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MATH2811 - Assignment n°1

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Problem 1 -

$$\delta_t = \begin{cases} 0.01 + 0.04t & \text{if } t \geq 0.5 \\ 0.05 - 0.04t & \text{if } t < 0.5 \end{cases}$$

Interest $= 10000\, a(3) - 10000\, a(1) + 5000\, a(2)\Big|_{t \geq 1} - 5000$

$$a(3) = e^{\int_0^3 \delta_s \, ds} = e^{\int_0^{0.5} 0.05 - 0.04t + \int_{0.5}^{3} 0.01 + 0.04t}$$

$$= e^{\left(0.05t - \frac{0.04t^2}{2}\right)\Big|_0^{0.5} + \left(0.01t + \frac{0.04t^2}{2}\right)\Big|_{0.5}^{3}}$$

$$= e^{0.015 + 0.20} = e^{0.215}$$

$$a(1) = e^{0.015 + \left(0.01t + \frac{0.04t^2}{2}\right)\Big|_{0.5}^{1}} = e^{0.015 + 0.02}$$

$$= e^{0.035}$$

$$a(2)\Big|_{t \geq 1} = e^{\left(0.01t + \frac{0.04t^2}{2}\right)\Big|_0^2} = e^{0.1}$$

Thus, interest:

$$10000 \times e^{0.215} - 10000\, e^{0.035} + 5000\, e^{0.1} - 5000$$

$$\approx 12\,398.62 - 10\,356.20 + 5525.85 - 5000$$

$$\approx \underline{\underline{2568.27}}.$$

Problem 2 b)

Lets compute the interest rates of all funds:

fund A: $\delta_t = 0.048$ thus $(\delta_t)^{-1} = 0.048t$

$a(2) = e^{0.096}$, $a(0.5) = e^{0.024}$

$$i = \frac{e^{0.096}}{e^{0.024}} - 1 = e^{0.072} - 1 \approx 0.7466$$

fund B: $m = 12$ at 5%

$$a(t) = \left(1 + \frac{0.05}{12}\right)^{12t} = \left(\frac{12.05}{12}\right)^{12t}$$

$$i = \frac{a(2)}{a(0.5)} - 1 = \frac{\left(\frac{12.05}{12}\right)^{24}}{\left(\frac{12.05}{12}\right)^{6}} - 1 = \left(\frac{12.05}{12}\right)^{18} - 1$$

≈ 0.7772

fund C: $m = 2$, with 5.2% over first 2 years:

$$a(t) = \left(1 + \frac{0.052}{2}\right)^{2t} = 1.026^{2t}$$

$$i = \frac{a(2)}{a(0.5)} - 1 = \frac{1.026^4}{1.026} - 1 = 1.026^3 - 1$$

≈ 0.8005

Taking into account the circumstances of this investment (starting at $t = 0.5$ and stopping at $t = 2$), we can see that fund C has the highest interest rate thus it should be considered.

P2 b)

- find A:

$$a(3) = e^{0.048t|_0^3} = e^{0.144}$$

$$a(1) = e^{0.048t|_0^1} = e^{0.048}$$

$$i_{[1,3]} = e^{0.144 - 0.048} - 1 \approx 0.1008$$

- find B:

$$i_{[1,3]} = \frac{\left(\frac{12.05}{12}\right)^{36}}{\left(\frac{12.05}{12}\right)^{12}} - 1 = \left(\frac{12.05}{12}\right)^{24} - 1$$

$$\approx 0.1049$$

find C:

The accumulation we used previously doesn't apply in this case anymore:

$$a(t) \begin{cases} 1.026^{2t} & \text{if } t \leq 2 \\ \cancel{\quad} \left(1.026^{4}\right)\left(1.023^{2(t-2)}\right) & \end{cases}$$

$$a(1) = 1.026^2 \approx$$

$$a(3) = 1.026^4 \cdot 1.026^2 \approx 1.15969$$

$$i_{[1,3]} = \frac{1.15969}{1.05268} - 1 \approx 0.10165$$

If we take the average of all 3 funds:

$$i_{Total}[1,3] = 0.1008 \times 0.3 + 0.1049 \times 0.4 + 0.10165 \times 0.3$$

$$= 0.03024 + 0.04196 + 0.030495$$

$$= \boxed{0.102695}$$

Q3\ We have

$$I = Kd_{[0,n]}$$

$$\cdot\ 370 = 4630\left(1 - \frac{1}{a(n)}\right)$$

Let $a(n) = \left(1 - \frac{d}{m}\right)^{-mn}$

We pick $n = 1$ and $m = 2$.

$$a(1) = \left(1 - \frac{d}{2}\right)^{-2}$$

$$\cdot\ 370 = 4630\left(1 - \frac{1}{\left(1 - \frac{d}{2}\right)^{-2}}\right) \iff \frac{370}{4630} = 1 - \frac{1}{\left(1 - \frac{d}{2}\right)^{-2}}$$

$$\iff \frac{370}{4630} = \cancel{1}\ 1 - \left(1 - \frac{d}{2}\right)^2$$

$$\sqrt{1 - \frac{370}{4630}} = 1 - \frac{d}{2} \iff \left(\sqrt{1 - \frac{370}{4630}} - 1\right) \times (-2) = d$$

$d = 0.0816$: The effective discount.

We know that for compound discount:

$$d_n = 1 - \left(1 - \frac{d^{(m)}}{m}\right)^m$$

$$\iff d^{(m)} = -m\left((1 - d_n)^{\frac{1}{m}} - 1\right)$$

$$= -2\left[(1 - 0.0816)^{\frac{1}{2}} - 1\right]$$

$$= 0.0833$$

Let's now find the interest paid by B:

$I = Kd.$

$$I = 10000 \times \left(1 - \frac{1}{a(2t_2)}\right).$$

$$a(2t_2) = \left(1 - \frac{0.0833}{2}\right)^{-4} = 1.1855$$

$$I = 10000 \times \left(1 - \frac{1}{1.1855}\right) = 1564.74$$

Q3 b) Let $m = 2$ and $d_2 = 2 \times d^{(m)} = 2 \times 0.0833 = 0.1666$.

$$a(2^{1/2}) = \left(1 - \frac{0.1666}{2}\right)^{-4} \approx 1.4161$$

$$I_2 = 10000 \times \left(1.4161 - \frac{1}{\cancel{\text{[illegible]}}}\right) = \cancel{\text{[illegible]}}$$

4161.00

Total repayment amount:

~~[illegible] = 21 2938.85~~

$10000 + 4161 = \boxed{14161}$

Bonus Problem 2:

- A: $a(3) = \left(1 + \frac{0.06}{2}\right)^6 = 1.03^6 = 1.1940$

- B. $a(t) = \left(1 + \frac{0.058}{m}\right)^{2m}$ $\left(1 + \frac{i^{(m)}}{m}\right)^{m(t-2)}$

if $i^{(m)} = 5.8\%$:

$$a(3) = \left(1 + \frac{0.058}{m}\right)^{3m}.$$

let: $m = 2$

$$a(3) = \left(1 + \frac{0.058}{2}\right)^6 = 1.029^6 \approx 1.1871$$

let $m = 12$:

$$a(3) = \left(1 + \frac{0.058}{12}\right)^{36} \approx 1.1896$$

We can see that even with an m value of 12, ~~the~~ Mr B's growth remain's lower than Mr A. Thus it must be that $i^{(m)}$ is higher than 5.8% in order ~~to~~ for Mr B to have a greater amount than Mr A after 3 years.

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