q = -e$ and $m_e = 9.11 \times 10^{-31} \text{ kg}$. $K_a = 0$ since the electron is released from rest.

$$\text{EXECUTE: } -\frac{keq_1}{r_{1a}} - \frac{keq_2}{r_{2a}} = -\frac{keq_1}{r_{1b}} - \frac{keq_2}{r_{2b}} + \frac{1}{2}m_e v_b^2.$$

$$E_a = K_a + U_a = k(-1.60 \times 10^{-19} \text{ C}) \left(\frac{(3.00 \times 10^{-9} \text{ C})}{0.250 \text{ m}} + \frac{(2.00 \times 10^{-9} \text{ C})}{0.250 \text{ m}} \right) = -2.88 \times 10^{-17} \text{ J}.$$

$$E_b = K_b + U_b = k(-1.60 \times 10^{-19} \text{ C}) \left(\frac{(3.00 \times 10^{-9} \text{ C})}{0.100 \text{ m}} + \frac{(2.00 \times 10^{-9} \text{ C})}{0.400 \text{ m}} \right) + \frac{1}{2}m_e v_b^2 = -5.04 \times 10^{-17} \text{ J} + \frac{1}{2}m_e v_b^2.$$

$$\text{Setting } E_a = E_b \text{ gives } v_b = \sqrt{\frac{2}{9.11 \times 10^{-31} \text{ kg}} (5.04 \times 10^{-17} \text{ J} - 2.88 \times 10^{-17} \text{ J})} = 6.89 \times 10^6 \text{ m/s}.$$

EVALUATE: $V_a = V_{1a} + V_{2a} = 180 \text{ V}$. $V_b = V_{1b} + V_{2b} = 315 \text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

23.17. IDENTIFY: The potential at any point is the scalar sum of the potentials due to individual charges.

SET UP: $V = kq/r$ and $W_{ab} = q(V_a - V_b)$.

$$\text{EXECUTE: (a) } r_{a1} = r_{a2} = \frac{1}{2} \sqrt{(0.0300 \text{ m})^2 + (0.0300 \text{ m})^2} = 0.0212 \text{ m}. \quad V_a = k \left(\frac{q_1}{r_{a1}} + \frac{q_2}{r_{a2}} \right) = 0.$$

$$\text{(b) } r_{b1} = 0.0424 \text{ m}, \quad r_{b2} = 0.0300 \text{ m}.$$

$$V_b = k \left(\frac{q_1}{r_{b1}} + \frac{q_2}{r_{b2}} \right) = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{+2.00 \times 10^{-6} \text{ C}}{0.0424 \text{ m}} + \frac{-2.00 \times 10^{-6} \text{ C}}{0.0300 \text{ m}} \right) = -1.75 \times 10^5 \text{ V}.$$

$$\text{(c) } W_{ab} = q_3(V_a - V_b) = (-5.00 \times 10^{-6} \text{ C})[0 - (-1.75 \times 10^5 \text{ V})] = -0.875 \text{ J}.$$

EVALUATE: Since $V_b < V_a$, a positive charge would be pulled by the existing charges from a to b , so they would do positive work on this charge. But they would repel a negative charge and hence do negative work on it, as we found in part (c).

23.18. IDENTIFY: The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

SET UP: The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

EXECUTE: (a) (i) Since both charges have the same sign, there are no points for which the potential is zero.

(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but V is never zero.

(b) (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.

(ii) Between the two charges, the fields point in the same direction, so E cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence E is not zero anywhere.

EVALUATE: It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

23.19. IDENTIFY: Apply $V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$.

SET UP: The locations of the charges and points A and B are sketched in Figure 23.19.

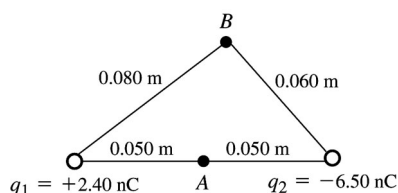


Figure 23.19

EXECUTE: (a) $V_A = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{A1}} + \frac{q_2}{r_{A2}} \right)$.

$$V_A = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{+2.40 \times 10^{-9} \text{ C}}{0.050 \text{ m}} + \frac{-6.50 \times 10^{-9} \text{ C}}{0.050 \text{ m}} \right) = -737 \text{ V}.$$

(b) $V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_{B1}} + \frac{q_2}{r_{B2}} \right)$.

$$V_B = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{+2.40 \times 10^{-9} \text{ C}}{0.080 \text{ m}} + \frac{-6.50 \times 10^{-9} \text{ C}}{0.060 \text{ m}} \right) = -704 \text{ V}.$$

(c) **IDENTIFY and SET UP:** Use $W_{a \rightarrow b} = q(V_a - V_b)$ and the results of parts (a) and (b) to calculate W .

EXECUTE: $W_{B \rightarrow A} = q(V_B - V_A) = (2.50 \times 10^{-9} \text{ C})[-704 \text{ V} - (-737 \text{ V})] = +8.2 \times 10^{-8} \text{ J}.$

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point B) to lower potential (point A).

23.20. IDENTIFY and SET UP: Apply conservation of energy: $K_a + U_a = K_b + U_b$. Use $V = U/q_0$ to express U in terms of V .

(a) EXECUTE: $K_1 + qV_1 = K_2 + qV_2$, $q(V_2 - V_1) = K_1 - K_2$; $q = -1.602 \times 10^{-19}$ C.

$$K_1 = \frac{1}{2} m_e v_1^2 = 4.099 \times 10^{-18} \text{ J}; \quad K_2 = \frac{1}{2} m_e v_2^2 = 2.915 \times 10^{-17} \text{ J}. \quad \Delta V = V_2 - V_1 = \frac{K_1 - K_2}{q} = 156 \text{ V}.$$

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1 = 2.915 \times 10^{-17}$ J, $K_2 = 0$. $V_2 - V_1 = \frac{K_1 - K_2}{q} = -182$ V.

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.21. IDENTIFY: We want to relate the electric field and potential.

SET UP and EXECUTE: (a) $V_B > V_A$. The electric field points from higher to lower potential (from B to A), so E_y is negative.

(b) We want $|E_y|$. $|E_y|(y_B - y_A) = 12.0$ V, so $|E_y| = (12.0 \text{ V})/(0.0700 \text{ m}) = 171$ V/m.

(c) $V_C - V_B = E_y(y_C - y_B) = (-171 \text{ V/m})(0.100 \text{ m}) = -17.1$ V.

EVALUATE: The $V_C = V_B - 17.1$ V/m, so $V_B > V_C$.

23.22. IDENTIFY: For a point charge, $E = \frac{k|q|}{r^2}$ and $V = \frac{kq}{r}$.

SET UP: The electric field is directed toward a negative charge and away from a positive charge.

EXECUTE: (a) $V > 0$ so $q > 0$. $\frac{V}{E} = \frac{kq/r}{k|q|/r^2} = \left(\frac{kq}{r}\right)\left(\frac{r^2}{kq}\right) = r$. $r = \frac{4.98 \text{ V}}{16.2 \text{ V/m}} = 0.307$ m.

(b) $q = \frac{rV}{k} = \frac{(0.307 \text{ m})(4.98 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.70 \times 10^{-10}$ C.

(c) $q > 0$, so the electric field is directed away from the charge.

EVALUATE: The ratio of V to E due to a point charge increases as the distance r from the charge increases, because E falls off as $1/r^2$ and V falls off as $1/r$.

23.23. (a) IDENTIFY and EXECUTE: The direction of \vec{E} is always from high potential to low potential so point b is at higher potential.

(b) IDENTIFY and SET UP: Apply $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$ to relate $V_b - V_a$ to E .

EXECUTE: $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E dx = E(x_b - x_a)$.

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240 \text{ V}}{0.90 \text{ m} - 0.60 \text{ m}} = 800 \text{ V/m}$$

(c) SET UP and EXECUTE: $W_{b \rightarrow a} = q(V_b - V_a) = (-0.200 \times 10^{-6} \text{ C})(+240 \text{ V}) = -4.80 \times 10^{-5} \text{ J}$.

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point b) to low potential (point a).

23.24. IDENTIFY: This problem deals with the potential and energy of point charges.

SET UP and EXECUTE: (a) We want $V_A - V_B$. The negative sphere loses kinetic energy as it goes from A to B , so it is gaining potential energy. Thus it has higher potential energy at B than at A . A negative charge is accelerated from low to high potential by the field, so since $U_B > U_A$, it follows that $V_B < V_A$.

Thus A has higher potential. Energy conservation gives $U_A + K_A = U_B + K_B$, so

$U_A - U_B = K_B - K_A = q(V_A - V_B)$. Using the given numbers gives $V_A - V_B = +100$ V.

(b) Since $V_A > V_B$, the electric field points from A to B , which is in the $+x$ direction. Thus $E_x \Delta x = \Delta V$.

$$\text{So } E_x = \frac{\Delta V}{\Delta x} = \frac{100 \text{ V}}{0.400 \text{ m}} = +250 \text{ V/m.}$$

EVALUATE: Careful! Positive charges are accelerated from high to low potential, but negative charges are accelerated from low to high potential.

23.25. IDENTIFY: This problem involves the potential energy of point charges.

SET UP and EXECUTE: (a) We want the work. $W = -\Delta U = -Q\Delta V = -Q(V_0 - V_\infty) = -QV_0$. Using

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \text{ with } q_1 = q_2 = q \text{ and } r_1 = r_2 = r \text{ gives } W = \frac{-Qq}{4\pi\epsilon_0} \left(\frac{1}{r} + \frac{1}{r} \right). \text{ Using } r = 0.400 \text{ m and the}$$

given values gives $W = 0.0539 \text{ J}$.

(b) We want the speed at the origin. $W = K = \frac{1}{2}mv^2$ gives $v = \sqrt{2W/m}$. Using W from part (a) and the given values gives $v = 3.00 \text{ m/s}$.

EVALUATE: At the origin the force on the sphere is zero, but it does not stop. In fact, its kinetic energy would be converted back to electric potential energy and it would move out to infinity.

23.26. IDENTIFY: This problem involves the potential of a small charged sphere.

SET UP and EXECUTE: We want the charge on the sphere. We need to relate $V_A - V_B$ to $1/r$ to interpret

the graph. $V_A - V_B = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_A} - \frac{q}{r} \right)$. The slope should be $-kq$ and the y -intercept $\frac{q}{4\pi\epsilon_0 r_A}$.

$$q = -4\pi\epsilon_0 (\text{slope}) = -4\pi\epsilon_0 (-18.0 \text{ V} \cdot \text{m}) = +2.00 \text{ nC}.$$

EVALUATE: Since the y -intercept is $\frac{q}{4\pi\epsilon_0 r_A}$, we could use it as a check if we knew its value.

23.27. IDENTIFY: The potential at any point is the scalar sum of the potential due to each shell.

SET UP: $V = \frac{kq}{R}$ for $r \leq R$ and $V = \frac{kq}{r}$ for $r > R$.

EXECUTE: (a) (i) $r = 0$. This point is inside both shells so

$$V = k \left(\frac{q_1}{R_1} + \frac{q_2}{R_2} \right) = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{6.00 \times 10^{-9} \text{ C}}{0.0300 \text{ m}} + \frac{-9.00 \times 10^{-9} \text{ C}}{0.0500 \text{ m}} \right).$$

$$V = +1.798 \times 10^3 \text{ V} + (-1.618 \times 10^3 \text{ V}) = 180 \text{ V}.$$

(ii) $r = 4.00 \text{ cm}$. This point is outside shell 1 and inside shell 2.

$$V = k \left(\frac{q_1}{r} + \frac{q_2}{R_2} \right) = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{6.00 \times 10^{-9} \text{ C}}{0.0400 \text{ m}} + \frac{-9.00 \times 10^{-9} \text{ C}}{0.0500 \text{ m}} \right).$$

$$V = +1.348 \times 10^3 \text{ V} + (-1.618 \times 10^3 \text{ V}) = -270 \text{ V}.$$

(iii) $r = 6.00 \text{ cm}$. This point is outside both shells.

$$V = k \left(\frac{q_1}{r} + \frac{q_2}{r} \right) = \frac{k}{r} (q_1 + q_2) = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}{0.0600 \text{ m}} [6.00 \times 10^{-9} \text{ C} + (-9.00 \times 10^{-9} \text{ C})]. \quad V = -450 \text{ V}.$$

(b) At the surface of the inner shell, $r = R_1 = 3.00 \text{ cm}$. This point is inside the larger shell,

so $V_1 = k \left(\frac{q_1}{R_1} + \frac{q_2}{R_2} \right) = 180 \text{ V}$. At the surface of the outer shell, $r = R_2 = 5.00 \text{ cm}$. This point is outside the smaller shell, so

$$V = k \left(\frac{q_1}{r} + \frac{q_2}{R_2} \right) = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{6.00 \times 10^{-9} \text{ C}}{0.0500 \text{ m}} + \frac{-9.00 \times 10^{-9} \text{ C}}{0.0500 \text{ m}} \right).$$

$V_2 = +1.079 \times 10^3 \text{ V} + (-1.618 \times 10^3 \text{ V}) = -539 \text{ V}$. The potential difference is $V_1 - V_2 = 719 \text{ V}$. The inner shell is at higher potential. The potential difference is due entirely to the charge on the inner shell.

EVALUATE: Inside a uniform spherical shell, the electric field is zero so the potential is constant (but *not* necessarily zero).

