TTT4120 Digital Signal Processing

Problem Set 2

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Contents

Problem 1 (3 points)

Two signals $x[n]$ and $y[n]$ are given by

$$x[n] = \begin{cases} 2 & n = 0 \\ 1 & n = \pm 1 \\ 0 & \text{otherwise}, \end{cases} \qquad y[n] = \begin{cases} 1 & -M \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the Fourier transform of $x[n]$ is given by... and sketch it for $\omega \in [-\pi, \pi]$.

$$X(\omega) = 2 + 2\cos\omega$$

We have that

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

In our case we can write

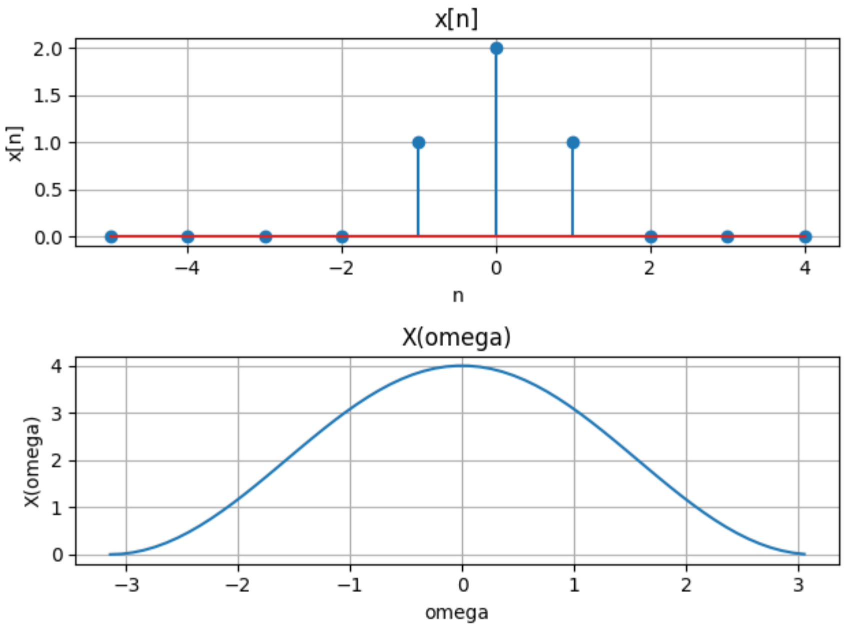
$$X(\omega) = \sum_{n=-1}^{1} x[n]e^{-j\omega n} = -e^{j\omega} + 2 + e^{-j\omega}$$

From Euler's formula we get that

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

by combining these two we get:

$$X(\omega) = 2 + 2\cos\omega$$



(b) Show that the Fourier transform of y[n] is given by

$$Y(\omega) = \frac{\sin\left(\omega\left(M + \frac{1}{2}\right)\right)}{\sin\left(\frac{\omega}{2}\right)}$$

and sketch it for $M = 10$ **and** $\omega \in [-\pi, \pi]$

$$Y(\omega) = \sum_{n=-M}^{M} e^{-j\omega n}$$

This is a finite geometric series. By using the general formula for a geometric series

$$\sum_{k=m}^{n} ar^k = \begin{cases} a(n-m+1) & \text{if } r = 1 \\ \frac{a\left(r^m - r^{n+1}\right)}{1-r} & \text{if } r \neq 1 \end{cases}$$

we get

$$Y(\omega) = \frac{e^{j\omega M} - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$$

Problem 2 (2 points)

Problem 3 (2 points)

Problem 4 (2 points)

Problem 5 (2 points)

Appendix