

Subject index

When a term is referred to more than one page, number(s) in **bold** refer to the page(s) containing the definition or technical details.

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