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Please indicate your answers by entering the option ((i), (ii), (iii) or (iv)) where asked.

You should append the completed document as a pdf with your type written worked solutions and upload to Blackboard by Friday 22nd of February 2019.

Q 2.31

Part (a):

(i) 4

(ii) 13

(iii) 26

(iv) 18

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Your Answer (i)-(iv): (ii)
```

Proof:

$$A = \begin{pmatrix} 1 & 5 & 4 \\ 2 & 3 & 6 \\ 1 & 1 & 1 \end{pmatrix}$$
$$Det(A) = 1 * [3 - 6] - 5 * [2 - 6] + 4 * [2 - 3] = -3 + 20 - 4 = 13$$

Part (b):

(i) 0

(ii) 12

(iii) 7

(iv) 4

```
Your Answer (i)-(iv): (i)
```

Proof:

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix} = \begin{pmatrix} 13 & 14 & 15 & 16 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 13 & 14 & 15 & 16 \\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13} \\ 9 & 10 & 11 & 12 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 13 & 14 & 15 & 16 \\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13} \\ 0 & \frac{4}{13} & \frac{8}{13} & \frac{12}{13} \\ 1 & 2 & 3 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 13 & 14 & 15 & 16 \\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13} \\ 0 & \frac{4}{13} & \frac{8}{13} & \frac{12}{13} \\ 0 & \frac{12}{13} & \frac{24}{13} & \frac{36}{13} \end{pmatrix} = \begin{pmatrix} 13 & 14 & 15 & 16 \\ 0 & \frac{12}{13} & \frac{24}{13} & \frac{36}{13} \\ 0 & \frac{4}{13} & \frac{8}{13} & \frac{12}{13} \\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13} \end{pmatrix}$$
$$= \begin{pmatrix} 13 & 14 & 15 & 16 \\ 0 & \frac{12}{13} & \frac{24}{13} & \frac{36}{13} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{8}{13} & \frac{16}{13} & \frac{24}{13} \end{pmatrix}$$

We have found using fundamental matrix operations that an entire row is made of 0's. Thus we can say that the determinant will be 0.

$$Det(B) = 0$$

MATLAB Code :

The question specifically asks for MATLAB code, however we were still asked to do the matrix operations typed. Here is the MATLAB code for it.

```
Determinant([1 5 4; 2 3 6; 1 1 1]);
Determinant([1 2 3 4; 5 6 7 8; 9 10 11 12; 13 14 15 16]);

function D = Determinant (A)
    sizeMatrix = size(A);
    if sizeMatrix(1) ~= sizeMatrix(2) || sizeMatrix(1) <= 1
        disp("Error: matrix needs to be n*n or bigger than 1*1");
        return;
    end
    D = det(A);
    disp(D);
end
```

Q 3.2

Part (a):

(i) 0.1241

(ii) 0.8125

(iii) 0.074995

(iv) 0.003462

```
Your Answer (i)-(iv): (ii)
```

Justification - Bisection method

We have

$a \ = \ 0$

$b \ = \ 1$

$f(a) \ = \ 0 \ - \ 2e^{-0} \ = \ -2[negative]$

$f(b) \ = \ 1 \ - \ 2e^{-1} \ = \ 0.26424[positive]$

Following the steps of the algorithm we have:

$x_{NS1} \ = \ \frac{0+1}{2} \ = \ 0.5$

$f(x_{NS1}) \ = \ 0.5 \ - \ 2^{-0.5} \ = \ -0.71306[negative]$

We will now use the interval [0.5, 1]

$x_{NS2} \ = \ \frac{0.5+1}{2} \ = \ 0.75$

$f(x_{NS2}) \ = \ 0.75 \ - \ 2^{-0.75} \ = \ -0.1947[negative]$

We will now use the interval [0.75, 1]

$x_{NS3} \ = \ \frac{0.75+1}{2} \ = \ 0.875$

$f(x_{NS3}) \ = \ 0.875 \ - \ 2^{-0.875} \ = \ 0.0412[positive]$

We will now use the interval [0.75, 0.875]

$x_{NS4} \ = \ \frac{0.75+0.875}{2} \ = \ 0.8125$

Thus the answer is 0.8125 : (ii)

Part (b):

(i) 0.72481

(ii) 0.85261

(iii) 0.62849

(iv) 0.17238

