TTT4120 Digital Signal Processing

Problem Set 4

Peter Pham

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Contents

Problem 1 - Contact potential

At equilibrium, there can be no net hole or electron current in a p-n junction. Use the criteria

$$J_n(x) = q\left[\mu_n n(x)\mathscr{E}(x) + D_n \frac{dn(x)}{dx}\right] = 0$$

to show that the contact potential V_0 is related to the doping concentrations by

$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}.$$

We can start with *Poisson's equation*

$$\frac{d\mathscr{E}(x)}{dx} = \frac{q}{\epsilon}\left(p - n + N_d^+ - N_a^-\right)$$

If we neglect the contribution of carriers (p-n) to the space charge, we have two regions of constant space change and at the same time assuming complete ionization:

$$\begin{aligned} \frac{d\mathscr{E}}{dx} &= \frac{q}{\epsilon} N_d, && 0 < x < x_{n0} \\ \frac{d\mathscr{E}}{dx} &= -\frac{q}{\epsilon} N_a, && -x_{p0} < x < 0 \end{aligned}$$

Furthermore we can find $\mathscr{E}$ by integrating either part of the equations above

$$\begin{aligned} \int_{\mathscr{E}_0}^{0} d\mathscr{E} &= \frac{q}{\epsilon}\int_0^{x_{n0}} N_d dx, && 0 < x < x_{n0} \\ \int_0^{\mathscr{E}_0} d\mathscr{E} &= -\frac{q}{\epsilon}\int_{-x_{p0}}^{0} N_a dx, && -x_{p0} < x < 0 \end{aligned}$$

Therefore, the maximum calue of the electric field is

$$\mathscr{E}_0 = -\frac{q}{\epsilon} N_d x_{n0} = -\frac{q}{\epsilon} N_a x_{p0}$$

From the definition of electric field $\mathscr{E}(x) = -\frac{dV(x)}{dx}$ we get

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx} \quad \text{or} \quad -V_0 = \int_{-x_{p0}}^{x_{n0}} \mathscr{E}(x) dx$$

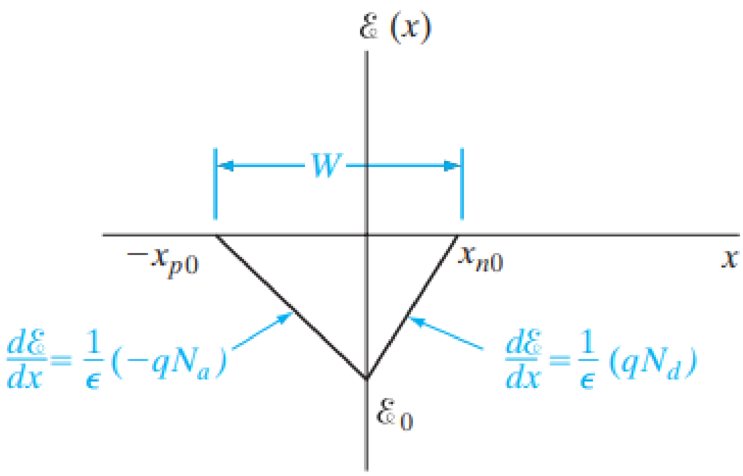


Figure 1: the electric field distribution, where the reference direction for $\mathscr{E}$ is arbitrarily taken as the +x-direction.

from figure 1 we can se that The contact potential are related to the width of the depletion region, which gives us

$$V_0 = \frac{1}{2}\mathscr{E}_0 W = \frac{1}{2}\frac{q}{\epsilon}N_d x_{n0} W$$

Since the balance of charge requirement is $x_{n0}N_d = x_{p0}N_a$, and W is simply $x_{p0} + x_{n0}$, we can write $x_{n0} = WN_a/(N_a + N_d)$:

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}\frac{N_a N_d}{N_a + N_d}W^2$$

