Samuel - 17333946 ST3009 Weekly assignment 1
Pehit

Q1 a) The first letter can be chosen from any of 10, the second from any of 9 ... if we continue this logic we note that the 10 letters can be ordered in any of $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 10! = 3628800$.

b) The number of permutations with ef together are: $9!$
with fe together: $9!$

So we have a total of $9! + 9! = 725760$ permutations

c) We have a total of 6 letters, 3 A's arrangable in $3!$ different orders, 2 N's in $2!$ and one B in $1!$ (trivial)
So we have:

$$\frac{6!}{3!2!1!} = 60$$

d) We have a total of 5 letters and need to pick 3 thus we get

$$\binom{5}{3} = \frac{5!}{2!3!} = \frac{120}{12} = 10.$$

Q2 a) We can obtain one of 6 possibilities at every throw. Since we have 4 throws, we then have $6 \times 6 \times 6 \times 6 = 6^4 = 1296$ possibilities.

b) We want exactly 2 3's.
which means we have:
3 3 _ _ , 3 _ 3 _ , 3 _ _ 3,
_ 3 3 _ , _ 3 _ 3 , _ _ 3 3 .

We notice that we have 6 cases and require the empty spots to not be a 3.
We then have:

$$6 \times 5^2 = 6 \times 25 = \underline{150}$$

c) We know that there are 150 strings sequences obtain exactly two 3's.

Reasoning in the same way as the previous question, we find that there are
$\binom{4}{1} \times 5^1 = 4 \times 5 = 20$ strings
with exactly three 3's.
finally, there is only 1 string with 4 3's thus:

150+20+1 = 171 of the strings obtain at least two 3's.

Question 3 a)
There are 8 cards where any 2 cards could be the same, thus, ~~one~~ ~~def~~ since we have 4 suits then we get:

$$\frac{8!}{2!2!2!2!} = 2520.$$

Q3 b) For any card dealt, you can match it with a card from the 3 other suits, since our deck is composed of 2 decks like we get
$\binom{4}{2} = 6$ distinct pair of cards.
(or more simply 3 times 2 since we use a double deck).

c) Out of the 8 cards only 4 are considered good. Since we have 2 ases of hearts and 2 of diamond and the order does not matter then we have 3 "good" sets of good cards:
- 2 hearts
- 2 diamonds
- 1 heart & 1 diamond.

So 3 is the answer.