TTT4120 Digital Signal Processing

Problem Set 1

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Contents

Problem 1 - Conductivity

a)

The conductivity of a semiconductor will depend on the doping concentration. Express the conductivity σ as a function of

$$\tilde{n} = \frac{n_0}{n_i}$$

where n_i is the intrinsic electron concentration, n_0 is the elecron concentration at thermal equilibrium.

The general equation for conductivity due to electron is

$$\sigma = qn_0\mu_n$$

from the given function we have that

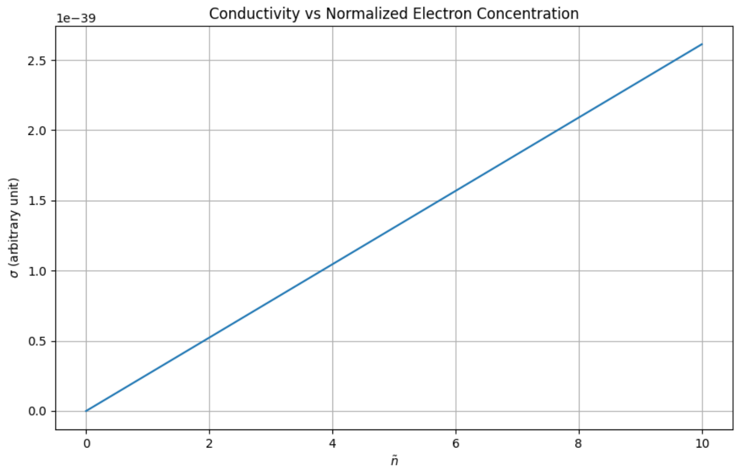
$$n_0 = \tilde{n} \cdot n_i$$

This gives us

$$\sigma = q(\tilde{n} \cdot n_i)\mu_n$$

where q is the charge of an electron, $\tilde{n}$ is the electron concentration normalized to the intrinsic value n_i the intrinsic carrier concentration, μ_n is the electron mobility.

Use the values of μ_n and μ_p for Si and plot the function for $\tilde{n} \in [0, 10]$. Give an intuitive explanation for the shape of the function. Do not consider the variations in the mobility with the doping concentration.



We can se that the relation between the conductivity σ and the electron concentration normalized to the intrinsic value is linear. It makes sense as the the conductivity increases when you n-dope the Si

b)

Show that the minimum conductivity is given by

$$\sigma_{\min} = 2qn_i\sqrt{\mu_n\mu_p}$$

The current density can be written in terms of mobility as

$$J_x = qn\mu_n\mathscr{E}_x$$

If both electrons and holes participate, then we must modify the equation to

$$J_x = q\left(n\mu_n + p\mu_p\right)\mathscr{E}_x = \sigma\mathscr{E}_x$$

by combining this with the product of electron and hole concentration and assuming that the conductivity is at minimum when $n_0 = p_0 = n_i$.

$$n_0p_o = n_i^2$$

$$p_0 = \frac{n_i^2}{n_0}$$

Substitute this into σ:

$$\sigma = q(n_0\mu_n + \frac{n_i^2}{n_0}\mu_p)$$

for σ to be minimized, $\frac{d\sigma}{dn_0} = 0$

$$\frac{d\sigma}{dn_0} = q(\mu_n - \frac{n_i^2\mu_p}{n_0^2}) = 0$$

$$\mu_n = \frac{n_i^2\mu_p}{n_0^2}$$

$$n_0^2 = n_i^2\frac{\mu_p}{\mu_n}$$

$$n_0 = n_i\sqrt{\frac{\mu_p}{\mu_n}}$$

$$n_0 = n_i\sqrt{\frac{\mu_p}{\mu_n}}$$

$$p_0 = n_i\sqrt{\frac{\mu_n}{\mu_p}}$$

We substitute these values into the expression for σ:

$$\sigma = q(n_0\mu_n + p_0\mu_p)$$

$$\sigma = q(n_i\sqrt{\frac{\mu_p}{\mu_n}}\mu_n + n_i\sqrt{\frac{\mu_n}{\mu_p}}\mu_p)$$

$$\sigma = qn_i(\sqrt{\mu_n\mu_p} + \sqrt{\mu_n\mu_p})$$

$$\sigma_{\min} = 2qn_i\sqrt{\mu_n\mu_p}$$

Hence, the minimum conductivity $\sigma_{\min}$ is given by $2qn_i\sqrt{\mu_n\mu_p}$ when $n_0 = n_i\sqrt{\frac{\mu_p}{\mu_n}}$.

