CS2010: Data Structures and Algorithms II

Strings

Ivana.Dusparic@scss.tcd.ie

Outline of String algorithms

› String sorting algorithms
  – Exploit properties of strings to speed up sorting
  – Radix sort

› Tries
  – Data structures for searching with string keys – more efficient than general purpose ones previously discussed, eg hashtables, search trees

› Substring search
  – Interesting problem- multiple approaches illustrating different algorithm design techniques

› Regular expressions
  – Searching for incomplete patterns rather than exact substrings

› Data compression
  – Save storage, faster network transfer
  – Run length encoding, Huffman compression

What are strings?

› Sequences of characters
  – Text
  – Genome sequences

› What are characters then?
  – In C – char data type is 8 bit integer, 7-bit ascii, 256 characters max
  – In Java – char data type, 16 bit unsigned int, range '\u0000' to '\uffff' (0 to 65,535)

java.lang.String

› Immutable sequence of characters

› Implements Comparable

› Implements CharSequences
  – Methods length() – number of characters
  – charAt(i) – returns the i-th character

  – Concatenation- concatenate one string to the end of another java.lang.String
  – Substring – extract a subsequence of characters

Immutable?

› String cannot be modified
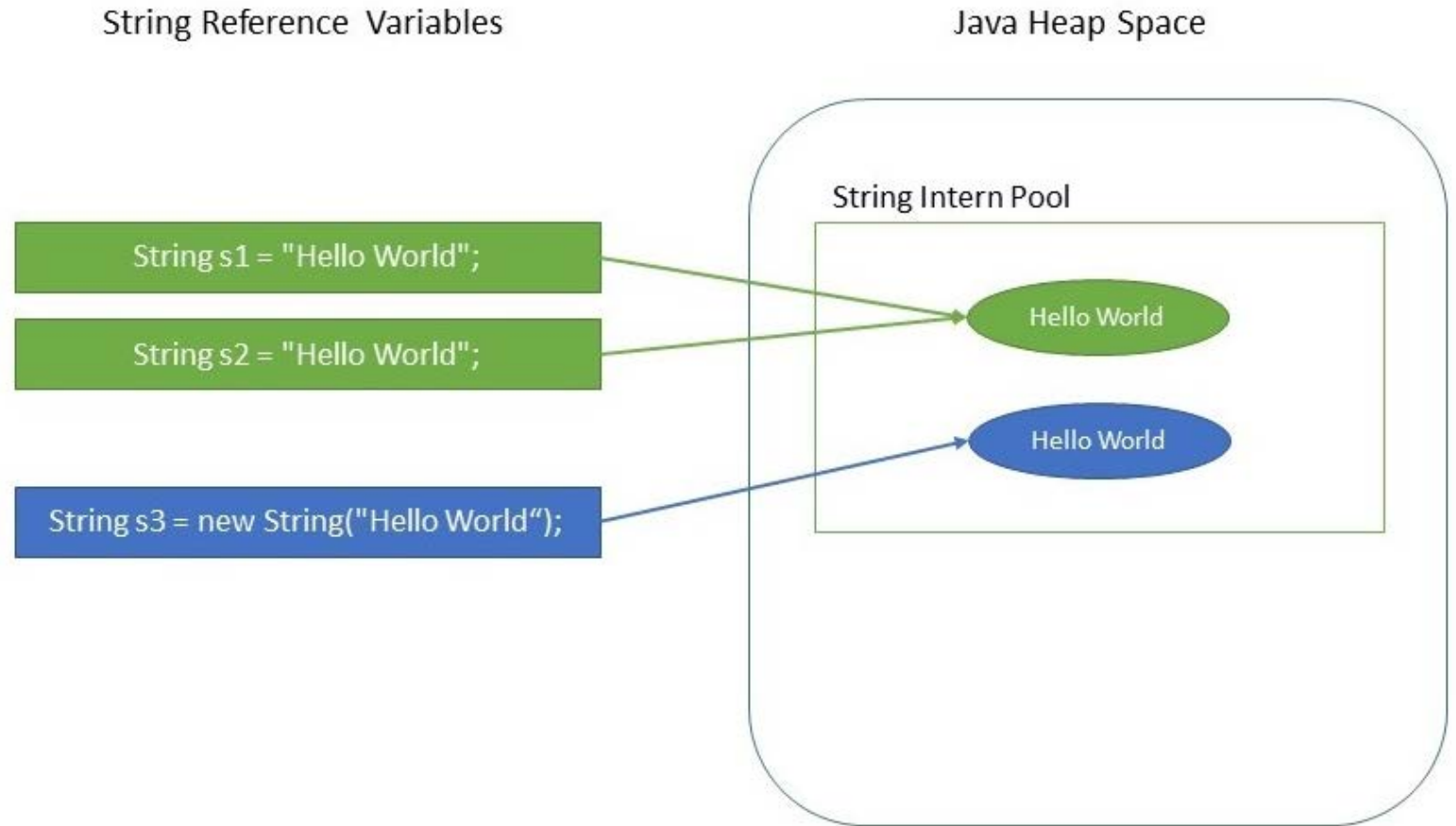
› What happens here then?

String test = "blah";

test = "meh";

System.out.println(test); //what does this print?

New string is created and reference test now points to it instead of the old one – reference to "blah" text has been lost, it's still unchanged somewhere in memory

Memory leaks/garbage collection

String pool

String Reference Variables

Java Heap Space

String Intern Pool

String s1 = "Hello World";

String s2 = "Hello World";

String s3 = new String("Hello World");

Hello World

Hello World

https://www.baeldung.com/java-string-immutable

Other implications of immutability

› Security
  – Parameters in many methods which could introduce vulnerability – security threats, eg network connection is passed a string – it could be modified to connect to a different machine, or a modified file name can be passed in etc

› Thread-safe
  – No need for synchronisation if shared between threads – no thread can modify it

› Can be used as keys in symbol tables

› Can calculate and save hashcode – efficiency

› https://www.baeldung.com/java-string-immutable

String implementation

char [] value

int length - saved for efficiency so it doesn't have to be calculated every time we need it

int hash -   calculated as s[0]*31^(n-1) + s[1]*31^(n-2) + … + s[n-1]

Cost of String operations

› How long does it take to reverse a String?

```
public static String reverse(String s)
{
    String rev = "";
    for (int i = s.length() - 1; i >= 0; i--)
        rev += s.charAt(i);          ←——— quadratic time
    return rev;
}
```

› Alternative – mutable sequence of characters, StringBuilder

```
public static String reverse(String s)
{
    StringBuilder rev = new StringBuilder();
    for (int i = s.length() - 1; i >= 0; i--)
        rev.append(s.charAt(i));     ←——— linear time
    return rev.toString();
}
```

