TTT4120 Digital Signal Processing

Problem Set 6

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Contents

Problem 1: Relationship between DTFT and DFT [3 points]

Consider the following sequence of length N_x,

$$x(n) = \begin{cases} 0.9^n & n = 0, \ldots, N_x - 1, \\ 0 & \text{otherwise.} \end{cases}$$

where $N_x = 28$. We would like to analyze the sequence in the frequency domain.

(a) Compute the spectrum $X(f)$ of $x(n)$ using the DTFT and plot its magnitude for $f \in [0, 1)$.

We start by using the formula for DTFT

$$X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n}$$

given that $N_x = 28$ we just need to sum from $n = 0$ to $n = 27$

$$X(f) = \sum_{n=0}^{27} x(n) e^{-j2\pi f n}$$

using the sum of a finite geometric series

$$S = \frac{a\left(1 - r_x^N\right)}{1 - r}$$

where $a = 1$, $r = ae^{-j2\pi f}$

This gives us

$$X(f) = \frac{1 - \left(ae^{-j2\pi f}\right)^{28}}{1 - ae^{-j2\pi f}}$$

To find the magnitude we need to take the absolute value of $X(f)$

$$|X(f)| = |\frac{1 - \left(ae^{-j2\pi f}\right)^{28}}{1 - ae^{-j2\pi f}}|$$

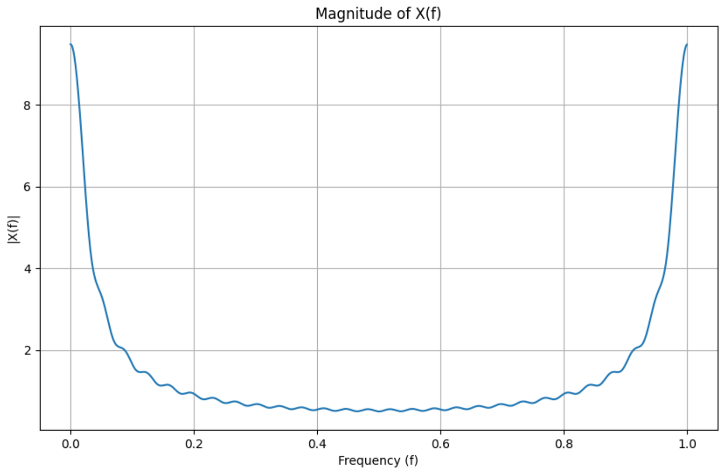

Figure 1: The magnitude of $X(f)$ for $f \in [0, 1)$

(b) Use the function fft to compute $X(k) = DFT\{x(n)\}$ with the DFT length equal to $N_x/4, N_x/2, N_x$ and $2N_x$.

