13th Triennial World Congress, San Francisco, USA



# AN INTAKE MANIFOLD MODEL FOR SPARK IGNITION ENGINES

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Abstract: In this paper a model of the air flow and pressure inside the intake manifold of spark ignition engines is derived from mass and momentum balance equations. The model is capable to predict the physical phenomena inside the engine cycle. Linearization around a nominal point derived from a Mean Value Model allows to determine the tranfer functions relating the throttle angle variation and the poppet valve lift to pressure and air flow in the manifold. The knowledge of these tranfer functions can be the starting point for the design of more accurate pressure control schemes.

Keywords: Modeling, Automotive, Spark ignition engine control, Air intake.

## 1. INTRODUCTION

The development of accurate engine models is motivated by the necessity of improving engine performances, expressed in terms of driveability, fuel consumption and pollutant emissions, by means of more and more sophisticated control strategies. A traditional approach to spark ignition engine modeling is to resort to the so-called Mean Value Models (MVM), see e.g. Dobner (1980), Aquino (1981), Hendricks and Sorenson (1990), Chaumerliac *et al.* (1994), Siviero *et al.* (1995). MVM are aimed at describing the engine dynamics with good accuracy without considering cycle variations. In general MVM are able to predict the main dynamics of the relevant engine variables, namely the intake manifold pressure, the fuel flow-rate inside the cylinder and the crank shaft speed with respect to variations of the engine inputs, that is the throttle plate angle, the spark advance angle, the injected fuel and the brake torque. Then their relatively easy structure, typically they are constituted by three (nonlinear) differential equations and a set of algebraic equations, makes MVM particularly suited for the control design phase.

A substantial evolution with respect to the use of MVM should consist in developing simple models suitable to describe the main physical phenomena inside the cycle for the various engine subsystems, the air intake system, the fuel supply system and the combustion process. In this direction, the purpose of this paper is to present a model for the air intake subsystem able to represent the acoustic phenomena. In particular, the model is first developed starting from mass and momentum balance equations and by considering the boundary conditions imposed by the throttle and the poppet valves. Then these relations are linearized around a nominal point derived by means of a MVM. Finally the model is given a discrete time form where the dependence of air flow and pressure variations upon throttle angle variations and poppet valve lift is represented in terms of discrete time transfer functions whose poles depend on the engine operating conditions. By analyzing the characteristics of these transfer functions it is then possible to predict the dynamics of the pressure and flow waves inside the manifold. It is believed that this possibility will be of increasing importance in the development of future engine control strategies where the acceleration pedal will be uncoupled from the throttle and the main goal of the control system will be to perform an accurate closed-loop control of the intake pressure by directly acting on the throttle valve, see Bidan *et al.* (1995).

The paper is organized as follows: in Section 2 the adopted notation is introduced. Section 3 briefly summarizes the MVM subsequently used for linearization. In Section 4 the intake manifold model is presented, while simulation results are reported in Section 5.

## 2. SYMBOLS

| | |
|---|---|
| $p$ | intake mean manifold air pressure |
| $p_{man}$ | instantaneous intake manifold air pressure |
| $p_{cyl}$ | pressure inside the cylinder |
| $p_{exh}$ | exhaust gas pressure |
| $p_{amb}$ | ambient pressure |
| $\dot{m}$ | intake mean manifold air flow |
| $\dot{m}_a$ | intake manifold air flow |
| $\dot{m}_f$ | injected fuel mass flow |
| $u$ | air speed inside the manifold |
| $c$ | speed of sound |
| $L$ | intake manifold length |
| $\theta$ | spark advance angle |
| $\lambda$ | relative air/fuel ratio |
| $n$ | crank shaft speed |
| $I$ | total moment of inertia loading engine |
| $H_u$ | fuel heating value |
| $V_d$ | engine displacement |
| $V$ | manifold-port passage volume |
| $\eta_v$ | volumetric efficiency |
| $\eta_b$ | brake thermal efficiency |
| $T_{man}$ | intake manifold air temperature |
| $R$ | gas constant |
| $P_b$ | brake power |
| $\alpha$ | throttle plate angle |
| $\alpha_0$ | throttle plate when tightly closed against the throttle bore |
| $C_t$ | flow coefficient of throttle valve |
| $C_d$ | flow coefficient of poppet valve |
| $D$ | throttle bore diameter |
| $D_v$ | poppet bore diameter |
| $A$ | intake manifold cross section area |
| $K$ | ratio of specific heats (1.4 for the air). |

