
# Macroeconomic Theory

## Assignment II

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# 1 Problem 1 (25 points)

Use the dataset PS1.csv for this question. You may assume that the frequency is annual. Assume that the model that generates the data is given by

$$y_t = \mu(1-\rho) + \rho y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

Estimate this model using the Kalman Filter & Maximum Likelihood. Compare your results with those you obtain from OLS estimation. Do you see important differences?

## Kalman loglikelihood Function

Step 1: Define model:

$y_t = \mu(1-\rho) + \rho y_{t-1} + \varepsilon_t$ , $\varepsilon_t \sim N(0,\sigma^2)$

$E[y_t] = \mu(1-\rho) + \rho E[y_{t-1}]$ since stationary, $E[y_t] = E[y_{t-1}]$.

$E[y_t] = \mu$.

Then we can demean:

$\to y_t - \mu = \rho(y_{t-1} - \mu) + \varepsilon_t$.

let $y_t - \mu = \tilde{y}_t$ , $y_{t-1} - \mu = \tilde{y}_{t-1}$

we get $\tilde{y}_t = \rho \tilde{y}_{t-1} + \varepsilon_t$.

Formula: $\begin{cases} \alpha_t = T\alpha_{t-1} + RU_t & U_t \sim N(0,Q) \\ y_t = Z\alpha_t + HW_t + \varepsilon_t & \varepsilon_t \sim N(0,H) \end{cases}$

since we get $\begin{cases} \alpha_t = \rho\alpha_{t-1} + \varepsilon_t. \\ \tilde{y}_t = \alpha_t \end{cases}$ , $\varepsilon_t \sim N(0,\sigma^2)$

So, $\to \begin{cases} T=\rho \ , \ R=1 \ , \ U_t = \varepsilon_t \ , \ Q = \sigma^2 \\ Z=1 \ , \ r=0 \ , \ \varepsilon_t = 0 \ . \ H = 0. \end{cases}$

Step 2: Initialization.

$a_0 = 0$

$P_0 = I_1 = 1$ initial state variance

Step 3: Time-0 Prediction.

transition prediction:

$$a_{1|0} = T a_0$$

$$P_{1|0} = T P_0 T' + RQR'$$

measurement prediction.

$$E_0[y_1] = Z a_{1|0}$$

$$V_0[y_1] = Z P_{1|0} Z' + H$$

Prediction Error

Expectation: $v_1 = y_1 - Z a_{1|0} = y_1 - T a_0$

Variance $F_1 = Z P_{1|0} Z' + H$

Step 4: Log-Likelihood. Calculation

$$\text{new} \ln L(y_1; \Theta) = -\frac{1}{2}\left(\ln(2\pi) + \ln(F_1) + \frac{v_1^2}{F_1}\right)$$

$$\ln L_K = \sum_{t=1}^{K} + \text{new} \ln L$$

Step 5: Kalman Gain Calculation

$$K_1 = \frac{P_{1|0} Z'}{F_1}$$

Step 6: Update

$$a_1 = a_{1|0} + K_1 v_1$$

$a_1$ updated better state estimate with observation combination

$$P_1 = P_{1|0} - K_1 Z P_{1|0}.$$

$P_1$ updated. variance. estimate.

Step 7. Iteration. Step 3 - Step 6.

Get $\sum_{t=1}^{T} \ln L_t$, sum (LL) at every iteration.

## Max Kalman Log-likelihood

f min search. (@(x) - kalman function ($\tilde{y}$, x), initial x )

where x is the set of parameters of $\rho$, $\sigma^2$

Results:

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Maximum Likelihood Estimation (MLE) results:
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$\hat{\rho}_{MLE}$ = 0.9551 0.0102 = $\hat{\sigma}^2_{MLE}$

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Ordinary Least Squares (OLS) results:
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$\hat{\rho}_{OLS}$ = 0.9554 0.0102 = $\hat{\sigma}^2_{OLS}$.

