# ELEC5650 Homework3

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## Problem 1

Denote

$$g[C,R](X) = h \circ \tilde{g}(X)$$
$$h(X) = AXA' + Q$$
$$\tilde{g}(X) = X - XC'[CXC' + R]^{-1}CX$$
$$C_{1,2,...,n} = [C_1; C_2; ...; C_n]$$
$$R_{1,2,...,n} = \begin{bmatrix} R_1 & & & \\ & R_2 & & \\ & & .. & \\ & & & R_n \end{bmatrix}$$

We define function $g, \tilde{g}$ with parameters $R_{1,2,...,n}, C_{1,2,..,n}$ as $g_{1,2,...,n}, \tilde{g}_{1,2,...,n}$

(1) From the diagrams, we have the steady state

$$\bar{P} = g_{1,2,3,4,5}(\bar{P}) \tag{1}$$

and the error covariance for each diagram are

$$P_1 = \tilde{g}_1 \circ g_{1,2,3,4}(\bar{P})$$
$$P_2 = g_{1,4}(\bar{P})$$

Omitting $R, C$, because $\tilde{g}$ can be written as $\tilde{g}(X) = (X^{-1} + C'RC)^{-1}$, because $R_i > 0\ \forall i$, we can know $P_2 = g(\bar{P}) > 0$; also, $\tilde{g}(\bar{P}) > 0$, $g(\bar{P}) = h \circ \tilde{g} > 0$, similarly, $P_1 = \tilde{g} \circ h \circ \tilde{g}(\bar{P}) > 0$.

(2) From the lecture, we know tbar

$$g_{1,2,3,4}(\bar{P}) \geq g_{1,2,3,4,5}(\bar{P}) = \bar{P}$$
$$\tilde{g}_1(\bar{P}) \geq \tilde{g}_{1,4}(\bar{P}) = P_2$$

so we have

$$\begin{aligned} P_1 &= \tilde{g}_1 \circ g_{1,2,3,4}(\bar{P}) \geq \tilde{g}_1(\bar{P}) \\ &\geq \tilde{g}_{1,4}(\bar{P}) = P_2 \end{aligned}$$

Hence, $P_1 \geq P_2$ proved.

(3) The plotted figure is shown below, and from the listed code, we can obtain that $P_1 = 1.0110, P_2 = 0.5583$.