- 23.28. IDENTIFY and SET UP:** Outside a solid conducting sphere $V = k \frac{q}{r}$. Inside the sphere the potential is constant because $E = 0$, and it has the same value as at the surface of the sphere.

EXECUTE: (a) This is outside the sphere, so $V = \frac{kq}{r} = \frac{k(3.50 \times 10^{-9} \text{ C})}{0.480 \text{ m}} = 65.6 \text{ V}$.

(b) This is at the surface of the sphere, so $V = \frac{k(3.50 \times 10^{-9} \text{ C})}{0.240 \text{ m}} = 131 \text{ V}$.

(c) This is inside the sphere. The potential has the same value as at the surface, 131 V.

EVALUATE: All points of a conductor are at the same potential.

- 23.29. (a) IDENTIFY and SET UP:** The electric field on the ring's axis is given by $E_x = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2 + a^2)^{3/2}}$.

The magnitude of the force on the electron exerted by this field is given by $F = eE$.

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring.

From Example 23.11, $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$, where a is the radius of the ring.

EXECUTE: $x_a = 30.0 \text{ cm}$, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$.

Thus $\frac{1}{2}mv^2 = U_a - U_b$.

And $U = qV = -eV$ so $v = \sqrt{\frac{2e(V_b - V_a)}{m}}$.

$$V_a = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x_a^2 + a^2}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{24.0 \times 10^{-9} \text{ C}}{\sqrt{(0.300 \text{ m})^2 + (0.150 \text{ m})^2}}.$$

$$V_a = 643 \text{ V}.$$

$$V_b = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x_b^2 + a^2}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{24.0 \times 10^{-9} \text{ C}}{0.150 \text{ m}} = 1438 \text{ V}.$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602 \times 10^{-19} \text{ C})(1438 \text{ V} - 643 \text{ V})}{9.109 \times 10^{-31} \text{ kg}}} = 1.67 \times 10^7 \text{ m/s}.$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

- 23.30. IDENTIFY:** For an isolated conducting sphere, all the excess charge is on its outer surface. For points outside the sphere, it behaves like a point-charge at its center, and the electric field is zero inside the sphere.

SET UP: Use V at 1.20 m to find V at the surface. $V = k\frac{q}{r}$. We don't know the charge on the sphere, but we know the potential 1.20 m from its center.

EXECUTE: Take the ratio of the potentials: $\frac{V_{\text{surface}}}{V_{1.20 \text{ m}}} = \frac{kq/(0.400 \text{ m})}{kq(1.20 \text{ m})} = \frac{1.20}{0.400} = 3.00$, so

$$V_{\text{surface}} = (3.00)(24.0 \text{ V}) = 72.0 \text{ V}.$$

The electric field is zero inside the sphere, so the potential inside is constant and equal to the potential at the surface. So at the center $V = 72.0 \text{ V}$.

EVALUATE: An alternative approach would be to use the given information to find the charge on the sphere. Then use that charge to calculate the potential at the surface. The potential is 72.0 V at *all* points inside the sphere, not just at the center. Careful! Just because the electric field inside the sphere is zero, it does not follow that the potential is zero there.

- 23.31. IDENTIFY:** If the small sphere is to have its *minimum* speed, it must just stop at 8.00 cm from the surface of the large sphere. In that case, the initial kinetic energy of the small sphere is all converted to electrical potential energy at its point of closest approach.

SET UP: $K_1 + U_1 = K_2 + U_2$. $K_2 = 0$. $U_1 = 0$. Therefore, $K_1 = U_2$. Outside a spherical charge distribution the potential is the same as for a point charge at the location of the center of the sphere, so $U = kqQ/r$. $K = \frac{1}{2}mv^2$.

EXECUTE: $U_2 = \frac{kqQ}{r_2}$, with $r_2 = 12.0 \text{ cm} + 8.0 \text{ cm} = 0.200 \text{ m}$. $\frac{1}{2}mv_1^2 = \frac{kqQ}{r_2}$.

$$v_1 = \sqrt{\frac{2kqQ}{mr_2}} = \sqrt{\frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(3.00 \times 10^{-6} \text{ C})(5.00 \times 10^{-6} \text{ C})}{(6.00 \times 10^{-5} \text{ kg})(0.200 \text{ m})}} = 150 \text{ m/s}.$$

EVALUATE: If the small sphere had enough initial speed to actually penetrate the surface of the large sphere, we could no longer treat the large sphere as a point charge once the small sphere was inside.

- 23.32. IDENTIFY:** For a line of charge, $V_a - V_b = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$. Apply conservation of energy to the motion

of the proton.

SET UP: Let point a be 18.0 cm from the line and let point b be at the distance of closest approach, where $K_b = 0$.

EXECUTE: (a) $K_a = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27} \text{ kg})(3.50 \times 10^3 \text{ m/s})^2 = 1.02 \times 10^{-20} \text{ J}$.

(b) $K_a + qV_a = K_b + qV_b$. $V_a - V_b = \frac{K_b - K_a}{q} = \frac{-1.02 \times 10^{-20} \text{ J}}{1.60 \times 10^{-19} \text{ C}} = -0.06397 \text{ V}$.

$$\ln(r_b/r_a) = \left(\frac{2\pi\epsilon_0}{\lambda} \right) (-0.06397 \text{ V}).$$

$$r_b = r_a \exp\left(\frac{2\pi\epsilon_0 (-0.06397 \text{ V})}{\lambda} \right) = (0.180 \text{ m}) \exp\left(-\frac{2\pi\epsilon_0 (0.06397 \text{ V})}{5.00 \times 10^{-12} \text{ C/m}} \right) = 0.0883 \text{ m} = 8.83 \text{ cm}.$$

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

- 23.33. IDENTIFY:** For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$.

EXECUTE: (a) Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a) = (8.50 \times 10^{-6} \text{ C/m})(2 \times 9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \ln\left(\frac{10.0 \text{ cm}}{6.00 \text{ cm}}\right).$$

$$\Delta V = 7.82 \times 10^4 \text{ V} = 78,200 \text{ V} = 78.2 \text{ kV}.$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

- 23.34. IDENTIFY:** The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.

SET UP: Using $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(r_b/r_a)$ and solving for r_b , we have $r_b = r_a e^{2\pi\epsilon_0 \Delta V / \lambda}$.

EXECUTE: The exponent is $\frac{\left(\frac{1}{2 \times 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}\right)(175 \text{ V})}{15.0 \times 10^{-9} \text{ C/m}} = 0.648$, which gives

$$r_b = (2.50 \text{ cm}) e^{0.648} = 4.78 \text{ cm}.$$

The distance above the *surface* is $4.78 \text{ cm} - 2.50 \text{ cm} = 2.28 \text{ cm}$.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV , even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each r is the distance from the *center* of the cylinder, not from the surface.

- 23.35. IDENTIFY:** The electric field of the line of charge does work on the sphere, increasing its kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$ and $K_1 = 0$. $U = qV$ so $qV_1 = K_2 + qV_2$. $V = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_0}{r}\right)$.

EXECUTE: $V_1 = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_0}{r_1}\right)$. $V_2 = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_0}{r_2}\right)$.

$$K_2 = q(V_1 - V_2) = \frac{q\lambda}{2\pi\epsilon_0} \left(\ln\left(\frac{r_0}{r_1}\right) - \ln\left(\frac{r_0}{r_2}\right) \right) = \frac{\lambda q}{2\pi\epsilon_0} (\ln r_2 - \ln r_1) = \frac{\lambda q}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right).$$

$$K_2 = \frac{(3.00 \times 10^{-6} \text{ C/m})(8.00 \times 10^{-6} \text{ C})}{2\pi(8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2))} \ln\left(\frac{4.50}{1.50}\right) = 0.474 \text{ J}.$$

EVALUATE: The potential due to the line of charge does *not* go to zero at infinity but is defined to be zero at an arbitrary distance r_0 from the line.

- 23.36. IDENTIFY and SET UP:** For oppositely charged parallel plates, $E = \sigma/\epsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

EXECUTE: (a) $E = \frac{\sigma}{\epsilon_0} = \frac{47.0 \times 10^{-9} \text{ C/m}^2}{\epsilon_0} = 5310 \text{ N/C}$.

(b) $V = Ed = (5310 \text{ N/C})(0.0220 \text{ m}) = 117 \text{ V}$.

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

EVALUATE: The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles, the work, which is force times distance, doubles and the potential difference doubles.

- 23.37. IDENTIFY and SET UP:** Use $\Delta V = Ed$ to relate the electric field between the plates to the potential difference between them and their separation. The magnitude of the force this field exerts on the particle is given by $F = qE$. Use $W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l}$ to calculate the work.

EXECUTE: (a) Using $\Delta V = Ed$ gives $E = \frac{V_{ab}}{d} = \frac{360 \text{ V}}{0.0450 \text{ m}} = 8000 \text{ V/m}$.

(b) $F = |q|E = (2.40 \times 10^{-9} \text{ C})(8000 \text{ V/m}) = +1.92 \times 10^{-5} \text{ N}$.

(c) The electric field between the plates is shown in Figure 23.37.

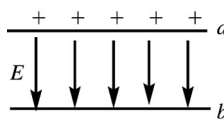


Figure 23.37

The plate with positive charge (plate a) is at higher potential. The electric field is directed from high potential toward low potential (or, \vec{E} is from $+$ charge toward $-$ charge), so \vec{E} points from a to b . Hence the force that \vec{E} exerts on the positive charge is from a to b , so it does positive work.

$W = \int_a^b \vec{F} \cdot d\vec{l} = Fd$, where d is the separation between the plates.

$W = Fd = (1.92 \times 10^{-5} \text{ N})(0.0450 \text{ m}) = +8.64 \times 10^{-7} \text{ J}$.

(d) $V_a - V_b = +360 \text{ V}$ (plate a is at higher potential).

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40 \times 10^{-9} \text{ C})(-360 \text{ V}) = -8.64 \times 10^{-7} \text{ J}$.

EVALUATE: We see that $W_{a \rightarrow b} = -(U_b - U_a) = U_a - U_b$.

- 23.38. IDENTIFY and SET UP:** $V_{ab} = Ed$ for parallel plates.

EXECUTE: $d = \frac{V_{ab}}{E} = \frac{1.5 \text{ V}}{1.0 \times 10^{-6} \text{ V/m}} = 1.5 \times 10^6 \text{ m} = 1.5 \times 10^3 \text{ km}$.

EVALUATE: The plates would have to be nearly a thousand miles apart with only a AA battery across them! This is a small field!

- 23.39. IDENTIFY:** The potential of a solid conducting sphere is the same at every point inside the sphere because $E = 0$ inside, and this potential has the value $V = q/4\pi\epsilon_0 R$ at the surface. Use the given value of E to find q .

SET UP: For negative charge the electric field is directed toward the charge.

For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \frac{|q|}{4\pi\epsilon_0 r^2}$ and $|q| = 4\pi\epsilon_0 Er^2 = \frac{(3800 \text{ N/C})(0.200 \text{ m})^2}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.69 \times 10^{-8} \text{ C}.$

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking

∞ as zero, is $V = \frac{q}{4\pi\epsilon_0 r} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(-1.69 \times 10^{-8} \text{ C})}{0.200 \text{ m}} = -760 \text{ V}$ at the surface of the

sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be -760 V .

EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

- 23.40. IDENTIFY:** The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the potential is that of a point-charge, $V = Q/(4\pi\epsilon_0 R)$.

EXECUTE: (a) Solving for Q and substituting the numbers gives

$$Q = 4\pi\epsilon_0 RV = (0.125 \text{ m})(3750 \text{ V})/(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) = 5.21 \times 10^{-8} \text{ C} = 52.1 \text{ nC}.$$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 3.75 kV .

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

- 23.41. IDENTIFY and SET UP:** For a solid metal sphere or for a spherical shell, $V = \frac{kq}{r}$ outside the sphere and

$V = \frac{kq}{R}$ at all points inside the sphere, where R is the radius of the sphere. When the electric field is radial, $E = -\frac{\partial V}{\partial r}$.

EXECUTE: (a) (i) $r < r_a$: This region is inside both spheres. $V = \frac{kq}{r_a} - \frac{kq}{r_b} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right).$

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell.

$$V = \frac{kq}{r} - \frac{kq}{r_b} = kq\left(\frac{1}{r} - \frac{1}{r_b}\right).$$

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for a point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_a} - \frac{q}{r_b} \right)$ and $V_b = 0$, so $V_{ab} = \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{r_a} - \frac{1}{r_b} \right).$

(c) Between the spheres $r_a < r < r_b$ and $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right).$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{q}{4\pi\epsilon_0} \frac{\partial}{\partial r} \left(\frac{1}{r} - \frac{1}{r_b} \right) = +\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{V_{ab}}{\left(\frac{1}{r_a} - \frac{1}{r_b} \right) r^2}.$$

(d) Since $E_r = -\frac{\partial V}{\partial r}$, $E = 0$, since V is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} - \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = \frac{1}{4\pi\epsilon_0} \frac{(q-Q)}{r}. \text{ All potentials inside the outer shell are just shifted by an}$$

amount $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c)

do not change. However, now that the potential does vary outside the spheres, there is an electric field

$$\text{there: } E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r} \left(\frac{kq}{r} + \frac{-kQ}{r} \right) = \frac{kq}{r^2} \left(1 - \frac{Q}{q} \right) = \frac{k}{r^2} (q - Q).$$

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.

