1.6 Solving Uniform Acceleration Problems

Now that you have learned the definitions and basic equations associated with uniform acceleration, it is possible to extend your knowledge so that you can solve more complex problems. In this section, you will learn how to derive and use some important equations involving the following variables: initial velocity, final velocity, displacement, time interval, and average acceleration. Each equation derived will involve four of these five variables and thus will have a different purpose. It is important to remember that these equations only apply to uniformly accelerated motion.

The process of deriving equations involves three main stages:

1. State the given facts and equations.
2. Substitute for the variable to be eliminated.
3. Simplify the equation to a convenient form.

The derivations involve two given equations. The first is the equation that defines average acceleration, $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$. A second equation can be found by applying the fact that the area under the line on a velocity-time graph indicates the displacement. **Figure 1** shows a typical velocity-time graph for an object that undergoes uniform acceleration from an initial velocity ($\vec{v}_i$) to a final velocity ($\vec{v}_f$) during a time (Δt). The shape of the area under the line is a trapezoid, so the area is $\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$. (The area of a trapezoid is the product of the average length of the two parallel sides and the perpendicular distance between them.)

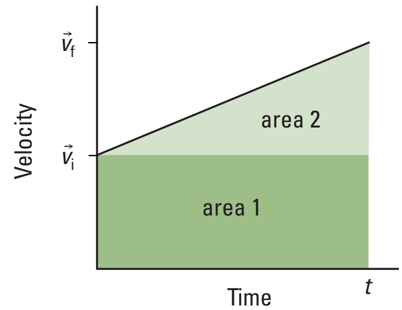

Figure 1
A velocity-time graph of uniform acceleration

Notice that the defining equation for the average acceleration has four of the five possible variables ($\Delta\vec{d}$ is missing), and the equation for displacement also has four variables ($\vec{a}_{av}$ is missing). These two equations can be combined to derive three other uniform acceleration equations, each of which involves four variables. (Two such derivations are shown next, and the third one is required in a section question.)

To derive the equation in which $\vec{v}_f$ is eliminated, we rearrange the defining equation of acceleration to get

$$\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$$

Substituting this equation into the equation for displacement eliminates $\vec{v}_f$.

$$\begin{aligned}\Delta\vec{d} &= \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t \\ &= \frac{1}{2}(\vec{v}_i + \vec{v}_i + \vec{a}_{av}\Delta t)\Delta t \\ &= \frac{1}{2}(2\vec{v}_i + \vec{a}_{av}\Delta t)\Delta t \\ \Delta\vec{d} &= \vec{v}_i\Delta t + \frac{\vec{a}_{av}(\Delta t)^2}{2}\end{aligned}$$

Next, we derive the equation in which Δt is eliminated. This derivation is more complex because using vector notation would render the results invalid. (We would encounter mathematical problems if we tried to multiply two vectors.) To overcome this problem, only magnitudes of vector quantities are used,

DID YOU KNOW ?

An Alternative Calculation

In **Figure 1**, the area can also be found by adding the area of the triangle to the area of the rectangle beneath it.

$$\begin{aligned}\Delta\vec{d} &= \text{area 1} + \text{area 2} \\ &= lw + \tfrac{1}{2}bh \\ &= \vec{v}_i\Delta t + \tfrac{1}{2}(\vec{v}_f - \vec{v}_i)\Delta t \\ &= \vec{v}_i\Delta t + \tfrac{1}{2}\vec{v}_f\Delta t - \tfrac{1}{2}\vec{v}_i\Delta t \\ \Delta\vec{d} &= \tfrac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t\end{aligned}$$

and directions of vectors involved will be decided based on the context of the situation. This resulting equation is valid only for one-dimensional, uniform acceleration. From the defining equation for average acceleration,

$$\Delta t = \frac{v_f - v_i}{a_{av}}$$ which can be substituted into the equation for displacement

$$\Delta d = \frac{1}{2}(v_f + v_i)\Delta t$$

$$= \frac{1}{2}(v_f + v_i)\left(\frac{v_f - v_i}{a_{av}}\right)$$ $(v_f + v_i)$ and $(v_f - v_i)$ are factors of the difference of two squares

$$\Delta d = \frac{v_f^2 - v_i^2}{2a_{av}}$$

or $2a_{av}\Delta d = v_f^{\,2} - v_i^2$

Therefore, $v_f^{\,2} = v_i^2 + 2a_{av}\Delta d$

The equations for uniform acceleration are summarized for your convenience in **Table 1**. Applying the skill of unit analysis to the equations will help you check to see if your derivations are appropriate.

