Image Registration Using Blur-Invariant Phase Correlation

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***Abstract*—In this paper, we propose an image registration method, which is invariant to centrally symmetric blur. The method utilizes the phase of the images and has its roots on phase correlation (PC) registration. We show how the even powers of the normalized Fourier transform of an image are invariant to centrally symmetric blur, such as motion or out-of-focus blur. We then use these results to propose blur-invariant phase correlation. The method has been compared to PC registration with excellent results. With a subpixel extension of PC registration, the method achieves subpixel accuracy for even heavily blurred images.**

***Index Terms*—Image alignment, image blurring, Fourier transform.**

I. Introduction

REGISTRATION of two-dimensional (2-D) images acquired from the same scene at different times, from different viewpoints, or different sensors is a fundamental process in image processing. It is needed before further analysis and fusion of the images. Typical applications include image mosaicing, super-resolution, and the fusion of multimodal images in the fields like remote sensing, medicine, and computer vision.

Comprehensive surveys about the widely studied 2-D image registration can be found in [1] and [2]. Image registration methods can be divided into the feature and area based methods. The former attempts to match the features of salient details of images while the latter attempts to match the whole images, also called template matching.

In practical applications, images contain various degradations due to imperfect imaging conditions including blur, which can result from atmospheric turbulence, out-of-focus, or relative motion between the camera and the scene. When the degradation process is modeled as a linear shift-invariant system, the relation between an ideal image $f(x,y)$ and an observed image $g(x,y)$ is given by

$$g(x,y)=f(x,y)*h(x,y)+n(x,y) \tag{1}$$

where $h(x,y)$ is the point spread function (PSF) of the system, $n(x,y)$ is additive noise, and $*$ denotes 2-D convolution. The point spread function $h(x,y)$ represents blur while other degradations are captured by the noise term $n(x,y)$.

With few exceptions, all the image registration methods are sensitive to blur, which may result in inaccurate registration. To the best of our knowledge, the only existing blur-invariant 2-D registration methods are based on the blur-invariant features (BIF) originally proposed by Flusser and Suk in [3]. They used the spatial BIFs based on image moments for matching a template to a larger blurred image. In [4], complex forms of the spatial BIFs are used as descriptors representing the neighborhood of a control point in feature based registration. In both papers, also rotation invariance was demonstrated. In [5], the spatial BIFs of [3] are used similarly for feature based registration of X-ray images. In that paper, rotation invariance is obtained by calculating the BIFs for every rotation angle and subpixel accuracy by interpolating the distance measure. The main shortcomings of these approaches based on the BIFs are that their computation is slow and that they have to be calculated at least once for every possible translation.

The blur-invariant phase correlation (BIPC) registration method for 2-D images, proposed in this paper, is based on phase correlation (PC) and similarly can be used to register images differing locally by translation. The method is invariant to centrally symmetric blur just like the BIFs proposed in [3], but it can be computed efficiently using FFT.

First, in Section II we briefly review the PC image registration method as its properties are similar to the proposed method. Then, in Section III our BIPC method is derived followed by the experimental results in Section IV and the conclusions in Section V.

II. Phase Correlation Method

The PC image registration method was first proposed in [6]. The method is based on the Fourier shift theorem, which states that if two images f_1 and f_2 differ only by displacement (x_0,y_0), namely

$$f_2(x,y)=f_1(x-x_0,y-y_0) \tag{2}$$

their Fourier transforms F_1 and F_2 are related by

$$F_2(u,v)=F_1(u,v)e^{-i(ux_0+vy_0)}. \tag{3}$$

This means that the images f_1 and f_2 have the same Fourier magnitude, while the phase difference is directly related to their spatial displacement. As the Fourier transform $F(u,v)$ itself is a complex function and can be written by its magnitude and argument, namely

$$F(u,v)=|F(u,v)|\,e^{-i\phi(u,v)} \tag{4}$$

it turns out that the normalized cross power spectrum of the two images defined as

$$S(u,v)=\frac{F_2(u,v)F_1^*(u,v)}{|F_2(u,v)F_1^*(u,v)|}=e^{-i(ux_0+vy_0)} \tag{5}$$

where * denotes complex conjugate, has the phase corresponding to the phase difference of the images f_1 and f_2. The

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inverse Fourier transform of (5) is $\delta(x-x_0, y-y_0)$, which is the Dirac delta function centered at (x_0, y_0) corresponding to the spatial shift between the images f_1 and f_2. The location of the very sharp peak can be found easily. The method is invariant to linear intensity changes and is robust to narrow band noise. Thus, it is well suited for images from different sensors and for varying illumination conditions [6].

