Project Proposal fro CSE 528 - Orbifold Tutte Embedding

Xuan Li
SBU ID: 111676019

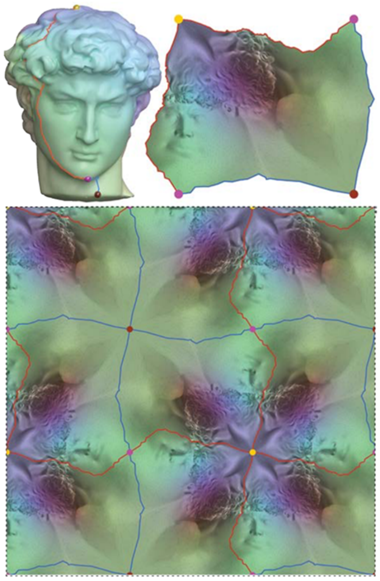

Fig. 1. An example of tiling of $\mathbb{R}^2$

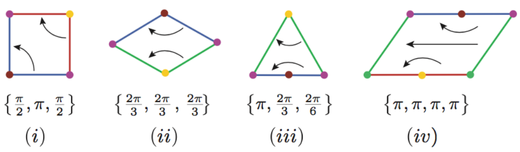

Fig. 2. 4 kinds of 2-dimension Euclidean orbifolds.

I. Introduction

This project is based on [1].

Injective parameterizations of surface meshes are vital for many applications in Computer Graphics, Geometry Processing and related fields. Only a few algorithms are guaranteed to produce injective parameterizations that are also globally optimal in some well-defined sense. The main approach known to provide such a guarantee is Tuttes embedding and its generalization to convex combination maps (CCM). However, CCM is currently limited to injective parameterizations of disk-type and toric surfaces, leaving the arguably most common case of spherical surfaces untreated. And this paper is to handle this problem.

This paper proposed the orbifold-Tutte planar embedding. It is a generalization of Tuttes embedding and CCM to other topologies. It can bijectively maps the original surface to a canonical, topologically equivalent, two-dimensional flat surface with cone singularities, called a Euclidean orbifold.

Orbifold is a generalization of the concept of manifold. Manifolds are locally euclidean, and orbifolds are locally euclidean under some group. There are 17 2-dimensional Euclidean orbifolds in total, which are corresponding to 17 wall paper groups. These orbifolds can be treated as the quotient space of $\mathbb{R}^2$ by some paper group. So $\mathbb{R}^2$ can be tiled seamlessly with the base domain of these orbifolds (i.e. Fig. 1). This is the key point in proving the injectiveness of orbifold Tutte embedding.

Out of all Euclidean orbifold, four of them have spherical topology (Fig. 2). This project will use these four orbifolds to achieve flat embedding of sphere-type surface.

The pipline of embedding can be summarized as follows:

1) Choose some cone vertices and slice through them to get a disk-type surface.
2) Solve a sparse linear system to get the global injective map from sliced surface to $\mathbb{R}^2$ which realizes the bijective map from original surface to its corresponding orbifold.

II. Weekly Plan

This project will reimplement the algorithm method of this paper and apply it to texture mapping. I will separate the whole software system into three components: view port with texture mapping function, mesh operations (for example, slice the mesh) and the sparse linear system solver. The weekly plan is as follows:

1) By week 2: Read the paper carefully to figure out the details of the background theory and the algorithm.
2) By week 4: Build a UI system which can show surface mesh and its texture.
3) By week 6: Implemented all mesh operations that the algorithm needed, including slicing the mesh along a line.
4) By week 8: Build the sparse solver for the algorithm.
5) By week 10: Test and improve the whole system.

If time is enough, I'll also implement the algorithms in [2] and [3]. They applied orbifold to different background geometries.

References

[1] Aigerman, N., & Lipman, Y. (2015). Orbifold Tutte Embeddings. ACM Transactions on Graphics (TOG) - Proceedings of ACM SIGGRAPH Asia 2015, 34(6).

[2] Aigerman, N., & Lipman, Y. (2016). Hyperbolic orbifold tutte embeddings. ACM Transactions on Graphics (TOG) - Proceedings of ACM SIGGRAPH Asia 2016, 35(6).

[3] Aigerman, N., Kovalsky, S. Z., & Lipman, Y. (2017). Spherical Orbifold Tutte Embeddings. ACM Transactions on Graphics (TOG), 36(4).