Table 1 Equations for Uniformly Accelerated Motion

Variables involved	General equation	Variable eliminated
$\vec{a}_{av}$, $\vec{v}_f$, $\vec{v}_i$, Δt	$\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$	$\Delta\vec{d}$
$\Delta\vec{d}$, $\vec{v}_i$, $\vec{a}_{av}$, Δt	$\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{\vec{a}_{av}(\Delta t)^2}{2}$	$\vec{v}_f$
$\Delta\vec{d}$, $\vec{v}_i$, $\vec{v}_f$, Δt	$\Delta\vec{d} = \vec{v}_{av}\Delta t$ or $\Delta\vec{d} = \dfrac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$	$\vec{a}_{av}$
$\vec{v}_f$, $\vec{v}_i$, $\vec{a}_{av}$, $\Delta\vec{d}$	$v_f^2 = v_i^2 + 2a_{av}\Delta d$	Δt
$\Delta\vec{d}$, $\vec{v}_f$, Δt, $\vec{a}_{av}$	$\Delta\vec{d} = \vec{v}_f\Delta t - \dfrac{\vec{a}_{av}(\Delta t)^2}{2}$	$\vec{v}_i$

Sample Problem 1

Starting from rest at $t = 0.0$ s, a car accelerates uniformly at 4.1 m/s^2 [S]. What is the car's displacement from its initial position at 5.0 s?

Solution

$\vec{v}_i = 0.0$ m/s

$\vec{a}_{av} = 4.1$ m/s^2 [S]

$\Delta t = 5.0$ s

$\Delta\vec{d} = ?$

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}_{av}\Delta t^2$$
$$= (0.0 \text{ m/s})(5.0 \text{ s}) + \frac{1}{2}(4.1 \text{ m/s}^2 [\text{S}])(5.0 \text{ s})^2$$
$$\Delta\vec{d} = 51 \text{ m [S]}$$

The car's displacement at 5.0 s is 51 m [S].

In Sample Problem 1, as in many motion problems, there is likely more than one method for finding the solution. Practice is necessary to help you develop skill in solving this type of problem efficiently.

Sample Problem 2

An Olympic diver falls from rest from the high platform. Assume that the fall is the same as the official height of the platform above water, 10.0 m. At what velocity does the diver strike the water?

Solution

Since no time interval is given or required, the equation to be used involves v_f^2, so we will not use vector notation for our calculation. Both the acceleration and the displacement are downward, so we choose downward to be positive.

$v_i = 0.0$ m/s
$\Delta d = +10.0$ m
$a_{av} = +9.8 \text{ m/s}^2$
$v_f = ?$

$$v_f^2 = v_i^2 + 2a_{av}\Delta d$$
$$= (0.0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(10.0 \text{ m})$$
$$v_f^2 = 196 \text{ m}^2/\text{s}^2$$
$$v_f = \pm 14 \text{ m/s}$$

The diver strikes the water at a velocity of 14 m/s [↓].

Activity 1.6.1

Human Reaction Time

Earth's acceleration due to gravity can be used to determine human reaction time (the time it takes a person to react to an event that the person sees). Determine your own reaction time by performing the following activity. Your partner will hold a 30 cm wooden ruler or a metre stick at a certain position, say the 25 cm mark, in such a way that the ruler is vertically in line with your thumb and index finger (**Figure 2**). Now, as you look at the ruler, your partner will drop the ruler without warning. Grasp it as quickly as possible. Repeat this several times for accuracy, and then find the average of the displacements the ruler falls before you catch it.

