TFE4146 - Semiconductor Devices - Fall 2023

Problem Set 6

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Contents

Problem 1 - Zener Breakdown

Consider a reversed biased p-n junction where the doping concentrations are $N_A = N_D = 4 \cdot 10^{18}$ cm^{-3}. Let V_r denote the reverse bias voltage, $V_j = V_0 + V_r$ the total voltage difference across the p-n junction and let W denote the width of the depletion region.

a) Explain the difference between avalanche and Zener breakdown.

We start by explaining them sepparately, and then comparing the two phenomens:

- **Zener breakdown:**

  When a heavily doped junction is reverse biased, the bands become crossed at a relative low voltage. If the barrier separating these two band is narrow, then tunneling of electrons can occour as shown in figure 1 (b). The basic requirements for tunneling current are a large number of electrons separated from a large number of empty states by a narrow barrier of finite height

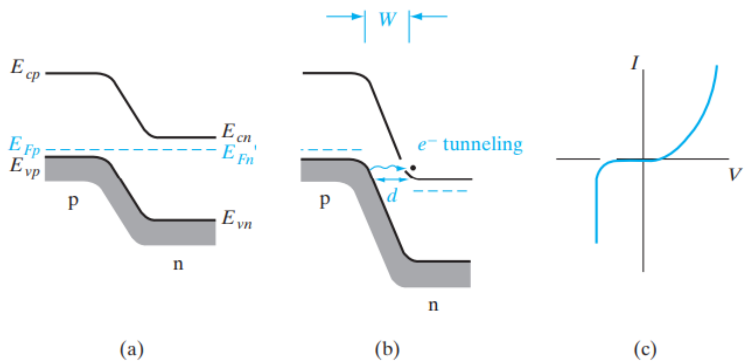
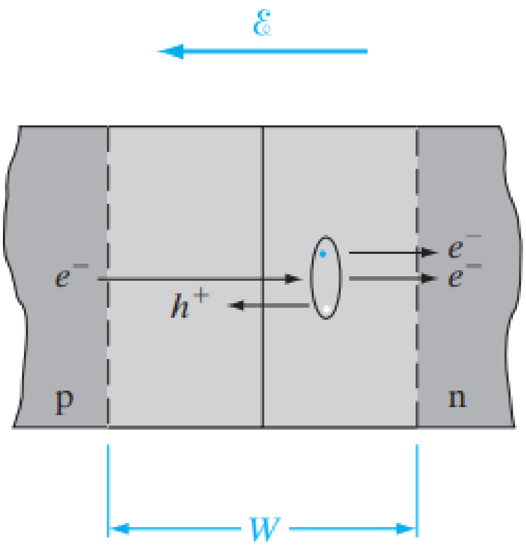


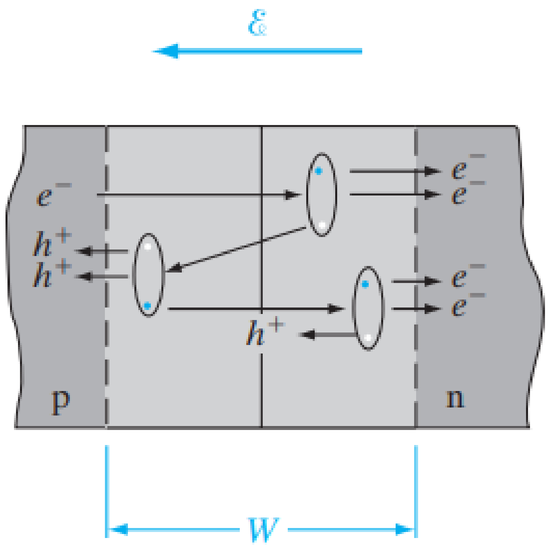
Figure 1: The Zener effect:(a) heavily doped junction at equilibrium; (b) reverse with electron tunneling from p to n; (c) $I-V$ characteristic.

