

ST 3009 Statistical methods for Computer Science

Week 2 questions - Samuel Petit - 17333946

Question 1 -

a) The space consists of all possible combinations that can be obtained from throwing a die 3 times. There are 6 different possibilities per throw so we get:

$$6 \times 6 \times 6 = 6^3 = \underline{\underline{216}}$$

b) The number of ways where no die in the 3 die falls on 2 is:

$$5 \times 5 \times 5 = 5^3 = 125.$$

This leaves $216 - 125 = 91$ different outcomes where there is at least one 2.

Thus the probability of at least one 2 being rolled is:

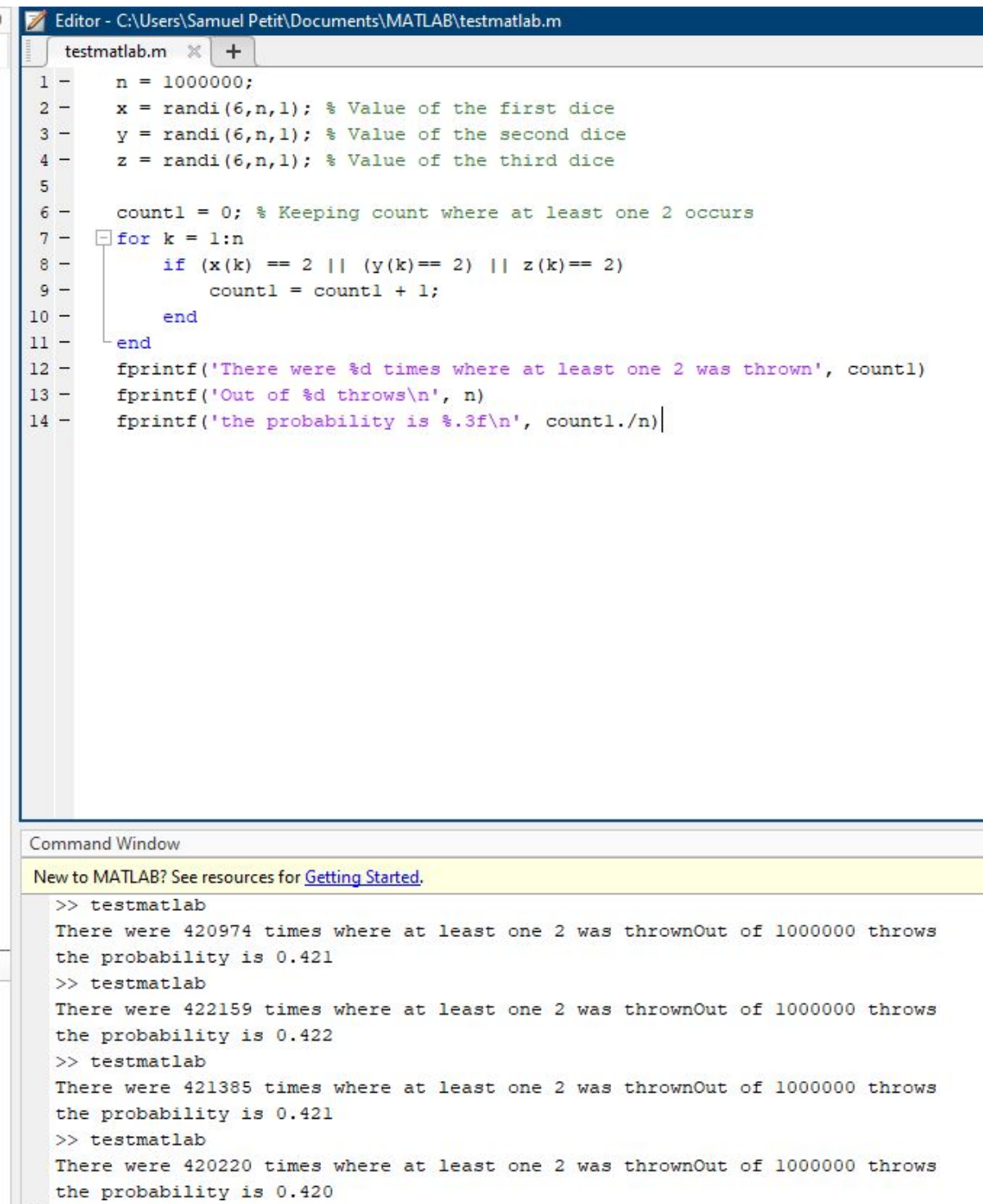
$$\frac{91}{216} \approx \underline{\underline{0.4213}}.$$

d) To get a sum of 17, you need 6, 5, 5 in any order, thus there are:

$$\frac{3!}{2!} = 3 \text{ different possibilities}$$

$$\text{We then get } P(\text{Sum is } 17) = \frac{3}{216} \approx \underline{\underline{0.0139}}$$

