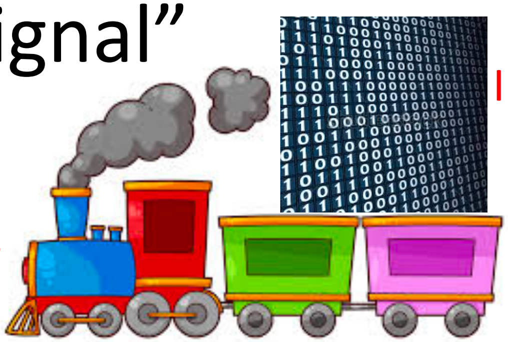
Signal Modulation

Signal Modulation

- Signal modulation allows to add information to a "carrier signal"



Information

Carrier

- The carrier signal can be of different type:

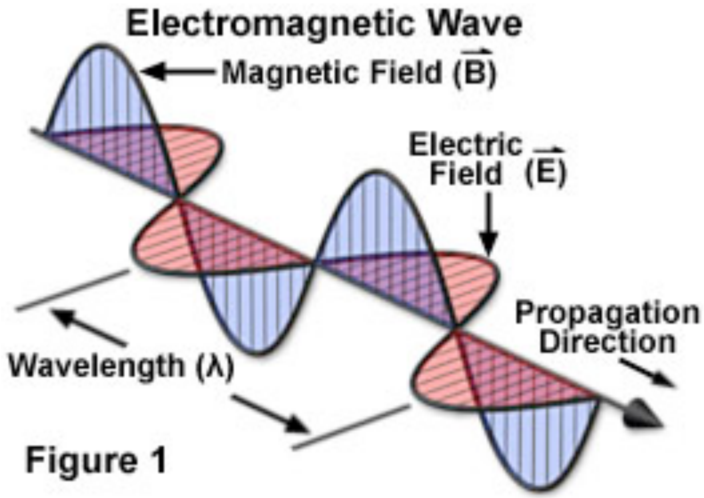


Figure 1

Modulation

- Modulating means changing some property of the carrier signal.
- Exmples:
  - For smoke signals I change the presence (Amplitude) of smoke
  - For visual light signal I change the amplitude of light (on-off) to transmit in Morse code
  - For an electromagnetic signal I can change the amplitude, frequency, phase or polarization.

Modulation of electric signal

- A sine wave has three parameters: Amplitude, phase and frequency.
- I can modulate a carrier sine wave by changing one of these parameters.

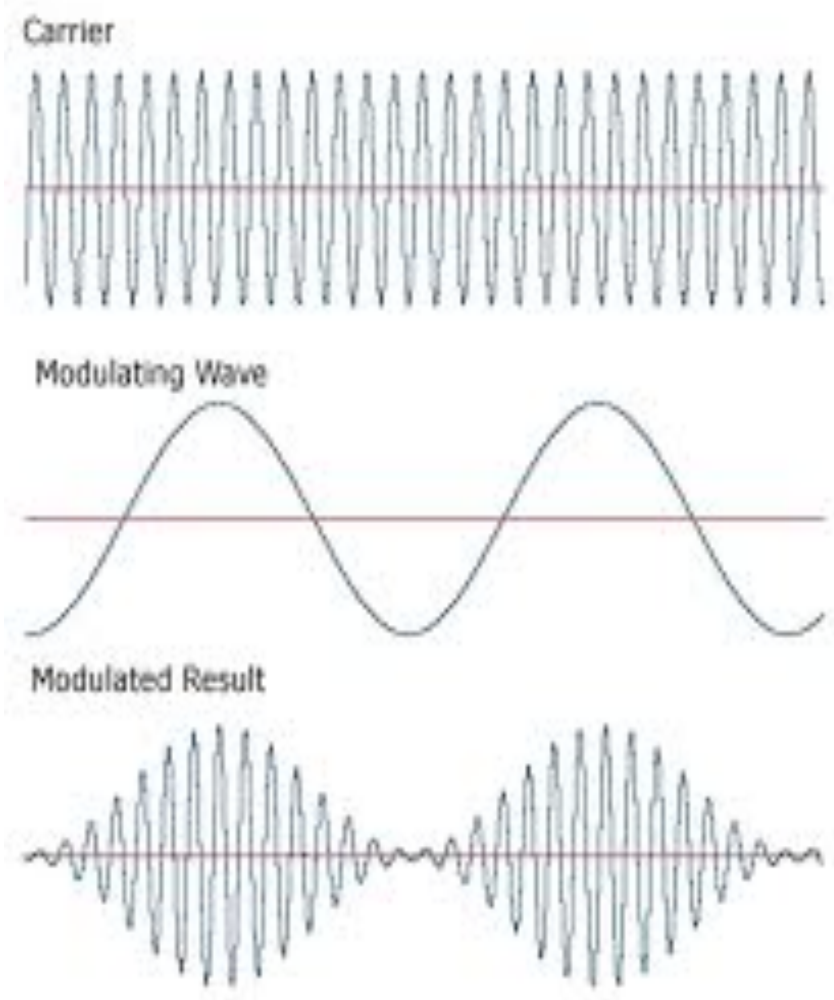
$$s(t) = A \cdot \sin(2\pi ft + \varphi)$$

Amplitude modulation

- This is the carrier sine wave

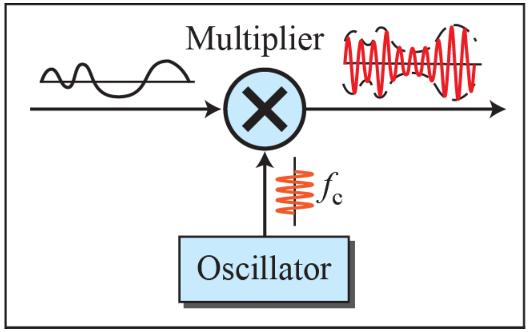
- This is the signal containing information

- This is the modulated signal



Amplitude modulation

- Amplitude modulation is obtained by simply multiplying the information signal by the carrier
- If s(t) is my signal carrying the information, and c(t) is my carrier:
- The modulated signal M(t) = s(t) x c(t)
- This is used in AM radio



Example of amplitude modulation

- My signal is a composite periodic signal, approximating a square wave with two frequencies:
  - a fundamental frequency of 1Hz, and an harmonic of 3 Hz

$$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)$$

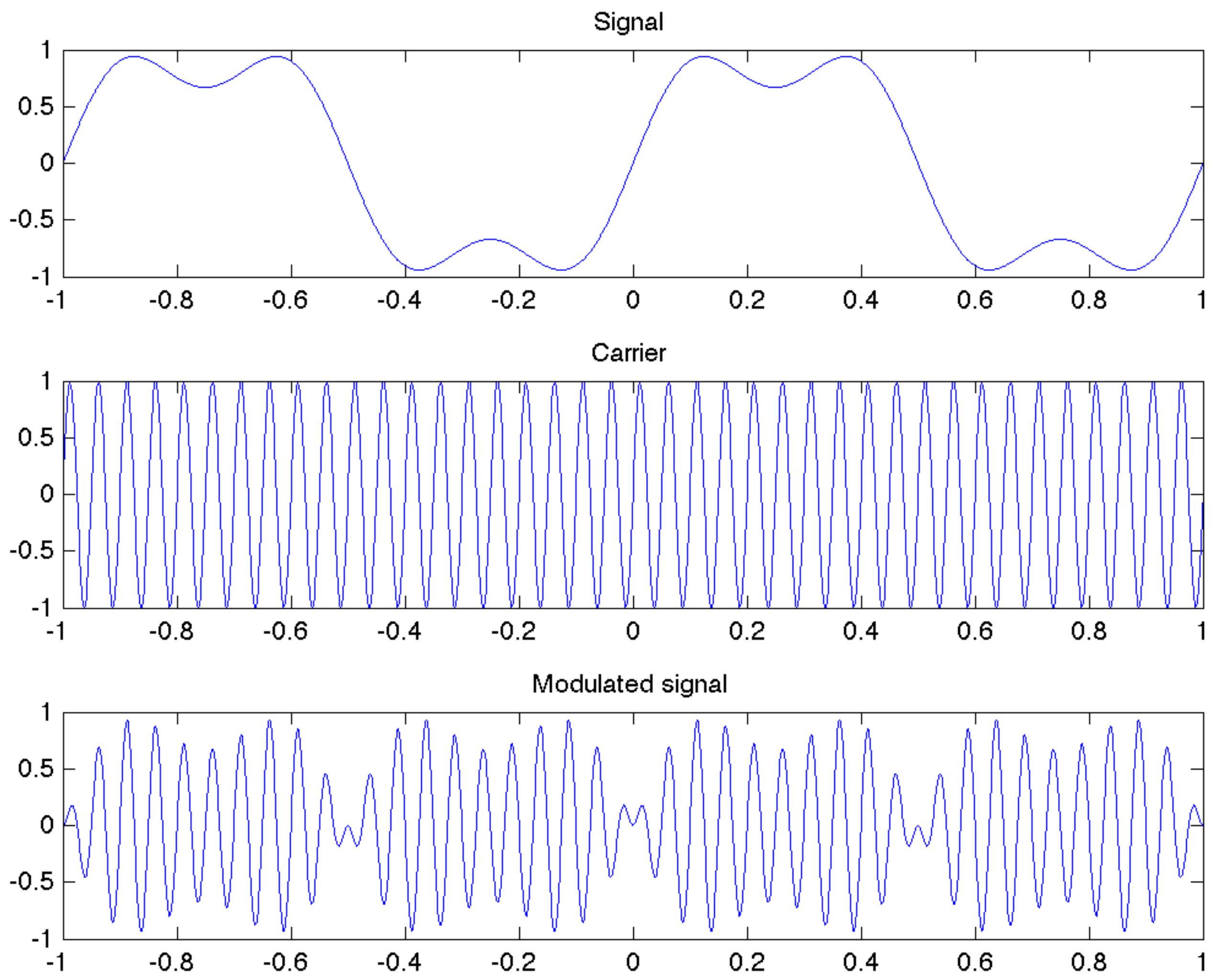
- My carrier is a sine wave of frequency 20Hz

$$c(t) = \sin(2\pi 20t)$$

- My modulated signal is:

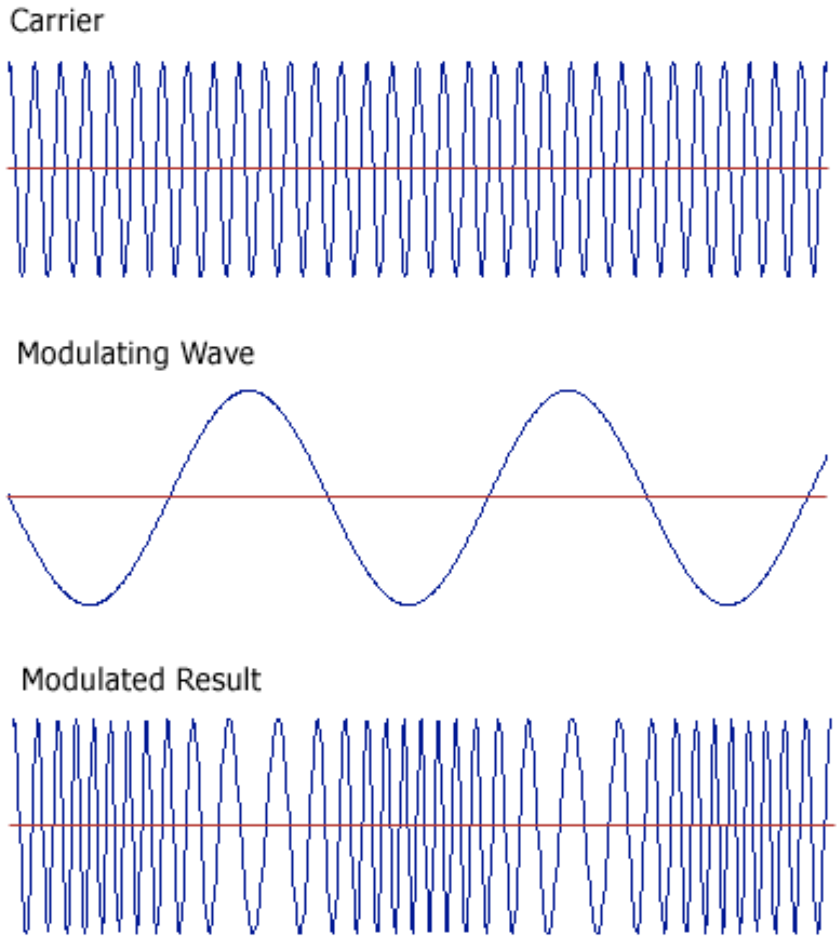
$$m(t) = \left(1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)\right) \cdot \sin(2\pi 20t)$$

Example of amplitude modulation



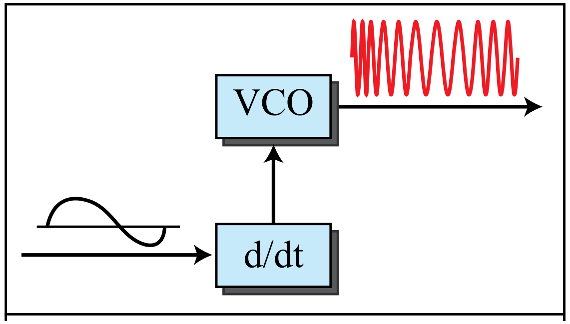
Phase modulation

- Another possibility is to change the phase of the carrier sine wave
  - This is the carrier sine wave
  - This is the signal containing information
  - This is the modulated signal



Phase modulation

- Phase modulation is obtained by changing the phase of the carrier using the signal
- If $s(t)$ is my signal carrying the information, and $\sin(2\pi f_c t)$ is my carrier:
- The modulated signal $M(t) = \sin(2\pi f_c t + s(t))$



- This is not used very often in analog signal, frequency modulation is instead used.

Example of phase modulation

- My signal is a composite periodic signal, approximating a square wave with two frequencies:
  - a fundamental frequency of 1Hz, and an harmonic of 3 Hz

$$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)$$

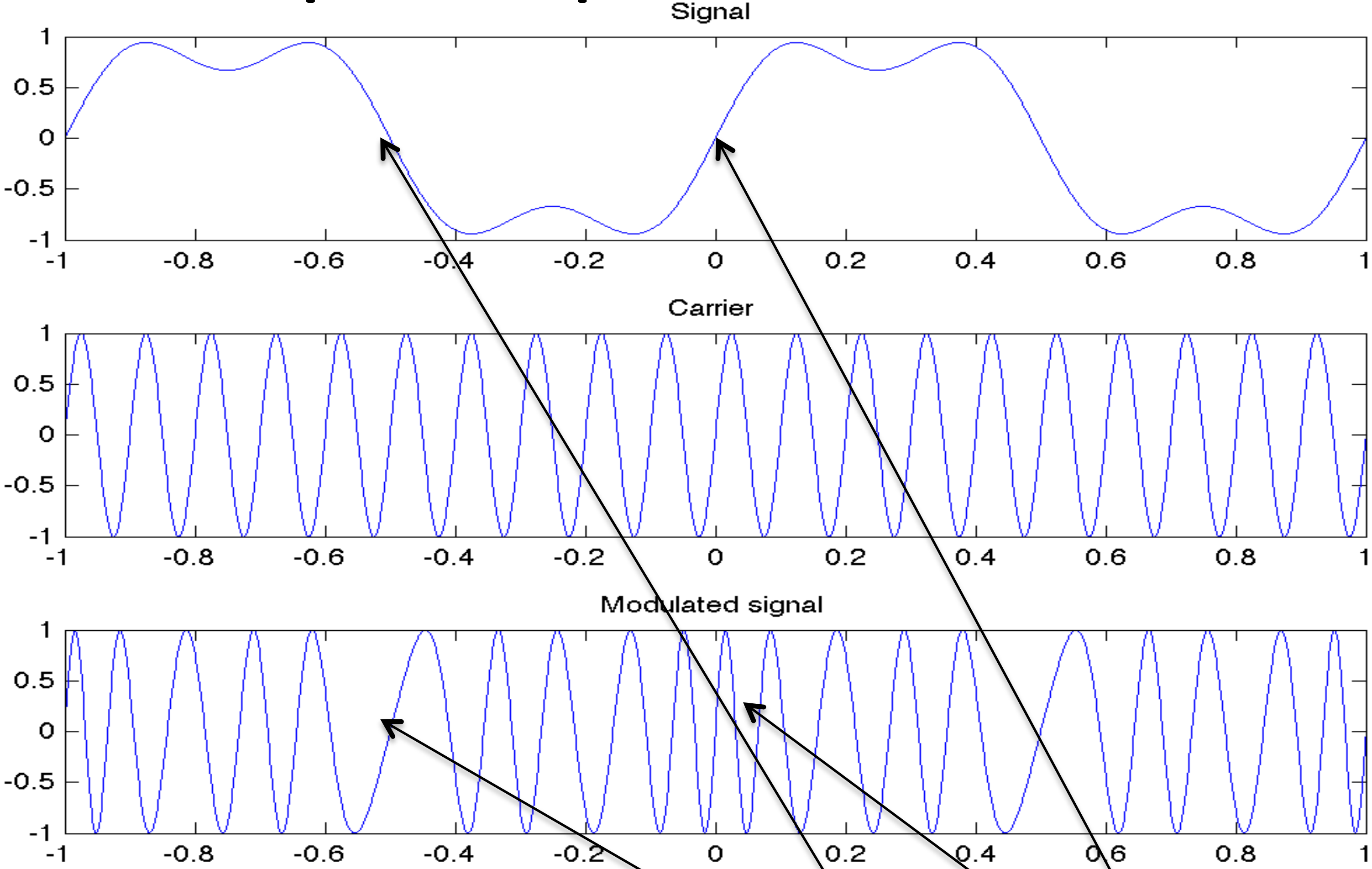
- My carrier is a sine wave of frequency 10Hz

$$c(t) = \sin(2\pi 10t)$$

- My modulated signal is (α is the phase modulation index):

$$m(t) = \sin\left(2\pi 10t + \alpha\left(1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)\right)\right)$$

