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Stallions

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Contents

Contest (1)

template.hpp

26 lines

```
// hash = 85ed39

#include <bits/stdc++.h>
using namespace std;

#define FOR(i, a, b) for(int i = (a); i < (b); i
    ++)
#define RFOR(i, a, b) for(int i = (a) - 1; i >= (
    b); i--)
#define SZ(a) int(a.size())
#define ALL(a) a.begin(), a.end()
#define PB push_back
#define MP make_pair
#define F first
#define S second

typedef long long LL;
typedef vector<int> VI;
typedef pair<int, int> PII;
typedef double db;

int main()
{
  ios::sync_with_stdio(0);
  cin.tie(0);

  return 0;
}
```

compilation.txt

2 lines

```
g++ -O2 -std=c++17 -Wno-unused-result -Wshadow -
    Wall -o %e %e.cpp
g++ -std=c++17 -Wshadow -Wall -o %e %e.cpp -
    fsanitize=address -fsanitize=undefined -
    D_GLIBCXX_DEBUG -g
```

s.sh

6 lines

```
for((i = 0; ; i++)) do
  echo $i
  ./gen $i > in
  diff -w <(./a < in) <(./brute < in) || break
  [ $? == 0 ] || break
done
```

hash.sh

1 lines

```
cpp -dD -P -fpreprocessed $1 | tr -d '[:space:]'|
     md5sum |cut -c-6
```

1.1 Rules

Don't code solution without proof.

Try to find counter-tests.

Discuss realisation, try to assist.

Freeze time: Discuss how much problem we need/want to solve. At beginning (and after AC) discuss situation and what to do.

1.2 Troubleshoot

Pre-submit

F9. Write a few manual test cases. Calculate time and memory complexity. Check limits. Check overflows, size of arrays, clearing mutitestcases, uninitialized variables.

Wrong answer

F9. Print your solution! Read your code. Check Pre-submit. Are you sure your algorithm works? Think about precision errors and hash collitions. Have you understood the problem correctly? Write brute and generator.

Runtime error

F9. Print your solution! Read your code. F9 with generator.

Time limit exceeded

What is the complexity of your algorithm? Are you copying a lot of unnecessary data? (References) Do you have any possible infinite loops? How big is the input and output? (consider scanf) Avoid vector, map. (use arrays/unordered_map)

Memory limit exceeded

Calculate memory usage with stack in recurtion.

1.2.1 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.

Data Structures (2)

dsu.hpp

25926a, 31 lines

```
struct DSU
{
  int n;
  VI p, sz;

  void init(int _n)
  {
    n = _n;
    p.resize(n);
    iota(ALL(p), 0);
    sz.assign(n, 1);
  }
  int find(int v)
  {
    if (v == p[v])
      return v;
    return p[v] = find(p[v]);
  }
  bool unite(int u, int v)
  {
    u = find(u);
    v = find(v);
    if (u == v)
      return false;
    if (sz[u] > sz[v])
      swap(u, v);
    p[u] = v;
    sz[v] += sz[u];
    return true;
  }
};
```

fenwick.hpp

9b9ab2, 45 lines

```
struct Fenwick
{
  int n;
  vector<LL> v;

  void init(int _n)
  {
    n = _n;
    v.clear();
    v.assign(n, 0);
  }

  void upd(int i, int x)
  {
    for (; i < n; i |= (i + 1))
      v[i] += x;
  }

  LL query(int i)
  {
    LL ans = 0;
    for (; i >= 0; i = (i & (i + 1)) - 1)
      ans += v[i];
    return ans;
  }

  // returns n if sum(a) < x
  int lowerBound(LL x)
  {
    LL sum = 0;
    int i = -1;
    int lg = 31 - __builtin_clz(n);
    while (lg >= 0)
    {
      int j = i + (1 << lg);
      if (j < n && sum + v[j] < x)
      {
        sum += v[j];
        i = j;
      }
      lg--;
    }
    return i + 1;
  }
};
```

fenwick.txt

20 lines

```
Minimum on segment:
1) Use two Fenwick trees with n = 2^k.
You can use if n > 1:
n = 1 << (32 - __builtin_clz(n - 1));
2) One tree for normal array and one for reversed
3) When querying for minimum on the segment
only consider segments [(i & (i + 1)), i]
from trees that are COMPLETELY inside [l, r]

Fenwick tree for adding on segment (prefixes):
1) Use 2 arrays: mult and add
2) upd(int i, int updMult, int updAdd)
default Fenwick update.
3) add x on segment [l, r]:
  upd(l, x, -x * (l - 1));
  upd(r, -x, x * r);
4) to calculate sum on prefix r:
  sumAdd and sumMult - default Fenwick sum
  st - initial value of r
  ans = st * sumMult + sumAdd
```

treap.hpp

Description: uncomment in split for explicit key or in merge for implicit priority. Minimum and reverse queries.

368d1d, 150 lines

```
mt19937 rng;

struct Node
{
  int l, r;
  int x, y;
  int cnt, par;
  int rev, mn;

  Node(int value)
  {
    l = r = -1;
    x = value;
    y = rng();
    cnt = 1;
    par = -1;
    rev = 0;
    mn = value;
  }
};

struct Treap
{
  vector<Node> a;
  void init(int n)
  {
    a.clear();
    a.reserve(n);
  }

  int getCnt(int v)
  {
    if (v == -1)
      return 0;
    return a[v].cnt;
  }
  int getMn(int v)
  {
    if (v == -1)
      return INF;
    return a[v].mn;
  }
```

```
  int newNode(int val)
  {
    a.PB(Node(val));
    return SZ(a) - 1;
  }
  void upd(int v)
  {
    if (v == -1)
      return;
    // important!
    a[v].cnt = getCnt(a[v].l) +
    getCnt(a[v].r) + 1;

    a[v].mn = min(a[v].x,
    min(getMn(a[v].l), getMn(a[v].r)));
  }
  void reverse(int v)
  {
    if (v == -1)
      return;
    a[v].rev ^= 1;
  }
  void push(int v)
  {
    if (v == -1 || a[v].rev == 0)
      return;
    reverse(a[v].l);
    reverse(a[v].r);
    swap(a[v].l, a[v].r);
    a[v].rev = 0;
  }
  PII split(int v, int cnt)
  {
    if (v == -1)
      return {-1, -1};
    push(v);
    int left = getCnt(a[v].l);
    PII res;
    // elements a[v].x == val will be in right
        part
    // if (val <= a[v].x)
    if (cnt <= left)
    {
      if (a[v].l != -1)
        a[a[v].l].par = -1;
      // res = split(a[v].l, val);
      res = split(a[v].l, cnt);
      a[v].l = res.S;
      if (res.S != -1)
        a[res.S].par = v;
      res.S = v;
    }
    else
    {
      if (a[v].r != -1)
        a[a[v].r].par = -1;
      // res = split(a[v].r, val);
      res = split(a[v].r, cnt - left - 1);
      a[v].r = res.F;
      if (res.F != -1)
        a[res.F].par = v;
      res.F = v;
    }
    upd(v);
    return res;
  }
  int merge(int v, int u)
  {
    if (v == -1) return u;
    if (u == -1) return v;
    int res;
    // if ((int)(rng() % (getCnt(v) + getCnt(u)))
         < getCnt(v))
    if (a[v].y > a[u].y)
    {
      push(v);
      if (a[v].r != -1)
        a[a[v].r].par = -1;
      res = merge(a[v].r, u);
      a[v].r = res;
      if (res != -1)
        a[res].par = v;
      res = v;
    }
    else
    {
      push(u);
      if (a[u].l != -1)
        a[a[u].l].par = -1;
      res = merge(v, a[u].l);
      a[u].l = res;
      if (res != -1)
        a[res].par = u;
      res = u;
    }
    upd(res);
    return res;
  }
  // returns index of element [0, n)
  int getIdx(int v, int from = -1)
  {
    if (v == -1)
      return 0;
    int x = getIdx(a[v].par, v);
    push(v);
    if (from == -1 || a[v].r == from)
      x += getCnt(a[v].l) + (from != -1);
    return x;
  }
};
```

ordered-set.hpp

12 lines

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
typedef tree<int, null_type, less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update>
    ordered_set;

/*
example: ordered_set s; s.insert(47);
s.order_of_key(k); - returns number of elements
    less then k
s.find_by_order(k); - returns iterator to k-th
    element or s.end()
s.count() does not exist.
*s.end() doesn't trigger runtime error. returns 0
     if compiled using f8
 */
```

sparse-table.hpp

ab1869, 38 lines

```
struct SparseTable
{
  VI t[LOG];
  VI lg;
  int n;

  void init(int _n)
  {
    n = _n;
    lg.resize(n + 1);
    FOR(i, 2, n + 1)
      lg[i] = lg[i / 2] + 1;

    FOR(j, 0, LOG)
      t[j].assign(n, INF);
  }
```

```
  void build(const VI& v)
  {
    FOR (i, 0, SZ(v)) t[0][i] = v[i];

    FOR (j, 1, LOG)
    {
      int len = 1 << (j - 1);
      FOR (i, 0, n - (1 << j) + 1)
      {
        t[j][i] = min(t[j - 1][i],
        t[j - 1][i + len]);
      }
    }
  }
  // [l, r)
  int query(int l, int r)
  {
    int i = lg[r - l];
    return min(t[i][l], t[i][r - (1 << i)]);
  }
};
```

convex-hull-trick.hpp

3f9166, 74 lines

```
struct Line
{
  LL a, b, xLast;
  Line() {}
  Line(LL _a, LL _b): a(_a), b(_b) {}
  bool operator<(const Line& l) const
  {
    return MP(a, b) < MP(l.a, l.b);
  }
  bool operator<(int x) const
  {
    return xLast < x;
  }
  __int128 getY(__int128 x) const
  {
    return a * x + b;
  }
  LL intersect(const Line& l) const
  {
    assert(a < l.a);
    LL dA = l.a - a, dB = b - l.b, x = dB / dA;
    if (dB < 0 && dB % dA != 0)
      x--;
    return x;
  }
};
struct ConvexHull: set<Line, less<>>
{
  bool needErase(iterator it, const Line& l)
  {
    LL x = it->xLast;
    if (it->getY(x) > l.getY(x))
      return false;
    if (it == begin())
      return it->a >= l.a;
    x = prev(it)->xLast + 1;
    return it->getY(x) < l.getY(x);
  }
  void add(LL a, LL b)
  {
    Line l(a, b);
    auto it = lower_bound(l);
    if (it != end())
    {
      LL x = it == begin() ? -LINF :
          prev(it)->xLast;
      if ((it == begin()
        || prev(it)->getY(x) >= l.getY(x))
        && it->getY(x + 1) >= l.getY(x + 1))
        return;
    }
    while (it != end() && needErase(it, l))
      it = erase(it);
    while (it != begin()
      && needErase(prev(it), l))
      erase(prev(it));
    if (it != begin())
    {
      auto itP = prev(it);
      Line itL = *itP;
      itL.xLast = itP->intersect(l);
      erase(itP);
      insert(itL);
    }
    l.xLast = it == end() ? LINF :
      l.intersect(*it);
    insert(l);
  }
  LL getMaxY(LL x)
  {
    return lower_bound(x)->getY(x);
  }
};
```

Graphs (3)

3.1 Decompositions

centroid.hpp

19ecf3, 51 lines

```
VI g[N];
int sz[N];
bool usedC[N];

int dfsSZ(int v, int par)
{
  sz[v] = 1;
  for (auto to : g[v])
  {
    if (to != par && !usedC[to])
      sz[v] += dfsSZ(to, v);
  }
  return sz[v];
}

void build(int u)
{
  dfsSZ(u, -1);
  int szAll = sz[u];
  int pr = u;
  while (true)
  {
    int v = -1;
    for (auto to : g[u])
    {
      if (to == pr || usedC[to])
        continue;
      if (sz[to] * 2 > szAll)
      {
        v = to;
        break;
      }
    }
    if (v == -1)
      break;
    pr = u;
    u = v;
  }
  int cent = u;
  usedC[cent] = true;

  // here calculate f(cent)

  for (auto to : g[cent])
```

```
  {
    if (!usedC[to])
    {
      build(to);
    }
  }
}
```

hld.hpp

Description: run dfsSZ(root, -1, 0) and dfsHLD(root, -1, root) to build HLD. Vertex v has index tin[v]. To update on path use process as in get(). Uses values in vertices.

40c18a, 67 lines