Substring operation

› Java 6 vs Java 7 (and up)

String x = "abcdef";

x = x.substring(1,3);

System.out.println(x);

https://www.programcreek.com/2013/09/the-substring-method-in-jdk-6-and-jdk-7/

Substring operation in Java 6

› Cheap (cost of 1)

› Memory leaks – unused portion of the string cant be garbage collected

› Points to same value

› Count modified

› Offset modified

The heap

char[] value
int count
int offset

String object

a b c d e f
0 1 2 3 4 5

x

char[] value
int count
int offset

String object
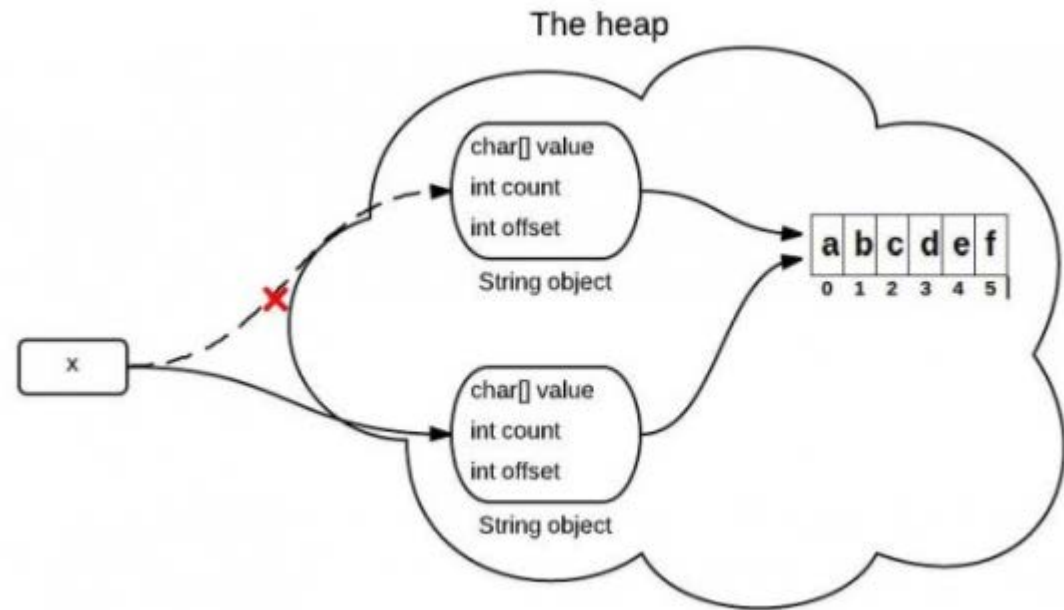
Substring operation in Java 7 (now)

› Cost of N, new string needs to be created-characters copied into it

› Old string can be garbage collected

The heap

char[] value
int count
int offset

String object

a b c d e f
0 1 2 3 4 5

x

char[] value
int count
int offset

String object

b c
0 1
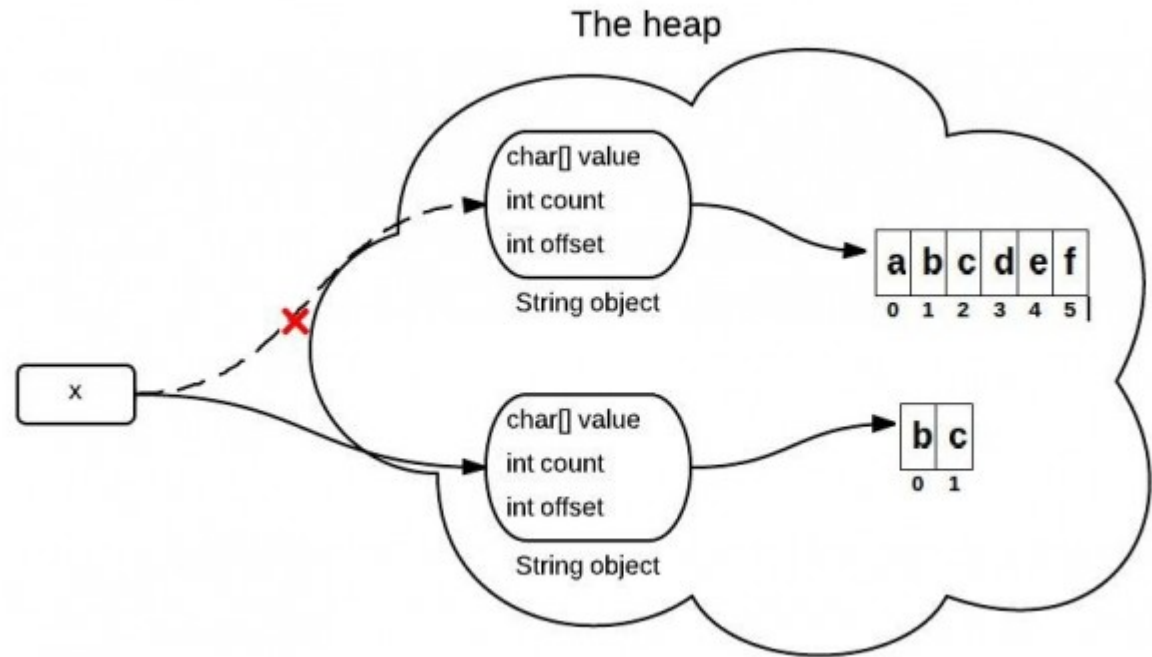
Comparing 2 Strings

Q. How many character compares to compare two strings of length W?

p	r	e	f	e	t	c	h
0	1	2	3	4	5	6	7
p	r	e	f	i	x	e	s

Running time. Proportional to length of longest common prefix.
- Proportional to W in the worst case.
- But, often sublinear in W.

Different Alphabets

› Performance depends on the size of the alphabet (i.e., unique characters)

Digital key. Sequence of digits over fixed alphabet.

