MA2C03: ASSIGNMENT 3
DUE BY THURSDAY, MARCH 14
AT LECTURE OR IN THE MATHS OFFICE ROOM 0.6

Please write down clearly both your name and your student ID number on everything you hand in. Please attach a cover sheet with a declaration confirming that you know and understand College rules on plagiarism. Details can be found on `http://tcd-ie.libguides.com/plagiarism/declaration`.

1) (10 points) Let (V, E) be the graph with vertices a, b, c, d, e, f, g, and h, and edges ab, bc, cd, de, ae, ac, ad, af, cg, and dh.

(a) Is this graph connected? Justify your answer.
(b) Does this graph have an Eulerian trail? Justify your answer.
(c) Does this graph have an Eulerian circuit? Justify your answer.
(d) Does this graph have a Hamiltonian circuit? Justify your answer.
(e) Is this graph a tree? Justify your answer.

2a) (5 points) Prove that the condition 'a graph (V, E) has no pendant vertices' is necessary for the graph (V, E) to possess a Hamiltonian circuit.

(b) (5 points) Prove that the condition 'a graph (V, E) has no pendant vertices' is not sufficient for the graph (V, E) to possess a Hamiltonian circuit by giving an example of a graph (V, E) with no pendant vertices that has no Hamiltonian circuit.

3) Consider the connected graph with vertices A, B, C, D, E, F, G, H, I, J, K, L, M and N and with edges, listed with associated costs, in the following table:

AD	EI	HI	GH	IN	DE	AB	EF	LM	HL	AE	CF	HK
1	1	1	1	1	2	2	2	2	2	3	3	3
GJ	JL	BF	EG	KI	KL	BE	EH	MN	KM	FI	BC	DG
3	3	4	4	4	4	5	5	5	6	6	7	8

(a) (2 points) Draw the graph and label each edge with its cost.
(b) (9 points) Determine the minimum spanning tree generated by Kruskal's Algorithm, where that algorithm is applied with the queue specified in the table above. For each step of the algorithm, write down the edge that is added.

(c) (9 points) Determine the minimum spanning tree generated by Prim's Algorithm, starting from the vertex E, where that algorithm is applied with the queue specified in the table above. For each step of the algorithm, write down the edge that is added.

4) (10 points) Let $(\mathcal{V}, \mathcal{E})$ be the directed graph with vertices A, B, C, D, and E and edges (A, C), (A, D) (B, A), (B, C), (C, C), (D, D), (E, D), (B, E), (E, B), and (E, A).

(a) Draw this graph.
(b) Write down this graph's adjacency matrix.
(c) Give an example of an isomorphism φ from the graph $(\mathcal{V}, \mathcal{E})$ to itself such that $\varphi(C) = D$. Note that an isomorphism of directed graphs should also respect the direction of the edges.

5) Recall that in lecture we only defined the notion of connectivity for undirected graphs. The purpose of this problem is to develop the same notion for directed graphs.

(a) (3 points) Give a definition of what it means for a directed graph to be connected. Justify your answer by explaining why your definition works.
(b) (7 points) Write down an algorithm that checks whether a directed graph is connected. Explain how your algorithm implements the definition you gave in part (a).

6) (10 points) Let R be a relation on a set $V = \{a, b, c, d, e\}$ given by

$$R = \{(a, a), (a, e), (b, e), (b, b), (b, c), (a, d), (b, d), (c, c), (d, c), (d, d), (e, e), (e, a), (e, c)\}.$$

(a) Using the one-to-one correspondence between relations on finite sets and directed graphs, draw the directed graph corresponding to the relation R.
(b) Is R an equivalence relation? Justify your answer.
(c) If R is not an equivalence relation, which ordered pairs would have to be added to R to make it into an equivalence relation?