Experimental Observation of the Universal Gravitation Constant Using a Cavendish-Laser Apparatus

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The universal gravitational constant G serves an important role in Newton's law of universal gravitation as a scaling factor, aiding in determining the gravitational force between two objects. Using a Cavendish-laser apparatus setup of two pairs of masses, one small and one large, we initiated the torsion pendulum to twist from the gravitational attraction between the small and the large masses. By measuring the angular torsion of the string, which was indicated by the amount of distance the reflected point of a laser oscillated, we calculated G using a derived formula. Here we present a measurement of $G = (5.41 \pm 0.01)E - 11\frac{Nm^2}{kg^2}$ with the uncertainty determined through a spread-based uncertainty analysis determined across 5 trials. We found an 18.95% percent error compared to the actual value of $G = 6.6743E - 11\frac{Nm^2}{kg^2}$. Given our suspicion of a systematic error present within our lab apparatus, we conclude with remarks on improving the experiment.

I. INTRODUCTION

The value of G has interested physicists ever since Newton proposed his law of universal gravitation in 1686, with the first formal measurement of G being made by Henry Cavendish in 1798 to a 1% accuracy using a torsion balance experiment. [1] While very accurate, there occurred some discrepancies, perhaps from the internal friction within the torsion fibers. [1] More recently in 2000, Jens Gundlach from the University of Washington utilized a turntable apparatus that could bypass this error, yielding a measurement of $G = (6.674255 \pm 0.000092)E - 11\frac{Nm^2}{kg^2}$.[1]

Currently, the accepted value of G upheld by the Committee on Data for Science and Technology is $(6.67430 \pm 0.00015)E - 11\frac{Nm^2}{kg^2}$. [2] Given its applicability in fields involving gravity, the value of G is integral to fields like astronomy and relativistic physics. As such, we seek to provide a measurement of the value of G using a Cavendish-laser apparatus by observing the damped oscillatory motion of the torsion pendulum.

II. THEORY

As briefly mentioned above, the apparatus of this lab experiment rests upon a torsion pendulum experiencing two external torques, which are produced by the gravitational attractions between a small mass m attached to the torsion pendulum and a large mass M. The oscillating motion, which decays due to internal friction, can then be modeled as a damped oscillator. This motion is accounted for by a laser that is sent to a mirror attached to the torsion pendulum, which is then reflected onto a photodetector that tracks the motion of the light.

Starting with the ordinary differential equation for a damped oscillator,

$$I\ddot{\theta} + C\dot{\theta} + \kappa\theta = 0,$$

whereby I is the moment of inertia, C is the damping constant, and κ is the torsion constant given by $\kappa = I\omega^2 + I\alpha^2 = I\omega^2 + \frac{c^2}{4I}$. Without delving too deep into the derivations, $\kappa = 2ml^2(\omega^2 + \alpha^2)$.

Furthermore, knowing Newton's law of universal gravitation,

$$F_g = G\frac{Mm}{r^2},$$

whereby F_g is the force of gravity, G is the universal gravitation, M and m the large and small masses, respectively, and r is the distance between M and m.

As we know this F_g is the force responsible for the torque on the pendulum and $\kappa\theta$ is the restoring torque of the pendulum,

$$\kappa\theta = 2\frac{GMm}{r^2}l,$$

whereby l is the lever arm using the small angle approximation.

Combining these derivations along with the geometry of the apparatus given by Figure 1 below,

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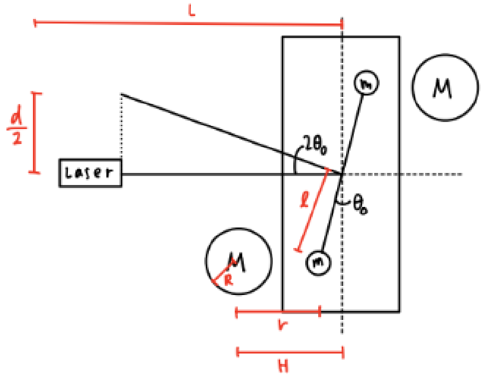

FIG. 1: Schematic Diagram of the Lab Apparatus.

we can solve for G,

$$G = \frac{ld(\omega^2 + \alpha^2)}{4ML}(H - l(\frac{d}{4L}))^2 \tag{1}$$

whereby d is the maximum amplitude of the laser tracked by the photodetector, L the distance from the pendulum to the laser, H the distance from M to the axis perpendicular to the axis of rotation, and l the level arm length. Table I in the Apparatus & Procedure section quantifies these constants.