## 3. THE MEAN VALUE MODEL

The aim of this Section is to present the Mean Value Model (MVM) used in the following to derive the steady-state engine conditions subsequently used for linearization. The MVM here considered (see Siviero *et al.*, 1995) is described by:

$$\dot{p}(t) = \frac{n(t)V_d\eta_v p(t)}{120V} + \frac{RT_{man}(t)}{V}\dot{m}(t) \tag{1}$$

$$\dot{n}(t) = -\frac{P_b(t)}{n(t)I} + \frac{H_u\eta_b}{n(t)I}\dot{m}_f(t) \tag{2}$$

In this model the engine temperature is assumed to be at the steady-state and the film fluid dynamics has been ignored , being useless for the following developments. Furthermore, $\eta_v$ and $\eta_b$ are functions of $(n,p,p_{exh})$, $(n,p,p_{exh},\lambda,\theta)$ respectively, see again Siviero *et al.* (1995).

In eq. (1), the air mass flow rate inside the manifold $\dot{m}(t)$ equals that through the throttle plate and is given by (see e.g. Heywood (1988), Siviero *et al.* (1995), Bidan *et al.* (1995)):

$$\dot{m}(t) = C_t\frac{\Pi}{4}D^2\frac{p_{amb}}{\sqrt{RT_{amb}}}\beta_1(\alpha)\beta_2[p(t)] \tag{3}$$

where, for conventional throttle valves:

$$\beta_1(\alpha) = \left(1-\frac{\cos(\alpha)}{\cos(\alpha_o)}\right) + \frac{2}{\Pi}\left[\frac{a}{\cos(\alpha)}\left(\cos^2(\alpha) - a^2\cos^2(\alpha_o)\right)^{\frac{1}{2}}\right] +$$

$$+\frac{2}{\Pi}\left[\frac{\cos(\alpha)}{\cos(\alpha_o)}\arcsin\left(a\frac{\cos(\alpha_o)}{\cos(\alpha)}\right) - a\left(1-a^2\right)^{\frac{1}{2}} - \arcsin(a)\right]$$

$a$ is a geometric constant and

$$\beta_2[p(0,t)] = \begin{cases} \sqrt{\dfrac{2k}{k-1}\left[\left(\dfrac{p(t)}{p_{amb}}\right)^{\frac{2}{k}} - \left(\dfrac{p(t)}{p_{amb}}\right)^{\frac{k+1}{k}}\right]} & \text{if } \dfrac{p(t)}{p_{amb}} \geq \left(\dfrac{2}{k+1}\right)^{\frac{k}{k-1}} \\ \sqrt{k}\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} & \text{otherwise} \end{cases}$$

From (1)-(3) and from the expressions of $\eta_v$ and $\eta_b$, it is possible to compute, for constant inputs $(\alpha,\lambda,\theta,P_b)$, the steady-state values $\bar{n}$, $\bar{p}$ and $\bar{\dot{m}}$ of the mean crank shaft speed $n(t)$, of the mean intake manifold air pressure $p(t)$ and of the mean air flow $\dot{m}(t)$.

With reference to the MVM (1)-(3), it is worth noting that, in realistic situations, the pressure dynamics is faster than the dynamics of $n(t)$, this can readily be shown by analyzing the eigenvalues of the linearized model obtained from (1)-(3), see Hendricks and Vesterholm (1992). Then, in view of this frequency decoupling, in the following it will be assumed that small perturbations of the throttle angle $\alpha$

with respect to a nominal value $\bar{\alpha}$ induce variations in pressure and flow which can be computed assuming $n(t)=\bar{n}$, or with a slowly varying $n$.