23.42. IDENTIFY: By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.

(a) SET UP: No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.

EXECUTE: Since we must do work on the positive charge, it gains potential energy, so the potential increases.

(b) SET UP: Since the electric field is uniform and is equal to $\sigma/2\epsilon_0$, we have $\Delta V = Ed = \frac{\sigma}{2\epsilon_0} d$.

EXECUTE: Solving for d gives

$$d = \frac{2\epsilon_0 \Delta V}{\sigma} = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.00 \text{ V})}{6.00 \times 10^{-9} \text{ C/m}^2} = 0.00295 \text{ m} = 2.95 \text{ mm}.$$

EVALUATE: Since the spacing of the equipotential surfaces ($d = 2.95 \text{ mm}$) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

23.43. IDENTIFY and SET UP: Use $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = -\frac{\partial V}{\partial z}$ to calculate the components of \vec{E} .

EXECUTE: $V = Axy - Bx^2 + Cy$.

$$\text{(a) } E_x = -\frac{\partial V}{\partial x} = -Ay + 2Bx.$$

$$E_y = -\frac{\partial V}{\partial y} = -Ax - C.$$

$$E_z = \frac{\partial V}{\partial z} = 0.$$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$$E_z = 0 \text{ everywhere.}$$

$$E_y = 0 \text{ at } x = -C/A.$$

And E_x is also equal to zero for this x , any value of z and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: V doesn't depend on z so $E_z = 0$ everywhere.

23.44. IDENTIFY: Apply $E_x = -\frac{\partial V}{\partial x}$ and $E_y = -\frac{\partial V}{\partial y}$ to find the components of \vec{E} , then use them to find its magnitude and direction. $V(x, y) = Ax^2y - Bxy^2$.

SET UP: $E = \sqrt{E_x^2 + E_y^2}$ and $\tan \theta = E_y/E_x$.

EXECUTE: First find the components of \vec{E} : $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(Ax^2y - Bxy^2) = -(2Axy - By^2)$.

Now evaluate this result at the point $x = 2.00$ m, $y = 0.400$ m using the given values for A and B .

$$E_x = -[2(5.00 \text{ V/m}^3)(2.00 \text{ m})(0.400 \text{ m}) - (8.00 \text{ V/m}^3)(0.400 \text{ m})^2] = -6.72 \text{ V/m}.$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(Ax^2y - Bxy^2) = -(Ax^2 - 2Bxy). \text{ At the point } (2.00 \text{ m}, 0.400 \text{ m}), \text{ this is}$$

$$E_y = -[(5.00 \text{ V/m}^3)(2.00 \text{ m})^2 - 2(8.00 \text{ V/m}^3)(2.00 \text{ m})(0.400 \text{ m})] = -7.20 \text{ V/m}.$$

Now use the components to find the magnitude and direction of \vec{E} .

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-6.72 \text{ V/m})^2 + (-7.20 \text{ V/m})^2} = 9.85 \text{ V/m}.$$

$\tan \theta = E_y/E_x = (-7.20 \text{ V/m})/(-6.72 \text{ V/m})$, which gives $\theta = 47.0^\circ$. Since both components are negative, the vector lies in the third quadrant in the xy -plane and makes an angle of $47.0^\circ + 180.0^\circ = 227.0^\circ$ with the $+x$ -axis.

EVALUATE: V is a scalar but \vec{E} is a vector and has components.

23.45. IDENTIFY: Exercise 23.41 shows that $V = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$ for $r_a < r < r_b$ and

$$V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right).$$

SET UP: $E = \frac{kq}{r^2}$, radially outward, for $r_a \leq r \leq r_b$.

$$\begin{aligned} \text{EXECUTE: (a) } V_{ab} &= kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500 \text{ V gives } q = \frac{500 \text{ V}}{k\left(\frac{1}{0.012 \text{ m}} - \frac{1}{0.096 \text{ m}}\right)} = 7.62 \times 10^{-10} \text{ C} \\ &= 0.762 \text{ nC.} \end{aligned}$$

(b) $V_b = 0$ so $V_a = 500$ V. The inner metal sphere is an equipotential with $V = 500$ V. $\frac{1}{r} = \frac{1}{r_a} + \frac{V}{kq}$.

$V = 400$ V at $r = 1.45$ cm, $V = 300$ V at $r = 1.85$ cm, $V = 200$ V at $r = 2.53$ cm, $V = 100$ V at $r = 4.00$ cm, $V = 0$ at $r = 9.60$ cm. The equipotential surfaces are sketched in Figure 23.45.

EVALUATE: (c) The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller r , where the electric field is largest.

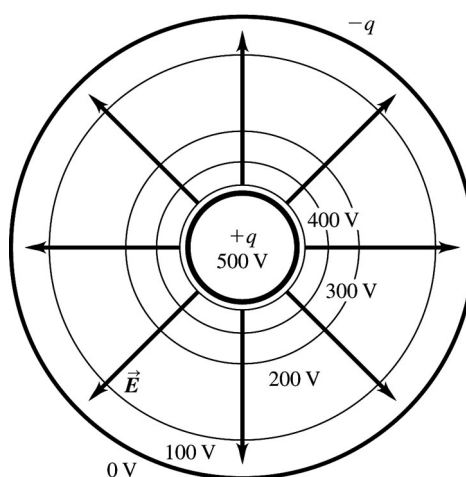


Figure 23.45

- 23.46. IDENTIFY:** As the sphere approaches the point charge, the speed of the sphere decreases because it loses kinetic energy, but its acceleration increases because the electric force on it increases. Its mechanical energy is conserved during the motion, and Newton's second law and Coulomb's law both apply.

SET UP: $K_a + U_a = K_b + U_b$, $K = \frac{1}{2}mv^2$, $U = kq_1q_2/r$, $F = kq_1q_2/r^2$, and $F = ma$.

EXECUTE: Find the distance between the two charges when $v_2 = 25.0$ m/s.

$$K_a + U_a = K_b + U_b.$$

$$K_a = \frac{1}{2}mv_a^2 = \frac{1}{2}(4.00 \times 10^{-3} \text{ kg})(40.0 \text{ m/s})^2 = 3.20 \text{ J}.$$

$$K_b = \frac{1}{2}mv_b^2 = \frac{1}{2}(4.00 \times 10^{-3} \text{ kg})(25.0 \text{ m/s})^2 = 1.25 \text{ J}.$$

$$U_a = k \frac{q_1q_2}{r_a} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C})(2.00 \times 10^{-6} \text{ C})}{0.0600 \text{ m}} = 1.498 \text{ J}.$$

$$U_b = K_a + U_a - K_b = 3.20 \text{ J} + 1.498 \text{ J} - 1.25 \text{ J} = 3.448 \text{ J}. \quad U_b = k \frac{q_1q_2}{r_b} \text{ and}$$

$$r_b = \frac{kq_1q_2}{U_b} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C})(2.00 \times 10^{-6} \text{ C})}{3.448 \text{ J}} = 0.02607 \text{ m}.$$

$$F_b = \frac{kq_1q_2}{r_b^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-6} \text{ C})(2.00 \times 10^{-6} \text{ C})}{(0.02607 \text{ m})^2} = 132.3 \text{ N}.$$

$$a = \frac{F}{m} = \frac{132.3 \text{ N}}{4.00 \times 10^{-3} \text{ kg}} = 3.31 \times 10^4 \text{ m/s}^2.$$

EVALUATE: As the sphere approaches the point charge, its speed decreases but its acceleration keeps increasing because the electric force on it keeps increasing.

23.47. IDENTIFY: $U = k \left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} \right).$

SET UP: In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate x , so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200 \text{ m} - x$.

EXECUTE: (a)

$$U = k \left(\frac{(4.00 \text{ nC})(-3.00 \text{ nC})}{(0.200 \text{ m})} + \frac{(4.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})} + \frac{(-3.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})} \right) = -3.60 \times 10^{-7} \text{ J}.$$

(b) If $U = 0$, then $0 = k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{x} + \frac{q_2 q_3}{r_{12} - x} \right)$. Solving for x we find:

$$0 = -60 + \frac{8}{x} - \frac{6}{0.2 - x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074 \text{ m}, 0.360 \text{ m}. \text{ Therefore, } x = 0.074 \text{ m} \text{ since it}$$

is the only value between the two charges.

EVALUATE: U_{13} is positive and both U_{23} and U_{12} are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$.

For $x = 0.074 \text{ m}$, $U_{13} = +9.7 \times 10^{-7} \text{ J}$, $U_{23} = -4.3 \times 10^{-7} \text{ J}$ and $U_{12} = -5.4 \times 10^{-7} \text{ J}$. It is true that $U = 0$ at this x .

23.48. IDENTIFY: We are dealing with the electric field and potential of point charges.

SET UP: $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ and $E = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$. Let $\frac{1}{4\pi\epsilon_0}$ be k .

EXECUTE: (a) We want to find where $E = 0$. To cancel, the fields must be in opposite directions and have equal magnitudes. This can happen only for $x > 0.200 \text{ m}$. Call d the distance between the -4.00 nC charge and the point where $E = 0$. The magnitudes are equal so $\frac{k(8.00 \text{ nC})}{(d + 0.400 \text{ m})^2} = \frac{k(4.00 \text{ nC})}{d^2}$. Solving

for d gives $d = 1.17 \text{ m}$.

(b) We want to find where $V = 0$. V can be zero when the potentials have equal magnitude but opposite sign. This occurs only for points closer to the -4.00 nC charge than to the 8.00 nC charge, which is between the charges and for $x > 0.200 \text{ m}$.

Between the charges: Call x the distance from the -4.00 nC charge. Equating the magnitudes of the potentials gives $\frac{k(8.00 \text{ nC})}{0.400 \text{ m} - x} = \frac{k(4.00 \text{ nC})}{x}$. $x = 0.067 \text{ m}$.

For $x > 0.200 \text{ m}$: Call d the distance between the -4.00 nC charge and the point of cancellation.

$$\frac{k(8.00 \text{ nC})}{d + 0.400 \text{ m}} = \frac{k(4.00 \text{ nC})}{d}. \quad d = 0.400 \text{ m}. \quad x = 0.400 \text{ m} + 0.200 \text{ m} = 0.600 \text{ m}.$$

(c) E could be zero only for $x > 0.200 \text{ m}$, which is at $x = 1.17 \text{ m}$ (as we found), so E is not equal to zero at any of these points.

EVALUATE: This problem is a good illustration of the need to analyze before using any equations. Doing so allows us to decide which regions to consider for E to be zero and for V to be zero before doing any unnecessary calculations.

23.49. IDENTIFY: This problem involves potential difference and electric fields.

SET UP: For a sheet of charge $E_\sigma = \sigma / 2\epsilon_0$, and for a line of charge $E_\lambda = \lambda / 2\pi\epsilon_0 r$. Make a sketch showing the fields, as in Fig. 23.49. $V_B - V_A$ is due to the two fields. Between the sheet and the line the fields are in opposite directions and on the other side of the line they are in the same direction, as shown in the figure. $V_A - V_B = \Delta V_\sigma + \Delta V_\lambda$.

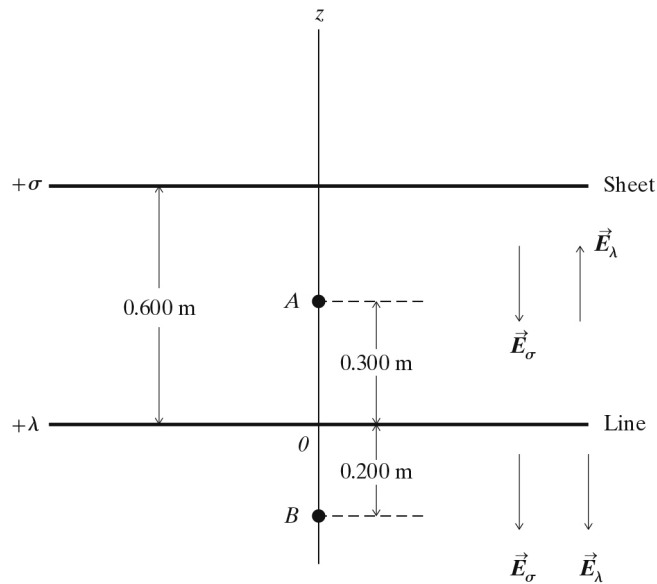


Figure 23.49

EXECUTE: For the sheet: E_σ is uniform, so

$$V_A - V_B = \Delta V_\sigma = E_\sigma \Delta z = \frac{\sigma}{2\epsilon_0} \Delta z = \frac{8.00 \mu\text{C}/\text{m}^2}{2\epsilon_0} (0.500 \text{ m}) = 2.260 \times 10^5 \text{ V}.$$

For the line: The field due to the line reverses direction as we cross the line. So the potential change in going from 0.200 m to the line is the opposite of the potential change in going from the line to 0.200 m on the other side. Thus the only potential change that does not cancel is in going from 0.300 m to 0.200 m on the upper side of the line. Therefore

$$\Delta V_\lambda = \int_{0.300 \text{ m}}^{0.200 \text{ m}} E_z dz = \int_{0.300 \text{ m}}^{0.200 \text{ m}} \frac{\lambda}{2\pi\epsilon_0 z} dz = \frac{\lambda}{2\pi\epsilon_0} \ln(2/3).$$

This gives $-3.645 \times 10^4 \text{ V}$.

