

MA2C03: ASSIGNMENT 2
DUE BY THURSDAY, JANUARY 31
AT LECTURE OR IN THE MATHS OFFICE ROOM 0.6

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1) (20 points)

- (a) Describe the formal language over the alphabet $\{a, b, c\}$ generated by the context-free grammar whose non-terminals are $\langle S \rangle$ and $\langle A \rangle$, whose start symbol is $\langle S \rangle$, and whose production rules are the following:

- (1) $\langle S \rangle \rightarrow a\langle S \rangle$
- (2) $\langle S \rangle \rightarrow b\langle A \rangle$
- (3) $\langle A \rangle \rightarrow b\langle A \rangle$
- (4) $\langle A \rangle \rightarrow c\langle A \rangle$
- (5) $\langle A \rangle \rightarrow c$
- (6) $\langle S \rangle \rightarrow a$

In other words, describe the structure of the strings generated by this grammar.

- (b) Is this grammar regular? Justify your answer.
- (c) Is this grammar in normal form? If it is not in normal form, then modify it to make it be in normal form. Explain why it generates the same language after your modifications.
- (d) Write down a regular expression that gives the language from part (a) and justify your answer.

2) (10 points) Let L be the language consisting of all binary numbers divisible by 4. Note that any binary number starting with 0 and containing more than one symbol is considered improper and should be rejected.

- (a) Draw a deterministic finite state acceptor that accepts the language L . Carefully label all the states including the starting state and the finishing states as well as all the transitions. Make sure you justify it accepts all strings in the language L and no others.

- (b) Devise a regular grammar in normal form that generates the language L . Be sure to specify the start symbol, the non-terminals, and all the production rules. Make sure you justify it generates all strings in the language L and no others.
- 3) (10 points) Let M be the language over the alphabet $\{a, b, c\}$ given by $M = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ } j = i + k\}$.
- (a) Use the Pumping Lemma to show this language is not regular.
- (b) Write down the production rules of a context-free grammar that generates exactly M .
- 4) (20 points) Let (V, E) be the graph with vertices a, b, c, d, e, f, g , and h , and edges $ab, bc, cd, de, ae, ac, ad, af, cg$, and dh .
- (a) Draw this graph.
- (b) Write down this graph's incidence table and its incidence matrix.
- (c) Write down this graph's adjacency table and its adjacency matrix.
- (d) Is this graph complete? Justify your answer.
- (e) Is this graph bipartite? Justify your answer.
- (f) Is this graph regular? Justify your answer.
- (g) Does this graph have any regular subgraph? Justify your answer.
- (h) Give an example of an isomorphism φ from the graph (V, E) to itself satisfying that $\varphi(c) = d$.
- (i) Is the isomorphism from part (h) unique or can you find another isomorphism ψ that is distinct from φ but also satisfies that $\psi(c) = d$? Justify your answer.