V_0 can be written in terms of the doping concentrations

$$W = \left[\frac{2\epsilon kT}{q^2}\left(\ln\frac{N_a N_d}{n_i^2}\right)\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

by combining the two equations above we get

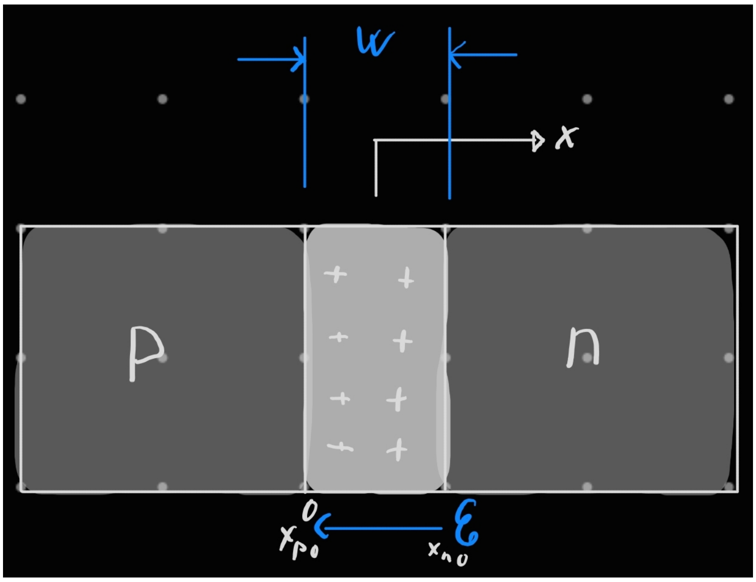
$$V_0 = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$$

Problem 2 - Depletion Region

a) Explain the *depletion approximation*.

The depletion approximation is the assumption of carrier depletion within W and neutrality outside W

b) Draw an illustration of the transition region in a $p^+ - n$ junction.



c) Use Poisson's equation to show that for a $\mathrm{p}^+ - \mathrm{n}$ junction, the contact potential is related to the width of the depletion region, W, by

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}N_d W^2.$$

if we continue from the equation from the previous problem:

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}N_d x_{n0} W$$

We can also calculate the penetration of the transition region into the n materials by

$$x_{n0} = \frac{WN_a}{N_a + N_d}$$

if assume that $N_a >> N_d$ then we get $X_{n0} \approx W$ and

$$V_0 = \frac{1}{2}\frac{q}{\epsilon}N_d W^2$$

Problem 3 - Equilibrium Junctions

a) Write equations that relate the charge of the depleted region, the electrical field strength and the electrical potential across a p-n junction.

The charge of the depleted charge $\rho(x)$, electrical field strenght $\mathscr{E}(x)$ and electrical potential $V(x)$ can be given by starting with the definition of electric field

$$\mathscr{E}(x) = -\frac{dV(x)}{dx}$$

combined with *Poisson's equation*

$$\frac{d\mathscr{E}(x)}{dx} = \frac{q}{\epsilon}\left(p - n + N_d^+ - N_a^-\right)$$

we get

$$-\frac{d^2V(x)}{dx^2} = \frac{d\mathscr{E}(x)}{dx} = \frac{q}{\epsilon}\left(p - n + N_d^+ - N_a^-\right)$$

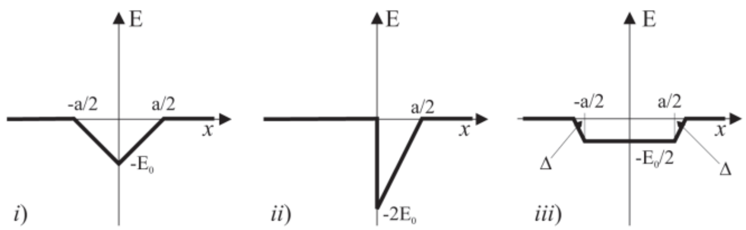


Figure 2: Electrical field strength profiles for three different p-n junctions in thermal equilibrium.

b) Calculate and sketch the potential $V(x)$ for each of the three profiles $\mathscr{E}(x)$ in Figure 1. Use $V(0) = 0$ and apply the variables given in the figure. Assume that $\Delta \ll a$ when you calculate $V(x)$ in $iii)$.
Then sketch the electron bands for the p-n junction $E_{c,v}(x)$ for each of $i) - iii)$. Indicate the approximate position of the Fermi level when sketching the diagrams.
Determine the contact potential V_0 in all of the three p-n junctions. Δ must be included in the calculations of V_0 in $iii)$.

