EE5175: Image Signal Processing

Lab-7

1. Assume a Markov-1 process with covariance matrix R of size 8 × 8 and $\rho = 0.91$. Compute the Energy Packing Efficiency and De-correlation Efficiency of the Walsh-Haddamard Transform and Discrete Cosine Transform for the above process. What is your observation about the eigenvectors of R in relation to the DCT basis?

2. Find $\beta^2 R^{-1}$, where $\beta^2 = \dfrac{1-\rho^2}{1+\rho^2}$. Does $\beta^2 R^{-1}$ have a tridiagonal structure?. Is it close to the tridiagonal matrix Q given by,

$$\mathbf{Q} = \begin{bmatrix} 1-\alpha & -\alpha & 0 & 0 & 0 & 0 & 0 & 0 \\ -\alpha & 1 & -\alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & -\alpha & 1 & -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & 1 & -\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & -\alpha & 1 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & -\alpha & 1 & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & -\alpha & 1 & -\alpha \\ 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 1-\alpha \end{bmatrix}$$

where $\alpha = \dfrac{\rho}{1+\rho^2}$

Try diagonalizing $\beta^2 R^{-1}$ and Q using the DCT matrix. What is your observation.?

3. Compute SVD for the given 8 × 8 image $\mathbf{g}$ (provided in `imageFile.mat` and also given below) using the following steps:

   (a) Perform eigen-value decomposition of $\mathbf{g}^T\mathbf{g}$ and $\mathbf{g}\mathbf{g}^T$.

   (b) Find the singular value matrix $\mathbf{\Sigma}$.

   (c) Reconstruct the image using $\mathbf{\Sigma}$ and the eigen-vector matrices.

4. Remove one singular value at a time from $\mathbf{\Sigma}$ and reconstruct the image ($\widehat{\mathbf{g}_k}$). Compute $\|\mathbf{g}-\widehat{\mathbf{g}_k}\|^2$ and compare it with the sum of the squares of the first k singular values.

$$\text{Image } \mathbf{g} = \begin{bmatrix} 255 & 255 & 255 & 255 & 255 & 255 & 255 & 255 \\ 255 & 255 & 255 & 100 & 100 & 100 & 255 & 255 \\ 255 & 255 & 100 & 150 & 150 & 150 & 100 & 255 \\ 255 & 255 & 100 & 150 & 200 & 150 & 100 & 255 \\ 255 & 255 & 100 & 150 & 150 & 150 & 100 & 255 \\ 255 & 255 & 255 & 100 & 100 & 100 & 255 & 255 \\ 255 & 255 & 255 & 255 & 50 & 255 & 255 & 255 \\ 50 & 50 & 50 & 50 & 255 & 255 & 255 & 255 \end{bmatrix}$$

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