NAME: **Samuel Petit**

STUDENT NUMBER: **17333946**

Please indicate your answers by entering the option ((i), (ii), (iii) or (iv)) where asked. You should append the completed document as a pdf with your typewritten worked solutions including MATLAB code) and upload to Blackboard.

Q 4.23

(i)

L =

1.5000	0	0	0
-2.0000	1.0000	0	0
0.5000	1.0000	1.5000	0
-2.0000	3.5000	-0.5000	1.0000

U =

4.0000	-1.0000	3.0000	2.0000
0	-1.0000	3.0000	0.5000
0	0	2.0000	1.0000
0	0	0	3.0000

(ii)

L =

1.0000	0	0	0
-2.0000	1.0000	0	0
0.5000	1.5000	1.0000	0
-2.0000	3.0000	-0.5000	1.0000

U =

4.0000	-1.0000	3.0000	2.0000
0	-2.0000	3.0000	0.5000
0	0	4.0000	2.0000
0	0	0	3.0000

(iii)

L =

1.5000	0	0	0
-2.0000	1.0000	0	0
0.5000	1.0000	1.0000	0
-2.0000	2.0000	-0.5000	1.0000

U =

3.0000	-1.5000	3.0000	2.0000
0	-2.0000	3.0000	0.5000
0	0	4.0000	2.5000
0	0	0	1.0000

(iv)

L =

1.5000	0	0	0
-2.0000	1.5000	0	0
0.5000	1.5000	1.5000	0
-2.0000	3.0000	-0.5000	1.5000

U =

4.0000	-1.0000	3.0000	2.0000
0	-2.0000	3.0000	0.5000
0	0	4.0000	2.0000
0	0	0	2.0000

Your Answer ((i) – (iv)): ii

MATLAB code :

```
A = [4,-1,3,2; -8, 0, -3, -3.5; 2, -3.5, 10, 3.75; -8, -4, 1, -0.5];
[L, U] = lu_decomposition(A);
disp(L);
disp(U);

function [lower_triangle, upper_triangle] = lu_decomposition(A)
    nb_rows = size(A, 1);
    lower_triangle = eye(nb_rows); % L is identity
    for index = 1 : nb_rows
        lower_triangle(index + 1 : nb_rows, index) = A(index + 1 : nb_rows,
index) / A(index, index);
        for l = index + 1 : nb_rows
            A(l, :) = A(l, :) - lower_triangle(l, index) * A(index, :);
        end
    end
```

```
    upper_triangle = A;
end
```

Output given by the program:

```
  1.0000       0       0       0
 -2.0000  1.0000       0       0
  0.5000  1.5000  1.0000       0
 -2.0000  3.0000 -0.5000  1.0000

  4.0000 -1.0000  3.0000  2.0000
       0 -2.0000  3.0000  0.5000
       0       0  4.0000  2.0000
       0       0       0  3.0000
```

Q 5.17

You need only to indicate the best team and the worst team (from teams 1 to 6).

Your Answers: Best 2 and 5 (same score) Worst 1

Matlab code:

```
A = ([0 0 0 1 0 0; 1 0 1 0 1 1; 0 1 0 0 1 0; 1 1 0 0 1 0; 1 1 1 0 0 1; 1 0
0 0 1 0]);

[V, D] = eig(A);
disp(V);
```

Output :

Columns 1 through 3

```
  0.1761 + 0.0000i   0.3379 + 0.0000i   0.0000 + 0.0000i
  0.5155 + 0.0000i  -0.1443 + 0.0000i   0.0000 + 0.0000i
  0.3938 + 0.0000i  -0.7555 + 0.0000i  -0.7071 + 0.0000i
  0.4611 + 0.0000i   0.1290 + 0.0000i   0.0000 + 0.0000i
  0.5155 + 0.0000i  -0.1443 + 0.0000i  -0.0000 + 0.0000i
  0.2642 + 0.0000i   0.5068 + 0.0000i   0.7071 + 0.0000i
```

Columns 4 through 6

```
 -0.5773 - 0.0000i  -0.5773 + 0.0000i   0.5774 + 0.0000i
 -0.0000 + 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i
  0.0000 - 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  0.5774 + 0.0000i   0.5774 + 0.0000i  -0.5773 + 0.0000i
 -0.0000 + 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i
  0.5773 + 0.0000i   0.5773 - 0.0000i  -0.5774 + 0.0000i
```

Using column 1 :

0.1761
0.5155
0.3938
0.4611
0.5155
0.2642

We can see that team 2 and 5 have the same (best) score and team 1 has the worst score.

Q 6.3

(i) $b = 4.6831 \times 10^{-8}, \quad m = 0.022, \quad population(1985) = 1014\ million$
(ii) $b = 4.8932 \times 10^{-8}, \quad m = 0.022, \quad population(1985) = 1024\ million$
(iii) $b = 4.6931 \times 10^{-8}, \quad m = 0.012, \quad population(1985) = 1038\ million$
(iv) $b = 4.9932 \times 10^{-8}, \quad m = 0.014, \quad population(1985) = 1042\ million$

Your Answer ((i)-(iv)): **iii**

Written justification on next page

We have to use the form $y = ae^{bx}$ where y is p (population), a and b are unknowns and x is the year.

$$y = ae^{bx}$$

$$\Leftrightarrow \log(y) = \log(a) + bx$$

Let $A = \log(a)$, $Y = \log(y)$, $B = b$, $X = x$

$$\Leftrightarrow Y = A + BX$$

We then find the following:

$$\Sigma Y = nA + B\Sigma X$$
$$\Sigma XY = nAX + B\Sigma X^2$$

Let's find:

$$\Sigma X = 1900 + \dots + 2010 = 13800$$
$$\Sigma Y = \Sigma \log(y) = \log(200) + \dots + \log(1370)$$
$$= 47.3756.$$
$$\Sigma X^2 = 1900^2 + \dots + 2010^2 = 27214000$$
$$\Sigma XY = 1900 \cdot \log(200) + \dots + 2010 \cdot \log(1370)$$
$$= 9338.4885$$

We then find the following 2 equations:

$47.318557 = 7A + 13800B$

$93384.488 = 13800A + 2721400B$

Solving these for A and B gives us:

$A = -16.8746$

$B = 0.0119884 \approx 0.012$

$A = \ln(a) \Leftrightarrow a = e^{A} = e^{-16.8746}$

$a = 4.69304 \times 10^{-8}$

$B = b = 0.012$.

Applying this to $y = ae^{bx}$

gives us:

$p = 4.69304 \times 10^{-8} e^{0.012x}$

Thus for $x = 1985$:

$p = 4.69304 \times 10^{-8} e^{0.012 \times 1985} = 1038.75$