Theorem 1. $\varphi_{\mathcal{N}(0,1)}(t) = e^{-\frac{t^2}{2}} \qquad \varphi_{\mathcal{N}(a,\sigma^2)}(t) = e^{ita-\frac{t^2\sigma^2}{2}}$

Theorem 2. *X random variable in $\mathbb{R}^d$, $\mathbb{E}|X_1|^{k_1}\dots|X_d|^{k_d} < \infty$, then $\frac{\partial^{k_1}}{\partial t_1^{k_1}}\dots\frac{\partial^{k_d}}{\partial t_d^{k_d}}\varphi_X(t)$ exists and equals $i^{|k|}\mathbb{E}X_1^{k_1}\dots X_d^{k_d}$.*

Remark 3. $X_1,\dots,X_d$ independent, then $\varphi_{X_1+\dots+X_d}(t) = \varphi_{X_1}(t)\dots\varphi_{X_d}(t)$, but opposite is not true in general.

Theorem 4. *$X_1,\dots,X_d$ independent iff $\forall_{t\in\mathbb{R}^d}\varphi_{(X_1,\dots,X_d)}(t) = \varphi_{X_1}(t_1)\dots\varphi_{X_d}(t_d)$.*

Theorem 5 (Lévy-Cramer)**.** *1. If μ_n,μ probability measures on $\mathbb{R}^d$ and $\mu_n \implies \mu$, then $\forall_{t\in\mathbb{R}^d}\varphi_{\mu_n}(t)\to\varphi_\mu(t)$.*

2. If μ_n probability measure on $\mathbb{R}^d$ and there exists function $\varphi:\mathbb{R}^d\to\mathbb{C}$ such that $\forall_{t\in\mathbb{R}^d}\varphi_{\mu_n}(t)\to\varphi(t)$ and φ is continuous at 0, then there exists a probability measure μ on $\mathbb{R}^d$ such that $\varphi=\varphi_\mu$ and $\mu_n \implies \mu$.

Corollary 6. $\mu_n \implies \mu$ iff $\varphi_{\mu_n}\to\varphi_\mu$ pointwise.

Theorem 7 (Inverse Fourier Theorem)**.** *Suppose that μ is a probability measure on $\mathbb{R}^d$ and $\varphi_\mu \in L^1(\mathbb{R}^d)$ such that $\int_{\mathbb{R}^d}|\varphi_\mu(x)|\,dx<\infty$, then μ has the density g given by the formula*

$$g(x) = \frac{1}{(2\pi)^d}\int_{\mathbb{R}^d}\varphi_\mu(t)e^{-i\langle t,x\rangle}dt.$$