DFT and IDFT Example

Calculate the DFT of the sequence *s(n)* = [1, –1, 1, –1, 1, –1, 1, –1]

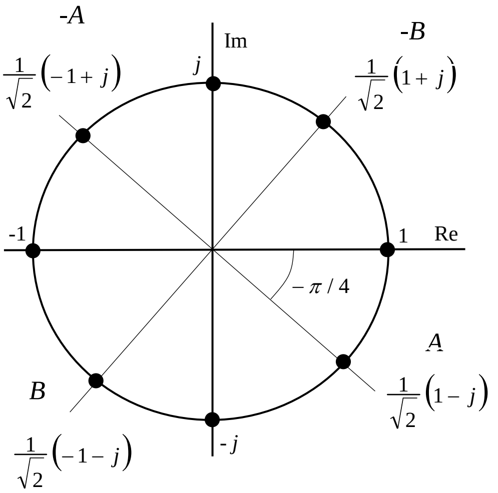
We need to use $S(k) = \sum_{n=0}^{N-1} s(n) e^{-j(2\pi/N)kn}$ for k = 0, … 7

Note that the term $W_N^{\,kn} = e^{-j(2\pi/N)kn}$ is called the twiddle factor

For k = 0, the term $W_N^{\,kn} = e^{-j(2\pi/N)0n}$ = 1 so that $S(0) = \sum_{n=0}^{N-1} s(n)$ = 1–1+1–1+ 1–1+1–1 = 0

This is equal to the first row of the square twiddle factor matrix (shown below) multiplied by the column vector form of *s(n)* to give *S(0)*.

For k = 1, the term $e^{-j(2\pi/N)kn} = e^{-j(\pi/4)n}$



$$
\begin{array}{cc}
 & \begin{array}{cccccccc} n=0 & & & & & & & n=7 \\ \downarrow & & & & & & & \downarrow \end{array} \qquad s(n)\downarrow \\
\begin{array}{r} k=0\rightarrow \\ \\ \\ S(k)= \\ \\ \\ \\ k=7\rightarrow \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & A & -j & B & -1 & -A & j & -B \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & B & j & A & -1 & -B & -j & -A \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -A & -j & -B & -1 & A & j & B \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & -B & j & -A & -1 & B & -j & A
\end{bmatrix}
\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}
=
\begin{bmatrix} S(0) \\ S(1) \\ S(2) \\ S(3) \\ S(4) \\ S(5) \\ S(6) \\ S(7) \end{bmatrix}
\end{array}
$$

This can be calculated easily using the unit circle in the complex plane as shown. The values of $e^{-j(\pi/4)n}$ for n = 0, 1, … 7 are simply read off the unit circle by stepping around it in steps of -π/4. If these values are written out as the second row of the matrix, then when this row is multiplied by the column vector form of *s(n)* we get *S(1)* as shown.

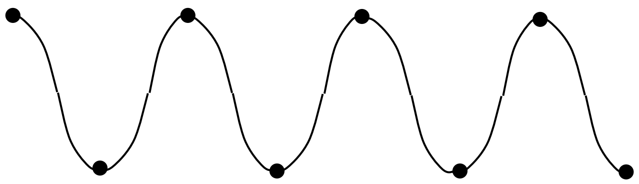
For k = 2, the term $e^{-j(2\pi/N)kn} = e^{-j(\pi/2)n}$ and we're now stepping around the unit circle in steps of -π/2 which results in the third row of the matrix.

Similarly, for k = 3 we're stepping in steps of -3π/4 and so on.

We have labelled some of the twiddle factors as *A* and *B* for convenience and it's a good idea to rationalise the matrix product terms further before substituting the actual values of *A* and *B*. That is, for k = 1, the second row by [*s(n)*]' is: 1 –A –*j* –B –1 +A +*j* +B = 0. Similarly, all the other terms collapse to zero except *S(4)* which is 8.

Therefore the DFT of *s(n)* = [1, –1, 1, –1, 1, –1, 1 –1] is *S(k)* = [0, 0, 0, 0, 8, 0, 0, 0]

What does this mean ?



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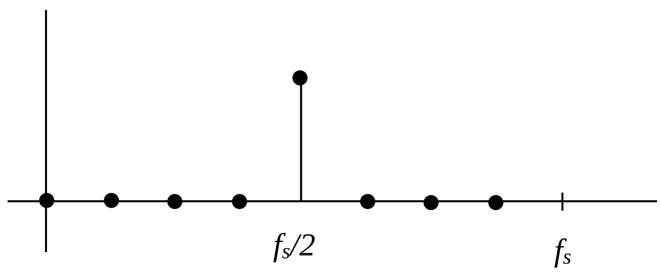

It means that all of the energy is centred on half the sampling frequency.

Using a similar approach to that above, verify that the IDFT of *S(k)* is equal to *s(n)*.