Kalman Filter Application in Mean-Variance Portfolio Management

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Overview

Introduction

Portfolio optimization is the process of selecting the best portfolio (asset distribution), out of the set of all portfolios being considered, according to some objective. The objective typically maximizes factors such as expected return, and minimizes costs like financial risk.

Suppose investor distribute the portfolio ω_k on the market X_k at time k to $k+1$, then the relative return is

$$r_k = \frac{X_{k+1} - X_k}{X_k}$$

and the return of this portfolio is

$$R = r_k \omega_k$$

Mean-Variance Portfolio Optimization

- Mean-Variance Portfolio Optimization minimize the exposure to risk under a certain level of expected returns, or a trade-off between risk and return expectations.
- Suppose we want to allocate the asset using a portfolio ω_k, then the expected mean and covariance of return are

$$\mu_k = (H_{k-1} + 1)\omega_k - 1$$
$$\sigma_k^2 = \omega_k{}' S_k \omega_k$$

where $H_{k-1} = [r_1(t_{k-1})\ r_2(t_{k-1})\ \dots\ r_n(t_{k-1})]$ is average returns over n risky assets up to time t_{k-1}, S_k is the historical sample covariance matrix of expected returns from n-risky assets observed up to time t_k.

Mean-Variance Portfolio Optimization

The optimization problem of MV Portfolio Optimization can be written as

$$\underset{\omega_k}{\arg\min}\ (1-\lambda)\sigma_k^2 - \lambda\mu_k{}^{a}$$

$$\text{subject to} \sum_{i}^{n} \omega_k^i = 1, \omega_k^i \geq 0$$

[a] This formulation is equivalent to the formulation from Markowitz (1952)

The $\omega_k^i \geq 0$ constraint means no short-selling, investor never borrows stocks from broker and sell them.
Specially, when $\lambda = 0$, the optimal solution is

$$\omega_k^\star = \frac{\mathbf{1}' S_k^{-1}}{\mathbf{1}' S_k^{-1} \mathbf{1}}$$

Mean-Variance Portfolio Management with Kalman Filter

- Though Mean-Variance optimization can get optimal solution in every time stamp, it may suffer from potential fluctuations of portfolio weights over time, or portfolio turnovers.
- In actual market, trading or transaction may have certain cost, saying 50 basis point.
- Portfolio turnover is defined by DeMiguel et al. (2009) as

$$Turnover(k) = \sum_{i=1}^{n} |\omega_k^i - \omega_{k-1}^i|$$

- Can a Kalman Filter model provide higher net returns, after adjustments for transaction costs are included, versus a comparable MV optimal model?

Kalman Filter Model

Assume a discrete time linear dynamic system, considering the following state-space form

$$x_{k+!} = A_k x_k + w_k$$
$$y_k = x_k + v_k$$
$$w_k \sim N(0, Q_k), v_k \sim N(0, R_k)$$
$$x_0 = \frac{1}{n}\mathbf{1}, P_0 = 0.1\mathbf{I}$$

where A_k depends on the Capital Asset Pricing Model (CAPM) model we use. KF model is initialized as uniformly distributed assert and an identity P_0.

Transition Model with 3-Factor CAMP Model

The transition model is

$$\hat{x}_{k|k-1} = A_{k-1}\hat{x}_{k-1|k-1}$$
$$P_{k|k-1} = A_{k-1}P_{k-1|k-1}A'_{k-1}$$

We begin by assuming that the previous M-months of m-daily portfolio returns can be estimated by an ordinary least squares (OLS) model

$$z_{k-1} = G_{k-1}F_{k-1} + \epsilon_{k-1}$$
$$G_{k-1} = [1_M \; r_{MKT}(t_{k-1}) \; r_{SMB}(t_{k-1}) \; r_{HML}(t_{k-1})]$$
$$F_{k-1} = [\alpha \; \beta_{MKT} \; \beta_{SMB} \; \beta_{HML}]$$

F_k is the coefficient we want to estimate, G_k is provided by Ken French's dataset.

