Intelligent systems - KBS

- Reasoning techniques for uncertainty
  - Teory of Bayes – probabilistic method
  - Theory of certainty
  - Theory of possibility (fuzzy logic)

Heuristic methods

Intelligent systems – KBS – Fuzzy systems

- Theory of possibility
- Content and design
- Typology
- Tools
- Advantages and limits

Intelligent systems – KBS – Fuzzy systems

Teoria posibilităţii (logica fuzzy)

- Why fuzzy?
  - Problem: translate in C++ code the following sentences:
    - Georgel is tall.
    - It is cold outside.

- When fuzzy is important?
  - Natural queries
  - Knowledge representation for a KBS
  - Fuzzy control – then we dead by imprecise phenomena (noisy phenomena)

Intelligent systems – KBS – Fuzzy systems

Remember the components of a KBS

- Knowledge base → knowledge representation
  - Formal logic (formal languages)
    - Definition
      - Science of formal principles for rationing
    - Components
      - Syntax – atomic symbols used by language and the constructing rules of the language
      - Semantic – associates a meaning to each symbol and a truth value (true or false) to each rule
      - Syntactic inference – rules for identifying a subset of logic expressions → theorems (for generating new expressions)
    - Typology
      - True value
        - Dual logic
        - Polyvalent logic
      - Basic elements
        - Classic → primitives = sentences (predicates)
        - Probabilistic → primitives = random variables
      - Working manner
        - Propositional logic → declarative propositions and fix or unique objects (Ionica is student)
        - First-order logic → declarative propositions, predicates and quantified variables, unique objects or variables associated to a unique object
  - Rules
  - Semantic nets
- Inference engine

Intelligent systems – KBS – Fuzzy systems

Theory of possibility – a little bit of history

- Parminedes (400 B.C.)
- Aristotle
  - "Law of the Excluded Middle“ – every sentence must be True or False
- Plato
  - A third region, between True and False
  - Forms the basis of fuzzy logic
- Lukasiewicz (1900)
  - Has proposed an alternative and sistematic approach related to bi-valent logic of Aristotle – trivalent logic: true, false or possible
- Lotfi A. Zadeh (1965)
  - Mathematical description of fuzzy set theory and fuzzy logic: truth functions takes values in [0,1] (instead of {True, False})
    - He as proposed new operations in fuzzy logic
    - He has considered the fuzzy logic as a generalisation of the classic logic
  - He has written the first paper about fuzzy sets

Theory of possibility

- Fuzzy logic
  - Generalisation of Boolean logic
  - Deals by the concept of partial truth
    - Classical logic – all things are expressed by binary elements
      - 0 or 1, white or black, yes or no
    - Fuzzy logic – gradual expression of a truth
      - Values between 0 and 1

- Logic vs. algebra
  - Logical operators are expressed by using mathematical terms (George Boole)
    - Conjunction = minimum ➔ $a \wedge b = \min(a, b)$
    - Disjunction = maximum ➔ $a \vee b = \max(a, b)$
    - Negation = difference ➔ $\neg a = 1 - a$

Intelligent systems – KBS – Fuzzy systems

Remember: KBS - design

- Knowledge base
  - Content
    - Specific information
      - Facts – correct affirmations
      - Rules – special heuristics that generate knowledge
  - Aim
    - Store all the information (facts, rules, solving methods, heuristics) about a given domain (taken from some experts)
- Inference engine
  - Content
    - Rules for generating new information
    - Domain-independent algorithms
    - Brain of a KBS
  - Aim
    - Help to explore the KB by reasoning for obtaining solutions, recommendations or conclusions

Intelligent systems – KBS – Fuzzy systems

Content and design

- Main idea
  - Cf. to certainty theory:
    - *Popescu is tall*
  - Cf. to uncertainty theory
    - Cf. to probability theory
      - *There is 80% chance that Popescu is young*
    - Cf. fuzzy logic

  - Cf. teoriei informaţiilor certe
    - *Popescu este tânăr*
  - Cf. teoriei informaţiilor incerte
    - Cf. teoriei probabilităţilor:
      - *Există 80% şanse ca Popescu să fie tânăr*
    - Cf. logicii fuzzy:
      - *Popescu's degree of membership to the group of young people is 0.80*

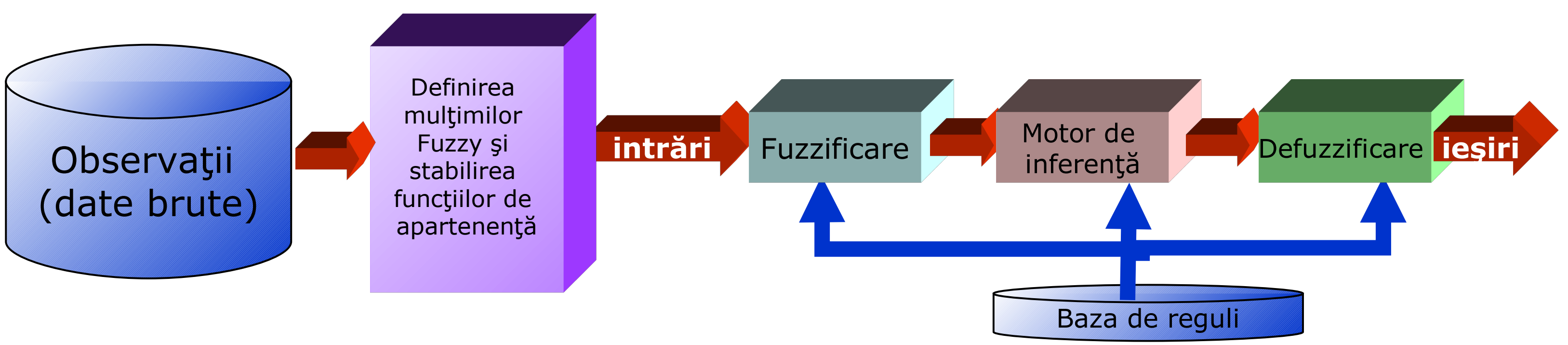
- Necessity
  - Real phenomena involve fuzzy sets
  - Example
    - *The room's temperature can be:*
      - *low,*
      - *Medium or*
      - *high*

    - These sets of possible temperatures can overlap
      - A temperature can belong to more classes (groups) depends on the person that evaluates that temperature

Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic)
  - Fuzzy facts (fuzzy sets)
    - Definition
    - Representation
    - Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
    - Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
    - Hedges
  - Fuzzy variables
    - Definition
    - Properties

- Establish the fuzzy variables and the fuzzy sets based on membership functions

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

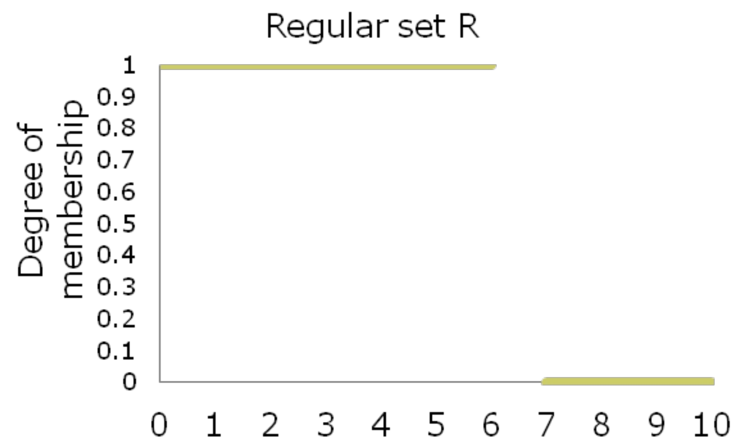
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **definition**
  - Set definition – 2 possibilities:
    - By enumeration of elements
      - Ex. *Set of students = {Ana, Maria, Ioana}*
    - By specifying a property of elements
      - Ex. Set of even numbers =*{x| x = 2n, where n = 2k}*
  - Characteristic function μ for a set
    - Let X a universal set and x an element of this set (x∊X)
    - Classical logic
      - Let R a sub-set of X: R⊂X, R – regular set
      - Every element x belong to set R
      - μ_R : X → {0, 1}, where $$\mu_R(x) = \begin{cases} 1, & x \in R \\ 0, & x \notin R \end{cases}$$
    - Fuzzy logic
      - Let F a sub-set of X (a univers) : F⊂X, F – fuzzy set
      - Every elemt x belongs to F by a given degree of membership $\mu_F(x)$
      - μ_F : X → [0, 1], $\mu_F(x)=g$, where g∊ [0,1] – membership degree of x to F
      - g = 0 ➔ not-belong
      - g = 1 ➔ belong
      - A fuzzy set = a pair (F, μ_F), where $$\mu_F(x) = \begin{cases} 1, & \text{if } x \text{ is totaly in } F \\ 0, & \text{if } x \text{ is not in } F \\ \in (0,1) & \text{if } x \text{ is part of } F (x \text{ is a fuzzy number}) \end{cases}$$

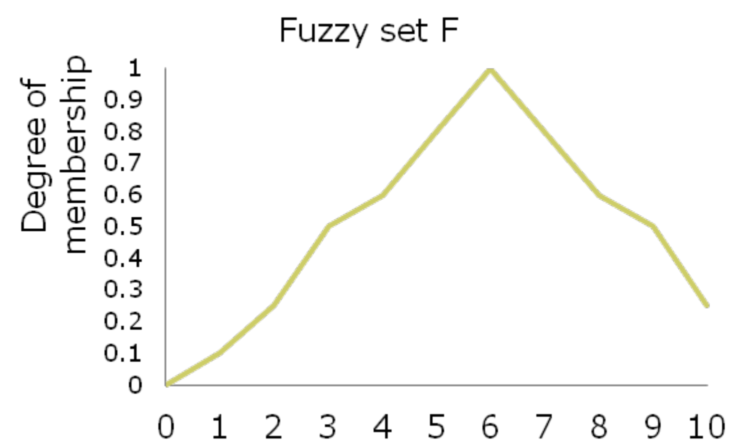
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **definition**
  - Example 1
    - X –set of natural numbers < 11
    - R – set of natural numbers < 7
    - F – set of natural numbers that are neighbours of 6

x	$\mu_R(x)$	$\mu_F(x)$
0	1	0
1	1	0.1
2	1	0.25
3	1	0.5
4	1	0.6
5	1	0.8
6	1	1
7	0	0.8
8	0	0.6
9	0	0.5
10	0	0.25

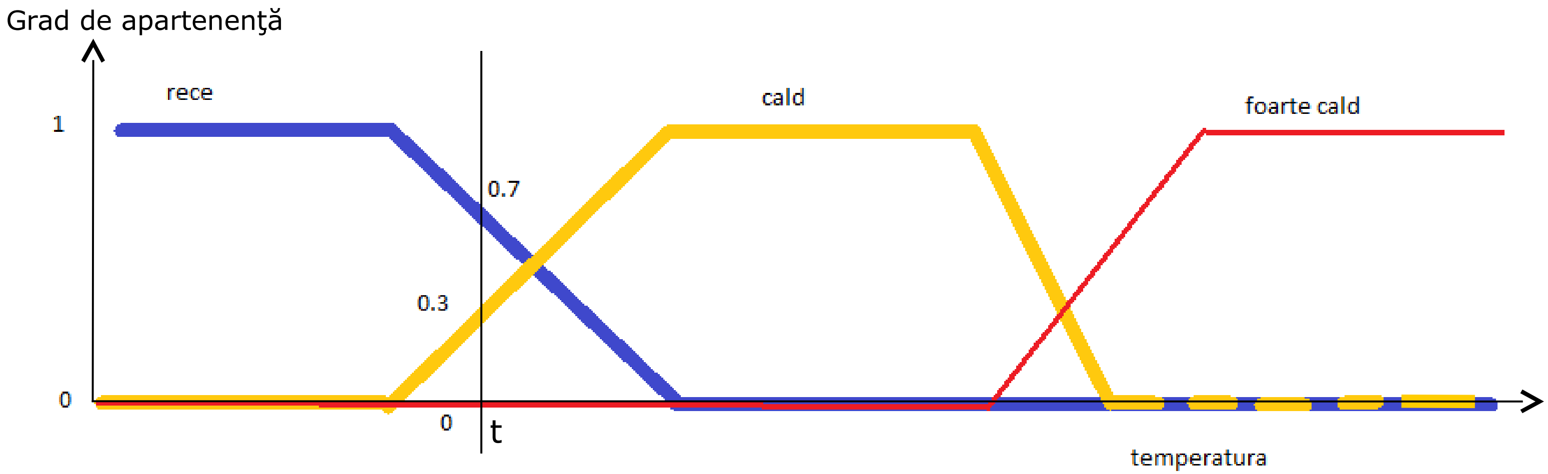




Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

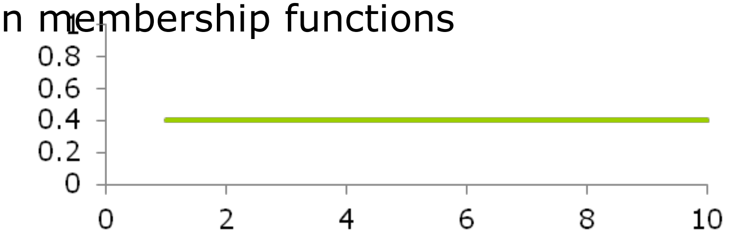
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **definition**
  - Example 2
    - A temperature *t* can have 3 truth values:
      - Red (0): is not hot
      - Orange (0.3): warm
      - Blue (0.7): cold



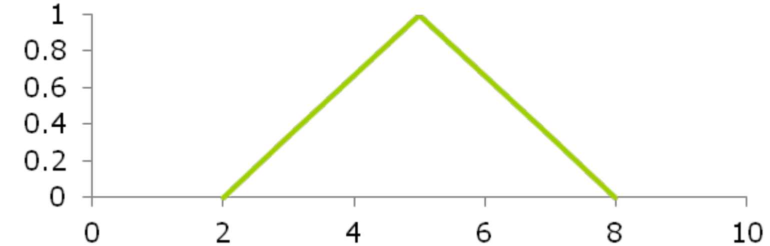
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

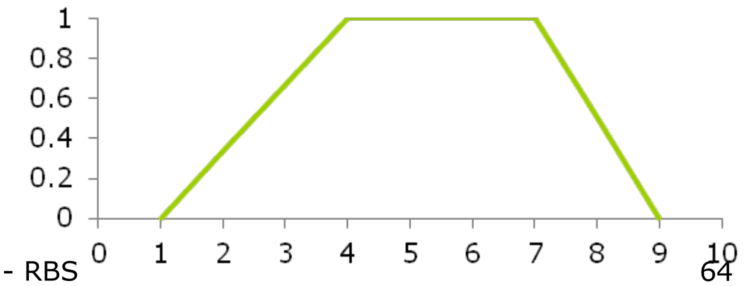
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **representation**
  - Regular sets
    - Exact limits → Venn diagrams
  - Fuzzy sets
    - Gradual limits → representations based on membership functions
      - Singular
        - *μ(x) = s*, where s is a scalar



      - Triangular
        - $\mu(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, 1, \frac{c-x}{c-b}\right\}\right\}$



      - Trapezoidal
        - $\mu(x) = S(x) = \max\left\{0, \min\left\{\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right\}\right\}$

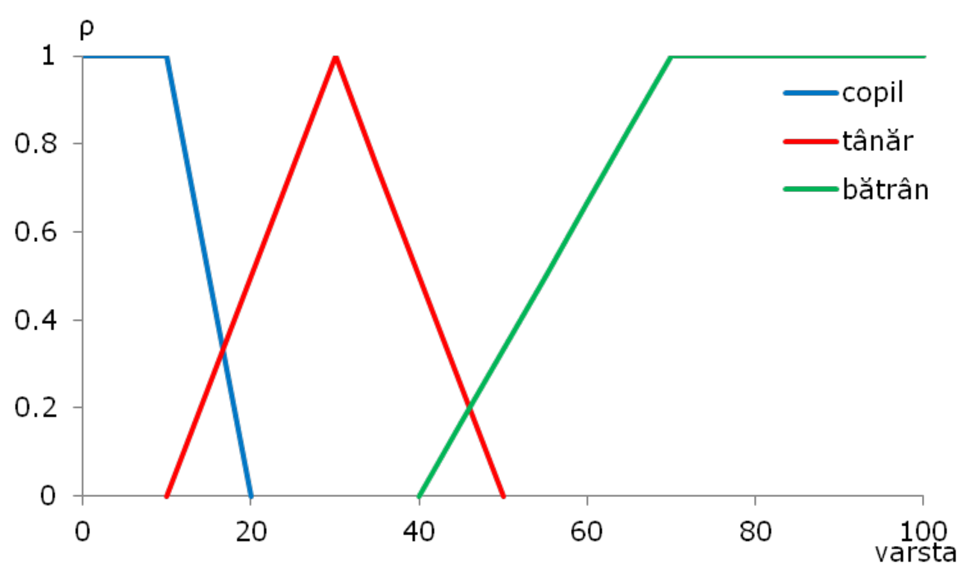


      - Z function
        - *μ(x) = 1- S(x)*
      - Π function
        - $\mu(x) = \Pi(x) = \begin{cases} S(x), & \text{if } x \le c \\ Z(x), & \text{if } x > c \end{cases}$

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **representation**
  - Example
    - *Age of a person*



Intelligent systems – KBS – Fuzzy systems

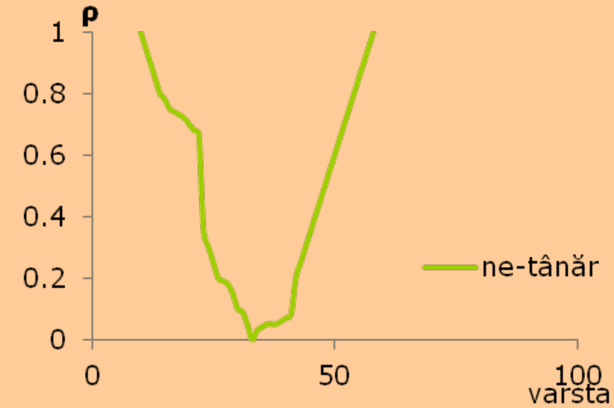
Content and design → fuzzification of input data

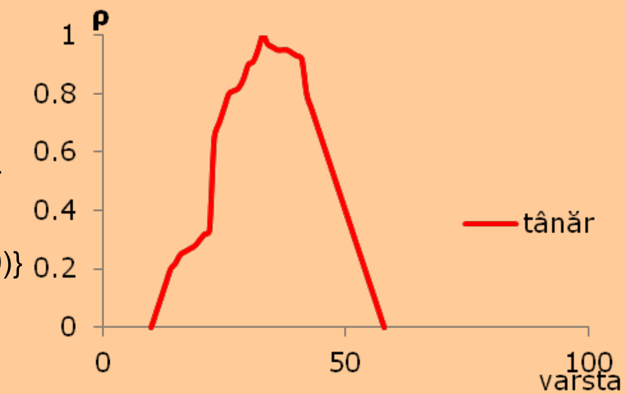
- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - complement
  - Containment
  - Intersection
  - Union
  - Equality
  - Algebraic product
  - Algebraic sum

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - Complement
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - B is complement of A (B= ┐A) if:
      - $\mu_B(x)=\mu_{\neg A}(x)=1-\mu_A(x)$ for all $x \in X$
    - Example:
      - *Old persons (based on their age)*
        - A={(30,0), (40, 0.2), (50, 0.4), (60, 0.6), (70, 0.8), (80, 1)}
      - *Young persons (that are not old) (based on their age)*
        - ┐A={(30,1), (40, 0.8), (50, 0.6), (60, 0.4), (70, 0.2), (80, 0)}





Intelligent systems – KBS – Fuzzy systems

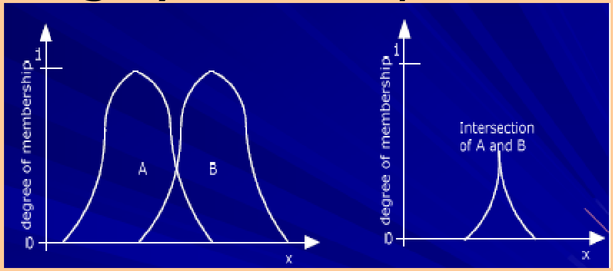
Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - Containment
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - B is a subset of A ($B \subset A$) if:
      - $\mu_B(x) \leq \mu_A(x)$ for all $x \in X$
    - Example
      - *Old persons (based on their age)*
        - A={(60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}
      - *Very old persons (based on their age)*
        - B={(60, 0.6), (65, 0.67) (70, 0.8), (75, 0.8), (80, 0.95)}

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - intersection
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - C – a fuzzy set (with universe X)
    - **C** is an intersection of A and B if:
      - $\mu_C(x)=\mu_{A\cap B}(x)=\min\{\mu_A(x), \mu_B(x)\}=\mu_A(x)\cap\mu_B(x)$ for all $x\in X$

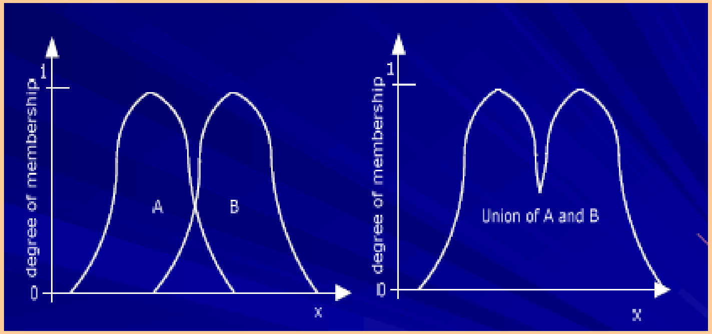


    - Example
      - *Old persons (based on their age)*
        - A={(30,0) (40, 0.1) (50, 0.2) (60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}
      - *Middle-age persons*
        - B={(30,0.1) (40, 0.2) (50, 0.6) (60, 0.5), (65, 0.2) (70, 0.1), (75, 0), (80, 0)}
      - *Old and middle age persons*
        - C={(30,0) (40, 0.1) (50, 0.2) (60, 0.5), (65, 0.2) (70, 0.1), (75, 0), (80, 0)}

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - union
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - C – a fuzzy set (with universe X)
    - C is the union of A nad B if:
      - $\mu_C(x)=\mu_{A\cup B}(x)=\max\{\mu_A(x), \mu_B(x)\}=\mu_A(x) \cup \mu_B(x)$ for all $x\in X$

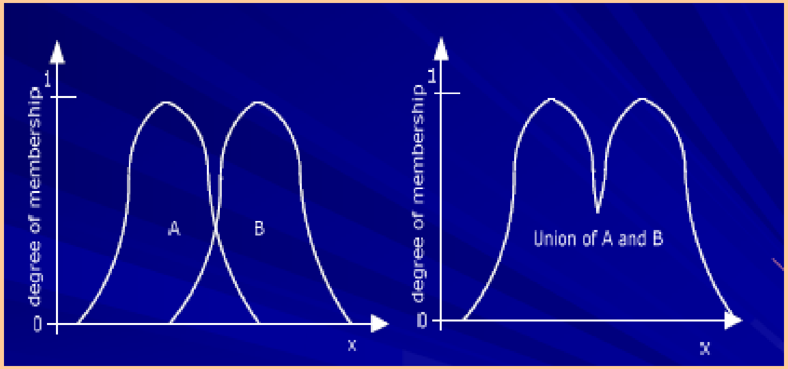


    - Example
      - *Old persons (based on their age)*
        - A={(30,0) (40, 0.1) (50, 0.2) (60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}
      - *Middle-age persons*
        - B={(30,0.1) (40, 0.2) (50, 0.6) (60, 0.5), (65, 0.2) (70, 0.1), (75, 0), (80, 0)}
      - *Old or middle-age persons*
        - C={(30,0.1) (40, 0.2) (50, 0.6) (60, 0.6), (65, 0.7) (70, 0.8), (75, 0.9), (80, 1)}

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **operations**
  - Equality, product and algebraic sum
    - X – a universe
    - A – a fuzzy set (with universe X)
    - B – a fuzzy set (with universe X)
    - C – a fuzzy set (with universe X)



    - B is equal to A (B=A) if:
      - $\mu_B(x)=\mu_A(x)$ for all $x \in X$
    - C is the product of A and B (C=A*B) if:
      - $\mu C(x)=\mu A*B(x)=\mu A(x)*\mu B(x)$ for all $x \in X$
    - C is the sum of A and B (C=A+B) if:
      - $\mu C(x)=\mu A+B(x)=\mu A(x)+\mu B(x)$ for all $x \in X$

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **properties**
  - Asociativity
  - Commutativity
  - Distributivity
  - Transitivity
  - Idem potency
  - Identity
  - Involution

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **hedges**

  - Main idea
    - Modifiers, adjectives or adverbs that change the truth values of sentences
      - Ex. *Very*, *less*, *much*, *more*, *close*, etc.
    - Change the shape of fuzzy sets
    - Can act on
      - Fuzzy numbers
      - Truth values
      - Membership functions
    - Heuristics

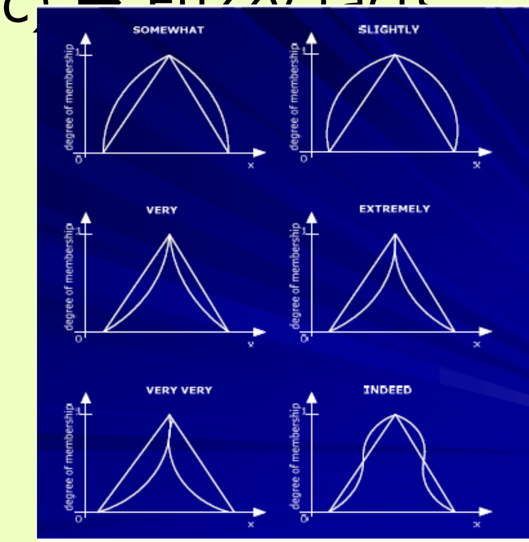
  - Utility
    - Closer to the natural language → subjectivism
    - Evaluation of linguistic variables

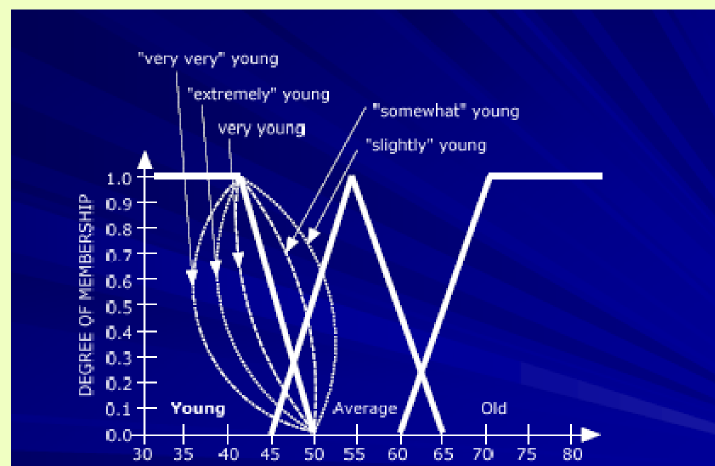
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy facts (fuzzy sets) → **hedges**
  - Typology
    - *Hedges* that reduce the truth value (produce a concentration)
      - *Very* $\mu_{A\_very}(x)=(\mu_A(x))^2$
      - *Extremly* $\mu_{A\_extremly}(x)=(\mu_A(x))^3$
      - *Very very* $\mu_{A\_very\_very}(x)=(\mu_{A\_foarte}(x))^2=(\mu_A(x))^4$
    - *Hedges* that increase the truth value (produce a dilatation)
      - *Somewhat* $\mu_{A\_somewhat}(x)=(\mu A(x))^{1/2}$
      - *slightly* $\mu_{A\_slightly}(x)=(\mu A(x))^{1/3}$
    - *Hedges* cthat intensify the truth value
      - *indeed*

$$\mu_{A\_indeed}(x)=\begin{cases} 2(\mu_A(x))^2, & if\ 0\le\mu_A(x)\le 0.5 \\ 1-2(1-\mu_A(x))^2, & if\ 0.5\le\mu_A(x)\le 1 \end{cases}$$





Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

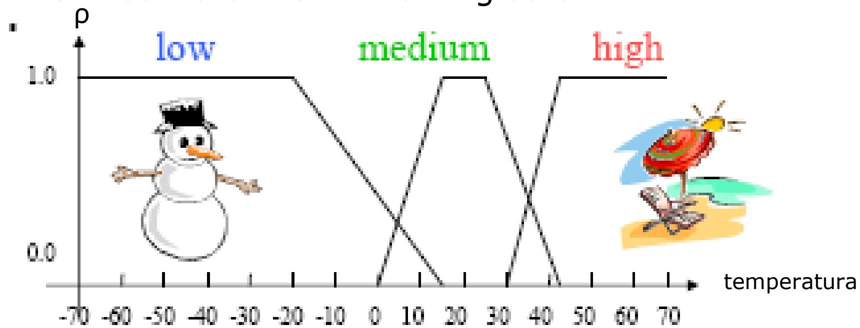
- Elements from probability theory (fuzzy logic)
  - Fuzzy facts (fuzzy sets)
    - Definition
    - Representation
    - Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
    - Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
    - Hedges
  - **Fuzzy variables**
    - **Definition**
    - **Properties**

    - Establish the fuzzy variables and the fuzzy sets based on membership functions

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

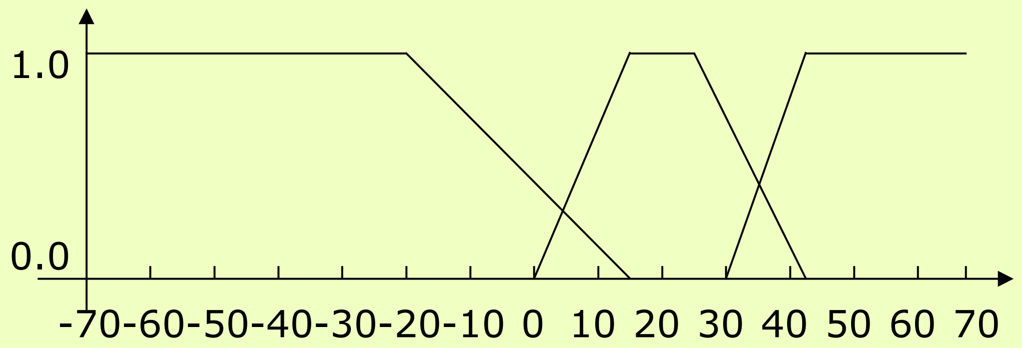
- Elements from probability theory (fuzzy logic) → Fuzzy variables → **definition**
  - A fuzzy variable is defined by $V = \{x, l, u, m\}$, where:
    - x – name of symbolic variable
    - L – set of possible labels for variable x
    - U – universe of the variable
    - M – semantic regions that define the meaning of labels from L (membership functions)
  - Membership functions
    - Subjective assessment
      - The shape of functions is defined by experts
    - Ad-hoc assessment
      - Simple functions that can solve the problem
    - Assessment based on distributions and probabilities of information extracted from measurements
    - Adapted assessment
      - By testing
    - Automated assessment
      - Algorithms utilised for defining functions based on some training data
  - Example
    - X = Temperature
    - L = {low, medium, high}
    - U = $\{x \in X \mid -70^\circ \leq x \leq +70^\circ\}$
    - M =



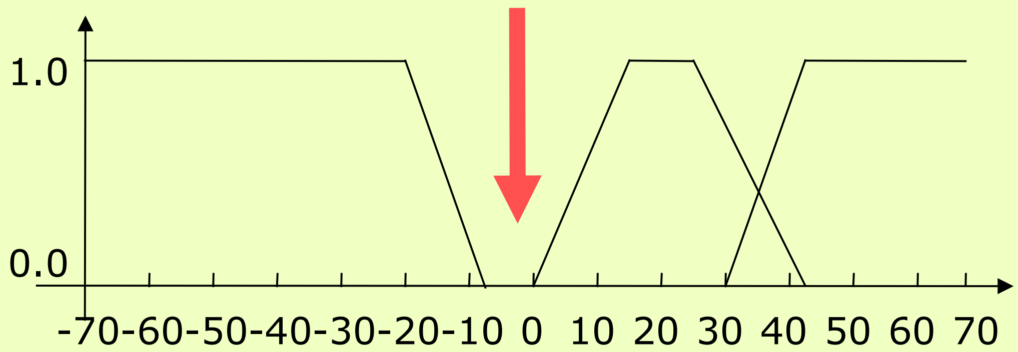
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy variables → **properties**
  - Completeness
    - A fuzzy variable V is complete if for all $x \in X$ there is a fuzzy set A such as $\mu_A(x)>0$



Complete

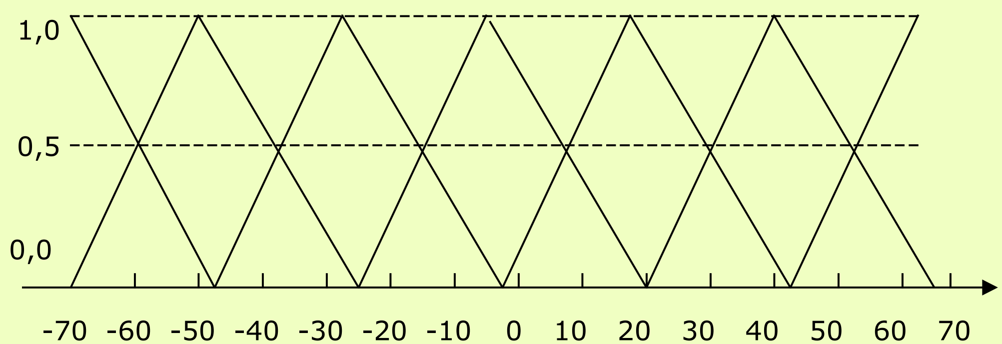


Incomplete

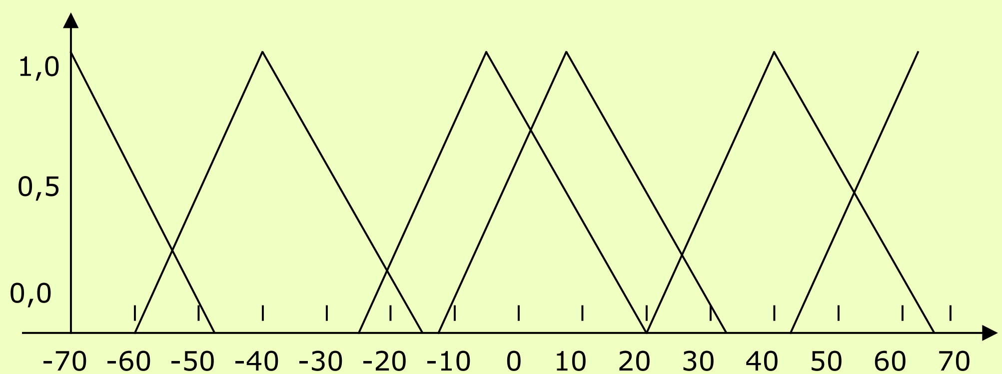
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy variables → **properties**
  - Unit partition
    - A fuzzy variable V forms a unit partition if for all input values x we have $\sum_{i=1}^{p} \mu_{A_i}(x) = 1$
    - where p is the number of sets that x belongs to
    - There are no rules for defining 2 neighbour sets
      - Usually, the overlap is between 25% şi 50%



Unit partition



Non-unit partition

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic) → Fuzzy variables → **properties**
  - Unit partition
    - A complete fuzzy variable can be transformed into a unit partition:

$$\mu_{\hat{A}_i}(x) = \frac{\mu_{A_i}(x)}{\sum_{j=1}^{p} \mu_{A_j}(x)} \; for \; i = 1, \ldots, p$$

Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- Elements from probability theory (fuzzy logic)
  - Fuzzy facts (fuzzy sets)
    - Definition
    - Representation
    - Operations – complements, containment, intersection, reunion, equality, algebraic product, algebraic sum
    - Properties – associativity, commutativity, distributivity, transitivity, idempotency, identity, involution
    - Hedges
  - Fuzzy variables
    - Definition
    - Properties

- **Establish the fuzzy variables and the fuzzy sets based on membership functions**

Intelligent systems – KBS – Fuzzy systems

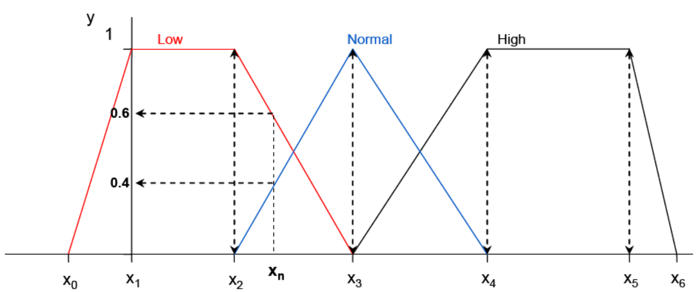
Content and design → fuzzification of input data

- Mechanism
  - Establish the raw (input and out[put) data of the system

  - Define membership functions for each input data
    - Each membership function has associated a quality label – linguistic variable
    - A raw variable can have associated one or more linguistic variables
    - Example
      - Raw variable: temperature T
      - Linguistic variable: law →A1, medium → A2, high → A3

  - Transform each raw input data into a linguistic data → fuzzification
    - Establish the fuzzy set of that raw input data
    - How?
      - For a given raw input determine the membership degree for each possible set

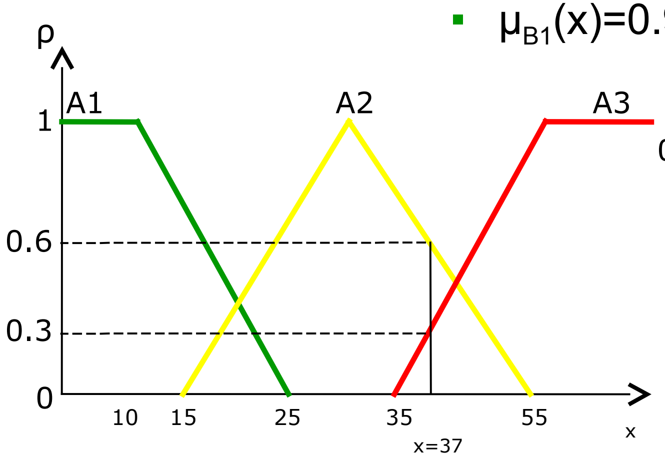
    - Example
      - T $(=x_n)$ $=5°$
      - $A_1 \rightarrow \mu_{A1}(T) = 0.6$
      - $A_2 \rightarrow \mu_{A2}(T) = 0.4$

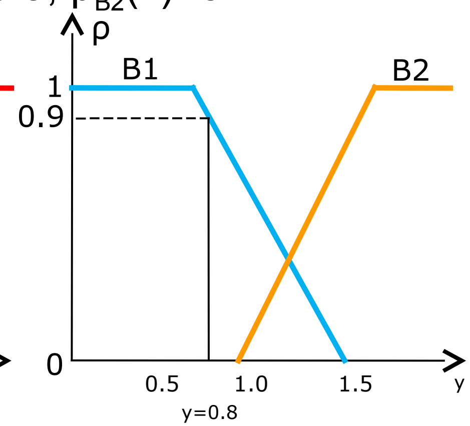


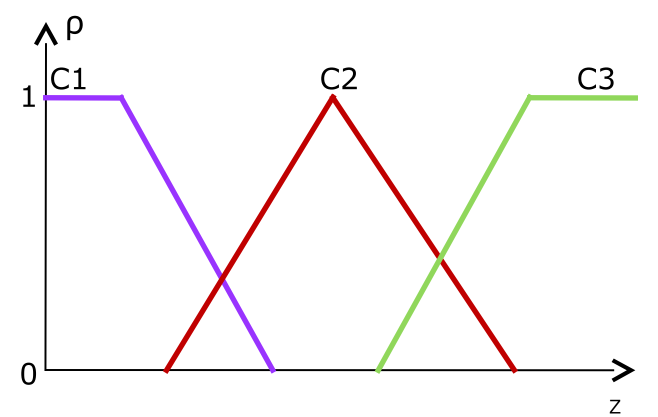
Intelligent systems – KBS – Fuzzy systems

Content and design → fuzzification of input data

- **Mechanism**
  - Example - air conditioner device
    - Inputs :
      - x (temperature – cold, normal, hot) and
      - y (humidity – small, large)
    - Outputs:
      - z (machine power – law, medium, high)
    - Input data:
      - Temperature x = 37
        - $\mu_{A1}(x)=0$, $\mu_{A2}(x)=0.6$, $\mu_{A3}(x)=0.3$
      - Humidity y = 0.8
        - $\mu_{B1}(x)=0.9$, $\mu_{B2}(x)=0$



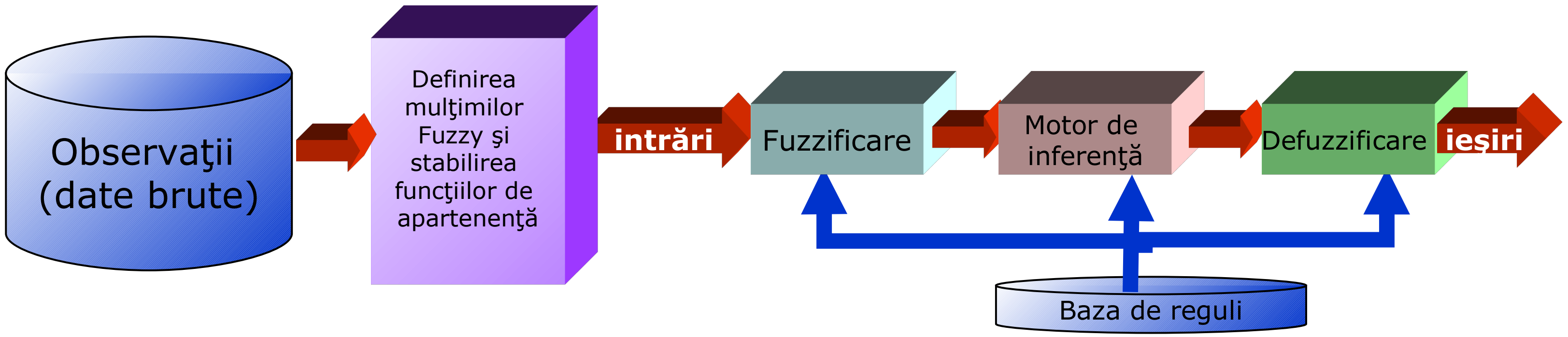




Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
    - Raw inputs and outputs
    - Fuzzification of inputs and outputs
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - **Construct a base of rules – by an expert**
    - **Decision matrix**
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
  - Definition
    - Linguistic constructions
      - Affirmative sentences: A
      - Conditional sentences: if A then B
    - Where A and B are (collections of) sentences that contain linguistic variables
      - A – premise of the rule
      - B – consequence of the rule
  - Typology
    - Non-conditional
      - x is (in) A_i
      - Eg. *Save the energy*
    - Conditional
      - If x is (in) A_i then z is (in) C_k
      - If x is (in) A_i and y is (in) B_j, then z is (in) C_k
      - If x is (in) A_i or y is (in) B_j, then z is (in) C_k

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
  - Example

	Rules of classical logic	Rules of fuzzy logic
R_1	*If temperature is -5, then is cold*	*If temperature is law, then is cold*
R_2	*If temperature is 15, then is warm*	*If temperature is medium, then is warm*
R_3	*If temperature is 35, then is hot*	*If temperature is high, then is hot*

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
- Database of fuzzy rules
  - R_{11}: if x is A_1 and y is B_1 then z is C_u
  - R_{12}: if x is A_1 and y is B_2 then z is C_v
  - ...
  - R_{1n}: : if x is A_1 and y is B_n then z is C_x
  - R_{21}: if x is A_2 and y is B_1 then z is C_x
  - R_{22}: if x is A_2 and y is B_2 then z is C_z
  - ...
  - R_{2n}: if x is A_2 and y is B_n then z is C_v
  - ...
  - R_{m1}: if x is A_m and y is B_1 then z is C_x
  - R_{m2}: if x is A_m and y is B_2 then z is C_v
  - ...
  - R_{mn}: if x is A_m and y is B_n then z is C_u

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Rules
- Properties
  - Completeness
    - A database of fuzzy rules is complete
      - If all input values have associated a value between 0 and 1
      - If all fuzzy variable are complete
      - If used fuzzy sets have a non-compact support
  - Consistency
    - A set of fuzzy rules is inconsistent if two rules have the same premises and different consequences
      - If x in A and y in B then z in C
      - If x in A and y in B then z in D
- Problems of the database
  - Rule's explosion
    - #of rules increases exponential whit the # of input variables
    - # of input set combinations is
      - Where the i^{th} variable is composed by p_i sets

$$P = \prod_{i=1}^{n} p_i$$

Intelligent systems – KBS – Fuzzy systems

Content and design → Construct a base of rules – by an expert

- Decision matrix of the knowledge database
- Example – air conditioner device
  - Inputs :
    - x (temperature – cold, normal, hot) and
    - y (humidity – small, large)
  - Outputs:
    - z (machine power – law, constant, high)
  - Rules:
    - *If temperature is normal and humidity is small then the power is constant*

		Input data y	
		Small	Large
Input data x	Cold	Law	Constant
	Normal	Constant	High
	Hot	High	High

Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
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      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - **Evaluate the rules**
    - **Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules**
  - Aggregate the results
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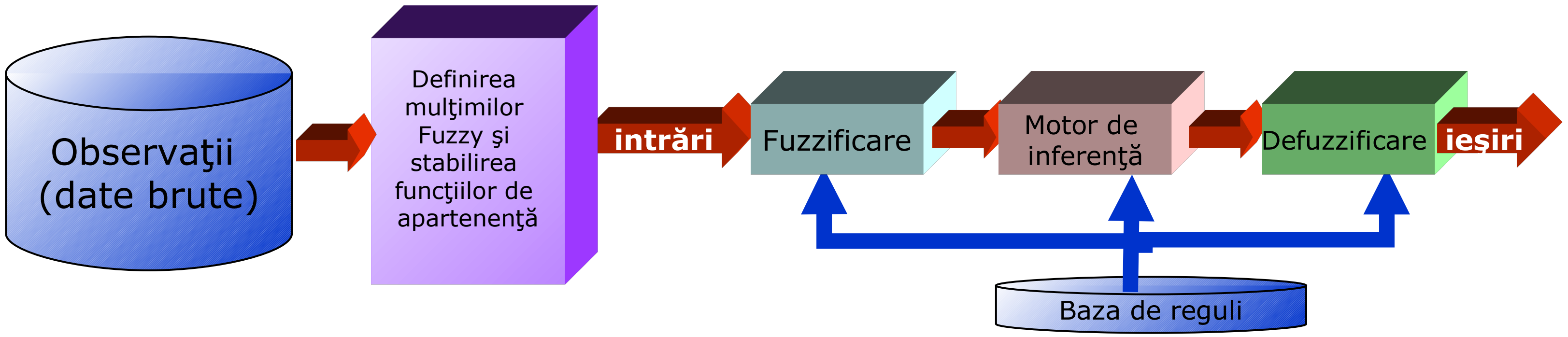
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference)

- Which rules are firstly evaluated?
  - Fuzzy inference
    - Rules are evaluated in **parallel** , each rules contributing to the shape of the final result
    - Resulted fuzzy sets are de-fuzzified **after all the rules** have been evaluated

  Remember
  - Forward inference
    - For a given state of problem, collect the required information and apply the possible rules
  - Backward inference
    - Identify the rules that determine the final state and apply only that rules (if it is possible)

- How the rules are evaluated?
  - Evaluation of causes
  - Evaluation of consequences

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference)

- Evaluation of causes
  - For each premise of a rule (*if s is (in) A*) establish the membership degree of raw input data to all fuzzy sets
  - A rule can have more premises linked by logic operators *AND, OR* or *NOT* → use fuzzy operators
    - Operator *AND* → intersection (minimum) of 2 sets
      - $\mu_{A \cap B}(x) = min\{\mu_A(x), \mu_B(x)\}$
    - Operator *OR* → union (maximum) of 2 sets
      - $\mu_{A \cup B}(x) = max\{\mu_A(x), \mu_B(x)\}$
    - Operator *NOT* → negation (complement) of a set
      - $\mu_{\neg A}(x) = 1 - \mu_A(x)$
  - The result of premise's evaluation
    - Degree of satisfaction
    - Other names:
      - Rule's firing strength
      - Degree of fulfillment

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference)

- Evaluation of consequences
  - Determine the results
    - Establish the membership degree of variables (involved in the consequences) to different fuzzy sets
  - Each output region must be de-fuzzified in order to obtain crisp value
  - Based on the consequence's type
    - Mamdani model – consequence of rule: "output variable belongs to a fuzzy set"
    - Sugeno model – consequence of rule: "output variable is a crisp function that depends on inputs"
    - Tsukamoo model – consequence of rule: "output variable belongs to a fuzzy set folowing a monotone membership function"

Intelligent systems – KBS – Fuzzy systems

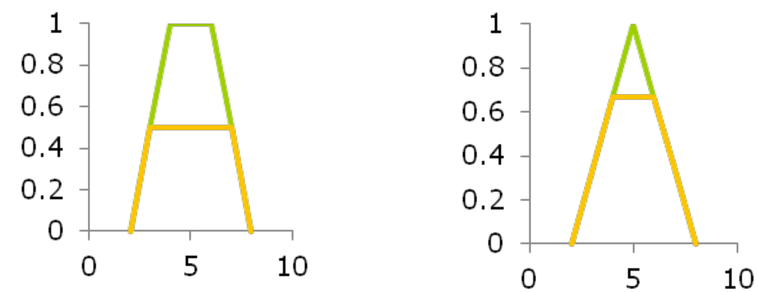
Content and design → rule evaluation (fuzzy inference) → **Evaluation of consequences**

- Mamdani model
  - Main idea:
    - consequence of rule: "output variable belongs to a fuzzy set"
    - Result of evaluation is applied for the membership function of the consequence
    - Example
      - ***if x is in A and y is in B, then z is in C***
  - Typology (based on how the results is applied on the membership function of the consequence)
    - Clipped fuzzy sets
    - Scaled fuzzy sets

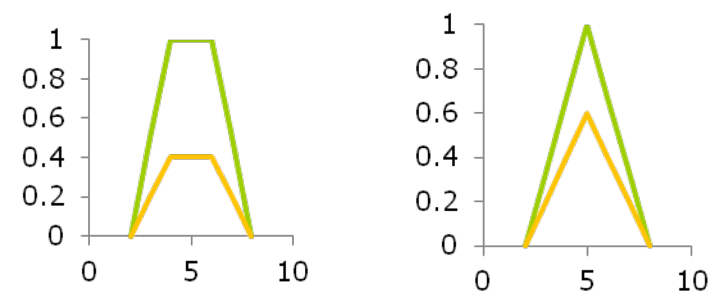
Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference) →
Evaluation of consequences

- Mamdani model
  - Typology (based on how the results is applied on the membership function of the consequence)
    - Clipped fuzzy sets
      - Membership function of the consequence is cut at the level of the result's truth value
      - Advantage → easy to compute
      - Disadvantage → some information are lost



    - Scaled fuzzy sets
      - Membership function of the consequence is adjusted by scaling (multiplication) at the level of the result's truth value
      - Advantage → few information is lost
      - Disadvantage → complicate computing



Intelligent systems – KBS – Fuzzy systems

- Content and design → rule evaluation (fuzzy inference) → **Evaluation of consequences** → Mamdani model
  - Example – air conditioner device
    - Inputs :
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      - y (humidity – small, large)
    - Outputs:
      - z (machine power – law, constant, high)
    - Input data:
      - Temperature x = 37
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      - Humidity y = 0.8
        - $\mu_{B1}(x)=0.9$, $\mu_{B2}(x)=0$

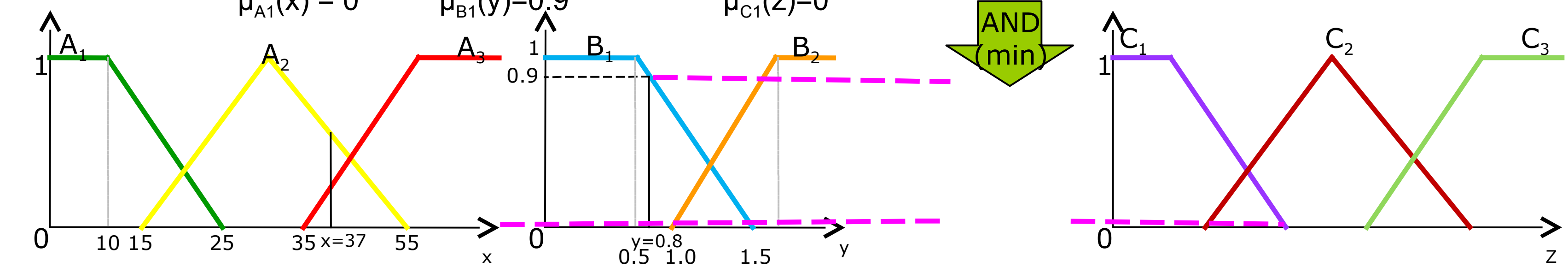
		Input data y	
		Small	Large
Input data x	Cold	Law	Constant
	Normal	Constant	High
	Hot	High	High

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation → Evaluation of consequences → Mamdani model

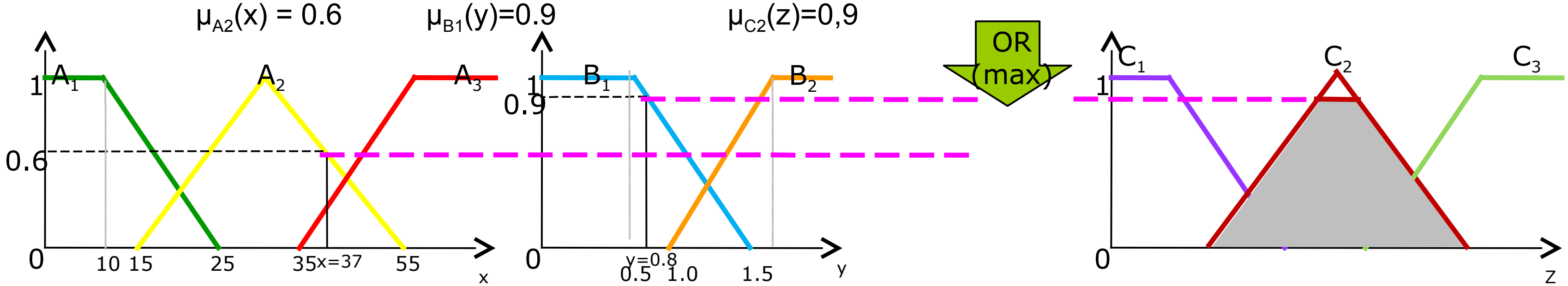
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



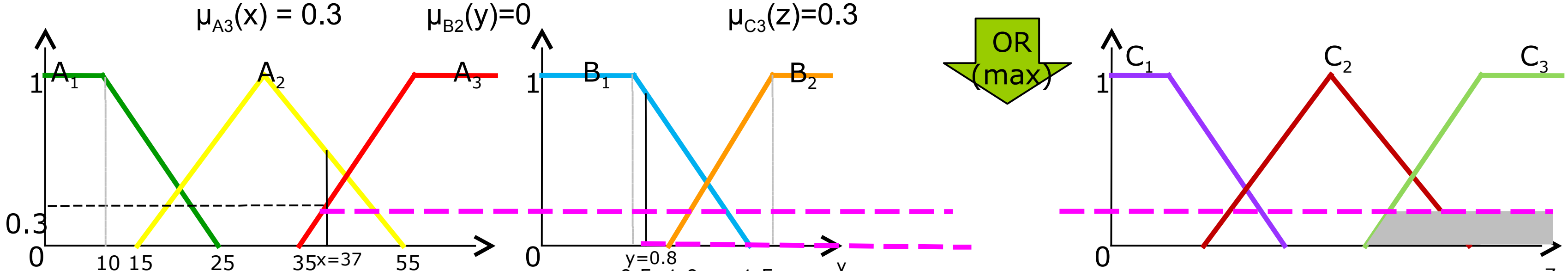
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference) → **Evaluation of consequences**

- Sugeno model
  - Main idea
    - consequence of rule: "output variable is a crisp function that depends on inputs"
    - Example

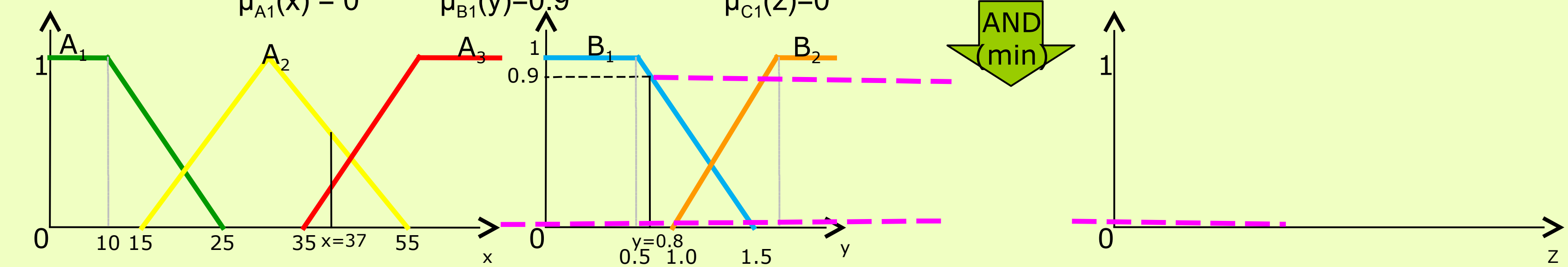
      If x is in A and y is in B then z is f(x,y)
    - Typology (based on charactersitics of f(x,y))
      - Sugeno model of degree 0 → if (f(x,y) = k – constant (membership function of the consequences are singleton – a fuzzy set whose membership functions have value 1 for a single (unique) point of the universe and 0 for all other points)
      - Sugeno model of degree 1→ if f(x,y) = ax + by+c

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation → Evaluation of consequences → Sugeno model

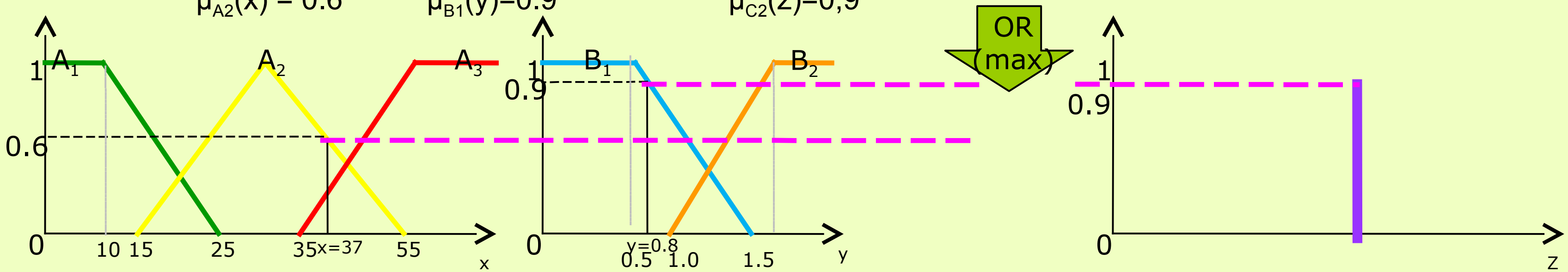
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



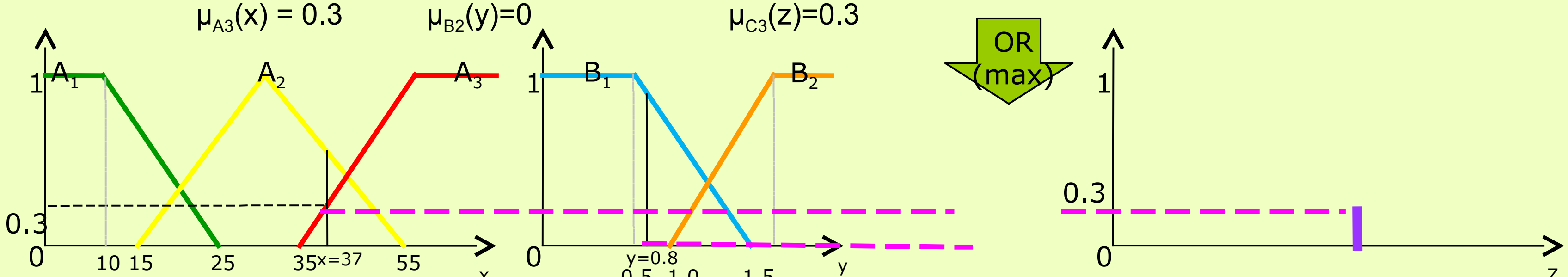
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0,9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation (fuzzy inference) → **Evaluation of consequences**

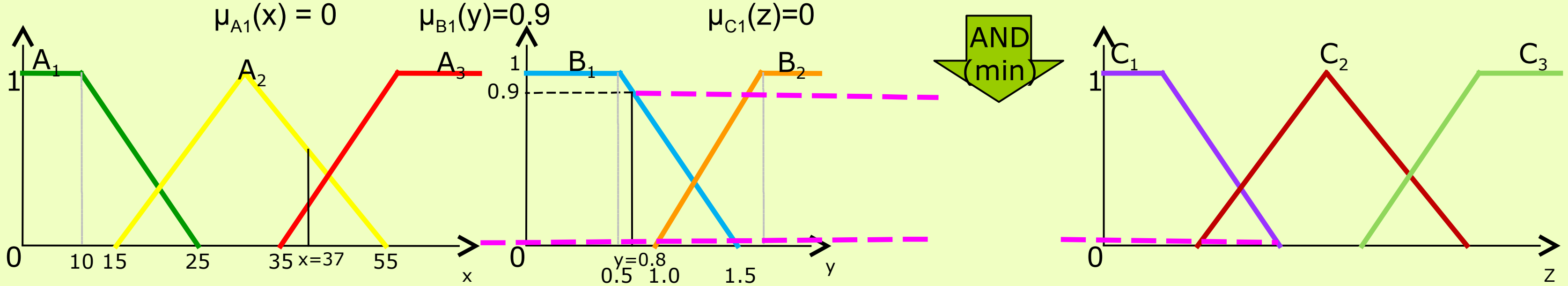
- Tsukamoto model
  - Main idea
    - consequence of rule: “output variable belongs to a fuzzy set folowing a monotone membership function”
      - A crisp value is obtained as output → *rule’s firing strength*

Intelligent systems – KBS – Fuzzy systems

Content and design → rule evaluation → Evaluation of consequences → Tsukamoto model

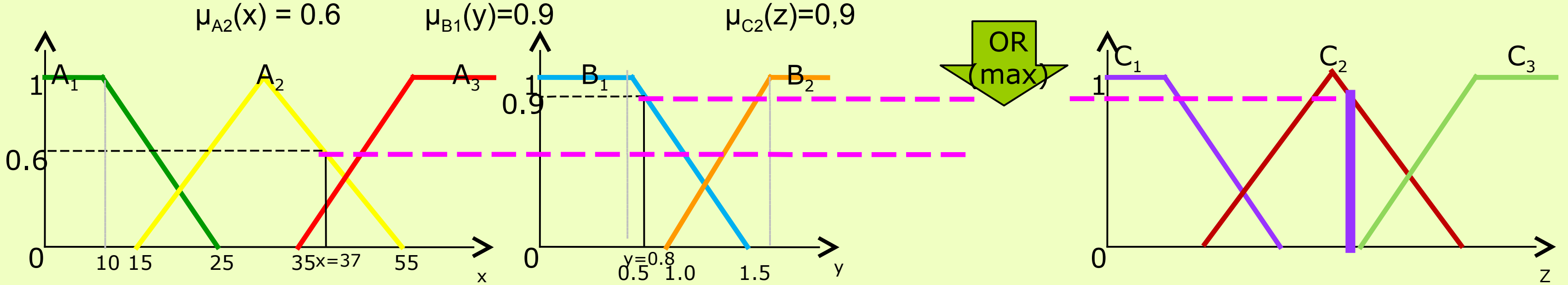
R1: if x is in A_1 and y is in B_1 then z is in C_1

$\mu_{A1}(x) = 0$ $\mu_{B1}(y)=0.9$ $\mu_{C1}(z)=0$



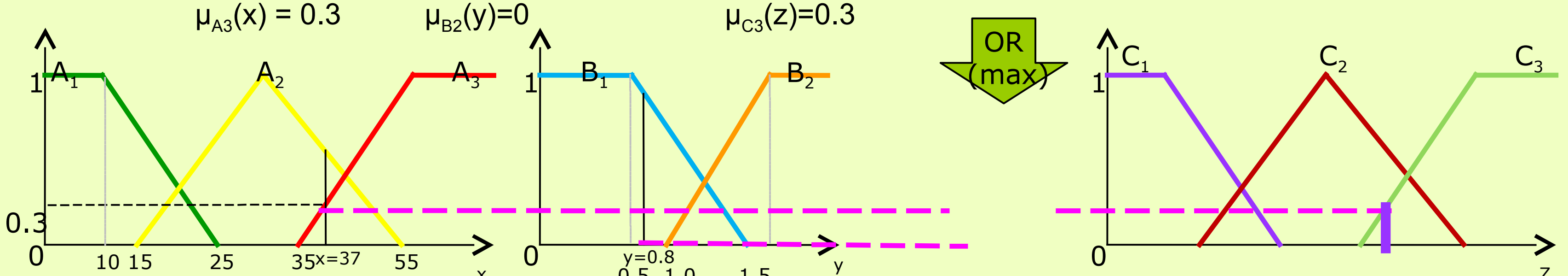
R2: if x is in A_2 or y is in B_1 then z is in C_2

$\mu_{A2}(x) = 0.6$ $\mu_{B1}(y)=0.9$ $\mu_{C2}(z)=0{,}9$



R3: if x is in A_3 or y is in B_2 then z is in C_3

$\mu_{A3}(x) = 0.3$ $\mu_{B2}(y)=0$ $\mu_{C3}(z)=0.3$



Intelligent systems – KBS – Fuzzy systems

Content and design

- Steps for constructing a fuzzy system
  - Define the inputs and the outputs – by an expert
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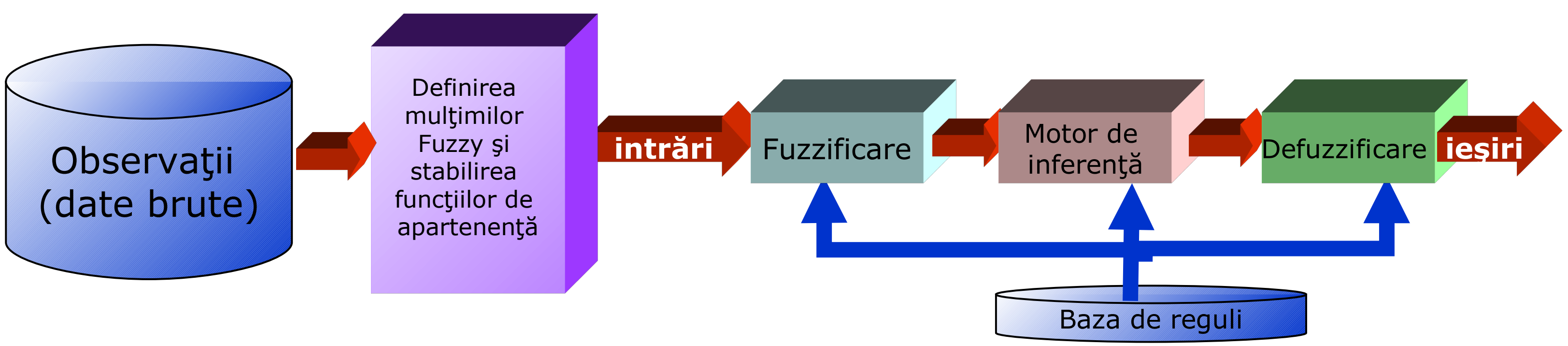
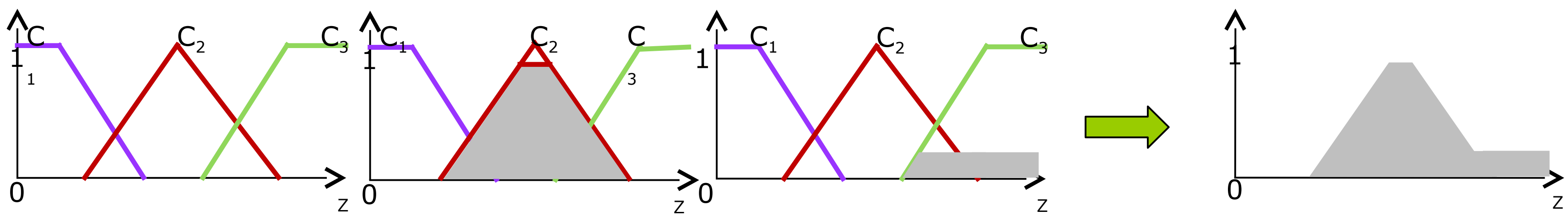
      - Fix the fuzzy variables and fuzzy sets based on membership functions
  - Construct a base of rules – by an expert
    - Decision matrix
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - **Aggregate the results**
  - Defuzzificate the result
  - Interpret the result



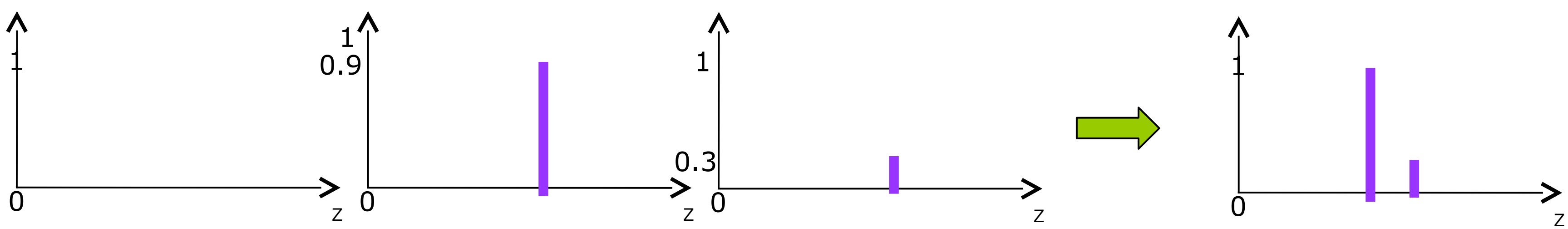
Intelligent systems – KBS – Fuzzy systems

Content and design → **Aggregate the results**

- Union of outputs for all the applied rules
- Consider the membership functions for all the consequences and combine them into a single fuzzy set (a single result)
- Aggregation process have as
  - Inputs → membership functions (clipped or scaled) of the consequences
  - Outputs → a fuzzy set of the output variable
- Example
  - Mamdani



  - Sugeno



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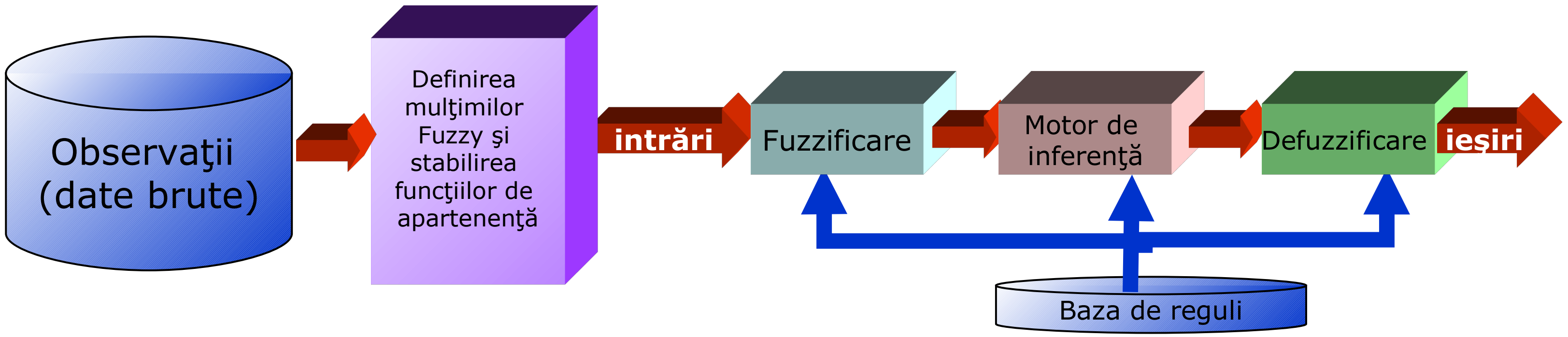
  - Interpret the result



Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification

- Main idea
  - Transform the fuzzy result into a crisp (raw) value
  - Inference → obtain some fuzzy regions for each output variable
  - Defuzzification → transform each fuzzy region into a crisp value
- Methods
  - Based on the gravity center
    - COA – Centroid Area
    - BOA – *Bisector of area*
  - Based on maximum of membership function
    - MOM - *Mean of maximum*
    - SOM - *Smallest of maximum*
    - LOM - *Largest of maximum*

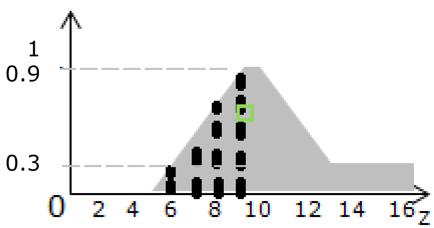
Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification → methods

- COA – Centroid Area
  - Identify the z point from the middle of aggregated set

$$COG = \frac{\sum_{i=0}^{n} x_i \mu_A(x_i)}{\sum_{i=0}^{n} \mu_A(x_i)} \text{ sau } COG = \frac{\int_{i=0}^{n} x_i \mu_A(x_i)}{\int_{i=0}^{n} \mu_A(x_i)}$$

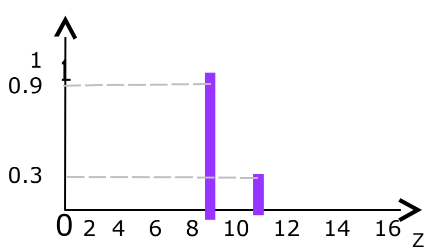
  - Example
    - Mamdani model → estimation of COA by using a sample of n points (x_i, i =1,2,..., n) of the resulted fuzzy set



$$COA = \frac{5*0+6*0.3+7*0.5+8*0.7+9*0.9+10*0.9+11*0.7+12*0.5+13*0.3+14*0.3+15*0.3+16*0.3}{0+0.3+0.5+0.7+0.9+0.9+0.7+0.5+0.3+0.3+0.3+0.3}$$

$$COA \cong 13.7$$

    - Sugeno or Tsukamoto model → COA becomes a weighted average of m crisp values obtained by applying all m rules



$$COA = \frac{9*0.9+11*0.3}{0.9+0.3}$$

$$COA \cong 9.5$$

Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification → methods

- BOA – Bisector of area
  - Identify the point z that determine the splitting of aggregated set in 2 parts of equal area

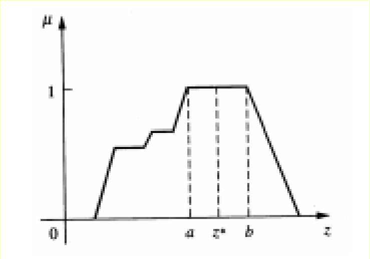
$$BOA = \int_{\alpha}^{z} \mu_A(x)dx = \int_{z}^{\beta} \mu_A(x)dx,$$
$$\text{where } \alpha = \min\{x \mid x \in A\} \text{ and } \beta = \min\{x \mid x \in A\}$$

Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification → methods

- MOM - *Mean of maximum*
  - Identify the point z that represents the mean of that points (from the aggregated set) that have a maximum membership function

$$MOM = \frac{\sum_{x_i \in \max\mu} x_i}{|max\mu|}, \text{where } max\mu = \mu^* = \{x \mid x \in A, \mu(x) = maxim\}$$



- SOM - *Smallest of maximum*
  - Identify the smallest point z (from the aggregated set) that have a maximum membership function

- LOM - *Largest of maximum*
  - Identify the largest point z (from the aggregated set) that have a maximum membership function

Intelligent systems – KBS – Fuzzy systems

Content and design → defuzzification

- Main idea
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  - Inference → obtain some fuzzy regions for each output variable
  - Defuzzification → transform each fuzzy region into a crisp value
- Methods
  - Based on the gravity center
    - COA – Centroid Area
    - BOA – *Bisector of area*
  - Based on maximum of membership function
    - MOM - *Mean of maximum*
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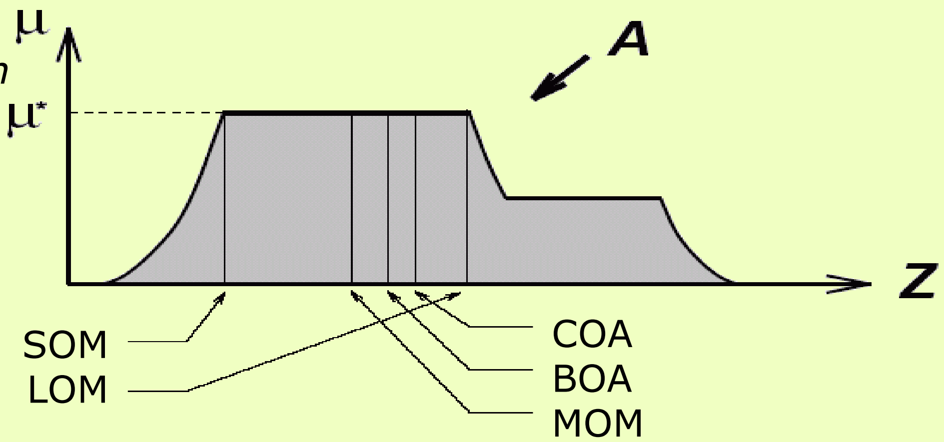

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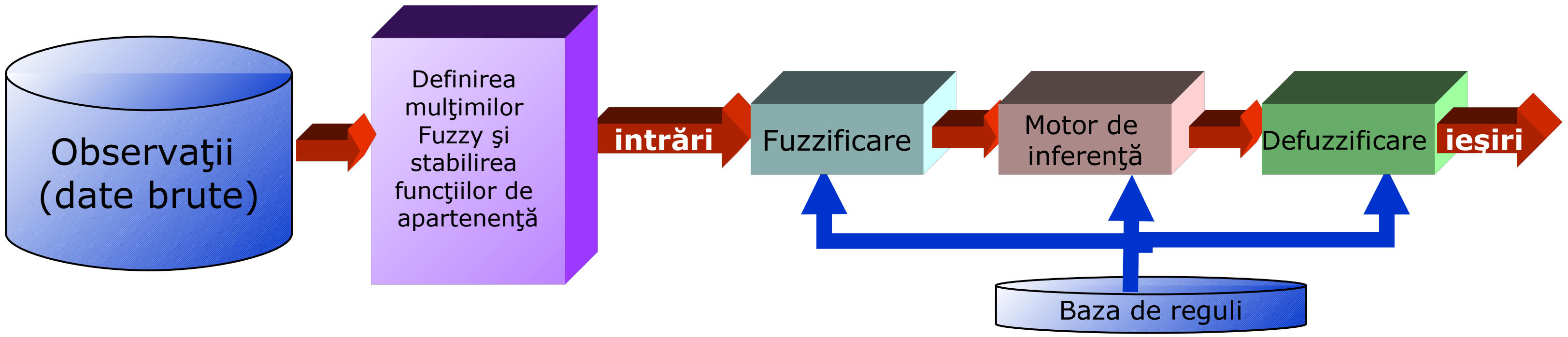
  - Evaluate the rules
    - Inference – transform the fuzzy inputs into fuzzy outputs by applying all the rules
  - Aggregate the results
  - Defuzzificate the result
  - **Interpret the result**



Intelligent systems – KBS – Fuzzy systems

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    - Fuzzification of inputs and outputs
  - Construct a base of rules – by an expert
  - Evaluate the rules
  - Aggregate the results
  - Defuzzificate the result
  - Interpret the result

Domain of expert

Inference engine

Intelligent systems – KBS – Fuzzy systems

- Advantages
  - Imprecision and real-world approximations can be expressed through some rules
  - Easy to understand, to test and to maintain
  - Robustness → can operate when rules are not so clear
  - Require few rules then other KBSs
  - Rules are evaluated in parallel

- Disadvantages
  - Require many simulations and tests
  - Do not automatically learn
  - It is difficult to identify the most correct rules
  - There is not mathematical model

Intelligent systems – KBS – Fuzzy systems

Applications

- Space control
  - Altitude of satellites
  - Setting the planes
- Auto-control
  - Automatic transmission, traffic control, anti-breaking systems
- Business
  - Decision systems, personal evaluation, fond management, market predictions, etc
- Industry
  - Energy exchange control, water purification control
  - pH control, chemical distillation, polymer production, metal composition
- Electronic devices
  - Camera exposure, humidity control. Air conditioner, shower setting
  - Freezer setting
  - Washing machine setting

Intelligent systems – KBS – Fuzzy systems

Applications

- Nourishment
  - Cheese production
- Military
  - Underwater recognition, infrared image recognition, vessel traffic decision
- Navy
  - Automatic drivers, route selection
- Medical
  - Diagnostic systems, pressure control during anesthesia, modeling the neuropathology results of Alzheimer patients
- Robotics
  - Kinematics (arms)

Review

- KBSs
  - Computation systems where knowledge database and inference engine overlap
- KBSs can work
  - In certainty environment
    - LBS
    - RBS
  - In uncertainty environments
    - Bayes systems
      - Rules have associated some probabilities
    - Systems based on certainty factors
      - Fact and rules have associated certainty factors
    - Fuzzy systems
      - Fact have associated degree of membership to some sets