Figure 2
Determining reaction time

(a) Knowing that the ruler accelerates from rest at 9.8 m/s^2 [↓] and the displacement it falls before it is caught, calculate your reaction time using the appropriate uniform acceleration equation.

(b) Compare your reaction time with that of other students.

(c) Assuming that your leg reaction time is the same as your hand reaction time, use your calculated value to determine how far a car you are driving at 100 km/h would travel between the time you see an emergency and the time you slam on the brakes. Express the answer in metres.

(d) Repeat the procedure while talking to a friend this time. This is to simulate distraction by an activity, such as talking on a hand-held phone, while driving.

SUMMARY Solving Uniform Acceleration Problems

- Starting with the defining equation of average acceleration, $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$, and a velocity-time graph of uniform acceleration, equations involving uniform acceleration can be derived. The resulting equations, shown in **Table 1**, can be applied to find solutions to a variety of motion problems.

Section 1.6 Questions

Understanding Concepts

1. In an acceleration test for a sports car, two markers 0.30 km apart were set up along a road. The car passed the first marker with a velocity of 5.0 m/s [E] and passed the second marker with a velocity of 33.0 m/s [E]. Calculate the car's average acceleration between the markers.
2. A baseball travelling at 26 m/s [fwd] strikes a catcher's mitt and comes to a stop while moving 9.0 cm [fwd] with the mitt. Calculate the average acceleration of the ball as it is stopping.
3. A plane travelling at 52 m/s [W] down a runway begins accelerating uniformly at 2.8 m/s^2 [W].
   (a) What is the plane's velocity after 5.0 s?
   (b) How far has it travelled during this 5.0 s interval?
4. A skier starting from rest accelerates uniformly downhill at 1.8 m/s^2 [fwd]. How long will it take the skier to reach a point 95 m [fwd] from the starting position?
5. For a certain motorcycle, the magnitude of the braking acceleration is $|4\vec{g}|$. If the bike is travelling at 32 m/s [S],
   (a) how long does it take to stop?
   (b) how far does the bike travel during the stopping time?
6. (a) Use the process of substitution to derive the uniform acceleration equation in which the initial velocity has been omitted.
   (b) An alternative way to derive the equation in which the initial velocity, $\vec{v}_i$, is eliminated is to apply the fact that the area on a velocity-time graph indicates displacement. Sketch a velocity-time graph (like the one in **Figure 1** of section 1.6) and use it to derive the equation required. (*Hint:* Find the area of the large rectangle on the graph, subtract the area of the top triangle, and apply the fact that $\vec{v}_f - \vec{v}_i = \vec{a}\Delta t$.)

7. A car travelling along a highway must uniformly reduce its velocity to 12 m/s [N] in 3.0 s. If the displacement travelled during that time interval is 58 m [N], what is the car's average acceleration? What is its initial velocity?

Applying Inquiry Skills

8. Make up a card or a piece of other material such that, when it is dropped in the same way as the ruler in Activity 1.6.1, the calibrations on it indicate the human reaction times. Try your calibrated device.
9. Design an experiment to determine the maximum height you can throw a ball vertically upward. This is an outdoor activity, requiring the use of a stopwatch and an appropriate ball, such as a baseball. Assume that the time for the ball to rise (or fall) is half the total time. Your design should include the equations you will use and any safety considerations. Get your teacher's approval and then perform the activity. After you have calculated the height, calculate how high you could throw a ball on Mars. The magnitude of the acceleration due to gravity on the surface of Mars is 3.7 m/s^2.

Making Connections

10. How could you use the device suggested in question 8 as a way of determining the effect on human reaction time of taking a cold medication that causes drowsiness?

Reflecting

11. This chapter involves many equations, probably more than any other chapter in this text. Describe the ways that you and others in your class learn how to apply these equations to solve problems.
12. Visual learners tend to like the graphing technique for deriving acceleration equations, and abstract learners tend to like the substitution technique. Which technique do you prefer? Why?