上海财经大学《 常微分方程 》课程模拟试卷一答案

一、

1. $y = \pm 1$

2. $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{-M} = \phi(y)$

3. $y' = p(x)y + q(x), \quad e^{-\int p(x)dx}$

4. 存在常数 L>0,对 $\forall (x, y_1), (x, y_2)$，有 $\left| f(x, y_1) - f(x, y_2) \right| \le L \left| y_1 - y_2 \right|$

5. $[-\frac{1}{2}, \frac{1}{2}]$

6. 无

7. $\Phi'(t)\Phi^{-1}(t)$

8. $e^{\lambda t}\left\{E+(A-\lambda E)t+\cdots \frac{t^{n-1}}{(n-1)!}(A-\lambda E)^{n-1}\right\}$

9. $W'(t)=-a_1(t)W(t), \quad W(t_0)e^{-\int_{t_0}^{t}a_1(s)ds}$

二、

1. D　2. A　3. B 4. C

三、

1.

$$ydy=\frac{2x}{1+x^2}dx, \quad \frac{1}{2}y^2=\ln(1+x^2)+C$$

2.

$$M=x^2e^x-y, \quad N=x$$

$$\frac{\partial M}{\partial y}=-1, \quad \frac{\partial N}{\partial x}=1, \quad \frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=-\frac{2}{x}, \quad \mu(x)=\frac{1}{x^2}$$

方程化为 $e^x dx+\frac{xdy-ydx}{x^2}=0$

通解为 $e^x+\frac{y}{x}=C$

3. 特征方程为 $\lambda^3-1=0, \lambda_1=1, \lambda_{2,3}=\frac{-1\pm\sqrt{3}i}{2}$

由于 $\lambda=\pm i$ 不是特征根，故方程的特解为 $x^*=A\cos t+B\sin t$

方程的通解为

$$x(t)=e^{-\frac{1}{2}t}(c_1\cos\frac{\sqrt{3}}{2}t+c_2\sin\frac{\sqrt{3}}{2}t)+c_3e^t-\frac{1}{2}(\cos t+\sin t)$$

4.

$$(yy'+x^2)=0$$
$$yy'+x^2=c_1, \quad yy'=c_1-x^2$$
$$\frac{1}{2}y^2=c_1x-\frac{1}{3}x^3+c_2$$
$$y^2=r_1x+r_2-\frac{2}{3}x^3$$

四、

（1）$A=\begin{pmatrix}11 & -25\\ 4 & -9\end{pmatrix},\quad |A-\lambda E|=(\lambda-1)^2=0,\lambda=1(\text{二重})$

方程组的解形如 $X(t)=(R_0+R_1t)e^t$

$$\begin{cases}(A-E)R_0=R_1\\ (A-E)^2R_0=0\end{cases}$$

$$A-E=\begin{pmatrix}10 & -25\\ 4 & -10\end{pmatrix},(A-E)^2=0$$

$$R_0=\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix},\text{则}R_1=\begin{pmatrix}10\\4\end{pmatrix},\begin{pmatrix}-25\\-10\end{pmatrix}$$

$$X_1(t)=\begin{pmatrix}1+10t\\4t\end{pmatrix}e^t,\quad X_2(t)=\begin{pmatrix}-25t\\1-10t\end{pmatrix}e^t$$

通解为

$$X(t)=e^t\begin{pmatrix}1+10t & -25t\\ 4t & 1-10t\end{pmatrix}C$$

（2）$\phi(t)=e^t\begin{pmatrix}1+10t\\4t\end{pmatrix}$

五、

$$y' = -\frac{1}{5}(y'' + 4y), y'' + 5y' + 4y = 0$$
$$\lambda^2 + 5\lambda + 4 = 0, \lambda = -1, \lambda = -4$$

方程通解为 $y = c_1 e^{-x} + c_2 e^{-4x}$

又

$$y(0) = 1, y'(0) = 0$$
$$c_1 = \frac{4}{3}, c_2 = -\frac{1}{3}$$

所求函数为 $y = \frac{4}{3}e^{-x} - \frac{1}{3}e^{-4x}$