ST3009 Statistical methods for Computer Science

Week 2 questions - Samuel Petit - 17333946

Question 1 -

a) The space consists of all possible combinations that can be obtained from throwing a die 3 times. There are 6 different possibilities per throw so we get:

$$6 \times 6 \times 6 = 6^3 = \underline{\underline{216}}$$

b) The number of ways where no die in the 3 die falls on 2 is:

$$5 \times 5 \times 5 = 5^3 = 125.$$

This leaves $216 - 125 = 91$ different outcomes where there is at least one 2.

Thus the probability of at least one 2 being rolled is:

$$\frac{91}{216} \approx \underline{\underline{0.4213}}.$$

d) To get a sum of 17, we need 6 6 5 in any order, thus there are: $\frac{3!}{2!} = 3$ different possibilities

We then get $P(\text{Sum is } 17) = \frac{3}{216} \approx \underline{\underline{0.0139}}$

Question 1 c -

Editor - C:\Users\Samuel Petit\Documents\MATLAB\testmatlab.m

testmatlab.m

```
n = 1000000;
x = randi(6,n,1); % Value of the first dice
y = randi(6,n,1); % Value of the second dice
z = randi(6,n,1); % Value of the third dice

count1 = 0; % Keeping count where at least one 2 occurs
for k = 1:n
    if (x(k) == 2 || (y(k)== 2) || z(k)== 2)
        count1 = count1 + 1;
    end
end
fprintf('There were %d times where at least one 2 was thrown', count1)
fprintf('Out of %d throws\n', n)
fprintf('the probability is %.3f\n', count1./n)
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> testmatlab
There were 420974 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.421
>> testmatlab
There were 422159 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.422
>> testmatlab
There were 421385 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.421
>> testmatlab
There were 420220 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.420
```

Question 1e

Let A be the event: the sum of 2 die is 11.

There are only 2 possible outcomes for this event: 5 6 and 6 5.

here the ~~entire~~ sample space is 6^2

Thus we have:

$$P(A) = \frac{2}{6^2} = \frac{2}{36} = \frac{1}{18} = \underline{\underline{0.0556}}.$$

Question 2 a)

We get the answer with the following calculations

P(get a 1 first throw) × P(get a 5 or 6 side die)

+ P(do not get a 1 on first throw) × P(get a 5 on a 20 side die)

$$= \frac{1}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{20} = \frac{1}{6^2} + \frac{5}{120}$$

$$\simeq 0.0694.$$

b) In this case, since the two events are still independent we can use a similar approach:

P(2nd throw is a 15) = P(1st throw is not 1) P(2nd throw is 15)

$$= \frac{5}{6} \times \frac{1}{20} = \frac{5}{120} = \frac{1}{24} \simeq 0.0417$$

Q3 - Let A be the event: the suspect has the characteristic and B the event: the inspector is convinced ~~the~~ suspect is guilty.

Let's find $P(B|A)$.

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)P(A) + P(A|B^c)\,P(B^c)}$$

$$= \frac{0.6 \times 1}{0.6 \times 1 + 0.2 \times 0.4} = \frac{0.6}{0.6 + 0.08} = \frac{0.6}{0.68} \approx \underline{0.882}$$

Question 4 on next page.

Question 4

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testmatlab.m

```matlab
% Values from the question sheet
prior = [
    0.05 0.1 0.05 0.05;
    0.05 0.1 0.05 0.05;
    0.05 0.05 0.1 0.05;
    0.05 0.05 0.1 0.05;
];
p_2bars = [
    0.75 0.95 0.75 0.05;
    0.05 0.75 0.95 0.75;
    0.01 0.05 0.75 0.95;
    0.01 0.01 0.05 0.75;
];
size = 16; % amount of values in each matrix
e = 0; % evidence value
result = zeros(4,4); % result matrix
% We apply bayes theorem to the problem
% Similarly to question 2 we use marginalisation to find P(O)
% start by computing the evidence value (to do so we iterate through
% the available values & sum their matching product).
for i = 1:size
        prior_and_2bars = prior(i) * p_2bars(i);
        e = e + prior_and_2bars;
end
% We then use the evidence value to compute the final results
for i = 1:size
    result(i) = prior(i) * p_2bars(i) / e;
end
disp(result); % display results
```

Command Window

```
>>
>>
>>
>>
>>
>>
>>
>> testmatlab
    0.0744    0.1885    0.0744    0.0050
    0.0050    0.1488    0.0942    0.0744
    0.0010    0.0050    0.1488    0.0942
    0.0010    0.0010    0.0099    0.0744
```