

A Study of Identity Based Encryption Systems

Samuel Petit

3rd Year Integrated Computer Science student at Trinity College Dublin, the University of Dublin

Email: petits@tcd.ie

Abstract—The abstract goes here.

I. INTRODUCTION

We will be studying the concept of Identity Based Encryption (IBE), a type of asymmetric (or public key) encryption system. It is characterised with the fact that one of its keys identifies the recipient (for example an email address). We will explain how such a system is possible and compare it to a more widely known encryption scheme: RSA. Perhaps, we will then be able to understand why IBE is not as widely used as RSA is today.

A. Introduction to Public Key Cryptography

Public Key Cryptography, also called Asymmetric Cryptography is an encryption scheme. In other words, it is a method for encrypting messages. Unlike symmetric cryptography which uses the same key for both encrypting and decrypting messages, asymmetric cryptography uses two different keys such that one is used for encrypting and the other for decrypting information. In this context, a key is a string of characters that is used in some mathematical formula to turn a information (a message for example) such that it is impossible to understand in its encrypted form. Similarly, we would then also use a key to map the encrypted information back to its original, usable form. In asymmetric encryption, we typically call the encryption key the Public Key, and the decryption key the Private Key. The first asymmetric key cryptosystem was published in 1976 by Whitfield Diffie and Martin Hellman, previously, all useful modern encryption system used symmetric key encryption systems. While both systems still have their usages in today's world, asymmetric key encryption systems are now used on a daily basis throughout the world. With systems such as both HTTP over TLS and HTTP over SSL protocols, digitally signed files, bitcoin, encrypted messaging services and many other all using some form of asymmetric encryption.

1) *How it works:* Put simply, let's say we had a public key K_{public} and $K_{private}$. We would then obtain a ciphertext *cipher* with the following formula:

$$cipher = K_{public}(message)$$

Similarly, we obtain the original message from the ciphertext with the following formula:

$$message = K_{private}(cipher)$$

2) *Requirements for public key algorithms:* In order for a Public key encryption scheme to be safe, we have a few requirements. The first one being ease of setup: it should be computationally easy to generate a pair of Public and Private keys.

Encrypting a message should also be computationally easy, this means that a sender X, with a message to send M and knowing the recipient's public key should be able to compute the ciphertext fairly easily.

Similarly, decrypting a message should be computationally easy, thus meaning that a receiver Y, with a ciphertext that was encrypted using his own public key should be able to obtain the original message using an easy computation.

In terms of keys, it should be impossible to obtain the private key from a public key, since these are, as its name suggests, public this would be a massive security issue.

Finally, it should also be impossible to find a message from the encrypted text and the public key used to obtain the ciphertext.

3) *Different implementations of Asymmetric Encryption:* Many popular protocols and systems can be used as examples of working asymmetric encryption. To start with, Identity Based Encryption (IBE) uses a set of asymmetric keys. Other popular protocols or systems include the Diffie-Hellman key exchange protocol which is used to exchange cryptographic keys over a non secure channel. RSA is a very popular cryptosystem which includes algorithms and functions to compute a set of keys as well as handling encryption and decryption. There are also multiple mathematical methods that are used by different encryption schemes, these include elliptic curves, Bilinear maps or Weil and Tate Pairings to name some of the most commonly used ones.

B. Introduction to Identity Based Encryption

Identity based encryption (IBE), is a type of public key encryption where the public key is an arbitrary string which represents information about a user's identity, for example, an email address.

1) *Brief history about IBE:* The concept of using meaningful public keys dates back to 1984 where Adi Shamir introduced the concept, it was quite a radical idea due to the fact that public keys are typically mathematically related to the private key. The main motivation for this effort was to remove the need of a public key distribution infrastructure that is required by all public key encryption schemes at the time. He was able to find a system for identity based signatures but was not able to find a working scheme for IBE itself. It is

only later in 2001 that three schemes for IBE were found, the two most common ones being found by Boneh and Franklin who's method relies on elliptic curves, and Clifford Cocks who's scheme relies on the RSA setting.

2) *IBE typical implementation:* As stated previously, in a Identity Based Encryption system, the public key can be any arbitrary string, this fundamentally changes from other implementations where public keys can be meaningful instead of a very large mathematical number. However this raises many questions such as: if my public key is a very simple string, how do I find the corresponding private key to decrypt messages, and what is stopping anyone else from finding that same private key and decrypting my messages? Let's have a look at how IBE schemes work.

