CS1003 Mathematics, Taylor Series Tutorial Sheet 2

Q1. Use the definition of a Taylor series to find the Taylor series about 2 for the function $f(x) = x^{-1}$. You are not expected to find the range of validity for this series.

Q2. Use the binomial series to find the Taylor series about 0 for the funciton $f(x) = (1+x)^5$, and state a range of validity for the series.

Q3. Use the binomial series to find the Taylor series about 0 for the function $f(x) = \dfrac{1}{\sqrt[3]{1+x}}$, simplifying each coefficient.

Q4. By writing 1.25 as $1 + 0.25$, use the Taylor series for the function $f(x) = \ln(1+x)$ about 0 to find a value for $\ln(1.25)$ to three decimal places.

Q5. Find the Taylor series about 0 for the function $f(x) = \dfrac{1}{(1-4x^2)}$. Determine a range of validity for this Taylor Series.

Q6. By replacing x by $x-2$ in the Taylor series about 0 for $\ln(1+x)$, find the Taylor series about 2 for the function $\ln(x-1)$. Determine a range of validity for this Taylor series.

Q7. Find the quartic Taylor Polynomial about 0 for the function

$$f(x) = e^x \sin(-x)$$

evaluating each coefficient.

Q8. Find the Taylor series about 0 for the function

$$\ln \frac{(1+x)}{(1-x)} = \ln(1+x) - \ln(1-x)$$

and determine the range of validity for this series.

Q9. Find the value of x such that $1.25 = \dfrac{(1+x)}{(1-x)}$. Hence use the Taylor series found in Q8. to find a value for $\ln(1.25)$ to three decimal places.

Q10. Comment on whether the method of evaluating $\ln(1.25)$ used in Q9. is better than the method used in Q4.