Plant Models Faithful to Botanical Structure and Development

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Abstract:
Some very impressive results have been obtained in the past few years in plants and trees image synthesis. Some algorithms are largely based on the irregularity and fuzziness of the objects, and use fractals, graftals or particle systems. Others focus on the branching pattern of the trees with emphasis on morphology. Our concern here is the faithfulness of the models to the botanical nature of trees and plants. We present a model which integrates botanical knowledge of the architecture of the trees: how they grow, how they occupy space, where and how leaves, flowers or fruits are located, etc. The very first interest of the model we propose is its great richness: the same procedural methods can produce "plants" as different as weeping willows, fir trees, cedar trees, frangipani trees, poplars, pine trees, wild cherry trees, herbs, etc. Another very important benefit one can derive from the model is the integration of time which enables viewing the aging of a tree (possibility to get different pictures of the same tree at different ages, accurate simulation of the death of leaves and branches for example). The ease to integrate physical parameters such as wind, the incidence of factors such as insects attacks, use of fertilizers, plantation density, and so on makes it a useful tool for agronomy or botany.

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1. Introduction.

The past few years have seen much effort devoted to image generation of natural phenomena by procedural methods. In this context, our interest lies in the generation of trees and plants images. Some very impressive results have already been obtained, using particular branching patterns (Kawaguchi [11], Aono and Kunii [1]), graftals (Smith [23]), particle systems (Reeves and Blau [18]), fractals (Oppenheimer [16]), extensions of graftals (Prusinkiewicz [17]) or combinatorics of trees (Eyrolles, Françon and Viennot [5]).

Our concern here is to produce images of plants and trees which should be faithful to their botanical nature and so to build a model which should include the known botanical laws which explain plants' growth and architecture. Such a concern is shared by Aono and Kunii in [1] but their study is limited to the cases of monopodial, dichotomic or ternary branching patterns which prove to be insufficient for the representation of a rich variety of plants and trees. In [7], Fournier proposed a taxonomy for the modelling of natural phenomena, categorizing the different models as empirical, purely physical, morphological, structural, impressionistic or self-models. As he noted in his course notes, our approach can be seen as a structural model; it can also be seen as a kind of physical model with botanical laws instead of physical ones as our approach integrates as much "knowledge" about the plant and its environment as possible: age of the plant, growing conditions, physics of the branches (for example, in our approach branches are bent by gravity or wind strength as opposed to their simulation by inhibitors and attractors by Aono and Kunii [1]).

From a botanical standpoint, the growth of plants has been studied by several authors. An extensive bibliography can be found in [24] and [17].

A "macroscopic" study of plant shapes more directly related to their "architecture" can be found in Fischer and Honda's paper ([6]) in the case of Terminalia; their work has been taken into account for image synthesis by Aono and Kunii [1].

A macroscopic approach can also be found in one of the authors' thesis (De Reffye [19]) where the mathematical model is applied to the simulation of the growth of the coffee-tree. The model has been enriched since and its domain of relevance widely enlarged; the aim of the present paper is to present its potential consequences in image synthesis of trees and plants.

As we want to emphasize here the topologic, not geometric model, its functioning and relation to the botanical and physical reality, we will not deal in this paper with important but somewhat different problems related to the graphical aspect of the image such as the smoothing of the limbs, the precise shape of the trunk, the rendering of the bark's texture, etc. These problems have been studied in the case of the maple tree by Bloomenthal ([3]).

The general idea of the method is to model the activity of buds at dicretized times: a bud, at a given clock signal can

- either become a flower and die (and disappear),
- or go into sleep (pause, break),
- or become a so called internode at the extremity of which one or several leaves appear with new so called lateral buds at their axil and a new so called apical bud at the end of the internode,
- or die (and disappear).

These events occur according to specific stochastic laws characteristic for each variety and each species. The geometric parameters, such as the length and diameter of an internode or the branching angles are also calculated according to specific stochastic characteristic laws.

These simulations rely on recent work on plant architecture.

The plan of the paper is as follows. In section 2, we give several simplified notions from botany on which the stochastic growth model explained in section 3 is founded. In section 4, we deal with the method used to simulate the growth of plants, and in section 5 we show how the model can be used to simulate other phenomena. The next section explains how the visualization is done and the last section gives results and conclusions.

