Theorem 1 (CTL in the iid case)**.** *$X_1, X_2, \ldots$ iid random variables, $\mathbb{E}X_1 = a$, $\mathrm{Var}(X_1) = \sigma^2 \in (0, \infty)$, then $\frac{X_1+\ldots+X_n-na}{\sqrt{n}\sigma} \implies \mathcal{N}(0,1)$.*

Corollary 2 (de Moivre-Laplace)**.** $X_n \sim \mathrm{Bin}(n,p)$, then $\frac{X_n-np}{\sqrt{np(1-p)}} \implies \mathcal{N}(0,1)$.

Definition 3 (triangular array)**.** $(X_{n,k})_{n=1,2,\ldots;1\leqslant k\leqslant k_n}$ is a *triangular array of random variables* if $\forall_n X_{n,1}, \ldots, X_{n,k_n}$ are independent.

Theorem 4 (CLT – Lindeberg's version)**.** *$(X_{n,k})$ a triangular array, $S_n = X_{n,1} + \ldots + X_{n,k_n}$,*

1. $\mathbb{E}S_n \to m$
2. $\mathrm{Var}(S_n) \to \sigma^2$
3. Lindeberg's condition: $\forall_{\varepsilon>0} \sum_{k=1}^{k_n} \mathbb{E}\left(|X_{n,k} - \mathbb{E}X_{n,k}|^2 \mathbb{1}_{\{|X_{n,k}-\mathbb{E}X_{n,k}|>\varepsilon\}}\right) \to 0$

Then $S_n \implies \mathcal{N}(m, \sigma^2)$.

Lemma 5. *Lindeberg's condition implies*

- $\forall_{t>0} \lim_{n\to\infty} \mathbb{P}(\max_{1\leqslant k\leqslant k_n} |X_{n,k}| \geqslant t) = 0,$
- $\lim_{n\to\infty} \max_{1\leqslant k\leqslant k_n} \mathrm{Var}(X_{n,k}) \to 0.$

Theorem 6 (Feller)**.** *Suppose that $(X_{n,k})$ is a triangular array, $\sum_{k=1}^{k_n} \mathbb{E}X_{n,k} \to m$, $\sum_{k=1}^{k_n} \mathrm{Var}(X_{n,k}) \to \sigma^2$, and $S_n = \sum_{k=1}^{k_n} X_{n,k} \implies \mathcal{N}(m, \sigma^2)$, then if $\max_{1\leqslant k\leqslant k_n} \mathrm{Var}(X_{n,k}) \to 0$, then Lideberg's condition is satisfied.*