MA2C03: TUTORIAL 18 PROBLEMS
TURING MACHINES

1) Consider the language over the binary alphabet $A = \{0, 1\}$ given by $L = \{0^m 1^{2m} \mid m \in \mathbb{N}\}$.

(a) Use the Pumping Lemma to show L is not a regular language.

(b) Write down the algorithm of a Turing machine that recognizes L. Process the following strings according to your algorithm: ϵ, 01, 011, and 010.

(c) Write down the transition diagram of the Turing machine from part (a) carefully labelling the initial state, the accept state, the reject state, and all the transitions specified in your algorithm.

(d) Is the language L finite, countably infinite, or uncountably infinite? Justify your answer.

Solution: 1) (a) If L is a regular language, then it has a pumping length p. In order to consider just one case, we work with $w = 0^p 1^{2p} \in L$. According to the Pumping Lemma, w is to be decomposed as xuy, where $|u| \geq 1$ and $|xu| \leq p$. Since $|xu| \leq p$, u can only consist of zeroes. Let $u = 0^{n_1}$, for some $n_1 \geq 1$. Clearly, $xu^2y \notin L$ as $xu^2y = 0^{p+n_1}1^{2p}$, so the length of the first sequence of zeroes is not one half that of the second sequence of zeroes violating the pattern of the language.

(b) Here is the algorithm for recognising $L = \{0^m 1^{2m} : m \in \mathbb{N}\}$.

(1) If there is a blank in the first cell, ACCEPT. If there is anything else, apart from 0, then REJECT.
(2) If 0 is in the current cell, delete it, then move right to the first 1.
(3) If there is no first 1, REJECT. Otherwise change 1 to x.
(4) Move to the leftmost non blank symbol. If 0, go to step 2. If 1, REJECT. If x, go to step 5. If y, go to step 6.
(5) Delete x, move right to the nearest 1. If none, REJECT. Otherwise change it to y and go to step 4.
(6) Move right to the rightmost non blank character. If anything but y is found, REJECT. Otherwise, ACCEPT.

Here is how the following strings are treated:

- ϵ is accepted immediately.

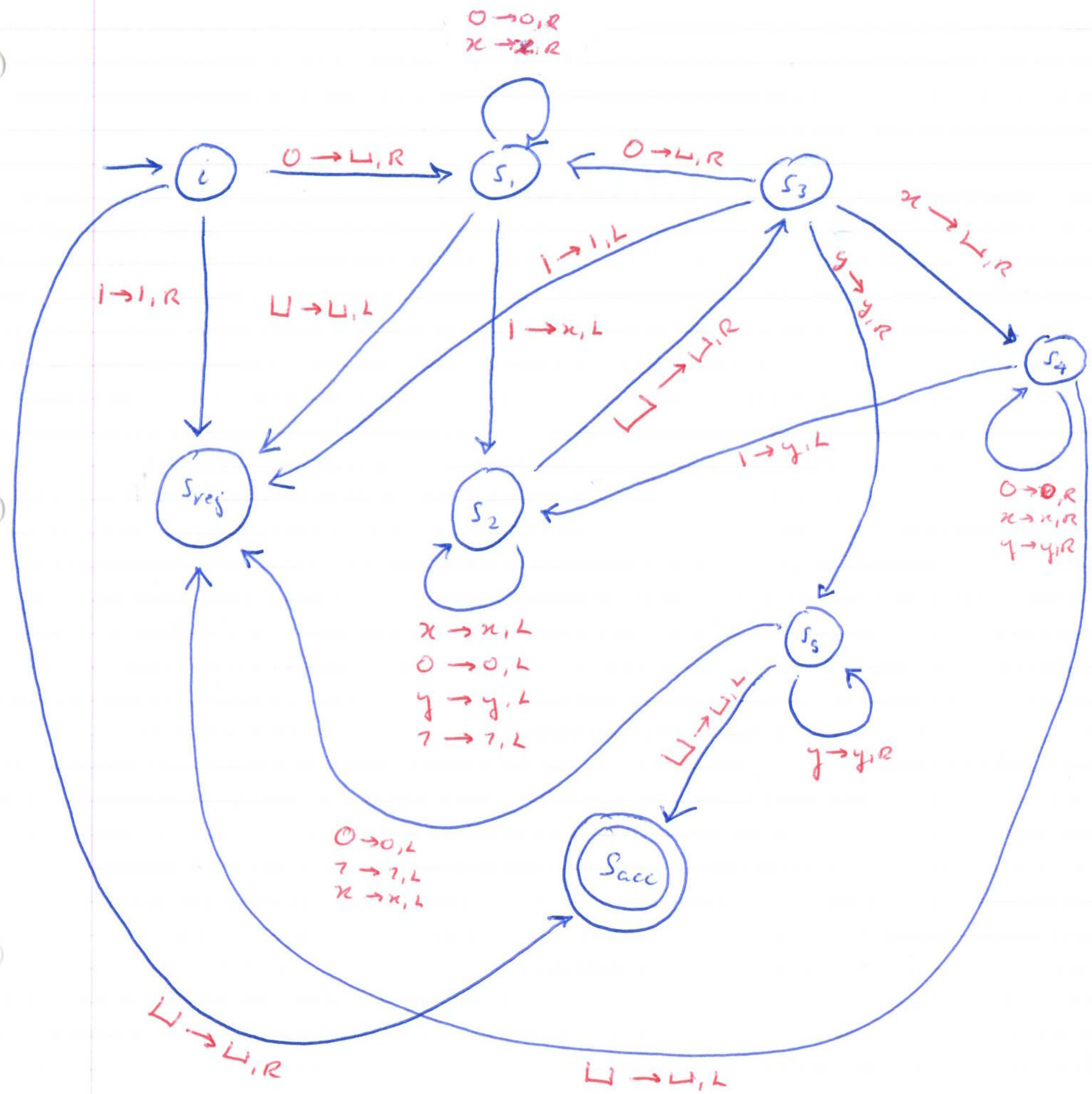
- $01 \to \sqcup 1 \to \sqcup x \to \sqcup\sqcup \to$ REJECT.
- $011 \to \sqcup 11 \to \sqcup x1 \to \sqcup\sqcup 1 \to \sqcup\sqcup y \to$ ACCEPT.
- $010 \to \sqcup 10 \to \sqcup x0 \to \sqcup\sqcup 0 \to$ REJECT.

(c) The transition diagram for

$$T = (\{i, s_1, s_2, s_3, s_4, s_5, s_{\text{acc}}, s_{\text{rej}}\}, \{0,1\}, \{0,1,x,y,\sqcup\}, t, i, s_{\text{acc}}, s_{\text{rej}})$$

is at the end of the solution set, along with an example of an accepted string.

(d) The language L is countably infinite. Consider the function $f : \mathbb{N} \to L$ given by $f(m) = 0^m 1^{2m}$. It is easy to see that f is both injective and surjective hence bijective. Therefore, L is in one-to-one correspondence with $\mathbb{N}$, hence L is countably infinite.



E.g

	0 0 1 1 1 1	i
	⊔ 0 1 1 1 1	s_1
	0 x 1 1 1	s_2
	0 x 1 1 1	s_3
	⊔ x 1 1 1	s_1
	x x 1 1	s_2
	x x 1 1	s_3
	⊔ x 1 1	s_4
	x y 1	s_2
	x y 1	s_3
	⊔ y 1	s_4
	y y	s_2
	y y	s_3
		s_{acc} !