Radix. Number of digits R in alphabet.

name	R()	lgR()	characters
BINARY	2	1	01
OCTAL	8	3	01234567
DECIMAL	10	4	0123456789
HEXADECIMAL	16	4	0123456789ABCDEF
DNA	4	2	ACTG
LOWERCASE	26	5	abcdefghijklmnopqrstuvwxyz
UPPERCASE	26	5	ABCDEFGHIJKLMNOPQRSTUVWXYZ
PROTEIN	20	5	ACDEFGHIKLMNPQRSTVWY
BASE64	64	6	ABCDEFGHIJKLMNOPQRSTUVWXYZabcdef ghijklmnopqrstuvwxyz0123456789+/
ASCII	128	7	*ASCII characters*
EXTENDED_ASCII	256	8	*extended ASCII characters*
UNICODE16	65536	16	*Unicode characters*

Key-Indexed Counting

Review: summary of the performance of sorting algorithms

Frequency of operations.

algorithm	guarantee	random	extra space	stable?	operations on keys
insertion sort	$\frac{1}{2}N^2$	$\frac{1}{4}N^2$	1	✔	compareTo()
mergesort	$N \lg N$	$N \lg N$	N	✔	compareTo()
quicksort	$1.39\,N \lg N^*$	$1.39\,N \lg N$	$c \lg N$		compareTo()
heapsort	$2\,N \lg N$	$2\,N \lg N$	1		compareTo()

* probabilistic

Lower bound. $\sim N \lg N$ compares required by any compare-based algorithm.

Q. Can we do better (despite the lower bound)?

A. Yes, if we don't depend on key compares. ⟵ use array accesses to make R-way decisions (instead of binary decisions)

Key-indexed counting

› Basis for other more complex sorting algorithms
  – LSD and MSD (least and most significant digit)

› Specialized sorting algorithm which works best when the following conditions are met:
  – Input consists of collection of n items
  – Maximum possible value of each of the individual item is K

Key-indexed counting: assumptions about keys

Assumption. Keys are integers between 0 and $R - 1$.

Implication. Can use key as an array index.

Applications.

- Sort string by first letter.
- Sort class roster by section.
- Sort phone numbers by area code.
- Subroutine in a sorting algorithm. [stay tuned]

Remark. Keys may have associated data $\Rightarrow$ can't just count up number of keys of each value.

Input		sorted result	
name	*section*	*(by section)*	
Anderson	2	Harris	1
Brown	3	Martin	1
Davis	3	Moore	1
Garcia	4	Anderson	2
Harris	1	Martinez	2
Jackson	3	Miller	2
Johnson	4	Robinson	2
Jones	3	White	2
Martin	1	Brown	3
Martinez	2	Davis	3
Miller	2	Jackson	3
Moore	1	Jones	3
Robinson	2	Taylor	3
Smith	4	Williams	3
Taylor	3	Garcia	4
Thomas	4	Johnson	4
Thompson	4	Smith	4
White	2	Thomas	4
Williams	3	Thompson	4
Wilson	4	Wilson	4

keys are small integers

Key-indexed counting demo

Goal. Sort an array a[] of N integers between 0 and $R-1$.

$R = 6$

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

use a for 0
     b for 1
     c for 2
     d for 3
     e for 4
     f for 5

Key-indexed counting demo

Goal. Sort an array a[] of N integers between 0 and $R - 1$.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

count requencies

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

offset by 1
[stay tuned]

r	count[r]
a	0
b	2
c	3
d	1
e	2
f	1
-	3

Key-indexed counting demo

Goal. Sort an array a[] of N integers between 0 and $R - 1$.
- Count frequencies of each letter using key as index.
- **Compute frequency cumulates which specify destinations.**
- Access cumulates using key as index to move items.
- Copy back into original array.

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

compute cumulates

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	0
b	2
c	5
d	6
e	8
f	9
	12

6 keys < d, 8 keys < e
so d's go in a[6] and a[7]

Key-indexed counting demo

Goal. Sort an array a[] of N integers between 0 and $R - 1$.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- **Access cumulates using key as index to move items.**
- Copy back into original array.

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

move
Items

i	a[i]
0	d
1	a
2	c
3	f
4	f
5	b
6	d
7	b
8	f
9	b
10	e
11	a

r	count[r]
a	2
b	5
c	6
d	8
e	9
f	12
-	12

i	aux[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

Key-indexed counting demo

Goal. Sort an array a[] of N integers between 0 and $R - 1$.

- Count frequencies of each letter using key as index.
- Compute frequency cumulates which specify destinations.
- Access cumulates using key as index to move items.
- Copy back into original array.

```
int N = a.length;
int[] count = new int[R+1];

for (int i = 0; i < N; i++)
    count[a[i]+1]++;

for (int r = 0; r < R; r++)
    count[r+1] += count[r];

for (int i = 0; i < N; i++)
    aux[count[a[i]]++] = a[i];

for (int i = 0; i < N; i++)
    a[i] = aux[i];
```

copy
back

i	a[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

r	count[r]
a	2
b	5
c	6
d	8
e	9
f	12
-	12

i	aux[i]
0	a
1	a
2	b
3	b
4	b
5	c
6	d
7	d
8	e
9	f
10	f
11	f

Key-indexed counting: analysis

Proposition. Key-indexed takes time proportional to $N+R$.

Proposition. Key-indexed counting uses extra space proportional to $N+R$.