Question 1 c -



The image shows a MATLAB environment with a script editor and a command window. The script, named 'testmatlab.m', simulates 10,000,000 throws of three dice (x, y, z) and counts how many times at least one die shows a 2. The command window shows the results of four separate runs of the script, each displaying the count of occurrences and the calculated probability.

```
Editor - C:\Users\Samuel Petit\Documents\MATLAB\testmatlab.m
testmatlab.m  X  +
1 - n = 1000000;
2 - x = randi(6,n,1); % Value of the first dice
3 - y = randi(6,n,1); % Value of the second dice
4 - z = randi(6,n,1); % Value of the third dice
5
6 - count1 = 0; % Keeping count where at least one 2 occurs
7 - for k = 1:n
8 -     if (x(k) == 2 || (y(k)== 2) || z(k)== 2)
9 -         count1 = count1 + 1;
10 -     end
11 - end
12 - fprintf('There were %d times where at least one 2 was thrown', count1)
13 - fprintf('Out of %d throws\n', n)
14 - fprintf('the probability is %.3f\n', count1./n)|

Command Window
New to MATLAB? See resources for Getting Started.

>> testmatlab
There were 420974 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.421
>> testmatlab
There were 422159 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.422
>> testmatlab
There were 421385 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.421
>> testmatlab
There were 420220 times where at least one 2 was thrownOut of 1000000 throws
the probability is 0.420
```


Question 1e

Let A be the event: the sum of 2 die is 11.

There are only 2 possible outcomes for this event: 5 6 and 6 5.

here the ~~size~~ sample space is 6^2

Thus we have:

$$P(A) = \frac{2}{6^2} = \frac{2}{36} = \frac{1}{18} = \underline{\underline{0.0556}}.$$

Question 2 a)

We get the answer with the following calculations

$$P(\text{get a 1 first throw}) \times P(\text{get a 5 or a 6 side die}) \\ + P(\text{do not get a 1 on first throw}) \times P(\text{get a 5 or a 2 side die})$$

$$= \frac{1}{6} \times \frac{2}{6} + \frac{5}{6} \times \frac{1}{20} = \frac{1}{6^2} + \frac{5}{120} \\ \approx 0.0694.$$

b) In this case, since the two events are still independent we can use a similar approach!

$$P(\text{2nd throw is a 1}) = P(\text{1st throw is not 1}) \\ = \frac{5}{6} \times \frac{1}{20} = \frac{5}{120} = \frac{1}{24} \approx 0.0417$$