## 2 Problem 2 (30 points)

An economy is populated by a continuum of atomistic and identical households that maximize their lifetime utility given by

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\theta \ln c_t + (1-\theta)\ln(1-l_t)\right)$$

Households own capital and rent it to firms at a rate $r_t$. They earn a wage $w_t$ for every hour worked. They accumulate capital according to the law of motion

$C_t + i_t = W_t l_t + r_t k_t$

$$k_{t+1} = i_t + (1-\delta)k_t$$

The economy is also composed by a continuum of identical firms that use capital and labor to produce a final good according to the production function

$$y_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha}$$

(a) (10 Points) Find the equilibrium conditions of the model. Then, provide the definition of equilibrium in this economy.

**Household Problem:**

$$\max_{C_t, l_t, k_{t+1}} U = E_0 \sum_{t=0}^{\infty} \beta^t \left[\theta \ln C_t - (1-\theta)\ln(1-l_t)\right]$$

$$\text{s.t.} \quad \left.\begin{aligned} C_t + i_t &= W_t l_t + r_t k_t \\ k_{t+1} &= i_t + (1-\delta)k_t \end{aligned}\right\} \rightarrow C_t + k_{t+1} = W_t l_t + (r_t + 1-\delta)k_t.$$

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\theta \ln C_t + (1-\theta)\ln(1-l_t) + \lambda_t \left(W_t l_t + (r_t + 1-\delta)k_t - C_t - k_{t+1}\right)\right]$$

$$\frac{\partial \mathcal{L}}{\partial C_t}: \quad \frac{\theta}{C_t} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial l_t}: \quad \frac{1-\theta}{1-l_t} = \lambda_t W_t$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}}: \quad E_t\left[\beta^{\cancel{t+1}} \lambda_{t+1}(r_{t+1} + 1 - \delta)\right] = \cancel{\beta^t}\lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t}: \quad C_t + k_{t+1} = W_t l_t + (r_t + 1 - \delta)k_t.$$

$$\Rightarrow \begin{cases} \dfrac{\theta W_t}{C_t} = \dfrac{1-\theta}{1-l_t} & ① \\ \beta C_t E_t[r_{t+1} + 1 - \delta] = E_t[C_{t+1}] & ② \end{cases}$$

√

**Firm Problem:**

$$\Pi_t = Y_t - w_t l_t - r_t k_t.$$

$$\Rightarrow \max_{k_t, l_t} \Pi_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha} - w_t l_t - r_t k_t$$

$$\frac{\partial \Pi_t}{\partial k_t}: \quad \alpha k_t^{\alpha-1} l_t^{1-\alpha} e^{z_t} = r_t. \qquad (3)$$

$$\frac{\partial \Pi_t}{\partial l_t}: \quad (1-\alpha) k_t^{\alpha} l_t^{-\alpha} e^{z_t} = w_t. \qquad (4)$$

**Market Clearing:**

$$\begin{cases} c_t + k_{t+1} = w_t l_t + (r_t + 1 - \delta) k_t. & \text{from HH.} \\ Y_t - w_t l_t - r_t k_t = 0 & \text{from firm.} \end{cases}$$

$$\Rightarrow Y_t = c_t + k_{t+1} - k_t(1-\delta). \longrightarrow Y_t = c_t + i_t$$

$$e^{z_t} k_t^{\alpha} l_t^{1-\alpha} = c_t + k_{t+1} - k_t(1-\delta)$$

**Define Equlibrium:**

Given $k_0$, and eq$^t$ consists in household allocations $\{\hat{c}_t, \hat{l}_t, \hat{k}_{t+1}\}_{t=0}^{\infty}$

firm allocations $\{\hat{l}_t, \hat{k}_t\}_{t=0}^{\infty}$

and prices $\{\hat{w}_t, \hat{r}_t\}_{t=0}^{\infty}$

Given above household allocation, firm allcation and prices solve household problem, firm problem and market clearing above.