2. A few simplified notions from Botany.

The growth of a plant is the result of the evolution of some specific cellular tissues (internal part of the bud), the so called *meristems.* A bud can, at a given time, die (abort), and it will not produce anything any longer, or it can give birth to a flower, or an inflorescence (and then the bud dies) or to an internode.

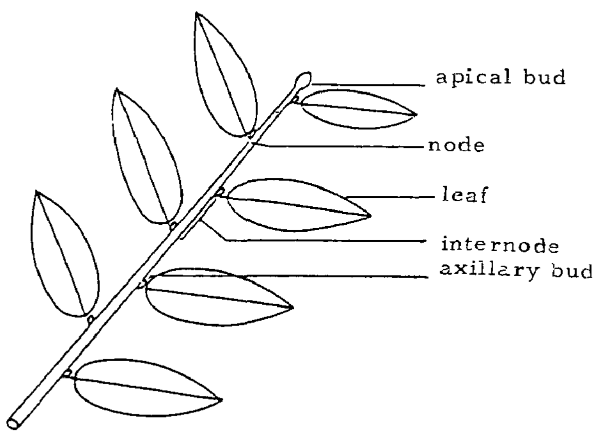


Figure 1: The leaves'axis.

The *leaves' axis* is the fundamental element of the architectural approach. It is the result of the activity of the bud situated at its tip, which is called the *apical bud.* It is made of a series of internodes; an *internode* is a (part of a) stem made of a ligneous material at the tip of which one can find one or several leaves. An internode's length can be very short (around 1 mm) in the case of some plants such as the fir tree, but can also measure up to several tenth of cms in the case of bamboo. Between two internodes there is a *node* which bears leaves and buds; each node bears at least one leaf (it is the symptom of the existence of a node); at each leaf's axil, one finds a so-called *axillary bud.* These notions are illustrated on Figure 1.

An axillary stem can either grow immediately (*sylleptic ramification*) or with some delay (*proleptic ramification*).

The growth of the leaves' axes of a plant is the result of the evolution of their apical buds. A central notion for the model is the notion of *growth unit* which is a sequence of internodes and nodes produced (usually in a very short period of time) by the apical bud of the previous node. We will distinguish between two cases: short growth units with few internodes (one sometimes) and long growth units which are made of numerous internodes (each of which is usually short).

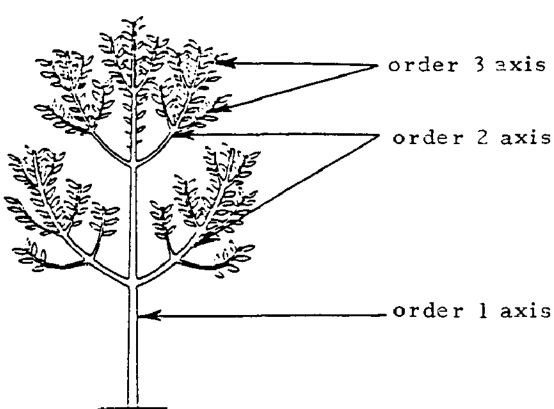


Figure 2: The notion of order of an axis.

Another important notion is the notion of the *order* of an axis (see Figure 2). The *order 1 axis* of a plant is the sequence of growth units such that each of these growth units is born of the apical bud of the previous one, and such that the first one of the axis is grown out of the seed of the plant. An *order i axis*, for $i>1$, is a sequence of growth units such that the first internode of the sequence is born of an axillary bud on an order $i-1$ axis, called the *bearing axis.*

In the absence of traumatisms, the relative position of the lateral buds (and, as a consequence of the leaves) of a node with respect to the lateral buds of the previous node follow regular laws known for each variety of each species and each order; this phenomenon is called *phyllotaxy* . The *spiraled* and *distic* cases are illustrated on Figure 3.

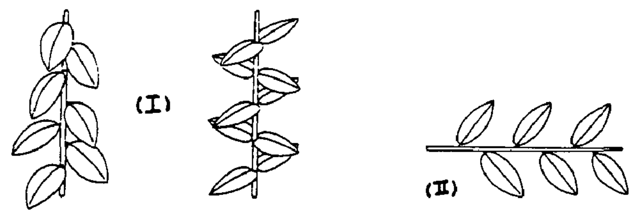


Figure 3: Phyllotaxy: (I) spiraled, (II) distic.

As regards the ramification process, in a growth unit, one can encounter (see Figure 4):

- *continuous ramification*: every node of an axis is the root of an axis of greater order,
- *rhythmic ramification*: some nodes (but not all of them) are the root of an axis of greater order, or
- *diffuse ramification*: the nodes forming roots of an axis of greater order are located at random.

