Physically-based Simulation
Exercise 1

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October 8, 2017

1 Analytic solution and results analysis

1.1 Analytic solution

To find the parameters c_1 and c_2 we use the inital values from the given codebase. The starting position ($t = 0$) is at $y = -1$.

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t) - L - \frac{mg}{k} \tag{1}$$

By differentiating this equation with respect to t we get.

$$y'(t) = c_1 e^{\alpha t}(\alpha \cos(\beta t) - \beta \sin(\beta t)) + c_2 e^{\alpha t}(\alpha \sin(\beta t) + \beta \cos(\beta t)) \tag{2}$$

By inserting $t = 0$ and $y_0 = -1$ into (1) we get:

$$-1 = c_1 e^{\alpha 0} \cos(\beta 0) + c_2 e^{\beta 0} \sin(\beta 0) - L - \frac{mg}{k} \tag{3}$$

$$-1 = c_1 e^{\alpha 0} - L - \frac{mg}{k} \tag{4}$$

$$c_1 = -1 + L + \frac{mg}{k} \tag{5}$$

To calculate c_2 we replace the starting speed with $y'(0) = 0$.

$$0 = c_1 e^{\alpha 0}(\alpha \cos(\beta 0) - \beta \sin(\beta 0)) + c_2 e^{\alpha 0}(\alpha \sin(\beta 0) + \beta \cos(\beta 0)) \tag{6}$$

$$0 = c_1 \alpha \cos(\beta 0) + c_2 \beta \cos(\beta 0) \tag{7}$$

$$0 = c_1 \alpha + c_2 \beta \tag{8}$$

$$c_2 = \frac{-c_1 \alpha}{\beta} \tag{9}$$

Subsituting c_1 to ge the value for c_2.

$$c_2 = -\frac{\alpha}{\beta}(-1 + L + \frac{mg}{k}) \tag{10}$$

1.2 Error convergence analysis

1.2.1 Velocity change table:

step	euler	symplectic-euler	midpoint	backwards-euler
0.500000				
0.250000	1.5	1.5	77.33	0.64
0.125000	3.44	3.44	−1.06	−3.51
0.062500	4.09	4.09	5	1.24
0.031250	4.15	4.15	6.78	3.25
0.015630	4.1	4.1	7.45	4.04
0.007810	4.06	4.06	7.75	4.17
0.003910	4.03	4.03	7.85	4.13
0.001950	4.02	4.02	8.67	4.07
0.000980	4.01	4.01	6.67	4.04

1.2.2 Displacement table:

step	euler	symplectic-euler	midpoint	backwards-euler
0.500000				
0.250000	7.38	4.53	4.92	−1.53
0.125000	−5.38	5.5	7.58	1.68
0.062500	1.7	5.11	8.26	6.64
0.031250	3.13	4.62	8.26	−23.33
0.015630	3.62	4.32	8.17	0.75
0.007810	3.83	4.16	8.09	2.85
0.003910	3.91	4.08	8.1	3.51
0.001950	3.96	4.04	8.2	3.78
0.000980	3.98	4.02	7.69	3.89

1.2.3 Analysis:

Based on the top two tables, we can see that the explicit, sympletic- and backward-euler converges with $O(h^2)$ and the midpoint-method with $O(h^3)$.

1.3 Stability analysis

1.3.1 Damping: 0

step	euler	symplectic-euler	midpoint	backwards-euler	analytic
0.000100	2.2	2.2	2.2	2.2	2
0.000200	2.22	2.2	2.2	2.2	2
0.000400	2.32	2.2	2.2	2.2	2
0.000800	$8.47 \cdot 10^{34}$	2.2	2.2	2.19	2
0.001600	∞	2.2	2.2	2.19	2
0.003200	∞	2.2	2.2	2.19	2
0.006400	∞	2.2	2.22	2.19	2
0.012800	∞	2.2	$1.89 \cdot 10^{307}$	2.18	2
0.025600	∞	2.2	$1.84 \cdot 10^{307}$	2.16	2
0.051200	∞	2.2	∞	2.14	2.01

1.3.2 Damping: 0.5

step	euler	symplectic-euler	midpoint	backwards-euler	analytic
0.000100	2.14	2.14	2.14	2.14	2
0.000200	2.14	2.14	2.14	2.14	2
0.000400	2.14	2.14	2.14	2.14	2
0.000800	2.14	2.14	2.14	2.14	2
0.001600	2.14	2.14	2.14	2.14	2
0.003200	2.14	2.14	2.14	2.14	2
0.006400	2.15	2.14	2.14	2.14	2
0.012800	2.15	2.14	2.14	2.13	2
0.025600	2.16	2.14	2.14	2.13	2
0.051200	3.26	2.14	2.14	2.12	2.01

1.3.3 Analysis:

Two methods tend to gain energy over the time (explicit euler and midpoint euler), the others seem relatively stable and do not gain energy by doubling the step-sizes. The more accurate a method is, the more stable it seems to be.
After adding a 0.5 damping, the results changed significantly. Most of the methods are stable now, but still the explicit euler seems to gain energy after doubling the steps 10 times. If one starts with higher step-sizes (0.0512), then only the backward-euler and the analytic solution are stable.