(i)

We can start by redefining the electric field

$$\mathscr{E}(x) = \begin{cases} -\mathscr{E}_0\left(1+\frac{2}{a}x\right) & \text{for} \quad \frac{-a}{2} \leq x \leq 0 \\ -\mathscr{E}_0\left(1-\frac{2}{a}x\right) & \text{for} \quad 0 \leq x \leq \frac{a}{2} \end{cases}$$

Furthermore we can determine the voltage by sepparating into two ranges $-\frac{a}{2} \geq x \geq 0$ and $0 \geq x \geq \frac{a}{2}$ and integrate it sepparately for both these two areas. this gives two equation

$$V(x) - V\left(-\frac{a}{2}\right) = \mathscr{E}_0 \int_{-a/2}^{x} \left(1+\frac{2}{a}x'\right) \mathrm{d}x' = \frac{a}{4}\mathscr{E}_0\left(1+\frac{2}{a}x\right)^2$$

and

$$V(x) - V(0) = \mathscr{E}_0 \int_{0}^{x} \left(1-\frac{2}{a}x'\right) \mathrm{d}x' = -\frac{a}{4}\mathscr{E}_0\left(1-\frac{2}{a}x\right)^2 + \frac{a}{4}\mathscr{E}_0$$

As the problem states that $V(0) = 0$, we get

$$V\left(-\frac{a}{2}\right) = -\frac{a}{4}\mathscr{E}_0 \quad \text{and} \quad V\left(\frac{a}{2}\right) = \frac{a}{4}\mathscr{E}_0$$

This results in

$$V(x) = \begin{cases} -\frac{a}{4}\mathscr{E}_0 & \text{for} \quad x < -\frac{a}{2} \\ -\frac{a}{4}\mathscr{E}_0\left[1-\left(1+\frac{2}{a}x\right)^2\right] & \text{for} \quad -\frac{a}{2} \leq x \leq 0 \\ -\frac{a}{4}\mathscr{E}_0\left[\left(1-\frac{2}{a}x\right)^2-1\right] & \text{for } 0 \leq x \leq \frac{a}{2} \\ \frac{a}{4}\mathscr{E}_0 & \text{for} \quad x > \frac{a}{2} \end{cases}$$

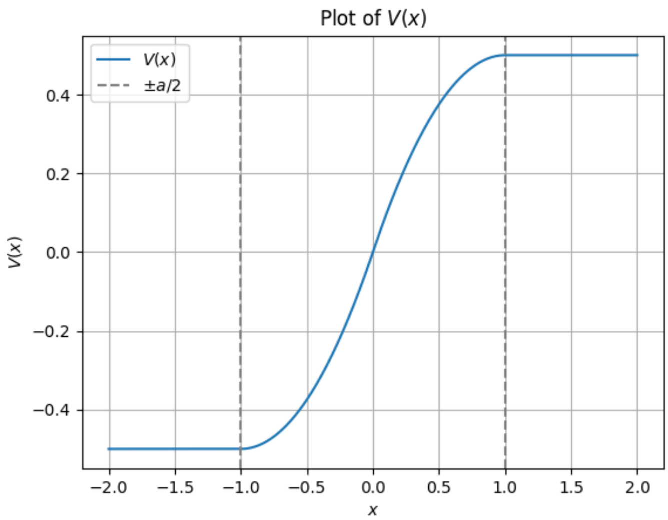


Figure 3: potential $V(x)$, where $\mathscr{E}_0 = 1$ and $a = 2$

(ii)

Here we can write the $\mathscr{E}(x)$ as

$$\mathscr{E}(x) = -2\mathscr{E}_0 \left(1 - \frac{2}{a}x\right) \quad \text{for} \quad 0 \le x \le \frac{a}{2}$$

Here we only need to look at the voltage V for $x \in \left[0, \frac{a}{2}\right]$, this gives usepackage

$$V(x) - V(0) = 2\mathscr{E}_0 \int_0^x \left(1 - \frac{2}{a}x'\right) \mathrm{d}x' = -\frac{a}{2}\mathscr{E}_0 \left(1 - \frac{2x}{a}\right)^2 + \frac{a}{2}\mathscr{E}_0$$

as $V = 0$ we end up with

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ -\frac{a}{2}\mathscr{E}_0 \left[\left(1 - \frac{2}{a}x\right)^2 - 1\right] & \text{for} \quad 0 \le x \le \frac{a}{2} \\ \frac{a}{2}\mathscr{E}_0 & \text{for} \quad x > \frac{a}{2} \end{cases}$$