Stable? ✓

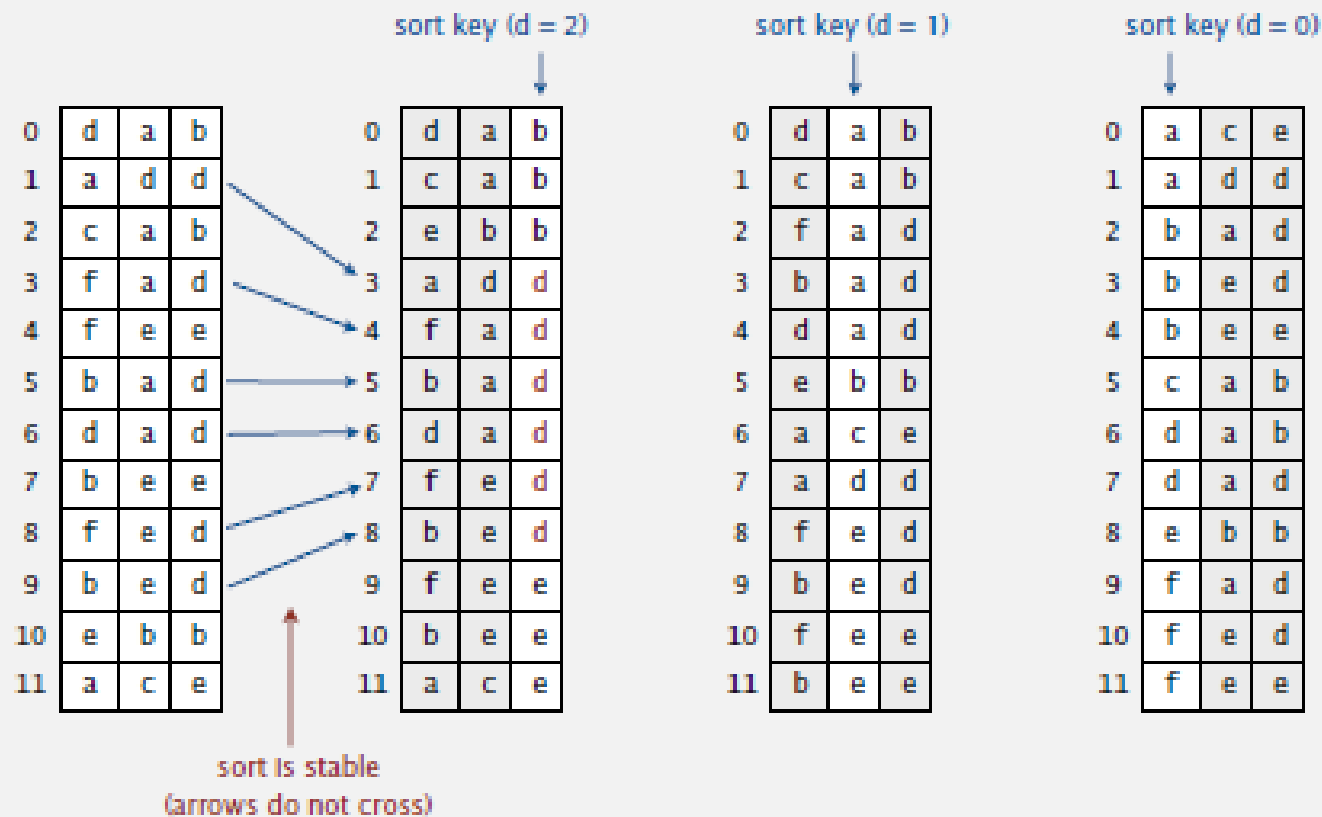|					
a[0]	Anderson	2	Harris	1	aux[0]
a[1]	Brown	3	Martin	1	aux[1]
a[2]	Davis	3	Moore	1	aux[2]
a[3]	Garcia	4	Anderson	2	aux[3]
a[4]	Harris	1	Martinez	2	aux[4]
a[5]	Jackson	3	Miller	2	aux[5]
a[6]	Johnson	4	Robinson	2	aux[6]
a[7]	Jones	3	White	2	aux[7]
a[8]	Martin	1	Brown	3	aux[8]
a[9]	Martinez	2	Davis	3	aux[9]
a[10]	Miller	2	Jackson	3	aux[10]
a[11]	Moore	1	Jones	3	aux[11]
a[12]	Robinson	2	Taylor	3	aux[12]
a[13]	Smith	4	Williams	3	aux[13]
a[14]	Taylor	3	Garcia	4	aux[14]
a[15]	Thomas	4	Johnson	4	aux[15]
a[16]	Thompson	4	Smith	4	aux[16]
a[17]	White	2	Thomas	4	aux[17]
a[18]	Williams	3	Thompson	4	aux[18]
a[19]	Wilson	4	Wilson	4	aux[19]

Radix Sorts – LSD and MSD

Radix sorts

› Non-comparative integer sorting algorithm

› Sorts data with integer keys by grouping keys by the individual digits which share the same significant position and value.

› LSD radix sort
  – short keys come before longer keys
  – keys of the same length are sorted lexicographically
  – Ie, normal order of integer representations, such as the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

› MSD radix sort
  – lexicographic order, which is suitable for sorting strings, such as words, or fixed-length integer representations
  – A sequence such as "b, c, d, e, f, g, h, i, j, ba" would be lexicographically sorted as "b, ba, c, d, e, f, g, h, i, j"
  – (1 to 10 would be output as 1, 10, 2, 3, 4, 5, 6, 7, 8, 9,)

LSD Sort

LSD Sort – Sort by Least Significant Digit first

LSD string (radix) sort.

- Consider characters from right to left.
- Stably sort using d^{th} character as the key (using key-indexed counting).

0	d	a	b
1	a	d	d
2	c	a	b
3	f	a	d
4	f	e	e
5	b	a	d
6	d	a	d
7	b	e	e
8	f	e	d
9	b	e	d
10	e	b	b
11	a	c	e

sort key (d = 2)

0	d	a	b
1	c	a	b
2	e	b	b
3	a	d	d
4	f	a	d
5	b	a	d
6	d	a	d
7	f	e	d
8	b	e	d
9	f	e	e
10	b	e	e
11	a	c	e

sort key (d = 1)

0	d	a	b
1	c	a	b
2	f	a	d
3	b	a	d
4	d	a	d
5	e	b	b
6	a	c	e
7	a	d	d
8	f	e	d
9	b	e	d
10	f	e	e
11	b	e	e

sort key (d = 0)

0	a	c	e
1	a	d	d
2	b	a	d
3	b	e	d
4	b	e	e
5	c	a	b
6	d	a	b
7	d	a	d
8	e	b	b
9	f	a	d
10	f	e	d
11	f	e	e

sort is stable
(arrows do not cross)

LSD exercise

Provide the trace of sorting the array of strings given in a table below using LSD sort, providing an equivalent table for each of the 3 passes of the algorithm.

A	R	M
C	A	T
C	A	B
A	R	T
B	U	G
B	U	S

LSD stability and correctness

LSD string sort: correctness proof

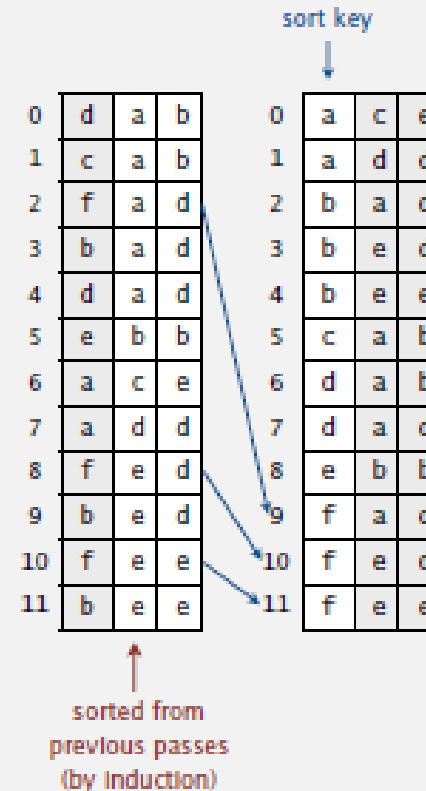**Proposition.** LSD sorts fixed-length strings in ascending order.

Pf. [by induction on i]
After pass i, strings are sorted by last i characters.
- If two strings differ on sort key,
  key-indexed sort puts them in proper
  relative order.
- If two strings agree on sort key,
  stability keeps them in proper relative order.

