TTT4120 Digital Signal Processing

Problem Set 3

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Contents

Problem 1 - Symbols

According to the notation and conventions of Streetman, explain the following physical parameters:

Table 1: i) Table of Notations, Parameters, and Units

Notation	Explanation of the Parameter	Unit
n, p	Concentration of electrons in the conduction band and holes in the valence band	cm^{-3}
n_0, p_0	Equilibrium concentrations of electrons and holes	cm^{-3}
n_i, p_i	Intrinsic concentrations of electrons and holes	cm^{-3}
n_n, p_n	Equilibrium concentrations of electrons and holes in n-type	cm^{-3}
n_p, p_p	Equilibrium concentrations of electrons and holes in p-type	cm^{-3}
$\delta n, \delta p$	Excess electron and hole concentrations	cm^{-3}
$\Delta n, \Delta p$	Excess electron and hole concentrations at the edge of the transition region	cm^{-3}

Table 2: ii) Table of Notations, Parameters, and Units

Notation	Explanation of the Parameter	Unit
$N(E)$	Density of electron states at a given energy level	$\frac{states}{eV \cdot cm^3}$
N_v, N_c	Effective density of states at the edge of the valence and the conduction band	$\frac{states}{eV \cdot cm^3}$
N_a, N_d	Concentration of acceptors and donors	cm^{-3}
N_a^-, N_d^+	Concentration of ionized acceptors and donors	cm^{-3}

Table 3: iii) Table of Notations, Parameters, and Units

Notation	Explanation of the Parameter	Unit
E	Energy	J, eV
E_v, E_c	Valence and conduction band edges	J, eV
E_g	Band gap energy	J, eV
E_F, E_i	Fermi energy and intrinsic level	J, eV
F_n, F_p	Quasi-Fermi levels for electrons and holes	J, eV

Table 4: iv)-x) Table of Notations, Parameters, and Units

Notation	**Explanation of the Parameter**	**Unit**
B, E	Base and emitter of a BJT	–
D, D_n, D_p	Diffusion coefficient for dopants, electrons, and holes	cm^2/s
L_n, L_p	Electron and hole diffusion lengths	cm
μ_n, μ_p	Electron and hole mobility	$cm^2/(V \cdot s)$
τ_n, τ_p	Recombination lifetime for electrons and holes	s
J, J_x, J_n, J_p	Current density for total, x-direction, electron, and hole	A/cm^2
σ, ρ	Conductivity and resistivity or charge density	$S/cm, \Omega \cdot cm$ or C/cm^3

Problem 2 - Quasi-Fermi Levels and Carrier Lifetime

Consider an n-doped Si sample with $N_D = 10^{15}$ cm^{-3}.

a) Calculate the position of the fermi level E_F relative to the intrinsic position E_i

From

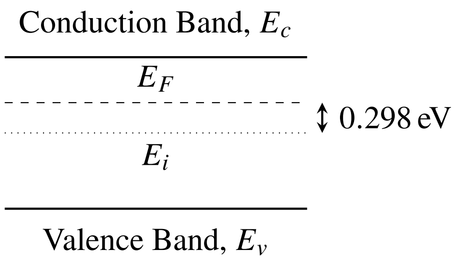
$$n_0 = n_i e^{(E_F - E_i)/kT}$$

we get

$$E_F - E_i = kT \ln \frac{n_0}{n_i}$$

Assuming we are at 300K we get

$$E_F - E_i = 8.62 \cdot 10^{-5} \cdot 300 \cdot \ln \frac{10^{15}}{10^{10}} = 0.298eV$$



b) The sample is steadily illuminated, causing an optical generation rate of $g_{op} = 10^{19}$ cm^{-3}/s. The carrier lifetime is $\tau_p = \tau_n = 100$ ns. Calculate the steady-state carrier concentrations n and p, as well as the relative positions of the quasi-fermi levels $F_n - E_i$ and $F_p - E_i$. Compare to the result from a).

