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Math assignment #2

i - $A \cup B =$

B

	0	1
A	1	1

$A \bar{\cup} (A \bar{\cup} B)$

$A \bar{\cup} B = \overline{A \cup B} =$

B

	1	0
A	0	0

$\boxed{A \cup B \neq A \bar{\cup} (A \bar{\cup} B)}$

$A \bar{\cup} (A \bar{\cup} B)$
$= \overline{A \cup (A \bar{\cup} B)} =$

B

	0	0
A	0	0

ii - $\overline{A \cap B} =$

B

	1	1
A	1	0

$\bar{A} \bar{\cup} (B \bar{\cup} A)$
$= \overline{\bar{A} \cup (A \bar{\cup} B)}$
$=$

	0	0
A	1	1

$\overline{A \cap B} \neq \bar{A} \bar{\cup} (B \bar{\cup} A)$

iii - $A \bar{\cup} (B \bar{\cup} C)$:

- $B \bar{\cup} C =$

B

	1	0	0	0
A	1	0	0	0

C

$A \bar{\cup} (B \bar{\cup} B) =$

B

	0	1	1	1
A	0	0	0	0

C

- $A \bar{\cup} B =$

B

	1	1	0	0
A	0	0	0	0

C

$A \bar{\cup} (B \bar{\cup} C)$
$\neq (A \bar{\cup} B) \bar{\cup} C$

$(A \bar{\cup} B) \bar{\cup} C =$

B

	0	0	0	1
A	1	0	0	1

C

b- A: Archaeological
B: Botany
C: Choral

		B			
	0	6	2	3	
A	2	2	5	10	
		C			

i - 10 students belong to the Archaeological society and botany society but not the choral.

ii - 19 students belong to the Archaeological Society

2 - i -

p	q	$(p \to q) \to p$		
F	F	T	F	F
F	T	T	F	F
T	F	F	T	T
T	T	T	T	T

NOT a tautology.

ii

p	q	r	$(p \to q \wedge r) \to (p \to q) \wedge (p \to r)$					
F	F	F	T	F	T	T	T	T
F	F	T	T	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	F	T	F	F	F
T	F	T	F	F	T	F	F	T
T	T	F	F	F	T	T	F	F
T	T	T	F	T	T	T	T	T

is a tautology.

b- $P \vee (C \to \neg S)$

$P \to \neg C$

C

$S \to P$

check:

$P \vee (C \to \neg S) \wedge (P \to \neg C) \wedge C \to (S \to P)$

1	premise	$P \vee (C \to \neg S)$	
2	premise	$P \to \neg C$	
3	premise	C	
4	negated conclusion	$\neg (S \to P)$	
5		S	$\alpha(4)$
6		$\neg P$	$\alpha(4)$
7		C	$\alpha(3)$
8		$(C \to \neg S)$	$\beta(6, 1)$
9		$\neg C$	$\beta(8, 5)$
		$\perp$	

hence, the argument is valid

3- a

1	$TB \to BL \to MA$		premises
2	$MA \wedge FD \to \neg GH$		
3	$\neg GJ \to FD \wedge GH$		
4	$\neg (GH \to FD \vee GJ)$		negated conclusion
5	GH	$\alpha(4)$	
6	$\neg (FD \vee GJ)$	$\alpha(4)$	
7	$\neg FD$	$\alpha(6)$	
8	$\neg GJ$	$\alpha(6)$	
9	$(FD \wedge GH)$	$\beta(3, 8, 7)$	
10	FD	$\alpha(9)$	
	$\perp$		

Hence the argument is valid

b

1	$TB \to BL \to MA$		premises
2	$MA \land FD \to \neg GH$		
3	$\neg GJ \to FD \land GH$		
4	$\neg (TB \land \neg GH \to \neg MA \to GJ)$		negated conclusion
5	$TB \land \neg GH$	$\alpha(4)$	
6	$\neg (\neg MA \to GJ)$	$\alpha(4)$	
7	TB	$\alpha(5)$	
8	$\neg GH$	$\alpha(5)$	
9	$\neg MA$	$\alpha(6)$	
10	$\neg GJ$	$\alpha(6)$	
11	$FD \land GH$	$\beta(3, 10)$	
12	FD	~~$\beta(10, 3)$~~ $\alpha(11)$	
13	GH	~~$\beta(10, 8)$~~ $\alpha(11)$	
14	$\perp$		

hense the argument is valid.