(b) (10 Points) Assume that $\delta = 0.025$, $\alpha = 0.3$ and a net return of capital equal to 1%. Find the steady state of the model. Note that to find it, you will need to calibrate $\beta$ and $\theta$. For $\theta$, you can set it so that the steady state amount of worked hours is equal to 1/3. Show your work.

since net return: $r-\delta = 0.01$ $\quad \delta = 0.025$

$r = 0.035 \rightarrow 1+r-\delta = 1.01$

$l = \frac{1}{3}$.

$$\begin{cases} \frac{\theta W_t}{C_t} = \frac{1-\theta}{1-l_t} & \text{① HH} \\ \beta C_t E_t[r_{t+1}+1-\delta] = E_t[C_{t+1}] & \text{② HH.} \\ \alpha k_t^{\alpha-1} l_t^{1-\alpha} e^{z_t} = r_t. & \text{③ firm} \\ (1-\alpha) k_t^{\alpha} l_t^{-\alpha} e^{z_t} = W_t. & \text{④ firm.} \\ e^{z_t} k_t^{\alpha} l_t^{1-\alpha} = C_t + K_{t+1} - K_t(1-\delta) & \text{⑤ RC.} \\ Y_t = e^{z_t} K_t^{\alpha} l_t^{1-\alpha} & \text{⑥ Production} \end{cases} \xrightarrow[e^{z_t}=1]{\text{In S.S}} \begin{cases} \frac{\theta W}{C} = \frac{1-\theta}{1-\frac{1}{3}} & ① \\ 1.01 \beta . C = C & ② \\ 0.3 K^{-0.7} \frac{1}{3}^{0.7} = 0.035 & ③ \\ 0.7 K^{0.3} \frac{1}{3}^{-0.3} = W & ④ \\ K^{0.3} \frac{1}{3}^{0.7} = C + 0.025K & ⑤ \\ Y = K^{0.7} \left(\frac{1}{3}\right)^{0.7}. \end{cases}$$

$\triangle$ $K_{t+1} = i_t + (1-\delta) K_t$.

$\triangle$ $i = \delta K$.

6 equations. solve 6 unknown: $\theta, w, c, \beta, K, y$.

By Matlab:

$$\Rightarrow \begin{cases} \beta = 0.99 \\ C = 0.658 \\ K = 7.17 \\ \theta = 0.359 \\ w = 1.76 \\ y = 0.837 \end{cases}$$

$\triangle$ $i = 0.179$.

(c) (10 points) Log-linearize the equilibrium system of equations. Show your work.

$$\begin{cases} \frac{\theta w_t}{c_t} = \frac{1-\theta}{1-l_t} & ① \\ \beta c_t E_t[r_t + 1 - \delta] = E_t[c_{t+1}] & ② \\ \alpha k_t^{\alpha-1} l_t^{1-\alpha} e^{z_t} = r_t. & ③ \\ (1-\alpha) k_t^{\alpha} l_t^{-\alpha} e^{z_t} = w_t. & ④ \\ e^{z_t} k_t^{\alpha} l_t^{1-\alpha} = c_t + k_{t+1} - k_t(1-\delta) \;/\; y_t = c_t + k_{t+1} - k_t(1-\delta) & ⑤ \\ y_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha} & ⑥. \end{cases}$$

} HH. (① ②)

} firm. (③ ④)

RC. (⑤)

production. (⑥)

$-\ln c_t$

①. $\frac{\theta w_t}{c_t} = \frac{1-\theta}{1-l_t}$:

1. log $\quad \ln\theta + \ln w_t - \ln c_t = \ln(1-\theta) - \ln(1-l_t)$

2. sub s.s.

$$\ln\theta - \ln\theta + \ln w_t - \ln\bar{w} - \ln c_t - -\ln\bar{c} = \ln(1-\theta) - \ln(1-\theta) - \ln(1-l_t) - -\ln(1-\bar{l})$$

$$\tilde{w}_t - \ln\hat{c}_t = -(\ln(1-l_t) - \ln(1-\bar{l}))$$

$$\text{since: } \ln(1-l_t) \approx \ln(1-\bar{l}) - \frac{1}{1-\bar{l}}(l_t - \bar{l})$$

$$\approx \ln(1-\bar{l}) - \frac{\bar{l}}{1-\bar{l}} \frac{(l_t-\bar{l})}{\bar{l}}$$

