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ST3009 Weekly assignment 10

Question 1 a

X is continuous thus $P(X = 0.5) = 0$.

b) $P(0.25 \leq x \leq 0.5) = \cancel{P(0.5) - P(0.25)}$

$= P(X < 0.5) - P(X < 0.25)$
$= 0.5 - 0.25 = \underline{\underline{0.25}}$

c) $P(-1 \leq x \leq 0.5) = P(x \leq 0.5)$

$= \underline{\underline{0.5}}$

Question 2.

a) if $x < 0$ then:

$F(y) = \int_{-\infty}^{y} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx$

$= \int_{-\infty}^{0} 0\,dx = 0.$

if $0 \leq x \leq 2$ then:

$$F(y) = \int_{-\infty}^{y} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{y} f(x)\,dx = 0 + \int_{0}^{y} f(x)\,dx$$

$$= \int_{0}^{y} x\,dx = \frac{x^2}{2}\Big|_{0}^{y}$$

$$= \frac{y^2}{2} - \frac{0^2}{2} = \frac{y^2}{2}.$$

Since $f_X(x) = 0$ when $x > 2$,

we have:

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{y^2}{2} & 0 \leq y \leq 2 \\ 1 & y > 2 \end{cases}$$

Q2 b)

$$P(0.5 \leq \cancel{X} \leq 10) = P(X \leq 10) - P(X \leq 0.5)$$

$$= 1 - \frac{0.5^2}{2} = 1 - \frac{0.25}{2} = \underline{\underline{0.875}}$$

Question 3.

We have:

$$f_{XY}(x, y) = P_{\cancel{x}}(X \leq x) P(Y \leq y)$$

$$f_X(x) = \frac{e^{-|x|}}{2}$$

$$f_Y(y) = e^{-2|y|}$$

We also know that $F_X(x) = P(X \leq x)$ and $F_Y(x) = P(Y \leq y)$.

Let's find F_X and F_Y.

- $F_X(x_a) = \int_{-\infty}^{x_a} \frac{e^{-|x_b|}}{2} dx_b$

$= \int_0^{x_a} \frac{e^{-x_b}}{2} dx_b = \frac{1}{2} e^{-x_b} \Big|_0^{x_a}$

$= \frac{1}{2} e^{-x_a} - \frac{1}{2} e^{-0} = \frac{1}{2} e^{-x_a} - 0.5$

- $F_Y(y) = \int_{-\infty}^{y} e^{-2|x|} dx = \int_0^y e^{-2x} dx$

$= e^{-2x} \Big|_0^y = e^{-2y} - e^{-2\times 0}$

$= e^{-2y} - e^0 = e^{-2y} - 1.$

Thus:

$$f_{XY}(x, y) = \left(\frac{1}{2} e^{-x} - 0.5\right)\left(e^{-2y} - 1\right)$$

with $x, y > 0$.

If $x, y \in \mathbb{R}$ then:

$$f_{XY}(x, y) = \left(\frac{1}{2} e^{-|x|} - 0.5\right)\left(e^{-2|y|} - 1\right)$$

3b) with $f_{XY}(x,y) = \frac{e^{-|xy|}}{2}$

$f_X(x) = \frac{e^{-|x|}}{2}$

$f_Y(y) = e^{-2|y|}$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$= \frac{\frac{e^{-|xy|}}{2}}{\frac{e^{-|x|}}{2}} = \frac{e^{-|x|}\,e^{-|y|}}{e^{-|x|}}$$

$$= \underline{\underline{e^{-|y|}}}.$$

3c) $$f_{Y|X}(y|x) = \frac{f_{X|Y}(x,y)\,f_Y(y)}{f_X(x)}$$

$$\Leftrightarrow \frac{f_{Y|X}(y|x)\,f_X(x)}{f_Y(y)} = f_{X|Y}(x,y)$$

$$\Leftrightarrow f_{X|Y}(x,y) = \frac{e^{-|y|} \times \frac{e^{-|x|}}{2}}{e^{-2|y|}}$$

$$\Leftrightarrow f_{X|Y}(x,y) = \frac{e^{-|y|} \times e^{-|x|}}{2e^{2}\,e^{-|y|}} = \frac{e^{-|x|}}{2e^{2}}$$

Thus: $f_{X|Y}(x|y) = (2e^2)^{-1} e^{-|x|}$

Question 4.

a) The joint pdf of independent variables is equal to the product of the marginal pdfs. Thus:

$$f_{Z|X} = \prod_{i=1}^{m} e^{-2|\theta y^{(i)} - x^{(i)}|}$$

b) Gradient ascent is essentially like gradient descent with the difference that ascent ~~takes~~ ~~for~~ is the process of maximizing while descent is the process of minimizing.

Thus by taking the loss function for gradient descent and multiplying it by -1 we can then look for a maximum value:

$$\frac{1}{m} \sum_{i=1}^{m} -1 \times \log(1 + e^{-y\theta^T x^{(i)}}).$$

We can then apply the exact same steps as the gradient descent using the above loss function to obtain a value for θ which maximizes the PDF from a.