Finding the Mass of the Neutron from a Hydrogen-Deuterium Discharge Lamp using Spectroscopy

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An electron emits a photon of characteristic wavelength when it drops from one level of orbital to the other. In hydrogen, it is described through the Balmer series. More specifically, the hydrogen Balmer series accounts for orbital level changes higher than $n = 2$, that is $3 \to 2$, $4 \to 2$, etc. These changes, which have associated distinct wavelengths, can be used to study the atomic composition of the atoms that those electrons orbit, meaning that depending on the composition, there will be a slight change in those wavelengths. We use this mechanism to our advantage by comparing the wavelengths from hydrogen and deuterium to measure the mass of the neutron (m_n). Here we show a final measurement of m_n to be $(2.031 \pm 0.005)E-27$ kg using spread-based error analysis and a weighted average from each line of the Balmer series. Although the known value of $m_n = 1.674E-27$ kg, our measurement presents a valuable insight on the limitations of measuring m_n as we inquire higher levels of the Balmer series, as well as ways on improving the measurement given the available experimental setup.

I. INTRODUCTION

The first major, collective discussion of the neutron began in 1933 in the 7th Solvay Conference, particularly on the mass of the neutron in comparison to that of the proton.[1] The first measurement of m_n was made in 1932, when James Chadwick estimated the mass to be around 1.005 Da to 1.008 Da (1 Da = 1.6605E-27 kg).[1] While numerous other measurements followed, the final accurate measurement was made in 1935 by Chadwick and Goldhaber, with the accepted value of $m_n = 1.00866$ Da, or $m_n = 1.674E-27$ kg.[1] The mass of the neutron, as well as neutron in itself, has opened doors to a wide branch of sciences, like neutron physics. By studying how neutrons interacts with other particles and forces, scientists have found applications of neutron physics to fields like nuclear physics and particle physics.[2] In light of this importance, the purpose of this laboratory experiment was to measure the mass of the neutron, m_n, by performing spectroscopy on a hydrogen-deuterium mix discharge lamp and exploring each line within the Balmer series.

II. THEORY

For hydrogen, which consists of a proton and an electron, the associated energy held in the electron at a given orbital level is given by the formula $E_n = -\frac{E_R}{n^2}$, whereby $E_R = 13.6eV$ is the Rydberg energy and n is the orbital level.[3] We can then claim that in the hydrogen Balmer series, when an electron emits energy and goes from a higher to a lower orbital level, the change in energy becomes $\Delta E = E_n - E'_n$. Using the formula $\lambda = \frac{1240\text{eV}*\text{nm}}{\Delta E}$ and converting from 1nm = $10\mathring{A}$, we can find the associated wavelengths of hydrogen Balmer series. We can formally recognize this as the wavelengths of a particular Balmer series transition, or λ_H; in this experiment we will only consider up to the first four transitions.

For deuterium, which consists of a proton, a neutron, and an electron, the formula for the wavelengths must be adjusted. This is done by using a reduced mass formula for deuterium. Without going into too much detail, the calibrated Rydberg energy for deuterium comes out to be $E_{RD} = 13.6029850746$eV. We can then follow the standard procedure for hydrogen to find the characteristic wavelength at each of the Balmer line for deuterium, which can be called λ_D. When we compare now the wavelengths of deuterium to that of hydrogen, we can recognize that there is a small but important difference. We will call this $\Delta\lambda$.

In the experiment, this difference will be visible in the form of a two, ideally distinct peaks, in which the distance between the two peaks will be $\Delta\lambda$. We will use $\Delta\lambda$ and use the known constants of the mass of election m_e, mass of proton m_p, and the relevant Balmer series transition wavelength of hydrogen λ_H to calculate for m_n. The formula provided as part of this lab is

$$m_n = \frac{m_e}{\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}} - m_p \tag{1}$$

III. APPARATUS & PROCEDURE

The apparatus used for the experiment involved a discharge lamp consisting of a hydrogen-deuterium mix. Once the excited electrons "discharged" photons, the lights were then focused via a lens, after which the focused beam was sent into the detector. The detector was composed of a monochromator with a rotatable diffraction grating, a slit of adjustable width, and a sensor of adjustable sensitivity. The focused light would then hit

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the rotating diffraction grating, which we could then control to select which wavelength the monochromator sent onto the sensor. This is particularly important as it allows us to precisely select the wavelength of the Balmer transition to be analyzed. The slit would control how much light enters into the sensor, meaning the range and intensity of light registering in the sensor. The sensitivity of the sensor controls the voltage difference between the metal plates in the photoamplifier; for the purposes of this lab we did not go beyond 12. All these components must be balanced to create clean data.

