TTT4120 Digital Signal Processing

Problem Set 1

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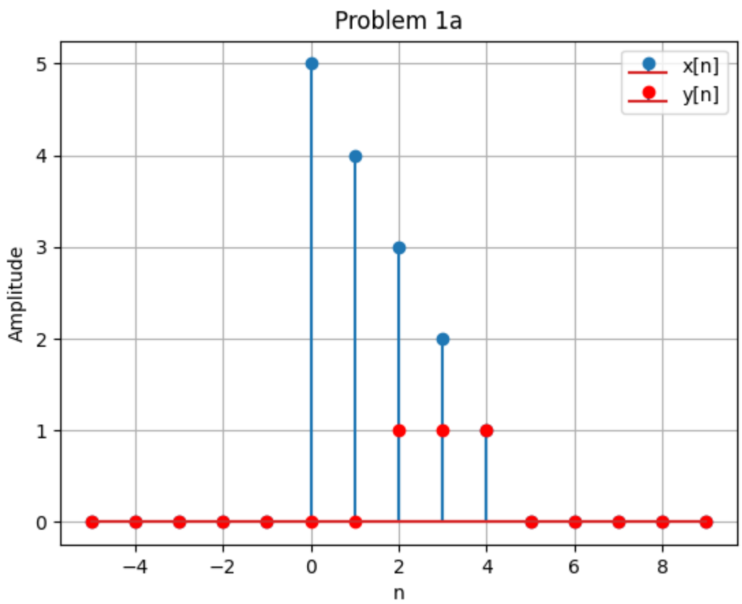
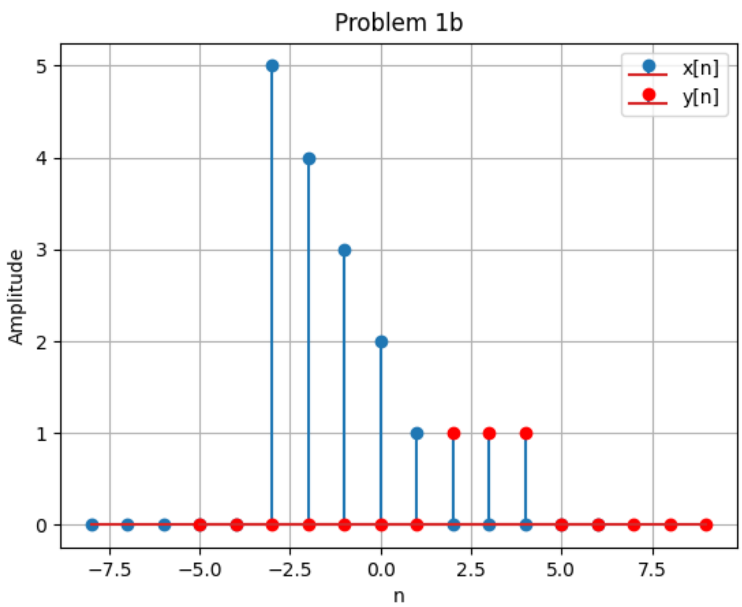
Contents

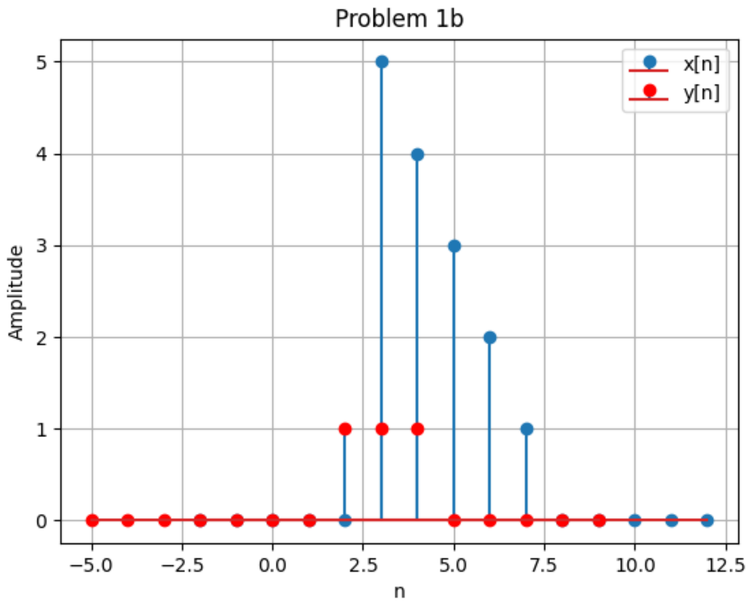
Problem 1 (2 points)

