Maths

1. *Over all real numbers, find the minimum value of a positive real number, y such that*

$$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

Solution

$$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

Re-writing the equation as a function of x

$$f(x) = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

Finding the first derivative of the function

$$f(x) = ((x+6)^2 + 25)^{\frac{1}{2}} + ((x-6)^2 + 121)^{\frac{1}{2}}$$

$$f'(x) = (((x+6)^2 + 25)^{-\frac{1}{2}}) + (1/2((x-6)^2 + 121)^{-\frac{1}{2}})$$

$$f'(x) = (x+6).\frac{1}{((x+6)^2 + 25)^{\frac{1}{2}}} + (x-6).\frac{1}{((x-6)^2 + 121)^{\frac{1}{2}}}$$

Finding the second derivative,

$$f''(x) = \frac{25}{((x+6)^2 + 25)^{\frac{3}{2}}} + \frac{121}{((x-6)^2 + 121)^{\frac{3}{2}}}$$

Now, we equate the first derivative to zero

$$f'(x) = \frac{(x+6)}{((x+6)^2 + 25)^{\frac{1}{2}}} + \frac{x-6}{((x-6)^2 + 121)^{\frac{1}{2}}} = 0$$

Simplifying

$$\frac{x(x^2 - 12x + 157)^{\frac{1}{2}} + x(x^2 + 12x + 61)^{\frac{1}{2}} + 6(x^2 - 12x + 157)^{\frac{1}{2}} - 6(x^2 + 12x + 61)^{\frac{1}{2}}}{(x^2 + 12x + 61)^{\frac{1}{2}}(x^2 - 12x + 157)^{\frac{1}{2}}} = 0$$

$$x(x^2 - 12x + 157)^{\frac{1}{2}} + x(x^2 + 12x + 61)^{\frac{1}{2}} + 6(x^2 - 12x + 157)^{\frac{1}{2}} - 6(x^2 + 12x + 61)^{\frac{1}{2}} = 0$$

$$x = -\frac{9}{4} = -2.25$$

Now, to find the minimum value, value of x is inserted into the original expression for y

$$y = \sqrt{(-2.25 + 6)^2 + 25} + \sqrt{(-2.25 - 6)^2 + 121} = 20$$

Minimum value of y=20