- **Avalanche breakdown:**

  When a junction is lightly doped, the electron tinneling is negligible. Therefore the breakdown mechanism involves the *impact ionization*. The normal lattice-scattering events can result in the creation of electron-hole pairs if the carrier being scattered has sufficient energy. This might occour when electric field $\mathscr{E}$ in the transition region is large, resulting in an electron accelerating to a high kinetic energy that leads to ionizing collision with the lattice as seen in figure 2b.



(a) a single ionizing collision by an incoming electron in the depletion region of the junction;



(b) primary, secondary, and tertiary collisions.

Figure 2: Electron-hole pairs created by impact ionization

- Diffrence between avalanche and Zener breakdown:

Avalanche Breakdown	**Zener Breakdown**
Occurs in lightly doped junctions.	Occurs in heavily doped junctions.
Initiated by carriers acquiring enough energy from the electric field to ionize other carriers, leading to a chain reaction and a rapid increase in current.	Caused by the quantum tunneling of carriers.
Non-destructive in nature unless the junction undergoes prolonged high current.	Typically occurs at a well-defined voltage and is utilized in Zener diodes for voltage regulation.
Breakdown voltage increases with an increase in temperature (positive temperature coefficient).	Breakdown voltage decreases with an increase in temperature (negative temperature coefficient).

Table 1: Differences between Avalanche and Zener Breakdown

b) Assume the critical field strength for Zener breakdown is $\mathrm{E}_B = 10^6$ V cm^{-1}. Find the reverse bias voltage V_r for which Zener breakdown occurs, i.e. when the maximum value of the electric field reaches E_B.

Given:
Relative permittivity in Si : $\epsilon_r = 11.8$
Permittivity in vacuum: $\epsilon_0 = 8.85 \cdot 10^{-14}$ F cm^{-1}
Intrinsic charge carrier density in Si : $n_i = 1.5 \cdot 10^{10}$ cm^{-3}**.**

As the width of the depletion region given by

$$W = \left[\frac{2\epsilon\,(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} \qquad \text{(with bias)}$$

where $V = -V_r$ and V_0 is given by

$$V_0 = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$$

This gives us

$$V_0 = \frac{1.38 \cdot 10^{-23} \cdot 300}{1.6 \cdot 10^{-19}}\ln\frac{\left(4 \cdot 10^{18}\right)^2}{\left(1.5 \cdot 10^{10}\right)^2} = 1.00V$$

The electric field across the depletion region is

$$\mathscr{E} = \frac{V_j}{W}$$

At zener breakdown, $\mathscr{E} = \mathscr{E}_B$

This gives us

$$\begin{aligned}
\frac{V_0 + V_r}{\left[\frac{2\epsilon(V_0 - V)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2}} &= \mathscr{E}_B \\
\frac{(V_0 + V_r)^2}{\frac{2\epsilon(V_0 + V_r)}{q}\left(\frac{N_a + N_d}{N_a N_d}\right)} &= \mathscr{E}_B^2 \\
V_0 + V_r &= \mathscr{E}_B^2 \frac{2\epsilon}{q}\left(\frac{N_a + N_d}{N_a N_d}\right) \\
V_r &= \mathscr{E}_B^2 \frac{2\epsilon}{q}\left(\frac{N_a + N_d}{N_a N_d}\right) - V_0 \\
V_r &= \left(10^6\right)^2 \frac{2 \cdot 11.8 \cdot 8.85 \cdot 10^{-14}}{1.6 \cdot 10^{-19}}\left(\frac{2 \cdot \left(4 \cdot 10^{18}\right)}{\left(4 \cdot 10^{18}\right)^2}\right) - 1 \\
V_r &= 5.53V
\end{aligned}$$

Problem 2 - Transient and A-C Conditions

Consider a $\mathrm{p}^+ - \mathrm{n}$ **junction, where** $Q_p(t)$ **denotes the time-dependent excess hole charge in the neutral n-region.**

a) Explain intuitively why $Q_p(t)$ and the diode current $i(t)$ is related by

$$i(t) = \frac{Q_p(t)}{\tau_p} + \frac{dQ_p(t)}{dt}$$

This relationship can be understood in two parts:

1. **Steady-State Recombination Current:**

   The term $\frac{Q_p(t)}{\tau_p}$ represents the recombination current in the base region. Intuitively, this means that the charge carriers (electrons and holes) stored in the base region will recombine and annihilate each other at a rate that is inversely proportional to the lifetime τ_p.The longer the lifetime, the slower the recombination. This term essentially says, "if we leave the stored charge $Q_p(t)$ in the base for a time τ_p, all of it would recombine." So, the rate at which it's recombining at any instant is given by the fraction $\frac{Q_p(t)}{\tau_p}$.