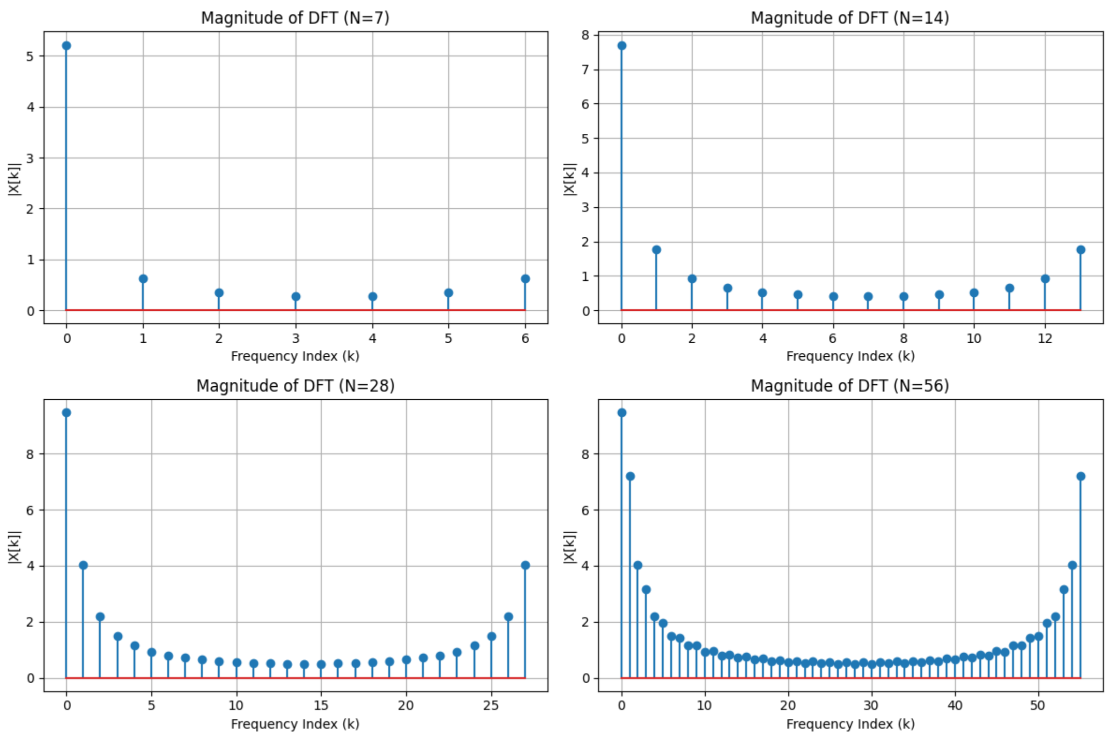

Figure 2: Plot of $X(f)$ with the DFT length equal to $N_x/4, N_x/2, N_x$ and $2N_x$.

(c) What is the relationship between the DFT index k and the normalized frequency f ? Find f that corresponds to the $k = 1$ for the four DFTs computed in (b).

The relationship between the DFT index k and the normalized frequency f is given by

$$f = \frac{k}{N}$$

For $k = 1$ and $N = 7$

$$f = \frac{1}{7}$$

For $k = 1$ and $N = 14$

$$f = \frac{1}{14}$$

For $k = 1$ and $N = 28$

$$f = \frac{1}{28}$$

For $k = 1$ and $N = 56$

$$f = \frac{1}{56}$$

(d) Plot the magnitude of each DFT (use stem) together with the magnitude of the DTFT (use plot) as a function of f. What is the relationship between DFT and DTFT for the different DFT lengths? Explain the results.