The idea involves a Private Key Generator (PKG), the PKG would be responsible for generating private keys for the users. It would also generate a pair of master keys (public key and private key), which are used for encryption and decryption. The master public key is used for message encryption and the master private key is used to compute all users private key.

The master public key, after its generation, would be published to all the users of the system, for a user to encrypt a message, he would use both the master public key and the ID of the recipient (for example it could be an email address) to obtain the ciphertext.

Then decryption would work by using the private key associated to the ID. These are all computed by the PKG so the user would then have to contact the PKG and retrieve his private key, only then is he able to decrypt the ciphertext. In order to obtain a private key from the PKG, a user would have to prove to the PKG that he is the owner of the key he is requesting, then the key would be sent through a secure channel.

Signing a message would be very similar, a user would obtain his private key from the PKG and encrypt the message using the key and send the signature along with the ciphertext. The recipient can then verify the signature by decrypting it using both the sender's ID and the master public key.

3) *Actions required by the system:* There are 4 algorithms involved in this system, these are the setup algorithm which generates the pair of master public and private keys. The extract action generates a private key for a specific public key string (for example, generate the public key for bob's email address bob@bob.com). Then encryption is the action of obtaining the ciphertext from a message, it uses both the recipient ID and the master public key. Finally, decryption happens by decrypting the ciphertext using the private key generated in the extract action.

4) *Mathematical theory:* As mentioned previously, there are different schemes using different mathematical concepts, let's focus here on Elliptic curve based schemes. This topic is quite maths heavy thus I will try to focus on the high level mathematical theory here without going into too much detail.

An elliptic curve is defined by an equation of the form

$$y^2 = x^3 + ax + b$$

A property of elliptic curves is that they are symmetrical about the x-axis. The reason these curves can be used for cryptography purposes is thanks to the fact that, say I were to compute xP , where x is an integer and P is a point on an elliptic curve. We would then obtain the point Q on the elliptic curve, this is done by finding the intersection to a line starting from P (note that there are an infinite amount of lines going through a single point) and obtaining it's symmetric point on the other side of the x axis, essentially adding P to itself, we would then do this operation x times in order to obtain Q .

Assuming I know the original point P , the parameters of the used curve, and the result point X , it is impossible to obtain x (the amount of times P was added to itself, or the amount of executed operations). Thus giving us a variant to the Diffie Hellman problem.

Finally, it is worth noting that we can have a IBE system which uses an Bilinear map $e : G_1 \times G_1 \rightarrow G_2$ so long as the Computational Diffie-Hellman problem in G_1 is hard, meaning we can't map back to the group G_1 from G_2 . One way to do so in the current elliptic curve system we are considering is to use the Weil pairing on an elliptic curve. Ideally the Bilinear map should be efficiently computable.

Let's take a look at the high level maths behind the 4 algorithms which enable encryption and decryption of messages using these concepts we just saw.

The setup would then pick an elliptic curve, a secret s and a point on the elliptic curve P . The system then publishes P and $P \times s$ as the master public key.

Encryption would then happen by hashing the ID attribute (for example bob's email address) to a point on the elliptic curve (let our hash function be $h = g^x$ from some x). Thus hashing bob's ID would set $x = ID$. We obtain a point of the curve by doing so since g is the generator for our group. Let our group be Z_q^* , of prime order q , then g would be the generator for Z_q^* , this would mean that for any integer value k , $g^k \in Z_q^*$.

Once we have hashed bob's ID, we would pick a random integer r , and compute $k = e(r * h, s * P)$

We can then send our message $E_k[M]$ using the key k and also send r .

In order to obtain the private key, we need to ask the PKG for it. The PKG will compute $s * ID$ and return the result, once again, ID is the point on the elliptic curve from our previous hash function. Knowing the private key for our ID we can then decrypt the message by finding k first: $k = e(s * ID, r * P)$ Which will enable us to decrypt the ciphertext. Note that this is possible thanks to the property for the elliptic curve that $e(aX, bY) = e(bX, aY)$ and because $s * ID$ is known.

It is worth noting that Cocks's scheme was based on the RSA setting thus we will go through how that setting works below.

