Physically-based Simulation
Exercise 1

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1 Analytic solution and results analysis

1.1 Analytic solution

To find the parameters c_1 and c_2 we use the inital values from the given codebase. The starting position ($t = 0$) is at $y = -1$.

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t) - L - \frac{mg}{k} \tag{1}$$

By differentiating this equation with respect to t we get.

$$y'(t) = c_1 e^{\alpha t}(\alpha \cos(\beta t) - \beta \sin(\beta t)) + c_2 e^{\alpha t}(\alpha \sin(\beta t) + \beta \cos(\beta t)) \tag{2}$$

By inserting $t = 0$ and $y_0 = -1$ into (1) we get:

$$-1 = c_1 e^{\alpha 0} \cos(\beta 0) + c_2 e^{\beta 0} \sin(\beta 0) - L - \frac{mg}{k} \tag{3}$$

$$-1 = c_1 e^{\alpha 0} - L - \frac{mg}{k} \tag{4}$$

$$c_1 = -1 + L + \frac{mg}{k} \tag{5}$$

To calculate c_2 we replace the starting speed with $y'(0) = 0$.

$$0 = c_1 e^{\alpha 0}(\alpha \cos(\beta 0) - \beta \sin(\beta 0)) + c_2 e^{\alpha 0}(\alpha \sin(\beta 0) + \beta \cos(\beta 0)) \tag{6}$$

$$0 = c_1 \alpha \cos(\beta 0) + c_2 \beta \cos(\beta 0) \tag{7}$$

$$0 = c_1 \alpha + c_2 \beta \tag{8}$$

$$c_2 = \frac{-c_1 \alpha}{\beta} \tag{9}$$

Subsituting c_1 to ge the value for c_2.

$$c_2 = -\frac{\alpha}{\beta}(-1 + L + \frac{mg}{k}) \tag{10}$$

1.2 Error convergence analysis

TODO

1.3 Stability analysis

TODO