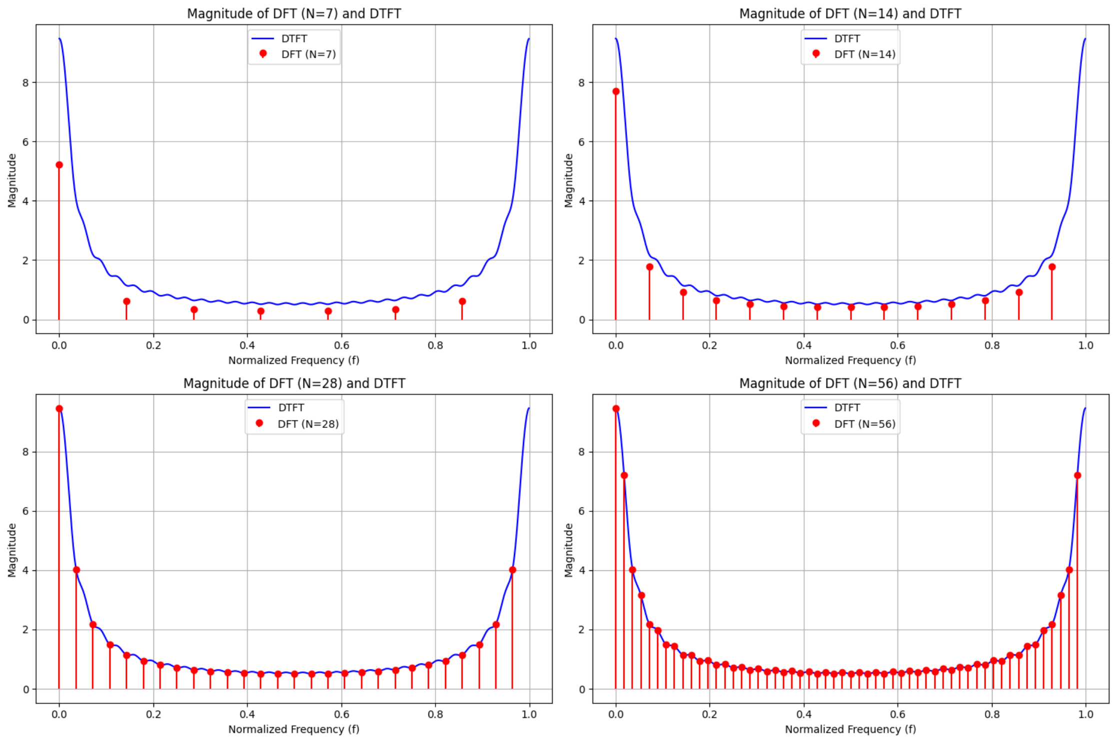

Figure 3: Plot of the magnitude of each DFT together with the magnitude of the DTFT as a function of f.

We can se that we get a better representation of the DTFT the larger N gets for DTF.

(e) Why is it sufficient to compute the values of the DTFT and DFT corresponding only to the frequency range $f \in [0, 0.5]$ for any real signal?

As we can se on the plots from c), the values becomes mirrored (close to at high N for DTF) on both sides of $f = 0.5$. ALso for any real signal, there wont be any negative frequences, therefore we dont need to compute the values of DTFT and DFT in the frequency range $f \in [0.5, 1]$ as this will represent the negative frequencies.

Problem 2: Linear Convolution [2.5 points]

The sequence $x(n)$ from Problem 1 is filtered through a FIR filter with unit sample response given by

$$h(n) = \begin{cases} 1 & n = 0, \ldots, N_h - 1, \\ 0 & \text{otherwise} , \end{cases}$$

where $N_h = 9$.

(a) Use Matlab to compute and plot the output signal $y(n)$ in the time domain. Useful Matlab functions: ones, conv, stem. What is the length of $y(n)$, N_y ? Does it agree with theory? Explain.

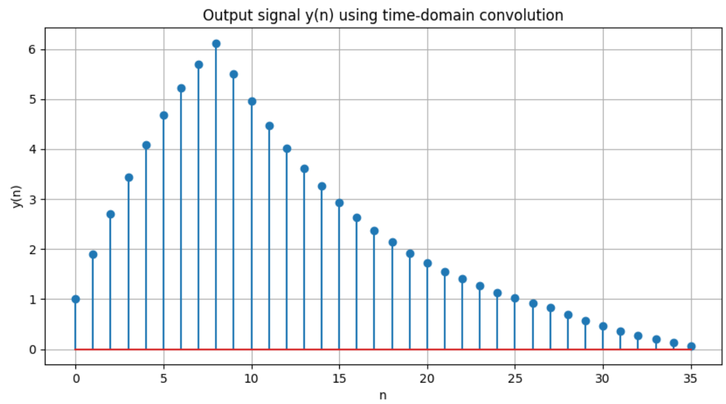

Figure 4: Plot of the output signal $y(n)$ in the time domain.

Length of $y(n)$, $N_y = 36$ this corresponds nice witht the theory as the lenght will be $N_x + N_h - 1$

(b) Use Matlab to compute and plot the output signal $y(n)$ via the frequency domain using DFT/IDFT. Useful Matlab functions: fft, ifft.

How should the DFT/IDFT lengths be chosen in order to obtain exact values of $y(n)$ using the above algorithm? Explain why.

The lenghts of the $DTF/IDFT$ should be as long as the lenght of $y(n)$, $N_y = 36$.

Run your Matlab program with DFT/IDFT lengths set to $N_y/4$, $N_y/2$, N_y and $2N_y$. Plot each result together with the output signal obtained in (a) (use different colors) and compare. Explain your observations.