$$\approx \ln(1-\bar{l}) - \frac{\bar{l}}{1-\bar{l}} \check{l}_t$$

$$\rightarrow \hat{w}_t - \ln\hat{c}_t = -\left(\cancel{\ln(1-\bar{l})} - \frac{\bar{l}}{1-\bar{l}}\hat{l}_t - \cancel{\ln(1-\bar{l})}\right)$$

$$\hat{w}_t - \ln\hat{c}_t = \frac{\bar{l}}{1-\bar{l}}\tilde{l}_t$$

②. $\quad \beta c_t E_t[r_t + 1 - \delta] = E_t[c_{t+1}] \quad ②$

For simple, I don't write $E_t[\ ]$ during the calculation

1. log.
$$\ln\beta + \ln C_t + \ln(r_{t+1}-\delta) = \ln C_{t+1}$$

2. sub s.s.
$$\check{C}_t + \ln(r_{t+1}-\delta) - \ln(\bar{r}+1-\delta) = \check{C}_{t+1}$$

$$-\check{C}_t = \beta\, \check{r}_{t+1} r^* - \check{C}_{t+1}$$

$$\check{C}_t = E[\check{C}_{t+1}] - \beta r^* E[\check{r}_{t+1}]$$

③.
$$\alpha k_t^{\alpha-1} l_t^{1-\alpha} e^{z_t} = r_t.$$

/ de $y_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha}.$

$$\Rightarrow \alpha \frac{\cancel{k_t^{\alpha-1}}\, \cancel{l_t^{1-\alpha}}\, \cancel{e^{z_t}}}{k_t^{\alpha}\, \cancel{l_t^{1-\alpha}}\, \cancel{e^{z_t}}} = \frac{r_t}{y_t.}$$

$$\alpha \frac{y_t}{k_t} = r_t.$$

1. log.
$$\ln\alpha + \ln y_t - \ln k_t = \ln r_t.$$

sub. s.s.
$$\ln y_t - \ln\bar{y} - (\ln k_t - \ln\bar{k}) = \ln r_t - \ln\bar{r}$$

$$\check{y}_t - \check{k}_t = \check{r}_t$$

④.
$$(1-\alpha) k_t^{\alpha} l_t^{-\alpha} e^{z_t} = W_t.$$

/ de $y_t = e^{z_t} k_t^{\alpha} l_t^{1-\alpha}.$

$$\frac{(1-\alpha)\cancel{k_t^{\alpha}}\, \cancel{l_t^{-\alpha}}\, \cancel{e^{z_t}}}{\cancel{k_t^{\alpha}}\, l_t \cancel{l_t^{-\alpha}}\, \cancel{e^{z_t}}} = \frac{W_t}{y_t.}$$

$$(1-\alpha)\frac{y_t}{l_t} = W_t.$$

1. log.
$$\ln(1-\alpha) + \ln y_t - \ln l_t = \ln W_t$$

2. s.s
$$\check{y}_t - \check{l}_t = \check{W}_t$$

⑤. $y_t = c_t + k_{t+1} - k_t(1-\delta)$

1. $\bar{y}\frac{y_t}{\bar{y}} = \bar{c}\frac{c_t}{\bar{c}} + \bar{k}\frac{k_{t+1}}{\bar{k}} - (1-\delta)\bar{k}\frac{k_t}{\bar{k}}$

$\bar{y}e^{\ln\frac{y_t}{\bar{y}}} = \bar{c}e^{\ln\frac{c_t}{\bar{c}}} + \bar{k}e^{\ln\frac{k_{t+1}}{\bar{k}}} - (1-\delta)\bar{k}e^{\ln\frac{k_t}{\bar{k}}}$

$\bar{y}e^{\ln y_t - \ln\bar{y}} = \bar{c}e^{\ln c_t - \ln\bar{c}} + \bar{k}e^{\ln k_{t+1} - \ln\bar{k}} - (1-\delta)\bar{k}e^{\ln k_t - \ln\bar{k}}$

$\bar{y}e^{\tilde{y}_t} = \bar{c}e^{\tilde{c}_t} + \bar{k}e^{\tilde{k}_{t+1}} - (1-\delta)\bar{k}e^{\tilde{k}_t}$

since $e^{\tilde{y}_t} \approx (1+\tilde{y}_t)$ $\quad e^{\tilde{c}_t} \approx (1+\tilde{c}_t)$ $\quad e^{\tilde{k}_t} \approx (1+\tilde{k}_t)$

$\bar{y}(1+\tilde{y}_t) = \bar{c}(1+\tilde{c}_t) + \bar{k}(1+\tilde{k}_{t+1}) - (1-\delta)\bar{k}(1+\tilde{k}_t)$

2. sub. s.s.