These kinds of ramifications are functions of the order of the axis for a given variety and species. A *monopod* is a ramified system which includes a unique number *1* axis and a finite number of axes of higher order; if the orders go up to *k* (included), the monopod is said an *order k monopod.*

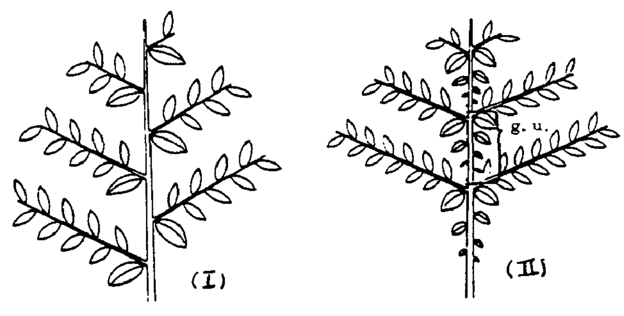


Figure 4: Ramification (I) continuous, (II) rhythmic

The geometric trend of an axis with respect to its bearing axis is also an important parameter; it is characterized usually by a general trend. If the latter is horizontal the development is said *plagiotropic*, if it is vertical it is said *orthotropic* (see Figure 15). An order 1 axis is usually orthotropic. The trend frequently effects the phyllotaxy: orthotropy is usually associated with a spiraled phyllotaxy whereas plagiotropy is associated with distic phyllotaxy.

Some plants do not grow as monopods: when the apical bud of an order *i* axis dies, some axillary buds of the previous node produce an axis whose behaviour is of an order *i* instead of $i+1$ axis. Such a behaviour is called *sympodial growth.* A similar phenomenon can be observed after the pruning of a tree (see figure 5 and Photo 2). The observed phenomenon is called *traumatic reiteration.*

Another phenomenon alters the shape of "old" trees; it is called *reiteration* (see [4], [15]). It accounts for the following behaviour: an axillary bud can produce an axis which behaves as an order *1* axis (a new "young" tree) or, in a few cases as an axis of greater order. These are "natural" occurences of recursion! Reiteration is still very badly understood from a botanical point of view and will be ignored here except in the case of traumatic reiteration.

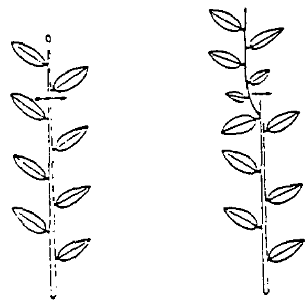


Figure 5: Traumatic reiteration.

Tree architecture has been studied rather recently from a scientific point of view, in a systematic way. Botanists Hallé, Oldeman and Tomlinson ([9]) studied the growth of tropical trees and sifted out the notion of *architectural model*, which can be seen as a growth strategy to occupy space. A surprisingly large variety of plants and trees grow and build their shape in the same kind of way, although the results seem very different from one to another. Altogether, there are only 23 different architectural types, called architectural models. The classification relies mainly on the presence/absence of sympodial growth, on ramification, continuous growth or not, and development trend (plagiotropy/orthotropy). Let us describe briefly four of these, among the most frequently encountered:

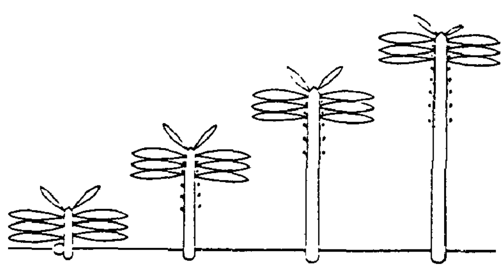


Figure 6: Corner model.

- in *Corner's model* (see Figure 6), there is one order *1* axis and no ramification at all. These plants are monopods; examples include coconut trees (see Photo 8) and date palm trees (see Figure 16).

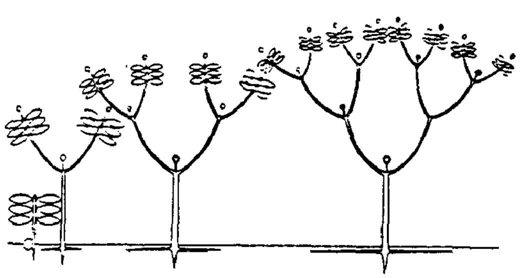


Figure 7: Leeuwenberg model.