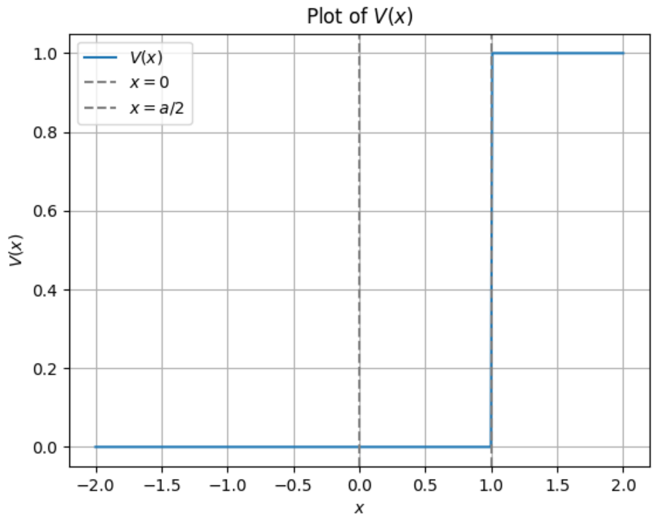


Figure 4: potential $V(x)$, where $\mathscr{E}_0 = 1$ and $a = 2$

(iii)

As $\Delta \ll a$ in figure 2, then we can calculate as if the electric field strenght profile has a rectangular form. Tgis gives us a simplified equation

$$\mathscr{E}(x) = \frac{\mathscr{E}_0}{2} \text{ for } \quad \frac{-a}{2} \leq x \leq \frac{a}{2}$$

this gives us

$$V(x) - V\left(-\frac{a}{2}\right) = \frac{\mathscr{E}_0}{2} \int_{-a/2}^{x} \mathrm{d}x = \frac{\mathscr{E}_0}{2}\left(x + \frac{a}{2}\right)$$

and we end up with

$$V(x) = \begin{cases} -\frac{a}{4}\mathscr{E}_0 & \text{for} \quad x < -\frac{a}{2} \\ \frac{x}{2}\mathscr{E}_0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \\ \frac{a}{4}\mathscr{E}_0 & \text{for} \quad x > \frac{a}{2} \end{cases}$$

