

Question 1

(i) implies (ii):

$$P(A \cap B) = P(A|B) * P(B) = P(A) * P(B)$$

(ii) implies (i):

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) * P(B)}{P(B)} = P(A)$$

Question 2

a

$$P(gw|ps) = \frac{28}{28 + 40} = \frac{28}{68}$$

Counts $gw \cap \neg ps$ and $\neg gw \cap \neg ps$ are not useful in this situation.

b

$$P(ps|gw) = \frac{28}{28 + 2} = \frac{28}{30}$$

Counts $gw \cap \neg ps$ and $\neg gw \cap \neg ps$ are not useful in this situation.

Question 3

Question 4

$$P(cool : +) = \frac{62 + 108}{38 + 292} = \frac{170}{330} = 0.52 \quad (1)$$

$$P(cool : + | noisy : +) = \frac{62}{62 + 38} = \frac{62}{100} = 0.62 \quad (2)$$

We have $P(cool : +) \neq P(cool : + | noisy : +)$ so the variables are not independant.

Question 5

$$P(\text{cool} : + | \text{open} : +) = \frac{90}{100} = 0.9 \quad (3)$$

$$P(\text{cool} : + | \text{open} : +, \text{noisy} : +) = \frac{54}{64} = 0.9 \quad (4)$$

We have $P(\text{cool} : + | \text{open} : +) = P(\text{cool} : + | \text{open} : +, \text{noisy} : +)$ so cool: + is conditionally independent of noisy: + given open: +.

$$P(\text{cool} : + | \text{open} : -) = \frac{80}{400} = \frac{8}{40} \quad (5)$$

$$P(\text{cool} : + | \text{open} : -, \text{noisy} : +) = \frac{8}{40} \quad (6)$$

We have $P(\text{cool} : + | \text{open} : -) = P(\text{cool} : + | \text{open} : -, \text{noisy} : +)$ so cool: + is conditionally independent of noisy: + given open: -.