$\bar{y}\tilde{y}_t = \bar{c}\tilde{c}_t + \bar{k}\tilde{k}_{t+1} + \bar{k}\tilde{k}_{t+1} - (1-\delta)\bar{k}\tilde{k}_t$

⑥. $y_t = e^{z_t}k_t^{\alpha}l_t^{1-\alpha}$

1. log

$\ln y_t = z_t + \alpha\ln k_t + (1-\alpha)\ln l_t.$

2. sub s.s.

$\tilde{y}_t = z_t - \bar{z} + \alpha\tilde{k}_t + (1-\alpha)\tilde{l}_t$

$\Rightarrow \tilde{y}_t = \tilde{z} + \alpha\tilde{k}_t + (1-\alpha)\tilde{l}_t$ special : $z_t - \bar{z} \approx \tilde{z}$.

⑦ $k_{t+1} = i_t + (1-\delta)k_t$.

$\bar{k}\frac{k_{t+1}}{\bar{k}} = \bar{i}\frac{i_t}{\bar{i}} + (1-\delta)\bar{k}\frac{k_t}{\bar{k}}$

$\bar{k}\tilde{k}_t = \bar{i}\tilde{i}_t + (1-\delta)\bar{k}\tilde{k}_t$

# 3 Problem 3 (45 points)

An economy is populated by a continuum of atomistic and identical households that maximize their lifetime utility given by

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

Households own capital and rent it to firms at a rate $r_t$. Capital utilization is now a choice variable. This is, households, in addition to decide how much capital to accumulate, also decide the rate $u_t$ at which capital is utilized. They accumulate capital according to the law of motion

$$C_t + i_t = r_t u_t k_t + W_t.$$

$$k_{t+1} = i_t + (1 - \delta(u_t))k_t,$$

where $\delta(u_t) = \delta_0 + \delta_1 u_t^{\omega}$.

Households earn a wage $w_t$, and they are endowed with one unit of time (they do not care about leisure).

The economy is also composed by a continuum of identical firms that use capital and labor to produce a final good according to the production function

$k_t$ $l_t$.

$$y_t = e^{z_t}(u_t k_t)^{\alpha} l_t^{1-\alpha}$$

Note that firms do not decide the utilization rate of capital. Instead, they decide the *effective* units of capital $u_t k_t$.

(a) (10 Points) Find the equilibrium conditions of the model.

HH Problem

$$\max_{C_t, U_t, K_{t+1}} U = E_0 \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$\text{S.t.} \begin{cases} C_t + i_t = W_t + r_t U_t K_t. \\ k_{t+1} = i_t + (1 - \delta_0 - \delta_1 U_t^{\omega}) K_t \end{cases} \rightarrow C_t + K_{t+1} = W_t + (r_t U_t + 1 - \delta_0 - \delta_1 U_t^{\omega}) K_t$$

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t + \lambda_t \left( W_t + (r_t U_t + 1 - \delta_0 - \delta_1 U_t^{\omega}) \underline{K_t} - C_t - \underline{K_{t+1}} \right) \right]$$

$$\frac{\partial \mathcal{L}}{\partial C_t}: \quad \frac{\cancel{\beta^t}}{C_t} = \cancel{\beta^t} \lambda_t \qquad \frac{1}{C_t} = \lambda_t. \qquad ⓐ$$

$$\frac{\partial \mathcal{L}}{\partial U_t}: \quad \cancel{\beta^t} \cancel{\lambda_t} r_t = \cancel{\beta^t} \cancel{\lambda_t} \delta_1 \omega U_t^{\omega - 1} \qquad r_t = \delta_1 \omega U_t^{\omega - 1} \qquad ②$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}}: \cancel{\beta^{t+1}}\lambda_{t+1}(r_{t+1}u_{t+1}+1-\delta_0-\delta_1 u_{t+1}^{\omega}) = \cancel{\beta^{t}}\lambda_t \quad (b)$$

$$\beta E_t[\lambda_{t+1}(r_{t+1}u_{t+1}+1-\delta_0-\delta_1 u_{t+1}^{\omega})] = \lambda_t.$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t}: \quad c_t + k_{t+1} = w_t + (r_t u_t + 1 - \delta_0 - \delta_1 u_t^{\omega})k_t$$