5) *Reasons for IBE:* Now that we know more about how an IBE scheme works, let's take a look at why it exists, what such a system enables. The first notable information is that such a system does not need a public key distribution system, the master public key is known to all users and thus sending an encrypted message to Bob's email address is as simple

as using the exact same email address along with the master public key for encryption. Meaning there is no need to manage large numbers of public keys such that a typical public key infrastructure would require.

Secondly, the central system can decrypt messages locally and because of that, it can enable for systems with minimal setup for its users, requiring no installation and instead they would simply ask for the message to the central system. Note that this is not necessarily the best solution however it is a possibility with IBE.

Certain encrypted system require a key escrow for multiple reasons, one might be due to government regulations, in an IBE system, the master private key can generate all keys thus enabling a key escrow.

IBE also enables delegation of keys where, for example, a company employee can decrypt certain emails, perhaps another employee requires to read those exact same emails, the same private key can be used by both employees.

Finally, this scheme enables further research into this area of cryptography.

6) *Security of IBE*: This specific IBE scheme relies on elliptic curves with the use of bilinear maps thus making it follow some variant of the Diffie-Hellman problem. That is, as of today, it is easy to compute values from operands to obtain a result, but it is impossible to obtain the operands from the result. For example, the use of pairings such as Weil and Tate Pairings in order to obtain Bilinear maps make it so that computing aX is easy but finding a given X and Xa is computationally not feasible.

Secondly, the default IBE scheme does not account for expiration of revocation of keys, meaning that theoretically, someone obtaining a private key for a specific ID will enable that person to decrypt all past and future messages sent to that specific recipient. This is very dangerous and can be avoided quite easily, one way to do so is by appending an expiration date to the public key, that is our ID (for example Bob's email address) and a date: *bob@bob.com — expiration date*. Bob will then be able to use a private key until it expires and then will have to request a new one to the PKG. The time each key is valid can then be variable based on how one decides to implement the system, theoretically a key could be valid for years or minutes.

7) *Improvements thanks to IBE*: While identity based encryption has its flaws, what it did is enable further research. From IBE was derived what is called attribute based encryption (or ABE), a system designed for use with biometric values.

Secondly, both IBE and ABE were the basis for functionnal encryption, which takes the theories from both methods and generalises their ideas.

8) *Comparing to a commonly used public key encryption scheme*: Let's now see how a very popular system such as the RSA encryption scheme works, that way we will be able to compare both systems and try to understand why IBE is not currently commonly used.

C. An Overview of RSA

1) Key generation:

```

 $p \leftarrow \text{prime}()$  {prime() returns a prime number}
 $q \leftarrow \text{prime}(N)$ 
 $n \leftarrow pq$ 
 $\phi(n) \leftarrow (p-1)(q-1)$ 
 $e \leftarrow \text{coprime}(\phi(n))$  {coprime returns a value coprime to  $\phi$  where  $1 < e < \phi$ }
 $d \equiv e^{-1} \text{mod } \phi(n)$ 

```

To explain a bit more about this algorithm, we start by generating randomly two prime numbers p and q . We obtain n from their product, n is used as part of the encryption and decryption function as we will see soon. Then we use Euler's totient function such as to count the positive integers up to $(p-1)(q-1)$, we could alternatively use Carmichael's totient function here with a slight change in algorithm, the same keys would be generated regardless. We then pick a value e such that it is positive, smaller than our value obtained from Euler's totient $\phi(n)$ function and coprime to it. We have then obtained the public key $\{e, n\}$. To compute our private key with simply compute the modular multiplicative inverse of e modulo $\phi(n)$ ($d \equiv e^{-1} \text{mod } (\phi(n))$) to obtain our private key: $\{d, n\}$.

2) *Encryption*: Given a public key $\{e, n\}$, we can then very simply encrypt a message M . This is done simply by computing:

$$C = M^e \text{ mod } n$$

This part is fairly simple as it only contains a single operation, though keep in mind that in practice e could be a very large number so M^e could be demanding on the device.

3) *Decryption*: Finally, for decryption, given that we have a private key $\{d, n\}$, and a ciphertext which was encrypted using the private key's corresponding public key, we can obtain the original message M using a formula very similar to that of the encryption:

$$M = C^d \text{ mod } n$$

4) *Computing Signatures*: RSA supports the use for signature. Signatures are used in order to make sure a message was sent by a specific person.