- in *Leeuwenberg's model* (see Figure 7), the apical buds systematically die after one growth unit so that the growth is sympodial; examples include the frangipani tree (see Photo 13) and the mistletoe.
- in *Massart's model* (see Figure 8), the order *1* axis is orthotropic, the other ones are plagiotropic. These plants are monopods with a rhythmic ramification growth; examples include fir trees (Photo 1), spruce trees and cedar trees (Photo 10).
- in *Rauh's model* (see Figure 9), every axis is orthotropic. These plants are monopods with a rhythmic ramification growth; examples include poplar trees (Photo 4), aspen trees (Photo 3), pine trees (Photo 5) and fruit trees of temperate regions (Photos 6, 7 and 9).

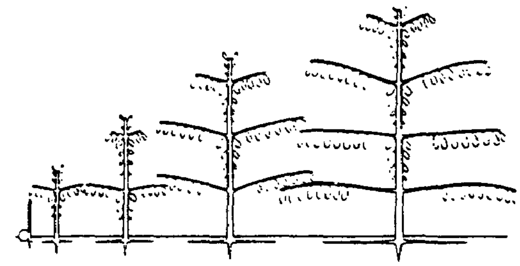


Figure 8: Massart model.

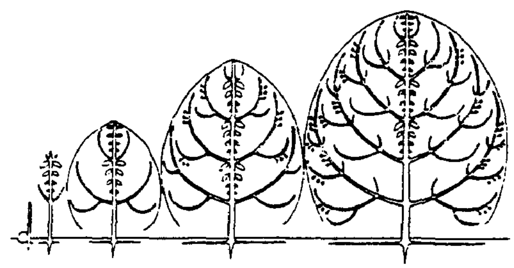


Figure 9: Rauh model.

To end this botany section, let us mention that the same architectural concepts are relevant to other botanical phenomena: the rhizome of some herbs can be considered as an underground monopod; an inflorescence can be viewed as a monopod whose apical buds are replaced by flowers. This will be used in section 4 for simulation purposes.

3. The growth model.

Our approach is based on a mathematical simulation of botanical architectural models which originated in Philippe de Reffye's thesis [19]. It is based on a botanically accurate simulation of the functioning of meristems. Although related botanical studies are of a qualitative nature, the proposed model is quantitative.

In this section, we will ignore the sexual organs of the plants (flowers and inflorescences).

As shown by botanical studies, three important phenomena characterize the functioning of meristems:

- growth,
- ramification,
- mortality.

They heavily depend on time so that models of plant architectures have to integrate time in growth, ramification and mortality processes.

In de Reffye's mathematical macroscopic model of plants' growth, the unit of discretized time is the time taken by the growth of a growth unit, this length of time being supposed constant for the axes a given order of a given plant. As a consequence, axes of different order of the same plant can grow with different speeds (see Figure 14).

Moreover, each bud is given two probabilities (stochastic parameters of the model): its probability to abort, and its probability to make a break, i.e. to wait during a unit of time without growing (if it is not dead). The "non-abort" probability of an axillary bud is, in fact, a ramification probability, that is the probability of birth of an axillary stem which is the start of a new axis.

Let us call the age of a node or a bud its date of birth according to the clock of the model, with time *0* being the time when the seed begins to grow; the dimension of a node or a bud is its age relative to the birthday of its axis.

We will give now several examples of the functioning of the model, starting with a very trivial one.

Example 0: Let us draw a growth unit as a straight segment and suppose that the seed has produced a growth unit at time *1*. Then, at time 2, if the break and death probabilities are different from *0*, the 3 cases depicted on

Figure 10 can occur.

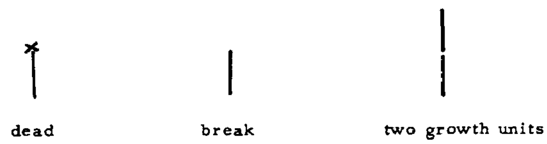


Figure 10: Growth model. Example 0.

To explain the branching process, let us recall that at each node two types of axillary buds can appear. Those which correspond to "standard" branching can give birth to an new axis of greater (by one) order; reiteration buds give birth to a reiteration. Moreover, these axillary buds become visible only if the apical bud is not dead or in a break phase. The number of buds of each type can be calculated by using a probability law called the branching law, but is usually constant for axes of a given order of a given plant (in our simulations).