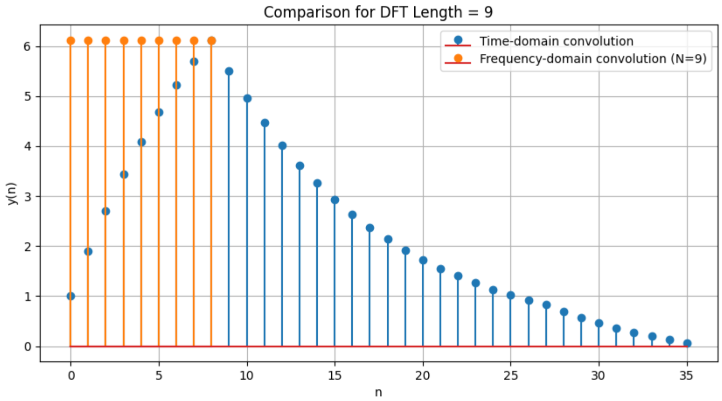

Figure 5: $\frac{N_y}{4}$

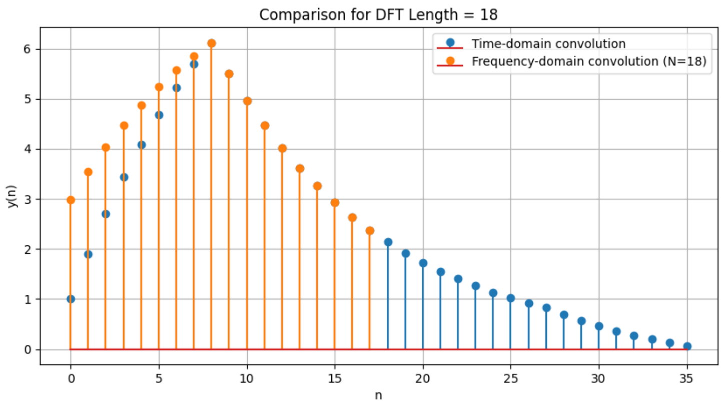

Figure 6: $\frac{N_y}{2}$

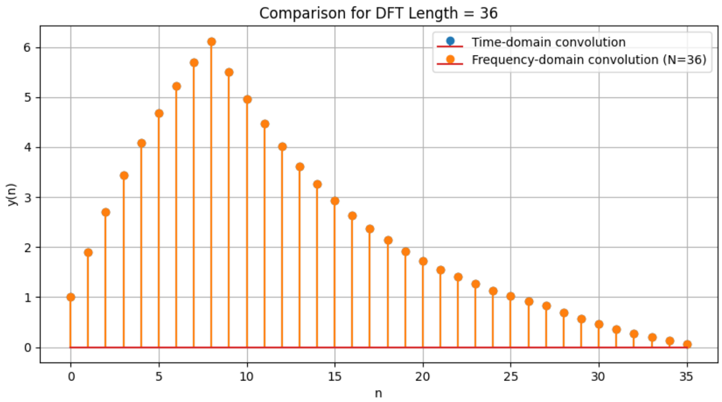

Figure 7: N_y

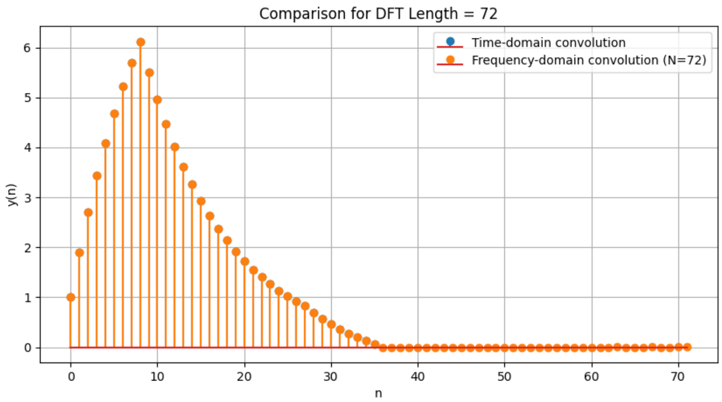

Figure 8: $2N_y$

Had to do in 4 sepparate plots so the stems wouldn't overlap and hide each other. I did it in python as i have had a lot of issues using MatLab on Linux.

