$13 \cdot 30 = 430$ $\quad +1.(-1.(-70)+13.(-30))$ $\quad 70-430$ $\quad -380$ $\quad -400$

$-2.(-1.(60)+1.(30)-10.(-4))$

$-2.(-60+30+40)$

-20

Ciência da Computação Prof. Tiago J. Arruda

Matrizes e Determinantes[1]

1. **(2,5 pt.)** Considere as matrizes $A = (a_{ij})_{2\times 2}$, tal que $a_{ij} = \begin{cases} 3i - j^2, & i = j \\ j + 2, & i \neq j \end{cases}$, e $B = (b_{ij})_{2\times 2}$, onde $b_{ij} = i - 2j$. Determine:

   (a) **(0,5 pt.)** A e B

   (b) **(0,5 pt.)** $-2A + 3B$

   (c) **(0,5 pt.)** $A^2 - BA^t$

   (d) **(0,5 pt.)** $\det(B^6) - \mathrm{tr}(A + B)$

   (e) **(0,5 pt.)** $\det(2A^{-1}B^t)$

2. **(2,0 pt.)** Resolva os exercícios abaixo.

   (a) **(1,0 pt.)** Calcule x, y e z para que $A = \begin{pmatrix} \frac{1}{z} + 3 & 2x & x + 1 \\ x^2 & 2 - \frac{y^2}{2} & -4x \\ 1 & x^3 & 2 - \sqrt{2y} \end{pmatrix}$ seja *antissimétrica*, isto é, $A = -A^t$.

   (b) **(1,0 pt.)** Para quais valores de t a matriz $B = \begin{pmatrix} 12 - t & 0 & 3t \\ 13t & -26 & 39 \\ 0 & 6 & 0 \end{pmatrix}$ é *invertível*?

   $+ \quad - \quad +$

3. **(2,0 pt.)** Use o *desenvolvimento de Laplace* para calcular $\begin{vmatrix} 2 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 4 & 30 \\ 7 & 1 & 0 & 1 & -10 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 15 \end{vmatrix}$.

4. **(2,5 pt.)** Considere a equação matricial

$$A - BX = 2C$$

onde todas as matrizes são *quadradas* e de mesma ordem 3×3.

$2.\begin{vmatrix} 0 & 0 & 4 & 30 \\ 1 & 0 & 1 & -10 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 15 \end{vmatrix} - 1.\begin{vmatrix} -2 & 0 & 4 & 30 \\ 7 & 0 & 1 & -10 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 15 \end{vmatrix}$

   (a) **(0,5 pt.)** Determine X em função das matrizes A, B e C e comente se é necessária alguma imposição à matriz B.

   (b) **(2,0 pt.)** Sendo $A = \begin{pmatrix} 1 & 2 & 6 \\ 2 & -3 & 4 \\ 6 & 4 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 4 & 1 & 8 \end{pmatrix}$ e $C = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}$, determine a matriz X e calcule o seu *traço*.

5. **(1,0 pt.)** Sabendo que $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 14$, calcule o valor de $\begin{vmatrix} x_3 + 2y_3 & x_3 - y_3 & z_3 \\ x_2 + 2y_2 & x_2 - y_2 & z_2 \\ x_1 + 2y_1 & x_1 - y_1 & z_1 \end{vmatrix}$.

$39 \cdot 12 = 468$ $\quad 468 \div 13 = 36$

$13 \cdot 4 = 52$, $52 \cdot 48 = 2496$, $2496 + 169 = 2665$, $13 \cdot 13 = 169$

$55 \cdot 55 = 3025$, $43 \cdot 43 = 1225$

[1] Coloque o nome completo nas folhas de prova e escreva o resultado final das questões à caneta. Respostas sem resolução e/ou justificativa não serão consideradas. Não é permitido o uso de quaisquer equipamentos eletrônicos. Data da Avaliação: 26/03/2025

$2.\left(-1.\begin{vmatrix} 0 & 4 & 30 \\ 1 & 1 & -10 \\ 1 & 0 & 15 \end{vmatrix}\right) - 1.\left(-1\begin{vmatrix} -2 & 4 & 30 \\ 7 & 1 & -10 \\ -1 & 0 & 15 \end{vmatrix}\right)$

Petrônio Dias de Carvalho Júnior

9,0

1) $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ $\quad a_{ij} = \begin{cases} 3i - j^2, & i = j \\ j + 2, & i \neq i \end{cases}$