The total potential difference is $2.26 \times 10^5 \text{ V} - 3.645 \times 10^4 \text{ V} = +1.90 \times 10^5 \text{ V}$.

$V_A = V_B + 1.90 \times 10^5 \text{ V}$, so A is at a higher potential than B .

EVALUATE: It may appear that we skipped over an important point, namely that the potential becomes infinite if the line has no thickness. But any real line has some (but very small) thickness, so the potential difference between 0.200 m above the line can cancel the potential difference below the line without any infinities coming into play.

- 23.50. IDENTIFY:** Two forces do work on the sphere as it falls: gravity and the electrical force due to the sheet. The energy of the sphere is conserved.

SET UP: The gravity force is mg , downward. The electric field of the sheet is $E = \frac{\sigma}{2\epsilon_0}$ upward, and

the force it exerts on the sphere is $F = qE$. The sphere gains kinetic energy $K = \frac{1}{2}mv^2$ as it falls.

EXECUTE: $mg = 4.90 \times 10^{-6} \text{ N}$. $E = \frac{\sigma}{2\epsilon_0} = \frac{8.00 \times 10^{-12} \text{ C/m}^2}{2(8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2))} = 0.4518 \text{ N/C}$. The electric force is $qE = (7.00 \times 10^{-6} \text{ C})(0.4518 \text{ N/C}) = 3.1626 \times 10^{-6} \text{ N}$, upward. The net force is downward, so the sphere moves downward when released. Let $y = 0$ at the sheet. $U_{\text{grav}} = mgy$. For the electric force, $\frac{W_{a \rightarrow b}}{q} = V_a - V_b$. Let point a be at the sheet and let point b be a distance y above the sheet. Take $V_a = 0$. The force on q is qE , upward, so $\frac{W_{a \rightarrow b}}{q} = Ey$ and $V_b = -Ey$. $U_b = -E y q$. $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$. $y_1 = 0.400 \text{ m}$, $y_2 = 0.100 \text{ m}$. $K_2 = U_1 - U_2 = mg(y_1 - y_2) - E(y_1 - y_2)q$. $K_2 = (5.00 \times 10^{-7} \text{ kg})(9.80 \text{ m/s}^2)(0.300 \text{ m}) - (0.4518 \text{ N/C})(0.300 \text{ m})(7.00 \times 10^{-6} \text{ C})$. $K_2 = 1.470 \times 10^{-6} \text{ J} - 0.94878 \times 10^{-6} \text{ J} = 0.52122 \times 10^{-6} \text{ J}$. $K_2 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.52122 \times 10^{-6} \text{ J})}{5.00 \times 10^{-7} \text{ kg}}} = 1.44 \text{ m/s}$.

EVALUATE: Because the weight is greater than the electric force, the sphere will accelerate downward, but if it were light enough the electric force would exceed the weight. In that case it would never get closer to the sheet after being released. We could also solve this problem using Newton's second law and the constant-acceleration kinematics formulas. $a = F/m = (mg - qE)/m$ gives the acceleration. Then we use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_{0x} = 0$ to find v .

23.51. IDENTIFY and SET UP: Treat the gold nucleus as a point charge so that $V = k \frac{q}{r}$. According to

conservation of energy we have $K_1 + U_1 = K_2 + U_2$, where $U = qV$.

EXECUTE: Assume that the alpha particle is at rest before it is accelerated and that it momentarily stops when it arrives at its closest approach to the surface of the gold nucleus. Thus we have $K_1 = K_2 = 0$, which implies that $U_1 = U_2$. Since $U = qV$ we conclude that the accelerating voltage must be equal to the voltage at its point of closest approach to the surface of the gold nucleus. Therefore

$$V_a = V_b = k \frac{q}{r} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{79(1.60 \times 10^{-19} \text{ C})}{(7.3 \times 10^{-15} \text{ m} + 2.0 \times 10^{-14} \text{ m})} = 4.2 \times 10^6 \text{ V}.$$

EVALUATE: Although the alpha particle has kinetic energy as it approaches the gold nucleus this is irrelevant to our solution since energy is conserved for the whole process.

23.52. IDENTIFY: The charged particles repel each other and therefore accelerate away from one another, causing their speeds and kinetic energies to continue to increase. They do not have equal speeds because they have different masses. The mechanical energy and momentum of the system are conserved.

SET UP: The proton has charge $q_p = +e$ and mass $m_p = 1.67 \times 10^{-27} \text{ kg}$. The alpha particle has charge $q_a = +2e$ and mass $m_a = 4m_p = 6.68 \times 10^{-27} \text{ kg}$. We can apply both conservation of energy and

conservation of linear momentum to the system. $a = \frac{F}{m}$, where $F = k \frac{|q_1 q_2|}{r^2}$.

EXECUTE: *Acceleration:* The maximum force and hence the maximum acceleration occurs just after they are released, when $r = 0.225 \text{ nm}$. $F = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \frac{(2)(1.60 \times 10^{-19} \text{ C})^2}{(0.225 \times 10^{-9} \text{ m})^2} = 9.09 \times 10^{-9} \text{ N}$.

$$a_p = \frac{F}{m_p} = \frac{9.09 \times 10^{-9} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 5.44 \times 10^{18} \text{ m/s}^2; \quad a_a = \frac{F}{m_a} = \frac{9.09 \times 10^{-9} \text{ N}}{6.68 \times 10^{-27} \text{ kg}} = 1.36 \times 10^{18} \text{ m/s}^2. \text{ The}$$

acceleration of the proton is larger by a factor of m_a/m_p .

Speed: Conservation of energy says $U_1 + K_1 = U_2 + K_2$. $K_1 = 0$ and $U_2 = 0$, so $K_2 = U_1$.

$$U_1 = k \frac{q_1 q_2}{r} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(2)(1.60 \times 10^{-19} \text{ C})^2}{0.225 \times 10^{-9} \text{ m}} = 2.05 \times 10^{-18} \text{ J}, \text{ so the total kinetic energy of}$$

the two particles when they are far apart is $K_2 = 2.05 \times 10^{-18} \text{ J}$. Conservation of linear momentum says how this energy is divided between the proton and alpha particle. $p_1 = p_2$. $0 = m_p v_p - m_a v_a$ so

$$v_a = \left(\frac{m_p}{m_a} \right) v_p.$$

$$K_2 = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_a v_a^2 = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_a \left(\frac{m_p}{m_a} \right)^2 v_p^2 = \frac{1}{2} m_p v_p^2 \left(1 + \frac{m_p}{m_a} \right).$$

$$v_p = \sqrt{\frac{2K_2}{m_p(1 + (m_p/m_a))}} = \sqrt{\frac{2(2.05 \times 10^{-18} \text{ J})}{(1.67 \times 10^{-27} \text{ kg})(1 + \frac{1}{4})}} = 4.43 \times 10^4 \text{ m/s}.$$

$$v_a = \left(\frac{m_p}{m_a} \right) v_p = \frac{1}{4} (4.43 \times 10^4 \text{ m/s}) = 1.11 \times 10^4 \text{ m/s}. \text{ The maximum acceleration occurs just after they are}$$

released. The maximum speed occurs after a long time.

EVALUATE: The proton and alpha particle have equal momentum, but proton has a greater acceleration and more kinetic energy.

23.53. (a) IDENTIFY: Apply the work-energy theorem.

SET UP: Points a and b are shown in Figure 23.53a.

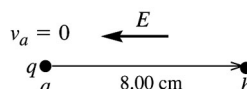


Figure 23.53a

$$\text{EXECUTE: } W_{\text{tot}} = \Delta K = K_b - K_a = K_b = 4.35 \times 10^{-5} \text{ J}.$$

The electric force F_E and the additional force F both do work, so that $W_{\text{tot}} = W_{F_E} + W_F$.

$$W_{F_E} = W_{\text{tot}} - W_F = 4.35 \times 10^{-5} \text{ J} - 6.50 \times 10^{-5} \text{ J} = -2.15 \times 10^{-5} \text{ J}.$$

EVALUATE: The forces on the charged particle are shown in Figure 23.53b.

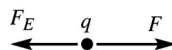


Figure 23.53b

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force F is in the direction of the displacement, so it does positive work.

(b) IDENTIFY and SET UP: For the work done by the electric force, $W_{a \rightarrow b} = q(V_a - V_b)$.

EXECUTE: $V_a - V_b = \frac{W_{a \rightarrow b}}{q} = \frac{-2.15 \times 10^{-5} \text{ J}}{7.60 \times 10^{-9} \text{ C}} = -2.83 \times 10^3 \text{ V}.$

EVALUATE The starting point (point a) is at $2.83 \times 10^3 \text{ V}$ lower potential than the ending point (point b). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

(c) IDENTIFY: Calculate E from $V_a - V_b$ and the separation d between the two points.

SET UP: Since the electric field is uniform and directed opposite to the displacement $W_{a \rightarrow b} = -F_E d = -qEd$, where $d = 8.00 \text{ cm}$ is the displacement of the particle.

EXECUTE: $E = -\frac{W_{a \rightarrow b}}{qd} = -\frac{V_a - V_b}{d} = -\frac{-2.83 \times 10^3 \text{ V}}{0.0800 \text{ m}} = 3.54 \times 10^4 \text{ V/m}.$

EVALUATE: In part (a), W_{tot} is the total work done by both forces. In parts (b) and (c) $W_{a \rightarrow b}$ is the work done just by the electric force.

- 23.54. IDENTIFY:** The net force on q_0 is the vector sum of the forces due to the two charges. Coulomb's law applies.

SET UP: $F = k \frac{|q_1 q_2|}{r^2}$, $W_{a \rightarrow b} = q(V_a - V_b)$, $V = k \frac{q}{r}$.

EXECUTE: **(a)** The magnitude of the force on q_0 due to each of the two charges at opposite corners of the square is $F = k \frac{|q_1 q_2|}{r^2} = k(5.00 \mu\text{C})(3.00 \mu\text{C})/(0.0800 \text{ m})^2 = 21.07 \text{ N}$. Adding the two forces vectorially gives the net force $F_{\text{net}} = (21.07 \text{ N})\sqrt{2} = 29.8 \text{ N}$. The direction is from A to B since both charges attract q_0 . Figure 23.54 shows this force.

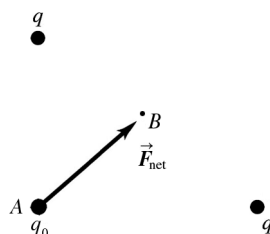


Figure 23.54

(b) At point B the two forces on q_0 are in opposite directions and have equal magnitudes, so they add to zero: $F_{\text{net}} = 0$.

(c) For each charge, $W_{A \rightarrow B} = q(V_A - V_B)$, so for both we must double this. Using $V = k \frac{q}{r}$ and

simplifying we get $W_{A \rightarrow B} = 2q(V_A - V_B) = 2kqq_0 \left(\frac{1}{r_A} - \frac{1}{r_B} \right)$. Putting in $q_0 = -3.00 \mu\text{C}$, $q = 5.00 \mu\text{C}$, $r_A =$

0.0800 m , and $r_B = 0.0400\sqrt{2} \text{ m}$, we get $W_{A \rightarrow B} = +1.40 \text{ J}$. The work done on q_0 by the electric field is positive since it this charge moves from A to B in the direction of the force. The charge loses potential energy as it gains kinetic energy. But since q_0 is negative, it moves to a point of higher potential.

EVALUATE: Positive charges accelerate toward lower potential, but negative charges accelerate toward higher potential.

- 23.55. IDENTIFY and SET UP:** Calculate the components of \vec{E} using $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and

$$E_z = -\frac{\partial V}{\partial z}, \text{ and use } \vec{F} = q\vec{E}.$$

EXECUTE: (a) $V = Cx^{4/3}$.

$$C = V/x^{4/3} = 240 \text{ V}/(13.0 \times 10^{-3} \text{ m})^{4/3} = 7.85 \times 10^4 \text{ V/m}^{4/3}.$$

$$\text{(b) } E_x(x) = -\frac{\partial V}{\partial x} = -\frac{4}{3}Cx^{1/3} = -(1.05 \times 10^5 \text{ V/m}^{4/3})x^{1/3}.$$

The minus sign means that E_x is in the $-x$ -direction, which says that \vec{E} points from the positive anode toward the negative cathode.

$$\text{(c) } \vec{F} = q\vec{E} \text{ so } F_x = -eE_x = \frac{4}{3}eCx^{1/3}.$$

Halfway between the electrodes means $x = 6.50 \times 10^{-3} \text{ m}$.

$$F_x = \frac{4}{3}(1.602 \times 10^{-19} \text{ C})(7.85 \times 10^4 \text{ V/m}^{4/3})(6.50 \times 10^{-3} \text{ m})^{1/3} = 3.13 \times 10^{-15} \text{ N}.$$

F_x is positive, so the force is directed toward the positive anode.

EVALUATE: V depends only on x , so $E_y = E_z = 0$. \vec{E} is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

- 23.56. IDENTIFY:** We model the finger as a parallel-plate capacitor.

SET UP and EXECUTE: (a) Estimate: 2 mm

(b) We want the field E . Assuming a uniform field, $V = Ed = (3 \text{ MV/m})(2 \text{ mm}) = 6 \text{ kV}$.

