Q1 Form the augmented matrix

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 1 & -1 & -3 & 0 \end{pmatrix}$$

$$\begin{array}{rrrrr} R2: & 2 & 1 & 0 & 3 \\ -2R1: & -2 & -2 & -2 & -4 \\ \hline & 0 & -1 & -2 & -1 \end{array}$$

$$\begin{array}{rrrrr} R3: & 1 & -1 & -3 & 0 \\ -R1: & -1 & -1 & -1 & -2 \\ \hline & 0 & -2 & -4 & -2 \end{array}$$

$$\xrightarrow[R3-R1]{R2-2R1} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & -2 & -4 & -2 \end{pmatrix}$$

$$\xrightarrow{(-1)\times R2} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & -2 & -4 & -2 \end{pmatrix}$$

$$\begin{array}{rrrrr} R1: & 1 & 1 & 1 & 2 \\ -R2: & 0 & -1 & -2 & -1 \\ \hline & 1 & 0 & -1 & 1 \end{array}$$

$$\xrightarrow[R3+2R2]{R1-R2} \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{rrrrr} R3: & 0 & -2 & -4 & -2 \\ 2R2: & 0 & 2 & 4 & 2 \\ \hline & 0 & 0 & 0 & 0 \end{array}$$

This is in reduced row echelon form and corresponds to the linear system

$$x - z = 1$$
$$y + 2z = 1$$

Let $z = t$, $t \in \mathbb{R}$ then $x = 1 + t$ and $y = 1 - 2t$.

Hence the system has an infinite number of solutions of the form

$$x = 1 + t, \quad y = 1 - 2t, \quad z = t, \quad t \in \mathbb{R}.$$

Q2 Form the augmented matrix

$$\left(\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{rrrrrrr} R1: & 1 & 1 & 1 & 1 & 0 & 0 \\ -R2: & 0 & -1 & -1 & 0 & -1 & 0 \\ \hline & 1 & 0 & 0 & 1 & -1 & 0 \end{array}$$

$$\xrightarrow{R1-R2} \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{rrrrrrr} R2: & 0 & 1 & 1 & 0 & 1 & 0 \\ -R3: & 0 & 0 & -1 & 0 & 0 & -1 \\ \hline & 0 & 1 & 0 & 0 & 1 & -1 \end{array}$$

$$\xrightarrow{R2-R3} \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right)$$

Hence the required inverse matrix is $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

$$\det \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} + 0 + 0 = 1(1-0) = 1$$

matrix of cofactors $= \begin{pmatrix} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \end{pmatrix}$

$= \begin{pmatrix} 1-0 & -(0-0) & (0-0) \\ -(1-0) & (1-0) & -(0-0) \\ (1-1) & -(1-0) & (1-0) \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$

Hence the required inverse matrix is $\frac{1}{1} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

Q3 $\begin{vmatrix} 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 5 & 0 & 0 & 7 \end{vmatrix} = 0\,[\quad] - 1 \begin{vmatrix} 3 & 0 & 2 \\ 0 & 2 & 1 \\ 5 & 0 & 7 \end{vmatrix} - 1 \begin{vmatrix} 3 & 0 & 2 \\ 0 & 1 & 1 \\ 5 & 0 & 7 \end{vmatrix} - 0\,[\quad]$

$= -1\left(3\,[2(7) - 1(0)] - 0\,[\quad] + 2\,[0 - 2(5)]\right)$

$\quad -1\left(3\,[1(7) - 1(0)] - 0\,[\quad] + 2\,[0(0) - 1(5)]\right)$

$= -3\ (14) - 2\,(-10) - 3\,(7) - 2\,(-5)$

$= -42 + 20 - 21 + 10$

$= -33$

Q4

For $f(x) = e^{3x}$ we have

$$\begin{aligned} f(x) &= e^{3x} & \qquad f(-1) &= e^{-3} \\ f'(x) &= 3e^{3x} & \qquad f(-1) &= 3e^{-3} \\ f''(x) &= 9e^{3x} & \qquad f(-1) &= 9e^{-3} \\ f^{(3)}(x) &= 27e^{3x} & \qquad f(-1) &= 27e^{-3} \end{aligned}$$

The cubic Taylor Polynomial about 0 for $f(x) = e^{3x}$ is

$$\begin{aligned} p(x) &= f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \frac{f^{(3)}(-1)}{3!}(x+1)^3 \\ &= e^{-3} + 3e^{-3}(x+1) + \frac{9e^{-3}}{2!}(x+1)^2 + \frac{27e^{-3}}{3!}(x+1)^3 \\ &= \frac{1}{e^3} + \frac{3}{e^3}(x+1) + \frac{9}{2e^3}(x+1)^2 + \frac{9}{2e^3}(x+1)^3 \end{aligned}$$

Q5

$$1 - 5x + 25x^2 - 125x^3$$

So we get

$$g(x) = x^2 - 5x^3 + 25x^4 - 125x^5$$