2. **Rate of Change of Stored Charge:**

   The term $\frac{dQ_p(t)}{dt}$ represents the rate of change of the stored charge in the base region. This is intuitive because any change in the stored charge $Q_p(t)$ will directly affect the current flowing through the diode. If the stored charge is increasing (i.e., $\frac{dQ_p(t)}{dt} > 0$), this implies that more carriers are entering the base than leaving, which increases the diode current. Conversely, if the stored charge is decreasing (i.e., $\frac{dQ_p(t)}{dt} < 0$), this means more carriers are leaving the base than entering, which reduces the diode current.

To tie everything together:

- The diode current is affected both by the charge currently stored in the base region and by how that charge is changing with time.
- The first term gives the contribution from the steady-state recombination of stored charge, while the second term gives the contribution from the dynamic change in stored charge.

b) Find the stored charge as a function of time when a forward current I is suddenly switched on at time $t = 0$. In other words, let

$$i(t) = \begin{cases} 0 & \text{for } t < 0 \\ I & \text{for } t \geq 0' \end{cases} .$$

Assume $Q_p(t) = 0$ for all $t < 0$ and find $Q_p(t)$ for $t \geq 0$.

We begin with Laplace transform

$$\mathscr{L}\{i(t)\} = \mathscr{L}\left\{\frac{Q_p(t)}{\tau_p}\right\} + \mathscr{L}\left\{\frac{dQ_p(t)}{dt}\right\}$$

For the term $\mathscr{L}\left\{\frac{Q_p(t)}{\tau_p}\right\}$:

$$\mathscr{L}\left\{\frac{Q_p(t)}{\tau_p}\right\} = \frac{1}{\tau_p}\mathscr{L}\left\{Q_p(t)\right\}$$

For the term $\mathscr{L}\left\{\frac{dQ_p(t)}{dt}\right\}$, using the property:

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

where $F(s)$ is the Laplace transform of $f(t)$ and $f(0)$ is the initial value of $f(t)$, we get:

$$\mathcal{L}\left\{\frac{dQ_p(t)}{dt}\right\} = s\mathcal{L}\left\{Q_p(t)\right\} - Q_p(0)$$

For the term $\mathscr{L}\{i(t)\}$:

$$\mathscr{L}\{i(t)\} = \mathscr{L}\{Iu(t)\}$$

where $u(t)$ is the unit step function

$$\mathscr{L}\{i(t)\} = \frac{I}{s}$$

Combining the three results:

$$\frac{I}{s} = \frac{1}{\tau_p}\mathcal{L}\left\{Q_p(t)\right\} + s\mathcal{L}\left\{Q_p(t)\right\} - Q_p(0)$$

If we let $Q_p(s)$ be the Laplace transform of $Q_p(t)$, and apply the assumption of $Q_p(t) = 0$ for $t \leq 0$, then:

$$\frac{I}{s} = \left(\frac{1}{\tau_p} + s\right) Q_p(s)$$
$$\frac{I}{s\left(\frac{1}{\tau_p} + s\right)} = Q_p(s)$$
$$\frac{I\tau_p}{s(s\tau_p) + 1} = Q_p(s) = \frac{A}{s} + \frac{B}{s\tau_p + 1}$$

this gives us

$$I\tau_p = A\left(s\tau_p + 1\right) + Bs$$

for $s = 0$

$$I\tau_p = A$$

to find B:

$$I\tau_p - A\left(s\tau_p + 1\right) = Bs$$
$$-I\tau_p\left(s\tau_p\right) = Bs$$
$$B = -I\tau_p^2$$

