The Driver's Dilemma: The Nash Equilibrium Behind Large Car Preferences

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October 24, 2025

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Roadmap

Motivation and Question

- **SUVs/light trucks sales accounted for ∼75%** in the US (2021, excl. commercial use).
  - Individually beneficial: Convenience and safety.
  - Socially costly: Higher risk to other drivers and pedestrians (Anderson & Auffhammer, 2014; Tyndall, 2021; Winston & Yan, 2021 etc.).

⇒ **Individually rational behavior** can generate a **socially inefficient** outcome.

- This paper provides a game-theoretical framework to analyze this,
  - compares policy tools from price/quantity regulation perspectives.
  - extends the analysis to dynamic environments.

Roadmap

Model Setup

- N drivers and $\Omega - N$ pedestrians in the model.
  λ denotes the ratio of drivers:

$$N = \lambda \Omega.$$

- Only drivers choose type of car $\theta \in \{L, S\}$: Large (L) or Small (S).
  k denote the ratio of L:

$$\begin{aligned} &\text{Large Car:} \quad && kN, \\ &\text{Small Car:} \quad && (1-k)N \end{aligned}$$

Model Setup

- Driver i's utility function has a quasi-linear form in which price enters linearly and other components affect utility nonlinearly:

$$
\begin{aligned}
u_i^{\theta}(k, N) = & \alpha_1 \ln(1+\phi_{s_i}) + \alpha_2 \ln(1+\psi_{s_i}) - \alpha_3\, p_{s_i} \\
& - \alpha_4 q\, \ln(1+\mathbb{E}[D_{s_i}(s_{-i})|k, N]) + \alpha_5 \ln(1+O_{s_i}).
\end{aligned}
$$

where ϕ is objective safety benefit; ψ represents subjective perception of safety; p is price; D is expected damage when colliding with driver $-i$; O is other utility; q is probability of accident; and α are weights of each component.

  - This reflects, except for price, **the marginal utility/disutility diminish.**
  - We only consider collision between two cars.

Model Setup

- Expected Damage $D_\theta(s_{-i})$ assumes below ordering reflecting empirical evidence (Anderson & Auffhammer, 2014):

$$D_S(L) > D_L(L) = D_S(S) > D_L(S)$$

- Conditioning the ratio of large cars and drivers, the expected damage is given by:

$$\mathbb{E}[D_L(s_{-i})|k, N] = D_L(L)\frac{kN-1}{N-1} + D_L(S)\frac{(1-k)N}{N-1},$$
$$\mathbb{E}[D_S(s_{-i})|k, N] = D_S(L)\frac{kN}{N-1} + D_S(S)\frac{(1-k)N-1}{N-1}.$$

Model Setup

- Per-capita social welfare is defined as:

$$W_{pc}(k, \Omega, \lambda) := \lambda u_i^{\theta}(k, N) + (1-\lambda) u_P(k)$$

where $u_P(k)$ represent a representative pedestrian utility.

  - Since we restrict attention to the case where two cars may collide, we have four possible combination assuming the following order:

$$u_P(S,S) > u_P(L,S) = u_P(L,S) > u_P(L,L)$$

- Then, We can express the utility functions in terms of k:

$$\begin{aligned} u_i^{\theta}(k,N) &= k u_i^{L}(k,N) + (1-k) u_i^{S}(k,N), \\ u_P(k) &= k^2 u_P(L,L) + 2k(1-k) u_P(L,S) + (1-k)^2 u_P(S,S). \end{aligned}$$

Calibration for Numerical Simulation

Notation	Value	Notation	Value
ϕ_L	0.8	$D_S(S) = D_L(L)$	0.5
ϕ_S	0.5	$O_L = O_S$	0.2
ψ_L	0.9	q	0.04
ψ_S	0.4	α_j	1 (all j)
p_L	0.7	$u_P(S, S)$	1.0
p_S	0.3	$u_P(L, S) = u_P(S, L)$	0.4
$D_S(L)$	0.9	$u_P(L, L)$	0.1
$D_L(S)$	0.2	λ	0.59