A computer would then produce data, in which intensity as a function of wavelength was generated. The produced graph was two, ideally distinct, peaks. On the computer, there were four functionalities. The first was "Start Run," which would prompt the setup to start collecting data. The second was "Mark," which allowed the user to manually select two "marks." In the experiment we selected the beginning of the first peak and the end of the second peak. The third was "Stop Run," which would stop data collection. The fourth, "Save Run," saved the data in .csv file, in which the first column included the two marks and the second column had the intensity values. Each of these .csv files were then organized by Balmer line folders during exporting. I will provide a schematic diagram below to visualize the apparatus setup.

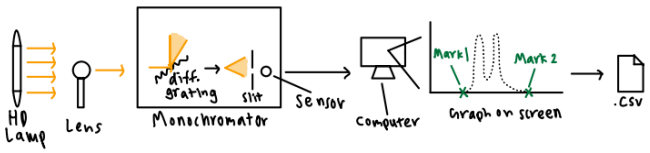

FIG. 1: Schematic diagram of the lab apparatus.

The production of the two peaks is possible given that we are recording a specific Balmer transition of hydrogen and deuterium. From the discussion of theory, we know that the Rydberg energy of hydrogen is slightly less than that of deuterium, $E_R < E_{RD}$, meaning that the $\Delta\lambda$ of hydrogen will be less than that of deuterium given a specific transition. This means that the wavelength of hydrogen is slightly contracted, or shorter, compared to that of deuterium, making the peak of hydrogen appear slightly left to that of deuterium. Because the sensor records intensity as a function of wavelength, then, there should be a concentration of intensity at both transitional wavelengths, producing a two-peaks shape of the graph.

All collection of data took place in a dark room to minimize unwanted light entering the detector. For the selected wavelengths on the monochromator (WM), we used the following values, adjusting for the ~ 500nm offset. I will also include the number of trials (T), detector sensitivity (DS), and the slit width (SW).

Balmer Transition	WM (nm)	T	DS	SW (microns)
$3 \to 2$	6000	11	8	40
$4 \to 2$	4300	6	11	32
$5 \to 2$	3800	5	12	22
$6 \to 2$	3600	1	12	22

TABLE I: Values used for each Balmer transition. Note the first row will be referred as Balmer 1, the second row Balmer 2, etc. for the sake of simplicity.

IV. OBSERVATIONS AND ANALYSIS

Once all data was imported to Python, we generated the two peaks graph. Given the two marks, we interpolated to create the x-values necessary to generate plot and to search the value of the wavelength at the peak. As we generated 23 plots, we will only present one characteristic graph from each Balmer line.

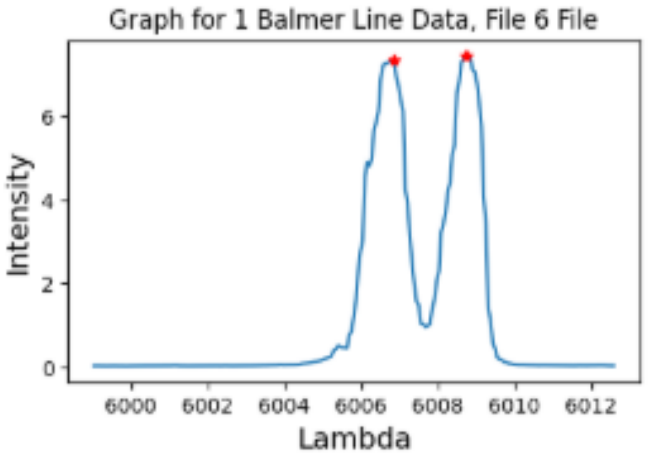

FIG. 2: Characteristic graph for Balmer 1. Hydrogen peak is the left while the deuterium peak is the right. $\Delta\lambda$ between the two peaks is also clearly visible.

