Primes

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Fundamental Property of Natural Numbers, N

Any non empty set of Natural numbers has a least element.

A Natural number is a non-negative integer, i.e.

$$\mathbb{N} = \{ n | n \in \mathbb{Z} \land n \ge 0 \}$$

Prime Numbers

A natural number, p, is prime if it has exactly two divisors, 1 and p.

The first dozen primes are:

A natural number, n, is composite if it is not prime, i.e. n has more than two factors.



Smallest Factor is Prime

The smallest factor $(\neq 1)$ of a number, n, is prime.

The set of factors $(\neq 1)$ of n is a non empty set of natural numbers. Let p be its least element, then 1 . If <math>p is not prime, then p has a factor, k, such that 1 < k < p. Since k a factor of p and p is a factor of n then k is a factor $(\neq 1)$ of n that is less than p which was assumed to be the least factor of n. p is prime.

Infinity of Primes

The set of primes is infinite, i.e. there is no greatest prime.

Proof.

Assume the set of primes is finite i.e. the set of all primes is $\{p_1, \ldots, p_k\}$.

Let $n = 1 + p_1 * p_2 * \cdots * p_k$. Then n > 1 and has a least factor, p, which is prime.

The number, n, leaves a remainder, 1, when divided by p_1 and p_2 ... and p_k and so none of the elements in $\{p_1, \ldots, p_k\}$ is a factor of n and so p is a prime not in the set $\{p_1, \ldots, p_k\}$, a contradiction as $\{p_1, \ldots, p_k\}$ was assumed the set of all primes

Determine if n is prime

To determine if a natural number, n, is prime, check for (prime) divisors/factors (>1) up to the \sqrt{n} . If there are no factors (>1) then the number, n, is prime.

Example: Check if 199 is prime. Since $14^2=196<199<15^2=225$, check all the primes up to 14 as divisors of 199.

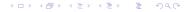
Notation: a|b "a divides b" or "a is a factor of b" Also, $a \not\mid b$ "a does not divide b".

2 /199, 3 /199, 5 /199,

$$199 = 7 * 28 + 3 : 7 / 199,$$

 $199 = 11 * 18 + 1 : 11 / 199,$
 $199 = 13 * 15 + 4 : 13 / 199,$

∴ 199 is prime.



Find prime factors of *n*

Find Prime Factors

To determine the prime factors of a natural number, n: start with the the lowest prime, 2, and divide the number by the lowest prime divisor giving a quotient and repeatedly check the prime divisors of the quotient.

Example: Find prime factors of 980.

$$\frac{980}{2} = 490$$
, $\frac{490}{2} = 245$, $\frac{245}{5} = 49$, $\frac{49}{7} = 7$, $\frac{7}{7} = 1$::

$$980 = 2 * 490 = 2^2 * 245 = 2^2 * 5 * 49 = 2^2 * 5 * 7^2$$
 i.e.

$$980 = 2^2 * 5 * 7^2$$



Rules of Thumb for divisibility

- n is divisible by 2 or 2 is a factor of n
 if the last (least significant) digit is divisible by 2
- n is divisible by 3
 if the digit sum is divisible by 3.
- n is divisible by 5
 if the last digit is divisible by 5 i.e. the last digit is 0 or 5.
- n is divisible by 7
 Check if n leaves a 0 remainder i.e. n mod 7 = 0.
- n is divisible by 11
 Take the alternating sum of the digits in the number, reading from left to right. If that is divisible by 11, so is the original number.

Example: n = 2728 has alternating sum of digits 2 - 7 + 2 - 8 = -11. Since -11 is divisible by 11, so is 2728.