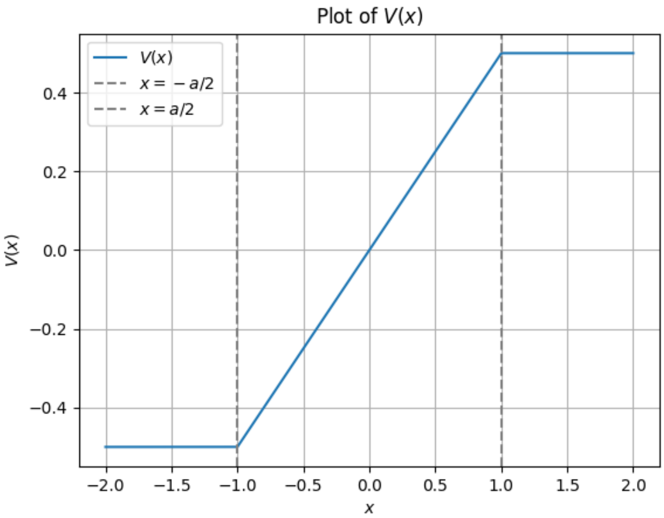


Figure 5: potential $V(x)$, where $\mathscr{E}_0 = 1$ and $a = 2$

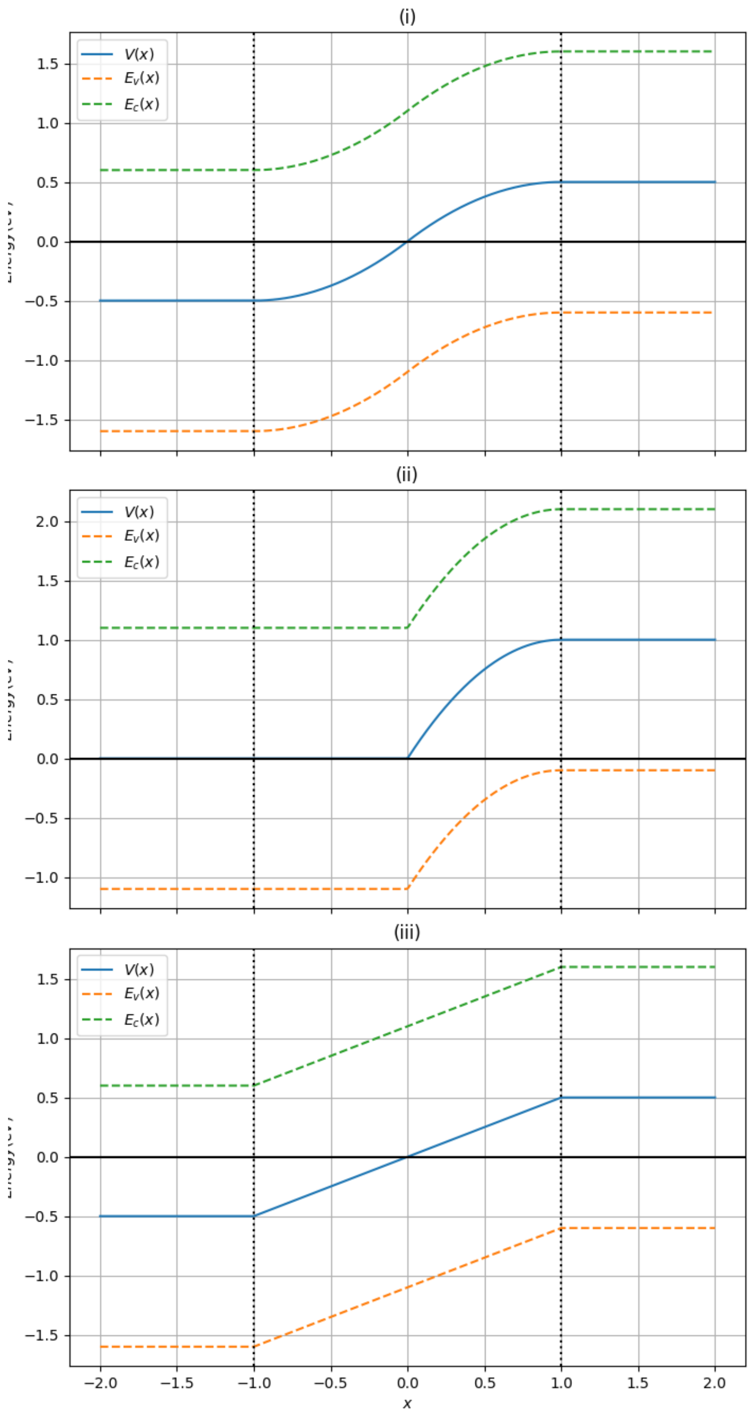


Figure 6: Electron bands for the p-n junction $E_{c,v}(x)$ for each of $i) - iii)$

by watching the area below the field profiles in figure 2 we can se that

$$
\begin{aligned}
(i) \quad & V_0 = \frac{a}{2}\mathscr{E}_0 \\
(ii) \quad & V_0 = \frac{a}{2}\mathscr{E}_0 \\
(iii) \quad & V_0 = \frac{1}{2}\mathscr{E}_0(a + \Delta)
\end{aligned}
$$

c) Calculate and sketch $\rho(x)$ for all profiles in Figure 2. Δ must be included in the calculations in iii). Sketch the dopant profile for all three cases assuming ideal diode conditions.

by using Poissons equation

$$\rho(x) = \varepsilon \frac{d\mathscr{E}(x)}{dx}$$

we get

(i)

$$
\rho(x) = \begin{cases}
0 & \text{for} \quad x < -\frac{a}{2} \\
-\frac{2}{a}\varepsilon\mathscr{E}_0 & \text{for} \quad -\frac{a}{2} \le x \le 0 \\
\frac{2}{a}\varepsilon\mathscr{E}_0 & \text{for} \quad 0 \le x \le \frac{a}{2} \\
0 & \text{for} \quad x > \frac{a}{2}
\end{cases}
$$

