Petit

Math Assignment #3

Samuel

1. $\sum_{x=1}^{3}\left(\sum_{y=1}^{3} x \times y\right) = \sum_{n=1}^{3} n \times \sum_{m=1}^{3}$

$= 1\times(1+2+3) + 2\times(1+2+3) + 3\times(1+2+3)$

$= 1\times(6) + 2\times 6 + 3\times 6 = 6+12+18$

$= 36.$

2. $938\times x + 242\times y = \gcd(938, 242)$

$938 = 3\times 242 + 212$

$242 = 212\times 1 + 30$

$212 = 30\times 7 + 2$

$30 = 2\times 15 + 0$

$\gcd(938, 242) = 2.$

Reversing calculations

$2 = 212\times 1 - 30\times 7$

$= 212\times 1 - (242 - 212\times 1)\times 7$

$= 212\times 8 + 242(-7) = 242(-7) + 212\times 8$

$= 242(-7) + (938 - 242\times 3)\times 8$

$= 938\times 8 + 242(-7) + 242(-24)$

$= 938\times 8 + 242(-31).$

$x = 8. \qquad y = -31.$

3- $23 \times_{31} x = 1$.

- $31 = 23 \times 1 + 8$.
  $23 = 8 \times 2 + 7$.
  $8 = 7 \times 1 + 1$
  $7 = 7 \times 1 + 0$.
  $\gcd(31, 23) = 1$.

- $8 - 7 \times 1 = 1$
  $\Leftrightarrow$ $8 - (23 - 8 \times 2) = 1$
  $\Leftrightarrow$ $8 \times 23 + 8 \times 2 = 1$ $\Leftrightarrow 1 = 8 \times 3 - 23$

$8 = 31 - 23 \times 1$
$1 = 3 \times (31 - 23 \times 1) - 23$
$\therefore 1 = 3 \times 31 + 23(-3) - 23$
$1 = 3 \times 31 - 4 \times 23$

$23^{-1} \bmod 31 \equiv -4 \equiv 27$.
$23 \times x = 1 \pmod{43}$

$x = 27$.