The first step for verifying signatures is for the sender to generate one. Using the senders private key and the message to send we can obtain the signature S :

$$S = M^d \text{ mod } n$$

We would then send the signature S with the ciphertext to the recipient. Note that we must respect $M < n$ here too.

5) *Verifying signatures*: The person receiving the message can then verify the identity of the sender. If we use the senders public key in the following formula to obtain the original message, then we can be certain of the senders identity.

$$M = S^e \text{ mod } n$$

Once again, $M < n$ is required.

6) *Security*: RSA's security relies on the fact that factoring numbers is complicated and expensive. For instance, let's assume that we have a public key $\{e, n\}$, we could in theory find the values for p and q from the keygeneration algorithm by factoring n (recall that $n = pq$). Then all we would have to do is use Euler's theorem to obtain $\phi(n)$ and from there we could find e and finally obtain the private key d by solving the equation $ed = 1 \pmod{\phi(n)}$.

In practise though, we pick very large values for p and n , factoring very large prime numbers is very hard and trying to obtain p and q using a brute force method would be very expensive.

D. Mathematical Theorems enabling RSA

Goal: Explain that at the core of public key encryption, underlies many important mathematical concepts.

1) *Discrete logarithms*: The difficulty of breaking a Public Key encryption system is based on the Discrete Logarithm problem (DLP). Given a multiplicative group G_n^* of order n , Let's then define the element α such that it is a subgroup of G_n^* (i.e. $\alpha \subseteq G_n^*$). We then have α , a subgroup of G_n^* , note that α is also cyclic of order n (same as G_n^*).

The Discrete Logarithm Problem is then defined as, given $\beta \in \alpha$, find x such that $0 \leq x \leq n - 1$.

$$\alpha^x \equiv \beta \pmod{p} \quad (1)$$

It is important to note that not the discrete logarithm problem is not always hard to solve. In the context of encryption we choose a group Z_p^* with p a prime number which makes this problem very hard and expensive to solve. It is also worth noting that not all primes are safe to use and some primes make this problem much easier to solve, using methods such as the Pohlig–Hellman algorithm. In order to be safe, one can follow the rule that a prime must be of the form $2p + 1$, where p is a large prime number.

2) *Prime numbers*: As we have started to introduce prime numbers earlier, we can't help but notice that prime numbers are at the core of many concepts being used in the types of asymmetric encryptions we are covering here. There is a reason for that, it is the fact that primes are very easy to multiply together, however factorising a number into two prime numbers is extremely computer intensive. Much more so than it would be if the numbers weren't primes.

So then, how do we check that a number is prime ? Do we try all possible factors until we know we have a prime number? This approach would work, however, it would be terribly expensive when we are generating very large primes. Thankfully, there are mathematical theorems we can use to make this check easier.

Two famous mathematical methods to check if a number is prime are Miller–Rabin and Fermat's primality test. Fermat's little theorem is the basis for Fermat's primality test. It states that given, a , an integer and p a prime number where a is not divisible by p , we have:

$$a^p \equiv a \pmod{p}$$

Thanks to this equation, we can use it to find Fermat's Primality test:

$$a^{p-1} \equiv 1 \pmod{p}$$

We use it by picking two integers a and p such that a is not divisible by p . If the equality holds, then p is a prime number.

The way we generate large prime numbers is by generating integers and testing if the generated integer is prime. We can compute the probability of picking a prime number given a size. For instance if we wanted to find a prime number a with a bit size of 2^{1024} . We would then have a probability of picking a prime number of

$$P(a \text{ is prime}) = \frac{2}{\ln(2^{1024})} = \frac{2}{1024 \ln(2)} = 0.00281776375$$

This probability is based on the more general Prime number theorem which statistically describes the distribution of prime numbers. Its distribution is described as such:

$$\pi(N) \sim \frac{1}{\log(N)}$$

Where $\pi(N)$ is the prime counting function: it computes the amount of primes that are less than or equal to N .