The variables we are measuring from the equation is α and ω. We found these through the model equation for a damped oscillator,

$$V(t) = Ae^{-\alpha t}\cos(\omega t + \phi) + B, \tag{2}$$

whereby A is the amplitude of the oscillation, α the decay constant, ω the angular frequency, ϕ the phase offset, B the vertical offset, and t is the independent variable for the position function $V(t)$.

III. APPARATUS & PROCEDURE

The apparatus for this lab experiment includes a torsion pendulum consisting of a thin metal rod with two small masses m on each end, in which the rod is suspended in the air at the middle by a thin string. This system lies inside a rotatable platform holding two large masses M, whereby by rotating the platform, such that m and M are close to one another, we induce the string to twist due to the torque generated by the gravitational attraction between m and M, which then oscillate in a twisting motion due to the restoring force of the string. This twisting motion is recorded by a photodetector, which has been aligned on some horizontal axis for the photodetector to track, following the light from a laser reflected from a mirror on the string.

We initially began by measuring the constants d, L, H, M, and l, which has been recorded in table I below. For the measurement of M, the scale we utilized had a 1000 g offset, which has been accounted for in the table. Note that there is no uncertainty measurement because this lab experiment conducted a spread-based uncertainty analysis. We deemed a spread-based uncertainty analysis to provide a more relevant uncertainty measurement, as the calculation of G depends more on the parameters we find through model fitting and less so on the uncertainty of the individual length measurements, which can also be easily mitigated by using high-precision measurement tools.

Constant	Measurement
d	0.061 m
L	2.06 m
H	0.05 m
M	1.503 kg
l	0.0405 m

TABLE I: Descriptions of the Constants.

To set the torsion pendulum in motion, we rotated the platform to bring M closer to m. Because of the sensitivity of the string and to prevent any unwanted swings of the pendulum itself, we carefully moved the platform. To record data, we restarted the program on the computer connected to the photodetector, and after some time greater than two hours, we came back to repeat the same process. For every trial, we measured d (or more descriptively, $\frac{d}{2}$) using two tapes to reference the equilibrium position and the maximum amplitude of the position of the light.

The ideal time periods to conduct this lab were during late evening and early morning hours as to minimize any vibrations in the room. All data consisting of 6 trials in .csv format were exported and analyzed in Python.

IV. OBSERVATIONS AND ANALYSIS

The raw graph with the best fit plot of Equation 2 is as shown below in Figure 2. All model fitting occurred through the Model and Parameters packages from lmfit.

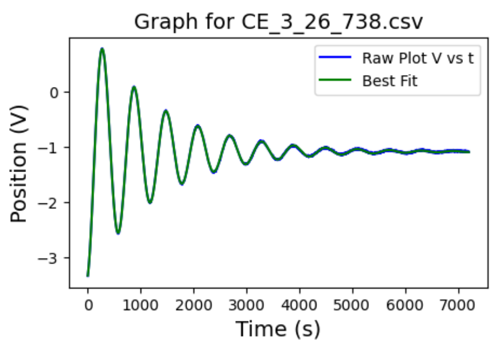

FIG. 2: Raw and Best Fit Plots.

A note should be made, however, that although we started with 6 trials, we had to remove one of the trials due to poor fitting, as illustrated below in Figure 3. We suspect that the sharp vertical line in the first oscillation, likely caused by unwanted motion in the room, caused the poor fit.

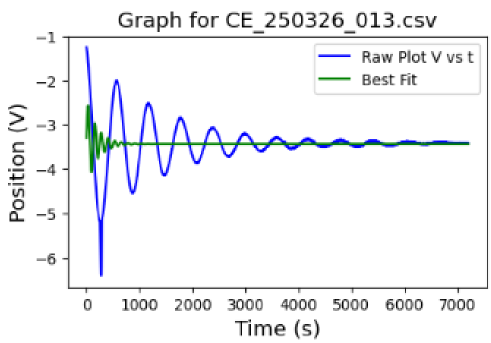

FIG. 3: Example of a Poor Fit Plot.

After extracting the α and ω parameters from the lmfit Fit Report and calculating G with Equation 1, we conducted spread-based uncertainty analysis for the G values with

$$u = \frac{S}{\sqrt{N}},$$

whereby u is the uncertainty, S the standard deviation, and N the number of trials (5). For the final presented value of G, we took the mean of the G values. An important note should be made, however, which is that we only obtained one d measurement, which forced us to use one d measurement for the G calculations. Following this process, we report our final presented value of G to be $(5.41 \pm 0.01)E - 11\frac{Nm^2}{kg^2}$. Compared to the accepted value of $G = 6.6743E - 11\frac{Nm^2}{kg^2}$,[2] we found a 18.95% percent error.

We now graphically present our G measurements with the uncertainty reported from our spread-based error analysis as illustrated by Figure 4.

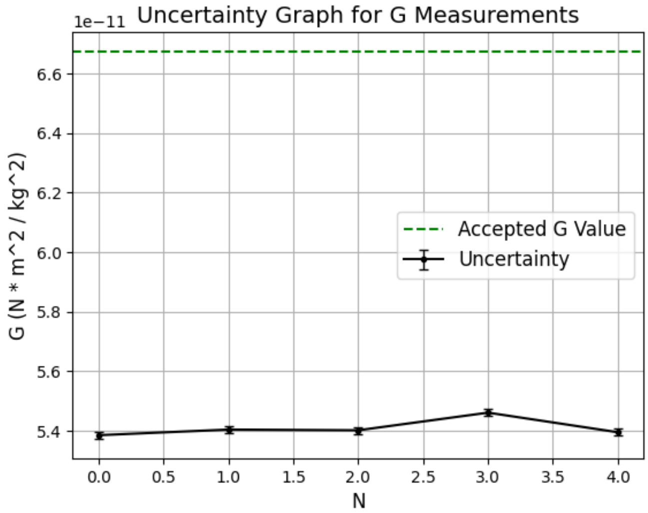

FIG. 4: G Values with the Uncertainty Value as the Error Bar.

Although our low number of trials $N = 5$ provides a limited evaluation of G, we can observe from Figure 4 that there is likely a systematic error within the lab apparatus. This is given by the fact that the spread in the G values is extremely low and the high precision of the G measurements that are consistently away from the accepted G value bolsters the suspicion that there may be something that causes this consistent offset, perhaps in the measured constants (e.g., lengths) or in the parameters from lmfit. Furthermore, given this high precision, we can also note that there is little random error, which, in the context of the lab experiment in which the apparatus is left to run on its own, is consistent with the type of lab experiment we are conducting.

V. CONCLUSIONS AND OUTLOOK

In this lab experiment, we sought the measurement of G using a Cavendish-laser apparatus, with the final presented value of $G = (5.41 \pm 0.01)E - 11\frac{Nm^2}{kg^2}$. Although the 18.95% percent error is discouraging, in the context of the possibility of a systematic error, the high precision of the G values in Figure 4 show that the data collection process itself was very consistent. To this end, we propose several methods of improving the experimental setup.

First, attaining the d measurements corresponding to each trial could aid in improving the G measurements, as there will likely be variations in d depending on how close the two masses M and m are together. This could help in reducing the consistent offset away from the true value of G, as if our d value used for all the other trials is inaccurate, it could cause the entire G calculations to be systematically shifted. Second, measuring the two masses M separately could also help in reducing the systematic uncertainty. Because we assumed the two masses M to be of the same mass for the simplicity of calculation, the difference in mass between the two, which would translate to different torque on each end of the torsion pendulum, could contribute to the consistent offset in the G values. Third, accounting for all the different interactions between the components in the apparatus, such as that between m and M on opposite ends that result in a countertorque, could provide a more accurate calculation of G. While we suspect that this may be minimal due to the fact that $F_g \propto \frac{1}{r^2}$, introducing these nuanced relationships may help improve our measurements. Lastly, while this is not directly related to addressing the systematic error, ensuring a disturbance-free lab environment, particularly for actions that cause vibrations in the apparatus (e.g., turning the M platform such that it shakes the pendulum, closing the door too harshly), would help improve the usability of the data, particularly for the lm-fit fitting, which would not only improve the number of trials N but also improve the quality of the fitted parameters used for the G calculations. By taking the above measures, we could improve both data quality and mitigate potential sources of systematic errors.

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[1] The Eöt-Wash Group, Gravitational constant, `https://www.npl.washington.edu/eotwash/gravitational-constant`, n.d., Accessed: 2025-04-14.

[2] National Institute of Standards and Technology (NIST), Newtonian constant of gravitation, `https://physics.nist.gov/cgi-bin/cuu/Value?bg`, 2022, Accessed: 2025-04-15.