$$Q_p(s) = \frac{I\tau_p}{s} - \frac{I\tau_p^2}{s\tau_p + 1}$$

$$Q_p(t) = I\tau_p\left(1 - e^{-\frac{t}{\tau_p}}\right) u(t)$$

c) At any time during the transient, the junction voltage $v(t)$ is related to $\Delta p_n(t)$ by

$$\Delta p_n(t) = p_n\left(e^{\frac{qv(t)}{kT}} - 1\right)$$

Use the quasi steady-state approximation

$$\delta p_n\left(x_n, t\right) = \Delta p_n(t) e^{-x_n/L_p}$$

to find the junction voltage $v(t)$ as a function of time for the turn-on transient. Assume the neutral n-region is long compared to L_p. Sketch $v(t)$ and verify that your result is reasonable for $t \to \infty$.

Using the quasi steady-state approximation

$$\delta p_n\left(x_n, t\right) = \Delta p_n(t) e^{-x_n/L_p}$$

we have for the stored charge at any instant

$$Q_p(t) = qA \int_0^{\infty} \Delta p_n(t) e^{-x_n/L_p} dx_n = qAL_p \Delta p_n(t)$$

Relating $\Delta p_n(t)$ to $v(t)$ we have

$$\begin{aligned}
\Delta p_n(t) &= p_n\left(e^{qv(t)/kT} - 1\right) = \frac{Q_p(t)}{qAL_p} \\
e^{qv(t)/kT} &= \frac{Q_p(t)}{qAL_p p_n} + 1 \\
v(t) &= \frac{kT}{q} \ln\left(\frac{I\tau_p\left(1 - e^{-\frac{t}{\tau_p}}\right) u(t)}{qAL_p p_n} + 1\right)
\end{aligned}$$

Using arbitary values for the variables we get

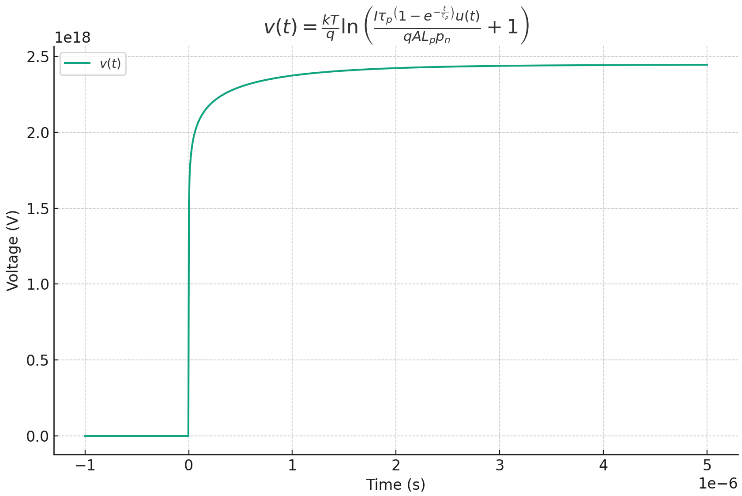


Figure 3: Plot of $v(t)$

Problem 3 - Narrow base diode

Assume that a $p^+ - n$ diode is built with an n-region width l smaller than a hole diffusion length, i.e. $l < L_p$. This is the so-called narrow base diode. Since for this case holes are injected into a short n-region under forward bias, we cannot use the assumption $\delta p_n (x_n = \infty) = 0$ as a boundary condition for the steady-state diffusion equation. Instead, assume that the minority carrier concentration is forced to zero at $x_n = l$ by an ohmic contact to the external circuit.

a) Solve the diffusion equation for holes in the neutral n-region to obtain

$$\delta p_n (x_n) = \frac{\Delta p_n \left[e^{(l-x_n)/L_p} - e^{(x_n-l)/L_p} \right]}{e^{l/L_p} - e^{-l/L_p}}$$

Sketch $\delta p_n (x_n)$ for different values of l/L_p.