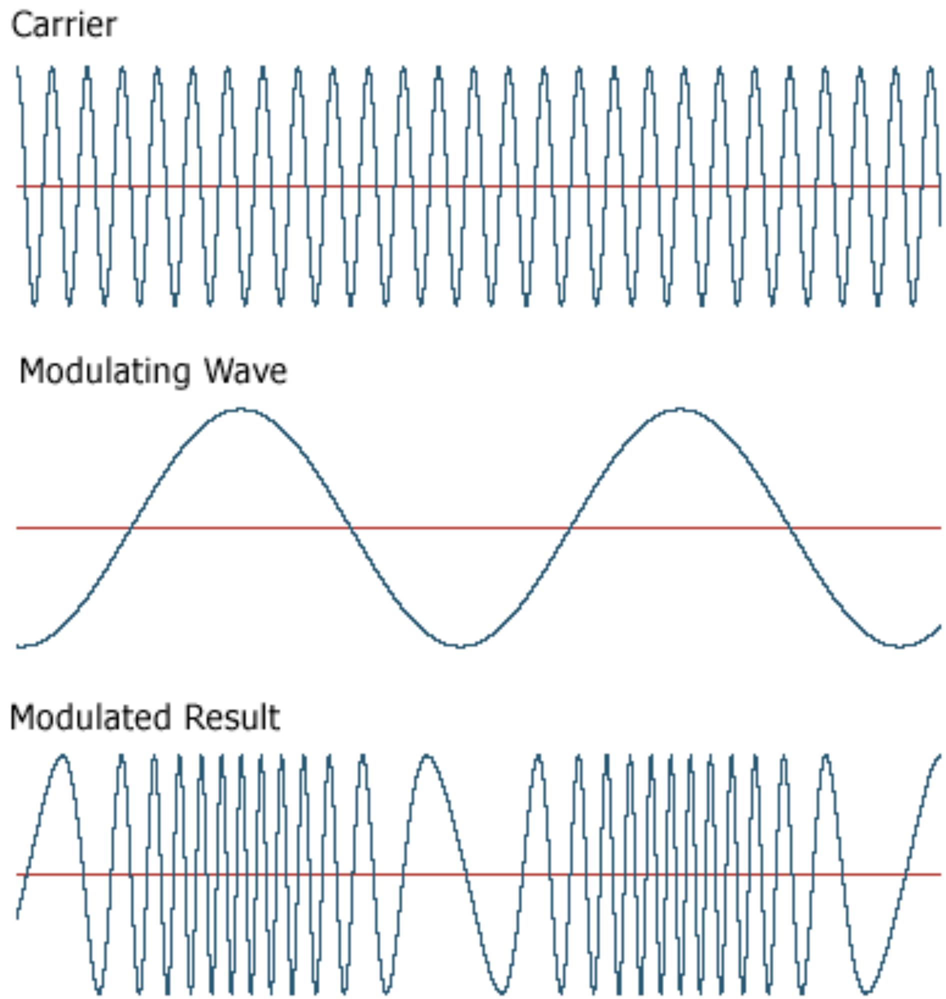
Example of phase modulation



Notice that the changes occur where the **rate of change** of the signal is higher (negative or positive)

Frequency modulation

- Another possibility is to change the frequency of the carrier sine wave
  - This is the carrier sine wave
  - This is the signal containing information
  - This is the modulated signal

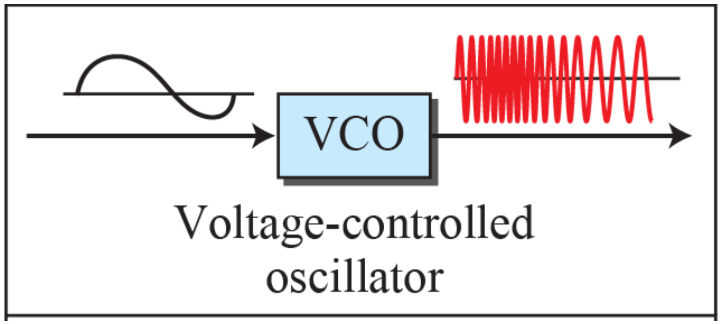


Frequency modulation

- Frequency modulation is obtained by changing the frequency of the carrier using the signal
- Notice that the frequency is the mathematical derivative of the phase, or inversely the phase is the integral of the frequency

- If s(t) is my signal carrying the information, and $\sin(2\pi f_c t)$ is my carrier:
- The modulated signal: $m(t) = \sin\left(2\pi f_c t + \int_0^t s(t)\,dt\right)$

- This is used in FM radio

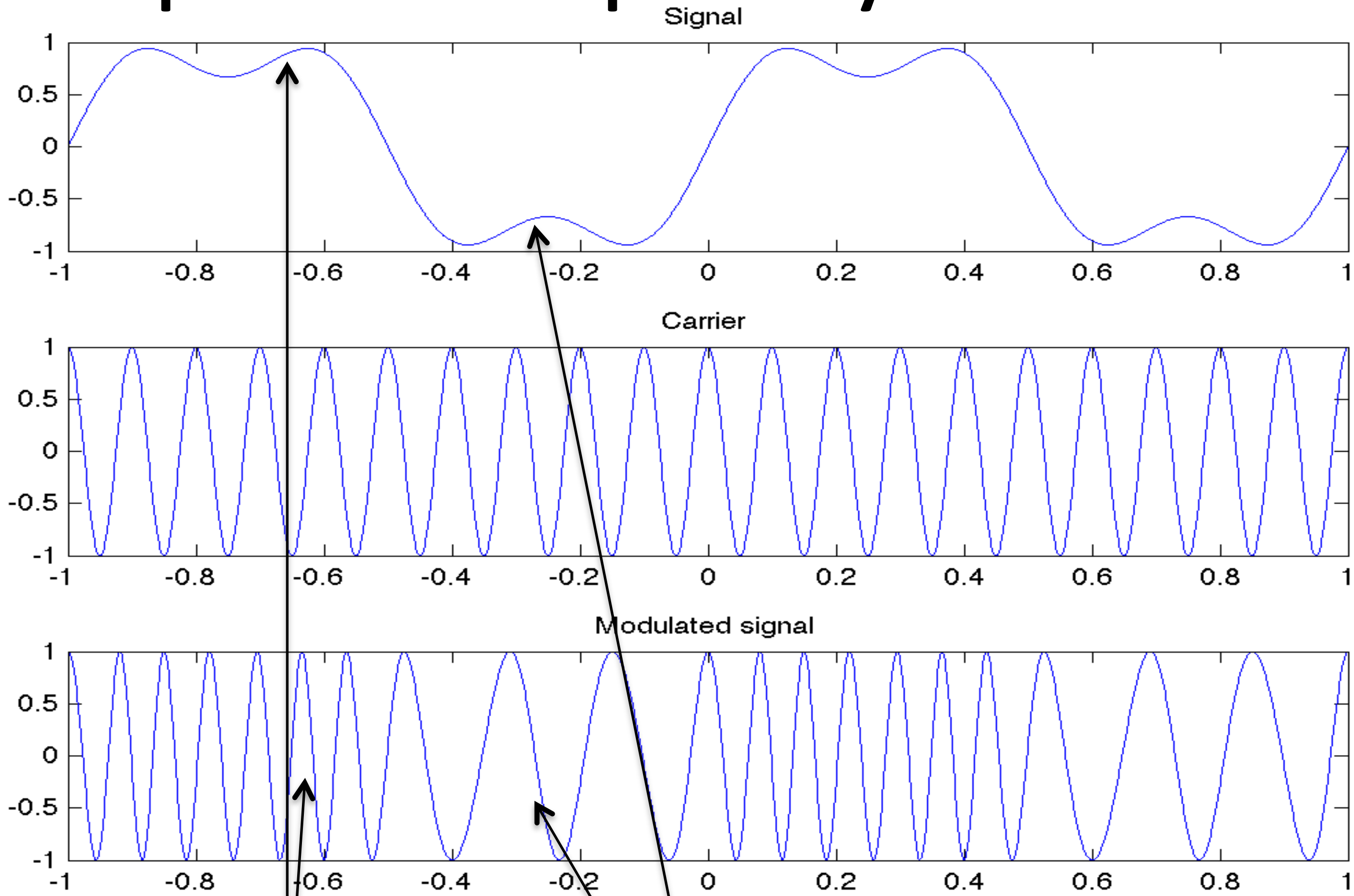


Example of frequency modulation

- My signal is a composite periodic signal, approximating a square wave with two frequencies:
  - a fundamental frequency of 1Hz, and an harmonic of 3 Hz $$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)$$
- My carrier is a sine wave of frequency 10Hz $$c(t) = \sin(2\pi 10t)$$
- My modulated signal is (β is the frequency modulation index):

$$m(t) = \sin\left(2\pi 10t + \beta \int_0^t \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t) \cdot dt\right)$$

Example of frequency modulation



Notice that the changes occur where the signal is higher (positive or negative)

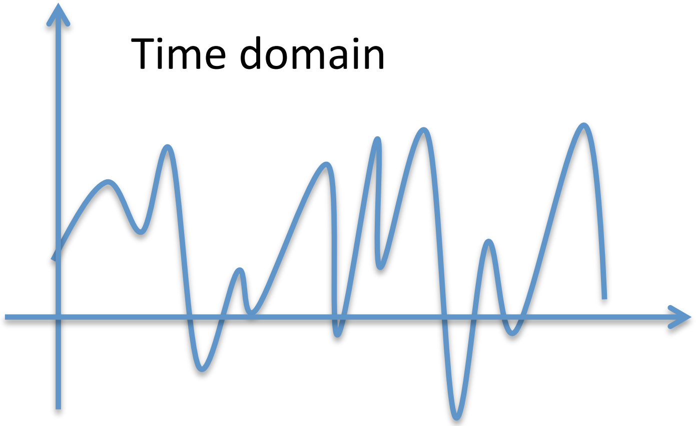
The dark (negative) side of the spectrum

