CS1003 Mathematics, Taylor Series Tutorial Sheet 1

Q1. Find the linear Taylor polynomial about 1 for the function $f(x) = \dfrac{1}{x^2}$.

Q2. Find the linear Taylor polynomial about 0 for the function $f(x) = (8+x)^{\frac{1}{3}}$. Hence find an approximate value for $\sqrt[3]{8.01}$ and use your calculator to find the associated remainder to six decimal places.

Q3. Find the linear Taylor polynomial about 0 for each of the functions below. Use the polynomial to find an approximation for $f(0.02)$, and use your calculator to find the value of the associated remainder to eight decimal places.

(a) $f(x) = e^{-x}$

(b) $f(x) = (4-x)^{\frac{1}{2}}$ $(x < 4)$

Q4. Find the linear Taylor polynomial about 1 for each of the functions below:

(a) $f(x) = e^{-x}$

(b) $f(x) = \frac{1}{1+x}$

Q5. (a) Use the linear Taylor polynomial about 1 from Q4 above to find an approximate value for the reciprocal of 2.01. Use your calculator to find the value of the associated remainder to eight decimal places.

(b) Find the linear Taylor polynomial about 0 for $f(x) = \dfrac{1}{(1+x)^3}$. Use your answer to find an approximate value for the reciprocal of 1.01. Use your calculator to find the value of the associated remainder to eight decimal places.

Q6. Find the quadratic Taylor polynomial about 0 for each of the functions below. Use the polynomial to find an approximation for $f(0.01)$, and use your calculator to find the value of the associated remainder to eight decimal places.

(a) $f(x) = e^{\frac{x}{2}}$

(b) $f(x) = x\cos x$

Q7. (a) Show that the linear Taylor polynomial about 0 for the function

$$f(x) = (1+x)^k, \qquad \text{where} \quad k > 0,$$

is

$$p(x) = 1 + kx.$$

(b) Use the polynomial p from part (a) with $k = \frac{1}{2}$ and $x = 0.01$ to find an approximate value for $\sqrt{1.01}$. Use your calculator to find, to six decimal places, the value of the associated remainder.

Q8. Find the quadratic Taylor polynomial about 0 for the function $f(x) = \tan x$. Hence find an approximate value for $\tan(-0.1)$, and use your calculator to find the associated remainder to six decimal places.

Q9. Find the Taylor polynomial of degree n about 0 for the function

$$f(x) = \frac{1}{1-x}.$$

Q10. Find the cubic Taylor polynomial about $\frac{1}{6}\pi$ for the function $f(x) = \sin x$.

Q11. Find the quintic Taylor polynomial for each of the following functions about the given point

(a) $f(x) = x^6$ about 1

(b) $f(x) = \ln(1-x)$ about 0

(c) $f(x) = \sin x$ about $\frac{1}{4}\pi$

(d) $f(x) = (1+x)^{\frac{1}{2}}$ about 0

Q12. By expressing 10 as $9(1+\frac{1}{9})$ and using your result from the previous question, evaluate $\sqrt{10}$ to three decimal places.

Q13. Find the quartic Taylor polynomial about 0 for each of the functions below.

(a) $f(x) = \cos(2x)$

(b) $f(x) = \ln\left(\frac{1}{1+x}\right)$

(c) $f(x) = \sqrt{1-x}$

Q14. (a) Find the quintic Taylor polynomial about π for the function $f(x) = \cos x$.

(b) Find the quintic Taylor polynomial about e for the function $f(x) = \ln x$.

Q15. (a) Given that the Taylor polynomial of degree n about 0 for the function $f(x) = \ln(1+x)$ is

$$p_n(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots + \frac{(-1)^{n+1}}{n}x^n,$$

for $n = 1, 2, 3, \ldots$ calculate the value of $\ln(1.05)$ to four decimal places.

(b) Given that the Taylor polynomial of degree $2n$ about 0 for the function $f(x) = \cos(2x)$ can be expressed as

$$p_{2n}(x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + (-1)^n\frac{(2x)^{2n}}{(2n)!},$$

for $n = 0, 1, 2, \ldots$, calculate the value of $\cos(0.2)$ to four decimal places.