Transition Model with 3-Factor CAMP Model

The actual portfolio return can also be written as

$$z_{k-1} = H_{k-1}\hat{x}_{k-1|k-1} = [r_1(t_{k-1})\ r_2(t_{k-1})\ \ldots\ r_n(t_{k-1})]\hat{x}_{k-1|k-1}$$

So the estimated coefficients F_k is

$$F_{k-1} = G_{k-1}^{+} H_{k-1}\hat{x}_{k-1|k-1}$$

where G_k^{+} is pseudoinverse of G_k.

The prediction of portfolio returns is found as a column vector of length of additional days predicted in the next month

$$\hat{z}_k = G_k F_k = [1_{M+1}\ r_{MKT}(t_k)\ r_{SMB}(t_k)\ r_{HML}(t_k)]$$

Transition Model with 3-Factor CAMP Model

The predicted portfolio weight $\hat{x}_{k|k}$ is solved by

$$\hat{z}_k = H_k \hat{x}_{k-1|k-1}$$

This yields,

$$\hat{x}_{k|k-1} = H_k^+ \hat{z}_k = [H_k^+ G_k G_{k-1}^+ H_{k-1}] \hat{x}_{k-1|k-1}$$

- Determine regression coefficients F_k based on actual returns from previous portfolio $\hat{x}_{k-1|k-1}$;
- Apply regression coefficients to determine k predicted portfolio returns;
- Estimate $\hat{x}_{k|k-1}$ using ordinary least squares with knowledge of $\hat{z}_k$.

Measurement Model with MV Optimization

MV Optimization is directly utilized as measurement, by assigning $y_k = \omega_k^\star$

$$\arg\min_{\omega_k} \; (1-\lambda)\sigma_k^2 - \lambda\mu_k$$
$$\text{subject to} \sum_i^n \omega_k^i = 1, \omega_k^i \geq 0$$

So the measurement update is

$$K_k = P_{k|k-1}[P_{k|k-1} + R]^{-1}$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k[y_k - \hat{x}_{k|k-1}]$$
$$P_{k|k} = P_{k|k-1} - K_k P_{k|k-1}$$

Noted that for all $\hat{x}_{k|k-1}, \hat{x}_{k|k}$, x should be re-normalized to 1 after update.

Empirical Results

Experiment on 17 industry dataset [1], selected 552 months, number of industries $n = 17$, and assuming the cost per transaction $c = 0.5\%$.

The comparison between two different risk level $\lambda = 0$, $\lambda = 0.5$ are conducted.

Risk Level, λ	KF	OPT
0	12.1%	10.5%
0.5	31.3%	34.8%

Table: Return statistics

[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Empirical Results

	Portfolio	Cumulative Return	Annual Return	Annual standard deviation
$\lambda = 0$	KF gross	9762%	10.5%	13.8%
	OPT gross	7037%	9.72%	13.1%
	KF net	6968%	9.70%	13.8%
	OPT net	5252%	9.04%	13.1%
$\lambda = 0.5$	KF gross	21572%	12.40%	15.1%
	OPT gross	22223%	12.48%	14.9%
	KF net	9026%	10.31%	15.2%
	OPT net	8421%	10.15%	15.0%

Table: Turnover at different risk level

Figure

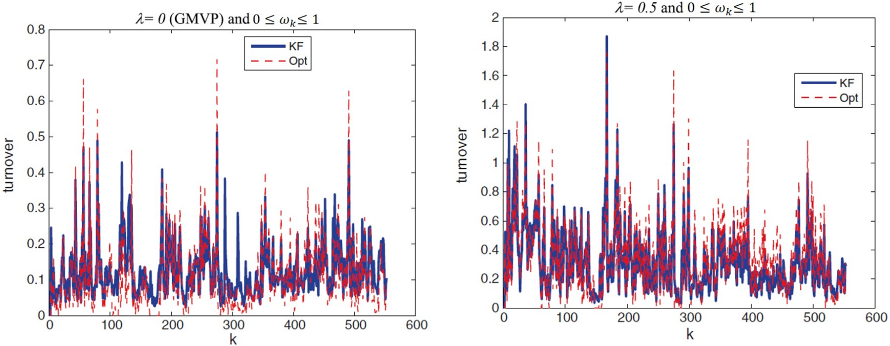

Figure: KF portfolio turnover and optimal portfolio turnover with different risk level

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