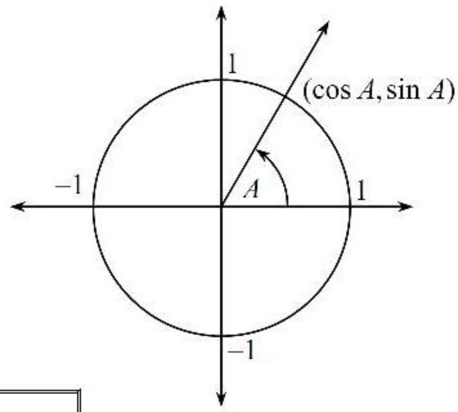
Trigonometry

Definitions

$$\tan A = \frac{\sin A}{\cos A} \qquad \cot A = \frac{1}{\tan A}$$

$$\sec A = \frac{1}{\cos A} \qquad \operatorname{cosec} A = \frac{1}{\sin A}$$



Trigonometric ratios of certain angles

A (degrees)	0°	90°	180°	270°	30°	45°	60°
A (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos A$	1	0	−1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\sin A$	0	1	0	−1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\tan A$	0	not defined	0	not defined	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Basic identities

$$\cos^2 A + \sin^2 A = 1$$

$$\cos(-A) = \cos A$$

$$\sin(-A) = -\sin A$$

$$\tan(-A) = -\tan A$$

Compound angle formulae

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double angle formulae

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)$$

$$\sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

Products to sums and differences

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

Sums and differences to products

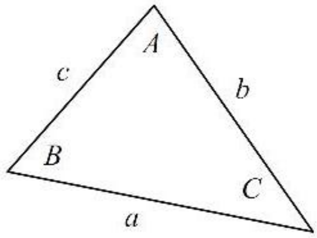
$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

Trigonometry of the triangle



Area = $\frac{1}{2}ab\sin C$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

In a right-angled triangle,

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

Algebra

Roots of quadratic equation: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial theorem: $(x+y)^n = \sum_{r=0}^{n}\binom{n}{r}x^{n-r}y^r = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots \binom{n}{r}x^{n-r}y^r + \cdots + \binom{n}{n}y^n$

De Moivre's theorem: $[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$ or $(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}(n\theta)$

Inverse of matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$: $\quad \dfrac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, where $\det A = ad - bc$

Sequences and series

Arithmetic sequence or series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Geometric sequence or series

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$S_\infty = \dfrac{a}{1-r}$, where $|r| < 1$

Number sets – notation

Natural numbers: $\mathbf{N} = \{1, 2, 3, 4, \cdots\}$

Whole numbers: $\mathbf{W} = \{0, 1, 2, 3, 4, \cdots\}$

Integers: $\mathbf{Z} = \{\cdots -3, -2, -1, 0, 1, 2, 3, \cdots\}$

Rational numbers: $\mathbf{Q} = \left\{ \frac{p}{q} \,\middle|\, p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0 \right\}$

Real numbers: $\mathbf{R}$

Complex numbers: $\mathbf{C} = \left\{ a + bi \,\middle|\, a \in \mathbf{R}, b \in \mathbf{R}, i^2 = -1 \right\}$

Calculus

Differentiation

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\ln x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}
a^x	$a^x \ln a$
$\cos x$	$-\sin x$
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\cos^{-1} \frac{x}{a}$	$-\frac{1}{\sqrt{a^2 - x^2}}$
$\sin^{-1} \frac{x}{a}$	$\frac{1}{\sqrt{a^2 - x^2}}$
$\tan^{-1} \frac{x}{a}$	$\frac{a}{a^2 + x^2}$

Product rule

$$y = uv \quad \Rightarrow \quad \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient rule

$$y = \frac{u}{v} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain rule

$$y = u(v(x)) \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dv}\frac{dv}{dx}$$

Newton-Raphson Iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Maclaurin series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(r)}(0)}{r!}x^r + \cdots$$

Taylor series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \cdots + \frac{h^r}{r!}f^{(r)}(x) + \cdots$$

Integration

Constants of integration omitted.

$f(x)$	$\int f(x)dx$
$x^n, (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$a^x \quad (a > 0)$	$\frac{a^x}{\ln a}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\frac{1}{\sqrt{a^2 - x^2}} \quad (a > 0)$	$\sin^{-1}\frac{x}{a}$
$\frac{1}{x^2 + a^2} \quad (a > 0)$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Solid of revolution about *x*-axis

$$\text{Volume} = \int_{x=a}^{x=b} \pi y^2 dx$$

Standard Taylor Series about 0

$$
\begin{aligned}
\sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 + \dots, \quad \text{for } x \in \mathbf{R} \\
\cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \dots, \quad \text{for } x \in \mathbf{R} \\
e^x &= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots, \quad \text{for } x \in \mathbf{R} \\
\ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots, \quad \text{for } -1 < x < 1 \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \dots, \quad \text{for } -1 < x < 1 \\
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots,
\end{aligned}
$$

$$\text{for } -1 < x < 1 \text{ and for any } \alpha \in \mathbf{R}.$$