## 4. THE INTAKE MANIFOLD MODEL

### 4.1 Mass and momentum balance equations

In order to derive the model, we can consider the manifold as a pipe with two regulating elements which impose boundary conditions. By ignoring thermal effects and friction, the flow can be assumed to be isoentropic inside the manifold, this is motivated by noting that pressure dynamics is faster than the thermal one. Under these assumptions, and by omitting for ease of notation the dependence on $t$ and $x$ of the variables, the mass balance equation:

$$\frac{A}{\bar{c}^2}\frac{\partial p_{man}}{\partial t}+\frac{\partial \dot{m}_a}{\partial x}=0 \tag{4}$$

and the momentum balance equation:

$$\frac{\partial \dot{m}_a}{\partial t}+2\bar{u}\frac{\partial \dot{m}_a}{\partial x}+\left(1-\frac{\bar{u}^2}{\bar{c}^2}\right)A\frac{\partial p_{man}}{\partial x}=0 \tag{5}$$

fully specify the manifold model. Denoting by $\Delta y(t)$ the variation of signal $y(t)$ around its nominal value $\bar{y}$, linearization of eqs. (4), (5) leads to:

$$\frac{A}{\bar{c}^2}\frac{\partial \Delta p_{man}}{\partial t}+\frac{\partial \Delta\dot{m}_a}{\partial x}=0$$

$$\frac{\partial \Delta\dot{m}_a}{\partial t}+2\bar{u}\frac{\partial \Delta\dot{m}_a}{\partial x}+\left(1-\frac{\bar{u}^2}{\bar{c}^2}\right)A\frac{\partial \Delta p_{man}}{\partial x}=0$$

By applying the Laplace transform to these equations one obtains:

$$\frac{A}{\bar{c}^2}s\Delta p_{man}(x,s)+\frac{d\Delta\dot{m}_a(x,s)}{dx}=0 \tag{6}$$

$$s\Delta\dot{m}_a(x,s)+2\bar{u}\frac{d\Delta\dot{m}_a(x,s)}{dx}+\left(1-\frac{\bar{u}^2}{\bar{c}^2}\right)A\frac{d\Delta p_{man}(x,s)}{dx}=0 \tag{7}$$

The solution of system (6), (7) leads to:

$$\Delta\dot{m}_a(x,s)=k_1(s)\cdot e^{-\frac{sx}{\bar{c}+\bar{u}}}+k_2(s)\cdot e^{\frac{sx}{\bar{c}-\bar{u}}} \tag{8}$$

$$\Delta p_{man}(x,s)=\frac{\bar{c}^2}{A}\left(\frac{k_1(s)}{\bar{c}+\bar{u}}\cdot e^{-\frac{sx}{\bar{c}+\bar{u}}}-\frac{k_2(s)}{\bar{c}-\bar{u}}\cdot e^{\frac{sx}{\bar{c}-\bar{u}}}\right) \tag{9}$$

where $k_1(s)$ and $k_2(s)$ have to be determined by imposing boundary conditions.

### 4.2 Boundary conditions

The boundary conditions of eqs. (8) and (9) are represented by the flow equations through the throttle and the poppet valves. The first one has already been described by eq. (3) where $p(t)$ and $\dot{m}(t)$ must be substituted by $p_{man}(0,t)$ and $\dot{m}_a(0,t)$.

As for the flow through the poppet valve, it is expressed by:

$$\dot{m}_a(L,t)=\frac{C_d\Pi D_v L_v p_{man}(L,t)}{\sqrt{RT_{man}}}\beta_{22}\left[p_{man}(L,t)\right] \tag{10}$$

where

$$\beta_{22}\left[p_{man}(L,t)\right]=\begin{cases}\sqrt{\dfrac{2k}{k-1}\left[\left(\dfrac{p_{cyl}(t)}{p_{man}(L,t)}\right)^{\frac{2}{k}}-\left(\dfrac{p_{cyl}(t)}{p_{man}(L,t)}\right)^{\frac{k+1}{k}}\right]} & if\ \ \dfrac{p_{cyl}(t)}{p_{man}(L,t)}\geq\left(\dfrac{2}{k+1}\right)^{\frac{k}{k-1}}\\ \sqrt{k}\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} & otherwise\end{cases}$$

while the poppet valve lift $L_v$ is a periodic signal which depends on the crank shaft speed $n$, i.e. $L_v=L_v(n)$. This function can be viewed as the sum of a constant term $\bar{L}_v(n)$ and a zero-mean periodic signal $\delta L_v(n)$. Since during filling the pressure inside the cylinder can be assumed constant, in order to achieve congruence of this model with the MVM of Section 3, we can now derive the value of $\bar{L}_v$ by imposing that, for $n(t)=\bar{n}$ and $p(t)=\bar{p}$, the mean air flow through the throttle in eq. (10) equals $\bar{\dot{m}}$.

