Definition 1 (characteristic function)**.** $\varphi_\mu(t) = \int_{\mathbb{R}^d} e^{i\langle t,x\rangle} d\mu(x)$
$\varphi_X(t) = \varphi_{\mu_X}(t) = \mathbb{E}e^{i\langle t,x\rangle}$

Proposition 2. *$\varphi_X(0) = 1$ and $|\varphi_X(t)| \leqslant 1, t \in \mathbb{R}^d$*

Proposition 3. *φ_X uniformly continuous on $\mathbb{R}^d$.*

Proposition 4. *φ_X is a nonnegatively determined function on $\mathbb{R}^d$.*

Theorem 5 (Bochner)**.** *A function $\varphi : \mathbb{R}^d \to \mathbb{C}$ is a characteristic function of a random d-dimensional vector iff $\varphi(0) = 1$, φ is continuous and φ is nonnegatively determined, i.e. $(\varphi(t_i - t_j))_{i,j}$ is nonnegatively determined for any t_i's.*

Proposition 6. *$\varphi_{AX+b}(t) = e^{i\langle b,t\rangle}\varphi_X(A^T t)$, in particular $\varphi_{-X}(t) = \varphi_X(-t) = \overline{\varphi_X(t)}$.*

Proposition 7. *X real random variable, if $\mathbb{E}|X|^k < \infty$, $k \in \mathbb{Z}_+$, then $\varphi_X \in C^k(\mathbb{R})$ and $\varphi_X^{(k)}(t) = i^k \mathbb{E}X^k e^{itX}$.*

Remark 8. Existence of φ'_X does not imply $\mathbb{E}|X| < \infty$.

Proposition 9. *If μ, ν probability measures on $\mathbb{R}^d$ such that $\varphi_\mu = \varphi_\nu$, then $\mu = \nu$.*

Proposition 10. *$X_1, \ldots, X_n$ independent random d-dimensional variables, then $\varphi_{X_1+\ldots+X_n}(t) = \varphi_{X_1}(t)\ldots\varphi_{X_n}(t)$.*