(a) Sketch $x[n]$ and $y[n]$

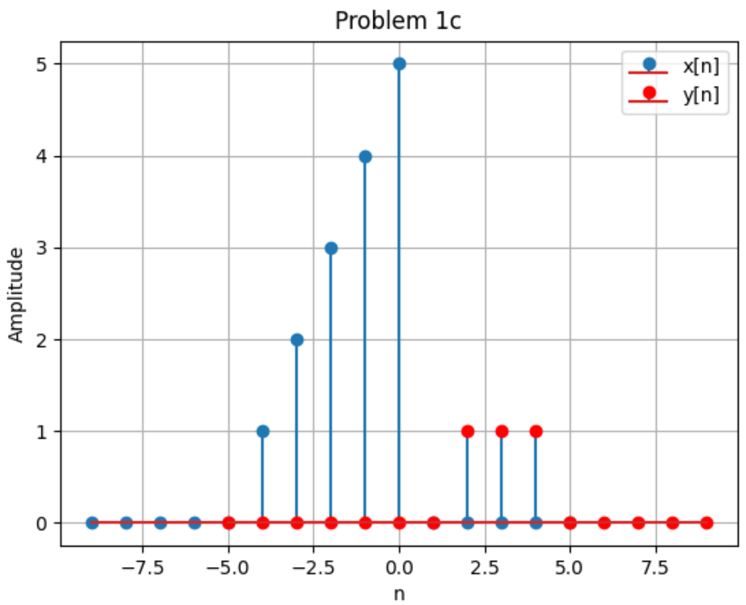


(b) Sketch $x[n-k]$ for $k = 3$ and $k = -3$.

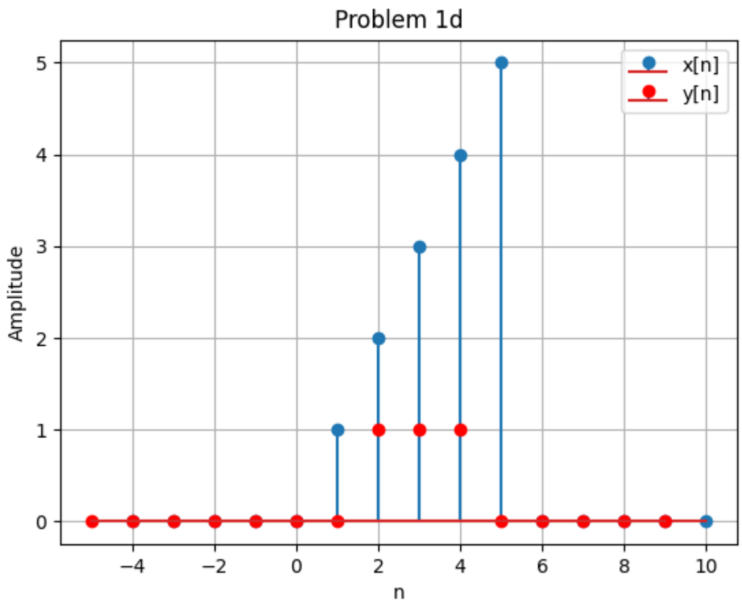



Problem 1b
x[n]
y[n]
Amplitude
n
5
4
3
2
1
0
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5