Problem 3: Spectral Estimation [2.5 points]

In this problem we will use DFT to estimate the spectrum of a discrete-time signal based on finite signal segments. The following signal is considered

$$x(n) = \sin(2\pi f_1 n) + \sin(2\pi f_2 n),$$

where $f_1 = 7/40$ **and** $f_2 = 9/40$**.**

(a) Sketch the magnitude spectrum of the sampled signal, $|X(f)|$ for $f \in [0, 0.5]$.

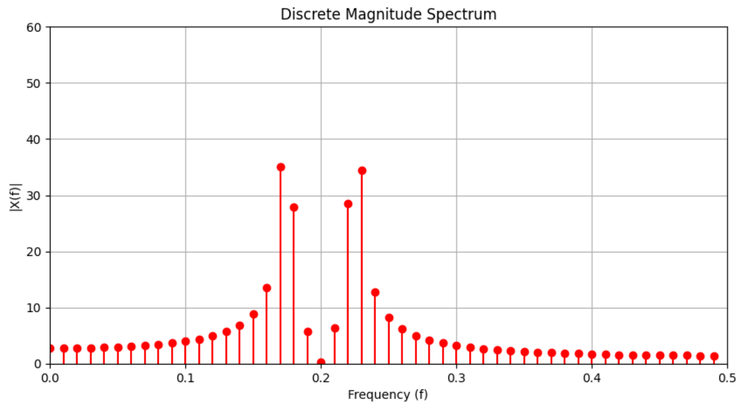

Figure 9: magnitude spectrum of the sampled signal, $|X(f)|$ for $f \in [0, 0.5]$.

(b) Use Matlab to generate a segment of length 100 of the signal $x(n)$.

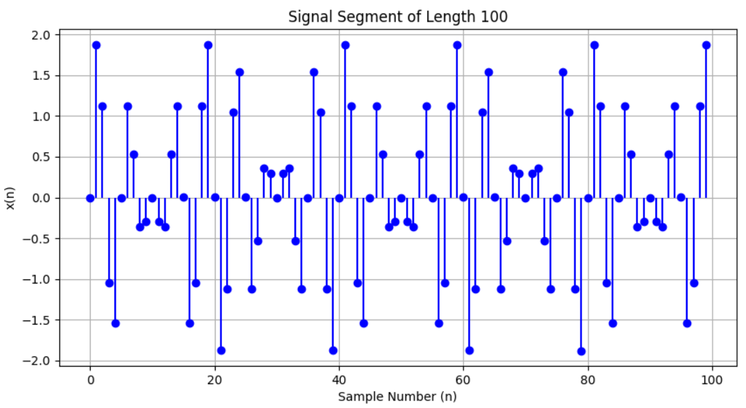

Figure 10: Segment of length 100 of the signal $x(n)$.

Use DFT of length 1024 to estimate the spectrum $X(f)$ based on this signal segment.

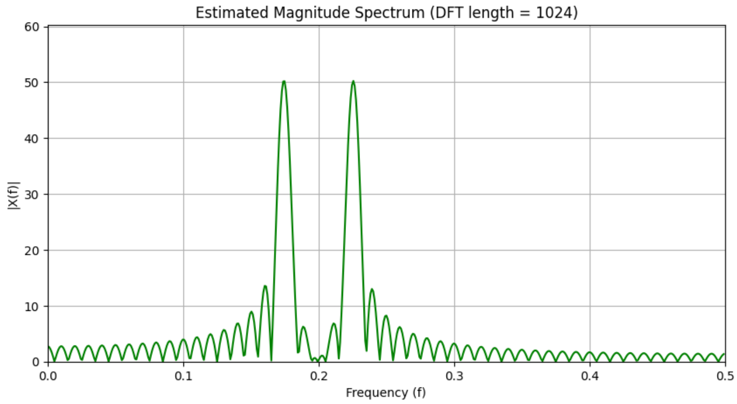

Figure 11: DFT of length 1024 to estimate the spectrum $X(f)$.