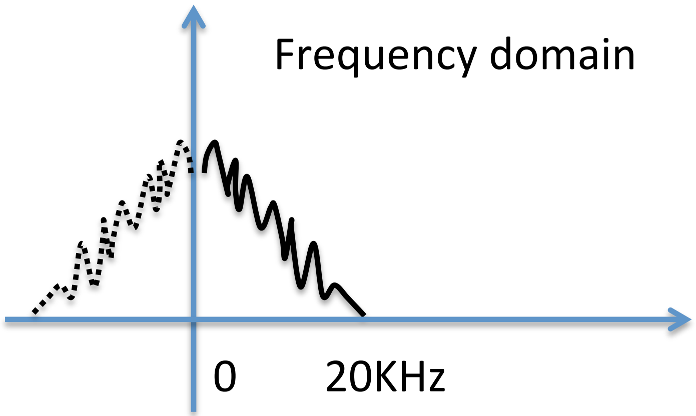
- The Fourier integral creates a mirror image of the positive frequency for the negative frequencies.
- This is true also for the fourier Series, which can be expressed as:

$$F(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}, \qquad e^{inx} = \cos(nx) + i\sin(nx)$$

Negative spectrum

- Any spectrum will always have a negative side which mirrors the positive side





- This doesn't really matter when we work on baseband, as the negative frequencies don't have a real physical meaning…
- … BUT…

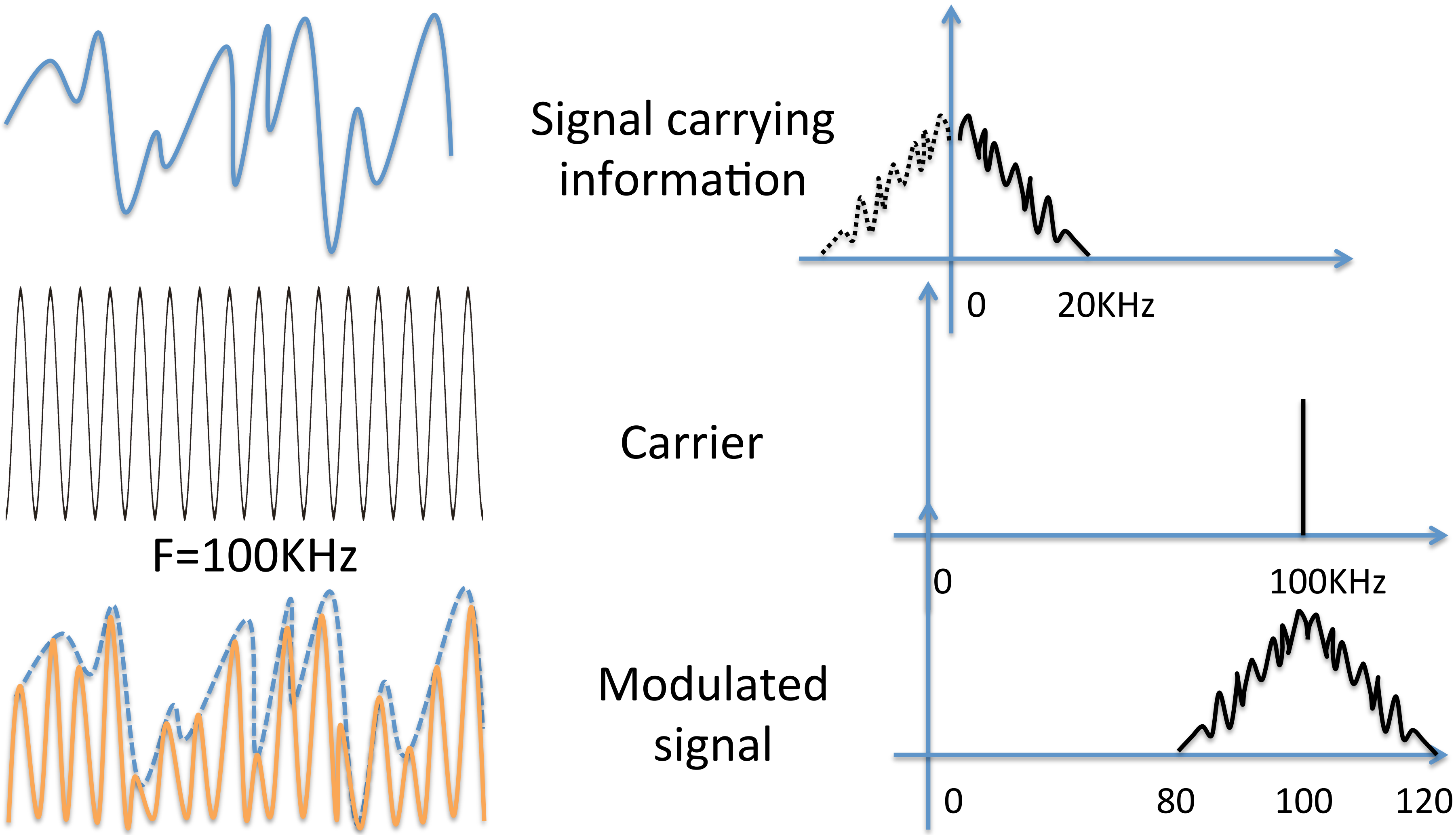
Effects of modulation

- Modulating a signal, shifts its spectrum towards higher frequencies.

- The spectrum of the signal carrying information becomes centered around the frequency of the carrier wave.

Spectrum of a modulated signal

Time domain **Spectrum domain**



Example of spectrum of modulated signal

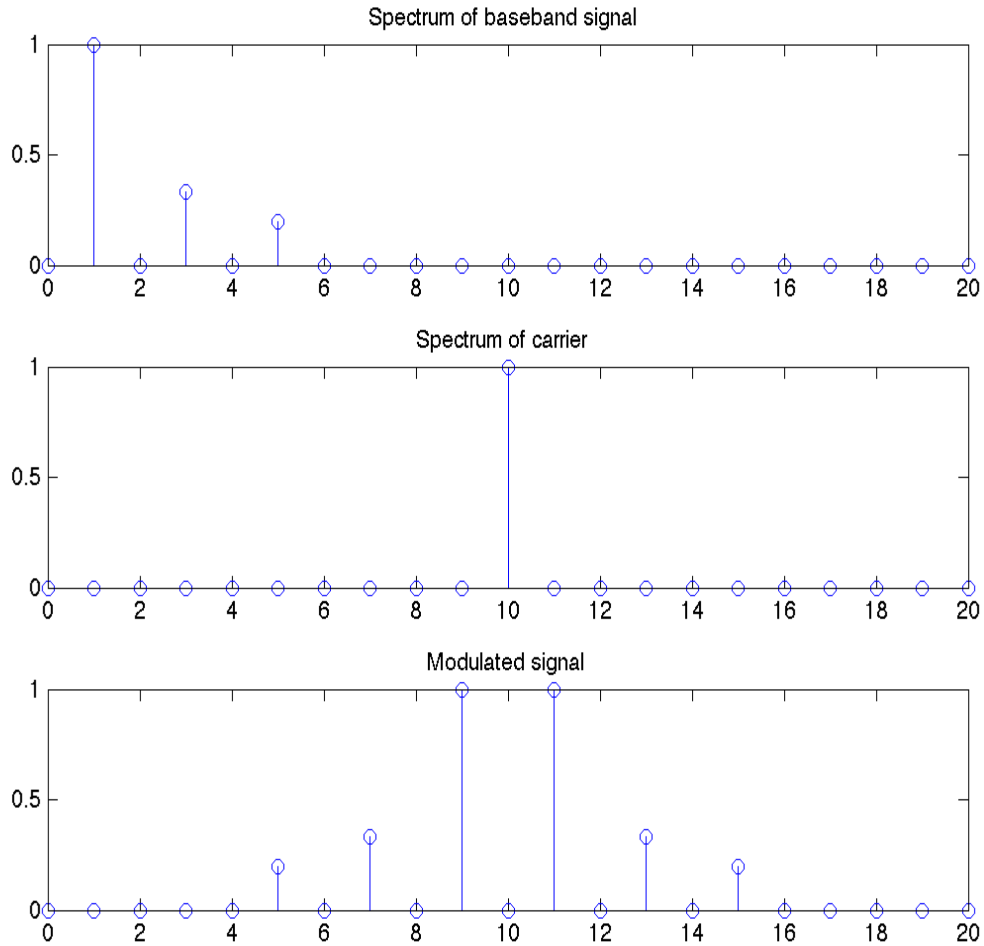
- Signal:

$$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t) + \frac{1}{5}\sin(2\pi 5t)$$

- Carrier:

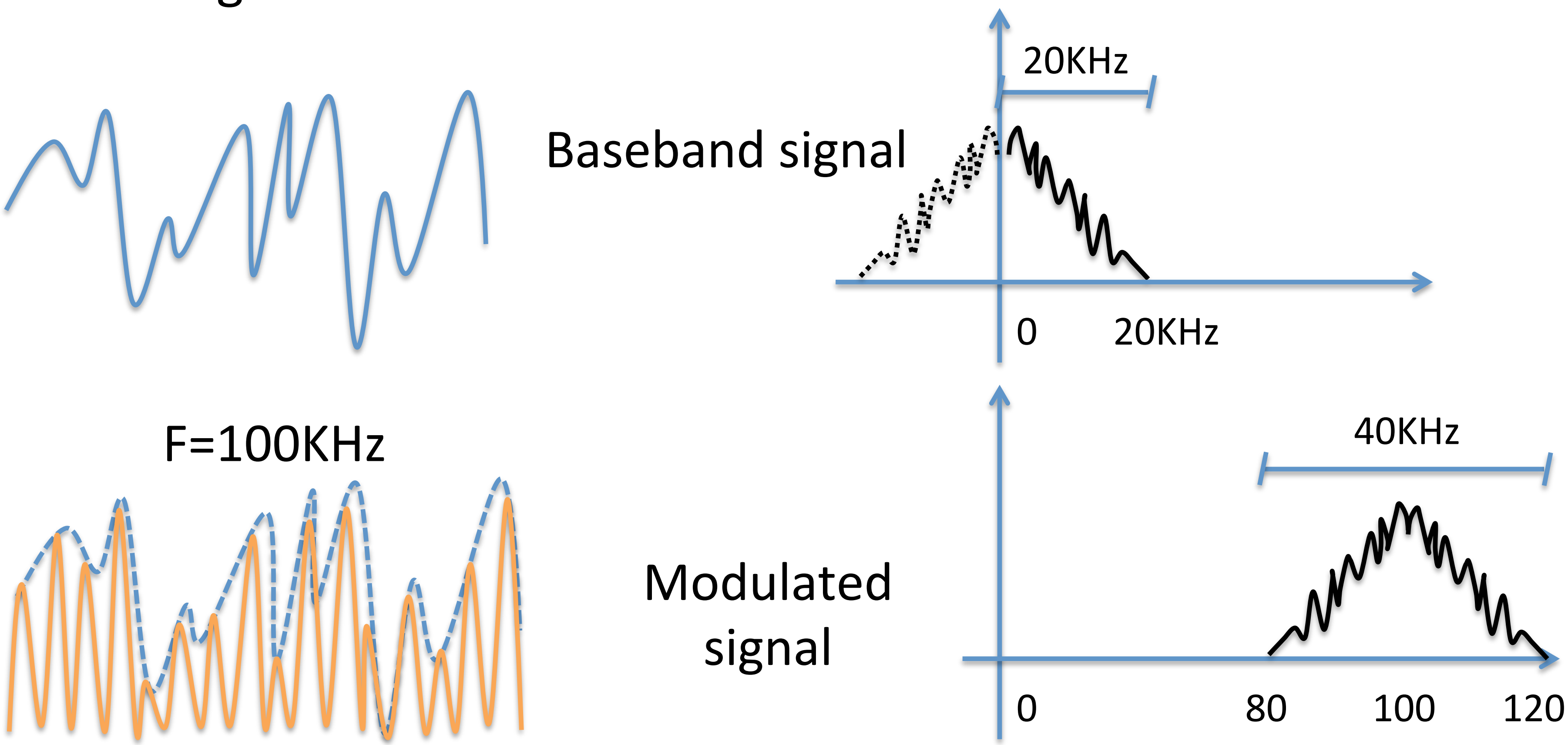
$$c(t) = \sin(2\pi 10t)$$

- The spectrum of the amplitude modulated, is the same as the signal but centered at the carrier frequency



Bandwidth of a modulated signal

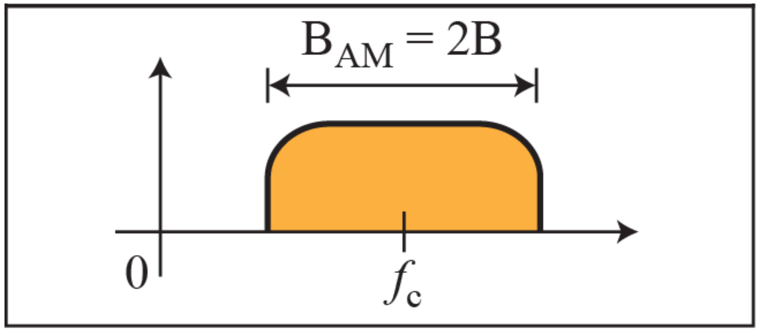
- When a signal is modulated the negative side of the spectrum is moved to the positive side and becomes ‘real’.
- Thus this part also needs to be accounted for.
- For example for amplitude modulation, the bandwidth of the signal is the double of that in the baseband



Bandwidth occupied by amplitude modulation

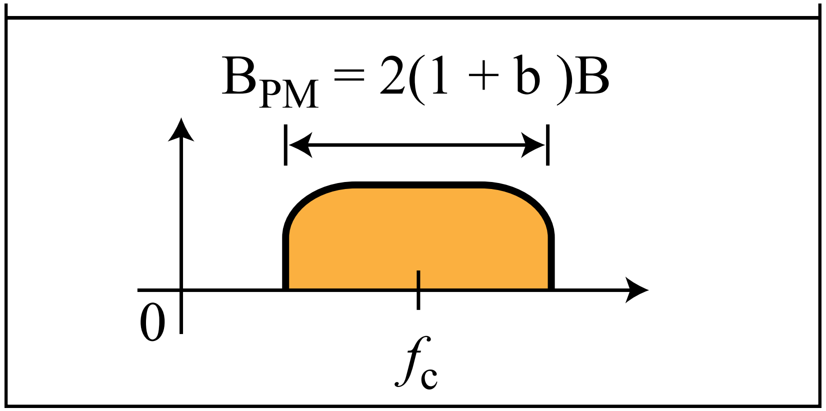
- Seen in the frequency domain, amplitude modulations simply shifts the baseband signal to the carrier frequency
- The band is simply double of the baseband signal (remember that a baseband signal also has a negative side of the spectrum that is exposed when the signal gets shifted towards higher frequencies).
- If B_m is the bandwidth of the modulated signal and B_b that of the baseband signal:

$$\mathbf{B_m=2B_b}$$



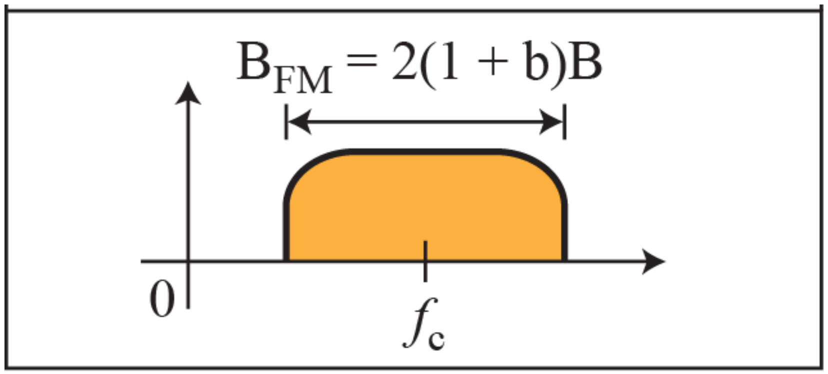
Bandwidth occupied by phase modulation

- Phase modulation shifts the baseband signal to the frequency of the carrier.
- However it also expands the bandwidth:
- Approximately, $\mathbf{B_m=2(1+\beta)B_b}$, where β is between 1 and 3



Bandwidth occupied by frequency modulation

- Frequency modulation shifts the baseband signal to the frequency of the carrier.
- However it also expands the bandwidth:
- Approximately, $\mathbf{B_m=2(1+\beta)B_b}$, where β is about 4



