Reading Week Assignment CS1003

(Do All Three Questions)

1.

(a) Let the set operator, $\bar{\cup}$ be defined so that

$$X\bar{\cup}Y = \overline{X \cup Y}$$

where the set $\overline{X}$ is the complement of the set X .
Determine by Karnaugh Map whether:

   i. $A \cup B = A\bar{\cup}(A\bar{\cup}B)$
  ii. $\overline{A \cap B} = \overline{A}\bar{\cup}(B\bar{\cup}A)$
 iii. $A\bar{\cup}(B\bar{\cup}C) = (A\bar{\cup}B)\bar{\cup}C$

(b) A survey was made of a group of 30 students concerning the 3 societies:
Archaeological, Botany and Choral.
All the members of the group of 30 students belong to at least one society.
• 6 students belong to only the Choral society.
• 5 students belong to all 3 societies.
• 2 students belong to the Archaeological and Choral societies but not to the Botany society.
• 15 students belong to the Choral society.
• 2 students belong only to the Archaeological society.
• 3 students belong only to the Botany society.

   i. How many students belong to the Archaeological society and Botany society but not the Choral society?
  ii. How many students belong to the Archaeological society?

2.

(a) Determine using truth tables or otherwise, whether the following are Tautologies

   i. $(p \rightarrow q) \rightarrow p$

   ii. $(p \rightarrow q \wedge r) \rightarrow (p \rightarrow q) \wedge (p \rightarrow r)$

(b) Determine by Truth Table or otherwise, whether the following argument is valid

The programmer is careful or if the program crashes then the specification is not clear.
If the programmer is careful then the program does not crash.
The program crashes.
$\therefore$
If the specification is clear then the programmer is careful.

Abbreviate:
P: The programmer is careful.
S: The specification is clear.
C: The program crashes.

3. **The Tardy Bus Problem**
   The following three statements are given as premises:
   If Bill takes the bus then, Bill misses his appointment if the bus is late.
   Bill does not go home, if Bill misses his appointment and Bill feels downcast.
   If Bill does not get the job then, Bill feels downcast and Bill goes home.

   Determine, by the use of KE Deduction, whether each of the following 2 conjectures can be inferred from the premises.

   (a) *If Bill goes home then he either feels downcast or he gets the job..*
   (b) *If Bill takes the bus and he does not go home then, he gets the job if he does not miss his appointment.*

   Abbreviations:

TB:	*Bill Takes the Bus.*
MA:	*Bill Misses his Appointment*
BL:	*The Bus is Late.*
GH:	*Bill Goes Home.*
FD:	*Bill Feels Downcast.*
GJ:	*Bill Gets the Job.*

   Translations of Premises

P1:	$TB \rightarrow BL \rightarrow MA$
P2:	$MA \wedge FD \rightarrow \neg GH$
P3:	$\neg GJ \rightarrow FD \wedge GH$

   Translation of Conjectures.

(a)	$GH \rightarrow FD \vee GJ$
(b)	$TB \wedge \neg GH \rightarrow \neg MA \rightarrow GJ$

KE_Deduction Rules

$\alpha - rules$				
Premise	$\neg\neg P$	$P \wedge Q$	$\neg(P \vee Q)$	$\neg(P \rightarrow Q)$
Conclusion	P	P	$\neg P$	P
Conclusion		Q	$\neg Q$	$\neg Q$

$\beta - rules$				
Premise	$P \vee Q$	$\neg(P \wedge Q)$	$P \rightarrow Q$	$P \rightarrow Q$
Premise	$\neg P$	P	P	$\neg Q$
Conclusion	Q	$\neg Q$	Q	$\neg P$

Branching Rule

$$\bigwedge$$
$$P \quad \neg P$$