Proposition. LSD sort is stable.
Pf. Key-indexed counting is stable.

sort key

0	d	a	b	0	a	c	e
1	c	a	b	1	a	d	d
2	f	a	d	2	b	a	d
3	b	a	d	3	b	e	d
4	d	a	d	4	b	e	e
5	e	b	b	5	c	a	b
6	a	c	e	6	d	a	b
7	a	d	d	7	d	a	d
8	f	e	d	8	e	b	b
9	b	e	d	9	f	a	d
10	f	e	e	10	f	e	d
11	b	e	e	11	f	e	e

sorted from
previous passes
(by induction)

LSD in Java

```java
public class LSD
{
    public static void sort(String[] a, int W)          ← fixed-length W strings
    {
        int R = 256;                                     ← radix R
        int N = a.length;
        String[] aux = new String[N];

        for (int d = W-1; d >= 0; d--)                   ← do key-indexed counting
        {                                                    for each digit from right to left
            int[] count = new int[R+1];
            for (int i = 0; i < N; i++)
                count[a[i].charAt(d) + 1]++;
            for (int r = 0; r < R; r++)                  ← key-indexed counting
                count[r+1] += count[r];
            for (int i = 0; i < N; i++)
                aux[count[a[i].charAt(d)]++] = a[i];
            for (int i = 0; i < N; i++)
                a[i] = aux[i];
        }
    }
}
```

Performance wrt other sorting algorithms

Frequency of operations.

algorithm	guarantee	random	extra space	stable?	operations on keys
insertion sort	$\frac{1}{2} N^2$	$\frac{1}{4} N^2$	1	✔	compareTo()
mergesort	$N \lg N$	$N \lg N$	N	✔	compareTo()
quicksort	$1.39 \, N \lg N$ [*]	$1.39 \, N \lg N$	$c \lg N$		compareTo()
heapsort	$2 \, N \lg N$	$2 \, N \lg N$	1		compareTo()
LSD sort [†]	$2 \, W (N + R)$	$2 \, W (N + R)$	$N + R$	✔	charAt()

[*] probabilistic
[†] fixed-length W keys

LSD Performance

› Key indexed counting
  – 11n+4R+1
    › Initialize arrays n+R+1
    › First loop 3n
    › Second loop 3R
    › Third loop 5n
    › Fourth loop 2n

› LSD
  – 10wn+n+WR
  – W x LSD apart from third loop which is just 1 x rather than LSD x
  – R usually much smaller than N, so proportional to wn – linear!

Example - sorting large integer arrays

String sorting interview question

Problem. Sort one million 32-bit integers.

Ex. Google (or presidential) interview.

Which sorting method to use?
- Insertion sort.
- Mergesort.
- Quicksort.
- Heapsort.
- LSD string sort.

LSD Sorting large integer arrays

› https://algs4.cs.princeton.edu/51radix/LSD.java.html

› Break up 32-bit integer in 4 8-bit characters

› Use bit shifting and masking to isolate characters to sort by

› What if strings are not same length? MSD!

MSD Sort

MSD – Sort by Most Significant Digit first

› Does simply reversing LSD work?
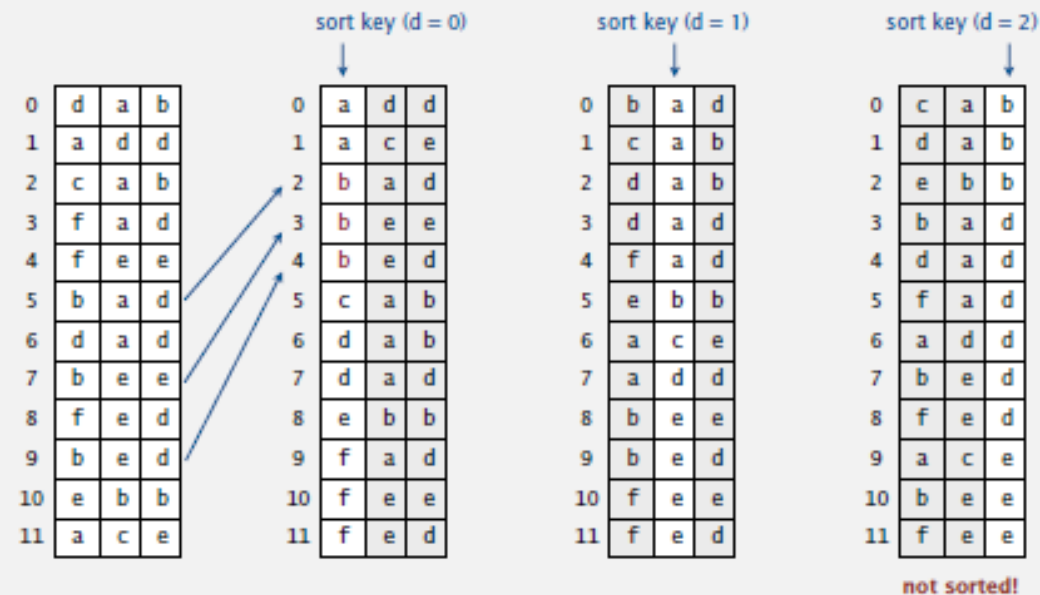
MSD string sort

› Similar to quicksort

› Partition array into R (radix) pieces according to the first character (most significant digit) using key-indexed counting

› Recursively sort all strings that start with each character (key-indexed counts delineate subarrays to sort)

