FOURIER ANALYSIS OF REAL SIGNALS

For a periodic signal of arbitrary period $T \neq 2\pi$ the Fourier series representation becomes:

$$s(t)=\frac{a_0}{2}+a_1\cos\left(\frac{2\pi}{T}\right)t+b_1\sin\left(\frac{2\pi}{T}\right)t+a_2\cos\left(\frac{4\pi}{T}\right)t+b_2\sin\left(\frac{4\pi}{T}\right)t+\cdots+a_n\cos\left(\frac{2n\pi}{T}\right)t+b_n\sin\left(\frac{2n\pi}{T}\right)t+\cdots$$

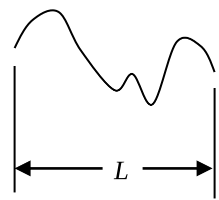
where

$$a_n=\frac{2}{T}\int_{-T/2}^{T/2}s(t)\cos\left(\frac{2n\pi}{T}\right)tdt \qquad \text{for} \qquad n=0,1,2,\cdots\infty$$

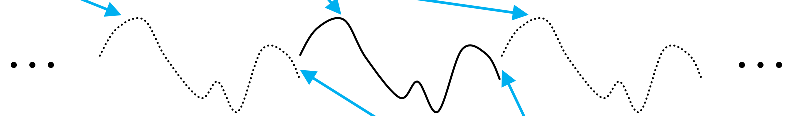
and

$$b_n=\frac{2}{T}\int_{-T/2}^{T/2}s(t)\sin\left(\frac{2n\pi}{T}\right)tdt \qquad \text{for} \qquad n=1,2,\cdots\infty$$

Also, in practice, most real information bearing signals of interest are of finite length and are not necessarily periodic as shown right.
We can still analyze such signals using the Fourier series representation. However, to perform such a Fourier analysis we have to impose an assumption on our finite length signal. This assumption is called ***periodic extension*** meaning that we assume that the signal is in fact infinitely long and is composed of the original signal padded with an infinite number of duplicate copies of itself as shown below.



Finite duration Non-periodic signal



Signal made infinitely long and periodic by periodic extension

Note that extending the signal in this way introduces discontinuities at the edges of each duplicate segment of the original. These discontinuities manifest themselves in the frequency domain as an unwanted spectral spreading known as ***spectral leakage***. How would you minimize leakage in Fourier analysis of signals?

Ex. Write a Matlab/Octave m-file to demonstrate spectral leakage associated with Fourier analysis.

Hint: First write a function m-file to generate a sampled sinewave. Write this m-file in such a way that you can have it generate an integer number of cycles of the required sinewave OR a non-integer number of cycles of the required sinewave. Applying Fourier analysis to an integer number of cycles will result in no spectral leakage whereas applying Fourier analysis to a non-integer number of cycles will result in leakage.

Note 1: Include explanatory comments in your m-file

Note 2: You can use the Matlab or Octave *fft* function to compute the Fourier transform.

Q: How does this topic relate to Power Electronics which some of you are studying? (EE313)