Petit Samuel CS1003 Homework I.

Please note this paper instead of my first submission as it solved the last question in a different method from what was asked. Thank you.

Petit Samuel CS1003 Homework 1

Before I start I just wanted to say that I am French and I went to highschool in France so I may not have a few of the reflexes (such as the . instead of a , for example). Thank you to let me know if you notice any of these and I will try my best to forget my french habits.

Q1 - i - A determinant is calculated by : det = ad - bc . With a, b, c and d, reels numbers of a square matrix of the form : $Y = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

We then have:

- $\det(A) = 3 \times 2 - 4 \times 1 = 6 - 4 = 2.$
- $\det(B) = 3 \times 1 - 4 \times 2 = 3 - 8 = -5.$

ii - Let's calculate the first side of the equation first.

$$(AB)^T = \left(\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \right)^T = \begin{pmatrix} (3 \ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (3 \ 1)\begin{pmatrix} 4 \\ 3 \end{pmatrix} \\ (4 \ 2)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (4 \ 2)\begin{pmatrix} 4 \\ 3 \end{pmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} 5 & 15 \\ 8 & 22 \end{pmatrix}^T = \begin{pmatrix} 5 & 8 \\ 15 & 22 \end{pmatrix}.$$

Now lets work with the second part of the equation.
Lets first transpose A and B.

$A^T = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$, $B^T \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

We can now find $B^T A^T$.

$$B^T A^T = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} (1 \ 2)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (1 \ 2)\begin{pmatrix} 4 \\ 2 \end{pmatrix} \\ (4 \ 3)\begin{pmatrix} 3 \\ 1 \end{pmatrix} & (4 \ 3)\begin{pmatrix} 4 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 8 \\ 15 & 22 \end{pmatrix} = (AB)^T.$$

The 2 results are the same. The equation $(AB)^T = B^T A^T$ is true.

Q2 - i -

- $$AB = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & 5 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \ 2 \ 3)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} & (1 \ 2 \ 3)\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} & (1 \ 2 \ 3)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \\ (2 \ 1 \ 3)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} & (2 \ 1 \ 3)\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} & (2 \ 1 \ 3)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \\ (1 \ 1 \ 2)\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} & (1 \ 1 \ 2)\begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix} & (1 \ 1 \ 2)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2+4+3 & 5+2+6 & 3+2+6 \\ 4+2+3 & 10+1+6 & 6+1+6 \\ 2+2+2 & 5+1+4 & 3+1+4 \end{pmatrix} = \begin{pmatrix} 9 & 13 & 11 \\ 9 & 17 & 13 \\ 6 & 10 & 8 \end{pmatrix}.$$

- $$BA = \begin{pmatrix} 2 & 5 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (2 \ 5 \ 3)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (2 \ 5 \ 3)\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} & (2 \ 5 \ 3)\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \\ (2 \ 1 \ 1)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (2 \ 1 \ 1)\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} & (2 \ 1 \ 1)\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \\ (1 \ 2 \ 2)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} & (1 \ 2 \ 2)\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} & (1 \ 2 \ 2)\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2+10+3 & 4+5+3 & 6+15+6 \\ 2+2+1 & 4+1+1 & 6+3+2 \\ 1+4+2 & 2+2+2 & 3+6+4 \end{pmatrix} = \begin{pmatrix} 15 & 12 & 27 \\ 5 & 6 & 11 \\ 7 & 6 & 13 \end{pmatrix}$$

ii - $AB \neq BA$. Because matrix multiplication is not commutative.

Q3 - first I am going to turn this system of linear equations in the form $A\vec{x} = \vec{b}$.

We then have: $A \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ -1 & 1 & 2 \end{pmatrix}$

$\vec{x} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $\vec{b} \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$.

Lets now make the augmented matrix $(A\vec{b})$.

$(A\vec{b}) = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 1 & 3 \\ -1 & 1 & 2 & 6 \end{pmatrix}$.

Lets now compute it into reduced row echelon form using elementary row operations.

1.
$$\begin{array}{llccccc} & \text{row } 2 & 2 & 3 & 1 & 3 \\ + & -2 \times \text{row}_1 & -2 & -2 & -2 & -4 \\ \hline & \text{new row } 2 & 0 & 1 & -1 & -1 \end{array}$$

$$\begin{array}{llcccc} & 1 \times r_1 & 1 & 1 & 1 & 2 \\ + & r_3 & -1 & 1 & 2 & 6 \\ \hline & \text{new } r_3 & 0 & 2 & 3 & 8 \end{array}$$

We now have $B = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 2 & 3 & 8 \end{pmatrix}$

2.
$$\begin{array}{lcccc} r_1 & 1 & 1 & 1 & 2 \\ -1 \times r_2 & 0 & -1 & 1 & 1 \\ \hline \text{new } r_1 & 1 & 0 & 2 & 3 \end{array}$$

$$\begin{array}{lcccc} r_3 & 0 & 2 & 3 & 8 \\ -2 \times r_2 & 0 & -2 & 2 & 2 \\ \hline \text{new } r_3 & 0 & 0 & 5 & 10 \end{array}$$

We now have: $B = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 5 & 10 \end{pmatrix}$.

$\frac{1}{5} \times r_3 : 0 \quad 0 \quad 1 \quad 2$.

$B = \begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$.

$$\begin{array}{lrrrr} r_2 & 0 & 1 & -1 & -1 \\ + r_3 & 0 & 0 & 1 & 2 \\ \hline \text{new } r_2 & 0 & 1 & 0 & 1 \end{array}$$

We now have the reduced row echelon form.

$$\begin{array}{lrrrr} r_1 & 1 & 0 & 2 & 3 \\ -2 \times r_3 & 0 & 0 & -2 & -4 \\ \hline \text{new } r_1 & 1 & 0 & 0 & -1 \end{array}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

We can now solve the system:

- $1 \times x + 0y + 0z = -1$
  $\iff x = 1.$
- $0 \times x + 1 \times y + 0 \times z = 1$
  $\iff y = 1.$
- $0 \times x + 0 \times y + 1 \times z = 2$
  $\iff z = 2.$