MA2C03: ASSIGNMENT 4
DUE BY THURSDAY, APRIL 4
AT LECTURE OR IN THE MATHS OFFICE ROOM 0.6

Please write down clearly both your name and your student ID number on everything you hand in. Please attach a cover sheet with a declaration confirming that you know and understand College rules on plagiarism. Details can be found on `http://tcd-ie.libguides.com/plagiarism/declaration`.

1) (20 points)

(a) Is $\{(x \in \mathbb{R} \mid \cos x = 1\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

(b) Is $\{x \in \mathbb{R}^+ \mid \log x \in \mathbb{R}\backslash\mathbb{Q}\}$ finite, countably infinite, or uncountably infinite? Justify your answer. The set $\mathbb{R}^+$ is the set of all positive real numbers.

(c) Is $\bigcup_{n=1}^{10} \left\{ \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = n^2\} \cap \{(x, y) \in \mathbb{R}^2 \mid y^2 - x^4 = 0\} \right\}$ finite, countably infinite, or uncountably infinite? Justify your answer.

(d) Is the language L_{DFA} defined in lecture finite, countably infinite, or uncountably infinite? Justify your answer. Recall that

$$L_{DFA} = \{\langle B, w\rangle \mid B \text{ is a DFA that accepts input string } w\}$$

2) (20 points)

(a) Consider the language over the binary alphabet $A = \{0, 1\}$ given by $L = \{0^m 1^{3m} \mid m \in \mathbb{N}\}$. Write down the algorithm of a Turing machine that recognizes L. Process the following strings according to your algorithm: ϵ, 01, 011, 0110, and 0111.

(b) Write down the transition diagram of the Turing machine from part (a) carefully labelling the initial state, the accept state, the reject state, and all the transitions specified in your algorithm.

3) (10 points) Write down the algorithm of an enumerator that prints out EXACTLY ONCE every string in the language $L = \{7m+2 \mid m \in \mathbb{N}\}$ over the alphabet $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

4) (20 points)

(a) Prove that the language

$$L_{RG} = \{\langle G, w\rangle \mid G \text{ is a regular grammar that generates string } w\}$$

is Turing-decidable.

(b) Prove that $L_{CFG} \setminus L_{RG}$ is Turing-decidable, where L_{CFG} was defined in lecture as

$$L_{CFG} = \{\langle G, w\rangle \mid G \text{ is a CFG that generates string } w\}.$$