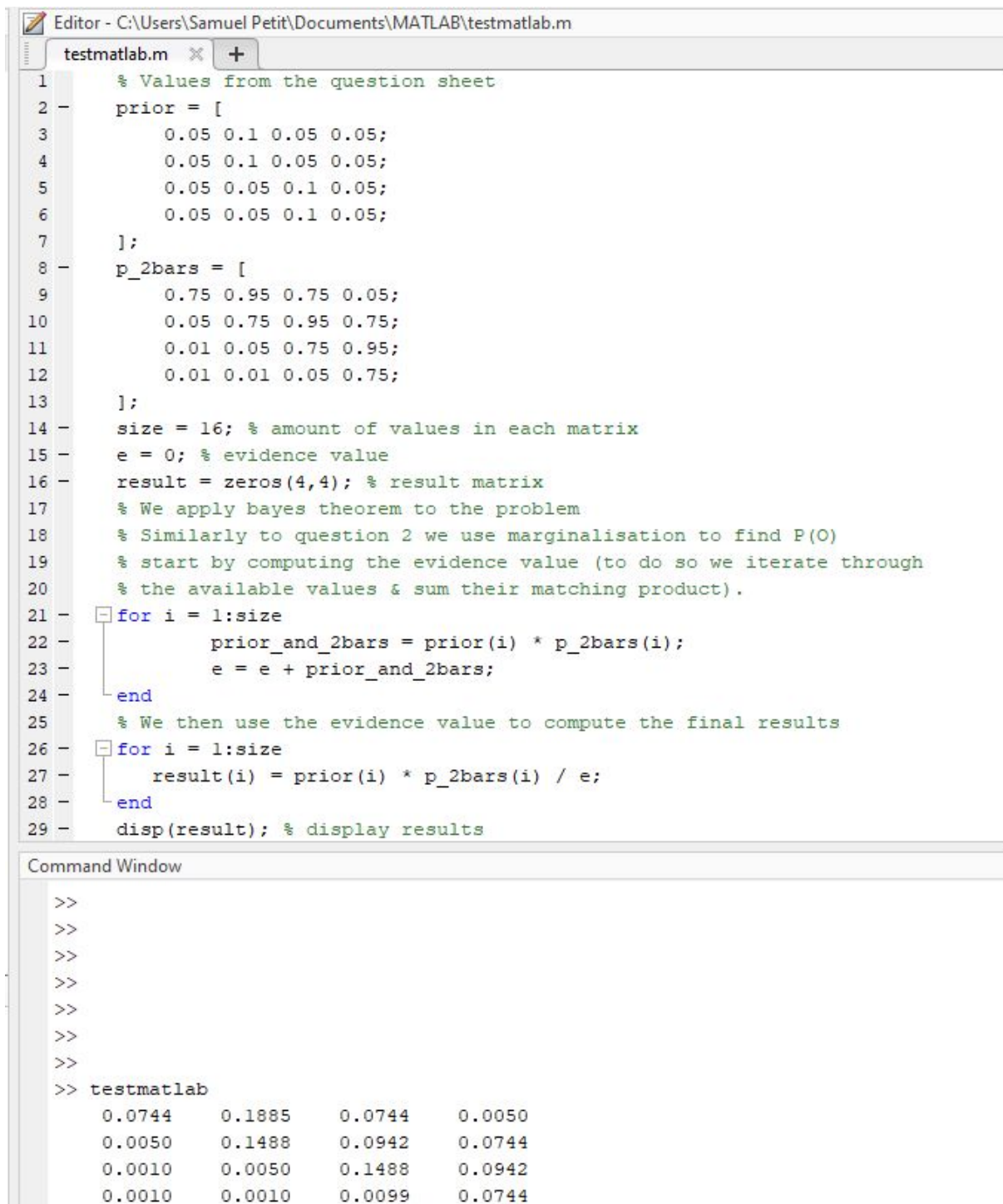
Q3 - Let A be the event: the suspect has the characteristic and B the event: the inspector is convinced the suspect is guilty.

Let's find $P(B|A)$.

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$
$$= \frac{0.6 \times 1}{0.6 \times 1 + 0.2 \times 0.4} = \frac{0.6}{0.6 + 0.08} = \frac{0.6}{0.68} \approx 0.882$$

Question 4 on next page.

Question 4



The image shows a MATLAB Editor window with a script named `testmatlab.m` and a Command Window below it. The script implements a Bayesian inference process using marginalization to find the posterior probabilities $P(O)$.

```
1 % Values from the question sheet
2 - prior = [
3     0.05 0.1 0.05 0.05;
4     0.05 0.1 0.05 0.05;
5     0.05 0.05 0.1 0.05;
6     0.05 0.05 0.1 0.05;
7 ];
8 - p_2bars = [
9     0.75 0.95 0.75 0.05;
10    0.05 0.75 0.95 0.75;
11    0.01 0.05 0.75 0.95;
12    0.01 0.01 0.05 0.75;
13 ];
14 - size = 16; % amount of values in each matrix
15 - e = 0; % evidence value
16 - result = zeros(4,4); % result matrix
17 % We apply bayes theorem to the problem
18 % Similarly to question 2 we use marginalisation to find P(O)
19 % start by computing the evidence value (to do so we iterate through
20 % the available values & sum their matching product).
21 - for i = 1:size
22     prior_and_2bars = prior(i) * p_2bars(i);
23     e = e + prior_and_2bars;
24 - end
25 % We then use the evidence value to compute the final results
26 - for i = 1:size
27     result(i) = prior(i) * p_2bars(i) / e;
28 - end
29 - disp(result); % display results
```

The Command Window shows the execution of the script, displaying the resulting 4x4 matrix of posterior probabilities:

```
>>
>>
>>
>>
>>
>>
>>
>> testmatlab
    0.0744    0.1885    0.0744    0.0050
    0.0050    0.1488    0.0942    0.0744
    0.0010    0.0050    0.1488    0.0942
    0.0010    0.0010    0.0099    0.0744
```