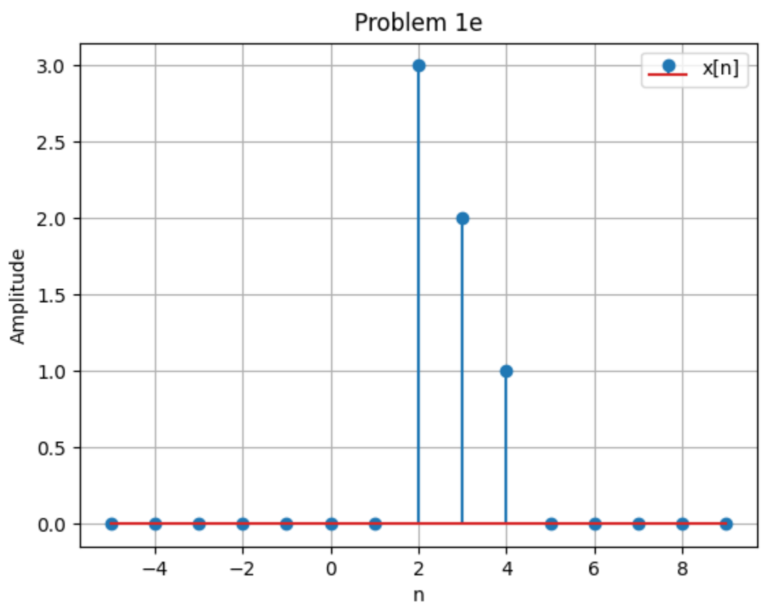
(c) Sketch $x[-n]$.


Problem 1c
x[n]
y[n]
Amplitude
n
5
4
3
2
1
0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5

(d) Sketch $x[5-n]$.



(e) Sketch $x[n] \cdot y[n]$.



(f) Express the signal $x[n]$ by using the unit sample sequence $\delta[n]$.

$$x[n] = 5\delta[n] + 4\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + 1\delta[n-4]$$

(g) Express the signal $y[n]$ by using the unit step signal $u[n]$.

$$y[n] = u[n-2] - u[n-5]$$

(h) Compute the energy of the signal $x[n]$.

The energy og the signal $x[n] = 55$ Solved in python

Python Code

```
import matplotlib.pyplot as plt
import numpy as np

# Problem 1:
def x_n(n):
    if 0 <= n <= 4:
        return 5 - n
    else:
        return 0

def y_n(n):
    if 2 <= n <= 4:
        return 1
    else:
        return 0

n_values = np.arange(-5, 10)  # A range of n values
x_values = [x_n(n) for n in n_values]
y_values = [y_n(n) for n in n_values]
plt.figure()


# (a) Sketch x[n] and y[n]
```

```
def problem_1a():
    plt.stem(n_values, x_values, label="x[n]")
    plt.stem(n_values, y_values, label="y[n]", markerfmt='ro')
    plt.xlabel('n')
    plt.ylabel('Amplitude')
    plt.legend()
    plt.title('Problem 1a')
    plt.grid(True)
    plt.show()
problem_1a()

# (b) Sketch x[n-k] for k=3 and k=-3
def problem_1b1():
    plt.stem(n_values-3, x_values, label="x[n]")
    plt.stem(n_values, y_values, label="y[n]", markerfmt='ro')
    plt.xlabel('n')
    plt.ylabel('Amplitude')
    plt.legend()
    plt.title('Problem 1b')
    plt.grid(True)
    plt.show()
problem_1b1()
def problem_1b2():
    plt.stem(n_values+3, x_values, label="x[n]")
    plt.stem(n_values, y_values, label="y[n]", markerfmt='ro')
    plt.xlabel('n')
    plt.ylabel('Amplitude')
    plt.legend()
    plt.title('Problem 1b')
    plt.grid(True)
    plt.show()
problem_1b2()

# (c) Sketch x[-n]
def problem_1c():
    plt.stem(-1*n_values, x_values, label="x[n]")
    plt.stem(n_values, y_values, label="y[n]", markerfmt='ro')
    plt.xlabel('n')
    plt.ylabel('Amplitude')
    plt.legend()
    plt.title('Problem 1c')
```

```
    plt.grid(True)
    plt.show()
problem_1c()

# (d) Sketch x[5-n]
def problem_1d():
    plt.stem(-1*n_values+5, x_values, label="x[n]")
    plt.stem(n_values, y_values, label="y[n]", markerfmt='ro')
    plt.xlabel('n')
    plt.ylabel('Amplitude')
    plt.legend()
    plt.title('Problem 1d')
    plt.grid(True)
    plt.show()
problem_1d()

# (e) Sketch x[n] * y[n]
def problem_1e():
    def x_n_y_n(n):
        return x_n(n)*y_n(n)
    xy_values = [x_n_y_n(n) for n in n_values]

    plt.stem(n_values, xy_values, label="x[n]")
    plt.xlabel('n')
    plt.ylabel('Amplitude')
    plt.legend()
    plt.title('Problem 1e')
    plt.grid(True)
    plt.show()
problem_1e()



# (h) Compute the energy of the signal x[n]

def problem_1h():
    e=0
    for n in n_values:
        e+=x_n(n)*x_n(n)

    print("The energy og the signal x[n] = ",e)
```

```
problem_1h()

# Energy = sum of (x[n])^2 for all n
```

