Notes on Schemes

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1 Few properties of schemes

Remark 1.1 (Keywords)**.** The definitions of *connected, connected component, (ir)reducible, irreducible component, quasicompact, generization, specialization, generic point, Noetherian topological space,* and *closed point.*

1.1 Reduced schemes

A scheme X is saied to be *reduced* if $\mathcal{O}_X(U)$ is *reduced* for every open set U of X.

- *Reducedness* is a **stalk–local** property: A scheme is reduced iff none of the stalks have nonzero nilpotents. Hence if f and g are two functions (global sections of $\mathcal{O}_X$) on a reduced scheme that agree at all points, then $f = g$. (Hint: consider $\mathcal{O}_X \hookrightarrow \prod_{p \in U} \mathcal{O}_{X,\mathfrak{p}}$)

Example 1.1. If a ring A is reduced, then $\operatorname{Spec}(A)$ is reduced. This implies $\mathbb{A}^n_k$ and $\mathbb{P}^n_k$ are reduced.

Example 1.2. The schemes $\operatorname{Spec} k[x,y]/(y^2, xy)$ is non-reduced. Show $\big(k[x,y]/(y^2,xy)\big)_x$ has no nonzero nilpotent elements. The only point of $\operatorname{Spec} k[x,y]/(y^2,xy)$ with a non-reduced stalk is the origin.

Remark 1.2. Reducedness is not in general an open condition.