With very straightforward hypotheses on the probabilities involved, a few simulations can be easily done. Let us suppose, for example, that the maximum order is *3* (this can be obtained when the ramification probability of axillary buds on order *3* axes is equal to *0*, or when the buds on order *4* axes die systematically). With one axillary bud on the last node of each growth unit, with no axillary bud elsewhere, with the standard ramification probability equal to *1* and with growth speed identical on every axis, one can obtain the results of examples 1 to 4 (the geometry is very straightforward) below.

Example 1 (see Figure 11): break and death probabilities equal to *0*.

Example 2 (see Figure 12): break probability equal to *0*, death probabilities different from *0*.

Example 3 (see Figure 13): death probability equal to *0*, break probability equal to *0* on order *1* axis, different from *0* and large on the other axes.

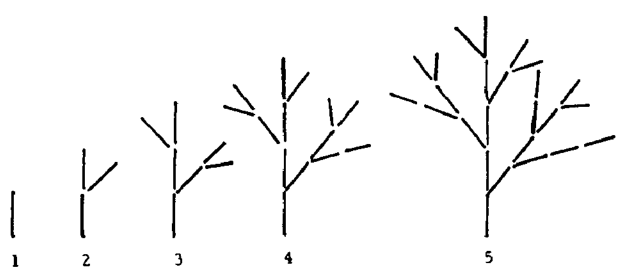


Figure 11: Growth model. Example 1 (age 1, 2, 3, 4, 5).

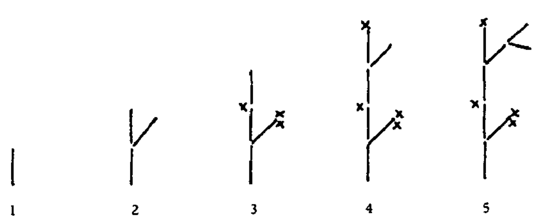


Figure 12: Growth model. Example 2 (age 1, 2, 3, 4, 5).

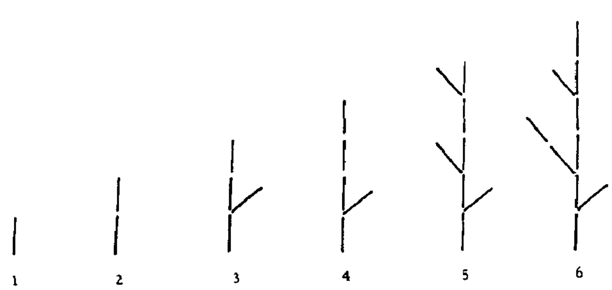


Figure 13: Growth model. Example 3 (age 1, 2, 3, 4, 5, 6).

Example 4: order *1* and *2* axes only, one internode per growth unit, 2 axillary buds per node on axis *1*, distic phyllotaxy, death probability equal to *0*, break probability equal to *0* on order *1* axis, different from *0* on order *2* axes, growth speed ratio (order *1* axis over order 2 axis) equal to *r*. For ages around 20 units of time on the order *1* axis, with a geometry similar to that of the robusta coffee tree, and fall of the leaves simulated, the architectures shown on Figure 14 can be obtained.

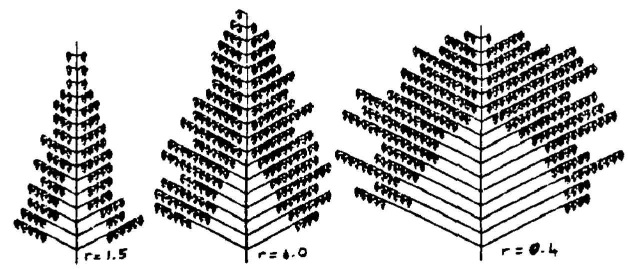


Figure 14: Growth of order *1* and order *2* axes, with growth ratio *r* (with break).

In the growth model, it is supposed that the probabilities are only functions of the age, dimension and order of the bud, for a given variety and species. It has been proved experimentally, for example, that the mortality of an apical bud is a monotonic increasing function of its dimension for a given order. As another example, the probability for a bud to make a break can be chosen as a monotonic increasing function of age, for a given order. For image synthesis purposes these hypotheses prove to be sufficient (even *0* and *1* probabilities produce a wide variety of forms). For a greater fidelity to botany, one has to take into account the dependence on other parameters such as the number of nodes in a growth unit.