(ii)

$$
\rho(x) = \begin{cases}
0 & \text{for} \quad x < 0 \\
\frac{4\varepsilon}{a}\mathscr{E}_0 & \text{for} \quad 0 \le x \le \frac{a}{2} \\
0 & \text{for} \quad x > \frac{a}{2}
\end{cases}
$$

(iii)

$$
\rho(x) = \begin{cases}
0 & \text{for} \quad < -\frac{a+2\Delta}{2} \\
-\frac{\varepsilon}{2\Delta}\mathscr{E}_0 & \text{for} \quad -\frac{a+2\Delta}{2} \le x < -\frac{a}{2} \\
0 & \text{for} \quad -\frac{a}{2} \le x \le \frac{a}{2} \\
\frac{\varepsilon}{2\Delta}\mathscr{E}_0 & \text{for} \quad \frac{a}{2} < x \le \frac{a+2\Delta}{2} \\
0 & \text{for} \quad x > \frac{a+2\Delta}{2}
\end{cases}
$$

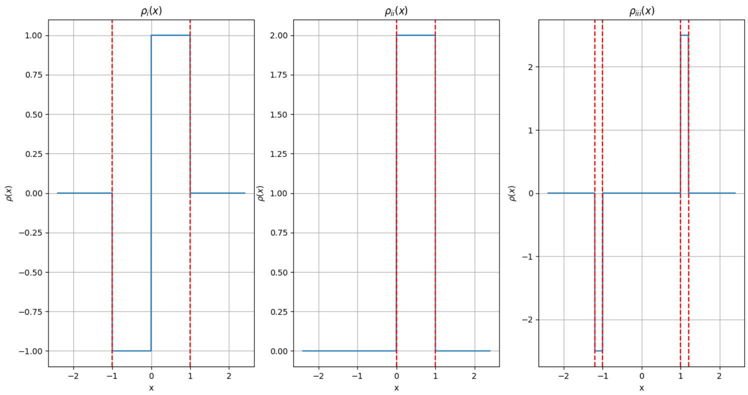


Figure 7: $\rho(x)$ for all profiles in Figure 2.

by using the equation that relate the charge of the depleted region, the electrical field strength and the electrical potential across a p-n junction from 3a we get

(i)

$$\begin{aligned} N_a(x) &= \tfrac{2\varepsilon}{qa}\mathscr{E}_0 \quad \text{for} \quad x < 0 \\ N_d(x) &= \tfrac{2\varepsilon}{qa}\mathscr{E}_0 \quad \text{for} \quad x > 0 \end{aligned}$$

(ii)

$$\begin{aligned} N_a(x) &\gg \tfrac{4\varepsilon}{qa}\mathscr{E}_0 \quad \text{for} \quad x < 0 \\ N_d(x) &= \tfrac{4\varepsilon}{qa}\mathscr{E}_0 \quad \text{for} \quad x > 0 \end{aligned}$$

(iii)

$$\begin{aligned} N_a(x) &= \tfrac{\varepsilon}{2q\Delta}\mathscr{E}_0 \quad \text{for} \quad x < -\tfrac{a}{2} \\ N_d(x) &= 0 \quad \text{for} \quad \tfrac{-a}{2} \leq x \leq \tfrac{a}{2} \\ N_d(x) &= \tfrac{\varepsilon}{2q\Delta}\mathscr{E}_0 \quad \text{for} \quad x > \tfrac{a}{2} \end{aligned}$$

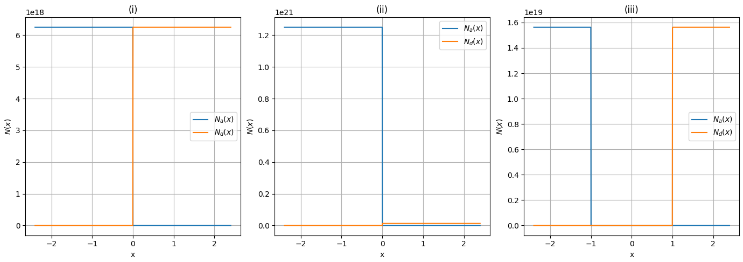


Figure 8: The dopant profile for all three cases assuming ideal diode conditions.

Problem 4 - Numerical Simulations