Week #9 Exercises

Qs. #1

1. Simplify (without a calculator)

$$\frac{\sqrt{5}-1}{\sqrt{5}+1}+\frac{\sqrt{5}+1}{\sqrt{5}-1}$$

2. $(+k : low \leq k \leq high \wedge P(k) : f(k)) =$
$g(low) + g(low+1) + \cdots + g(high)$
where $g(k) = if\ P(k)\ then\ f(k)\ else\ 0$
Example: ($\mathbb{N}$ is the set of Natural numbers)
$(+k \in \mathbb{N} : 1 \leq k \leq 9 \wedge odd(k) : k) = 1+3+5+7+9 = 25$
Calculate:
$(+k \in \mathbb{N} : 0 \leq k \leq 20 \wedge prime(k) : k)$
where
$prime(k)$ iff k has exactly two divisors, itself and 1.

Qs. #2

The functions, $(a\ div\ b)$ and $(a\ mod\ b)$ are defined so that, for $b \neq 0$,

$$a = b * (a\ div\ b) + (a\ mod\ b)\ \wedge 0 \leq (a\ mod\ b) < |b|$$

Determine

- $123\ div\ 10$ and $123\ mod\ 10$
  i.e. determine $123\ div\ 10$ and $123\ mod\ 10$ so that
  $123 = 10 * (123\ div\ 10) + (123\ mod\ 10) \wedge$
  $0 \leq (123\ mod\ 10) < 10$
  is satisfied.
- $(-123)\ div\ 10$ and $(-123)\ mod\ 10$
  i.e. determine $(-123)\ div\ 10$ and $(-123)\ mod\ 10$ so that
  $123 = 10 * ((-123)\ div\ 10) + ((-123)\ mod\ 10) \wedge$
  $0 \leq ((-123)\ mod\ 10) < 10$
  is satisfied.

Qs. 2 Cont'd

- $123\ div\ (-10)$ and $123\ mod\ (-10)$
  i.e. determine $123\ div\ (-10)$ and $123\ mod\ (-10)$ so that
  $123 = 10 * (123\ div\ (-10)) + (123\ mod\ (-10)) \wedge$
  $0 \leq (123\ mod\ (-10)) < 10$
  is satisfied.

- $(-123)\ div\ (-10)$ and $(-123)\ mod\ (-10)$
  i.e. determine $(-123)\ div\ (-10)$ and $(-123)\ mod\ (-10)$ so that
  $-123 = (-10) * ((-123)\ div\ (-10)) + ((-123)\ mod\ (-10)) \wedge$
  $0 \leq ((-123)\ mod\ (-10)) < 10$
  is satisfied.