III. Blur-Invariant Phase Correlation

In this section, the theory of BIPC is derived. The derivation has similarities with the derivation of the frequency domain BIFs in [3].

If noise is neglected, (1) can be expressed in the Fourier domain using the convolution theorem by

$$G(u,v) = F(u,v) \cdot H(u,v) \tag{6}$$

and in the phasor form by

$$G(u,v) = |G(u,v)|\, e^{-i\phi_g(u,v)}. \tag{7}$$

If the Fourier transform $G(u,v)$ is normalized by its magnitude, only the complex exponential containing the phase remains, namely

$$\frac{G(u,v)}{|G(u,v)|} = e^{-i\phi_g(u,v)} = e^{-i(\phi_f(u,v)+\phi_h(u,v))} \tag{8}$$

where $\phi_f(u,v)$ is the phase of the original image and $\phi_h(u,v)$ the phase of the blur PSF.

Since $h(x,y)$ is assumed to be centrally symmetric, its Fourier transform $H(u,v)$ is real and its phase $\phi_h(u,v)$ has only two possible values

$$\phi_h(u,v) = 0 \vee \phi_h(u,v) = \pi. \tag{9}$$

It follows from this and from the periodicity of the complex argument that the equality

$$\begin{aligned}\left\{e^{-i\phi_g(u,v)}\right\}^{2n} &= e^{-i2n\phi_g(u,v)} \\ &= e^{-i2n\phi_f(u,v)} e^{-i2n\phi_h(u,v)} \\ &= \left\{e^{-i\phi_f(u,v)}\right\}^{2n}\end{aligned} \tag{10}$$

holds for any integer n.

Thus, any even power of the normalized Fourier transform, i.e. $\{e^{-i\phi_g(u,v)}\}^{2n}$, is invariant with respect to the convolution of the original image with any centrally symmetric PSF.

It can be seen that (5) is made of two terms which are similar to the left hand side of (8). Consequently, any even power of the normalized cross power spectrum (5) of two observed images $g_1 = f_1 * h_1$ and $g_2 = f_2 * h_2$, where $*$ denotes 2-D convolution, is invariant with respect to convolution of the original images f_1 and f_2 with any centrally symmetric PSFs h_1 and h_2, namely

$$\begin{aligned}S_n(u,v) &= \left(\frac{G_2(u,v)G_1^*(u,v)}{|G_2(u,v)|\,|G_1^*(u,v)|}\right)^{2n} \\ &= e^{-i2n\phi_{g_2}(u,v)} e^{i2n\phi_{g_1}(u,v)} \\ &= e^{-i2n\phi_{f_2}(u,v)} e^{i2n\phi_{f_1}(u,v)}\end{aligned} \tag{11}$$

where n is some integer.

If we assume that the images f_1 and f_2 differ only by displacement (x_0, y_0) and apply the Fourier shift theorem in (2) and (3) to the right hand side of the (11), we get

$$\begin{aligned}S_n(u,v) &= e^{-i2n\phi_{f_1}(u,v)} e^{-i2n(ux_0+vy_0)} e^{i2n\phi_{f_1}(u,v)} \\ &= e^{-i2n(ux_0+vy_0)}.\end{aligned} \tag{12}$$

The inverse Fourier transform of (12) is $\delta(x-2nx_0, y-2ny_0)$, which is the Dirac delta function centered at $(2nx_0, 2ny_0)$ corresponding to the spatial shift between the original images f_1 and f_2 multiplied by $2n$. This means that the inverse Fourier transform of (12) can be used to obtain the displacement between images and that the result is invariant to centrally symmetric blur, such as motion or out-of-focus blur, in one or both of the images. The resolution of the displacement vector is also increased by the factor $2n$. BIPC preserves the desired properties of PC, mentioned in Section II.