Problem 2 (2 points)

(a) Which physical frequencies F_1 can f_1 correspond to if $F_s = 6000Hz$?

As the sampling frequency need to be twice as high as the frequency after sampling we get from the equation:

$$-\frac{F_s}{2} \leq f \leq \frac{F_s}{2}$$

this gives us:

$$-3000Hz \leq f \leq 3000Hz$$

(b) Use Matlab or Python to generate a sequence of length 4 seconds of $x[n]$.

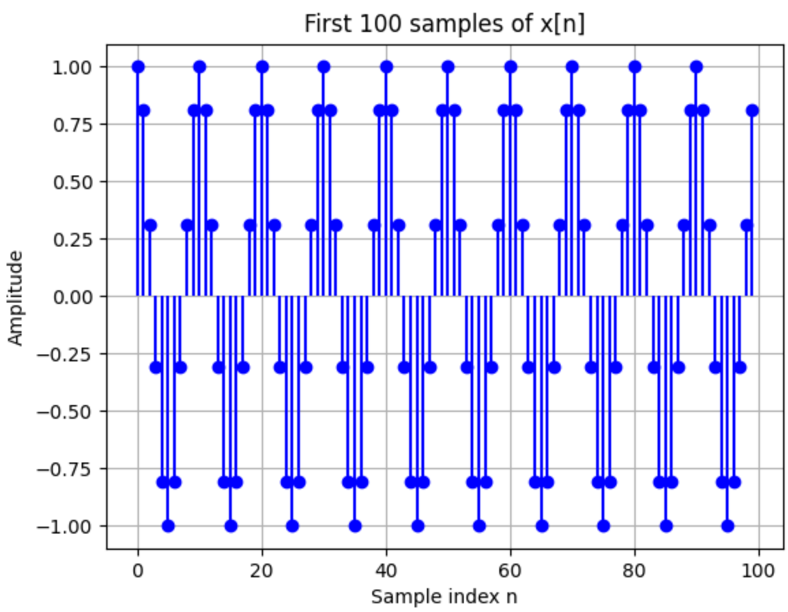


Figure 1: *
Generated sequence of lenght 4 seconds of $x[n]$ where $A = 1$, $f_1 =$, $F_s = 6000Hz$ and $T = 4s$

(c) Use the Matlab command soundsc or the Python command sounddevice.play to listen to the harmonic when the normalized frequency $f_1 = 0.3$and the sampling rate F_s is given by respectively 1000Hz, 3000Hz and 12000Hz. Comment on what you hear.

When listening to a higher normalized frequencies the pitch did also sound higher.

(d) Now assume a fixed sampling rate $F_s = 8000Hz$ while the physical frequency F_1 is respectively 1000Hz, 3000Hz and 6000Hz. Comment on what you hear. Relate it to the corresponding normalized frequency f_1.

In this case the pitch was diffrent where the highest pitch was 3000 Hz, but then i have difficulty deciding whenever if 1000Hz og 6000Hz are the highest pitch. When i played the tones again with the corresponding normalized frequency f_1 the pitch sounded accordingly to the frequencies 6000Hz being the highest and 1000Hz being the lowest.

Problem 3 (2 points)

I will write a bit more on problem (3(a)) for future usage, the rest will use the same methods only shorter.

Linearity

To decide whenever a system is linear or not we have to check if the principle of superposition is satisfied.