$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ $\quad b_{ij} = i - 2j$

0,5

a) $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ $\quad B = \begin{pmatrix} -1 & -3 \\ 0 & -2 \end{pmatrix}$

0,5

b) $-2A = \begin{pmatrix} -4 & -8 \\ -6 & -4 \end{pmatrix} + 3B = \begin{pmatrix} -3 & -9 \\ 0 & -6 \end{pmatrix} = \begin{pmatrix} -7 & -17 \\ -6 & -10 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ $\quad A^2 = \begin{pmatrix} 4+12 & 8+8 \\ 6+6 & 12+4 \end{pmatrix} = \begin{pmatrix} 16 & 16 \\ 12 & 16 \end{pmatrix}$

$B = \begin{pmatrix} -1 & -3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$ $\quad B.A^t = \begin{pmatrix} -2-12 & -3-6 \\ 0-8 & 0-4 \end{pmatrix} = \begin{pmatrix} -14 & -9 \\ -8 & -4 \end{pmatrix}$

$A^2 - BA^t = \begin{pmatrix} 30 & 25 \\ 20 & 20 \end{pmatrix}$

0,5

d) $B = \begin{pmatrix} -1 & -3 \\ 0 & -2 \end{pmatrix}$

$\det = +2$

$trA = 2+2 = 4$

$trB = -1-2 = -3$

$tr(A+B) = 4-3 = 1$

$\det B^6 \; tr\, A+B$

$64 - 1 = 63$

0,5

$2^6 = 64$

2) $\det A = -8$

$\det A^{-1} = 1/-8$

$\det(2A^{-1}) = 2^2 . \det A^{-1}$

$\det(2A^{-1}) = 4 . -1/8$

$\det(2A^{-1}) = -1/2$

$\det(B) = \det(B^t) = 2$

$\det(2A^{-1}) . \det(B^t)$

$-1/2 \; . \; 2 = -1$

0,5

2) a - $A = \begin{pmatrix} \frac{1}{z}+3 & 2x & x+1 \\ x^2 & 2-\frac{y^2}{2} & -4x \\ 1 & x^3 & 2-\sqrt{2y} \end{pmatrix}$ $\quad a_{ij} = -a_{ji}$

1,0

$\begin{pmatrix} -\frac{1}{z}-3 & -4 & 1 \\ 4 & -2+\frac{y^2}{2} & -8 \\ 1 & -8 & -2+\sqrt{2y} \end{pmatrix}$

$a_{12} \quad 2x = -x^2$

$a_{13} \quad x+1 = -1$

$a_{23} \quad -4x = x^3$

$x+1=-1$

$x=-2$

$X = -2$

$Y = 2$

$Z = -1/3$

$-2+\frac{y^2}{2} = 2-\frac{y^2}{2}$

$-4 = -\frac{2y^2}{2}$

$-4 = -y^2$

$Y = 2$

$-\frac{1}{z}-3 = \frac{1}{z}+3$

$-\frac{2}{z} = 6$

$-2 = 6Z$

$Z = -1/3$

b - $B = \begin{pmatrix} 12-t & 0 & 3t \\ 13t & -26 & 39 \\ 0 & 6 & 0 \end{pmatrix}$

$+ \quad - \quad +$

$-6 \cdot \begin{vmatrix} 12-t & 3t \\ 13t & 39 \end{vmatrix}$

$-6\,(468-39t-39t^2)$

$-78\,(-3t^2-3t+36)$

$+78 \cdot 3\,(+t^2+t-12)$

0,8

?

$1-4\cdot1\cdot-12$

$1+48 = 49$

$\frac{1 \pm \sqrt{49}}{2} = \frac{50}{2} = 25$

ou

$-\frac{48}{2} = 24$

$X \neq 25$

$X \neq -24$

$12 \cdot 39 = 108 + 36 = 144$

$144 \div 13 = 48$

$13 \cdot 4 = 52$

$32 \cdot 48 = 416 + 208 = 2496$

$2496 + 169 = 2665$

$15 \cdot 7 = 105$

3) $\begin{array}{c} + \quad - \quad + \quad - \quad + \\ \begin{vmatrix} 2 & 1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 4 & 30 \\ 7 & 1 & 0 & 1 & -10 \\ 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 15 \end{vmatrix} \end{array}$