The excess carrier concentration can be written as

$$\delta n = \delta p = g_{op} \tau_n$$

this gives us

$$\delta n = \delta p = 10^{19} \cdot 10^{-7} = 10^{12} cm^{-3}$$

The steady-state carrier concentrations n and p can be written as

$$n = n_0 + \delta n$$
$$p = p_0 + \delta p$$

Now that the subscripts are removed, we cant use equilibrium equation $n_0 p_0 = n_i^2$ as $np \neq n_i^2$

$$n_0 \approx N_D = 10^{15}$$
$$p_0 = \frac{n_i^2}{n_0} = \frac{\left(10^{10}\right)^2}{10^{15}} = 10^5$$

this gives us

$$n = 10^{15} + 10^{12} \approx 10^{15}$$
$$p = 10^5 + 10^{12} \approx 10^{12}$$

from

$$n = n_i e^{\frac{F_n - E_i}{kT}}$$

we get

$$F_n - E_i = kT \ln \frac{n}{n_i}$$
$$F_n - E_i = 0.0259 \ln \left(\frac{10^{15}}{10^{10}}\right)$$
$$F_n - E_i = 0.298 eV$$

using

$$p = n_i e^{\frac{E_i - F_p}{kT}}$$

we get

$$E_i - F_p = kT \ln \frac{p}{n_i}$$
$$E_i - F_p = 0.0259 \ln \left(\frac{10^{12}}{10^{10}} \right)$$
$$E_i - F_p = 0.119eV$$

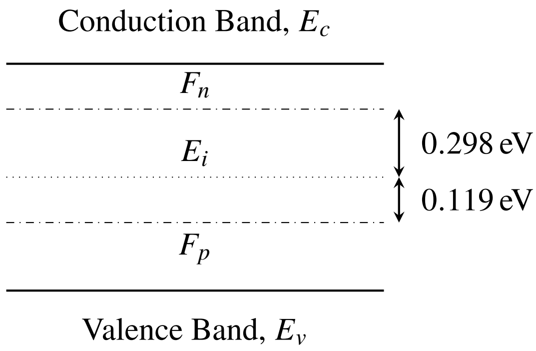


Figure 1: Energy band diagram showing Fermi level (E_F), intrinsic level (E_i), and quasi-Fermi levels (F_n and F_p) with their separations.

c) Calculate the time it takes from the light is turned off until the hole concentration is 10% higher than its equilibrium value.

To calculate when the hole concentration $p = 1.1p_0 = 1.1 \cdot 10^5$ we must find out $\delta p(t)$

$$\delta p(t) = \Delta p e^{-\frac{t}{tau_p}}$$

as the carrier lifetime is $\tau_p = \tau_n = 100$ ns, and $\Delta p = 10^5$ we get

$$\delta p(t) = 10^5 e^{-\frac{t}{10^{-7}}} = 1.1 \cdot 10^5$$
$$-\frac{t}{10^{-7}} = \ln \left(\frac{1.1 \cdot 10^5}{10^5} \right)$$
$$t = \ln \left(\frac{1.1 \cdot 10^5}{10^5} \right) \cdot \left(-10^{-7} \right)$$
$$t = 9.53 \cdot 10^{-9} s$$

Problem 3 - The Haynes-Shockley Experiment

Figure 2 shows the experimental setup for the Haynes-Shockley experiment. The length of the sample is $L_0 = 75$ mm **and the points (1) and (2) are separated by** $L = 30$ mm**. The supplied voltages are** $E_0 = 100$ V **and** $E_2 = 15$ V **and the resistors have the values** $R_1 = 10\text{k}\Omega$ **and** $R_2 = 7\text{k}\Omega$ **respectively.**