(c) $E = \sigma/\epsilon_0$, so $\sigma = \epsilon_0 E = \epsilon_0 (3 \text{ MV/m}) \approx 30 \mu\text{C/m}^2$.

(d) Estimate: $\frac{1}{2} \text{ cm}$ by $\frac{1}{2} \text{ cm} = 0.25 \text{ cm}^2$.

(e) $q = \sigma A = (30 \mu\text{C/m}^2)(0.25 \text{ cm}^2) = 0.02 \text{ pC}$.

(f) $q = ne$, so $n = q/e = (0.02 \text{ pC})/e = 10^5$ electrons.

EVALUATE: A voltage of 6 kV seems dangerous, but only a small amount of charge is involved.

- 23.57. IDENTIFY:** $U = \frac{kq_1q_2}{r}$.

SET UP: Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of q and $-q$ separated by d , 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of q and $-q$ separated by $\sqrt{3}d$.

$$\text{EXECUTE: (a) } U = kq^2 \left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d} \right) = -\frac{12kq^2}{d} \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) = -1.46q^2/\pi\epsilon_0 d.$$

EVALUATE: (b) The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

- 23.58. IDENTIFY:** We are modeling electrical power lines as conducting cylinders.

SET UP and EXECUTE: (a) Estimate: About 22 ft $\approx 7.0 \text{ m}$.

$$\text{(b) We want the linear charge density } \lambda. \Delta V = \int E_r dr = \int_{0.02 \text{ m}}^{7 \text{ m}} \frac{\lambda}{2\pi\epsilon_0 r} dr = \frac{\lambda}{2\pi\epsilon_0} \ln(7/0.02). \text{ Solving}$$

for λ using $V = 22 \text{ kV}$ gives $\lambda \approx 210 \text{ nC/m}$ which rounds to 200 nC/m .

$$\text{(c) We want } E. E = \frac{\lambda}{2\pi\epsilon_0 r}. \text{ Using } \lambda = 210 \text{ nC/m and } r = 7 \text{ m gives } E \approx 540 \text{ V/m}.$$

EVALUATE: The field in (c) is not very strong compared to many fields in physics labs.

23.59. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$.

The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.59.

EXECUTE: $T \cos \theta = mg$ and $T \sin \theta = F_e$ gives

$$F_e = mg \tan \theta = (1.50 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan(30^\circ) = 0.0085 \text{ N}.$$

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085 \text{ N})(0.0500 \text{ m})}{8.90 \times 10^{-6} \text{ C}} = 47.8 \text{ V}.$$

EVALUATE: $E = V/d = 956 \text{ V/m}$. $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46 \times 10^{-9} \text{ C/m}^2$.

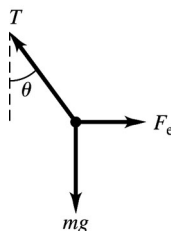


Figure 23.59

23.60. IDENTIFY: Outside a uniform spherical shell of charge, the electric field and potential are the same as for a point-charge at the center. Inside the shell, the electric field is zero so the potential is constant and equal to its value at the surface of the shell. The net potential is the scalar sum of the individual potentials.

SET UP: $V = k\frac{q}{r}$. Call V_1 the potential due to the inner shell and V_2 the potential due to the outer shell.

$$V_{\text{net}} = V_1 + V_2.$$

EXECUTE: (a) At $r = 2.50 \text{ cm}$, we are inside both shells. V_1 is the potential at the surface of the inner shell, so $V_1 = kq_1/R_1$; and V_2 is the potential at the surface of the outer shell, so $V_2 = kq_2/R_2$. The net potential is

$$V_{\text{net}} = kq_1/R_1 + kq_2/R_2 = k(q_1/R_1 + q_2/R_2).$$

$$V_{\text{net}} = (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)[(3.00 \mu\text{C})/(0.0500 \text{ m}) + (-5.00 \mu\text{C})/(0.150 \text{ m})] = 2.40 \times 10^5 \text{ V} = 240 \text{ kV}.$$

(b) At $r = 10.0 \text{ cm}$, we are outside the inner shell but still inside the outer shell. The inner shell now is equivalent to a point-charge at its center, so the net potential is

$$V_{\text{net}} = kq_1/r + kq_2/R_2 = k(q_1/r + q_2/R_2).$$

$$V_{\text{net}} = k[(3.00 \mu\text{C})/(0.100 \text{ m}) + (-5.00 \mu\text{C})/(0.150 \text{ m})] = -30.0 \text{ kV}.$$

(c) At $r = 20.0 \text{ cm}$, we are outside both shells, so both are equivalent to point-charges at their center. So

$$V_{\text{net}} = kq_1/r + kq_2/r = k(q_1 + q_2)/r = k(-2.00 \mu\text{C})/(0.200 \text{ m}) = -89.9 \text{ kV}.$$

EVALUATE: $E = 0$ inside a spherically symmetric shell, but that does not necessarily mean that $V = 0$ there. It only means that $V_a - V_b = 0$ for any two points inside the shell, so V is constant.

23.61. (a) IDENTIFY: The potential at any point is the sum of the potentials due to each of the two charged conductors.

SET UP: For a conducting cylinder with charge per unit length λ the potential outside the cylinder is given by $V = (\lambda/2\pi\epsilon_0) \ln(r_0/r)$ where r is the distance from the cylinder axis and r_0 is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for V applies to both. This problem says to take $r_0 = b$.

EXECUTE: For the hollow tube of radius b and charge per unit length $-\lambda$: outside

$$V = -(\lambda/2\pi\epsilon_0)\ln(b/r); \text{ inside } V = 0 \text{ since } V = 0 \text{ at } r = b.$$

For the metal cylinder of radius a and charge per unit length λ :

$$\text{outside } V = (\lambda/2\pi\epsilon_0)\ln(b/r), \text{ inside } V = (\lambda/2\pi\epsilon_0)\ln(b/a), \text{ the value at } r = a.$$

$$(i) \ r < a; \text{ inside both } V = (\lambda/2\pi\epsilon_0)\ln(b/a).$$

$$(ii) \ a < r < b; \text{ outside cylinder, inside tube } V = (\lambda/2\pi\epsilon_0)\ln(b/r).$$

$$(iii) \ r > b; \text{ outside both the potentials are equal in magnitude and opposite in sign so } V = 0.$$

$$(b) \text{ For } r = a, V_a = (\lambda/2\pi\epsilon_0)\ln(b/a).$$

$$\text{For } r = b, V_b = 0.$$

$$\text{Thus } V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a).$$

(c) IDENTIFY and SET UP: Use $E_r = -\frac{\partial V}{\partial r}$ to calculate E .

$$\text{EXECUTE: } E = -\frac{\partial V}{\partial r} = -\frac{\lambda}{2\pi\epsilon_0} \frac{\partial}{\partial r} \ln\left(\frac{b}{r}\right) = -\frac{\lambda}{2\pi\epsilon_0} \left(\frac{r}{b}\right) \left(-\frac{b}{r^2}\right) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}.$$

(d) The electric field between the cylinders is due only to the inner cylinder, so V_{ab} is not changed, $V_{ab} = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

EVALUATE: The electric field is not uniform between the cylinders, so $V_{ab} \neq E(b-a)$.

23.62. IDENTIFY: The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives

$$E(r) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}.$$

$$\text{SET UP: } a = 145 \times 10^{-6} \text{ m}, \ b = 0.0180 \text{ m}.$$

$$\text{EXECUTE: } E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r} \text{ and}$$

$$V_{ab} = E \ln(b/a) r = (2.00 \times 10^4 \text{ N/C})(\ln(0.018 \text{ m}/145 \times 10^{-6} \text{ m}))0.012 \text{ m} = 1157 \text{ V}.$$

EVALUATE: The electric field at any r is directly proportional to the potential difference between the wire and the cylinder.

23.63. IDENTIFY and SET UP: Use $\vec{F} = q\vec{E}$ to calculate \vec{F} and then $\vec{F} = m\vec{a}$ gives \vec{a} . $E = V/d$.

EXECUTE: (a) $\vec{F}_E = q\vec{E}$. Since $q = -e$ is negative \vec{F}_E and \vec{E} are in opposite directions; \vec{E} is upward

so \vec{F}_E is downward. The magnitude of E is $E = \frac{V}{d} = \frac{22.0 \text{ V}}{0.0200 \text{ m}} = 1.10 \times 10^3 \text{ V/m} = 1.10 \times 10^3 \text{ N/C}$. The

magnitude of F_E is $F_E = |q|E = eE = (1.602 \times 10^{-19} \text{ C})(1.10 \times 10^3 \text{ N/C}) = 1.76 \times 10^{-16} \text{ N}$.

(b) Calculate the acceleration of the electron produced by the electric force:

$$a = \frac{F}{m} = \frac{1.76 \times 10^{-16} \text{ N}}{9.109 \times 10^{-31} \text{ kg}} = 1.93 \times 10^{14} \text{ m/s}^2.$$

EVALUATE: This acceleration is much larger than $g = 9.80 \text{ m/s}^2$, so the gravity force on the electron can be neglected. \vec{F}_E is downward, so \vec{a} is downward.

(c) IDENTIFY and SET UP: The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

EXECUTE: x-component: $v_{0x} = 6.50 \times 10^6$ m/s; $a_x = 0$; $x - x_0 = 0.060$ m; $t = ?$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ and the } a_x \text{ term is zero, so } t = \frac{x - x_0}{v_{0x}} = \frac{0.060 \text{ m}}{6.50 \times 10^6 \text{ m/s}} = 9.231 \times 10^{-9} \text{ s.}$$

y-component: $v_{0y} = 0$; $a_y = 1.93 \times 10^{14}$ m/s²; $t = 9.231 \times 10^{-9}$ s; $y - y_0 = ?$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2. \quad y - y_0 = \frac{1}{2}(1.93 \times 10^{14} \text{ m/s}^2)(9.231 \times 10^{-9} \text{ s})^2 = 0.00822 \text{ m} = 0.822 \text{ cm.}$$

(d) IDENTIFY and SET UP: The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

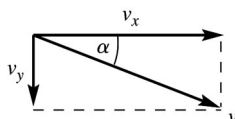


Figure 23.63

EXECUTE:

$$v_x = v_{0x} = 6.50 \times 10^6 \text{ m/s (since } a_x = 0 \text{).}$$

$$v_y = v_{0y} + a_y t.$$

$$v_y = 0 + (1.93 \times 10^{14} \text{ m/s}^2)(9.231 \times 10^{-9} \text{ s}).$$

$$v_y = 1.782 \times 10^6 \text{ m/s.}$$

$$\tan \alpha = \frac{v_y}{v_x} = \frac{1.782 \times 10^6 \text{ m/s}}{6.50 \times 10^6 \text{ m/s}} = 0.2742 \text{ so } \alpha = 15.3^\circ.$$

EVALUATE: The greater the electric field or the smaller the initial speed the greater the downward deflection.

(e) IDENTIFY and SET UP: Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

EXECUTE: x-component: $v_{0x} = 6.50 \times 10^6$ m/s; $a_x = 0$; $x - x_0 = 0.120$ m; $t = ?$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ and the } a_x \text{ term is zero, so } t = \frac{x - x_0}{v_{0x}} = \frac{0.120 \text{ m}}{6.50 \times 10^6 \text{ m/s}} = 1.846 \times 10^{-8} \text{ s.}$$

y-component: $v_{0y} = 1.782 \times 10^6$ m/s (from part (b)); $a_y = 0$; $t = 1.846 \times 10^{-8}$ s; $y - y_0 = ?$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782 \times 10^6 \text{ m/s})(1.846 \times 10^{-8} \text{ s}) = 0.0329 \text{ m} = 3.29 \text{ cm.}$$

EVALUATE: The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is 0.822 cm + 3.29 cm = 4.11 cm. The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

23.64. IDENTIFY: The charge on the plates and the electric field between them depend on the potential difference across the plates.

SET UP: For two parallel plates, the potential difference between them is $V = Ed = \frac{\sigma}{\epsilon_0} d = \frac{Qd}{\epsilon_0 A}$.

EXECUTE: (a) Solving for Q gives $Q = \epsilon_0 AV/d = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.030 \text{ m})^2(25.0 \text{ V})}{0.0050 \text{ m}}$.

$$Q = 3.98 \times 10^{-11} \text{ C} = 39.8 \text{ pC.}$$

(b) $E = V/d = (25.0 \text{ V})/(0.0050 \text{ m}) = 5.00 \times 10^3 \text{ V/m}$.

(c) **SET UP:** Energy conservation gives $\frac{1}{2}mv^2 = eV$.

EXECUTE: Solving for v gives $v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60 \times 10^{-19} \text{ C})(25.0 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} = 2.96 \times 10^6 \text{ m/s}$.

EVALUATE: Typical voltages in student laboratory work run up to around 25 V, so typical reasonable values for the charge on the plates is about 40 pC and a reasonable value for the electric field is about 5000 V/m, as we found here. The electron speed would be about 3 million m/s.

23.65. (a) IDENTIFY and SET UP: Problem 23.61 derived that $E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}$, where a is the radius of the

inner cylinder (wire) and b is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab} . Use this expression to calculate E at the specified r .