$2 \cdot \begin{vmatrix} 0 & 0 & 4 & 30 \\ 1 & 0 & 1 & -10 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 15 \end{vmatrix}$ $-1 \cdot \begin{vmatrix} -2 & 0 & 4 & 30 \\ 7 & 0 & 1 & -10 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 15 \end{vmatrix}$

$+ \quad - \quad + \quad -$

$2 \cdot 1 \cdot \begin{vmatrix} 0 & 4 & 30 \\ 1 & 1 & -10 \\ 1 & 0 & 15 \end{vmatrix}$ $-1 \cdot 1 \cdot \begin{vmatrix} -2 & 4 & 30 \\ 7 & 1 & -10 \\ -1 & 0 & 15 \end{vmatrix}$

$$\begin{array}{r} 380 \\ -260 \\ \hline 120 \end{array} \qquad \begin{array}{r} 95 \\ \cdot 4 \\ \hline 380 \end{array}$$

$2.1.(-4.(25)+30.(-1))-1.(-2.(15)-4.(95)+30.(+1)$

$2.(-100-30)-1.(-30-380+30)$

$2.-130+380=120$

$det = 120$ 2,0

4) $A - BX = 2C$ 3x3

0,2

a- $-BX = 2C - A$ $\qquad X = (-2C + A).B^{-1}$

$BX = -2C + A$

"A matriz B deve ter determinante diferente de zero"

b- $A = \begin{pmatrix} 1 & 2 & 6 \\ 2 & -3 & 4 \\ 6 & 4 & -1 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 4 & 1 & 8 \end{pmatrix}$ $\quad C = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}$

$-2C = \begin{pmatrix} -4 & -2 & -6 \\ -2 & 0 & -4 \\ -6 & -4 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 6 \\ 2 & -3 & 4 \\ 6 & 4 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$

$det\,B = (2.-2)+(-1.0)+(3.1)$

$-4+3=-1$

$C_B = \begin{pmatrix} -2 & 0 & 1 \\ -5 & 4 & 2 \\ 2 & -1 & -1 \end{pmatrix}$

2,0

$B^{-1} = -1/1 . \begin{pmatrix} -2 & -5 & 2 \\ 0 & 4 & -1 \\ 1 & 2 & -1 \end{pmatrix}$

$tr(X) = -6 + 12 - 3 = +3$

$\begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix} . \begin{pmatrix} 2 & 5 & -2 \\ 0 & -4 & 1 \\ -1 & -2 & 1 \end{pmatrix} \Rightarrow \quad X = \begin{pmatrix} -6 & -15 & 6 \\ 0 & 12 & -3 \\ 3 & 6 & -3 \end{pmatrix}$

$\begin{array}{r} 144 \,|\underline{13} \\ 24 \;\; 48 \end{array}$

$\begin{array}{r} 13 \\ \cdot 4 \\ \hline 52 \end{array}$

$\begin{array}{r} 2496 \\ +169 \\ \hline 2665 \end{array}$

$\begin{array}{r} 15 \\ \cdot 7 \\ \hline 105 \end{array}$

1

-8

$-2+\sqrt{2}Y$

5) $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 14$

$$\begin{vmatrix} x_3 + 2y_3 & x_3 - y_3 & z_3 \\ x_2 + 2y_2 & x_2 - y_2 & z_2 \\ x_1 + 2y_1 & x_1 - y_1 & z_1 \end{vmatrix} = 14$$

$$\begin{vmatrix} x_3 & x_3 & z_3 \\ x_2 & x_2 & z_2 \\ x_1 & x_1 & z_1 \end{vmatrix} \qquad \begin{vmatrix} x_3 & -y_3 & z_3 \\ x_2 & -y_2 & z_2 \\ x_1 & -y_1 & z_1 \end{vmatrix} \qquad \begin{vmatrix} 2y_3 & x_3 & z_3 \\ 2y_2 & x_2 & z_2 \\ 2y_1 & x_1 & z_1 \end{vmatrix}$$

det = 0   det = - 14   det = 28

~~0,7~~   0,5

$$\begin{vmatrix} 2y_3 & -y_3 & z_3 \\ 2y_2 & -y_2 & z_2 \\ 2y_1 & -y_1 & z_1 \end{vmatrix}$$

det = 0