TTT4120 Digital Signal Processing

Problem Set 4

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Contents

Problem 1 (2 points)

Given a filter with transfer function

$$H(z) = \frac{1}{1 - az^{-1}}$$

(a) Draw the pole-zero plot for the filter given $a = 0.9$ and $a = -0.9$. Determine the filter type for two filters? Explain using the pole-zero plot.

We can rewrite the transfer function as

$$H(z) = \frac{z}{z - a}$$

This gives us 1 zero when z=0 for both case $a = 0.9$ and $a = -0.9$, we however get a pole at $z = 0.9$ when $a = 0.9$ and a pole at $z = -0.9$ when $a = -0.9$

this gives us the pole zero plots:

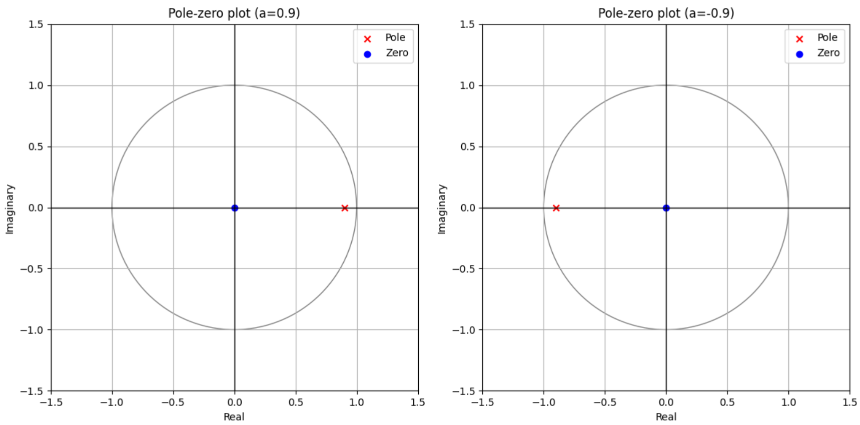

Figure 1: pole-zero plot for the filter given $a = 0.9$ and $a = -0.9$.

Here we can see that when $a = 0.9$, $H(z)$ will increase as z increase, this indicates that the it is a low pass filter. The opposite happens when $a = -0.9$ as the pole is on the negative side of the imaginary axis. We can also se this in the magnitude plots

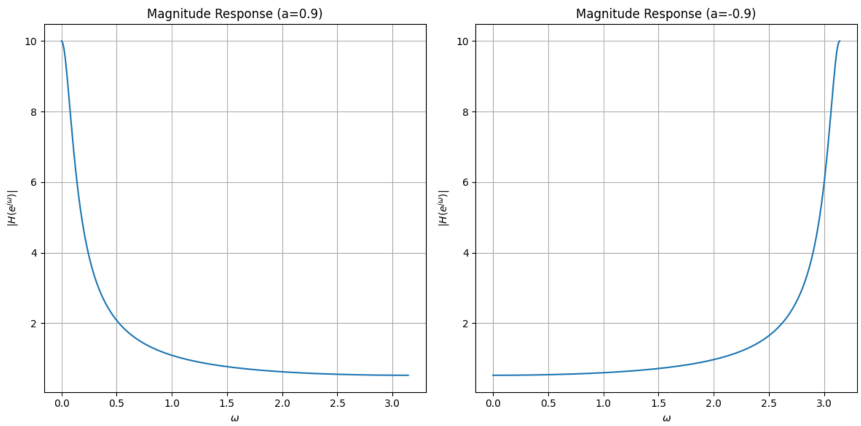

Figure 2: Magnitude plot for the filter given $a = 0.9$ and $a = -0.9$.

(b) Verify the results in 1(a) with pezdemo. The demo can be downloaded from the course home page.

Problem 2 (2 points)

Consider a causal digital filter with transfer function

$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}$$

(a) Find the transfer function of the inverse filter of $H(z)$

As $H(z) = \frac{B(z)}{A(z)}$, we can find transfer function of the inverse filter of $H(z)$ by switching the numerator with the denominator $H^{-1}(z) = \frac{A(z)}{B(z)}$. This gives us

$$H^{-1}(z) = \frac{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}{1}$$

We can further rewrite too

$$H^{-1}(z) = \frac{\left(z - \frac{1}{2}\right)\left(z + \frac{1}{2}\right)}{z}$$

(b) Is the inverse filter stable? Justify the answer.

In this filter we can se that we get a pole when $z = 0$, as the pole is inside the unit circle we know that the system is stable. However this implies that the system gets an infinite gain at $z = 0$ or at DC this is not realizable. This make sthe filter not stable in a practical sense.

(c) Is the inverse filter a minimum-phase filter?

The definition for a miminum-phase system is whenever all zeros and poles are inside the unit circe. This stands true for this filter as the zeros are at $z = \frac{1}{2}$ and $z = -\frac{1}{2}$.

(d) Does the inverse filter have a linear phase characteristics? Justify your answer.

As the zeros doesn't have a corresponding symmetric counterpart, and the 2 we have aren't symmetric with respect to the unit circle, the inverse filter does not have a linear phase charasteristics.

Problem 3 (2 points)

In the recording/mastering of sound signals or during playback, it is often desired to alter the characteristics of the sound at different frequencies. For example, we may wish to highlight the lower/middle frequencies, while we may wish to reduce the presence of high frequencies.This can be done by using so-called 'shelving' filters. Figure 3 shows a low-frequency shelving filter implementation. The filter $A(z)$ is :

$$A(z) = \frac{\alpha - z^{-1}}{1 - \alpha z^{-1}}$$

The parameters α and K are used to *tune* the filter.

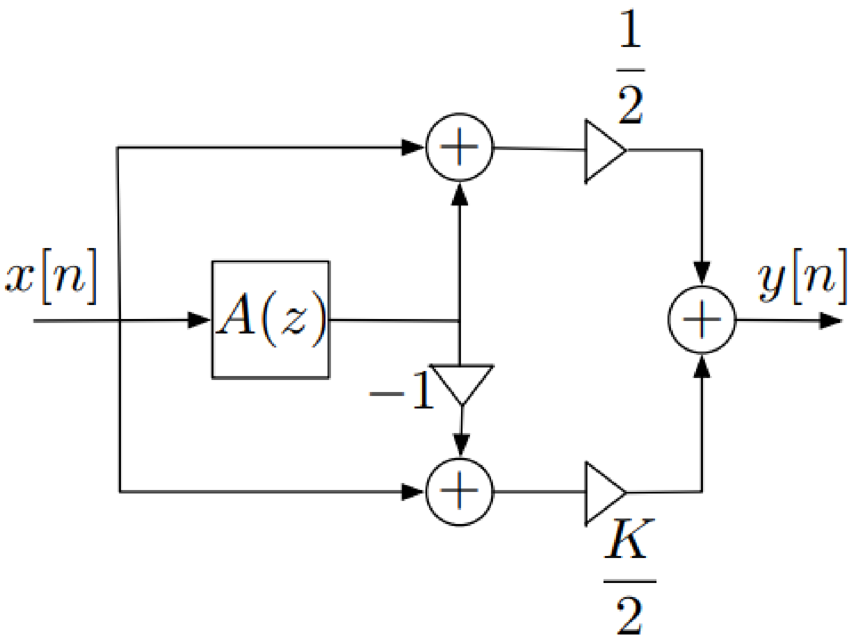

Figure 3: Low-frequency shelving filter

title

Problem 4 (4 points)

Appendix