

EE5175: Image Signal Processing

Non-local means filtering

In this experiment, we will implement non-local means (NLM) filtering algorithm for the application of denoising.

You are given a noisy image, \mathbf{g} (`krishna_0_001.png`), corresponding to a latent image, \mathbf{f} (`krishna.png`), corrupted with additive Gaussian noise of mean 0 and variance 0.001. Your task is to apply NLM filtering on \mathbf{g} following the steps in the given pseudocode to arrive at the denoised image, $\hat{\mathbf{f}}$.

The parameters of the algorithm are the search neighbourhood radius W , the similarity neighbourhood radius W_{sim} and the filter parameter σ_{NLM} . A radius of W at a pixel denotes a window size of $(2W + 1) \times (2W + 1)$ around that pixel. The same applies to W_{sim} .

Q1 Show plots between the PSNR between \mathbf{f} and $\hat{\mathbf{f}}$ (y-axis) for different NLM filter parameter values $\sigma_{NLM} = 0.1$ to 1.0 in steps of 0.1 (x-axis) for the following search radius and similarity radius settings:

- (a) $W = 5$, $W_{sim} = 3$,
- (b) $W = 10$, $W_{sim} = 3$.

Show two plots in the same window with two different colours corresponding to (a) and (b). Compare the PSNR plots with the baseline PSNR between the noisy image \mathbf{g} and the latent image \mathbf{f} .

Q2 We will now compare NLM filtering with the traditional Gaussian filtering. Denoise \mathbf{g} using space-invariant Gaussian filter with $\sigma_g = 0.1$ to 1.0 in steps of 0.1 having a kernel window size of 11×11 for all σ_g values. Calculate the PSNR between the denoised images and \mathbf{f} . Add this plot to the plot window in Q1.

For the following filtering settings: (a) $W = 5$, $W_{sim} = 3$, $\sigma_{NLM} = 0.5$ for the NLM filtering, and (b) $\sigma_g = 1.0$ for Gaussian filtering, and at the following pixel locations \mathbf{p} : (i) row = 63, column = 93, and (ii) row = 77, column = 118, (total four combinations), do **Q3** and **Q4**.

Q3 Show the 11×11 filter (kernel) as an image. Comment.

Q4 Show the 11×11 image patch from the noisy image and the denoised images. Comment.

Use the 'InitialMagnification' option in `imshow` command with a value greater than 100 (say, 1000) to display larger pixel sizes for patches and kernels.

Pseudocode

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Read the noisy image  $\mathbf{g}$  and the latent image  $\mathbf{f}$  in the intensity range  $[0, 1]$ .
for every pixel position  $\mathbf{p}$  do
    // Obtain similarity neighbourhood around  $\mathbf{p}$ 
    Take the RGB patch  $\mathcal{N}_p$  around  $\mathbf{p}$  of radius  $W_{sim}$  in the image  $\mathbf{g}$ 
    Vectorize the patch  $\mathcal{N}_p$  as a column vector  $\mathbf{V}_p$ 

    // Form the filter  $\mathbf{w}_p$  at pixel  $\mathbf{p}$ .
    // It can be formed as a 1D vector and visualized as a 2D matrix.
    // We form a single filter for all three colour components.
    for every pixel position  $\mathbf{q}$  around  $\mathbf{p}$  within radius  $W$  do
        // Obtain similarity neighbourhood around  $\mathbf{q}$ 
        Take the RGB patch  $\mathcal{N}_q$  around  $\mathbf{q}$  of radius  $W_{sim}$  in the image  $\mathbf{g}$ 
        Vectorize the patch  $\mathcal{N}_q$  as a column vector  $\mathbf{V}_q$ 

        The value of the filter  $\mathbf{w}_p$  for the position  $\mathbf{q}$  is given by

        
$$\mathbf{w}_p(\mathbf{q}) = \exp(-(\mathbf{V}_p - \mathbf{V}_q)^T(\mathbf{V}_p - \mathbf{V}_q)/\sigma_{NLM}^2)$$


    end for
    Normalize  $\mathbf{w}_p \leftarrow \mathbf{w}_p / \sum \mathbf{w}_p$ 

    // Obtain search neighbourhood patch around  $\mathbf{p}$ 
    Take the RGB patches  $\mathcal{N}_p^W(\mathbf{R})$ ,  $\mathcal{N}_p^W(\mathbf{G})$ ,  $\mathcal{N}_p^W(\mathbf{B})$  around  $\mathbf{p}$  of radius  $W$  in
    the image  $\mathbf{g}$  separately
    Vectorize them as column vectors  $\mathbf{V}_p^W(\mathbf{R})$ ,  $\mathbf{V}_p^W(\mathbf{G})$ ,  $\mathbf{V}_p^W(\mathbf{B})$ 

    // Calculate the filtered output at pixel  $\mathbf{p}$ 
    // Use the same filter for all colour channels
    The intensity at the output pixel  $\mathbf{p}$  for each colour channel is given by

    
$$\begin{aligned} \hat{\mathbf{f}}(\mathbf{p}, \mathbf{R}) &= \mathbf{V}_p^W(\mathbf{R})^T \mathbf{w}_p \\ \hat{\mathbf{f}}(\mathbf{p}, \mathbf{G}) &= \mathbf{V}_p^W(\mathbf{G})^T \mathbf{w}_p \\ \hat{\mathbf{f}}(\mathbf{p}, \mathbf{B}) &= \mathbf{V}_p^W(\mathbf{B})^T \mathbf{w}_p \end{aligned}$$


    // Calculate the PSNR
    // MSE : Mean Squared Error
    // PSNR : Peak Signal-to-Noise Ratio
    // The operation here assumes  $\mathbf{f}$  and  $\hat{\mathbf{f}}$  are column vectors.
    
$$\text{MSE} = (\mathbf{f} - \hat{\mathbf{f}})^T(\mathbf{f} - \hat{\mathbf{f}}) / (\text{total number of pixels including all colour channels})$$

    
$$\text{PSNR} = 10 * \log_{10}(1 / \text{MSE})$$

end for

```

Note:

1. For any two vectors, the inner product $\mathbf{a}^T \mathbf{b}$ calculates the element-wise multiplication followed by addition, and $\mathbf{a}^T \mathbf{a}$ results in the sum of the square of each element.
2. The general formula for PSNR is $10 \cdot \log_{10}(\text{MAX} \cdot \text{MAX} / \text{MSE})$, where MAX is the maximum image intensity value. We use MAX=1 in this experiment.

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