Roadmap

Individual Decision Rule

A driver chooses a large car only when the utility of L exceeds that from S.

$$u_i^L - u_i^S > 0$$

Nash Equilibrium

Proposition 1

If the following parameter condition holds for all $k \in [0,1]$:

$$\alpha_1 \ln\left(\frac{1+\phi_L}{1+\phi_S}\right) + \alpha_2 \ln\left(\frac{1+\psi_L}{1-\psi_S}\right) + \alpha_5 \ln\left(\frac{1+O_L}{1+O_S}\right) > \alpha_3(p_L - p_S) + \alpha_4 q \ln\left(\frac{1+\mathbb{E}[D_L(s_{-1}|k,N)]}{1+\mathbb{E}[D_S(s_{-i}|k,N)]}\right),$$

then for any number of homogeneous drivers $N \geq 2$, the unique pure-strategy Nash equilibrium is for all drivers to choose large vehicles: $k^* = 1$

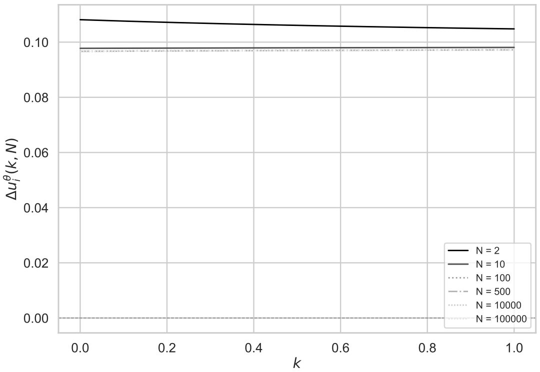
- The left-hand side captures **the utility premium of a large car**, while the right-hand side captures **the total burden associated with the vehicle**.
- The contemporary automobile market—particularly the widespread popularity of SUVs and pickup trucks—provides clear evidence that this inequality holds in the US.
- With the calibrated value, the inequality holds as well.

Social Welfare

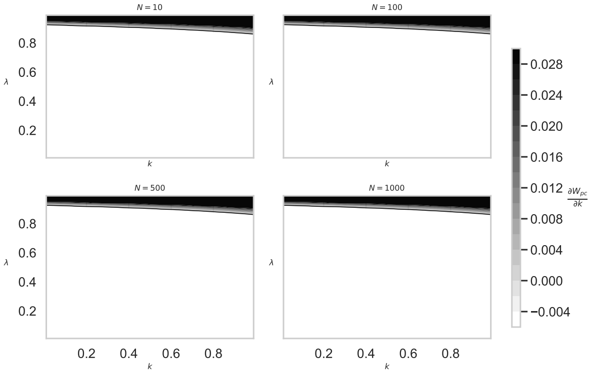
Proposition 2

If λ is below a sufficiently high threshold $\bar{\lambda}$, then the per-capita social welfare $W_{pc}(k)$ in the homogeneous N-driver model is strictly decreasing in k. Therefore, the socially optimal outcome is: $k^{\text{opt}} = 0$.

- As long as the proportion of pedestrians is not too small, the loss in **pedestrian utility outweighs the gain in driver utility** as more drivers choose large vehicles.
- The threshold $\bar{\lambda}$ lies approximately between 0.871 and 0.933 as $N \to \infty$ and $k \in [0, 1]$.



(a) Variation of Δu_i^θ



(b) Positive Area (Dark) of $\frac{\partial W_{pc}}{\partial k}$

Figure: Numerical Illustration of Proposition 1 and 2

Roadmap

Introducing Heterogeneity

- To introduce heterogeneity, we simplify the expression of driver's utility:

$$u_i^\theta = \alpha_{\text{safe},i} V_{\text{safe}}^\theta - \alpha_{\text{price},i}\, p_\theta + \alpha_{\text{other},i} \ln(1 + O_\theta)$$

  where V_{safe}^θ represents the aggregated safety utility component:

$$V_{\text{safe}}^\theta = \ln(1 + \phi_\theta) + \ln(1 + \psi_\theta) - q \ln(1 + \mathbb{E}[D_\theta | k, N]).$$

- Individual heterogeneity is then characterized by a single parameter, the preference ratio r_i:

$$r_i \equiv \frac{\alpha_{\text{safe},i}}{\alpha_{\text{price},i}},$$

  assuming $r_i \sim N(\mu_r, \sigma_r^2)$.