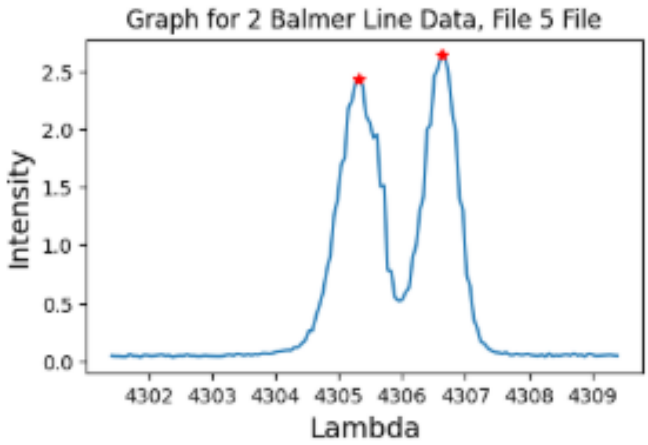

FIG. 3: Characteristic graph for Balmer 2. Hydrogen peak is the left while the deuterium peak is the right. $\Delta\lambda$ between the two peaks is also clearly visible.

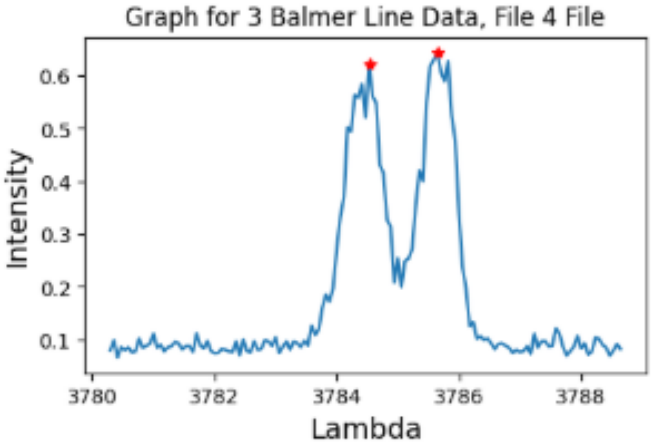

FIG. 4: Characteristic graph for Balmer 3. Hydrogen peak is the left while the deuterium peak is the right. $\Delta\lambda$ between the two peaks is also clearly visible.

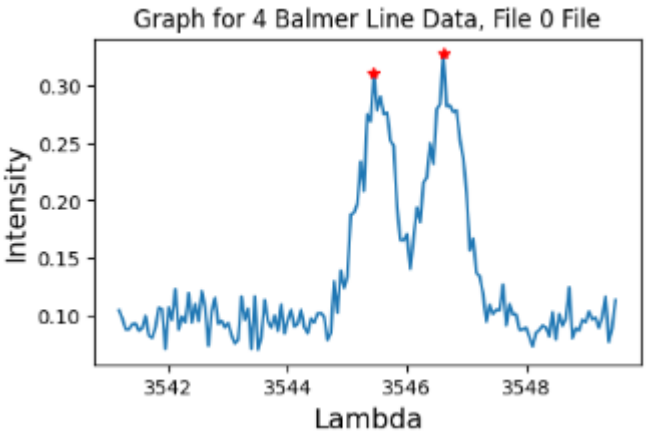

FIG. 5: Characteristic and only graph for Balmer 4. Hydrogen peak is the left while the deuterium peak is the right. $\Delta\lambda$ between the two peaks is also clearly visible.

As evident, we made an attempt to find the value of the wavelength associated with the two peaks, marked by the two red stars, with Numpy's peak-finding functionality, but were unable to do so, likely due to a small error in the parameters. Since we need the values associated with the two peaks to calculate $\Delta\lambda$, in those cases in which we lacked two peaks, we excluded from later analysis. This led to 21 out of the total 23 graphs having the respective peaks values being analyzed for m_n, as one from Balmer 1 and 3 failed to produce two peaks.

Before calculating for m_n, however, an important calibration had to be made for the formula:

$$m_n = \frac{m_e}{\frac{m_e}{m_p} - \frac{\Delta\lambda}{\lambda_H}} - m_p \tag{2}$$

That is, we must adjust the λ_H value for the ~ 500nm offset. We did this by finding the average wavelength value for each Balmer line, which we defined to be the value of the first peak of the graph. We then subtracted them from the known values of λ_H at the respective Balmer line to find the average offset caused by the apparatus. We then subtracted this average offset value from the known λ_H values to find the corrected λ_H values to be used. For your reference, these are the corrected λ_H values.

Balmer Line	Corrected λ_H (nm)
1	6006.8417
2	4305.3417
3	$3784.5416\overline{9}$
4	$3545.7416\overline{9}$

TABLE II: Corrected λ_H values.