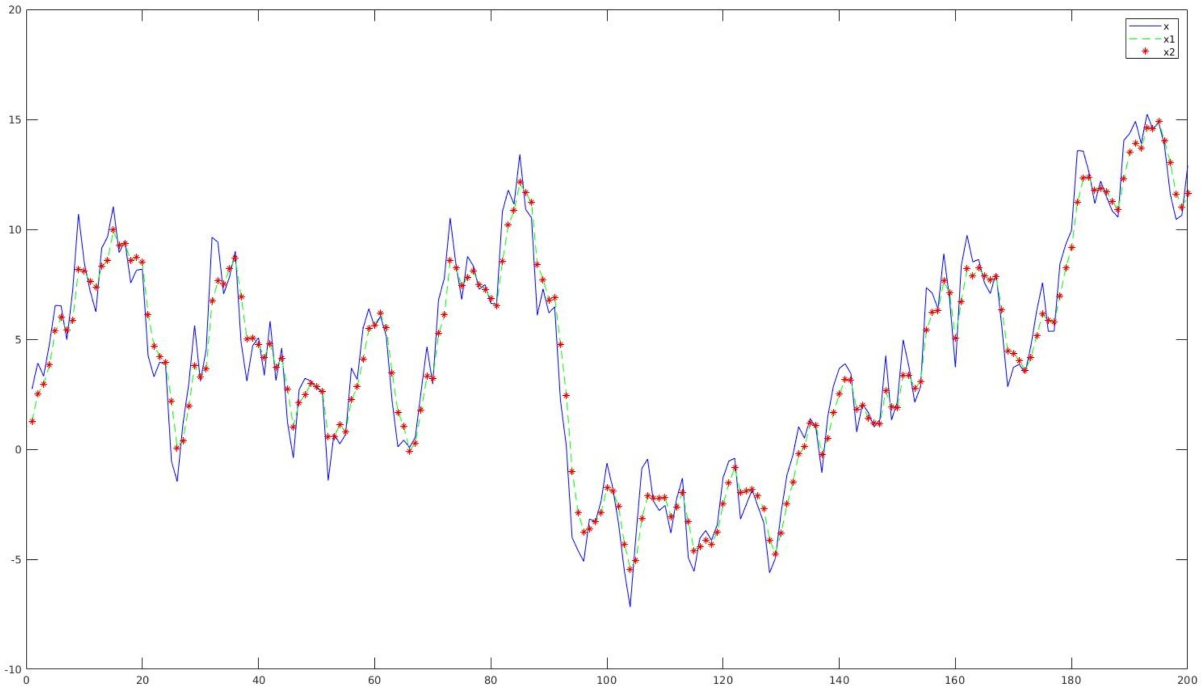


Figure 1: Plot results of $x_k, \hat{x}_k^1, \hat{x}_k^2$

```
function [x_next, P_next] = KalmanFilter(x, y, P, A, C, R, Q)
    x_ = A * x;
    P_ = A * P * A' + Q;
    k = P * C * inv(C' * P_ * C + R);
    x_next = x_ + k * (y - C * x_)';
    P_next = P_ - k * C' * P_;
end
```

Listing 1: KalmanFilter.m

```
%% system parameters
n = 200;
A = 0.99;
Q = 5;
C1 = 2;
R1 = 5;
C2 = 2 * ones(1,2);
R2 = 5 * eye(2);
C4 = 2 * ones(1,4);
R4 = 5 * eye(4);
C5 = 2 * ones(1,5);
R5 = 5 * eye(5);

%% initial state
x0 = 1;
P0 = 1;
x = zeros(n+1,1);
```

```
y = zeros(n+1,5);
x(1) = x0;
% estimator 1
x1 = zeros(n+1,1);
P1 = zeros(n+1,1);
x1(1) = x0;
P1(1) = P0;
% estimator 2
x2 = zeros(n+1,1);
P2 = zeros(n+1,1);
x2(1) = x0;
P2(1) = P0;

%% measurement y
for i = 1:n
    w = wgn(1,1,5);
    x(i+1) = A * x(i) + w;
    v = wgn(1,5,5);
    y(i+1,:) = C5 * x(i+1) + v;
end

%% esimator 1
for i = 1:n
    % when t-2, 5 sensor measurement
    if i > 2
        [x1(i-1), P1(i-1)] = KalmanFilter(x1(i-2), y(i-1, :), P1(i-2), A, C5, R5, Q);
    end
    % t-1, 4 sensor measurement
    if i > 1
        [x1(i), P1(i)] = KalmanFilter(x1(i-1), y(i, 1:4), P1(i-1), A, C4, R4, Q);
    end
    % t, 1 sensor measurement
    [x1(i+1), P1(i+1)] = KalmanFilter(x1(i), y(i+1, 1), P1(i), A, C1, R1, Q);
end

for i = 1:n
    % t-1, 5 sensor measurement
    if i > 1
        [x2(i), P1(i)] = KalmanFilter(x2(i-1), y(i, :), P2(i-1), A, C5, R5, Q);
    end
    % t, 2 sensor measurement
    [x2(i+1), P2(i+1)] = KalmanFilter(x2(i), y(i+1, 1:2), P2(i), A, C2, R2, Q);
end

t = 1:n;
figure;
plot(t, x(t+1), 'b');
hold on
plot(t, x1(t+1), 'g--');
hold on
plot(t, x2(t+1), 'r*');
```

```
hold on
legend('x', 'x1', 'x2');

%% solve dare
P = dare(A, C5, Q, R5);           % steady state
P1 = P - P * C4 * inv(C4' * P * C4 + R4) * C4' * P;
P1 = A * P1 * A' + Q;
P1 = P1 - P1 * C1 * inv(C1' * P1 * C1 + R1) * C1' * P1;
P2 = P - P * C2 * inv(C2' * P * C2 + R2) * C2' * P;
```

Listing 2: hw3.m