The linearization of eq.(3) around $\bar{\alpha}$ and $\bar{p}$ leads to

$$\Delta\dot{m}_a(0,t)=r_1\,\Delta p_{man}(0,t)+r_2\,\Delta\alpha \tag{11}$$

where

$$r_1=C_t\frac{\Pi}{4}D^2\frac{p_{amb}}{\sqrt{RT_{amb}}}\beta_1(\bar{\alpha})\left.\frac{\partial\beta_2}{\partial p_{man}(0,t)}\right|_{\bar{p}}$$

with

$$\left.\frac{\partial\beta_2}{\partial p_{man}}\right|_{\bar{p}} = \frac{1}{2}\left(\frac{2k}{k-1}\right)^{\frac{1}{2}}\left(\left(\frac{\bar{p}}{p_{amb}}\right)^{\frac{2}{k}} - \left(\frac{\bar{p}}{p_{amb}}\right)^{\frac{k+1}{k}}\right)^{-\frac{1}{2}}$$

$$\left[\left(\frac{1}{p_{amb}}\right)^{\frac{2}{k}}\frac{2}{k}(\bar{p})^{\frac{2-k}{k}} - \left(\frac{1}{p_{amb}}\right)^{\frac{k+1}{k}}\frac{k+1}{k}(\bar{p})^{\frac{1}{k}}\right]$$

if $\frac{\bar{p}}{p_{amb}} \geq \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$ and $\left.\frac{\partial\beta_2}{\partial p_{man}}\right|_{\bar{p}} = 0$ otherwise,

and

$$r_2 = C_t\frac{\Pi}{4}D^2\frac{p_{amb}}{\sqrt{RT_{amb}}}\beta_2(\bar{p})\left.\frac{\partial\beta_1(\alpha)}{\partial\alpha}\right|_{\bar{\alpha}}$$

with ($a$ is again a geometric constant):

$$\left.\frac{\partial\beta_1(\alpha)}{\partial(\alpha)}\right|_{\bar{\alpha}} = \frac{\sin(\bar{\alpha})}{\cos(\alpha_o)} + \frac{2}{\Pi}\left[a\frac{\sin(\bar{\alpha})}{\cos^2(\bar{\alpha})}\left(\cos^2(\bar{\alpha}) - a^2\cos^2(\alpha_o)\right)^{\frac{1}{2}}\right] +$$

$$+\frac{2}{\Pi}\left[-a\sin(\bar{\alpha})\left(\cos^2(\bar{\alpha}) - a^2\cos^2(\alpha_o)\right)^{-\frac{1}{2}}\right] +$$

$$+\frac{2}{\Pi}\left[\frac{\sin(\bar{\alpha})}{\cos(\alpha_o)}\arcsin\left(a\frac{\cos(\alpha_o)}{\cos(\bar{\alpha})}\right) + a\frac{\sin(\bar{\alpha})}{\cos(\bar{\alpha})}\left(1-\left(a\frac{\cos(\alpha_o)}{\cos(\bar{\alpha})}\right)^2\right)^{-\frac{1}{2}}\right]$$

The Laplace transform of (11) is then given by:

$$\Delta\dot{m}_a(0,s) = r_1\,\Delta p_{man}(0,s) + r_2\,\Delta\alpha(s) \qquad (12)$$