The intrinsic conductivity σ_i for Si can be calculated using the equation:

$$\sigma_i = qn_i(\mu_n + \mu_p)$$

for silicon:

Parameter	Value
μ_n (Electron mobility in $cm^2/V\text{-}s$)	1350
μ_p (Hole mobility in $cm^2/V\text{-}s$)	450
n_i (Intrinsic carrier concentration in cm^{-3})	1.5×10^{10}
q (Elementary charge in C)	1.6×10^{-19}
σ_i (Intrinsic conductivity in S/cm)	4.32×10^{-2}
σ_{min}(Minimum conductivity in S/cm)	3.74×10^{-2}

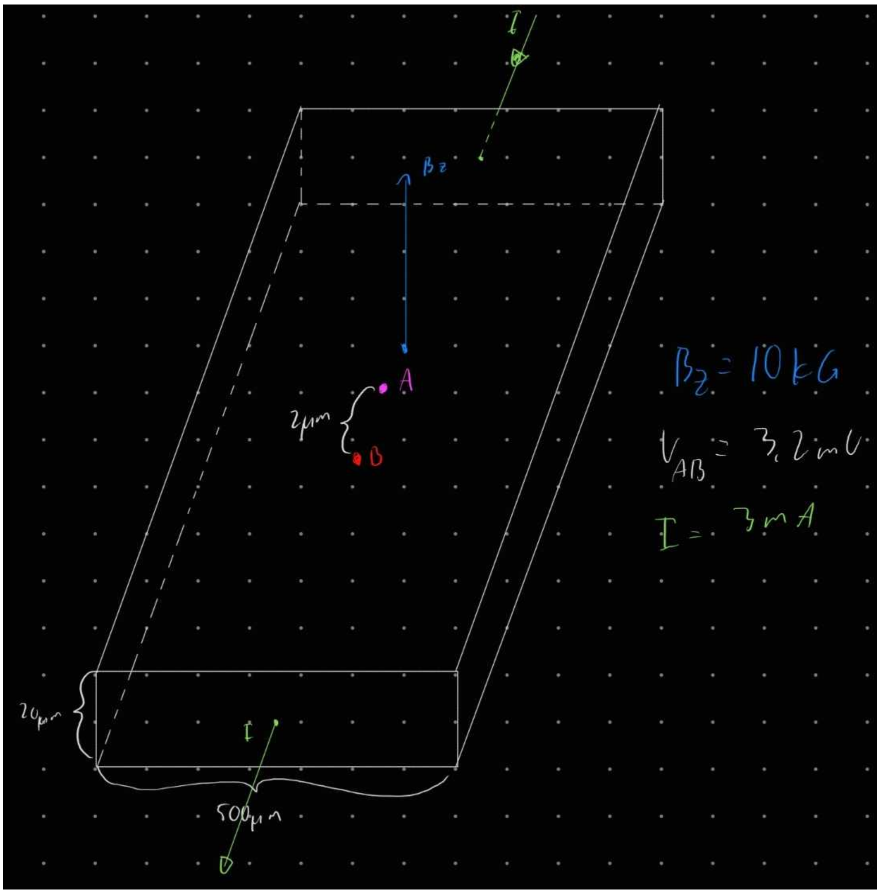
Table 1: Typical values for silicon and calculated intrinsic conductivity

