Ex.1) We define $\begin{bmatrix} s1\ (t) \\ s2\ (t) \end{bmatrix} = \Sigma_t$

$$\Sigma_1 = A\,\Sigma_0 + Bu(0)$$

initaly $s1(0) = s2(0) = 0$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0.95 \end{bmatrix} = A$$

$$\Sigma_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0.1 \end{bmatrix} = B$$

Also having the constraints $\left.\begin{matrix} s1(N) = 10 \\ s2(N) = 0 \end{matrix}\right\} \rightarrow \begin{bmatrix} 10 \\ 0 \end{bmatrix} = d$

Now Finding the pattern for $Cx = d$

$$\Sigma_1 = Bu_0$$

$$\Sigma_2 = A\Sigma_1 + Bu_1 = A(ABu_0 + Bu_1) + Bu_2$$

$$\vdots$$

$$\Sigma_N = A^{N-1}Bu_0 + A^{N-2}u_1 + \dots + AB+u_1 + Bu_0 \Rightarrow$$

$\Rightarrow$

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$$Cx = d \Rightarrow$$

$$\Rightarrow \left[A^{N-1}B, A^{N-2}B, \dots B\right] \begin{bmatrix} u_0 \\ \vdots \\ \vdots \\ u_{N-1} \end{bmatrix} = \begin{bmatrix} 10 \\ \\ 0 \end{bmatrix}$$

then i made a method in python to compute C and therefore solve the least square and find u.

Now to find $s1, s2$:

From the first equation ⇒

$$\Sigma(t+1) = S_1(t+1) = s1(t) + s2(t)$$

$$s2(t+1) = 0.95\, s2(t) + 0.1\, u(t)$$

And then plotting everything in python.

Also to solve least squares for $\left\| I \cdot u_n - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_N \right\|_2$ i used scipy.
subj to $Cx = d$

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Question b)

In [1]:

```
import numpy as np
from scipy import linalg
from scipy.linalg import lapack
import matplotlib.pyplot as plt
```

function to compute the C matrix

In [2]:

```
def final_C(N):

    d=np.array([[10],[0]])
    A=np.array([[1,1],[0,0.95]])
    B=np.array([[0],[0.1]])
    C=[A for i in range(N-1)]

    k=np.arange(N-2,-1,-1)
    for j in range(N-1):

        for i in range(k[j]):
            C[j]=np.dot(A,C[j])



    C=[np.dot(C[i],B) for i in range(N-1)]
    C.append(B)

    new_C=np.zeros((2,N))

    for i in range(2):
        for j in range(N):
            new_C[i,j]=C[j][i]

    return new_C
```

In [3]:

```
final_C(29)
```

Out[3]:

```
array([[1.52434623, 1.49931182, 1.47295981, 1.44522085, 1.41602195,
        1.38528626, 1.35293291, 1.31887675, 1.28302816, 1.24529279,
        1.20557136, 1.16375933, 1.11974666, 1.07341754, 1.02465004,
        0.97331583, 0.91927982, 0.86239982, 0.80252612, 0.73950118,
        0.67315914, 0.60332541, 0.52981622, 0.45243812, 0.3709875 ,
        0.28525   , 0.195     , 0.1       , 0.        ],
       [0.02378269, 0.02503441, 0.02635201, 0.02773896, 0.0291989 ,
        0.03073569, 0.03235335, 0.03405616, 0.03584859, 0.03773536,
        0.03972143, 0.04181203, 0.04401267, 0.04632912, 0.0487675 ,
        0.05133421, 0.05403601, 0.05688001, 0.05987369, 0.06302494,
        0.06634204, 0.06983373, 0.07350919, 0.07737809, 0.08145062,
        0.0857375 , 0.09025   , 0.095     , 0.1       ]])
```

Compute u vector with least squares function from scipy package.