In steady state cases the diffusion equations can be written as

$$\frac{d^2 \delta p}{dx^2} \equiv \frac{\delta p}{L_p^2}$$

Given the boundary conditions:

1. At $x_n = 0$:$\delta p_n(o) = \Delta p_n$
2. At $x_n = l$:$\delta p_n(l) = 0$

The solution to this equation has the form

$$\delta p(x) = C_1 e^{\frac{x}{L_p}} + C_2 e^{-\frac{x}{L_p}}$$

using the boundary conditions we get

$$\begin{aligned} \Delta p_n &= C_1 e^{\frac{0}{L_p}} + C_2 e^{-\frac{0}{L_p}} \\ C_2 &= \Delta p_n - C_1 \end{aligned}$$

$$
\begin{aligned}
0 &= C_1 e^{\frac{l}{L_p}} + C_2 e^{-\frac{l}{L_p}} \\
C_2 &= \frac{-C_1 e^{\frac{l}{L_p}}}{e^{-\frac{l}{L_p}}} \\
C_2 &= -C_1 e^{\frac{2l}{L_p}}
\end{aligned}
$$

$$
\begin{aligned}
\Delta p_n - C_1 &= -C_1 e^{\frac{2l}{L_p}} \\
\Delta p_n &= C_1 - C_1 e^{\frac{2l}{L_p}} \\
\Delta p_n &= C_1 \left(1 - e^{\frac{2l}{L_p}}\right) \\
C_1 &= \frac{\Delta p_n}{1 - e^{\frac{2l}{L_p}}}
\end{aligned}
$$

$$
C_2 = -\frac{\Delta p_n e^{\frac{2l}{L_p}}}{1 - e^{\frac{2l}{L_p}}}
$$

This gives us the solution

$$
\begin{aligned}
\delta p_n(x_n) &= \frac{\Delta p_n e^{\frac{x_n}{L_p}}}{1 - e^{\frac{2l}{L_p}}} - \frac{\Delta p_n e^{\frac{2l}{L_p}} e^{-\frac{x_n}{L_p}}}{1 - e^{\frac{2l}{L_p}}} \\
\delta p_n(x_n) &= \frac{\Delta p_n \left[e^{\frac{x_n}{L_p}} - e^{\frac{2l}{L_p}} e^{-\frac{x_n}{L_p}}\right]}{1 - e^{\frac{2l}{L_p}}} \\
\delta p_n(x_n) &= \frac{\Delta p_n \left[e^{\frac{x_n}{L_p}} e^{-\frac{l}{L_p}} - e^{\frac{2l}{L_p}} e^{-\frac{l}{L_p}} e^{-\frac{x_n}{L_p}}\right]}{e^{-\frac{l}{L_p}} - e^{\frac{2l}{L_p}} e^{-\frac{l}{L_p}}} \\
\delta p_n(x_n) &= \frac{\Delta p_n \left[e^{\frac{x_n - l}{L_p}} - e^{\frac{l - x_n}{L_p}}\right]}{e^{-\frac{l}{L_p}} - e^{\frac{l}{L_p}}} \\
\delta p_n(x_n) &= \frac{\Delta p_n \left[e^{\frac{l - x_n}{L_p}} - e^{\frac{x_n - l}{L_p}}\right]}{e^{\frac{l}{L_p}} - e^{-\frac{l}{L_p}}}
\end{aligned}
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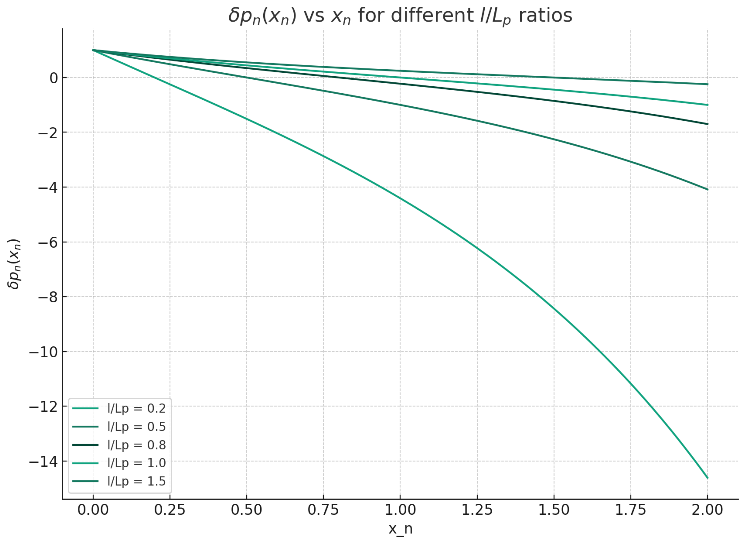