MSD String sort
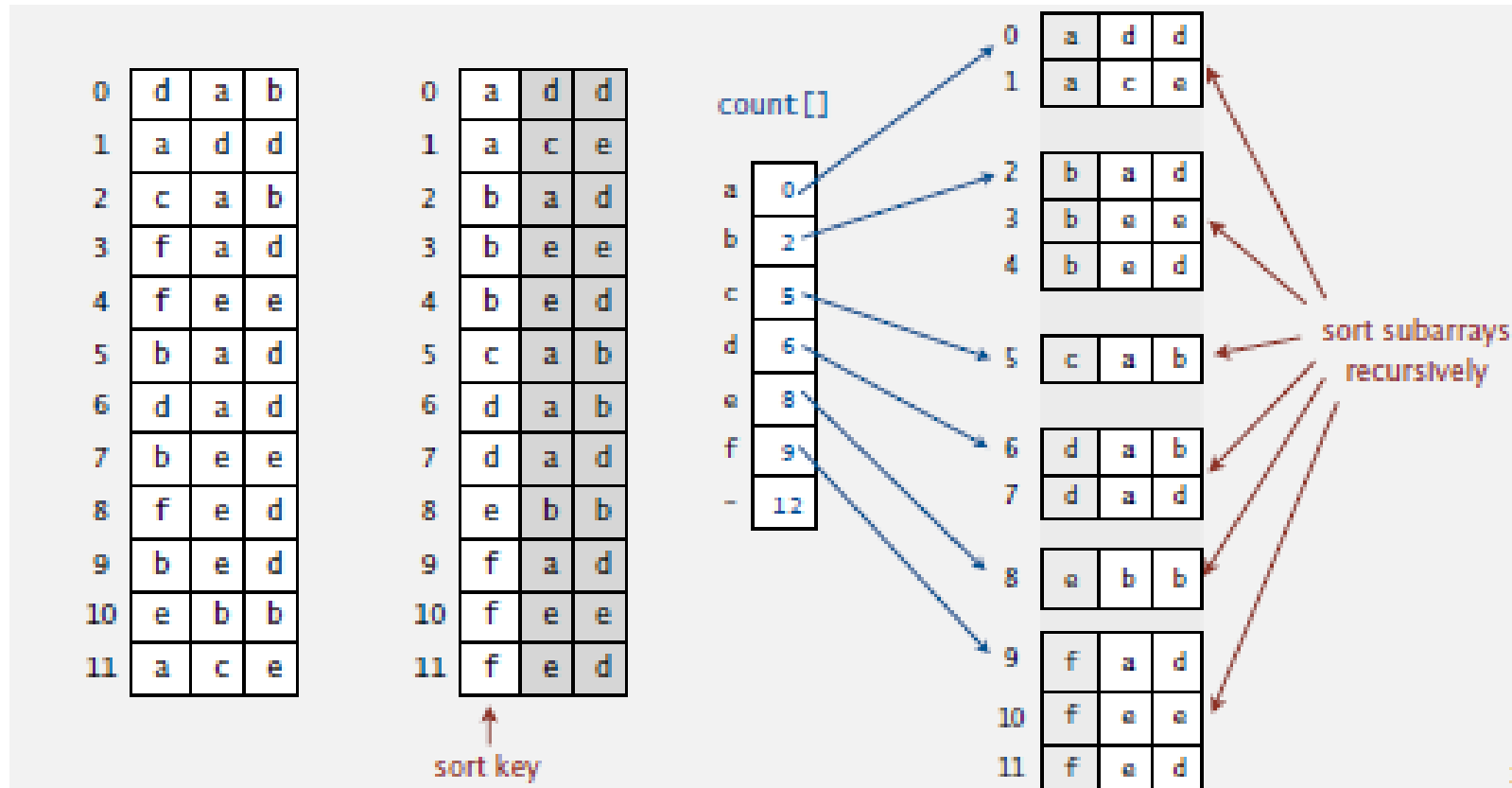
MSD string sort: example

input

she	are	are	are	are	are	are	are	are
sells	by	by	by	by	by	by	by	by
seashells	she	sells	seashells	sea	sea	sea	seas	sea
by	sells	seashells	sea	seashells	seashells	seashells	seashells	seashells
the	seashells	sea	seashells	seashells	seashells	seashells	seashells	seashells
sea	sea	sells	sells	sells	sells	sells	sells	sells
shore	shore	seashells	sells	sells	sells	sells	sells	sells
the	shells	she	she	she	she	she	she	she
shells	she	shore	shore	shore	shore	shore	shore	shore
she	sells	shells	shells	shells	shells	shells	shore	shells
sells	surely	she	she	she	she	she	she	she
are	seashells	surely	surely	surely	surely	surely	surely	surely
surely	the	the	the	the	the	the	the	the
seashells	the	the	the	the	the	the	the	the

need to examine every character in equal keys

end of string goes before any char value

							output
are	are	are	are	are	are	are	are
by	by	by	by	by	by	by	by
sea	sea	sea	sea	sea	sea	sea	sea
seashells	seashells	seashells	seashells	seashells	seashells	seashells	seashells
seashells	seashells	seashells	seashells	seashell	seashells	seashells	seashells
sells	sells	sells	sells	sells	sells	sells	sells
sells	sells	sells	sells	sells	sells	sells	sells
she	she	she	she	she	she	she	she
shore	sshore	shore	shells	she	she	she	she
shells	hells	shells	she	shells	shells	shells	shells
she	she	she	shore	shore	shore	shore	shore
surely	surely	surely	surely	surely	surely	surely	surely
the	the	the	the	the	the	the	the
the	the	the	the	the	the	the	the

Trace of recursive calls for MSD string sort (no cutoff for small subarrays, subarrays of size 0 and 1 omitted)

Variable length Strings

Treat strings as if they had an extra char at end (smaller than any char).

why smaller?

0	s	e	a	-1						
1	s	e	a	s	h	e	l	l	s	-1
2	s	e	l	l	s	-1				
3	s	h	e	-1						
4	s	h	e	-1						
5	s	h	e	l	l	s	-1			
6	s	h	o	r	e	-1				
7	s	u	r	e	l	y	-1			

she before shells

```
private static int charAt(String s, int d)
{
    if (d < s.length()) return s.charAt(d);
    else return -1;
}
```

C strings. Have extra char '\0' at end ⇒ no extra work needed.

MSD String sort – Java Implementation

```java
public static void sort(String[] a)
{
    aux = new String[a.length];
    sort(a, aux, 0, a.length - 1, 0);
}

private static void sort(String[] a, String[] aux, int lo, int hi, int d)
{
    if (hi <= lo) return;
    int[] count = new int[R+2];
    for (int i = lo; i <= hi; i++)
        count[charAt(a[i], d) + 2]++;
    for (int r = 0; r < R+1; r++)
        count[r+1] += count[r];
    for (int i = lo; i <= hi; i++)
        aux[count[charAt(a[i], d) + 1]++] = a[i];
    for (int i = lo; i <= hi; i++)
        a[i] = aux[i - lo];

    for (int r = 0; r < R; r++)
        sort(a, aux, lo + count[r], lo + count[r+1] - 1, d+1);
}
```

recycles aux[] array but not count[] array
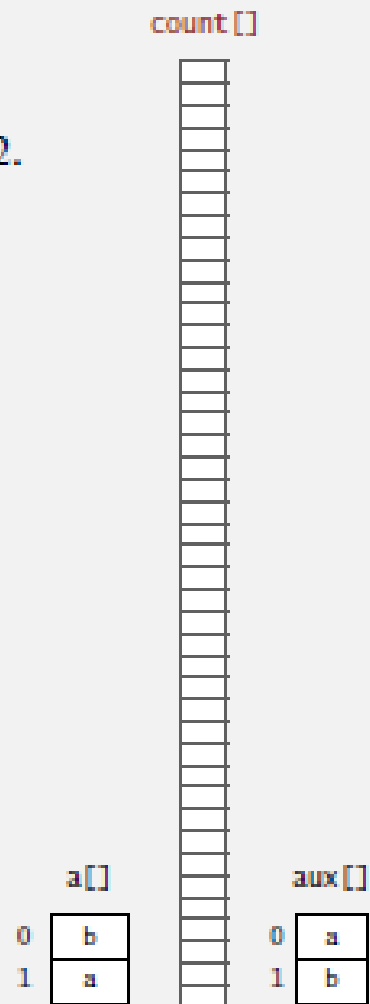
key-indexed counting

sort R subarrays recursively

MSD – improvements

Observation 1. Much too slow for small subarrays.

