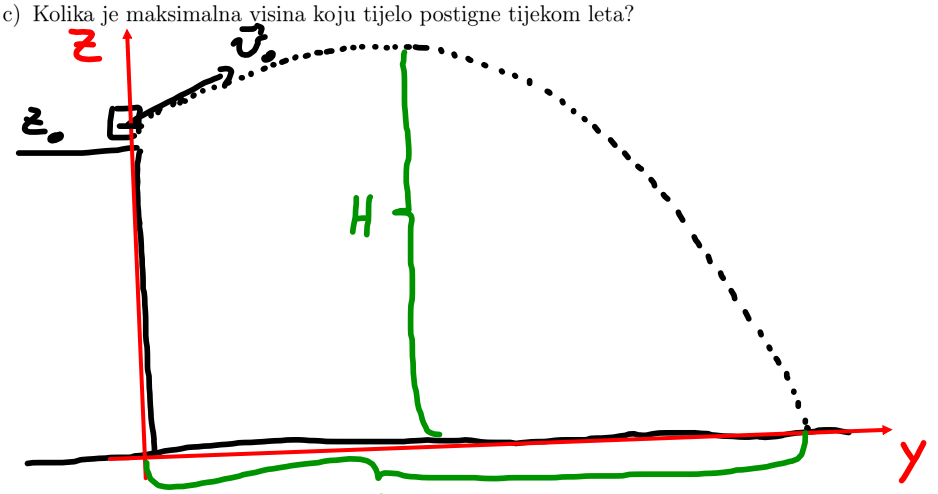
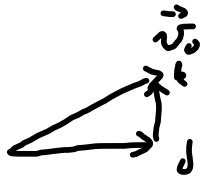
3.1. Tijelo je bačeno koso prema gore pod kutom od 30° prema horizontali početnom brzinom iznosa 20 ms^{-1} s visine 10 m iznad tla. Izračunajte (zanemarite otpor zraka):

a) Vrijeme udarca tijela o tlo.
b) Domet tijela.
c) Kolika je maksimalna visina koju tijelo postigne tijekom leta?



$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$



$$\vec{r}_0 = z_0\vec{k} = 10\,m\,\vec{k}$$

$$\vec{v}_0 = v_0\cdot\cos\alpha\cdot\vec{j} + v_0\sin\alpha\,\vec{k}$$

$$\vec{a} = \vec{g} = -g\vec{k}$$

$$\vec{r}(t) = z_0\vec{k} + v_0\cos\alpha\cdot t\,\vec{j} + v_0\sin\alpha\,t\vec{k} - \frac{1}{2}gt^2\vec{k}$$

$$= (v_0\cos\alpha\cdot t)\vec{j} + \left(z_0 + v_0\sin\alpha\cdot t - \frac{1}{2}gt^2\right)\vec{k} \;/\cdot\vec{k}$$

$z = 0$ TADA TIJELO UDARI O TLO

$$z(t) = z_0 + v_0\sin\alpha\,t - \frac{1}{2}gt^2 = 0$$

$$-\frac{1}{2}gt^2 + v_0\sin\alpha\,t + z_0 = 0$$

$$ax^2 + b\cdot x + c = 0$$

$$t_{1,2} = \frac{v_0\sin\alpha \pm \sqrt{(v_0\sin\alpha)^2 + 2gz_0}}{g}$$

$$= \frac{20\,ms^{-1}\sin 30° \pm \sqrt{\left(20\frac{m}{s}\sin 30\right)^2 + 2\cdot 9{,}81\frac{m}{s^2}\cdot 10\,m}}{9{,}81\,ms^{-2}}$$

$t_1 = 2{,}77\,s$ $\qquad$ $t_2 = -0{,}74\,s$

O.K. ✓

b) DOMET

$\vec{r}(t) = (v_0 \cos\alpha \cdot t)\vec{j} + (z_0 + v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2)\vec{k} \,/\cdot \vec{j}$

$\vec{j}\cdot\vec{j} = 1$

$\vec{j}\cdot\vec{k} = 0$

$y(t) = v_0 \cos\alpha \cdot t$

$y(t = 2{,}77_0) = v_0 \cos\alpha \cdot 2{,}77_0$

↖ VRIJEME PADA TIJELA

$D = y(t = 2{,}77_0) = 20\frac{m}{s} \cdot \cos 30^\circ \cdot 2{,}77_0$

$= 47{,}98\ m$

$D = v_y \cdot t$

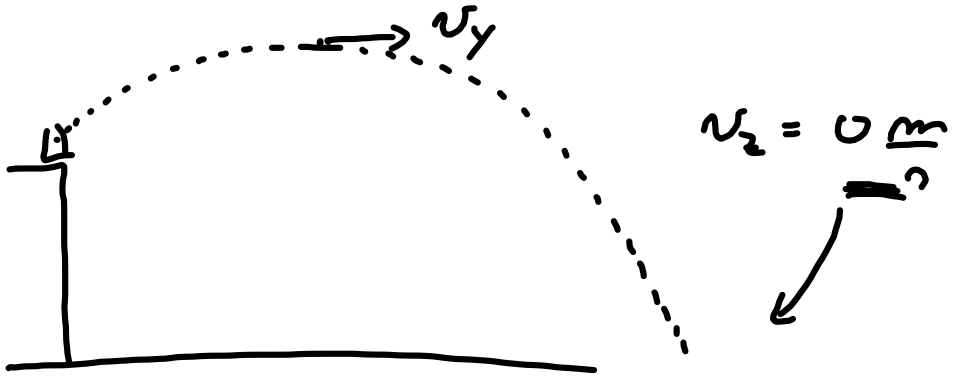
$\vec{v}(t) = \frac{d}{dt}(\vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}t^2)$

$= (\vec{v}_0 + \frac{1}{2}\vec{g}\cdot 2\cdot t)$

$= v_0 \cos\alpha \vec{j} + v_0 \sin\alpha \vec{k} - g t \vec{k}$

$= \underbrace{v_0 \cos\alpha}_{v_y}\vec{j} + \underbrace{(v_0 \sin\alpha - g t)}_{v_z}\vec{k}$ (*)

$D = v_y \cdot t = v_0 \cos\alpha \cdot t$



$$(*) \rightarrow \quad v_z = v_0 \sin\alpha - g t$$

$$0 = v_0 \sin\alpha - g t$$

$$g t = v_0 \sin\alpha$$

$$t_H = \frac{v_0 \sin\alpha}{g}$$

$$\vec{r}(t) = (v_0 \cos\alpha \cdot t)\vec{j} + (z_0 + v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2)\vec{k}$$

$$z(t) = z_0 + v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2$$

$$z(t = t_H) = H = z_0 + v_0 \sin\alpha \left(\frac{v_0 \sin\alpha}{g}\right) - \frac{1}{2} g \left(\frac{v_0 \sin\alpha}{g}\right)^2$$

$$H = z_0 + \frac{v_0^2 \sin^2\alpha}{g} - \frac{1}{2} \frac{v_0^2 \sin^2\alpha}{g}$$

$$= z_0 + \frac{1}{2} \frac{v_0^2 \sin^2\alpha}{g}$$