Following this we solved for the m_n, using the above corrected λ_H values and the known values of $m_p = 1.67262192595E-27$ kg and $m_e = 9.1093837139E-31$ kg. Note that my raw calculations had a problem with a

negative being assigned in front of the calculated value. However, this is likely a negative being misassigned in the code, and given the reasonable magnitude we took the absolute value of the calculations. This process left 10, 6, 4, and 1 calculations of m_n for each Balmer line, which sum to 21 as expected.

Balmer 1 n_m	Balmer 2 n_m	Balmer 3 n_m	Balmer 4 n_m
1.9917696E-27	2.0747851E-27	1.9997426E-27	2.0043892E-27
1.9886076E-27	2.0295292E-27	1.9628565E-27	
1.9844996E-27	2.0595552E-27	2.0432191E-27	
2.0229902E-27	2.0693931E-27	2.0508286E-27	
1.9916046E-27	2.0570959E-27		
2.0022780E-27	2.0400439E-27		
2.0275988E-27			
2.0571906E-27			
1.9978844E-27			
2.0096263E-27			

TABLE III: m_n values for each Balmer line, with units in kilograms. Note that I only presented the first 7 decimal values out of the 16 decimal places from Python for formatting purposes. Later analysis was done with the full values in Python.

Given this value, we conducted a spread-based error analysis, whereby we used the following formula to present a "final" measurement of m_n from each Balmer line ($m_{n,\text{mean}}$) with the associated uncertainty value (u), whereby S is the standard deviation and N is the number of m_n values in the given Balmer line. In the context of spread-based error analysis, the standard deviation, which tells how spread out a given dataset is relative to the average, as well as the number of samples N, are crucial to defining the uncertainty for a given dataset, which in our case is for Balmer lines 1, 2, 3, and 4. Note that for Balmer line 4, we said there is a ± 0.0 uncertainty, which is due to the fact that there is only one size, so $S = 0$, making $u = 0$. This means that due to there only being a single data point, there is little statistical significance of this data.

$$m_{n,\text{mean}} = \sum_{i=1}^{n} m_{n,\text{i}} \tag{3}$$

$$u = \frac{S}{\sqrt{N}} \tag{4}$$

Balmer line	"Final" $m_{n,\text{mean}} \pm u$ kg
1	2.0074049731341815E-27±6.799461605960585E-30
2	2.0550671209550456E-27±6.446752578922886E-30
3	2.0141617229046862E-27±1.7728730138094855E-29
4	2.0043892234276273E-27±0.0

TABLE IV: "Final" calculations of m_n with the associated uncertainty u. Note on the "Final" because this is not our final stated value of m_n. Therefore, they have not been rounded to conventions yet.

For the final stated value of m_n, we calculated the weighted average for the first three Balmer lines. This was intentional as the fourth Balmer line has an uncertainty $u = 0.0$, which would give the m_n of the fourth Balmer line an infinite weight. We used the following formula.

$$m_{n,\text{best}} = \sum_{i=1}^{n} \frac{m_{n,\text{i}} \frac{1}{u_i^2}}{\sum_{i=1}^{n} \frac{1}{u_i^2}} \tag{5}$$

$$u_{best} = \sqrt{\frac{1}{\sum_{i=1}^{n} \frac{1}{u_i^2}}} \tag{6}$$

This yields the final m_n calculation with its uncertainty of

$$(2.031 \pm 0.005)E - 27 \text{ kg}.$$

As a discussion point on uncertainty, we present an error bar graph for Balmer 1, 2, and 3 lines. The fourth Balmer line is not presented as there was only one sample and there can be no meaningful presentation of uncertainty from one data point in a spread-based error analysis. For the error bars on each individual points, we used S, the standard deviation, for each Balmer line. The table below summarizes the specifics of the error bar graph of Balmer line, the number of samples (N), and the standard deviation (S).

Balmer line	N	S (kg)
1	10	6.799461605960585E-30
2	6	6.446752578922886E-30
3	4	1.7728730138094855E-29

TABLE V: Number of samples N and the standard deviation S for each Balmer line.