```
VI g[N];
int sz[N];
int h[N];
int p[N];
int top[N];
int tin[N];
int tout[N];
int t = 0;

void dfsSZ(int v, int par, int hei)
{
  sz[v] = 1;
  h[v] = hei;
  p[v] = par;
  for (auto& to : g[v])
  {
    if (to == par)
      continue;
    dfsSZ(to, v, hei + 1);
    sz[v] += sz[to];
    if (g[v][0] == par || sz[g[v][0]] < sz[to])
      swap(g[v][0], to);
  }
}
void dfsHLD(int v, int par, int tp)
{
  tin[v] = t++;
  top[v] = tp;
  FOR (i, 0, SZ(g[v]))
  {
    int to = g[v][i];
    if (to == par)
      continue;
    if (i == 0)
      dfsHLD(to, v, tp);
    else
      dfsHLD(to, v, to);
  }
  tout[v] = t - 1;
}
LL get(int u, int v)
{
  LL res = 0;
  while(true)
  {
    int tu = top[u];
    int tv = top[v];
    if (tu == tv)
    {
      int t1 = tin[u];
      int t2 = tin[v];
      if (t1 > t2)
        swap(t1, t2);
      // query [t1, t2] both inclusive
      res += query(t1, t2);
      break;
    }
    if (h[tu] < h[tv])
    {
      swap(tu, tv);
      swap(u, v);
    }
    res += query(tin[tu], tin[u]);
    u = p[tu];
  }
  return res;
}
```

biconnected-components.hpp

Description: Colors the edges so that the vertices, connected with the same color are still connected if you delete any vertex.
Time: $\mathcal{O}(m)$

b28bb5, 136 lines

```
struct Graph
{
  vector<PII> edges;
  vector<VI> g;

  VI used, par;
  VI tin, low, inComp;
  int t = 0, c = 0;
  vector<int> st;

  // components of vertices
  // a vertex can be in several components
  vector<VI> verticesCol;
  // components of edges
  vector<VI> components;
  // col[i] - component of the i-th edge
  VI col;

  int n, m;

  void init(int _n, int _m)
  {
    n = _n;
    m = _m;

    edges.assign(m, {0, 0});
    g.assign(n, {});

    used.assign(n, false);
    par.assign(n, -1);

    tin.assign(n, 0);
    low.assign(n, 0);
    inComp.assign(n, 0);

    t = c = 0;

    components.clear();
    col.assign(m, -1);
  }

  void addEdge(int a, int b, int i)
  {
    assert(0 <= a && a < n);
    assert(0 <= b && b < n);
    assert(0 <= i && i < m);

    edges[i] = MP(a, b);
    g[a].PB(i);
    g[b].PB(i);
  }

  void addComp()
  {
    unordered_set<int> s;
    s.reserve(7 * SZ(components[c]));
    for (auto e : components[c])
    {
      s.insert(edges[e].F);
      s.insert(edges[e].S);
      inComp[edges[e].F] = true;
      inComp[edges[e].S] = true;
    }
    verticesCol.PB(VI(ALL(s)));
  }
```

```
void dfs(int v, int p = -1)
{
  used[v] = 1;
  par[v] = p;
  low[v] = tin[v] = t++;
  int cnt = 0;
  for (auto e : g[v])
  {
    int to = edges[e].F;
    if (to == v)
      to = edges[e].S;

    if (p == to) continue;
    if (!used[to])
    {
      cnt++;
      st.PB(e);
      dfs(to, v);

      low[v] = min(low[v], low[to]);

      if ((par[v] == -1 && cnt > 1) ||
      (par[v] != -1 && low[to] >= tin[v]))
      {
        components.PB({});
        while (st.back() != e)
        {
          components[c].PB(st.back());
          col[st.back()] = c;

          st.pop_back();
        }
        components[c].PB(st.back());
        addComp();
        col[st.back()] = c++;

        st.pop_back();
      }
    }
    else
    {
      low[v] = min(low[v], tin[to]);
      if (tin[to] < tin[v])
        st.PB(e);
    }
  }
}
void build()
{
  FOR (i, 0, n)
  {
    if (used[i]) continue;
    dfs(i, -1);
    if (st.empty()) continue;
    components.PB({});
    while (!st.empty())
    {
      int e = st.back();
      col[e] = c;
      components[c].PB(e);
      st.pop_back();
    }
    addComp();
    c++;
  }
  FOR (i, 0, n)
    if (!inComp[i])
      verticesCol.PB(VI(1, i));
  }
};
```

3.2 Maximum matching

kuhn.hpp

Time: 0.6s for $|V| = 10^5, |E| = 2 * 10^5$

f09721, 85 lines

```
struct Graph
{
  int szL, szR;
  // edges from the left to the right, 0-indexed
  vector<VI> g;
  VI mateForR, mateForL, usedL;

  void init(int l, int r)
  {
    szL = l, szR = r;
    g.resize(szL);
    mateForL.resize(szL);
    usedL.resize(szL);

    mateForR.resize(szR);
  }

  void addEdge(int from, int to)
  {
    assert(0 <= from && from < szL);
    assert(0 <= to && to < szR);

    g[from].PB(to);
  }
  int iter;
  bool kuhn(int v)
  {
    if (usedL[v] == iter) return false;
    usedL[v] = iter;
    shuffle(ALL(g[v]), rng);
    for(int to : g[v])
    {
      if (mateForR[to] == -1)
      {
        mateForR[to] = v;
        mateForL[v] = to;
        return true;
      }
    }
    for(int to : g[v])
    {
      if (kuhn(mateForR[to]))
      {
        mateForR[to] = v;
        mateForL[v] = to;
        return true;
      }
    }
    return false;
  }
  int doKuhn()
  {
    fill(ALL(mateForR), -1);
    fill(ALL(mateForL), -1);
    fill(ALL(usedL), -1);

    int res = 0;
    iter = 0;

    while(true)
    {
      iter++;

      VI order(szL);
      iota(ALL(order), 0);
      shuffle(ALL(order), rng);

      bool ok = false;
      for(int v : order)
      {
        if (mateForL[v] == -1)
        {
          if (kuhn(v))
          {
```

```
            ok = true;
            res++;
          }
        }
      }
      if (!ok) break;
    }
    return res;
  }
};
```

edmonds-blossom.hpp

Description: Finds the maximum matching in a graph Time complexity: $O(n^2m)$

490491, 133 lines

```
struct Graph
{
  int n;
  vector<VI> g;
  VI label, first, mate;

  void init(int _n)
  {
    n = _n;
    g.clear();
    g.resize(n + 1);
    label.resize(n + 1);
    first.resize(n + 1);
    mate.resize(n + 1, 0);
  }
  void addEdge(int u, int v)
  {
    assert(0 <= u && u < n);
    assert(0 <= v && v < n);
    u++;
    v++;
    g[u].PB(v);
    g[v].PB(u);
  }
  void augmentPath(int v, int w)
  {
    int t = mate[v];
    mate[v] = w;
    if (mate[t] != v)
      return;
    if (label[v] <= n)
    {
      mate[t] = label[v];
      augmentPath(label[v], t);
      return;
    }
    int x = label[v] / (n + 1);
    int y = label[v] % (n + 1);
    augmentPath(x, y);
    augmentPath(y, x);
  }
  int findMaxMatching()
  {
    FOR(i, 0, n + 1)
      assert(mate[i] == 0);
    int mt = 0;
    DSU dsu;
    FOR(u, 1, n + 1)
    {
      if (mate[u] != 0)
        continue;
      fill(ALL(label), -1);
      iota(ALL(first), 0);
      dsu.init(n + 1);
      label[u] = 0;
      dsu.unite(u, 0);
      queue<int> q;
      q.push(u);
      while (!q.empty())
      {
        int x = q.front();
        q.pop();
        for (int y: g[x])
        {
          if (mate[y] == 0 && y != u)
          {
            mate[y] = x;
            augmentPath(x, y);
            while (!q.empty())
              q.pop();
            mt++;
            break;
          }
          if (label[y] < 0)
          {
            int v = mate[y];
            if (label[v] < 0)
            {
              label[v] = x;
              dsu.unite(v, y);
              q.push(v);
            }
          }
          else
          {
            int r = first[dsu.find(x)],
              s = first[dsu.find(y)];
            if (r == s)
              continue;
            int edgeLabel = (n + 1) * x + y;
            label[r] = label[s] = -edgeLabel;
            int join;
            while (true)
            {
              if (s != 0)
                swap(r, s);
              r = first[dsu.find(label[mate[r]])
                ];
              if (label[r] == -edgeLabel)
              {
                join = r;
                break;
              }
              label[r] = -edgeLabel;
            }
            for (int z: {x, y})
            {
              for (int v = first[dsu.find(z)];
                v != join;
                v = first[dsu.find(
                  label[mate[v]])])
              {
                label[v] = edgeLabel;
                if (dsu.unite(v, join))
                  first[dsu.find(join)] = join;
                q.push(v);
              }
            }
          }
        }
      }
    }
    return mt;
  }
  int getMate(int v)
  {
    assert(0 <= v && v < n);
    v++;
    int u = mate[v];
    assert(u == 0 || mate[u] == v);
    u--;
    return u;
  }
};
```

3.3 Flows

dinic.hpp

86349e, 97 lines

```cpp
struct Graph
{
  struct Edge
  {
    int from, to;
    LL cap, flow;
  };

  int n;
  vector<Edge> edges;
  vector<VI> g;
  VI d, p;

  void init(int _n)
  {
    n = _n;
    edges.clear();
    g.clear();
    g.resize(n);
    d.resize(n);
    p.resize(n);
  }
  void addEdge(int from, int to, LL cap)
  {
    assert(0 <= from && from < n);
    assert(0 <= to && to < n);
    assert(0 <= cap);
    g[from].PB(SZ(edges));
    edges.PB({from, to, cap, 0});
    g[to].PB(SZ(edges));
    edges.PB({to, from, 0, 0});
  }
  int bfs(int s, int t)
  {
    fill(ALL(d), -1);
    d[s] = 0;
    queue<int> q;
    q.push(s);
    while (!q.empty())
    {
      int v = q.front();
      q.pop();
      for (int e : g[v])
      {
        int to = edges[e].to;
        if (edges[e].flow < edges[e].cap
          && d[to] == -1)
        {
          d[to] = d[v] + 1;
          q.push(to);
        }
      }
    }
    return d[t];
  }
  LL dfs(int v, int t, LL flow)
  {
    if (v == t || flow == 0)
      return flow;
    for (; p[v] < SZ(g[v]); p[v]++)
    {
      int e = g[v][p[v]], to = edges[e].to;
      LL c = edges[e].cap, f = edges[e].flow;
      if (f < c
        && (to == t || d[to] == d[v] + 1))
      {
        LL push = dfs(to, t, min(flow, c - f));
        if (push > 0)
        {
          edges[e].flow += push;
          edges[e ^ 1].flow -= push;
          return push;
        }
      }
    }
    return 0;
  }
  LL flow(int s, int t)
  {
    assert(0 <= s && s < n);
    assert(0 <= t && t < n);
    assert(s != t);
    LL flow = 0;
    while (bfs(s, t) != -1)
    {
      fill(ALL(p), 0);
      while (true)
      {
        LL f = dfs(s, t, LINF);
        if (f == 0)
          break;
        flow += f;
      }
    }
    return flow;
  }
};
```

min-cost-flow.hpp

f26cf1, 110 lines

```cpp
struct Graph
{
  struct Edge
  {
    int from, to;
    int cap, flow;
    LL cost;
  };

  int n;
  vector<Edge> edges;
  vector<VI> g;
  vector<LL> d;
  VI p, w;

  void init(int _n)
  {
    n = _n;
    edges.clear();
    g.clear();
    g.resize(n);
    d.resize(n);
    p.resize(n);
    w.resize(n);
  }
  void addEdge(int from, int to,
    int cap, LL cost)
  {
    assert(0 <= from && from < n);
    assert(0 <= to && to < n);
    assert(0 <= cap);
    assert(0 <= cost);
    g[from].PB(SZ(edges));
    edges.PB({from, to, cap, 0, cost});
    g[to].PB(SZ(edges));
    edges.PB({to, from, 0, 0, -cost});
  }
  pair<int, LL> flow(int s, int t)
  {
    assert(0 <= s && s < n);
    assert(0 <= t && t < n);
    assert(s != t);
    int flow = 0;
    LL cost = 0;
    while (true)
    {
      fill(ALL(d), LINF);
      fill(ALL(p), -1);
      fill(ALL(w), 0);
```

```
    queue<int> q1, q2;
    w[s] = 1;
    d[s] = 0;
    q2.push(s);
    while (!q1.empty() || !q2.empty())
    {
      int v;
      if (!q1.empty())
      {
        v = q1.front();
        q1.pop();
      }
      else
      {
        v = q2.front();
        q2.pop();
      }
      for (int e : g[v])
      {
        if (edges[e].flow == edges[e].cap)
          continue;
        int to = edges[e].to;
        LL newDist = d[v] + edges[e].cost;
        if (newDist < d[to])
        {
          d[to] = newDist;
          p[to] = e;
          if (w[to] == 0)
            q2.push(to);
          else if (w[to] == 2)
            q1.push(to);
          w[to] = 1;
        }
      }
      w[v] = 2;
    }
    if (p[t] == -1)
      break;
    int curFlow = INF;
    LL curCost = 0;
    for (int v = t; v != s;)
    {
      int e = p[v];
      curFlow = min(curFlow,
      edges[e].cap - edges[e].flow);
      curCost += edges[e].cost;
      v = edges[e].from;
    }
    for (int v = t; v != s;)
    {
      int e = p[v];
      edges[e].flow += curFlow;
      edges[e ^ 1].flow -= curFlow;
      v = edges[e].from;
    }
    flow += curFlow;
    cost += curCost * curFlow;
  }
  return {flow, cost};
}
```

3.3.1 Recover

Min cut To find the min-cut use search from vertex S on not saturated edges. Original edges from used vertices to unused is in min-cut.

Min vertex cover A min vertex cover can be found only in bipartite graphs. The minimum number of the vertex to cover all **edges** is equal to the size of matching. To restore min vertex cover, make a directed graph.

- matched edges direct from R to L
- unmatched edges direct from L to R

From unmathced vertices in left part start traversal. Cover have vertices from matching:

- unvisited vertices in L
- visited vertices in R

Max independent set A max independent set can be found only in bipartite graphs. It is the complement of the min vertex cover.

Min edge cover A min edge cover can be found only in bipartite graphs. Minimum edges to cover all vertices are possible to find only in graphs without isolated vertices. Using one edges in the matching we cover two vertices, and any other vertices we cover using one edge for each.

DAG pathes In DAG you can find a minimum number of non-intersecting paths that cover all vertices. Duplicate vertices and make a bipartite graph with edges $u_L \to v_R$. Edges in the matching are edges in paths.

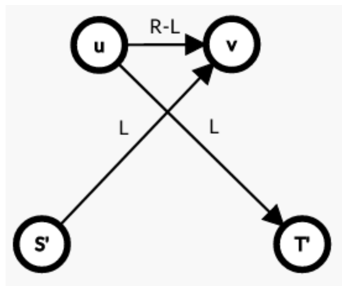
Flow with lower bound

https://atcoder.jp/contests/abc285/editorial/5535

On the resulting graph, accumulate maximum flow in the following order:

- from S' to T'
- from S' to T
- from S to T'
- from S to T.

An $S - T$ flow that satisfies the minimum capacities exists if and only if, for all outgoing edges from S' and incoming edges to T', the flow and capacity are equal.



Binary optimization

$$\sum_i a_i x_i + \sum_i b_i \overline{x_i} + \sum_{i,j} c_{ij} x_i \overline{x_j} \to \min$$

If $a_i \leq b_i$, add an edge from S to i of capacity $b_i - a_i$ and add a_i to the answer.

Otherwise, add an edge from i to T of capacity $a_i - b_i$ and add b_i to the answer.

Add an edge from i to j of capacity c_{ij}.

Add the $S - T$ minimum cut to the answer.

3.4 Dominator tree

dominator-tree.hpp

Description: works for cyclic graphs. *par* - parent in dfs. *p* - parent in dsu. *val* - vertex with min sdom in dsu. *dom* - immidiate dominator. *sdom* - semidominator, min vertex with alternate path. *bkt* - vertices with this sdom. $dom[root] = -1$. $dom[v] = -1$ if v is unreachable.

Time: $\mathcal{O}(n)$

c9472a, 117 lines

```
struct Graph
{
  int n;
  vector<VI> g, gr, bkt;
  VI par, used, p, val, sdom, dom, tin;
  int T;
  VI ord;

  void init(int _n)
  {
    n = _n;
    g.resize(n);
    gr.resize(n);
    bkt.resize(n);
    par.resize(n);
    used.resize(n);
    p.resize(n);
    val.resize(n);
    sdom.resize(n);
    dom.resize(n);
    tin.resize(n);
  }

  void addEdge(int u, int v)
  {
    g[u].PB(v);
    gr[v].PB(u);
  }

  int find(int v)
  {
    if (p[v] == v)
      return v;
    int y = find(p[v]);
    if (p[y] == y)
      return v;
    if (tin[sdom[val[p[v]]]] < tin[sdom[val[v]]])
      val[v] = val[p[v]];
    p[v] = y;
    return y;
  }
  int get(int v)
  {
    find(v);
    // return vertex with min sdom
    return val[v];
  }

  void dfs(int v, int pr)
  {
    tin[v] = T++;
    used[v] = true;
    ord.PB(v);
    par[v] = pr;
    for (auto to : g[v])
    {
      if (!used[to])
        dfs(to, v);
    }
  }

  void build(int s)
  {
    FOR (i, 0, n)
    {
      used[i] = false;
      sdom[i] = i;
      dom[i] = -1;
      p[i] = i;
      val[i] = i;
      bkt[i].clear();
    }
    ord.clear();
    T = 0;

    dfs(s, -1);

    RFOR(i, SZ(ord), 0)
    {
      int v = ord[i];
      for (auto from : gr[v])
      {
        // don't consider unreachable vertices
        if (!used[from])
          continue;
        // find min sdom
        if (tin[sdom[v]] > tin[sdom[get(from)]])
        {
          sdom[v] = sdom[get(from)];
        }
      }
      if (v != s)
        bkt[sdom[v]].PB(v);
      for (auto y : bkt[v])
      {
        int u = get(y);
        // if sdoms equals then this is dom
        // else we will find it later
        if (sdom[y] == sdom[u])
          dom[y] = sdom[y];
        else dom[y] = u;
      }
      // add vertex to dsu
      if (par[v] != -1)
        p[v] = par[v];
    }

    for (auto v : ord)
    {
      if (v == s || dom[v] == -1)
        continue;
      if (dom[v] != sdom[v]) dom[v] = dom[dom[v
        ]];
    }
  }

};
```

3.5 Sqrt problems

3-cycles.hpp

Description: finds all triangles in a graph. *cnt++* respond to the triangle v, u, w.

Time: $\mathcal{O}(m \cdot \sqrt{m})$

61be84, 42 lines

```
int triangles(int n, vector<PII> edges)
{
  vector<VI> g(n);
  int m = SZ(edges);
  VI deg(n, 0);
  FOR(i, 0, m)
  {
    auto [u, v] = edges[i];
    assert(0 <= u && u < n);
    assert(0 <= v && v < n);
    deg[u]++;
    deg[v]++;
  }
```

```
  FOR (i, 0, m)
  {
    auto [u, v] = edges[i];
    if (MP(deg[u], u) < MP(deg[v], v))
      g[u].PB(v);
    else
      g[v].PB(u);
  }
  int cnt = 0;
  VI used(n, 0);
  FOR (v, 0, n)
  {
    for (auto u : g[v])
      used[u] = 1;
    for (auto u : g[v])
    {
      for(auto w : g[u])
      {
        if (used[w])
        {
          cnt++;
        }
      }
    }
    for (auto u : g[v])
      used[u] = 0;
  }
  return cnt;
}
```

4-cycles.hpp

Description: sort d and add break to speed up. With breaks works 0.5s for m = $5 \cdot 10^5$.
Time: $\sum_{uv \in E} min(d_u, d_v) = \mathcal{O}\left(m \cdot \sqrt{m}\right)$

e2c43b, 20 lines

```
LL rect(int n)
{
  LL cnt4 = 0;
  vector<PII> d(n);
  FOR (v, 0, n) d[v] = MP(SZ(g[v]), v);
  VI L(n);
  FOR (v, 0, n)
  {
    for (auto u : g[v])
      if (d[u] < d[v])
        for (auto y : g[u])
          if (d[y] < d[v])
            cnt4 += L[y], L[y]++;
    for (auto u : g[v])
      if (d[u] < d[v])
        for (auto y : g[u])
          L[y] = 0;
  }
  return cnt4;
}
```

Strings (4)

aho-corasick.hpp

c51141, 67 lines

```
const int AL = 26;

struct Node
{
  int p;
  int c;
  int g[AL];
  int nxt[AL];
  int link;

  Node(int _c, int _p)
  {
    c = _c;
    p = _p;
    fill(g, g + AL, -1);
    fill(nxt, nxt + AL, -1);
    link = -1;
  }
};

struct AC
{
  vector<Node> a;
  void init(int n)
  {
    a.reserve(n);
    a.PB(Node(-1, -1));
  }
  int addStr(const string& s)
  {
    int v = 0;
    FOR (i, 0, SZ(s))
    {
      // change to [0 AL)
      int c = s[i] - 'a';
      if (a[v].nxt[c] == -1)
      {
        a[v].nxt[c] = SZ(a);
        a.PB(Node(c, v));
      }
      v = a[v].nxt[c];
    }
    return v;
  }
  int go(int v, int c)
  {
    if (a[v].g[c] != -1)
      return a[v].g[c];

    if (a[v].nxt[c] != -1)
      a[v].g[c] = a[v].nxt[c];
    else if (v != 0)
      a[v].g[c] = go(getLink(v), c);
    else
      a[v].g[c] = 0;

    return a[v].g[c];
  }
  int getLink(int v)
  {
    if (a[v].link != -1)
      return a[v].link;
    if (v == 0 || a[v].p == 0)
      return 0;
    return a[v].link=go(getLink(a[v].p), a[v].c);
  }
};
```

automaton.hpp

8c531f, 65 lines

```
const int AL = 26;

struct Node
{
  int g[AL];
  int link;
  int len;
  int cnt;
  Node()
  {
    fill(g, g + AL, -1);
    link = -1;
    len = -1;
    cnt = 1;
  }
};

struct Automaton
{
  vector<Node> a;
  int head;
  void init(int n)
  {
    a.reserve(2 * n);
    a.PB(Node());
    head = 0;
  }
  void add(char c)
  {
    // change to [0 AL)
    int ch = c - 'a';
    int nhead = SZ(a);
    a.PB(Node());
    a[nhead].len = a[head].len + 1;
    int cur = head;
    head = nhead;
    while (cur != -1 && a[cur].g[ch] == -1)
    {
      a[cur].g[ch] = head;
      cur = a[cur].link;
    }
    if (cur == -1)
    {
      a[head].link = 0;
      return;
    }
    int p = a[cur].g[ch];
    if (a[p].len == a[cur].len + 1)
    {
      a[head].link = p;
      return;
    }
    int q = SZ(a);
    a.PB(Node());
    a[q] = a[p];
    a[q].cnt = 0;
    a[q].len = a[cur].len + 1;
    a[p].link = a[head].link = q;
    while (cur != -1 && a[cur].g[ch] == p)
    {
      a[cur].g[ch] = q;
      cur = a[cur].link;
    }
  }
};
```

suffix-array.hpp

ed9bcc, 61 lines

```
void countSort(VI& p, const VI& c)
{
  int n = SZ(p);
  VI cnt(n);
  FOR (i, 0, n)
    cnt[c[i]]++;
  VI pos(n);
  FOR (i, 1, n)
    pos[i] = pos[i - 1] + cnt[i - 1];
  VI p2(n);
```

```
  for (auto x : p)
  {
    int i = c[x];
    p2[pos[i]++] = x;
  }
  p = p2;
}

VI suffixArray(const string& t)
{
  // add symbol smaller than all s[i]
  string s = t + "$";
  int n = SZ(s);
  VI p(n), c(n);
  iota(ALL(p), 0);
  sort(ALL(p), [&](int i, int j)
  {
    return s[i] < s[j];
  });
  int x = 0;
  c[p[0]] = 0;
  FOR (i, 1, n)
  {
    if (s[p[i]] != s[p[i - 1]])
      x++;
    c[p[i]] = x;
  }
  int k = 0;
  while ((1 << k) < n)
  {
    FOR (i, 0, n)
      p[i] = (p[i] - (1 << k) + n) % n;

    countSort(p, c);

    VI c2(n);
    PII pr = {c[p[0]], c[(p[0] + (1 << k)) % n]};
    FOR (i, 1, n)
    {
      PII nx={c[p[i]], c[(p[i] + (1 << k)) % n]};
      c2[p[i]] = c2[p[i - 1]];
      if (pr != nx)
        c2[p[i]]++;
      pr = nx;
    }
    c = c2;
    k++;
  }
  p.erase(p.begin());
  return p;
}
```

lcp.hpp

72ff1e, 24 lines

```
VI lcpArray(const string& s, const VI& sa)
{
  int n = SZ(s);
  VI rnk(n);
  FOR (i, 0, n)
    rnk[sa[i]] = i;
  VI lcp(n - 1);
  int h = 0;
  FOR (i, 0, n)
  {
    if (h > 0)
      h--;
    if (rnk[i] == 0)
      continue;
    int j = sa[rnk[i] - 1];
    for (; j + h < n && i + h < n; h++)
    {
      if (s[j + h] != s[i + h])
        break;
    }
    lcp[rnk[i] - 1] = h;
  }
  return lcp;
}
```

z.hpp

e27ac7, 23 lines

```
VI zFunction(const string& s)
{
  int n = SZ(s);
  VI z(n);

  int l = 0;
  int r = 0;
  FOR (i, 1, n)
  {
    z[i] = 0;
    if (i <= r)
      z[i] = min(r - i + 1, z[i - l]);

    while(i + z[i] < n && s[i + z[i]] == s[z[i]])
      z[i]++;
    if(i + z[i] - 1 > r)
    {
      r = i + z[i] - 1;
      l = i;
    }
  }
  return z;
}
```

prefix.hpp

500608, 16 lines

```
VI prefixFunction(const string& s)
{
  int n = SZ(s);
  VI p(n);
  p[0] = 0;
  FOR (i, 1, n)
  {
    int j = p[i - 1];
    while(j != 0 && s[i] != s[j])
      j = p[j - 1];

    if (s[i] == s[j]) j++;
    p[i] = j;
  }
  return p;
}
```

manacher.hpp

Description: $d1_i$ – half-length of odd length palindrome with center in i. $d2_i$ – half-length of even length palindrome if i is right center of it. half-length = (length + 1) / 2

6ef9bb, 39 lines

```
pair<VI, VI> manacher(const string& s)
{
    int n = SZ(s);
  VI d1(n), d2(n);
    int l = -1;
    int r = -1;
    FOR (i, 0, n)
    {
        if (i <= r)
            d1[i] = min(r - i + 1,
              d1[l + (r - i)]);
        while (i + d1[i] < n && i - d1[i] >= 0
          && s[i + d1[i]] == s[i - d1[i]])
            d1[i]++;
        if (i + d1[i] - 1 > r)
        {
            r = i + d1[i] - 1;
            l = i - (d1[i] - 1);
        }
    }
    l = -1;
    r = -1;
    FOR (i, 0, n)
```

```
    {
        if (i <= r)
            d2[i] = min(r - i + 1,
              d2[l + (r - i) + 1]);
        while (i + d2[i] < n
          && i - (d2[i] + 1) >= 0
          && s[i + d2[i]] == s[i - (d2[i] + 1)])
            d2[i]++;
        if (i + d2[i] > r)
        {
            r = i + d2[i] - 1;
            l = i - d2[i];
        }
    }
    return {d1, d2};
}
```

palindromic-tree.hpp

2e0b47, 62 lines

```
const int AL = 26;

struct Node
{
  int to[AL];
  int link;
  int len;
  Node(int _link, int _len)
  {
    fill(to, to + AL, -1);
    link = _link;
    len = _len;
  }
};
struct PalTree
{
  string s;
  vector<Node> a;
  int last;

  void init(string t)
  {
    a.clear();
    a.reserve(2 * SZ(t));
    a.PB(Node(-1, -1));
    a.PB(Node(0, 0));
    last = 1;
    s = t;
  }
  void add(int idx)
  {
    // change t o [ 0 AL)
    int ch = s[idx] - 'a';

    int cur = last;
    while (cur != -1)
    {
      int pos = idx - a[cur].len - 1;
      if (pos >= 0 && s[pos] == s[idx])
        break;
      cur = a[cur].link;
    }
    if (a[cur].to[ch] == -1)
    {
      a[cur].to[ch] = SZ(a);
      int link = a[cur].link;
      while (link != -1)
      {
        int pos = idx - a[link].len - 1;
        if (pos >= 0 && s[pos] == s[idx])
          break;
        link = a[link].link;
      }
      if (link == -1)
        link = 1;
      else
        link = a[link].to[ch];
      a.PB(Node(link, a[cur].len + 2));
    }
    last = a[cur].to[ch];
  }
};
```

Geometry (5)

In general, try to build programs that are resistant to the oddities of floating-point numbers. Imagine that some evil demon is slightly modifying every result you compute in the way that is most likely to make your program fail. And try to write clean code that is clearly correct at first glance. If you need long explanations to justify why your program will not fail, then it is more likely that your program will in fact fail.

Victor Lecomte, Handbook of geometry for competitive programmers

point.hpp

009dbe, 112 lines

```
struct Pt
{
  db x, y;
  Pt operator+(const Pt& p) const
  {
    return {x + p.x, y + p.y};
  }
  Pt operator-(const Pt& p) const
  {
    return {x - p.x, y - p.y};
  }
  Pt operator*(db d) const
  {
    return {x * d, y * d};
  }
  Pt operator/(db d) const
  {
    return {x / d, y / d};
  }
};
// Returns the squared absolute value
db sq(const Pt& p)
{
  return p.x * p.x + p.y * p.y;
}
// Returns the absolute value
db abs(const Pt& p)
{
  return sqrt(sq(p));
}
// Returns -1 for negative numbers, 0 for zero,
// and 1 for positive numbers
int sgn(db x)
{
  return (EPS < x) - (x < -EPS);
}
// Returns 'p' rotated counter-clockwise by 'a'
Pt rot(const Pt& p, db a)
{
  db co = cos(a), si = sin(a);
  return {p.x * co - p.y * si,
    p.x * si + p.y * co};
}
// Returns 'p' rotated counter-clockwise by 90
Pt perp(const Pt& p)
{
  return {-p.y, p.x};
}
// Returns the dot product of 'p' and 'q'
db dot(const Pt& p, const Pt& q)
{
  return p.x * q.x + p.y * q.y;
}
// Returns the angle between 'p' and 'q'
db angle(const Pt& p, const Pt& q)
{
  return acos(clamp(dot(p, q) / abs(p) /
    abs(q), (db)-1.0, (db)1.0));
}
// Returns the cross product of 'p' and 'q'
db cross(const Pt& p, const Pt& q)
{
  return p.x * q.y - p.y * q.x;
}
// Positive if R is on the left side of PQ,
// negative on the right side,
// and zero if R is on the line containing PQ
db orient(const Pt& p, const Pt& q, const Pt& r)
{
  return cross(q - p, r - p) / abs(q - p);
}
// Checks if a polygon 'v' is convex
bool isConvex(const vector<Pt>& v)
{
  bool hasPos = false, hasNeg = false;
  int n = SZ(v);
  FOR(i, 0, n)
  {
    int o = sgn(orient(v[i], v[(i + 1) % n],
      v[(i + 2) % n]));
    hasPos |= o > 0;
    hasNeg |= o < 0;
  }
  return !(hasPos && hasNeg);
}
// Checks if argument of 'p' is in [-pi, 0)
bool half(const Pt& p)
{
  assert(sgn(p.x) != 0 || sgn(p.y) != 0);
  return sgn(p.y) == -1 ||
    (sgn(p.y) == 0 && sgn(p.x) == -1);
}
// Polar sort of vectors in 'v' around 'o'
void polarSortAround(const Pt& o, vector<Pt>& v)
{
  sort(ALL(v), [o](const Pt& p, const Pt& q)
  {
    bool hp = half(p - o), hq = half(q - o);
    if (hp != hq)
      return hp < hq;
    int s = sgn(cross(p, q));
    if (s != 0)
      return s == 1;
    return sq(p - o) < sq(q - o);
  });
}
// Example:
// cout << a + b << " " << a - b << "\n";
ostream& operator<<(ostream& os, const Pt& p)
{
  return os << "(" << p.x << "," << p.y << ")";
}
```

line.hpp

b6de60, 58 lines

```
struct Line
{
  // Equation of the line is dot(n, p) + c = 0
  Pt n;
  db c;
  Line (const Pt& _n, db _c): n(_n), c(_c) {}
  // The line containing two points 'p' and 'q'
  Line(const Pt& p, const Pt& q):
    n(perp(q - p)), c(-dot(n, p)) {}
  // The "positive side": dot(n, p) + c > 0
  // The "negative side": dot(n, p) + c < 0
  db side(const Pt& p) const
  {
    return dot(n, p) + c;
  }
  // Returns the distance from 'p'
  db dist(const Pt& p) const
  {
    return abs(side(p)) / abs(n);
```

```
  }
  // Returns the squared distance from ‘p‘
  db sqDist(const Pt& p) const
  {
    return side(p) * side(p) / (db)sq(n);
  }
  // Returns the perpendicular line through ‘p‘
  Line perpThrough(const Pt& p) const
  {
    return {p, p + n};
  }
  // Compares ‘p‘ and ‘q‘ by their projection
  bool cmpProj(const Pt& p, const Pt& q) const
  {
    return sgn(cross(p, n) - cross(q, n)) < 0;
  }
  // Returns the orthogonal projection of ‘p‘
  Pt proj(const Pt& p) const
  {
    return p - n * side(p) / sq(n);
  }
  // Returns the reflection of ‘p‘ by the line
  Pt refl(const Pt& p) const
  {
    return p - n * 2 * side(p) / sq(n);
  }
};
// Checks if ‘l1‘ and ‘l2‘ are parallel
bool parallel(const Line& l1, const Line& l2)
{
  return sgn(cross(l1.n, l2.n)) == 0;
}
// Returns the intersection point
Pt inter(const Line& l1, const Line& l2)
{
  db d = cross(l1.n, l2.n);
  assert(sgn(d) != 0);
  return perp(l2.n * l1.c - l1.n * l2.c) / d;
}
```

segment.hpp

687634, 44 lines

```
// Checks if ‘p‘ is in the disk of diameter [ab]
bool inDisk(const Pt& a, const Pt& b,
  const Pt& p)
{
  return sgn(dot(a - p, b - p)) <= 0;
}
// Checks if ‘p‘ lies on segment [ab]
bool onSegment(const Pt& a, const Pt& b,
  const Pt& p)
{
  return sgn(orient(a, b, p)) == 0
    && inDisk(a, b, p);
}
// Checks if the segments [ab] and [cd] intersect
// properly (their intersection is one point
// which is not an endpoint of either segment)
bool properInter(const Pt& a, const Pt& b,
  const Pt& c, const Pt& d)
{
  db oa = orient(c, d, a);
  db ob = orient(c, d, b);
  db oc = orient(a, b, c);
  db od = orient(a, b, d);
  return sgn(oa) * sgn(ob) == -1
    && sgn(oc) * sgn(od) == -1;
}
// Returns the distance between [ab] and ‘p‘
db segPt(const Pt& a, const Pt& b, const Pt& p)
{
  Line l(a, b);
  assert(sgn(sq(l.n)) != 0);
  if (l.cmpProj(a, p) && l.cmpProj(p, b))
    return l.dist(p);
  return min(abs(p - a), abs(p - b));
}
// Returns the distance between [ab] and [cd]
db segSeg(const Pt& a, const Pt& b, const Pt& c,
  const Pt& d)
{
  if (properInter(a, b, c, d))
    return 0;
  return min({segPt(a, b, c), segPt(a, b, d),
      segPt(c, d, a), segPt(c, d, b)});
}
```

polygon.hpp

18c5da, 237 lines

```
// Returns the area of triangle abc
db areaTriangle(const Pt& a, const Pt& b,
  const Pt& c)
{
  return abs(cross(b - a, c - a)) / 2.0;
}
// Returns the area of polygon ‘v‘
db areaPolygon(const vector<Pt>& v)
{
  db area = 0.0;
  int n = SZ(v);
  FOR(i, 0, n)
    area += cross(v[i], v[(i + 1) % n]);
  return abs(area) / 2.0;
}
// Checks if point ‘a‘ is inside the convex
// polygon ‘v‘. Returns true if on the boundary.
// ‘v‘ must not contain duplicated vertices
bool inConvexPolygon(const vector<Pt>& v,
  const Pt& a)
{
  if (sgn(orient(v.back(), v[0], a)) < 0
    || sgn(orient(v[0], v[1], a)) < 0)
    return false;
  int i = lower_bound(v.begin() + 2, v.end(),
    a, [&](const Pt& p, const Pt& q)
  {
    return sgn(orient(v[0], p, q)) > 0;
  }) - v.begin();
  return sgn(orient(v[i - 1], v[i], a)) >= 0;
}
// Returns true if ‘p‘ is at least as high as ‘a‘
bool above(const Pt& a, const Pt& p)
{
  return sgn(p.y - a.y) >= 0;
}
// Checks if [pq] crosses the ray from ‘a‘
bool crossesRay(const Pt& a, const Pt& p,
  const Pt& q)
{
  return sgn((above(a, q) - above(a, p))
    * orient(a, p, q)) == 1;
}
// Checks if point ‘a‘ is inside the polygon
// If ‘strict‘, false when ‘a‘ is on the boundary
bool inPolygon(const vector<Pt>& v, const Pt& a,
  bool strict = true)
{
  int numCrossings = 0;
  int n = SZ(v);
  FOR(i, 0, n)
  {
    if (onSegment(v[i], v[(i + 1) % n], a))
      return !strict;
    numCrossings +=
      crossesRay(a, v[i], v[(i + 1) % n]);
  }
  return numCrossings & 1;
}
// Returns the counter-clockwise convex hull
vector<Pt> convexHull(vector<Pt> v)
{
  if (SZ(v) <= 1)
```

```
    return v;
  sort(ALL(v), [](const Pt& p, const Pt& q)
  {
    int dx = sgn(p.x - q.x);
    if (dx != 0)
      return dx < 0;
    return sgn(p.y - q.y) < 0;
  });
  vector<Pt> lower, upper;
  for (const Pt& p : v)
  {
    while (SZ(lower) > 1
      && sgn(orient(lower[SZ(lower) - 2],
      lower.back(), p)) <= 0)
      lower.pop_back();
    while (SZ(upper) > 1
      && sgn(orient(upper[SZ(upper) - 2],
      upper.back(), p)) >= 0)
      upper.pop_back();
    lower.PB(p);
    upper.PB(p);
  }
  reverse(ALL(upper));
  lower.insert(lower.end(), upper.begin() + 1,
    prev(upper.end()));
  return lower;
}
// Returns the indices of tangent points
PII tangetsToConvexPolygon(const vector<Pt>& v,
  const Pt& p)
{
  int n = SZ(v), i = 0;
  while (sgn(orient(p, v[i], v[(i + 1) % n]))
    * sgn(orient(p, v[i],
    v[(i + n - 1) % n])) > 0)
    i++;
  int s1 = 1, s2 = -1;
  if (sgn(orient(p, v[i], v[(i + 1) % n]))
    == s1 || sgn(orient(p, v[i],
    v[(i + n - 1) % n])) == s2)
    swap(s1, s2);
  PII res;
  int l = i, r = i + n - 1;
  while (r - l > 1)
  {
    int m = (l + r) / 2;
    if (sgn(orient(p, v[i], v[m % n])) != s1
      && sgn(orient(p, v[m % n],
      v[(m + 1) % n])) != s1)
      l = m;
    else
      r = m;
  }
  res.F = r % n;
  l = i;
  r = i + n - 1;
  while (r - l > 1)
  {
    int m = (l + r) / 2;
    if (sgn(orient(p, v[i], v[m % n])) == s2
      || sgn(orient(p, v[m % n],
      v[(m + 1) % n])) != s2)
      l = m;
    else
      r = m;
  }
  res.S = r % n;
  return res;
}
// Returns the Minkowski sum of two convex
// polygons
vector<Pt> minkowskiSum(const vector<Pt>& v1,
  const vector<Pt>& v2)
{
  auto comp = [](const Pt& p, const Pt& q)
  {
    return sgn(p.x - q.x) < 0
      || (sgn(p.x - q.x) == 0
      && sgn(p.y - q.y) < 0);
  };
  int i1 = min_element(ALL(v1), comp)
    - v1.begin();
  int i2 = min_element(ALL(v2), comp)
    - v2.begin();
  vector<Pt> res;
  int n1 = SZ(v1), n2 = SZ(v2),
    j1 = 0, j2 = 0;
  while (j1 < n1 || j2 < n2)
  {
    const Pt& p1 = v1[(i1 + j1) % n1];
    const Pt& q1 = v1[(i1 + j1 + 1) % n1];
    const Pt& p2 = v2[(i2 + j2) % n2];
    const Pt& q2 = v2[(i2 + j2 + 1) % n2];
    if (SZ(res) >= 2 && onSegment(
      res[SZ(res) - 2], p1 + p2,
      res.back()))
      res.pop_back();
    res.PB(p1 + p2);
    int s = sgn(cross(q1 - p1, q2 - p2));
    if (j1 < n1 && (j2 == n2 || s > 0
      || (s == 0 && (SZ(res) < 2
      || sgn(dot(res.back()
      - res[SZ(res) - 2],
      q1 + p2 - res.back())) > 0))))
      j1++;
    else
      j2++;
  }
  if (SZ(res) > 2
    && onSegment(res[SZ(res) - 2], res[0],
    res.back()))
    res.pop_back();
  return res;
}
// Returns the counter-clockwise ordered vertices
// of the half-plane intersection. Returns empty
// if the intersection is empty. Adds a bounding
// box to ensure a finite area
vector<Pt> hplaneInter(vector<Line> lines)
{
  const db C = 1e9;
  lines.PB({{-C, C}, {-C, -C}});
  lines.PB({{-C, -C}, {C, -C}});
  lines.PB({{C, -C}, {C, C}});
  lines.PB({{C, C}, {-C, C}});
  sort(ALL(lines), []
    (const Line& l1, const Line& l2)
  {
    bool h1 = half(l1.n), h2 = half(l2.n);
    if (h1 != h2)
      return h1 < h2;
    int p = sgn(cross(l1.n, l2.n));
    if (p != 0)
      return p > 0;
    return sgn(l1.c / abs(l1.n)
      - l2.c / abs(l2.n)) < 0;
  });
  lines.erase(unique(ALL(lines), parallel),
    lines.end());
  deque<pair<Line, Pt>> d;
  for (const Line& l : lines)
  {
    while (SZ(d) > 1 && sgn(l.side(
      (d.end() - 1)->S)) < 0)
      d.pop_back();
    while (SZ(d) > 1 && sgn(l.side(
      (d.begin() + 1)->S)) < 0)
      d.pop_front();
    if (!d.empty() && sgn(cross(
      d.back().F.n, l.n)) <= 0)
```

```
      return {};
    if (SZ(d) < 2 || sgn(d.front().F.side(
      inter(l, d.back().F))) >= 0)
    {
      Pt p;
      if (!d.empty())
      {
        p = inter(l, d.back().F);
        if (!parallel(l, d.front().F))
          d.front().S = inter(l,
            d.front().F);
      }
      d.PB({l, p});
    }
  }
  vector<Pt> res;
  for (auto [l, p] : d)
  {
    if (res.empty()
      || sgn(sq(p - res.back())) > 0)
      res.PB(p);
  }
  return res;
}
```

circle.hpp

b2218a, 114 lines

```
// Returns the circumcenter of triangle abc
Pt circumCenter(const Pt& a, Pt b, Pt c)
{
  b = b - a;
  c = c - a;
  assert(sgn(cross(b, c)) != 0);
  return a + perp(b * sq(c) - c * sq(b))
    / cross(b, c) / 2;
}
// Returns circle-line intersection points
vector<Pt> circleLine(const Pt& o, db r,
  const Line& l)
{
  db h2 = r * r - l.sqDist(o);
  if (sgn(h2) == -1)
    return {};
  Pt p = l.proj(o);
  if (sgn(h2) == 0)
    return {p};
  Pt h = perp(l.n) * sqrt(h2) / abs(l.n);
  return {p - h, p + h};
}
// Returns circle-circle intersection points
vector<Pt> circleCircle(const Pt& o1, db r1,
  const Pt& o2, db r2)
{
  Pt d = o2 - o1;
  db d2 = sq(d);
  if (sgn(d2) == 0)
  {
    assert(sgn(r2 - r1) != 0);
    return {};
  }
  db pd = (d2 + r1 * r1 - r2 * r2) / 2;
  db h2 = r1 * r1 - pd * pd / d2;
  if (sgn(h2) == -1)
    return {};
  Pt p = o1 + d * pd / d2;
  if (sgn(h2) == 0)
    return {p};
  Pt h = perp(d) * sqrt(h2 / d2);
  return {p - h, p + h};
}
// Finds common tangents (outer or inner)
// If there are 2 tangents, returns the pairs of
// tangency points on each circle (p1, p2)
// If there is 1 tangent, the circles are tangent
// to each other at some point p, res contains p
// 4 times, and the tangent line can be found as
// line(o1, p).perpThrough(p)
// The same code can be used to find the tangent
// to a circle through a point by setting r2 to 0
// (in which case 'inner' doesn't matter)
vector<pair<Pt, Pt>> tangents(const Pt& o1,
  db r1, const Pt& o2, db r2, bool inner)
{
  if (inner)
    r2 = -r2;
  Pt d = o2 - o1;
  db dr = r1 - r2, d2 = sq(d),
    h2 = d2 - dr * dr;
  if (sgn(d2) == 0 || sgn(h2) < 0)
  {
    assert(sgn(h2) != 0);
    return {};
  }
  vector<pair<Pt, Pt>> res;
  for (db sign : {-1, 1})
  {
    Pt v = (d * dr + perp(d) * sqrt(h2)
      * sign) / d2;
    res.PB({o1 + v * r1, o2 + v * r2});
  }
  return res;
}
// Returns the smallest enclosing circle of 'v'
pair<Pt, db> welzl(vector<Pt> v)
{
  int n = SZ(v), k = 0, idxes[2];
  mt19937 rng;
  shuffle(ALL(v), rng);
  Pt c = v[0];
  db r = 0;
  while (true)
  {
    FOR(i, k, n)
    {
      if (sgn(abs(v[i] - c) - r) > 0)
      {
        swap(v[i], v[k]);
        if (k == 0)
          c = v[0];
        else if (k == 1)
          c = (v[0] + v[1]) / 2;
        else
          c = circumCenter(
            v[0], v[1], v[2]);
        r = abs(v[0] - c);
        if (k < i)
        {
          if (k < 2)
            idxes[k++] = i;
          shuffle(v.begin() + k,
            v.begin() + i + 1, rng);
          break;
        }
      }
    }
    while (k > 0 && idxes[k - 1] == i)
      k--;
    if (i == n - 1)
      return {c, r};
  }
}
}
```

closest-pair.hpp

8696b6, 26 lines

```
// Returns the distance of the closest points
db closestPair(vector<Pt> v)
{
  sort(ALL(v), [](const Pt& p, const Pt& q)
  {
    return sgn(p.x - q.x) < 0;
  });
  set<pair<db, db>> s;
```

```
  int n = SZ(v), ptr = 0;
  db h = 1e18;
  FOR(i, 0, n)
  {
    for (auto it = s.lower_bound(
      MP(v[i].y - h, v[i].x)); it != s.end()
      && sgn(it->F - (v[i].y + h)) <= 0; it++)
    {
      Pt q = {it->S, it->F};
      h = min(h, abs(v[i] - q));
    }
    for (; sgn(v[ptr].x - (v[i].x - h)) <= 0;
      ptr++)
      s.erase({v[ptr].y, v[ptr].x});
    s.insert({v[i].y, v[i].x});
  }
  return h;
}
```

dual-graph.hpp

9c69cd, 153 lines

```
vector<Pt> vertices;

struct Edge
{
  // cross(vertices[j] - vertices[i], l.n) > 0
  int i, j;
  Line l;
  Edge(int _i, int _j, const Line& _l):
    i(_i), j(_j), l(_l)
  {
    assert(0 <= i && i < SZ(vertices));
    assert(0 <= j && j < SZ(vertices));
    assert(i != j);
  }
  Edge(int _i, int _j): Edge(_i, _j,
    {vertices[_i], vertices[_j]}) {}
};

vector<Edge> edges;

struct Comparator
{
  static int vertexIdx;
  db getY(const Line& l) const
  {
    return -(l.n.x * vertices[vertexIdx].x
      + l.c) / l.n.y;
  }
  bool operator()(int i, int j) const
  {
    auto [u1, v1, l1] = edges[i];
    auto [u2, v2, l2] = edges[j];
    if (u1 == vertexIdx && u2 == vertexIdx)
      return sgn(cross(l1.n, l2.n)) > 0;
    if (v1 == vertexIdx && v2 == vertexIdx)
      return sgn(cross(l1.n, l2.n)) < 0;
    int dy = sgn(getY(l1) - getY(l2));
    assert(dy != 0);
    return dy < 0;
  }
};
int Comparator::vertexIdx;

DSU dsu;

// Returns the dual graph to the planar graph
// given by `vertices` and `edges`. Vertex
// `dsu.find(i)` in the dual graph corresponds to
// the face under `edges[i]`
vector<unordered_set<int>> buildDualGraph()
{
  const db ROT_ANGLE = 4;
  int n = SZ(vertices), m = SZ(edges);
  for (Pt& p : vertices)
    p = rot(p, ROT_ANGLE);
  VI idxesVertices(n), invIdxes(n);
  iota(ALL(idxesVertices), 0);
  sort(ALL(idxesVertices), [](int i, int j)
  {
    return vertices[i].x < vertices[j].x;
  });
  vector<Pt> sortedVertices(n);
  FOR(i, 0, n)
  {
    sortedVertices[i] =
      vertices[idxesVertices[i]];
    invIdxes[idxesVertices[i]] = i;
  }
  vertices = sortedVertices;
  vector<VI> edgesL(n), edgesR(n);
  FOR(k, 0, m)
  {
    auto& [i, j, l] = edges[k];
    i = invIdxes[i];
    j = invIdxes[j];
    l.n = rot(l.n, ROT_ANGLE);
    if (i > j)
    {
      swap(i, j);
      l.n = l.n * (-1);
      l.c *= -1;
    }
    edgesL[j].PB(k);
    edgesR[i].PB(k);
  }
  vertices.PB({-INF, INF});
  vertices.PB({INF, INF});
  edges.PB({n, n + 1});
  dsu.init(m + 1);
  set<int, Comparator> s;
  s.insert(m);
  vector<PII> edgesDual;
  FOR(i, 0, n)
  {
    Comparator::vertexIdx = i;
    Comparator comparator;
    int upper = -1, lower = -1;
    if (!edgesL[i].empty())
    {
      assert(i > 0);
      sort(ALL(edgesL[i]), comparator);
      auto it =
        s.lower_bound(edgesL[i][0]);
      lower = edgesL[i][0];
      for (int e : edgesL[i])
      {
        assert(*it == e);
        assert(next(it) != s.end());
        edgesDual.PB({e, *next(it)});
        it = s.erase(it);
      }
      assert(it != s.end());
      upper = *it;
    }
    if (!edgesR[i].empty())
    {
      assert(i + 1 < n);
      sort(ALL(edgesR[i]), comparator);
      if (upper == -1)
      {
        upper =
          *s.lower_bound(edgesR[i][0]);
      }
      int prv = -1;
      for (int e : edgesR[i])
      {
        s.insert(e);
        if (prv != -1)
          edgesDual.PB({e, prv});
        prv = e;
```

```
      }
      edgesDual.PB({upper,
        edgesR[i].back()});
      dsu.unite(edgesL[i].empty() ? upper :
        lower, edgesR[i][0]);
    }
    else
    {
      assert(upper != -1 && lower != -1);
      dsu.unite(upper, lower);
    }
  }
  vector<unordered_set<int>> dualGraph;
  dualGraph.resize(m + 1);
  for (auto& [u, v] : edgesDual)
  {
    u = dsu.find(u);
    v = dsu.find(v);
    dualGraph[u].insert(v);
    dualGraph[v].insert(u);
  }
  return dualGraph;
}
```

Math (6)

6.1 Number-theoretic algorithms

gcd.hpp

Description: $ax + by = d, gcd(a, b) = \|d\| \to (d, x, y)$.
Minimizes $\|x\| + \|y\|$. And minimizes $\|x - y\|$ for $a > 0$, $b > 0$.

5d0cbe, 16 lines

```
tuple<int, int, int> gcdExt(int a, int b)
{
  int x1 = 1, y1 = 0;
  int x2 = 0, y2 = 1;
  while (b)
  {
    int k = a / b;
    x1 -= k * x2;
    y1 -= k * y2;
    a %= b;
    swap(a, b);
    swap(x1, x2);
    swap(y1, y2);
  }
  return {a, x1, y1};
}
```

fast-chinese.hpp

Description: $x\%p_i = m_i, \text{lcm}(p) \le 10^{18}, p \le 10^9 \to x$ or -1.
Not tested on good tests
Time: $\mathcal{O}(n \log(\text{lcm}(p_i)))$

cadfb1, 25 lines

```
LL fastChinese(VI m, VI p)
{
  assert(SZ(m) == SZ(p));
  LL aa = p[0];
  LL bb = m[0];
  FOR(i, 1, SZ(m))
  {
    int b = (m[i] - bb % p[i] + p[i]) % p[i];
    int a = aa % p[i];
    int c = p[i];

    int x, y;
    int d = gcd(a, c, x, y);
    if(b % d != 0)
      return -1;
    a /= d;
    b /= d;
    c /= d;
    b = b * (LL)x % c;

    bb = aa * b + bb;
    aa = aa * c;
  }
  return bb;
}
```

chinese.hpp

Description: Calculate result % mod.
Not tested on good tests
Time: $\mathcal{O}(n^2)$

ff1bca, 36 lines

```
int chinese(VI m, VI p)
{
  int n = SZ(m);
  FOR(i, 1, n)
  {
    LL a = 1;
    LL b = 0;
    RFOR(j, i, 0)
    {
      b = (b * p[j] + m[j]) % p[i];
      a = a * p[j] % p[i];
    }
    b = (m[i] - b + p[i]) % p[i];

    int c = p[i];
    int x, y;
    int d = gcd(a, c, x, y);

    if(b % d != 0)
      return -1;
    a /= d;
    b /= d;
    c /= d;

    b = b * x % c;
    m[i] = b;
    p[i] = c;
  }
  int res = m[n - 1] % mod;
  RFOR(i, n - 1, 0)
  {
    res = mult(res, p[i]);
    res = add(res, m[i]);
  }
  return res;
}
```

miller-rabin.hpp

Description: to speed up change candidates to at least 4 random values rng()
use __int128 in mult

394bc8, 33 lines

```
VI candidates = {2, 3, 5, 7, 11, 13, 17, 19, 23,
    29, 31, 47};
bool millerRabin(LL n)
{
  if (n == 1)
    return false;
  if (n == 2 || n == 3)
    return true;
  LL d = n - 1;
  int s = __builtin_ctzll(d);
  d >>= s;

  for (LL b : candidates)
  {
    if (b >= n)
      break;
    b = binpow(b, d, n);
    if (b == 1)
      continue;
    bool ok = false;
    FOR (i, 0, s)
    {
      if (b + 1 == n)
      {
        ok = true;
        break;
      }
      b = mult(b, b, n);
    }
    if (!ok)
      return false;
  }
  return true;
}
```

pollard.hpp

Description: uses Miller-Rabin test. rho finds divisor of n. use __int128 in mult.
Time: $\mathcal{O}\left(n^{1/4} \cdot \log n\right)$.

53da5d, 62 lines

```
LL f(LL x, LL c, LL n)
{
  return add(mult(x, x, n), c, n);
}

LL rho(LL n)
```

```
{
  const int iter = 47 * pow(n, 0.25);
  while (true)
  {
    LL x0 = rng() % n;
    LL c = rng() % n;
    LL x = x0;
    LL y = x0;
    LL g = 1;
    FOR (i, 0, iter)
    {
      x = f(x, c, n);
      y = f(y, c, n);
      y = f(y, c, n);
      g = gcd(abs(x - y), n);
      if (g != 1)
        break;
    }
    if (g > 1 && g < n)
      return g;
  }
}
VI primes = {2, 3, 5, 7, 11, 13, 17, 19, 23};

vector<LL> factorize(LL n)
{
  vector<LL> ans;

  for (auto p : primes)
  {
    while (n % p == 0)
    {
      ans.PB(p);
      n /= p;
    }
  }
  queue<LL> q;
  q.push(n);

  while (!q.empty())
  {
    LL x = q.front();
    q.pop();
    if (x == 1)
      continue;
    if (millerRabin(x))
      ans.PB(x);
    else
    {
      LL y = rho(x);
      q.push(y);
      q.push(x / y);
    }
  }
  return ans;
}
```

6.2 Matrices

gaussian.hpp

Description: if there is no solution, returns an empty vector. Otherwise, returns any solution.

497b9b, 45 lines

```
VI solveLinearSystem(vector<VI> a, VI b)
{
  int n = SZ(b), m = SZ(a[0]);
  FOR(i, 0, n)
    a[i].PB(b[i]);
  int p = 0;
  VI pivots;
  FOR(j, 0, m)
  {
    if (a[p][j] == 0)
    {
      int l = -1;
      FOR(i, p, n)
        if (a[i][j] != 0)
          l = i;
      if (l == -1)
        continue;
      swap(a[p], a[l]);
    }
    int inv = binpow(a[p][j], mod - 2);
    FOR(i, p + 1, n)
    {
      int c = mult(a[i][j], inv);
      FOR(k, j, m + 1)
        updSub(a[i][k], mult(c, a[p][k]));
    }
    pivots.PB(j);
    p++;
    if (p == n)
      break;
  }
  FOR(i, p, n)
    if (a[i].back() != 0)
      return {};
  VI x(m);
  RFOR(i, p, 0)
  {
    int j = pivots[i];
    x[j] = a[i].back();
    FOR(k, j + 1, m)
      updSub(x[j], mult(a[i][k], x[k]));
    x[j] = mult(x[j], binpow(a[i][j], mod - 2));
  }
  return x;
}
```

6.3 Linear programming

simplex.hpp

Description: $c^T x \to \max$, $Ax \leq b$, $x \geq 0$.

03c648, 142 lines

```
typedef vector<db> VD;

struct Simplex
{
  void pivot(int l, int e)
  {
    assert(0 <= l && l < m);
    assert(0 <= e && e < n);
    assert(abs(a[l][e]) > EPS);
    b[l] /= a[l][e];
    FOR(j, 0, n)
      if (j != e)
        a[l][j] /= a[l][e];
    a[l][e] = 1 / a[l][e];
    FOR(i, 0, m)
    {
      if (i != l)
      {
        b[i] -= a[i][e] * b[l];
        FOR(j, 0, n)
          if (j != e)
            a[i][j] -= a[i][e] * a[l][j];
        a[i][e] *= -a[l][e];
      }
    }
    v += c[e] * b[l];
    FOR(j, 0, n)
      if (j != e)
        c[j] -= c[e] * a[l][j];
    c[e] *= -a[l][e];
    swap(nonBasic[e], basic[l]);
  }
  void findOptimal()
  {
    VD delta(m);
    while (true)
    {
      int e = -1;
```

```
      FOR(j, 0, n)
        if (c[j] > EPS && (e == -1 || nonBasic[j]
            < nonBasic[e]))
          e = j;
      if (e == -1)
        break;
      FOR(i, 0, m)
        delta[i] = a[i][e] > EPS ? b[i] / a[i][e]
            : LINF;
      int l = min_element(ALL(delta)) - delta.
          begin();
      if (delta[l] == LINF)
      {
        // unbounded
        assert(false);
      }
      pivot(l, e);
    }
  }
  void initializeSimplex(const vector<VD>& _a,
      const VD& _b, const VD& _c)
  {
    m = SZ(_b);
    n = SZ(_c);
    nonBasic.resize(n);
    iota(ALL(nonBasic), 0);
    basic.resize(m);
    iota(ALL(basic), n);
    a = _a;
    b = _b;
    c = _c;
    v = 0;
    int k = min_element(ALL(b)) - b.begin();
    if (b[k] > -EPS)
      return;
    nonBasic.PB(n);
    iota(ALL(basic), n + 1);
    FOR(i, 0, m)
      a[i].PB(-1);
    c.assign(n, 0);
    c.PB(-1);
    n++;
    pivot(k, n - 1);
    findOptimal();
    if (v < -EPS)
    {
      // infeasible
      assert(false);
    }
    int l = find(ALL(basic), n - 1) - basic.begin
        ();
    if (l != m)
    {
      int e = -1;
      while (abs(a[l][e]) < EPS)
        e++;
      pivot(l, e);
    }
    n--;
    int p = find(ALL(nonBasic), n) - nonBasic.
        begin();
    assert(p < n + 1);
    nonBasic.erase(nonBasic.begin() + p);
    FOR(i, 0, m)
      a[i].erase(a[i].begin() + p);
    c.assign(n, 0);
    FOR(j, 0, n)
    {
      if (nonBasic[j] < n)
        c[j] = _c[nonBasic[j]];
      else
        nonBasic[j]--;
    }
    FOR(i, 0, m)
    {
      if (basic[i] < n)
      {
        v += _c[basic[i]] * b[i];
        FOR(j, 0, n)
          c[j] -= _c[basic[i]] * a[i][j];
      }
      else
        basic[i]--;
    }
  }
  pair<VD, db> simplex(const vector<VD>& _a,
      const VD& _b, const VD& _c)
  {
    initializeSimplex(_a, _b, _c);
    assert(SZ(a) == m);
    FOR(i, 0, m)
      assert(SZ(a[i]) == n);
    assert(SZ(b) == m);
    assert(SZ(c) == n);
    assert(SZ(nonBasic) == n);
    assert(SZ(basic) == m);
    findOptimal();
    VD x(n);
    FOR(i, 0, m)
      if (basic[i] < n)
        x[basic[i]] = b[i];
    return {x, v};
  }
private:
  int m, n;
  VI nonBasic, basic;
  vector<VD> a;
  VD b;
  VD c;
  db v;
};
```

6.4 Assignment problem

hungarian.hpp

0baccf, 63 lines

```
LL hungarian(const vector<vector<LL>>& a)
{
  int n = SZ(a), m = SZ(a[0]);
  assert(n <= m);
  vector<LL> u(n + 1), v(m + 1);
  VI p(m + 1, n), way(m + 1);
  FOR(i, 0, n)
  {
    p[m] = i;
    int j0 = m;
    vector<LL> minv(m + 1, LINF);
    vector<int> used(m + 1);
    while (p[j0] != n)
    {
      used[j0] = true;
      int i0 = p[j0], j1 = -1;
      LL delta = LINF;
      FOR(j, 0, m)
      {
        if (!used[j])
        {
          int cur = a[i0][j] - u[i0] - v[j];
          if (cur < minv[j])
          {
            minv[j] = cur;
            way[j] = j0;
          }
          if (minv[j] < delta)
          {
            delta = minv[j];
            j1 = j;
          }
        }
      }
```

```
      assert(j1 != -1);
      FOR(j, 0, m + 1)
      {
        if (used[j])
        {
          u[p[j]] += delta;
          v[j] -= delta;
        }
        else
          minv[j] -= delta;
      }
      j0 = j1;
    }
    while (j0 != m)
    {
      int j1 = way[j0];
      p[j0] = p[j1];
      j0 = j1;
    }
  }
  VI ans(n + 1);
  FOR(j, 0, m)
    ans[p[j]] = j;
  LL res = 0;
  FOR(i, 0, n)
    res += a[i][ans[i]];
  assert(res == -v[m]);
  return res;
}
```

6.5 Polynomials and FFT

fft.hpp

Description: $GEN^{\frac{LEN}{2}} = mod - 1$. Comments for complex. $mod = 9223372036737335297, GEN = 3^{\frac{mod-1}{LEN}}, LEN \le 2^{24}$.

d24e3f, 97 lines

```
const int mod = 998244353;

int add(int a, int b)
{
  return a + b < mod ? a + b : a + b - mod;
}
int sub(int a, int b)
{
  return a - b >= 0 ? a - b : a - b + mod;
}
int mult(int a, int b)
{
  return (LL)a * b % mod;
}
int binpow(int a, int n)
{
  int res = 1;
  while(n)
  {
    if(n & 1)
      res = mult(res, a);
    a = mult(a, a);
    n /= 2;
  }
  return res;
}

const int LEN = 1 << 23;
const int GEN = 31;
const int IGEN = binpow(GEN, mod - 2);

//void init()
//{
//  db phi = (db)2 * acos(-1.) / LEN;
//  FOR(i, 0, LEN)
//    pw[i] = com(cos(phi * i), sin(phi * i));
//}

void fft(VI& a, bool inv)
{
  int lg = __builtin_ctz(SZ(a));
  FOR(i, 0, SZ(a))
  {
    int k = 0;
    FOR(j, 0, lg)
      k |= ((i >> j) & 1) << (lg - j - 1);
    if(i < k)
      swap(a[i], a[k]);
  }
  for(int len = 2; len <= SZ(a); len *= 2)
  {
    int ml = binpow(inv ? IGEN : GEN, LEN / len);
    //int diff = inv ? LEN - LEN / len : LEN /
        len;
    for(int i = 0; i < SZ(a); i += len)
    {
      int pw = 1;
      //int pos = 0;
      FOR(j, 0, len / 2)
      {
        int v = a[i + j];
        int u = mult(a[i + j + len / 2], pw);
        // * pw[pos]
        a[i + j] = add(v, u);
        a[i + j + len / 2] = sub(v, u);

        pw = mult(pw, ml);
        //pos = (pos + diff) % LEN;
      }
    }
  }
  if(inv)
  {
    int m = binpow(SZ(a), mod - 2);
    FOR(i, 0, SZ(a))
      a[i] = mult(a[i], m);
  }
}

VI mult(VI a, VI b)
{
  int sz = 0;
  int sum = SZ(a) + SZ(b) - 1;
  while((1 << sz) < sum) sz++;
  a.resize(1 << sz);
  b.resize(1 << sz);

  fft(a, false);
  fft(b, false);

  FOR(i, 0, SZ(a))
    a[i] = mult(a[i], b[i]);

  fft(a, true);
  a.resize(sum);
  return a;
}
```

inverse.hpp

Description: Calculate $a^{-1} \% x^k$.

a4673f, 32 lines

```
VI inverse(const VI& a, int k)
{
  assert(SZ(a) == k && a[0] != 0);
  if(k == 1)
    return {binpow(a[0], mod - 2)};

  VI ra = a;
  FOR(i, 0, SZ(ra))
    if(i & 1)
      ra[i] = sub(0, ra[i]);

  int nk = (k + 1) / 2;
  VI t = mult(a, ra);
```

```
  t.resize(k);

  FOR(i, 0, nk)
    t[i] = t[2 * i];

  t.resize(nk);
  t = inverse(t, nk);
  t.resize(k);

  RFOR(i, nk, 1)
  {
    t[2 * i] = t[i];
    t[i] = 0;
  }

  VI res = mult(ra, t);
  res.resize(k);
  return res;
}
```

exp-log.hpp

Description: Calculate $\log(a)\%x^k$ and $\exp(a)\%x^k$.

33cb46, 52 lines

```
VI deriv(const VI& a, int k)
{
  VI res(k);
  FOR(i, 0, k)
    if(i + 1 < SZ(a))
      res[i] = mult(a[i + 1], i + 1);
  return res;
}

VI integr(const VI& a, int k)
{
  VI res(k);
  RFOR(i, k, 1)
    res[i] = mult(a[i - 1], inv[i]);
  res[0] = 0;
  return res;
}

VI log(const VI& a, int k)
{
  assert(a[0] == 1);
  VI ml = mult(deriv(a, k), inverse(a, k));
  return integr(ml, k);
}

VI exp(VI a, int k)
{
  assert(a[0] == 0);
  VI Qk = {1};
  int pw = 1;
  while(pw <= k)
  {
    pw *= 2;

    Qk.resize(pw);
    VI lnQ = log(Qk, pw);

    FOR(i, 0, SZ(lnQ))
    {
      if(i < SZ(a))
        lnQ[i] = sub(a[i], lnQ[i]);
      else
        lnQ[i] = sub(0, lnQ[i]);
    }
    lnQ[0] = add(lnQ[0], 1);

    Qk = mult(Qk, lnQ);
  }
  Qk.resize(k);
  return Qk;
}
```

modulo.hpp

Description: Modulo returns $[\frac{a}{b}]$ and $a\%b$

4ccc23, 37 lines

```
void removeLeadingZeros(VI& a)
{
  while(SZ(a) > 0 && a.back() == 0)
    a.pop_back();
}
pair<VI, VI> modulo(VI a, VI b)
{
  removeLeadingZeros(a);
  removeLeadingZeros(b);
  //be careful with this case
  assert(SZ(a) != 0 && SZ(b) != 0);

  int n = SZ(a), m = SZ(b);
  if(m > n)
    return MP(VI{}, a);

  reverse(ALL(a));
  reverse(ALL(b));

  VI d = b;
  d.resize(n - m + 1);
  d = mult(a, inverse(d, n - m + 1));
  d.resize(n - m + 1);
  reverse(ALL(a));
  reverse(ALL(b));
  reverse(ALL(d));

  VI res = mult(b, d);
  res.resize(SZ(a));
  FOR(i, 0, SZ(a))
    res[i] = sub(a[i], res[i]);

  removeLeadingZeros(d);
  removeLeadingZeros(res);
  return MP(d, res);
}
```

multipoint-eval.hpp

Description: Function *build* calculates the products of $x - x_i$. Function *solve* calculates the values of $q(x)$ in $x_0, \ldots, x_{n-1}$. 1. Call $build(0, 0, n)$. 2. Call $solve(0, 0, n, q)$.

d753bb, 34 lines

```
int x[LEN];
VI p[2 * LEN];

void build(int v, int tl, int tr)
{
  if(tl + 1 == tr)
  {
    p[v] = {sub(0, x[tl]), 1};
    return;
  }
  int tm = (tl + tr) / 2;
  build(2 * v + 1, tl, tm);
  build(2 * v + 2, tm, tr);

  p[v] = mult(p[2 * v + 1], p[2 * v + 2]);
}
int ans[LEN];
void solve(int v, int tl, int tr, const VI& q)
//q != q % p[0] -> wa
{
  if(SZ(q) == 0)
    return;
  if(tl + 1 == tr)
  {
    ans[tl] = q[0];
    return;
  }
  int tm = (tl + tr) / 2;
  solve(2 * v + 1, tl, tm,
  modulo(q, p[2 * v + 1]).S);
```

```
  solve(2 * v + 2, tm, tr,
  modulo(q, p[2 * v + 2]).S);
}
```

6.5.1 Newton's method

Usable to find the solution of equation $F(Q) = 0$.

For example $F(Q) = x \cdot Q^2 + A - Q = 0$.

Newton's method approximates the solution of the equation using the formula:

$$Q_{k+1} = Q_k - \frac{F(Q_k)}{F'(Q_k)}, where\ F' = \frac{dF}{dQ}$$

Example of the derivative: $F'(Q) = 2 \cdot x \cdot Q - 1$.

Keep in mind that $|Q_k| = 2^k$.

6.5.2 Specific FFT

FFT with doubles Move comments from code here.

FFT in 2D The complexity is $O(nm(\log n + \log m))$. The main problem to resize the matrix. You must add vectors of some size.

D-and-C FFT Using D-and-C to calculate DP table. (For example $DP[i] = sum(DP[j] \cdot DP[i-j])$)

By the time we compute the values for the segment $[l, r)$, the following conditions are already met:

- The values for $[0, l)$ on the DP table is already determined.
- The sum of contributions from $[0, l)$ through $[l, r)$ is already applied to the DP table in $[l, r)$.

When calculate the values for the segment $[l, r)$ do:

- Calculate the values for the segment $[l, m)$ recursively.
- Calculate the contributions from $[l, m)$ to $[m, r)$.
- Calculate the values for the segment $[m, r)$ recursively.

6.5.3 Interpolation

When $x_0, x_1, \ldots, x_d$ and $y_0, y_1, \ldots, y_d$ are given (where x_i are pairwise distinct), a polynomial $f(x)$ of degree no more than d such that $f(x_i) = y_i (i = 0, \ldots, d)$ is uniquely determined.

Lagrange polynomial

Lagrange basis polynomial: $L_i(x) = \prod_{j \neq i} \frac{x - x_j}{x_i - x_j}$.

$f(x) = y_0 L_0(x) + y_1 L_1(x) + \cdots + y_d L_d(x)$.

Newton polynomial

Divided differences:

$[y_i] = y_i$

$[y_i, y_{i+1}] = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$

$[y_i, y_{i+1}, \ldots, y_{j-1}, y_j] = \frac{[y_{i+1}, \ldots, y_{j-1}, y_j] - [y_i, y_{i+1}, \ldots, y_{j-1}]}{x_j - x_i}$.

Newton basis polynomial: $N_i(x) = \prod_{j=0}^{i-1}(x - x_j)$.

$f(x) = [y_0] N_0(x) + \cdots + [y_0, y_1, \ldots, y_d] N_d(x)$.

6.6 Linear recurrence

berlekamp-massey.hpp

Description: Finds a sequence of integers $c_1, \ldots, c_d$ of the minimum length $d \geq 0$ such that $a_i = \sum_{j=1}^{d} c_j a_{i-j}$.

866c28, 36 lines

```
VI berlekampMassey(const VI& a)
{
  VI c = {1}, bp = {1};
  int l = 0, b = 1, x = 1;
  FOR(j, 0, SZ(a))
  {
    assert(SZ(c) == l + 1);
    int d = a[j];
    FOR(i, 1, l + 1)
      updAdd(d, mult(c[i], a[j - i]));
    if (d == 0)
    {
      x++;
      continue;
    }
    VI t = c;
    int coef = mult(d, binPow(b, mod - 2));
    if (SZ(bp) + x > SZ(c))
      c.resize(SZ(bp) + x);
    FOR(i, 0, SZ(bp))
      updSub(c[i + x], mult(coef, bp[i]));
    if (2 * l > j)
    {
      x++;
      continue;
    }
    l = j + 1 - l;
    bp = t;
    b = d;
    x = 1;
  }
  c.erase(c.begin());
  for (int& ci : c)
    ci = mult(ci, mod - 1);
  return c;
}
```

bostan-mori.hpp

Description: computes the n-th term of a given linearly recurrent sequence $a_i = \sum_{j=1}^{d} c_j a_{i-j}$. Time complexity: $O(d \log d \log n)$.

966fbd, 41 lines

```
int bostanMori(const VI& c, VI a, LL n) {
  int k = SZ(c);
  assert(SZ(a) == k);
  int m = 1 << (33 - __builtin_clz(k));
  assert(m >= 2 * k + 1);
  VI q(k + 1);
  q[0] = 1;
  FOR(i, 0, k)
    q[i + 1] = sub(0, c[i]);
  VI p = mult(a, q);
  p.resize(m);
  FOR(i, k, m)
    p[i] = 0;
  q.resize(m);
  VI qMinus;
  while (n)
  {
    qMinus = q;
    for (int i = 1; i <= k; i += 2)
      qMinus[i] = sub(0, qMinus[i]);
    fft(qMinus, false);
    fft(p, false);
    fft(q, false);
    FOR(i, 0, m)
      p[i] = mult(p[i], qMinus[i]);
    fft(p, true);
    FOR(i, 0, m)
      q[i] = mult(q[i], qMinus[i]);
```

```
    fft(q, true);
    FOR(i, 0, k)
      p[i] = p[2 * i + (n & 1)];
    FOR(i, k, m)
      p[i] = 0;
    FOR(i, 0, k + 1)
      q[i] = q[2 * i];
    FOR(i, k + 1, m)
      q[i] = 0;
    n >>= 1;
  }
  return mult(p[0], binpow(q[0], mod - 2));
}
```

6.7 Convolutions

conv-xor.hpp

Description: $c_{i \oplus j} += a_i * b_j$.

b80d13, 24 lines

```
void convXor(VI& a, int k)
{
  FOR(i, 0, k)
    FOR(j, 0, 1 << k)
      if((j & (1 << i)) == 0)
      {
        int u = a[j];
        int v = a[j + (1 << i)];
        a[j] = add(u, v);
        a[j + (1 << i)] = sub(u, v);
      }
}
VI multXor(VI a, VI b, int k)
{
  convXor(a, k);
  convXor(b, k);
  FOR(i, 0, 1 << k)
    a[i] = mult(a[i], b[i]);
  convXor(a, k);
  int d = inv(1 << k);
  FOR(i, 0, 1 << k)
    a[i] = mult(a[i], d);
  return a;
}
```

conv-or.hpp

Description: $c_{i \vee j} += a_i * b_j$.

e4e659, 21 lines

```
void convOr(VI& a, int k, bool inverse)
{
  FOR(i, 0, k)
    FOR(j, 0, 1 << k)
      if((j & (1 << i)) == 0)
      {
        if(inverse)
          updSub(a[j + (1 << i)], a[j]);
        else
          updAdd(a[j + (1 << i)], a[j]);
      }
}
VI multOr(VI a, VI b, int k)
{
  convOr(a, k, false);
  convOr(b, k, false);
  FOR(i, 0, 1 << k)
    a[i] = mult(a[i], b[i]);
  convOr(a, k, true);
  return a;
}
```

6.8 Numerical methods

golden-section-search.hpp

4c0990, 27 lines

```
db goldenSectionSearch(db l, db r)
{
  const db c = (-1 + sqrt(5)) / 2;
  const int M = 474;
  db m1 = r - c * (r - l), fm1 = f(m1),
    m2 = l + c * (r - l), fm2 = f(m2);
  FOR(i, 0, M)
  {
    if (fm1 < fm2)
    {
      r = m2;
      m2 = m1;
      fm2 = fm1;
      m1 = r - c * (r - l);
      fm1 = f(m1);
    }
    else
    {
      l = m1;
      m1 = m2;
      fm1 = fm2;
      m2 = l + c * (r - l);
      fm2 = f(m2);
    }
  }
  return (l + r) / 2;
}
```

6.8.1 Simpson's rule

n – even number, $h = \frac{b-a}{n}$, $x_i = a + ih$

$$\int_a^b f\left(x\right)\mathrm{d}x \approx \frac{h}{3}\left[f\left(x_0\right) + 4\sum_{i=1}^{\frac{n}{2}} f\left(x_{2i-1}\right) + 2\sum_{i=1}^{\frac{n}{2}-1} f\left(x_{2i}\right) + f\left(x_n\right)\right]$$

6.9 Runge-Kutta 4th Order Method for Ordinary Differential Equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x,y), \qquad y(0) = y_0, \qquad x_{i+1} - x_i = h,$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h,$$

$$k_1 = f(x_i, y_i), \qquad k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h),$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h), \qquad k_4 = f(x_i + h, y_i + k_3 h).$$

Various (7)

7.1 NP complete

Number of solutions to 2-SAT.

Formulas (8)

8.1 Modular formulas

8.1.1 Calculation of $a^b \mod m$

if $b \geq \phi(m)$ then value $a^b \mod m$ equals to the value $a^{[b \mod \phi(m)] + \phi(m)} \mod m$.

8.1.2 Generators

Generator exist only for $n = 1, 2, 4, p^k, 2p^k$ for odd primes p and positive integer k.

g is generator for modulo n if any comprime with n can be represented as $\left[g^i \mod n\right], 0 \leq i < \phi(n)$.

To find generator:

- find $\phi(n)$ and $p_1, ..., p_m$ — prime factors of $\phi(n)$
- g is generator only if $g^{\frac{\phi(n)}{p_j}} \mod n \neq 1$ for each j
- check $g = 2, 3, 4, ..., p-1$

8.1.3 Wilson

p is prime if and only if $(p-1)! = (p-1) \mod p$.

8.1.4 Quadratic residue

q is quadratic residue modulo p if there exist integer x that $x^2 = q \mod p$. If p is odd prime then there exists $\frac{p+1}{2}$ residues (including 0).

Legendre symbol is equal to 0 if q is divisible by p, equal to 1 if q is quadratic residue, and -1 otherwise:

$$\left(\frac{q}{p}\right) = q^{\frac{p-1}{2}} (mod p)$$

Jacobi symbol (Legendre symbol for all p):

$$\left(\frac{q}{p}\right) = \prod \left(\frac{q}{p_i}\right)^{\alpha_i}$$

8.2 Number Theory

8.2.1 Mobius

$$g(n) = \sum_{d|n} f(d) \iff f(n) = \sum_{d|n} \mu(d) g(\frac{n}{d})$$

$$M(n) = \sum_{k=1}^{n} \mu(k) \iff \sum_{n=1} x M([\frac{x}{n}]) = 1$$

8.2.2 Catalan

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k} = \frac{1}{n+1} C_{2n}^n = C_{2n}^n - C_{2n}^{n-1}$$

8.2.3 Binomials

$$\sum_{k=0}^{n} C_n^k = 2^n \qquad \sum_{k=0}^{m} C_{n+k}^k = C_{n+m+1}^m$$

$$\sum_{m=0}^{n} C_m^k = C_{n+1}^{k+1} \qquad \sum_{k=0}^{n} (C_n^k)^2 = C_{2n}^n$$

$$\sum_{j=0}^{k} C_m^j C_{n-m}^{k-j} = C_n^k \qquad \sum_{j=0}^{m} C_m^j C_{n-m}^{k-j} = C_{n+1}^{k+1}$$

$$\sum_{k=0}^{n} C_{n-k}^k = F_{n+1}$$

8.2.4 Fibonacci

$$F_1 = F_2 = 1 \qquad F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$$

$$F_n = F_{n-1} + F_{n-2} \qquad F_{n+1} F_{n-1} - F_n^2 = (-1)^n$$

$$gcd(F_m, F_n) = F_{gcd(n,m)}$$

$$F_n = \frac{(\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n}{\sqrt{5}}$$

8.2.5 Stirling

$S(n,k)$ — number of ways to divide n element into k non-empty groups.
$S(n,n) = 1,\ n \geq 0$
$S(n,0) = 0,\ n > 0$
$S(n,k) = S(n-1,k-1) + S(n-1,k) * k$.

$B_n = \sum S(n,k)$ from $n = 0$:

1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597, 27644437, 190899322, 1382958545, 10480142147, 82864869804,...

8.2.6 Burnside's lemma

Let G be a finite group that acts on a set X.

The *orbit* of an element x in X is the set of elements in X to which x can be moved by the elements of G. The orbit of x is denoted by $G \cdot x$:

$$G \cdot x = \{g \cdot x \mid g \in G\}.$$

For each g in G, let X^g denote the set of elements in X that are fixed by g (also said to be left invariant by g), that is, $X^g = \{x \in X \mid g \cdot x = x\}$. Burnside's lemma asserts the following formula for the number of orbits, denoted $|X/G|$:

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$

8.3 Math

8.3.1 Cayley's theorem

Let $T_{n,k}$ be the number of labelled forests on n vertices with k connected components, such that vertices $1, 2, \ldots, k$ all belong to different connected components. Then $T_{n,k} = k \cdot n^{n-k-1}$.

8.4 List of integrals

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C$$
$$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$
$$\int \frac{\mathrm{d}x}{\sqrt{x^2+a}} = \ln\left|x+\sqrt{x^2+a}\right| + C$$
$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$
$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

8.5 Taylor series

$$f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k + o((x-x_0)^n)$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} \qquad \ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$$
$$\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} \qquad \sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$$

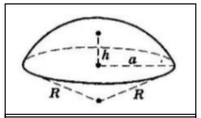
8.6 Geometry

8.6.1 Trigonometry

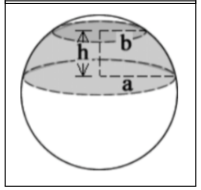
$$\sin(v+w) = \sin v\cos w + \cos v \sin w$$
$$\sin(v-w) = \sin v\cos w - \cos v \sin w$$
$$\tan(v+w) = \tfrac{\tan v + \tan w}{1-\tan v\tan w}$$
$$\sin v + \sin w = 2\sin\tfrac{v+w}{2}\cos\tfrac{v-w}{2}$$
$$\cos v + \cos w = 2\cos\tfrac{v+w}{2}\cos\tfrac{v-w}{2}$$

8.6.2 Ball

$$a = \sqrt{h*(2R-h)}$$
$$V = \pi * h^2(R-\tfrac{h}{3})$$



$$V = \tfrac{1}{6}\pi h(3a^2+3b^2+h^2)$$
$$R = \sqrt{\tfrac{((a-b)^2+h^2)((a+b)^2+h^2)}{4h^2}}$$



8.6.3 Pick's theorem

Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points inside, and let b be the number of integer points on boundary. Then the area $S = i + \frac{b}{2} - 1$.

8.6.4 Ptolemy's theorem

If the cyclic quadrilateral is $ABCD$, then $AC \cdot BD = AB \cdot CD + AD \cdot BC$.

8.6.5 Ceva's theorem

Given a triangle $\triangle ABC$ with a point P inside the triangle, continue lines AP, BP, CP to hit BC, CA, AB at D, E, F, respectively. Ceva's theorem states that $\frac{AF}{FB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA} = 1$.

8.6.6 Simson line

Given a triangle $\triangle ABC$ and a point P on its circumcircle, the three closest points to P on lines AB, AC, and BC are collinear. The line through these points is the Simson line of P.

8.6.7 Euler line

The line on which the orthocenter, triangle centroid, circumcenter, and a number of other important triangle centers lie.

8.6.8 Platonic solids

Polyhedron	Vertices	Edges	Faces
tetrahedron	4	6	4
cube	8	12	6
octahedron	6	12	8
dodecahedron	20	30	12
icosahedron	12	30	20