Computing velocity and position

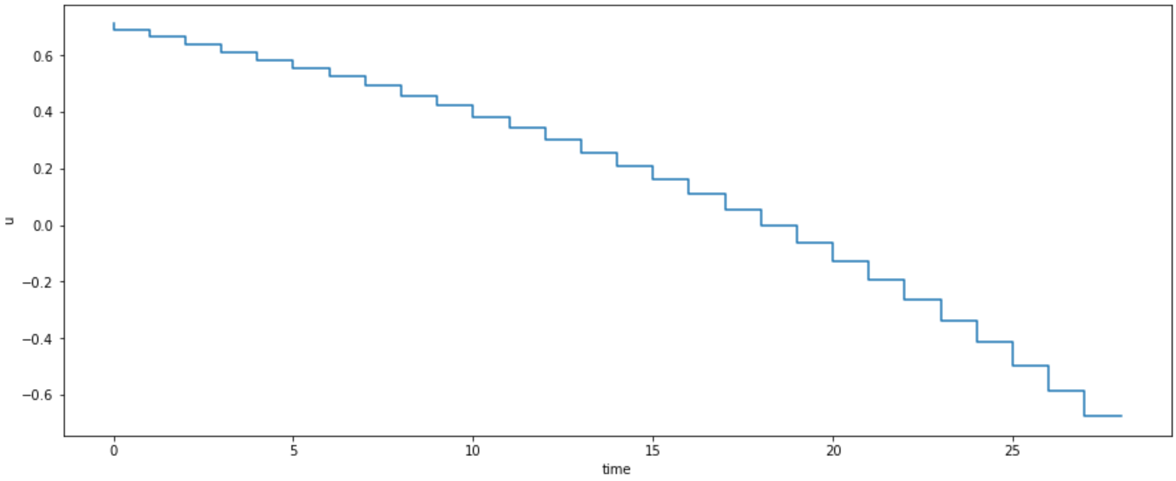
In [4]:

```
def new_u(N):
    I=np.identity(N)
    d=np.array([[10],[0]])
    u = lapack.dgglse(I, final_C(N), np.zeros(N), d)

    vel=np.zeros(N+1)
    position=np.zeros(N+1)
    for i in range(0,N):
        position[i+1]=position[i]+vel[i]
        vel[i+1]=0.95*vel[i]+0.1*u[3][i]

    return u[3],vel,position
```

Plotting u,velocity and position

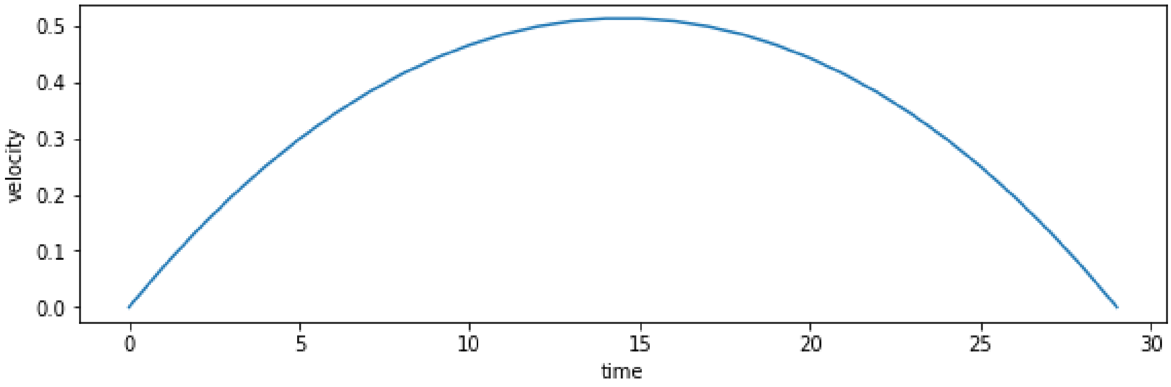
In [5]:

```
plt.figure(figsize = (15,6))
plt.step(range(29),new_u(29)[0])
plt.xlabel("time")
plt.ylabel("u")
plt.show()
```



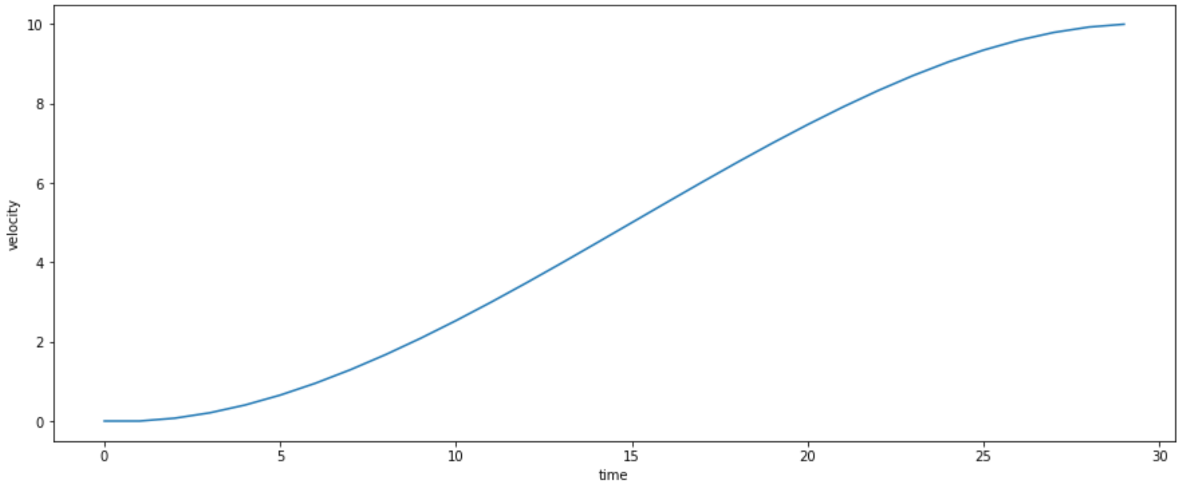
In [6]:

```
plt.figure(figsize = (10,3))
plt.plot(range(30),new_u(29)[1])
plt.xlabel("time")
plt.ylabel("velocity")
plt.show()
```