- Each function call needs its own `count[]` array.
- ASCII (256 counts): 100x slower than copy pass for $N=2$.
- Unicode (65,536 counts): 32,000x slower for $N=2$.

Observation 2. Huge number of small subarrays
because of recursion.

count []

a[]

| 0 | b |
| 1 | a |

aux []

| 0 | a |
| 1 | b |

Yep, you guessed it – cutoff to Insertion sort

Cutoff to insertion sort

Solution. Cutoff to insertion sort for small subarrays.

- Insertion sort, but start at d^{th} character.

```java
private static void sort(String[] a, int lo, int hi, int d)
{
    for (int i = lo; i <= hi; i++)
        for (int j = i; j > lo && less(a[j], a[j-1], d); j--)
            exch(a, j, j-1);
}
```

- Implement less() so that it compares starting at d^{th} character.

```java
private static boolean less(String v, String w, int d)
{
    for (int i = d; i < Math.min(v.length(), w.length()); i++)
    {
        if (v.charAt(i) < w.charAt(i)) return true;
        if (v.charAt(i) > w.charAt(i)) return false;
    }
    return v.length() < w.length();
}
```

MSD Performance

Frequency of operations.

algorithm	guarantee	random	extra space	stable?	operations on keys
insertion sort	$\frac{1}{2} N^2$	$\frac{1}{4} N^2$	1	✔	compareTo()
mergesort	$N \lg N$	$N \lg N$	N	✔	compareTo()
quicksort	$1.39 \, N \lg N$ *	$1.39 \, N \lg N$	$c \lg N$		compareTo()
heapsort	$2 \, N \lg N$	$2 \, N \lg N$	1		compareTo()
LSD sort †	$2 \, W (N+R)$	$2 \, W (N+R)$	$N+R$	✔	charAt()
MSD sort ‡	$2 \, W (N+R)$	$N \log_R N$	$N + D \, R$	✔	charAt()

D = function-call stack depth
(length of longest prefix match)

* probabilistic
† fixed-length W keys
‡ average-length W keys

MSD improvements - American flag sort

› Analogy to Dutch national flag, partition array into many "stripes"

› In-place variant of radix sort that distributes items into hundreds of buckets

› How?

› Cut off to insertion sort

› Replaces recursion with explicit stack

› In-place, eliminate auxiliary array (no stability)

MSD vs Quicksort for String sorting

Disadvantages of MSD string sort.
- Extra space for aux[].
- Extra space for count[].
- Inner loop has a lot of instructions.
- Accesses memory "randomly" (cache inefficient).

Disadvantage of quicksort.
- Linearithmic number of string compares (not linear).
- Has to rescan many characters in keys with long prefix matches.

doesn't rescan
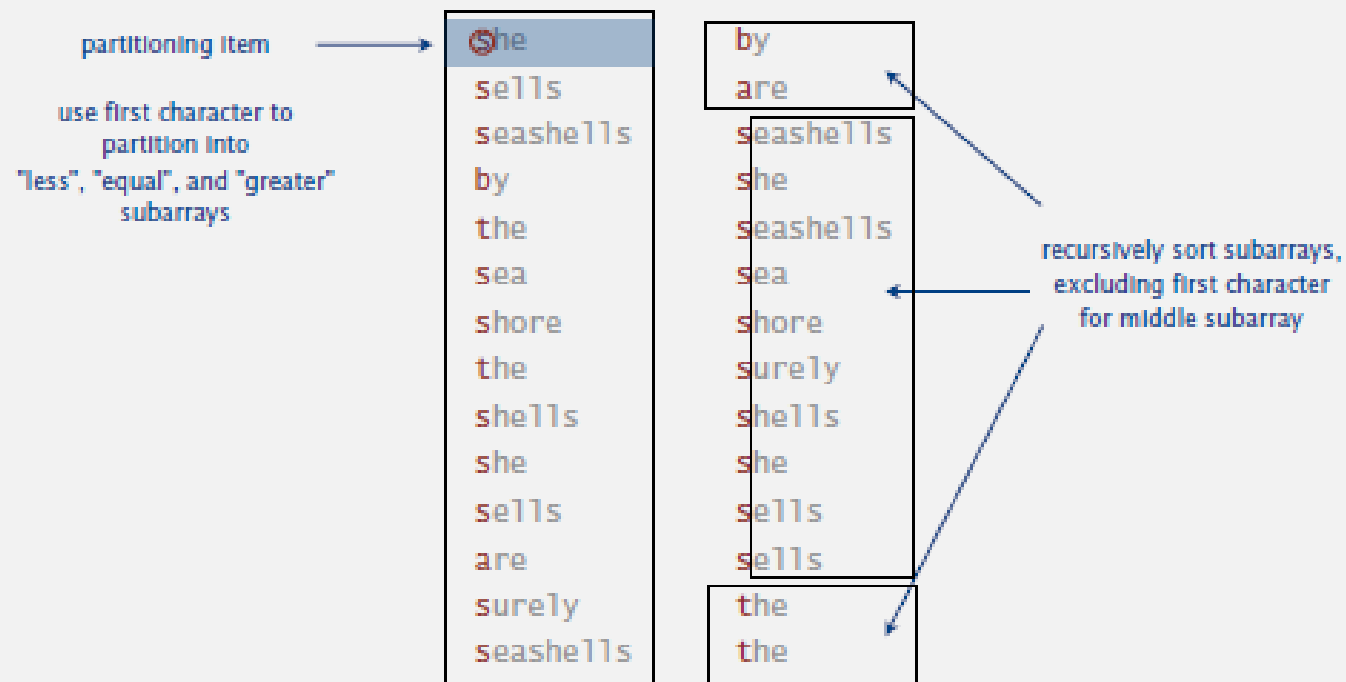characters

tight inner loop,
cache friendly

Goal. Combine advantages of MSD and quicksort.

