MA2C03: TUTORIAL 2 PROBLEM SHEET

1) Prove that $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$ for all sets A, B, and C.

Solution: We prove inclusion in both directions by the criterion for proving equality of sets that we stated at the beginning of the course.

"$\subseteq$" For every $x \in A \setminus (B \setminus C)$, $x \in A$ and $x \notin (B \setminus C)$. The second condition amounts to $x \notin B$ or $x \in C$. Therefore, we have $(x \in A)$ and $((x \notin B)$ or $(x \in C))$. We know the connective *and* distributes with respect to the connective *or*, so we get $(x \in A$ and $x \notin B)$ or $(x \in A$ and $x \in C)$. In other words, we have gotten $x \in (A \setminus B)$ or $x \in A \cap C$. The connective *or* translates to union, so we have $x \in (A \setminus B) \cup (A \cap C)$. We have thus proven that $A \setminus (B \setminus C) \subseteq (A \setminus B) \cup (A \cap C)$ as needed.

"$\supseteq$" For every $x \in (A \setminus B) \cup (A \cap C)$, $x \in (A \setminus B)$ or $x \in (A \cap C)$. Therefore, $(x \in A$ and $x \notin B)$ or $(x \in A$ and $x \in C)$. We thus have $x \in A$ and $(x \notin B$ or $x \in C)$. The second condition amounts to $x \notin (B \setminus C)$ by the de Morgan laws. We have shown $x \in A \setminus (B \setminus C)$ for every $x \in (A \setminus B) \cup (A \cap C)$. Therefore, $(A \setminus B) \cup (A \cap C) \subseteq A \setminus (B \setminus C)$.

Please note that **NO Veitch or Venn diagrams** will be accepted as valid solutions. On a homework set or exam you are advised to follow this procedure when solving a problem that asks you to prove some statement in set theory.

2) Let A be the set of all people who have ever lived. For $x, y \in A$, xRy if and only if x and y were born less than one week apart. Determine:

(i) Whether or not the relation R is *reflexive*: Yes, it is reflexive as xRx must hold. A person is born less than a week from herself or himself.

(ii) Whether or not the relation R is *symmetric*: Yes, R is symmetric since xRy means x was born less than a week apart from y, which in turn means y was born less than a week apart from x, i.e. yRx holds.

(iii) Whether or not the relation R is *anti-symmetric*: No, R is symmetric, so $xRy \Rightarrow yRx$, which means xRy and yRx are both true at the same time without necessarily implying that $x = y$. Any two people born less than a week apart but not on the same day assigned to x and y provides a counterexample.

(iv) Whether or not the relation R is *transitive*: No, use as a counterexample a set of three people x, y, and z, where x and y are born five days apart, y and z are also born five days apart, but x and z are born ten days apart. Therefore, xRy and yRz both hold, but xRz is false contradicting transitivity.

(v) Whether or not the relation R is an *equivalence relation*: No, since R is not transitive.

(vi) Whether or not the relation R is a *partial order*: No, since R is neither anti-symmetric nor transitive.