Samuel Petit - CSU44061 Machine Learning
Dataset # id:13–3785.6-78

Question 1

Code for all questions provided in the appendix.

Part a

Reading in the data was done using code provided in the assignment (using the pandas package). I mapped the data using standardisation:

$$x_j = \frac{x_j - \mu_j}{\sigma_j}$$

Where

$$\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$$

and

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(x_j^{(i)} - \mu)^2}$$

This aims to make the mean of the dataset 0 and the standard deviation 1. Standardisation in this case gives me values in range -1.7 to 1.7 for both features x and outputs y which is good enough for me to work with.

Gradient descent is done by repeating the following set of steps:

$$\delta_0 := -\frac{2\alpha}{m}\sum_{i=1}^{m}(h(x^{(i)}) - y^{(i)})$$

$$\delta_1 := -\frac{2\alpha}{m}\sum_{i=1}^{m}(h(x^{(i)}) - y^{(i)})x^{(i)}$$

$$\theta_0 = \theta_0 + \delta_0; \theta_1 = \theta_1 + \delta_1$$

Where θ_i is the coefficient for a feature $x^{(i)}$, α is the learning rate and m is the amount of datapoints. Finally, $h(x) = \theta^T x$ is our model. This enables us to retrieve a set of coefficients θ_0 and θ_1 which minimise the cost function (to a local minimum), thus making the predictions of the model more accurate.

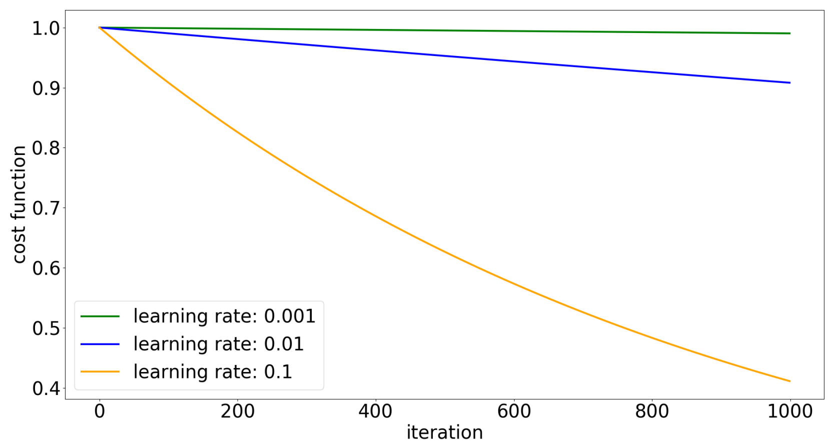
For this part of the exercise I decided to execute this algorithm 200 times to get an overview of its performance.

Part b

i.

Using code provided in the appendix, we obtain the following plot for learning rates $\alpha = 0.1$, $\alpha = 0.01$ and $\alpha = 0.001$. I chose to use 1000 iterations in this case as I thought it made a more interesting plot to look at, given that the cost function is still going down after 1000 iterations with a learning rate of 0.1.

In this case the bigger the learning rate, the faster the cost function gets reduced using gradient descent. The values are obtained by computing the cost function for every iteration of the gradient descent using the current set of coefficients θ_0 and θ_1.



ii.

After training the linear regression model we obtain the following parameter values: $\theta_0 = -2.5002977321421088e - 17$; $\theta_1 = 0.8797086791404171$. This gives us the linear model:

$$h_\theta(x) = \theta_0 + \theta_1 x \approx 0 + 0.8797 * x \approx 0.8797x$$

iii.

I chose the mean μ as the constant value of the baseline model as it will reduce the cost function as much as possible. We obtain the following results:

Baseline model:

$$J(\theta_0, \theta_1) = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2 = 0.9999999999999989 \approx 1$$

Trained model (using $\alpha = 0.5$ and 1000 iterations):

$$J(\theta_0, \theta_1) = 0.4123744218292116 \approx 0.4124$$

Our goal is to minimise the cost function $J(\theta_0, \theta_1)$. In this case we notice that there is a considerable difference between both values of approximately 0.5876 (note that values are standardised thus it makes for an important increase in precision).

iv.

Using sklearn, we split the features and outputs of the dataset into two categories, training and testing sets. We then create a linear model with which we train our model using the training data. We can then extract the following parameters:

$$h_\theta(x) = 0.00648141 + 0.94980041x \approx 0.0065 + 0.9498x$$

The cost function for this model is:

$$J(\theta_0, \theta_1) = 0.11706807 \approx 0.1171$$

Remember the cost function for our previous model gave us a value of $J(\theta_0, \theta_1) \approx 0.4124$ The difference in precision is of 0.29533. We can conclude that the sklearn trained model is a much more accurate model (once again, values are standardised).