3) *Square and multiply*: Something you may have noticed is that some of these operations require computing many exponents, given the fact that we generally pick very large numbers for encryptions, finding a way to compute massive exponents in a way that is more manageable for computers would help efficient computations quite drastically. This is exactly what the square and multiply algorithm enables, it is essentially an algorithm for computing exponents iteratively. This algorithm relies on the fact that exponent value can be broken down into the multiplication of multiple terms:

$$x^n = \begin{cases} x(x^2)^{\frac{n-1}{2}}, & \text{if } n \text{ is odd} \\ x(x^2)^{\frac{n}{2}}, & \text{if } n \text{ is even} \\ 1, & \text{if } n = 0 \end{cases}$$

This algorithm can be implemented such that is it based on binary values, by using the bit values to determine which powers are computed. Here is a quick example using x^5 . 5 in binary is 101. Iterating bit by bit:

Initialisation: result = 1

Step 1: result = result²; ($= x^0$); bit 1 is 1; result = result x ; ($= x^1$)

Step 2: result = result²; ($= x^2$); bit 2 is 0, so there is no computation this step.

Step 3: result = result²; ($= x^4$); bit 3 is 1, result = result x ; ($= x^5$)

The pattern from this example is fairly straightforward, for every bit iteration, square our current result, then multiply by x if the bit at the current position is 1.

Note that side attacks are possible on this particular algorithm, due to the fact that based on the iteration being a 0 or a 1 a single or two operations are executed. For instance if a hacker could access the power drawn by the device overtime then we could notice that more power would be drawn to

compute an iteration of this algorithm with a bit value of 1 in comparison to a 0. In some cases we can monitor these from the frequencies emitted by a device and thus we may be able to obtain a private key using such methods.

There are ways to counter this, known as padding (explained later and add a bit here).

4) *Chinese remainder theorem*: The Chinese remainder theorem plays a big role in optimising the amount of work required for decrypting a message as well as signature verification.

Remember from the RSA setup stage that we compute $n = pq$, where n is part of the public key. Also remember that decryption requires a ciphertext C and a private key $\{d, n\}$ to obtain the plain message M . $M = C^d \mod n$.

We can use the Chinese remainder theorem to split the computation of modular exponentiation, in the case of decryption using p and q .

$$M_P = M \mod P$$

$$M_Q = M \mod Q$$

We can even push the optimisation further:

$$\begin{aligned} M_P = M \mod P &= (C^d \mod n) \mod p \\ &= C^d \mod p = C^{D \mod (p-1)} \mod p \end{aligned}$$

We can do these simplifications thanks to the fact that $n = pq$ thus

$$(C^d \mod n) \mod p = C^d \mod p$$

And thanks to Fermat's little theorem giving us the final optimisation. Note that $D \mod (p-1)$ can be computed on setup instead of during decryption thus making this approach much faster for message decryption.

Once we have computed both M_q and M_p , we then need to find such that M satisfies both:

$$M \equiv (C^d \mod n) \mod q$$

$$M \equiv (C^d \mod n) \mod p$$

Using the fact that p and q are relatively prime and because of the Chinese remainder theorem, we can directly assume that:

$$M \equiv (C^d \mod n) \mod pq = M \equiv C^d \mod pq$$

Thus obtaining the decrypted message M which is what we were looking for.

There are many other mathematical theorems and optimisation used in the implementation of RSA, however this should serve as an introduction to some of the main ideas.

E. Comparisons between IBE and RSA

Let's now go through the many pros and cons of the IBE schemes we have covered here and, where appropriate, compare to the functioning of the RSA encryption scheme.

1) *Advantages of IBE*: The first obvious advantage to IBE is the fact that we use meaningful public keys. This removes the need for public key distribution system that is typically in place in most public key encryption systems. For example, RSA typically distributes public keys using Certificate Authorities (CA), these authorities essentially sign public keys using their own private key and returns the signature. A user can use that signature to verify a public key is in fact the one we think it is and not some attacker's public key for example. Put simply, they essentially verify the owner of the certificate, ensure they are who they say they are and that the provided information is valid, this means the users put a trust in these CAs to provide accurate public keys. In short: a key distribution infrastructure is complex, expensive and being able to use a public key that the user knows without looking it up is a great advantage.

Users can send messages to a recipient who has not contacted the PKG yet, meaning that we can encrypt messages for users who yet do not exist. This is not the case in RSA as messages are encrypted using the recipient's public key, in order for a user to have a public key, that user would have to generate (so go through the setup algorithm) and that key would have to be inserted inside CAs databases of public keys.

If an IBE scheme accounts for private key revocation, then it is possible to enable for sending messages that are only decryptable at some time in the future, this is also not the case with RSA as keys are not generated by a central system.

We've also mentioned that message decryption can happen server-side, meaning this could enable for applications with minimal setup for users.

