Problem 1 -

The amount of k^{th} payment is :

$400 < 300(1.01)^k$

$\Leftrightarrow \log_{1.01}\left(\frac{4}{3}\right) < k$

$28.911 < k$

$$\max(400, 300(1.01)^k) = \begin{cases} 400 & \text{if } k \leq 28 \\ 300(1.01)^k & \text{if } k > 28 \end{cases}$$

We can split it into 2 annuities: one which pays 400 at the end of k^{th} month where $k = 0, 1, \ldots 28$. And the second one which pays $300(1.01)^k$ where the first payment is delayed by $\frac{28}{12} = 2.416$ years.

$a(t) = 1.2027^t$.

$$i = \frac{a\left(\frac{k}{12}\right) - a\left(\frac{k-1}{12}\right)}{a\left(\frac{k-1}{12}\right)} = 1.2027^{\frac{1}{12}} - 1 = 0.01549$$

~~Transforming the cashflow into an arithmetic progression.~~

a_k

It seems that for every payment k, the amount of repayment increases by 3 thus we take

$P = 300(1.01)^1 = 303.$

$Q = 3$

$n \to \infty$.

We have $PV = \underbrace{(I_{303,3}\, a)_{\overline{28}|0.01549}}_{\text{Annuity 1 P.V.}} + \frac{400 a_{\overline{28}|0.01549}}{a(28)}$

$$PV = £\,303\, a_{\overline{28}|0.01549} + \frac{3}{0.01549}\left(a_{\overline{28}|0.01549} - \frac{28}{(1.01549)^{28}}\right) + \frac{\frac{400}{0.01549}}{1.2027^{28/12}}$$

$$= 303\,(22.5789) + 193.67\,(22.5789 - 18.2077) + 16787.0679$$

$$= 6841.4067 + 846.7058 + 16787.0679$$

$$= 24475.18047.$$

b) let's use the above formulas with an updated i value:

$$a(t) = 1.1^{t}$$

$$i = 1.1^{\frac{1}{12}} - 1 = 0.00797$$

$$PV = 303\, a_{\overline{28}|0.00797} + \frac{3}{0.00797}\left(a_{\overline{28}|0.00797} - \frac{28}{(1.00797)^{28}}\right) + \frac{\frac{400}{0.00797}}{1.1^{28/12}}$$

$$PV = 303\,(25) + 378.41\,(25 - 22.419) + 40180$$

$$PV = 7575 + 971.514 + 40180 = \underline{48726.514}$$