Appendix

Util functions file:

```python
import math


def h(coef1, coef2, feature):
    # Returns the value for the model with provided coefs and feature
    return coef1 + coef2 * feature


def get_median(value):
    median = 0
    length = len(value)
    for i in range(length):
        median += value[i][0]
    return median / length


def cost_function(x, y, coef1, coef2):
    length = len(x)
    current_sum = 0
    for i in range(length):
        tmp = h(coef1, coef2, x[i][0]) - y[i][0]
```

```python
        tmp = tmp * tmp
        current_sum += tmp
    return current_sum / length


def scaling_factor(value):
    # Compute the scaling factor for data standardisation
    # In this case the scaling factor is the std deviation
    length = len(value)
    valsSum = 0
    median = get_median(value)
    for i in range(length):
        valsSum += ((value[i][0] - median) ** 2) / length
    return math.sqrt(valsSum)


def gradient_descent(x, y, rate, iterations):
    # Find coefs that minimize our cost function using gradient descent
    coef1 = 0
    coef2 = 0
    length = len(x)
    deltas = []
    for _ in range(iterations):
        deltas.append(cost_function(x, y, coef1, coef2))
        sum1 = gradient_descent_sum(x, y, coef1, coef2, False)
        sum2 = gradient_descent_sum(x, y, coef1, coef2, True)
        gamma1 = ((-2 * rate) / length) * sum1
        gamma2 = ((-2 * rate) / length) * sum2
        coef1 += gamma1
        coef2 += gamma2
    return {
        "coef_1": coef1,
        "coef_2": coef2,
        "deltas": deltas,
    }


def gradient_descent_sum(x, y, coef1, coef2, multiply):
    # Computes the sum part for the gradient descent algorithm
    length = len(x)
    current_sum = 0
    for i in range(length):
        tmp = h(coef1, coef2, x[i][0]) - y[i][0]
        if multiply == True:
            tmp = tmp * x[i][0]
        current_sum += tmp
```

```
    return current_sum / length
```

Main code file:

```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import math
# Import our custom functions from the utils file
from util import get_median, scaling_factor, cost_function, gradient_descent
from sklearn.linear_model import LinearRegression
from sklearn import preprocessing

# Read data from CSV
df = pd.read_csv("dataset.csv", comment='#')
# Extract columns from data
x = np.array(df.iloc[:, 0])
x = x.reshape(-1, 1)
y = np.array(df.iloc[:, 1])
y = y.reshape(-1, 1)
# Convert x to floats
x = x.astype(float)

# normalise x data
median = get_median(x)
scaling = scaling_factor(x)
length = len(x)
for i in range(length):
    value = x[i][0]
    normalised = (value - median) / scaling
    x[i][0] = normalised

# normalise y data
median = get_median(y)
scaling = scaling_factor(y)
length = len(y)
for i in range(length):
    value = y[i][0]
    normalised = (value - median) / scaling
    y[i][0] = normalised

# For Q1 use 200 iterations and 0.1 learning rate
nb_iterations = 200
coefs = gradient_descent(x, y, 0.1, nb_iterations)

print("Q1, coefficients: ", coefs["coef_1"], coefs["coef_2"])
```

```
# Q2 i: use gradient descent on learning rates 0.1, 0.01, 0.001.
nb_iterations = 1000
xaxis = range(nb_iterations)
coefs_001 = gradient_descent(x, y, 0.001, nb_iterations)
coefs_01 = gradient_descent(x, y, 0.01, nb_iterations)
coefs_1 = gradient_descent(x, y, 0.1, nb_iterations)

# Plot the cost function evolution
plt.rc('font', size=30)
plt.rcParams['figure.constrained_layout.use'] = True
plt.plot(xaxis, coefs_001["deltas"], color='green', linewidth=3)
plt.plot(xaxis, coefs_01["deltas"], color='blue', linewidth=3)
plt.plot(xaxis, coefs_1["deltas"], color='orange', linewidth=3)
plt.xlabel("iteration")
plt.ylabel("cost function")
plt.legend(["learning rate: 0.001", "learning rate: 0.01", "learning rate: 0.1"]
# Plot is shown at the end of program executiong
# as it blocks execution otherwise.

# Q2 ii: use learning rate of 0.5
coefs_05 = gradient_descent(x, y, 0.5, nb_iterations)

print("Q2 ii parameter values after training: ",
      coefs_05["coef_1"], coefs_05["coef_2"])

print("Q2 iii cost function for trained model: ", coefs_05["deltas"][199])
print("Q2 iii cost function for baseline model (using mean): ",
      cost_function(x, y, get_median(y), 0))

# Q3, reload values from dataset to start over
# Read data from CSV
df = pd.read_csv("dataset.csv", comment='#')
# Extract columns from data
x = np.array(df.iloc[:, 0])
x = x.reshape(-1, 1)
y = np.array(df.iloc[:, 1])
y = y.reshape(-1, 1)
# Convert x to floats
x = x.astype(float)

# Standardise data x and y.
x = preprocessing.scale(x)
y = preprocessing.scale(y)

# Separate features into training / test sets.
Xtrain = x[:-20]
```

```python
Xtest = x[-20:]

# Separate targets as well.
Ytrain = y[:-20]
Ytest = y[-20:]

# Train the model
regr = LinearRegression()
regr.fit(Xtrain, Ytrain)

# Predict using the test set
y_pred = regr.predict(Xtest)

# Print coefficients and cost function output.
print("Q3")
print('Sklearn model Coefficients: ', regr.coef_, regr.intercept_)
print('Sklearn model Cost function: ',
      cost_function(x, y, regr.intercept_, regr.coef_))


# Show plot for Q2 i.
plt.show()
```