Definition 1 (characteristic function). $\varphi_\mu(t) = \int_{\mathbb{R}^d} e^{i\langle t,x\rangle} d\mu(x)$

$\varphi_X(t) = \varphi_{\mu_X}(t) = \mathbb{E}e^{i\langle t,x\rangle}$

Proposition 2. $\varphi_X(0) = 1$ *and* $|\varphi_X(t)| \leqslant 1, t \in \mathbb{R}^d$

Proposition 3. φ_X *uniformly continuous on* $\mathbb{R}^d$.

Proposition 4. φ_X *is a nonnegatively determined function on* $\mathbb{R}^d$.

Theorem 5 (Bochner). *A function* $\varphi : \mathbb{R}^d \to \mathbb{C}$ *is a characteristic function of a random d-dimensional vector iff* $\varphi(0) = 1$, φ *is continuous and* φ *is nonnegatively determined, i.e.* $(\varphi(t_i - t_j))_{i,j}$ *is nonnegatively determined for any* t_i*'s.*

Proposition 6. $\varphi_{AX+b}(t) = e^{i\langle b,t\rangle}\varphi_X(A^T t)$, *in particular* $\varphi_{-X}(t) = \varphi_X(-t) = \overline{\varphi_X(t)}$.

Proposition 7. X *real random variable, if* $\mathbb{E}|X|^k < \infty$, $k \in \mathbb{Z}_+$, *then* $\varphi_X \in C^k(\mathbb{R})$ *and* $\varphi_X^{(k)}(t) = i^k \mathbb{E}X^k e^{itX}$.

Remark 8. Existence of φ'_X does not imply $\mathbb{E}|X| < \infty$.

Proposition 9. *If* μ, ν *probability measures on* $\mathbb{R}^d$ *such that* $\varphi_\mu = \varphi_\nu$, *then* $\mu = \nu$.

Proposition 10. $X_1, \ldots, X_n$ *independent random d-dimensional variables, then* $\varphi_{X_1+\ldots+X_n}(t) = \varphi_{X_1}(t)\ldots\varphi_{X_n}(t)$.