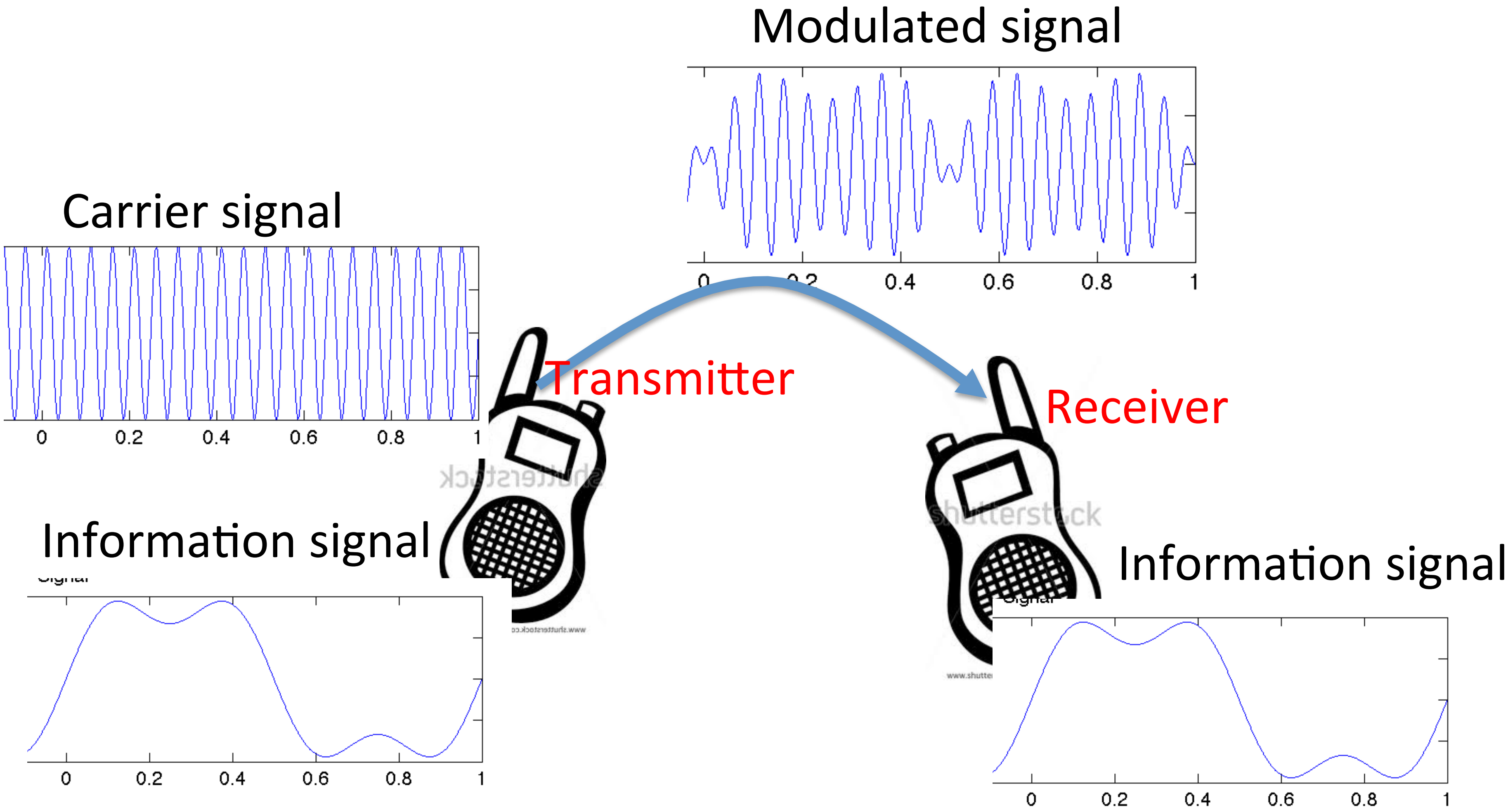
Demodulation

- Demodulation is the process by which a modulated signal is converted back into its original non-modulated version.

- A demodulator extracts the signal from the carrier, converting it back into baseband.

- For example once at the receiver, an FM radio signal is demodulated and converted from 100MHz to baseband. The demodulated signal is then amplified and sent to the speakers.

Demodulation example

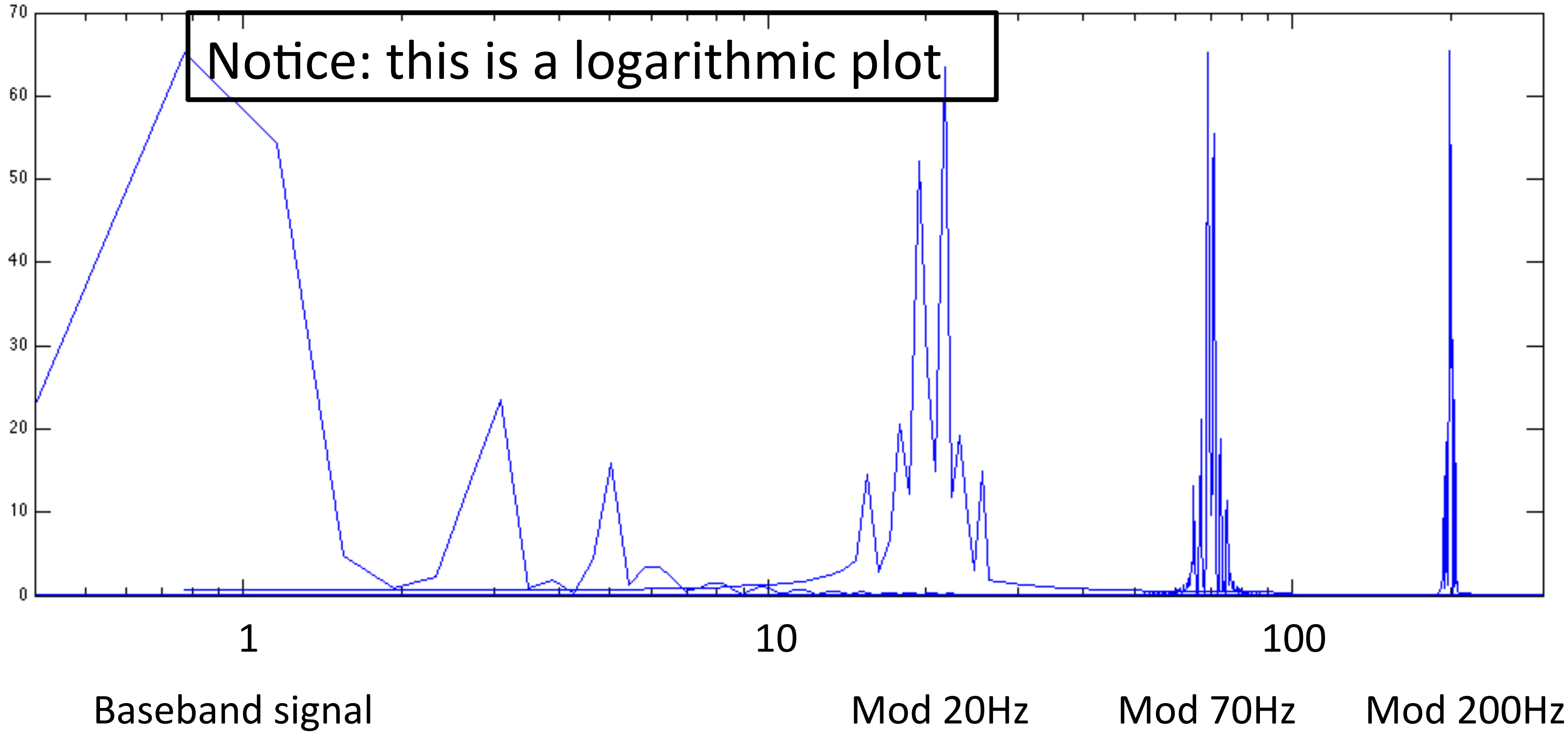


Why do we need modulation?

- There are a number of reasons to modulate a "baseband" signal into a carrier wave.
  - Higher frequencies have more bandwidth available
  - Transmission over wireless medium is more efficient at high frequency and requires smaller antennas
  - Multiple channels can fit into the same wire or wireless medium (Frequency multiplexing)

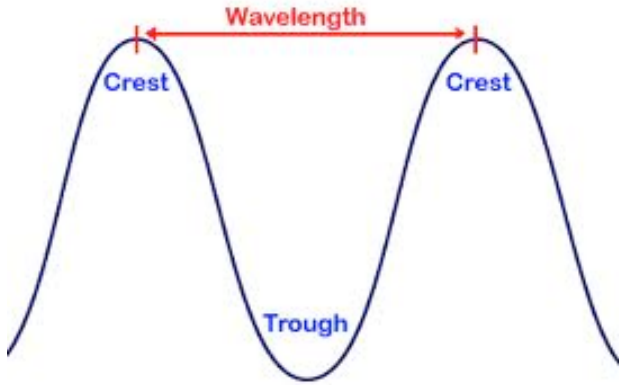
Higher frequencies have more bandwidth

- One of the biggest advantages of using higher frequencies is that there is much more bandwidth available.



Wavelength definition

- Wavelength is the distance in meter between two crests of a sine wave
- In the time domain we called it period, but in the space domain is called wavelength (λ) and is measured in meters.



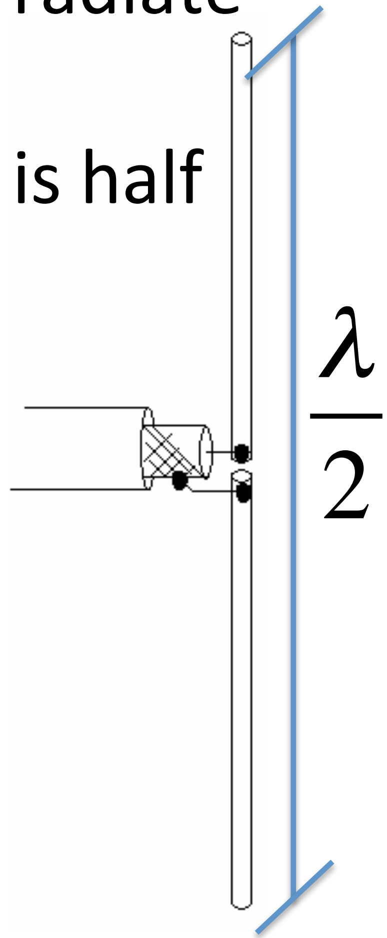
- Relation between wavelength and frequency:

$$\lambda = \frac{c}{f}$$

Where C is the speed of light in the medium considered, and is always slower than the speed of light in the vacuum $C_0 \approx 3 \times 10^8$ m/s

Antennas are shorter at higher frequencies

- Antennas are more practical and tend to radiate more at higher frequencies.
- The ideal size of a simple dipole antenna is half the wavelength.