EXECUTE: Midway between the wire and the cylinder wall is at a radius of

$$r = (a + b)/2 = (90.0 \times 10^{-6} \text{ m} + 0.140 \text{ m})/2 = 0.07004 \text{ m}.$$

$$E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r} = \frac{50.0 \times 10^3 \text{ V}}{\ln(0.140 \text{ m}/90.0 \times 10^{-6} \text{ m})(0.07004 \text{ m})} = 9.71 \times 10^4 \text{ V/m}.$$

(b) **IDENTIFY and SET UP:** The magnitude of the electric force is given by $F = |q|E$. Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$.

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0 \times 10^{-9} \text{ kg})(9.80 \text{ m/s}^2)}{9.71 \times 10^4 \text{ V/m}} = 3.03 \times 10^{-11} \text{ C}.$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

23.66. (a) IDENTIFY: Calculate the potential due to each thin ring and integrate over the disk to find the potential. V is a scalar so no components are involved.

SET UP: Consider a thin ring of radius y and width dy . The ring has area $2\pi y dy$ so the charge on the ring is $dq = \sigma(2\pi y dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance x from the ring is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2 + y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0} \frac{y dy}{\sqrt{x^2 + y^2}}.$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{y dy}{\sqrt{x^2 + y^2}} = \frac{\sigma}{2\epsilon_0} \left[\sqrt{x^2 + y^2} \right]_0^R = \frac{\sigma}{2\epsilon_0} (\sqrt{x^2 + R^2} - x).$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2 + R^2} = x(1 + R^2/x^2)^{1/2} \approx x(1 + R^2/2x^2) \text{ so } \sqrt{x^2 + R^2} - x \approx R^2/2x.$$

$$\text{Then } V \approx \frac{\sigma}{2\epsilon_0} \frac{R^2}{2x} = \frac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \frac{Q}{4\pi\epsilon_0 x}, \text{ as expected.}$$

(b) **IDENTIFY and SET UP:** Use $E_x = -\frac{\partial V}{\partial x}$ to calculate E_x .

EXECUTE: $E_x = -\frac{\partial V}{\partial x} = -\frac{\sigma}{2\epsilon_0} \left(\frac{x}{\sqrt{x^2 + R^2}} - 1 \right) = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right).$

EVALUATE: Our result agrees with the results of Example 21.11.

- 23.67. IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell of radius r and thickness dr is $dq = \rho 4\pi r^2 dr$, and $\rho = Q/(4/3 \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3 \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq , where V is the potential due to q , which is $q/4\pi\epsilon_0 r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int Vdq = \int \frac{q}{4\pi\epsilon_0 r} dq = \int_0^R \frac{\rho \frac{4}{3} \pi r^3}{4\pi\epsilon_0 r} (\rho 4\pi r^2 dr) = \frac{3}{5} \left(\frac{1}{4\pi\epsilon_0} \frac{Q^2}{R} \right)$$

where we have substituted $\rho = Q/(4/3 \pi R^3)$ and simplified the result.

EVALUATE: For a point charge, $R \rightarrow 0$ so $U \rightarrow \infty$, which means that a point charge should have infinite self-energy. This suggests that either point charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

- 23.68. IDENTIFY:** Divide the rod into infinitesimal segments with charge dq . The potential dV due to the segment is $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$. Integrate over the rod to find the total potential.

SET UP: $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a d\theta$.

EXECUTE: $dV = \frac{1}{4\pi\epsilon_0} \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{a} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\pi a} \frac{dl}{a} = \frac{1}{4\pi\epsilon_0} \frac{Q d\theta}{\pi a}.$

$$V = \frac{1}{4\pi\epsilon_0} \int_0^\pi \frac{Q d\theta}{\pi a} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a}.$$

EVALUATE: All the charge of the ring is the same distance a from the center of curvature.

- 23.69. IDENTIFY and SET UP:** The sphere no longer behaves as a point charge because we are inside of it. We know how the electric field varies with distance from the center of the sphere and want to use this to find the potential difference between the center and surface, which requires integration.

$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$. The electric field is radially outward, so $\vec{E} \cdot d\vec{l} = E dr$.

EXECUTE: For $r < R$: $E = \frac{kQr}{R^3}$. Integrating gives

$$V = -\int_\infty^R \vec{E} \cdot d\vec{r}' - \int_R^r \vec{E} \cdot d\vec{r}' = \frac{kQ}{R} - \frac{kQ}{R^3} \int_R^r r' dr' = \frac{kQ}{R} - \frac{kQ}{R^3} \frac{1}{2} r'^2 \Big|_R^r = \frac{kQ}{R} + \frac{kQ}{2R} - \frac{kQr^2}{2R^3} = \frac{kQ}{2R} \left[3 - \frac{r^2}{R^2} \right].$$
 At

the center of the sphere, $r = 0$ and $V_1 = \frac{3kQ}{2R}$. At the surface of the sphere, $r = R$ and $V_2 = \frac{kQ}{R}$. The

potential difference is $V_1 - V_2 = \frac{kQ}{2R} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(4.00 \times 10^{-6} \text{ C})}{2(0.0500 \text{ m})} = 3.60 \times 10^5 \text{ V}.$

EVALUATE: To check our answer, we could actually do the integration. We can use the fact that

$$E = \frac{kQr}{R^3} \text{ so } V_1 - V_2 = \int_0^R E dr = \frac{kQ}{R^3} \int_0^R r dr = \frac{kQ}{R^3} \left(\frac{R^2}{2} \right) = \frac{kQ}{2R}.$$

23.70. IDENTIFY: The potential at the surface of a uniformly charged sphere is $V = \frac{kQ}{R}$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume are conserved.

EXECUTE: (a) $V = \frac{kQ}{R} = \frac{k(-3.60 \times 10^{-12} \text{ C})}{6.50 \times 10^{-4} \text{ m}} = -49.8 \text{ V}.$

(b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2} R = 8.19 \times 10^{-4} \text{ m}$ and the new charge is $Q_{\text{new}} = 2Q = -7.20 \times 10^{-12} \text{ C}$. The new potential is

$$V_{\text{new}} = \frac{kQ_{\text{new}}}{R_{\text{new}}} = \frac{k(-7.20 \times 10^{-12} \text{ C})}{8.19 \times 10^{-4} \text{ m}} = -79.0 \text{ V}.$$

EVALUATE: The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\frac{2}{2^{1/3}} = 2^{2/3} \approx 1.6$.

23.71. IDENTIFY: Slice the rod into thin slices and use $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ to calculate the potential due to each

slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point P are shown in Figure 23.71a.

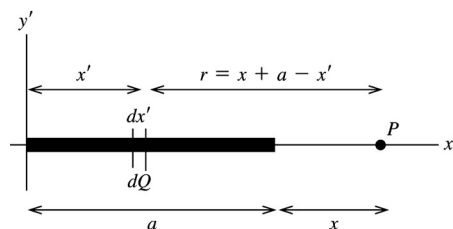


Figure 23.71a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance x given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx' . Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point P . The potential at P due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0} \left(\frac{dQ}{r} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \left(\frac{dx'}{x + a - x'} \right).$$

Compute the total V at P due to the entire rod by integrating dV over the length of the rod ($x' = 0$ to $x' = a$):

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a} \int_0^a \frac{dx'}{(x + a - x')} = \frac{Q}{4\pi\epsilon_0 a} [-\ln(x + a - x')]_0^a = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x + a}{x}\right).$$

EVALUATE: As $x \rightarrow \infty$, $V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x}{x}\right) = 0$.

(b) SET UP: An infinitesimal slice of the rod and its distance from point R are shown in Figure 23.71b.

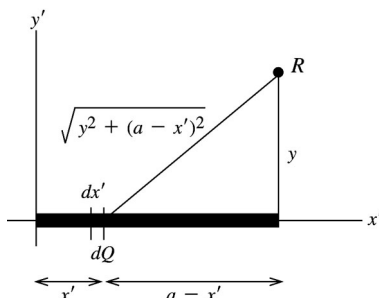


Figure 23.71b

$dQ = (Q/a)dx'$ as in part (a).

Each slice dQ is a distance $r = \sqrt{y^2 + (a - x')^2}$ from point R .

EXECUTE: The potential dV at R due to the small slice dQ is

$$dV = \frac{1}{4\pi\epsilon_0} \left(\frac{dQ}{r} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q}{a} \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a} \int_0^a \frac{dx'}{\sqrt{y^2 + (a - x')^2}}.$$

In the integral make the change of variable $u = a - x'$; $du = -dx'$

$$V = -\frac{Q}{4\pi\epsilon_0 a} \int_a^0 \frac{du}{\sqrt{y^2 + u^2}} = -\frac{Q}{4\pi\epsilon_0 a} \left[\ln\left(u + \sqrt{y^2 + u^2}\right) \right]_a^0.$$

$$V = -\frac{Q}{4\pi\epsilon_0 a} \left[\ln y - \ln(a + \sqrt{y^2 + a^2}) \right] = \frac{Q}{4\pi\epsilon_0 a} \left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right) \right].$$

(The expression for the integral was found in Appendix B.)

EVALUATE: As $y \rightarrow \infty$, $V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{y}{y}\right) = 0$.

(c) SET UP: part (a): $V = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{x+a}{x}\right) = \frac{Q}{4\pi\epsilon_0 a} \ln\left(1 + \frac{a}{x}\right)$.

From Appendix B, $\ln(1+u) = u - u^2/2 \dots$, so $\ln(1+a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \left(\frac{a}{x}\right) = \frac{Q}{4\pi\epsilon_0 x}$. For large x , V becomes the potential of a point charge.

$$\text{part (b): } V = \frac{Q}{4\pi\epsilon_0 a} \left[\ln\left(\frac{a + \sqrt{a^2 + y^2}}{y}\right) \right] = \frac{Q}{4\pi\epsilon_0 a} \ln\left(\frac{a}{y} + \sqrt{1 + \frac{a^2}{y^2}}\right).$$

From Appendix B, $\sqrt{1 + a^2/y^2} = (1 + a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \dots$

Thus $a/y + \sqrt{1 + a^2/y^2} \rightarrow 1 + a/y + a^2/2y^2 + \dots \rightarrow 1 + a/y$. And then using $\ln(1+u) \approx u$ gives

$$V \rightarrow \frac{Q}{4\pi\epsilon_0 a} \ln(1 + a/y) \rightarrow \frac{Q}{4\pi\epsilon_0 a} \left(\frac{a}{y} \right) = \frac{Q}{4\pi\epsilon_0 y}.$$

EVALUATE: For large y , V becomes the potential of a point charge.

23.72. IDENTIFY: We are dealing with a charged bar in an external electric field.

SET UP and EXECUTE: (a) We want V as a function of y . E is constant so $V_y - V_0 = -E_y y$, which gives

$$V(y) = V_0 - Ey.$$

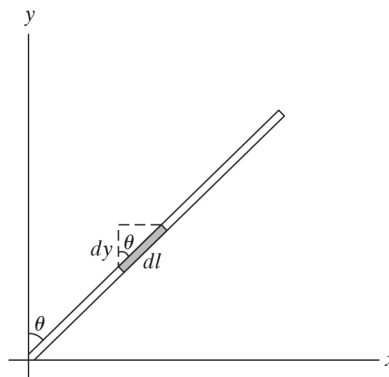


Figure 23.72

(b) We want the potential energy. Refer to Fig. 23.72. $U = \int V dq$. $dq = \lambda dl$ with $\lambda = Q/L$, and

$$dl \cos \theta = dy. \quad U = \int_0^{L \cos \theta} (V_0 - Ey) (Q/L) \frac{dy}{\cos \theta} = \frac{Q}{L} \left(V_0 y - \frac{Ey^2}{2} \right) \bigg|_0^{L \cos \theta} = Q \left(V_0 - \frac{EL \cos \theta}{2} \right).$$

(c) We want V_0 so $U = 0$ when $\theta = 0^\circ$. $Q \left(V_0 - \frac{EL \cos \theta}{2} \right) = 0$ gives $V_0 = EL/2$.

(d) We want the angular speed ω at $\theta = 0^\circ$ if the bar is released from rest at $\theta = 90^\circ$. Energy conservation gives $U_0 + K_0 = U_{90} + K_{90}$. We can neglect gravity, so $\frac{1}{2} I \omega^2 = QV_0$. Using the result from

(c) and $I = \frac{1}{3} ML^2$ gives $\omega = \sqrt{\frac{3QE}{ML}}$.

(e) We want the frequency f of oscillation. Apply $\Sigma \tau_z = I \alpha_z$. $-QE \frac{L}{2} \sin \theta = \frac{1}{3} ML^2 \frac{d^2 \theta}{dt^2}$. This gives

$$\frac{d^2 \theta}{dt^2} = - \left(\frac{3QE}{2ML} \right) \sin \theta \approx - \left(\frac{3QE}{2ML} \right) \theta \text{ for small amplitude oscillations. } \omega = \sqrt{\frac{3QE}{2ML}}, \text{ so}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{3QE}{2ML}}.$$

EVALUATE: This situation is very similar to an upside-down pendulum.

23.73. IDENTIFY: We are modeling forces in the atomic nucleus.

SET UP and EXECUTE: (a) We want the potential energy U . $U = U_{\text{spring}} + U_{\text{electric}}$, so

$$U = \frac{1}{2}kd^2 + \frac{1}{4\pi\epsilon_0}\frac{e^2}{d}.$$

(b) We want the equilibrium separation d_0 . When U is a minimum, $dU/dd = 0$. Taking the derivative of

the energy in (a) gives $kd - \frac{1}{4\pi\epsilon_0}\frac{e^2}{d^2} = 0$. Solve for $d = d_0$: $d_0 = \left(\frac{e^2}{4\pi\epsilon_0 k}\right)^{1/3}$.