(a) (b): $$\beta E_t\left[\frac{1}{c_{t+1}}(r_{t+1}u_{t+1}+1-\delta_0-\delta_1 u_{t+1}^{\omega})\right] = \frac{1}{c_t} \quad (1).$$

from statement $$i_t = k_{t+1} - (1-\delta_0-\delta_1 u_t^{\omega})k_t \quad (3).$$

Firm. Problem

$$\max_{k_t, l_t} \ \Pi = e^{z_t}(u_t k_t)^{\alpha} l_t^{1-\alpha} - w_t l_t - r_t u_t k_t$$

$$\frac{\partial \Pi}{\partial k_t}: \alpha e^{z_t} u_t^{\alpha} k_t^{\alpha-1} l_t^{1-\alpha} = r_t u_t \longrightarrow \alpha e^{z_t} u_t^{\alpha} k_t^{\alpha-1} = r_t u_t \quad (4)$$

$$\frac{\partial \Pi}{\partial l_t}: (1-\alpha) e^{z_t} u_t^{\alpha} k_t^{\alpha} l_t^{-\alpha} = w_t. \quad \xrightarrow{\text{since } l_t=1} \quad (1-\alpha) e^{z_t} u_t^{\alpha} k_t^{\alpha} = w_t. \quad (5)$$

from statement.

$$y_t = e^{z_t}(u_t k_t)^{\alpha} l_t^{1-\alpha}. \xrightarrow{l_t=1} y_t = e^{z_t}(u_t k_t)^{\alpha}$$

Market Clearing.

$$\begin{cases} c_t + k_{t+1} = w_t + (r_t u_t + 1 - \delta_0 - \delta_1 u_t^{\omega})k_t \\ y_t = w_t + r_t u_t k_t. \end{cases}$$

$$\rightarrow, \quad y_t + \cancel{w_t} + r_t u_t k_t + (1-\delta_0-\delta_1 u_t^{\omega})k_t = c_t + k_{t+1} + \cancel{w_t} + r_t u_t k_t.$$

$$y_t = c_t + k_{t+1} + (1-\delta_0-\delta_1 u_t^{\omega})k_t. \quad (6).$$

Define Equlibrium:

Given $k_0$, and eq$^{t}$ consists in household allocations $\{\hat{c}_t, \hat{u}_t, \hat{k}_{t+1}, \hat{i}_t\}_{t=0}^{\infty}$

firm allocations $\{\hat{k}_t\}_{t=0}^{\infty}$

and prices $\{\hat{w}_t, \hat{r}_t\}_{t=0}^{\infty}$

Given above household allocation, firm allocation and prices solve household problem, firm problem and market clearing above.

(b) (10 Points) Assume $\beta = 0.99$, $\alpha = 0.3$, $\delta_0 = 0$, $\delta_1 = 0.03$, $\omega = 2$, and a net return of capital equal to 1% in steady state. Find the steady state prices and allocations of this economy.

$$
\begin{cases}
\beta E_t\left[\frac{1}{C_{t+1}}\left(r_{t+1}u_{t+1}+1-\delta_0-\delta_1 u_{t+1}^{\omega}\right)\right]=\frac{1}{C_t} & ① \\
r_t=\delta_1\omega u_t^{\omega-1} & ② \\
i_t=k_{t+1}-(1-\delta_0-\delta_1 u_t^{\omega})k_t & ③ \\
\alpha e^{z_t}u_t^{\alpha-1}k_t^{\alpha-1}=r_t & ④ \\
(1-\alpha)e^{z_t}u_t^{\alpha}k_t^{\alpha}=w_t. & ⑤ \\
y_t=e^{z_t}(u_tk_t)^{\alpha} & \text{production } ⑥. \\
y_t=c_t+i_t \,/\, y_t=C_t+k_{t+1}+(1-\delta_0-\delta_1u_t^{\omega})k_t & ⑦ \text{ R.C.}
\end{cases}
$$

