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Homework 11

Q1 - I will first turn the system of linear equations to the form $A\vec{x} = \vec{b}$.

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

Let's now create the augmented matrix $(A:b)$.

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 1 & -1 & -3 & 0 \end{pmatrix}.$$

I will now turn this matrix into reduced row echelon form.

$$\begin{array}{l} R2 \to R2 - 2R1 \\ R3 \to R3 - R1 \end{array} \longrightarrow \begin{array}{c} \\ \\ \\ \end{array} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -2 & -1 \\ 0 & -2 & -4 & -2 \end{pmatrix} \begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

$$\begin{array}{l} R1 \\ R2 \\ R3 \end{array} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & -2 & -4 & -2 \end{pmatrix} \xleftarrow{R2 \times (-1)}$$

$$\begin{array}{l} R1 \Rightarrow R1 - R2 \\ R3 \to R3 + 2R2 \end{array} \longrightarrow \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let's now solve the system:

we have:

$$x - z = 1$$
$$y + 2z = 1$$

by deciding $x = t$ where t is any real number

$$\boxed{\begin{array}{l} z = t - 1. \\ y = 1 - 2z = 3 - 2t. \end{array}}$$

and $x = t$.

Q2- I will first create the augmented matrix (B:I).

(with $B = \begin{pmatrix} 2 & 4 & 1 \\ 3 & 3 & 2 \\ 4 & 1 & 4 \end{pmatrix}$ and I the 3x3 identity matrix).

$$\left(\begin{array}{ccc|ccc} 2 & 4 & 1 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} R1 \\ R2 \\ R3 \end{array} \quad \xleftarrow{\frac{1}{2} R1}$$

$R2 \to R2 - 3R1$
$R3 \to R3 - 4R1$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -3 & \frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & -7 & 2 & -2 & 0 & 1 \end{array}\right)\begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

$$\begin{array}{l} R1 \\ R2 \\ R3 \end{array}\left(\begin{array}{ccc|ccc} 1 & 2 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & -7 & 2 & -2 & 0 & 1 \end{array}\right) \quad \xleftarrow{-\frac{1}{3} R2}$$

$R1 \to R1 - 2R2$
$R3 \to R3 + 7R2$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{6} & -\frac{1}{2} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{5}{6} & \frac{3}{2} & -\frac{7}{3} & 1 \end{array}\right)\begin{array}{l} R1 \\ R2 \\ R3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{5}{6} & -\frac{1}{2} & \frac{2}{3} & 0 \\ 0 & 1 & -\frac{1}{6} & \frac{1}{2} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{array}\right) \quad \xleftarrow{\frac{6}{5} R3}$$

$R1 \to R1 - \frac{5}{6} R3$
$R2 \to R2 + \frac{1}{6} R3$

$$\longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 3 & -1 \\ 0 & 1 & 0 & \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ 0 & 0 & 1 & \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{array}\right)$$

we then have the inverse of the matrix:

$$\begin{pmatrix} -2 & 3 & -1 \\ \frac{4}{5} & -\frac{4}{5} & \frac{1}{5} \\ \frac{9}{5} & -\frac{14}{5} & \frac{6}{5} \end{pmatrix}.$$

Q3 - I will first write this set of equations to the form $A\vec{x} = \vec{b}$.

$$\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}.$$

with $A = \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}$. $\det(A) = 9 + 2 = 11$.

I will now calculate the determinants of A_1 and A_2, which are the matrix A with the first and second columns replaced by $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$:

$A_1 \begin{pmatrix} 5 & -1 \\ 1 & 3 \end{pmatrix}$, $A_2 \begin{pmatrix} 3 & 5 \\ 2 & 1 \end{pmatrix}$.

$$\det(A_1) = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 15 + 1 = 16.$$

$$\det(A_2) = \begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} = 3 - 10 = -7.$$

We then have:

$$x = \frac{\det(A_1)}{\det(A)} = \frac{16}{11}.$$

$$y = \frac{\det(A_2)}{\det(A)} = -\frac{7}{11}.$$