Analogously, the linearization of eq.(10) leads to:

$$\Delta\dot{m}_a(L,t) = r_3\,\Delta p_{man}(L,t) + r_4\,\delta L_v(\bar{n}) \qquad (13)$$

where

$$r_3 = \frac{C_d\pi D_v L_v}{\sqrt{RT_{man}}}\left[\ \beta_{22}(p_{man}) + p_{man}\frac{\partial\beta_{22}(p_{man})}{\partial p_{man}}\ \right]_{\bar{p}}$$

with $\left.\frac{\partial\beta_{22}}{\partial p_{man}(L,t)}\right|_{\bar{p}}$ either zero or

$$\left.\frac{\partial\beta_{22}}{\partial p_{man}(L,t)}\right|_{\bar{p}} = \frac{1}{2}\left(\frac{2k}{k-1}\right)^{\frac{1}{2}}\left(\left(\frac{p_{cyl}}{\bar{p}}\right)^{\frac{2}{k}} - \left(\frac{p_{cyl}}{\bar{p}}\right)^{\frac{k+1}{k}}\right)^{-\frac{1}{2}}\cdot$$

$$\cdot\left[-\frac{2}{k}p_{cyl}^{\frac{2}{k}}\bar{p}^{-\frac{2+k}{k}} + \frac{k+1}{k}p_{cyl}^{\frac{k+1}{k}}\bar{p}^{-\frac{1+2k}{k}}\right]$$

and

$$r_4 = \frac{C_d\Pi D_v}{\sqrt{RT_{man}}}\bar{p}\beta_{22}(\bar{p})$$

The Laplace transform of eq.(13) is :

$$\Delta\dot{m}_a(L,s) = r_3\,\Delta p_{man}(L,s) + r_4\,\Delta L_v(s) \qquad (14)$$

*4.3 Evaluation of $k_1(s)$ and $k_2(s)$*

From eqs.(12) and (14), it is now possible to derive $k_1(s)$ and $k_2(s)$ in (8) and (9). In particular, letting:

$$z^+ = \frac{\bar{c}^2}{A(\bar{c}+\bar{u})} \quad ; \quad z^- = \frac{\bar{c}^2}{A(\bar{c}-\bar{u})}$$

eqs (8) and (9) can be written as:

$$\Delta\dot{m}_a(x,s) = k_1(s)\cdot e^{-\frac{sx}{\bar{c}+\bar{u}}} + k_2(s)\cdot e^{\frac{sx}{\bar{c}-\bar{u}}} \qquad (15)$$

$$\Delta p_{man}(x,s) = k_1(s)\cdot z^+\cdot e^{-\frac{sx}{\bar{c}+\bar{u}}} - k_2(s)\cdot z^-\cdot e^{\frac{sx}{\bar{c}-\bar{u}}} \qquad (16)$$

and, by imposing (12) and (14) one obtains:

$$k_1(s) = \frac{\frac{r_2}{1+r_1z^-}\Delta\alpha(s) - \frac{r_4}{1+r_3z^-}\cdot e^{-\frac{sL}{\bar{c}-\bar{u}}}\Delta L_v(s)}{\frac{1-r_1z^+}{1+r_1z^-} - \frac{1-r_3z^+}{1+r_3z^-}\cdot e^{-st_o}} \qquad (17)$$

$$k_2(s) = \frac{\frac{r_2}{1-r_1z^+}\Delta\alpha(s) - \frac{r_4}{1-r_3z^+}\cdot e^{\frac{sL}{\bar{c}+\bar{u}}}\Delta L_v(s)}{\frac{1+r_1z^-}{1-r_1z^+} - \frac{1+r_3z^-}{1-r_3z^+}\cdot e^{st_o}} \qquad (18)$$

where

$$t_o = \frac{2L\bar{c}}{\bar{c}^2 - \bar{u}^2}$$

*4.4 Discrete time form of pressure and flow dynamics*

During filling $\bar{c} >> \bar{u}$ ($\bar{c} \cong 340$ *m/s*, $\bar{u} \cong 30$ *m/s*), then one can assume:

$$t_o \cong 2\frac{L}{\bar{c}}$$

$$z^+ \cong \frac{\bar{c}}{A} \quad ; \quad z^- \cong \frac{\bar{c}}{A} \quad ; \quad z^+ = z^- = z_c = \frac{\bar{c}}{A}$$