```
Your Answer (i)-(iv): (ii)
```

Justification - Secant method

We have :

$$f(x) \ = \ x \ - \ 2e^{-x}$$
$$x_0 \ = \ 0$$
$$x_1 \ = \ 1$$

Calculating until x_4:

$x_2 \ = \ 1 \ - \ \frac{f(1)*(0-1)}{f(0)-f(1)} \ = \ 1 \ - \ \frac{-0.26424}{-2.26424} \ = \ 0.8833$
$x_3 \ = \ 0.8833 \ - \ \frac{f(0.8833)*(1-0.8833)}{f(1)-f(0.8833)} \ = \ 0.8833 \ - \ \frac{0.05647*0.1167}{0.26424-0.05647} \ = \ 0.851582$
$x_4 \ = \ 0.851582 \ - \ \frac{f(0.851582)*(0.8833-0.851582)}{f(0.8833)-f(0.851582)} \ = \ 0.851582 \ - \ \frac{-6.0169*10^{-5}}{0.058367} \ = \ 0.85261$

Thus the answer is 0.85261 : (ii)

Part ©:

(i) 0.65782

(ii) 0.59371

(iii) 0.45802

(iv) 0.85261

```
Your Answer (i)-(iv): (iv)
```

Justification - Newton’s Method

We have:

$x_{i+1} \ = \ x_i \ - \ \frac{f(x_i)}{f'(x_i)}$

The function:

$f(x) \ = \ x \ - \ 2e^{-x}$

$f'(x) \ = \ 1 \ + \ 2e^{-x}$

Using : $x_1 \ = \ 1$

$x_2 \ = \ 1 \ - \ \frac{1-2e^{-1}}{1+2e^{-1}} \ = \ 0.847777$
$x_3 \ = \ 0.847777 \ - \ \frac{0.847777-2e^{-0.847777}}{1+2e^{-0.847777}} \ = \ 0.853$

Q 4.24

(i)

Inverse(a)=

-0.7143 0.0 1.4286

0.2571 0.1000 0.2857

-0.2286 -0.2000 0.8571

Inverse(b)=

1.6667 2.8889 -2.2222 1.0000

0.0 0.3333 -0.3333 0.0

-0.3333 -0.4444 0.1111 0.0

1.5000 2.0000 -1.5000 0.5000

(ii)

Inverse(a)=

0.7243 0.0 1.3286

1.2571 0.1000 0.2757

-0.2386 -0.2010 0.9571

Inverse(b)=

1.6677 2.9889 3.2222 1.01700

0.3433 -0.3433 0.3333 0.00371

-0.3433 -0.2879 0.2111 0.0

1.2400 2.0120 -1.5783 0.5600

(iii)

Inverse(a)=

0.7143 0.003 2.3276

1.2671 0.1100 0.3759

-0.2486 -0.2110 0.9771

Inverse(b)=

1.6877 3.9789 3.2002 2.01800

0.3533 -0.4433 0.3333 0.02371

-0.3443 -0.2999 0.3121 0.0382

1.2420 3.0130 -1.5733 0.5610

(iv)

Inverse(a)=

0.8343 1.01 1.3336

2.2572 0.1003 0.3857

-0.2486 -0.2110 0.9671

Inverse(b)=

1.6777 4.9889 3.2232 1.11700

0.3443 -0.3443 0.3233 0.07371

-0.3443 -0.2979 0.3211 0.07800

1.2480 2.1220 -1.5883 0.5621