HH (①–③), firm (④–⑥)

$$\xrightarrow[e^{z_t}=1]{\text{in s.s}}$$

$$
\begin{cases}
\beta\left[ru+1-\delta_0-\delta_1u^{\omega}\right]=1 \\
r=\delta_1\omega u^{\omega-1} \\
i=(\delta_0+\delta_1u^{\omega})k \\
r=\alpha u^{\alpha-1}k^{\alpha-1} \\
w=(1-\alpha)u^{\alpha}k^{\alpha} \\
y=u^{\alpha}k^{\alpha} \\
y=c+i
\end{cases}
$$

since $\beta=0.99$, $\alpha=0.3$, $\delta_0=0$, $\delta_1=0.03$, $\omega=2$.

$$
\Rightarrow
\begin{cases}
ru+1-0.03u^2=\frac{1}{0.99} & ① \\
r=0.06u & ② \\
i=(0.03u^2)k & ③ \\
r=0.3(uk)^{-0.7} & ④ \\
w=0.7(uk)^{0.3} & ⑤ \\
y=(uk)^{0.3}. & ⑥ \\
y=c+i. & ⑦.
\end{cases}
$$

7 equations

7 unknowns: $r, u, k, i, w, y, c$

$-\delta_1u^{\omega}+ur$

$-\delta_1u^{\omega}+\delta_1\omega u^{\omega-1}$

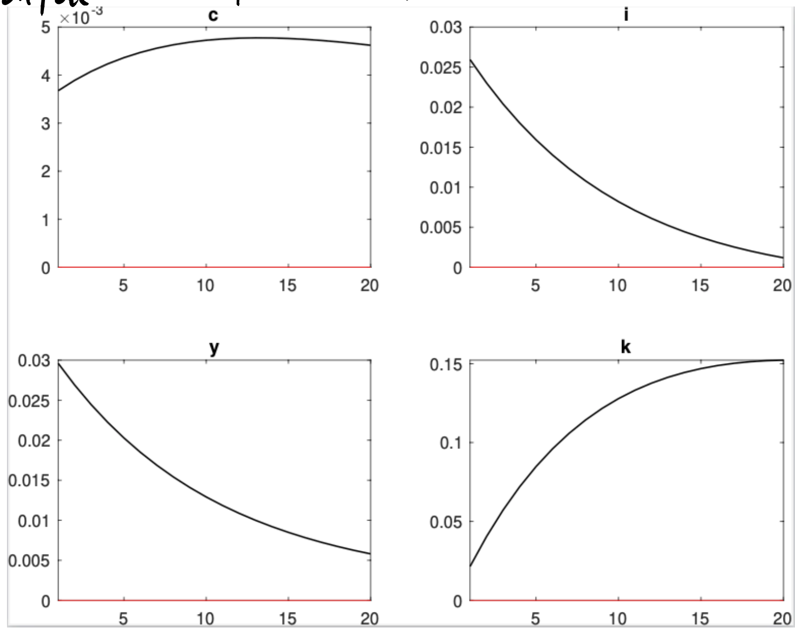
by MatLab:

$$
\begin{cases}
r=0.035 \rightarrow \triangle \\
u=0.580 \\
k=37.376 \\
i=0.378 \\
w=1.762 \\
y=2.517 \\
c=2.139.
\end{cases}
$$

(c) (10 Points) Assume that $z_t = 0.9z_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim \mathcal{N}(0, 0.01^2)$. Use Dynare to compute the impulse response functions to a one standard deviation productivity shock of consumption, investment, output, and capital.

use S.S. we get from b) to Dynare.

IRF output with productivity shock




(d) (15 Points) Recompute your impulse response functions but now assume $\delta_0 = 0.025$ and $\delta_1 = 0$. Compare your results with those of part (c). Remember to be specific and provide economic intuition for your results.

① use new parameters to calculate new S.S.