Figure 4: Plot of $\delta p_n(x_n)$ for different values of l/L_p.

b) Show that the current in the diode is

$$I = \left(\frac{qAD_p p_n}{L_p}\coth\left(\frac{l}{L_p}\right)\right)\left(e^{qV/kT}-1\right).$$

Compare the magnitude of the current of this narrow base diode to that of a long $p^+ - n$ **junction.**
Given:

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The hole current density, J_p, is the sum of the drift and diffusion current densities, but in the neutral n-region, the drift current is zero because there is no electric field. Therefore, the hole current is given by the diffusion current:

$$J_p = -qD_p\frac{d\delta p_n}{dx_n}$$

Using the hole concentration, $\delta p_n(x_n)$, we derived in part a:

Differentiating $\delta p_n(x_n)$ with respect to x_n :

$$\frac{d\delta p_n}{dx_n} = \frac{\Delta p_n\left[-\frac{1}{L_p}e^{\frac{l-x_n}{L_p}} - \frac{1}{L_p}e^{\frac{x_n-l}{L_p}}\right]}{e^{\frac{l}{L_p}} - e^{-\frac{l}{L_p}}}$$

Let's substitute this expression into the equation for J_p to determine the hole current density:

$$J_p = qD_p \frac{\Delta p_n \left[\frac{1}{L_p} e^{\frac{l-x_n}{L_p}} + \frac{1}{L_p} e^{\frac{x_n-l}{L_p}} \right]}{e^{\frac{1}{L_p}} - e^{-\frac{1}{L_p}}}$$

At the $p^+ - n$ junction, $x_n = 0.$ So:

$$J_p(0) = qD_p \frac{\Delta p_n \left[\frac{1}{L_p} e^{\frac{1}{L_p}} + \frac{1}{L_p} e^{-\frac{1}{L_p}} \right]}{e^{\frac{1}{L_p}} - e^{-\frac{1}{L_p}}}$$

Now, the total current I is J_p multiplied by the cross-sectional area, A

$$I = AJ_p(0)$$

Substituting the given relation for $\coth(x)$:

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The expression for the current I in the diode is:

$$I = \left(\frac{qAD_p p_n}{L_p} \cdot \frac{\left(e^{\frac{1}{L_p}} + e^{-\frac{1}{L_p}} \right)}{e^{\frac{1}{L_p}} - e^{-\frac{l}{L_p}}} \right) \left(e^{\frac{qV}{kT}} - 1 \right)$$

Using the given relation for $\coth(x)$:

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

The above equation can be simplified as:

$$I = \left(\frac{qAD_p p_n}{L_p} \cdot \coth\left(\frac{l}{L_p} \right) \right) \left(e^{\frac{qV}{kT}} - 1 \right)$$

Which matches the desired result.

Now, let's compare the magnitude of the current of this narrow base diode to that of a long $p^+ - n$ junction.

For a long $p^+ - n$ junction, the current is given by:

$$I = \frac{qAD_p p_n}{L_p} \left(e^{\frac{qV}{kT}} - 1 \right)$$

Comparing the two expressions for I, we can see that the current in the narrow base diode is modified by a factor of:

$$\coth\left(\frac{l}{L_p}\right)$$

The hyperbolic cotangent function, $\coth(x)$, approaches 1 as x approaches 0 , and as x becomes large, $\coth(x)$ approaches 1 . This means that for a very short n-region (l is small compared to L_p), the current in the narrow base diode will be close to that of a long $p^+ - n$ junction. As l increases, the factor $\coth\left(\frac{l}{L_p}\right)$ will continue to be close to 1 , meaning the current will remain close to that of the long junction.

In conclusion, the current in the narrow base diode is modified by the factor $\coth\left(\frac{l}{L_p}\right)$, but for many practical cases, it will be close to the current in a long $p^+ - n$ junction.