Advanced Computer Linguistics - Assignment 1
Samuel Petit - 17333946

Question 1

(i) implies (ii):

$$P(A \cap B) = P(A|B) * P(B) = P(A) * P(B)$$

(ii) implies (i):

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) * P(B)}{P(B)} = P(A)$$

Question 2

a

$$P(gw|ps) = \frac{28}{28 + 40} = \frac{28}{30}$$

Counts $gw \cap \neg ps$ and $\neg gw \cap \neg ps$ are not useful in this situation.

b

$$P(ps|gw) = \frac{28}{28 + 2} = \frac{28}{168}$$

Counts $gw \cap \neg ps$ and $\neg gw \cap \neg ps$ are not useful in this situation.

Question 3

a

We have:

$$P(vmel) = 0.01$$

$$P(dbi|vmel) = 0.95$$

$$P(dbi|\neg vmel) = 0.01$$

Let's compute helper values for doing marginalization:

$$P(\neg vmel) = 1 - P(vmel) = 1 - 0.01 = 0.99$$

$$P(dbi \cap vmel) = P(dbi|vmel) * p(vmel) = 0.95 * 0.01 = 0.095$$

$$P(dbi \cap \neg vmel) = P(dbi|\neg vmel) * p(\neg vmel) = 0.01 * 0.99 = 0.099$$

We can now compute $P(vmel|dbi)$ and $P(\neg vmel|dbi)$:

$$P(vmel|dbi) = \frac{P(dbi \cap vmel)}{P(dbi \cap vmel) + P(dbi \cap \neg vmel)} = \frac{0.095}{0.095 + 0.099} = \frac{95}{194}$$

$$P(\neg vmel|dbi) = \frac{P(dbi \cap \neg vmel)}{P(dbi \cap vmel) + P(dbi \cap \neg vmel)} = \frac{0.099}{0.095 + 0.099} = \frac{99}{194}$$

We find that $P(vmel|dbi) < P(\neg vmel|dbi)$ so $\neg vmel$ is the best guess.

b

We have:

$$P(vmel) = 0.15$$
$$P(dbi|vmel) = 0.95$$
$$P(dbi|\neg vmel) = 0.01$$

Let's compute helper values for doing marginalization:

$$P(\neg vmel) = 1 - P(vmel) = 1 - 0.15 = 0.85$$

$$P(dbi \cap vmel) = P(dbi|vmel) * p(vmel) = 0.95 * 0.15 = 0.1425$$
$$P(dbi \cap \neg vmel) = P(dbi|\neg vmel) * p(\neg vmel) = 0.01 * 0.85 = 0.0085$$

We can now compute $P(vmel|dbi)$ and $P(\neg vmel|dbi)$:

$$P(vmel|dbi) = \frac{P(dbi \cap vmel)}{P(dbi \cap vmel) + P(dbi \cap \neg vmel)} = \frac{0.1425}{0.1425 + 0.0085} = \frac{1425}{1510} = \frac{285}{302}$$

$$P(\neg vmel|dbi) = \frac{P(dbi \cap \neg vmel)}{P(dbi \cap vmel) + P(dbi \cap \neg vmel)} = \frac{0.0085}{0.1425 + 0.0085} = \frac{85}{1510} = \frac{17}{302}$$

We find that $P(vmel|dbi) > P(\neg vmel|dbi)$ so $vmel$ is the best guess.

c

We have:

$$P(vmel) = 0.01$$
$$P(dbi|vmel) = 0.95$$
$$P(dbi|\neg vmel) = 0.001$$

Let's compute helper values for doing marginalization:

$$P(\neg vmel) = 1 - P(vmel) = 1 - 0.01 = 0.99$$

$$P(dbi \cap vmel) = P(dbi|vmel) * p(vmel) = 0.95 * 0.01 = 0.0095$$
$$P(dbi \cap \neg vmel) = P(dbi|\neg vmel) * p(\neg vmel) = 0.001 * 0.99 = 0.00099$$

We can now compute $P(vmel|dbi)$ and $P(\neg vmel|dbi)$:

$$P(vmel|dbi) = \frac{P(dbi \cap vmel)}{P(dbi \cap vmel) + P(dbi \cap \neg vmel)} = \frac{0.0095}{0.0095 + 0.00099} = \frac{950}{1049}$$

$$P(\neg vmel|dbi) = \frac{P(dbi \cap \neg vmel)}{P(dbi \cap vmel) + P(dbi \cap \neg vmel)} = \frac{0.00099}{0.0095 + 0.00099} = \frac{99}{1049}$$

We find that $P(vmel|dbi) > P(\neg vmel|dbi)$ so $vmel$ is the best guess.

Question 4

$$P(cool : +) = \frac{62 + 108}{38 + 292} = \frac{170}{330} = 0.52 \tag{1}$$

$$P(cool : +|noisy : +) = \frac{62}{62 + 38} = \frac{62}{100} = 0.62 \tag{2}$$

We have $P(cool : +) \neq P(cool : +|noisy : +)$ so the variables are not independant.

Question 5

$$P(cool : +|open : +) = \frac{90}{100} = 0.9 \tag{3}$$

$$P(cool : +|open : +, noisy : +) = \frac{54}{64} = 0.9 \tag{4}$$

We have $P(cool : +|open : +) = P(cool : +|open : +, noisy : +)$ so cool: + is conditionally independant of noisy: + given open: +.

$$P(cool : +|open : -) = \frac{80}{400} = \frac{8}{40} \tag{5}$$

$$P(cool : +|open : -, noisy : +) = \frac{8}{40} \tag{6}$$

We have $P(cool : +|open : -) = P(cool : +|open : -, noisy : +)$ so cool: + is conditionally independant of noisy: + given open: -.