(c) We want k . Solve the result of (b) for k and use $d_0 = 1.00 \text{ fm} = 1.00 \times 10^{-15} \text{ m}$, giving $k = 2.30 \times 10^{17} \text{ N/m}$.

(d) We want the stored energy. $U_0 = \frac{1}{2}kd_0^2 + \frac{1}{4\pi\epsilon_0}\frac{e^2}{d_0}$. Using d_0 and k from above gives

$$U_0 = 3.45 \times 10^{-13} \text{ J} = 2.16 \text{ MeV}.$$

(e) We want the speed v . $U_0 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$, so $v = \sqrt{U_0/m} = 1.44 \times 10^7 \text{ m/s}$.

EVALUATE: $v/c = 1.44/3.0 = 0.048$, so $v \approx 5\%$ of the speed of light.

23.74. IDENTIFY and SET UP: For points outside of them, the spheres behave as though all the charge were concentrated at their centers. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

EXECUTE: (a) $E_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R_1^2}$, $V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{R_1} = R_1 E_1$.

(b) Two conditions must be met:

1) Let q_1 and q_2 be the final charges of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation).

2) Let V_1 and V_2 be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$.

$$q_1 R_2 = q_2 R_1 = (Q_1 - q_1) R_1.$$

This gives $q_1 = (R_1/[R_1 + R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1 + R_2]) = Q_1(R_2/[R_1 + R_2])$.

(c) $V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{R_1} = \frac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$ and $V_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{R_2} = \frac{Q_1}{4\pi\epsilon_0(R_1 + R_2)}$, which equals V_1 as it should.

(d) $E_1 = \frac{V_1}{R_1} = \frac{Q_1}{4\pi\epsilon_0 R_1(R_1 + R_2)}$. $E_2 = \frac{V_2}{R_2} = \frac{Q_1}{4\pi\epsilon_0 R_2(R_1 + R_2)}$.

EVALUATE: Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

23.75. IDENTIFY: Apply conservation of energy, $K_a + U_a = K_b + U_b$.

SET UP: Assume the particles initially are far apart, so $U_a = 0$. The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

EXECUTE: Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$$9.50 \text{ MeV} = \frac{k(2e)(82e)}{r} \quad \text{and} \quad r = \frac{k(164)(1.60 \times 10^{-19} \text{ C})^2}{(9.50 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 2.48 \times 10^{-14} \text{ m}.$$

EVALUATE: The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

23.76. IDENTIFY and SET UP: Apply $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$. $\frac{W_{a \rightarrow b}}{q_0} = V_a - V_b$ and $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$.

EXECUTE: (a) $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -2Ax\hat{i} + 6Ay\hat{j} - 2Az\hat{k}.$

(b) A charge is moved in along the z -axis. The work done is given by

$$W = q \int_{z_0}^0 \vec{E} \cdot \hat{k} dz = q \int_{z_0}^0 (-2Az) dz = +(Aq)z_0^2. \text{ Therefore,}$$

$$A = \frac{W_{a \rightarrow b}}{qz_0^2} = \frac{6.00 \times 10^{-5} \text{ J}}{(1.5 \times 10^{-6} \text{ C})(0.250 \text{ m})^2} = 640 \text{ V/m}^2.$$

(c) $\vec{E}(0,0,0.250) = -2(640 \text{ V/m}^2)(0.250 \text{ m})\hat{k} = -(320 \text{ V/m})\hat{k}.$

(d) In every plane parallel to the xz -plane, y is constant, so $V(x,y,z) = Ax^2 + Az^2 - C$, where

$$C = 3Ay^2. \quad x^2 + z^2 = \frac{V+C}{A} = R^2, \text{ which is the equation for a circle since } R \text{ is constant as long as we}$$

have constant potential on those planes.

(e) $V = 1280 \text{ V}$ and $y = 2.00 \text{ m}$, so $x^2 + z^2 = \frac{1280 \text{ V} + 3(640 \text{ V/m}^2)(2.00 \text{ m})^2}{640 \text{ V/m}^2} = 14.0 \text{ m}^2$ and the radius

of the circle is 3.74 m .

EVALUATE: In any plane parallel to the xz -plane, \vec{E} projected onto the plane is radial and hence perpendicular to the equipotential circles.

23.77. IDENTIFY and SET UP: We know that the potential is of the mathematical form $V(x,y,z) = Ax^l + By^m + Cz^n + D$. We also know that $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = -\frac{\partial V}{\partial z}$. Various measurements are given

in the table with the problem in the text.

EXECUTE: (a) First get A , B , C , and D using data from the table in the problem.

$$V(0, 0, 0) = 10.0 \text{ V} = 0 + 0 + 0 + D, \text{ so } D = 10.0 \text{ V}.$$

$$V(1.00, 0, 0) = A(1.00 \text{ m})^l + 0 + 0 + 10.0 \text{ V} = 4.00 \text{ V}, \text{ so } A = -6.0 \text{ V} \cdot \text{m}^{-l}.$$

$$V(0, 1.00, 0) = B(1.00 \text{ m})^m + 10.0 \text{ V} = 6.0 \text{ V}, \text{ so } B = -4.0 \text{ V} \cdot \text{m}^{-m}.$$

$$V(0, 0, 1.00 \text{ m}) = C(1.00 \text{ m})^n + 10.0 \text{ V} = 8.0 \text{ V}, \text{ so } C = -2.0 \text{ V} \cdot \text{m}^{-n}.$$

Now get l , m , and n .

$$E_x = -\frac{\partial V}{\partial x} = -lAx^{l-1}, \text{ and from the table we know that } E_x(1.00, 0, 0) = 12.0 \text{ V/m. Therefore}$$

$$-l(-6.0 \text{ V} \cdot \text{m}^{-l})(1.00 \text{ m})^{l-1} = 12.0 \text{ V/m}.$$

$$l(6.0 \text{ V} \cdot \text{m}^{-l}) = 12.0 \text{ V/m}.$$

$$l = 2.0.$$

$$E_y = -\frac{\partial V}{\partial y} = -mBy^{m-1}.$$

$$E_y(0, 1.00, 0) = -m(-4.0 \text{ V} \cdot \text{m}^{-m})(1.00 \text{ m})^{m-1} = 12.0 \text{ V/m}.$$

$$m = 3.0.$$

$$E_z = -\frac{\partial V}{\partial z} = -nCz^{n-1}.$$

$$E_z(0, 0, 1.00) = -n(-2.0 \text{ V} \cdot \text{m}^{-n})(1.00 \text{ m})^{n-1} = 12.0 \text{ V/m}.$$

$$n = 6.0.$$

Now that we have l , m , and n , we see the units of A , B , and C , so

$$A = -6.0 \text{ V/m}^2.$$

$$B = -4.0 \text{ V/m}^3.$$

$$C = -2.0 \text{ V/m}^6.$$

Therefore the equation for $V(x, y, z)$ is

$$V = (-6.0 \text{ V/m}^2)x^2 + (-4.0 \text{ V/m}^3)y^3 + (-2.0 \text{ V/m}^6)z^6 + 10.0 \text{ V}.$$

(b) At $(0, 0, 0)$: $V = 0$ and $E = 0$ (from the table with the problem).

At $(0.50 \text{ m}, 0.50 \text{ m}, 0.50 \text{ m})$:

$$V = (-6.0 \text{ V/m}^2)(0.50 \text{ m})^2 + (-4.0 \text{ V/m}^3)(0.50 \text{ m})^3 + (-2.0 \text{ V/m}^6)(0.50 \text{ m})^6 + 10.0 \text{ V} = 8.0 \text{ V}.$$

$$E_x = -\frac{\partial V}{\partial x} = -(-12.0 \text{ V/m}^2)x = (12.0 \text{ V/m}^2)(0.50 \text{ m}) = 6.0 \text{ V/m}.$$

$$E_y = -\frac{\partial V}{\partial y} = -3(-4.0 \text{ V/m}^3)y^2 = (12 \text{ V/m}^3)(0.50 \text{ m})^2 = 3.0 \text{ V/m}.$$

$$E_z = -\frac{\partial V}{\partial z} = -(-12.0 \text{ V/m}^6)z^5 = (12.0 \text{ V/m}^6)(0.50 \text{ m})^5 = 0.375 \text{ V/m}.$$

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(6.0 \text{ V/m})^2 + (3.0 \text{ V/m})^2 + (0.375 \text{ V/m})^2} = 6.7 \text{ V/m}.$$

At $(1.00 \text{ m}, 1.00 \text{ m}, 1.00 \text{ m})$:

Follow the same procedure as above. The results are $V = -2.0 \text{ V}$, $E = 21 \text{ V/m}$.

EVALUATE: We know that l , m , and n must be greater than 1 because the components of the electric field are all zero at $(0, 0, 0)$.

23.78. IDENTIFY and SET UP: Energy is conserved and the potential energy is $U = k \frac{q_1 q_2}{r}$. $K_1 + U_1 = K_2 + U_2$.

EXECUTE: (a) Energy conservation gives $K_1 + 0 = K_2 + U_2$.

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + k \frac{qQ}{x} \rightarrow v^2 = v_0^2 - \frac{2kqQ}{m} \cdot \frac{1}{x}.$$

On a graph of v^2 versus $1/x$, the graph of this equation will be a straight line with y -intercept equal to v_0^2

and slope equal to $-\frac{2kqQ}{m}$.

(b) With the given equation of the line in the problem, we have $v^2 = 400 \text{ m}^2/\text{s}^2 - (15.75 \text{ m}^3/\text{s}^2) \frac{1}{x}$. As x

gets very large, $1/x$ approaches zero, so $v_0 = \sqrt{400 \text{ m}^2/\text{s}^2} = 20 \text{ m/s}$.

(c) The slope is $-\frac{2kqQ}{m} = -15.75 \text{ m}^3/\text{s}^2$, which gives

$$Q = -m(\text{slope})/2kq = -(4.00 \times 10^{-4} \text{ kg})(-15.75 \text{ m}^3/\text{s}^2)/[2k(5.00 \times 10^{-8} \text{ C})] = +7.01 \times 10^{-6} \text{ C} = +7.01 \mu\text{C}.$$

(d) The particle is closest when its speed is zero, so

$$v^2 = 400 \text{ m}^2/\text{s}^2 - (15.75 \text{ m}^3/\text{s}^2) \frac{1}{x} = 0, \text{ which gives } x = 3.94 \times 10^{-2} \text{ m} = 3.94 \text{ cm}.$$

EVALUATE: From the graph in the problem, we see that v^2 decreases as $1/x$ increases, so v^2 decreases as x decreases. This means that the positively charged particle is slowing down as it gets closer to the sphere, so the sphere is repelling it. Therefore the sphere must be positively charged, as we found.

23.79. IDENTIFY: When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

SET UP: The volume of the drop is related to its radius r by $V = \frac{4}{3}\pi r^3$.

EXECUTE: (a) $F_g = mg = \frac{4\pi r^3}{3}\rho g$. $F_e = |q|E = |q|V_{AB}/d$. $F_e = F_g$ gives $|q| = \frac{4\pi}{3} \frac{\rho r^3 g d}{V_{AB}}$.

(b) $\frac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t$ gives $r = \sqrt{\frac{9\eta v_t}{2\rho g}}$. Using this result to replace r in the expression in part (a) gives

$$|q| = \frac{4\pi}{3} \frac{\rho g d}{V_{AB}} \left[\sqrt{\frac{9\eta v_t}{2\rho g}} \right]^3 = 18\pi \frac{d}{V_{AB}} \sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

(c) We use the values for V_{AB} and v_t given in the table in the problem and the formula

$$|q| = 18\pi \frac{d}{V_{AB}} \sqrt{\frac{\eta^3 v_t^3}{2\rho g}} \text{ from (c). For example, for drop 1 we get}$$

$$|q| = 18\pi \frac{1.00 \times 10^{-3} \text{ m}}{9.16 \text{ V}} \sqrt{\frac{(1.81 \times 10^{-5} \text{ N} \cdot \text{s/m}^2)^3 (2.54 \times 10^{-5} \text{ m/s})^3}{2(824 \text{ kg/m}^3)(9.80 \text{ m/s}^2)}} = 4.79 \times 10^{-19} \text{ C. Similar calculations}$$

for the remaining drops gives the following results:

Drop 1: $4.79 \times 10^{-19} \text{ C}$

Drop 2: $1.59 \times 10^{-19} \text{ C}$

Drop 3: $8.09 \times 10^{-19} \text{ C}$

Drop 4: $3.23 \times 10^{-19} \text{ C}$

(d) Use $n = q/e_2$ to find the number of excess electrons on each drop. Since all quantities have a power of 10^{-19} C , this factor will cancel, so all we need to do is divide the coefficients of 10^{-19} C . This gives

Drop 1: $n = q_1/q_2 = 4.79/1.59 = 3$ excess electrons

Drop 2: $n = q_2/q_2 = 1$ excess electron

Drop 3: $n = q_3/q_2 = 8.09/1.59 = 5$ excess electrons

Drop 4: $n = q_4/q_2 = 3.23/1.59 = 2$ excess electrons

(e) Using $q = -ne$ gives $e = -q/n$. All the charges are negative, so e will come out positive. Thus we get

Drop 1: $e_1 = q_1/n_1 = (4.79 \times 10^{-19} \text{ C})/3 = 1.60 \times 10^{-19} \text{ C}$

Drop 2: $e_2 = q_2/n_2 = (1.59 \times 10^{-19} \text{ C})/1 = 1.59 \times 10^{-19} \text{ C}$

Drop 3: $e_3 = q_3/n_3 = (8.09 \times 10^{-19} \text{ C})/5 = 1.62 \times 10^{-19} \text{ C}$

Drop 4: $e_4 = q_4/n_4 = (3.23 \times 10^{-19} \text{ C})/2 = 1.61 \times 10^{-19} \text{ C}$

The average is

$$e_{\text{av}} = (e_1 + e_2 + e_3 + e_4)/4 = [(1.60 + 1.59 + 1.62 + 1.61) \times 10^{-19} \text{ C}]/4 = 1.61 \times 10^{-19} \text{ C}.$$

EVALUATE: The result $e = 1.61 \times 10^{-19} \text{ C}$ is very close to the well-established value of $1.60 \times 10^{-19} \text{ C}$.