In [7]:

```
plt.figure(figsize = (15,6))
plt.plot(range(30),new_u(29)[2])
plt.xlabel("time")
plt.ylabel("velocity")
plt.show()
```



Question c)

In [49]:

```
print(new_u(2)[0])
print(new_u(3)[0])
```

```
[100. -95.]
[ 50.    2.5 -47.5]
```

finding energy for each step

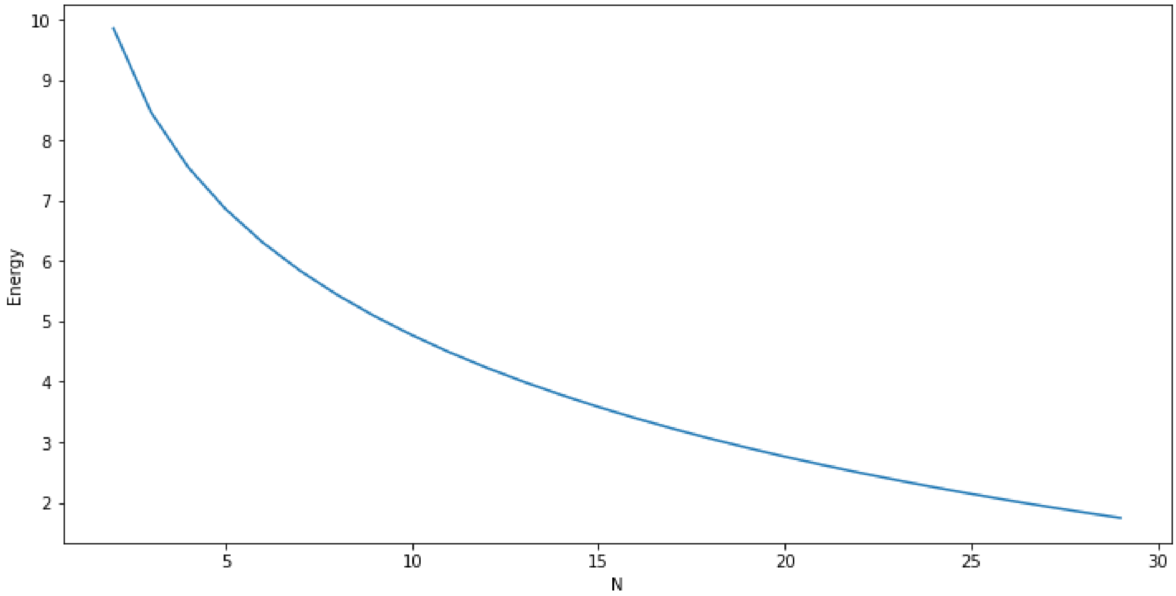
In [55]:

```
all_sums=[]
summ=0
for N in range(2,30):
    summ=0
    total_new_u=new_u(N)[0]
    for j in range(len(total_new_u)):
        summ+=pow((total_new_u[j]),2)
    all_sums.append(summ)
all_sums=[np.log(x) for x in all_sums]
```

Plotting energy:

In [57]:

```
plt.figure(figsize=(12,6))
plt.plot(range(2,30),all_sums)
plt.xlabel("N")
plt.ylabel("Energy")
plt.show()
```



Question 4)

In [ ]:

```
#res = minimize(Energy,u, constraints=())
```

Question d) For question d we use:

$$\begin{bmatrix} \hat{x} \\ z \end{bmatrix} = \begin{bmatrix} 2AA^T & C^T \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2A^Tb \\ d \end{bmatrix}$$

but in python, i failed to compute it correctly (i used minimum from scipy) so i erased it.

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