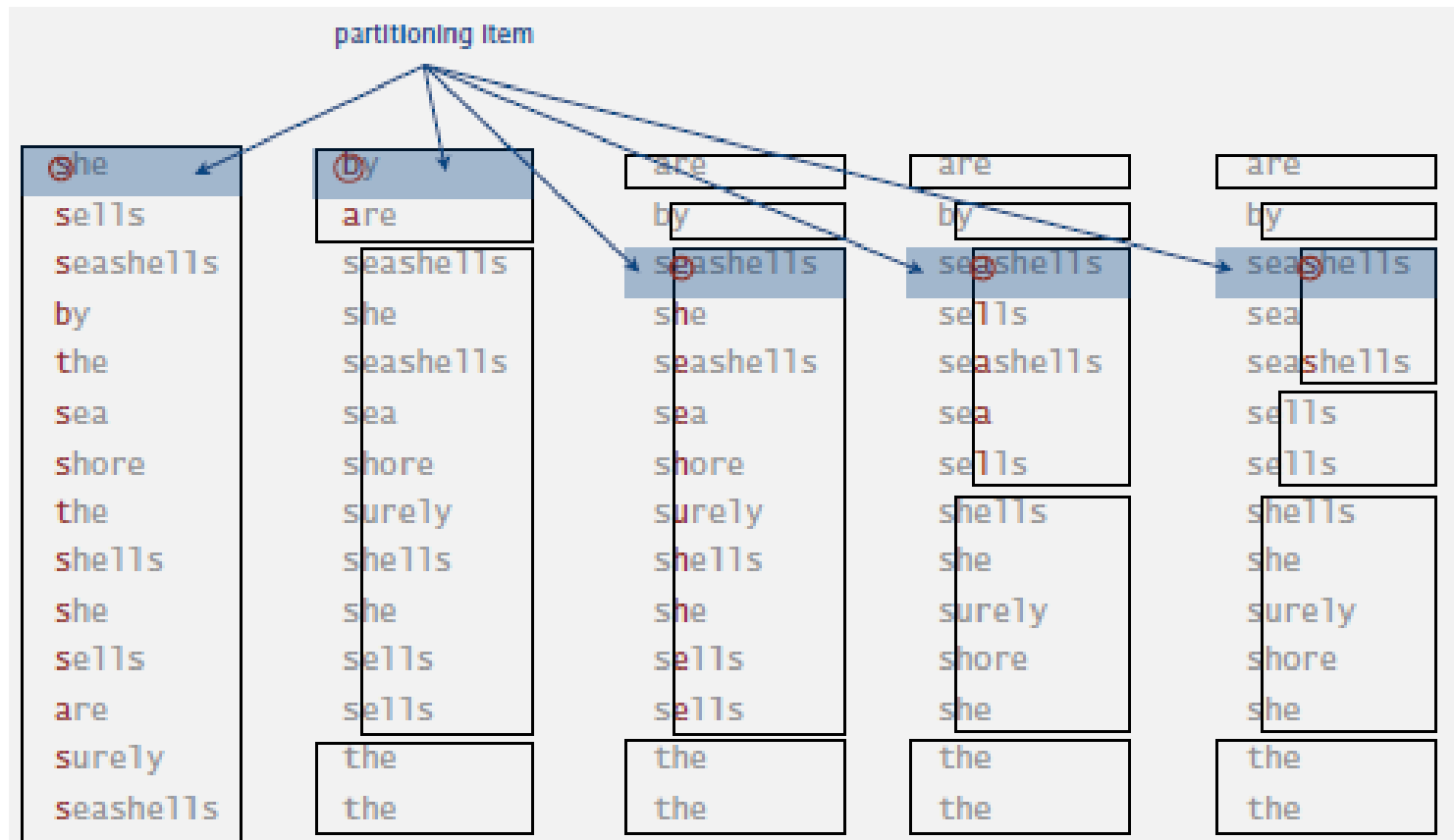› 3-way string quicksort – addressed inefficiency of both

3-way string quicksort

Overview. Do 3-way partitioning on the dth character.
- Less overhead than R-way partitioning in MSD string sort.
- Does not re-examine characters equal to the partitioning char.
  (but does re-examine characters not equal to the partitioning char)

partitioning item →

use first character to partition into "less", "equal", and "greater" subarrays

| She |
| sells |
| seashells |
| by |
| the |
| sea |
| shore |
| the |
| shells |
| she |
| sells |
| are |
| surely |
| seashells |

| by |
| are |
| seashells |
| she |
| seashells |
| sea |
| shore |
| surely |
| shells |
| she |
| sells |
| sells |
| the |
| the |

recursively sort subarrays, excluding first character for middle subarray

3- way string quicksort

Trace of first few recursive calls for 3-way string quicksort (subarrays of size 1 not shown)

3-way string quicksort in java

› charAt instead of compare

```
private static void sort(String[] a)
{   sort(a, 0, a.length - 1, 0);   }

private static void sort(String[] a, int lo, int hi, int d)
{
    if (hi <= lo) return;                                    3-way partitioning
    int lt = lo, gt = hi;                                    (using d^th character)
    int v = charAt(a[lo], d);
    int i = lo + 1;
    while (i <= gt)
    {                                                        to handle variable-length strings
        int t = charAt(a[i], d);
        if       (t < v) exch(a, lt++, i++);
        else if (t > v) exch(a, i, gt--);
        else             i++;
    }

    sort(a, lo, lt-1, d);
    if (v >= 0) sort(a, lt, gt, d+1);    ←——— sort 3 subarrays recursively
    sort(a, gt+1, hi, d);
}
```

Summary of string sorts

Frequency of operations.

algorithm	guarantee	random	extra space	stable?	operations on keys
insertion sort	$\frac{1}{2} N^2$	$\frac{1}{4} N^2$	1	✔	compareTo()
mergesort	$N \lg N$	$N \lg N$	N	✔	compareTo()
quicksort	$1.39 \, N \lg N$ *	$1.39 \, N \lg N$	$c \lg N$		compareTo()
heapsort	$2 N \lg N$	$2 N \lg N$	1		compareTo()
LSD sort †	$2 \, W \, (N + R)$	$2 \, W \, (N + R)$	$N + R$	✔	charAt()
MSD sort †	$2 \, W \, (N + R)$	$N \log_R N$	$N + D R$	✔	charAt()
3-way string quicksort	$1.39 \, W N \lg R$ *	$1.39 \, N \lg N$	$\log N + W$		charAt()

String sorting algorithms – when to use which?

› Insertion
  – Small arrays, arrays in (almost)order
› Quick
  – General purpose when space is tight
› Merge
  – General purpose stable
› 3-way quick
  – Large number of equal keys
› LSD
  – Short fixed-length strings
› MSD
  – Random strings
› 3-way string quicksort
  – General purpose, strings with long pre-fix matches

Check if 2 strings are anagrams

› two words are anagrams if they contain the same characters
 – "abc" and "cba" are anagrams