CSU44061 Machine Learning - Week 8
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Code for all questions provided in the appendix.

Question i

Part a

In order to do so there are 2 problems to solve: - knowing how many iterations to apply the kernel within the image (for columns and rows) - For each iteration, compute the resulting value to place in the output

To do so, I start by computing both:

$$rows = \#rows_{image} - \#rows_{kernel} + 1$$

$$columns = \#columns_{image} - \#columns_{kernel} + 1$$

Then, given that both *rows* and *columns* are positive numbers, we know the amount of positions to place the kernel within the image on both the x and y axis.

The first step is then to iterate over both ranges, that is:

```
x_positions = len(image) - len(kernel) + 1
y_positions = len(image[0]) - len(kernel[0]) + 1
# Make sure kernel and image sizes are valid
assert x_positions > 0 and y_positions > 0,
        "image should not be smaller than kernel"
# Iterate over kernel positions within the image
for i in range(x_positions):
    for j in range(y_positions):
        # Compute the output point here
```

Let's now look at the second problem to solve, that is, given a certain x and y iteration (here represented as i and j), compute the resulting value to position in the output matrix.

To do so, I iterate over the kernel matrix rows and columns such as to compute:

$$output_{i,j} = \sum_{k=0}^{\#kernel_{rows}} \sum_{l=0}^{\#kernel_{columns}} kernel^{(k,l)} * image^{(i+k,j+l)}$$

That it, for each point in the kernel, we multiply it to its corresponding point (using the offset of i and j), once we have done so for all kernel points we have the result value for the point i, j of the output matrix.

In code this looks like:

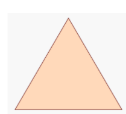
```
sum = 0
for k in range(len(kernel)):
    for l in range(len(kernel[0])):
        sum = sum + (kernel[k][l] * image[i + k][j + l])
```

Finally, all we need to do now is use this sum to form the current output row and finally use the row to build the output matrix. All together:

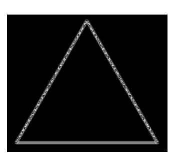
```
def convolutional_layer(image, kernel):
    x_positions = len(image) - len(kernel) + 1
    y_positions = len(image[0]) - len(kernel[0]) + 1
    assert x_positions > 0 and y_positions > 0,
            "image should not be smaller than kernel"
    result = []
    for i in range(x_positions):
        row = []
        for j in range(y_positions):
            sum = 0
            for k in range(len(kernel)):
                for l in range(len(kernel[0])):
                    sum = sum + (kernel[k][l] *
                                image[i + k][j + l])
            row.append(sum)
        result.append(row)
    return result
```

Part b

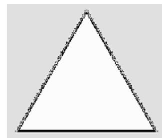
I chose the picture of a triangle found online:



Using the code provided in the assignment and the above picture as an input. I can apply the 2 provided kernels to my image, we obtain the following outputs:
Kernel 1:



Kernel 2:



1 Question ii

Part a

The downloaded code uses the following ConvNet code:

```
model.add(Conv2D(16, (3,3), padding='same',
        input_shape=x_train.shape[1:], activation='relu'))
model.add(Conv2D(16, (3,3), strides=(2,2),
                    padding='same', activation='relu'))
model.add(Conv2D(32, (3,3), padding='same',
                                activation='relu'))
model.add(Conv2D(32, (3,3), strides=(2,2),
                    padding='same', activation='relu'))
model.add(Dropout(0.5))
model.add(Flatten())
model.add(Dense(num_classes, activation='softmax',
            kernel_regularizer=regularizers.l1(0.0001)))
```

We notice a list of sequential steps are added, in order we find:

- A 2D Convolution Layer which use 16 filters to learn from. a kernel size of 3x3, it also uses padding such as to make outputs the same dimensions as the input, finally it also uses 'relu' validation (or rectified linear activation function) is a method which will map the output to 0 if it is negative.
- We then find another 2D Convolution Layer with 16 filters, 3x3 size, uses padding to keep the same output size and 'relu' activation. The difference being that it uses a Stride of size 2x2 such as to downsample (it determines how much the window moves by - the default being 1x1)
- We then find the exact same 2 steps as we've seen above, in the same order with the difference that the filter size is set to 32 instead of 16.
- We then find a Droupout step, which sets randomly input values to 0 with a rate of 0.5 here. This helps to prevent overfitting. It will also update non-0 values such as to keep the sum of values consistent.
- We then have a Flatten step which simply maps the data to be flat
- The final step is a Dense step which is a step for densely connected neural network layers, it uses the activation function provided such as to form its output. It uses 10 units (or classes) as specified in the downloaded code,

L1 regularisation and the activation function is set to 'softmax' which outputs the values of a probability density function.