```
Your Answer (i)-(iv): (i)
```

Justification

Matrix a

$$inverse(a) = \begin{pmatrix} -1 & 2 & 1 \\ 2 & 2 & -4 \\ 0.2 & 1 & 0.5 \end{pmatrix}^{-1}$$

Using an augmented 3*3 Matrix

$$\begin{bmatrix} -1 & 2 & 1 & | & 1 & 0 & 0 \\ 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0.2 & 1 & 0.5 & | & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ -1 & 2 & 1 & | & 1 & 0 & 0 \\ 0.2 & 1 & 0.5 & | & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0 & 3 & -1 & | & 1 & \frac{1}{2} & 0 \\ 0.2 & 1 & 0.5 & | & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0 & 3 & -1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 0.8 & 0.9 & | & 0 & -0.1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0 & 3 & -1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1.16666 & | & -0.26666 & -0.23333 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0 & 3 & -1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1.16666 & | & -0.26666 & -0.23333 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0 & 3 & -1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & | & -0.22857 & -0.2 & 0.85714 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 & | & 0 & 1 & 0 \\ 0 & 3 & 0 & | & 0.77142 & 0.3 & 0.85714 \\ 0 & 0 & 1 & | & -0.22857 & -0.2 & 0.85714 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 0 & | & -0.91428 & 0.2 & 3.42857 \\ 0 & 3 & 0 & | & 0.77142 & 0.3 & 0.85714 \\ 0 & 0 & 1 & | & -0.22857 & -0.2 & 0.85714 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & 0 & | & -0.91428 & 0.2 & 3.42857 \\ 0 & 1 & 0 & | & 0.25714 & 0.1 & 0.28571 \\ 0 & 0 & 1 & | & -0.22857 & -0.2 & 0.85714 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 & | & -1.42857 & 0 & 2.85714 \\ 0 & 1 & 0 & | & 0.25714 & 0.1 & 0.28571 \\ 0 & 0 & 1 & | & -0.22857 & -0.2 & 0.85714 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & | & -0.71428 & 0 & 1.42857 \\ 0 & 1 & 0 & | & 0.25714 & 0.1 & 0.28571 \\ 0 & 0 & 1 & | & -0.22857 & -0.2 & 0.85714 \end{bmatrix}$$

Thus the inverse is :

$$inverse(a) \ = \ \begin{pmatrix} -0.71428 & 0 & 1.42857 \\ 0.25714 & 0.1 & 0.28571 \\ -0.22857 & -0.2 & 0.85714 \end{pmatrix}$$

Matrix b

$$inverse(b) \ = \ \begin{pmatrix} -1 & -2 & 1 & 2 \\ 1 & 1 & -4 & -2 \\ 1 & -2 & -4 & -2 \\ 2 & -4 & 1 & -2 \end{pmatrix}^{-1}$$

Using an augmented 4*4 Matrix

$$
\begin{aligned}
&\left[\begin{array}{cccc|cccc}
-1 & -2 & 1 & 2 & 1 & 0 & 0 & 0 \\
1 & 1 & -4 & -2 & 0 & 1 & 0 & 0 \\
1 & -2 & -4 & -2 & 0 & 0 & 1 & 0 \\
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
1 & 1 & -4 & -2 & 0 & 1 & 0 & 0 \\
1 & -2 & -4 & -2 & 0 & 0 & 1 & 0 \\
-1 & -2 & 1 & 2 & 1 & 0 & 0 & 0
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & 3 & -\frac{9}{2} & -1 & 0 & 1 & 0 & -\frac{1}{2} \\
1 & -2 & -4 & -2 & 0 & 0 & 1 & 0 \\
-1 & -2 & 1 & 2 & 1 & 0 & 0 & 0
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & 3 & -\frac{9}{2} & -1 & 0 & 1 & 0 & -\frac{1}{2} \\
0 & 0 & -\frac{9}{2} & -1 & 0 & 0 & 1 & -\frac{1}{2} \\
-1 & -2 & 1 & 2 & 1 & 0 & 0 & 0