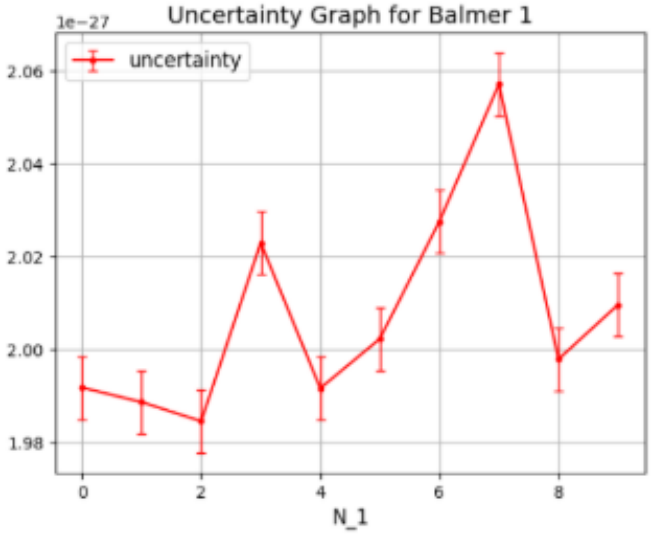

FIG. 6: Error bar graph for Balmer 1.

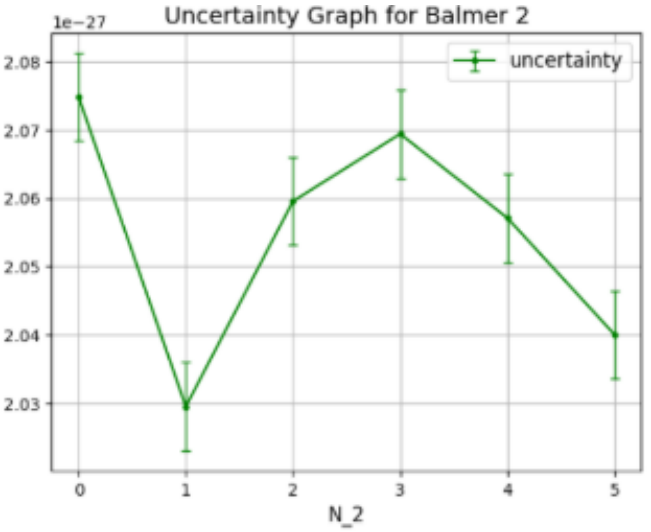

FIG. 7: Error bar graph for Balmer 2.

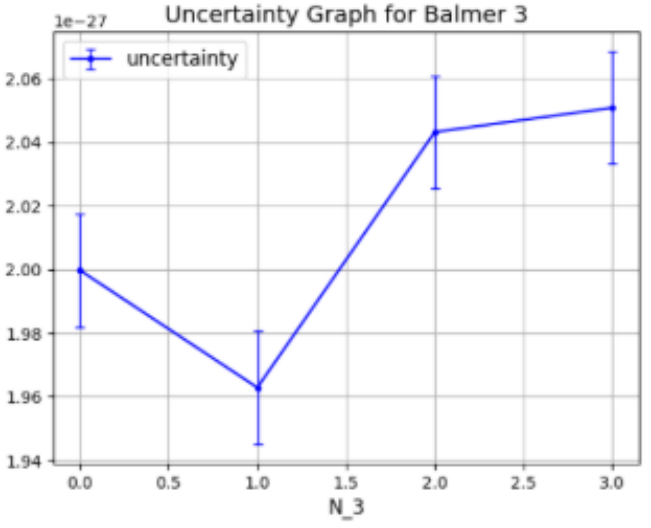

FIG. 8: Error bar graph for Balmer 3.

FIG. 9: Error bar graphs for Balmer lines 1, 2, and 3.

From the graphs, it is observable that the individual data points for Balmer 1 fall very frequently within the error bars (the standard deviation, S), less frequently for those in Balmer 2, and significantly less for those in Balmer 3. It is clear that the number of measurements, N, has a significant impact on how precise our individual m_n calculation values are, as Balmer 1 shows higher precision than Balmer 2, which in turn shows higher than Balmer 3. This can also be generalized to the discussion on the viability of data from higher Balmer lines, as even from the two peaks graphs 2, 3, 4, and 5, that the figures tend to have more noise as they go up the Balmer line. Furthermore, from table V we can see that the standard deviation for Balmer line 3 is a whole order of magnitude higher than that of Balmer 1 and 2, which we can see reflected in the noise levels of the graphs, as we would expect a noisier graph to produce a higher uncertainty in the final calculation of m_n, as is the case here. The noise component of the graphs is directly tied to the detector sensitivity having to become higher for higher Balmer lines, as evident in table I, in which detector sensitivity (DS) values were set higher to accommodate for the weaker spectral lines in the higher Balmer lines, or transitions.

V. CONCLUSIONS AND OUTLOOK

In this lab experiment, we conducted spectroscopy on each Balmer transition for hydrogen and deuterium to inquire the small difference, $\Delta\lambda$, in the characteristic wavelengths to solve for the mass of neutron, m_n. We produced graphs for each of the measurements we made. We also organized the m_n calculations for each Balmer line and used spread-based error analysis to find the mean and the associated uncertainty value. We then brought the first three Balmer line calculations by taking the weighted average to find the final presented value of $m_n = (2.031 \pm 0.005)E - 27$ kg, and accompanied the three calculations with an error bar graph for each.

We can now compare our final presented m_n value to the accepted m_n value of $m_n = 1.674E - 27$ kg (in our analysis, we used the full decimals of $m_n = 1.674927471E - 27$) using the percent error formula.

$$\text{Percent Error} = \left|\frac{\text{Measured } m_n - \text{Accepted } m_n}{\text{Accepted } m_n}\right| * 100\% \tag{7}$$

There was a 20.732% difference in our final presented value of m_n compared to the accepted value of m_n. We can use this percent error as a beginning point of the potential methods of improving future measurements with the same lab apparatus.

The first method of improvement is the way how the two marks are added by the experimenter. Sometimes, it was clear, simply reading the .csv file, that the marks could have been placed at some other corresponding intensity. While this difference may be small because we

are measuring a quantity so small, seemingly minuscule differences could affect the overall calculation of m_n, as the two marks directly affect the interpolation as well as the finding of $\Delta\lambda$ from the two peaks, which are assigned their λ value from the interpolation. If there was a way to add the marks following data collection, that is, have the apparatus collect wavelength values for all corresponding intensity values, we could ensure a more recording of marks, and resulting peaks, data. From another perspective, our parameters for finding peaks in the graphs could also be improved. By ensuring a more thorough collection of the two peaks data, we ensure a higher number of sample size N that can be used for the uncertainty analysis as wel as the calculation of m_n.

The second method of improvement involves a stronger control of external light sources from affecting the detector readings. Particularly in the higher Balmer lines, when detector sensitivity values were larger, we noted how even a faint light near the detector could negatively impact the usability of the data. By ensuring a darker environment, perhaps a secondary curtain around the lamp, lens, and the detector, we could introduce less noise to the data.

The third method of improvement simply requires more trials. We discussed in the pervious section of how the small number of sample size N negatively affects the precision of the presented calculations, as well as finding the accurate mean and uncertainty value for each Balmer line. Furthermore, since we are doing a spread-based error analysis followed by a weighted average, ensuring a more reliable mean and standard deviation by increasing N should not only improve the accuracy of the final presented value but also provide a more ideal uncertainty u value. Compared to our final presented value of m_n, $2.031E-27$ kg, we recognize how our uncertainty value, $u = 0.005E-27$ kg, is orders of magnitude below the final presented value. This reflects an overly optimistic reporting of the uncertainty in our final m_n value.

Following these improvements in the future could address the relatively large percent error reported for this experiment. However, to this end, there were some components of the lab outside of control. For example, we were unable to verify if the offset that occurred in the lab is due to a mechanical error, which would likely then be consistent for every trial, or something else. Furthermore, the uncertainty in our making of two marks are not confirmable whether it is consistent across all measurements. Additionally, if there were any contamination present within the discharge lamp, then the location of those two marks may become inaccurate as the peaks would have shown an inaccurate representation of the peaks had there been simply hydrogen and deuterium in the lamp. These are particularly noteworthy in interpreting our presented u value, as a spread-based uncertainty analysis does not account well for the errors systematically present in the apparatus. Perhaps, next time a propagation-based error analysis that considers the effects of the individual uncertainties on the outcome of the m_n formula can be utilized, which can be accompanied by modifying our equation for m_n to include those components.

Acknowledgments

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[1] Wikipedia contributors, Neutron, 2025.
[2] J. D. Rogers, Physics Procedia **43**, 1 (2013).
[3] J. Taylor, C. Zafiratos, and M. A. Dubson, *Modern Physics for Scientists and Engineers*, Pearson, 2nd edition, 2004.