CS1003 Sample IN-CLASS TEST 2

All questions carry equal marks

Q1If possible, solve the following linear system using Gaussian Elimination:

$$\begin{aligned} x+y+z &= 2 \\ 2x+y &= 3 \\ x-y-3z &= 0 \end{aligned}$$

Q2 Find the inverse of the following matrix using elementary row operations

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Verify your solution using the matrix of cofactors method.

Q3 Find the determinant of the following 4×4 matrix:

$$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 5 & 0 & 0 & 7 \end{pmatrix}$$

Q4 Find the cubic Taylor Polynomial about -1 for the function e^{3x}.

Q5 Use the standard Taylor series to find the first four non-zero terms of the Taylor series about 0 for the function

$$f(x) = \frac{1}{1+5x}$$

Hence write down the first four non-zero terms of the Taylor series aobut 0 for the function

$$g(x) = \frac{x^2}{1+5x}$$

Q4
For $f(x) = e^{3x}$ we have

$$\begin{aligned} f(x) &= e^{3x} & f(-1) &= e^{-3} \\ f'(x) &= 3e^{3x} & f(-1) &= 3e^{-3} \\ f''(x) &= 9e^{3x} & f(-1) &= 9e^{-3} \\ f^{(3)}(x) &= 27e^{3x} & f(-1) &= 27e^{-3} \end{aligned}$$

The cubic Taylor Polynomial about 0 for $f(x) = e^{3x}$ is

$$\begin{aligned} p(x) &= f(-1) + f'(-1)(x+1) + \frac{f''(-1)}{2!}(x+1)^2 + \frac{f^{(3)}(-1)}{3!}(x+1)^3 \\ &= e^{-3} + 3e^{-3}(x+1) + \frac{9e^{-3}}{2!}(x+1)^2 + \frac{27e^{-3}}{3!}(x+1)^3 \\ &= \frac{1}{e^3} + \frac{3}{e^3}(x+1) + \frac{9}{2e^3}(x+1)^2 + \frac{9}{2e^3}(x+1)^3 \end{aligned}$$

Q5

$$1 - 5x + 25x^2 - 125x^3$$

So we get

$$g(x) = x^2 - 5x^3 + 25x^4 - 125x^5$$