- So if we consider the vacuum:
  - f=100KHz ➔ $\frac{\lambda}{2} = \frac{3 \cdot 10^8}{2 \cdot 10^5} = 1.5 \cdot 10^3 = 1.5km$
  - F=100MHz ➔ $\frac{\lambda}{2} = \frac{3 \cdot 10^8}{2 \cdot 10^8} = 1.5 \cdot 10^0 = 1.5m$
  - F=10 GHz ➔ $\frac{\lambda}{2} = \frac{3 \cdot 10^8}{2 \cdot 10^{10}} = 1.5 \cdot 10^{-2} = 1.5cm$

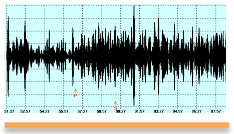
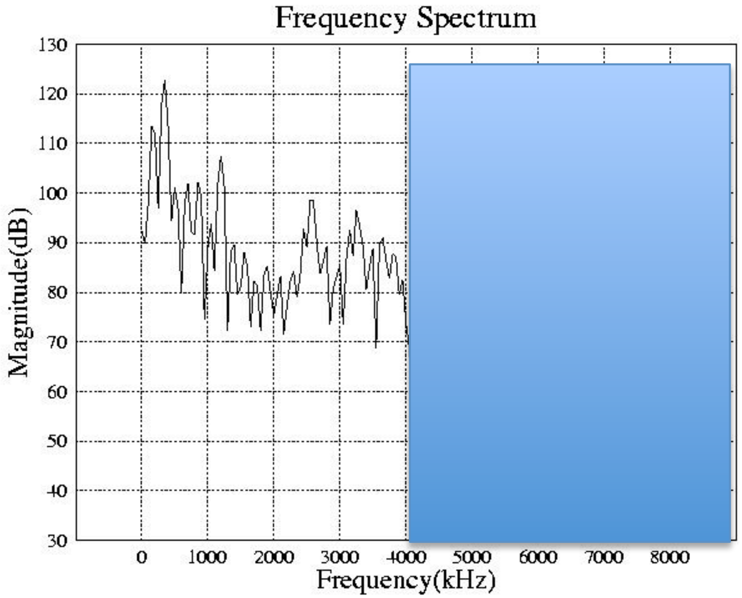
Multiplexing !

- **Modulating information channels at different frequencies allows to transmit them at the same time over the same medium without interfering.**

Example: Telephone conversation

- The microphone in the telephone transforms our speech into an electrical signal.

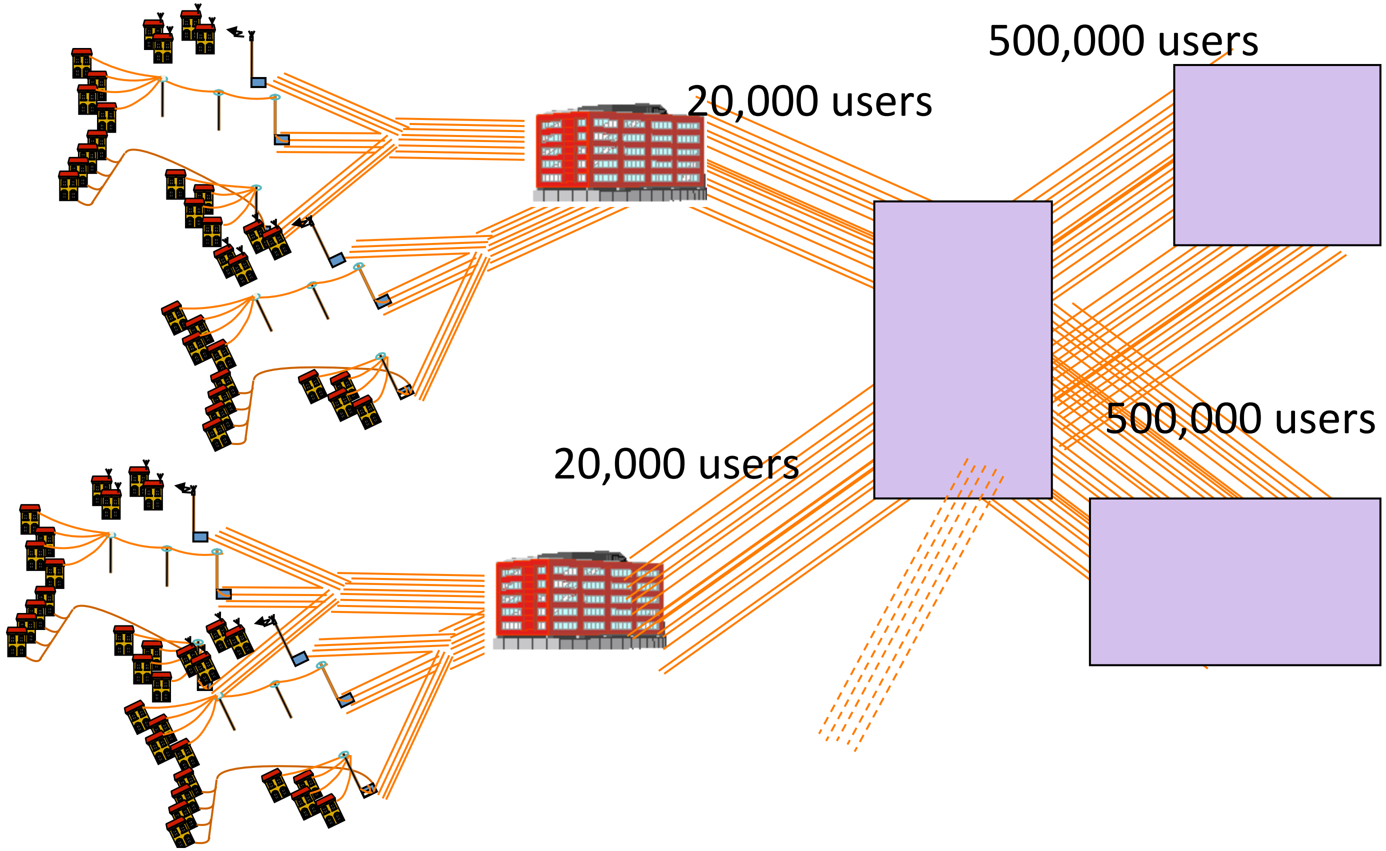


- The telephone system was designed to cut all frequencies above 4KHz.
- This is done at the central office
- The voice quality is deteriorated, but the speech is clearly intelligible

Hypothetic example

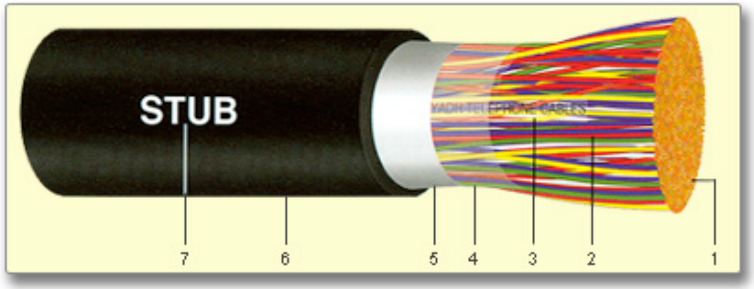
- Each phone connects with a twisted pair to the central office. **If** each conversation stays on a different wire, at the metro nodes we **would have** hundred of thousand of wires…



This is not feasible, so this solution cannot be used and a better solution is required

Hypothetic example

- Even if we used highly compressed copper cables

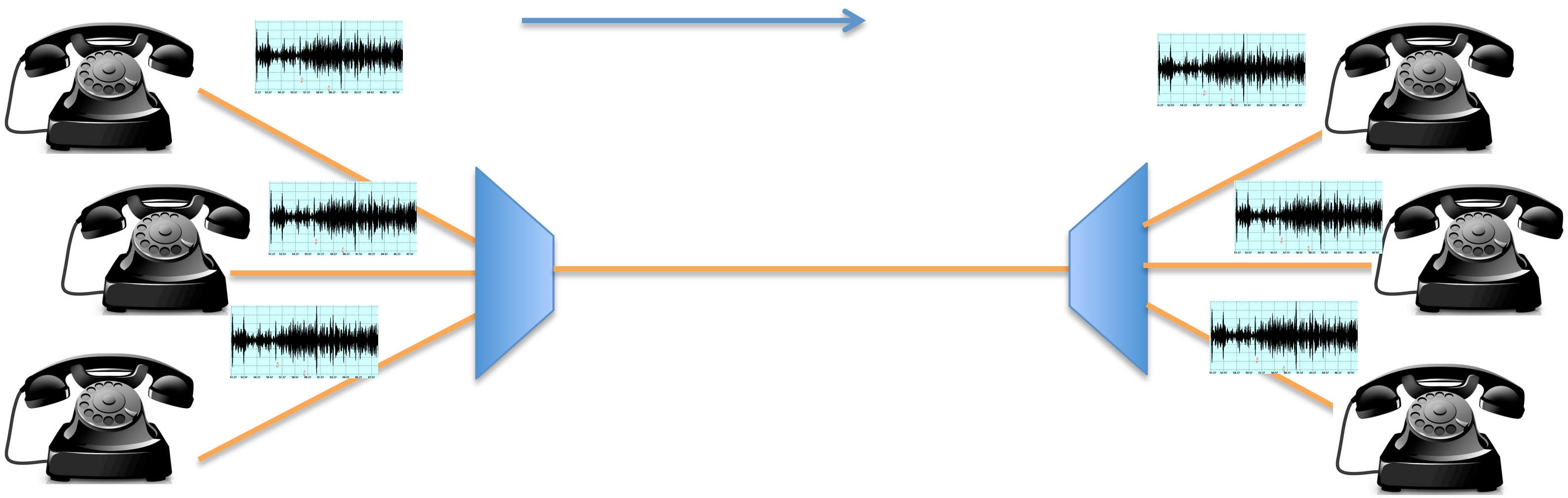


- 400 wires can fit in a cable that is 4.5cm in diameter, and weight 3 tons per Km
- For 500,000 we would require 1250 such cables, with a size of about 2 meters in diameter and a weight of 3750 tons per km
- And 500,000 is a relatively small number… think of countries like the US…

