Correlation and Causality

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Why causality matters

Because correlation is a proxy.

[Vig]

Why causality matters

Because A / B testing is not always possible.

[ERSS+13]

Simpson's paradox: cautionary tales

Simpson's paradox: a phenomenon in probability and statistics in which a trend appears disappears or reverses depending on grouping of data. [Wik], [PGJ16]

Example: University of California, Berkeley 1973 admission figures

	Men		Women	
	Applicants	Admitted	Applicants	Admitted
Total	8442	**44%**	4321	35%

[FPP98]

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	*825*	62%	108	**82%**
B	*560*	63%	25	**68%**
C	325	**37%**	*593*	34%
D	417	33%	375	**35%**
E	191	**28%**	*393*	24%
F	373	6%	341	**7%**

[BHO75]

A brief, biased history of causality

- Aristotle, 384 - 322 BC
- Isaac Newton, 1643 - 1727 AD
- David Hume, 1711 - 1776 AD
- Francis Galton, 1822 - 1900 AD, Karl Pearson, 1857 - 1936 AD
- Judea Pearl, b. 1936 AD

Counterfactuals and causality

Ideal: Intervention + Multiverse $\rightarrow$ Causality

Examples:

- Medical treatment (e.g. kidney stone treatment)
- Social outomes (e.g. university admissions)
- Business outcomes (e.g. click-through rate, hit rate)

In-practice:

- Correlation: approximate multiverse by comparing intervention at t to result at $t-1$
- Random population: approximate multiverse by splitting sample well
- A / B testing: random populations A / B + intervention in one

Counterfactual example: hit rate for insurance

Variables:

- product_type: Client line of business
- days: Number of days to generate quote
- rating: Binary indication of client risk
- hit: Binary, 1 for success (binding the quote), 0 for failure

Fake data:

product_type	days	rating	hit
property	3	1	0
financial	2	1	0
financial	1	1	0
financial	0	0	1
financial	0	1	0

Counterfactual example: hit rate for insurance

Variables:

- product_type: Client line of business
- days: Number of days to generate quote
- rating: Binary indication of client risk
- hit: Binary, 1 for success (binding the quote), 0 for failure
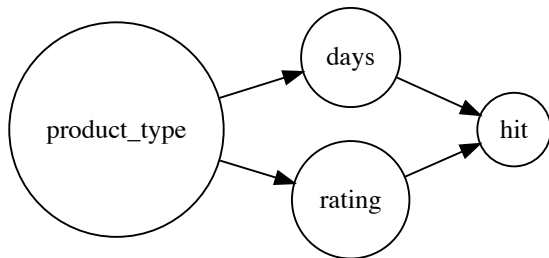
Non-counterfactual approach: condition and query

Goal: estimate effect of days on hit.

Calculate
- $P(\mathrm{hit} = 1|\mathrm{days} = 0) - P(\mathrm{hit} = 1|\mathrm{days} = 1)$,
- $P(\mathrm{hit} = 1|\mathrm{days} = 1) - P(\mathrm{hit} = 1|\mathrm{days} = 2)$,
- ...

From exercise Jupyter notebook:

days	prob
0	0.534270
1	0.449109
2	0.318932
3	0.168999

The Structural Causal Model

The definitions in following slides are from [Pea07], [PGJ16].

Definition
A *structural causal model M* consists of two sets of variables U, V and a set of functions F, where

- U are considered *exogenous*, or background variables,
- V are the *causal* variables, i.e. that can be manipulated, and
- F are the functions that represent the process of assigning values to elements of V based on other values in U, V, e.g. $v_i = f(u, v)$.

We denote by G the graph induced on U, V by the functions F, and call it the *causal graph* of (U, V, F).

Hit rate example: $U = \{\text{product\_type}, \text{rating}\}$, $V = \{\text{days}, \text{hit}\}$, $F \leftrightarrow$ sample from conditional probabilty tables in directed graphical model.

Formalizing interventions: the intuition of "do"

For business application, quantity of interest is not $P(\text{hit} = 1 | \text{days} = d)$, but intervention

$$P(\text{hit} = 1 | \text{do}(\text{days} = d))$$
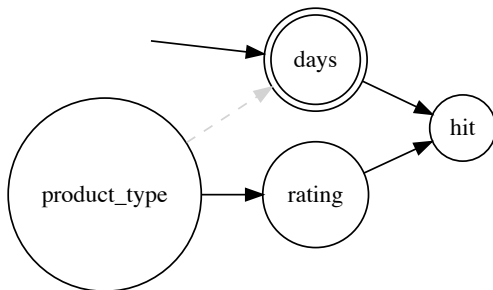
.

Formalizing interventions: the intuition of "do"

For business application, quantity of interest is effect of intervention / counterfactual

Not $P(\text{hit} = 1|\text{days} = d)$ but $P(\text{hit} = 1|\text{do}(\text{days} = d))$

$$G =$$
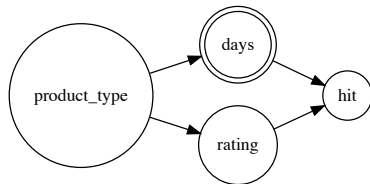
$$G' = G_{\underline{\text{days}}} =$$
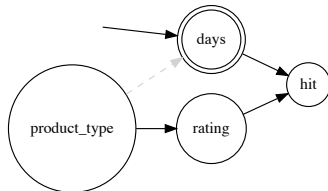
Formalizing interventions: the intuition of "do"

First, find quantities unchanged between G and $G' = G_{\underline{\text{days}}}$
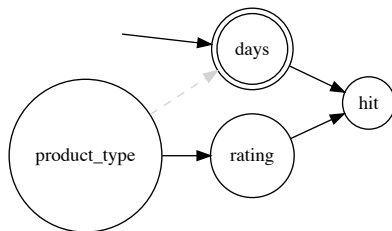
$$P_{G'}(\text{product\_type} = p, \text{rating} = r)$$
$$= P_G(\text{product\_type} = p, \text{rating} = r) \tag{1}$$
$$P_{G'}(\text{hit} = 1 | \text{product\_type} = p, \text{rating} = r)$$
$$= P_G(\text{hit} = 1 | \text{product\_type} = p, \text{rating} = r) \tag{2}$$

Formalizing interventions: the intuition of "do"

$P(\text{hit} = 1|\text{do}(\text{days}) = d)$

$= P_{G'}(\text{hit} = 1|\text{days} = d)$, by definition

$= \sum_{p,r} P_{G'}(\text{hit} = 1|\text{days} = d, \text{product\_type} = p, \text{rating} = r)$

$\qquad P_{G'}(\text{product\_type} = p, \text{rating} = r|\text{days} = d)$, by total probability

$= \sum_{p,r} P_{G'}(\text{hit} = 1|\text{days} = d, \text{product\_type} = p, \text{rating} = r)$

$\qquad P_{G'}(\text{product\_type} = p, \text{rating} = r)$, by substitution

$= \sum_{p,r} P_{G}(\text{hit} = 1|\text{days} = d, \text{product\_type} = p, \text{rating} = r)$

$\qquad P_{G}(\text{product\_type} = p, \text{rating} = r)$, our *adjustment* formula

References: [PGJ16], [Pro]
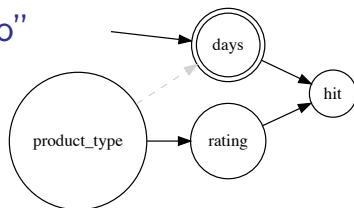
Causal hit rate

$P(\text{hit} = 1 | \text{days} = d)$

days	prob
0	0.534270
1	0.449109
2	0.318932
3	0.168999

$P(\text{hit} = 1 | \text{do}(\text{days} = d))$

days	prob
0	0.555426
1	0.409581
2	0.274834
3	0.196311

Judea Pearl's Rules of Causality

Let X, Y, Z and W be arbitrary disjoint sets of nodes in a DAG G. Let $G_{\underline{X}}$ be the graph obtained by removing all arrows pointing into (nodes of) X. Denote by $G_{\overline{X}}$ the graph obtained by removing all arrows pointing out of X. If, e.g. we remove arrows pointing out of X and into Z, we the resulting graph is denoted by $G_{\underline{X}\overline{Z}}$

Rule 1: Insertion / deletion of observations

$$P(y|\mathrm{do}(x), z, w) = P(y|\mathrm{do}(x), w) \text{ if } (Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}}}$$

Rule 2: Action / observation exchange

$$P(y|\mathrm{do}(x), \mathrm{do}(z), w) = P(y|\mathrm{do}(x), z, w) \text{ if } (Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}\underline{Z}}}$$

Rule 3: Insertion / deletion of actions

$$P(y|\mathrm{do}(x), \mathrm{do}(z), w) = P(y|\mathrm{do}(x), w) \text{ if } (Y \perp\!\!\!\perp Z | X, W)_{G_{\overline{X}\overline{Z(W)}}},$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\overline{X}}$.

Special cases of the causal rules

By judicious setting of sets of nodes to be empty, we obtain some useful corollaries of the causal rules.

Rule 1': Insertion / deletion of observations, with $W = \emptyset$

$$P(y|\mathrm{do}(x), z) = P(y|\mathrm{do}(x)) \text{ if } (Y \perp\!\!\!\perp Z|X)_{G_{\overline{X}}}$$

Rule 2': Action / observation exchange, with $X = \emptyset$

$$P(y|\mathrm{do}(z), w) = P(y|z, w) \text{ if } (Y \perp\!\!\!\perp Z|W)_{G_{\underline{Z}}}$$

Rule 3': Insertion / deletion of actions, with $X, W = \emptyset$

$$P(y|\mathrm{do}(z)) = P(y) \text{ if } (Y \perp\!\!\!\perp Z)_{G_{\overline{Z}}}$$

Special cases of the causal rules

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Rule 1': Insertion / deletion of observations, with $W = \emptyset$

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Rule 3': Insertion / deletion of actions, with $X, W = \emptyset$

$$P(y|\mathrm{do}(z)) = P(y) \text{ if } (Y \perp\!\!\!\perp Z)_{G_{\overline{Z}}}$$

$\implies$ d-separation + causal rules = *adjustment formulas*: do queries as normal queries.

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[FPP98] D. Freedman, R. Pisani, and R. Purves, *Statistics*, W.W. Norton, 1998.

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