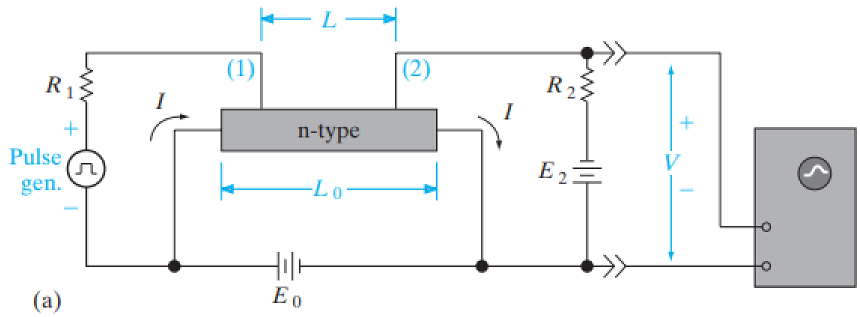


Figure 2: The setup for the Haynes-Shockley experiment

a) Explain how we can use this experiment to determine the mobility and diffusion coefficient for holes in the n-type semiconductor.

The basic principles of the Haynes-Shockley experiment are:

1. A pulse of holes is created in a n-type bar that contain an electric field.
2. As the pulse drifts in the field and spreads out by diffusion, the excess hole concentration is monitored at som point down the bar.
3. The time required for the holes to drift a given distance in the field gives a measure of the mobility.
4. The spreading of the pulse during a given time is used to calculate the diffusion coefficient.

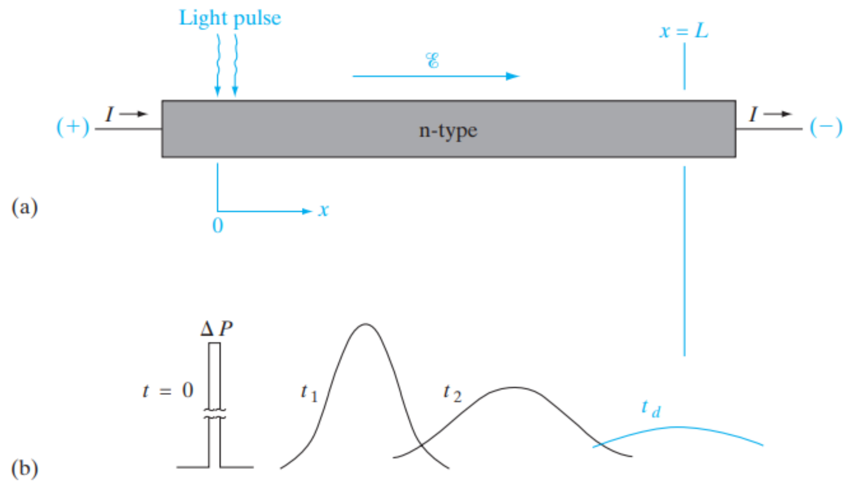


Figure 3: Drift and diffusion of a hole pulse in an n-type bar: (a) sample geometry; (b) position and shape of the pulse for several times during its drift down the bar.

b) A very short pulse of holes is injected into the sample at point (1) at time $t = 0$. Because of the applied voltage E_0, the pulse of excess holes will be swept from (1) to (2). The voltage signal seen on the oscilloscope is shown in figure 4 . The signal peak arrives after $t_d = 200$ ns.Calculate the mobility μ_p.

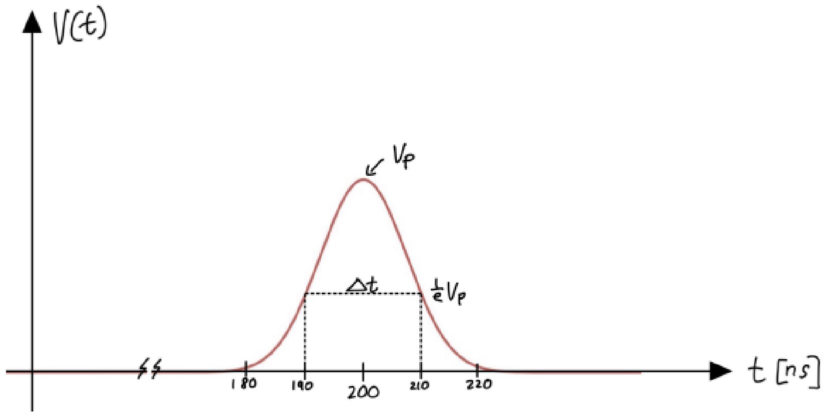


Figure 4: The voltage signal seen on the oscilloscope

The mobility μ is given by

$$\mu_p = \frac{V_d}{\mathscr{E}}$$

where V_d is the drift velocity given by

$$V_d = \frac{L}{t_d} = 1.5 \cdot 10^5 \frac{m}{s}$$

the $\mathscr{E}$-field is given by

$$\mathscr{E} = \frac{E_0}{L_0} = \frac{100}{0.075} = 1333.3 \frac{V}{m}$$

This gives us

$$\mu_p = \frac{1.5 \cdot 10^5}{1333.3}$$
$$\mu_p = 112.5 \frac{m^2}{V \cdot s}$$

c) Assume $t_d << \tau_p$ such that recombination of charge carriers can be ignored. Show that the diffusion coefficient is given by

$$D_p = \frac{(\Delta t L)^2}{16 t_d^3},$$

and calculate D_p for this particular sample.Hint: Ignoring recombination, the diffusion equation, which describes the broadening of the pulse, reduces to

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2}$$

which has the solution

$$\delta p(x,t) = \frac{\Delta P}{2\sqrt{\pi D_p t}} e^{\frac{-x^2}{4 D_p t}}$$

As the pulse drifts in the $\mathscr{E}$ field it aslo spreads out by diffusion. By examin the case of diffusion of a pulse without drift, neglecting recombination, the equation which the hole distribution must satisfy is the time-dependent diffusion equation

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_n}$$

For the case of negligible recombination, we can rewrite the diffusion equation as

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2}$$

The function which satisfies this qeuation is called a *gaussioan distribution*,

$$\delta p(x,t) = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}} \right] e^{-x^2/4D_p t}$$

where ΔP is the number of holes per unit area created over a negligibly small distance at $t = 0$. The factor in brackets indicates that the peak value of the pulse at $(x = 0)$ decreases with time. We can redefine the peak value of the pulse as $\widehat{\delta p}$ at any given time, we can use the equationa bove too calculate D_p at some point x. The most convenient choice is $\frac{\Delta x}{2}$ as this is where Δp is down by $\frac{1}{e}$ of its peak value. There fore we can write

$$e^{-1}\widehat{\delta p} = \widehat{\delta p} e^{-(\Delta x/2)^2/4D_p t_d}$$
$$D_p = \frac{(\Delta x)^2}{16 t_d}$$

Problem 4 - The Einstein Relation

At equilibrium, no net current flows in a semiconductor. Any fluctuation that would begin a diffusion current also sets up an electric field which redistributes carriers by drift. Use this fact to show that the diffusion coefficient and the mobility is related by

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

We start with the definition of electric field

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx}$$

At equilibrium we can set the equation below to zero.

$$J_p(x) = q\mu_p p(x)\mathscr{E}(x) - qD_p\frac{dp(x)}{dx}$$

this gives us

$$\mathscr{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}$$

by using

$$p_0 = n_i e^{\left(E_i - E_f\right)/kT}$$

for $p(x)$ we get

$$\mathscr{E}(x) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right)$$

as the equilibrium Fermi level doesn't vary with x the derived equals to 0.