Multiplexing

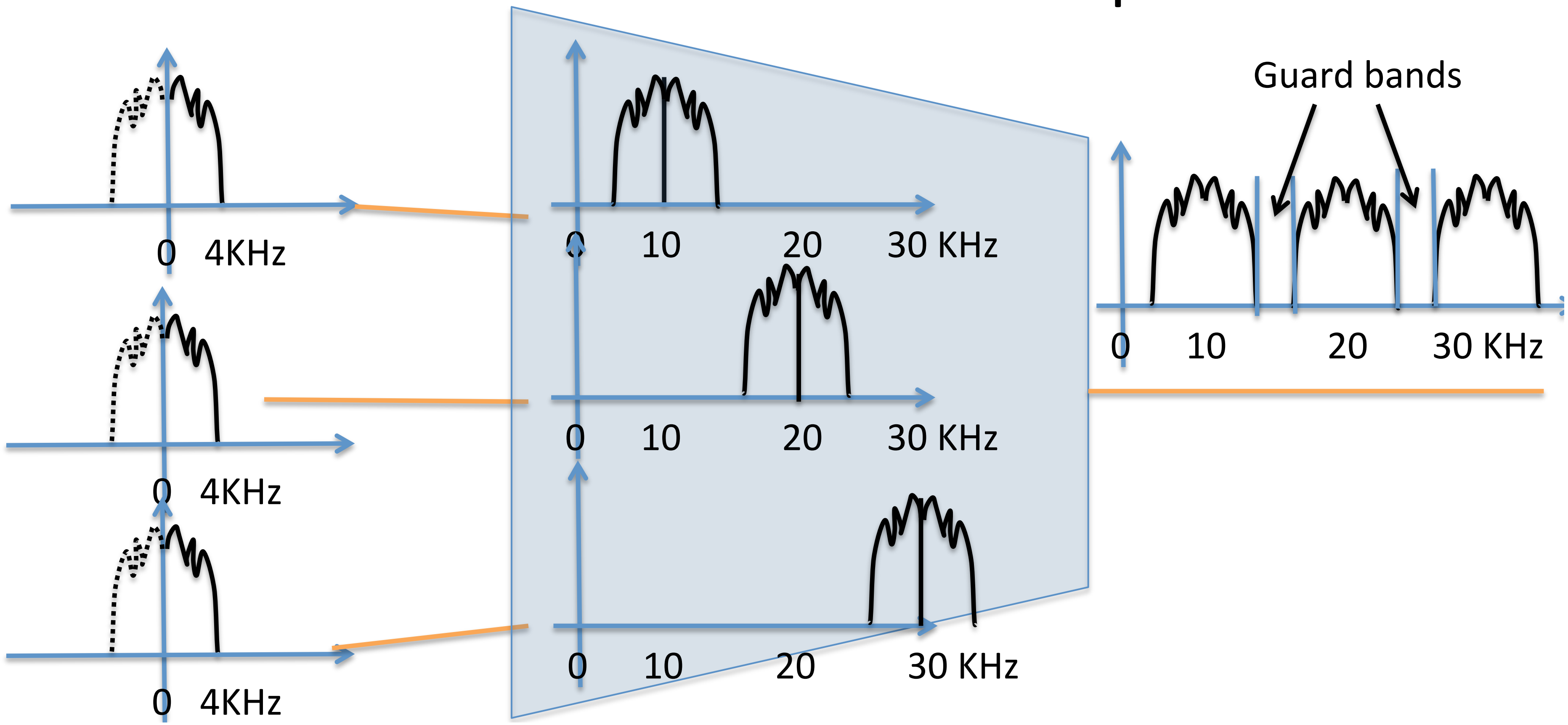
Is there a better way to transmit multiple channels (i.e., signals from multiple users)?

Can we squeeze more than one signal into the same wire and then recover each signal at the other end?



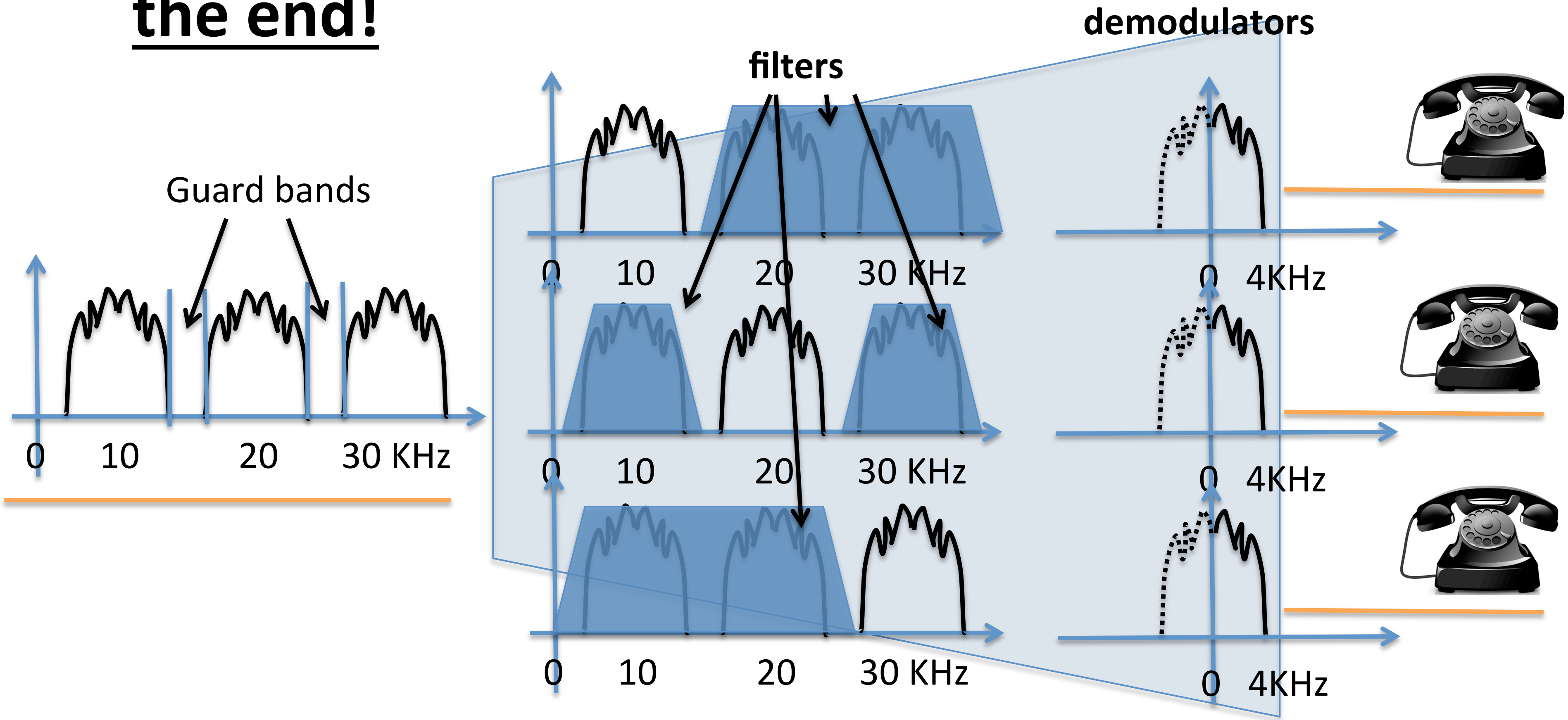
Frequency Division Mutliplexing

- Yes, we can, thanks to modulation!
- Let's take three phone signals, and modulate them with carriers of different frequencies:



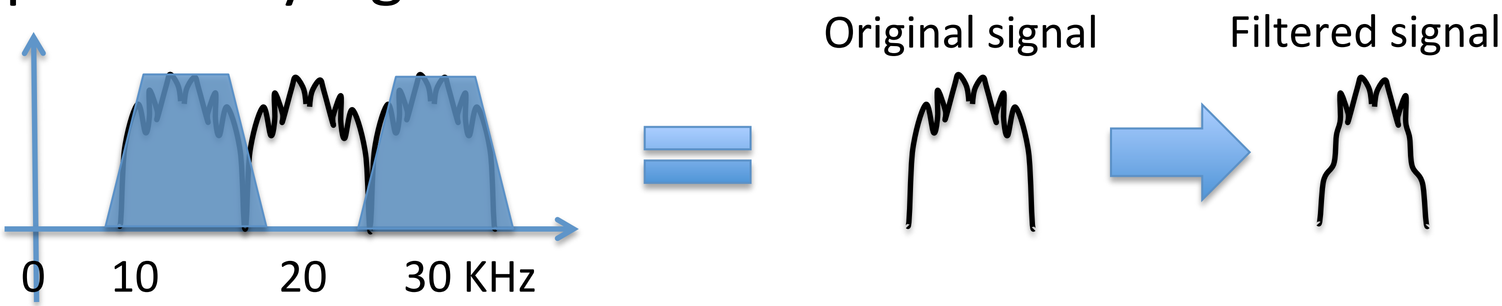
Demultiplexing

- **<u>Important: as far as the frequencies don't overlap, I can separate (filter) the signals at the end!</u>**



Guard bands

- Guard bands separate the channels by a certain amount.
- They are required because filters are never perfect square.
- If there was no guard band, the filters would cut off part of my signal



- In addition better filters are more expensive, so there is a trade-off between wasting space in guard bands and reducing the cost of the filters

Which modulations can be used if I want to transmit multiple signals through frequency division multiplexing?

A. Amplitude Modulation
B. Phase modulation
C. Frequency modulation