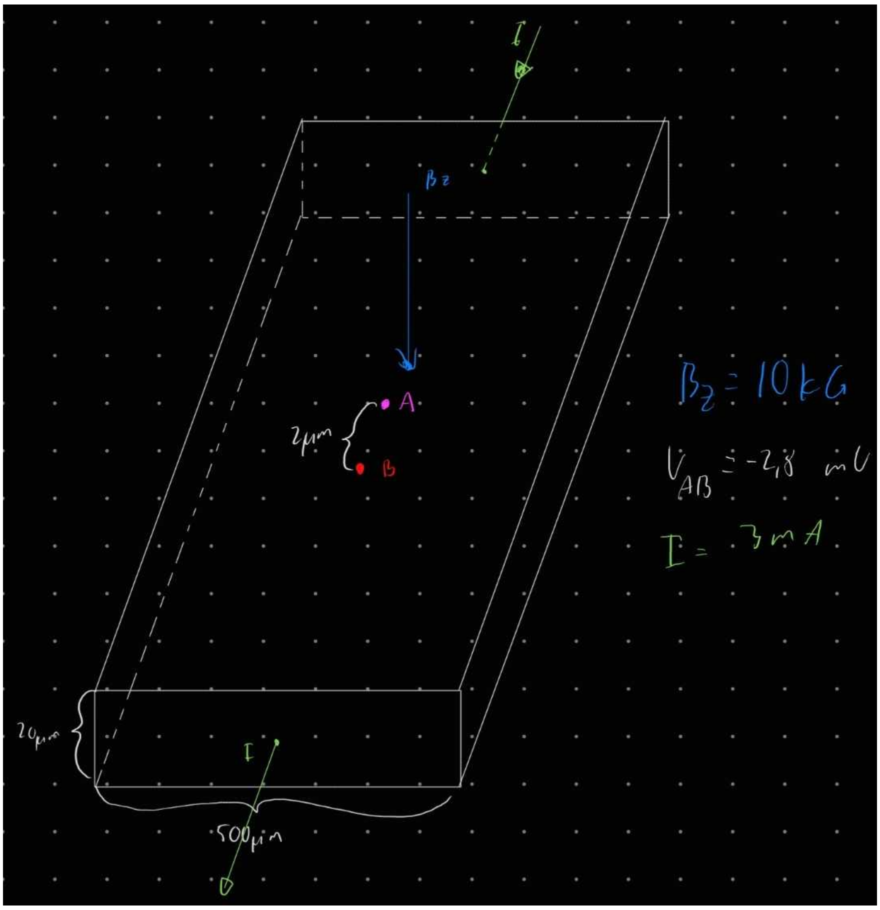
when comparing the two conductivities we can se that extrinsic doesn't necessary mean that we get a more conductivity.

Problem 2 - The Hall Experiment

Hall measurements are made on a p-type semiconductor bar $500\mu\mathrm{m}$ **wide and** $20\mu\mathrm{m}$ **thick. The Hall contacts** A **and** B **are displaced** $2\mu\mathrm{m}$ **with respect to each other in the direction of a current flow of** 3 mA**. The voltage between** A **and** B **with a magnetic field of** $10\mathrm{kG}$ $\left(1\mathrm{kG} = 10^{-5}\ \mathrm{Wb}/\mathrm{cm}^2\right)$ **pointing out of the plane of the sample is** $3.2\mathrm{mV}$**. When the magnetic field direction is reversed, the voltage changes to** $-2.8\mathrm{mV}$**.**

a) Draw a figure that illustrate the setup and includes all the given information. Make sure you get the correct orientation of the contacts A and B.





b) Calculate the hole concentration and mobility.

we can get the hole concentration from the equation

$$p_0 = \frac{I_x \mathscr{B}_z}{qtV_{AB}}$$

$$p_0 = \frac{0.003 \times 10^{-4}}{1.6 \times 10^{-19} \times 2 \times 10^{-5} 0.0031} = 3.02 \times 10^{19} cm^{-3}$$

the resistivity is given by

$$\rho = \frac{\frac{V_{AB}}{I_x}}{\frac{L}{wt}}$$

$$\rho = \frac{\frac{0.0031}{0.003}}{\frac{2 \times 10^{-6}}{2 \times 10^{-5} \times 5 \times 10^{-4}}} = 0.00516 \Omega m$$

the mobility is given by

$$\mu_n = \frac{1}{\rho q p_o}$$

$$\mu_n = \frac{1}{0.00516 \times 1.6 \times 10^{-19} \times 3.02 \times 10^{19}} = 40.11 \frac{cm^2}{Vs}$$