- This framework includes the homogeneous model as a special case where $\sigma_r = 0$ and $\mu_r = 1$.

Numerical Solution

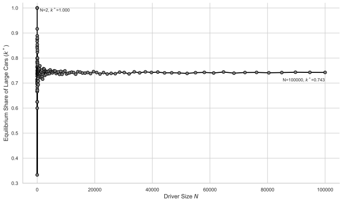
Definition 1

In the heterogeneous model, the Nash Equilibrium is a proportion of L drivers, k^*, such that no individual driver wishes to change his/her vehicle choice, given that a k^* proportion of other drivers have chosen L cars.

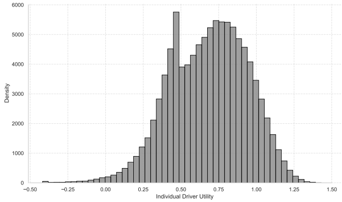
- The equilibrium proportion k^* must satisfy the following fixed-point condition:

$$k^* = \frac{1}{N} \sum_{i=1}^{N} \mathbb{1} \left\{ u_i^L(k^*, N) > u_i^S(k^*, N) \right\}$$

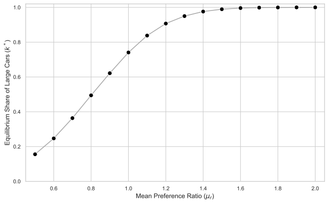
- This problem does not have a closed-form analytical solution and must be solved numerically.
- The numerical solution, using the baseline parameters and setting the preference distribution to $\mu_r = 1.0$ and $\sigma_r = 0.3$ for a driver population of $N = 100,000$, yields a stable Nash Equilibrium at $k^* \approx 0.743$.



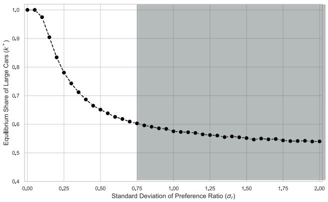
(a) Driver size N variation



(b) Utility Distribution at $k^* = 0.743$



(c) Mean Preference Ratio μ_r variation



(d) Std. of Preference Ratio σ_r variation

Dynamics

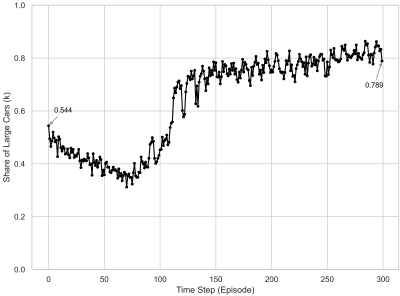
- Under the heterogeneous setting, we simulate four combinations of sequential games: **perfect information** vs. **imperfect information**, and **individual sequential game** vs. **group sequential game**. The results converge to equilibria of k^* between $= 0.7416$ and 0.7425.

- Moreover, the results remain consistent under the **evolutionary game** framework (see details in the paper).

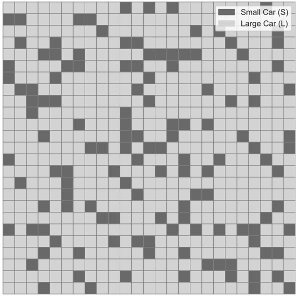
- All of the above models assume **fully rational individuals**. What if drivers are not fully rational?

- We therefore develop an **Agent-Based Model (ABM)** using a **reinforcement learning mechanism**. This allows us to test the robustness of the model under **bounded rationality** and **peer effects**.

