## JF Maths CS1003

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| Time\Day | **Mon.** | **Tues.** | **Wed.** |
|---|---|---|---|
| | **Lectures** | | |
| **9am** | CS1003(LB08) | | |
| **10pm** | CS1003 (Joly ) | CS1003 (LB04) | |
| **3pm** | | | |
| **Tutorials** | | | |
| **12noon** | | LB 120 | |
| **1pm** | | | LB 04 |
| **5pm** | | | LB 08 |

## This Term

- Set Theory
- Logic
- Number Theory

## Set Operations

$A \cup B$ is $A$ union $B$
$A \cap B$ is $A$ intersection $B$
$\overline{A}$ is the complement of $A$ i.e. the elements that are not in $A$ relative to a Universal Set.

Define a new set operator:

$$X \hat{} Y = X \cup \overline{Y}$$

Does $A\hat{}(B \cap C) = (A\hat{}B)\hat{}C$ ?

# Set Theory

A language college consists of students that study French, German or Spanish. In the college, 280 students study French, 254 students study German and 280 students study Spanish.
97 students study French as well as German,
152 students study French as well as Spanish and
138 students study German as well as Spanish.
73 students study all the three languages.

How many students are there in the language college?

# Number Sets

## Number Sets:

$\mathbb{N} = \{0, 1, 2 \dots\}$ – Natural Numbers
$\mathbb{Z} = \{\cdots -2, -1, 0, 1, 2 \dots\}$ – Integers
$\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z} \textit{ and } b \neq 0\}$ – Rationals (Fractions)
$\mathbb{R} = \textit{Real Numbers}$

# Set Size

Use $|S|$ for the number of elements in the set $S$
i.e. $|S|$ is the size of $S$.
Some sets have finite size, some sets are infinite.
All the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ are infinite.
Since $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$ , does it make sense to conclude that these sets have different sizes.
For example:
Is $|\mathbb{N}| < |\mathbb{Q}|$ since between any two natural numbers there is an infinite number of Rational numbers.?
Is $|\mathbb{N}| < |\mathbb{R}|$ since between any two natural numbers there is an infinite number of Reals?
Also, a Real number in decimal notation may have an infinite number of decimal digits
e.g. $\pi = 3.14159265358979323846264338327950288419\ldots$
**Note**: From Calculus, $\pi = 4(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots)$

# Logic

In Logic, one can check if an argument is valid. Consider the following argument about the existence of Superman.

If Superman was able and willing to prevent evil, he would do so.
If Superman was unable to prevent evil, he would be powerless.
Superman does not prevent evil.
if Superman was unwilling to prevent evil, he would be malevolent.
If Superman exists then he is neither powerless nor malevolent.
$\therefore$(therefore)
Superman does not exist

# Number Theory

As in Computer Science, the symbol, $*$, is used for multiplication. Sometimes in Maths, the symbol $\times$ is used for multiplication and so $a * b = a \times b$. In Computer Science, $\times$ is not used for multiplication as $\times$ is similar to x.
In Maths, $ab$ can stand for $a \times b$ (e.g. $(a+b)^2 = a^2 + 2ab + b^2$) but this is not done in Computer Science.
Let $a, b \in \mathbb{R}$. Finding a solution, for $x$, in an equation such as $a * x = b$ is straigthforward.
It is not straightforward when using Modular Arithmetic.
Notation:
$a *_n b = (a * b)\ mod\ n$
e.g. $4 *_7 5 = (4 * 5)\ mod\ 7 = 6$.
Find a solution for $x$ in $3 *_7 x = 4$.
Is there solution for $x$ in $3 *_6 x = 4$?