In some cases, also PC is invariant to convolutional degradations. In [6] it has been noticed that if both images undergo the same convolutional degradation, the phase difference matrix remains unchanged. Also, if the images are convolved by different one or two dimensional exact Gaussian PSFs, the phase difference remains unchanged. This can be explained by the fact that the Fourier transform of a Gaussian is also a Gaussian resulting always in $\phi_h(u,v) = 0$ in (9). This was also verified by the experiments with a discrete approximation of a Gaussian PSF.

Although the higher positive powers $(n > 1)$ produce BIPC with finer resolution, the best accuracy in practice, in the presence of noise, is obtained by using the value $n = 1$ and then refining the result with the methods used also with PC.

IV. Experimental Results

We performed several image registration experiments on the BIPC with comparisons to PC and the BIFs. In all the experiments, we have used the value $n = 1$ in (11), and performed the necessary windowing and zero-padding in the spatial domain before Fourier transforming the images. The subpixel accuracy for both PC and BIPC is obtained by applying the approach proposed in [7].

In the first experiment, we registered pairs of blurred, noisy, and translated images of size 300×300 taken from either 400×400 image in Fig. 1. Each image in a pair was motion blurred in a different and random direction, and the other image was corrupted by additive Gaussian noise resulting in a peak signal-to-noise ratio (PSNR) of 34 dB. For subpixel translation we had to perform interpolation. Translations were randomly generated in the range $[-50, 50]$. Fig. 2 summarizes the results when the blur length is increased in steps of one, and the experiment is repeated a thousand times for each blur length and for each of the images in Fig. 1 (Blur length zero corresponds to PSF length one). As can be seen in Fig. 2, the root mean square (RMS) registration error (in pixels) in the case of PC increases as the length of blur increases. On the contrary, BIPC seems to perform nearly equally despite blur, and we really achieve subpixel registration accuracy for even heavily blurred images. This is slightly surprising, taken that the situation is realistic, i.e. the images are not cyclic, which would result in exact invariance. The explanation is the large size of the images compared to the blur length, which is also the case typically in practice.

It seems that without blurring, when the only degradation is noise, PC performs slightly better than BIPC. For this reason, in the second experiment, we decided to blur the images only a little and investigate what is the limit for the length of the blur for each noise level after which one should use BIPC instead of

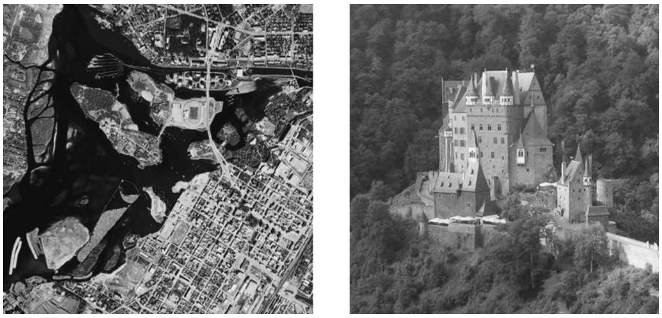

Fig. 1. Images used in the experiments.

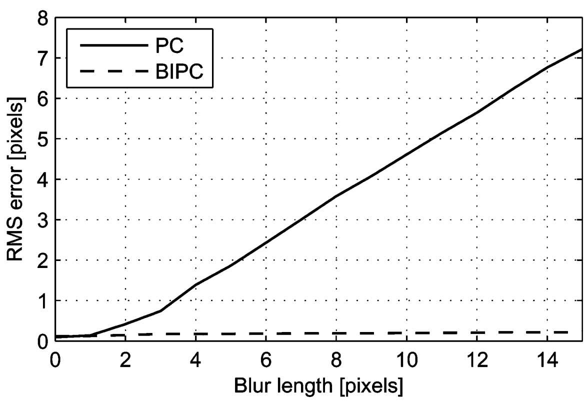

Fig. 2. RMS registration error of motion blurred images as a function of blur length using PC or BIPC (PSNR 34 dB).