Problem 3 - Invariance of the Fermi Level at Equilibrium

Consider the figure below showing 2 materials in contact at equilibrium. $N_1(E)$ and $N_2(E)$ denotes the density of states in material 1 and 2 and $f(E)$ is the probability of a state being filled at energy E in each material.Let r_{12} and r_{21} denote the rate of transfer of electrons from material 1 to material 2 and from material 2 to material 1 , respectively.

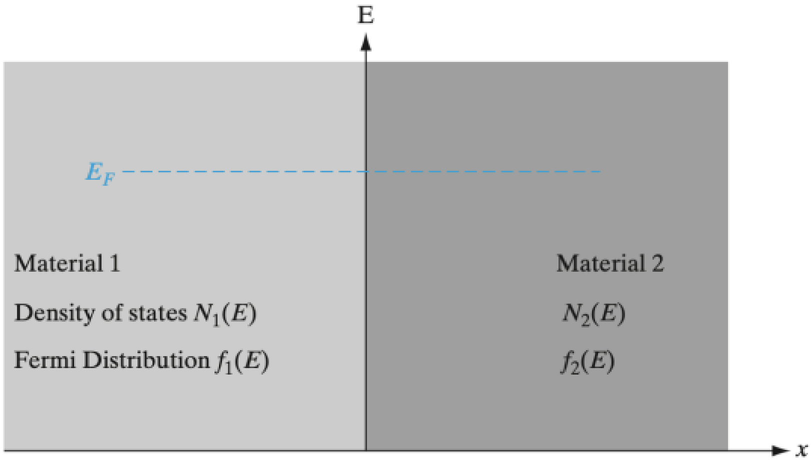


Figure 1: *
Two materials in intimate contact at equilibrium.

a) Explain in your own words why

$$r_{12} \propto N_1(E) f_1(E) \cdot N_2(E) \left[1 - f_2(E)\right]$$

and

$$r_{21} \propto N_2(E) f_2(E) \cdot N_1(E) \left[1 - f_1(E)\right] .$$

We can start by explaining the symbols

Symbol	Notation	Description
r_{12}	Rate	rate of transition from state 1 to state 2.
r_{21}	Rate	rate of transition from state 2 to state 1.
$N_1(E)$	Density of States	density of states at energy E for state 1.
$N_2(E)$	Density of States	density of states at energy E for state 2.
$f_1(E)$	Fermi–Dirac distribution	probability that a given energy state in state 1 is occupied by an electron.
$f_2(E)$	Fermi–Dirac distribution	probability that a given energy state in state 2 is occupied by an electron.
E	Energy	energy of the system.

Table 2: Symbols, Notation, and Descriptions.

The equation states that at a given Energy, the rate of transfer for electrons from 1 to 2 is proportional to the number of filled states a E in material 1 times the number of empty states at E in material 2 and vice versa. This is because we assume that there is no current, and therefore no charge transport. Electrons can't appear from thin air, the does also mean that the rate of transfer for electrons must be balanced.

b) In thermal equilibrium, there is, by definition, no net current, no net transfer of electrons and no net transfer of energy. Use this fact and the equations above to show that the fermi level must be constant throughout materials in intimate contact.

when in thermal equilibrium these must be equal

$$N_1(E)f_1(E) \cdot N_2(E)\left[1 - f_2(E)\right] = N_2(E)f_2(E) \cdot N_1(E)\left[1 - f_1(E)\right]$$

this results In

$$f_1(E) = f_2(E)$$

Therefore we can say that the fermienergy $E_{F1} = E_{F2}$