Then, by defining:

$$a_1 = \frac{r_2}{1 + r_1 z_c} \quad ; \quad a_2 = \frac{r_4}{1 + r_3 z_c}$$

$$a_3 = \frac{r_2}{1 - r_1 z_c} \quad ; \quad a_4 = \frac{r_4}{1 - r_3 z_c}$$

from (15)-(18) one has:

$$\Delta p_{man}(x,s) = z_c \left[ \frac{a_1 a_3 a_4 \Delta\alpha(s) - a_2 a_3 a_4 \cdot e^{-\frac{sL}{\bar{c}}} \Delta L_v(s)}{a_1 a_4 - a_2 a_3 \cdot e^{-st_o}} \cdot e^{-\frac{sx}{\bar{c}}} \right] +$$
$$+ z_c \left[ \frac{a_1 a_2 a_3 \Delta\alpha(s) - a_1 a_2 a_4 \cdot e^{\frac{sL}{\bar{c}}} \Delta L_v(s)}{a_1 a_4 \cdot e^{st_o} - a_2 a_3} \cdot e^{\frac{sx}{\bar{c}}} \right] \tag{19}$$

$$\Delta \dot{m}_a(x,s) = \frac{a_1 a_3 a_4 \Delta\alpha(s) - a_2 a_3 a_4 \cdot e^{-\frac{sL}{\bar{c}}} \Delta L_v(s)}{a_1 a_4 - a_2 a_3 \cdot e^{-st_o}} \cdot e^{-\frac{sx}{\bar{c}}} +$$
$$- \frac{a_1 a_2 a_3 \Delta\alpha(s) - a_1 a_2 a_4 \cdot e^{\frac{sL}{\bar{c}}} \Delta L_v(s)}{a_1 a_4 \cdot e^{st_o} - a_2 a_3} \cdot e^{\frac{sx}{\bar{c}}} \tag{20}$$

Eqs. (19) and (20) represent the final model of the intake manifold and allow one to compute the variations of air flow and pressure inside the manifold as functions of the throttle plate angle variations and of the poppet lift variations. To this regard, it is worth recalling that an air flow variation induces a variation in the crank shaft speed, which, in turn, modifies the poppet valve opening. However this phenomenon is characterized by a slower dynamics than the air flow and pressure dynamics.

A simpler model can now be obtained by giving to eqs. (19), (20) a discrete time form. To this regard define the sample time $t_c = {}^{t_o}\!/_2$ and let $Z = e^{s\frac{t_o}{2}} = e^{s\frac{L}{\bar{c}}}$. Note that $t_c$ is the transfer time of a pressure wave inside the manifold.

With these positions, from eqs.(19) and (20) one directly obtains:

$$\Delta p_{man}(0,Z) = z_c a_3 \frac{Z^2 + \frac{a_2}{a_4}}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta\alpha - z_c a_2 \frac{\left(1 + \frac{a_3}{a_1}\right) Z}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta L_v \tag{21}$$

$$\Delta p_{man}(L,Z) = z_c a_3 \frac{\left(1 + \frac{a_2}{a_4}\right) Z}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta\alpha - z_c a_2 \frac{Z^2 + \frac{a_3}{a_1}}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta L_v \tag{22}$$

and

$$\Delta \dot{m}_a(0,Z) = a_3 \frac{Z^2 - \frac{a_2}{a_4}}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta\alpha + a_2 \frac{\left(1 - \frac{a_3}{a_1}\right) Z}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta L_v \tag{23}$$

$$\Delta \dot{m}_a(L,Z) = a_3 \frac{\left(1 - \frac{a_2}{a_4}\right) Z}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta\alpha + a_2 \frac{Z^2 - \frac{a_3}{a_1}}{Z^2 - \frac{a_2 a_3}{a_1 a_4}} \Delta L_v \tag{24}$$

The discrete time model of eqs.(21)-(24) describes flow and pressure waves inside the manifold. These waves can be due to various causes. In fact, the system poles are either real and symmetric with respect to the origin or complex. Then they can lead to oscillatory transients of $\dot{m}_a$ and $p_{man}$ for throttle angle variations. Moreover, the signal $\delta L_v$ is periodic by definition and produces by itself the oscillations usually observed also in steady-state conditions.