ABM Results



(a) Evolution of Large Vehicle Share



(b) Final Grid $t = 300$

- After around 100 iter., k^* varies within the 0.75–0.80 range
- No remarkable clustering emerges in the final outcome due to influence of global market share.

Roadmap

Policy Tools

- We discuss policy tools such as price-based (tax/subsidy), and quantity-based (quotas, permits) in the paper.
- Here, we show only the revenue-neutral tax and subsidy regime which is one of the most effective policy tool:
  - Let the social planner impose a tax $\tau > 0$ on each L vehicle and provide a subsidy $\delta > 0$ for each S vehicle. The effective prices for drivers, p'_L and p'_S, are now given by:
  $$p'_L = p_L + \tau, \quad p'_S = p_S - \delta.$$
  - Revenue-neutrality implies $N_L \cdot \tau = N_S \cdot \delta.$, leading:
  $$\delta = \frac{k}{1-k}\tau.$$
  - Considering no-deviation scenario from S, we can set:
  $$\delta = \frac{\tau}{N-1}.$$

Homogeneous Model

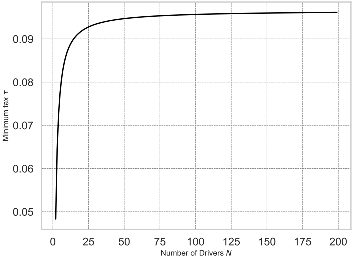


Figure: Minimum Tax τ on L Cars for Social Optimum

- In homogeneous driver model, numerical exercise gives that the minimum value of τ^* increases with N, but gradually converges to approximately 0.096.
- Lump-sum tax of around 0.096 is sufficient to achieve the socially optimal outcome.

Heterogeneous Model

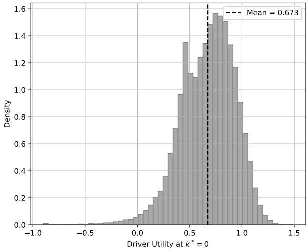
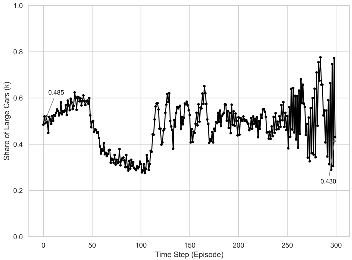


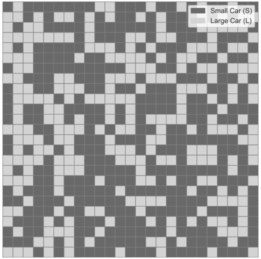
Figure: Utility Distributions at $k^* = 0$ in the Heterogeneous Model

- In heterogeneous model, the required tax can be as high as $\tau^*_{het} \approx 0.764$ to reach $k^* = 0$ due to an extreme outlier in the utility distribution at $k^* = 0$.
- While achieving a perfect social optimum $(k = 0)$ is theoretically possible, the cost of doing so may be politically and economically infeasible.

Agent-Based Model



(a) Evolution of Large Vehicle Share with Tax



(b) Final Grid with Tax $t = 300$

- When we impose $\tau = 0.096$ (value of homogeneous model), at $t = 300$, k stands at 0.43—still below 0.5—but given the high volatility observed near the end.
- The applied value of $\tau = 0.096$ does not impose a sufficiently strong individual deterrent effect on agents, thereby allowing for persistent fluctuations.

Concluding Remarks

- The paper shows a coordination failure in a game-theoretic framework such that individually rational choices can lead to a socially inefficient outcome.
- the equilibrium of the model with heterogeneous driver preferences (large-vehicle share $\approx 74\%$) closely matches the observed market outcome.
- When ignoring the ongoing administrative costs borne by the government, under $k = 0$, the tax–subsidy scheme and the strict quota are equivalent in terms of social welfare, and both are higher welfare than the emission permit system.
- When heterogeneity among drivers is taken into account, the tax required to achieve the socially optimal share of large vehicles becomes very high. Therefore, it is crucial to consider the distribution of driver preferences when designing policies.

Thank you!