All the probability laws involved can be measured experimentally; this has been done in several cases (for example the coffee-tree) for agronomic purposes, and the measures validate the model (cf. [20]). Moreover, with different choices of the probability laws, one can obtain the architectural types of Hallé, Oldeman and Tomlinson's classification.

4. Growth simulation.

Computer simulation of the growth of plants according to our model is possible as the model is a numerical one. Results of the first experiments appear in [20] and [21].

In order to simulate the growth of a plant, the following parameters have to be given:

- the age,
- the clocks or growth speeds of the axes,
- the number of possible buds at each node, as a function of order,
- the probabilities of death, pause, ramification and reiteration given as functions of age, dimension and order (see section 3). These probabilities are sufficient to generate images of great realism; a rather wide variety of forms can even be obtained with probabilities restricted to 0 and 1. The probability laws chosen for the simulation can either be the result of experimental measures or calculated from mathematical laws taken as hypotheses, or completely arbitrary, according to the objective of the simulation (although, up to now, the model has been used to give more faithful images of plants, it would be of interest, and very easy, to simulate the effect of "strange" growth laws). Most of the time, the probabilities are the same for all the buds of axes of a given order of a given plant; usually, the laws are uniform or exponential laws.
- the type of the growth unit and the number of nodes per growth unit, given as functions of the same parameters as the previous probabilities (if one seeks still greater fidelity to botany, one has to define these parameters as random variables which depend on various other parameters (see [10]));
- the geometric parameters of an internode: length and diameter as a function of the same parameters and of the age of the plant for the diameter (for cylindrical internodes) or diameters (for cone-shaped internodes);
- for every axis order, the development trend (orthotropy or plagiotropy), the insertion angle with the bearing axis (with the ground for order *1* axis), the phyllotaxy, which is also characterized by an angle. For a greater fidelity to botany, one has, as has been indicated above for the growth unit parameters, to define the insertion angle and the phyllotaxy as random variables, functions of various parameters and to consider the development trend as composite (see [10]).

The insertion angle and the development trend are used to smooth out the shape of an axis according to its trend. For example, in the case of a vertical order *1* axis bearing an order *2* axis, the orthotropic and plagiotropic case are dealt with as shown in Figure 15.

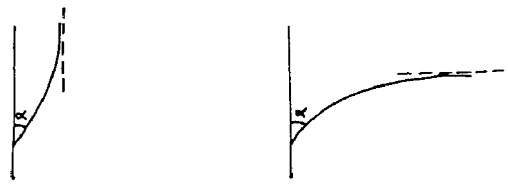


Figure 15: Development trend of a tree (left:orthotropy, right: plagiotropy).

In order to obtain images with great fidelity to nature, the choice of the important parameters and their experimental measurement has to be done with particular care. This usually requires a good knowledge of botany and of the model.

The simulation consists in going through each instant of discretized time from the birth of the plant to the given age, and, for each of these instants, to consider all the buds which are alive at that moment. Each bud undergoes a mortality test; if necessary, a break test; and lastly, if necessary, a ramification test. According to the results of these tests, the bud can be suppressed from the set of living buds or a new internode can (if there is no break of the growth at that point) be created (with a new apical bud and new axillary buds with a given order and geometric position).

The simulation can be expressed in pseudo-code as follows:

```
for each clock signal do
  for each bud which is still alive do
  {order, age, dimension, position, etc. are known attributes of the bud}
    if bud doesn't die then
      if bud doesn't make a pause then
        create internode
          {with position in space}
        create apical bud
        for each possible bud do
          if ramification then create axillary buds
                              {with age, order and dimension}
        endfor
      endif
    endif
  endfor
endfor
```

Up to now, such a simulation of time has not been used in the actual implementation of the algorithm because of its requirements in memory space. A prefix traversal of the tree has been preferred: the axillary buds of a node are numbered $1, 2, \ldots, k$ in any order, the apical bud is numbered $k+1$; a prefix traversal is the chain of prefix traversals of the "trees" stemmed from bud i, for $i=1,\ldots, k$. This kind of traversal can be used for simulation purposes since, because of the hypotheses, when the traversal visits a bud, by previous processing its age, dimension, order (and, possibly others structural parameters if needed) and position are known. Since, in nature, the order of an axis is very rarely bigger than five, the size of the stack used for the simulation of the traversal is moderate.