## 5. SIMULATION RESULTS

In order to illustrate the characteristics of the linearized model (21)-(24) some simulations have been made with reference to an engine characterized by the following parameters: $V = 1600$ cm$^3$; $V_d = 3000$ cm$^3$; $D = 5$ cm; $C_t = 0.83$; $C_d = 0.8$; $D_v = 3$ cm; $a = 0.14$. Linearization has been performed around an idle speed condition defined by $\bar{n} = 870$ rpm; $\bar{p} = 370$ mbar; $\bar{\dot{m}} = 11$ g/sec; $p_{cyl} = 100$ mbar; $T_{man} = 310$ K.

Starting from this operating point, the throttle valve ($\Delta\alpha$) has been progressively opened according to a truncated ramp-type profile. As for $\Delta L_v$, it has been chosen as the

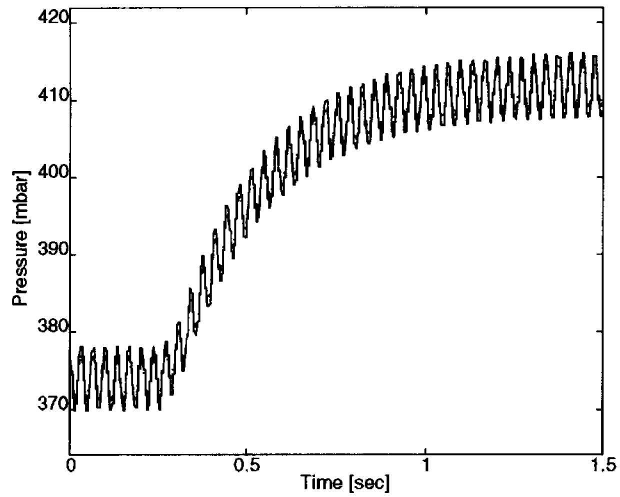


**Figure 1:** Pressure variations at the throttle and at the poppet valves due to a variation of the throttle angle.

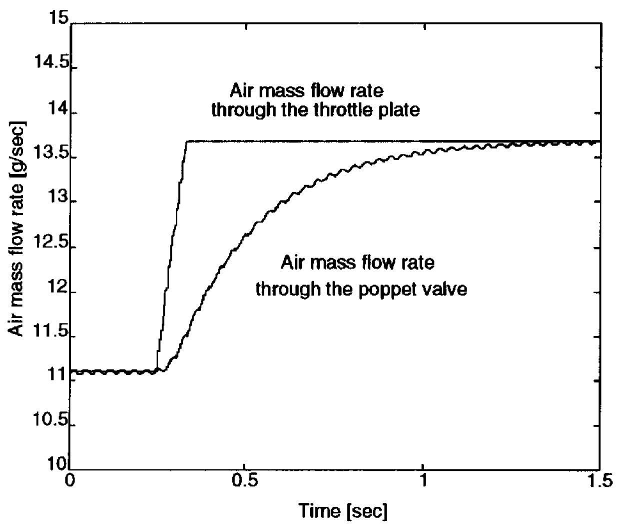


**Figure 2:** Air mass flow rates due to a variation of the throttle angle.

periodic signal $\Delta L_v(t) = \sin\left(\frac{4.\pi.\bar{n}}{30}\right)$, in order to simulate the periodic opening and closing of the poppet valves. The transients of the pressure at the throttle and poppet positions are reported in Fig. 1 which clearly shows the mean variation of the pressure and the superimposed waves due to the movements of the poppet valves. In Fig. 2 the air mass flow rate through the throttle and the mean flow rate through the poppet valve are also reported.

## 6. CONCLUDING REMARKS

In this paper a model of the intake manifold of spark ignition engines has been developed. The model is derived from physical considerations and is able to represent the main phenomena inside the engine cycle. Linearization of mass and momentum balance equations is performed around a nominal operating condition which can be computed by means of Mean Value Models. It is believed that the model can be of interest in the synthesis of more sophisticated control strategies for spark ignition engines. Further research is needed in this direction: first experimental validation with real data is required to assess the validity of the proposed approach. A second stream of future research will be to develop a model for the phenomena inside the cylinders which can cope with the intake model here presented. The availability of this model would lead to a precise description of the overall engine dynamics during the engine cycles.

## Acknowledgments

The authors are indebted to Prof. Maffezzoni for helpful discussions. The paper has been financially supported by MURST.

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