2) *Drawbacks of IBE*: The first notable drawback is that, by definition of the IBE scheme we've considered, there exists a set of master public and private keys. This means that the PKG is in a position of generating all private keys and decrypting all messages that go through the system. This is known as a key escrow, it can, in some cases be advantageous as a law may require for such a system or in an enterprise environment a key escrow may be desirable. Another way this could be seen as an advantage is in some cases this can allow for recovering encrypted data in the event of the loss of a private key by a user. Finally, another way this is an advantage is for delegation of keys, where in some cases multiple persons may require to read the same messages.

However, for a public application, this requires great trust in the system and thus by definition is not widely adopted in systems such as private channels or encrypted messaging systems. The much more widely used RSA is much preferred in such applications, even though it requires a key distribution system. It is also a concern as, theoretically, if the master private key was stolen, then an attacker would be in a position of decrypting all messages, this would remove all security that was previously in the system. This problem can be solved by adding key revocation, however it is worth noting that the scheme we have studied here does not enable for key revocation or expiration. In RSA, none of this is an issue since there exists no master key that enables somehow the

decryption of all messages, in fact, in RSA if a single key is cracked, then only a single user will be impacted. This is also the case for IBE however, cracking the main private key would mean the decryption of all users message becomes possible. In short, IBE, by definition requires a lot of trust in the system while it is not the case for RSA.

Another drawback is that, in an IBE scheme, users need to be given their private key. This can be solved by using secure communication channels, or even giving a physical copy of the private key on a usb key for example, however it is a drawback nonetheless since a private key is very sensible information (more so than a single message) and systems like RSA do not require to do so, a private key is generated on the device it is used.

F. Attacking a Public Key Encryption system

Many possible attacks are possible on the encryption schemes we've covered. For example RSA, being deterministic is not very safe until we introduce concepts such as padding data.

Implementations on devices of encryption or decryption methods can lead to security issues, for example the square and multiply algorithm, when not randomised is prone to side attacks due to the fact that a bit value of 1 or 0 will change drastically the time and electricity power required.

Given that such issues are dealt with, other possible attacks on the systems we've seen here include: Brute force, that is, trying out all the combinations in order to find the private key.

Theoretically, it could be possible to derive the private key from the public key, it just hasn't been proven mathematically that it is or it is not possible to do so. We've mentioned that our schemes use the Diffie-Hellman assumption (or some variant of that assumption) meaning that they will be safe as long as the assumption stands.

Another attack worth noting is the probable message attack, given that the public keys are known to attackers, one could try to encrypt all messages until there is a match.

II. CONCLUSION

We have seen that an Identity Encryption system introduces new concepts to the world of cryptography. It also pushed research further by inspiring researchers to discover what is possible from such a system where the public key is meaningful. One of the reasons that public key encryption systems were originally not widely used is due to the requirement of a public key distribution system, IBE solves that problem however that solution comes with its drawbacks. Mainly the fact that a key escrow exists by definition in the system without having the choice is the main drawback for IBE, it requires too much trust in the system for public applications. On the other hand, some argue that IBE is a better compromise between security and complexity for users, the main question being whether users see it that way or not. Perhaps, for the moment, IBE's advantages are more suited for specific environments such as use within an enterprise for example. A lot of research is currently being done on this topic, so perhaps one day a variant of such an IBE scheme could become the industry standard for encryption.

APPENDIX

[1] Ben Lynn June "ON THE IMPLEMENTATION OF PAIRING-BASED CRYPTOSYSTEMS", page 1 to 45.

[2] Dan Boneh, Matthew Franklin "Identity-Based Encryption from the Weil Pairing"

[3] Clifford Cocks, "An Identity Based Encryption Scheme based on Quadratic Residues"

[4] Stanford IBE Secure Email Implementation <https://crypto.stanford.edu/ibe/download.html>

[5] Donghoon Chang, Amit Kumar Chauhan, Sandeep Kumar, Somitra Kumar Sanadhya "Revocable Identity-based Encryption from Codes with Rank Metric"

[6] Keita Emura, Shuichi Katsumata, Yohei Watanabe "Identity-Based Encryption with Security against the KGC: A Formal Model and Its Instantiations"

[7] Nico Döttling, Sanjam Garg "Identity-Based Encryption from the Diffie-Hellman Assumption"