$\beta = 0.99$ $\alpha = 0.3$ $\delta_0 = 0.025$ $\delta_1 = 0$ $\omega = 2$

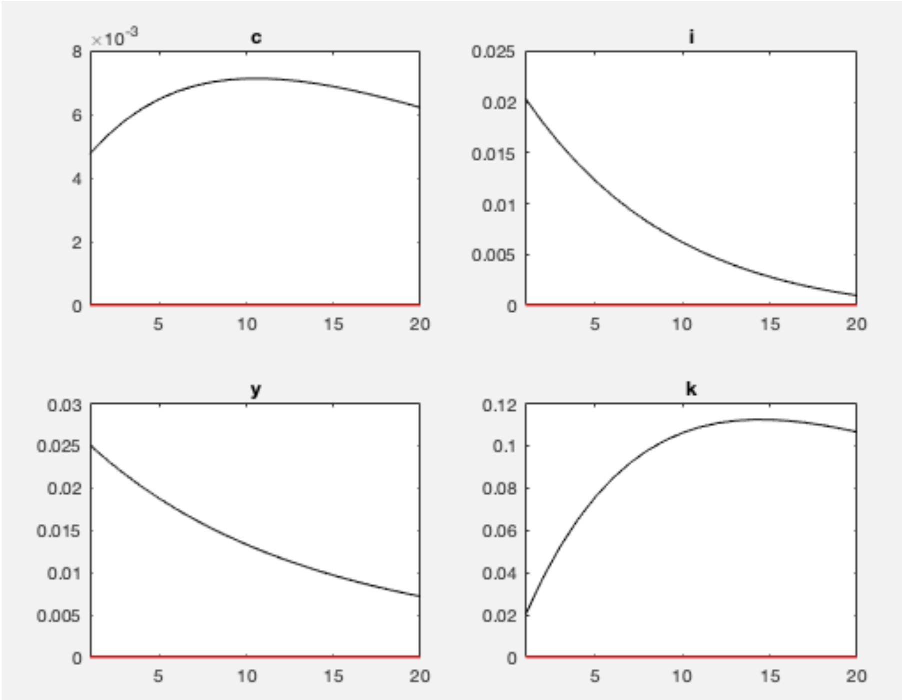
$k_{t+1} = i_t + (1 - \delta_0 - \delta_1 U_t^{\omega}) K_t \longrightarrow K_{t+1} = i_t + (1 - \delta_0) K$ since $\delta_1 = 0$

Thus, the HH and firm will not use $U_t$ to maximize their utility

we let $U_t = 1$ in steady state.

```
Steady-State Solution:
r = 0.035000
K = 21.524504
Y = 2.511192
W = 1.757835
I = 0.538113
C = 1.973080
Z = 0.000000
```

② Dynare




Compare:

3.c

$$\begin{cases} r = 0.035 \\ u = 0.580 \\ k = 37.376 \\ i = 0.378 \\ w = 1.762 \\ y = 2.517 \\ c = 2.139 \end{cases}$$

3.d.

$$\begin{cases} r = 0.035 & \approx \\ u = 1 & \uparrow \\ K = 21.525 & \downarrow \\ i = 0.538 & \uparrow \\ w = 1.758 & \approx \\ y = 2.511 & \approx \\ c = 1.973. & \downarrow. \end{cases}$$

$k_{t+1} = i_t + (1 - 0.03 u_t^2) k_t$

$k_{t+1} = i_t + (1 - 0.025) K$, constant depreciation rate

As shown above, investment increases,
capital and consumption decrease,
production, wage, rate are very close.

The different depreciation function is the reason of this pattern:
the higher capital utilization rate always brings higher capital depreciation rate. As a result, people will invest less, hold capital more.
On the other hand, in the second model, the depreciation rate doesn't related to the capital utilization rate, as a result, people invest more.

In addition, for IMF:

In captical utilization related depreciation economy, the instant shock in investment, captial, is strong than constant depreciation economy. However less strong in consumption.

Because the model 1 allow the people use switch capital utilization to modify their investment and holding capital. However, in constant depreciate capital economy are more rely to derectly modify investment to react the shock.