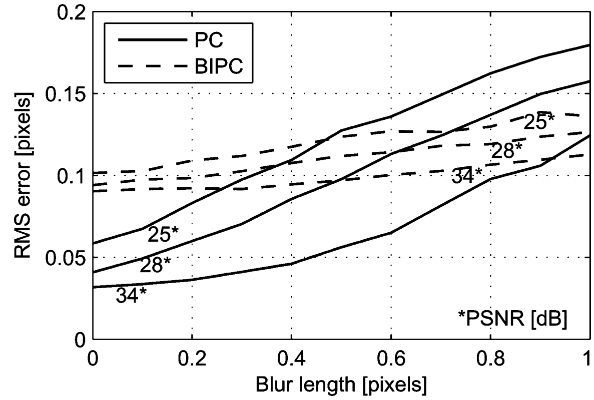

Fig. 3. Rms registration error of motion blurred images as a function of blur length using PC or BIPC for three different noise levels. The blur length is small so that the intersection of the PC and BIPC RMSE curves can be found.

Fig. 4. Image of quarters used in the third experiment.

PC. In this experiment, we used a high-resolution (9 Mpix) version of the left image in Fig. 1 and performed the translation in integer pixel accuracy after which we downsampled the image resulting in subpixel shifts. In this way, we avoided the subpixel interpolation, which would create additional blur. Fig. 3 shows the resulting RMS registration error when the blur length is in the range [0, 1] and the experiment is repeated a thousand times for each blur length in steps of 0.1. The curves for both PC and BIPC are shown for three different noise levels. As expected, when noise is added, the RMS registration error for each method increases. Without blurring, the RMSE in the case of PC is around 0.05 pixels and in the case of BIPC 0.1 pixels. The limit after which it is better to use BIPC instead of PC seems to be between the blur lengths of 0.5 and 0.9. In practice, one should always use BIPC if there is a possibility of some motion blur existing in the images. In the presence of noise, ordinary PC seems to be more robust for very large translations, i.e. in the case of only a small overlap, provided that blurring is subtle. When the PSNR is less than 28 dB, BIPC often fails if the translations are larger than a quarter of the image size.

In the third experiment, the performance of BIPC was compared to the BIFs. In our experiments we used the spatial version of the BIFs of order seven that are used also in [3]. The spatial BIFs exhibit similar invariance to blurring as BIPC, but they are far slower to compute. So, the aim was to show that BIPC can perform similarly with less computation. It is practically impossible to register large images using the BIFs as it would be too slow, and because one image should be included into the second. So, we registered a small template of size $T \times T$ to a larger blurred and noisy image of size $N \times N$. Similar template registration was done also in [3]. We repeated the experiment 100 times by picking a template $(T = 60)$ from a random subpixel location of the image $(N = 200)$, which is shown in Fig. 4. Before registration, the larger image was blurred by motion blur in a random direction with a blur length of eight and corrupted by additive Gaussian noise resulting in a PSNR of 34 dB. Template matching using BIPC or PC can be made faster by correlating the $T \times T$ template with a larger $B \times B$ blocks of the $N \times N$ image $(N \geq B \geq T)$. We used overlapping blocks so that the template is always included as a whole exactly in one block. The highest correlation result among the blocks is chosen to be the correct one. We did not use subpixel registration accuracy in this experiment. Fig. 5 shows the results as a percentage of correct registrations (error in the range $[-1, 1]$ pixels) for different block sizes B. The value $B = 60$ corresponds to a situation were the correlation is calculated once for every possible translation. In the case of the BIFs, $B = 60$ always and thus its result is a constant 98%. As can be seen, BIPC performs even better compared to BIFs when the block size $B \leq 80$. Fig. 5 shows also that the result of BIPC degrades faster than the result of PC when B is increased and the relative overlap of the template and the block becomes smaller.