Additivity: $T(x_1[n] + x_2[n]) = T(x_1[n]) + T(x_2[n])$

Time-Invariance

A system is time-invariant if a time shift in the input signal results in an identical time shift in the output signal. Mathematically, this property can be expressed as:

$$T(x[n-k]) = y[n-k] \tag{1}$$

where $T(x[n]) = y[n]$.

Causality

A system is causal if the output at any time n depends only on the present and past input values but not on future input values. Mathematically, this can be expressed as:

$$y[n] = T(x[n], x[n-1], x[n-2], \ldots)$$

(a) $y[n] = x[n] - x^2[n-1]$

Linearity

Additivity: If we have two signals $x_1[n]$ and $x_2[n]$. The systemresponse of $x_1[n] + x_2[n]$ would be:

$$\begin{aligned} y[n] &= (x_1[n] + x_2[n]) - (x_1[n-1] + x_2[n-1])^2 && (2) \\ &= x_1[n] + x_2[n] - (x_1^2[n-1] + 2x_1[n-1]x_2[n-1] + x_2^2[n-1]) && (3) \end{aligned}$$

This is not equal to $y_1[n] + y_2[n]$ where $y_1[n] = x_1[n] - x_1^2[n-1]$ and $y_2[n] = x_2[n] - x_2^2[n-1]$

Therefore This system is not linear

Time-Invariance

If we consider an input $x[n-k]$, the systemresponse would be:

$$y[n] = x[n-k] - (x[n-k-1])^2$$

This is exactly the output $y[n]$ shifted by k samples, assuming $T(x[n]) = y[n]$

Causality

In this system, the outpyt $y[n]$ at any time demepds on the current $x[n]$ and the past input $x[n-1]$, this means that the system is causal.

Summary

- The system is not linear.
- The system is time-invariant.
- The system is causal.

(b) $y[n] = nx[n] + 2x[n-2]$

- **Linearity**, two signals $x_1[n]$ and $x_2[n]$:

$$y[n] = nx_1[n] + nx_2[n] + 2x_1[n-2] + 2x_2[n-2])$$

  This is equal to $y_1[n]+y_2[n]$ where $y_1[n] = nx_1[n]-2x_1[n-1]$ and $y_2[n] = nx_2[n]-2x_2[n-1]$

- **Time-Invariance**, consider an input $x[n-k]$

$$y[n] = nx[n-k] - 2x[n-k-2]$$

  this is not the same as $y[n]$ shifted by k samples.

- **Causality**: The same as before, the output depends of a past input $x[n-2]$

Summary

- The system is linear.
- The system is not time-invariant.
- The system is causal.

(c) $y[n] = x[n] - x[n-1]$

- **Linearity**, two signals $x_1[n]$ and $x_2[n]$:

$$y[n] = nx_1[n] + nx_2[n] + 2x_1[n-2] + 2x_2[n-2])$$

This is equal to $y_1[n]+y_2[n]$ where $y_1[n] = nx_1[n]-2x_1[n-1]$ and $y_2[n] = nx_2[n]-2x_2[n-1]$

- **Time-Invariance**, consider an input $x[n-k]$

$$y[n] = nx[n-k] - 2x[n-k-2]$$

this is not the same as $y[n]$ shifted by k samples.

- **Causality**: The same as before, the output depends of a past input $x[n-2]$

Summary

- The system is linear.
- The system is not time-invariant.
- The system is causal.

(d) $y[n] = nx[n] + 2x[n-2]$

- **Linearity**, two signals $x_1[n]$ and $x_2[n]$:

$$y[n] = nx_1[n] + nx_2[n] + 2x_1[n-2] + 2x_2[n-2])$$

This is equal to $y_1[n]+y_2[n]$ where $y_1[n] = nx_1[n]-2x_1[n-1]$ and $y_2[n] = nx_2[n]-2x_2[n-1]$

- **Time-Invariance**, consider an input $x[n-k]$

$$y[n] = nx[n-k] - 2x[n-k-2]$$

this is not the same as $y[n]$ shifted by k samples.

- **Causality**: The same as before, the output depends of a past input $x[n-2]$

Summary

- The system is linear.
- The system is not time-invariant.
- The system is causal.