Plot the estimated magnitude spectrum for $f \in [0, 0.5]$.

Repeat the above with segment lengths equal to 1000,30 and 10 .

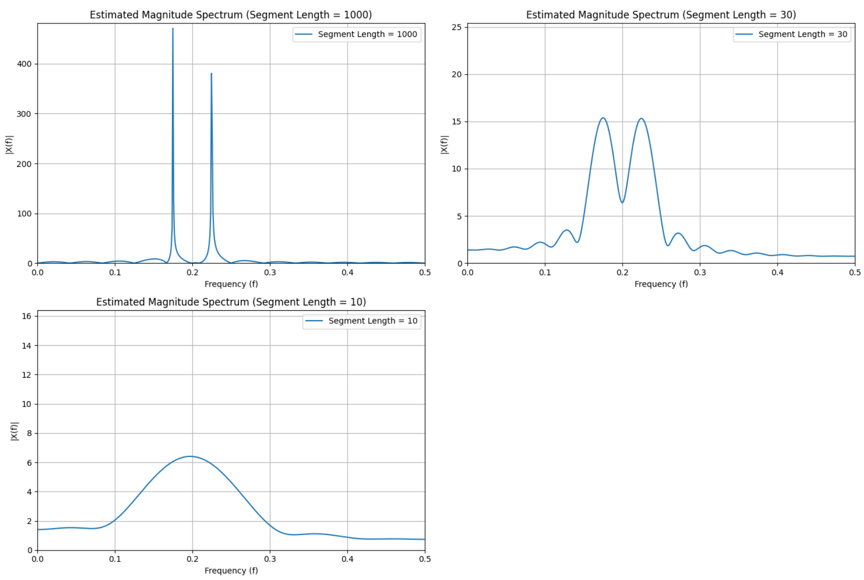

Figure 12: Plot of the estimated magnitude spectrum for $f \in [0, 0.5]$ with segment lengths equal to 1000,30 and 10.

Compare with the sketch in (a) and explain the similarities and differences.

It does look wuite similar as in (a) wt an increased N, We can se that at a very high segment lenght we can se that f_1 is more dominant than f_2

(c) Repeat (b) with segment of length 100 of the signal $x(n)$, using DFT lengths equal to 256 and 128 . What effect does the DFT length have on spectral estimation.

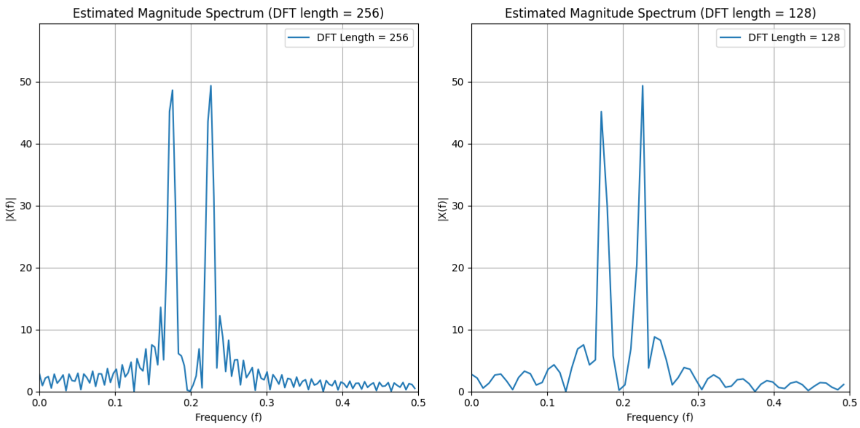

Figure 13: Plot of the estimated magnitude spectrum for $f \in [0, 0.5]$using DFT lengths equal to 256 and 128

As the DFT length increases, the frequency resolution improves. In the plots, you can observe that the spectrum with DFT length 256 provides a more detailed representation compared to the one with DFT length 128.

Problem 4: Fast Fourier Transform (FFT) [2 points]