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & 3 & -\frac{9}{2} & -1 & 0 & 1 & 0 & -\frac{1}{2} \\
0 & 0 & -\frac{9}{2} & -1 & 0 & 0 & 1 & -\frac{1}{2} \\
0 & -4 & \frac{3}{2} & 1 & 1 & 0 & 0 & \frac{1}{2}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & -4 & \frac{3}{2} & 1 & 1 & 0 & 0 & \frac{1}{2} \\
0 & 0 & -\frac{9}{2} & -1 & 0 & 0 & 1 & -\frac{1}{2} \\
0 & 3 & -\frac{9}{2} & -1 & 0 & 1 & 0 & -\frac{1}{2}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & -4 & \frac{3}{2} & 1 & 1 & 0 & 0 & \frac{1}{2} \\
0 & 0 & -\frac{9}{2} & -1 & 0 & 0 & 1 & -\frac{1}{2} \\
0 & 0 & -\frac{27}{8} & -\frac{1}{4} & \frac{3}{4} & 1 & 0 & -\frac{1}{8}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & -4 & \frac{3}{2} & 1 & 1 & 0 & 0 & \frac{1}{2} \\
0 & 0 & -\frac{9}{2} & -1 & 0 & 0 & 1 & -\frac{1}{2} \\
0 & 0 & 0 & \frac{1}{2} & \frac{3}{4} & 1 & -\frac{3}{4} & \frac{1}{4}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & -4 & \frac{3}{2} & 1 & 1 & 0 & 0 & \frac{1}{2} \\
0 & 0 & -\frac{9}{2} & -1 & 0 & 0 & 1 & -\frac{1}{2} \\
0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & -4 & \frac{3}{2} & 1 & 1 & 0 & 0 & \frac{1}{2} \\
0 & 0 & -\frac{9}{2} & 0 & \frac{3}{2} & 2 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & -2 & 0 & 0 & 0 & 1 \\
0 & -4 & \frac{3}{2} & 0 & -\frac{1}{2} & -2 & \frac{3}{2} & 0 \\
0 & 0 & -\frac{9}{2} & 0 & \frac{3}{2} & 2 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2}
\end{array}\right] \\
=&\left[\begin{array}{cccc|cccc}
2 & -4 & 1 & 0 & 3 & 4 & -3 & 2 \\
0 & -4 & \frac{3}{2} & 0 & -\frac{1}{2} & -2 & \frac{3}{2} & 0 \\
0 & 0 & -\frac{9}{2} & 0 & \frac{3}{2} & 2 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2}
\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \left[\begin{array}{cccc|cccc} 2 & -4 & 1 & 0 & 3 & 4 & -3 & 2 \\ 0 & -4 & \frac{3}{2} & 0 & -\frac{1}{2} & -2 & \frac{3}{2} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \\
&= \left[\begin{array}{cccc|cccc} 2 & -4 & 0 & 0 & \frac{10}{3} & \frac{40}{9} & -\frac{28}{9} & 2 \\ 0 & -4 & 0 & 0 & 0 & -\frac{4}{3} & \frac{4}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \\
&= \left[\begin{array}{cccc|cccc} 2 & -4 & 0 & 0 & \frac{10}{3} & \frac{40}{9} & -\frac{28}{9} & 2 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \\
&= \left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & \frac{10}{3} & \frac{52}{9} & -\frac{40}{9} & 2 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2} \end{array}\right] \\
&= \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{5}{3} & \frac{26}{9} & -\frac{20}{9} & 1 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{4}{9} & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 1 & \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2} \end{array}\right]
\end{aligned}
$$

We find the inverse matrix on the right side of the extended matrix:

$$
inverse(b) = \begin{pmatrix} \frac{5}{3} & \frac{26}{9} & -\frac{20}{9} & 1 \\ 0 & \frac{1}{3} & -\frac{1}{3} & 0 \\ -\frac{1}{3} & -\frac{4}{9} & \frac{1}{9} & 0 \\ \frac{3}{2} & 2 & -\frac{3}{2} & \frac{1}{2} \end{pmatrix}
$$