The prefix traversal we used up to now is not suited to an animation of growth as it imposes that the whole plant is calculated again at each age used in the animation. Moreover, with such a traversal, one cannot take into account some significant botanical phenomena such as growth obstruction (growing of mortality or break, for example) due to fixed obstacles or to part of the plant against another part, or the effect of the shade cast by the tree on its own buds. A more accurate simulation of growth and its application to obstruction is in progress.

The growth simulation program manages the tests of death, break or ramification by calling a random number generator and the laws procedures for the probabilities. It also manages the geometric parameters. Its output is a file whose records' fields include a tag characteristic of the component (short or long internode, leaf,...), a position in space, and two 3D directions for the component.

The described generator is an architecture generator; for ease of exposition we have not mentioned, up to now, how to deal with the simulation of other phenomena. This will be done in the next section.

5. Other simulations.

Autumn's fall of leaves can be very easily simulated; one has only to define a life length for leaves, which is usually taken as constant for a given plant.

In a similar way, botanists know the laws that rule the birth of flowers and inflorescences, their life length and the fruits' life-time. The accordance with the botanical model insures that the leaves and fruits are inserted in the right places on the branches (according to the age of the tree). This implies that the model is also accurate when one looks at a precise part of the tree, instead of looking at its overall shape.

As for the pruning of trees, it is simulated by saying that when all the buds of the same branch are dead the branch itself is dead and falls after some time.

Lastly, the calculation of the bending of a branch under the action of forces like gravity or wind can be done according to the theory of material strength (see [20], [25]); it is done using the geometrical shape of internodes and the elasticity parameter (Young's modulus) which characterizes the wood of the branch. As regards wind effects no images are included here but an animation is in progress.

Although in a few cases other models exist ([8], [2]), the same plant generator can also be diverted to output parts of a plant instead of its overall architecture (according to [7] our modeling for image synthesis purposes is, in this case, structural but not physical). For example, an inflorescence is considered by the plant generator as a small tree bearing axes of order at most *2* or *3*, very regular, the last node on every axis bearing a flower. In a similar way, the sepals, petals, stamens and stigmas of a flower can be generated as specific internodes or growth units (for another approach, see [12]). A palm tree's palm can also be generated as

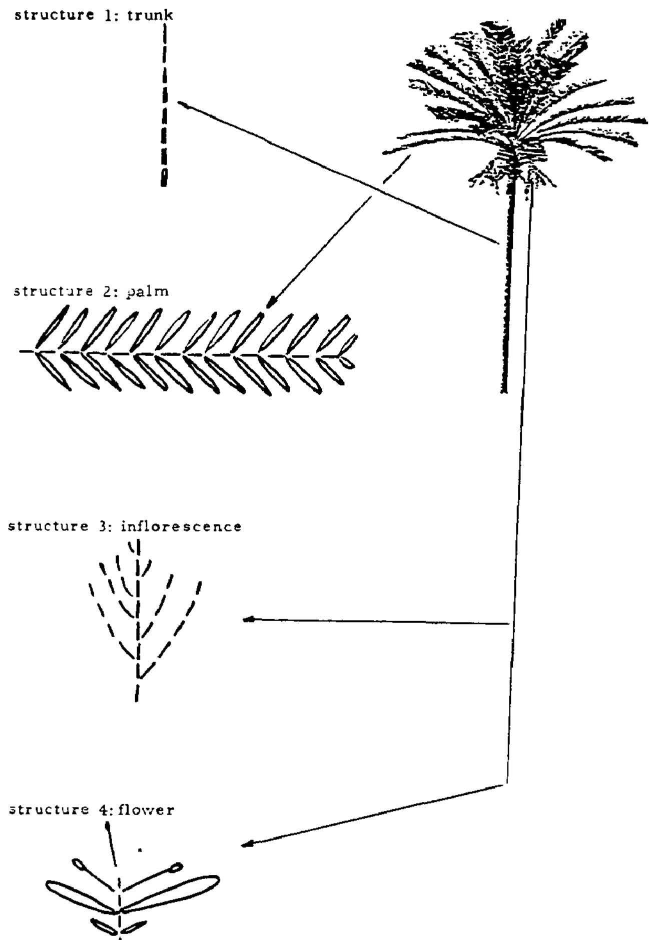
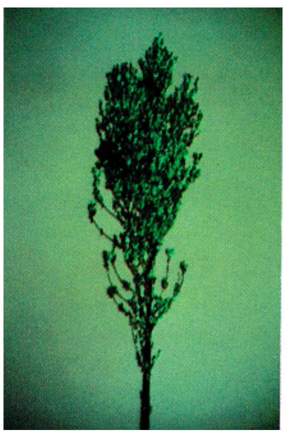
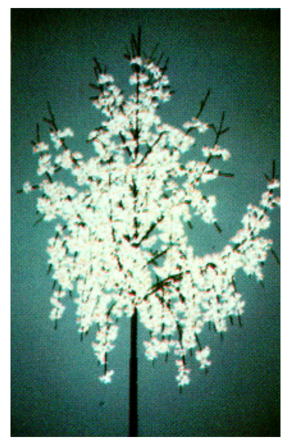


Figure 16: Four structures used for a date palm tree.





