MA2C03: TUTORIAL 12 PROBLEMS
GRAPH THEORY

1) Let (V, E) be the graph with vertices a, b, c, d, and e and edges ab, bd, be, ac, cd, and ae.

(a) Draw this graph.
(b) Is this graph connected? Justify your answer.
(c) What is the minimum number of edges you would have to remove for the resulting subgraph to have two connected components? Justify your answer.
(d) What about three connected components? Justify your answer.
(e) What about four connected components? Justify your answer.
(f) What about five connected components? Justify your answer.
(g) Give an example of a shortest possible circuit in the graph. Justify your answer.
(h) Give an example of a longest possible circuit in the graph. Justify your answer.
(i) Does this graph have an Eulerian trail? Justify your answer.
(j) Does this graph have an Eulerian circuit? Justify your answer.

Solution: Let (V, E) be the graph with vertices a, b, c, d, and e and edges ab, bd, be, ac, cd, and ae.

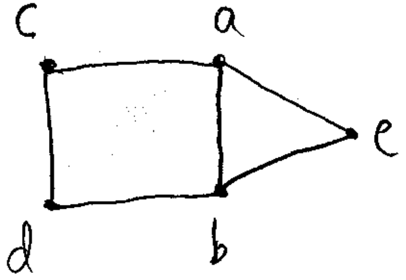
(a) The graph is drawn at the end of the solutions.
(b) The graph is connected as there is a walk from every vertex to every other vertex.
(c) Two edges: removing ae and be gives the component consisting of the vertex e alone and the component consisting of abcd.
(d) Three edges: removing ac, cd, and bd from the original graph gives the component consisting of vertex c alone, the component consisting of vertex d alone, and the component that is the triangle consisting of vertices a, b, and e.
(e) Five edges: the three we removed before (ac, cd, and bd) as well as two edges of the triangle to disconnect it, say ae and be.
(f) Six edges: as the graph has five vertices, we need to remove all the edges for the vertices by themselves to constitute the five components.
(g) The shortest possible circuit is aeba. We know that a circuit is a trail, so it cannot repeat edges. If the circuit were to pass through just two vertices, it would have to return to the initial vertex along

the same edge, which is not allowed. As a result, the shortest circuit in a graph passes through at least three distinct vertices. For this graph, we have a circuit aeba, which passes through exactly three vertices.

(h) The longest possible circuit is eacdbe. We are using five out of the six edges of the graph. We cannot use the 6th edge ab because according to Corollary 1 to the theorem we proved in class on Wednesday, January 23, if a circuit passes through a vertex v, then the number of edges of the circuit incident to v is even. If we were to use all edges of the graph in a circuit, then vertices a and b would have 3 edges of the circuit incident to each of them.

(i) Yes, it has an Eulerian trail: acdbeab.

(j) No, as we saw in the previous part of the question, using up all the edges gives a trail and not a circuit.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