If we choose E_i as a reference we can relate the electric field to this reference by

$$\mathscr{E}(x) = -\frac{d\mathscr{V}(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i}{(-q)}\right] = \frac{1}{q}\frac{dE_i}{dx}$$

we get

$$\frac{D_p}{\mu_p} = \frac{kT}{q}$$

Problem 5 - Diffusion

Consider figure 3 showing a steady state hole distribution $p(x)$, which is the result of an injection of holes at $x = 0$. Assume the length of the bar is infinite and let the excess hole concentration at $x = 0$ be denoted by $\delta p(0) = \Delta p$.

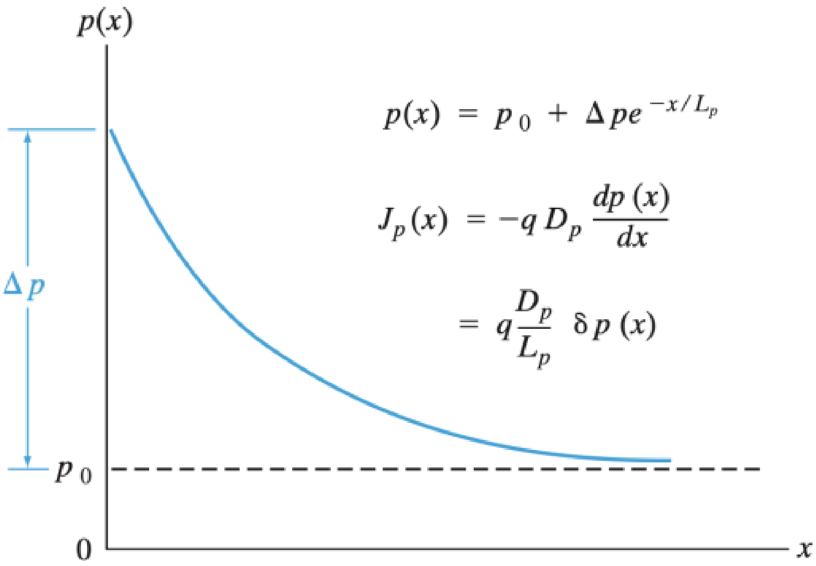


Figure 5: Injection of holes at $x = 0$, giving rise to a steady state hole distribution $p(x)$. Figure from Streetman.

a) State the steady state diffusion equation for holes and find the solution under the given assumptions.

The steady state diffusion equartions for holes is

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} \equiv \frac{\delta p}{L_p^2}$$

where $L_p \equiv \sqrt{D_p \tau_p}$ is called the hole *diffusion lenght*. As we have steady state, we no longer need partial derivatives. The solution to this equation has the form

$$\delta p(x) = C_1 e^{\frac{x}{L_p}} + C_2 e^{-\frac{x}{L_p}}$$

as we expect the distrobution of excess holes to decay to zero for large values of x because of recombination, and $\delta p = \Delta p$ at $x = 0$ we end up with

$$\delta p(x) = \Delta p e^{-\frac{x}{L_p}}$$

b) Calculate the probability that a hole injected at $x = 0$ will recombine in the infinitesimally small interval between x and $x + dx$.

The probability that a hole injected at $x = 0$ survives to x without recombination is $\frac{\delta p(x)}{\Delta p} = e^{-\frac{x}{L_p}}$
From this we get

$$\frac{\delta p(x) - \delta p(x + dx)}{\delta p(x)} = \frac{-(d\delta p(x)/dx)dx}{\delta p(x)} = \frac{1}{L_p}dx$$

Thus the total probability that a hole injected at $x = 0$ will recombine in a given dx is the product of the two probabilities:

$$\left(e^{-x/L_p}\right)\left(\frac{1}{L_p}dx\right) = \frac{1}{L_p}e^{-x/L_p}dx$$

c) Show that L_p is the average distance a hole will diffuse before recombining.

By using the averaging thechnique

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)P(x)dx$$

we get

$$\langle x \rangle = \int_0^{\infty} x\frac{e^{-x/L_p}}{L_p}dx = L_p$$