Part b

Section i

By simply running the program and reading the output from the console we find the metrics required by the question:

- This model has a total of: 37,146 parameters
- The layer with the most parameters is the Dense layer, which has 20,490 parameters.
- When I ran the code I found that the accuracy on the test data was of 0.5, on the train data I obtain an accuracy of 0.62. We find that the accuracy does drop by some considerable amount compared with the train data ($0.62 - 0.5 = 0.12$), however that makes sense as the test data is data the model hasn't seen before, unlike the train.

I trained a DummyRegressor model to use as a baseline model for comparison. This model predicts the most commonly seen value, it is very straightforward to train:

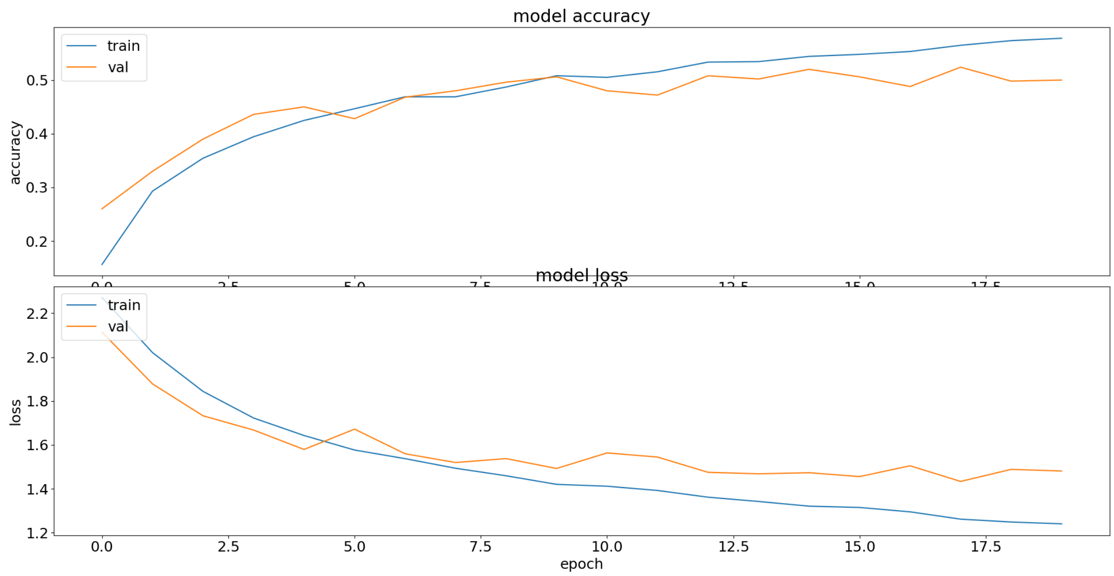
```
dummy_model = DummyRegressor().fit(x_train, y_train)
```

I then use the same methods as included in the provided code to obtain an accuracy metric, we find the that baseline model has an accuracy of 0.1 for both the test and train data. We can clearly confirm that our previous model is doing very well in comparison, with an accuracy increase of $0.5 - 0.1 = 0.4$ on the test data and $0.62 - 0.1 = 0.52$ on the test data.

Section ii

The graph we obtain the history variable in the provided code is the following:

Train size	Train time	Accuracy train	Accuracy test
5K	41.17s		
10K	81.21s		
20K	158.01s		
40K	307.86s		



Looking at the second part of the graph which plots the loss on the y axis and epoch on the x axis can be used to make some observations about over and underfitting. We notice that during the first few epoch (training iterations) the loss reduces by a noticeable amount quite fast. We could say that the model was underfitting towards the start and started tuning more parameters as we increase the epoch number.

However, we get to a point where the loss somewhat stabilises and starts tu fluctuate, becoming quite unpredictable (it seems to have a high variance). We notice a form of overfitting where we continue training the model and tuning parameters with no gain in the models performance.

Appendix

Part iii

Training this same model using bigger amounts of training data points, I measure the training time and accuracy for each of these models. We obtain the following metrics: