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# Stallions

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## Contest (1)

```
template.hpp27 lines

//hash = 7d0184

#include <bits/stdc++.h>
using namespace std;

#define FOR(i, a, b) for(int i = (a); i < (b); i++)
#define RFOR(i, a, b) for(int i = (a) - 1; i >= (b); i--)
#define SZ(a) int(a.size())
#define ALL(a) a.begin(), a.end()
#define PB push_back
#define MP make_pair
#define F first
#define S second

typedef long long LL;
typedef vector<int> VI;
typedef pair<int, int> PII;
typedef double db;

int main()
{
    ios::sync_with_stdio(0);
    cin.tie(0);
```

```
1    cout << fixed << setprecision(15);
2
3    return 0;
4}

4 compilation.txt2 lines
10 g++ -O2 -std=c++17 -Wno-unused-result -Wshadow -Wall -o %e %e.cpp
13 g++ -std=c++17 -Wshadow -Wall -o %e %e.cpp -fsanitize=address -fsanitize=undefined -D_GLIBCXX_DEBUG -g
16
18 s.sh6 lines
22 for((i = 0; ; i++)) do
22     echo $i
    ./gen $i > in
    diff -w <(/a < in) <(/brute < in) || break
    [ $? == 0 ] || break
done

hash.sh1 lines
cpp -dD -P -fpreprocessed $1 | tr -d '[:space:]' | md5sum | cut -c-6

troubleshoot.txt59 lines
Pre-submit:
F9.
Write a few manual test cases.
Calculate time and memory complexity. Check limits.
Check overflows, size of arrays, clearing mutitestcases, uninitialized variables.

Wrong answer:
F9.
Print your solution!
Read your code.
Check Pre-submit.
Are you sure your algorithm works?
Think about precision errors and hash collisions.
Have you understood the problem correctly?
Write brute and generator.

If you don't know what to do:
Are you clearing all data structures between test cases?
```

Can your algorithm handle the whole range of input?

Read the full problem statement again.

Do you handle all corner cases correctly?

Any uninitialized variables?

Any overflows?

Confusing N and M, i and j, etc.?

What special cases have you not thought of?

Are you sure the STL functions you use work as you think?

Add some assertions, maybe resubmit.

Go for a small walk, e.g. to the toilet.

Is your output format correct? (including whitespace)

Rewrite your solution from the start or let a teammate do it.

Runtime error:

F9.

Print your solution!

Read your code.

F9 with generator.

If you don't know what to do:

Invalidated pointers or iterators?

Are you reading or writing outside the range of any vector?

Any assertions that might fail?

Any possible division by 0? (mod 0 for example)

Have you tested all corner cases locally?

Any uninitialized variables?

Any possible infinite recursion?

Are you using too much memory?

Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:

What is the complexity of your algorithm?

Are you copying a lot of unnecessary data? (References)

Do you have any possible infinite loops?

How big is the input and output? (consider scanf)

Avoid vector, map. (use arrays/unordered\_map)

What do your teammates think about your algorithm?

?

Memory limit exceeded:

Calculate memory usage with stack in recursion.

# Data Structures (2)

dsu.hpp

2de4ff, 34 lines

```
struct DSU
{
    int n;
    VI p;
    VI sz;

    void init(int _n)
    {
        n = _n;
        sz.assign(n, 1);
        p.resize(n);
        iota(ALL(p), 0);
    }

    int find(int v)
    {
        if (v == p[v])
            return v;
        return p[v] = find(p[v]);
    }

    bool unite(int u, int v)
    {
        u = find(u);
        v = find(v);
        if (u == v)
            return false;
        if (sz[u] > sz[v])
            swap(u, v);
        p[u] = v;
        sz[v] += sz[u];
        return true;
    }
};
```

Fenwick.hpp

d4ebda, 43 lines

```
struct Fenwick
{
    int n;
    vector<LL> v;

    void init(int _n)
    {
        n = _n;
        v.assign(n, 0);
    }
};
```

```
void add(int i, int x)
{
    for (; i < n; i = (i + 1) | i)
        v[i] += x;
}

LL sum(int i)
{
    LL ans = 0;
    for (; i >= 0; i = (i & (i + 1)) - 1)
        ans += v[i];
    return ans;
}

int lower_bound(LL x)
{
    LL sum = 0;
    int i = -1;
    int lg = 31 - __builtin_clz(n);
    while (lg >= 0)
    {
        int j = i + (1 << lg);
        if (j < n && sum + v[j] < x)
        {
            sum += v[j];
            i = j;
        }
        lg--;
    }
    return i + 1;
}
};
```

Fenwick.txt

20 lines

Minimum on segment:  
1) Use two Fenwick trees with  $n = 2^k$ .  
You can use if  $n > 1$ :  
 $n = 1 << (32 - \text{__builtin\_clz}(n - 1))$ ;  
2) One tree for normal array and one for reversed  
3) When querying for minimum on the segment  
only consider segments  $[(i \& (i + 1)), i]$   
from trees that are COMPLETELY inside  $[l, r]$

Fenwick tree for adding on segment (prefixes):  
1) Use 2 arrays: mult and add  
2) upd(int i, int updMult, int updAdd)  
default Fenwick update.  
3) add x on segment  $[l, r]$ :  
     $\text{upd}(l, x, -x * (1 - 1))$ ;

```
upd(r, -x, x * r);
4) to calculate sum on prefix r:
sumAdd and sumMult - default Fenwick sum
st - initial value of r
ans = st * sumMult + sumAdd
```

treap.hpp

Description: uncomment in split for explicit key or in merge for implicit priority.

925cdb, 145 lines

mt19937 rng;

```
struct Node
{
    int l, r;
    int x;
    int y;
    int cnt;
    int par;
    int rev;
    int mn;

    void init(int value)
    {
        l = r = -1;
        x = value;
        y = rng();
        cnt = 1;
        par = -1;
        rev = 0;
        mn = value;
    }
};

struct Treap
{
    Node A[N];
    int sz = 0;

    int getCnt(int v)
    {
        if (v == -1)
            return 0;
        return A[v].cnt;
    }

    int getMn(int v)
    {
        if (v == -1)
            return INF;
        return A[v].mn;
    }
};
```

```

int newNode(int val)
{
    A[sz].init(val);
    return sz++;
}

void upd(int v)
{
    if (v == -1)
        return;
    A[v].cnt = getCnt(A[v].l) +
    getCnt(A[v].r) + 1;

    A[v].mn = min(A[v].x,
    min(getMn(A[v].l), getMn(A[v].r)));
}

void reverse(int v)
{
    if (v == -1)
        return;
    A[v].rev ^= 1;
}

void push(int v)
{
    if (v == -1 || A[v].rev == 0)
        return;
    reverse(A[v].l);
    reverse(A[v].r);
    swap(A[v].l, A[v].r);
    A[v].rev = 0;
}

PII split(int v, int cnt)
{
    if (v == -1)
        return {-1, -1};
    push(v);
    int left = getCnt(A[v].l);
    PII res;
    // if (val <= A[v].x)
    if (cnt <= left)
    {
        if (A[v].l != -1)
            A[A[v].l].par = -1;
        res = split(A[v].l, cnt);
        A[v].l = res.second;
        if (res.second != -1)
            A[res.second].par = v;
        res.second = v;
    }
    else
    {

```

```

        if (A[v].r != -1)
            A[A[v].r].par = -1;
        // split(v, val)
        res = split(A[v].r, cnt - left - 1);
        A[v].r = res.first;
        if (res.first != -1)
            A[res.first].par = v;
        res.first = v;
    }
    upd(v);
    return res;
}

int merge(int v, int u)
{
    if (v == -1) return u;
    if (u == -1) return v;
    int res;
    // if (rng() % (getCnt(v) + getCnt(u)) < getCnt(v))
    if (A[v].y > A[u].y)
    {
        push(v);
        if (A[v].r != -1)
            A[A[v].r].par = -1;
        res = merge(A[v].r, u);
        A[v].r = res;
        if (res != -1)
            A[res].par = v;
        res = v;
    }
    else
    {
        push(u);
        if (A[u].l != -1)
            A[A[u].l].par = -1;
        res = merge(v, A[u].l);
        A[u].l = res;
        if (res != -1)
            A[res].par = u;
        res = u;
    }
    upd(res);
    return res;
}

int getIdx(int v, int from = -1)
{
    if (v == -1)
        return 0;
    int x = getIdx(A[v].par, v);
    if (from == -1 || A[v].r == from)

```

```

        x += getCnt(A[v].l) + 1;
    push(v);
    return x;
}
};

```

## ordered-set.hpp

8 lines

```

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using namespace std;
typedef tree<int, null_type, less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update>
    ordered_set;
// example: ordered_set s; s.insert(47);
// s.order_of_key(k); — returns number of
// elements less then k
// s.find_by_order(k); — returns iterator to k-th
// element or s.end()
// s.count() does not exist.

```

## sparse-table.hpp

7fdd30, 30 lines

```

int lg[N + 1];

struct SparseTable
{
    int t[N][LOG];

    void init(const VI& v)
    {
        lg[1] = 0;
        FOR (i, 2, N + 1) lg[i] = lg[i / 2] + 1;
        FOR (i, 0, N) FOR (j, 0, LOG) t[i][j] = INF;
        FOR (i, 0, SZ(v)) t[i][0] = v[i];

        FOR (j, 1, LOG)
        {
            int len = 1 << (j - 1);
            FOR (i, 0, N - (1 << j))
            {
                t[i][j] = min(t[i][j - 1],
                    t[i + len][j - 1]);
            }
        }
    }

    int query(int l, int r)
    {
        int i = lg[r - l + 1];

```

```

    return min(t[l][i], t[r - (1 << i) + 1][i]);
}
} st;

```

## convex-hull-trick.hpp

06db0c, 74 lines

```

struct Line
{
    LL a, b, xLast;
    Line() {}
    Line(LL _a, LL _b): a(_a), b(_b) {}
    bool operator<(const Line& l) const
    {
        return MP(a, b) < MP(l.a, l.b);
    }
    bool operator<(int x) const
    {
        return xLast < x;
    }
    __int128 getY(__int128 x) const
    {
        return a * x + b;
    }
    LL intersect(const Line& l) const
    {
        assert(a < l.a);
        LL dA = l.a - a, dB = b - l.b, x = dB / dA;
        if (dB < 0 && dB % dA != 0)
            x--;
        return x;
    }
};

struct ConvexHull: set<Line, less<>>
{
    bool needErase(iterator it, const Line& l)
    {
        LL x = it->xLast;
        if (it->getY(x) > l.getY(x))
            return false;
        if (it == begin())
            return it->a >= l.a;
        x = prev(it)->xLast + 1;
        return it->getY(x) < l.getY(x);
    }
    void add(LL a, LL b)
    {
        Line l(a, b);
        auto it = lower_bound(l);
        if (it != end())
        {

```

```

            LL x = it == begin() ? -LINF :
                prev(it)->xLast;
            if ((it == begin()
                || prev(it)->getY(x) >= l.getY(x))
                && it->getY(x + 1) >= l.getY(x + 1))
                return;
        }
        while (it != end() && needErase(it, l))
            it = erase(it);
        while (it != begin()
            && needErase(prev(it), l))
            erase(prev(it));
        if (it != begin())
        {
            auto itP = prev(it);
            Line lIt = *itP;
            lIt.xLast = itP->intersect(l);
            erase(itP);
            insert(lIt);
        }
        l.xLast = it == end() ? LINF :
            l.intersect(*it);
        insert(l);
    }
    LL getMaxY(LL x)
    {
        return lower_bound(x)->getY(x);
    }
};

```

# Graphs (3)

## 3.1 Decompositions

### centroid.hpp

**Description:** dfsSZ calculates size of subtrees not going to usedc[v] vertices

226f45, 35 lines

```

void build(int cent)
{
    dfsSZ(cent, -1);
    int szAll = sz[cent];
    int pr = cent;
    while (true)
    {
        int v = -1;
        for (auto to : g[cent])
        {
            if (to == pr || usedc[to])

```

```

            continue;
            if (sz[to] * 2 > szAll)
            {
                v = to;
                break;
            }
        }
        if (v == -1)
            break;
        pr = cent;
        cent = v;
    }
    usedc[cent] = true;

    // here calculate f(cent)

    for (auto to : g[cent])
    {
        if (!usedc[to])
        {
            build(to);
        }
    }
}

```

### HLD.hpp

**Description:** run dfsSZ(root, -1, 0) and dfsHLD(root, -1, root) to build HLD. Vertex v has index tin[v]. To update on path use process as in get().

0031c1, 66 lines

```

VI g[N];
int sz[N];
int h[N];
int p[N];
int top[N];
int tin[N];
int tout[N];
int t = 0;

void dfsSZ(int v, int par, int hei)
{
    sz[v] = 1;
    h[v] = hei;
    p[v] = par;
    for (auto& to : g[v])
    {
        if (to == par)
            continue;
        dfsSZ(to, v, hei + 1);
        sz[v] += sz[to];
        if (g[v][0] == par || sz[g[v][0]] < sz[to])

```

```

        swap(g[v][0], to);
    }
}
void dfsHLD(int v, int par, int tp)
{
    tin[v] = t++;
    top[v] = tp;
    FOR (i, 0, SZ(g[v]))
    {
        int to = g[v][i];
        if (to == par)
            continue;
        if (i == 0)
            dfsHLD(to, v, tp);
        else
            dfsHLD(to, v, to);
    }
    tout[v] = t - 1;
}
LL get(int x, int y)
{
    LL res = 0;
    while(true)
    {
        int tx = top[x];
        int ty = top[y];
        if (tx == ty)
        {
            int t1 = tin[x];
            int t2 = tin[y];
            if (t1 > t2)
                swap(t1, t2);
            res += query(t1, t2);
            break;
        }
        if (h[tx] < h[ty])
        {
            swap(tx, ty);
            swap(x, y);
        }
        res += query(tin[tx], tin[x]);
        x = p[tx];
    }
    return res;
}

```

biconnected-components.hpp

2d79e1, 83 lines

```

struct Graph
{
    vector<PII> edges;

```

```

    vector<VI> g;

    VI tin, low;
    VI col;
    VI par;
    VI used;
    int t = 1, c = 1;
    vector<int> st;

    int n, m;

    void init(int _n, int _m)
    {
        n = _n;
        m = _m;

        edges.assign(m, {0, 0});
        g.assign(n, {});
        tin.assign(n, 0);
        used.assign(n, 0);
        par.assign(n, -1);
        used.assign(n, 0);

        t = c = 1;
    }

    void addEdge(int a, int b, int i)
    {
        assert(0 <= a && a < n);
        assert(0 <= b && b < n);
        assert(0 <= i && i < m);

        edges[i] = MP(a, b);
        g[a].PB(i);
        g[b].PB(i);
    }

    void dfs(int v, int p = -1)
    {
        used[v] = 1;
        par[v] = p;
        low[v] = tin[v] = t++;
        int cnt = 0;
        for (auto e : g[v])
        {
            int to = edges[e].F;
            if (to == v)
                to = edges[e].S;

            if (p == to) continue;

```

```

if (!used[to])
{
    cnt++;
    st.PB(e);
    dfs(to, v);

    low[v] = min(low[v], low[to]);

    if ((par[v] == -1 && cnt > 1) ||
        (par[v] != -1 && low[to] >= tin[v]))
    {
        while (st.back() != e)
        {
            col[st.back()] = c;
            st.pop_back();
        }
        col[st.back()] = c++;
        st.pop_back();
    }
}
else
{
    low[v] = min(low[v], tin[to]);
    if (tin[to] < tin[v])
        st.PB(e);
}
}
};

```

## 3.2 Flows

kuhn.hpp

Time: 0.6s for  $|V| = 10^5$ ,  $|E| = 2 * 10^5$ 

39cb20, 93 lines

```

struct Graph
{
    int L, R;
    //edges from left to right in 0 indexing
    vector<VI> g;
    VI mt, P, U;

    void init(int l, int r)
    {
        L = l, R = r;
        g.resize(L);
        P.resize(L);
        U.resize(L);

        mt.resize(R);
    }
}

```

```

void addEdge(int from, int to)
{
    assert(0 <= from && from < L);
    assert(0 <= to && to < R);

    g[from].PB(to);
}

int iter;
bool kuhn(int v)
{
    if (U[v] == iter) return false;
    U[v] = iter;
    random_shuffle(ALL(g[v]));
    for(int to : g[v])
    {
        if (mt[to] == -1)
        {
            mt[to] = v;
            P[v] = to;
            return true;
        }
    }
}

for(int to : g[v])
{
    if (kuhn(mt[to]))
    {
        mt[to] = v;
        P[v] = to;
        return true;
    }
}

return false;
}

int doKuhn()
{
    fill(ALL(mt), -1);
    fill(ALL(P), -1);
    fill(ALL(U), -1);

    int res = 0;
    iter = 0;
    VI order(L);
    iota(ALL(order), 0);
    random_shuffle(ALL(order));

    while(true)
    {
        iter++;

```

```

        bool ok = false;
        for(int v : order)
        {
            if (P[v] == -1)
                if (kuhn(v))
                {
                    ok = true;
                    res++;
                }
        }
        if (!ok) break;
    }
    return res;
};

//Minimum number of vertex to cover edges
//equal to size of matching

//Maximal independent set
//is complement of vertex cover

//To restore cover make directed graph
//matched edges l <- r, not matched l -> r
//dfs from vl : P[vl] == -1
//cover: used[r] != -1 && mt[r] != -1
//      used[l] == -1 && P[l] != -1

```

dinic.hpp

6afa18, 93 lines

```

struct Graph
{
    struct Edge
    {
        int from, to;
        LL cap, flow;
    };

    int _n;
    vector<Edge> edges;
    vector<VI> g;
    VI d, p;

    Graph() : _n(0) {}
    Graph(int n) : _n(n), g(n), d(n), p(n) {}

    void addEdge(int from, int to, LL cap)
    {
        assert(0 <= from && from < _n);
        assert(0 <= to && to < _n);
        assert(0 <= cap);

```

```

        g[from].PB(SZ(edges));
        edges.PB({from, to, cap, 0});
        g[to].PB(SZ(edges));
        edges.PB({to, from, 0, 0});
    }

    int bfs(int s, int t)
    {
        fill(ALL(d), -1);
        d[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty())
        {
            int v = q.front();
            q.pop();
            for (int e : g[v])
            {
                int to = edges[e].to;
                if (edges[e].flow < edges[e].cap
                    && d[to] == -1)
                {
                    d[to] = d[v] + 1;
                    q.push(to);
                }
            }
        }
        return d[t];
    }

    LL dfs(int v, int t, LL flow)
    {
        if (v == t || flow == 0)
            return flow;
        for (; p[v] < SZ(g[v]); p[v]++)
        {
            int e = g[v][p[v]], to = edges[e].to;
            LL c = edges[e].cap, f = edges[e].flow;
            if (f < c && (to == t || d[to] == d[v] + 1))
            {
                LL push = dfs(to, t, min(flow, c - f));
                if (push > 0)
                {
                    edges[e].flow += push;
                    edges[e ^ 1].flow -= push;
                    return push;
                }
            }
        }
    }
}

```

```

    return 0;
}

LL flow(int s, int t)
{
    assert(0 <= s && s < _n);
    assert(0 <= t && t < _n);
    assert(s != t);
    LL flow = 0;
    while (bfs(s, t) != -1)
    {
        fill(ALL(p), 0);
        while (true)
        {
            LL f = dfs(s, t, LINF);
            if (f == 0)
                break;
            flow += f;
        }
    }
    return flow;
};

```

## min-cost-flow.hpp

Sa8605, 103 lines

```

struct Graph
{
    struct Edge
    {
        int from, to;
        int cap, flow;
        LL cost;
    };

    int _n;
    vector<Edge> edges;
    vector<VI> g;
    vector<LL> d;
    VI p, w;

    Graph(): _n(0) {}
    Graph(int n): _n(n), g(n), d(n), p(n), w(n) {}

    void addEdge(int from, int to, int cap, LL cost)
    {
        assert(0 <= from && from < _n);
        assert(0 <= to && to < _n);
        assert(0 <= cap);
        assert(0 <= cost);
        g[from].PB(SZ(edges));
    }
};

```

```

edges.PB({from, to, cap, 0, cost});
g[to].PB(SZ(edges));
edges.PB({to, from, 0, 0, -cost});
}

pair<int, LL> flow(int s, int t)
{
    assert(0 <= s && s < _n);
    assert(0 <= t && t < _n);
    assert(s != t);
    int flow = 0;
    LL cost = 0;
    while (true)
    {
        fill(ALL(d), LINF);
        fill(ALL(p), -1);
        fill(ALL(w), 0);
        queue<int> q1, q2;
        w[s] = 1;
        d[s] = 0;
        q2.push(s);
        while (!q1.empty() || !q2.empty())
        {
            int v;
            if (!q1.empty())
            {
                v = q1.front();
                q1.pop();
            }
            else
            {
                v = q2.front();
                q2.pop();
            }
            for (int e : g[v])
            {
                if (edges[e].flow == edges[e].cap)
                    continue;
                int to = edges[e].to;
                LL newDist = d[v] + edges[e].cost;
                if (newDist < d[to])
                {
                    d[to] = newDist;
                    p[to] = e;
                    if (w[to] == 0)
                        q2.push(to);
                    else if (w[to] == 2)
                        q1.push(to);
                    w[to] = 1;
                }
            }
        }
    }
}

```

```

    }
    w[v] = 2;
}
if (p[t] == -1)
    break;
int curFlow = INF;
LL curCost = 0;
for (int v = t; v != s;)
{
    int e = p[v];
    curFlow = min(curFlow,
        edges[e].cap - edges[e].flow);
    curCost += edges[e].cost;
    v = edges[e].from;
}
for (int v = t; v != s;)
{
    int e = p[v];
    edges[e].flow += curFlow;
    edges[e ^ 1].flow -= curFlow;
    v = edges[e].from;
}
flow += curFlow;
cost += curCost * curFlow;
}
return {flow, cost};
}
};

```

## 3.3 Flows text

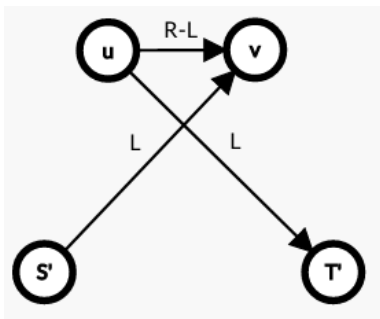
<https://atcoder.jp/contests/abc285/editorial/5535>

On the resulting graph, accumulate maximum flow in the following order:

- from  $S'$  to  $T'$
- from  $S'$  to  $T$
- from  $S$  to  $T'$
- from  $S$  to  $T$ .

An  $S - T$  flow that satisfies the minimum capacities exists if and only if, for all outgoing edges from  $S'$  and incoming edges to  $T'$ , the flow and capacity are equal.





$$\sum_i a_i x_i + \sum_i b_i \bar{x}_i + \sum_{i,j} c_{ij} x_i \bar{x}_j \rightarrow \min$$

If  $a_i \leq b_i$ , add an edge from  $S$  to  $i$  of capacity  $b_i - a_i$  and add  $a_i$  to the answer.

Otherwise, add an edge from  $i$  to  $T$  of capacity  $a_i - b_i$  and add  $b_i$  to the answer.

Add an edge from  $i$  to  $j$  of capacity  $c_{ij}$ .

Add the  $S - T$  minimum cut to the answer.

### 3.4 Specific

#### hungarian.hpp

0baccf, 63 lines

```
LL hungarian(const vector<vector<LL>>& a)
{
    int n = SZ(a), m = SZ(a[0]);
    assert(n <= m);
    vector<LL> u(n + 1), v(m + 1);
    VI p(m + 1, n), way(m + 1);
    FOR(i, 0, n)
    {
        p[m] = i;
        int j0 = m;
        vector<LL> minv(m + 1, LINF);
        vector<int> used(m + 1);
        while (p[j0] != n)
        {
            used[j0] = true;
            int i0 = p[j0], j1 = -1;
            LL delta = LINF;
            FOR(j, 0, m)
            {
                if (!used[j])
                {
                    int cur = a[i0][j] - u[i0] - v[j];
```

```
                    if (cur < minv[j])
                    {
                        minv[j] = cur;
                        way[j] = j0;
                    }
                    if (minv[j] < delta)
                    {
                        delta = minv[j];
                        j1 = j;
                    }
                }
            }
            assert(j1 != -1);
            FOR(j, 0, m + 1)
            {
                if (used[j])
                {
                    u[p[j]] += delta;
                    v[j] -= delta;
                }
                else
                {
                    minv[j] -= delta;
                }
            }
            j0 = j1;
        }
        while (j0 != m)
        {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        }
    }
    VI ans(n + 1);
    FOR(j, 0, m)
        ans[p[j]] = j;
    LL res = 0;
    FOR(i, 0, n)
        res += a[i][ans[i]];
    assert(res == -v[m]);
    return res;
}
```

#### edmonds-blossom.hpp

76c9ac, 127 lines

```
struct Graph
{
    int n;
    vector<VI> g;
    VI label, first, mate;

    Graph() {}
```

```
    Graph(int _n): n(_n), g(_n + 1), label(_n + 1),
        first(_n + 1), mate(_n + 1) {}

    void addEdge(int u, int v)
    {
        assert(0 <= u && u < n);
        assert(0 <= v && v < n);
        u++;
        v++;
        g[u].PB(v);
        g[v].PB(u);
    }

    void augmentPath(int v, int w)
    {
        int t = mate[v];
        mate[v] = w;
        if(mate[t] != v)
            return;
        if(label[v] <= n)
        {
            mate[t] = label[v];
            augmentPath(label[v], t);
            return;
        }
        int x = label[v] / (n + 1);
        int y = label[v] % (n + 1);
        augmentPath(x, y);
        augmentPath(y, x);
    }

    int findMaxMatching()
    {
        FOR(i, 0, n + 1)
            assert(mate[i] == 0);
        int mt = 0;
        DSU dsu;
        FOR(u, 1, n + 1)
        {
            if(mate[u] == 0)
            {
                fill(ALL(label), -1);
                iota(ALL(first), 0);
                dsu.init(n + 1);
                label[u] = 0;
                dsu.unite(u, 0);
                queue<int> q;
                q.push(u);
                while(!q.empty())
                {
                    int x = q.front();
                    q.pop();
```

```

for(int y: g[x])
{
    if(mate[y] == 0 && y != u)
    {
        mate[y] = x;
        augmentPath(x, y);
        while(!q.empty())
            q.pop();
        mt++;
        break;
    }
    if(label[y] < 0)
    {
        int v = mate[y];
        if(label[v] < 0)
        {
            label[v] = x;
            dsu.unite(v, y);
            q.push(v);
        }
    }
}
else
{
    int r = first[dsu.find(x)], s =
        first[dsu.find(y)];
    if(r != s)
    {
        int edgeLabel = (n + 1) * x + y;
        label[r] = label[s] = -edgeLabel;
        int join;
        while(true)
        {
            if(s != 0)
                swap(r, s);
            r = first[dsu.find(label[mate[r]
                ])]);
            if(label[r] == -edgeLabel)
            {
                join = r;
                break;
            }
            label[r] = -edgeLabel;
        }
        for(int z: {x, y})
        {
            for(int v = first[dsu.find(z)];
                v != join;
                v = first[dsu.find(label[mate
                    [v]])])
            {

```

```

                label[v] = edgeLabel;
                if (dsu.unite(v, join))
                    first[dsu.find(join)] =
                        join;
                q.push(v);
            }
        }
    }
}
return mt;
}
int getMate(int v)
{
    assert(0 <= v && v < n);
    v++;
    int u = mate[v];
    assert(u == 0 || mate[u] == v);
    u--;
    return u;
}
};

```

# Strings (4)

## Aho-Corasick.hpp

e15aeb, 70 lines

```
const int AL = 26;
```

```
struct Node
{
    int p;
    int c;
    int g[AL];
    int nxt[AL];
    int link;

    void init()
    {
        c = -1;
        p = -1;
        fill(g, g + AL, -1);
        fill(nxt, nxt + AL, -1);
        link = -1;
    }
};
```

```
struct AC
{
    Node A[N];
    int sz;
    void init()
    {
        A[0].init();
        sz = 1;
    }
    int addStr(const string& s)
    {
        int v = 0;
        FOR (i, 0, SZ(s))
        {
            int c = s[i] - 'a'; // change to [0 AL)
            if (A[v].nxt[c] == -1)
            {
                A[v].nxt[c] = sz;
                A[sz].init();
                A[sz].c = c;
                A[sz].p = v;
                sz++;
            }
            v = A[v].nxt[c];
        }
        return v;
    }
};
```

```

    }
    int go(int v, int c)
    {
        if (A[v].g[c] != -1)
            return A[v].g[c];

        if (A[v].nxt[c] != -1)
            A[v].g[c] = A[v].nxt[c];
        else if (v != 0)
            A[v].g[c] = go(getLink(v), c);
        else
            A[v].g[c] = 0;

        return A[v].g[c];
    }
    int getLink(int v)
    {
        if (A[v].link != -1)
            return A[v].link;
        if (v == 0 || A[v].p == 0)
            return 0;
        return A[v].link=go(getLink(A[v].p), A[v].c);
    }
} A;
```

## automaton.hpp

0e3aee, 60 lines

```
const int AL = 26;
struct Node
{
    int g[AL];
    int link;
    int len;
    int cnt;
    void init()
    {
        fill(g, g + AL, -1);
        link = -1;
        len = -1;
    }
};
struct Automaton
{
    Node A[N * 2];
    int sz;
    int head;
    void init()
    {
        sz = 1;
        head = 0;
        A[0].init();
    }
};
```

```

    }
    void add(char c)
    {
        int ch = c - 'a'; // change to [0 AL)
        int nhead = sz++;
        A[nhead].init();
        A[nhead].len = A[head].len + 1;
        int cur = head;
        head = nhead;
        while (cur != -1 && A[cur].g[ch] == -1)
        {
            A[cur].g[ch] = head;
            cur = A[cur].link;
        }
        if (cur == -1)
        {
            A[head].link = 0;
            return;
        }
        int p = A[cur].g[ch];
        if (A[p].len == A[cur].len + 1)
        {
            A[head].link = p;
            return;
        }
        int q = sz++;
        A[q] = A[p];
        A[q].len = A[cur].len + 1;
        A[p].link = A[head].link = q;
        while (cur != -1 && A[cur].g[ch] == p)
        {
            A[cur].g[ch] = q;
            cur = A[cur].link;
        }
    }
};
```

## suffix-array.hpp

d69c2d, 60 lines

```
void countSort(VI& p, const VI& c)
{
    int n = SZ(p);
    VI cnt(n);
    FOR (i, 0, n)
        cnt[c[i]]++;
    VI pos(n);
    FOR (i, 1, n)
        pos[i] = pos[i - 1] + cnt[i - 1];
    VI p2(n);
    for (auto x : p)
    {

```

```

    int i = c[x];
    p2[pos[i]++] = x;
}
p = p2;
}

VI suffixArray(const string& t)
{
    string s = t + "$";
    int n = SZ(s);
    VI p(n), c(n);
    FOR (i, 0, n) p[i] = i;
    sort(ALL(p), [&](int i, int j)
    {
        return s[i] < s[j];
    });
    int x = 0;
    c[p[0]] = 0;
    FOR (i, 1, n)
    {
        if (s[p[i]] != s[p[i - 1]])
            x++;
        c[p[i]] = x;
    }
    int k = 0;
    while ((1 << k) < n)
    {
        FOR (i, 0, n)
            p[i] = (p[i] - (1 << k) + n) % n;

        countSort(p, c);

        VI c2(n);
        PII pr = {c[p[0]], c[(p[0] + (1 << k)) % n]};
        FOR (i, 1, n)
        {
            PII nx={c[p[i]], c[(p[i] + (1 << k)) % n]};
            c2[p[i]] = c2[p[i - 1]];
            if (pr != nx)
                c2[p[i]]++;
            pr = nx;
        }
        c = c2;
        k++;
    }
    p.erase(p.begin());
    return p;
}

```

## lcp.hpp

72ff1e, 24 lines

```

VI lcpArray(const string& s, const VI& sa)
{
    int n = SZ(s);
    VI rnk(n);
    FOR (i, 0, n)
        rnk[sa[i]] = i;
    VI lcp(n - 1);
    int h = 0;
    FOR (i, 0, n)
    {
        if (h > 0)
            h--;
        if (rnk[i] == 0)
            continue;
        int j = sa[rnk[i] - 1];
        for (; j + h < n && i + h < n; h++)
        {
            if (s[j + h] != s[i + h])
                break;
        }
        lcp[rnk[i] - 1] = h;
    }
    return lcp;
}

```

## z.hpp

e27ac7, 23 lines

```

VI zFunction(const string& s)
{
    int n = SZ(s);
    VI z(n);

    int l = 0;
    int r = 0;
    FOR (i, 1, n)
    {
        z[i] = 0;
        if (i <= r)
            z[i] = min(r - i + 1, z[i - 1]);

        while(i + z[i] < n && s[i + z[i]] == s[z[i]])
            z[i]++;
        if(i + z[i] - 1 > r)
        {
            r = i + z[i] - 1;
            l = i;
        }
    }
    return z;
}

```

}

## prefix.hpp

500608, 16 lines

```

VI prefixFunction(const string& s)
{
    int n = SZ(s);
    VI p(n);
    p[0] = 0;
    FOR (i, 1, n)
    {
        int j = p[i - 1];
        while(j != 0 && s[i] != s[j])
            j = p[j - 1];

        if (s[i] == s[j]) j++;
        p[i] = j;
    }
    return p;
}

```

## manacher.hpp

**Description:** d1[i] - half-length of odd length palindrome with center in i. d2[i] - half-length of even length palindrome if i is right center of it.

2f1541, 39 lines

```

int d1[N], d2[N];

void manacher(const string& s)
{
    int n = SZ(s);
    int l = -1;
    int r = -1;
    FOR (i, 0, n)
    {
        if (i <= r)
            d1[i] = min(r - i + 1,
                        d1[l + (r - i)]);
        while (i + d1[i] < n && i - d1[i] >= 0
            && s[i + d1[i]] == s[i - d1[i]])
            d1[i]++;
        if (i + d1[i] - 1 > r)
        {
            r = i + d1[i] - 1;
            l = i - (d1[i] - 1);
        }
    }
    l = -1;
    r = -1;
    FOR (i, 0, n)
    {

```

```

    if (i <= r)
        d2[i] = min(r - i + 1,
            d2[l + (r - i) + 1]);
    while (i + d2[i] < n
        && i - (d2[i] + 1) >= 0
        && s[i + d2[i]] == s[i - (d2[i] + 1)])
        d2[i]++;
    if (i + d2[i] > r)
    {
        r = i + d2[i] - 1;
        l = i - d2[i];
    }
}
}

```

## palindrome-tree.hpp

c4e179, 64 lines

```

struct Node
{
    int to[AL];
    int link;
    int len;
    void clear()
    {
        fill(to, to + AL, -1);
        link = -1;
        len = -1;
    }
};

struct PalTree
{
    string s;
    vector<Node> A;
    int sz;
    int last;

    void init(string t)
    {
        A.resize(2 * SZ(t));
        A[0].clear();
        A[1].clear();
        A[1].len = 0;
        A[1].link = 0;
        sz = 2;
        last = 1;
        s = t;
    }
    void add(int idx)
    {
        int cur = last;
        while (cur != -1)

```

```

{
    int pos = idx - A[cur].len - 1;
    if (pos >= 0 && s[pos] == s[idx])
        break;
    cur = A[cur].link;
}
assert(cur != -1);
if (A[cur].to[s[idx] - 'a'] == -1)
{
    A[cur].to[s[idx] - 'a'] = sz;
    A[sz].clear();
    A[sz].len = A[cur].len + 2;
    int link = A[cur].link;
    while (link != -1)
    {
        int pos = idx - A[link].len - 1;
        if (pos >= 0 && s[pos] == s[idx])
            break;
        link = A[link].link;
    }
    if (link == -1)
        link = 1;
    else
        link = A[link].to[s[idx] - 'a'];
    A[sz].link = link;
    sz++;
}
last = A[cur].to[s[idx] - 'a'];
}
} pt;

```

# Geometry (5)

In general, try to build programs that are resistant to the oddities of floating-point numbers. Imagine that some evil demon is slightly modifying every result you compute in the way that is most likely to make your program fail. And try to write clean code that is clearly correct at first glance. If you need long explanations to justify why your program will not fail, then it is more likely that your program will in fact fail.

Victor Lecomte, Handbook of geometry for competitive programmers

geometry.hpp

f709e0, 386 lines

```
struct Pt
{
    db x, y;
    Pt operator+(const Pt& p) const
    {
        return {x + p.x, y + p.y};
    }
    Pt operator-(const Pt& p) const
    {
        return {x - p.x, y - p.y};
    }
    Pt operator*(db d) const
    {
        return {x * d, y * d};
    }
    Pt operator/(db d) const
    {
        return {x / d, y / d};
    }
};
// Returns the squared absolute value
db sq(const Pt& p)
{
    return p.x * p.x + p.y * p.y;
}
// Returns the absolute value
db abs(const Pt& p)
{
    return sqrt(sq(p));
}
// Returns -1 for negative numbers, 0 for zero,
// and 1 for positive numbers
int sgn(db x)
```

```
{
    return (EPS < x) - (x < -EPS);
}
// Returns 'p' rotated counter-clockwise by 'a'
Pt rot(const Pt& p, db a)
{
    db co = cos(a), si = sin(a);
    return {p.x * co - p.y * si,
            p.x * si + p.y * co};
}
// Returns 'p' rotated counter-clockwise by 90
Pt perp(const Pt& p)
{
    return {-p.y, p.x};
}
// Returns the dot product of 'p' and 'q'
db dot(const Pt& p, const Pt& q)
{
    return p.x * q.x + p.y * q.y;
}
// Returns the angle between 'p' and 'q'
db angle(const Pt& p, const Pt& q)
{
    return acos(clamp(dot(p, q) / abs(p) /
                      abs(q), (db)-1.0, (db)1.0));
}
// Returns the cross product of 'p' and 'q'
db cross(const Pt& p, const Pt& q)
{
    return p.x * q.y - p.y * q.x;
}
// Positive if R is on the left side of PQ,
// negative on the right side,
// and zero if R is on the line containing PQ
db orient(const Pt& p, const Pt& q, const Pt& r)
{
    return cross(q - p, r - p) / abs(q - p);
}
// Checks if a polygon 'v' is convex
bool isConvex(const vector<Pt>& v)
{
    bool hasPos = false, hasNeg = false;
    int n = SZ(v);
    FOR(i, 0, n)
    {
        int o = sgn(orient(v[i], v[(i + 1) % n],
                          v[(i + 2) % n]));
        hasPos |= o > 0;
        hasNeg |= o < 0;
    }

    return !(hasPos && hasNeg);
}
// Checks if argument of 'p' is in [-pi, 0)
bool half(const Pt& p)
{
    assert(sgn(p.x) != 0 || sgn(p.y) != 0);
    return sgn(p.y) == -1 ||
           (sgn(p.y) == 0 && sgn(p.x) == -1);
}
// Polar sort of vectors in 'v' around 'o'
void polarSortAround(const Pt& o, vector<Pt>& v)
{
    sort(ALL(v), [o](const Pt& p, const Pt& q)
    {
        bool hp = half(p - o), hq = half(q - o);
        if (hp != hq)
            return hp < hq;
        int s = sgn(cross(p, q));
        if (s != 0)
            return s == 1;
        return sq(p - o) < sq(q - o);
    });
}
// Returns the distance of the closest points
db closestPair(vector<Pt> v)
{
    sort(ALL(v), [](const Pt& p, const Pt& q)
    {
        return sgn(p.x - q.x) < 0;
    });
    set<pair<db, db>> s;
    int n = SZ(v), ptr = 0;
    db h = 1e18;
    FOR(i, 0, n)
    {
        for (auto it = s.lower_bound(
            MP(v[i].y - h, v[i].x)); it != s.end()
            && sgn(it->F - (v[i].y + h)) <= 0; it++)
        {
            Pt q = {it->S, it->F};
            h = min(h, abs(v[i] - q));
        }
        for (; sgn(v[ptr].x - (v[i].x - h)) <= 0;
            ptr++)
            s.erase({v[ptr].y, v[ptr].x});
        s.insert({v[i].y, v[i].x});
    }
    return h;
}
// Example:
```

```
// cout << a + b << " " << a - b << "\n";
ostream& operator<<(ostream& os, const Pt& p)
{
    return os << "(" << p.x << "," << p.y << ")";
}

struct Line
{
    // Equation of the line is dot(n, p) + c = 0
    Pt n;
    db c;
    // The line containing two points 'p' and 'q'
    Line(const Pt& p, const Pt& q):
        n(perp(q - p)), c(-dot(n, p)) {}
    // The "positive side": dot(n, p) + c > 0
    // The "negative side": dot(n, p) + c < 0
    db side(const Pt& p) const
    {
        return dot(n, p) + c;
    }
    // Returns the distance from 'p'
    db dist(const Pt& p) const
    {
        return abs(side(p)) / abs(n);
    }
    // Returns the squared distance from 'p'
    db sqDist(const Pt& p) const
    {
        return side(p) * side(p) / (db)sq(n);
    }
    // Returns the perpendicular line through 'p'
    Line perpThrough(const Pt& p) const
    {
        return {p, p + n};
    }
    // Compares 'p' and 'q' by their projection
    bool cmpProj(const Pt& p, const Pt& q) const
    {
        return sgn(cross(p, n) - cross(q, n)) < 0;
    }
    // Returns the orthogonal projection of 'p'
    Pt proj(const Pt& p) const
    {
        return p - n * side(p) / sq(n);
    }
    // Returns the reflection of 'p' by the line
    Pt refl(const Pt& p) const
    {
        return p - n * 2 * side(p) / sq(n);
    }
};
```

```
// Checks if 'l1' and 'l2' are parallel
bool parallel(const Line& l1, const Line& l2)
{
    return sgn(cross(l1.n, l2.n)) == 0;
}

// Returns the intersection point
Pt inter(const Line& l1, const Line& l2)
{
    db d = cross(l1.n, l2.n);
    assert(sgn(d) != 0);
    return perp(l2.n * l1.c - l1.n * l2.c) / d;
}

// Checks if 'p' is in the disk of diameter [ab]
bool inDisk(const Pt& a, const Pt& b,
            const Pt& p)
{
    return sgn(dot(a - p, b - p)) <= 0;
}

// Checks if 'p' lies on segment [ab]
bool onSegment(const Pt& a, const Pt& b,
               const Pt& p)
{
    return sgn(orient(a, b, p)) == 0
        && inDisk(a, b, p);
}

// Checks if the segments [ab] and [cd] intersect
// properly (their intersection is one point
// which is not an endpoint of either segment)
bool properInter(const Pt& a, const Pt& b,
                 const Pt& c, const Pt& d)
{
    db oa = orient(c, d, a);
    db ob = orient(c, d, b);
    db oc = orient(a, b, c);
    db od = orient(a, b, d);
    return sgn(oa) * sgn(ob) == -1
        && sgn(oc) * sgn(od) == -1;
}

// Returns the distance between [ab] and 'p'
db segPt(const Pt& a, const Pt& b, const Pt& p)
{
    Line l(a, b);
    assert(sgn(sq(l.n)) != 0);
    if (l.cmpProj(a, p) && l.cmpProj(p, b))
        return l.dist(p);
    return min(abs(p - a), abs(p - b));
}

// Returns the distance between [ab] and [cd]
db segSeg(const Pt& a, const Pt& b, const Pt& c,
           const Pt& d)
```

```
{
    if (properInter(a, b, c, d))
        return 0;
    return min({segPt(a, b, c), segPt(a, b, d),
               segPt(c, d, a), segPt(c, d, b)});
}

// Returns the area of triangle abc
db areaTriangle(const Pt& a, const Pt& b,
                const Pt& c)
{
    return abs(cross(b - a, c - a)) / 2.0;
}

// Returns the area of polygon 'v'
db areaPolygon(const vector<Pt>& v)
{
    db area = 0.0;
    int n = SZ(v);
    FOR(i, 0, n)
        area += cross(v[i], v[(i + 1) % n]);
    return abs(area) / 2.0;
}

// Returns true if 'p' is at least as high as 'a'
bool above(const Pt& a, const Pt& p)
{
    return sgn(p.y - a.y) >= 0;
}

// Checks if [pq] crosses the ray from 'a'
bool crossesRay(const Pt& a, const Pt& p,
                const Pt& q)
{
    return sgn((above(a, q) - above(a, p))
               * orient(a, p, q)) == 1;
}

// Checks if point 'a' is inside a polygon
// If 'strict', false when 'a' is on the boundary
bool inPolygon(const vector<Pt>& v, const Pt& a,
               bool strict = true)
{
    int numCrossings = 0;
    int n = SZ(v);
    FOR(i, 0, n)
    {
        if (onSegment(v[i], v[(i + 1) % n], a))
            return !strict;
        numCrossings +=
            crossesRay(a, v[i], v[(i + 1) % n]);
    }
    return numCrossings & 1;
}

// Returns the counter-clockwise convex hull
```

```

vector<Pt> convexHull(vector<Pt> v)
{
    sort(ALL(v), [](const Pt& p, const Pt& q)
    {
        int dx = sgn(p.x - q.x);
        if (dx != 0)
            return dx < 0;
        return sgn(p.y - q.y) < 0;
    });
    vector<Pt> lower, upper;
    for (const Pt& p : v)
    {
        while (SZ(lower) > 1
            && sgn(orient(lower[SZ(lower) - 2],
                lower.back(), p)) < 0)
            lower.pop_back();
        while (SZ(upper) > 1
            && sgn(orient(upper[SZ(upper) - 2],
                upper.back(), p)) > 0)
            upper.pop_back();
        lower.PB(p);
        upper.PB(p);
    }
    reverse(ALL(upper));
    lower.insert(lower.end(), upper.begin() + 1,
        prev(upper.end()));
    return lower;
}

// Returns the circumcenter of triangle abc
Pt circumCenter(const Pt& a, Pt b, Pt c)
{
    b = b - a;
    c = c - a;
    assert(sgn(cross(b, c)) != 0);
    return a + perp(b * sq(c) - c * sq(b))
        / cross(b, c) / 2;
}

// Returns circle-line intersection points
vector<Pt> circleLine(const Pt& o, db r,
    const Line& l)
{
    db h2 = r * r - l.sqDist(o);
    if (sgn(h2) == -1)
        return {};
    Pt p = l.proj(o);
    if (sgn(h2) == 0)
        return {p};
    Pt h = perp(l.n) * sqrt(h2) / abs(l.n);
    return {p - h, p + h};
}

```

```

// Returns circle-circle intersection points
vector<Pt> circleCircle(const Pt& o1, db r1,
    const Pt& o2, db r2)
{
    Pt d = o2 - o1;
    db d2 = sq(d);
    if (sgn(d2) == 0)
    {
        assert(sgn(r2 - r1) != 0);
        return {};
    }
    db pd = (d2 + r1 * r1 - r2 * r2) / 2;
    db h2 = r1 * r1 - pd * pd / d2;
    if (sgn(h2) == -1)
        return {};
    Pt p = o1 + d * pd / d2;
    if (sgn(h2) == 0)
        return {p};
    Pt h = perp(d) * sqrt(h2 / d2);
    return {p - h, p + h};
}

// Finds common tangents (outer or inner)
// If there are 2 tangents, returns the pairs of
// tangency points on each circle (p1, p2)
// If there is 1 tangent, the circles are tangent
// to each other at some point p, res contains p
// 4 times, and the tangent line can be found as
// line(o1, p).perpThrough(p)
// The same code can be used to find the tangent
// to a circle through a point by setting r2 to 0
// (in which case 'inner' doesn't matter)
vector<pair<Pt, Pt>> tangents(const Pt& o1,
    db r1, const Pt& o2, db r2, bool inner)
{
    if (inner)
        r2 = -r2;
    Pt d = o2 - o1;
    db dr = r1 - r2, d2 = sq(d),
        h2 = d2 - dr * dr;
    if (sgn(d2) == 0 || sgn(h2) < 0)
    {
        assert(sgn(h2) != 0);
        return {};
    }
    vector<pair<Pt, Pt>> res;
    for (db sign : {-1, 1})
    {
        Pt v = (d * dr + perp(d) * sqrt(h2)
            * sign) / d2;
        res.PB({o1 + v * r1, o2 + v * r2});
    }
}

```

```

    }
    return res;
}

```



# Math (6)

gcd.hpp	e001bc, 16 lines
<pre>int gcd(int a, int b, int&amp; x, int&amp; y) {     x = 1, y = 0;     int x2 = 0, y2 = 1;     while (b)     {         int k = a / b;         x -= k * x2;         y -= k * y2;         a %= b;         swap(a, b);         swap(x, x2);         swap(y, y2);     }     return a; }</pre>	
fast-chinese.hpp	9b392a, 25 lines
<p><b>Description:</b> <math>x\%p_i = m_i, lcm(p) &lt; 10^{18}, p &lt; 10^9</math>. no solution -&gt; return -1 <b>Time:</b> <math>\mathcal{O}(nlog)</math></p>	
<pre>LL FastChinese(VI m, VI p) {     assert(SZ(m) == SZ(p));     LL aa = p[0];     LL bb = m[0];     FOR(i, 1, SZ(m))     {         int b = (m[i] - bb % p[i] + p[i]) % p[i];         int a = aa % p[i];         int c = p[i];          int x, y;         int d = gcd(a, c, x, y);         if(b % d != 0)             return -1;         a /= d;         b /= d;         c /= d;          b = b * x % c;         m[i] = b;         p[i] = c;     }     LL res = m[n - 1];     RFOR(i, n - 1, 0)     {         res *= p[i];         res += m[i];     }     return res; }</pre>	
gauss.hpp	e404cd, 50 lines
<p><b>Description:</b> a[i].back() is right side element <b>Time:</b> <math>\mathcal{O}(m^2 * n)</math></p>	
<pre>VI Gauss(vector&lt;VI&gt; a) {     int n = SZ(a);     if(n == 0)         return {};     int m = SZ(a[0]) - 1; //number of variables</pre>	

assert(n >= m);
<pre>int vars = m; FOR(i, 0, m) {     if (a[i][i] == 0)     {         //for double find row         //with max abs value         int row = -1;         FOR(k, i + 1, n)         {             if (a[k][i] != 0)                 row = k;         }         if(row == -1)         {             //variable i can be any             vars--;             continue;         }         swap(a[i], a[row]);     }     int d = inv(a[i][i]);     FOR(k, i + 1, n)     {         int c = mult(a[k][i], d);         FOR(j, 0, m + 1)             updSub(a[k][j], mult(c, a[i][j]));     } } FOR(i, vars, n)     if(a[i].back() != 0)         cout &lt;&lt; "No solution\n"; VI x(m); RFOR(i, m, 0) {     x[i] = a[i].back();     FOR(j, i + 1, m)         updSub(x[i], mult(a[i][j], x[j]));     x[i] = mult(x[i], inv(a[i][i])); } return x; }</pre>
miller-rabin.hpp
<p><b>Description:</b> to speed up change candidates to at least 4 random values rng() use __int128 in mult</p>
62f1a5, 33 lines

```

VI candidates = {2, 3, 5, 7, 11, 13, 17, 19, 23,
  29, 31, 47};
bool MillerRabin(LL a)
{
    if (a == 1)
        return false;
    if (a == 2 || a == 3)
        return true;
    LL d = a - 1;
    int s = __builtin_ctzll(d);
    d >>= s;

    for (LL b : candidates)
    {
        if (b >= a)
            break;
        b = binpow(b, d, a);
        if (b == 1)
            continue;
        bool ok = false;
        FOR (i, 0, s)
        {
            if (b + 1 == a)
            {
                ok = true;
                break;
            }
            b = mult(b, b, a);
        }
        if (!ok)
            return false;
        return true;
    }
}

```

### pollard.hpp

**Description:** uses Miller-Rabin test. rho finds divisor of n. use \_\_int128 in mult. works in  $O(n^{1/4} * \log n)$ .

28f253, 62 lines

```

LL f(LL x, LL c, LL n)
{
    return add(mult(x, x, n), c, n);
}

```

```

LL rho(LL n)
{
    const int iter = 47 * sqrt(sqrt(n));
    while (true)
    {
        LL x0 = rng() % n;
        LL c = rng() % n;

```

```

LL x = x0;
LL y = x0;
LL g = 1;
FOR (i, 0, iter)
{
    x = f(x, c, n);
    y = f(y, c, n);
    y = f(y, c, n);
    g = gcd(abs(x - y), n);
    if (g != 1)
        break;
}
if (g > 1 && g < n)
    return g;
}
}
VI primes = {2, 3, 5, 7, 11, 13, 17, 19, 23};

vector<LL> factorize(LL n)
{
    vector<LL> ans;

    for (auto p : primes)
    {
        while (n % p == 0)
        {
            ans.PB(p);
            n /= p;
        }
    }
    queue<LL> q;
    q.push(n);

    while (!q.empty())
    {
        LL x = q.front();
        q.pop();
        if (x == 1)
            continue;
        if (MillerRabin(x))
            ans.PB(x);
        else
        {
            LL y = rho(x);
            q.push(y);
            q.push(x / y);
        }
    }
    return ans;
}

```

### simplex.hpp

**Description:**  $c^T x \rightarrow \max, Ax \leq b, x \geq 0$ .

3805fb, 142 lines

```

struct Simplex
{
private:
    int m, n;
    VI nonBasic, basic;
    vector<vector<db>> a;
    vector<db> b;
    vector<db> c;
    db v;

public:
    void pivot(int l, int e)
    {
        assert(0 <= l && l < m);
        assert(0 <= e && e < n);
        assert(abs(a[l][e]) > EPS);
        b[l] /= a[l][e];
        FOR(j, 0, n)
            if (j != e)
                a[l][j] /= a[l][e];
        a[l][e] = 1 / a[l][e];
        FOR(i, 0, m)
        {
            if (i != l)
            {
                b[i] -= a[i][e] * b[l];
                FOR(j, 0, n)
                    if (j != e)
                        a[i][j] -= a[i][e] * a[l][j];
                a[i][e] *= -a[l][e];
            }
        }
        v += c[e] * b[l];
        FOR(j, 0, n)
            if (j != e)
                c[j] -= c[e] * a[l][j];
        c[e] *= -a[l][e];
        swap(nonBasic[e], basic[l]);
    }

    void findOptimal()
    {
        vector<db> delta(m);
        while (true)
        {
            int e = -1;
            FOR(j, 0, n)
                if (c[j] > EPS && (e == -1 || nonBasic[j] < nonBasic[e]))

```

```

    e = j;
    if (e == -1)
        break;
    FOR(i, 0, m)
        delta[i] = a[i][e] > EPS ? b[i] / a[i][e]
        : LINF;
    int l = min_element(ALL(delta)) - delta.
        begin();
    if (delta[l] == LINF)
    {
        // unbounded
        assert(false);
    }
    pivot(l, e);
}

void initializeSimplex(const vector<vector<db>
>>& _a, const vector<db>& _b, const vector<
db>& _c)
{
    m = SZ(_b);
    n = SZ(_c);
    nonBasic.resize(n);
    iota(ALL(nonBasic), 0);
    basic.resize(m);
    iota(ALL(basic), n);
    a = _a;
    b = _b;
    c = _c;
    v = 0;
    int k = min_element(ALL(b)) - b.begin();
    if (b[k] > -EPS)
        return;
    nonBasic.PB(n);
    iota(ALL(basic), n + 1);
    FOR(i, 0, m)
        a[i].PB(-1);
    c.assign(n, 0);
    c.PB(-1);
    n++;
    pivot(k, n - 1);
    findOptimal();
    if (v < -EPS)
    {
        // infeasible
        assert(false);
    }
    int l = find(ALL(basic), n - 1) - basic.begin
        ();
    if (l != m)

```

```

{
    int e = -1;
    while (abs(a[l][e]) < EPS)
        e++;
    pivot(l, e);
}
n--;
int p = find(ALL(nonBasic), n) - nonBasic.
    begin();
assert(p < n + 1);
nonBasic.erase(nonBasic.begin() + p);
FOR(i, 0, m)
    a[i].erase(a[i].begin() + p);
c.assign(n, 0);
FOR(j, 0, n)
{
    if (nonBasic[j] < n)
        c[j] = _c[nonBasic[j]];
    else
        nonBasic[j]--;
}
FOR(i, 0, m)
{
    if (basic[i] < n)
    {
        v += _c[basic[i]] * b[i];
        FOR(j, 0, n)
            c[j] -= _c[basic[i]] * a[i][j];
    }
    else
        basic[i]--;
}
}
pair<vector<db>, db> simplex(const vector<
vector<db>>& _a, const vector<db>& _b,
const vector<db>& _c)
{
    initializeSimplex(_a, _b, _c);
    assert(SZ(a) == m);
    FOR(i, 0, m)
        assert(SZ(a[i]) == n);
    assert(SZ(b) == m);
    assert(SZ(c) == n);
    assert(SZ(nonBasic) == n);
    assert(SZ(basic) == m);
    findOptimal();
    vector<db> x(n);
    FOR(i, 0, m)
        if (basic[i] < n)
            x[basic[i]] = b[i];

```

```

    return {x, v};
}
};

```

## Convolutions (7)

fft.hpp

44d94c, 87 lines

```

const int mod = 998244353;

int add(int a, int b)
{
    return (a + b < mod) ? (a + b) : (a + b - mod);
}
int sub(int a, int b)
{
    return (a - b >= 0) ? (a - b) : (a - b + mod);
}
int mult(int a, int b)
{
    return a * (LL) b % mod;
}
int binpow(int a, int n)
{
    int res = 1;
    while(n)
    {
        if(n & 1)
            res = mult(res, a);
        a = mult(a, a);
        n /= 2;
    }
    return res;
}

const int LEN = 1 << 23;
const int GEN = 31;
const int IGEN = binpow(GEN, mod - 2);

void fft(VI& a, bool inv)
{
    int lg = 0;
    while((1 << lg) < SZ(a)) lg++;
    FOR(i, 0, SZ(a))
    {
        int x = 0;
        FOR(j, 0, lg)
            x |= ((i >> j) & 1) << (lg - j - 1);
        if(i < x)
            swap(a[i], a[x]);
    }

```

```

}
for(int len = 2; len <= SZ(a); len *= 2)
{
    int ml = binpow(inv ? IGEN : GEN, LEN / len);
    for(int i = 0; i < SZ(a); i += len)
    {
        int pw = 1;
        FOR(j, 0, len / 2)
        {
            int v = a[i + j];
            int u = mult(a[i + j + len / 2], pw);

            a[i + j] = add(v, u);
            a[i + j + len / 2] = sub(v, u);

            pw = mult(pw, ml);
        }
    }
}
if(inv)
{
    int m = binpow(SZ(a), mod - 2);
    FOR(i, 0, SZ(a))
        a[i] = mult(a[i], m);
}
}

VI mult(VI a, VI b)
{
    int sz = 0;
    int sum = SZ(a) + SZ(b) - 1;
    while((1 << sz) < sum) sz++;
    a.resize(1 << sz);
    b.resize(1 << sz);

    fft(a, 0);
    fft(b, 0);

    FOR(i, 0, SZ(a))
        a[i] = mult(a[i], b[i]);

    fft(a, 1);
    a.resize(sum);
    return a;
}

```

## inverse.hpp

a4673f, 32 lines

```

VI inverse(const VI& a, int k)
{
    assert(SZ(a) == k && a[0] != 0);

```

```

if(k == 1)
    return {binpow(a[0], mod - 2)};

VI ra = a;
FOR(i, 0, SZ(ra))
    if(i & 1)
        ra[i] = sub(0, ra[i]);

int nk = (k + 1) / 2;
VI t = mult(a, ra);
t.resize(k);

FOR(i, 0, nk)
    t[i] = t[2 * i];

t.resize(nk);
t = inverse(t, nk);
t.resize(k);

RFOR(i, nk, 1)
{
    t[2 * i] = t[i];
    t[i] = 0;
}

VI res = mult(ra, t);
res.resize(k);
return res;
}

```

## exp-log.hpp

5549eb, 52 lines

```

VI deriv(const VI& a, int k)
{
    VI res(k);
    FOR(i, 0, k)
        if(i + 1 < SZ(a))
            res[i] = mult(a[i + 1], i + 1);
    return res;
}

VI integr(const VI& a, int k)
{
    VI res(k);
    RFOR(i, k, 1)
        res[i] = mult(a[i - 1], inv[i]);
    res[0] = 0;
    return res;
}

VI log(const VI& a, int k)

```

```

{
    assert(a[0] == 1);
    VI ml = mult(deriv(a, k), inverse(a, k));
    return integr(ml, k);
}

VI exp(VI a, int k)
{
    assert(a[0] == 0);

    VI Qk = {1};
    int pw = 1;
    while(pw <= k)
    {
        pw *= 2;

        Qk.resize(pw);
        VI lnQ = log(Qk, pw);

        FOR(i, 0, SZ(lnQ))
        {
            if(i < SZ(a))
                lnQ[i] = sub(a[i], lnQ[i]);
            else
                lnQ[i] = sub(0, lnQ[i]);
        }
        updAdd(lnQ[0], 1);

        Qk = mult(Qk, lnQ);
    }
    Qk.resize(k);
    return Qk;
}

```

## modulo.hpp

8b6a95, 34 lines

```

void removeLeadingZeros(VI& a)
{
    while(SZ(a) > 0 && a.back() == 0)
        a.pop_back();
}

pair<VI, VI> modulo(VI a, VI b)
{
    //assert(a.back() != 0 && b.back() != 0);
    int n = SZ(a), m = SZ(b);
    if(m > n)
        return MP(VI{}, a);

    reverse(ALL(a));
    reverse(ALL(b));

```

```

VI d = b;
d.resize(n - m + 1);
d = mult(a, inverse(d, n - m + 1));
d.resize(n - m + 1);

```

```

reverse(ALL(a));
reverse(ALL(b));
reverse(ALL(d));

```

```

VI res = mult(b, d);
res.resize(SZ(a));
FOR(i, 0, SZ(a))
    res[i] = sub(a[i], res[i]);

```

```

removeLeadingZeros(d);
removeLeadingZeros(res);
return MP(d, res);

```

```

}

```

## multipoint-eval.hpp

8f6f41, 33 lines

```

int x[LEN];
VI P[2 * LEN];

void build(int v, int tl, int tr)
{
    if(tl + 1 == tr)
    {
        P[v] = {sub(0, x[tl]), 1};
        return;
    }
    int tm = (tl + tr) / 2;
    build(2 * v + 1, tl, tm);
    build(2 * v + 2, tm, tr);

    P[v] = mult(P[2 * v + 1], P[2 * v + 2]);
}

int ans[LEN];
void solve(int v, int tl, int tr, const VI& Q)
//Q != Q % P[0] -> wa
{
    if(SZ(Q) == 0)
        return;
    if(tl + 1 == tr)
    {
        ans[tl] = Q[0];
        return;
    }
    int tm = (tl + tr) / 2;
    solve(2 * v + 1, tl, tm,

```

```

modulo(Q, P[2 * v + 1]).S);
solve(2 * v + 2, tm, tr,
modulo(Q, P[2 * v + 2]).S);
}

```

## newton.hpp

9ffaac, 50 lines

```

VI newton(VI a, int k)
{
    //c_n = a_n + sum(i = 0, n - 1) c_i * c_{n-1-i}
    //Q = A + x * Q * Q
    //F(Q) = Q - x * Q * Q - A
    //F'(Q) = 1 - 2 * x * Q

    VI Qk = {a[0]};
    int pw = 1;
    while(pw <= k)
    {
        assert(SZ(Qk) == pw);
        pw *= 2;

        VI F1(pw);
        F1[0] = 1;
        FOR(i, 0, pw / 2)
            F1[i + 1] = sub(0, mult(2, Qk[i]));
        //F' = 1 - 2 * x * Q

        VI F = mult(Qk, Qk);
        F.resize(pw);
        RFOR(i, pw, 1)
            F[i] = sub(0, F[i - 1]);
        F[0] = 0; // F = -x * Q*Q

        FOR(i, 0, pw / 2)
            F[i] = add(F[i], Qk[i]);
        //F = Q - x * Q * Q
        FOR(i, 0, min(pw, SZ(a)))
            F[i] = sub(F[i], a[i]);
        //F = Q - x * Q * Q - A

        F = mult(F, inverse(F1, pw));
        F.resize(pw);

        FOR(i, 0, pw)
            F[i] = sub(0, F[i]); // -F/F'
        FOR(i, 0, pw / 2)
            F[i] = add(F[i], Qk[i]); // Q - F/F'

        //new Qk = Qk - F(Qk) / F'(Qk) mod(x ^ pw)
        Qk = F;
    }
}

```

```

Qk.resize(k);
return Qk;
}

```

## berlekamp-massey.hpp

866c28, 36 lines

```

VI berlekampMassey(const VI& a)
{
    VI c = {1}, bp = {1};
    int l = 0, b = 1, x = 1;
    FOR(j, 0, SZ(a))
    {
        assert(SZ(c) == l + 1);
        int d = a[j];
        FOR(i, 1, l + 1)
            updAdd(d, mult(c[i], a[j - i]));
        if (d == 0)
        {
            x++;
            continue;
        }
        VI t = c;
        int coef = mult(d, binPow(b, mod - 2));
        if (SZ(bp) + x > SZ(c))
            c.resize(SZ(bp) + x);
        FOR(i, 0, SZ(bp))
            updSub(c[i + x], mult(coef, bp[i]));
        if (2 * l > j)
        {
            x++;
            continue;
        }
        l = j + 1 - l;
        bp = t;
        b = d;
        x = 1;
    }
    c.erase(c.begin());
    for (int& ci : c)
        ci = mult(ci, mod - 1);
    return c;
}

```

## botsan-mori.hpp

74e03a, 29 lines

```

// c - coefficients c[1], ..., c[k] but 0-index
// a - initial values a[0], a[1], ..., a[k-1]
int botsanMori(VI c, VI a, LL n) {
    int k = SZ(c);
    assert(SZ(a) == k);
    VI q(k + 1);

```

```

q[0] = 1;
FOR(i, 0, k)
    q[i + 1] = sub(0, c[i]);

VI p = mult(a, q);
p.resize(k);
while (n) {
    VI qMinus = q;
    for (int i = 1; i <= k; i += 2)
        qMinus[i] = sub(0, qMinus[i]);

    VI newP = mult(p, qMinus);
    VI newQ = mult(q, qMinus);
    FOR(i, 0, k)
        p[i] = newP[2 * i + (n & 1)];

    FOR(i, 0, k + 1)
        q[i] = newQ[2 * i];

    n >>= 1;
}
return mult(p[0], binPow(q[0], mod - 2));
}

```

## conv-xor.hpp

8ce066, 12 lines

```

void convXor(VI& a, int k)
{
    FOR(i, 0, k)
        FOR(j, 0, 1 << k)
            if((j & (1 << i)) == 0)
            {
                int u = a[j];
                int v = a[j + (1 << i)];
                a[j] = u + v;
                a[j + (1 << i)] = u - v;
            }
}

```

## conv-and.hpp

b8d23e, 12 lines

```

void convAnd(VI& a, int k, bool inverse)
{
    FOR(i, 0, k)
        FOR(j, 0, 1 << k)
            if((j & (1 << i)) == 0)
            {
                if(inverse)
                    a[j] -= a[j + (1 << i)];
                else
                    a[j] += a[j + (1 << i)];
            }
}

```

```

    }
}

conv-or.hpp
6ffc0, 12 lines
void convOr(VI& a, int k, bool inverse)
{
    FOR(i, 0, k)
        FOR(j, 0, 1 << k)
            if((j & (1 << i)) == 0)
            {
                if(inverse)
                    a[j + (1 << i)] -= a[j];
                else
                    a[j + (1 << i)] += a[j];
            }
}

```

# Various (8)

mobius.hpp

fba6c5, 19 lines

```
void mobius()
{
    fill(pr, pr + N, 1);
    fill(mu, mu + N, 1);
    pr[1] = false;
    FOR (i, 2, N)
    {
        if (!pr[i])
            continue;
        mu[i] = mod - 1;
        for (int j = 2 * i; j < N; j += i)
        {
            pr[j] = false;
            if (j % (i * i) == 0)
                mu[j] = 0;
            mu[j] = mult(mu[j], mod - 1);
        }
    }
}
```

triangles.hpp

**Description:** finds all triangles in a graph. Should take vector of edges and EMPTY graph g. In line cnt++ we find triangle v, u, w.  
**Time:**  $\mathcal{O}(m * \text{sqrt}(m))$

a22d8b, 30 lines

```
int triangles(int n, int m)
{
    FOR (i, 0, m)
    {
        auto [u, v] = edges[i];
        if (MP(deg[u], u) < MP(deg[v], v))
            g[u].PB(v);
        else
            g[v].PB(u);
    }
    int cnt = 0;
    FOR (v, 0, n)
    {
        for (auto u : g[v])
            used[u] = 1;
        for (auto u : g[v])
        {
            for(auto w : g[u])
            {
                if (used[w])
                {
```

```
                cnt++;
            }
        }
        for (auto u : g[v])
            used[u] = 0;
    }
    return cnt;
}
```

ternary.hpp

3db54c, 29 lines

```
const db phi = (3. - sqrt(5.0)) / 2.;
db get(db L, db R)
{
    db M1, M2, v1, v2;
    M1 = L + (R - L) * phi;
    M2 = R - (R - L) * phi;
    v1 = f(M1);
    v2 = f(M2);
    FOR (i, 0, 74)
    {
        if (v1 > v2) // for minimum
        {
            L = M1;
            M1 = M2;
            v1 = v2;
            M2 = R - (R - L) * phi;
            v2 = f(M2);
        }
        else
        {
            R = M2;
            M2 = M1;
            v2 = v1;
            M1 = L + (R - L) * phi;
            v1 = f(M1);
        }
    }
    return L; // or f(L);
}
```

lader-nim.txt

8 lines

Players have stone piles of size a0, a1, ..., an. In one move player can take 0 < x <= ai stones from i-th pile and move them to (i-1)-th pile.

In this game you can forget about even piles. Take stones from odd is equal to remove in NIM

and from even equal to add in NIM. Adding in NIM useless.

# Formulas (9)

## 9.1 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$
$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

## 9.2 Triangles

Area:  $A = \sqrt{p(p - a)(p - b)(p - c)}$

$$A = \frac{abc}{4R} = pr$$

Length of median:  $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

## 9.3 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \quad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \quad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln |\cos ax|}{a} \quad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \quad \int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)$$

Integration by parts:

$$\int_a^b f'(x)g(x)dx = [f(x)g(x)]_a^b - \int_a^b f(x)g'(x)dx$$

## 9.4 Series

$$f(x) = \sum_{n=0}^{\inf} \frac{f^{(n)}(a) * (x - a)^n}{n!}$$

$$e^x = \sum \frac{x^n}{n!}$$

$$\ln(1-x) = -\sum \frac{x^n}{n}, (-1 \leq x < 1)$$

9.5 Number Theory

9.5.1 Mobius

for  $n \geq 1$   $g(n) = \sum_{d|n} f(d)$

then  $f(n) = \sum_{d|n} \mu(d)g(n/d)$

$$M(n) = \sum_{k=1}^n \mu(k), \sum_{n=1} xM([x/n]) = 1$$

9.5.2 Catalan

$$C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$$

$$C_n = \frac{1}{n+1} C_{2n}^m$$

$$C_n = C_{2n}^n - C_{2n}n - 1$$

9.5.3 Binomials

$$\sum_{k=0}^n C_n^k = 2^n$$

$$\sum_{m=0}^n C_m^k = C_{n+1}^{k+1}$$

$$\sum_{k=0}^m C_{n+k}^k = C_{n+m+1}^m$$

$$\sum_{k=0}^n (C_n^k)^2 = C_{2n}^n$$

$$\sum_{j=0}^k C_m^j C_{n-m}^{k-j} = C_n^k$$

$$\sum_{j=0}^m C_m^j C_{n-m}^{k-j} = C_{n+1}^{k+1}$$

$$\sum_{k=0}^n C_{n-k}^k = F_{n+1}$$

9.6 Runge-Kutta 4th Order Method for Ordinary Differential Equations

$$\frac{dy}{dx} = f(x,y), y(0) = y_0$$

$$x_{i+1} - x_i = h$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1h)$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2h)$$

$$k_4 = f(x_i + h, y_i + k_3h)$$

9.7 Burnside’s lemma

Let  $G$  be a finite group that acts on a set  $X$ .

The *orbit* of an element  $x$  in  $X$  is the set of elements in  $X$  to which  $x$  can be moved by the elements of  $G$ . The orbit of  $x$  is denoted by  $G \cdot x$ :

$$G \cdot x = \{g \cdot x \mid g \in G\}.$$

For each  $g$  in  $G$ , let  $X^g$  denote the set of elements in  $X$  that are fixed by  $g$  (also said to be left invariant by  $g$ ), that is,  $X^g = \{x \in X \mid g \cdot x = x\}$ . Burnside’s lemma asserts the following formula for the number of orbits, denoted  $|X/G|$ :

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$$