Photos

Previous page, from left to right:
- first row
  Photo 1: Fir tree.
  Photo 2: Pruned tree (with traumatic reiterations).
  Photo 3: Top of a poplar (aspen).
- second row
  Photo 4: Italy poplar.
  Photo 5: Pine tree.
  Photo 6: Approximate wild cherry tree with flowers.
- third row
  Photo 7: Another approximate wild cherry tree with fruits.
  Photo 8: Coconut tree.
  Photo 9: Fruit tree in spring.

This page:
- top left
  Photo 10: Young cedar tree.
- right column, from top to bottom:
  Photo 11: Weeping willow without leaves.
  Photo 12: Weeping willow with leaves.
  Photo 13: Frangipani tree.
  Photo 14: Herbs and trees.
  Photo 15: Herbs and poplars in autumn.

a growth unit with specific internodes (see figure 16). As a last example, let us mention that an underground rhizome can be calculated as a monopod with a rather simple architecture; at its nodes, herbs, which are monopods with another architecture, grow.

6. Visualization.

The plant generator's output is a file of records whose fields are the following ones: a tag which characterizes a botanical component (short or long internode, leaf, flower, fruit, and, if one enlarges the simulation, cf. sect. 5, rhizome internode, palm internode, petal internode, sepal internode, etc.) and geometric information needed to position the component in space.

A form taken from a library of forms is associated with every tag: for example, straight cylinders, hexagonal cylinders or straight truncated cones for internodes, polygons for leaves or flower petals,.... Moreover, the library of forms can be parametrized by age. For the photos we generated, we only used cones with hexagonal bases or even segments for internodes; for leaves, petals or sepals, we used an assembly of at most 4 convex plane polygons (in a few cases a vector is sufficient) with an associated normal if needed. Small fruits are made with a few vectors. To each form a color is associated.

The visualization is then the visualization of a set of vectors and faces. Basic rudimentary methods have then been used for the rendering: perspective, no antialiasing, uniform color for faces,... In a few cases a light source was used to change the polygons' luminosity according to the angle between the source direction and the normal to the polygon. Hidden surface removal is done by the use of a Z-buffer.

The use of such primitive rendering techniques proved to be sufficient for obtaining images with great realism: the richness of forms is such that antialiasing or texture is usually not necessary; the only exception is for close-ups of a part of a tree, of its trunk or of one of its branches. Usually using a rough Z-buffer (with a unique depth for all the pixels of a leaf for example) is enough.

Let us mention that the complexity of the structure grows rapidly with the age of the plant. A few tens of components are enough to describe a small plant, but several hundreds of thousands are needed to describe tall trees. In such cases, the computer time needed to generate the model and visualize can be important (a few tens of minutes instead of a few minutes for a small tree).

7. Conclusion.

We have presented a model for the growth of plants and trees which incorporates botanical knowledge of their architecture.

The very first interest of the model is its great richness: the same procedural methods can produce "plants" as different as weeping willows, fir trees, cedar trees, frangipani trees, poplars, pine trees, wild cherry trees, herbs, etc.

Another very important benefit from the model is the integration of time which enables to view the aging of a tree (accurate simulation of the death of leaves and branches for example). Using the model, a short movie showing the evolution of a tree from birth to death has been produced.

Let us also stress that part of the study has been developed for agronomic purposes: mathematical models of the architecture of plants and trees turn out to be very useful for studying land crops; the production is very often dependent on plant growth and architecture. The incidence of factors such as insect attacks, use of fertilizers, planting density, etc. can be studied using the architectural models combined with other mathematical models. This ability to put together different models should also be investigated for image synthesis purposes.

Another very promising study would be to simulate the growth with "arbritary" non botanic parameters in order to obtain non existing architectures, strange shapes, etc. In doing so, one could benefit from the richness of the model for animation purposes.

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