23.80. IDENTIFY: We want the potential due to an annulus. For this we need to use calculus.

SET UP: Fig. 23.80 shows the mathematical set up. $V = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{D}$, where $D = \sqrt{z^2 + r^2}$, $dq = \sigma dA$,

and $dA = 2\pi r dr$.

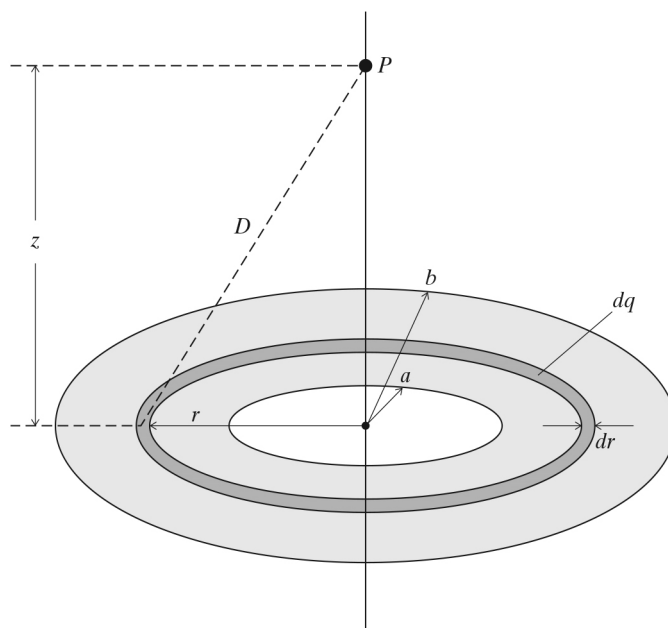


Figure 23.80

EXECUTE: (a) $V = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{D} = \int_a^b \frac{\sigma 2\pi r dr}{4\pi\epsilon_0 \sqrt{z^2 + r^2}} = \frac{\sigma}{4\epsilon_0} \int_a^b \frac{2r dr}{\sqrt{z^2 + r^2}}$. To integrate, let $u = z^2 + r^2$ so

$du = 2r dr$. The integral is then of the form $\int \frac{du}{u^{1/2}} = 2u^{1/2}$, which gives $V = \frac{\sigma}{4\epsilon_0} \left(2\sqrt{z^2 + r^2} \right) \Big|_a^b$.

$$V = \frac{\sigma}{2\epsilon_0} \left(\sqrt{z^2 + b^2} - \sqrt{z^2 + a^2} \right).$$

(b) We want E_z . Use $E_z = -dV/dz$ for the result from (a), giving $\vec{E} = \frac{\sigma z}{2\epsilon_0} \left(\frac{1}{\sqrt{z^2 + a^2}} - \frac{1}{\sqrt{z^2 + b^2}} \right) \hat{k}$.

(c) Let $a \rightarrow 0$: $E_z \rightarrow \frac{\sigma z}{2\epsilon_0} \left(\frac{1}{z} - \frac{1}{\sqrt{z^2 + b^2}} \right)$.

Let $b \rightarrow \infty$: The second term approaches zero, so $E_z \rightarrow \frac{\sigma z}{2\epsilon_0} \left(\frac{1}{z} \right) = \frac{\sigma}{2\epsilon_0}$, which is the field for an infinite sheet of charge.

(d) We want the potential at the origin. At the origin, $z = 0$, so $V(0) = \frac{\sigma}{2\epsilon_0} (b - a)$.

$$\sigma = \frac{Q}{A} = \frac{Q}{\pi b^2 - \pi a^2} = \frac{Q}{\pi(b^2 - a^2)}, \text{ so } V(0) = \frac{Q}{2\pi\epsilon_0(b^2 - a^2)}(b - a) = \frac{Q}{2\pi\epsilon_0(b + a)}.$$

Using $a = 5.00$ cm, $b = 10.0$ cm, and $Q = 1.00$ μC gives $V(0) = 120$ kV.

(e) We want the speed v . Energy conservation gives $qV = \frac{1}{2}mv^2$, so $v = \sqrt{2qV/m}$. Using $V = 120$ kV

and $m = 0.00100$ kg gives $v = 15.5$ m/s.

EVALUATE: The electric field is *not* an inverse square in z and the potential is *not* an inverse z .

23.81. IDENTIFY: We are dealing with a heart cell that is modeled as a cylindrical shell of charge.

SET UP: $E = \frac{\lambda}{2\pi\epsilon_0 r}$.

EXECUTE: (a) We want the charge Q . Using $E = \frac{\lambda}{2\pi\epsilon_0 r}$ with $\lambda = Q/L$, we have

$$\Delta V = \int E_r dr = \int \frac{\lambda}{2\pi\epsilon_0} dr = \int_a^b \frac{Q/L}{2\pi\epsilon_0 r} dr = \frac{Q/L}{2\pi\epsilon_0} \ln(b/a). \text{ Solving for } Q \text{ and using } \Delta V = 90.0 \text{ mV}, a =$$

$9.0 \mu\text{m}$, $b = 10.0 \mu\text{m}$, and $L = 100 \mu\text{m}$ gives $Q = 4.75 \times 10^{-15} \text{ C}$.

(b) We want the electric field just inside the membrane at $r = a = 9.0 \mu\text{m}$. Using $E = \frac{Q/L}{2\pi\epsilon_0 a}$ gives $E = 94.9 \text{ kV/m}$.

(c) We want to find out how much charged moved across the cell wall. The potential went from -90.0 mV to $+20.0 \text{ mV}$, so $\Delta V = 110 \text{ mV}$. Using information from (a) gives $Q = \frac{(2\pi\epsilon_0)(\Delta V)L}{\ln(b/a)}$.

With $\Delta V = 110 \text{ mV}$ and the given numbers, we get $Q = 5.81 \times 10^{-15} \text{ C}$.

(d) We want to know how many Na^+ ions crossed the boundary. $N = \frac{5.81 \times 10^{-15} \text{ C}}{1.60 \times 10^{-19} \text{ C/ion}} = 36,300$.

EVALUATE: Note how small the charges, fields, and potentials are in cellular electrical processes. By contrast the electric field of the proton in a hydrogen atom at the location of the electron is approximately $6 \times 10^{11} \text{ V/m}$.

23.82. IDENTIFY: Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\frac{\partial V}{\partial x}$.

SET UP: Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate z along the x -axis, relative to the center of the tube.

EXECUTE: (a) For an infinitesimal slice of the finite cylinder, we have the potential

$$dV = \frac{k dQ}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L} \frac{dz}{\sqrt{(x-z)^2 + R^2}}. \text{ Integrating gives}$$

$$V = \frac{kQ}{L} \int_{-L/2}^{L/2} \frac{dz}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L} \int_{-L/2-x}^{L/2-x} \frac{du}{\sqrt{u^2 + R^2}} \text{ where } u = x - z. \text{ Therefore,}$$

$$V = \frac{kQ}{L} \ln \left[\frac{\sqrt{(L/2-x)^2 + R^2} + (L/2-x)}{\sqrt{(L/2+x)^2 + R^2} - L/2-x} \right] \text{ on the cylinder axis.}$$

$$(b) \text{ For } L \ll R, V \approx \frac{kQ}{L} \ln \left[\frac{\sqrt{(L/2-x)^2 + R^2} + L/2-x}{\sqrt{(L/2+x)^2 + R^2} - L/2-x} \right] \approx \frac{kQ}{L} \ln \left[\frac{\sqrt{x^2 - xL + R^2} + L/2-x}{\sqrt{x^2 + xL + R^2} - L/2-x} \right].$$

$$V \approx \frac{kQ}{L} \ln \left[\frac{\sqrt{1 - xL/(R^2 + x^2)} + (L/2-x)/\sqrt{R^2 + x^2}}{\sqrt{1 + xL/(R^2 + x^2)} + (-L/2-x)/\sqrt{R^2 + x^2}} \right] = \frac{kQ}{L} \ln \left[\frac{1 - xL/2(R^2 + x^2) + (L/2-x)/\sqrt{R^2 + x^2}}{1 + xL/2(R^2 + x^2) + (-L/2-x)/\sqrt{R^2 + x^2}} \right].$$

$$V \approx \frac{kQ}{L} \ln \left[\frac{1 + L/2\sqrt{R^2 + x^2}}{1 - L/2\sqrt{R^2 + x^2}} \right] = \frac{kQ}{L} \left(\ln \left[1 + \frac{L}{2\sqrt{R^2 + x^2}} \right] - \ln \left[1 - \frac{L}{2\sqrt{R^2 + x^2}} \right] \right).$$

$$V \approx \frac{kQ}{L} \frac{2L}{2\sqrt{x^2 + R^2}} = \frac{kQ}{\sqrt{x^2 + R^2}}, \text{ which is the same as for a ring.}$$

$$(c) E_x = -\frac{\partial V}{\partial x} = \frac{2kQ \left(\sqrt{(L-2x)^2 + 4R^2} - \sqrt{(L+2x)^2 + 4R^2} \right)}{\sqrt{(L-2x)^2 + 4R^2} \sqrt{(L+2x)^2 + 4R^2}}.$$

EVALUATE: For $L \ll R$ the expression for E_x reduces to that for a ring of charge, $E_x = \frac{kQx}{(x^2 + R^2)^{3/2}}$, as shown in Example 23.14.

23.83. IDENTIFY: Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

EXECUTE: $mv_1b = mv_2r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2}mv_2^2 + \frac{kq_1q_2}{r_2}$. $E_1 = 11 \text{ MeV} = 1.76 \times 10^{-12} \text{ J}$. r_2 is the

distance of closest approach. Substituting in for $v_2 = v_1 \left(\frac{b}{r_2} \right)$ we find $E_1 = E_1 \frac{b^2}{r_2^2} + \frac{kq_1q_2}{r_2}$.

$(E_1)r_2^2 - (kq_1q_2)r_2 - E_1b^2 = 0$. For $b = 10^{-12} \text{ m}$, $r_2 = 1.01 \times 10^{-12} \text{ m}$. For $b = 10^{-13} \text{ m}$, $r_2 = 1.11 \times 10^{-13} \text{ m}$. And for $b = 10^{-14} \text{ m}$, $r_2 = 2.54 \times 10^{-14} \text{ m}$.

EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases.

23.84. IDENTIFY and SET UP: The He ions are first accelerated toward the center and then accelerated away from the center, but always in the same direction. During the first acceleration, their charge is $-e$, and during the second acceleration it is $+2e$. The work-energy theorem gives $\Delta K = q\Delta V$. Call V the voltage at the center.

EXECUTE: (a) Toward the center: $\Delta K = q\Delta V = eV$.

Away from the center: $\Delta K = q\Delta V = 2eV$.

The ions gain 3.0 MeV of kinetic energy, so $eV + 2eV = 3.0 \text{ MeV}$.

$3eV = 3.0 \text{ MeV}$.

$V = +1.0 \text{ MV}$, since the e cancels. This is choice (d).

EVALUATE: The negative He^- ions are accelerating to higher potential, and the positive He^{++} ions are accelerating toward lower potential.

23.85. IDENTIFY and SET UP: Conservation of energy gives $K = U_{\text{electric}} = k \frac{q_1q_2}{r}$.

EXECUTE: Solve for Q : $Q = rK/kq = (10 \times 10^{-15} \text{ m})(3.0 \text{ MeV})/(2ek) = 1.67 \times 10^{-18} \text{ C}$. In terms of e , this is $Q = (1.67 \times 10^{-18} \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 10.4e \approx 11e$, so choice (b) is best.

EVALUATE: If $Q = 11e$, the atom is sodium (Na), which has an atomic mass of 23, compared to 4 for He. So it is reasonable to assume that the nucleus does not move appreciably, since it is about 6 times more massive than the He.

23.86. IDENTIFY and SET UP: The potential changes by 6.0 MV over a distance of 12 m. $E_{\text{av}} = \frac{\Delta V}{\Delta x}$.

EXECUTE: $E_{\text{av}} = \frac{\Delta V}{\Delta x} = (6.0 \text{ MV})/(12 \text{ m}) = 0.50 \times 10^6 \text{ V/m} = 500,000 \text{ V/m}$, which is choice (c).

EVALUATE: The actual variation of the field may be somewhat complicated, but the average value gives a good idea of a typical electric field in such apparatus.