The computation of BIPC is fast because it only requires two FFT's (size of $T + B$) plus some extra operations per block. In the case of the BIFs, a large number of spatial moments of different orders has to be calculated to get the final invariants. Table I shows the number of arithmetic operations needed when the block size B of BIPC is changed. As can be seen, BIPC requires one tenth of the operations of the order seven BIFs (BIF7)

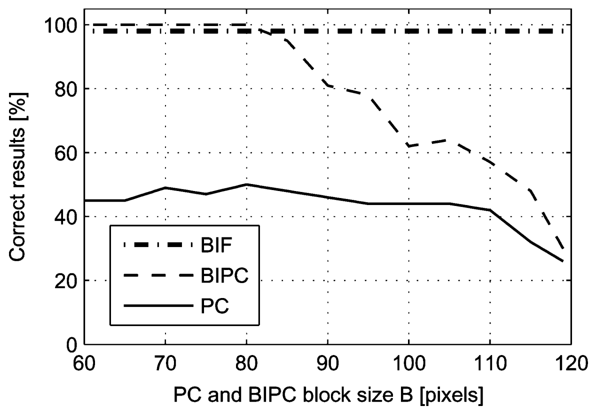

Fig. 5. Percentage of correct registrations of a 60 × 60 template to a 200 × 200 image using different methods. In the case of PC and BIPC the template is correlated with a block of size $B \times B$. For the BIFs, the block size is always 60 × 60 (PSNR 34 dB).

TABLE I
NUMBER OF ARITHMETIC OPERATIONS NEEDED TO REGISTER A 60 × 60 TEMPLATE TO A 200 × 200 IMAGE USING BIFS OF ORDERS 5 OR 7 AND BIPC PROCESSING A $B \times B$ SIZE BLOCK OF THE IMAGE AT TIME

Method	BIF7	BIF5	BIPC			
Block Size B	60	60	60	80	100	120
Number of Operations ($\times 10^6$)	14000	6500	1200	3.8	1.3	0.78

and one sixth of the operations of the order five BIFs (BIF5) even in the slowest case when $B = 60$. When B becomes larger, the BIPC method becomes even faster.

In the final experiment, we used real images. The images were captured using a vibrating video camera so that motion blur was created on some of the images. In this case, it is impossible to evaluate which method is better in the subpixel level. So, we compared the BIPC and PC methods using heavily but nearly symmetrically blurred images so that the errors were large. In Fig. 6, a typical registration result is shown obtained using BIPC. The estimated translation is (19.26,116.85), which seems to be very accurate. For the same image pair, PC estimated the translation to be (13.31,122.81), which is clearly incorrect. If the blur is strongly nonsymmetric, also BIPC will fail.

Fig. 6. Example of the results of the fourth experiment with real blurred images. Images contain motion blur and are registered using BIPC. The result seems to be good. For the same image pair PC produced a result that was noticeable incorrect.

V. CONCLUSION

In this paper, we have shown how the even powers of the normalized Fourier transform of an image are invariant to centrally symmetric blur. Based on this, we have derived the analytical expressions for BIPC, which can be used to register images containing centrally symmetric blur. BIPC is based on PC and exhibits similar desired properties such as robustness to brightness changes and fast calculation using FFT. Similar to PC, BIPC can only deal with image translation.

Our results show that registration can be made much more accurately using BIPC than PC when images contain motion blur. As demonstrated, with the subpixel extension of PC [7], BIPC can produce subpixel registration accuracy even in the case of heavy blur. If it is possible that the images contain motion blur, BIPC should always be used as it performs better compared to PC if the blur length exceeds one pixel. The performance difference for sharp images is negligible, but even a blur of a few pixels can lead to significant error in the case of PC.

The BIPC method was also compared to the BIFs proposed in [3], which have been used for registration of blurred images. BIPC is much faster to compute and the other image must not be fully included in the other as is the case with the BIFs. Taken these arguments and the fact that BIPC can achieve at least similar registration accuracy as the BIFs, it should be preferred.

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